Tight and Compact MILP Formulation of Start-Up and Shut-Down Ramping in Unit Commitment

Journal:	*IEEE Transactions on Power Systems*
Manuscript ID:	TPWRS-00080-2012
Manuscript Type:	Transactions
Date Submitted by the Author:	26-Jan-2012
Complete List of Authors:	Morales-España, German; Universidad Pontificia Comillas, Instituto de Investigación Tecnológica Latorre, Jesus; Comillas Pontifical University, Institute for Research in Technology Ramos, Andres; Universidad Pontificia Comillas, Instituto de Investigacion Tecnologica
Technical Topic Area :	Power system operations, Power system markets
Key Words:	Start-up & shut-down ramps, Mixed-integer linear programming, Unit commitment, Thermal units

SCHOLARONE™
Manuscripts

Tight and Compact MILP Formulation of Start-Up and Shut-Down Ramping in Unit Commitment

German Morales-España, *Student Member, IEEE,* Jesus M. Latorre, *Member, IEEE,* and Andres Ramos

***Abstract*—The Start-Up (SU) & Shut-Down (SD) ramps of thermal units must be considered in Unit Commitment (UC) formulations to accurately represent the unit's power production. However, these ramps are usually ignored because they will considerably increase the computational burden of the UC problem. This paper presents a Mixed-Integer Linear Programming (MILP) formulation of SU & SD power trajectories of thermal units where the computational burden is considerably decreased in comparison to formulations commonly found in the literature. This is because the proposed formulation is i) tighter, i.e. the relaxed solution is nearer to the optimal integer solution; and ii) more compact, i.e. it needs fewer constraints, variables and non-zero elements in the constraint matrix. The proposed formulation is illustrated employing the self-UC problem faced by a thermal unit. Results show that not considering SU & SD ramps changes the commitment decisions causing a negative economic impact. We provide computational results comparing the proposed formulation with others found in the literature. The computation time was dramatically reduced as a natural consequence of the considerably tighter and more compact formulation.**

***Index Terms*—Mixed-integer linear programming, start-up & shut-down ramps, thermal units, unit commitment.**

NOMENCLATURE

THE main definitions and notation used are presented in this section for quick reference. Upper-case letters are used for denoting parameters and sets; and lower-case letters for variables and indexes.

A. Definitions

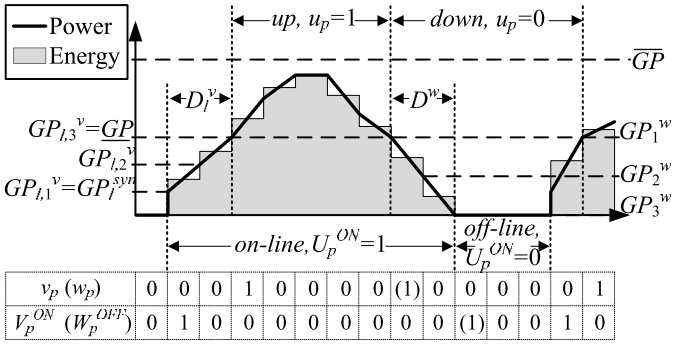
Before starting, we show some definitions given to different states of the thermal unit and the difference between them. The following terminology is used in this paper to reference the different unit operation states, see Fig. 1.

on-line the unit is synchronized with the system.
off-line the unit is not synchronized with the system.
up the unit is producing above its minimum output. During the *up* state, the unit output is controllable.
down the unit is producing below its minimum output, either *off-line, s*tarting up or shutting down.

B. Indexes and Sets

$l \in L$ Type of start-up, running from 1 (the hottest) to N_L (the coldest).

The authors are with the Institute for Research in Technology (IIT) of the School of Engineering (ICAI), Universidad Pontificia Comillas, Madrid, España (e-mail: german.morales@iit.upcomillas.es; g.morales.espana@gmail.com; jesus.latorre@iit.upcomillas.es; andres.ramos@iit.icai.upcomillas.es).



v_p (w_p)	0	0	0	1	0	0	0	0	(1)	0	0	0	0	0	1
V_p^{ON} (W_p^{OFF})	0	1	0	0	0	0	0	0	0	0	(1)	0	0	1	0

Fig. 1. Operation states of a thermal unit, including SU & SD ramps.

$p \in P$ Hourly periods in the time horizon, running from 1 to N_P hours.

C. Variables

All the variables listed here are non-negative continuous variables with the exception of u_p, which is the only binary variable in the proposed formulation.

gp_p Power output at the end of period p, production above the minimum output [MW].
ge_p Energy production during period p [MWh].
u_p Commitment of the unit during period p $\{0,1\}$. Binary variable which is equal to 1 if the unit is *up* and 0 if *it is down*, see Fig. 1.
v_p Start-up in period p $[0,1]$. Continuous variable which takes the value of 1 if the unit starts up in period p and 0 otherwise, see Fig. 1.
w_p *S*hut-down in period p $[0,1]$. Continuous variable which takes the value of 1 if the unit shuts down in period p and 0 otherwise, see Fig. 1.
$\delta_{p,l}$ Start-up type l in period p $[0,1]$. Continuous variable which takes the value of 1 in the period where the unit starts up for the start-up type l and 0 otherwise.

D. Parameters

1) Cost/Price Data:

EP_p Forecasted price of energy in period p [$/MWh].
FC Fixed cost of the unit [$/h].
VC Variable cost of the unit [$/MWh].
StC^l Start-up cost for the start-up type l [$].
ShC Shut-down cost [$].

2) Technical Data:

$\overline{GP}$, $\underline{GP}$	Maximum and minimum power output [MW].
RU, RD	Ramp-up and ramp-down rates [MW/h].
TU, TD	Minimum up and down periods [h].
D_l^v	Start-up ramp duration for the start-up type l [h].
D^w	Shut-down ramp duration [h].
GP_l^{sync}	Power output at which the unit is synchronized [MW], see Fig. 1.
$GP_{l,i}^v$	Power output at the beginning of the i^{th} interval of the SU ramp process [MW], see Fig. 1.
$GP_{l,i}^w$	Power output at the beginning of the i^{th} interval of the SD ramp process [MW], see Fig. 1.
T_l^{DW}	Minimum number of periods that the unit must be down for the start-up type l [h].

I. Introduction

THE actual operation of power generation units must be considered in detail in order to rigorously model the generation scheduling of these units. Moreover, with the introduction of competition, accurate modeling and solutions of Unit Commitment (UC) problems are even more necessary to achieve efficiency and feasibility in energy production [1], [2].

Modeling ramping constraints is a fundamental issue for the accurate representation of technical operation of thermal units in UC problems, either in competitive or centralized environments. Ramping constraints restrict the capability of thermal units to increase or decrease their production over consecutive time periods in which a unit can be either in the dispatchable phase (up state), during the Start-Up (SU) or Shut-Down (SD) processes. Besides this, ramping constraints limit the contribution of thermal units to operating reserves. Therefore, an accurate modeling of thermal unit ramps in UC formulations is crucial to guarantee a feasible and optimal operation of these units.

Most of the literature on modeling constraints of thermal units deals with the unit operation above the minimum output [3], [4]. Units are considered to start/end their production at the minimum output while the SU and SD power trajectories are ignored, they are just represented by their associated costs in the objective function, also known as SU and SD costs. Some papers are aware of the importance of considering these ramps in the UC optimization problem. However, they do not include them because the resulting model will be considerably more complex causing prohibitive solving times [5]–[7]. On the other hand, due to the increasing penetration of wind generation nowadays, thermal units are being shut down and started up more often [8]. Therefore a detailed modeling of the SU and SD processes in UCs is required.

Lagrangian relaxation (LR) was the dominant optimization technique to solve UC problems through problem decomposition. However, the application of direct mixed-integer linear programming (MILP) to solving UC is growing significantly due to improvements on MILP solvers. Combining pure algorithmic speedup with the progress in computing machinery, solving MILPs has become around 100 million times faster in the last 20 years [9]. The main advantages of MILP over LR are that (i) it provides a more accurate measure of optimality, as well as (ii) a proven global optimal solution, and (iii) it is easier to modify the MILP model which enhances modeling capabilities and adaptability. However, the main disadvantage of MILP over LR is the memory requirement limiting thus its scalability [1], [10].

Several models have been proposed to deal with the ramp limits above the minimum output under different approaches, such as dynamic programing and LR [4], [11]. The SU and SD power trajectories are explicitly modeled under the LR approach in [12] and under the MILP framework in [13] and [14]. In [12] and [13], just one possible power trajectory for the SU process is modeled, while [14] considers different SU power trajectories depending on the unit's prior down time. The main drawback of the aforementioned formulations is the creation of large models which highly increase the complexity of UC problems and therefore making them unattractive for practical implementation. The MILP models proposed in [13] and [14] are large due to the introduction of many constraints in order to deal with the power trajectories above and below the minimum output. Apart from this, [14] needs many binary variables to model the different SU power trajectories.

The use of MILP-based UC formulations has been considerably increasing since around fifty years ago [15]. As computational and algorithmic power increases, so does the complexity of the MILP formulations, adding features like e.g. ramping constraints, minimum up and down times and exponential SU costs [3]. MILP Formulations should also be further improved so even more advanced and computationally demanding problems can be implemented, such as stochastic formulations [16], multiple simultaneous contingencies [17] and generation expansion planning [18]. Furthermore, improved formulations speed up the convergence to achieve optimal solutions. The need of optimal solutions is of crucial importance in competitive markets because two quasi-optimal solutions can produce significantly different payments to market players [1], [2].

Improving a MILP formulation allows a faster search for optimality by tightening (removing inefficient solutions from) the original feasible region. Tightening requires strong lower bounds for minimization problems. This means formulating the problem in such a way that the associated linear programming (LP) relaxation provides a good approximation of the value of the integer optimal solution. The time required for proving optimality is often prohibitive because the gap between the integer optimal solution and its associated LP relaxation is typically very large [19]. Furthermore, a poor lower bound provided by the LP relaxation will not be adequate to guide the search for good feasible solutions in the solving phase (branch-and-cut) of a standard MILP solver [20]. One common measure of the quality of a MILP formulation is the integrality gap, which is the difference (or ratio) between the optimal MILP solution and the LP relaxation. The most common way to decrease the integrality gap is by adding more constraints, and sometimes more variables, to the formulation which are redundant for the optimal integer solution, thus tightening the feasible region. A compact MILP formulation (few constraints, variables and

non-zero elements in the constraint matrix) usually has a weak LP relaxation [21]. The relaxation is then frequently tightened by a reformulation that involves a huge number of constraints and (sometimes) variables. However, the resulting expanded model must close the gap enough to be worth the extra time taken to solve the LP relaxations during the solving phase [22], [23]. In other words, usually, tightening a MILP formulation comes at the expense of expanding the model which implies extra time consumption. Therefore, creating tight and compact MILP formulations is a nontrivial task because the obvious formulations are commonly either very weak or very large [24].

Creating tight (or strong) MILP formulations has been widely researched [25], [26]. For the case of UC problems, there have been efforts on different specific aspects. In [27], a strong formulation of the minimum up/down time constraints is proposed by reformulating the constraints; in [28], a tighter linear approximation for quadratic generation costs is proposed; and [29] presents a new class of inequalities giving a tighter description of the feasible operating schedules for generators. The latter formulation proposes an expanded model that due to the resulting dimension, and in order to obtain computational benefits, the inequalities are iteratively introduced as cuts (when needed) during the branch-and-cut phase.

The main contributions of this paper are as follows:

1) A rigorous MILP formulation of SU and SD power trajectories is provided, taking into account the unit's previous down time and making a distinction between energy and power trajectories. This formulation is tighter compared to [13] and [14]. This means that the relaxed solution is closer to the optimal integer solution speeding up the search for optimality during the MILP solving process.
2) This MILP formulation is compact, hence overcoming the main disadvantage of previous models [13], [14]. If one power trajectory is modeled for the SU & SD ramps, then there is neither a need to increase the number of constraints nor a necessity to rise the number of variables in comparison to a formulation without SU & SD ramps. This formulation depends on less than half the constraint-matrix density and constraints in relation to [13]. Furthermore, when considering different SU trajectories, the formulation of this paper requires the introduction of fewer constraints and merely continuous variables compared to [14]. Moreover, the proposed formulation is easier to implement than the aforementioned ones, mainly because the problem is formulated in a conciser way.
3) The proposed formulation is simultaneously tight and compact. Tightening a MILP formulation usually comes at the expense of expanding the model, like [29] which introduces complicated constraints to tighten the UC formulation. As a natural consequence of tightness and compactness of the presented formulation, its computational burden is reduced in relation to [13] and [14].
4) This formulation of SU & SD ramps is suitable for any UC problem, either in centralized or competitive environments. Additionally, further model expansion will not require the introduction of numerous terms in the constraints in order to avoid conflicts between the up and down states, unlike [13] and [14].

In order to illustrate the effectiveness of the proposed formulation, the self-UC for a price-taker thermal generator is used. The objective of a thermal generator, in the self-UC, is to maximize the profits from selling energy in the day-ahead market, while satisfying all technical constraints.

The remainder of this paper is organized as follows: Section II presents the formulation of the SU and SD constraints in detail. Section III provides and discusses results from several case studies, where an impact of not considering the SU and SD ramps is shown and a comparison of the proposed formulation with [13] and [14] formulations is made. Finally, some relevant conclusions are drawn in Section IV.

II. Proposed Approach

This section presents the mathematical formulation of the SU & SD power trajectories. These constraints are valid for any UC formulation. With the purpose of illustrating how this formulation works, the objective function is formulated for the case of a price taker self-UC problem. This section is divided in three parts, Subsection II-A details the generic mathematical formulation; Subsection II-B shows how to obtain the on-line and off-line unit states after solving the optimization problem; and Subsection II-C presents how the formulation is simplified (ii) in case fast generators, which can ramp up/down in one hour, are considered; and (ii) in case that single SU ramps are modeled.

Hourly times intervals are considered, but it should be noted that the formulation can be easily adapted to handle shorter time periods. For the sake of simplicity, the reserve constraints are not considered, however they can be easily introduced in the model, the interested reader is referred to [30].

A. Mathematical Formulation

The different operation states of a thermal unit are presented in Fig. 1 [14]. Difference between up/down and on/off states is made, see Fig. 1 and definitions at the beginning of the paper. The proposed formulation is modeled as a function of up/down (u_p), SU (v_p) and SD (w_p) variables. However, variables depending on the on-/off-line states, on-line (U_p^{ON}), turn-on (V_p^{ON}) and turn-off (W_p^{OFF}), can be obtained after solving the optimization problem, see Subsection II-B. During the up period, the unit has the flexibility to follow any trajectory being bounded between the maximum and minimum output and by the ramping-rate limits. On the other hand, the power output when the unit is starting up or shutting down follows a predefined power trajectory. Unlike the SD ramp, the SU ramp trajectory depends on the unit's previous down time.

By considering the commitment variable u_p as up/down state, the generation output above and below $\underline{GP}$ can be managed independently. Therefore, the SU and SD power trajectories do not interfere in the constraints that contains the power output gp_p, such as ramping and max-/min- output limits. This is the main characteristic that makes the proposed

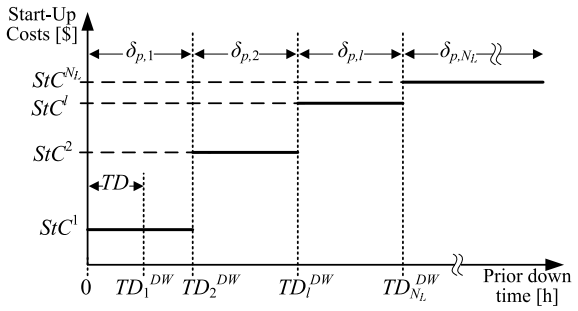


Fig. 2. Start-up costs as a function of the unit's previous down time.

formulation compact. Unlike [13] and [14] where many of the constraints contain summations of binary variables in order to avoid conflicts between the power output above and below $\underline{GP}$.

1) Start-Up Type: Different SU types are modeled depending on how long the unit has been down. The SU type $\delta_{p,l}$ is selected if the unit has been previously down within the interval $\left[T_l^{DW}, T_{l+1}^{DW}\right)$, see Fig. 2. Similarly to [31] and [14], new variables are introduced to select the SU type. The following constraint allow the selection of the SU type depending on how long the unit has been down

$$\delta_{p,l} \leq \sum_{i=T_l^{DW}}^{T_{l+1}^{DW}-1} w_{p-i} \quad \forall p \in \left[T_{l+1}^{DW}, N_P\right], \forall l \in [1, N_L) \qquad (1)$$

where the right side of (1) is equal to 1 if the unit has been down within the interval $\left[T_l^{DW}, T_{l+1}^{DW}\right)$ before hour p. Therefore, $\delta_{p,l}$ can only be activated ($\delta_{p,l} \leq 1$) if the unit has been previously down within this interval.

Note that (1) is not defined for the first hours. For the initial hours of the scheduling horizon, the SU activation depends on the unit state before the first hour $p = 1$, Appendix A details how the first SU types $\delta_{p,l}$ are obtained by the initial conditions of the unit.

The following constraint ensures that just one SU type is selected when the unit starts up:

$$\sum_{l=1}^{N_L} \delta_{p,l} = v_p \quad \forall p \qquad (2)$$

Equation (1) constrain all SU types excepting the coldest one δ_{p,N_L}. However, constraints (1) and (2) ensure $\delta_{p,N_L} = 1$ when the unit starts up, $v_p = 1$, and has been down for at least $T_{N_L}^{DW}$ hours. This is because (1) makes $\delta_{p,l} = 0$ for all $l \neq N_L$ and then (2) makes $\delta_{p,N_L} = 1$. In the case that more than one SU type variable can be activated ($\delta_{p,l} \leq 1$) then (2) together with the objective function ensure that the hottest, which is the cheapest, possible option is always selected. Therefore, just one variable is activated (equal to one) even when they are modeled as continuous variables. This is due to the convex (monotonically increasing) characteristic of the exponential SU costs of thermal units [3], see Fig. 2. In summary, when the unit has been down for a certain time, (1) ensures that hotter (than the correct) SU types are not selected and (2) together with the objective function guarantee that the correct SU type is chosen.

The constraint (1) is made even more compact by taking into account the minimum down constraint (minimum up/down time constraints are presented in the following Subsection II-A2). The hottest SU $\delta_{p,1}$ must be activated when the unit has been down within the interval $\left[0, T_2^{DW}\right)$. However, the minimum down constraint (4) ensures that the unit cannot be down for less than the minimum down hours TD. Therefore, the hottest SU is only possible within the interval $\left[TD, T_2^{DW}\right)$. By defining $T_1^{DW} = TD$, see Fig. 2, the constraint (1) together with the minimum down constraint (4) ensure that the hottest SU $\delta_{p,1}$ can be activated only when the unit has been down less than T_2^{DW} hours.

2) Minimum Up/Down Times: Constraints (3) and (4) ensure the minimum up and down times respectively [27]. This formulation has been compared with others and has shown a better performance [27], [29], [32].

$$\sum_{i=p-TU+1}^{p} v_i \leq u_p \quad \forall p \in [TU, N_P] \qquad (3)$$

$$\sum_{i=p-TD+1}^{p} w_i \leq 1 - u_p \quad \forall p \in [TD, N_P] \qquad (4)$$

The minimum down time TD in (4) contains the SD ramp duration (D^w), plus the hottest SU ramp duration (D_1^v), plus the minimum time that the unit must be off-line. Therefore, (4) is needed in the proposed formulation to avoid overlapping of the SU & SD ramps. Similarly, T_l^{DW} also contains the SU & SD ramp durations (D_l^v & D^w) as well as the off-line time required to activate the l SU type $\delta_{p,l}$.

Appendix A presents how the initial conditions force the unit to remain up/down during the first hours depending on the unit state before the scheduling horizon.

3) Logic for Commitment, Start-Up & Shut-Down: The commitment state of a thermal unit in each hour p depends on the previous hour state, the starting and stopping decisions for that hour. The following constraint can be found in models published about fifty years ago [15].

$$u_p - u_{p-1} = v_p - w_p \quad \forall p \qquad (5)$$

The unit commitment variable, u_p, must be modeled as binary. The SU, v_p, and SD, w_p, can be treated, in practice, as continuous variables. Once u_p is defined binary, v_p and w_p take 0/1 values [29].

4) Capacity Limits: The generation output of the unit above its minimum output is modeled as:

$$0 \leq gp_p \leq \left(\overline{GP} - \underline{GP}\right)(u_p - w_{p+1}) \quad \forall p \qquad (6)$$

Constraint (6) ensures the total power output to be equal to $\underline{GP}$ ($gp_p = 0$) at the beginning and at the end of a continuous up period. Unlike the power output, the energy can take different values than $\underline{GP}$ in the limits of an up period, as shown in Fig. 1. The SU (SD) trajectory ends (begins) at $\underline{GP}$ level, making thus the connection with the up period that starts (ends) at this level, see Fig. 1.

5) Ramping Limits: By modeling u_p as the up/down state and gp_p as the output above $\underline{GP}$, there are no conflicts between

the up and down states in the constraints involving g_p. This characteristic helps the proposed formulation to be compact, otherwise, in order to avoid conflicts between the up and down states, u_p, v_p and w_p need to be introduced in most of the constraints that involve gp_p, like in [13] and [14]. Therefore, the proposed formulation avoid the introduction of many variables in most of the equations, this is the case of the ramping constraints that just depend on the generation variables between two consecutive hours:

$$gp_p - gp_{p-1} \leq RU \quad \forall p \tag{7}$$

$$gp_{p-1} - gp_p \leq RD \quad \forall p \tag{8}$$

6) Energy Production: To obtain the complete energy produced by the unit, the energy during the SU & SD (see Fig. 1):

$$\begin{aligned} ge_p =& \underline{GP} \cdot u_p + \frac{gp_p + gp_{p-1}}{2} \\ &+ \sum_l \sum_{i=1}^{D_l^v} \frac{GP^v_{l,i+1} + GP^v_{l,i}}{2} \delta_{(p-i+D_l^v+1),l} \\ &+ \sum_{i=1}^{D^w} \frac{GP^w_{i+1} + GP^w_i}{2} w_{(p-i+1)} \qquad \forall p \end{aligned} \tag{9}$$

where the terms in the summations include the energy produced during the SU & SD procedures.

7) Objective Function: The goal of the price-taker producer in a self-UC is to maximize his profits during the planning period, which is the difference between the revenues and the total operating costs (10). These costs are the fixed and variable production costs, SU & SD costs. For the sake of simplicity, a linear production cost is used in this paper.

$$\begin{aligned} \max \sum_p \Big[& EP_p \cdot ge_p - \Big(FC \cdot u_p + VC \cdot ge_p \\ & + \sum_l StC_l^F \cdot \delta_{p,l} + ShC^F \cdot w_p \Big) \Big] \end{aligned} \tag{10}$$

The production cost, fixed plus variable, must be accounted during all the on-line period. Note that unlike the variable cost, the fixed cost considered in (10) ignores the SU & SD periods. This is because the fixed cost in (10) just multiplies the commitment during the up period u_p. In order to consider the fixed cost during the SU & SD periods, StC_l^F and ShC^F are introduced in (10) and defined as:

$$StC_l^F = StC_l + FC \cdot D_l^v \quad \forall l \tag{10a}$$

$$ShC^F = ShC + FC \cdot D^w \tag{10b}$$

B. Final Power Schedule

The complete energy profile, including SU & SD power trajectories, was presented in (9) but the complete power output has not been obtained yet. Furthermore, the unit state on/off is also missing. However, the (usual) information based on the on/off state and the total generation output can be obtained from the the decision variables used in the formulation. This information is obtained after solving the optimization problem, therefore with an insignificant computational cost.

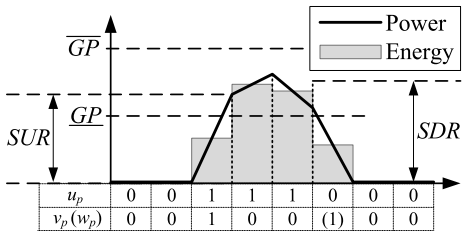


Fig. 3. SU & SD ramps for fast generators

1) Total Power Output: defined as GP_p, is obtained considering the power production below $\underline{GP}$ including the SU & SD trajectories:

$$\begin{aligned} GP_p =& \underline{GP}\,(u_p + v_{p+1}) + gp_p \\ &+ \sum_l \sum_{i=1}^{D_l^v} GP^v_{l,i} \cdot \delta_{(p-i+D_l^v+2),l} \\ &+ \sum_{i=2}^{Dw+1} GP_i^w \cdot w_{(p-i+2)} \qquad \forall p \end{aligned} \tag{11}$$

2) On/Off state: Similar to u_p, the on-line decision U_p^{ON} with 0/1 values (see Fig. 1) can be obtained from:

$$U_p^{ON} = \sum_l \sum_{i=1}^{D^v} \delta_{(p-i+D^v+1),l} + \sum_{i=1}^{D^w} w_{(p-i+1)} \quad \forall p \tag{12}$$

3) Turn-On and Turn-Off: Analogously to the SU (v_p) and SD (w_p) variables, the turn-on V_p^{ON} and turn-off W_p^{OFF} decisions (see Fig. 1) can be obtained from:

$$V_p^{ON} = \sum_l \delta_{(p+D_l^v),l} \quad \forall p \tag{13}$$

$$W_p^{OFF} = w_{(p-D^w)} \quad \forall p \tag{14}$$

similarly to these equations, a synchronization decision (as in [14]) can be obtained after solving the optimization problem without increasing the computational burden of the formulation.

C. Specific cases

The formulation can be modified depending on some specific characteristics. Two different cases are considered here: 1) the model of SU & SD ramps for fast generators, which can ramp up/down within one hour; and 2) the model simplification in case that just one SU trajectory is considered, as in [13].

1) Fast Generators: Fast generating technologies, like open cycle gas turbines, can ramp up from 0 to more than $\underline{GP}$ in one hour. Constraint (6) ensures that the unit starts and finishes the up period at $\underline{GP}$. This constraint as well as the energy output (9) must be modified to allow faster units to ramp up (down) from 0 (above $\underline{GP}$) to above $\underline{GP}$ (0). The new capacity limits

Table I
THERMAL UNIT DATA

Technical Information					Cost Coefficients		
$\overline{GP}$ [MW]	$\underline{GP}$ [MW]	RU/RD [MW/h]	TU/TD [h]	D^w [h]	FC [$]	VC [$/MWh]	ShC [$]
378,0	150,0	80	4	2	200	55	20

Start-Up Ramping Information				
SU Type l	StC_l [$]	GP_l^{syn} [MW]	D_l^v [h]	T_l^{DW} [h]
01	16	50	1	4
02	28	50	2	6
03	36	50	3	8
04	40	50	4	11
05	41	50	5	14

and energy constraints are then defined as:

$$0 \leq gp_p \leq \left(\overline{GP} - \underline{GP}\right) u_p - \left(\overline{GP} - SUR\right) v_p - \left(\overline{GP} - SDR\right) w_{p+1} \quad \forall p \tag{15}$$

$$ge_p = \frac{\underline{GP}\,(u_p + u_{p-1})}{2} + \frac{gp_p + gp_{p-1}}{2} \quad \forall p \tag{16}$$

where SUR and SDR are the maximum generation output that the unit can achieve when starting-up and shutting-down respectively, see Fig. 3. Finally, note that the whole production period is not included in u_p, see Fig. 3, therefore, the fixed cost of one hour is not considered. In order to take into account the fixed costs for the last power-production hour, the SD production cost must be modified using (10b) with a ramp duration $D^w = 1$ hour. On the other hand, the SU cost should not be modified given that u_p contains the SU ramp hour, as shown in Fig. 3.

2) Single-Ramp Formulation: When just one SU type is modeled, there is no need to introduce new variables ($\delta_{p,l}$) because there is no choice to be made. Therefore, (1) and (2) are not needed. The scheduled energy in (9) must be modified to be directly affected by the SU variable v_p:

$$ge_p = \underline{GP} \cdot u_p + \frac{gp_p + gp_{p-1}}{2} + \sum_{i=1}^{D^v} \frac{GP_{i+1}^v + GP_i^v}{2} v_{(p-i+D^v+1)} + \sum_{i=1}^{D^w} \frac{GP_{i+1}^w + GP_i^w}{2} w_{(p-i+1)} \quad \forall p \tag{17}$$

In relation to a formulation where no SU & SD ramps are considered, the single-ramp formulation neither introduces new variables nor new constraints in the model. There is just one constraint that is modified to take into account the energy produced during the SU & SD ramping processes.

III. TEST RESULTS

The proposed formulation is tested for the case of a price-taker self-UC for one hypothetical producer. The technical and economic data of the thermal unit, including five different SU ramps, are presented in Table I, and the expected electricity prices are shown in Table V in Appendix B. These data are based on information presented in [14]. As initial conditions, the unit has been up for 6 hours before the scheduling horizon and its initial power output is 200 MW. All tests in this paper were carried out using CPLEX 12.3 under GAMS [33] on an Intel i7 2.4 GHz with 4 GB of RAM memory. Problems are solved within 0% of optimality tolerance unless otherwise specified.

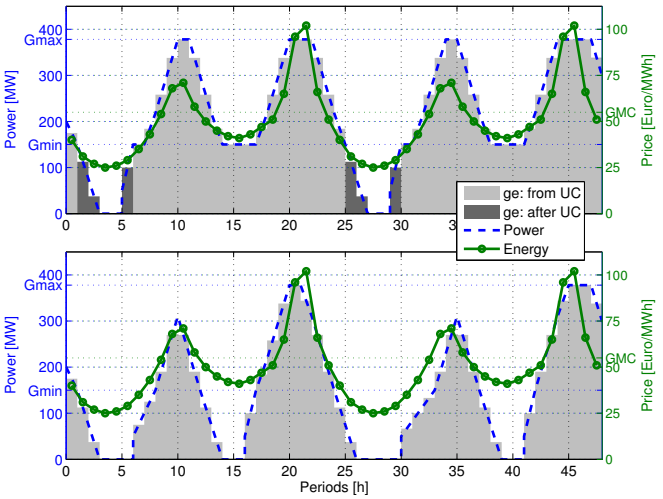


Fig. 4. Optimal generation scheduling for the Traditional formulation and with SU-&-SD-ramps in the up and bottom part of the figure respectively.

Table II
PRODUCER COSTS AND PROFITS

	Costs ($)	Revenues ($)	Profits ($)
Traditional	576417	617252	40835
SU & SD Ramps	402201	461674	59473
% of change	-30.22	-25.20	45.64

This section is divided in two parts. The first part shows the change in the commitment decision and its economic impact due to not considering SU & SD ramps. The second part presents a comparison of the proposed formulation with [13] and [14].

A. Scheduling and Economic Impact of Considering Ramps

In order to illustrate how the unit operation is affected by considering the SU & SD ramps, the case study has been solved with and without the ramp trajectories for a self-UC. The previous UC is solved for one generator with five different SU types and a time span of 48 hours.

A common practice to model the SU & SD processes is considering the SU & SD costs in the objective function, where the SU costs depend on how long the unit has been down (see Fig. 2). In other words, the power trajectories of these processes are ignored. The formulation without SU & SD power trajectories is here labeled as "Traditional". The impact on the commitment decision of this Traditional formulation is compared with the "formulation with SU & SD ramps" presented in Section II.

Unlike the Traditional formulation, considering the energy produced during the SU & SD ramps makes the formulation perceive revenues during the SU & SD ramping processes. To make a fair comparison between the two formulations, the energy during the SU & SD ramps is introduced to the

Traditional formulation. This is made, based on the formulation proposed in Section II, in the following way: (i) the Traditional formulation keeps the SU types $\delta_{p,l}$, then w_p and $\delta_{p,l}$ are used in the objective function to consider the SD and the exponential SU costs (see Fig. 2); (ii) the ramp trajectories are removed from (9) (by not including the summation terms); (iii) by solving this remaining model we obtain the results of the Traditional formulation; finally, (iv) after solving the UC problem, the summation terms in (9) together with $\delta_{p,l}$ are used to obtain the energy produced during the SU & SD ramping. Now, the total revenues for the Traditional formulation are obtained by adding the revenues obtained from the UC solution plus the revenues obtained from the energy produced during the SU & SD processes. These latter revenues are calculated by multiplying the electricity price by the energy produced during the SU & SD ramps.

Table V in Appendix B and Fig. 4 show the optimal power and energy schedules of the Traditional formulation and the one with SU & SD ramps. The optimal scheduling decision taken by the Traditional formulation around hours 15-17 and 39-41 was to produce at minimum output $\underline{GP}$ (G_{min} in Fig. 4) even when electricity prices are lower than the Generator Marginal Cost (GMC). This is a very common behavior, where producing at $\underline{GP}$ generates lower losses than shutting down and starting up the unit within a short period. On the other hand, when SU & SD ramps are considered, the optimal scheduling decision is to turn off the unit during these hours. The reason is that the SU and SD costs are offset by the revenues received from the energy produced during the SU & SD ramping processes. In short, unlike the formulation with SU & SD ramps, in the Traditional model, the SU and SD processes are perceived as pure losses in the objective function. Therefore, the optimum decision of the Traditional UC formulation is not to turn off the unit for short periods to avoid these losses.

The optimal scheduling decision of the two formulations is to turn off the unit during hours 4-5 and 28-29, see Fig. 4. However, the off-line time of the Traditional formulation is shorter than when SU & SD ramps are considered. For the Traditional formulation, the optimal decision is to turn off the unit for two hours and produce at minimum output during the following hour. According to the Traditional formulation, this decision is better than turning off the unit for a longer period, thus avoiding a more costly SU costs without receiving any income during the SU process.

The main problem of the Traditional formulation is that revenues during the SU and SD processes are not considered in the optimization problem. Therefore, there is a tendency to produce at least at minimum output, even when electricity prices are lower than GMC, and thus having some revenues that absorb the losses. This drawback is overcome by considering the SU & SD ramp production in the formulation.

Table II shows the difference between costs, revenues and profits between the solutions of the two formulations. As mentioned before, for the case of the Traditional model, the total revenue is obtained by adding the revenues due to the ramping process ($ 14800) to the revenues obtained from the optimal solution ($ 602452). For the example case, the profits when considering the SU & SD ramps are around 46% higher than when these ramps are not considered.

B. Comparing Different Formulations

The proposed formulation is compared with [13] and [14]. [13] proposes a formulation to deal with one SU and SD ramp trajectories whereas [14] deals with different SU trajectories depending on the unit's prior down time. In order to make the comparisons, the proposed formulation is implemented considering one, three and five different SU ramp types, and these models are labeled as R1, R3 and R5 respectively. The single ramp type model R1, described in Subsection II-C2, can be directly compared with the single-ramp formulation proposed in [13]. The model presented in [14] was implemented considering three SU ramp types, and hence it can be directly compared with R3. For comparisons, model R5 is presented thus the reader can observe the extra computational effort due to considering extra SU ramp trajectories.

Several case studies of different sizes were solved. The price profile of one day (see Table V in Appendix B) has been replicated to test horizons from 4 to 256 days. The unit data is presented in Table I, where the information for: the one-ramp models (R1 and [13]) is the SU ramp type 02; the three-ramp models (R3 and [14]) are the first three SU types; and the five-ramp model (R5) are the five SU types.

In order to compare all the formulations, [13] and [14] were implemented using the same objective function as the formulation presented here (see Section II), therefore, the only difference between formulations is the modeling of the SU & SD ramps. Table IV presents the optimal solutions of all the models for different time spans. As expected, the optimal solution of models R1 and [13] are the same, as well as the solution of models R3 and [14]. Interestingly, model R5, which considers five ramp types, also presents the same solution as R3 and [14]. This is because, in R5, ramps 04 and 05 were never activated for this example case (the unit was not down long enough).

Table III shows the dimension of all the models for the different case studies. Models R1 and [13] have the same number of variables, however [13] presents three times more binary variables than R1. This is because [13] defines the SU & SD variables as binary. The formulation in [13] also presents above 2 and 2.4 times more constraints and non-zeros elements, respectively, than the proposed formulation R1. These extra constraints and non-zero elements are needed in [13] to avoid conflicts between the power output above and below $\underline{GP}$.

As shown in Table III, [14] presents about 16 times more binary variables than the proposed formulation R3. Models R3 and R5 have more real variables than [14], however the total number of variables of R3 and R5 are smaller than the number of binary variables in [14]. R3 and R5 also present less than the half of the number of constraints than [14]. Furthermore, [14] presents up to 4.3 and 3 times more non-zero elements than R3 and R5 respectively. Similarly to [13], [14] introduce these extra variables and non-zero elements to deal with the different SU ramp types and to avoid conflicts between the

up and down states. Model R5 is slightly bigger than R3, regarding the number of variables and constraints, because R5 considers two more ramp types than R5. This also shows that the compact formulation is not increased considerable when considering more SU types. Finally, it is interesting to note that [13], which models one ramp trajectory and does not take into account exponential SU costs neither, has on average 1.27 and 1.15 times more constraints and non-zero variables than R5, which considers five different SU ramps and also exponential SU costs.

In summary, regarding models sizes, the proposed formulation solves a smaller problem with respect to [13] and [14] while achieving the same optimal results. However, as mentioned in the Introduction, what matters in a MILP formulation is how tight it is, which is measured by the integrality gap. In fact, a compact formulation usually presents a weak LP relaxation that can dramatically increase the MILP resolution time. The integrality gap, for a maximization problem, is defined as $(Z_{LP}-Z_{MILP})/Z_{MILP}$, where Z_{MILP} and Z_{LP} are the optimal values of the MILP and the relaxed LP respectively.

Table IV shows the integrality gaps of the different formulations. Compared to [13], the proposed one-ramp formulation R1 has improved (reduced) the integrality gap between 46% and 49%. Similarly, with respect to [14], R3 improves the integrality gap between 41% and 45%. It is also interesting to notice that, the bigger the problem (more days), the better the gap improvements between the proposed formulation and [13], [14]. Table IV the nodes explored during the branch-and-cut phase, these are usually decreased with tighter formulations. However, the use of cuts and heuristics, which also affects dramatically the performance, during the solving process are also influenced by the tightness [20]. Therefore, we will only comment about CPU times which offer a more complete view.

In conclusion, the proposed formulation is more compact (fewer constraints, total variables and non-zero elements) and tighter (smaller integrality gap) than the ones shown in [13] and [14], see Tables III and IV. As a natural consequence, the formulation presented in this paper achieves significantly faster resolution times than the other two formulations. The CPU times of the different study cases are presented in Table IV, where [13] and [14] are up to 22.4 and 10.1 slower than R1 and R3 respectively. It is also interesting to note that the formulation in [13], which models one ramp trajectory and does not take into account exponential SU costs neither, requires more time to solve the problem than R5, which considers five different SU ramps and also exponential SU costs.

Finally, Fig. 5 shows the convergence evolution, of the different formulations, to small optimality tolerances for the case study of 256 days. The proposed formulations, R1, R3 and R5, converge significantly faster than [13] and [14]. For common stopping criteria, which is within a tolerance interval $\left[10^{-2}, 10^{-4}\right]$, the time difference between the proposed formulation and the others is already significant. The faster convergence of the proposed formulation is mainly due to the tightness proposed formulation, as discussed in the Introduction. The other study cases present similar convergence behaviors to the ones shown in Fig. 5.

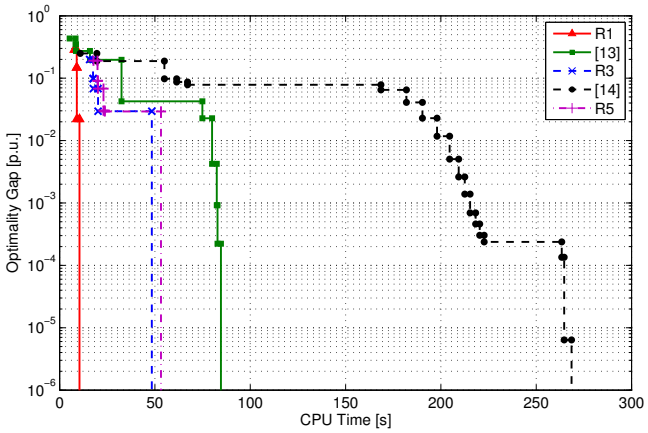


Fig. 5. Comparison of convergence evolution to optimal solutions.

IV. Conclusions

This paper presented an improved MILP formulation of the Start-Up and Shut-Down power trajectories of thermal units. This formulation is simultaneously tighter and more compact in comparison with equivalent formulations found in the literature. Consequently, its computation time was dramatically reduced. The proposed MILP formulation was analyzed in the context of a price taker self-UC problem. However, it can be directly applied to any UC problem, either in centralized or competitive environments. Several study cases were analyzed to show the previous improvements of this formulation over others found in the literature. Although SU & SD ramps are usually neglected, mainly because of the computation complexity, simulation results showed that ignoring them change the commitment decisions causing a negative economic impact.

Appendix

A. Initial Conditions

The behavior of the unit during the first hours is conditioned by the unit state before the scheduling horizon. Therefore, initial conditions must be considered to ensure the proper operation of the unit. Before running the optimization model, some variables must be fixed according to the initial conditions. The following parameters are needed to deal with the unit state during the first periods:

u_0	Initial commitment state $\{0, 1\}$.
TU_0	Number of hours that the unit has been up before the scheduling horizon.
TD_0	Number of hours that the unit has been down before the scheduling horizon.

1) Initial Minimum Up/Down *Times:* The minimum *up/down* times must be satisfied taking into account the unit state before the scheduling horizon. The following condition must be satisfied if $(TU_R + TD_R) \geq 1$:

$$u_p = u_0 \quad \forall p \in [1, TU_R + TD_R] \tag{18}$$

where TU_R and TD_R are the number of initial hours in which the unit must remain *up* or *down* at the beginning of the

Table III
DIMENSION COMPARISON OF DIFFERENT FORMULATIONS

Case	# of Constraints					# of Real Variables					# of Binary Variables				Non-zero elements				
(# of days)	R1	[13]	R3	[14]	R5	R1	[13]	R3	[14]	R5	R1	[13]	R3&R5	[14]	R1	[13]	R3	[14]	R5
4	668	1444	944	2395	1113	384	192	657	382	824	96	288	96	1540	3131	7515	4576	15050	6352
16	2684	5764	3824	9595	4569	1536	768	2673	1534	3416	384	1152	384	6148	12635	30267	18688	75808	26512
64	10748	23044	15344	38395	18393	6144	3072	10737	6142	13784	1536	4608	1536	24580	50651	121275	75136	318880	107152
256	43004	92164	61424	153595	73689	24576	12288	42993	24574	55256	6144	18432	6144	98308	202715	485307	300928	1291168	429712

Table IV
OPTIMAL SOLUTIONS AND COMPUTATIONAL EFFICIENCY COMPARISON

Case	Optimal Solution ($)		Integrality Gap (%)					# of Nodes					CPU Time (s)				
(# of days)	R1&[13]	R3,R5&[14]	R1	[13]	R3	[14]	R5	R1	[13]	R3	[14]	R5	R1	[13]	R3	[14]	R5
4	120250.5	118899.5	15.61	29.06	16.18	27.56	15.97	0	64	14	453	21	0.047	0.234	0.125	1.264	0.109
16	486382.5	475459.4	14.76	28.71	16.53	29.78	16.48	0	550	477	473	604	0.156	3.494	1.545	6.474	2.013
64	1950910.5	1901699.2	14.56	28.62	16.62	30.33	16.60	0	496	508	498	488	0.998	16.255	7.956	37.908	8.565
256	7809022.5	7606658.5	14.50	28.60	16.64	30.47	16.63	0	483	467	490	494	10.405	84.599	48.422	268.665	53.181

scheduling horizon. TU_R and TD_R can be mathematically expressed as a function of other initial values:

$$TU_R = \max\{0, (TU - TU_0)\, u_0\} \tag{18a}$$
$$TD_R = \max\{0, (TD - TD_0)\,(1 - u_0)\} \tag{18b}$$

Equation (18) fixes the unit state to up or down according to TU_R or TD_R respectively. Note that in (18), at least, one of the two parameters TU_R or TD_R is always zero according to the initial commitment state u_0, see (18a) and (18b).

2) Initial Start-Up Type: In case the unit was down before the scheduling horizon, the possible SU type for the first periods depends on the time that the unit had been down. Note that the constraint that binds the SU types $\delta_{p,l}$ (1) is not defined for the initial periods. The equation (19a) complements (1) taking into account the initial conditions if $TD_0 \geq 2$:

$$\delta_{p,l} = 0 \qquad \forall l \in [1, N_L)\,,\, p \in \left(T^{DW}_{l+1} - TD_0, T^{DW}_{l+1}\right) \tag{19a}$$

Finally, the following guarantees that the SU $\delta_{p,l}$ is not activated before the SU ramp duration D^v_l:

$$\delta_{p,l} = 0 \qquad \forall l,\, p \in [1, D^v_l] \tag{19b}$$

In other words, this condition ensures that if the unit is turned on in the first hour, the power output above $\underline{GP}$ comes after the ramp duration D^v_l. Therefore, the SU type $\delta_{p,l}$ is only possible after D^v_l hours.

B. Generator and Price Data

The expected electricity prices and optimal power schedules mentioned in Section III are shown in Table V, where sub-indexes T and R make reference to the Traditional and SU-&-SD-ramps formulations. The numbers in parenthesis are the power/energy that were included after solving the UC problem.

ACKNOWLEDGMENT

German Morales-España has been awarded an Erasmus Mundus Ph.D. Fellowship. The authors would like to express their gratitude towards all partner institutions within the programme as well as the European Commission for their support.

REFERENCES

[1] B. F. Hobbs, W. R. Stewart, R. E. Bixby, M. H. Rothkopf, R. P. O'Neill, and H.-p. Chao, "Why this book? new capabilities and new needs for unit commitment modeling," in *The Next Generation of Electric Power Unit Commitment Models*, B. F. Hobbs, M. H. Rothkopf, R. P. O'Neill, and H.-p. Chao, Eds. Boston: Kluwer Academic Publishers, 2002, vol. 36, pp. 1–14.

[2] R. Sioshansi, R. O'Neill, and S. S. Oren, "Economic consequences of alternative solution methods for centralized unit commitment in Day-Ahead electricity markets," *IEEE Transactions on Power Systems*, vol. 23, no. 2, pp. 344–352, May 2008.

[3] A. J. Wood and B. F. Wollenberg, *Power Generation, Operation, and Control*, 2nd ed. Wiley-Interscience, Jan. 1996.

[4] N. Padhy, "Unit commitment-a bibliographical survey," *Power Systems, IEEE Transactions on*, vol. 19, no. 2, pp. 1196–1205, 2004.

[5] J. Garcia-Gonzalez, A. San Roque, F. Campos, and J. Villar, "Connecting the intraday energy and reserve markets by an optimal redispatch," *Power Systems, IEEE Transactions on*, vol. 22, no. 4, pp. 2220–2231, 2007.

[6] J. Kiviluoma, M. O'Malley, A. Tuohy, P. Meibom, M. Milligan, B. Lange, H. Holttinen, and M. Gibescu, "Impact of wind power on the unit commitment, operating reserves, and market design," in *2011 IEEE Power and Energy Society General Meeting*. IEEE, Jul. 2011, pp. 1–8.

[7] P. Meibom, R. Barth, B. Hasche, H. Brand, C. Weber, and M. O'Malley, "Stochastic optimization model to study the operational impacts of high wind penetrations in ireland," *IEEE Transactions on Power Systems*, vol. 26, no. 3, pp. 1367–1379, Aug. 2011.

[8] N. Troy, E. Denny, and M. O'Malley, "Base-Load cycling on a system with significant wind penetration," *Power Systems, IEEE Transactions on*, vol. 25, no. 2, pp. 1088–1097, 2010.

[9] T. Koch, T. Achterberg, E. Andersen, O. Bastert, T. Berthold, R. E. Bixby, E. Danna, G. Gamrath, A. M. Gleixner, S. Heinz, A. Lodi, H. Mittelmann, T. Ralphs, D. Salvagnin, D. E. Steffy, and K. Wolter, "MIPLIB 2010," *Mathematical Programming Computation*, vol. 3, no. 2, pp. 103–163, Jun. 2011.

[10] T. Li and M. Shahidehpour, "Price-based unit commitment: a case of lagrangian relaxation versus mixed integer programming," *Power Systems, IEEE Transactions on*, vol. 20, no. 4, pp. 2015–2025, 2005.

[11] H. Y. Yamin, "Review on methods of generation scheduling in electric power systems," *Electric Power Systems Research*, vol. 69, no. 2-3, pp. 227–248, May 2004.

[12] C. Wang and S. M. Shahidehpour, "Effects of ramp-rate limits on unit commitment and economic dispatch," *IEEE Transactions on Power Systems*, vol. 8, no. 3, pp. 1341–1350, 1993.

Table V
PRICE DATA AND OPTIMAL GENERATION SCHEDULE

p	EP_p $/MWh	gp_R MW	ge_R MWh	gp_T MW	gp_T MWh	p	EP_p $/MWh	gp_R MW	ge_R MWh	gp_T MW	gp_T MWh	p	EP_p $/MWh	gp_R MW	ge_R MWh	gp_T MW	gp_T MWh	p	EP_p $/MWh	gp_R MW	ge_R MWh	gp_T MW	gp_T MWh
0	—	200	—	200	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
1	40	150	175	150	175	13	50	75	112.5	218	258	25	40	75	112.5	150	184	37	50	150	190	218	258
2	31	75	112.5	(75)	(112.5)	14	45	0	37.5	150	184	26	31	0	37.5	(75)	(112.5)	38	45	75	112.5	150	184
3	27	0	37.5	(0)	(37.5)	15	42	0	0	150	150	27	27	0	0	(0)	(37.5)	39	42	0	37.5	150	150
4	25	0	0	0	0	16	41	50	0	150	150	28	25	0	0	0	0	40	41	0	0	150	150
5	26	0	0	(50)	0	17	43	150	100	150	150	29	26	0	0	(50)	0	41	43	50	0	150	150
6	29	50	0	150	(100)	18	47	218	184	218	184	30	29	50	0	150	(100)	42	47	150	100	218	184
7	35	100	75	150	150	19	51	298	258	298	258	31	35	83.33	66.67	150	150	43	51	230	190	298	258
8	43	150	125	218	184	20	65	378	338	378	338	32	43	116.67	100	218	184	44	65	310	270	378	338
9	54	230	190	298	258	21	96	378	378	378	378	33	54	150	133.33	298	258	45	96	378	344	378	378
10	68	310	270	378	338	22	102	310	344	378	378	34	68	230	190	378	338	46	102	378	378	378	378
11	71	230	270	378	378	23	66	230	270	298	338	35	71	310	270	378	378	47	66	378	378	378	378
12	58	150	190	298	338	24	51	150	190	218	258	36	58	230	270	298	338	48	51	298	338	298	338

[13] J. Arroyo and A. Conejo, "Modeling of start-up and shut-down power trajectories of thermal units," *Power Systems, IEEE Transactions on*, vol. 19, no. 3, pp. 1562–1568, 2004.

[14] C. Simoglou, P. Biskas, and A. Bakirtzis, "Optimal Self-Scheduling of a thermal producer in Short-Term electricity markets by MILP," *Power Systems, IEEE Transactions on*, vol. 25, no. 4, pp. 1965–1977, 2010.

[15] L. L. Garver, "Power generation scheduling by integer Programming-Development of theory," *Power Apparatus and Systems, Part III. Transactions of the American Institute of Electrical Engineers*, vol. 81, no. 3, pp. 730–734, 1962.

[16] F. Bouffard, F. Galiana, and A. Conejo, "Market-clearing with stochastic security-part i: formulation," *Power Systems, IEEE Transactions on*, vol. 20, no. 4, pp. 1818–1826, 2005.

[17] A. Street, F. Oliveira, and J. M. Arroyo, "Contingency-Constrained unit commitment with n-K security criterion: A robust optimization approach," *IEEE Transactions on Power Systems*, vol. 26, no. 3, pp. 1581–1590, Aug. 2011.

[18] B. Palmintier and M. Webster, "Impact of unit commitment constraints on generation expansion planning with renewables," in *2011 IEEE Power and Energy Society General Meeting*. IEEE, Jul. 2011, pp. 1–7.

[19] T. Wu, L. Shi, J. Geunes, and K. Akartunalı, "On the equivalence of strong formulations for capacitated multi-level lot sizing problems with setup times," *Journal of Global Optimization*, May 2011, article in press.

[20] E. L. Johnson, G. L. Nemhauser, and M. W. Savelsbergh, "Progress in linear Programming-Based algorithms for integer programming: An exposition," *INFORMS Journal on Computing*, vol. 12, no. 1, pp. 2 –23, 2000.

[21] C. Barnhart, E. L. Johnson, G. L. Nemhauser, M. W. P. Savelsbergh, and P. H. Vance, "Branch-and-Price: column generation for solving huge integer programs," *Operations Research*, vol. 46, no. 3, pp. 316 –329, May.

[22] C. A. Phillips, J. Eckstein, and W. Hart, "Massively parallel Mixed-Integer programming: Algorithms and applications," in *Parallel Processing for Scientific Computing*, M. A. Heroux, P. Raghavan, and H. D. Simon, Eds. SIAM, Jan. 2006, pp. 323–340.

[23] F. Vanderbeck and L. A. Wolsey, "Reformulation and decomposition of integer programs," in *50 Years of Integer Programming 1958-2008*, M. Jünger, T. M. Liebling, D. Naddef, G. L. Nemhauser, W. R. Pulleyblank, G. Reinelt, G. Rinaldi, and L. A. Wolsey, Eds. Berlin, Heidelberg: Springer Berlin Heidelberg, 2010, pp. 431–502.

[24] J. Luedtke and G. L. Nemhauser, "Strategic planning with Start-Time dependent variable costs," *Operations Research*, vol. 57, no. 5, pp. 1250 –1261, Sep. 2010. [Online]. Available: http://or.journal.informs.org/content/57/5/1250.abstract

[25] L. Wolsey, "Strong formulations for mixed integer programs: valid inequalities and extended formulations," *Mathematical programming*, vol. 97, no. 1, p. 423–447, 2003.

[26] ——, "Strong formulations for mixed integer programming: A survey," *Mathematical Programming*, vol. 45, no. 1-3, pp. 173–191, Aug. 1989.

[27] D. Rajan and S. Takriti, "Minimum Up/Down polytopes of the unit commitment problem with Start-Up costs," IBM, Research Report RC23628, Jun. 2005. [Online]. Available: http://domino.research.ibm.com/library/cyberdig.nsf/1e4115aea78b6e7c85256b360066f0d4/cdcb02a7c809d89e8525702300502ac0?OpenDocument

[28] A. Frangioni, C. Gentile, and F. Lacalandra, "Tighter approximated MILP formulations for unit commitment problems," *IEEE Transactions on Power Systems*, vol. 24, no. 1, pp. 105–113, Feb. 2009.

[29] J. Ostrowski, M. F. Anjos, and A. Vannelli, "Tight mixed integer linear programming formulations for the unit commitment problem," *IEEE Transactions on Power Systems*, Aug. 2011, article in press.

[30] A. Tuohy, P. Meibom, E. Denny, and M. O'Malley, "Unit commitment for systems with significant wind penetration," *Power Systems, IEEE Transactions on*, vol. 24, no. 2, pp. 592–601, 2009.

[31] J. A. Muckstadt and R. C. Wilson, "An application of Mixed-Integer programming duality to scheduling thermal generating systems," *IEEE Transactions on Power Apparatus and Systems*, vol. PAS-87, no. 12, pp. 1968–1978, Dec. 1968.

[32] K. Hedman, R. O'Neill, and S. Oren, "Analyzing valid inequalities of the generation unit commitment problem," in *Power Systems Conference and Exposition, 2009. PSCE '09. IEEE/PES*, 2009, pp. 1–6.

[33] "The GAMS development corporation website," http://www.gams.com/, 2011. [Online]. Available: http://www.gams.com/



German Morales-España (S' 2010) received the B.Sc. degree in electrical engineering from the Universidad Industrial de Santander, Colombia, in 2007; the M.Sc. degree from the Delft University of Technology, The Netherlands, in 2010. He is now pursuing the (Ph.D.) Erasmus Mundus Joint Doctorate in Sustainable Energy Technologies and Strategies (SETS) hosted by the Universidad Pontificia Comillas, Spain; the Royal Institute of Technology, Sweden; and Delft University of Technology, The Netherlands. He is currently an assistant researcher at the Institute for Research in Technology (IIT) at the Universidad Pontificia Comillas, and he is also a member of the Research Group on Electric Power Systems (GISEL) at the Universidad Industrial de Santander. His areas of interest are power systems operation, economics and reliability; as well as power quality and intelligent protection.



Jesus M. Latorre (S'00-M'07) was born in Madrid, Spain, in 1977. He received the degree of Electronic Engineer in 2001 and the Ph.D. degree in November 2007, from Comillas Pontifical University, Madrid, Spain. He is currently a postdoctoral researcher at the Institute for Research in Technology, of the Comillas Pontifical University. His main interest areas include operations research and mathematical modeling, stochastic programming, parallel and distributed computing, algorithms and numerical methods,...



Andres Ramos received the degree of Electrical Engineering from Universidad Pontificia Comillas, Madrid, Spain, in 1982 and the Ph.D. degree in Electrical Engineering from Universidad Politécnica de Madrid, Madrid, Spain, in 1990. He is a Research Fellow at Instituto de Investigación Tecnológica, Madrid, Spain, and a Full Professor at Comillas' School of Engineering, Madrid, Spain, where he has been the Head of the Department of Industrial Organization. His areas of interest include the operation, planning, and economy of power systems and the application of operations research to industrial organization.