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Do oil prices impact on transportation? Evidence from random matrix theory

Antonio Garcia-Amate[1] · **Laura Molero-Gonzalez**[1] · **Miguel Angel Sanchez-Granero**[2] · **Juan Evangelista Trinidad-Segovia**[1]

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Abstract

Our economic system is based on the consumption of oil and its derivatives for a very long time. Far from ending this status quo, certain institutions warn that by 2050 the demand for fossil fuels will remain stable at around 70% out of total. Energy dependence on fossil fuels has an unprecedented impact on consumer prices, financial markets and several industries. One of these, transportation, is the largest oil-consuming industry, with more than 50 million barrels of crude oil consumed daily. According to this data, oil prices are expected to impact the transportation. However, the literature continues failing to give a clear answer about that. From Random Matrix Theory approach, the main goal is to analyze whether the price of oil, natural gas and their derivatives really affect the financial performance of two transportation sub-sectors (Passenger Transportation Services and Freight & Logistics Services) globally. Contrary to much of the previous literature, the extracted results reveal that the prices of oil, natural gas and its derivatives to not affect stock prices of the transportation industry. Through an approach not employed so far, the extracted results allow to continue the debate on the influence of oil prices, thus giving a different perspective that allows policy-makers, shareholders and investors to make a better decision on the industry under study.

Keywords Transport · Oil prices · RMT · APT · Stock prices · Brent

JEL Classification G12 · Q31 · G15

1 Introduction

According to the latest data published by the International Energy Agency (2023), fossil fuels continue to play an important role in the primary energy demand matrix, accounting for 80% of the total. Despite energy transition efforts to minimize fossil fuel consumption, organizations such as the Energy Information Administration (2022) warn that by 2050 the demand for fossil fuels will remain stable around 70%. Furthermore, by 2050, it is estimated that energy needs will increase by 15% due to population growth and living standards, which will cause fossil fuel consumption to remain stable (Exxon Mobil 2023). As a result of this dependence of energy on fossil fuels, the impact of the price of oil on the cost of living is relevant and will continue to be so in the future. For this reason, there is a growing need to better understand the performance of the oil price and how it affects different macroeconomic variables and financial markets.

Financial markets can be influenced by different macroeconomic variables, such as interest rates or exchange rates (Sadorsky 2001; Mensi et al. 2022). Other variables such as production cost, inflation, and investor confidence have been related to financial markets (Amano and Van Norden 1998). Among the variables that can affect

✉ Antonio Garcia-Amate
aga237@ual.es

Laura Molero-Gonzalez
lmg172@ual.es

Miguel Angel Sanchez-Granero
misanche@ual.es

Juan Evangelista Trinidad-Segovia
jetrini@ual.es

[1] Department of Economics and Business, University of Almeria, 04120 Almeria, Spain

[2] Department of Mathematics, University of Almeria, 04120 Almeria, Spain

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financial markets, the price of oil is considered an explanatory factor (Hamilton 2003). Sadorsky (1999) found that when oil prices rose, the returns of US stocks fell. This relationship is theoretically supported by three arguments. First, due to the increase in oil prices, production costs increase, thus reducing the company's cash flow and its return, and ultimately negatively affects the price of the shares (Filis 2010; Adekunle et al. 2020). Indirectly, a rise in oil prices increases inflation. High inflation generates a tight monetary policy in which interest rates will rise to reduce consumption. This will in turn make it more expensive for companies to finance themselves, affecting the discount rate and ultimately the return and the price of the shares (Basher and Sadorsky 2006; Bouri et al. 2023). Finally, considering that oil is a commodity, a rise in price will cause demand to fall, thus reducing the company's production and thus affecting the return and price of the shares (Edelstein and Kilian 2009).

Previous literature has deeply analyzed the impact of oil prices on financial markets, drawing different conclusions. For example, Chen et al. (1986) find no evidence that the oil price is an explanatory factor in the United States and Japan, respectively. Some authors suggest that the relationship may or may not be significant depending on whether a country is an oil importer or exporter, or whether developed or emerging markets are analyzed. For instance, for an oil exporting country like Saudi Arabia, Tausif et al. (2023) show that an increase in oil prices increases GDP, inflation, export value, and import value. Others, such as Shaharudin (2009) indicate that the price of oil affects not only the price of the share but also the distribution of dividends or retained earnings. It can be seen that, according to different results, the oil price cannot be considered as an explanatory factor in general terms. Therefore, it is concluded that the relationship should be analyzed according to different types of countries (oil importer vs. oil exporter) or sectors (oil producers vs. oil consumers). At the end of the twentieth century, Huang et al. (1996) noted that a significant causality between oil price and market was only found in the Oil and Gas (O&G) sector, finding no statistical significance in other sectors or in the aggregate market. Others, such as Saeed (2012) found that the oil price had an impact on specific sectors such as O&G, automobiles or electronics. A priori, oil-related sectors tend to have a significant impact on returns. Transportation is one of these sectors. According to the latest data published by the International Energy Agency (2022), the transport sector is the largest oil consumer, with 50.43 mb/d (million barrels per day), ahead of sectors such as petrochemicals, construction, or oil refineries. On the other hand, the"Stated Policies Scenario" shows an increase in demand to 56 mb/d in 2030. Therefore, the study of this sector is critical in order to understand in depth the relationship between oil prices and transport companies.

Taking into account oil consumption in the transportation sector, it is expected that the price of oil would influence the share price through cash flow due to the increase in production cost. However, the literature presents contradictory results, leading to fuzzy conclusions about this relationship. For example, while Çatik et al. (2020) warn that the price of oil negatively and significantly affects the return of the Turkish transport sector, El-Sharif et al. (2005) does not support a significant relationship for a set of British transport companies. Much of the literature focuses on specific countries or regions, resulting in a lack of unified results that would allow glimpses of conclusions that could be applied to the global transport sector. Previous literature has primarily focused on studying the Brent and West Texas Intermediate (WTI) benchmark prices. Both prices serve as the basis for a large portion of international crude oil contracts. Brent, on one hand, is the reference price used for marketing crude from North Sea basin, while WTI is used for contracts involving North American crude oil. This prominence has led previous authors to overlook testing other reference prices, such as those for gasoline, diesel, or natural gas. Finally, an over-focus on a particular subsector within the transport (e.g. rail or air transport, among others) may mean that there are other relevant subsectors that have not been considered. Our motivation lies in the perceived need for an in-depth study to provide the basis for global evidence on the relationship between the oil price (and its derivatives) and the transport sector.

The purpose of this study is to find out whether the price of oil (and its derivatives) has a significant impact over time on the returns of the companies belonging to the transport sector globally. The study employs a purely statistical approach to determine the number of statistically significant factors in explaining the cross-section of expected stock returns. In addition, we seek to test whether any of these factors are the market one. To achieve these goals, Random Matrix Theory statistical techniques have been applied (Molero González et al. 2023). Concretely, we have used the Onatski (2009) test, which is a null hypothesis test proposed to determine the number of factors in the generalized Dynamic Factor Model (DFM) of Forni et al. (2000). Data from 451 global companies in a wide range of subsectors has been analyzed, differentiating between Passenger Transportation Services and Freight and Logistics Services. Results show that for companies belonging to Passenger Transportation Services no factors are identified as statistically significant to explain the cross-section of stocks returns for a confidence level of 99%. If the confidence level is reduced to 95%, a three-factor model is identified by the test. One of these factors is the market factor. For the companies belonging to Freight & Logistic Services no factors are identified for a confidence level of 99%. For a lower confidence level (95%) still no factors are identified, except for small periods, where there is one significant factor: the market. In addition, we have tested whether different reference prices (Brent crude oil, Natural Gas, Diesel, Gasoline and Heating oil) could have an impact on companies' returns. Results show that none of them do.

The rest of the paper is structured as follows. Section 2 presents a comprehensive and in-depth review of the literature on previous research. Section 3 focuses on the methodology. Data and results are presented in Sect. 4. The paper ends with a brief discussion of the results, main conclusions and implications in Sect. 5.

2 Literature review

The versatility of crude oil, enabling it to be transformed into essential products ranging from gasoline to plastics, means there is no comparable substitute for this energy resource (Carson 2023). As a result, oil has become indispensable for many sectors such as industry, agriculture, or transportation, making it one of the most important commodity in the world (Alazzani and Wan-Hussin 2013; Mensi et al. 2014). A recent study published by Morning Consult (2023), notes that 90% of Americans considers that O&G plays a key role in the economy of the country. Furthermore, authors such as Chang (2023) note that volatility of oil prices can have a great impact on the stability of financial markets. In fact, previous literature has put a lot of effort into oil price forecasting, in order to better understand the explanatory factors of its high volatility (Deng et al. 2020; Cheng et al. 2022). Due to the high relevance of oil, many authors began to consider the impact of the oil price on the return of companies (Jones and Kaul 1996; Faff and Brailsford 1999; Sadorsky 2001; Jouini 2013; Balcilar et al. 2019; Dai and Zhu 2022). Jones and Kaul (1996) documented a negative impact of oil prices on the US, Canadian, Japanese, and UK markets. Kapusuzoglu (2011) found a similar long-term relationship for the Turkish market. However, when the literature begins to study the impact of oil prices on specific sectors, different results begin to emerge. For instance, Faff and Brailsford (1999) find that oil prices have a positive and significant impact on the Australian O&G industry, while sectors such as banking, transportation, and paper experience a negative impact when prices rise. However, Açikgöz and Alp (2023) find a positive relationship between oil prices and the Turkish transport industry. Through the analysis of 35 sectors globally, Nandha and Faff (2008) support the idea that the increase in oil prices negatively affects all sectors except mining and O&G, which it affects positively. Others such as Cong et al. (2008) did not find significance in the relationship between oil price and Chinese sectors, except for manufacturing and some O&G companies. For the oil shocks analyzed, the authors find that price volatility negatively affects these sectors. Based on these results, the need to differentiate between oil consumers and oil producers began to be considered.

Despite the fact that the literature has increased considerably in studying the impact of the oil price on financial markets at the sectoral level, there is no consensus in the results (Al-hajj et al. 2021). This is due to the type of relationship and the level of dependency that each industry may have on crude oil (Lee et al. 1995; Davis and Haltiwanger 2001; Lee and Ni 2002). For the financial sector in Jordan, Katircioglu et al. (2018) show that the financial performance of the firms is totally independent of oil price volatility. Phan et al. (2015) note that an increase in the oil price can have positive effects on producing sectors (O&G) and negative effects on consuming sectors (air transport, trucking, and chemical manufacturing). For example, sectors such as energy show an increase in return when the oil price increases, while other sectors such as food, manufacturing, real state or transportation experience a drop in return (McSweeney and Worthington 2008; Narayan and Sharma 2011). The latter is considered one of the main energy users, especially oil (Aggarwal et al. 2012; Mensi et al. 2014), so it deserves to be treated independently to know what the main results extracted from the literature are.

Maitra et al. (2021) demonstrate that there is a contagion effect between oil price fluctuations and the stock returns of logistics and international transportation companies. Through a sectoral study in Australia, McSweeney and Worthington (2008) note that the WTI has a significant impact on the banking, energy, retail and transportation sectors. A similar study in the United States between 1998 and 2006 conducted by Elyasiani et al. (2011) shows that the oil price coefficient is significant for equipment and air transportation. In line with this result, Shaeri and Katircioglu (2018) indicate that the WTI has a significant effect on the Dow Jones Transportation Average index. Other authors, such as Horobet et al. (2022) and Açikgöz and Alp (2023), find similar results for a sample of 25 airlines and the Turkish transportation sector, respectively. Despite these results, the previous literature also presents contradictory conclusions that invite a different alternative approach to the issue.

For the British transport sector, El-Sharif et al. (2005) find no significant impact on the price of oil. One explanation given by the author for this unusual result is that previous literature had studied the sector in countries such as the United States or Australia, where distances are generally greater, and this could affect the operating costs of companies based on these countries. An increase in costs may lead to reduced net income, which in turn could impact the company's market value. Other authors also found no causality between oil prices and the transport sector in Germany and the United Kingdom (Lee et al. 2012), and the Gulf Cooperation Council (GCC) countries (Hamdi et al. 2019). Mensi (2019) studies the Saudi transport sector and its relationship with Brent and WTI for the period 2006–2017, finding evidence of co-movement only during the financial crisis (2007–2009). More recently, Dar (2022) studied the price of the three largest Indian airlines and their relationship with Brent for the period 2005–2018. While it is true that in the short run there are very short time frames where the Brent is significant, in the long run airline stocks and Brent appear independent of each other. Brancaccio et al. (2023) find that the elasticity between fuel costs and transportation costs is low, ranging from 0.1 to 1.2, indicating that the impact of fuel prices on cargo costs is not significantly high. Similarly, Buljan and Badovinac (2023) demonstrate that for a set of 27 countries (2020–2023), crude oil prices and the Harmonized Index of Consumer Prices (HICP) for transportation services are positively and significantly related at the 5% level.

One of the gaps is the focus on a particular region or country. This makes it difficult to try to test different results to draw any kind of homogeneous conclusion. Some of the markets analyzed separately are: United States (Narayan and Sharma 2011; Phan et al. 2015; Shaeri and Katircioglu 2018; Saha 2022), Saudi Arabia (Mensi 2019), Australia (Faff and Brailsford 1999; McSweeney and Worthington 2008), GCC (Hamdi et al. 2019; Al-Fayoumi et al. 2023), Brazil (Ferreira et al. 2020), Iran (Ghoilpour 2011), United Kingdom (El-Sharif et al. 2005), Turkey (Çatik et al. 2020; Açikgöz and Alp 2023), and India (Dar 2022). To our knowledge, the only paper focused on the global transportation sector is the one published by Nandha and Brooks (2009). With a sample of 38 countries, the authors try to analyze the relationships of the transport sector with the WTI, concluding that the oil price has significant impact on the companies in the Developed, Europe, and G7 regions. However, this evidence does not apply to the Asia–Pacific, Emerging Markets and Latin America regions.

There is a scarcity of global studies focused on the oil price-transport sector relationship. On the other hand, many studies use monthly or weekly data. According to Tsuji (2018), it is crucial to measure the interaction between the oil price and the financial markets through high frequency (daily) data. Others, such as Al-Fayoumi et al. (2021) indicate that daily frequency can better extract market dynamics, making the implications more value-added for practitioners and policy-makers. Finally, previous literature works largely with a single oil benchmark price (Brent or WTI), which can reduce the robustness of the result. The motivation of the present study lies in reducing these gaps and to be able to contribute to the debate on the transport performance and its relationship with the oil price, analyzing from a statistical approach not applied so far and extracting robust results that allow drawing conclusions applicable to transport globally. Through the daily data of 332 companies and their relationship with five reference prices (Brent, Natural Gas, Low Sulphur Gasoil, RBOB Gasoline, and Heating Oil), the Onatski test (Onatski 2009), framed in the Random Matrix Theory, is carried out. The results for both samples (Passenger Transportation Services and Freight & Logistics Services) indicate that no reference price has a significant impact on stock prices.

3 Methodology

High-dimensional factor models with correlated idiosyncratic terms have been extensively studied. In finance, they make up the basis of the Arbitrage Pricing Theory (APT). One issue that has attracted a great deal of scholarly attention and which is very important to address is to determine how many factors there are.

When talking about high-dimensionality in statistics, it is well-known that the covariance matrix is distributed according to the Wishart distribution (Wishart 1928). If consider X a matrix of dimension $n\ x\ p$, with random entries i.i.d., according to a Gaussian probability distribution function (PDF), with mean zero and variance equals one; each vector $X_i \sim N_p(0, \Sigma)$, $i = 1, \ldots, n$. In statistics, $\mathbf{W} = \mathbf{X}^{\mathbf{T}}\mathbf{X}$ is said to have a p-variant Wishart distribution of n freedom degrees $W_p(n, \Sigma)$.

Within the field of RMT, it is said that matrices with structure $\mathbf{W}$ belong to the Wishart Orthogonal Ensemble (WOE). They are rotationally invariant under the Haar measure. A universal result is that regardless of the particular matrix X, when the number of columns is of the same order as the number of rows, and both dimensions grow without any limit, then the distribution of the eigenvalues of matrix $\mathbf{W}$ converges to the (Marchenko and Pastur 1967) law:

$$\rho(\lambda) = \frac{\sqrt{(\lambda_+ - \lambda)(\lambda - \lambda_-)}}{2\pi q\lambda} \tag{1}$$

where it is assumed that $\mathbf{\Sigma} = \mathbf{I}$.

When this theory is transposed to the financial context, X represents the data matrix of returns, where the first dimension represents the number of transactions days, and the second dimension the number of assets. In this way, **W** becomes the covariance matrix of p stock returns time series of length n. Thus, the Marchenko-Pastur law implies a bias in the estimation of the covariance matrix that increases as $q = p/n \rightarrow \infty$ when $n,p \rightarrow \infty$. On the other hand, the asset pricing problem relies on the factor models, having the covariance matrix as their central object. Consequently, it is needed to consider the effect of dimension to avoid bias in estimating the number of factors affecting stock returns.

To achieve this goal, one approach is to consider the Tracy and Widom (1994) distribution, which explains the behavior of the eigenvalues near the boundary of the Marchenko-Pastur law. This distribution helps to distinguish whether the eigenvalues are noise or true signals. By leveraging the Tracy-Widom distribution, hypothesis tests can be constructed to formally differentiate this bias.

In 2008, Onatski (2008) proved that the joint distribution function of the largest normalized centered eigenvalues of a complex p-dimensional Whisart matrix converges to the Tracy-Widom (Tracy and Widom 1994, 1996) distribution, as $p, n \rightarrow \infty$, so that $\frac{p}{n} = q \in (0, \infty)$. The author demonstrates how the theoretical result of his work can be used to find the set of common factors in the returns of stocks, with a confidence level of 95%. In 2009, Onatski (2009) developed a null hypothesis that there are k_0 factors, but less than $k_1 + 1$, in a generalized dynamic factor model (DFM). The test is based on a statistic $\widehat{R}$, proposed to determine the number of factors in the generalized DFM of Forni et al. (2000):

$$\widehat{R} = \max_{k_o < i \leq k_1} \frac{\widehat{\lambda}_i - \widehat{\lambda}_{i+1}}{\widehat{\lambda}_{i+1} - \widehat{\lambda}_{i+2}} \tag{2}$$

where $\widehat{\lambda}_i$ is the i-th largest eigenvalue of the smoothed periodogram estimate of the spectral density matrix of the data at a predetermined frequency. The author proved that the statistic of the test is asymptotically pivotal and has a distribution related to that of Tracy Widom type 2. Onatski tabulated the percentiles of the distribution, providing a set of critical values for the hypothesis test, which are shown in Online Resource 2.

The model considers a total of n observations with the structure $X_t = \Lambda(L)F_t + e_t$; where $\Lambda(L)$ is the polynomial matrix of size $p \times n$ in the delay operator L, F_t is a k-dimensional vector of factors at time t, and e_t is a p-dimensional vector of stationary correlated idiosyncratic terms.

A particular case occurs when $\Lambda(L)$ does not depend on L, but factors do no necessarily follow a white (Gaussian) noise process. In this case, the dynamic factor model is reduced to the approximate factor model of Chamberlain and Rothschild (1982) and the idiosyncratic terms are Gaussian and independent over time, so that we can prove the hypothesis of the approximated number of factors (in contrast to the dynamic one).

To determine the number of statistically significant factors, first, given the data matrix X, it is divided into two periods of time of equal size and the complex matrix $\widehat{X}_j$ is constructed:

$$\widehat{X}_j = X_j + iX_{j+\frac{n}{2}} \tag{3}$$

We then compute the eigenvalues $\widehat{\lambda}_i, \ldots, \widehat{\lambda}_p$ of

$$\frac{2}{n}\widehat{X}\widehat{X}^* \tag{4}$$

where $*$ denotes the transpose of the conjugate of the matrix. Third, the $\widehat{R}$ statistic is calculated through expression 1. And finally, the null hypothesis H_0 about the existence of k_0 factors is accepted or rejected, according to the value of $\widehat{R}$ and as indicated in the table of critical values provided by Onatski (2009).

The basis of the test consists of the idea of spectral separation and is related to the Tracy-Widom joint distribution for the Gaussian Orthogonal Ensemble (GUE) with $\beta = 2$ (Dyson 1962). When k is the actual number of factors, the first k eigenvalues are separated from the bulk and fall off the scale. The argument is based on the fact that the data matrix $\widehat{X}$ has a structure that is given by a factor model. The covariance matrix of the idiosyncratic errors is assumed to follow a Whisart distribution with complex entries. Then, since $\widehat{X}$ has a k-factor structure, the eigenvalues $\widehat{\lambda}_1, \ldots, \widehat{\lambda}_k$ diverge as $p, n \rightarrow \infty$; while $\widehat{\lambda}_{k+1}, \ldots, \widehat{\lambda}_p$ converge to the Marcenko-Pastur distribution with $\beta = 2$ (Marchenko and Pastur 1967), as they are complex entries. Thus, given that $\lambda_k \rightarrow \infty$ and that $\widehat{\lambda}_{k+1}$ and $\widehat{\lambda}_{k+2}$ remain bounded by approximation to the Marcenko-Pastur distribution, we have that the $\widehat{R}$-statistic diverges in the asymptotic regime $p, n \rightarrow \infty$ (Molero-Gonzalez et al. 2023).

In practical terms, the procedure to determine the number of factors consists on applying the Onatski's test iteratively, testing on each iteration a different number of factors until the null hypothesis is reached. In this way, it is assumed, a priori, that the actual number of factors (k) takes a value comprised between k_1 and k_2. Subsequently, we test $H_0 : k = k_1$ against $H_1 : k_1 < k \leq k_2$, with a significance level of α. If H_0 is accepted, it is concluded that the estimated number of factors is k_1 and the algorithm stopped. If, on the other hand, H_0 is rejected, we test $H_0 : k = k_1 + 1$ against $H_1 : k_1 + 1 < k \leq k_2$. This procedure is repeated until H_0 is accepted. The corresponding number of factors is considered to be the best estimation of the associate model.

Furthermore, following the methodology used in Molero-González et al. (2023), we have tested whether the identified factors are or not the market factor. To do so, we performed the experiment once more, applying the Onatski's test to the residual covariance matrix of the Sharpe model, instead of applying it to the covariance matrix of the returns. In other words, what we do is to remove the market factor and repeat the test without considering it. To remove the market factor instead of applying the test to the covariance matrix of returns (being $R_i = \alpha_i + \beta_i R_m + \varepsilon_i$), we apply it to the residual covariance matrix, and, thus, removing the market factor β_i.

4 Empirical application

4.1 Data

To achieve the objectives of this study, a broad, global, and deep sample is needed. According to the classification presented by the Refinitiv database (2025), the sample has been divided into Passenger Transportation Services and Freight & Logistics Services. We have established a company selection criterion, in which only those companies were taken in which the number of days with missing closing prices does not exceed 5% and their logarithmic returns were estimated. Thus, the number of companies, denoted by p, is 125 for the Passenger Transportation Services dataset, from the 184 companies that made up the initial sample; and $p = 207$ companies for the Freight & Logistics Services dataset, being the initial sample made of 267 companies. The study period considered starts in January 2008 and ends in November 2023.

In the context of random matrices, there is a determining parameter, which is given by the ratio between the number of stocks (p) and the number of observations $(n) : q = p/n$. In particular, the noise effect due to the finiteness of the sample is observed for values of $q \to 1$. In this study, we have considered only one value for $q = 1/2$. When considering $q = 1/8$, we talk about low dimensionality, which can be solved employing classical statistical techniques. We talk about high dimensionality when $q = 1/2$. This second scenario requires modern techniques, such as the Tracy-Widom and Onatski statistical tests, which fall within the scope of Random Matrix Theory. For the case of our study, we are going to focus, thus, on the high-dimensionality scenario.

We designed a sliding-window experiment for each dataset. In each window, the number of statistically significant factors was determined at 3 different confidence levels $\alpha \in \{0.01, 0.05, 0.10\}$. The number of windows for both datasets have been 230.

For the Passenger Transportation Services dataset, the number of observations (denoted by n) in each window is 250 days. For the Freight & Logistics Services dataset, $n = 414$ days.

For all subsectors, a skip of $\Delta n = 20$ days of transactions has been considered for successive windows; i.e., windows are shifted successively 20 days. We have considered an upper bound $k_2 = 8$. In this way, the alternative hypothesis has been tested to that limit. We consider that an eight-factor model is enough to cover most situations.

4.2 Results

Figure 1a shows the dynamic of the estimated number of factors for companies that belong to the Passenger Transportation Services subsector. Taking into account a level of significance of 99% (represented by the blue dots), the test does not identify any factors as statistically significant to explain the cross-section of stocks returns. There are two small periods (in mid-2021 and at the end of 2023), where one factor is identified as significant by the test. If we decrease the confidence level to 95%, up to three factors are identified by the test. Until the end of 2012, there is only one factor. Subsequently, we identified three factors until the end of 2013. From then until the end of 2016, no factors were identified. Then, there are two factors until the end of 2022. From this moment on, until the end of the study period, there are no significant explanatory factors. We found similar results if we consider a level of significance of 90%. Looking at Fig. 1a, and focusing on the green starts, we can see that the test identifies up to three factors.

Figure 1b shows the dynamic of the number of statistically significant factors if the market factor (equal weighted market) is removed. What we do here is to apply the Onatski test again, but now, instead of applying it to covariance matrix of the returns, we apply it to the residual covariance matrix of the Sharpe model; i.e., we remove the market and apply the test again to see how many factors the test identifies. As can be seen, if we remove the market, the test identifies one factor less for all periods and levels of confidence. In this way, where in 1a we found two factors, now in 1b there is only one, and so on.

Figure 2a shows the dynamic of the number of factors for the Freight & Logistics Services subsector. Taking into account a confidence level of 99%, there are no significant factors, with the exception of two small periods, at the beginning of 2021 and 2023, where the test identifies only one factor as statistically significant. Similar results are obtained when taking a confidence level of 95%. In general, the test does not identify any factors, with the exception of the periods from November 2011 to December 2012, and from 2016 to 2017, where there is one factor.

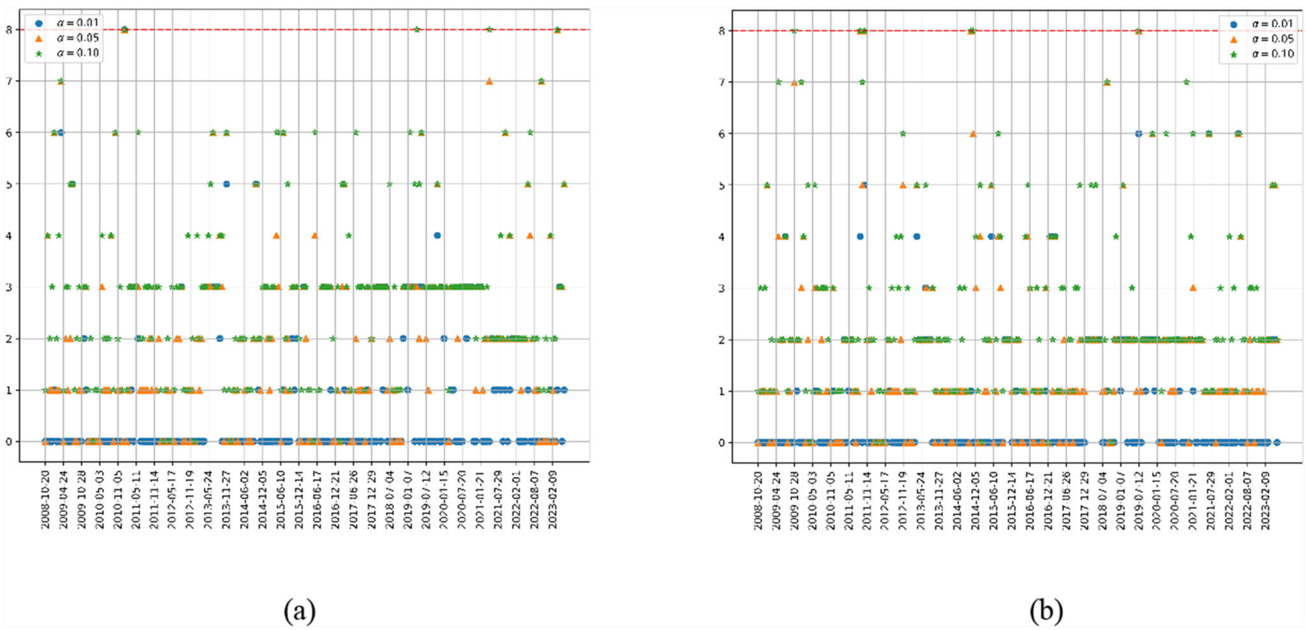


Fig. 1 Number of factors as a function of time for **a** the Passenger Transportation Services subsector stocks; **b** the residual covariance matrix of the Sharpe model of the Passenger Transportation Services subsector stocks. The parameter α denotes the significance level of the Onatski test, and the red dotted line represents the upper bound of the alternative hypothesis $k2$

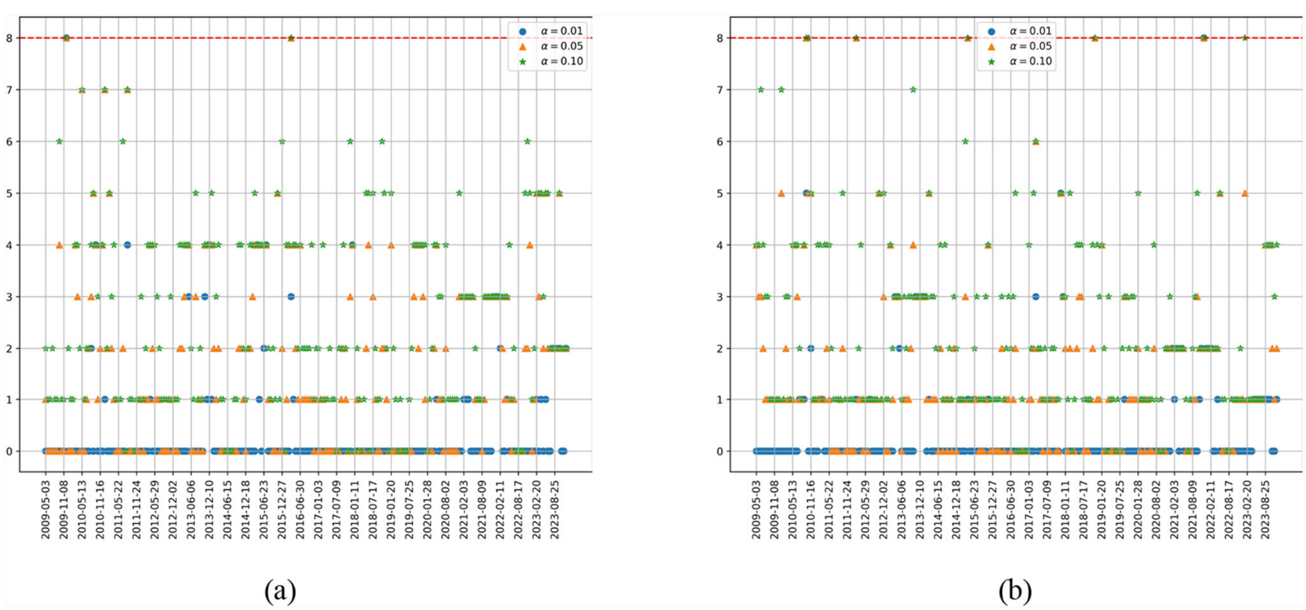


Fig. 2 Number of factors as a function of time for **a** the Freight & Logistics Services subsector stocks; **b** the residual covariance matrix of the Sharpe model of the Freight & Logistics Services subsector stocks. The parameter α denotes the significance level of the Onatski test, and the red dotted line represents the upper bound of the alternative hypothesis $k2$

If the level of confidence drops to 90%, the general trend is that there is only one factor for the study period, with small periods where up to four factors are identified.

As in the Passenger Transportation Services dataset, we have removed the market to prove whether the identified factor is the market factor. Figure 2b shows the results of Freight & Logistics Services subsector. It can be seen that for the periods when the test identified two, three and four factors in Fig. 2a, now there are one, two and three, respectively. These results are analogous to those of the Passenger Transportation Services dataset. However, there is a difference with respect to those and is that in the periods in which only one factor was identified, it does not disappear when the market is eliminated. This leads to the conclusion that the market is one of the factors identified by the test, but it is not the factor that exists when there is only one.

4.2.1 Testing oil prices as market factor for the transport companies

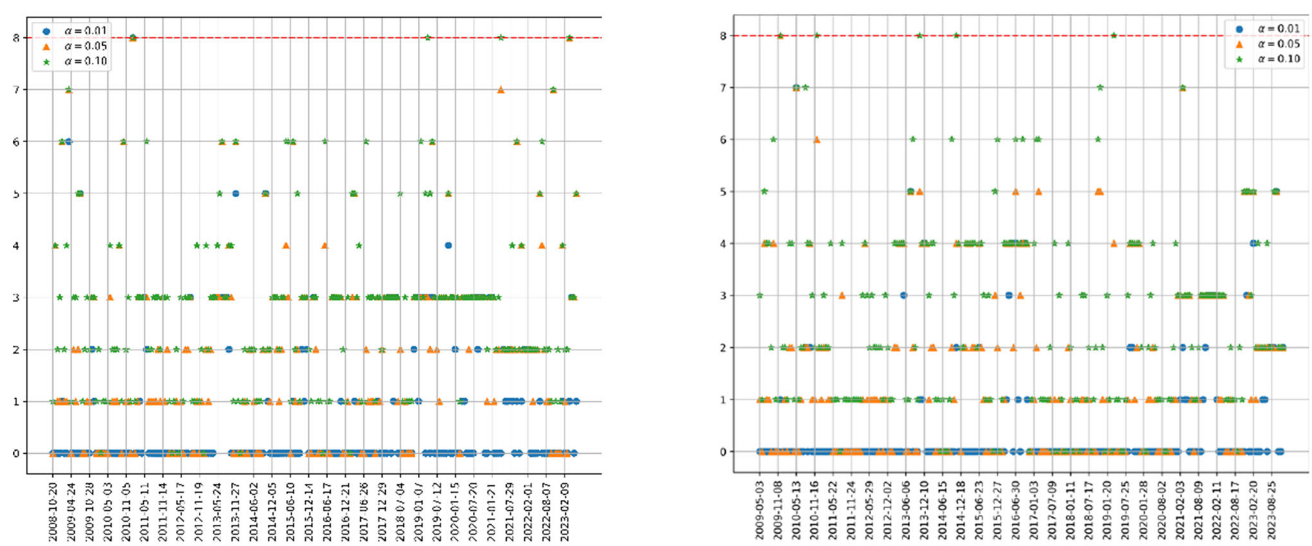
Going one step further, we have tested whether the Brent, Natural Gas, Low Sulphur Gasoil, RBOB Gasoline, and Heating Oil prices can be an explanatory factor of the cross-section of stocks returns. These reference prices have been selected due to their intensive use by the transportation industry. To do so, we repeated the experiment of the previous section, but taking these prices as the market factor (the experiment was performed once for each price). The results show that none of them play a role in explaining the cross-section of the return of the Passenger Transportation Services and Freight & Logistics Services subsectors.

Figures 3 and 4 show the results for both subsectors when the Brent price is taken as the market factor. As can be seen, the number of factors identified by the test does not change when the market factor is removed, leading one to conclude that the Brent price cannot be considered as the explanatory factor of these markets. The results for Natural Gas, Low Sulphur Gasoil, RBOB Gasoline, and Heating Oil prices are almost the same. They are shown in Online Resource 2.

5 Discussion

The oil price impact on the transport return is a matter of relevance and interest (Nandha and Brooks 2009). Oil is considered one of the key resources for the development of a country or region, in part because oil reference prices (e.g. gasoline or diesel) are indispensable for sectors such as transportation (Ferreira et al. 2020). Therefore, it is important to analyze deeply the exposure of transport to oil price (Mohanty et al. 2010). Although it is generally understood that high oil prices can cause transportation companies to suffer (Shaeri and Katircioglu 2018), we have observed how the previous literature yields contradictory results in that regard. With 332 global companies, and a methodological approach not applied so far in the analysis of the impact of oil price on transportation companies, we have been able to extract results that allow us to robustly contradict the general opinion that the literature has on the significant relationship between oil price and transport.

The results show that none of the five reference prices has a significant impact on the companies' return. On the other hand, the Onatski test (Onatski 2009) indicates that at 90% and 95% confidence levels up to 4 significant factors are observed, of which the reference market is one of them. However, none of these factors is the set of oil reference prices analyzed. At the 99% confidence level, no significant factors are identified that could affect the overall transportation sector. The results are in line with those obtained by previous authors such as El-Sharif et al. (2005)



(a) (b)

Fig. 3 Number of factors as a function of time, considering the Brent price as the market factor, for **a** the Freight and Logistics Services subsector stocks; **b** the residual covariance matrix of the Sharpe model of the Freight & Logistics Services subsector stocks. The parameter α denotes the significance level of the Onatski test, and the red dotted line represents the upper bound of the alternative hypothesis *k*2

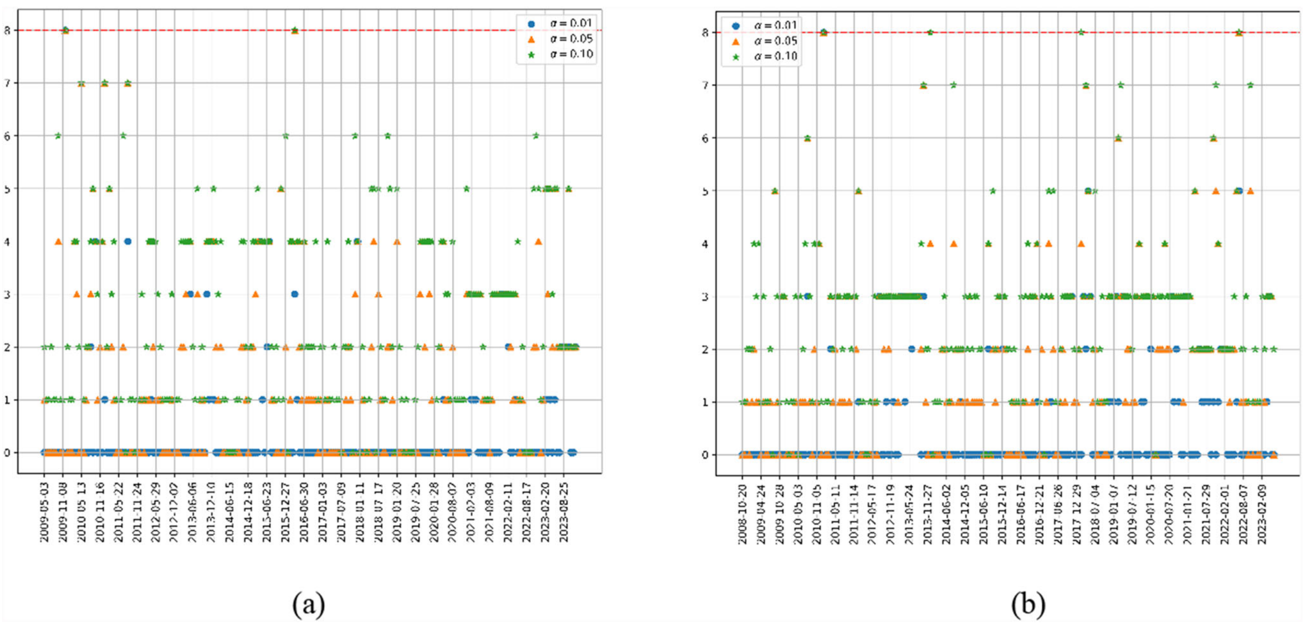


Fig. 4 Number of factors as a function of time, considering the Brent price as the market factor, for **a** the Passenger Transportation Services subsector stocks; **b** the residual covariance matrix of the Sharpe model of the Passenger Transportation Services subsector stocks. The parameter α denotes the significance level of the Onatski test, and the red dotted line represents the upper bound of the alternative hypothesis *k*2

or Mohanty et al. (2010), among others. One explanation for the result obtained may be the global nature of the study. Previous studies that have analyzed markets such as India (Dar 2022), Iran (Ghoilpour 2011), Brazil (Ferreira et al. 2020) or GCC (Hamdi et al. 2019) have not found significance in the oil price. Moreover, the only global study focused on transportation notes that there is no impact on the oil price in Asia–Pacific, Emerging Markets, and Latin America (Nandha and Brooks 2009). Therefore, the non-significance of price in a global sample focused on the transportation sector is not a questionable result.

6 Concluding remarks

The study sample, the number of benchmark prices tested and the novel methodology employed allow us to add value to the debate on whether or not oil benchmark prices really affect the transport. Through daily data, we have testedthe dynamics of transport companies globally, contradicting most of the results of the previous literature, but also giving support to certain results already presented by some authors. This paper provides a different view of the problem analyzed, addressing the issue with a novel methodology that allows us to extract reliable results from a global sample, leaving alive the debate. The results obtained can assist investors and fund managers in making investment decisions that may enhance their portfolio returns. Knowing how the market performs and predicting its movement is crucial.

As observed, conclusions regarding the impact of oil prices on transportation stock returns are highly varied. At first glance, it can be assumed that, since oil is an essential resource for transportation activities, an increase in oil prices may raise costs, thereby directly affecting net income and, consequently, market value. However, this assumption cannot be empirically validated on a global scale, as much of the previous research has focused on specific regions. This limitation may be due to the industry's sensitivity to more localized factors, such as government regulations, the type of transportation prevalent in the area, or the volume of trade. This study aims to address this limitation by showing that oil price and its derivatives are not significant factors in stock price performance from an international perspective. The study has also identified up to four other significant factors, one of which is the market. This finding, along with the insignificance of the five benchmark prices tested, adds theoretical value to the ongoing debate on whether benchmark prices impact a company's value.

From a practitioner standpoint, the findings provide deeper insights into how stocks in the transportation sector perform in response to oil price movements. For practitioners and business this is a crucial issue, as the results suggest they do not necessarily need to account for crude oil prices to create value for stakeholders—the ultimate goal of any business. A better understanding of this relationship enables business leaders to make decisions that contribute to business improvement without being constrained by potential future benchmark price movements. This allows decisions to be based on the company's own economic and financial criteria rather than on geopolitical issues or macroeconomic data that may not impact the market value of the sector. Additionally, these results offer significant added value for investors and fund managers. Knowing that transportation companies are independent of oil price movements allows for more informed investment decisions based on other types of company-specific financial information or growth expectations. This reduces the potential misconception that a rise in oil prices will automatically impact transportation stocks.

Although the contributions of the paper are significant, it is not free of limitations. While it is true that we consider a global sample, no distinction is made between countries or regions. As indicated in the previous literature, depending on the region, the significance of the oil price may differ (Nandha and Brooks 2009). Thus, the discrimination of the sample by region (e.g. through dummy variables) may yield different results that allow for a deeper understanding of the sector's performance. Second, as indicated by Aggarwal et al. (2012), the impact of the oil price can differ depending on the type of subsector within the transportation. The present study, although it considers a broad set of subsectors (see Online Resource 2), does not discriminate according to these, but only differentiates between Passenger Transportation Services and Freight & Logistics Services for data availability reasons. In this way, if it is discriminated by subsector (air, truck, rail, among others), it is possible impact on the relationship that could occur due to market shocks such as the financial crisis, the Russia-Ukraine war or the Covid-19 pandemic. This is an issue that so far the previous literature has not analyzed. A thorough analysis of the impact on the oil price-transport relationship could yield new conclusions that would allow us to know and predict the relationship.

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Data availability The datasets generated during and/or analyzed during the current study are not publicly available due to that is not public but are available from the corresponding author on reasonable request.

Declarations

Conflict of interest Not applicable.

Ethical approval Not applicable.

Human Participants and/or Animals Not applicable.

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