Value of Perfect Information of Spot Prices in the Joint Energy and Reserve Hourly Scheduling of Pumped Storage Plants

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***Abstract*—The value of perfect information of the day-ahead energy prices is studied in the context of the operation of a closed-loop and daily-cycle pumped-storage hydropower plant, participating in the spot market as a price-taker and in the secondary regulation reserve market as a price-maker, and also considering the secondary regulation energy due to the real-time use of reserves. Results show that the percentage of the profit loss with respect the maximum theoretical income due to the errors in forecasting the spot prices in the Iberian electricity market decreases if the plant also participates in the secondary regulation service.**

***Index Terms*—Pumped-Storage Plant, Secondary Regulation Service, Price Forecasting, Value of Perfect Information.**

NOMENCLATURE

Subscripts/Superscripts

d	superscript that indicates that the magnitude is related to generation/discharge
p	superscript that indicates that the magnitude is related to consumption/pumping
sec	superscript that indicates that the magnitude is related to the secondary regulation service (reserve or energy)
t	subscript for hourly period, running from 1 to T

Parameters

cSU^d	start-up cost in generating mode, €
cSU^p	start-up cost in pumping mode, €
δ^d	energy coefficient in generating mode, MW/hm^3/h
fv	target water volume in the last hour of the time horizon (in this study, the last hour of the day), hm^3
$\overline{g}^d$, $\underline{g}^d$	maximum and minimum technical power generation, MW
$\overline{g}^p$	maximum technical power consumption (there is no minimum power consumption as this study considers a pump-turbine unit with fixed speed in pumping mode), MW
l_t	time length of period t, 1 h
$\lambda_{D,t}$	day-ahead electricity market price, €/MWh
$\lambda^o_{S,t}$	the intercept of the linear approximation of a residual reserve curve, i.e. the secondary regulation reserve price when the residual reserve quantity of the system is zero, €/MW
$\lambda^m_{S,t}$	the slope of the linear approximation of a residual reserve curve, €/MW/MW
$\lambda_{up,t}$	upward secondary regulation energy price, €/MWh
$\lambda_{dw,t}$	downward secondary regulation energy price, €/MWh
M	a big number, 10^6
$\overline{\eta}^d$, $\underline{\eta}^d$	efficiency in generating mode at maximum and minimum water discharge, %
η^p	efficiency in pumping mode (only at maximum pumped water as this study considers a pump-turbine unit with fixed speed in the said mode), %
$\overline{q}^d$, $\underline{q}^d$	maximum and minimum technical water discharge, hm^3/h
$\overline{q}^p$	maximum technical pumped water (there is no minimum pumped water as this study considers a pump-turbine unit with fixed speed in pumping mode), hm^3/h
ρ^{up}_t, ρ^{dw}_t	percentage of the real-time use of the upward and downward secondary regulation reserves
R^{SM}_t	ratio between the upward and the total secondary regulation reserve, set by the TSO in advance
$\overline{v}$, $\underline{v}$	maximum and minimum water storage limits of the upper reservoir due to design characteristics, hm^3

Positive Variables

g^d_t, g^p_t	power generation and consumption, MW
q^d_t	total water discharge, which includes water for the real-time use of reserves, hm^3/h
q^p_t	pumped water, which does not include water for the real-time use of reserves as this study considers a pump-turbine unit with fixed speed in pumping mode, hm^3/h
qs^d_t	water discharge above the minimum technical limit, hm^3/h
$g^{sec,up}_t$	upward secondary regulation reserve, MW
$g^{sec,dw}_t$	downward secondary regulation reserve, MW
$\lambda_{S,t}$	clearing price in the secondary regulation reserve market, decided by the price-maker agent, €/MW
$e^{sec,up}_t$	upward secondary regulation reserve requested in real-time by the TSO, MWh

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$e_t^{sec,dw}$ downward secondary regulation reserve requested in real-time by the TSO, MWh

v_t water volume of the upper reservoir, hm^3

Binary Variables

u_t^d On/Off state in generating mode

u_t^p On/Off state in pumping mode

y_t^d 1 if the pump-turbine unit is started-up in generating mode, 0 otherwise

y_t^p 1 if the pump-turbine unit is started-up in pumping mode, 0 otherwise

ϕ_t 1 if there is more upward than downward secondary regulation reserve requested in real-time, 0 otherwise

I. Introduction

Competition has been introduced in many electricity power systems worldwide in order to achieve improvements of economic efficiencies.

The perfect information of the spot prices is not available. Its value depends on the quality of the forecasting tool that is used to forecast the prices but also depends on the technology of the power producer, being the impact in the pump-storage technology higher than in the conventional hydro and thermal technology [1].

The main goal of this paper is to study the value of perfect information (also called *profit loss*) of the spot prices in the context of the operation of conventional pumped-storage hydropower plants (PSHPs) participating in the spot market and the secondary regulation service (power and energy). For this purpose, a deterministic mixed integer quadratic programming model is proposed. In the literature, the value of perfect information of the day-ahead energy prices (VPIDM) has been studied in the context of the demand-side [2], of a thermal and hydro-based generation company [3] and of the operation of PSHPs participating in the spot market [1]. However, to authors' knowledge, there is no paper where the VPIDM is calculated including the secondary regulation service.

The objective function of the proposed model consists in maximizing the income of a PSHP participating in the spot market as a price-taker and in the secondary regulation reserve market as a price-maker, in the framework of the Iberian electricity market [4], [5]. In addition, the secondary regulation energy due to the real-time use of reserves is also considered. Note that the procedure for the procurement of secondary regulation is not exclusive of the Iberian system. For instance, a similar procedure is implemented in the Swiss system in the short-term (less than a week) and in the Dutch, Belgian, German or Danish electricity systems in the long-term (more than a week).

The proposed model, used to estimate the VPIDM of a realistic closed-loop and daily-cycle PSHP in the above-mentioned markets and services, is based on the one proposed in [6], which has been revised in order to consider the PSHP as a price-maker in the secondary regulation reserve market. For the purpose to estimate the VPIDM, the model was run, day by day, for a period of one year (the selected year is 2014), in order to obtain representative enough as well as useful results for studying the *profit loss* due to the errors in forecasting the spot prices.

The paper is organised as follows: the price-maker approach and the model formulation are described in Section II and III, respectively. The case study is described in Section IV. Section V presents the results and discussion and finally, conclusions are described in Section VI.

II. Price-maker Approach in the Secondary Regulation Reserve Market

The PSHP is considered in this study as a price-maker in the secondary regulation reserve market, i.e. we assume that the bids offered in the said market have a notable impact in the clearing price. For this purpose, we use the residual demand curves of the secondary reserve market (hereafter referred to as the residual reserve curves, RRCs), solid line in Fig. 1(b). The RRC of each hour can be obtained as the difference between the demand and supply functions of the upward reserve, Fig. 1(a). As the PSHP considered in the paper is assumed to be a new participant in the reserve market, the RRCs are supposed to be directly the historical RRCs of the system.

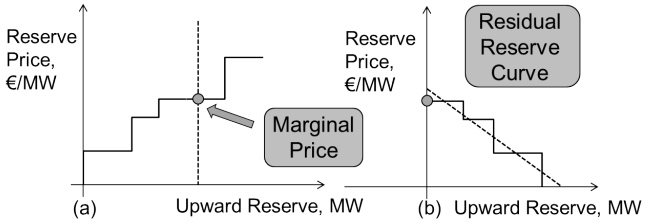


Figure 1. (a) Supply and demand functions (solid and dashed lines, respectively), and the marginal price of the market. (b) RRC of the upward reserve and its linear approximation (solid and dashed lines, respectively)

The price-maker approach makes the scheduling problem non-linear, because the income in the reserve market is the result of the marginal price multiplied by the market quota of the agent. Several approaches have been adopted in the literature to model the residual demand curve (it also applies to RRCs) and cope with the said nonlinearity [7]: 1) polynomial approximation, 2) piecewise linear approximation and 3) stepwise approximation. The method adopted in this work is a first order polynomial approximation (linear approximation) of the RRCs, dashed line in Fig. 1(b), composed by a slope and an intercept [8]. Therefore, the income in the reserve market is a quadratic function of the agent reserve offer. According to the results obtained in [7], the error due to the use of a linear RRC is expected to be lower than 2% of the estimated income.

The quadratic income in the reserve market under the price-maker approach is obtained as follows. Firstly, the non-linear income of the upward and downward reserves, I_S is directly the result of the total reserves multiplied by the marginal price (1). The reserve price $\lambda_{S,t}$ is obtained as a linear approximation of the RRC (2). In the Spanish electricity system, the transmission system operator (TSO) forces all

regulation zones to meet an hourly ratio R_t^{SM} between the upward and the total reserve (3). If (2) and (3) are included in (1), the income of the upward and downward reserves can be expressed as a quadratic function (4). Note that the linear approximation of a RRC only depends on the upward reserve and not on the downward reserve because when the former is decided, the latter is automatically defined by (3).

$$I_S = \sum_t \left(g_t^{sec,up} + g_t^{sec,dw} \right) \lambda_{S,t} \tag{1}$$

$$\lambda_{S,t} = \lambda_{S,t}^o + \lambda_{S,t}^m \cdot g_t^{sec,up} \quad \forall t \tag{2}$$

$$\frac{g_t^{sec,up}}{g_t^{sec,up} + g_t^{sec,dw}} = R_t^{SM} \quad \forall t \tag{3}$$

$$\begin{aligned} I_S &= \sum_t \left(g_t^{sec,up} + \frac{g_t^{sec,up}}{R_t^{SM}} - g_t^{sec,up} \right) \left(\lambda_{S,t}^o + \lambda_{S,t}^m \cdot g_t^{sec,up} \right) \\ &= \sum_t \frac{g_t^{sec,up}}{R_t^{SM}} \left(\lambda_{S,t}^o + \lambda_{S,t}^m \cdot g_t^{sec,up} \right) \end{aligned} \tag{4}$$

III. Model Formulation

The model formulation of a conventional PSHP with a single reversible Francis pump-turbine unit, considered as a price-taker in the spot market and as a price-maker in the reserve market is composed by equations (5)-(21) in addition to equations (2) and (3).

A. Objective Function

The objective function of the model consists in maximizing the net income of the PSHP in the spot market, and for the different "products" comprised within the secondary regulation service of the Spanish electricity system [5]. In (5), the first term expresses the net income in the spot market and the start-up costs in generating and pumping mode, respectively. The second term expresses the quadratic income for the upward and downward reserves put at disposal of the TSO (see Section II for details). And, finally, the third term expresses the net income for the secondary regulation energy.

$$\begin{aligned} Max \quad z = \sum_t \Big[& \left(g_t^d - g_t^p \right) \lambda_{D,t} - cSU^d \cdot y_t^d - cSU^p \cdot y_t^p \\ & + \frac{g_t^{sec,up}}{R_t^{SM}} \left(\lambda_{S,t}^o + \lambda_{S,t}^m \cdot g_t^{sec,up} \right) \\ & + \left(e_t^{sec,up} \cdot \lambda_{up,t} - e_t^{sec,dw} \cdot \lambda_{dw,t} \right) \Big] \end{aligned} \tag{5}$$

B. Constraints

1) Water Balance: The water balance in the upper reservoir, (6) does not include any water inflow because a closed-loop PSHP is considered in this article. In addition to this, limits are imposed to the water volume in all hours (7). The water balance and limits in the lower reservoir are not included in the proposed model formulation for the sake of clarity. Note that q_t^d is total water discharge, which includes water for the real-time use of reserves (11).

$$v_t = v_{t-1} + l_t \left(q_t^p - q_t^d \right) \quad \forall t \tag{6}$$

$$\underline{v} \leq v_t \leq \overline{v} \quad \forall t \tag{7}$$

A target water volume in the upper reservoir at the end of the time horizon (in this article, last hour of the day) is imposed by (8). The target water volume is deemed as input data to the model.

$$v_t = fv \quad \forall t \in \mathrm{T} \tag{8}$$

2) Hydropower Generation and Consumption: The generation/consumption curves of the pump-turbine unit used in the paper can be seen in Fig. 2 and are modelled with (9)-(13). The curves represent the relationship between water discharge and pumped water (on x-axes) with hydropower generation and consumption (on y-axes). The proposed formulation assumes that the upper reservoir has a negligible head variation in comparison with the available gross head. Hence, head dependency is not included in the model.

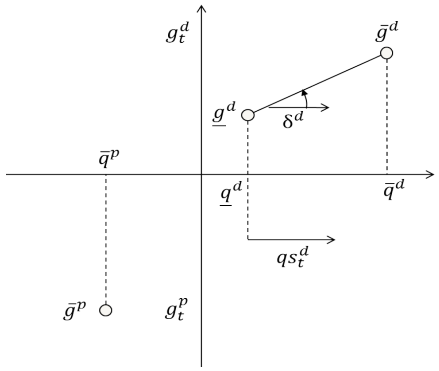


Fig. 2. Generation and consumption curves of a reversible Francis pump-turbine unit, considered in this study

The feasible operating range in generating mode (between maximum and minimum power generation) is ensured by (9) and (10). In addition, the total water discharge includes not only water for the energy committed in the spot market but also extra water discharge for the real-time use of the upward reserves and water that is kept in the upper reservoir due to the real-time use of the downward reserves (11). The consumption curve (a single operating point as a fixed-speed pump-turbine unit is assumed in this study) is modelled by (12) and (13).

$$g_t^d = u_t^d \cdot \underline{g}^d + \delta^d \cdot qs_t^d \quad \forall t \tag{9}$$

$$qs_t^d \leq u_t^d \left(\overline{q}^d - \underline{q}^d \right) \quad \forall t \tag{10}$$

$$q_t^d = u_t^d \cdot \underline{q}^d + qs_t^d + \frac{1}{\delta^d} \left(\rho_t^{up} \cdot g_t^{sec,up} - \rho_t^{dw} \cdot g_t^{sec,dw} \right) \quad \forall t \tag{11}$$

$$g_t^p = u_t^p \cdot \overline{g}^p \quad \forall t \tag{12}$$

$$q_t^p = u_t^p \cdot \overline{q}^p \quad \forall t \tag{13}$$

A reversible Francis pump-turbine unit is not able to operate in generating and pumping mode simultaneously (14). Finally, the start-up decisions are modelled by (15) in generating mode and by (16) in pumping mode.

$$u_t^d + u_t^p \leq 1 \quad \forall t \tag{14}$$

$$y_t^d \geq u_t^d - u_{t-1}^d \quad \forall t \tag{15}$$

$$y_t^p \geq u_t^p - u_{t-1}^p \quad \forall t \tag{16}$$

3) Secondary Regulation Service: The secondary regulation service (also called load-frequency control service) in the Spanish electricity system comprises the hourly power reserve, that is modelled with (3), (17) and (18), and the real-time use of reserves, i.e. the regulation energy, that is modelled with (19)-(21).

$$g_t^{sec,up} \leq u_t^d \cdot \overline{g}^d - g_t^d \quad \forall t \tag{17}$$
$$g_t^{sec,dw} \leq g_t^d - u_t^d \cdot \underline{g}^d \quad \forall t \tag{18}$$

$$e_t^{sec,up} - e_t^{sec,dw} = \rho_t^{up} \cdot g_t^{sec,up} - \rho_t^{dw} \cdot g_t^{sec,dw} \quad \forall t \tag{19}$$
$$0 \leq e_t^{sec,up} \leq \phi_t \cdot M \quad \forall t \tag{20}$$
$$0 \leq e_t^{sec,dw} \leq (1-\phi_t)\, M \quad \forall t \tag{21}$$

IV. Case Study

The case study calculates the VPIDM associated to several forecasting models of the spot prices in the year 2014.

A. Electric Power System Data

The electric power system data correspond to hourly values of: 1) the spot price, 2) the reserve price, 3) the upward and downward regulation energy prices, 4) the percentages of the real-time use of reserves and 5) the ratio between the upward and total reserves, R_t^{SM}.

Perfect knowledge is assumed in all data except in the spot prices. Five Cases are analysed, each corresponding to a different approach to forecast the spot prices, Table I. For instance, Case A assumes that the spot price in hour t of a certain day of 2014 is the average of the spot price in the said hour across all days in 2013. Note that Case F assumes perfect knowledge also in the spot prices, and that its results are the maximum theoretical income against which the results of the other Cases are checked.

The forecasting model of Case D is adjusted as follows. The time series is transformed into a regular and seasonal stationary process by means of the logarithmic transformation and the first order regular and seasonal differentiation. The rest of the parameters of the forecasting model are obtained by an iterative process investigating the autocorrelation and partial autocorrelation functions of the transformed time series and of the residuals of the fitted models. The parsimony criteria is used to identify the forecasting models: as few parameters as required to properly explain the data are selected.

The forecasting model of Case E corresponds to GARCH(1,1) and ARIMA(10,1,0)[1] with the forecast wind power of the system, published in the day D-1 by the TSO, as an explanatory variable. The forecasting model is fitted using a similar process as in Case D.

B. Technical Data

Technical data of the reversible Francis pump-turbine unit are presented in Table II. A constant gross head of 400 m and hydraulic losses of 3% of the gross head are assumed.

[1]The orders of the GARCH model and the order of the autorregressive part of the ARIMA model was selected from a trial and error approach

TABLE I
CASES TO FORECAST THE SPOT PRICE, AND THE MEAN ABSOLUTE PERCENTAGE ERROR (MAPE) THROUGHOUT 2014

Case	Spot price	MAPE
A	Mean hourly price of 2013	32.1%
B	Historical price of the previous week	25.3%
C	Historical price of the previous day	21.6%
D	SARIMA$(0,1,1)(1,1,1)^{24}$ model	23.8%
E	GARCH-ARIMAX model	17.8%
F	Perfect knowledge	No error

TABLE II
TECHNICAL DATA OF THE REVERSIBLE FRANCIS PUMP-TURBINE UNIT, POWER IN MW, FLOW IN m^3/s AND START-UP COSTS IN €

$\overline{q}^d$	$\overline{g}^d$	$\overline{\eta}^d$	$\underline{q}^d$	$\underline{g}^d$	$\underline{\eta}^d$
231.5	793	90%	99.5	287.9	76%

$\overline{q}^p$	$\overline{g}^p$	η^p	cSU^d	cSU^p
176.6	793	90%	2667	2118

In generating mode, the minimum water discharge and efficiencies at both maximum and minimum water discharges are calculated from typical performance curves in [9]. The efficiency in pumping mode is 90% [10]; this corresponds to a round-trip efficiency of 76.3 %. The start-up costs in generating and pumping modes have been calculated according to [11].

The upper reservoir has a storage capacity of 5 Mm^3 ($\overline{v} = 5$ Mm^3 and $\underline{v} = 0$ Mm^3), starting and finishing the day at the same water volume of 2.5 Mm^3.

C. Methodology

The VPIDM in each Case in the year 2014 is calculated by the following three steps, Fig. 3: 1) the optimal operation of the PSHP considering the forecast spot price is obtained with a daily sequential optimization, using the proposed model and the branch and cut algorithm in Cplex in a computer with a 2.4 GHz Intel Core i5-450M CPU and 4 GB of RAM memory (365 daily problems in each Case), 2) the actual net income in the spot market (DM Income) is calculated in a post-optimal simulation process from the optimal energy schedule in the spot market of the previous step and the historical hourly spot prices, and 3) the VPIDM in Cases A-E is the difference between the maximum theoretical income (Case F) and the actual total income in each Case A-E, which considers the DM Income from the previous step. Note that the methodology presented in Fig. 3 applies in both when the PSHP only participates in the spot market and when it also participates in the secondary regulation service, i.e., without and with the actual RRCs respectively.

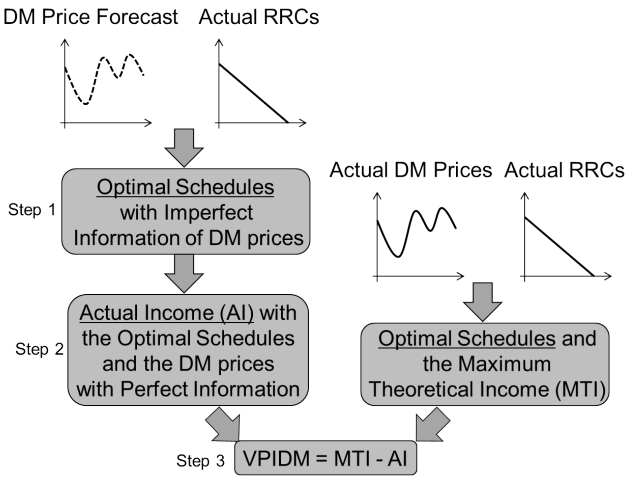


Figure 3. Methodology to obtain the value of perfect information of spot prices

V. Results and Discussion

The DM Income and the actual total income, both obtained after the above-mentioned post-optimal simulation process, and the VPIDM in € and in % of the maximum theoretical income are presented in Table III when the PSHP only participates in the spot market, and in Table IV when the PSHP also participates in the secondary regulation service (SRS).

TABLE III
Results participating the PSHP only in the spot market. Income and VPIDM in €

Case	DM Income	Total Income	VPIDM	VPIDM
A	9 164 408	6 458 701	6 652 213	50.7%
B	8 951 347	6 719 714	6 391 199	48.7%
C	6 572 766	4 360 279	8 750 635	66.7%
D	8 865 832	6 972 309	6 138 604	46.8%
E	9 642 518	8 343 340	4 767 573	36.4%
F	15 323 401	13 110 914	-	-

TABLE IV
Results participating the PSHP in the spot market and the secondary regulation service. Income and VPIDM in €

Case	DM Income	Total Income	VPIDM	VPIDM
A	2 486 626	15 718 582	9 156 444	36.8%
B	868 763	15 789 135	9 085 891	36.5%
C	-438 515	14 809 660	10 065 366	40.4%
D	1 457 727	16 220 402	8 654 624	34.8%
E	1 347 414	18 164 160	6 710 866	26.9%
F	11 408 606	24 875 026	-	-

Two comparisons are presented next: 1) the *profit loss* of several forecasting models of the spot prices and 2) the *profit loss* if the PSHP participates in both the spot market and the SRS, or if only participates in the spot market.

Regarding the first comparison, in all Cases, the *profit loss* due to the errors in forecasting the spot prices is significant. Case E obtains the results that are the closest to Case F: the *profit loss* is 26.9% and 36.4% if the PSHP participates or not in the SRS, respectively. In addition, Case C obtains the worst results: the *profit loss* is 40.4% and 66.7% if the PSHP participates or not in the SRS, respectively. However, as observed in [1], the MAPE does not explain the *profit loss* as can be concluded from the results of Case C with the second lowest MAPE (21.6%). Therefore, the so-called Rank Correlation, used in [12], is proposed as a future work to be used in order to analyse the *profit loss* because it only identifies proper peaks and valleys in a forecast with respect a time series, ignoring their absolute values.

Regarding the second comparison, if the PSHP also participates in the SRS, the *profit loss* in € increases in all Cases A-E. This result confirms one of those obtained in [6]: the PSHP is still using the SRS as the priority source of income, and the spot market as a subsidiary means to maximize the participation in the reserve market, even if the market agent is modelled as a price-marker in the reserve market. By contrast, if the PSHP also participates in the SRS, the *profit loss* in % decreases in all Cases A-E. This seemingly contradictory result is due to the participation in the SRS, increasing the actual total income and, at the same time, decreasing the actual income in the spot market but to a lower extent than the increase in the actual total income. Hence, the added value of the SRS in the operation of PSHP might be proven.

Despite the foregoing, the observed reduction of the *profit loss* in % should be confirmed by independently analysing the VPI of all uncertain variables for which a perfect knowledge was assumed in this paper, namely: the RRCs, the regulation energy prices and the real-time use of the reserves. This is proposed as a future work.

VI. Conclusions

The VPIDM, i.e. the *profit loss* due to the errors in forecasting the spot prices, in the operation of a closed-loop and daily-cycle PSHP, participating in the spot market as a price-taker and in the secondary regulation reserve market as a price-maker, and also considering the secondary regulation energy due to the real-time use of reserves is significant, 26.9% of the maximum theoretical income in the best Case analised, which forecasts the spot prices with a GARCH-ARIMAX model. However, the participation in the SRS reduces notably the *profit loss* in % of the maximum theoretical income, comparing to participate only in the spot market: from 36.4% to 26.9% in the best Case analised. In addition to all the mentioned future work, analysing the *profit loss* with more flexible PSHPs such as variable speed PSHPs and PSHPs in hydraulic short-circuit is established as the most promising.

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