Improving the B&B Search for Detailed Large-scale Hydrothermal Weekly Scheduling Problems

Ernesto Parrilla and Javier García-González, *Member*, *IEEE*

***Abstract*— This paper presents an optimization based algorithm to solve the weekly scheduling problem of a large-scale hydrothermal power system, formulated as a mixed-integer linear programming model (MILP). The main drawback of the MILP approach is the high computational burden required to solve large-size problems. The proposed algorithm overcomes this difficulty by providing an initial feasible and integer solution, which enhances the search of the Branch and Bound (B&B) over the space of feasible solutions. A detailed representation of thermal, pumped storage, and hydroelectric units is considered, taking into account the net head dependence of hydro plants by means of an under-relaxed iterative process. The presented algorithm has been applied to real-scale study cases, obtaining satisfactory results in computational time and optimality.**

***Index Terms*— Head dependent reservoirs, large-scale hydrothermal scheduling, mixed integer linear programming, power generation dispatch.**

NOMENCLATURE

Sets:

K, J, I, B	Set of periods, thermal, hydro and pumped storage units
Ω_i	Subset of units direct upstream of unit i

Parameters:

pd , ps	Penalty factors of demand deviations [€/MWh] and spillages [€·h/Hm^3]
dem_k	System demand requirement in period k [MWh]
l_k	Duration of period k [h]
str_k	Thermal spinning reserve in period k [MWh]
$\overline{P}_j$, $\underline{P}_j$	Output power limits of unit j [MW]
a_j, c_j, d_j, e_j	Start up [€], fixed [€/h], variable [€/MWh] and shutdown costs [€] of unit j
ur_{jk}, dr_{jk}	Ramp rate limits of unit j in period k [MW]
w_{ik}	Lateral inflows of unit i in period k [Hm^3/h]
$\overline{\theta}_{ik}$, $\underline{\theta}_{ik}$	Water rights of unit i in period k [Hm^3/h]
$\overline{q}_{ik}$, $\underline{q}_{ik}$	Outflow discharge limits [Hm^3/h]
q_{ik}^{mxef} , p_{ik}^{mxef}	Outflow [Hm^3/h] and output power [MW] at maximum efficiency point of unit i in period k
$\overline{v}_i$, $\underline{v}_i$	Reservoir capacity limits [Hm^3]
η_b	Efficiency of generation-pumped cycle of unit b
$\underline{vd}_b, \overline{vd}_b$	Downstream reservoir limits of unit b [MWh]
$\underline{vu}_b, \overline{vu}_b$	Upstream reservoir limits of unit b [MWh]

Variables:

t_{jk} , p_{jk}	Total and over minimum output power of unit j in period k [MW]
u_{jk}	Commitment decision of unit j in period k {0,1}
y_{jk} , z_{jk}	Start up and shutdown decisions of unit j in k
ph_{ik} , p_{jk}	Output power of units i and j in period k [MW]
h_{ik} , v_{ik}	Net hydraulic head [m] and reservoir level of unit i in period k [Hm^3]
q_{ik} , s_{ik}	Outflow and spillage discharge of unit i in period k [Hm^3/h]
q_{ik}^a , q_{ik}^b , q_{ik}^c	Outflow discharge under the minimum, over the minimum up to maximum efficiency outflow and over maximum efficiency outflow [Hm^3/h]
ens_k	Non-served energy [MWh]
vd_{bk}, vu_{bk}	Downstream and upstream energy reservoir levels of unit b [MWh]
ed_{bk}, ep_{bk}	Power generated and pumped by unit b [MWh]

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The authors are with the Instituto de Investigación Tecnológica (IIT), Universidad Pontifica Comillas de Madrid, c\ Alberto Aguilera 23, CP: 28015 Madrid, SPAIN. (Phone: + 34 915-422-800; fax: + 34915-423-176; e-mail: ernesto@iit.upco.es, javiergg@iit.upco.es).

I. INTRODUCTION

THIS paper presents an optimization based algorithm to solve the weekly scheduling problem of a large-scale hydro-thermal power system. The aim of the proposed algorithm is to determine the hourly output power of thermal, hydro and pumped storage units, and also, the start-up and shutdown decisions during the whole week.

Two main approaches can be found in the literature to solve this large-scale, non-linear, non-convex and mixed-integer problem [1, 2]. The first one is to simplify the real problem in order to apply classical optimization techniques, [3]. Among them, Lagrangian Relaxation (LR) is the most popular one [4-6]. However, the number of iterations required to find the optimal solution for a 168 hours scope can be very high.

Moreover, the duality gap makes necessary the use of *ex post* heuristic procedures in order to reach a feasible solution, and therefore, the optimality could deteriorate. Mixed Integer Linear Programming (MILP) is another well-accepted optimization technique [7]. Its main advantages are that new constraints can be easily added to the problem and non-linearities can be accurately considered by piecewise linear approximations. The Branch & Bound (B&B) method, used for solving MILP problems, is able to find the optimal solution within reasonable time and with high accuracy for small-scale problems. However, for very large systems, the size of the search tree can make very difficult, or even impossible, to find a feasible solution.

The second approach is based on heuristic techniques such as simulated annealing [8], evolutionary programming [9], tabu search, artificial neural networks [10], etc [11]. In this case, the scheduling problem can be formulated in a more realistic way, and therefore, head variation effect, non-linear cost functions, forbidden operating areas, etc. can be considered through successive simulations. The main drawback of this approach is that optimality of the solution cannot be guaranteed. In addition, the computational burden can also increase considerably for large-scale problems.

The question is whether it is more suitable to obtain the optimum solution of a simplified problem, or whether it is better an approximate solution of a more realistic one. In this paper a compromise between both approaches is chosen in order to obtain a better global result. In particular, the heuristic techniques are used only during the first steps to estimate an initial feasible integer solution. After that, taking this solution as the starting point, the original optimization problem is solved by the standard B&B method.

In short, major contributions of this paper with respect to previous and related works are the following ones:

- Existing MILP models fail when trying to solve directly the weekly scheduling of a large hydrothermal system with a detailed representation of both the generators and the temporal scope. The algorithm presented in this paper can overcome this difficulty providing an initial feasible integer solution, which enables the B&B to reach the optimum.
- Head dependency of hydro units is normally neglected in short-term hydrothermal models. In this paper, real input-output curves are taken into account obtaining more realistic schedules.

This paper is organized as follows. Section II explains the main idea of the resolution algorithm and its modeling assumptions. Section III presents the scheduling problem formulation assuming that the hourly input-output curves relating the output power and the water discharge of hydro units are known. Then, in section IV, the strategies used within the algorithm to reduce the size of the problem are justified. Section V describes the different steps of the algorithm, and in particular, how to estimate the evolution of the net heads which are necessary to build the hourly input-output curves of the head dependent units. After that, a realistic study case is presented in section VI. Finally, concluding remarks are given in section VII.

II. BASIC APPROACH

The idea of combining classical optimization techniques with heuristics methods is not new, and can be found in papers like [4, 5, 12]. However, as they are basically based on LR, a high number of iterations could still be required to solve the weekly problem, and for that reason, in this paper a MILP approach is preferred.

To cope with the main drawback of the MILP technique, an innovative heuristic and optimization-based algorithm has been developed improving significantly the search of the B&B over the space of feasible solutions.

In the B&B method, the lower bound of each sub-problem generated after branching is computed by solving the linear programming (LP) relaxation obtained by dropping the integrality constraints. In case this lower bound exceeds the best-known feasible integer solution (upper bound), or in case it is LP is infeasible, the node is pruned as no globally optimal solution can exist in its corresponding subspace. The search proceeds until all nodes have been solved or pruned, or until a specified threshold is met between the objective function value of the current best integer solution and the best possible one. To start the B&B, it is necessary to derive an initial upper bound, for instance $+\infty$. After that, feasible integer solutions obtained when solving the LP relaxed sub-problems can be used to update this upper bound. The number of nodes explored before obtaining the first feasible integer solution can be very high, and therefore only few nodes can be pruned until this occurs. This fact leads to a sharp increase in the size of the search tree, and so, a proportional rise in the computational burden. This is especially significant for very large systems where obtaining the first feasible integer solution can take a long execution time.

The aim of the algorithm presented in this paper is to provide a non-naive initial integer and feasible solution. This way, the provided starting point enhances considerably the B&B search as many more branches can be efficiently pruned during the first steps. Moreover, the available strategies for selecting the next sub-problem candidate can also take advantage of starting from a good feasible integer solution.

Regarding the generators representation, in this paper thermal units are modeled using the standard MILP formulation, which allows considering binary commitment decisions, start-up costs, ramp-rate limits, spinning reserve, etc [13].

Concerning the hydroelectric plants, a special emphasis has been done to preserve a detailed representation, taking into account cascade and head-dependent units, water rights constraints, outflow and output power limits, final reservoir level goals, etc. In this sense, one of the most difficult issues to overcome is the nonlinear relationship among the output power, the water discharge flow, and the net hydraulic head [13]. Depending on the particular structure of the hydro sub-

system this dependence might be neglected or not for short-term studies. In the Spanish system, there are several hydro chains formed by many but small reservoirs. In this case, even in the short-term, the head dependency has to be considered within the scheduling algorithm in order to obtain feasible and realistic results.

Basically, three main strategies can be found to cope with the head variation effect. The first one is to consider a constant average net head [4, 6]. The second one is to implement an iterative procedure where a fixed head is considered at each iteration, which is successively updated [7, 12]. The third strategy is to consider an analytical function and solve the problem using non-linear optimization techniques [5].

In this paper the second strategy has been chosen. In order to avoid oscillating solutions, the under-relaxation updating strategy proposed for pure hydro systems in [14], is extended successfully to the whole hydrothermal problem.

III. Problem Formulation

The problem presented in this section is stated assuming that the hourly input-output curves relating the output power and the water discharge of hydro units are known. The model is formulated as a traditional cost minimization problem, subject to a set of constraints that can be classified into four categories: thermal generation, hydropower subsystems, pumped storage units, and demand requirements.

The link between the weekly problem and the long-term can be done by both primal and dual coordination techniques [15, 16]. In this paper, primal coordination has been implemented where long-term signals are basically the target levels of the hydro reservoirs and pumped storage units at the end of the week.

A. Thermal unit constraints

The thermal unit model takes into account minimum and maximum output power, (1) and (2), logical commitment constraints (3), up and down ramp rate limits, (4) and (5), and thermal spinning reserve requirement (6).

$$t_{jk} = \underline{P}_j \cdot u_{jk} + p_{jk} \tag{1}$$

$$p_{jk} \le \left(\overline{P}_j - \underline{P}_j\right) \cdot u_{jk} \tag{2}$$

$$u_{jk} - u_{j(k-1)} = y_{jk} - z_{jk} \tag{3}$$

$$p_{j(k+1)} - p_{jk} \le ur_j \cdot \left(l_k + l_{(k+1)}\right)/2 \tag{4}$$

$$p_{jk} - p_{j(k+1)} \le dr_j \cdot \left(l_k + l_{(k+1)}\right)/2 \tag{5}$$

$$\sum_J \left(u_{jk} \cdot \overline{P}_j - t_{jk}\right) \ge str_k \tag{6}$$

Note that ramp rate constraints are formulated assuming that the duration of two consecutive periods k could be different.

B. Hydro unit constraints

The hydro model is formed by two different blocks. The first block deals with water continuity (7), reservoir level limits (8) , and water rights (9):

$$v_{i(k+1)} = v_{ik} + l_k \cdot \left[-q_{ik} - s_{ik} + w_{ik} + \sum_{I \in \Omega_i}^{I} \left(q_{ik} + s_{ik}\right)\right] \tag{7}$$

$$\underline{v}_{ik} \le v_{ik} \le \overline{v}_{ik} \tag{8}$$

$$\underline{\theta}_{ik} \le q_{ik} + s_{ik} \le \overline{\theta}_{ik} \tag{9}$$

The initial and final reservoir level conditions are forced by substituting in (7), the corresponding values for the first and last periods.

The second block of hydro constraints deals with the input-output curves modeling. In case the hourly net heads of the reservoirs were known, it would be possible to define the real hourly curves, that can be approximated by piecewise linear functions as shown in. Fig. 1. This approximation is built by joining three linear segments defined by the origin, the minimum output power point ($\underline{q}_{ik}$, $\underline{p}_{ik}$), the maximum efficiency operation point (q_{ik}^{mxef}, p_{ik}^{mxef}) and the maximum output power point ($\overline{q}_{ik}$, $\overline{p}_{ik}$).

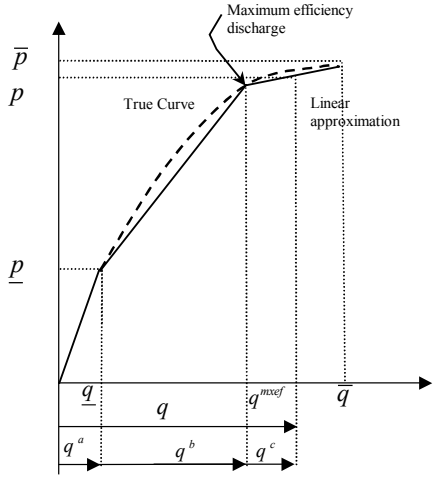

Fig. 1: Input-output curve and its linear approximation for a given net head.

Taking advantage of its concavity, this input-output curve approximation can be expressed by the following linear constraints:

$$q_{ik} = q_{ik}^a + q_{ik}^b + q_{ik}^c \tag{10}$$

$$q_{ik}^a \le \underline{q}_{ik} \tag{11}$$

$$q_{ik}^b \le q_{ik}^{mxef} - \underline{q}_{ik} \tag{12}$$

$$q_{ik}^c \le \overline{q}_{ik} - q_{ik}^{mxef} \tag{13}$$

$$\begin{aligned} ph_{ik} = {} & q_{ik}^a \cdot \left(\underline{p}_{ik} / \underline{q}_{ik}\right) + q_{ik}^b \cdot [(p_{ik}^{mxef} - \underline{p}_{ik})(q_{ik}^{mxef} - \underline{q}_{ik})] \\ & + q_{ik}^c \cdot [(\overline{p}_{ik} - p_{ik}^{mxef}) / (\overline{q}_{ik} - q_{ik}^{mxef})] \end{aligned} \tag{14}$$

C. Pumped storage units constraints

Pumped storage units are modeled by an equivalent energy representation [17] characterized by two reservoirs storing energy (MWh) instead of water (Hm^3). This model is stated by the following equations:

$$vu_{b(k+1)} = vu_{bk} + \left[ep_{bk} \cdot \eta_b - ed_{bk}\right] \cdot l_k \tag{15}$$

$$vd_{b(k+1)} = vd_{bk} + \left[ed_{bk} - ep_{bk} \cdot \eta_b\right] \cdot l_k \tag{16}$$

$$0 \le ep_{bk} \le \overline{ep}_{bk} \quad (17)$$

$$0 \le ed_{bk} \le \overline{ed}_{bk} \quad (18)$$

$$\underline{vd}_b \le vd_{bk} \le \overline{vd}_b \quad (19)$$

$$\underline{vu}_b \le vu_{bk} \le \overline{vu}_b \quad (20)$$

Equations (15) and (16) establish the energy balance between both reservoirs. Note that the pumping process is affected by an efficiency rate, η_b. Finally, the bounds for the pump power, the generated output power, and the stored energies at the reservoirs, are set in (18)-(20).

D. Demand requirements

The energy balance is stated in (21), where besides the thermal, hydro and pumped storage units, non-served energy is included (ens_k) to avoid infeasible solutions in case of production scarcity.

$$\left(\sum_{j=1}^{J} t_{jk} + \sum_{i=1}^{I} ph_{ik} + \sum_{b=1}^{B} (qt_{bk} - qb_{bk})\right) \cdot l_k + ens_k = dem_k \quad (21)$$

E. Objective function

The objective function is to minimize thermal operating costs including the fixed cost of having a generator running, the incremental cost, and the start-up and shutdown costs (22). Additional terms include the penalizations for spillages and non-served energy.

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$$\text{fo} = \sum_{k=1}^{K} \left(\begin{array}{l} \sum_{j=1}^{J} \left(a_j \cdot y_{jk} + \left(c_j \cdot u_{jk} + d_j \cdot p_{jk} \right) \cdot l_k + e_j \cdot z_{jk} \right) \\ + pd \cdot ens_k + \sum_{i=1}^{I} \left(ps \cdot s_{ik} \cdot l_k \right) \end{array} \right) \quad (22)$$

IV. Problem Reduction Strategies for Initial Solution

Three non-exclusive strategies are used within the proposed algorithm to reduce the size of the original problem, and thus, to make possible its resolution:

- Grouping consecutive hours in the same period.
- Relaxing binary variables.
- Fixing some variables after the analysis of the results obtained with a previously simplified problem.

The justification of applying these heuristics lies in the particular structure of the hydrothermal scheduling problem, where broadly speaking, the hard work is to find the optimal commitment decisions of the thermal units close to the marginal ones. For instance, before the optimization it could be ensured that a started nuclear unit will remain scheduled at its maximum capacity during the whole week, as its variable cost is smaller than any other technology. On the contrary, it is very probable that the most expensive fuel-oil unit will be "off" if cheaper generators are available to satisfy the system demand. In this sense, the LR approach presented in [18] shows that after a certain number of iterations, the solution of the subproblems oscillate among a reduced collection of possibilities, where unaltered variables are precisely the commitment decisions of the units with higher and lower cost values.

As a conclusion, in a high percentage of realistic cases, a LP relaxed optimization problem, even with a not very detailed temporal representation, could be enough to extract interesting conclusions about the commitment decision of such "cheap" and "expensive" units.

Hereafter, the proposed strategies are explained in detail.

A. Grouping consecutive hours

Instead of using hourly periods, grouping consecutive hours in the same time period allows to reduce notably the scale of the problem. However, to avoid oversimplification mistakes, some specific rules are proposed in this paper to drive this aggregation process.

First of all, it is necessary to perform a clustering algorithm to classify each one of the 168 hours in off-peak, mid-peak and peak hours. The attributes used in this cluster analysis are the system demand and the dynamic parameters of hydro units.

After that, it is necessary to decide the maximum duration permitted for a time period. For instance, during the first phases of the algorithm, longer periods would result in faster resolutions.

Finally, hours grouped in the same period have to be chronologically consecutive, must belong to the same cluster (off-peak, mid-peak and peak hours) and also, must belong to the same day. This way, formed periods will maintain a proper chronological structure. Moreover, non-desired averaging effects are avoided as only very similar hours are mixed together. Finally, the requirement of belonging to the same day is very useful when some daily constraints want to be added to the problem.

Fig. 2 shows an example of the resulting periods structure when hours of two consecutive days are grouped with a maximum length of three hours. Note that the original 48 hourly periods are transformed into 21 periods with a non-uniform duration up to three hours.

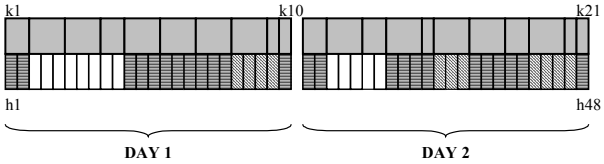

Fig. 2: Example of the strategy of grouping consecutive hours in the same period. The maximum duration allowed for new periods is 3 hours.

This strategy makes necessary to re-calculate the input-data according to the new temporal structure. For example, the aggregated system demand and the spinning reserve requirement of period k can be computed from their corresponding hourly values as follows:

$$dem_k = \sum_{h \in k} dem_h \quad (23)$$

$$str_k = \frac{\sum_{h \in k} str_h}{l_k} \tag{24}$$

Similar formulae can be used for the other parameters, as water inflows, water rights, reservoir limits, etc.

B. Relaxing binary variables

Relaxing binary variables (commitment and start-up decisions of thermal units) is a well-known strategy used in LP based unit-commitment models. Although these solutions may not be technically feasible, they can provide a good approximation of the final solution of the problem. Moreover, this approximate solution is usually integer feasible in a high percentage of the involved binary variables and its analysis might provide interesting conclusions about the operation of some units.

C. Fixing variables

The basic idea behind this strategy is to estimate the optimum value of some variables before the optimization, and to fix them in order to reduce the size of the problem (both for MILP and for LP resolutions). Obviously, the key point is to guess the right values. To do so, in this paper it is proposed to deduce them from the results obtained during the previous phases of the algorithm, where the resolution of different simplified problems could provide useful information.

The main risk of this strategy is to fix a wrong value. In this case, the obtained solution could be far away from the optimum or even become infeasible. For that reason this strategy is only used in the final steps of the proposed algorithm, once a good approximation of the weekly scheduling has been reached.

V. ALGORITHM DESCRIPTION

The developed algorithm involves four different steps (see Fig. 3), which are described hereafter.

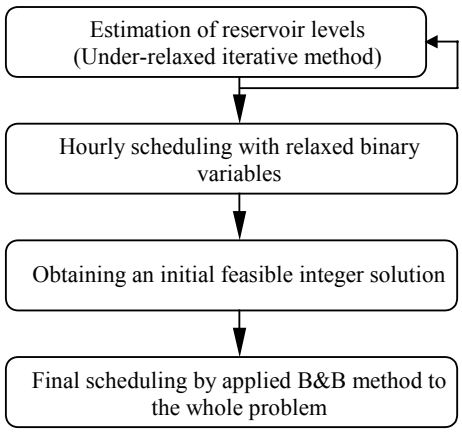

Fig. 3: Resolution algorithm scheme

A. Step 1:Estimation of reservoir net heads by an under-relaxed iterative procedure

The aim of this step is to estimate the hourly reservoir levels in order to get the best approximation of the real input-output curves. This is done by solving iteratively the problem presented in section II, but relaxing commitment variables of thermal units and grouping consecutive hours within periods up to 4 hours.

After each iteration (υ), hourly input-output curves are updated taking into account the net heads obtained from previous iterations. Assuming an average tailrace elevation, the net head can be expressed as a function of the forebay elevation and, therefore, as a function of the reservoir level. This way, the last execution provides new values for the reservoir levels v_{ik}, which can be used directly to update the net heads $h_{ik}^{\upsilon+1}$:

$$h_{ik}^{\upsilon+1} = \rho_i\left(v_{ik}\right) \tag{25}$$

However, this formula could produce diverging oscillations. For that reason, in this paper it is proposed to apply the under-relaxation technique presented in [14]. Let define the relaxation parameter $\alpha > 0$. The updated net heads can be obtained as:

$$h_{ik}^{\upsilon+1} = \rho_i\left(v_{ik}^{\upsilon+1}\right) = \rho_i\left(v_{ik}^{\upsilon} + \alpha \cdot \left[v_{ik} - v_{ik}^{\upsilon}\right]\right). \tag{26}$$

Note that (25) is just a particular case of (26) when $\alpha = 1$. The selection of the best under-relaxation factor is empiric and unfortunately, it can be case-dependent. However, a general rule can be stated: for the early stages of iterations lower values of the under-relaxation factor will help to avoid divergence, and as the iterations get closer to the converged state, values very close to 1 help to speed up the progress.

Finally, it is necessary to check if convergence has been reached. This can be achieved defining the following error ε:

$$\varepsilon = \frac{(v_{i'k} - v_{i'k}^{\upsilon})}{v_{i'k}^{\upsilon}} \tag{27}$$

where i' is the index of the reservoir in which the stored water in period k has the highest mismatch between two consecutive iterations. This first step of the algorithm finishes when the value of ε is smaller than a given tolerance (e.g. 1%) or when the iteration limit is reached.

From the obtained results, commitment decisions of thermal units producing at their maximum output power during the whole week are fixed to 1 in the following 2nd and 3rd steps of the algorithm.

B. Step 2: Hourly scheduling with relaxed binary variables

Once the reservoirs' levels have been estimated, the hourly input-output curves of hydro units can be approximated by piece-wise linear functions as described in section III.B.

In this step, the scheduling problem is solved considering an hourly temporal representation, a detailed modeling of the hydro, pumped storage and thermal units, but relaxing the binary variables related to thermal start-up and unit-commitment decisions.

After the optimization, if a thermal unit is running at its maximum power during a whole day, its commitment decision is fixed to 1 during this day for the optimization in the 3rd step.

Furthermore, the new hourly input-output curves are updated using the net heads obtained from the last optimization ($\alpha = 1$). Then, real output power of the hydro units are computed with this new input-output curves, considering the existence of minimum output power constraints. Finally, the resulting hydro schedule is fixed.

C. *Step 3: Obtaining an initial feasible integer solution*

In this step, the problem presented in section II is solved by the B&B method, where many variables have been fixed during previous steps. In fact, the problem is to find the optimal value of the output power and commitment decisions of some thermal units and all variables of pumped storage units. The size of this problem is smaller than the original one, and can be solved by a solver such as CPLEX 7.1.

The result of this optimization is an integer feasible solution of the weekly hydro-thermal scheduling, that in many real situations as the example case presented in section VI, is very close to the global optimum.

D. *Step 4: Final scheduling applying the B&B method to the whole problem*

The integer feasible solution obtained in the 3rd step can be used as the starting point for a new execution of the B&B applied to the whole problem, although assuming the same net heads previously obtained. In contrast with the 3rd step, in this case, no variables are fixed at all, and therefore the model recovers the flexibility previously lost to try to improve the integer and feasible initial solution.

VI. CASE STUDY AND RESULTS

The algorithm proposed in this paper has been implemented in GAMS, using the commercial solver CPLEX 7.1 for both the LP and MILP optimizations, and executed in a 256 MB, 666 MHz, Pentium III.

The model has been tested in a real size study case with very satisfactory results. The temporal scope covered is a whole week divided into hourly periods, starting on Saturday and finishing on the next Friday. The generation system comprises 44 thermal units, six pumped storage units, and 102 hydro plants together with 77 reservoirs, grouped in seven watersheds. The smallest watershed consists of six reservoirs and 14 hydro units, while the largest one comprises 14 reservoirs and 19 hydro units (see Fig. 4).

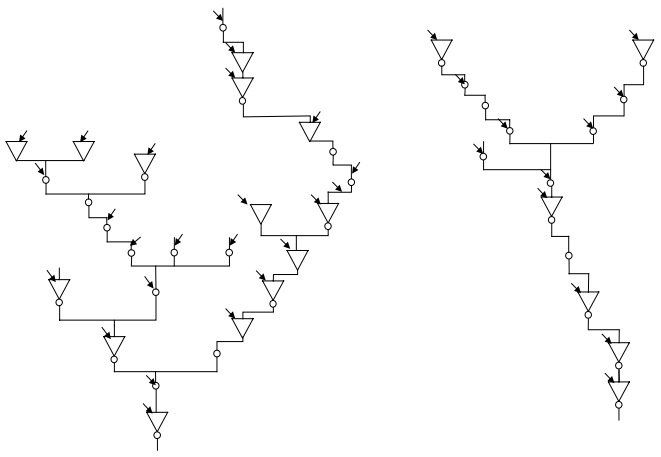

Fig. 4: The topology of the largest and the smallest hydro chains considered in the example case.

Lateral inflows are represented by arrows, generating units by circles and reservoirs by triangles. Run of the river plants can be identified as single circles while head dependent plants are represented by joint triangles and circles.

The original problem (219482 continuous variables, 7392 binary variables and 162693 equations), was impossible to be solved directly by the B&B even assuming that the net heads used to build the hydro input-output curves were known.

However, after applying the proposed algorithm, the initial feasible integer solution was obtained just in 25 minutes. Then, the 4th step of the algorithm required 15 minutes more to reach the final solution, with the *optcr*[1] CPLEX parameter set to 0.001 p.u.. The improvement of this final solution respect to the initial one was just a 0.015% better. Table I presents the size of the different problems solved within the algorithm, and also the value of the obtained objective functions.

TABLE I
SIZE OF THE PROBLEMS SOLVED WHTHIN THE ALGORITHM, AND F.O.

	STEP 1				STEP 2	STEP 3	STEP 4
	iter 1	iter 2	iter 3	iter 4			
Cont. var.	74516	74516	74516	74516	219482	42170	219482
Binary var.	2508	2508	2508	2508	4536	1608	7392
Equations	55403	55403	55403	55403	162693	40380	162693
f.o.	20719	20660	20661	20661	20963	20967	20964

Fig. 5 illustrates the weekly scheduling obtained with the presented algorithm, aggregated by power generation technologies. Note that nuclear and coal thermal units represent the base of the system production. On the other hand, pumped storage units show the typical behavior of this kind of groups, storing energy during off-peak hours and releasing it as needed during peak hours. Finally, it can be observed the peak shaving effect, especially for working days (h48 to h168), where the most expensive thermal units are substituted by hydro generation during peak hours. As a consequence, the thermal generation is quite smooth and remains almost constant, except for off-peak hours.

[1] Relative optimality criterion for a MIP problem.

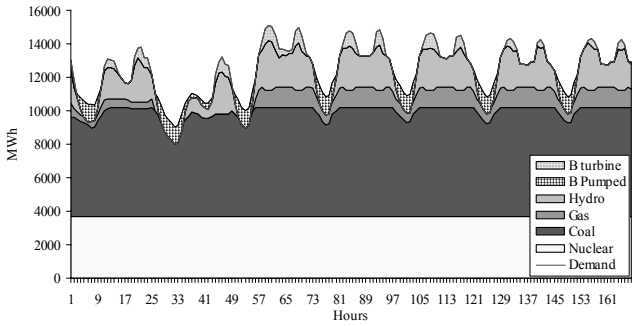

Fig. 5: Obtained weekly scheduling, aggregated by power generation technologies

Fig. 6 presents the schedule obtained during the first hours of Monday (h48 to h61). It is important to note that regarding the hydro units, only the hourly global productions of the watersheds (s1-s7) have been presented. The other more detailed hydro variables such as the generation of each unit, the reservoir management along the week, the outflow discharges, etc, have been omitted here due to space limitations.

	1	2	3	4	5	6	7	8	9	10	11	12	13
j1	1050	1050	1050	1050	1050	1050	1050	1050	1050	1050	1050	1050	1050
j2	950	950	950	950	950	950	950	950	950	950	950	950	950
j3	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000
j4	700	700	700	700	700	700	700	700	700	700	700	700	700
j5	500	500	500	500	500	500	500	500	500	500	500	500	500
j6	500	500	500	500	500	500	500	500	500	500	500	500	500
j7	400	400	400	400	400	400	400	400	400	400	400	400	400
j8	400	400	400	400	400	400	400	400	400	400	400	400	400
j9	510	510	510	510	510	510	510	510	510	510	510	510	510
j10	510	510	510	510	510	510	510	510	510	510	510	510	510
j11	535	535	535	535	535	365	535	535	535	535	535	535	535
j12	535	535	535	365	199	369	535	535	535	535	535	535	535
j13	350	350	350	350	350	350	350	350	350	350	350	350	350
j14	350	350	350	350	350	350	350	350	350	350	350	350	350
j15	350	350	350	350	350	350	350	350	350	350	350	350	350
j16	350	350	350	350	350	350	350	350	350	350	350	350	350
j17	300	240	180	125	125	180	240	300	300	300	300	300	300
j18	295	235	175	175	175	180	240	300	300	300	300	300	300
j19	176	176	120	120	122	178	234	290	300	300	300	300	300
j20	229	176	120	120	132	188	244	300	300	300	300	300	300
j21	0	0	0	0	0	0	0	260	315	370	315	260	315
j22	0	0	0	0	0	0	0	260	315	318	263	208	258
j23	0	0	0	0	0	0	0	0	213	271	239	291	233
j24	0	0	0	0	0	0	0	0	213	271	213	271	224
j25	0	0	0	0	0	0	0	0	0	0	0	0	0
j26	0	0	0	0	0	0	0	0	0	0	0	0	0
j27	0	0	0	0	0	0	0	0	0	0	0	0	0
j28	0	0	0	0	0	0	0	0	0	0	0	0	0
s1	272.2	165.2	165	52.44	52.41	52.38	164.2	270.7	399.5	399.3	629.9	631.2	631.2
s2	198.1	198.1	198.1	198.1	198.1	198.1	198.1	198.1	198.1	217.2	403	403	403
s3	0	0	0	0	0	0	0	0	29.78	48.5	48.51	48.52	29.76
s4	338.7	257.6	256.9	165.1	140.3	121.6	258.1	260.1	604.7	723.3	761	819.9	809.8
s5	179.4	124.7	124.7	124.7	124.7	124.7	124.7	124.7	252.7	252.5	252.5	252.2	251.9
s6	368.9	331.2	251	238.3	242.6	238.3	333.3	332.6	530	431	582.3	685.9	654
s7	63.95	63.94	63.93	63.92	63.91	63.89	63.88	63.87	102.1	104.5	120.9	128.7	152
b1	0	0	0	0	0	0	0	0	0	12	323	323	323
b2	0	0	0	0	0	0	0	0	0	232	232	232	232
b3	0	0	0	0	0	0	0	0	0	129	129	129	129
b4	74	0	0	0	0	0	0	0	0	103	103	103	103
b5	0	0	0	0	0	0	0	0	0	0	0	0	0
b6	100	0	0	0	0	0	0	0	0	30	116	116	116
b1	0	204	375	375	375	375	290	0	0	0	0	0	0
b2	0	0	237	270	270	270	0	0	0	0	0	0	0
b3	0	48	150	150	150	150	150	0	0	0	0	0	0
b4	0	120	120	120	120	120	120	0	0	0	0	0	0
b5	0	0	0	0	0	0	0	0	0	0	0	0	0
b6	0	135	135	135	135	135	135	0	0	0	0	0	0

Fig. 6: Monday first hours scheduling of thermal units, hydro subsystems and pumped storage units [MW]

Respect to the goodness of the 1st step of the algorithm used to estimate the net heads, in this example case the stop tolerance was fixed to 1%, allowing a maximum of 4 iterations. The error ε computed as in (27), did not fall below this tolerance, and therefore, the maximum number of iterations was reached. However, the proportion of reservoirs in which the error ε was above the tolerance for any period, was finally just a 0.98%. Table II presents the evolution of this proportion along the iterations. It can be seen that in the first iteration, where the reservoirs volumes are initialized assuming a linear trajectory between the initial and final levels, the 14.04% of cases have a mismatch above the tolerance. This percentage is sharply reduced in subsequent iterations, showing the validity of the under-relaxed procedure.

TABLE III
FIRST STEP CONVERGENCE EVOLUTION

Iteration	1	2	3	4
% mismatches over 1%	14.04%	2.23%	1.48%	0.98%

VII. CONCLUSIONS

The algorithm presented in this paper improves the search of the B&B method over the space of feasible solutions by providing an initial feasible integer solution of the weekly scheduling problem. This algorithm consists of four steps where different strategies are applied to reduce the size of the problem, taking advantage of intermediate results for subsequent optimizations.

The main conclusion derived from the test of the algorithm in real cases, is that it can be used successfully to find the hourly schedule of large hydrothermal power systems where a direct B&B fails to find a feasible solution. Moreover, the improvement of the objective function achieved after the last B&B execution is just slightly better (never over 0.5%) than the initial solution provided after 3rd step. This fact validates and justifies the problem reduction strategies applied within the algorithm.

Finally, the application of the under-relaxed technique during the 1st step of the algorithm has proven to be a suitable way to take into account the hydro plants net head dependence.

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BIOGRAPHIES



Ernesto Parrilla was Ernesto Parrilla Pozzy was born in Granada, Spain, in 1976.He received the degree of Electrical Engineer from the Universidad Pontificia de Comillas in 2000. Since July 2000, he has been a research assistant and Ph.D. candidate in IIT. His areas of interest include modeling and management of power systems and electric power market behaviors.



Javier García-González was born in Salamanca in 1972. He obtained his Industrial Engineering degree, specialized in Electricity, from the Universidad Politécnica de Cataluña (UPC), Barcelona, Spain in 1996. After that, he joined the Instituto de Investigación Tencológica (IIT), obtaining his PhD. degree in 2001. Currently he belongs to the research staff of the IIT and takes part in several research projects developed for the Spanish electric sector utilities.