



MÁSTER EN INGENIERÍA INDUSTRIAL

TRABAJO FIN DE MÁSTER

Effects of load modelling on Voltage Impasse Regions (*VIR*)

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Madrid

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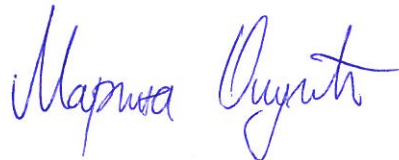
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Julio de 2019

Resumen

TÍTULO: Efecto del modelado de cargas sobre Voltage Impasse Regions (*VIR*)

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Voltage Impasse Region (*VIR*) es un fenómeno presente en los sistemas eléctricos de potencia cuya dinámica está descrita por un sistema de ecuaciones Diferenciales Algebraicas (DAE - Differential Algebraic Equations). *VIR* representa un área en el espacio de estado donde se pierde la causalidad de la tensión, es decir, el Jacobiano de las ecuaciones algebraicas se convierte en singular. En simulaciones realizadas en el dominio del tiempo (TDS - Time Domain Simulation), una vez que la trayectoria dinámica intersecta con *VIR*, la simulación deja de converger (no es capaz de encontrar ninguna solución). A partir de ese instante, no tiene sentido continuar con la simulación. Por esta razón, es importante entender los mecanismos que provocan la aparición de *VIR*. Se sabe que *VIR* está relacionado con el empleo de modelos de carga estáticos no lineales. Sin embargo, aún se desconoce cual es el efecto que múltiples cargas estáticas tienen sobre *VIR*.

Este Trabajo Fin de Máster (TFM) ha profundizado en el concepto de *VIR* llevando a cabo un estudio estructurado sobre el efecto que modelos estáticos de cargas tienen sobre *VIR*. Con este objetivo, este TFM propone una metodología cuasi-dinámica para trazar *VIR* en el espacio de estado definido por los ángulos internos de los generadores. Esta nueva metodología introduce un índice discreto llamado Voltage Impasse Region Flag (VIR_{flag}). VIR_{flag} permite localizar *VIR* porque determina si el sistema de ecuaciones algebraicas del DAE es solucionable o no. Para verificar la metodología se usa un sencillo modelo de un sistema eléctrico. Entonces, el índice VIR_{flag} es empleado para trazar *VIR* para diferentes combinaciones de cargas estáticas. También se examina la relación entre los problemas de convergencia de las simulaciones en el dominio del tiempo y la intersección de la trayectoria dinámica con *VIR*.

En conclusión, este TFM ha demostrado que la metodología propuesta es efectiva en la determinación de *VIR* independientemente del número de cargas no lineales incluidas en el modelo de un sistema eléctrico. Entre los modelos de carga estáticos exponenciales, las cargas de Potencia Constante (CP - Constant Power) son las que tienen una mayor influencia sobre la aparición y la forma de *VIR*. El modelo de Intensidad Constante (CC - Constant Current) induce áreas de *VIR* más pequeñas, mientras que el modelo de Impedancia Constante (CI - Constant Impedance) sólo altera la forma de *VIR* en presencia de otras cargas no lineales.

Palabras clave: colapso de tensión, dinámica de los sistemas eléctricos de potencia, metodología cuasi-dinámica, modelos de carga, Voltage Impasse Regions (*VIR*)

Abstract

TITLE: Effects of load modelling on Voltage Impasse Regions (*VIR*)

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Voltage Impasse Region (*VIR*) is a phenomenon in power systems whose dynamics are describe by a set of Differential Algebraic Equations (DAE). *VIR* denotes a state-space area where voltage causality is lost, i.e. the Jacobian of the algebraic part of DAE is singular. In a Time Domain Simulation (TDS) once system trajectories enter *VIR*, TDS experiences non-convergence of the solution. Then, there is no reason to continue with the simulation. This is why it is important to understand the mechanisms that introduce *VIR*. It is known that *VIR* appears in relation to static, non-linear load models. However, it remained unknown what the cumulative effect of several static, non-linear loads would be.

This master thesis has further expanded the concept of *VIR* by carrying out a structured study on how the load modelling affects *VIR*. For this purpose, this thesis proposes a quasi-dynamic methodology to map *VIR* in the relative rotor angle space. The methodology introduces a new discrete index called Voltage Impasse Region Flag (VIR_{flag}), which allows to determine if the algebraic equations of DAE are solvable or not and, thus, to locate *VIR*. A test system is used to test the proposed quasi-dynamic approach. The VIR_{flag} was first used to map *VIR* for various load combinations. Then, the relationship between TDS non-convergence issues and the intersection of a trajectory with *VIR* is examined to verify the proposed methodology.

The proposed method has been proved to be efficient in the determination of *VIR* regardless of the number of non-linear loads in the power system. Among the static exponential load models, the Constant Power (CP) load component has been identified as the one with the largest influence on *VIR* appearance and shape. The Constant Current (CC) loads induce "smaller" *VIR* areas and the Constant Impedance (CI) load can only alter the shape of *VIR* in the presence of non-linear load models.

Keywords: load models, power system dynamics, quasi-dynamic approach, voltage collapse, Voltage Impasse Regions (*VIR*)

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Chapter 1

Introduction

Since the 80's, electric power systems have experienced more stressed conditions in comparison with the past. An increase of demand and the growing interconnection of the electricity markets are, among others, some of the reasons for today's heavy loaded networks.

If a heavily loaded power system encounters a disturbance, there is a high probability that it results in instability. Among all the types of instability, voltage instability is one of the most common. After a disturbance occurs in a stressed system, voltage stability can be lost. This instability can lead the system into a voltage collapse. The voltage collapse denotes a quick drop of voltage magnitudes that, if the system controls fail to halt, results in a partial or a total blackout [1].

The Transmission System Operator (TSO) has the important task of guaranteeing the continuity and security of the electricity supply. Therefore, the understanding of the mechanisms and underlying reasons of voltage collapse is a major issue in the power systems literature. Furthermore, the increasing number of voltage collapse cases worldwide has turned this topic into one of the main concerns in the studies related to electric power systems.

The methods used for the analysis of voltage stability can be classified as static or dynamic methods. Static methods consist of load flow or small-signal stability studies. More commonly, power systems operate under slow changing conditions that can be analyzed by using a steady-state approach. That is why there has previously been a lot of work focused on voltage collapse from the static point of view.

Dynamics of multi-machine power systems are represented by a set of non-linear Differential Algebraic Equations (DAE). In dynamics, the stability of the system depends on the type of disturbance and on the initial operating conditions. If there is no equilibrium in the beginning, the system will obviously not be stable no matter the disturbance. There-

fore, it is important to ensure that the system is operating within its loadability limits, i.e. that there is a solution for the power flow equations. This demonstrates that the static and dynamic analyses of voltage stability should be complementary [2].

However, less research has been carried out in the dynamic framework. Most of the previous work has focused on the fast load dynamics (e.g. dynamics of induction motors, electronically controlled loads and HVDC converters [3]) and their contribution to short-term voltage instability. A minor number of studies connected a dynamic voltage collapse with the intersection of a dynamic trajectory with a so called “impasse surface” [4]. A recent paper [5] analyzed more concisely the nature of these “impasse surfaces” and introduced the concept of Voltage Impasse Regions (*VIR*). The study analyzed the influence that the modeling of one static, non-linear load connected to an N -bus system has on *VIR*. In that case, all the other loads were modeled as constant impedances. Starting from that point, this master thesis aims to analyze the cumulative effect of several static, non-linear loads and their interaction on *VIR*. For such a purpose, a new quasi-dynamic approach is proposed as a methodology for mapping the *VIR* in the relative rotor angle space.

This document is organized in six chapters. Chapter 1 makes a revision of the current state of the art and states the motivation and the main objectives of this master thesis. In Chapter 2, the Structure Preserving Model (SPM) of a power system (which is considered in this thesis) is presented along with the components of the network. Then, Chapter 3 includes a static analysis of voltage collapse: it revises the concepts of power flow and includes some examples of how the loadability limit is calculated. After that, in Chapter 4, the proposed quasi-dynamic methodology is introduced and its validity is tested for specific load modeling cases where only one load is modeled as a non-linear load. Chapter 5 presents the results obtained when *VIR* is mapped by the means of the new quasi-dynamic approach. By employing a simple 3-machine test system, Chapter 5 analyzes exemplary cases in which more than one load is modeled as a load of non-constant impedance type. The connection between the non-convergence issues in Time Domain Simulations (TDS) and *VIR* is also studied. Finally, Chapter 6 gathers the main conclusions of the work performed and presents some proposals for the future work.

1.1 State of the art

In the static framework, voltage stability is connected to the capacity of the network to transport real power. In these terms, loadability is defined as the maximum power that can be provided by the system before it loses stable equilibrium. Analytically, it is defined as the point where the load characteristic becomes tangent to the PU network curves. For constant power loads, the loadability limit coincides with the most extreme point of the curve, the so called “nose curve” [6]. The number of methods available to determine the loadability limits is large [7], but the majority of researchers focused on connecting the loadability limits with the bifurcation analysis. For instance, in [8], it was analytically demonstrated (by using the static bifurcation analysis) that the sensitivity of the PQ buses voltages is infinite with respect to the power flow parameters at the point of voltage collapse.

For TSOs, it is very useful to identify the proximity to a static voltage collapse through indices. The identification of the bifurcation point where the loadability limit is reached can be done by using Singular Value Decomposition (SVD) or eigenvalues analysis. In [9], it is proved that both techniques (if applied on the reduced load flow Jacobian with respect to the reactive power equations ($\mathbf{J}_{qv}$)) provide the same information regarding the proximity to a static voltage collapse. Other studies [10] compute the Minimum Singular Value ($\min(\text{SVD})$) for the Jacobian matrix and for other three important submatrices in order to assess the proximity to a static voltage collapse. The conclusion drawn in [10] was also that the $\min(\text{SVD})$ of the reduced Jacobian $\mathbf{J}_{qv}$ is the best voltage instability index. However, reference [9] abandons the SVD technique by favoring the use of the determinant of $\mathbf{J}_{qv}$ when the reduction is carried out with respect to the critical bus of a system. This determinant is considered by [9] as the best way to check the proximity to a voltage collapse.

In order to reduce the computational efforts needed for the calculation of the loading margin, the work from [9] also developed a technique to find the sensitivity of the loading margin to different parameters. According to that method, the loading margin can be computed by using linear or quadratic estimators and the information about the initial operating point. The static voltage analysis that studies the loading margins requires the calculation of power flow. Nonetheless, that study can also reflect the proximity to a dynamic voltage collapse. It can be mathematically demonstrated that the steady-state loadability limits are met in general at a Saddle-Node Bifurcation (SNB) point since the Jacobian of the algebraic equations coincides with the long-term Jacobian of the DAE system [11]. In the dynamic framework, voltage collapse has been connected with the loss of DAE solution. Only a few research works associated the voltage collapse (that occurs after a large disturbance) with the singularity of the algebraic equations of the respective system. In these works, the loss of DAE solution is commonly associated with the intersection of a post-

fault trajectory with an “impasse surface”. An impasse surface is a hypersurface defined by the points of the constraint manifold of the DAE system whose algebraic Jacobian is singular. Dynamic behavior of a system cannot be defined if a trajectory meets an impasse surface. In [4], the researchers presented more specific conditions that have to be fulfilled in order to guarantee the existence of voltage causality regions when Structure Preserving network Models (SPM) are used.

However, none of the previous work analyzed the nature of impasse surfaces in relation to load modeling. In [5], an explicit formulation is developed to define voltage collapse areas in the relative rotor angle space. For the three static load models (namely constant power, constant impedance and constant current load models), these “no voltage solution areas”, called Voltage Impasse Regions (*VIR*), are identified. **In *VIR*, there is no real, positive solution for the load voltage magnitude. So, if after a disturbance a dynamic trajectory ends up in *VIR*, the simulation will not converge.** Also, in line with the static efforts for calculating the proximity to a static voltage collapse, Voltage Collapse Indicators (*VCIs*) are calculated to provide a measure of the distance from a voltage collapse along a post-fault trajectory. The formulation for the constant impedance load model presented in [5] proves that, for this model, a *VIR* does not exist. These results justify the use of constant impedance model in Time Domain Simulations (TDS) to reduce computational issues. The case study employed in [5] shows that *VIR* mostly depends on the size and type of the load model that is considered. Regarding the load size, *VIR* becomes larger with the increase of the non-linear load. When it comes to the type of load model, the constant power model was shown to be the most risky one.

Furthermore, in [5] it is shown that the proximity to a *VIR* happens at the same time when the $\min(\text{SVD})$ approaches zero. *VCIs* are confirmed to be sufficiently precise as tools for the identification of impasse surfaces in the state-space. Therefore, *VCIs* are proposed as new indices which are less computationally demanding than the usually employed SVD. This master thesis takes the work from [5] as a starting point and aims to analyze how a higher number of loads and their modeling affect the properties of *VIR*.

1.2 Motivation

The non-linearity of electric power systems makes it difficult to analytically determine the expressions for *VIR*. Nonetheless, it is of great interest to identify the location of *VIR* in the relative rotor angle space to guarantee that a post-fault trajectory will not result in a voltage collapse. Thus, the motivation of this master thesis is to determine *VIR* by using a different approach from the analytical one used in [5]. This new approach will allow to extend the *VIR* concept to the case of multiple non-linear loads.

1.3 Thesis objectives

The aim of this master thesis is to study the effect of load interaction on *VIR* both in the terms of load modeling and the size of the loads. The main objectives of this master thesis are:

- To study the loadability limits of the considered system by using a parametric approach. The thesis considers a simple 3-machine system that is based on IEEE-9 bus system. Power flow calculations and the Newton-Raphson method are used for this purpose when the load is iteratively increased.
- To develop a new quasi-dynamic methodology for mapping *VIR* in the state-space for different combinations of static, exponential load models.
- To validate the proposed methodology by comparing the analytical expressions for one load (as derived in [5]) with the results obtained by the new quasi-dynamic approach.
- To perform a structured study about the influence of multiple non-linear loads on *VIR*.
- To establish the relationship between the loss of voltage causality, *VIR* and the singularity of the algebraic Jacobian $\mathbf{g}_y$.
- To perform TDS to verify the previously obtained results. By doing this it can be observed how large disturbances can affect the consistency of the DAE model.

Chapter 2

Theoretical background: power system modeling

Consider a generic multi-machine power system as the one that is shown in Fig. 2.1. The transmission network has a total of N ($k = 1, \dots, N$) buses, out of which the first n_g buses ($k = 1, \dots, n_g$) are the generator terminal buses.

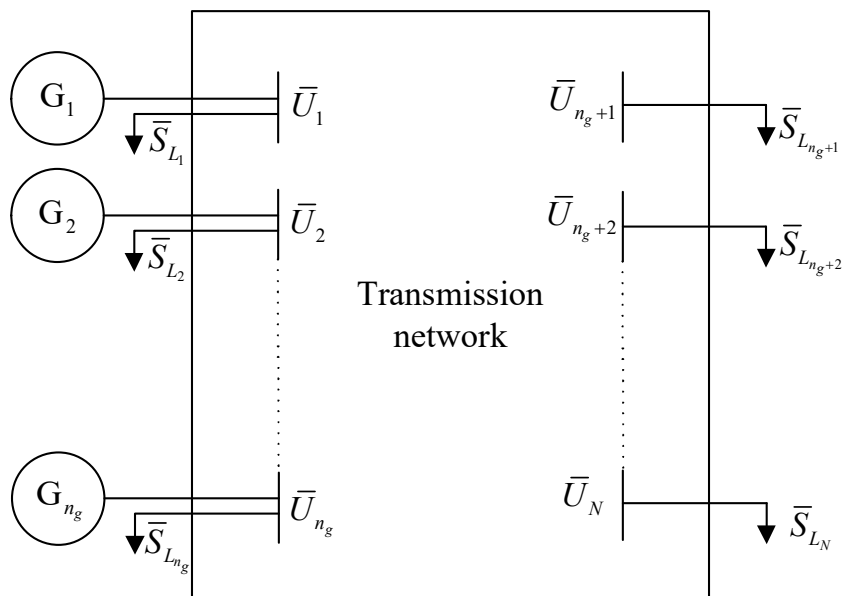


Figure 2.1: General model of a multi-machine power system

Regarding the modeling of the power system, in this master thesis it will be considered that:

1. Dynamics of each generator are described by the classical model.
2. Mechanical power of each generator is constant.
3. Loads are represented by static load models in exponential form.
4. Transmission network lines are represented by the π -equivalent model.

This chapter starts by revising the models of the different components of the power system (in Section 2.1). Afterwards, Section 2.2 describes the Structure Preserving network Model (SPM). SPM is, in a fact, the set of Differential Algebraic Equations (DAE) that relate to the aforementioned models of the system components.

2.1 Component modeling

2.1.1 Synchronous machines

Generators are represented by the classical model. This model is widely used in the transient stability studies because it reduces computational complexity.

In this model, the k^{th} generator is represented by a voltage source with a constant magnitude E'_{qk} behind the d -axis transient reactance x'_{dk} (which here also includes the transformer's reactance), as shown in Fig. 2.2.

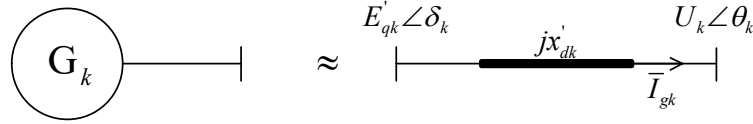


Figure 2.2: Classical model of a synchronous generator

In Fig. 2.2, δ_k is the angle of the generator's internal voltage $\overline{E}'_{qk}$, $\overline{I}_{gk}$ the current of the k^{th} generator, U_k is the magnitude of the terminal voltage at bus k and, θ_k is the phase angle of the voltage at bus k .

Then, the dynamics of the k^{th} generator are described by:

$$\begin{aligned}\dot{\delta}_k &= \omega_k; \\ \dot{\omega}_k &= \frac{1}{M_k} (P_{mk} - P_{ek} - D_k \omega_k),\end{aligned}\tag{2.1}$$

where:

$$P_{ek} = \frac{E'_{qk} U_k \sin(\delta_k - \theta_k)}{x'_{dk}}.\tag{2.2}$$

In Eqs. (2.1), P_{mk} denotes the mechanical power input of the k^{th} generator and P_{ek} is the electrical power output of the generator. Both the inertia of the machine (M_k) and the damping (D_k) are assumed to be constant.

The main advantage of the classical model is that it simplifies the dynamics of the generator by considering only the swing equation.

2.1.2 Loads

The exponential load model is a static model that expresses the power at any instant of time as an exponential function of the bus voltage magnitude at the same instant. Unlike dynamic load models, static load models do not consider past instants of time. Therefore, it is not necessary to use differential equations to represent exponential load models [12]. The Fig. 2.3 shows the connection of a generic load to some bus k . This load is modeled by its complex power $\bar{S}_{L_k}$.

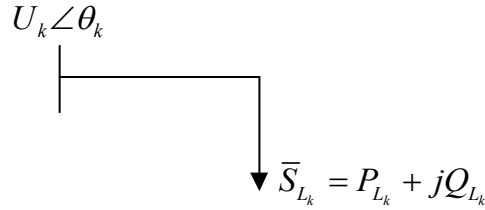


Figure 2.3: Load model

The exponential model of a load connected to bus k can be written in the form:

$$\begin{aligned} P_{L_k} &= P_{L_{k0}} \left(\frac{U_k}{U_{k0}} \right)^{mp}; \\ Q_{L_k} &= Q_{L_{k0}} \left(\frac{U_k}{U_{k0}} \right)^{mq}, \end{aligned} \quad (2.3)$$

where U_k is the instantaneous voltage magnitude at the terminal bus k , U_{k0} is the initial value of the voltage, and $P_{L_{k0}}$ and $Q_{L_{k0}}$ are the active and reactive powers demanded by the load when the voltage is U_{k0} .

The exponents mp and mq are the parameters of this load model. Depending on their value, different load characteristics can be modeled:

- with $mp = mq = 0$, the load model represents Constant Power (CP) characteristic;
- with $mp = mq = 1$, the load model represents Constant Current (CC) characteristic;
- with $mp = mq = 2$, the load model represents Constant Impedance (CI) characteristic.

The simple exponential load model is a type of static load model that allows to model a load as a constant impedance, constant current or constant power load depending on the value of the exponents mp and mq . A more general form of modeling a static, exponential load is

known as the ZIP load model. The ZIP load model is named like that due to the fact that the load is composed out of constant impedance (Z), constant current (I), and constant power (P) components at the same time. In the ZIP load model, active and reactive components of the load complex powers are expressed as:

$$\begin{aligned} P_{L_k} &= P_{L_{k0}} \left[k_{pz} \left(\frac{U_k}{U_{k0}} \right)^2 + k_{pi} \left(\frac{U_k}{U_{k0}} \right) + k_{pp} \right]; \\ Q_{L_k} &= Q_{L_{k0}} \left[k_{qz} \left(\frac{U_k}{U_{k0}} \right)^2 + k_{qi} \left(\frac{U_k}{U_{k0}} \right) + k_{qp} \right]. \end{aligned} \quad (2.4)$$

As shown in Eq. (2.4), the ZIP load model combines the three basic types of exponential load models. In this way, the parameters k_p and k_q define the fraction of each exponential load component:

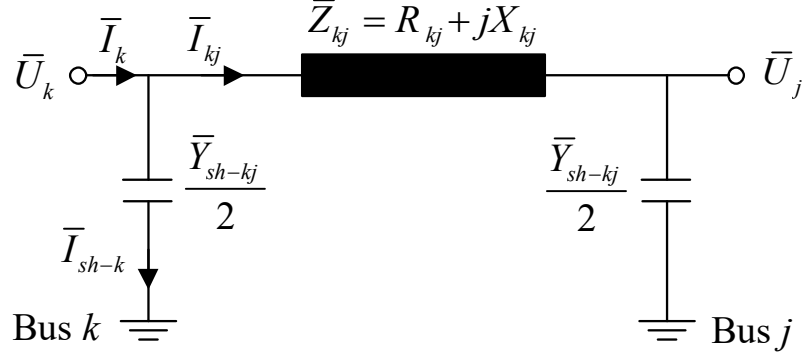
- k_{pz} and k_{qz} represent the active and reactive fraction of the constant impedance load;
- k_{pi} and k_{qi} represent the active and reactive fraction of the constant current load;
- k_{pp} and k_{qp} represent the active and reactive fraction of the constant power load.

It is important to remark that, in order to have a complete ZIP model, all the coefficients (k_{pz} , k_{qz} , k_{pi} , k_{qi} , k_{pp} and k_{qp}) have to be larger than zero in a way that $k_{pz} + k_{pi} + k_{pp} = k_{qz} + k_{qi} + k_{qp} = 1$ also holds.

2.1.3 Lines

The π -equivalent line model is a convenient model to use when it is not important to analyze in detail the performance of the transmission lines. This line model is typically used to represent medium long lines (with a length between 100 and 300 km). Compared to the model of short lines, this model takes into account the shunt capacitance of the lines.

The π -equivalent line model is depicted in Fig. 2.4, where $\bar{Z}_{kj}$ is the series impedance and $\bar{Y}_{sh-kj}$ is the total shunt admittance of the line.

Figure 2.4: π -equivalent line model

As it can be seen in Fig. 2.4, the shunt admittance ($\bar{Y}_{sh-kj}$) is divided into two equal parts that are placed at the two ends of the line that connects buses k and j .

In a case of having highly interconnected networks, it is common to use the admittance matrix ($\mathbf{Y}_{bus}$) to represent the network characteristics. For an N -bus system, the network admittance matrix $\mathbf{Y}_{bus}$ is a $N \times N$ matrix of the form:

$$\mathbf{Y}_{bus} = \begin{bmatrix} \bar{Y}_{11} & \bar{Y}_{12} & \dots & \bar{Y}_{1N} \\ \bar{Y}_{21} & \bar{Y}_{22} & \dots & \bar{Y}_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \bar{Y}_{N1} & \bar{Y}_{N2} & \dots & \bar{Y}_{NN} \end{bmatrix}. \quad (2.5)$$

$\mathbf{Y}_{bus}$ is derived from systematically applying Kirchhoff's current law to all the buses in a transmission network. In this manner, for $k = 1, \dots, N$ and $j = 1, \dots, N$; the elements of the $\mathbf{Y}_{bus}$ are given as:

- for $k = j$:

$$\bar{Y}_{kk} = G_{kk} + jB_{kk} = \sum_{\substack{j=1 \\ j \neq k}}^N \left(\bar{y}_{kj} + \frac{\bar{Y}_{sh-kj}}{2} \right); \quad (2.6)$$

- for $k \neq j$:

$$\bar{Y}_{kj} = G_{kj} + jB_{kj} = -\bar{y}_{kj}. \quad (2.7)$$

In Eq. 2.1.3 G_{kk} and B_{kk} represent the equivalent short-circuit conductance and susceptance of the k^{th} generator. In Eq. 2.1.3 G_{kj} and B_{kj} represent the transfer conductance and

susceptance between internal buses k and j . It has to be taken into account that $\bar{y}_{kj} = \frac{1}{\bar{Z}_{kj}}$. Therefore, the $(k, k)^{th}$ diagonal element of the $\mathbf{Y}_{\text{bus}}$ is equal to the sum of all the admittances that are connected to the bus k , whereas the $(k, j)^{th}$ element is equal to the line series admittance connected between buses k and j with the opposite sign.

2.2 Structure Preserving Model (SPM)

The general model that describes the dynamics of a power system is given by a set of Differential Algebraic Equations (DAE). The DAE model consists of n_x differential and n_y algebraic equations:

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{f}(\mathbf{x}, \mathbf{y}); \\ \mathbf{0} &= \mathbf{g}(\mathbf{x}, \mathbf{y}), \end{aligned} \quad (2.8)$$

where $\mathbf{x}$ is the vector of n_x state variables and $\mathbf{y}$ is the vector of n_y algebraic variables.

The power system model that is classically used for transient stability studies considers the loads as constant impedances that are then included in the admittance matrix. The main disadvantage of this Reduced Network Model (RNM) is that the network topology is lost [13]. Furthermore, from the load modeling point of view (which is of concern in this thesis) the RNM loses the behaviour of the power system components. Therefore, the Structure Preserving Model (SPM) [14] (where the loads are not included in the transmission system) is used here. In this way, the SPM can take into account the behavior of different types of static loads and, also, the power system model maintains its integrity. For the case of component modeling that has been considered in this thesis, the differential equations in SPM correspond to the dynamics of the generators. Hence, for this specific case, the differential equations from (2.8) are given by:

$$\begin{aligned} \dot{\delta}_k &= \omega_k; \\ \dot{\omega}_k &= \frac{1}{M_k}(P_{mk} - P_{ek} - D_k\omega_k), \quad \text{for } k = 1, \dots, n_g. \end{aligned} \quad (2.9)$$

Now, considering that $\mathbf{Y}_{\text{bus}}$ is the admittance matrix of the transmission network, and by making the physically reasonable assumption that the transmission network is lossless ($R_{kj} \ll X_{kj}$), the admittance matrix becomes $\mathbf{Y}_{\text{bus}} = j\mathbf{B}_{\text{bus}}$.

The expressions for the active and reactive powers injected in bus k depend on the type of the bus:

- for buses with generators connected to them, i.e. for $k = 1, \dots, n_g$:

$$\begin{aligned} P_k &= \sum_{j=1}^N B_{kj} U_k U_j \sin(\theta_k - \theta_j) + \frac{E'_{qk} U_k \sin(\theta_k - \delta_k)}{x'_{dk}}; \\ Q_k &= - \sum_{j=1}^N B_{kj} U_k U_j \cos(\theta_k - \theta_j) + \frac{U_k^2 - E'_{qk} U_k \cos(\theta_k - \delta_k)}{x'_{dk}}, \end{aligned} \quad (2.10)$$

- and for the other buses, i.e. for $k = (n_g + 1), \dots, N$:

$$\begin{aligned} P_k &= \sum_{j=1}^N B_{kj} U_k U_j \sin(\theta_k - \theta_j); \\ Q_k &= - \sum_{j=1}^N B_{kj} U_k U_j \cos(\theta_k - \theta_j). \end{aligned} \quad (2.11)$$

The algebraic equations that define the system reflect that the sum of all powers flowing into bus k must be equal to zero:

$$\begin{aligned} P_k + P_{L_k} &= 0; \\ Q_k + Q_{L_k} &= 0. \end{aligned} \quad (2.12)$$

In the balance equations of (2.12), P_{L_k} and Q_{L_k} are, respectively, the active and reactive powers of the load at bus k . These powers are represented by the exponential load model described in Section 2.1.2.

By defining the state variables $\mathbf{x}$ and the algebraic variables $\mathbf{y}$ as:

$$\begin{aligned} \mathbf{x} &= [\delta_1 \dots \delta_{n_g}, \omega_1 \dots \omega_{n_g}]^T; \\ \mathbf{y} &= [U_1 \dots U_N, \theta_1 \dots \theta_N]^T, \end{aligned} \quad (2.13)$$

the equations in (2.9) and (2.12) are given in the compact DAE form from (2.8).

This chapter has revised the set of DAE which describes the dynamical behavior of the multi-machine system from Fig. 2.1. The state variables $\mathbf{x}$ are connected with the algebraic variables $\mathbf{y}$ through Eq. (2.10). The SPM introduced allows to preserve the characteristics of the loads (included in the model through Eq. (2.12)).

The rest of chapters will also introduce some theoretical content, although their main aim is to study the voltage collapse from a static and a dynamic point of view.

Chapter 3

Static analysis

Voltage stability is a problem related to the maximum power that can be transferred. That is why the computation of the loadability limits is an important aspect of voltage stability studies. As demonstrated in [11], loadability limits coincide with bifurcation points, which are the points in a parameter-space at which system equilibria change their stability properties. This is also where a voltage collapse occurs from a static point of view.

This master thesis is interested in computing the loadability limits in order to ensure that the system is operating "far enough" from bifurcation points. In this way, it can be guaranteed that the voltage collapse phenomenon occurs due to the entrance of a post-fault trajectory in *VIR*, and not due to the proximity to a bifurcation point.

The number of methods to calculate the loadability limits of a system is large (for further information see Chapter 9 of [6]). In this thesis, an iterative procedure based on power flow calculations is used.

This chapter first presents the power flow problem in Section 3.1. Section 3.2 carries out a detailed review of the loadability concept. As a conclusion, Section 3.3 offers some results of the procedure that has been used to calculate the loadability limits. The test system used for all the case studies performed in this master thesis is introduced in Section 3.3, as well.

3.1 Power flow

The objective of a power flow calculation is to determine the steady state conditions of an electric power system. By solving the power flow, voltage magnitudes and voltage angles at all system buses are found for a certain set of system parameters.

3.1.1 Problem formulation

Let's observe a balanced power system with N buses as the one shown in Fig. 2.1. Consider that the lines are represented with the π -equivalent model described in Section 2.1.3.

The current injected in bus k can be calculated as the sum of all the currents leaving the bus either towards the line shunts or towards another bus:

$$\bar{I}_k = \sum_{j=1}^N \bar{Y}_{kj} \bar{U}_j, \quad (3.1)$$

where $\sum_{j=1}^N \bar{Y}_{kj}$ represents the sum of all the elements that belong to the k^{th} row of the admittance matrix $\mathbf{Y}_{bus}$.

Eq. (3.1) can be rewritten in terms of the complex power as:

$$\bar{S}_k = P_k + jQ_k = \bar{U}_k \bar{I}_k^* = \bar{U}_k \sum_{j=1}^N (G_{kj} - jB_{kj}) \bar{U}_j^*. \quad (3.2)$$

Taking into account that $\bar{U}_k = U_k(\cos\theta_k + j\sin\theta_k)$ and that $\theta_{kj} = \theta_k - \theta_j$, the active and reactive powers injected into bus k can be expressed in scalar terms as:

$$\begin{aligned} P_k &= U_k \sum_{j=1}^N U_j (G_{kj} \cos\theta_{kj} + B_{kj} \sin\theta_{kj}); \\ Q_k &= U_k \sum_{j=1}^N U_j (G_{kj} \sin\theta_{kj} - B_{kj} \cos\theta_{kj}). \end{aligned} \quad (3.3)$$

The power balance at each bus k can then be expressed as:

$$\begin{aligned} P_k - P_{net_k} &= 0; \\ Q_k - Q_{net_k} &= 0, \end{aligned} \quad (3.4)$$

where $P_{net_k} = \sum_{j=1}^N P_{kj} = P_{G_k} - P_{L_k}$ and $Q_{net_k} = \sum_{j=1}^N Q_{kj} = Q_{G_k} - Q_{L_k}$ stand for the net active and reactive powers that leave bus k towards other buses. In Eq. (3.4) P_{G_k} is

the active generation dispatch, Q_{G_k} is the reactive generated power, and P_{L_k} and Q_{L_k} the active and reactive load demands, all at bus k .

Each bus allows to formulate two equations but has four different variables (U_k , θ_k , P_{net_k} and Q_{net_k}). This means that the system of equations is undetermined. Therefore, it is necessary to fix two of the variables and to find the other two. According to the variables that are chosen to be fixed, the classification of the system buses can be made as:

- **PQ buses:** the net generated powers P_{net_k} and Q_{net_k} are assumed to be known. Thus, for this buses, P_{G_k} , Q_{G_k} , P_{L_k} and Q_{L_k} have some pre-set, known values. The unknown variables are the voltage magnitude (U_k) and angle (θ_k). Commonly, PQ buses are the buses with pure consumptions.
- **PU buses:** the net generated power P_{net_k} (i.e. P_{G_k} and P_{L_k}) and the voltage magnitude U_k are assumed to be known. The unknown variables are the reactive power (Q_{net_k}) and the voltage angle (θ_k). PU buses need to have some sort of voltage regulating device connected.
- **Slack buses:** the voltage magnitude U_k and angle θ_k are fixed, and the active and reactive powers injected into this bus are unknown variables. There is only one slack bus in every system and it compensates the losses in the system lines.

The N buses of the system can be classified as follows:

- n_{PQ} : number of PQ buses;
- n_{PU} : number of PU buses;
- $n_S = 1$ slack bus,

such that $N = n_{PQ} + n_{PU} + 1$.

It is possible to write the following non-linear equations:

- $n_{PQ} + n_{PU}$ active power balance equations;
- n_{PQ} reactive power balance equations,

and find the values of the variables:

- $n_{PQ} + n_{PU}$ unknown voltage angles θ_k ,
- n_{PQ} unknown voltage magnitudes U_k .

Table 3.1 summarizes the set of system equations that is used in the power flow calculations.

Bus type	Number	Balance equations		Unknown variables	
		$P_{net_k} = \sum_{j=1}^N P_{kj}$	$Q_{net_k} = \sum_{j=1}^N Q_{kj}$	U	θ
PU bus	n_{PU}	n_{PU}	0	0	n_{PU}
PQ bus	n_{PQ}	n_{PQ}	n_{PQ}	n_{PQ}	n_{PQ}
Slack bus	1	0	0	0	0
Total	N	$2n_{PQ} + n_{PU}$		$2n_{PQ} + n_{PU}$	

Table 3.1: Summary of the equations and unknown quantities in the load flow calculations

As the number of equations match the number of unknown variables, the system is solvable. Due to the non-linearity of the load flow equations, it is necessary to use numerical methods to solve the system. The Newton-Raphson method [15] is the one commonly used for this purpose. Once voltage magnitudes and angles at all the buses are found, the other quantities that are missing (such as the slack's active and reactive power generations, the injected reactive power at PU buses or the losses on the lines) can be easily computed.

3.1.2 Numerical method: Newton-Raphson

A general system of n_z non-linear equations and n_z independent variables $z_1, z_2, \dots, z_{n_z}$ is considered:

$$\begin{aligned}
 f_1(z_1, \dots, z_{n_z}) &= \eta_1; \\
 f_2(z_1, \dots, z_{n_z}) &= \eta_2; \\
 &\vdots \\
 f_{n_z}(z_1, \dots, z_{n_z}) &= \eta_{n_z},
 \end{aligned} \tag{3.5}$$

where $f_1, \dots, f_{n_z}$ are function of the variables $z_1, z_2, \dots, z_{n_z}$.

The implicit functions $g_1, \dots, g_{n_z}$ are defined as given below:

$$\begin{aligned}
 g_1(z_1, \dots, z_{n_z}) &= f_1(z_1, \dots, z_{n_z}) - \eta_1 = 0; \\
 g_2(z_1, \dots, z_{n_z}) &= f_2(z_1, \dots, z_{n_z}) - \eta_2 = 0; \\
 &\vdots \\
 g_{n_z}(z_1, \dots, z_{n_z}) &= f_{n_z}(z_1, \dots, z_{n_z}) - \eta_{n_z} = 0.
 \end{aligned} \tag{3.6}$$

The system of Eqs. from (3.6) can be written in the compact form as:

$$\mathbf{g}(\mathbf{z}) = \mathbf{f}(\mathbf{z}) - \boldsymbol{\eta} = \mathbf{0}. \quad (3.7)$$

The basis of the Newton-Raphson method is the expansion of (3.7) into the Taylor's series [16]. That expansion is then iteratively evaluated starting from an initial estimation $\mathbf{z}^{(0)}$ that evolves into a series of better guesses $\mathbf{z}^{(1)}, \mathbf{z}^{(2)}, \dots, \mathbf{z}^{(n_z)}$ until the system converges.

Considering that $\mathbf{z}^*$ is the solution of (3.7) such that $\mathbf{g}(\mathbf{z}^*) = \mathbf{0}$, and by taking into account that $\mathbf{z}^{(i)}$ is the estimation of $\mathbf{z}^*$, the estimation error can be defined as $\Delta \mathbf{z}^i = \mathbf{z}^* - \mathbf{z}^i$. In this way, Eq. (3.7) is redefined as:

$$\mathbf{g}(\mathbf{z}^*) = \mathbf{g}(\mathbf{z}^{(i)} + \Delta \mathbf{z}^{(i)}) = \mathbf{0}, \quad (3.8)$$

for which the application of the Taylor expansion gives:

$$\mathbf{g}(\mathbf{z}^*) = \mathbf{g}(\mathbf{z}^{(i)}) + \mathbf{JAC}^{(\mathbf{z}^{(i)})} \Delta \mathbf{z}^{(i)} = \mathbf{0}. \quad (3.9)$$

In Eq. (3.9), $\mathbf{JAC}$ is the Jacobian of the function $\mathbf{g}(\mathbf{z})$:

$$\mathbf{JAC}^{(\mathbf{z}^{(i)})} = \left[\frac{\partial \mathbf{g}(\mathbf{z})}{\partial \mathbf{z}} \right]_{\mathbf{z}=\mathbf{z}^{(i)}} = \begin{bmatrix} \frac{\partial g_1(\mathbf{z})}{\partial z_1} & \cdots & \frac{\partial g_1(\mathbf{z})}{\partial z_{n_z}} \\ \vdots & \ddots & \vdots \\ \frac{\partial g_{n_z}(\mathbf{z})}{\partial z_1} & \cdots & \frac{\partial g_{n_z}(\mathbf{z})}{\partial z_{n_z}} \end{bmatrix}_{\mathbf{z}=\mathbf{z}^{(i)}}. \quad (3.10)$$

Since from Eq. (3.9) it follows that $\Delta \mathbf{g}(\mathbf{z}^{(i)}) = \mathbf{0} - \mathbf{g}(\mathbf{z}^{(i)})$, the estimation error $\Delta \mathbf{z}^{(i)}$ can be cleared from Eq. (3.9) as:

$$\Delta \mathbf{z}^{(i)} = \mathbf{z}^* - \mathbf{z}^{(i)} = \left[\mathbf{JAC}^{(\mathbf{z}^{(i)})} \right]^{-1} \Delta \mathbf{g}(\mathbf{z}^{(i)}). \quad (3.11)$$

Furthermore, since $\mathbf{g}(\mathbf{z}^{(i)}) = \mathbf{f}(\mathbf{z}^{(i)}) - \boldsymbol{\eta}$, then $\Delta \mathbf{g}(\mathbf{z}^{(i)})$ is given by:

$$\Delta \mathbf{g}(\mathbf{z}^{(i)}) = \boldsymbol{\eta} - \mathbf{f}(\mathbf{z}^{(i)}) = -\mathbf{g}(\mathbf{z}^{(i)}). \quad (3.12)$$

As $\boldsymbol{\eta}$ is a constant, the Jacobian of $\mathbf{g}$ is the same as the Jacobian of $\mathbf{f}$. Hence:

$$\Delta \mathbf{z}^{(i)} = \begin{bmatrix} \Delta z_1^{(i)} \\ \vdots \\ \Delta z_{n_z}^{(i)} \end{bmatrix} = \begin{bmatrix} \frac{\partial f_1(\mathbf{z})}{\partial z_1} & \cdots & \frac{\partial f_1(\mathbf{z})}{\partial z_{n_z}} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_{n_z}(\mathbf{z})}{\partial z_1} & \cdots & \frac{\partial f_{n_z}(\mathbf{z})}{\partial z_{n_z}} \end{bmatrix}_{\mathbf{z}=\mathbf{z}^{(i)}}^{-1} \begin{bmatrix} \eta_1 - f_1(z_1^{(i)}, \dots, z_{n_z}^{(i)}) \\ \vdots \\ \eta_{n_z} - f_{n_z}(z_1^{(i)}, \dots, z_{n_z}^{(i)}) \end{bmatrix}. \quad (3.13)$$

Figure 3.1 summarizes the Newton-Raphson method. The underlying idea in this numerical method is that every iteration $i = i + 1$ will produce a solution $\mathbf{z}^*$ that is more accurate than the solution provided by the previous iteration i .

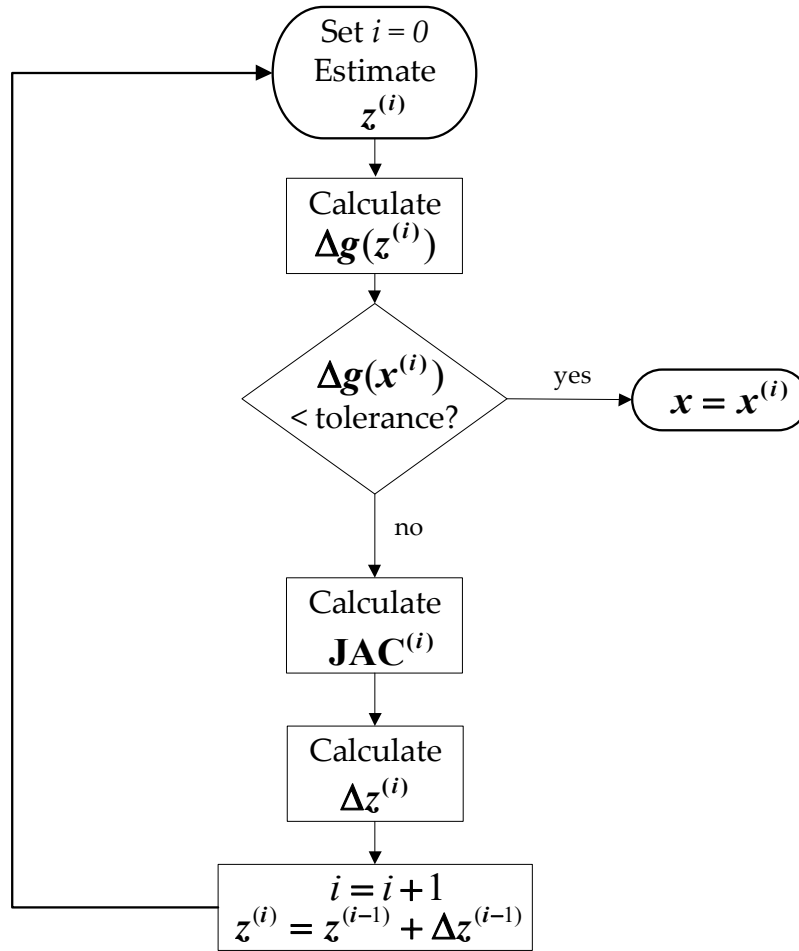


Figure 3.1: Iterative procedure used in the Newton-Raphson method

3.1.3 Problem resolution: Newton-Raphson applied to power systems

When applying the Newton-Raphson method to power flow calculations, Eqs. (3.4) have to be expressed in the terms of Eq. (3.7). In this way, the following vector functions are defined:

$$z = \begin{bmatrix} \theta \\ U \end{bmatrix} = \begin{bmatrix} \theta_1 \\ \vdots \\ \theta_N \\ U_1 \\ \vdots \\ U_N \end{bmatrix}; f(\theta, U) = \begin{bmatrix} f_P(\theta, U) \\ f_Q(\theta, U) \end{bmatrix} = \begin{bmatrix} P_1 \\ \vdots \\ P_N \\ Q_1 \\ \vdots \\ Q_N \end{bmatrix}; \eta = \begin{bmatrix} \eta_P \\ \eta_Q \end{bmatrix} = \begin{bmatrix} P_{net_1} \\ \vdots \\ P_{net_N} \\ Q_{net_1} \\ \vdots \\ Q_{net_N} \end{bmatrix}, \quad (3.14)$$

where θ is $(N - 1) \times 1$ vector and U is a $(N - n_{PU} - 1) \times 1$ vector.

By using the expression in (3.12), the power mismatches at system buses are:

$$\begin{aligned} \Delta P_k &= P_{net_k} - P_k, & \text{for } k \neq \text{slack bus}; \\ \Delta Q_k &= Q_{net_k} - Q_k, & \text{for } k \neq \text{slack bus and PU bus}, \end{aligned} \quad (3.15)$$

where $\Delta P = \eta_P - f_P(\theta, U)$ and $\Delta Q = \eta_Q - f_Q(\theta, U)$ represent the vectors of mismatches between the specified power value and the one obtained with (θ, U) in each iteration.

The Jacobian matrix of the system of equations $f(\theta, U)$ is given by:

$$\mathbf{JAC} = \begin{bmatrix} \frac{\partial f_P(\theta, U)}{\partial \theta} & \frac{\partial f_P(\theta, U)}{\partial U} \\ \frac{\partial f_Q(\theta, U)}{\partial \theta} & \frac{\partial f_Q(\theta, U)}{\partial U} \end{bmatrix} = \begin{bmatrix} \mathbf{H} & \mathbf{N}' \\ \mathbf{J} & \mathbf{L}' \end{bmatrix}, \quad (3.16)$$

where,

$$\begin{aligned} \mathbf{H} &\text{ is an } (n_{PQ} + n_{PU}) \times (n_{PQ} + n_{PU}) \text{ matrix;} \\ \mathbf{N}' &\text{ is an } (n_{PQ} + n_{PU}) \times (n_{PQ}) \text{ matrix;} \\ \mathbf{J} &\text{ is an } (n_{PQ}) \times (n_{PQ} + n_{PU}) \text{ matrix;} \\ \mathbf{L}' &\text{ is an } (n_{PQ}) \times (n_{PQ}) \text{ matrix.} \end{aligned}$$

Based on Eqs. (3.12)-(3.16) it holds that:

$$\begin{bmatrix} \Delta \theta P \\ \Delta U \end{bmatrix} = \begin{bmatrix} \mathbf{H} & \mathbf{N}' \\ \mathbf{J} & \mathbf{L}' \end{bmatrix}^{-1} \begin{bmatrix} \Delta \\ \Delta Q \end{bmatrix}. \quad (3.17)$$

To simplify the entries of $\mathbf{N}'$ and $\mathbf{L}'$ these two matrices are multiplied by $\mathbf{U}$. Eq. (3.17) is then rewritten as:

$$\begin{bmatrix} \Delta\theta \\ \frac{\Delta U}{U} \end{bmatrix} = \begin{bmatrix} \Delta\theta \\ \Delta U' \end{bmatrix} = \begin{bmatrix} \mathbf{H} & \mathbf{N} \\ \mathbf{J} & \mathbf{L} \end{bmatrix}^{-1} \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix}. \quad (3.18)$$

Finally, the entries of the modified Jacobian matrix from Eq. (3.18) are:

- for $k = j$

$$\begin{aligned} H_{kk} &= \frac{\partial P_k}{\partial \theta_k} = -Q_k - B_{kk}U_k^2; \\ N_{kk} &= U_k \frac{\partial P_k}{\partial U_k} = P_k + G_{kk}U_k^2; \\ J_{kk} &= \frac{\partial Q_k}{\partial \theta_k} = P_k - G_{kk}U_k^2; \\ L_{kk} &= U_k \frac{\partial Q_k}{\partial U_k} = Q_k - B_{kk}U_k^2, \end{aligned} \quad (3.19)$$

- for $k \neq j$

$$\begin{aligned} H_{kj} &= \frac{\partial P_k}{\partial \theta_j} = U_k U_j [G_{kj} \sin(\theta_{kj}) - B_{kj} \cos(\theta_{kj})]; \\ N_{kj} &= U_j \frac{\partial P_k}{\partial U_j} = U_k U_j [G_{kj} \cos(\theta_{kj}) + B_{kj} \sin(\theta_{kj})]; \\ J_{kj} &= \frac{\partial Q_k}{\partial \theta_j} = -U_k U_j [G_{kj} \cos(\theta_{kj}) + B_{kj} \sin(\theta_{kj})]; \\ L_{kj} &= U_j \frac{\partial Q_k}{\partial U_j} = U_k U_j [G_{kj} \sin(\theta_{kj}) - B_{kj} \cos(\theta_{kj})]. \end{aligned} \quad (3.20)$$

At the end of every iteration, the elements of $\mathbf{U}$ and $\boldsymbol{\theta}$ are updated:

$$\begin{aligned} \theta_k &= \theta_k + \Delta\theta_k && \text{for } k \neq \text{slack bus;} \\ U_k &= U_k (1 + \Delta U') && \text{for } k \neq \text{slack bus and PU bus.} \end{aligned} \quad (3.21)$$

3.2 Loadability

Imagine that a power system is described at an equilibrium by a set of n_u algebraic equations and n_u algebraic variables. If the vector $\mathbf{u}$ represents the n_u variables, the following system of equations can be defined:

$$\varphi(\mathbf{u}, \mathbf{p}) = 0, \quad (3.22)$$

where $\mathbf{p}$ is the vector of parameters.

Loadability limits are detected when a power system is subjected to smooth changes in the parameters $\mathbf{p}$ and the equilibrium point suddenly disappears. In a power system there are many different ways of reaching a loadability limit according to the change of the $\mathbf{p}$ parameters along a direction $\mathbf{d}$. By considering that μ denotes the size of the increase from an initial value of the parameters ($\mathbf{p}^o$), the change of $\mathbf{p}$ can be expressed as:

$$\mathbf{p} = \mathbf{p}^o + \mu \mathbf{d}. \quad (3.23)$$

The result that can be obtained from (3.22) depends on the set of equations employed. In this master thesis (in line with the definition provided in [6]) the loadability limit is defined as the operating point at which loads reach maximum values. A further increase in loads after these maximum values, does not yield a solution for (3.22). Hence, Eq. (3.22) represents the set of non-linear load flow equations of a power system, vector $\mathbf{u}$ contains the voltage magnitudes and angles of the system buses ($\mathbf{u} = \mathbf{z}$ in accordance to the naming used in Section 3.1.3), and the entries of $\mathbf{p}$ are the load demands.

Here, in order to calculate the loadability limits, the load demands are iteratively increased and the load flow equations are solved by using the Newton-Raphson numerical method. The steps followed are:

1. For a certain operating point, the set of non-linear power flow equations is solved. If the loadability limit has not been reached yet, the Newton-Raphson method will be able to converge, i.e. it will be able to find a solution of (3.22) in less than a pre-set number of iterations.
2. The loads are increased. This increase in the loads is evenly split between the generators in order to recreate a realistic situation in the operation of a power system.

These steps are repeated until the numerical method is not able to find a solution of the load flow equations. This last "solvable" operating point is the loadability limit. As the loadability limit coincides with the tip of the PU "nose curve" [11], this procedure allows to draw the upper part of the nose curve (i.e. the upper voltage solution).

Another way to find the loadability limit of a power system, is the approach that comes from small signal stability studies. In this case, a bifurcation analysis is carried out by examining the stability of the linearized DAE system. The type of bifurcation that is related to voltage collapse is the Saddle-Node Bifurcations (SNB) [17]. Once a SNB is encountered, the loadability limits can be easily identified because the necessary conditions for a SNB coincide with the necessary conditions for reaching a loadability limit [11].

3.3 Results

3.3.1 The case study

This thesis considers a simple 3-machine system that is based on IEEE-9-bus system. This 3-machine 3-load test system is depicted in Fig. 3.2.

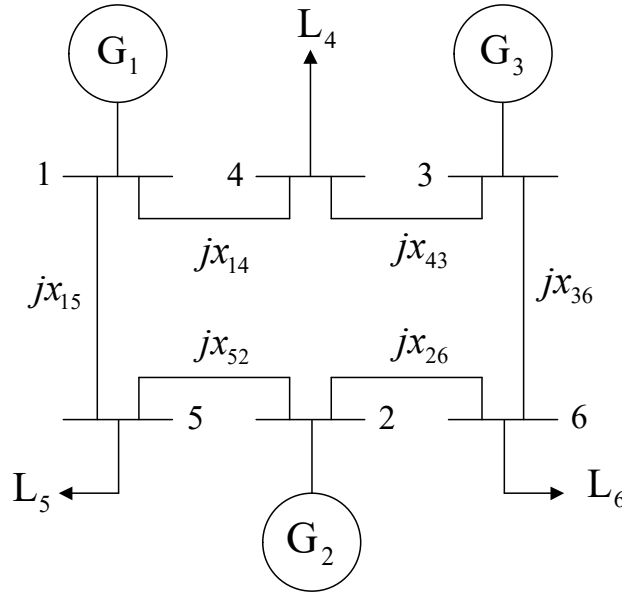


Figure 3.2: 3-machine 3-load test system

Regarding the modeling of the test system, the classical model has been used for generators. The loads are represented by exponential load models. In Fig. 3.2 the transformers are not shown, but their reactance (x_T) is included in x'_d . The transmission network lines, represented by the π -line model, are assumed to be lossless. For load flow calculation purposes, the terminal bus of the generator 1 has been chosen as the slack bus. The initial operating point of this system is presented in Table 3.2.

Bus	P_G (p.u.)	P_L (p.u.)	Q_L (p.u.)
1	0.67	0	0
2	1.63	0	0
3	0.85	0	0
4	0	0.9	0.3
5	0	1.25	0.5
6	0	1	0.35

Table 3.2: Initial operating point of the case study

3.3.2 Loadability limits

As explained in Section 3.2, the loadability limits are computed by iteratively increasing the loads. More concisely, only the active power of the three loads from the test system is varied. In this way, the loads remain inductive throughout the analysis. A change in the loads means that the dispatch of the generators has to change as well. As mentioned before, the load increment is split equally between the three generators, such that 1/3 of the total additional power required by the loads is added to each generator in every increase step.

In the test system, there are three loads that can be increased either individually or in a combined manner to get to the loadability limit. In line with the formulation from (3.23), the load increase can be expressed as:

$$\begin{bmatrix} P_{L_4} \\ P_{L_5} \\ P_{L_6} \end{bmatrix} = \begin{bmatrix} 0.9 \\ 1.25 \\ 1 \end{bmatrix} + \mu \mathbf{d} \quad (\text{p.u.}).$$

The vector $\mathbf{d}$ denotes the direction that is followed to reach the loadability limit. In other words, $\mathbf{d}$ reflects which of the load or loads are increased. For instance, if only the load at bus 4 is increased then $\mathbf{d} = [1 \ 0 \ 0]^T$. For the sake of simplicity, μ is considered to be constant in each increase step and it also has the same value for all the loads that are incremented.

The results of the loadability studies carried out for the test system are included in Table 3.3. This table shows the loadability limit in the terms of the total load increase compared to the initial operating point. The loadability limit is the last operating point for which the load flow calculations converge. At this point, the tip of the "nose curve" has been reached and in a real power system any further increase would trigger a voltage collapse. Note that here the total load increase is equal to the sum of the individual increments of all the loads.

Case	Direction of increase (d)			Total load increase (p.u.)	Ind. load increase (p.u.)		
	Load 4	Load 5	Load 6		ΔP_{L_4}	ΔP_{L_5}	ΔP_{L_6}
A	1	0	0	7.4	7.4	0	0
B	0	1	0	7.4	0	7.4	0
C	0	0	1	11	0	0	11
D	1	1	0	11	5.5	5.5	0
E	1	0	1	15.6	7.8	0	7.8
F	0	1	1	15.8	0	7.9	7.9
G	1	1	1	20.4	6.8	6.8	6.8

Table 3.3: Loadability results for various load increase directions

From the results of Table 3.3, several conclusions can be drawn:

- When increasing loads individually, the maximum loadability limit is reached when the load in bus 6 is the one that varies.
- The loadability is encountered "faster" (at lower loading levels) if fewer loads are incremented.
- The largest loadability limit (equal to the total added power) is obtained when all the loads are increased.

The iterative procedure used for calculating the loadability limits allows to compute the upper part of the PU "nose curve". For instance, Fig. 3.3 shows how the voltages of buses 4, 5 and 6 (U_4 , U_5 and U_6) change with the load power (P_L) when the three loads increase in each iteration. That is, the PU curve represented in Fig. 3.3 corresponds to case G from Table 3.3 with $d = [1 \ 1 \ 1]^T$. P_L denotes the total demanded power (P_L is the sum of the active powers of the loads at buses 4, 5 and 6).

As it can be seen in Fig. 3.3, the voltages of the load buses decrease slowly when the loads increase. In normal operating conditions, the operating point would lie on this upper part of the curve. When the loadability limit is reached, the numerical procedure (that is used for solving the power flow equations) does not converge. In small-signal stability analysis, reaching a loadability limit point implies getting to a bifurcation point, which further implies that a voltage collapse is likely to occur.

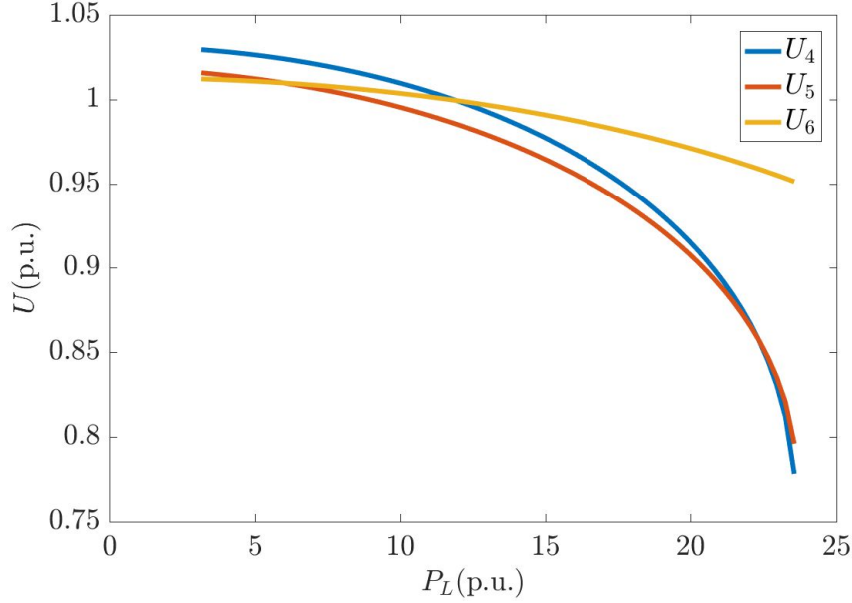


Figure 3.3: Upper parts of PU nose curves (case G)

There are other paths to reach a loadability limit aside from the main directions mentioned in Table 3.3. The plots in Fig. 3.4 depict the bus voltages U_4 , U_5 and U_6 as functions of the increase of the active powers of the loads at buses 4 and 5.

The resulting graphs are some kind of "PU surfaces" which are constructed with the multi-directional increase of certain loads. In the surfaces from Fig. 3.4, the three main directions have also been plotted out. These directions correspond to some of the cases from Table 3.3.

This chapter has presented the static aspect of voltage collapse. In static analysis, a voltage collapse is triggered by a loss of a load flow solution under parameter variations. Since the load flow is an approximation of the algebraic set of equations (in DAE model), it was possible to detect loadability limit as the point for which the numerical procedure employed to solve the power flow did not converge. The following chapters will examine the concept of losing a voltage solution from the dynamic point of view.

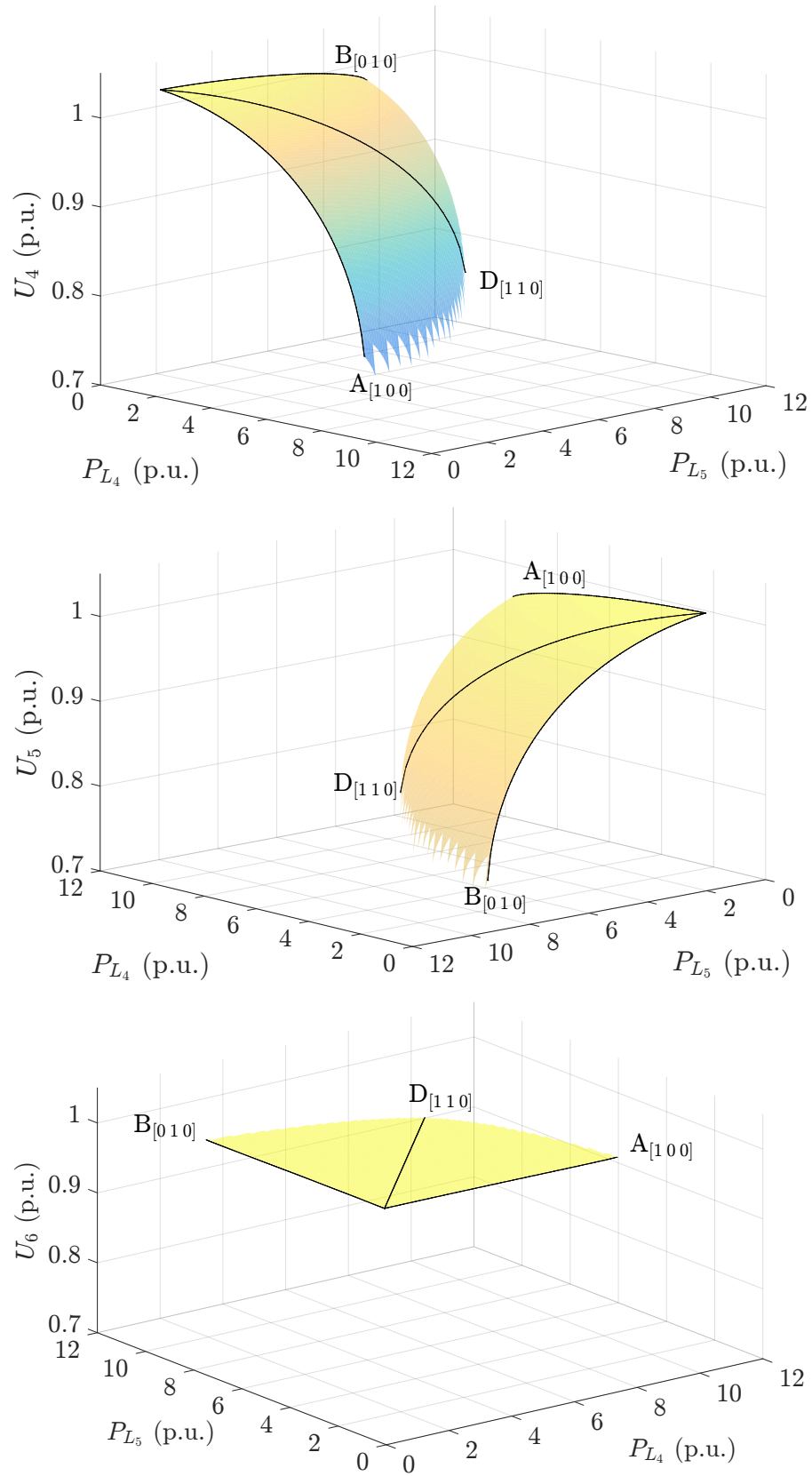


Figure 3.4: Upper parts of "PU surfaces" under increase of the loads at buses 4 and 5

Chapter 4

Quasi-dynamic approach

Among the objectives of this master thesis, one of them is to carry out a quasi-dynamic study with the purpose of finding *VIR* for different combinations of static, exponential load models. In order to confirm that the proposed quasi-dynamic approach fits that purpose, the analytical expressions of *VIR* that were derived in [5] will also be employed to find *VIR* for the test system considered in this master thesis. If the definition of *VIR* from the analytical expressions of [5] matches *VIR* that is found by the proposed quasi-dynamic approach, the newly introduced methodology can be verified.

This chapter is divided into three different sections. First, in Section 4.1, the analytical expressions of *VIR* from [5] are revised. Section 4.2 explains the quasi-dynamic methodology that is proposed in this thesis. Finally, Section 4.3 compares the two methodologies for finding *VIR* by using the simple 3-machine 3-load test system.

4.1 Analytical expressions of *VIR*

The aim of the study carried out in [5] was to describe how the load modeling affects the singularity regions (*VIR*) that appear in the rotor angle space after a disturbance.

The general model that describes the dynamics of a power system is given by the set of DAE from Eq. (2.8). When a dynamical system is expressed in terms of the state variables $\mathbf{x}$ and the algebraic variables $\mathbf{y}$, it is said that the system is written in its implicit form. If the algebraic variables can be eliminated from Eqs. (2.8), then the dynamical system is described in an explicit way by a set of differential equations which are functions of the state variables [18]. This set of differential equations then conforms a system of Ordinary Differential Equations (ODE):

$$\dot{\mathbf{x}} = \mathbf{F}(\mathbf{x}). \quad (4.1)$$

The solution of the set of non-linear differential equations can be obtained by Time Domain Simulation (TDS). TDS techniques consist of the numerical integration of the differential equations that describe the dynamical behavior of the system. However, according to the Implicit Function Theorem [18], the conversion of a DAE into its equivalent ODE system can only be made if the algebraic Jacobian ($\partial \mathbf{g} / \partial \mathbf{y}$) is non-singular. In [5], the singularity of the algebraic Jacobian in the neighborhood of a post-fault operating point is the starting point for identifying *VIR*.

Furthermore, [5] focused on defining *VIR* in a power system when all the loads except one are modeled as Constant Impedance (CI) loads. By using the closed form expressions for the voltage magnitudes, [5] analytically derived the conditions for *VIR* appearance in the relative rotor angle space. The derivation of the analytical expressions for *VIR* was possible due to the fact that in an n_g -machine N -bus system only one of the loads (the load in bus L) was not a CI load. In this way, the voltage of any k^{th} bus ($k = 1, 2, \dots, N$) can be written as a function of the non-linear load voltage at bus L as:

$$\bar{U}_k = -\bar{Z}_{kL} \left(\frac{\bar{S}_{LL}}{\bar{U}_L} \right)^* + \sum_{i=1}^{n_g} \bar{Z}_{ki} \frac{\bar{E}'_{qk}}{j x'_{dk}}. \quad (4.2)$$

All the parameters from Eq. (4.2) except the impedances have been introduced in the previous sections. In Eq. (4.2), $\bar{Z}_{kL}$ and $\bar{Z}_{ki}$ represent the elements of a modified impedance matrix that has been created for the purpose of calculating $\bar{U}_k$.

According to the previous formulation, *VIR* is defined as the union of all the combinations of relative rotor angles for which the voltage magnitudes in (4.2) do not have a real, positive solution. The work from [5], defined some Voltage Collapse Indicators (*VCI*s) which allow to measure the distance between a state-space location and *VIR*. With the rotor angle of the q^{th} machine taken as the reference, *VIR* has different expressions depending on the load model that is used at bus L :

- When the load at bus L is modeled as a Constant Power (CP) load:

$$VIR = \{(\delta_{1q}, \delta_{2q}, \dots, \delta_{n_gq}) : VCI_{CP} < 0\}. \quad (4.3)$$

- When the load at bus L is modeled as a Constant Current (CC) load:

$$VIR = \left\{ \begin{array}{ll} (\delta_{1q}, \delta_{2q}, \dots, \delta_{n_gq}) : VCI_{CC1} \leq 0 & \text{for } b_R \leq 0 \\ (\delta_{1q}, \delta_{2q}, \dots, \delta_{n_gq}) : VCI_{CC2} < 0 & \text{for } b_R > 0 \end{array} \right\}, \quad (4.4)$$

where $b_R \leq 0$ refers to an inductive load situation, and $b_R > 0$ corresponds to a capacitive load.

- When the load at bus L is also modeled as a Constant Impedance (CI) load:

$$VIR = \emptyset. \quad (4.5)$$

In this case, it is possible to use Reduced Network Models (RNM) [19], which avoid TDS problems. In [5], the voltage solution of load L is formulated also for this load case, just for verification purposes. As expected, it was demonstrated that for the CI load model, there is no loss of voltage causality, i.e. there can be no VIR .

However, whenever more than one static, non-linear load is employed, all the voltages from Eq. (4.2) become dependent on the voltages of the buses where non-linear loads are connected. In order to solve such a set of multi-dependent equations and identify VIR , it would be necessary to use numerical methods. Furthermore, for different power system configurations, the analytical expression of VIR would change. All in all, when more loads are of non-constant impedance type, it is not possible to find general expressions for VIR in the form of Eqs. (4.3) and (4.4). For this reason, a quasi-dynamic approach to find VIR is proposed and tested in this master thesis.

4.2 Quasi-dynamic approach

The dynamics of the system that will be considered in this master thesis follow the DAE from Eq. (2.8). For the system from Eq. (2.8), the state variables $\mathbf{x}$ and the algebraic variables $\mathbf{y}$ are defined as:

$$\begin{aligned} \mathbf{x} &= [\delta_1 \dots \delta_{n_g}, \omega_1 \dots \omega_{n_g}]^T; \\ \mathbf{y} &= [U_1 \dots U_N, \theta_1 \dots \theta_N]^T. \end{aligned} \quad (4.6)$$

The dynamics of the generators will be described by the differential equations in (2.9), whereas the state variables and the algebraic variables are connected by the algebraic equations from (2.12).

The methodology proposed in this master thesis follows a quasi-dynamic approach. In a TDS, the state variables vary according to the dynamics of the system. In the quasi-dynamic approach of this master thesis, the state variables are changed artificially and then fed into the algebraic equations. As the rotor angles of the generators are the only state variables that appear in the algebraic equations (see Eqs. in (2.10)), by choosing machine q as the reference, the elements of δ_q (which is the vector of relative rotor angles, i.e. $\delta_q = [\delta_{1q}, \delta_{2q}, \dots, \delta_{n_g q}]^T$) need to be changed. The algebraic equations are then solved for the set of state-space locations that are determined by δ_q . As those algebraic equations are non-linear, a numerical method is used to solve them. A solvable location is associated with value 1, and when the set of algebraic equations could not be solved the location is awarded value 0. This process is repeated iteratively until the neighborhood of a post-fault operating point (x_s^{post}) is examined. In this way, the proposed quasi-dynamic approach transforms the state-space into a matrix that contains ones (1) and zeros (0) with respect to the tested state-space locations.

The flow chart in Fig. 4.1 shows the steps that are followed to find *VIR* in one quadrant of a state-space area. MATLAB (R2018b) is the computational environment that is used in this master thesis and `fsolve` has been chosen among the functions available in MATLAB to solve the algebraic equations. The similar procedure can be followed to map the relative rotor angle space for other interesting quadrants (in an n_g -machine power system, *VIR* could be mapped in the $n_g - 1$ state-space). The explanation of the numerical procedure follows in the sequel.

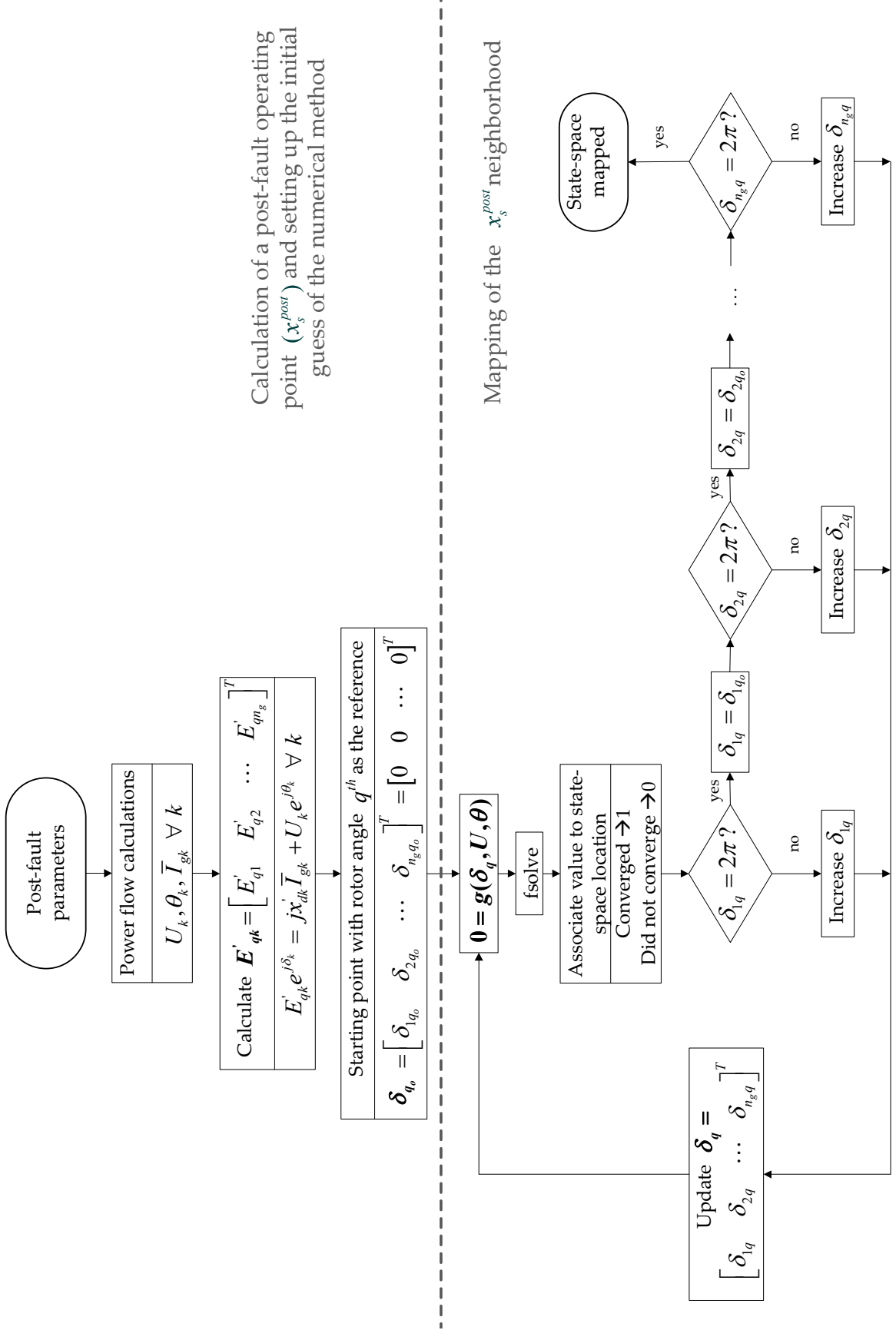


Figure 4.1: Flow chart of the proposed methodology for mapping one quadrant of the state-space

4.2.1 fsolve

The algebraic equations that need to be solved to determine VIR are non-linear. Hence, a numerical method has to be used. Among the numerical methods this master thesis chose to employ a MATLAB routine: `fsolve` [20].

The routine `fsolve` tries to solve systems of non-linear equations of several variables which are written in an implicit form.

Let's consider a set of $f = 1, 2, \dots, n_f$ non-linear, continuous functions F_{nl} . Given F_{nl} `fsolve` aims to find a vector x_f that makes all $F_{nl}(x_f) = 0$. Those solutions are called roots or zeros of F_{nl} . To start iterating, the `fsolve` routine requires an initial guess that should be near the root. It is of importance to remark that the selection of the initial guess when using `fsolve` can lead to different solutions. For a certain initial guess, `fsolve` tries to solve the system of equations by minimizing the sum of squares of the components of x_f . If the sum of squares is zero, the system of equations is considered to be solved. In the case of being successful, `fsolve` only provides one root.

The function `fsolve` has three different algorithms:

- **Trust-region:** This method is based on the idea of getting closer, at each iteration, to an objective function within a certain region at each iteration [21].
- **Trust-region dogleg:** This is the default algorithm used by `fsolve`. The algorithm is a variant of the Powell dogleg method described in [22]. It is specifically designed to solve non-linear equations. However, the limitation of the Trust-region dogleg is that it can only be used when the system of equations is square (when the number of equations equals the number of unknowns).
- **Levenberg-Marquardt:** This approach was mainly developed to solve the non-linear least square problem that rises when fitting a parameterized function to a group of measured data. When using the Levenberg-Marquardt algorithm, the system of equations does not need to be square [23].

Apart from the solution of the system of equations (x_f), `fsolve` provides the following output arguments:

- `fval`: which is the real vector containing the values of the objective function at the evaluated solution.
- `exitflag`: which is the integer number that informs of why `fsolve` stopped iterating. The value of `exitflag` can range between -3 and 1. Positive values of `exitflag` mean that `fsolve` succeeded in finding a solution, whether zero or a negative value means it did not.

- `output`: is a structure that contains the information about the optimization process (number of iterations taken, step size, exit message, etc.)
- `jacobian`: is the real matrix containing the Jacobian at the evaluated solution.

In this master thesis, the parameter of utmost interest is `exitflag`, which is taken as the basis for defining a new, discrete index: the Voltage Impasse Region Flag (VIR_{flag}). The index VIR_{flag} will be used in this master thesis to indicate if the application of `fsolve` to the system of algebraic equations at some state-space point $(\delta_{1q}, \delta_{2q}, \dots, \delta_{n_gq})$ converges or not. Convergence here means finding a solution of a set of algebraic equations inside the given maximum number of iterations within a pre-set tolerance. The system of algebraic equations is described by:

$$g(\delta_q, U, \theta) = 0, \quad (4.7)$$

By solving the system (4.7) for different values of $\delta_q = [\delta_{1q}, \delta_{2q}, \dots, \delta_{n_gq}]^T$, VIR_{flag} will indicate if the algebraic Jacobian is singular or not and, hence, if the solutions for U and θ are obtainable.

Therefore, VIR is defined as the area in the relative rotor angle space (with machine q^{th} taken as a reference) where VIR_{flag} is equal to zero:

$$VIR = \{(\delta_{1q}, \delta_{2q}, \dots, \delta_{n_gq}) : VIR_{flag} = 0\} \quad (4.8)$$

4.3 Confirmation of the methodology

In order to confirm that the quasi-dynamic approach has been correctly defined, the analytical expressions for one non-constant impedance load (that were derived in [5]) are applied to the test-system from Section 3.3.1.

In the case of the considered test system, there are three generators. By choosing machine 1 as the reference, δ_{21} and δ_{31} are increased and then used to solve the algebraic equations from (4.7). Due to the ratio of generator inertias, in this system it is expected that machines 2 and 3 separate from machine 1 (which has a larger inertia). Therefore, in this thesis, quadrant one of the relative rotor angle space is of utmost interest. The flow chart in Fig. 4.2 shows the steps followed for mapping VIR for the considered test system. It has to be taken into account that Fig. 4.2 is an example of the application of flowchart in Fig. 4.1 to a specific test system. Finally, if VIR defined by the analytical expressions from [5] matches the one obtained by the quasi-dynamic approach, the proposed methodology can be verified.

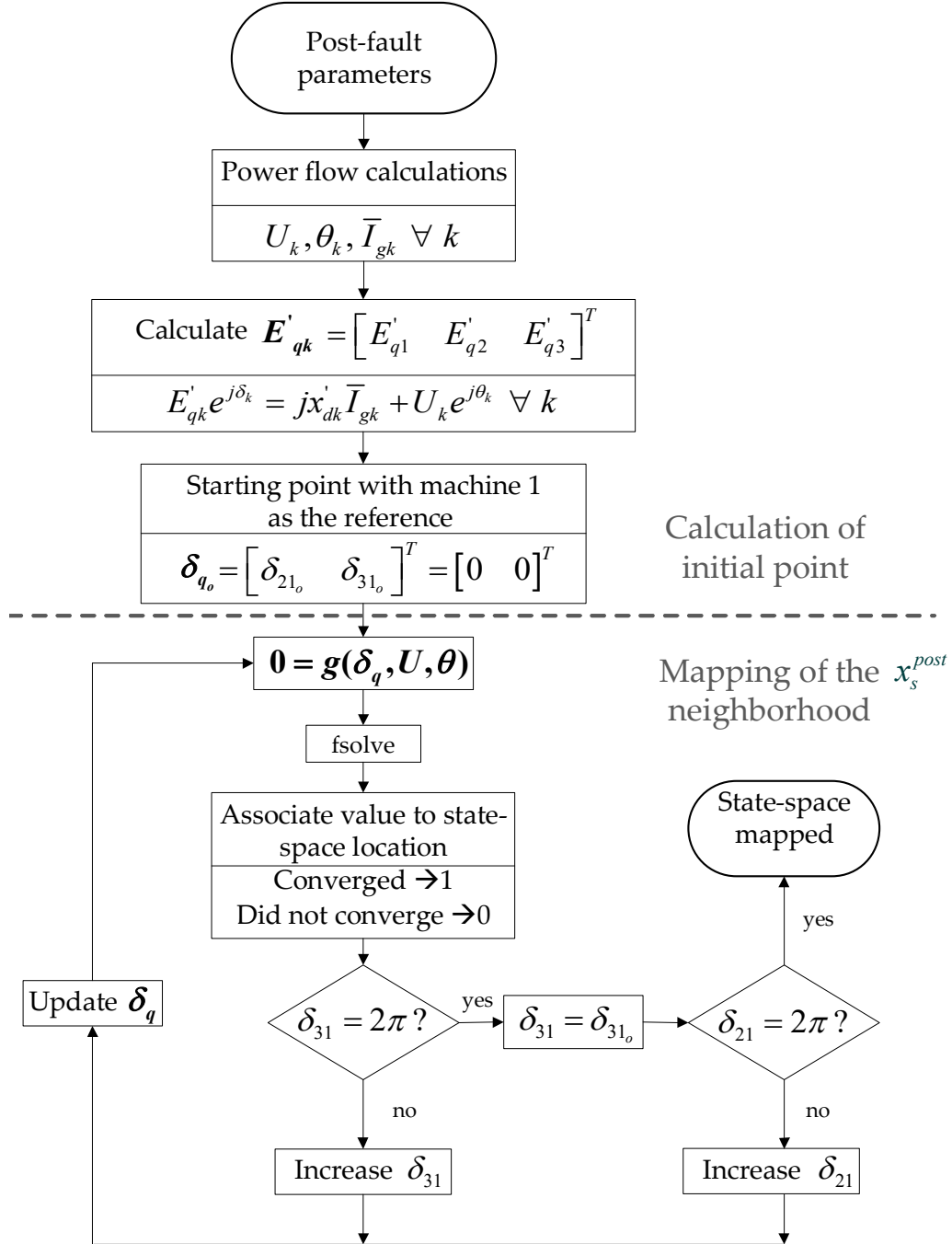


Figure 4.2: Flow chart of the proposed methodology for mapping the first quadrant of the state-space for the considered test system

As the considered test system is a 3-load system, only one of those three loads will be a non-linear load. In line with the exponents of the static load model (mp and mq from Section 2.1.2), the cases are named according to the following criteria:

- if a load is modeled with the Constant Power (CP) model, it will be marked with 0;
- if a load is modeled with the Constant Current (CC) model, it will be marked with 1;
- if a load is modeled with the Constant Impedance (CI) model, it will be marked with 2.

VIR is calculated by using both the quasi-dynamic approach and the analytical expressions from [5] for the cases in Table 4.1.

Load bus	4	5	6
Case	2	2	2
	2	2	1
	2	1	2
	1	2	2
	2	2	0
	2	0	2
	0	2	2

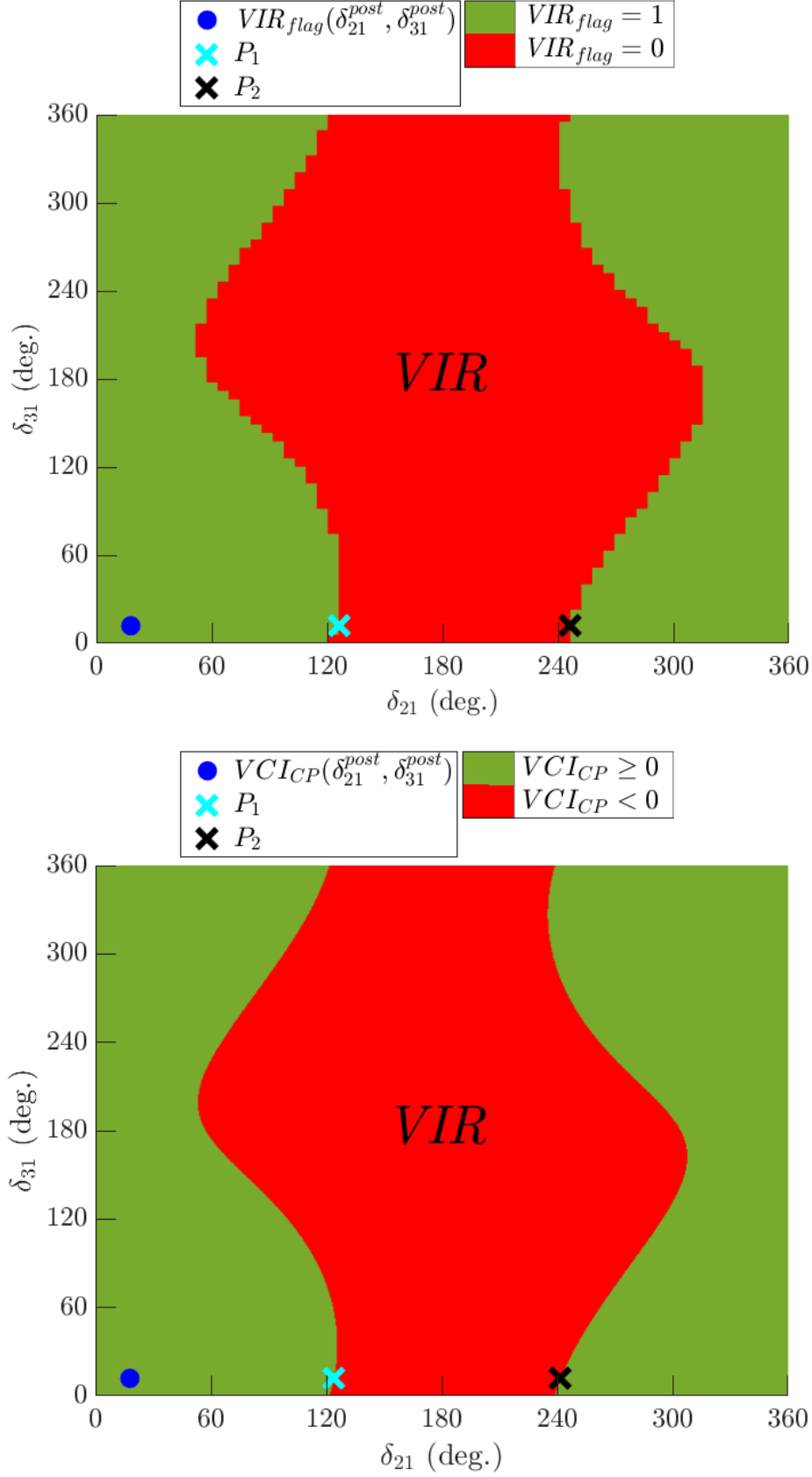
Table 4.1: Cases studied for the purpose of confirming the quasi-dynamic approach

The results have shown that the quasi-dynamic approach is sufficiently accurate in defining VIR : for all the cases from Table 4.1, VIR calculated by the expressions from [5] was almost the same as VIR obtained by the quasi-dynamic methodology. As an example, the results of three different loading cases are included below: Case 202, Case 212 (for inductive loads) and Case 212 (for capacitive loads). In all of the three cases, the load at bus 5 was modeled either as a CP or a CC load. All the cases considered the same post-fault parameters from Table 3.2 as the initial guess provided to `fsolve`.

• **Case 202 - load at bus 5 modeled as a CP load:**

For Case 202, the quasi-dynamic approach and the analytical expression from Eq. (4.3) are employed to map VIR in the relative rotor angle space. Fig. 4.3 shows the similarity in VIR no matter if it has been mapped by VIR_{flag} or by the VCI_{CP} from [5].

In Fig. 4.3, the post-fault operating point (dark blue circle) and two other points (P_1 and P_2) are marked. P_1 (cyan cross) and P_2 (black cross) are plotted in order to measure the precision of the quasi-dynamic approach. In Table 4.2, the coordinates

Figure 4.3: Comparison of *VIR* identified by VIR_{flag} and by VCI_{CP} (Case 202)

of P_1 and P_2 are included for comparison purposes. For both points $\delta_{31} = \delta_{31}^{post}$ and, thus, the precision is calculated with respect to the error of δ_{21} . As shown in Table 4.2, VIR_{flag} has a relative error of approximately 2 %.

Indices	$(\delta_{21}, \delta_{31})$ (deg.)	
	P_1	P_2
VIR_{flag}	(126.1, 11.9)	(246.1, 11.9)
VCI_{CP}	(123.8, 11.9)	(241.2, 11.9)
$\varepsilon_{\delta_{21}}$	1.86 %	2.03 %

Table 4.2: Coordinates and relative error of P_1 and P_2

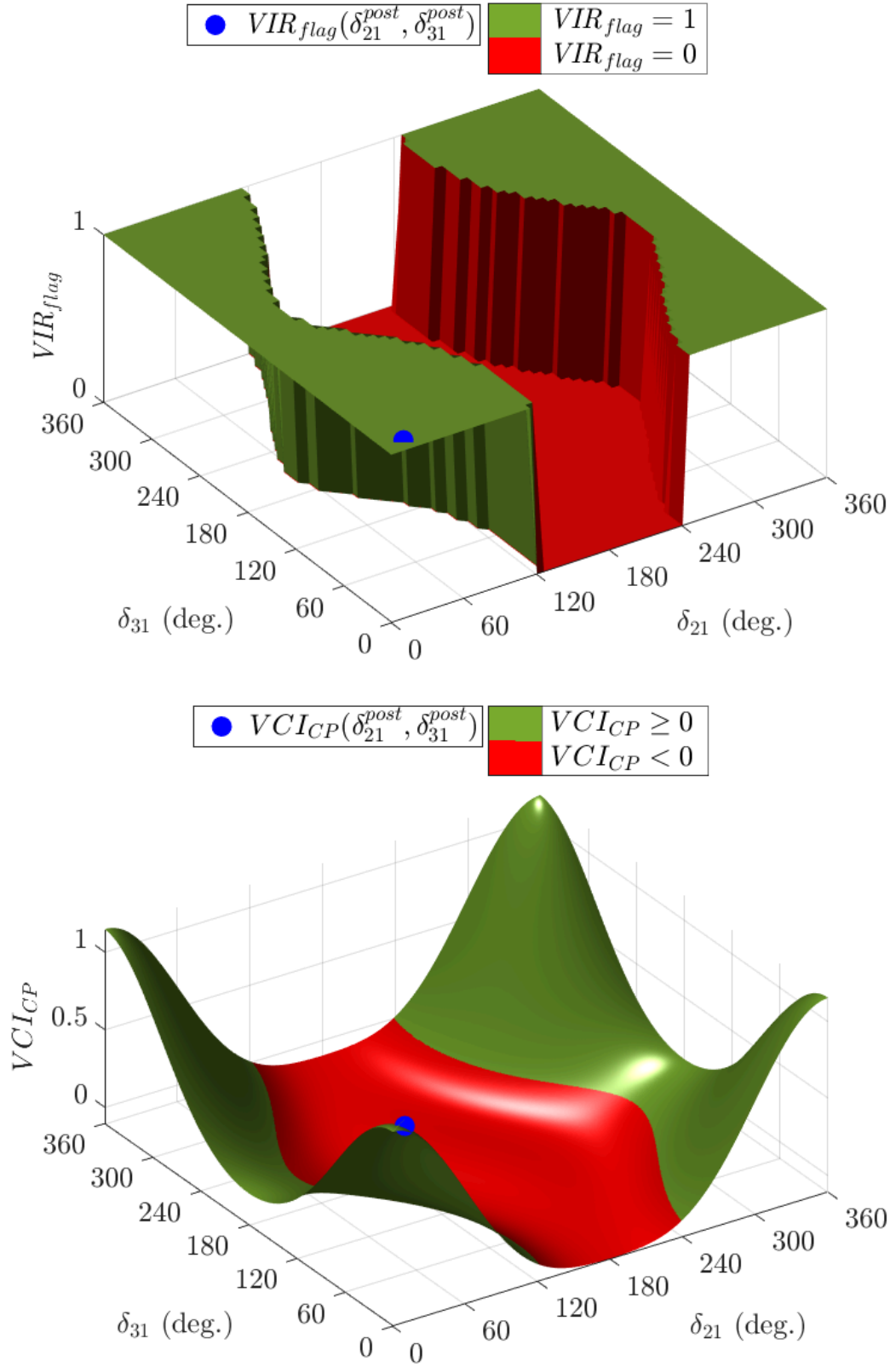
However, although it has been demonstrated that both the procedure developed in this master thesis and the one in [5] can determine VIR precisely enough, there are some differences between the indices that were used to identify VIR . In Fig. 4.4, the upper picture shows the value of VIR_{flag} in the neighborhood of $(\delta_{21}^{post}, \delta_{31}^{post})$. The lower picture of the same figure depicts the VCI_{CP} function. In both cases, the red areas correspond to VIR . Both graphs are plotted to underline that whereas VIR_{flag} is a discrete index (that can be 0 or 1), VCI_{CP} is a continuous index.

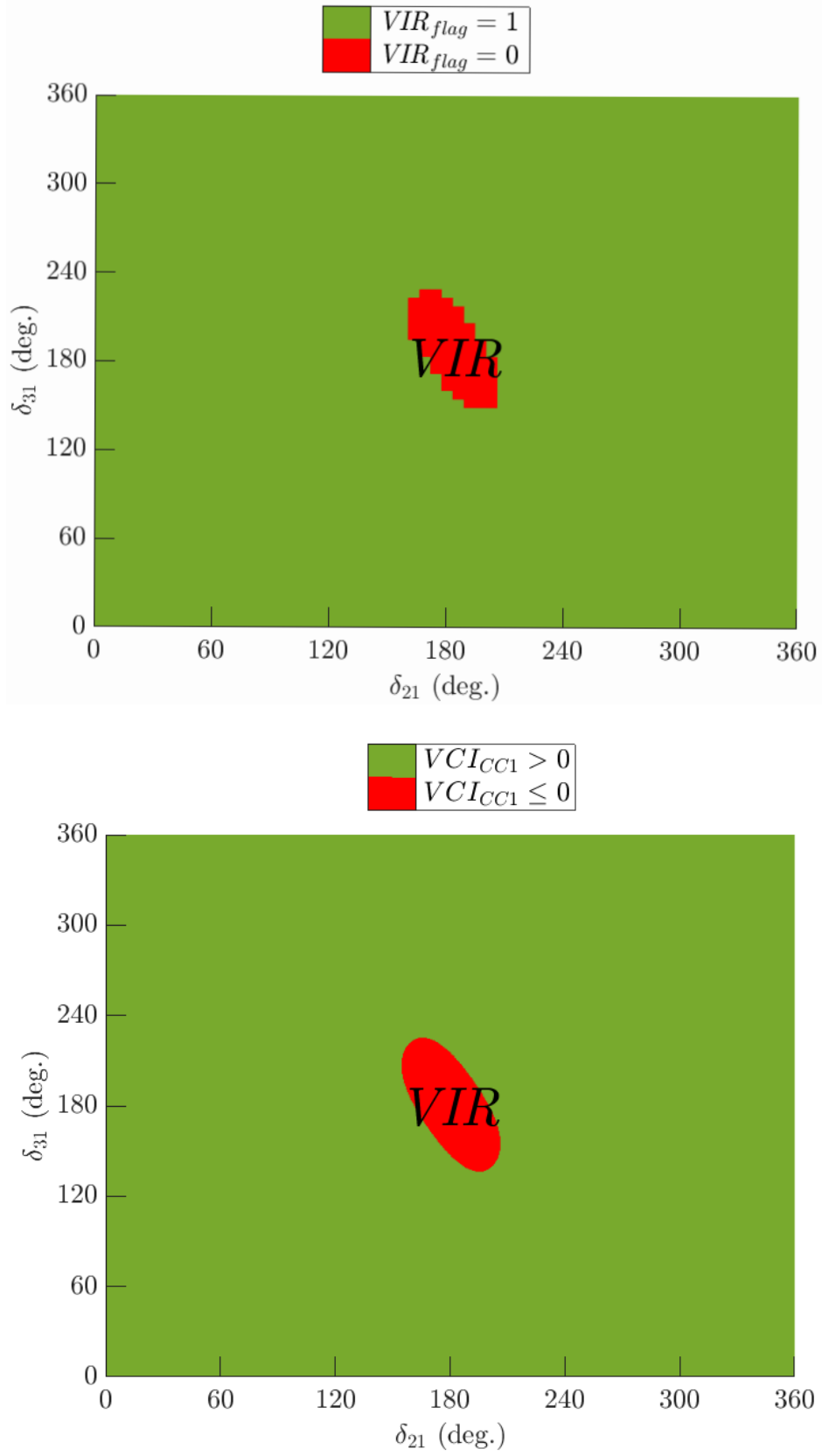
- **Case 202 - load at bus 5 modeled as an inductive CC load ($b_R \leq 0$):**

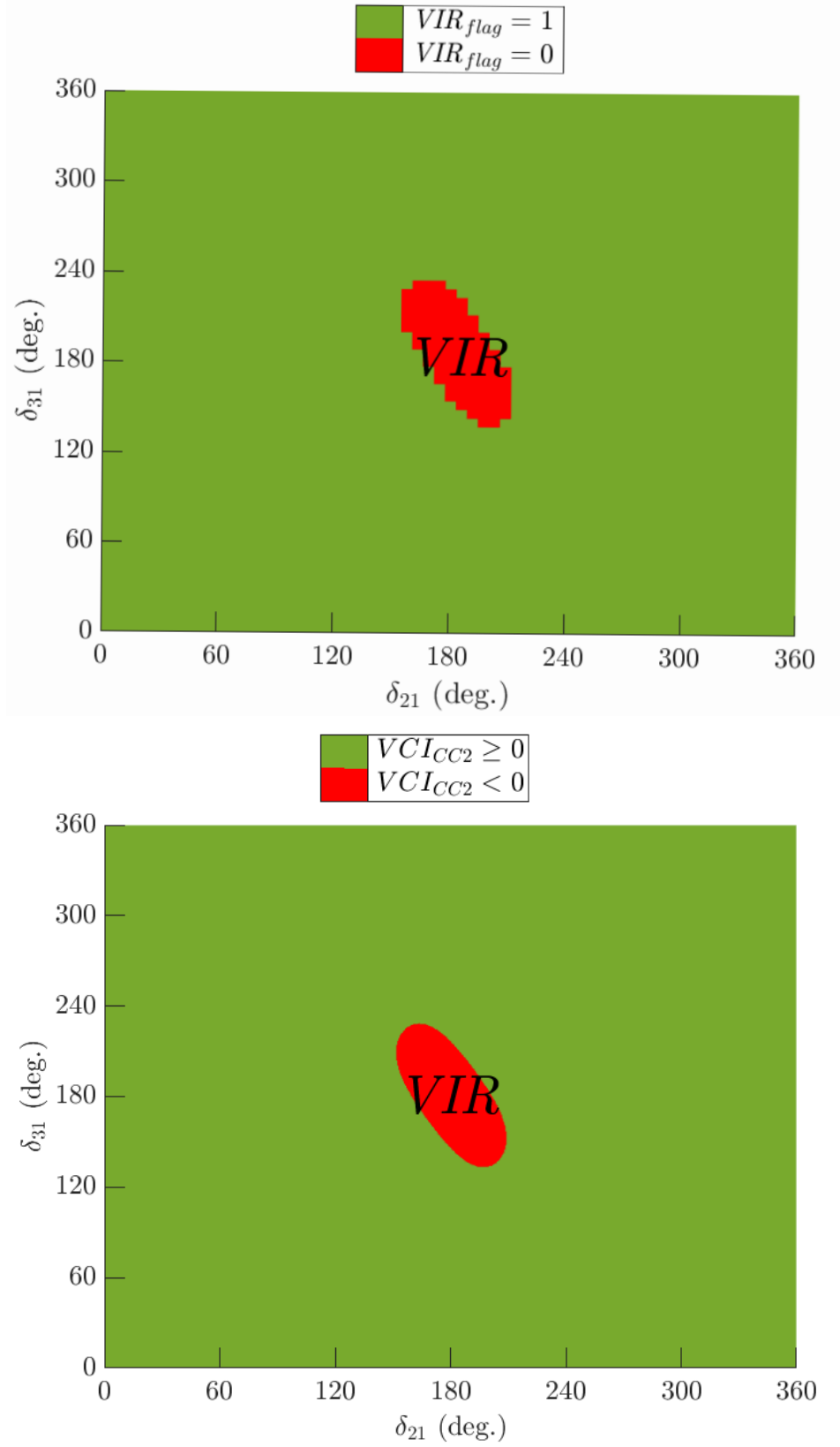
Fig. 4.5 depicts VIR calculated by both VIR_{flag} and by VCI_{CP} . The last one is calculated from Eq. (4.4) for $b_R \leq 0$. By comparing the two plots from Fig. 4.5 it can be concluded that the quasi-dynamic procedure for locating VIR has a satisfactory performance for the case of an inductive constant current load.

- **Case 202 - load at bus 5 modeled as a capacitive CC load ($b_R > 0$):**

Now, the load at bus 5 is turned into capacitive by making negative its corresponding reactive power. For this specific case, Fig. 4.6 is shown. Once more, the quasi-dynamic approach was confirmed as valid.

Figure 4.4: Comparison of the values of VIR_{flag} and VCI_{CP} (Case 202)

Figure 4.5: Comparison of *VIR* identified by VIR_{flag} and by VCI_{CC1} (Case 212, $b_R \leq 0$)

Figure 4.6: Comparison of *VIR* identified by VIR_{flag} and by VCI_{CC2} (Case 212, $b_R > 0$)

This chapter has confirmed that the quasi-dynamic approach is an adequate methodology to define *VIR* in the relative rotor angle space. It has introduced a new discrete index called Voltage Impasse Region Flag (VIR_{flag}) that allows to determine if the system of algebraic equations is solvable or not and, thus, to identify *VIR*. The next chapter will focus on studying *VIR* for different combinations of static, exponential load models. Cases where several loads are of non-linear type will be assessed and TDS will also be performed to study the entrance of a post-fault trajectory into *VIR*.

Chapter 5

Multi-non-linear loads and TDS

The work performed in [5] analyzed in which manner non-linear load models affect the *VIR*. In [5], the analytical expressions of *VIR* were derived for the power systems with one non-linear load. In that case, it was observed that *VIR* becomes larger when the non-linear load increases. The study also confirmed that the constant power load is the exponential load model that has the largest influence on *VIR*. Furthermore, the second important contribution of [5] was that it established the connection between TDS non-convergence issues and *VIR*.

The quasi-dynamic approach that has been proposed in this master thesis allows to locate *VIR* in the relative rotor angle space when more non-constant impedance loads are included. After having confirmed in the previous chapter that the quasi-dynamic approach performs well, Section 5.1 will use the proposed methodology to analyze how load interaction affects *VIR* in the terms of the exponential load model used and the size of the loads. In [5], the authors proposed studying the effects that more realistic load models have on *VIR* (as a topic for a future research). Hence, after having investigated the nature of *VIR* for the specific exponential load models (namely for the Constant Power (CP), Constant Current (CC) and Constant Impedance (CI) load models), a more general, static load model (the ZIP load model) is employed in Section 5.2. Finally, Section 5.3 will show the results of performing TDS for some of the cases that are included in Section 5.1.

5.1 Multi-non-linear loads

The test system considered in this thesis is a 3-machine 3-load power system, and the operating point used for the simulations is the one from Table 3.2. Both are used for the purpose of studying the effect of multi-non-linear loads on *VIR*. The methodology explained in Section 4.2 is employed to map *VIR* in the four quadrants of the relative rotor angle space.

In Section 4.3, Case 202 was assessed to confirm that the proposed quasi-dynamic approach was accurate in identifying the *VIR*. Now, Case 202 is modified in such way that the load at bus 4 is modeled by a Constant Current (CC) load instead of being modeled by a Constant Impedance (CI) load. Thus, it becomes Case 102. The plot in Fig. 5.1 shows *VIR* for Case 102 in the relative rotor angle space.

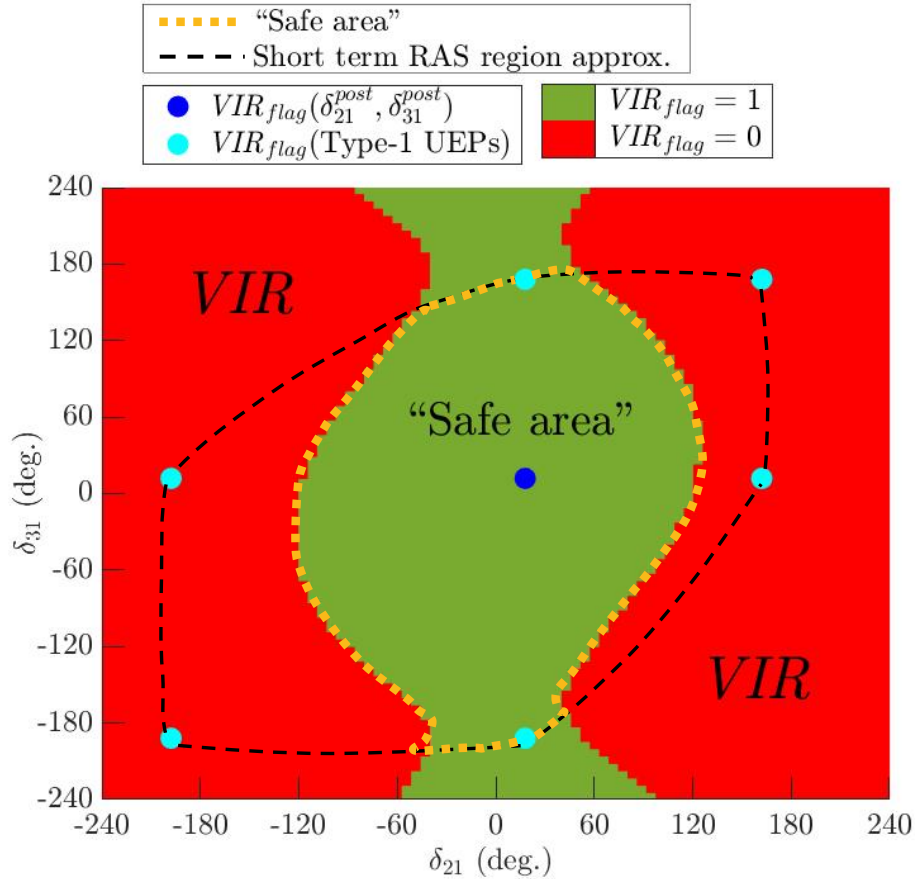


Figure 5.1: *VIR* identified by VIR_{flag} (Case 102)

As it can be seen, the *VIR* for Case 102 has been plotted for all the relative rotor angle combinations ranging from -240 to +240 (deg.). Six points, which are the approximations

of type-1 Unstable Equilibrium Points (UEPs) [24], are also marked. These points:

$$\begin{aligned} &(\pi - \delta_{21}^{post}, \delta_{31}^{post}); \quad (-\pi - \delta_{21}^{post}, \delta_{31}^{post}); \quad (\pi - \delta_{21}^{post}, \pi - \delta_{31}^{post}); \\ &(\delta_{21}^{post}, \pi - \delta_{31}^{post}); \quad (\delta_{21}^{post}, -\pi - \delta_{31}^{post}); \quad (-\pi - \delta_{21}^{post}, -\pi - \delta_{31}^{post}), \end{aligned} \quad (5.1)$$

provide an insight into the stability region that concerns short-term Rotor Angle Stability (RAS) [3]. In this way, one can get a picture about the interaction between the important short-term instability mechanisms (RAS and voltage collapse that is caused by static, non-linear loads).

Another interesting analysis to perform is to check *VIR* when there are no loads that are modeled as constant impedances. As an example, Case 101 is analysed and the *VIR* which occurs in this case is shown in Fig. 5.2.

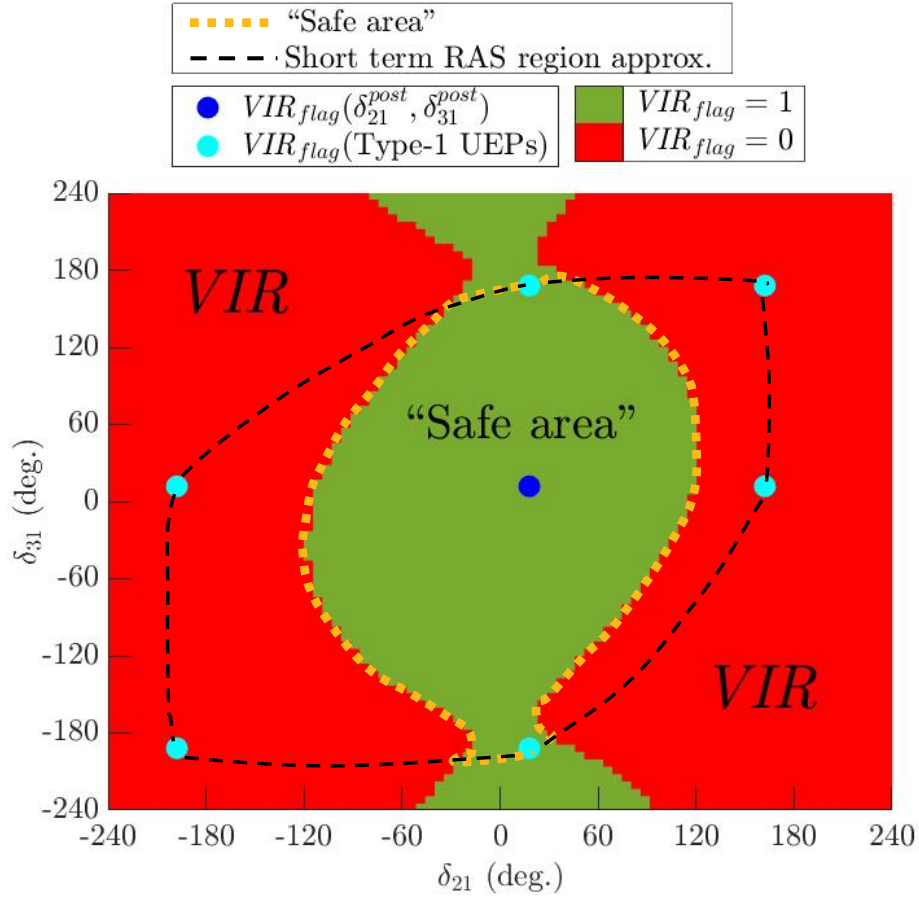


Figure 5.2: *VIR* identified by VIR_{flag} (Case 101)

By comparing the *VIR* from Case 102 and the *VIR* from Case 101, it is clear that both of them have a similar shape. However, the “safe, no-*VIR*” area of Case 101 is smaller than the “safe area” in Case 102. This result is in line with the conclusions obtained in [5]: *VIR* enlarges when non-linear loads are increased (and Case 101 includes a larger amount of non-linear loads (in (p.u.)) compared to Case 102). To confirm this, Case 202, Case 102 and Case 101 are plotted in Fig. 5.3. Table 5.1 is also included to show how the total amount of load is distributed among the different static load models.

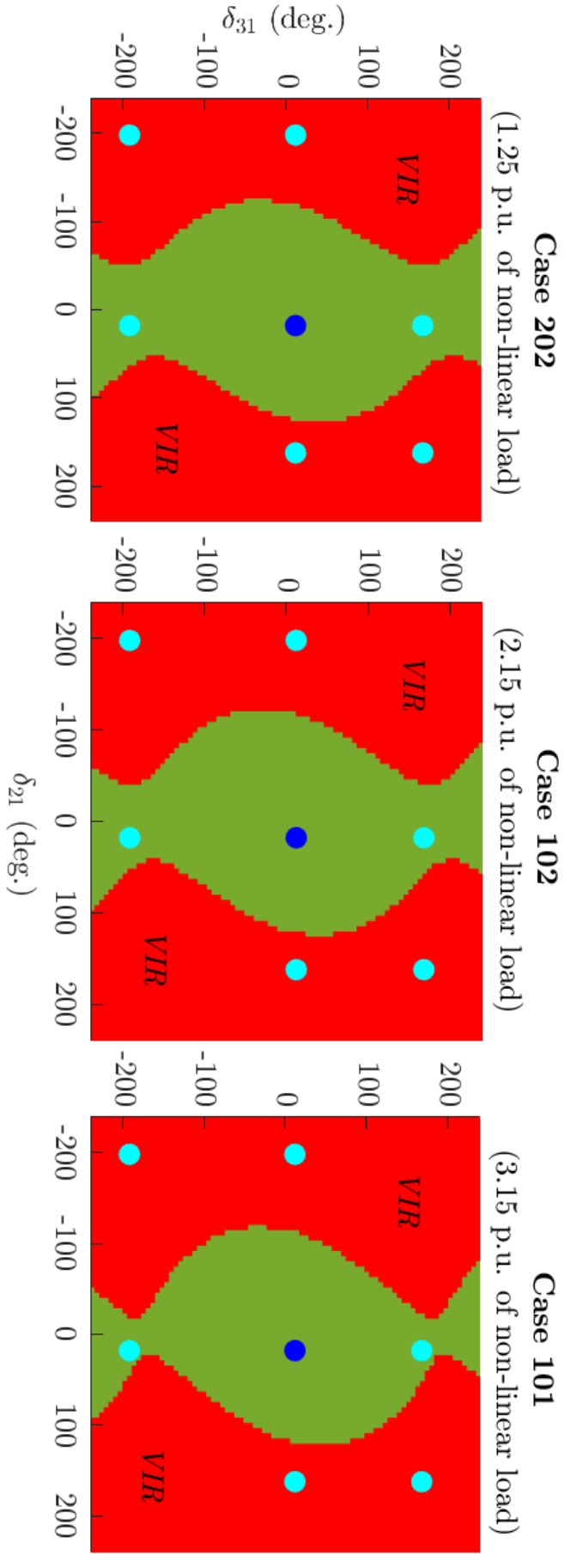


Figure 5.3: Comparison of VIR that are identified by VIR_{flaq} (Case 202, Case 102 and Case 101)

Case	CP load (p.u.)	CC load (p.u.)	CI load (p.u.)	Total non-linear load (p.u.)	Total load (p.u.)	% of non-linear load
202	1.25	0	1.9	1.25	3.15	39.68 %
102	1.25	0.9	1	2.15		68.25 %
101	1.25	1.9	0	3.15		100 %

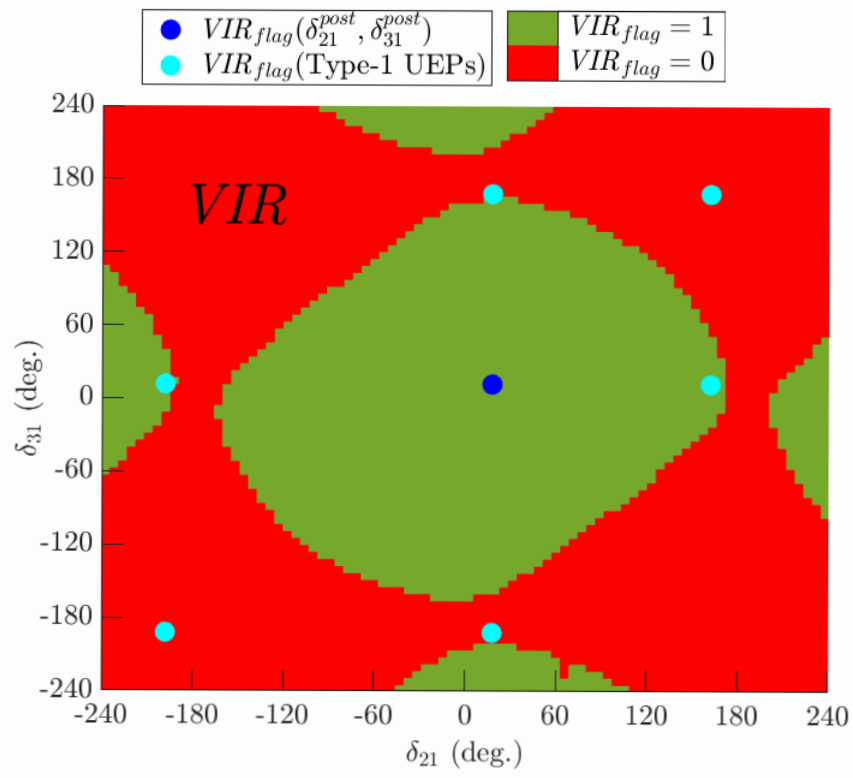
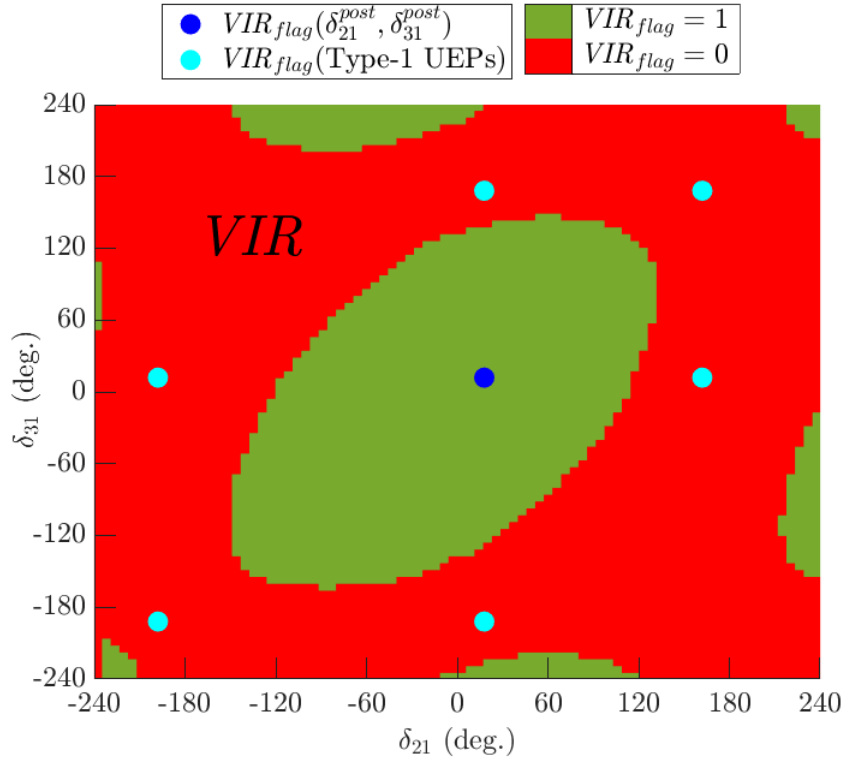
Table 5.1: Cases employed to study the effect that an increasing amount of non-linear load has on *VIR*

It has been demonstrated that the total amount of non-linear load affects the size of *VIR*. However, it is not clear in which manner the "topological location" of the non-linear loads in the power system changes *VIR*. To get an insight about this relationship, the three cases from Table 5.2 are compared.

Case	CP load (p.u.)	CC load (p.u.)	CI load (p.u.)	Total non-linear load (p.u.)	Total load (p.u.)
012	0.9	1.25	1	2.15	3.15
102	1.25	0.9	1	2.15	
210	1	1.25	0.9	2.25	

Table 5.2: Cases employed to analyze the effect of the location of Constant Power (CP) load on *VIR*

These three cases differ one from another in the sense of the position of the Constant Power (CP) load. The remaining loads are either of Constant Current (CC) or Constant Impedance (CI) type. However, in the three cases, the total amount of non-linear load is almost the same. Case 012 is shown in Fig. 5.4, Case 102 was depicted previously in Fig. 5.1, and Case 210 is shown in the sequel as Fig. 5.5.

Figure 5.4: *VIR* identified by VIR_{flag} (Case 012)Figure 5.5: *VIR* identified by VIR_{flag} (Case 210)

The comparison of Cases 012, 102 and 210 (which are depicted side by side in Fig. 5.6) suggests that the CP load model is the one that determines the shape of *VIR* in the state-space. In order to confirm this hypothesis, one last analysis is performed. In this analysis, the cases from Table 5.2 are modified in such way that those loads modeled as CI are instead modeled by the CC model, and vice versa (e.g. Case 012 becomes Case 021, Case 102 turns into 201, and Case 210 turns into 120). In this way, the initial and the transformed cases can be compared to see if the change in locations of the CC and CI loads in the power system has an effect on *VIR*.

The six aforementioned cases are depicted in Fig. 5.7. From the analysis of the pairs of plots, it is clear that the initial assumption about the importance of the CP load was right. As long as the load modeled as CP is the same, the main properties of *VIR* in the relative rotor angle space remain similar. From the three pairs of cases studied, Cases 012 and 021 are the ones that look less alike. More specific conclusions about the relationship between the location of different types of non-linear loads and the size/shape of *VIR* require a more extensive analysis that is planned as a part of a future study.

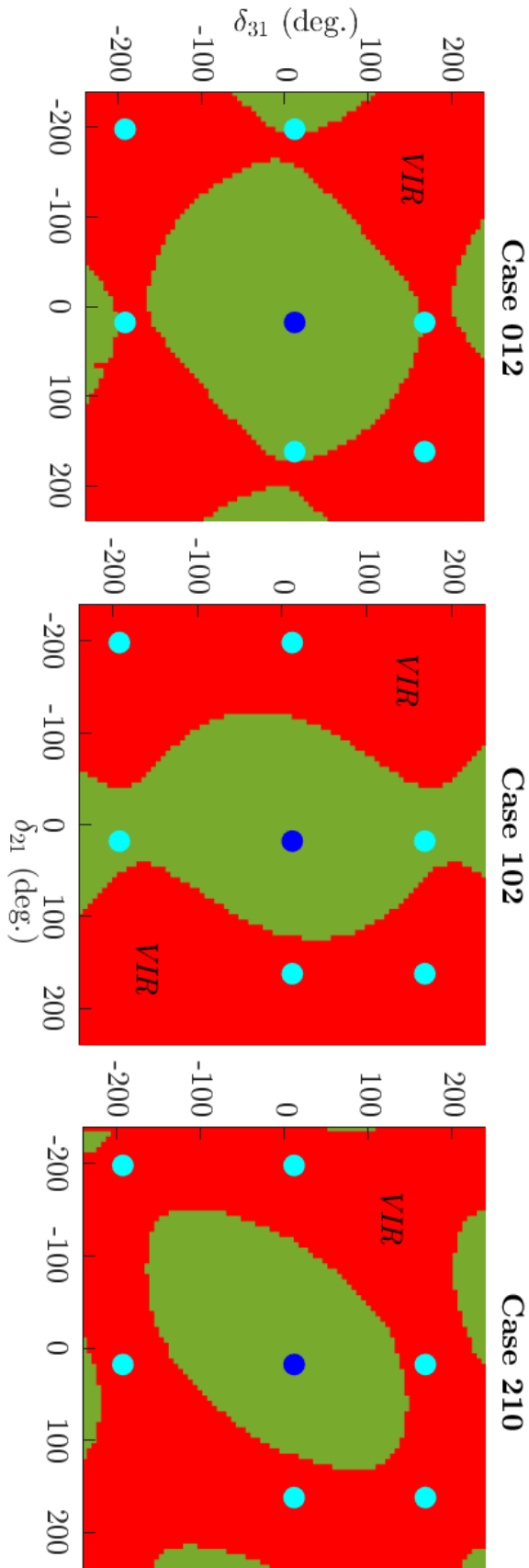


Figure 5.6: Comparison of *VIR* that are identified by VIR_{flag} : The effect of CP load model (Case 012, Case 102 and Case 210)

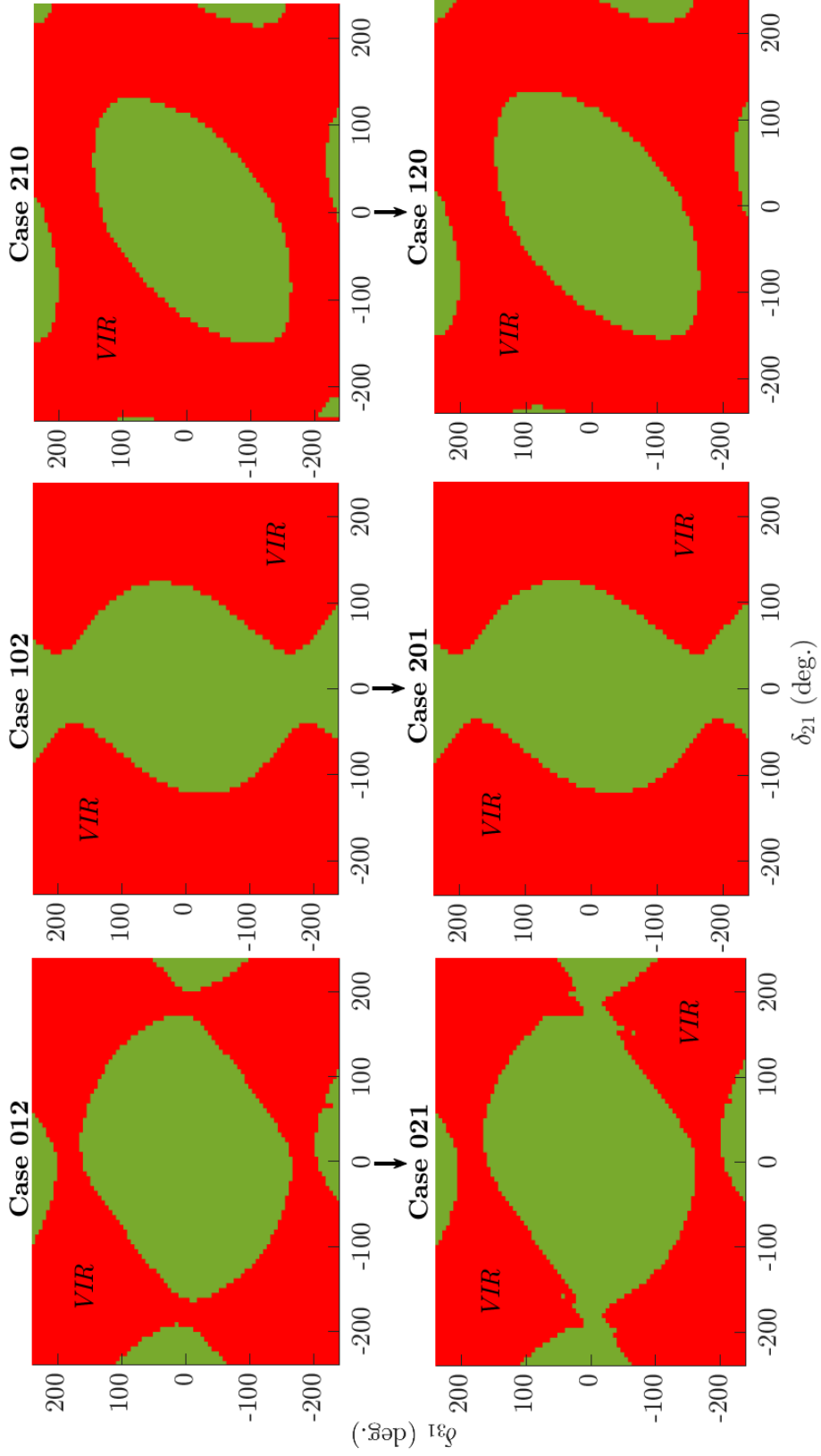


Figure 5.7: Comparison of VIR that are identified by VIR_{flag} : The effect of CC and CI load models (Case 012 and 021; Case 102 and 201; Case 210 and 120)

The performed analysis allows to draw the first conclusion about how the non-linear load interaction affects *VIR*:

Whenever more than one non-linear load is introduced into a power system, the Constant Power (CP) load is the one that has the largest impact on the shape of the *VIR*.

By comparing the pairs of the cases from Fig. 5.7 separately, it can be seen that the modification of the location of the CI and CC load models does not affect *VIR* as much as changing the location of the CP model.

When studying the effects of the load modeling on *VIR*, it is also interesting to analyze how the increase in the load demands affects *VIR*. Let's consider Case 102 that was shown in Fig. 5.1. By observing this case, it is going to be studied how *VIR* changes when the active powers of the loads are increased separately. The total active load demand is increased by 3 (p.u.). As before, when the active load is increased, every generator is forced to compensate for one third of the required active power.

The following three scenarios are analyzed:

- The Constant Current (CC) load at bus 4 is increased by 3 (p.u.);
- The Constant Power (CP) load at bus 5 is increased by 3 (p.u.);
- The Constant Impedance (CI) load at bus 6 is increased by 3 (p.u.).

The results of these scenarios are gathered in Fig. 5.8. When the three plots from Fig. 5.8 are compared, it can be seen that the *VIR* is different depending on the type of the load that has been incremented. In the case where the additional power is added to the demands that are modeled as non-linear loads, *VIR* becomes larger compared to the case of increased linear load. Furthermore, the addition of 3 (p.u.) of active power to the CP load (the load at bus 5) shrunk the “safe area” more than for the case in which the 3 (p.u.) were added to the CC load (the load at bus 4). Hence, it is possible to draw another important conclusion about the relationship between *VIR* and multiple non-linear loads:

As long as the post-fault equilibrium point remains within the loadability limits, the loading of the power system results in the increase of the size of *VIR*. The most influential load model is the Constant Power (CP) model, which is followed by the Constant Current (CC) one. The Constant Impedance (CI) load model does not increase the risk for the appearance of *VIR*. In a fact, the Constant Impedance load (CI) model affects the *VIR* only indirectly (in the presence of non-linear loads).

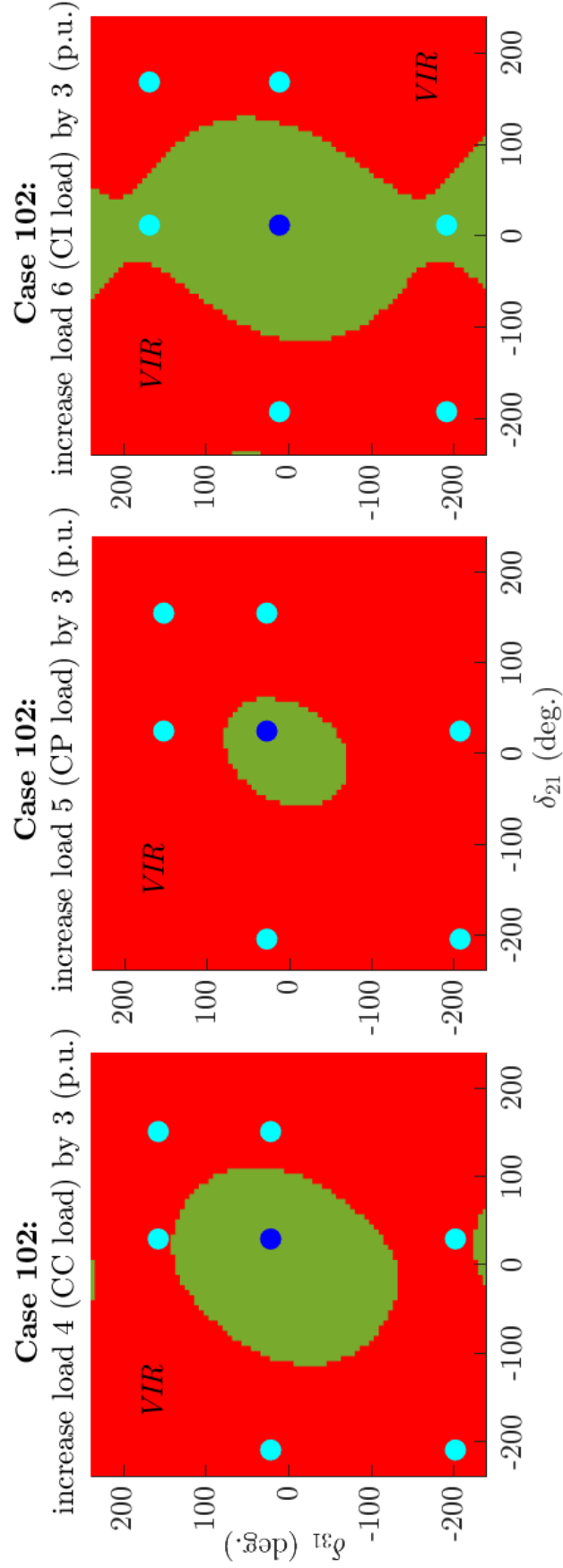


Figure 5.8: Comparison of VIR that are identified by VIR_{flag} : The effect of increasing the loading of a certain static load model (Case 102 when different loads are increased by 3 (p.u.))

5.2 ZIP load model

When considering the influence of load modeling on *VIR*, it is interesting to study the effect that the static, exponential load models have on *VIR* both individually and, also, as the parts of more general load models. The ZIP load model (which was presented in Section 2.1.2) is a more general form of modeling a demand as a static load.

To study how the ZIP load model affects *VIR*, an exemplary case is included below. Depending on the value of the parameters, the ZIP load reflects more the influence of a certain type of exponential load model (e.g. if $k_{pz} = k_{qz} = 0.9$ and $k_{pi} = k_{qi} = k_{pp} = k_{qp} = 0.05$, the ZIP model will tend to behave more as a constant impedance load). Only in the situations where all the parameters are equal to $1/3$, the three basic exponential load models are equally represented in the ZIP load model.

Let's consider a specific case of the test system from Fig. 3.2 where the loads at buses 4 and 5 are modeled as constant impedances, and the load at bus 6 is represented by the ZIP load model. It is assumed that the corresponding parameters of the active and reactive parts of the load complex power are always equal, for simplicity purposes (i.e. $k_{pz} = k_{qz}$; $k_{pi} = k_{qi}$; $k_{pp} = k_{qp}$). The three main scenarios are analyzed (in each one of them, a certain exponential load model is predominant):

- **Scenario ZIP_{CI}**: the predominant exponential model is the Constant Impedance (CI) part of the ZIP load ($k_{pz} = k_{qz} = k_{ZIP_{CI}}$);
- **Scenario ZIP_{CC}**: the predominant exponential model is the Constant Current (CC) part of the ZIP load ($k_{pi} = k_{qi} = k_{ZIP_{CC}}$);
- **Scenario ZIP_{CP}**: the predominant exponential model is the Constant Power (CP) part of the ZIP load ($k_{pp} = k_{qp} = k_{ZIP_{CP}}$).

The basic scenario is the case where all the exponential models have an equal representation in the ZIP load ($k_{ZIP_{CI}} = k_{ZIP_{CC}} = k_{ZIP_{CP}} = 1/3$). Then, in every scenario, the parameter of the predominant exponential model is increased. For instance, Table 5.3 shows how the parameters of Scenario ZIP_{CP} (where the predominant parameter is $k_{ZIP_{CP}}$) are modified.

	$k_{ZIP_{CI}}$	$k_{ZIP_{CC}}$	$k_{ZIP_{CP}}$
Scenario ZIP_{CP}	0.25	0.25	0.5
	0.15	0.15	0.7
	0.05	0.05	0.9

Table 5.3: Parameters in Scenario ZIP_{CP}

For the Scenarios ZIP_{CI} and ZIP_{CC} , the respective predominant parameters $k_{ZIP_{CI}}$ and $k_{ZIP_{CC}}$ are increased in the same way. The VIR ($VIR_{flag} = 0$) for all the scenarios is plotted in Fig. 5.9, including the VIR for the Initial Scenario in which $k_{ZIP_{CI}} = k_{ZIP_{CC}} = k_{ZIP_{CP}} = 1/3$. The last row of the plots in Fig. 5.9 (which corresponds to Scenario ZIP_{CP}) demonstrates how an increase in the parameter of the CP part of the ZIP model ($k_{ZIP_{CP}}$ was increased from 0.5 to 0.9) enlarges VIR . The first two rows in Fig. 5.9 depict Scenarios ZIP_{CI} and ZIP_{CC} . In both of these scenarios, $k_{ZIP_{CP}}$ has the same value, and it also decreases for the same amount for every increase of the predominant parameter (e.g. for the cases where $k_{ZIP_{CI}} = 0.5$ and $k_{ZIP_{CC}} = 0.5$, the value of $k_{ZIP_{CP}}$ is the same - 0.25). It is interesting to see how increasing $k_{ZIP_{CI}}$ and $k_{ZIP_{CC}}$ shrinks VIR (because $k_{ZIP_{CP}}$ decreases). Furthermore, in Scenario ZIP_{CI} and Scenario ZIP_{CC} the shape and size of VIR appear to be similar. However, VIR is actually bigger for the cases from Scenario ZIP_{CC} (as the non-linear part of the load is predominant in the ZIP model). This can be clearly seen by comparing the case where $k_{ZIP_{CI}} = 0.9$ in Scenario ZIP_{CI} with the respective case in Scenario ZIP_{CC} ($k_{ZIP_{CC}} = 0.9$). VIR is bigger in Scenario ZIP_{CC} because the sum of the non-linear parameters of the ZIP load model (i.e. the sum of $k_{ZIP_{CP}}$ and $k_{ZIP_{CC}}$) is greater. More concisely, in Scenario ZIP_{CC} the value of $k_{ZIP_{CP}} + k_{ZIP_{CC}} = 0.95$, whereas in Scenario ZIP_{CI} $k_{ZIP_{CP}} + k_{ZIP_{CC}} = 0.1$.

Therefore, the more general conclusions can be drawn about the effect that each component of the ZIP load model has on VIR :

- **The Constant Power (CP) part of the ZIP model has the major influence on the appearance of VIR . An increase in $k_{ZIP_{CP}}$ results in the enlargement of VIR in the relative rotor angle space.**
- **In those situations where $k_{ZIP_{CP}}$ has the same value, what defines the size of VIR is the value of the non-linear parameters. The larger is the sum of $k_{ZIP_{CP}}$ and $k_{ZIP_{CC}}$, the larger is also the VIR .**

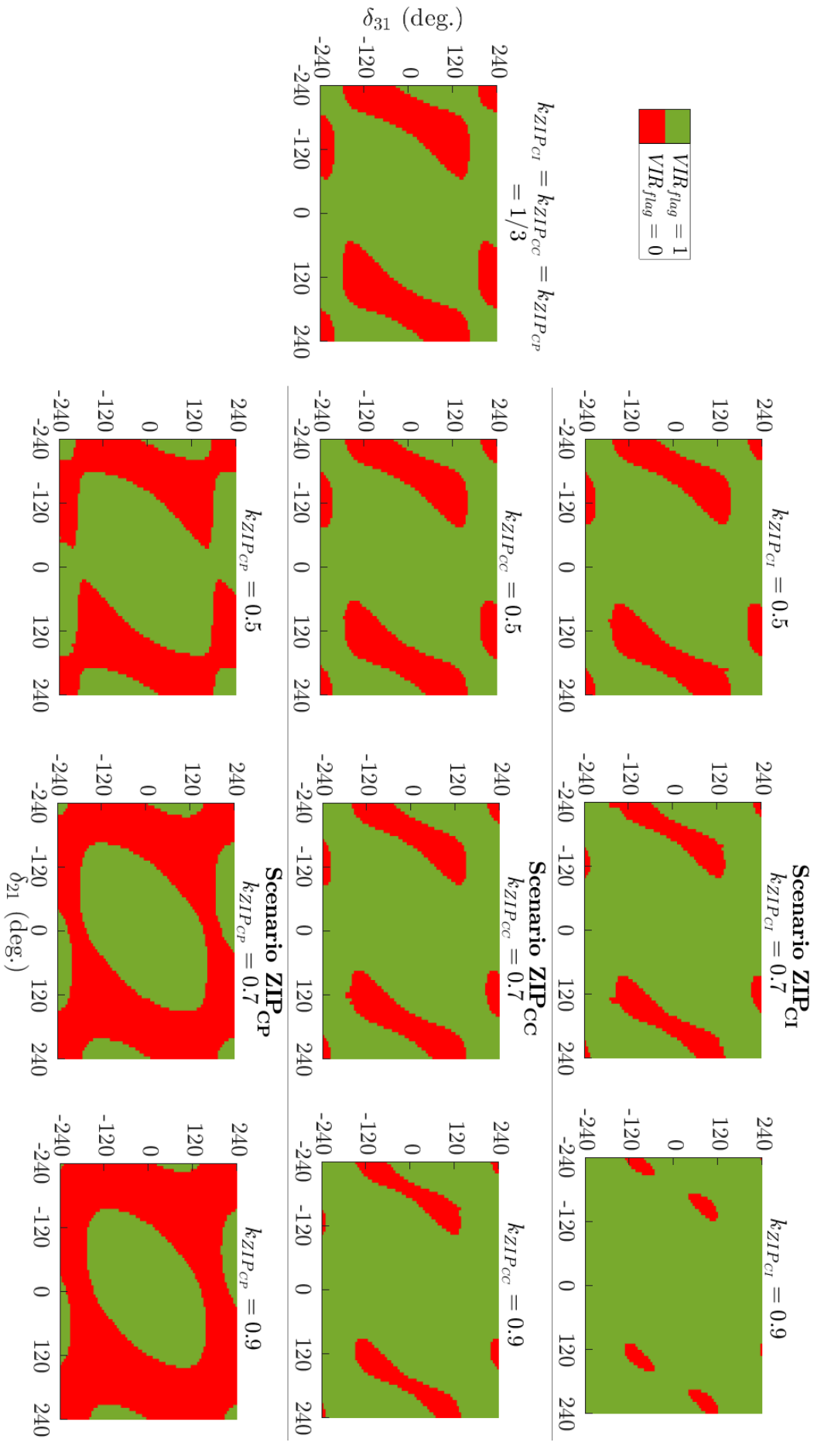


Figure 5.9: Comparison of VIR that are identified by VIR_{flag} : The effect of increasing the predominant parameter of the ZIP load model at bus 6 (Initial Scenario, Scenario ZIP_{cc}, Scenario ZIP_{cp})

If the predominant parameter is further increased up to 0.99, the scenarios represent the case in which the load at bus 6 is modeled almost completely by only one exponential component. In this way:

- The scenario with $k_{ZIP_{CI}} = 0.99$ from Scenario ZIP_{CI} is almost the same as Case 222 (where all the loads are modeled as CI loads)
- The scenario with $k_{ZIP_{CC}} = 0.99$ from Scenario ZIP_{CC} is almost the same as Case 221 (where the loads at buses 4 and 5 are modeled as CI loads, and the load at bus 6 as a CC load)
- The scenario with $k_{ZIP_{CP}} = 0.99$ from Scenario ZIP_{CP} is almost the same as Case 220 (where the loads at buses 4 and 5 are modeled as CI loads, and the load at bus 6 as a CP load)

Figure 5.10 depicts VIR for these three situations.

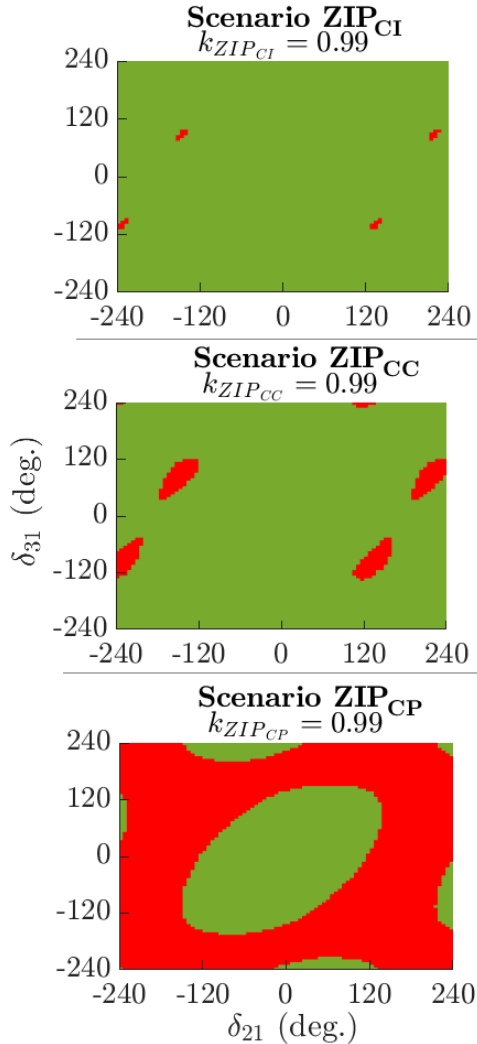


Figure 5.10: Comparison of VIR that are identified by VIR_{flag} : The predominant parameter is 0.99 when there is a ZIP load model at bus 6 (Scenario ZIP_{CI} , Scenario ZIP_{CC} and Scenario ZIP_{CP})

By comparing the last column of Fig. 5.9 with Fig. 5.10, it can be observed that the increase of the predominant parameter from 0.9 to 0.99 follows the same tendency as before (in Scenario ZIP_{CI} and Scenario ZIP_{CC} - VIR shrinks; in Scenario ZIP_{CP} - VIR enlarges). It is also remarkable that for Scenario ZIP_{CI} with $k_{ZIP_{CI}} = 0.99$ there is still a small presence of VIR : the 1 % of non-linearity in load 6 is the reason for this. If all the loads are modeled as CI, there is no VIR .

In order to prove that the MATLAB script (which was created to map VIR when there are ZIP load models in the system) performs accurately, the same case that was used to validate the quasi-dynamic methodology (i.e. Case 202) is going to be employed. The loads at buses 4 and 6 are modeled as CI and the load at bus 5 is represented by the ZIP load model. In this way, by increasing $k_{ZIP_{CP}}$ from 0.5 to 0.99, it can be analyzed how the transition from a CP predominant ZIP load model to a pure CP load affects VIR . The results from this analysis are shown in Fig. 5.11, where there is a total of six plots that depict VIR in the relative rotor angle space ranging from -240 to +240 deg. Table 5.4 includes all the scenarios/cases depicted in Fig. 5.11. Table 5.4 also shows how VIR is identified in each one of the cases and, also, it gives the percentage of the CP load in the ZIP load at bus 5.

Methodology to locate VIR	VIR identified by	Case/Scenario	% of CP component of load at bus 5
Quasi-dynamic approach	VIR_{flag}	Case 202	100 %
		$k_{ZIP_{CP}} = 0.5$	50 %
		$k_{ZIP_{CP}} = 0.7$	70 %
		$k_{ZIP_{CP}} = 0.9$	90 %
		$k_{ZIP_{CP}} = 0.99$	99 %
Anal. expr.	VCI_{CP}	Case 202	100 %

Table 5.4: Cases/Scenarios for the study of the transition from Scenario ZIP_{CP} to pure CP (for the load at bus 5)

As shown in the last column of Fig. 5.11, VIR areas from Case 202 (both identified by VIR_{flag} in the first row and by VCI_{CP} in the third row) have similar shapes and sizes as the VIR from Scenario ZIP_{CP} with $k_{ZIP_{CP}} = 0.99$. This demonstrates two things: **(i) that the MATLAB code for the ZIP load is correctly formulated and (ii) that the transition from a ZIP load (with a CP predominant component) to a pure CP load does not trigger abrupt changes in VIR .**

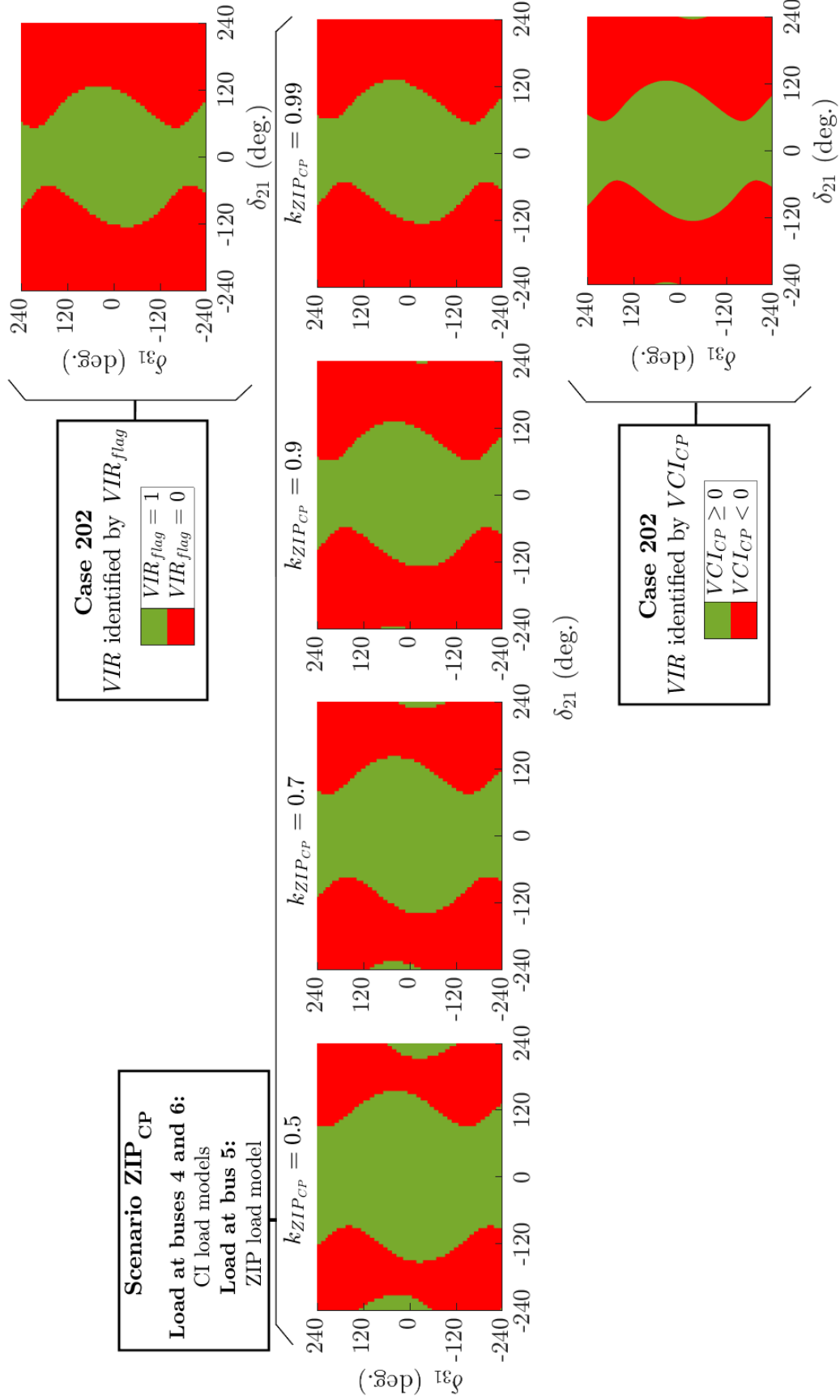


Figure 5.11: Comparison of VIR that are identified by VIR_{flag} : The transition of the load at bus 5 from Scenario ZIP_{CP} to pure CP

In all of the previously assessed cases, only one of the loads was modeled as a ZIP load model (the other two were CI loads). For the purpose of understanding how multiple ZIP loads affect *VIR*, all the three demands from the test system are modeled as ZIP loads. The Scenarios ZIP_{CI} , ZIP_{CC} and ZIP_{CP} are considered and, in each one of them, the corresponding predominant parameter are incremented. And once again, the Initial Scenario (where all of the parameters have the same value: $k_{\text{ZIP}_{\text{CI}}} = k_{\text{ZIP}_{\text{CC}}} = k_{\text{ZIP}_{\text{CP}}} = 1/3$) is also considered.

The results from this analysis, which are gathered in Fig. 5.12, allow to draw the same conclusions as before (e.g. the value of the parameter $k_{\text{ZIP}_{\text{CP}}}$ is directly proportional to the size of *VIR*). However, this time, there is a larger difference between the *VIR* from Scenario ZIP_{CI} and the *VIR* from Scenario ZIP_{CC} . By comparing the plots from $k_{\text{ZIP}_{\text{CI}}} = 0.9$ and $k_{\text{ZIP}_{\text{CC}}} = 0.9$, it is even more clear (than in Fig. 5.9) that the larger sum of $k_{\text{ZIP}_{\text{CC}}}$ and $k_{\text{ZIP}_{\text{CP}}}$ means larger *VIR*.

Regarding the shape of the *VIR* from Fig. 5.12, some comments can be made:

- All the plots show a periodical behaviour. Even when all the loads are modeled as ZIP loads, there is a symmetry of *VIR* in the state-space.
- There are some discontinuities in the *VIR* due to the numerical difficulties encountered by `fsolve` at different state-space locations.

All in all, Section 5.2 has performed (together with Section 5.1) a structured study about the influence of different static, non-linear loads on *VIR*. The main conclusion of this study is that the CP component of the load is the main cause of *VIR* at different post-fault state-space locations.

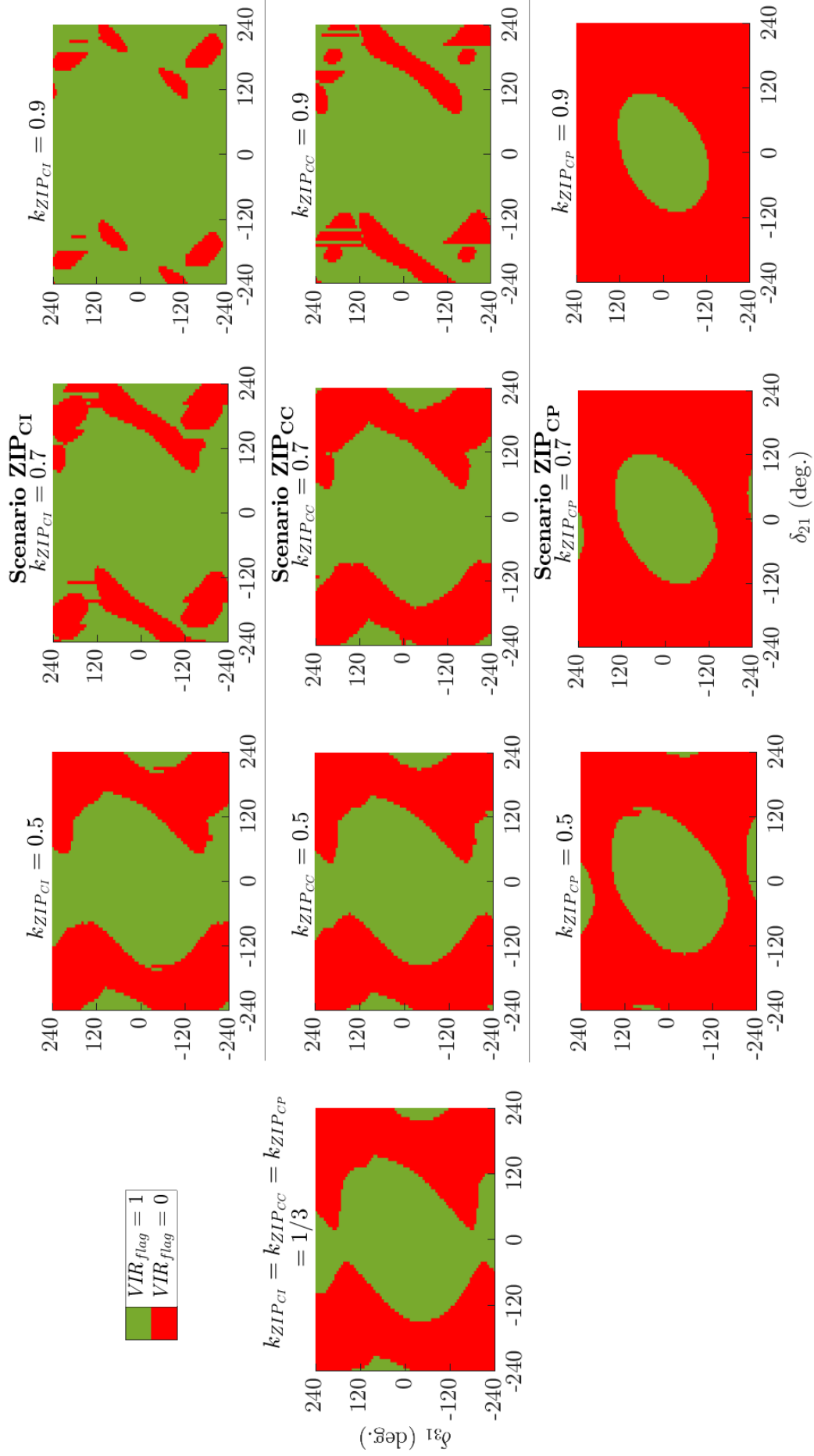


Figure 5.12: Comparison of VIR that are identified by VIR_{flag} : The effect of increasing the predominant parameter when all demands are modeled as ZIP loads (Initial Scenario, Scenario ZIP_{CI}, Scenario ZIP_{CC} and Scenario ZIP_{CP})

5.3 Time Domain Simulations (TDS)

One of the main contributions of the research from [5] was the determination of the connection between the non-convergence of TDS and the intersection of a post-fault trajectory with a *VIR*. In the study performed in [5], it was concluded that sometimes, when the TDS trajectory intersects with a *VIR* after a disturbance, the solver can not converge. In other occasions, the TDS solver resulted in a trajectory that seemingly reached the *VIR* boundary and then came back to the “safe area”. However, as soon as the dynamics of a DAE system get into *VIR*, the solution of the TDS can not be taken as a realistic representation of what would have happened in a real power system.

In this master thesis, TDS are carried out by simulating three-phase short circuits at different buses of the test system from Section 3.3.1. If t_e is the end time of the simulation, t_f is the time at which the fault occurs and t_{cl} represents the time at which the fault is cleared, the dynamics of the system in the pre-disturbance stage, during-disturbance stage and post-disturbance stage are given by:

- **Pre-disturbance stage:**

$$\begin{aligned}\dot{\mathbf{x}} &= \mathbf{f}^{pre}(\mathbf{x}, \mathbf{y}); \\ \mathbf{0} &= \mathbf{g}^{pre}(\mathbf{x}, \mathbf{y}),\end{aligned}\quad \text{for } 0 \leq t < t_f$$

- **During-disturbance stage:**

$$\begin{aligned}\dot{\mathbf{x}} &= \mathbf{f}^{dur}(\mathbf{x}, \mathbf{y}); \\ \mathbf{0} &= \mathbf{g}^{dur}(\mathbf{x}, \mathbf{y}),\end{aligned}\quad \text{for } t_f \leq t < t_{cl}$$

- **Post-disturbance stage:**

$$\begin{aligned}\dot{\mathbf{x}} &= \mathbf{f}^{post}(\mathbf{x}, \mathbf{y}); \\ \mathbf{0} &= \mathbf{g}^{post}(\mathbf{x}, \mathbf{y}),\end{aligned}\quad \text{for } t_{cl} \leq t < \infty$$

where the vector $\mathbf{x}$ represents the state variables and $\mathbf{y}$ are the algebraic variables. In the specific case of the test system from Section 3.3.1, the vectors $\mathbf{x}$ and $\mathbf{y}$ are respectively:

$$\begin{aligned}\mathbf{x} &= [\delta_1 \ \delta_2 \ \delta_3 \ \omega_1 \ \omega_2 \ \omega_3]^T = [\boldsymbol{\delta} \ \boldsymbol{\omega}]^T; \\ \mathbf{y} &= [U_1 \ U_2 \ U_3 \ U_4 \ U_5 \ U_6 \ \theta_1 \ \theta_2 \ \theta_3 \ \theta_4 \ \theta_5 \ \theta_6]^T = [\mathbf{U} \ \boldsymbol{\theta}]^T.\end{aligned}$$

For all the three simulation stages, the differential equations are defined by applying the expressions from (2.9) to the three generators. The algebraic equation for each of the system buses is obtained from Eq. (2.12). Since the machines have been modeled by the

classical model of the generator, the algebraic equations do not involve all of the state variables $\mathbf{x}$. Only the rotor angles $(\delta_1, \delta_2, \delta_3)$ appear in the algebraic equations. In this way, $\mathbf{g}(\mathbf{x}, \mathbf{y})$ becomes $\mathbf{g}(\boldsymbol{\delta}, \mathbf{U}, \boldsymbol{\theta})$.

It is also important to remark that, for the pre- and post-disturbance stages, the algebraic equations will remain the same, which does not hold for the during-fault stage. This is the case due to the fact that there will be no configurational changes when the fault is cleared (i.e. $x_s^{post} = x_s^{pre}$).

To simulate the three-phase short circuits, a large admittance is connected to the bus where the fault is considered. Thus, depending on the stage, the $\mathbf{Y}_{bus}$ matrix is different. Consider that $\mathbf{Y}_{bus}$ is the admittance matrix for the pre- and post-fault operating stages, and that a fault happens at bus f . Then, the transmission network is represented by the during-fault admittance matrix $\mathbf{Y}_{bus}^{dur}$, which is the $\mathbf{Y}_{bus}$ admittance matrix whose $(f, f)^{th}$ entry is modified as:

$$\bar{Y}_{ff}^{dur} = \bar{Y}_{ff} + j10^6.$$

In this master thesis, simulations are carried out by using a partitioned solution approach [25]. In each time step, the integration of the differential equations is done with the routine `ode45` [26] in MATLAB. The algebraic equations are solved by employing `fsolve`.

However, the partitioned solution approach is only applicable if the DAE system is well-defined along a post-fault trajectory. As previously explained in Section 4.1, the DAE system is well-defined (and it is then possible to convert the DAE into its equivalent ODE system) if the algebraic Jacobian $(\partial \mathbf{g} / \partial \mathbf{y})$ is non-singular everywhere in the state-space.

There are different ways to determine if a matrix is singular [27]. Some of the most common techniques are:

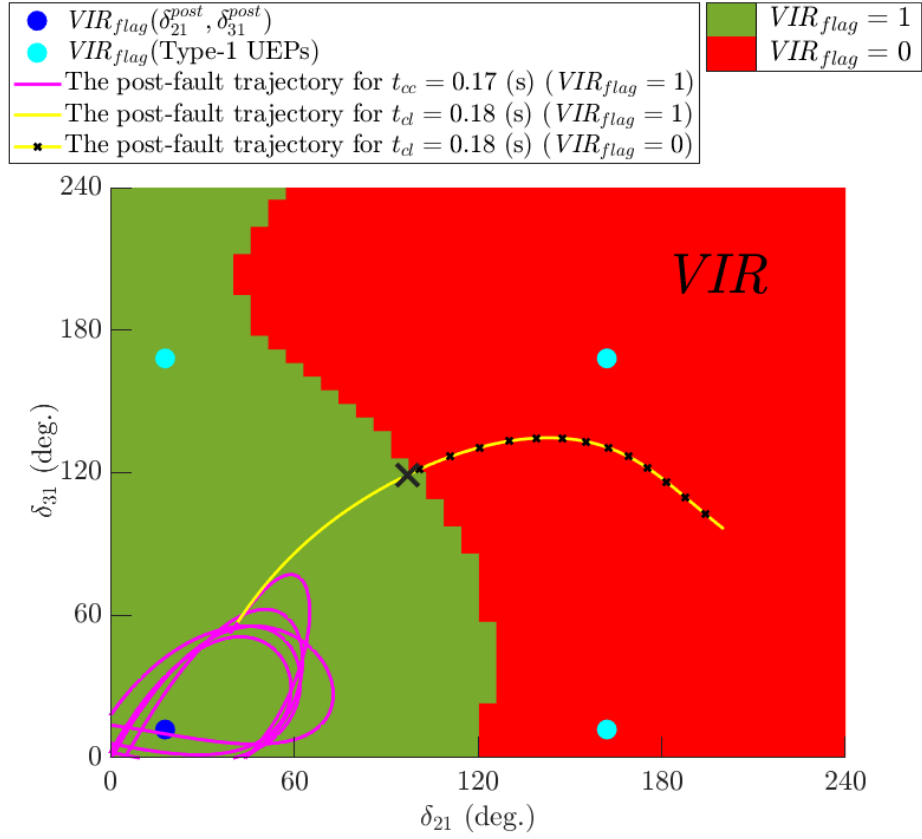
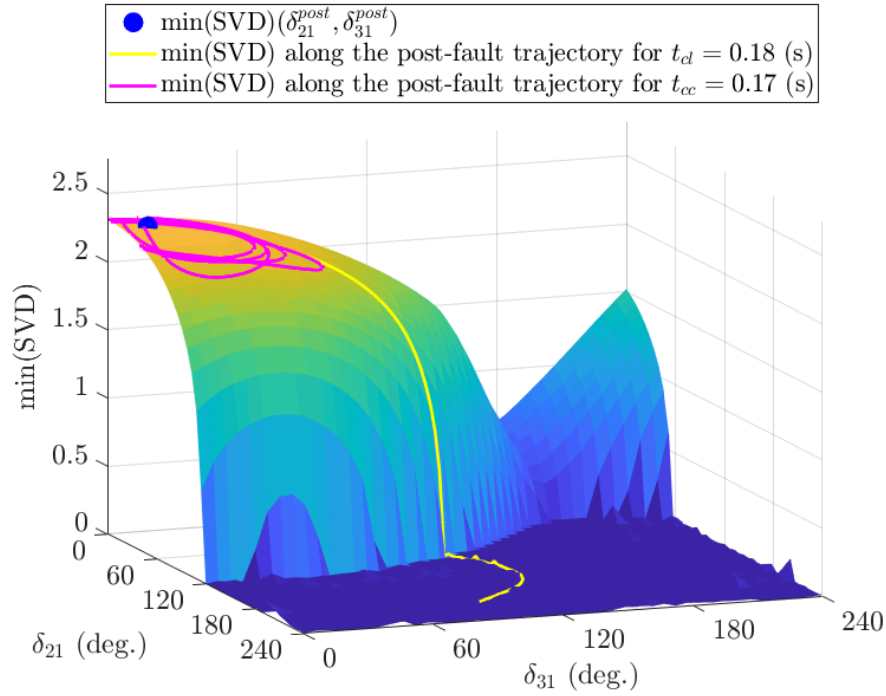
- **The calculation of the determinant of the matrix:** If the determinant is zero, the matrix is clearly not invertible, i.e. it is singular. However, computational environments such as MATLAB are inefficient when the determinant of large arrays is to be calculated. This drawback can result in drawing the wrong conclusions about whether a matrix is singular or not.
- **The calculation of the rank of the matrix:** The rank of a matrix is the maximum number of independent rows or columns. In square matrices of size $N \times N$ (such as the algebraic Jacobian that concerns this masters thesis), the matrix is non-singular if and only if its rank is N .
- **Singular Value Decomposition (SVD) [28]:** The closer the minimum singular value ($\min(\text{SVD})$) of a matrix is to zero, the closer the system is to singularity. This approach is widely used in voltage stability analysis for the purpose of investigating Jacobian singularity at the loadability limits [29].

As it has been outlined before, MATLAB is the computational environment used in this master thesis. MATLAB offers algorithms that can implement the aforementioned techniques to check for the singularity of a matrix. However, the SVD is the tool that is used as a basis for the remaining approaches. Hence, the min(SVD) will be employed here to detect if the algebraic Jacobian becomes singular somewhere along a trajectory.

5.3.1 Results: TDS and VIR

When it comes to connecting the non-convergence of TDS with the intersection of a post-fault dynamic trajectory with a VIR, this master thesis also focuses on studying the mechanisms that result in losing the stability in the short-term time scale. The two (in)stability mechanisms of importance here are short-term Rotor Angle Stability (RAS) (or transient stability) and the voltage instability that occurs as a result of the loss of voltage causality along a power swing. In transient stability, more stressed loading conditions typically lead to lower critical clearing times (t_{cc}). The critical clearing time is the maximum time for which a disturbance can be applied without the system losing its stability. Since it is expected that more stressed loading conditions also lead to VIR enlargement, critical clearing times could decrease even more due to VIR existence. For Case 102, the following three-phase short circuits are simulated:

- Fault at the generator terminal bus (bus 3):** In this case, the largest t_{cl} that provided successful TDS was $t_{cl} = t_{cc} = 0.17$ (s). This trajectory is plotted in Fig. 5.13 along with VIR_{flag} and the unstable trajectory which was obtained for $t_{cl} = 0.18$ (s). From this figure it can be observed that once the post-fault trajectory enters VIR that was predicted by VIR_{flag} , TDS fails to converge. The total duration of the TDS was 5 (s) for both the unstable and the stable trajectories. If the SVD analysis of the algebraic Jacobian is performed for the relative rotor angle space, the min(SVD) becomes zero exactly at the border of VIR, as shown in Fig. 5.14. Figure 5.14 also depicts the min(SVD) along the unstable post-fault trajectory. At the point where the algebraic Jacobian becomes singular, the TDS program fails to converge. Although the integrator of the TDS continues working after this point, the further results provided by the TDS are not a reliable representation of what would have happened in a real post-fault operation of a power system.


 Figure 5.13: VIR_{flag} and TDS trajectories (Case 102, fault at bus 3)

 Figure 5.14: Minimum Singular Value ($\min(\text{SVD})$) of the Jacobian (Case 102, fault at bus 3)

- **Fault at a pure consumption bus (bus 6):**

A three-phase short circuit has been carried out at bus 6. In this case, the system was able to remain stable for $t_{cl} = 0.105$ s.

In the beginning of Section 5.1, the comparison between Cases 202, 102 and 101 was used to show how *VIR* enlarges whenever the total amount of non-linear load increases. Now, the three-phase short circuits at buses 3 and 6 are also studied for the same cases in order to check how their t_{cc} changes with respect to the percentage of non-linear load. The t_{cc} results for Cases 202, 102 and 101 are included and compared in Table 5.5.

t_{cc} (s)		Case		
		202	102	101
% of non-linear load		39.6 %	68.2 %	100 %
Three-phase short circuit at bus	3	0.21	0.17	0.1
	6	0.15	0.105	0.103

Table 5.5: Critical clearing times (Cases 202, 102 and 101)

If the results from Table 5.5 are analyzed, it can be concluded that the increase in the total amount of non-linear load makes the critical clearing times decrease. With an increased portion of non-linear load, *VIR* enlarges, i.e. it approaches the post-fault equilibrium point such that voltage collapse is more likely to happen.

This chapter has analyzed the interaction of multiple static, exponential loads on *VIR*. The effect of the exponential load models on *VIR* has been studied both individually (each load is just of CP, CI or CC type) and in a combined way (with the ZIP load model). Also, it has been demonstrated that the TDS fail when a post-fault trajectory intersected with *VIR*. The final chapter of this document will summarize the main conclusions of the research. It will also present proposals for future studies.

Chapter 6

Conclusions and future work

Voltage instability problems are more and more frequent nowadays due to the increase in the loading of the power networks. Therefore, the dangerous, voltage-instability-induced situations cannot be underestimated and greater efforts should be put on studying the mechanisms of voltage collapse. Although there is a large amount of research that deals with voltage stability from the static point of view, the investigation of [5] is one of the few that tackled the dynamic voltage collapses that relate to non-linear modeling of static loads. The major contribution of [5] was that it asserted the influence of loads on the voltage collapse areas called Voltage Impasse Regions (*VIR*), and this master thesis has continued the research in that direction. The work of this master thesis is mostly academically oriented as it provides a new methodology for mapping the *VIR* when there are multiple static, exponential loads of non-linear type. However, such a study was necessary to be carried out before moving towards practical implications of *VIR* in large-scale power systems. This final chapter sums up the contributions and the main conclusions drawn in this master thesis (in Section 6.1). As a closure of the thesis, Section 6.2 identifies possible research challenges and proposes a plan for the future work.

6.1 Conclusions

This master thesis can be partitioned into two large segments with respect to the nature of voltage collapse. First, Chapter 3 revises the concepts of static voltage collapse. After that, Chapters 4 and 5 go deeper into the topic of voltage collapse and *VIR* appearance from the dynamic point of view. Regarding the static approach to voltage collapse, the conclusions of this master thesis are summarized as given below.

- Chapter 3 calculates the loadability limits of the considered test system by smoothly increasing the load demands while simultaneously solving the power flow equations by the means of the Newton-Raphson numerical method. An increase in the loading of the power system results in a drop of the voltages at the system buses. It has been demonstrated that, for the considered test system, the loadability limit is reached at lower loading levels if the loads are increased individually. The methodology used for calculating the loading margin is then employed to find several interesting "PU surfaces". These surfaces are built by increasing the loads in multiple directions.

In this way, the voltage collapse is associated with the failure of the numerical method to find a solution of the power flow equations at the loadability limit. This master thesis computes the loadability limits in order to ensure that the system is not operating too close to one of them. The aim is to guarantee that a voltage collapse situation triggered by the entrance of a dynamic trajectory into *VIR* is not mistaken for reaching the loadability limit of the power system.

The authors in [5] were the first to perform a detailed analysis on the nature of the "impasse surfaces" with respect to the static modeling of the loads. For such a purpose, [5] developed the analytical expressions to distinguish the areas where the voltage causality is lost (*VIR*) from those areas where the voltage causality is maintained. Those specific expressions allow to locate *VIR* for power systems where only one load is given by a non-linear exponential model. By continuing with the work from [5], the main contributions from Chapters 4 and 5 (regarding the dynamic aspects of voltage collapse) follow in the sequel.

- Chapter 4 presents and explains the quasi-dynamic methodology that is proposed in this master thesis to map *VIR* (in the relative rotor angle space). Such a novel approach has been capable to locate *VIR* accurately enough regardless of the number of non-linear loads.
- Chapter 4 also presents a new discrete index called Voltage Impasse Region Flag (VIR_{flag}) that allows to detect whether the algebraic part of the DAE system is solvable. By using VIR_{flag} , the relative rotor angle space is transformed into a matrix

where $VIR_{flag} = 1$ means that the numerical method has succeeded at a certain post-fault location, and $VIR_{flag} = 0$ is identified as the VIR (no voltage solution was found in that case). Chapter 4 also compares the performance of the newly introduced, discrete index VIR_{flag} with the continuous Voltage Collapse Indicators (VCI s) that were analytically defined in [5].

- In Chapter 5, the proposed quasi-dynamic approach has been used to study the cumulative effect of the static, non-linear loads on VIR . From the cases analyzed, it can be concluded that the Constant Power (CP) load has the largest impact on VIR (no matter if the CP load was employed individually or as a part of the ZIP load model). The inclusion of a larger amount of CP loads into a power system results in the increased size of the VIR . The VIR also grows along with the amount of the non-linear load. Additionally, it has been demonstrated that the Constant Impedance (CI) loads have an indirect effect on VIR when other non-linear loads are present in the power system. Larger VIR implies that these "no voltage solution areas" interfere more with the short-term Rotor Angle Stability (RAS) region. Then, the chance of losing the stability after a certain disturbance is higher in comparison with those situations where the boundary of VIR stays further away from the boundary of the RAS region.
- Chapter 5 has demonstrated (by simulating three-phase short circuits at different buses of the considered test system) that TDS fail to converge after a post-fault dynamic trajectory intersects with VIR . It is clearly shown how the algebraic Jacobian becomes singular (the $\min(\text{SVD})$ drops to zero) at the point along the trajectory where the solver is unable to find a solution of the algebraic equations.

The newly proposed quasi-dynamic methodology is a useful tool for identifying the VIR in the state-space. Whenever there is more than one non-linear load in the system, it becomes difficult to calculate the analytical expressions of VIR (as it was done in [5]). In those cases, the quasi-dynamic approach seems to be a good alternative for locating the VIR in the relative rotor angle space. In the quasi-dynamic approach, the state variables are artificially changed (these changes replace the actual dynamics of the differential part of the DAE model). The challenges at this point are: (i) how to make a proper choice of the numerical method that will be used to solve the set of the algebraic equations and; (ii) how to make the best initial guess for the voltage solutions. It is true that the quasi-dynamic approach is more robust compared to the analytical expressions from [5] (since the quasi-dynamic methodology allows to define the VIR for multiple non-linear loads). However, the discrete index VIR_{flag} does not provide such a clear idea about the proximity to a VIR as the VCI s from [5] do. On the other hand, the disturbances develop anyhow too fast to check for the proximity to a VIR during the disturbance itself. It is more important to know whether or not a VIR surrounds a post-fault equilibrium (and VIR_{flag} can provide this information) than to know the exact distance to VIR .

6.2 Future work

This master thesis has detected some research challenges related to *VIR* identification in the relative rotor angle space. There are some interesting research directions that are planned to be followed in the future:

- The study performed in Chapter 5 (that deals with the cumulative effect that non-linear loads have on *VIR*) mentions the impact of the topological location (in the system network) of the non-linear loads. However, it would be interesting to do a more extensive and structured investigation about the relationship between the non-linear load location and *VIR* properties.
- The numerical routine used in the thesis for solving the algebraic equations of the DAE system was `fsolve`. This routine from MATLAB has proved to be effective when identifying the *VIR* by the means of VIR_{flag} of the proposed quasi-dynamic approach. However, `fsolve` only provides one solution. A task for the future study could be to implement other numerical methods that can both perform well in finding *VIR* and that can also provide the upper and the lower voltage solutions. Such a method would be even more robust and it would also provide a better insight into *VIR*-related phenomena.
- The final aim is to understand the consequences of *VIR* existence for the case of large-scale power systems with more realistic modeling of the system components.

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