PROBABILISTIC BULK PRODUCTION COST MODEL BY A CONTINUATION MONTECARLO SIMULATION METHOD

Michel Rivier, Ignacio J Pérez-Arriaga

IIT, Universidad Pontificia Comillas, SPAIN.

INTRODUCTION

Probabilistic production cost models are the backbone of operation and capacity expansion studies of electric power systems. The greater availability of computation resources, the increased sophistication of the users and the emergence of new and more involved operation and planning problems, are posing very exacting requirements that cannot be satisfactorily met with the existing models. More specifically, these requirements typically include one or more of the following:

a) To account for the major uncertain factors (i.e., demand, hydrology and outages) in system operation and to reflect them in the probabilistic measures of performance of the power system (e.g., costs, reliability indices, capacity factors, etc).

b) To properly represent the chronological evolution of the power system and the associated operation decisions. This includes unit commitment considerations, limited hydro resources and quotas, and the actual system's planners response to unexpected events, such as outages, deviations of hydro inputs with respect to predicted values, or abnormal demand levels.

c) To explicitly include a reasonably accurate network representation, specifically when the particular application (e.g., the computation of wheeling costs) or the existence of active network operation constraints precisely needs this type of model.

Conventional production cost models such as Booth [1] cannot meet these requirements satisfactorily. Other more advanced models such as Rivier et al [2] (optimal DC power flow, but deterministic) or Jenkins [3] (chronological but single node) address some of these issues but not all. Only Montecarlo bulk simulation models such as MEXICO, see Dodu and Merlin [4], or preferably chronological ones such as one of the versions of SICRET, see Noferi et al [5], have all the required modelling capabilities. However, these models still take an excessive amount of time for most applications out of the possible range that may be considered.

This paper presents a novel approach that attempts to drastically reduce the computation cost of Montecarlo simulations of bulk power systems, without sacrificing modelling detail regarding network representation or chronology. The approach borrows both from techniques based on the continuation method, see Vojdani and Galiana [6], and from ideas germane to LP parametric programming, see Hax et al [7] and Luenberger [8] for instance. The method has been specifically designed to be used with power systems of large dimensions; however, so far it has been only tested at a prototype level. The emphasis of this paper is on the presentation of the theoretical aspects of the proposed approach. Extensive testing is still needed to show its computational capabilities.

An interesting additional feature of the proposed method is the consideration placed on the computation of different kinds of sensitivities, particularly those directly related with marginal costing. One of the outputs of the model is the expected value of the spot prices at each node of the network for the considered time interval.

METHODOLOGY

The Montecarlo simulation method consists of randomly generating possible scenarios or states of the system (defined by the availability status of the components, the demand level, the hydrology, etc) according to predefined probabilistic distribution functions. Then each scenario is evaluated to determine its operation cost and whether there exists any reliability problem (i.e., network violation or loss of load) before or after the appropriate corrective measures have been taken. This renders a credible representation of the possible system behavior, as well as cost and reliability measures that can be computed as easily as from a historical record of the system operation.

All existing Montecarlo simulation models must limit quite drastically the representation of the time response of the power system to changing operation conditions. This simplification not only includes network stability considerations, but also generating units start-up and ramp limitations, modifications to a scheduled unit committment, deviations from programmed water utilization, etc.

Typically, chronological Montecarlo simulation has been considered more computationally demanding than the non-chronological, or so called random, version. The reason for this is the higher number of samples that is typically required in order to reach a prescribed level of accuracy in the computation of the measures of performance of the system. This is unfortunate, since chronological simulation provides a better description of the actual system behavior, particularly for those systems with significant operation memory (i.e., coupling between different operation intervals), as a result of hydro storage or unit start-up limitations, for instance.

The core of the proposed approach is a chronological Montecarlo simulation, however it exploits the sequential nature of this simulation in order to try to reduce the computation time significantly. While in existing chronological Montecarlo simulation algorithms an optimization procedure has to be applied typically at hourly intervals in order to properly follow the operational changes in the simulated system, in the proposed method a discrete-event simulation is used to allow the algorithm to jump from one event (e.g.,

component outage or repair, start-up of a unit) to the next, therefore in principle avoiding the unnecessary costs of computing many intermediate stages.

Both the gradual changes in load and the discrete events result in changes in the optimal operation point of the power system; these changes are followed by the proposed algorithm, which detects the variables and restrictions of the LP solution that either become active or inactive. In this way repeated optimization is completely avoided, although the operation is guaranteed to be optimal at all times. The idea of taking advantage of the results of the previous hour optimization in order to facilitate the optimization of the next hour has already been suggested. This paper goes further by continuously following the optimal operation point through a wide variety of possible changes in the system conditions: load variations and failure, repair or maintenance of generation units and network components.

The power system operation is represented by an optimal power flow (OPF) with a DC network model that has been upgraded to include a piecewise linear approximation to the ohmic loss function. The resulting model is linear, therefore allowing the utilization of linear parametric techniques, see [7,8], which have been extended to take advantage of the specific characteristics of the model.

Modules that are external to this core simulation logic provide: a) the changes in the current status of the uncertain inputs, e.g., component availability, load level, hydrological inputs, etc ; b) the rules that are necessary to reproduce the operation decisions that are actually needed to follow these uncertain external inputs to the power system, see Fig. 1.

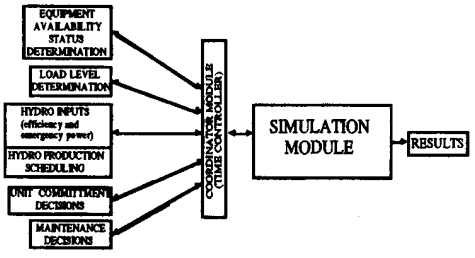


Figure 1. The simulation computer package.

THE SIMULATION ALGORITHM

THE POWER DISPATCH MODEL.

The short term operation of the power system is represented by a DC based optimal power flow with some enhancements, that include the following (see [2] for a detailed description):

i) The production schedule for each hydro plant is provided by some higher order function; therefore it is a data input to the model. However, it can be modified by an external module according to the development of the simulation, see Fig. 1. Deviations from this schedule are allowed, although any increases are penalized with a water opportunity cost; the hydro output of any plant has an upper limit, which is determined by its reservoir level.

ii) Unserved power at any bus is possible, although it is penalized with a user defined per unit cost. Due to the high value of this cost, the dispatch algorithm will have as the first priority the minimization of unserved power.

iii)The DC network model has been extended to include ohmic losses. Here, in order to avoid the iterative algorithm in [2], a piecewise linear approximation to the loss function for each bus has been employed.

It follows the mathematical formulation of the dispatch model:

$$\text{Minimize} \quad Z = \sum_{i=1}^{I} c_i g_i$$

$$\text{subject to} \quad \sum_{i=1}^{I} g_i + \sum_{i,j}^{I,J} TLFP_{i,j}(\theta_i - \theta_j) = \sum_{i=1}^{I} dm_i \qquad (1.a)$$

$$-Ym(\theta) + (g) = (dm) \qquad (1.b)$$

$$z = T(\theta) \qquad (1.c)$$

$$-\bar{z} \le z \le \bar{z} \qquad (1.d)$$

$$\underline{g} \le g \le \bar{g} \qquad (1.e)$$

where:

i: Bus number, i=1 ... I

i,j: End buses of a line.

d_i, θ_i, g_i, c_i: Voltage angle,total power output and per unit MW output cost at bus i.

dm_i: Load demand at bus i, which is modified to include a share (50%) of the ohmic losses of all the lines connected to bus i.

g, $\underline{g}$, $\bar{g}$;z,$\bar{z}$: Vectors of power outputs, max. and min. output limits at all buses; Vector of power flows and capacity limits of all the lines.

(g), (dm), (θ):Vectors of power outputs, modified load demands and voltage angles at all buses except for the slack bus s.

Y, T: Network admittance and admittance incidence bus-line matrices .

Ym: Network admittance matrix which is modified to include an ohmic loss factor.

$TLFP_{i,j}$, L: Coefficients of the linear approximation to the loss equation.

As indicated before, the objective of the proposed algorithm is to follow the operation point of the power system through any change in the already mentioned external inputs. First, it must be realized that the pair of coefficients TLFP and L are constant at each bus for any given operation point. Changes in the system operation point will eventually result in changes in these coefficients, in order to follow the piecewise linear loss function of each bus. Also note that the classical DC network equations must be modified accordingly, so that the Y-bus matrix becomes Ym = Y - 1/2 TLFP and a modified load dm = d + 1/2 L is employed.

Assume that the solution to the above optimization problem is known for some operation point. The important issue now is that changes in load level or in component availability can be represented by variations of the right hand side terms in the formulation of the preceding optimization problem.

In fact, it is easy to check that a load variation is simply represented by a change in the terms dm, while a modification in the availability status of a generation unit only implies to adapt its maximum or minimum output limit (also a right hand side term) to its new value. However, a change in the availability status of a line affects both its capacity limit and its corresponding admittance, with the latter not being represented by a right hand side term. The following trick may be used here: when two fictitious and opposite loads equal to the prefault flow in a line are added at the ends of the line, the optimal prefault and postfault operation points coincide (now with a new Ym-bus matrix). Then leading both fictitious loads to zero (which is again a variation of the right hand side term dm) results in the desired post fault optimal situation. Similarly the inclusion of a line in the system is simulated by adding two fictitious and opposite loads equal to the current value of $\gamma_{ij}(\theta_i-\theta_j)$ at the ends of the line, and then leading both loads to zero. All this can be accomplished by using LP parametric techniques.

LP PARAMETRIC PROGRAMMING

Linear programming parametric techniques for right hand side variations have been widely reported in the technical literature, see [7,8] for instance. Consider the standard LP problem in canonical form where slack variables have been added to turn all the inequalities into equalities:

$$\text{Minimize} \quad Z = c^t x$$

$$\text{s.t.} \quad A.x = b \qquad (2.a)$$

$$x \geq 0 \qquad (2.b)$$

with n+m variables (x) and m constraints. The classical solution to this standard LP problem will be briefly reviewed now. Later the proposed algorithm will be presented, which exploits some specific characteristics of the formulation of the power system model.

The optimal solution to (2) is known to reside in a vertex of the polyhedron that defines the feasible region. A vertex corresponds to n null variables (**non basic variables** x_{NB}) and m non null variables (**basic variables** x_B). To each variable x_i we assign its corresponding column-vector P_i in A; a P_i will be termed either basic or non basic at each iteration of the simplex method according to the status of x_i at that iteration.

The solution method, namely the simplex procedure, consists of moving from vertex to vertex so that Z is progressively decreased until the optimum vertex is eventually reached. Each new vertex is obtained by exchanging a basic variable for a non basic variable. Both are selected so that the objetive function Z is reduced by the exchange. The selection algorithm is based on the simplex tableau and the reduced (or relative) costs. The former contains the coordinates of all the formerly defined vectors P_i in terms of the basic vectors. The coordinates of the right hand side vector represent the current solution for x. A new vertex corresponds to a change of the basic vectors and the simplex-tableau is correspondingly updated. The reduced cost r_j for each variable x_j measures the difference betweeen its direct cost c_j and its replacement cost as a function of the basic variables. A positive relative cost for all the nonbasic vectors indicates that the vertex is the optimal one.

Let us denote by a_{ij} the coordinate of P_j for the basic vector P_i. The columns of the elements of the simplex-tableau corresponding to the slack variables represent the sensitivity of the basic variable with respect to the right hand side. That is, if the m vectors corresponding to the slack variables are successively placed after the n vectors corresponding to the remaining variables:

$$\Delta x_{Bi} = a_{i,n+k} \cdot \Delta b_k \qquad (3)$$

and these sensitivities are constant and valid as far as the feasibility of the solution is maintained, that is, as long as $\forall i,\ x_{Bi} \geq 0$. In fact, for all this interval the relative costs r_j are constant and then still positive, therefore indicating that track of the optimal point is maintained.

TIn order to go beyond the end of the interval it is necessary to determine which current non basic vector will become basic. Then it is needed to replace the basic vector whose variable has reached zero and which will become non basic.

An algorithm for selecting the correct variable can be deduced from both of the following considerations: the sensitivity of the new basic variable with respect to the right hand side variation must be positive since its initial value is zero, and all the new relative costs must also be positive (only the correct substitution will maintain all of them positive). An analysis of the updating process of the simplex-tableau leads us to chose the vector (variable) s so that

$$a_{is} < 0 \text{ and } -\frac{r_s}{a_{is}} \text{ is minimum} \qquad (4)$$

Once we have detected the variables to be exchanged in the basis, we proceed to update the simplex tableau, whose coefficients will again determine the new sensitivities with respect to the right hand side vector and so on.

Observe that, for a model such as (1), this process represents that the variables and the constraints of the LP problem become active or inactive as the right hand side changes.

REQUIRED ADAPTATIONS FOR LARGE SYSTEMS.

For large systems the computation of the whole simplex-tableau is very expensive. Then the so called reduced simplex method is employed, see [7,8] for instance, which handles only the data of the simplex-tableau that are strictly needed. For the standard LP formulation:

$$\text{Minimize} \quad Z = c^t.x$$

$$s.t. \quad A.x = b \qquad (5.a)$$

$$l \leq x \leq u \qquad (5.b)$$

where slack variables are used to convert all the constraints into equalities. By defining a non basic variable as a variable at one of its limits, (5) can be rewritten as

$$\begin{pmatrix} B & N \\ 0 & I \end{pmatrix}\begin{pmatrix} x_B \\ x_{NB} \end{pmatrix} = \begin{pmatrix} b \\ \text{lim} \end{pmatrix} \text{ where } A=(B\ N),\ \text{lim} = \begin{pmatrix} l \\ u \end{pmatrix} \qquad (6)$$

and the idea is to handle only reduced matrices such as B and N during the application of the simplex procedure. The optimality criterion is now determined by the dual variables σ of the constraints (5.b). At the optimal point σ must be positive when x=l and negative when x=u.

Since the dual variables are the optimal solution of the associated dual problem, σ is calculated as follows:

$$\begin{pmatrix} B^t & N^t \\ 0 & I \end{pmatrix}\begin{pmatrix} \pi \\ \sigma \end{pmatrix}=\begin{pmatrix} c_B \\ c_{NB} \end{pmatrix} \text{ that is } \begin{cases} \pi = B^{-1t}\, c_B \\ \sigma = c_{NB}-N^t\pi \end{cases} \qquad (7)$$

It is important to observe that the complete simplex-tableau presented in the previous subsection is reproduced by the matrix $B^{-1}.N$ except for the slight differences required to extend (2.b) to the more general formulation (5.b). We denote as a'_{ij} the elements of $B^{-1}.N$. Similarly σ is strongly related to the reduced cost vector r.

All these considerations lead to the following right hand side parametric algorithm:

1. Form the B and N matrices.

2. Calculate the sensitivities.

From (6)
$$\begin{cases} \Delta x_B = B^{-1}[\Delta b - N.\Delta lim] \\ \Delta x_{NB} = \Delta lim \end{cases} \qquad (8)$$

3.Obtain the vector leaving the basis

The above sensitivities for each basic variable determine when each one reaches its limit. Special care must be taken with the basic variables whose limits also vary. Assume it is the i-th the one vector leaving the basis.

4. Calculate a'_{ij} for all the non basic P_j.

They correspond to the i-th row or $B^{-1}.N$. Now solve $B^t.K = e_i$ with e_i being the i-th canonical vector of the basis, and then compute $K^t.N$.

5.Calculate σ_j for all the non basic P_j's as indicated in (7).

6. Obtain the vector that enters the basis.

Adapting (4) to the new terms a' and σ leads to the selection algorithm that is represented in fig. 2.

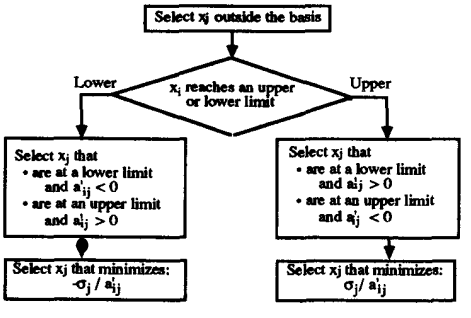


Figure 2. Variable entering the basis. Selection algorithm.

APPLICATION TO THE DISPATCH PROBLEM.

The dispatch optimization problem (1) can be stated as:

Minimize $Z = c^t.w$

s.t. $G1.w + G2.y = b1$ (9.a)

$b2 \le G3.y \le b3$ (9.b)

$lim1 \le w \le lim2$ (9.c)

We denote by w the vector of independent variables.

Solving (9.a) for y and substituting it in (9.b) renders the following formulation:

Minimize $Z = c^t.w$

s.t. $\underline{b} \le G\,w \le \bar{b}$ (10.a)

$\underline{lim} \le w \le \bar{lim}$ (10.b)

Adding the corresponding slack variables to either (9) or (10) leads to (5), so the previously described algorithm may be applied directly . However, the computational burden easily becomes significant for a realistic size system. Fortunately it can be reduced by taking advantage of the peculiarities of the problem.

The constraints (10.a) correspond to the capacity line limits and therefore they will be generally non active. Therefore the algorithm presented in the previous subsection will be modified to deal only with the active contraints and to eliminate the explicit representation of the slack variables so that the dimensions of the matrix that is strictly needed will be drastically reduced.

However, trying to deal only with the active constraints poses two problems that must be solved:

i) The sensitivities of the slack variables corresponding to non active constraints (needed to determine whether or not they reach their limits, i.e. if the corresponding constraint becomes active) are obtained through the sensitivities of both the active variables and the right hand side vector with respect to a common parameter variation.

ii) When such a slack variable vs_i is detected as leaving the basis, the process of detecting the new basic variable requires the computation of the a' values by solving $B^t.K = e_i$. Since this constraint was non active it is now ignored in the B matrix and the latter system is absurd. The solution is to add, just for this step of the algorithm, the corresponding coefficients of G (q and p) to B and N so that they become B* and N* (it doesn't affect any other step). Moreover it can be proved that solving $B^{*t}.K^* = e_i$ amounts to solving $B^t.K = -q$ and $K^{*t} = [1\ K]$, thus taking advantage of the current factorization of B.

The slack variables of the active constraints indicate when a constraint becomes active by entering the basis. We need then to obtain their coefficients a' and σ in order to apply the selection algorithm. However the analysis of their explicit representation shows that they only appear in the matrix N in the form of canonical columns, so that their corresponding a' and σ values can be recovered from the vectors K and $(-\pi)$ respectively, therefore avoiding their explicit representation.

In conclusion, it is possible to reduce the dimension of the matrices B and N drastically for this application.

Denote by x the vector of all the variables, the independent and slack ones. The resulting right hand side parametric algorithm is:

1. Form the B and N matrices with only the active constraints and ignoring the slack variables.

2. Sensitivity calculation

If we denote by Sb and Slim the sensitivities of the right hand side vector with respect to the parameter variation we obtain similarly to (8):

Solve for Sw_B: $B.Sw_B = Sb_A - N.Slim_A$ (11.a)

$Sw_{NB} = Slim_A$ (11.b)

$Svs_B = Slim_{NA} - G_{NA}.Sw$ (11.c)

$Svs_{NB} = 0$ (11.d)

where the sub-indices A and NA stand for active and non active respectively.

3. Obtain the vector leaving the basis

The procedure is the same one that was defined in the previous subsection, since we have already obtained the sensitivities of all the variables (independent and slack ones). Let be i the index of the vector leaving the basis.

4. Calculate a'_{ij} for all non basic P_j's.

According to the preceding discussion, the process is as represented in fig.3

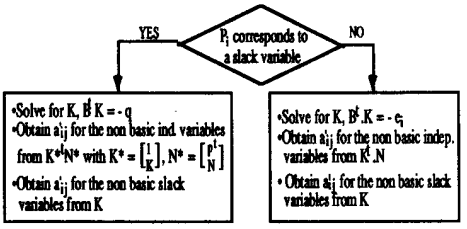


Figure 3. a'_{ij} calculation process.

5. Calculate σ_j for all the non basic P_j's.

According to the preceding discussion, the process is as follows:

Solve $B^t.\pi = c_B$

Obtain σ for the non basic independent variables through

$\sigma = c_{NB} - N^t.\pi$

Obtain σ from p for the non basic slack variables.

6. Obtain the vector entering the basis.

Same method in fig. 2, since we have obtained the a' and σ coefficients for all the non basic variables.

Finally the proposed algorithm will be explicitly applied to our problem formulation (1)

Solving (1.b) and (1.c) for θ and substituting it into (1.a) and (1.d) yields a formulation in the same format as (10):

Minimize $Z = c^t g$

$$\text{s.t.} \sum_{i=1}^{I} (1 - SPTRG_i)\, g_i = \sum_{i=1}^{I} (1 - SPTRG_i)\, dm_i \qquad (12.a)$$

$$T.Ym^{-1}(dm) - \bar{z} \le T.Ym^{-1}(g) \le T.Ym^{-1}(dm) + \bar{z} \qquad (12.b)$$

$$\underline{g} \le g \le \bar{g} \qquad (12.c)$$

Where SPTRG is solved from Ym.SPTRG=TLFT with

$$TLFT_i = \sum_{ij=1}^{IJ} TLFP_{ij}$$

The computation of the full matrix $T.Ym^{-1}$ is very expensive. However only the rows corresponding to the active contraints, that is, the saturated lines, must be calculated. The row vector α_{ij} corresponding to the line i,j is computed by solving the system

$$Ym^t.\alpha_{ij} = \gamma_{ij}.e_{ij} \qquad (13)$$

where γ_{ij} is the admitance of line i,j and e_{ij} is a vector with a dimension equal to the number of lines and such that all its elements are zero except for the i-th and j-th entries that are 1 and -1, respectively.

Remembering that we are interested in following the optimal solution for variations of the load or of the output generation limits, let us denote by Sd and Sg* their vector sensitivities with respect to a common parameter variation. Applied to the formulation stated in (12) the right hand side sensitivities become:

- $\sum_{i=1}^{I} (1 - SPTRG_i).Sd_i$ for the total balance equation.
- $\alpha.(Sd)$, with α representing the submatrix of $T.Ym^{-1}$ corresponding to the active constraints, for the line capacity limits.
- Sg* for the output generation limits.

To avoid the computation of the numerous rows of $T.Ym^{-1}$ corresponding to the inactive constraints that are necessary for the calculation of the sensitivities of their slack variables, the following process is followed:

From (1.b): solve $Ym.(S\theta) = (Sg) - (Sdm) = (Sg) - (Sd)$ (14.a)

From (1.c) and (1.d) $Svs = -T.(S\theta)$ (14.b)

where $(S\theta)$ is the vector of sensitivities of the voltage angles at all buses except for the slack.

The algorithm has the following steps:

1. Form the B and N matrices considering only the generation output variables, the total energy balance equation and the saturated line constraints.

2. Sensitivity calculation

For the power outputs (independent variables) g:

$$\text{Solve } B.Sg_B = \begin{bmatrix} \sum_{i=1}^{I} (1 - SPTRG_i) \\ \alpha(Sd) \end{bmatrix} - N.Sg^*_{NB} \qquad (15.a)$$

$Sg_B = Sg^*_{NB}$ (15.b)

For the slack variables:

Solve $Ym\,(S\theta) = (Sg) - (Sd)$ (15.c)

$Svs_B = T.(S\theta)$ (non saturated lines) (15.d)

$Svs_{NB} = 0$ (saturated lines) (15.e)

3. Obtain the vector leaving the basis.

From the sensitivities of all the variables and of all the limits, determine the first variable that reaches its limit.

The slack variables for the line flow limit constraints (1.d) are stated as:

$z + vs = \bar{z}$, so that their limits are: $0 \le vs \le 2\,\bar{z}$ (16)

Let i be the index of such a vector and i,k the corresponding line in case it is a slack vector.

4. Calculation of a'_{ij} for all non basic P_j's.

According to the previous discussion and from fig. 3.

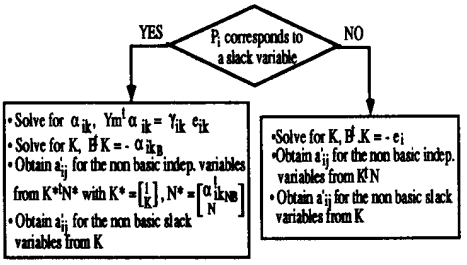


Figure 4. a'_{ij} calculation process.

5. Calculate σ_j for all the non basic P_j's.

$$\text{Solve } B^t.\pi = c_B \qquad (17.a)$$

$$\sigma = c_{NB} - N^t.\pi \qquad (17.b)$$

Obtain σ for the non basic slack variables from $(-\pi)$.

6. Obtain the vector entering into the basis.

The algorithm is represented in fig. 2.

7.Update the values of all the variables and return to 1.

The final simulation algorithm is slightly different because it has to deal with several generation units that are connected to a same node : it is supposed that they all have different production costs so that only one unit at a time is a candidate for entering or leaving the basis, therefore reducing the total number of variables appearing in the process. As explained before, the sign of a' allows one to know which generation interval must be considered as a candidate when no unit at this node is on the margin.

DISCUSSION ON THE POTENTIAL EFFICIENCY OF THE SIMULATION ALGORITHM.

The algorithm must solve one system of equations with the Ym matrix (two when a line constraint changes its activity status; a quite unfrequent event) and three systems with B which has a drastically reduced dimension (the number of saturated lines +1). Ym is factorized for each change of the system configuration (the failure/repair/maintenance of a line) or for each change of the interval losses when these are considered. In between the factorization of Ym is constant, thus greatly contributing to the efficiency of the algorithm, since Ym is large in general.

Note that updating B and N when a line reaches its capacity limit is immediate, since the row to be added is precisely the α_{ik} vector that has been already computed.

Once the computational burden of the parametric algorithm has been greatly reduced, the efficiency of the overall simulation algorithm will critically depend on the number of changes of the basis that are necessary to simulate an appropriate time interval. A priori this seems to depend on the specific power system under consideration and the theoretical comparison with the standard chronological approach may be difficult to establish.

It is apparent that a large amount of computing time a be saved if efficient heuristics are employed to reduce the candidates to be examined for entering or leaving the basis since, although the involved operations are very simple, the number of candidates may be very large for realistic power system models.

EXAMPLE

A very simple example will be used to illustrate the proposed algorithm. The system configuration is shown in Fig. 5 and Table 1 summarizes the generation and network data. For simplicity we consider a constant variable cost for the generation at each bus and ohmic losses are ignored. Also we suppose that the technical minima for all the generation units are assumed to be zero.

Table 1. Generation and network data.

Generation			Network		
Bus	Maximum output MW	Cost $/MW	Line	Suceptance p.u.	Capacity Limit MW
1	150	40	1-2	2.50	100
2	—	—	1-4	1.67	80
3	360	60	1-5	5.00	100
4	200	100	2-3	5.92	120
5	—	—	2-4	2.50	100
6	600	80	2-6	3.42	120
			3-5	10.09	220
			4-6	4.91	150

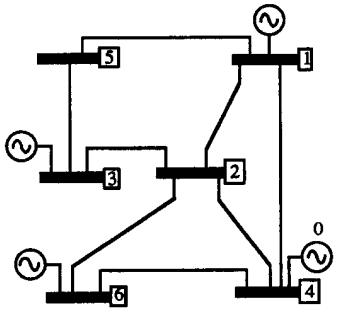


Figure 5. System configuration.

Two examples are provided. The first one illustrates the application of the algorithm for changes in load, while the second one shows how to deal with a change in the availability status of a component of the system, specifically for a line failure (or its disconnection for maintainability).

EXAMPLE 1: Simulation of operation under changes in load.

The total load demand Pd is chosen to be the independent parameter and thus the objective is to follow the optimal operating point as Pd varies. Pd will change according to the chronological load curve. In this example the optimal operating point will be followed for a specific interval of variation of Pd, e.g. from Pd= 160 MW to Pd= 752.863 MW, and the availability status of the system components will remain unchanged. As indicated before, the simulation of generation and line availability changes can be modeled by simple extensions of the techniques that are used to follow changes in load. The total load demand is distributed among the buses according to piece-wise constant load distribution factors

Sd_i. They are represented in Table 2.

Table 2. Load data.

Bus	Load for Pd=160 MW	Load Distribution Factors (Pd≤400) MW	Load for Pd=400 MW	Load Distribution Factors (Pd≥400) MW
1	11.200	0.070	28.000	0.144
2	64.000	0.400	160.000	0.222
3	16.000	0.100	40.000	0.000
4	24.000	0.150	60.000	0.277
5	44.800	0.280	112.000	0.355
6	0.000	0.000	0.000	0.000

When bus 1 is chosen to be the slack bus, the network admittance matrix Y and the admittance incidence bus-line matrix T are:

$$Y=\begin{bmatrix} 14.34 & -5.92 & -2.50 & 0.00 & -3.42 \\ -5.92 & 16.01 & 0.00 & -10.09 & 0.00 \\ -2.50 & 0.00 & 9.08 & 0.00 & -4.91 \\ 0.00 & -10.09 & 0.00 & 15.09 & 0.00 \\ -3.42 & 0.00 & -4.91 & 0.00 & 8.33 \end{bmatrix}$$

$$T=\begin{bmatrix} -2.50 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & -1.67 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & -5.00 & 0.00 \\ 5.92 & -5.92 & 0.00 & 0.00 & 0.00 \\ 2.50 & 0.00 & -2.50 & 0.00 & 0.00 \\ 3.42 & 0.00 & 0.00 & 0.00 & -3.42 \\ 0.00 & 10.09 & 0.00 & -10.09 & 0.00 \\ 0.00 & 0.00 & 4.91 & 0.00 & -4.91 \end{bmatrix}$$

The independent variables in the optimal dispatch problem are the generation outputs g_i, i=1...6 and the slack variables vs_{ij}, ij=1-2...4-6 are associated through (16) to the line flows, as presented in the previous section.

The values for all the variables corresponding to the optimal operating point for Pd = 160 MW are shown in Table 3. From them the steps of the simulation algorithm as the load grows within the specified interval are applied.

First interval ($160 \le Pd \le 380.6506$):

Step 1 Form the B and N matrices.

They are obtained by stating the active constraints in function of the non basic and basic independent variables. From Table 3 we deduce that the only constraint which is active is the total energy balance equation, which must always be active. Then

$$[1111]\begin{bmatrix} g3 \\ g1 \\ g4 \\ g6 \end{bmatrix}=\begin{bmatrix} Pd \\ \bar{g}1 \\ \bar{g}4 \\ \bar{g}6 \end{bmatrix}=\begin{bmatrix} 160 \\ 150 \\ 0 \\ 0 \end{bmatrix} \text{ that is, } B=[1] \text{ and } N=[111]$$

Step 2. Calculation of sensitivities.

For the independent variables: for the basic ones, from (15.a) with Sg*=0, α=0 and SPTRG=0, it is obtained that Sg3=1; for the non basic ones, from (15.b), $\begin{bmatrix} Sg1 \\ Sg4 \\ Sg6 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

For the slack variables, from (15.c) and (15.d):

$$(S\theta)=\begin{bmatrix} -0.0092 \\ 0.0711 \\ -0.0310 \\ 0.0290 \\ -0.0220 \end{bmatrix},\quad Svs_B=\begin{bmatrix} -0.0230 \\ -0.0517 \\ 0.1448 \\ 0.4752 \\ -0.0544 \\ -0.0439 \\ -0.4248 \\ 0.0439 \end{bmatrix}$$

Step 3. Identify the variable leaving the basis.

From the sensitivities of the non basic (independent and slack) variables, and from their limits (remembering that the lower and upper limits for vs_{ij} are respectively 0 and $2.\bar{z}_{ij}$) which do not depend on the parameter Pd, it is straightforward to prove that the first variable which reaches its limit (the upper one) is $vs_{2\text{-}3}$ for an increment ΔPd=220.6506 of Pd, when the line 2-3 reaches its capacity limit. Since Pd=380.6506 is lower at this point than 400, the 2-3 line saturation represents the next event to happen.

Step 4. Calculate $a'_{vs2\text{-}3,j}$ for all the non basic P_j's.

From Fig.4 and since the variable leaving the basis is an slack one, $Y^t.\alpha_{2\text{-}3} = \gamma_{2\text{-}3}.e_{2\text{-}3}$ is solved for:

$\alpha_{2\text{-}3}{}^t = [0.3649\ -0.4059\ 0.2664\ -0.2714\ 0.3068]$,

where $\alpha_{2\text{-}3B} = [-0.4059]$ and $\alpha_{2\text{-}3NB} = \begin{bmatrix} -0.0000 \\ 0.2664 \\ 0.3068 \end{bmatrix}$

Solving for K $B^t.K = -\alpha_{2\text{-}3B}$: $K = [0.4059]$

Then $K^* = \begin{bmatrix} 1 \\ 0.4059 \end{bmatrix}$, $N^* = \begin{bmatrix} 0 & 0.2664 & 0.3068 \\ 1 & 1 & 1 \end{bmatrix}$

and $\begin{bmatrix} a'vs2\text{-}3,g1 \\ a'vs2\text{-}3,g4 \\ a'vs2\text{-}3,g6 \end{bmatrix} = K^{*t}.N^* = \begin{bmatrix} 0.4059 \\ 0.6723 \\ 0.7127 \end{bmatrix}$

Step 5. Calculate σ_j for all the non basic P_j's.

From (17.a) with $c_B = [60]$: $\pi = [60]$

From (17.b) with $c_{NB} = \begin{bmatrix} 40 \\ 100 \\ 80 \end{bmatrix}$: $\sigma = \begin{bmatrix} {}^s g1 \\ {}^s g4 \\ {}^s g6 \end{bmatrix} = \begin{bmatrix} -20 \\ 40 \\ 20 \end{bmatrix}$

Step 6. Identify the variable entering the basis.

The following variable selection criteria are sequentially used as in Fig. 2 (upper limit case):

i) The independent variables out of the basis: g1,g4,g6

ii) { At a lower limit and with $a'vs2\text{-}3,j \ge 0$: g4,g6
   { At an upper limit and with $a'vs2\text{-}3,j \le 0$: none

iii) Finally the variable with the minimum value for $\sigma_j/a'vs2\text{-}3,j$: g6

Therefore g6 enters the basis (it will begin to generate a power output) by replacing $vs_{2\text{-}3}$. Now from the variable sensitivities it is straightforward to obtain the optimal operating point for Pd=380.6506MW; the corresponding optimal values for all the variables are shown in Table 3.

The algorithm is applied again to continue the simulation:.

Second interval.(380.6506 ≤ Pd ≤ 400)

Step 1 Form the B and N matrices

With respect to the previous situation a new constraint (the one corresponding to the saturated line 2-3) has now become active, and a new variable (g_6) is non basic. The coefficients of the new set of active constraints in function of the non basic and basic independent variables (i.e., the sensitivities of the 2-3 line flow with respect to the bus net injections) have been obtained already since they actually are the vector $\alpha_{2\text{-}3}$.

Therefore $B=\begin{bmatrix} 1 & 1 \\ -0.4059 & 0.3068 \end{bmatrix}$, $N=\begin{bmatrix} 1 & 1 \\ 0 & 0.2664 \end{bmatrix}$

Step 2. Calculation of sensitivities.

From (15) with $Sg^*=0$, $\alpha=\begin{bmatrix} \alpha_{2\text{-}3} \end{bmatrix}$, $\alpha.(Sd) = 0.069338$

and SPTRG=0 it is obtained that $\begin{bmatrix} Sg_3 \\ Sg_6 \end{bmatrix} = \begin{bmatrix} 0.3332 \\ 0.6668 \end{bmatrix}$,

$$\begin{bmatrix} Sg_1 \\ Sg_4 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix},\ (S\theta)=\begin{bmatrix} 0.0138 \\ 0.0138 \\ 0.0493 \\ -0.0094 \\ 0.1148 \end{bmatrix},\quad Svs_B=\begin{bmatrix} 0.0344 \\ 0.0824 \\ -0.0468 \\ 0 \\ 0.0889 \\ 0.3455 \\ -0.2332 \\ 0.3213 \end{bmatrix}$$

Step 3. Identify the variable leaving the basis.

From the sensitivities of the non basic and from their limits , it is straightforward to prove that the first variable which reaches its limit (the upper one) is $vs_{2\text{-}6}$ for an increment $\Delta Pd=369.9004$ of Pd, when the line 2-6 reaches its capacity limit. Since Pd=750.551 is lower at this point than 400 the next event is a change in the load distribution factors. In this case the next steps of the algorithm can be ignored.

From the variable sensitivities it is straightforward to obtain the optimal operating point for Pd = 400 MW; the corresponding optimal values for all the variables are shown in Table 3.

The algorithm is applied again to continue the simulation.

Third interval (400 ≤ Pd ≤ 752.863)

Step 1 Form the B and N matrices.

Neither the active-non active status of constraints nor the basic-non basic status of the variables have changed. Only (Sd) has been modified. Therefore B and N maintain their values.

Step 2. Calculation of sensitivities.

From (15) with $Sg^*=0$, $\alpha=\begin{bmatrix} \alpha_{2\text{-}3} \end{bmatrix}$, $\alpha.(Sd) = 0.05859$

and SPTRG=0 it is obtained that $\begin{bmatrix} Sg_3 \\ Sg_6 \end{bmatrix} = \begin{bmatrix} 0.3483 \\ 0.6517 \end{bmatrix}$,

$$\begin{bmatrix} Sg_1 \\ Sg_4 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix},\ (S\theta)=\begin{bmatrix} 0.0331 \\ 0.0331 \\ 0.0413 \\ -0.0015 \\ 0.1162 \end{bmatrix},\quad Svs_B=\begin{bmatrix} 0.0827 \\ 0.0690 \\ -0.0073 \\ 0 \\ 0.0207 \\ 0.2842 \\ -0.3483 \\ 0.3675 \end{bmatrix}$$

Step 3. Identify the variable leaving the basis.

From the sensitivities of the non basic and from their limits, it is straightforward to prove that the first variable which reaches its limit (the upper one) is g_3 for an increment $\Delta Pd=369.9004$ of Pd. Therefore the generation unit g_3 reaches its power output limit for Pd = 752.863. This is the next event to happen.

Step 4. Calculate $a'_{g_3,j}$ for all the non basic P_j's.

From Fig.4 and since the variable leaving the basis is not an slack one,

$B^t.K = e_{g_3} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ is solved for: $K = \begin{bmatrix} 0.4305 \\ -1.4031 \end{bmatrix}$

and then $\begin{bmatrix} a'_{g_3,g_1} \\ a'_{g_3,g_4} \end{bmatrix} = K^t.N = \begin{bmatrix} 0.4305 \\ 0.05671 \end{bmatrix}$

Next $a'_{g_3,j}$ for the non basic slack variables, that is $a'_{g_3,vs_{2\text{-}3}}$, is obtained from K. As $vs_{2\text{-}3}$ corresponds to the constraint represented in the second row of B and N, $a'_{g_3,vs_{2\text{-}3}}$ corresponds to the second value of K. Therefore $a'_{g_3,vs_{2\text{-}3}} = [-1.4031]$.

Step 5. Calculate σ_j for all the non basic P_j's.

From (17.a) with $c_B = \begin{bmatrix} 60 \\ 80 \end{bmatrix}$: $\pi = \begin{bmatrix} 71.3905 \\ 28.0623 \end{bmatrix}$

From (17.b) with $c_{NB}=\begin{bmatrix} 40 \\ 100 \end{bmatrix}$: $\sigma=\begin{bmatrix} s_{g_1} \\ s_{g_4} \end{bmatrix}=\begin{bmatrix} -31.3905 \\ 21.1293 \end{bmatrix}$

Next, σ_j for the non basic slack variables, that is $\sigma_{vs_{2\text{-}3}}$, is obtained from π. As $vs_{2\text{-}3}$ corresponds to the constraint represented in the second row of B and N, $\sigma_{vs_{2\text{-}3}}$ corresponds to the opposite of the second value of π. Therefore $\sigma_{vs_{2\text{-}3}} = [-28.0623]$

Step 6. Identify the variable entering the basis.

The following variable selection criteria are sequentially used as in Fig. 2 (upper limit case):

i) The independent variables out of the basis: $g_1, g_4, vs_{2\text{-}3}$

ii) $\begin{cases} \text{At a lower limit and with } a'_{g_3,j} \geq 0\text{: } g_4 \\ \text{At an upper limit and with } a'_{g_3,j} \leq 0\text{: } vs_{2\text{-}3} \end{cases}$

iii) Finally the variable with the minimum value for $\sigma_j/a'_{g_3,j}$: $vs_{2\text{-}3}$

Therefore $vs_{2\text{-}3}$ enters the basis (the line 2-3 will not be saturated any more) by replacing g_3.

Now from the variable sensitivities it is straightforward to obtain the optimal operating point for Pd = 752.863 MW; the corresponding optimal values for all the variables are shown in Table 3.

EXAMPLE 2: Simulation of a line failure

Now, it will be assumed that in the previous example the line 4-6 fails when Pd = 450 MW. From the optimal operating point for Pd = 400 MW (Table 3) and from the variable sensitivities obtained for the third interval, it is straightforward to determine the optimal operating point for Pd = 450 MW; the corresponding optimal values for all the variables are shown in Table 3.

As mentioned in a previous section, this optimal operating point coincides with the optimal operating point after the line has failed (although with different Y and T matrices), if two fictitious opposite loads are added to the end line buses. In this example since $z_{4\text{-}6}$ = -32.3926 MW, 32.3926 MW must be added to the load at bus 6 and substracted from the load at bus 4. Then, following the optimal operating point to bring down to zero these two extra loads, leads to the optimal operating point we are interested in. Therefore, the same process as in the previous example can be applied, with

$(Sd)^t = [Sd_2, Sd_3, Sd_4, Sd_5, Sd_6] = [\,0 \;\; 0 \;\; 1 \;\; 0 \;\; -1\,]$

and the following new network admittance matrix Y and admittance incidence bus-line matrix:

$$Y = \begin{bmatrix} 14.34 & -5.92 & -2.50 & 0.00 & -3.42 \\ -5.92 & 16.01 & 0.00 & -10.09 & 0.00 \\ -2.50 & 0.00 & 4.17 & 0.00 & 0.00 \\ 0.00 & -10.09 & 0.00 & 15.09 & 0.00 \\ -3.42 & 0.00 & 0.00 & 0.00 & 3.42 \end{bmatrix}$$

$$T = \begin{bmatrix} -2.50 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & -1.67 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & -5.00 & 0.00 \\ 5.92 & -5.92 & 0.00 & 0.00 & 0.00 \\ 2.50 & 0.00 & -2.50 & 0.00 & 0.00 \\ 3.42 & 0.00 & 0.00 & 0.00 & -3.42 \\ 0.00 & 10.09 & 0.00 & -10.09 & 0.00 \end{bmatrix}$$

Step 1 Form the B and N matrices.

The active constraints are the total energy balance equation and the saturation of line 2-3. The new coefficients of the latter in terms of the non basic (g_3,g_6) and basic(g_1,g_4) variables are obtained by solving for $\alpha_{2\text{-}3}$: $Y^t.\alpha_{2\text{-}3} = \gamma_{2\text{-}3}.e_{2\text{-}3}$;

$\alpha_{2\text{-}3}{}^t = [0.379 \;\; -0.3969 \;\; 0.2272 \;\; -0.2654 \;\; 0.379]$,

yielding $B = \begin{bmatrix} 1 & 1 \\ -0.3969 & 0.379 \end{bmatrix}$ and $N = \begin{bmatrix} 1 & 1 \\ 0 & 0.2272 \end{bmatrix}$

Step 2. Calculation of sensitivities.

From (15) with Sg^*=0, $\alpha = [\alpha_{2\text{-}3}]$, $\alpha.(Sd) = -0.1518$

and SPTRG=0 it is obtained that $\begin{bmatrix} Sg_3 \\ Sg_6 \end{bmatrix} = \begin{bmatrix} 0.1956 \\ -0.1956 \end{bmatrix}$,

$$\begin{bmatrix} Sg_1 \\ Sg_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, (S\theta) = \begin{bmatrix} 0.0585 \\ 0.0585 \\ -0.2047 \\ 0.0391 \\ 0.2937 \end{bmatrix}, Svs_B = \begin{bmatrix} 0.1463 \\ -0.3419 \\ 0.1956 \\ 0 \\ -0.6581 \\ 0.8044 \\ -0.1956 \end{bmatrix}$$

Step 3. Identify the variable leaving the basis.

From the sensitivities of the non basic and from their limits, it is straightforward to prove that the first variable which reaches its limit (the lower one) is $vs_{1\text{-}4}$ for an increment ΔPd=132.7868 of Pd, when the line 1-4 reaches its capacity limit. Since we only need to vary Pd by an amount ΔPd=32.3926 we only have to update the optimal operating point and the corresponding values of the variables (Table 3) according to the sensitivities above calculated.

Thus, it has been determined the optimal operating point for Pd = 450 MW after the failure of line 4-6. The normal process of tracking the optimal dispatch as the load varies can now be continued until the occurence of another event.

Table 3. Optimal values of the variables.

		EXAMPLE 1				EXAMPLE 2	
Total load MW	Pd	160	380.651	400	752.863	450 with 4-6	450 without 4-6
Independent variables MW	g_1	150.000	150.000	150.000	150.000	150.000	150.000
	g_2	10.000	230.651	237.098	360.000	254.513	260.845
	g_3	0.000	0.000	0.000	0.000	0.000	0.000
	g_4	0.000	0.000	12.902	242.863	45.487	39.151
Slack variables MW	$vs_{1\text{-}2}$	55.385	50.310	50.976	80.157	55.111	59.850
	$vs_{1\text{-}4}$	51.763	40.355	41.950	66.297	45.400	34.325
	$vs_{1\text{-}5}$	34.053	66.003	65.098	62.522	64.733	71.069
	$vs_{2\text{-}3}$	135.147	240.000	240.000	240.000	240.000	240.000
	$vs_{2\text{-}4}$	102.351	90.348	92.068	99.372	93.103	71.785
	$vs_{2\text{-}6}$	121.886	112.199	118.885	219.169	133.095	159.785
	$vs_{3\text{-}5}$	241.147	147.020	142.507	19.605	125.092	118.756
	$vs_{4\text{-}6}$	148.114	157.801	164.018	293.695	182.393	———

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