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Comparative analysis of profitability and volatility of AI, QC and mixed companies using fractional integration☆

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ABSTRACT

The growth of artificial intelligence (AI) and quantum computing (QC) firms has heightened interest in the financial behavior of these emerging technology sectors. This study applies fractional integration techniques to analyze the persistence of daily returns and volatility in companies specialized in AI, QC, and hybrid AI–QC activities. The results indicate that AI and QC firms exhibit similar degrees of long memory and deterministic behavior in returns, while hybrid companies display less persistent patterns. Volatility, in contrast, shows higher persistence across all groups, yet without significant structural differences. These findings provide novel evidence on the long term dependence underlying daily financial dynamics in high innovation industries, and highlight the usefulness of fractional models for characterizing the stability and risk profile of firms operating in transformative technological domains.

1. Introduction

Artificial Intelligence (AI), understood as the emulation of cognitive processes using advanced algorithms, and Quantum Computing (QC), based on the principles of quantum mechanics, are disruptive technologies which are currently impacting a number of sectors, including healthcare, logistics, energy, education, defense and finance. The convergence of artificial intelligence (AI) and quantum computing (QC) has attracted growing interest due to its potential to redefine the way in which highly complex computational problems are tackled. In this regard, Ahmadi (2023) notes that combining the machine learning capabilities of AI with the computational power of QC could transform strategic areas such as optimization, cryptography and drug discovery by substantially expanding the available solution space. From an economic and business perspective, AbuShanab (2024) argues that this technological

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synergy can enhance the competitiveness and resilience of small and medium-sized enterprises by providing more accurate analytical tools to manage uncertainty and anticipate scenarios of sustained recovery. At the same time, the interaction between AI and QC poses new challenges in terms of cybersecurity, particularly regarding the evolution of cryptographic schemes underpinning financial transactions and international capital flows (Radanliev, 2024). Rebentrost, Mohseni and Lloyd (2014) integrated binary classification algorithms with QC to expedite the processing of large data volumes, with applications in financial fraud detection and medical diagnostics. Meanwhile, Cao et al. (2019) examined how AI and QC algorithms enable the simulation of complex molecules, contributing to the design of new drugs and materials.

The financial sector is being transformed by the integration of AI and QC (Garg et al., 2025), enhancing predictive analytics and thereby facilitating better informed decisions (Dolu-Surabhi, 2024). Technologies such as blind quantum computing (BQC) and quantum neural networks (QNNs) enable the analysis and anticipation of stock market behavior (Iyer and Bakshi, 2024). These new technologies have given rise to new financial assets characterized by high volatility, whose dynamics challenge traditional models and demand a more detailed analysis of their temporal behavior.

The return and volatility of financial assets have been extensively examined using linear models such as the CAPM (Sharpe, 1964; Lintner, 1965; Mossin, 1966) and its multifactor extensions (Fama and French, 1993, 2015; Carhart, 1997), as well as with more advanced approaches such as ARCH/GARCH (Engle, 1982; Bollerslev, 1986). Recent years have seen the adoption of a long memory approach with fractional integration, as introduced by Granger (1980), (1981), Granger and Joyeux (1980) and Hosking (1981). Models such as the Autoregressive Fractionally Integrated Moving Average (ARFIMA), introduced by Sowell (1992) and the Fractionally Integrated Generalized Autoregressive Conditional Heteroskedasticity (FIGARCH) model of Baillie et al. (1996) permit the modelling of persistence in both mean and variance and are extensively used in financial analysis.

Given the growing interest in AI and QC firms, characterized by high expected returns and high risk, it is important to determine whether their returns and volatility series exhibit long memory, as posited by Baillie (1996) and Gil-Alana (2004), enabling a deeper understanding of long term dependence in these markets. In this context, the present study proposes to analyze the temporal dynamics of returns and volatility of AI and QC firms using fractional integration techniques with a deterministic trend. This approach enables the detection of persistent patterns without imposing rigid structures, thereby facilitating a better understanding of the financial framework of this emerging sector and providing useful tools for investors, analysts and economic policymakers.

The article is structured as follows: Section 2 provides a review of the literature on long memory in finance and its application to technological industries; Section 3 describes the fractional integration methodology, the data (dates, firms, sources) and the estimation procedure; in Section 4, the empirical results are presented, and Section 5 provides an analysis of the implications of our findings and avenues for future research.

2. Literature review

In their study of financial time series, Box and Jenkins (1976) applied ARIMA (AutoRegressive Integrated Moving Average) models to the dynamics of prices and returns, laying the foundations for short term return modelling. Observing that return variances were not constant over time, Engle (1982) introduced the ARCH (Autoregressive Conditional Heteroskedasticity) model to capture volatility clustering, and Bollerslev (1986) generalized this concept in the GARCH model, thereby allowing a more flexible representation of volatility persistence. Taylor (2008) developed stochastic volatility (SV) models as an alternative to the GARCH approach, while Poon and Granger (2003) conducted systematic comparisons of these models to improve the forecasting of market risk. Chatterjee et al. (2022) compared GARCH, GJR-GARCH (Glosten–Jagannathan–Runkle Generalized Autoregressive Conditional Heteroskedasticity) and EGARCH (Exponential Generalized Autoregressive Conditional Heteroskedasticity) with an LSTM (Long Short Term Memory) recurrent neural network in India's banking, technology and pharmaceutical sectors, finding that the relative efficacy of LSTMs versus GARCH models varies by industry, combining the networks' non-linear capacity with the asymmetry detection of EGARCH/GJR-GARCH.

On the other hand, in 1980, Granger (1980) introduced the concept of fractional integration, demonstrating that applying real-order differences d captures long-term dependencies in the data. Hosking (1981) formalized the fractional differencing operator $(1-L)^d$, thereby laying the foundations of the ARFIMA(p,d,q) model. In the financial sphere, Ding, Granger and Engle (1993) confirmed that series of absolute returns $|r_t|^k$ exhibit high autocorrelations even at very long lags, particularly as k approaches 1. To model conditional variance with long memory, Baillie, Bollerslev and Mikkelsen (1996) developed the FIGARCH model, which incorporates a fractional parameter into GARCH dynamics and produces a hyperbolic decay of volatility autocorrelation. Variants such as FIEGARCH and FIAPARCH (Fractionally Integrated Asymmetric Power ARCH) have become standard tools for markets exhibiting prolonged volatility clustering. Lahmiri and Bekiros (2021), employing ARFIMA for returns and FIGARCH for volatility across 45 cryptocurrencies and 16 international stock exchanges, found a significant increase in persistence during the COVID-19 pandemic.

Other authors, such as Tripathy (2022), analyzed the BRICS emerging markets (Brazil, Russia, India, China, South Africa) using GARCH, APARCH, ARFIMA and FIGARCH using daily data from 2000 to 2019, confirming dual long memory in mean and variance and emphasizing the implications for long term risk management. Joshi (2012) applied ARFIMA to returns and FIGARCH to the volatility of India's BSE and NSE indices, revealing long memory in volatility, more pronounced in the BSE, but not in returns.

A growing strand of the literature has applied fractional integration and fractional cointegration techniques directly to financial markets. Bollerslev, Osterrieder, Sizova and Tauchen (2013) analysed the long run relation between risk and return using intraday data for the S&P 500 and the VIX, showing that persistent components contain relevant predictive information for the dynamics of financial markets. In a related context, Caporale, Gil-Alana and Orlando (2015) examined linkages between the US and European stock markets using a fractional cointegration framework, providing evidence that long run dependence can also characterize relationships across

markets. These studies reinforce the usefulness of fractional methods for analysing persistence, dependence and predictability in financial markets, and provide a direct background for the present study.

A number of early studies provided evidence of long memory in financial markets: Diebold and Rudebusch (1989) detected it in the aggregate production in the United States; Lo (1991) estimated an order of $d \approx 0.3$ in the NYSE's monthly returns; Cheung, (1993) identified it in Asia-Pacific exchange rates and equity indices. Collectively, the literature confirms that fractional integration, both in its pure I(d) form and within ARFIMA/FIGARCH frameworks, is well suited to characterizing the long term dynamics of returns and volatility. However, to date no studies have systematically compared the behavior of the AI and QC sectors using these techniques, thereby justifying our analysis of their long term stability, predictability and risk profile.

Previous studies have focused on the application of ARIMA models and neural networks to forecast asset prices for future periods, as well as on comparing the various algorithms employed, including studies by Xing, Yan and Xie (2024) and Chlebus, Dyczko and Woźniak (2021), among others. This gap presents an opportunity to explore the stability and predictability of returns and volatility in these firms using fractional integration, thereby providing a deeper insight into their maturity and risk profile.

3. Methodology and data

3.1. Data

For data acquisition, the Python library *yfinance* was employed to download historical data from Yahoo Finance for the companies under study: Alphabet Inc. (Google), Microsoft Corporation, International Business Machines Corporation, Amazon.com, Inc., NVIDIA Corporation, Honeywell International Inc., IonQ, Inc., Rigetti Computing, Inc., Intel Corporation and Quantum Computing Inc.

The companies have been classified into three categories, as shown in Table 1, according to sector: Artificial Intelligence, Quantum Computing and firms operating in both sectors (although this classification may be subject to change in future owing to new corporate strategic decisions). The study period spanned from 1 January 2022–31 December 2024, daily, comprising a total of 752 observations per time series.

For each asset, two time series were computed: daily return and short term volatility. Daily return was defined as the difference in logarithms of the closing price, calculated as $R_t = \ln(P_t) - \ln(P_{t-1})$, where P_t denotes the closing price on day t. This formulation allows the relative price change to be expressed additively, which is particularly useful in time series analysis, since log-returns can be summed directly to obtain returns over longer horizons (Campbell, Lo and MacKinlay, 1997). The use of logarithmic returns offers several widely recognized theoretical and practical advantages in the financial literature. First, their symmetric behavior with respect to positive and negative variations enhances statistical interpretation compared with arithmetic returns, which may be distorted by large fluctuations (Cont, 2001). Second, log-returns tend to exhibit distributions closer to normality, favoring their application in models that require assumptions of homoscedasticity or residual normality (Tsay, 2010).

Short term volatility was computed as the rolling standard deviation of returns over a five-day window. This five-day window corresponds to a trading week, as these assets do not trade at weekends. In this sense, the resulting measure should be interpreted as a proxy for historical volatility over a short horizon based on recently observed returns, rather than as an alternative to indicators that are forward looking such as the VIX. This choice is consistent with the literature distinguishing between daily, weekly and monthly volatility components, where five-day horizons are commonly used to capture short-run persistence and rapid risk repricing (Corsi, 2009; Patton and Sheppard, 2015; Asgharian, Christiansen, and Hou, 2023). Table 2 presents a selection of statistical measures that illustrate the behavior of the analyzed series over time.

The comparative analysis of firms in the Artificial Intelligence, Quantum Computing and hybrid sectors reveals distinct risk–return profiles. AI companies stand out by combining positive average returns with contained volatility, suggesting an efficient performance that appeals to investors who are averse to risk. In contrast, quantum computing firms display high dispersion in both returns and volatility, reflecting their emergent and speculative nature. Hybrid companies exhibit greater stability, albeit with more modest returns. This aligns with the findings of previous studies linking technological maturity to lower risk and more predictable returns (Bharath and Dittmar, 2010; Hou et al., 2021).

Table 1
Companies and Primary Sector of Activity.

Ticker	Company Name	Primary Sector
GOOGL	Alphabet Inc. (Google)	Artificial Intelligence (AI)
MSFT	Microsoft Corporation	Artificial Intelligence (AI) and Quantum Computing (QC)
INTC	Intel Corporation	Artificial Intelligence (AI) and Quantum Computing (QC)
AMZN	Amazon.com, Inc.	Artificial Intelligence (AI)
NVDA	NVIDIA Corporation	Artificial Intelligence (AI)
HON	Honeywell International Inc.	Quantum Computing (QC)
IONQ	IonQ, Inc.	Quantum Computing (QC)
RGTI	Rigetti Computing, Inc.	Quantum Computing (QC)
QUBT	Quantum Computing Inc.	Quantum Computing (QC)
IBM	International Business Machines Corp.	Quantum Computing (& AI)

Note: IBM is predominantly quantum-focused and will therefore be classified within that group. Microsoft and Intel are strong in AI, with ongoing exploration and research in quantum computing; for this reason, they will be included in the AI group.

Table 2
Statistical measures.

Company	Average Profitability	Average Volatility	Max Profitability	Min Profitability	Max Volatility	Min Volatility	Category
AMZN	0.000347207	0.020850532	0.1269	-0.1514	0.0766	0	AI
GOOGL	0.00037141	0.018428059	0.0973	-0.0999	0.0601	0	AI
NVDA	0.002019415	0.030980959	0.2181	-0.1054	0.1046	0	AI
HON	0.000206117	0.0115625	0.0437	-0.0792	0.0393	0	Quantum
IONQ	0.001234707	0.052146338	0.2957	-0.1855	0.1598	0	Quantum
QUBT	0.002145346	0.059796011	0.6562	-0.5284	0.4525	0	Quantum
RGTI	0.00067008	0.067850399	0.4577	-0.4634	0.2614	0	Quantum
INTC	-0.00121369	0.023674301	0.1013	-0.3019	0.1408	0	Both
MSFT	0.000350532	0.015698005	0.0791	-0.0803	0.0471	0	Both
IBM	0.000817154	0.011373404	0.0906	-0.0861	0.0458	0	Both

This table presents the main descriptive statistical measures, including the mean, maximum, and minimum values of the data. Additionally, the data are classified and organized according to their corresponding categories.

These behavioral differences among the distinct company groups are displayed graphically in Figs. 1 and 2, using the mean values of the return and volatility series for each group as a reference.

Fig. 1 illustrates how the distribution of returns and volatility by technological group reflects key structural differences between the sectors. Quantum Computing firms exhibit high dispersion in both returns and volatility, evidencing a speculative profile characteristic of early-stage technologies. In contrast, Artificial Intelligence firms display a more compact and stable distribution, associated with

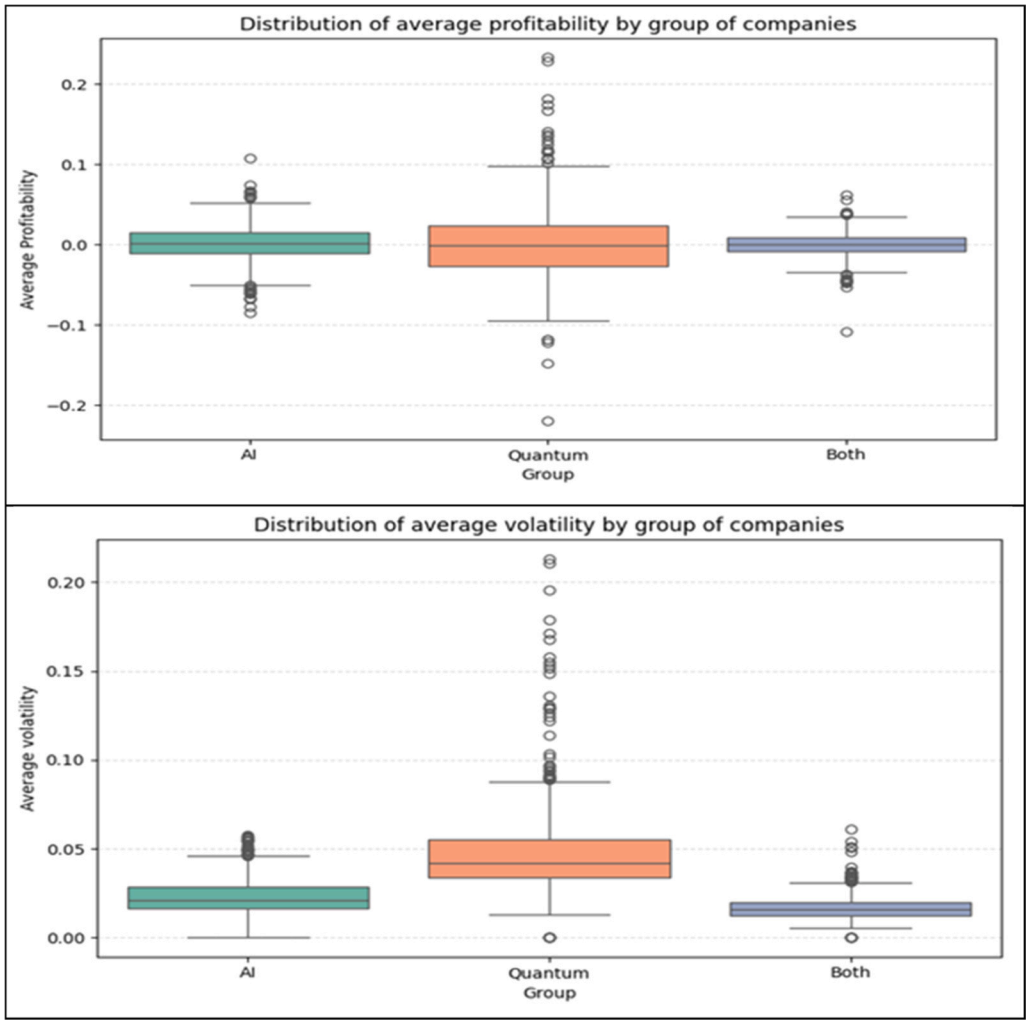

Fig. 1. Boxplot by company group. A combined plot is presented showing boxplots of profitability and volatility respectively, based on the mean values of companies grouped by category.

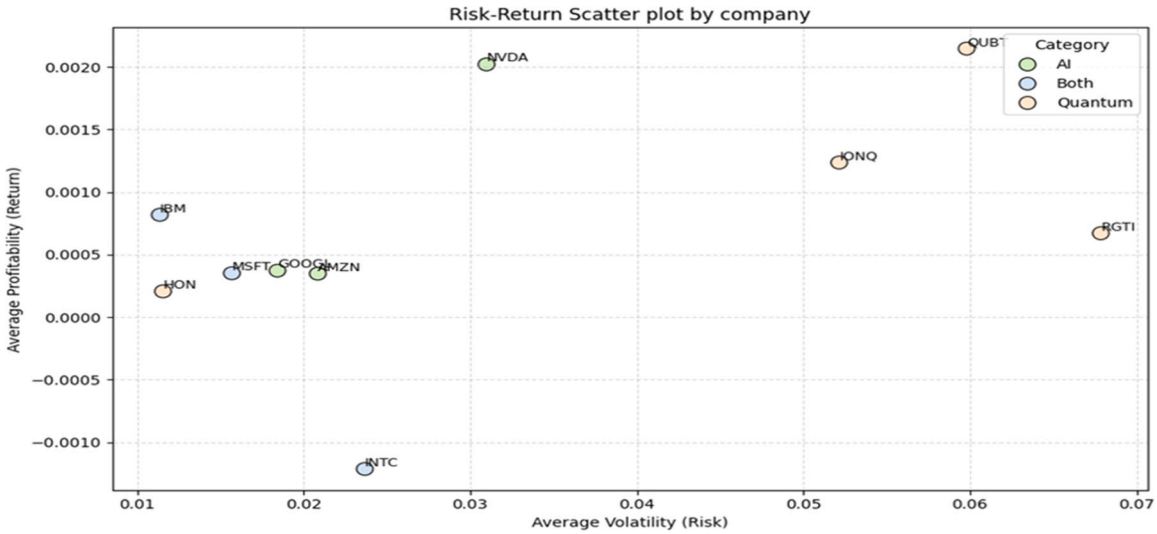

Fig. 2. Risk-return dispersion graph. A scatter plot is presented showing the relationship between profitability and volatility for each company. Companies are distinguished by groups, with a different colour assigned to each category.

their greater maturity and market adoption. Hybrid companies occupy an intermediate position on both measures. This pattern aligns with studies on the relationship between technological uncertainty and dispersion in financial performance (Pástor, Veronesi, 2009, Gu, Kelly and Xiu, 2020).

The risk–return chart (Fig. 2) reveals notable differences among companies based on their technological classification. Quantum Computing firms continue to exhibit higher volatility, although Honeywell International (HON) is an exception. By contrast, NVIDIA (NVDA), of the AI cohort, delivers significantly high returns alongside elevated risk, marking it as a high return yet high uncertainty option. QUBT, in the Quantum category, achieves the highest return of all, although with correspondingly high volatility. Firms combining both technologies, such as Microsoft (MSFT) and IBM, tend to occupy zones of lower risk and moderate returns, which may reflect more balanced or diversified strategies.

Although the visual analysis of the data through boxplots (Fig. 1) and scatterplots (Fig. 2) suggests greater dispersion in average return values within the quantum computing group, the statistical analysis does not support the existence of significant differences in mean return levels across the three technology groups. Specifically, the ANOVA test applied to the average daily returns revealed no statistically significant differences among the groups ($F = 0.288$; $p = 0.750$). This indicates that, despite differing internal variability, the mean returns of the groups do not differ significantly from a statistical standpoint.

In contrast, the analysis of average volatility (Fig. 2) reveals differences in both dispersion and mean levels across the groups. In this case, the ANOVA test yields a highly significant result ($F = 792.33$; $p \approx 0.000$), indicating statistically significant differences in average volatility among the three groups. Notably, companies in the quantum computing group exhibit greater variability and a higher average level of volatility compared to the other groups, suggesting a distinctly differentiated risk profile.

In order to validate these identified asset patterns, data from the period January to May 2025 were incorporated. Monthly averages of returns and volatility were calculated for each group of firms (QC, AI, and Both). The results indicate that, during this subperiod, QC firms exhibited returns within the interval [−0.006,0.0044], notably higher than those observed for the AI [−0.0018,0.0017] and Both [−0.0012,0.0017] groups.

A similar pattern is observed for volatility: QC firms reported values within the interval [0.000034,0.0041], while the AI and Both groups showed significantly lower ranges of [0.000016,0.001] and [0.000008,0.0011], respectively. While the period under consideration is shorter, these findings are in line with previous analyses.

3.2. Methodology

In this study, a fractional model with a deterministic trend was applied to two key variables: daily returns and estimated volatility (calculated as the five-day rolling standard deviation) of firms in the artificial intelligence and quantum computing sectors. The specification adopted is as follows:

$$y_t = \alpha + \beta\, t + x_t \text{ with } (1 - L)^d\, x_t = \varepsilon_t,\ t = 1, 2, \tag{1}$$

In this model, y_t denotes the observed data returns in one instance and volatility in the other. α and β are the coefficients corresponding to a constant term and a linear time trend respectively, while x_t is a stochastic component fractionally integrated of order d, driven by a process ε_t that may be white noise or exhibit weak autocorrelation according to the Bloomfield (1973) model. The Bloomfield model is regarded as a non-parametric description of the error term under weak autocorrelation: it lacks an explicit parametric form but is defined implicitly via its spectral density function,

$$f(\lambda;\sigma^2) = \frac{\sigma^2}{2\pi}\exp\left(2\sum_{r=1}^{m}\tau_r\cos(r\lambda)\right)$$

where σ^2 denotes the variance of the error term and m is a parameter indicating the highest of the Fourier frequencies. Bloomfield (1973) demonstrated that the logarithmic form of the spectral density function described above provides an excellent approximation to the spectrum of autoregressive (AR) structures. Moreover, this model is particularly well suited to the context of fractional differencing (see also Gil-Alana, 2004, 2008). An advantage of the Bloomfield specification is that it remains stationary for all values of m, unlike the AR model, while nonetheless producing an autocorrelation function that decays exponentially to zero, just as in an AR process.

This decomposition allows for the separation of three elements with distinct dynamics: a secular trend (potentially associated with processes of market maturity, technological shifts or speculative phenomena), a long memory stochastic component (reflecting structural persistence in returns or in perceived risk), and a short term innovation term.

In the case of returns, although the conventional assumption of stationarity is maintained, it is considered pertinent to empirically test for the possible presence of long range temporal dependence, given the highly speculative nature and potential informational inefficiency of emerging technology markets. The inclusion of a deterministic trend addresses the hypothesis that returns may display systematic bias over time, whether due to sustained expectations, bubble effects or protracted correction processes.

With regard to volatility, literature has robustly documented the existence of persistence and even long memory in the dynamics of financial risk. The proposed specification enables the modelling of such persistence without recourse to parametric conditional models (such as GARCH), while also allowing for the testing of a potential deterministic trend in the evolution of these firms' risk.

The estimation is conducted via likelihood function in the frequency domain, using a specific form of a testing approach developed in Robinson (1994) and widely used in empirical applications of fractional integration (GilAlana, Robinson, 1997; Abbritti et al., 2016, 2023; etc.).

4. Results

Tables 3 and 4 show the results of the analysis of the returns with white noise errors. In Table 3 we report the estimates of d and their associated 95% confidence bands for the three standard cases of 1) no deterministic terms, 2) with an intercept, and 3) with an intercept and a linear time trend, marking in bold for each series the best specification according to the t-values of the estimated coefficients. Table 4 reports the estimated values of the model in (1) based on the selected cases. We observe in these two tables that the estimates of d are either negative or positive but, in all cases, very close to zero. This implies that the series are stationary, meaning that the short memory hypothesis cannot be rejected. A value of $d < 0$ indicates that shocks revert rapidly, whereas a positive value of d close to zero suggests that shocks persist only over a short horizon.

In the same two tables, the coefficient of the time trend is significant for the three AI series (AMZN, GOOGL and NVDA) and bears a negative sign, suggesting that this market is more mature than that of the quantum computing firms (IONQ, QUBT and RGTI), which exhibit a positive trend characteristic of a market in an early stage with high potential. Although HON has been classified as a QC company, as were the three hybrids (INTC, MSFT and IBM), none of these require deterministic terms, which may indicate that the market has yet to find a clear way to value or position them, whether as AI or QC, thereby generating a degree of structural uncertainty and, consequently, an absence of a stable trend in their behavior.

The results in Table 5 are similar to those in Table 4, where the analysis is now carried out under the assumption of autocorrelated (Bloomfield) errors, except for the hybrid company MSFT, which in this case does exhibit a significant positive trend.

Table 3
Estimates of d. White noise errors.

Artificial Intelligence			
Series	No terms	An intercept	An intercept and a linear time trend
AMZN	-0.01 (-0.05, 0.05)	-0.01 (-0.05, 0.05)	**-0.02 (-0.07, 0.02)**
GOOGL	-0.03 (-0.09, 0.03)	-0.03 (-0.09, 0.03)	**-0.05 (-0.10, 0.02)**
NVDA	0.00 (-0.05, 0.05)	0.00 (-0.05, 0.05)	**-0.01 (-0.06, 0.04)**
Quantum Computing			
Series	No terms	An intercept	An intercept and a linear time trend
HON	**0.01 (-0.04, 0.07)**	0.01 (-0.04, 0.07)	0.01 (-0.05, 0.07)
IONQ	0.01 (-0.03, 0.07)	0.01 (-0.03, 0.07)	**0.00 (-0.04, 0.06)**
QUBT	0.05 (0.00, 0.11)	0.05 (0.00, 0.11)	**0.04 (-0.02, 0.10)**
RGTI	0.04 (0.00, 0.09)	0.04 (0.00, 0.09)	**0.03 (-0.02, 0.08)**
Artificial Intelligence & Quantum Computing			
Series	No terms	An intercept	An intercept and a linear time trend
INTC	**0.03 (-0.02, 0.10)**	0.03 (-0.02, 0.10)	0.03 (-0.02, 0.10)
MSFT	**-0.05 (-0.10, 0.01)**	-0.05 (-0.10, 0.01)	-0.05 (-0.10, 0.01)
IBM	**0.04 (-0.01, 0.10)**	0.04 (-0.01, 0.10)	0.03 (-0.02, 0.10)

The reported values are the estimates of d and the values in parenthesis indicate the 95% confidence bands. We marked in bold the selected specification for each series based on the t-values (unreported) of the coefficients associated to the intercept and the time trend.

Table 4
Estimated coefficients of the selected models in Table 3. White noise.

Artificial Intelligence			
Series	d (95% band)	Intercept (t-value)	Time trend (t-value)
AMZN	-0.02 (-0.07, 0.02)	-0.0023 (-2.43)	0.000006 (1.92)
GOOGL	-0.05 (-0.10, 0.02)	-0.0016 (-2.38)	0.000005 (1.95)
NVDA	-0.01 (-0.06, 0.04)	-0.0017 (-2.33)	0.000010 (1.81)
Quantum Computing			
Series	d (95% band)	Intercept (t-value)	Time trend (t-value)
HON	0.01 (-0.04, 0.07)	---	---
IONQ	0.00 (-0.04, 0.06)	-0.0079 (-1.85)	0.000024 (2.47)
QUBT	0.04 (-0.02, 0.10)	-0.0122 (-1.73)	0.000039 (2.42)
RGTI	0.03 (-0.02, 0.08)	-0.0156 (-2.22)	0.000044 (2.74)
Artificial Intelligence & Quantum Computing			
Series	d (95% band)	Intercept (t-value)	Time trend (t-value)
INTC	0.03 (-0.02, 0.10)	---	---
MSFT	-0.05 (-0.10, 0.01)	---	---
IBM	0.04 (-0.01, 0.10)	---	---

The values in column 2 indicate the estimates of *d* and the 95% confidence interval.
Columns 3 and 4 are the estimates of the intercept and time trend (and t-values). --- indicates lack of statistical significant.

The same analysis is carried out in Tables 6 and 7 for the volatility series, which has been measured as the five-day rolling standard deviation of returns. It shows that the estimates of *d* are substantially higher than in the case of returns. Indeed, they exhibit high persistence in all cases, and the null unit root hypothesis cannot be rejected for AMZN and QUBT. For the remaining series, the estimates lie in the interval [0.5, 1), implying non-stationary processes, albeit with mean reverting behavior. Overall, under the assumption of white noise errors, volatility displays non-stationary long memory; nevertheless, despite this non-stationarity, there is no discernible upward or downward trend in the series, indicating that the market does not perceive a systematic evolution of risk, even though a turbulent phase does not dissipate rapidly.

In Table 7, the results of the volatility analysis are presented under the assumption of autocorrelated (Bloomfield-type) errors. The first observation is that the estimates of *d* are markedly lower than in the case where errors are treated as white noise. Under the white noise assumption, the value of *d* absorbed much of the dependence. By accounting for autocorrelation, a cleaner estimate of the integration order is obtained. Correct specification of the error structure is therefore crucial for interpreting the order of integration. We also observe that, when allowing for autocorrelated errors, certain series, AMZN, HON and MSFT, show a deterministic trend with a negative coefficient, while RGTI also displays a trend, although with a positive coefficient (Table 7). Unlike the returns analysis, there is no clear behavioral divergence among the different company groups in this context.

5. Discussion and conclusions

The first conclusion of this work is that the model specification on the error term is important in the analysis of the data, particularly those related to series volatility. The results of the return analysis remain stable under different error term specifications, whereas those for volatility change markedly. This indicates that return behavior is more stable and thus provides clearer market signals. Volatility, by contrast, is more complex and sensitive to how the stochastic component is modelled, suggesting that perceived uncertainty is more ambiguous and depends on recent shocks and short term structures.

Secondly, the results indicate that the dynamics of returns vary across groups: firms specializing in AI and QC exhibit deterministic trends (either positive or negative), whereas hybrid firms show no clear directional pattern. This suggests that the market perceives and values return expectations differently depending on the type of technological specialization. In contrast, the analysis of volatility reveals a homogeneous temporal structure across all groups, characterized by long memory, which points to a shared perception of risk persistence. However, when examining average volatility levels, significant differences emerge, with QC firms displaying notably higher values. This apparent dissociation between temporal structure and average levels is further supported by the ANOVA results applied to monthly averages, which show no statistically significant differences in returns, but do reveal significant differences in volatility. These findings underscore the importance of analyzing financial behavior from multiple dimensions.

Thirdly, the case of hybrid firms stands out: their ambiguous return behavior (lack of trend) is not accompanied by lower risk exposure. This combination may hinder their positioning in the market, as they do not offer a clearly favorable return–risk profile compared to the other groups.

On the whole, the results suggest that the market has a clearer perception of expected returns, distinguishing them by technological specialization. In contrast, with respect to risk, although differences in average volatility levels are observed, the common temporal structure indicates greater uncertainty about its dynamic evolution.

It has been demonstrated that the error structure is crucial for estimating and interpreting the value of *d* in volatility series, whereas in the case of returns, the results remained stable. Under the assumption of autocorrelation, distinct patterns in volatility evolution may emerge among firms, an effect absent when errors are treated as white noise. This sensitivity of volatility to the error term specification constitutes a significant limitation, implying that findings on volatility dynamics must be interpreted with caution. Future studies incorporating models combining long memory and conditional heteroscedasticity could provide a more precise and robust

Table 5
Estimated coefficients of the selected models with autocorrelation.

Artificial Intelligence			
Series	d (95% band)	Intercept (t-value)	Time trend (t-value)
AMZN	-0.03 (-0.01, 0.06)	-0.0022 (-1.90)	0.000007 (1.99)
GOOGL	-0.14 (-0.21, −0.05)	-0.0015 (-2.18)	0.000005 (3.02)
NVDA	0.00 (-0.07, 0.08)	-0.0017 (-2.69)	0.000010 (1.73)
Quantum Computing			
Series	d (95% band)	Intercept (t-value)	Time trend (t-value)
HON	-0.06 (-0.15, 0.04)	---	---
IONQ	0.01 (-0.06, 0.10)	-0.0079 (-1.77)	0.000025 (2.33)
QUBT	-0.05 (-0.11, 0.02)	-0.0114 (-2.52)	0.000035 (3.41)
RGTI	0.05 (-0.03, 0.12)	-0.0157 (-2.01)	0.000045 (2.52)
Artificial Intelligence & Quantum Computing			
Series	d (95% band)	Intercept (t-value)	Time trend (t-value)
INTC	-0.02 (-0.09, 0.09)	---	---
MSFT	-0.12 (-0.18, −0.04)	-0.0007 (-2.00)	0.000003 (1.83)
IBM	0.00 (-0.09, 0.09)	0.0008 (1.91)	---

The values in column 2 indicate the estimates of *d* and the 95% confidence interval.
Columns 3 and 4 are the estimates of the intercept and time trend (and t-values). --- indicates lack of statistical significant.

Table 6
Estimated coefficients on the volatility series. White noise.

Artificial Intelligence			
Series	d (95% band)	Intercept (t-value)	Time trend (t-value)
AMZN	0.96 (0.86, 1.06)	---	---
GOOGL	0.84 (0.75, 0.93)	0.0215 (3.84)	---
NVDA	0.77 (0.69, 0.86)	0.0319 (3.70)	---
Quantum Computing			
Series	d (95% band)	Intercept (t-value)	Time trend (t-value)
HON	0.82 (0.72, 0.92)	0.0128 (3.59)	---
IONQ	0.87 (0.79, 0.96)	---	---
QUBT	1.02 (0.94, 1.11)	---	---
RGTI	0.88 (0.80, 0.97)	---	---
Artificial Intelligence & Quantum Computing			
Series	d (95% band)	Intercept (t-value)	Time trend (t-value)
INTC	0.85 (0.77, 0.95)	0.0168 (2.04)	---
MSFT	0.83 (0.74, 0.92)	0.0168 (3.76)	---
IBM	0.80 (0.72, 0.90)	0.0124 (3.04)	---

The values in column 2 indicate the estimates of *d* and the 95% confidence interval.
Columns 3 and 4 are the estimates of the intercept and time trend (and t-values). --- indicates lack of statistical significant.

Table 7
Estimated coefficients on the volatility series. Autocorrelation.

Artificial Intelligence			
Series	d (95% band)	Intercept (t-value)	Time trend (t-value)
AMZN	0.39 (0.24, 0.57)	0.0249 (8.81)	-0.000014 (-2.06)
GOOGL	0.45 (0.31, 0.61)	-0.0196 (7.86)	---
NVDA	0.52 (0.38, 0.68)	0.0326 (5.98)	---
Quantum Computing			
Series	d (95% band)	Intercept (t-value)	Time trend (t-value)
HON	0.34 (0.23, 0.49)	0.0144 (10.98)	-0.000006 (-2.17)
IONQ	0.56 (0.44, 0.75)	0.0428 (4.66)	---
QUBT	0.66 (0.55, 0.88)	0.0413 (2.27)	---
RGTI	0.60 (0.51, 0.72)	0.0166 (2.67)	0.000140 (1.94)
Artificial Intelligence & Quantum Computing			
Series	d (95% band)	Intercept (t-value)	Time trend (t-value)
INTC	0.49 (0.37, 0.67)	0.0208 (4.66)	---
MSFT	0.39 (0.27, 0.58)	0.0208 (10.42)	-0.000014 (-2.96)
IBM	0.41 (0.29, 0.57)	0.0199 (7.43)	---

The values in column 2 indicate the estimates of *d* and the 95% confidence interval.
Columns 3 and 4 are the estimates of the intercept and time trend (and t-values). --- indicates lack of statistical significant.

characterization. Additionally, the analysis presented in this paper also has further limitations. Specifically, it has focused exclusively on linear deterministic components. Nonetheless, nonlinear structures may also be accommodated within the *I(d)* framework. Examples are the Chebyshev polynomials in time, Fourier functions or neural networks. Similarly, structural breaks can coexist with the long memory properties of the data. This is important when dealing with long time series since long memory may be a spurious phenomenon caused by the presence of breaks not taken into account. Ongoing research is currently addressing these aspects to enhance the robustness and generalizability of the findings.

CRediT authorship contribution statement

Luis AlberikoGil-Alana: Investigation, Formal analysis, Data curation. **Carmen Lafuente:** Writing – original draft, Supervision, Methodology, Funding acquisition, Formal analysis, Conceptualization. **León Beleña:** Writing – original draft, Software, Project administration, Investigation, Funding acquisition, Data curation. **Alfonso Andrés Lafuente:** Writing – original draft, Visualization, Supervision, Software, Formal analysis, Data curation.

Authors statement

Alfonso Andrés Lafuente proposed the original idea; he wrote the intro and the literature review and also contributed with the conclusions.

Leon Beleña participated in the introduction, literature review and data analysis.

Luis Alberiko Gil-Alana conducted the empirical application and the interpretation of the results.

Carmen Lafuente collaborated with data analysis and interpretation of the results.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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