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Optimal Hydrothermal Dispatch Model for the National
Integrated System

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CONTENTS

1.	INTRODUCTION	1
2.	RECOMMENDATIONS	2
2.1	NUMBER OF HYDRO POWER PLANTS AND RESERVOIRS	2
2.2	NONLINEARITY	2
2.3	STOCHASTIC OPTIMIZATION ALGORITHM	3
2.4	STOCHASTICITY	5
3.	MODEL SCALABILITY	6
4.	COMPUTER IMPLEMENTATION	7
5.	CONCLUSIONS	7
6.	REFERENCES	8

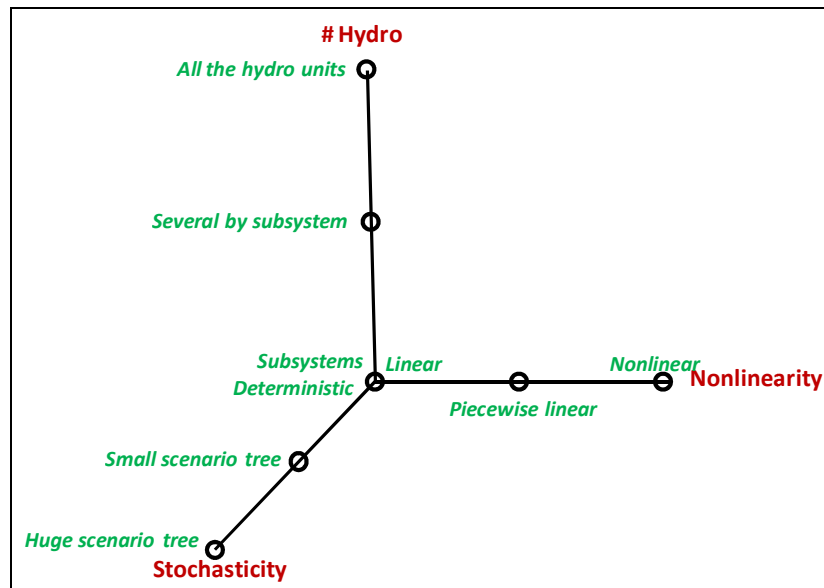
1. Introduction

A medium-term hydrothermal scheduling (MTHT) model determines the optimal operation of a hydrothermal system under stochastic inflows.

The time scope of the model is defined mainly by the hydro storage capability of the system. Several years (2 up to 5) are common in these models if the system has hyperannual storage capability.

In the next figure, we show a tridimensional representation of the major modeling decisions relevant for the development of a MTHT model for the Brazilian electric system.

Figure 1.1: Basic alternatives in the major modeling decisions.



According to this tridimensional representation, we specify in the following table the NEWAVE model [4] and the current model developed by the team of University of Campinas and Sao Paulo [7]. As shown in the table, these two models represent two extreme points of this figure. The main challenge is to obtain a balanced combination of the three major modeling decisions, i.e., to model the important hydro plants and reservoirs in each subsystem representing the nonlinearities by a piecewise linear approximation and with a big enough scenario tree to capture the stochasticity. None of these dimensions seems to be more important than other one so it is just a matter of playing with a reasonable balance among them. Therefore, we suggest the middle

points in each axis. However, the details of these main decisions have to be numerically tuned and adapted for the Brazilian system. For that, a flexible implementation of the mathematical model is required in order to accommodate changes easily.

Table 1.1: Some MTHT models used in Brazil.

	Stochasticity	Nonlinearity	# of hydro power plants
NEWAVE	Huge scenario tree	Linear	Subsystems
UNICAMP-USP	Deterministic	Nonlinear	All the hydro units
New Model	Small scenario tree	Piecewise linear	Several hydro units by subsystem

2. Recommendations

In this section, we are going to discuss each one of these major modeling decisions according to our experience.

2.1 Number of hydro power plants and reservoirs

The operation of hydro subsystems is, in general, very complex. Typically, hydro plants and reservoirs are located in cascades with dependency relations in the operation of the different reservoirs. Complex topological relations may connect them and therefore any model dealing specifically with hydro subsystems must consider as many hydro power plants as possible. Complex hydroelectric subsystems cannot be easily simplified without losing precision in their operation.

Before the introduction of the electricity market in Spain in 1998, Red Eléctrica planned the operation of the electric system by using a stochastic dynamic programming model called MITRE (http://www.doi.icae.upcomillas.es/simio/transpa/t_dp_ar.pdf). This model represented the main 35 hydro power plants with more than 100 MW of installed capacity of the Spanish hydro subsystem.

2.2 Nonlinearity

In the case of hydro subsystems, nonlinearity (and potentially nonconvexity) comes mainly from the power plant water head effects. Depending on the reservoir head variation, these effects are important or not. The production function of a hydro power plant can dramatically change depending of its water head.

Besides, with a constant value of the production function there are multiple hydro operation solutions in time and space. I.e., it is essentially the same to produce with one hydro plant or another or it is essentially the same to produce in any month.

In reference [6], we have analyzed for the Spanish hydrothermal system the importance of the nonlinearity and shown the main differences obtained in the operation of the reservoirs. As it is shown in this paper, the evolution of reservoir is smooth in the nonlinear approximation, as opposite to the oscillatory behavior of the linear approximation. Besides, the reservoir level is higher given that the model considers the improvement of the production function with the power plant water head.

The maximum sizes of the nonlinear problems we have solved with general-purpose solvers (as CONOPT, IPOPT, KNITRO and MINOS) are around 280000 constraints and 420000 variables. These problems take up to 2 hours of computing time having implemented the model very carefully for avoiding numerical problems. That means, for example:

- Avoiding superfluous constraints and variables
- A careful natural scaling of variables around one
- Simple expressions are used in the nonlinear constraints
- Running first a linear approximation of the problem to provide an initial point to the nonlinear solver.

Because of this experience, let us summarize some suggestions:

- Avoid superfluous nonlinearity, consider only the relevant one, use a linear approximation whenever it is possible
- Nonlinearity and stochasticity together are very limiting due to the size that the optimization problems may reach
- Linear solutions take far less computing time, one or two orders of magnitude, as seen in reference [6].

2.3 Stochastic optimization algorithm

For solving multistage stochastic linear optimization problems three main algorithms can be used:

1. Direct solution using simplex, dual simplex o interior point methods

The best method depends on the structure of the constraint matrix of the problem and on its size. Typically, the interior point method is suited for very large-scale problems although it requires more computer memory.

2. Nested Benders' decomposition

Benders' decomposition method solves a two-stage optimization problem by iterating between a master problem that provides first-stage candidate solutions and a subproblem that evaluates second-stage problems and gives feedback information by means of the dual variables. In this algorithm, all the subproblems are evaluated. In this case, several hundred of scenarios seem to be the maximum size of the tree.

Nested Benders' decomposition is the recursive use of Benders' decomposition for multistage problems. So, one iteration of the algorithm represents a forward pass from the first stage to the last stage providing water release decisions for the next stage (month) and a backward pass from the last stage to the first stage giving back the water values that will be used for building the future cost function (cost-to-go function).

Some improvements of Benders' decomposition have been reported in the literature that is worth to explore and quantify their impact. For example,

- Regarding the optimization method used for the subproblems

Simplex dual method is the initial candidate. Try simplex method or interior point method.

Warm start: Use of previous bases.

Use of initial point taken from the deterministic equivalent problem for a scenario.

- Advanced start procedures generate preliminary cuts prior to initiating a formal Benders algorithm
- Cut aggregation: trade-off between monocut or multicut formulations
- Tree partition or node aggregation avoid the decomposition inside the sub-tree and therefore improves SDDP convergence and decreases number of algorithm iterations

3. Stochastic dual dynamic programming (SDDP)

This algorithm is similar to the previous one (it has a forward and a backward pass) but instead of enumerating all the scenarios, these are sampled from the original distribution. In this case, no limit in the number of scenarios is explicitly needed.

The use of a recombining tree has important advantages in the backward pass of SDDP to reduce significantly the number of scenarios to evaluate without losing too much stochastic information, see [1].

SDDP is still a valid method for solving very large-scale stochastic problems but it needs to be improved to achieve better results.

After determining the optimal water releases, typically a simulation process, see [3], is run to obtain possible results of the electric system operation: reservoir levels, hydro plant outputs, marginal costs, etc.

A decision support model with a hierarchical use of a stochastic hydrothermal scheduling model whose results are introduced in a hydro simulation model can be seen in [5].

2.4 Stochasticity

In a stochastic optimization problem, the stochastic parameters are treated as discrete. The evolution of the stochastic parameters along the time is represented by means of a scenario tree. The tree represents in each node the decisions to be made and the instances of stochastic parameters. In the case of a MHT model stochasticity appears in natural hydro inflows. As hydro inflows belong to different reservoirs in the same or different basins, spatial and temporal correlation has to be preserved in the scenario tree representation. Therefore, a stochastic optimization model deals with a multivariate scenario tree that represents the values of the inflows for any reservoir and the probability of each scenario. Reference [2] presents several clustering algorithms extended to generate a multivariate scenario tree and their application for obtaining trees for natural hydro inflows. The neural gas clustering algorithm was selected as the best one.

Extreme scenarios (wet-wet, dry-dry, wet-dry, dry-wet) can be artificially introduced with a very low probability to be able to observe the behavior of the system for these cases while keeping the original statistical properties of the tree.

The first-stage optimal solutions have to be stable with respect to the scenario tree. That means they cannot change if we modify the structure of the tree. Therefore, the structure of the tree (number of branches at each stage, values of the multidimensional inflows for each scenario and probabilities of these scenarios) is part of the modeling process. The scenario tree has to be as large as possible, without over-explaining the stochasticity of the parameters, and given that, the stochastic

optimization problem has to be solvable. Consequently, the size depends of the algorithm used for solving the stochastic problem.

In an application of a weekly hydrothermal scheduling model for the Spanish electric system we have branched in X branches every month, X^{24} is the number of scenarios evaluated. We have tried $X=3$, $X=5$ and $X=7$ branches and 5 is good enough and avoid excessive computing time but at the same time capturing the stochasticity.

The representation of the stochasticity is done to consider the influence of potential outcomes of random parameters in the first stages decisions. For that reason, the number of scenarios generated has to be enough for observing inflow variability and, at the same time, providing stable information regarding the optimal decisions (e.g., the reservoir levels). When branching we need to balance between optimization and stochasticity representation with two possible extremes: big scenario tree and simplified hydro subsystem representation or small scenario tree and realistic hydro subsystem representation. At the end, the objective is building a tree as big as possible limited by the computing time of the optimization problem. As a general criterion, we must branch where there are huge variability of stochastic values, i.e., wet periods in hydro inflows. Other consideration is that short-term future (next weeks or months) will affect more that long-term future (two years ahead of present time). If the scope of the model is defined for several years branching in the first months will much more relevant than branching in the last years keeping controlled the computing time.

3. Model scalability

We suggest the use of simple analytical calculations to parameterize the size of the model as a function of the main attributes: number of periods P and of scenarios S , number of hydro H and of thermal T units, number of reservoirs R . Given these parameters, we can estimate the approximate number of constraints and variables by expressions like those shown in the following tables. Therefore, we can analyze the impact of changes in the values of the distinct parameters.

If we fix a target size of the model to approximately 2 million of constraints and/or variables, it will require around 2 GB of memory (1 MB for each 1000 constraints) to be solved directly and it can take up to 1 hour in a normal desktop to obtain the optimal solution with a powerful linear solver. This size will require the use of a 64 bits operating system.

Table 3.1: Main attributes of the electric system.

	Attributes
P	Periods
S	Scenarios

<i>T</i>	Thermal units
<i>H</i>	Hydro power plants
<i>R</i>	Hydro reservoirs

Table 3.2: Number of equations.

Equations		# of eqn.
Objective function	1	
Load balance	<i>PS</i>	
Water balance	<i>PSR</i>	
TOTAL		

Table 3.3: Number of variables.

Variables		# of var.
Output of each thermal unit	<i>PST</i>	
Output of each hydro power plant	<i>PSH</i>	
Reservoir level	<i>PSR</i>	
Reservoir water release	<i>PSR</i>	
TOTAL		

4. Computer implementation

The algebraic language GAMS (<http://www.gams.com/>) and Microsoft Excel are pillars in a computer implementation. GAMS allows a very easy coding of optimization problems with flexibility to define the scenario tree, the time periods and the topology of the hydro subsystem. Therefore, it allows straightforward adjusting of the characteristics of the model in the three decision dimensions. Microsoft Excel constitutes a user-friendly interface for data input and result output for any model that is going to be used in the context of any company.

Main solution algorithms are simplex, dual simplex and interior point. Typically interior point method is the best for very large-scale problems. Linear solvers like CPLEX, GUROBI or XPRESS are very good and are free under the academic program license (http://support.gams-software.com/doku.php?id=solver:academic_programs_by_solver_partners).

As an example for learning the GAMS coding we do, in the link (http://www.iit.upcomillas.es/~aramos/Ramos_CV.htm#ModelosAyudaDecision) it is a mockup of a medium-term hydrothermal scheduling model. Some explanation of this model is in the slides of this link (http://www.iit.upcomillas.es/~aramos/presentaciones/MOD_7_8.pdf).

5. Conclusions

The main recommendations to guide the development of a new model would be:

- Develop a flexible and easy-to-use model

These two characteristics are crucial if the model need to be tuned to decide the number of hydro plants and reservoirs to include for representing the hydro subsystem and the number of scenarios to consider in the scenario tree.

- Avoid data simplification as much as possible

Simplification of the hydro subsystem should be done as less as possible given its difficulty and complexity. On the contrary, a strong effort should be paid in the mathematical formulation and on the solution algorithm of the optimization model.

- Get rid of the nonlinear solvers

Nonlinear solvers are far less powerful and robust than the linear ones. If possible, the use of nonlinear solvers should be avoided by linearizing the nonlinear functions (typically due to the water head effect of the hydro power plants). If it is not possible it is necessary a very careful implementation of the nonlinear problem.

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