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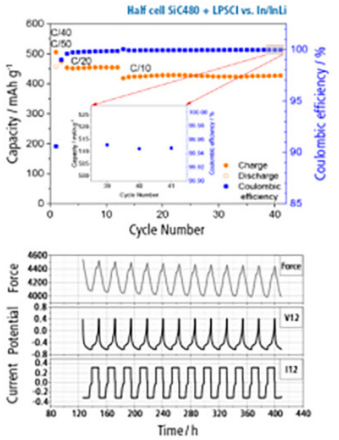
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Killing and homothetic initial data for general hypersurfaces

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Abstract. In this report we present a novel framework to address the homothetic Killing initial data problem for general hypersurfaces. After proving general identities on an abstract hypersurface $\mathcal{H}$ (of any causal character) relating any vector field η with its deformation tensor $\mathcal{L}_\eta g$ and also with the tensor $\mathcal{L}_\eta \nabla$, we are able to establish necessary conditions on $\mathcal{H}$ for the existence of a homothetic Killing vector in the spacetime where $\mathcal{H}$ is embedded. These conditions turn out to be also sufficient in different contexts. When $\mathcal{H}$ is spacelike, we recover the well-known homothetic KID equations [3, 4] in the language of hypersurface data. For two characteristic hypersurfaces, we generalize a previous result [2] (valid for Killings) to the homothetic case and, in addition, show that the equations are at the level of the initial data (i.e. they do not involve a priori spacetime quantities), thus putting the characteristic KID problem on equal footing with the spacelike KID problem. Other initial value problems, such as the smooth spacelike-null or the spacelike-null with corners, can be approached similarly.

1 General identities

Let $\{\mathcal{H}, \boldsymbol{\gamma}, \boldsymbol{\ell}, \ell^{(2)}, \mathbf{Y}\}$ be metric hypersurface data with dual contravariant data $\{P, n, n^{(2)}\}$. We refer to [6, 7, 8] for these concepts and associated geometric constructions. Assume it is (Φ, ξ)-embedded in a spacetime $(\mathcal{M}, g)$, and let η be a vector field on $\mathcal{M}$, $\mathcal{K} \coloneqq \mathcal{L}_\eta g$ and $\Sigma \coloneqq \mathcal{L}_\eta \nabla$. Given the vector $\mathcal{Q}^\mu \coloneqq g^{\alpha\beta}\Sigma[\eta]^\mu{}_{\alpha\beta} = \Box_g \eta^\mu + R^\mu{}_\nu \eta^\nu$, the following known identity holds [14]

$$\Box_g\big(\mathcal{K}_{\beta\nu} - \mu g_{\beta\nu}\big) - \nabla_{(\beta}\mathcal{Q}_{\nu)} - 2R_{\sigma(\beta}\big(\mathcal{K}^\sigma{}_{\nu)} - \mu\delta^\sigma_{\nu)}\big) + 2R_{\sigma\beta\mu\nu}\big(\mathcal{K}^{\sigma\mu} - \mu g^{\sigma\mu}\big) + 2\mathcal{L}_\eta R_{\beta\nu} = 0 \tag{1}$$

for any $\mu \in \mathbb{R}$. Assuming λ-vacuum and $\mathcal{Q} = 0$ this becomes a homogeneous PDE for the tensor $\mathcal{K}[\eta]_{\beta\nu} - \mu g_{\beta\nu}$ (provided $\mu\lambda = 0$) that may be used to prove $\mathcal{K} = \mu g$ on $\mathcal{M}$ (see [12, 3, 13] for Killings and [4] for homotheties). Introducing the tensors $\mathcal{K}_{ab} \coloneqq e^\alpha_a e^\beta_b \mathcal{K}_{\alpha\beta}$, $\mathfrak{K}_a \coloneqq e^\alpha_a \xi^\beta \mathcal{K}_{\alpha\beta}$, $\mathbb{k} \coloneqq \xi^\alpha \xi^\beta \mathcal{K}_{\alpha\beta}$, $\mathbb{\Sigma}_{ab} \coloneqq \xi^\mu e^\alpha_a e^\beta_b \Sigma_{\mu\alpha\beta}$, $\mathfrak{K}^{(2)}_a \coloneqq e^\alpha_a \xi^\beta \mathcal{L}_\xi \mathcal{K}_{\alpha\beta}$ and $\mathbb{k}^{(2)} \coloneqq \xi^\alpha \xi^\beta \mathcal{L}_\xi \mathcal{K}_{\alpha\beta}$, it follows that the tangential and transverse components of $\mathcal{Q}_\mu = \Sigma_\mu{}^\alpha{}_\alpha$, namely $\mathcal{Q}_c \coloneqq e^\mu_c \mathcal{Q}_\mu$ and $\mathfrak{q} \coloneqq \xi^\mu \mathcal{Q}_\mu$, can be written as [11]

$$\begin{aligned}\mathcal{Q}_c = {} & -2\mathbb{\Sigma}_{ca} n^a + 2\mathcal{L}_n \mathfrak{K}_c - 2P^{ab}\mathrm{U}_{bc}\mathfrak{K}_a + \big(2\kappa_n + \mathrm{tr}_P\,\mathbf{U}\big)\mathfrak{K}_c + 2\mathfrak{K}(n)\big(\mathrm{r}_c - \mathrm{s}_c + n^{(2)}\mathring{\nabla}_c \ell^{(2)}\big) \\ & + P^{ab}\mathring{\nabla}_a \mathcal{K}_{bc} - \frac{1}{2}P^{ab}\mathring{\nabla}_c \mathcal{K}_{ab} + \big(\mathrm{tr}_P\,\mathbf{Y} - n(\ell^{(2)})\big)\mathcal{K}_{cd}n^d - 2P^{db}(\mathrm{r}_b + \mathrm{s}_b)\mathcal{K}_{dc} + \mathbb{k}\mathring{\nabla}_c n^{(2)} \\ & + n^{(2)}\left(\mathfrak{K}^{(2)}_c + \big(\mathrm{tr}_P\,\mathbf{Y} - n(\ell^{(2)})\big)\mathfrak{K}_c - \frac{1}{2}\mathring{\nabla}_c \mathbb{k} + 2P^{ab}\mathrm{F}_{cb}\mathfrak{K}_a + 2\left(\mathrm{r}_c - \mathrm{s}_c + \frac{1}{2}n^{(2)}\mathring{\nabla}_c \ell^{(2)}\right)\mathbb{k}\right)\end{aligned} \tag{2}$$

and

$$\mathfrak{q} = \mathrm{tr}_P\,\mathbb{\Sigma} + n\big(\mathbb{k}\big) + 2\kappa_n \mathbb{k} - 2P^{ab}(\mathrm{r} + \mathrm{s})_a \mathfrak{K}_b - n(\ell^{(2)})\mathfrak{K}(n) + n^{(2)}\left(\frac{1}{2}\mathbb{k}^{(2)} - n(\ell^{(2)})\mathbb{k}\right), \tag{3}$$

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where $\mathbf{U} \coloneqq \frac{1}{2}\mathcal{L}_n\boldsymbol{\gamma} + \boldsymbol{\ell} \otimes_s dn^{(2)}$, $\mathbf{F} \coloneqq \frac{1}{2}d\boldsymbol{\ell}$, $\mathbf{s} \coloneqq \mathbf{F}(n,\cdot)$, $\mathbf{r} \coloneqq \mathbf{Y}(n,\cdot)$, $\kappa_n \coloneqq -\mathbf{r}(n)$ and $\mathring{\nabla}$ is the metric connection associated to $\{\boldsymbol{\gamma}, \boldsymbol{\ell}, \ell^{(2)}\}$. Computing the tangential components of the first transverse derivative of $\mathcal{K}$ in terms of hypersurface data one obtains the general identity

$$\begin{aligned}\Phi^\star\big(\mathcal{L}_\xi\mathcal{K}\big)_{ab} = \frac{2C}{n^{(2)}}\Big(\mathring{R}_{(ab)} - R_{ab} - 2\mathcal{L}_n\mathrm{Y}_{ab} - \Big(2\kappa_n + \mathrm{tr}_P\,\mathbf{U} - n^{(2)}\big(n(\ell^{(2)}) - \mathrm{tr}_P\,\mathbf{Y}\big)\Big)\mathrm{Y}_{ab}\\ +\mathring{\nabla}_{(a}\big(\mathrm{s}+2\mathrm{r}\big)_{b)} - 2\mathrm{r}_a\mathrm{r}_b + 4\mathrm{r}_{(a}\mathrm{s}_{b)} - \mathrm{s}_a\mathrm{s}_b - \big(\mathrm{tr}_P\,\mathbf{Y}\big)\mathrm{U}_{ab} + 2P^{cd}\mathrm{U}_{c(a}\big(2\mathrm{Y}+\mathrm{F}\big)_{b)d}\Big)\\ + 2C\Big(-2\mathrm{r}_{(a}\mathring{\nabla}_{b)}\ell^{(2)} + P^{cd}\big(\Pi_{ca}\Pi_{bd} + \Pi_{ac}\Pi_{db}\big) + \frac{1}{4}n^{(2)}\mathring{\nabla}_a\ell^{(2)}\mathring{\nabla}_b\ell^{(2)} - \frac{1}{2}n(\ell^{(2)})\mathrm{Y}_{ab}\\ -\frac{1}{2}\mathring{\nabla}_a\mathring{\nabla}_b\ell^{(2)}\Big) - 2\ell^{(2)}\big(n(C)\mathrm{Y}_{ab} + \mathring{\nabla}_a\mathring{\nabla}_bC\big) - 2\mathring{\nabla}_{(a}C\mathring{\nabla}_{b)}\ell^{(2)} + \big(\mathbb{k} - \bar{\eta}(\ell^{(2)})\big)\mathrm{K}_{ab}\\ + 2\mathcal{L}_{\bar{\eta}}\mathrm{Y}_{ab} - 2\left(\big(\mathcal{L}_{\bar{\eta}}\boldsymbol{\ell}\big)(n) - \mathfrak{K}(n)\right)\mathrm{Y}_{ab} - 2\mathring{\nabla}_{(a}\mathcal{L}_{\bar{\eta}}\ell_{b)} + 2\mathring{\nabla}_{(a}\mathfrak{K}_{b)},\end{aligned} \tag{4}$$

where $\mathring{R}$ is the Ricci tensor of $\mathring{\nabla}$, R is that of the spacetime, $\boldsymbol{\Pi} \coloneqq \mathbf{Y} + \mathbf{F}$, and $C, \bar{\eta}$ are defined by $\eta \overset{\mathcal{H}}{=} C\xi + \Phi_\star\bar{\eta}$.

2 Non-null case

First we restate the classic well-posedness theorem of the Einstein field equations [1, 5] in the language of hypersurface data.

Theorem 2.1. *Let $\{\mathcal{H}, \boldsymbol{\gamma}, \boldsymbol{\ell}, \ell^{(2)}, \mathbf{Y}\}$ be hypersurface data of dimension $\mathfrak{n}$ satisfying the constraint equations with cosmological constant $\lambda \in \mathbb{R}$ (see [6, Theorem 2] for their explicit form in terms of hypersurface data). Then there exists a unique maximal and globally hyperbolic λ-vacuum spacetime $(\mathcal{M}, g)$, embedding $\Phi : \mathcal{H} \hookrightarrow \mathcal{M}$ and rigging ξ such that $\{\mathcal{H}, \boldsymbol{\gamma}, \boldsymbol{\ell}, \ell^{(2)}, \mathbf{Y}\}$ is (Φ, ξ)-embedded on $(\mathcal{M}, g)$.*

To construct the candidate to Killing/homothetic vector the strategy is to integrate the equation $\Box\eta + \lambda\eta = 0$ with initial data $\eta|_{\mathcal{H}}$ and $\mathcal{L}_\xi\eta|_{\mathcal{H}}$ selected so that $\mathcal{K} \overset{\mathcal{H}}{=} \mu g$ and $\mathcal{L}_\xi\mathcal{K} \overset{\mathcal{H}}{=} \mu\mathcal{L}_\xi g$. Since (1) is then a homogeneous wave equation with trivial initial data it follows that $\mathcal{K} = \mu g$ everywhere on $(\mathcal{M}, g)$. This approach requires first of all being able to write down $\mathcal{K}_{ab}$ and $\big(\mathcal{L}_\xi\mathcal{K}\big)_{ab}$ in terms of hypersurface data in order to ensure $\mathcal{K}_{ab} = \mu\gamma_{ab}$ and $\big(\mathcal{L}_\xi\mathcal{K}\big)_{ab} = 2\mu\mathrm{Y}_{ab}$ at the abstract level. The former is simply

$$\mu\gamma_{ab} = 2C\mathrm{Y}_{ab} + 2\ell_{(a}\mathring{\nabla}_{b)}C + \mathcal{L}_{\bar{\eta}}\gamma_{ab}. \tag{5}$$

This equation will be called KID1. For the latter we use the general identity (4) and replace $\big(\mathcal{L}_\xi\mathcal{K}\big)_{ab}$ by $2\mu\mathrm{Y}_{ab}$, $\mathbb{k}$ by $\mu\ell^{(2)}$, $\mathfrak{K}$ by $\mu\boldsymbol{\ell}$ and R_{ab} by $\lambda\gamma_{ab}$. The resulting equation will be called KID2, and its explicit expression can be found in [11]. The hypersurface data version of the existence theorem for homothetic/Killing initial data is then the following [11].

Theorem 2.2. *Let $\{\mathcal{H}, \boldsymbol{\gamma}, \boldsymbol{\ell}, \ell^{(2)}, \mathbf{Y}\}$ be hypersurface data and $\lambda \in \mathbb{R}$ such that the constraint equations hold. Assume that $(C, \bar{\eta}) \in \mathcal{F}(\mathcal{H}) \times \mathfrak{X}(\mathcal{H})$ satisfy KID1 and KID2 with $\mu \in \mathbb{R}$ such that $\lambda\mu = 0$. Then there exists a unique maximal, globally hyperbolic λ-vacuum spacetime $(\mathcal{M}, g)$, embedding $\Phi : \mathcal{H} \hookrightarrow \mathcal{M}$, rigging ξ and vector field η such that $\{\mathcal{H}, \boldsymbol{\gamma}, \boldsymbol{\ell}, \ell^{(2)}, \mathbf{Y}\}$ is (Φ, ξ)-embedded on $(\mathcal{M}, g)$, $\eta|_{\Phi(\mathcal{H})} = C\xi + \Phi_\star\bar{\eta}$ and $\mathcal{L}_\eta g = \mu g$ on $\mathcal{M}$.*

Remark 2.3. *KID1 and KID2 are the hypersurface data version of the standard KID equations. The advantage of KID1-KID2 w.r.t their standard version is that the gauge is fully free, so it can be adapted to one's convenience. For instance, if one wants to construct a Killing/homothetic vector transverse to the initial hypersurface (but not necessarily normal) it may be easier to choose the gauge such that $C = 1$ and $\bar{\eta} = 0$, because then $\mathbf{Y} = 2\mu\boldsymbol{\gamma}$ and the constraint and KID equations take a much simpler form. Geometrically this gauge corresponds to choosing $\xi = \eta|_{\Phi(\mathcal{H})}$.*

3 Characteristic case

The characteristic Cauchy problem has been formulated recently in terms of detached data via the notion of *double null data* [10, 9], which consists on two null hypersurface data ($n^{(2)} = 0$) glued along their boundaries in an abstract way. For a single null hypersurface one can show that the tensor $\mathbb{\Sigma}$ satisfies transport evolution equations along the null generator n of the hypersurface.

Lemma 3.1. *Assume $\mathcal{K}_{ab}n^b = 0$, $\mathcal{K}_{ab} - \frac{\mathrm{tr}_P\,\mathcal{K}}{\mathfrak{n}-1}\gamma_{ab} = 0$ and $R_{\alpha\beta} = \lambda g_{\alpha\beta}$ in a neighbourhood of $\Phi(\mathcal{H})$ and let $\mu \in \mathbb{R}$ satisfying $\lambda\mu = 0$. Then,*

1. $\left(\mathcal{L}_n^{(2)} + \left(\frac{2\,\mathrm{tr}_P\,\mathbf{U}}{\mathfrak{n}-\mathfrak{1}} - \kappa_n\right)\mathcal{L}_n\right)\mathrm{tr}_P\,\mathcal{K} = 2\big(\mathrm{tr}_P\,\mathbf{U}\big)\mathbb{\Sigma}(n,n).$

2. *If moreover* $\mathcal{K}_{ab} = \mu\gamma_{ab}$ *and* $\mathfrak{K}(n) = \mu$*, then* $\mathcal{L}_n\big(\mathbb{\Sigma}_{ab}n^b\big) - \mathring{\nabla}_a\big(\mathbb{\Sigma}(n,n)\big) + \big(\operatorname{tr}_P \mathbf{U}\big)\mathbb{\Sigma}_{ab}n^b = 0$.

3. *If* $\mathcal{K}_{ab} = \mu\gamma_{ab}$ *and in addition* $\mathfrak{K} = \mu\boldsymbol{\ell}$ *and* $\mathbb{\Sigma}(n,\cdot) = 0$*, then*

$$2\mathcal{L}_n\big(\operatorname{tr}_P \mathbb{\Sigma}\big) + 2\big(\kappa_n + \operatorname{tr}_P \mathbf{U}\big)\operatorname{tr}_P \mathbb{\Sigma} = \big((\Bbbk - \mu\ell^{(2)})\operatorname{tr}_P \mathbf{U} + n\big(\Bbbk - \mu\ell^{(2)}\big)\big(\Bbbk - \mu\ell^{(2)}\big)\kappa_n\big) + \big(\Bbbk - \mu\ell^{(2)}\big)\mathcal{L}_n(\operatorname{tr}_P \mathbf{U}).$$

Combining this with (2)-(3) (for $n^{(2)} = 0$) one arrives at the following.

Theorem 3.2. *Let* $\{\mathcal{H}, \boldsymbol{\gamma}, \boldsymbol{\ell}, \ell^{(2)}, \mathbf{Y}\}$ *be null hypersurface data* (Φ, ξ)*-embedded in* $(\mathcal{M}, g)$ *with* $R_{\alpha\beta} = \lambda g_{\alpha\beta}$ *in a neighbourhood of* $\Phi(\mathcal{H})$ *and let* η *be a vector field on* $\mathcal{M}$ *satisfying* $\mathcal{Q} \overset{\mathcal{H}}{=} 0$*,* $\mathbb{\Sigma}(n,n) = 0$*,* $\mathcal{K}_{ab}n^b = 0$ *and* $\mathcal{K}_{ab} - \frac{\operatorname{tr}_P \mathcal{K}}{\mathfrak{n}-1}\gamma_{ab} = 0$*. Assume that* $\mathcal{H}$ *admits a cross-section and let* μ *be a constant satisfying* $\lambda\mu = 0$*. Then,*

1. *If* $P^{ab}\mathcal{K}_{ab} = (\mathfrak{n}-1)\mu$ *and* $\mathcal{L}_n\big(P^{ab}\mathcal{K}_{ab}\big) = 0$ *on a cross-section of* $\mathcal{H}$*, then* $\mathcal{K}_{ab} = \mu\gamma_{ab}$ *everywhere on* $\mathcal{H}$*. Moreover, if* $\mathfrak{K}(n) = \mu$ *on a cross-section, then* $\mathfrak{K}(n) = \mu$ *everywhere.*

2. *If in addition to item 1. one has* $\mathfrak{K}_c = \mu\ell_c$ *and* $\mathbb{\Sigma}_{bc}n^b = 0$ *on a cross-section of* $\mathcal{H}$*, then* $\mathfrak{K}_c = \mu\ell_c$ *and* $\mathbb{\Sigma}_{bc}n^b = 0$ *everywhere on* $\mathcal{H}$*.*

3. *Moreover, if in addition to items 1. and 2. one has* $\Bbbk = \mu\ell^{(2)}$ *and* $\operatorname{tr}_P \mathbb{\Sigma} = 0$ *on a cross-section of* $\mathcal{H}$*, then* $\Bbbk = \mu\ell^{(2)}$ *and* $\operatorname{tr}_P \mathbb{\Sigma} = 0$ *everywhere on* $\mathcal{H}$*.*

The equations obtained after writing $\mathbb{\Sigma}(n,n) = 0$, $\mathcal{K}_{ab}n^b = 0$ and $\mathcal{K}_{ab} - \frac{\operatorname{tr}_P \mathcal{K}}{\mathfrak{n}-1}\gamma_{ab} = 0$ in terms of hypersurface data will be called *null KID equations*. Given double null data $\{\mathcal{D}, \underline{\mathcal{D}}, \sigma, \phi\}$ several of the conditions of the previous theorem needed at an initial cross-section are redundant given the presence of a second transverse hypersurface. The non-redundant ones will be called *∂-KID equations*. The explicit form of both the null-KID equations and the ∂-KID equations can be found in [11]. A key property is that they can be written solely in terms of double null data.

Theorem 3.3. *Let* $\{\mathcal{D}, \underline{\mathcal{D}}, \sigma, \phi\}$ *be double null data satisfying the constraint equations and let* $(\bar{\eta}, C) \in \mathfrak{X}(\mathcal{H}) \times \mathcal{F}(\mathcal{H})$*,* $(\underline{\bar{\eta}}, \underline{C}) \in \mathfrak{X}(\underline{\mathcal{H}}) \times \mathcal{F}(\underline{\mathcal{H}})$ *satisfy* $\Psi(\bar{\eta}, C) \overset{\partial\mathcal{H}}{=} (\underline{\bar{\eta}}, \underline{C})$*. Assume both the null-KID equations and the ∂-KID equations are satisfied. Then, there exists a* λ*-vacuum development* $(\mathcal{M}, g)$ *of* $\{\mathcal{D}, \underline{\mathcal{D}}, \sigma, \phi\}$ *and a vector field* η *on* $\mathcal{M}$ *satisfying* $\mathcal{L}_\eta g = \mu g$ *such that* $\eta|_{\Phi(\mathcal{H})} = C\xi + \bar{\eta}$ *and* $\eta|_{\Phi(\underline{\mathcal{H}})} = C\xi + \underline{\bar{\eta}}$*.*

4 Other initial value problems

The techniques described above can be employed to solve other KID problems, for instance the smooth spacelike-characteristic KID problem or the spacelike-characteristic one with corners. In both cases, it turns out that the sufficient conditions to be imposed on the initial data are the spacelike KID equations (in the spacelike region) as well as the null KID equations (in the null one). No extra conditions on the transition surface are needed [11].

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