Multicentrality Analysis for Vulnerability Assessment in Electrical Distribution Network Topology Archetypes

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***Abstract*—As electricity distribution networks become increasingly complex due to factors such as the electrification of the demand or the integration of distributed generation, ensuring their resilience and reliability is a growing challenge for DNOs. In this project, the effectiveness of Multiple Centrality Analysis (MCA) in assessing vulnerabilities in electrical distribution networks was evaluated. It focuses on both purely topological centrality metrics and hybrid ones, that incorporate electrical properties. Using four real SPEN distribution networks of varying sizes and meshing levels, simulations were conducted under peak demand conditions. Contingency analysis was performed with OpenDSS to provide a benchmark, while NetworkX was used to calculate most centrality metrics.**

The study reveals that betweenness centrality, despite relying only on topological data, performs robustly as a rapid screening tool but tends to overestimate the criticality of some nodes. Hybrid metrics, particularly current-flow line betweenness centrality, usually outperformed purely topological metrics in identifying vulnerable nodes with improved accuracy. Both betweenness metrics effectively identify the most critical nodes, whereas discrimination among less critical nodes is less precise but less relevant practically. The methodology achieved significant computational savings compared to full contingency analysis, enabling efficient and scalable vulnerability screening for DNOs.

***Index Terms*—Multicentrality analysis, Vulnerability assessment, Electrical distribution networks.**

I. Introduction

THE resilience of electrical distribution networks has become a critical concern as the UK electricity system evolves towards greater decentralization and decarbonization. Distribution grids face challenges such as the increase of electrical demand, as well as the growing integration of distributed generation (DG). These changes increase system complexity and highlight the need for robust vulnerability assessment tools.

Conventional approaches, such as contingency analysis, are very computationally intensive and often impractical when applied to large scale distribution systems. As an alternative, centrality metrics have been widely proposed as a means of rapidly estimating node vulnerability based solely on the network's structure. However, the practical effectiveness of these metrics in accurately identifying critical nodes in real world power distribution systems remains uncertain. Most existing studies have focused on highly interconnected transmission grids or IEEE bus systems, which are fundamentally different from actual radial or weakly meshed distribution networks in both topology and operational characteristics.

Consequently, the predictive performance of centrality metrics established on benchmark systems cannot be guaranteed when applied directly to real distribution grids. This lack of validation in realistic network scenarios, along with the increasing complexity introduced by distributed generation and electrification, emphasizes the necessity for focused research to assess the suitability and reliability of these approaches for use in distribution networks.

In this context, Scottish Power Energy Networks (SPEN) has recognized the potential of these innovative methodologies and is interested in investigating their application to achieve a more efficient and scalable evaluation of the resilience of the distribution network. Therefore, this paper investigates the potential of Multiple Centrality Analysis (MCA), including both purely topological and hybrid metrics, to provide accurate and computationally efficient vulnerability assessments in real distribution networks. As such, the primary contributions of this paper are summarized as follows:

- The work investigates the performance and applicability of centrality metrics, including both purely topological and hybrid approaches, for vulnerability assessment in real SPEN electrical distribution networks. By focusing on real DSO networks rather than idealized or transmission grids, it addresses a key gap in practical deployment.
- The methodology develops and implements a comprehensive MCA framework, benchmarking the centrality based vulnerability metrics against a traditional contingency analysis model.
- The study evaluates the accuracy of different centrality metrics across the 4 selected case studies with varying topologies. It also provides practical insights into how MCA could be integrated into a DNO's network planning, supporting more effective and timely decision making for resilience enhancement.

II. Background and Literature review

This section summarizes how the increasing complexity of distribution networks is intensifying operational and computational challenges for DNOs, emphasizing the need for efficient vulnerability assessment tools. It also describes the traditional methods used for vulnerability assessment and some of the

Manuscript submitted August 15 2025.
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most promising new approaches, with a particular focus on MCA [1].

A. Vulnerability assessment and traditional approaches

Utilities use vulnerability frameworks to identify critical infrastructure and prioritize mitigation measures in those assets [2]. Vulnerability Assessment (VA) specifically focuses on determining the susceptibility of the network by analysing its components, their structure and their operational state. By this means, the impact of a failure on system stability or continuity of supply can be quantified and its management can be planned. Many companies use it to prioritize investments in assets whose failures cause the greatest impact, or optimize network design by evaluating different topologies (e.g. ring or radial) to reduce vulnerabilities.

VA in power systems has evolved according to the advancements in technology and grid characteristics. Below is an overview of the most widespread methods that have been deployed historically to carry out VA with key points and relevance to this study:

- Classical deterministic methods: During the 1980s and 1990s, these methods mainly revolved around contingency analysis (CA). $N - k$ contingency analysis simulates the failure of k components to test security of supply and system stability. CA requires running a power flow for each simulated contingency. This is a computationally intensive process, especially in large networks.
  The primary objective of CA is to ensure the system operates within prescribed voltage and thermal limits under outage conditions, typically allowing ±10% voltage variation [3] and avoiding conductor overloads [4]. CA remains a valuable and widely used tool however, it has several key limitations. It does not account for cascading failures, event probabilities or economic impacts. Consequently, it needs to be complemented with other approaches to ensure that it contemplates all the desired aspects.
- Probabilistic approaches: From the 1990s onwards, probabilistic risk assessments incorporated uncertainty into VA techniques, such as Fault Tree Analysis [5] and Monte Carlo Simulation. These probabilistic methods transformed metrics [6] like SAIDI and EENS from purely descriptive to predictive and economically meaningful indicators. However, they also have limitations. For example, SAIDI does not capture the social criticality of outages in different customer segments.

B. Future trends and emerging challenges

Electrical distribution networks have evolved from passive radial systems to complex active systems with bidirectional flows. This transformation is driven primarily by technological innovation, decarbonization policies, digitization and the decentralization of energy resources. Traditionally, distribution networks have always had a hierarchical radial structure. Their design was characterized by radial systems with unidirectional energy flows [7], as illustrated in Figure 1.

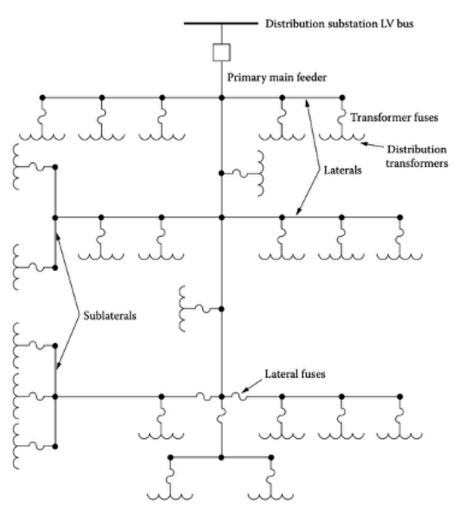

Fig. 1: Radial distribution network [7].

Electricity distribution networks are rapidly evolving towards smart grids, driven by the integration of DERs, electric vehicles and advanced metering technologies. This transformation has significantly increased the complexity of these networks, presenting operational and computational challenges for distribution network operators. Scottish Power, the second largest distribution network operator in the UK, is allocating approximately £360 million to the resilience of its network [8]. This investment is part of its RIIO-ED2 regulatory plan.

The rapid growth of DERs, including distributed PV generation, battery energy storage systems, EVs or heat pump water heaters, is reshaping traditional network operation [9]. DERs provide clear benefits such as emissions reduction and greater energy efficiency by generating close to consumption points. Nevertheless, their integration also introduces new operational challenges. High penetration of DERs can lead to reverse power flows [10], overvoltages during low load conditions [11] and protection system malfunctions [12], all of which impact network stability.

The deployment of EVs is accelerating worldwide, causing increased peak demand [13], network overload risks [14] and power quality issues, like voltage fluctuations and harmonics [15]. Similarly, heat pump water heaters can significantly increase electrical demand [16] and cause voltage instability [17]. These emerging complexities put distribution networks under greater stress, requiring network reinforcement and adaptive management strategies. Therefore, there is a critical need for efficient and scalable vulnerability assessment tools to be able to rapidly prioritize critical assets.

C. Alternative methods

As already explained above, the rapid growth of DERs and the increasing electrification of transport and heating are introducing significant challenges for the management of electrical distribution networks. To overcome these challenges, utilities want to develop faster, scalable and non-iterative approaches compared to the traditional ones.

Among the emerging methodologies are machine learning and AI driven models [18] using AMI data to predict vulnerabilities, graph based topological analyses [19] which calculate centrality metrics to rapidly identify structurally important components and hybrid methods [20] that combine physical grid models with topological insights.

Among these, centrality based approaches are particularly appealing for vulnerability analysis in future grids, since they use purely topological data and scale well as network size and complexity grow. The integration of graph based and multicentrality approaches presents a promising direction for fast, non-iterative and scalable VA, that could adequately complement those more computationally intensive traditional methodologies.

Multiple Centrality Analysis (MCA) is a general mathematical process that can be implemented in any network structure, regardless of what it represents. It aims to analyze the spatial distribution of centrality by representing the system as a primal graph [21] where nodes and edges (lines) correspond to physical intersections and connections, respectively.

However, "being central" [22] can have different meanings. Therefore, a single measure of centrality cannot capture all the important dimensions of the network. Because of this, it employs a set of distinct centrality metrics, such as betweenness or eigenvector centrality.

Although it was originally developed for applications in urban design, MCA has demonstrated versatility across various fields. For instance, in the identification of vulnerable ports in maritime supply chains through combined centrality rankings [23]. However, its direct application to electrical distribution networks remains limited. Among some of the few examples in electrical networks, [24] applied weighted MCA in transmission grids and [25] developed a metric called "electrical centrality" revealing highly electrically connected hub nodes that were not evident in the topological analysis.

Nonetheless, it is important to highlight that distribution networks possess unique structural characteristics very distinct from transmission grids and from IEEE test networks. They tend to exhibit predominantly radial or lightly meshed topologies with less redundancy. These structural differences may reduce the efficacy of traditional centrality metrics, motivating the need to critically evaluate MCA frameworks in real distribution grids.

Recent research has shown that topological centrality metrics provide useful initial screening, but may not fully capture operational vulnerabilities due to electrical constraints and power flow effects ([26], [27]). In contrast, hybrid centrality metrics that incorporate electrical parameters ([19], [20]), such as line impedances and node characteristics, have shown promise. However, both of this works use IEEE bus systems, but are not extensively tested in actual distribution networks.

This gap, combined with the limitations of traditional vulnerability assessment approaches, motivates the present project's focus on evaluating MCA performance within real distribution networks operated by SPEN, which exhibit lightly meshed structures.

D. MCA Theoretical framework

This project's MCA framework models the electrical distribution network as a graph $G = (V, E)$, where V represents the set of nodes (e.g., buses, substations, loads) and E the set of edges representing electrical lines. The adjacency matrix A of graph G is defined by $A_{ij} = 1$ if nodes i and j are connected, and 0 otherwise.

Centrality metrics quantify node importance within this network, each according to its definition. The study applies a combination of the following classical topological metrics and hybrid metrics based on electrical characteristics.

1) Classical topological centrality metrics: : These metrics evaluate node importance based solely on the structure of the network. They rely purely on the configuration of connections, without any reference to electrical properties of the system.

- **Degree Centrality** (DC) measures the number of direct connections a node has:
$$DC(i) = \frac{\sum_{j \in v} A_{ij}}{n-1} \tag{1}$$
Where A_{ij} represents the connectivity between nodes i and j in the adjacency matrix, A, and n is the total number of nodes in the network.
- **Betweenness Centrality** (BC) quantifies how often a node lies on shortest paths between node pairs:
$$BC(i) = \sum_{s \neq i \neq t} \frac{\sigma_{st}(i)}{\sigma_{st}} \tag{2}$$
with σ_{st} the total number of shortest paths from node s to t, and $\sigma_{st}(i)$ those passing through i.
- **Closeness Centrality** (CC) indicates inverse average shortest path distance from a node to all others:
$$CC(i) = \frac{1}{\sum_{j \in V} d(i,j)} \tag{3}$$
where $d(i,j)$ is the shortest path distance between nodes i and j.
- **Eigenvector Centrality** (EC) assigns relative scores based on connections to influential neighbours, defined as:
$$EC(i) = \frac{1}{\lambda} \sum_{j \in V} A_{ij} \cdot EC(j) \tag{4}$$
with λ being the respective eigenvalue of the adjacency matrix, A.
- **Clustering Coefficient** (CCoef) measures the tendency of nodes to cluster:
$$CCoef(i) = \frac{2 \cdot T(i)}{DC(i) \cdot (DC(i) - 1)} \tag{5}$$
with $T(i)$ being the number of triangles through node i and $DC(i)$ the degree centrality of that same node.

2) Hybrid centrality metrics: These metrics incorporate electrical parameters using the weighted Laplacian matrix L, calculated as the difference between the degree matrix and weighted adjacency matrix, with weights derived from line impedances. The pseudo-inverse L^+ of L is used to evaluate electrical distances.

- **Current-flow Closeness Centrality** (CF CC) extends classical closeness centrality by measuring the effective resistance distance (ERD) between a node and all other nodes in the network. The distance is defined as:

$$d_i = \frac{\sum_{k=1}^{n} Z_{ik}}{n} \tag{6}$$

Therefore, the metric is expressed as:

$$CF\,CC_i = \frac{n-1}{\sum_{k=1}^{n} Z_{ik}} \tag{7}$$

A central node for CF CC is a node that is joined to others by the shortest mean ERD. The shorter the total electrical distance, the greater its centrality.

- **Current-flow Line Betweenness Centrality** (CF Line BC) evaluates the importance of a line relative to power flow. For a line between nodes i and k and a source-target pair (s,t), powerflow is expressed as:

$$P_{ik}^{st} = \sum_{k=1}^{n} A_{ik} \cdot \left|\delta_i^{st} - \delta_k^{st}\right| = \sum_{k=1}^{n} A_{ik} \cdot \left|L_{is}^{+} - L_{it}^{+} - L_{ks}^{+} + L_{kt}^{+}\right| \tag{8}$$

Where $\delta_i^{st} = L_{is}^{+} - L_{it}^{+}$ and $\delta_k^{st} = L_{ks}^{+} - L_{kt}^{+}$ are the phase angle nodes of nodes i and k with the respective pseudo-inverse of L. After simplifying, applying it to every possible source and target pair and normalizing it:

$$CF\,Line\,BC_i = \frac{2 \cdot \sum_{s<t} P_{ik}^{st}}{n \cdot (n-1)} \tag{9}$$

This metric uniquely evaluates edges (lines), instead of nodes, thus identifying potential bottlenecks or overload points in the network.

III. Model development

To evaluate the effectiveness of MCA in identifying vulnerabilities within electrical distribution networks, a model was developed to systematically calculate a range of centrality metrics and validate them against an objective performance benchmark. This process involved implementing algorithms for both classical topological and hybrid centrality measures and constructing a comprehensive benchmark framework for performance comparison.

This section describes the methodology, including key design decisions. It presents the vulnerability assessment framework, covering model setup, algorithm implementation and validation procedures. The benchmark design is also detailed, specifying the operational scenarios, selected performance metrics and the development of comparison criteria.

A. Methodology development and benchmark selection

The project initially explored the use of purely topological centrality metrics to assess distribution network vulnerability, considering composite indices combining multiple measures with different weighting schemes. However, due to the subjectivity and network dependency of such composites, the focus shifted to evaluating individual centrality metrics. Recognizing the limitations of purely topological approaches, the study also incorporated hybrid centrality metrics that integrate electrical network characteristics to better capture operational vulnerabilities.

A critical step involved defining a clear, objective benchmark metric to evaluate the effectiveness of these centrality measures. Based on the operational context of SP Energy Networks (SPEN), vulnerability was defined by the network's ability to maintain reliable supply under disturbances. Metrics from the literature, like network efficiency, were found insufficient for distribution systems, as they fail to capture customer supply interruption directly. Therefore, Energy Not Served (ENS) was adopted as the principal vulnerability indicator, as it can quantify the total unsupplied energy due to faults or overloads through contingency analysis and power flow simulations. ENS provides a clear, objective and physically meaningful measure that reflects the actual loss of service experienced by customers. Hence, it serves as a robust reference for validating centrality based vulnerability assessments and guiding comparative analysis across network nodes.

B. MCA framework

The MCA model implemented in this project calculates the set of vulnerability metrics introduced in section II-D, encompassing both purely topological and hybrid centrality measures.

Topological metrics, such as degree and betweenness centrality, analyze the network's adjacency structure to identify structural vulnerabilities based solely on their connectivity patterns. Complementing this, hybrid metrics incorporate electrical parameters by using the weighted Laplacian matrix and its pseudo-inverse, capturing power flow and resistance characteristics that topological metrics overlook.

The purely topological centrality metrics were computed using the NetworkX [28] library. In contrast, hybrid metrics were calculated by directly implementing the mathematical formulations detailed in section II-D. Visualization tools including Plotly, Matplotlib and Seaborn were used to support interpretation of results and comparison of vulnerability indicators across nodes.

Model validation was performed using the IEEE 30 bus benchmark network. To ensure fidelity with the network model used in the comparative literature [27], the exact system data file was downloaded from the Pandapower repository. The successful reproduction of key centrality results and closely matching power flow simulations confirmed the framework's accuracy and established confidence before applying the approach to real SPEN distribution networks.

C. Case studies and operational conditions

To validate the vulnerability assessment framework, four LV distribution networks with diverse sizes and topologies, ranging from radial to meshed configurations, were selected for analysis. These areas were extracted from Scottish Power Energy Network's data and are presented below.

The Greasby network, located in the Wirral Peninsula and served by SP MANWEB, is a large and complex distribution system with a highly meshed topology. It includes 4 transformers, 1,127 lines and 1,126 nodes, of which 480 are loads. The Lanark network, managed by SP Distribution in central

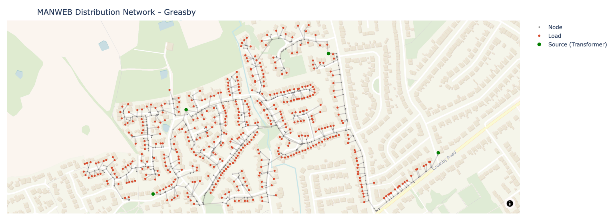

Fig. 2: Representation of the Greasby distribution network within SP MANWEB area.

and southern Scotland, is a medium-sized distribution system with a predominantly radial topology and some localized interconnections. It comprises 5 transformers, 847 lines and 847 nodes, including 356 loads.

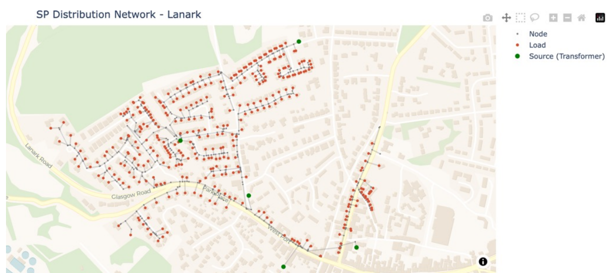

Fig. 3: Representation of the Lanark distribution network within SP MANWEB area.

The Whifflet subnetwork in Glasgow is a smaller, primarily radial distribution grid supplied by a single transformer. With 220 lines, 221 nodes and 70 loads, its simpler structure highlights vulnerability patterns associated with single points of failure.

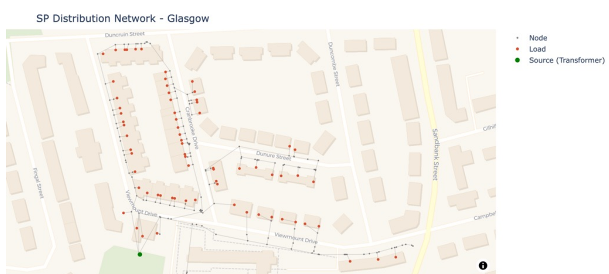

Fig. 4: Representation of the Whifflet distribution network within SP area.

The distribution network operated by SP MANWEB, shown in Figure 5, features a lightly meshed topology typical of the MANWEB area. It includes 4 transformers, 1,235 lines and nodes, of which 460 are loads. This network is more radial than the Greasby network but maintains some important redundancy and meshing.

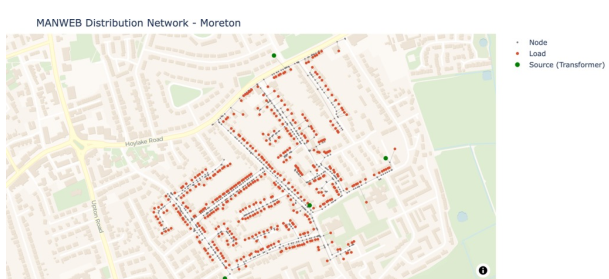

Fig. 5: Representation of the Moreton distribution network within SP area.

Regarding operational conditions, distribution network vulnerability is highly dependent on the demand scenario analyzed. Two representative scenarios were considered: peak demand, when high simultaneous consumption stresses network capacity and increases the risk of overloads and supply interruptions, and low demand, which can reveal issues like overvoltages, especially with high distributed generation.

This project concentrates on peak demand scenarios, as they represent the most critical and stressful conditions for LV grids. While this represents a critical operational condition, it should be acknowledged that distribution networks may exhibit different vulnerability patterns under other operational states. Hence, it is important to recognize the limitations of this approach, as other scenarios may introduce additional vulnerabilities that are not fully addressed in this study. However, this case was chosen due to several methodological and practical considerations.

Particularly, with the increasing electrification from EVs and heat pumps. Additionally, simulating node failures (CA) during peak demand aligns with nodal centrality metrics, enabling a direct comparison. While other scenarios may present different vulnerabilities, this focus provides relevant insight for practical network planning and contingency management in utility operations.

D. Benchmark model implementation

The project developed a realistic vulnerability benchmark for SPEN's distribution networks using OpenDSS with Python integration via OpenDSSDirect [29]. In this project, power flow analysis was employed as a fundamental tool for assessing network performance and identifying vulnerability to thermal overloads in SPEN's distribution network.

A contingency analysis model was implemented using OpenDSSDirect in Python. It focuses on single contingency (N-1) scenarios, where each and every node outage is simulated individually. By isolating the impact of individual element failures, it becomes possible to assign a distinct, quantitative vulnerability value to each node in the network. This is essential, just as MCA produces a centrality score for every node, the single contingency analysis yields a clear metric associated with the loss of each node, that can be directly compared. This methodological consistency supports the core objective of the benchmark framework, which is to enable a robust, node by node comparison between centrality based values and ENS increase outcomes derived from simulated outage scenarios.

The analysis iteratively isolates each node, calculates the baseline and post-contingency ENS, including isolated loads, and resets the network after each scenario. The outcome is a quantitative nodal ENS ranking consistent with MCA outputs.

Due to lack of customer load data, the CREST demand model was used to generate diverse residential peak demand profiles, which were randomly assigned to network loads. This approach enhances the realism and reliability of vulnerability assessment under critical peak demand conditions despite data limitations.

The model uses three main input datasets:

- Network topology and parameters provided by SPEN in OpenDSS format, including real connectivity and asset data.
- Geospatial node coordinates in .json format.
- Peak residential demand profiles, randomly assigned to network loads, each with an assumed power factor of 0.95.

The complete workflow for evaluating network vulnerability via ENS increase is detailed below in Figure 6.

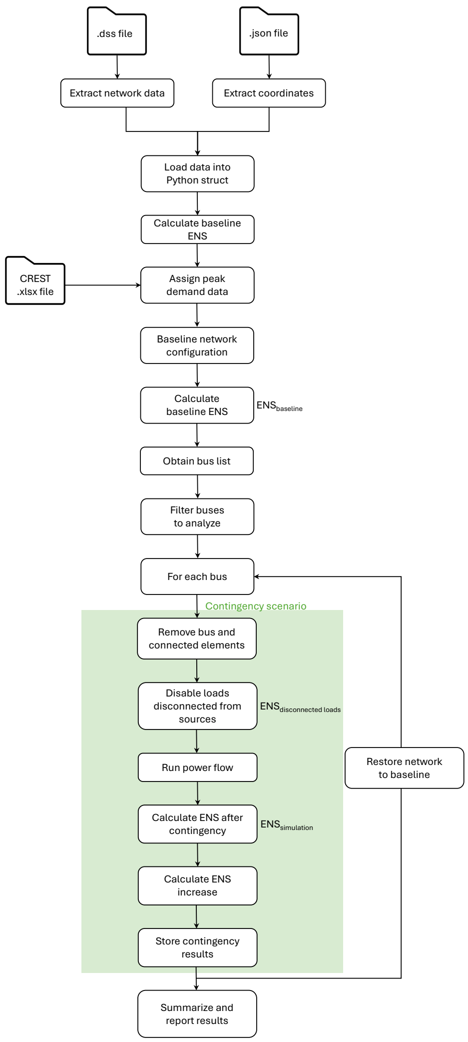

Fig. 6: Contingency analysis workflow for evaluating network vulnerability using ENS.

IV. Results and Analysis

This chapter presents the results from the modelling and simulations, comparing purely topological and hybrid centrality metrics with ENS values from contingency analysis. The analysis focuses primarily on the Greasby distribution network, illustrating how centrality measures correlate with node vulnerability to supply disruptions.

A. Topological metrics

As observed in Figure 7, most nodes exhibit low degree values, which reflects the limited connectivity that is characteristic of distribution networks. Results show that most nodes have low degree centrality with an average degree of 2.002, typical of radial distribution networks, where most nodes serve as simple endpoints and only a few act as main feeders or connection hubs.

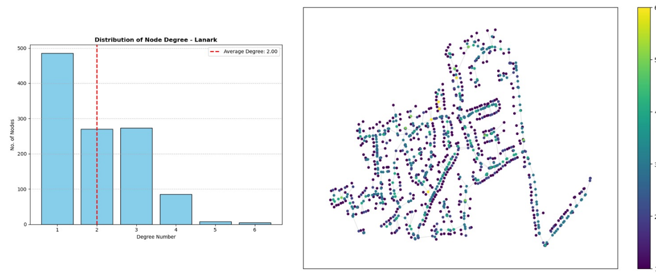

Fig. 7: Distribution of node degree in the Greasby distribution network.

Figure 8 compares five purely topological centrality metrics with node ENS values from failure simulations. It is worth noting that the clustering coefficient is mostly zero for nodes, reflecting the radial, tree-like topology of distribution networks with minimal local redundancy and high vulnerability to node disconnections. This contrasts with meshed transmission networks, as proved in the IEEE 30 bus validation, which exhibit higher clustering coefficients due to multiple paths and redundancy. Therefore, the metric is more meaningful for vulnerability assessment in more meshed systems.

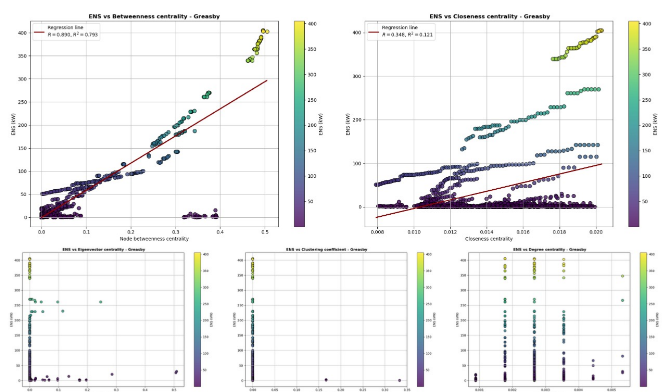

Fig. 8: Comparative scatter plots of five topological centrality metrics against ENS in the Greasby network.

Eigenvector centrality also reveals another fundamental limitation when applied to distribution networks. Most nodes cluster near zero and only a few have higher values that do not correlate well with ENS. This occurs because eigenvector centrality diminishes in robustness as network size increases and relies heavily on connections to other important nodes [30]. In extensive, sparse networks with low average degree, many nodes are peripheral, resulting in small eigenvector values.

Furthermore, the degree centrality plot shows distinct vertical bands due to discrete node degree values. While nodes with degree one tend to have low ENS, reflecting their role as loads affecting only single customers, overall there is no clear correlation between degree centrality and ENS. This discrepancy between ENS and degree centrality makes sense, because for example high degree nodes serving low demand areas may contribute little to vulnerability, whereas lower degree nodes near transformers or high demand loads can have significant ENS. This demonstrates that connectivity alone is insufficient to assess distribution network vulnerabilities.

Finally, it is worth looking more closely at both betweenness and closeness centrality since they both show some ability to predict vulnerable nodes, with betweenness exhibiting a strong positive correlation with ENS ($R^2 = 0.793$) and closeness showing weaker, noisier correlation ($R^2 = 0.121$). A significant number of nodes with high closeness scores cause little ENS when they fail, despite the fact that many other high closeness nodes correlate with heightened ENS. This suggests that closeness may occasionally overestimate criticality for well connected, but low demand nodes.

Betweenness centrality's performance suggests that the concept of intermediary importance, or more simply nodes that lie on shortest paths between other node pairs, translates meaningfully to operational vulnerability. Consequently, further analysis uses Spearman rank correlation to assess the consistency of node ranking between betweenness and ENS.

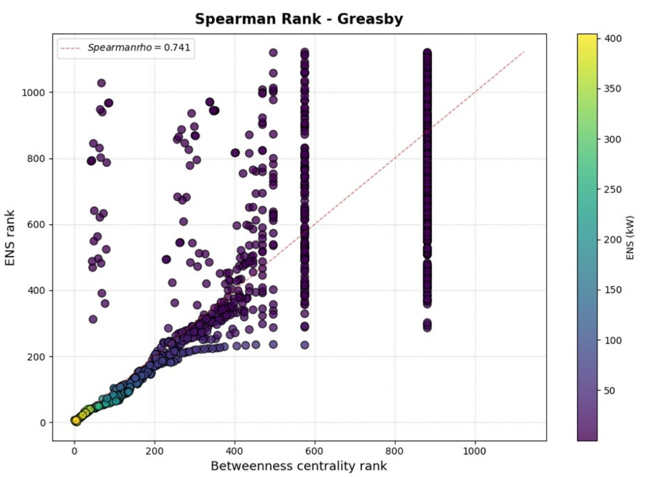

Fig. 9: Spearman rank correlation plot comparing node rankings by betweenness centrality and ENS in the Greasby network

A Spearman's ρ of 0.741 confirms a strong correlation between betweenness centrality and ENS rankings, validating betweenness as an effective vulnerability indicator. However, it tends to overestimate the criticality of a considerable amount of nodes, assigning similar high rankings to structurally less relevant nodes due to the network's large, sparse topology.

To gain a clearer understanding of this overestimation phenomenon, we have highlighted the most critical nodes and the overestimated nodes directly on the network plot. This visualization in Figure 10 allows to spatially examine whether there is a structural basis for the overestimation by betweenness centrality. From this, it appears that some of the nodes identified as overestimated correspond to key connecting lines or main streets. In that way, structurally, they represent important links and redundancy within the network topology.

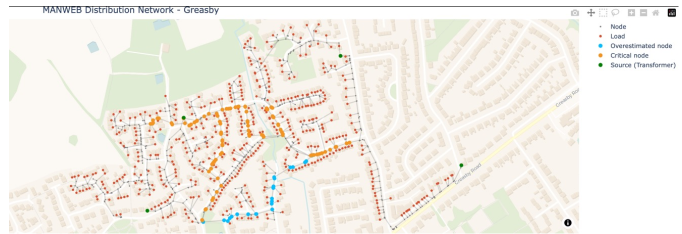

Fig. 10: Highlighted Greasby's network representation showing critical nodes and overestimated nodes

But, despite this structural significance, these connections may not be as electrically essential in terms of maintaining customer supply. In other words, while these nodes serve important topological roles by linking different areas, their failure does not necessarily translate into large scale supply interruptions reflected by the ENS metric. This divergence shows a fundamental limitation of purely topological metrics like betweenness centrality in distribution networks. They capture structural importance, but do not fully reflect the electrical operational impact of node failures.

These drawbacks highlight the need for a more sophisticated vulnerability assessment. Although betweenness centrality has shown impressive results in identifying electrically vulnerable nodes, as all critical nodes are identified correctly. Nonetheless, incorporating hybrid centrality metrics, which include electrical data, may further improve vulnerability detection.

B. Hybrid metrics

Current-flow (CF) centrality metrics integrate electrical properties into structural vulnerability analysis, offering a more physically grounded perspective. CF closeness centrality, the first hybrid metric analyzed, enhances the traditional topological measure by incorporating line impedances.

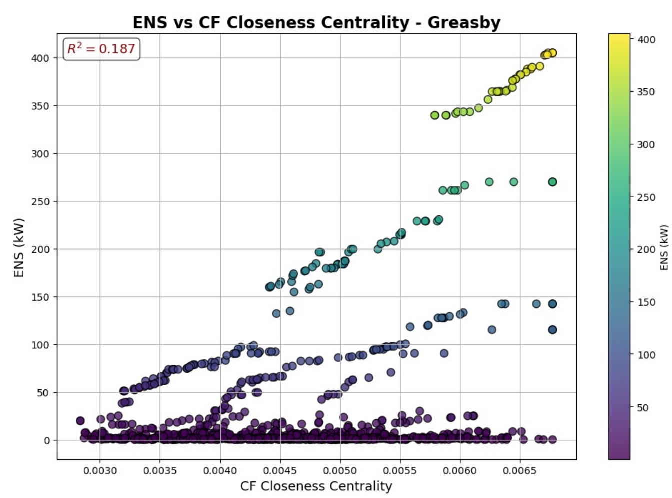

Fig. 11: Scatter plot of ENS against current-flow closeness centrality in the Greasby network.

The relationship between CF closeness centrality and ENS resembles that of standard closeness centrality, with only a modest improvement in the coefficient of determination from 0.121 to 0.187 overall. One possible explanation is that in distribution networks lines could have similar characteristics.

Therefore, the difference in impedance values is not enough for the weighted methodology to provide more accurate results.

Moreover, CF line betweenness centrality, which incorporates generator and load pairs, was mapped onto the Greasby network. Lines near source nodes and key connectors between network areas consistently show higher centrality values. This metric effectively captures the actual vulnerabilities in the electric grid, aligning well with the identified critical nodes.

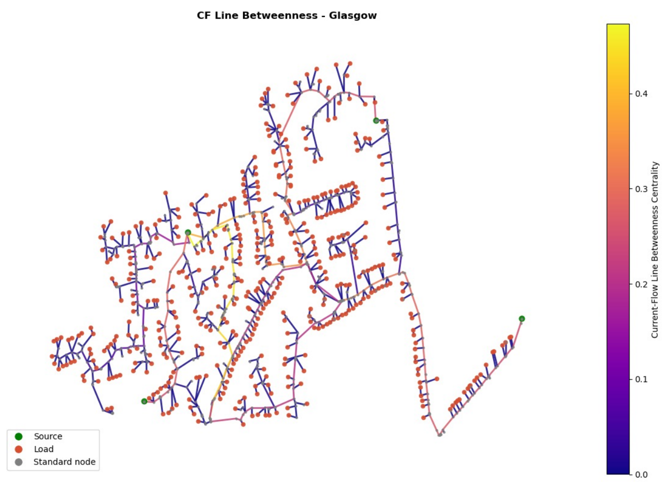

Fig. 12: Visualization of current-flow line betweenness centrality values in the Greasby network.

Since CF line betweenness is originally defined for lines, its values were aggregated to nodes by summing adjacent line scores to enable comparison with nodal metrics and ENS. The resulting analysis shows a coefficient of determination of 0.699, slightly lower than pure topological betweenness (0.793), but still indicates that higher CF betweenness scores align with higher ENS values.

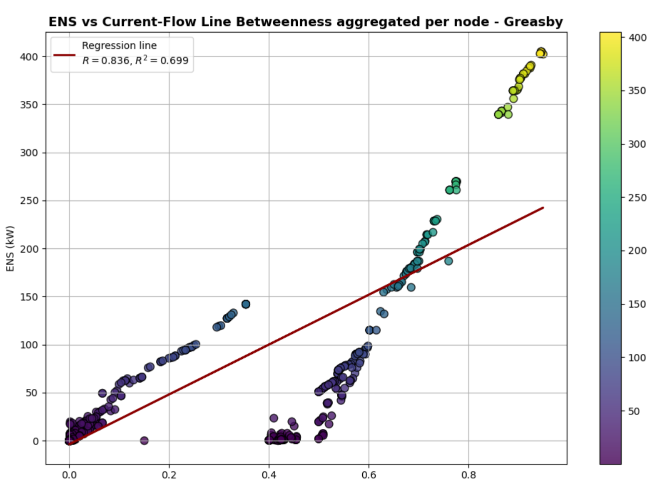

Fig. 13: Scatter plot of ENS against current-flow line betweenness centrality in the Greasby network.

However, nodes with higher CF betweenness score tend to have a higher ENS value, which indicates that this metric is good at assessing electrical vulnerability. Although there is an outlier group located towards the bottom right, which means that the metric overestimates vulnerability, it has not been assigned values as large as those of the truly most vulnerable nodes. Thus, even if the plot is slightly deviated, the metric seems to distinguish well among the most critical nodes. Therefore, a Spearman rank was also conducted to confirm this assumption.

In effect, Figure 14 shows excellent results at the identification of the most vulnerable nodes in the network. No nodes have been overestimated and compared to the most critical ones, in other words, the metric has not assigned similar values to electrically not so vulnerable nodes, as occurred with betweenness centrality in Figure 9.

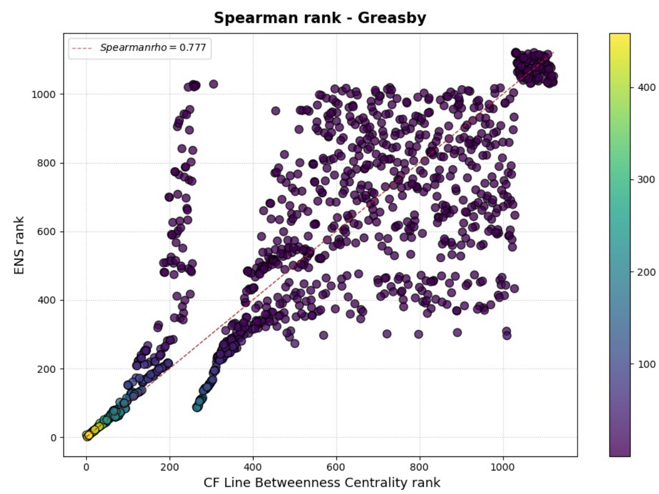

Fig. 14: Spearman rank correlation plot comparing node rankings by CF line betweenness centrality and ENS in the Greasby network.

As node criticality decreases, accuracy of CF line betweenness worsens with nodes dispersing above and below the diagonal, but unlike topological betweenness, it distinguishes less critical nodes better. Spearman's ρ improves from 0.741 to 0.777. But most importantly, all critical nodes are correctly identified, and there is no overestimation of non-critical nodes relative to truly vulnerable ones. Although precision diminishes for peripheral nodes, CF line betweenness excels at detecting the most vulnerable nodes, proving its value as an effective screening tool for distribution network vulnerability.

C. Computational efficiency and practical applications

The table below summarizes the computation times required for both traditional contingency analysis and the proposed MCA method, including the hybrid metrics, across the four distribution networks analysed in this study. MCA offers sig-

TABLE I: Computation time comparison between traditional approach and MCA

Distribution network	Contingency Analysis (s)	MCA (s)	Speed-up factor
Greasby	95.5	7	13.64
Moreton	103.2	6.1	16.92
Glasgow	3.7	0.2	18.5
Lanark	43.9	2.7	16.26

nificant computational savings compared to traditional contingency analysis, providing similar results up to 16 times faster. For large networks like SPEN's, conventional methods could require several days of computation, while MCA delivers results in a fraction of that time, saving approximately 92% of computational effort. This allows DNOs to quickly identify critical nodes for further analysis and ensures resources are focused where they are most valuable, making MCA an effective first stage screening tool or filter for large scale distribution networks.

V. Conclusions and Future work

The comparative analysis of centrality metrics revealed varying effectiveness in assessing vulnerability across distribution network topologies. Betweenness centrality, despite relying solely on topological data, showed robust performance but tended to overestimate some nodes' criticality. Nevertheless, it serves well as a rapid screening tool to identify a subset of potentially critical nodes, allowing focused detailed analyses.

Other standard topological metrics, such as clustering coefficient, eigenvector and degree centralities, showed limited applicability due to the radial and weakly connected nature of distribution networks. However, in more meshed MV networks, these metrics may hold more promise.

Hybrid centrality metrics, particularly CF line betweenness, outperformed purely topological measures by integrating electrical properties and network structure, making it the most effective metric for identifying vulnerable nodes. Both topological and hybrid betweenness metrics excelled in identifying the most critical nodes, which is the main operational concern, while less critical node rankings were less precise.

This suggests a two stage methodology where ideally CF line betweenness, but topological betweenness could also be an option, is used as an initial filter to rapidly identify critical nodes for deeper, computationally expensive analysis. Implementing MCA in this way could provide DNOs like Scottish Power with fast, routine vulnerability screening enabling a deeper study of significant nodes, resulting in a better prioritization of investments and faster responses to network issues.

Future improvements should include analyzing a wider range of operational scenarios beyond peak demand, such as low demand periods and varying distributed generation (DG) levels, to better capture network vulnerability nuances. Incorporating different network configurations and alternative vulnerability metrics beyond ENS, like loss of load probability or customer impact, could provide complementary insights, especially under conditions with high DG penetration where voltage and power flow issues arise. Applying the analysis to more meshed MV networks may enhance the effectiveness of topological metrics. A promising future direction could be to use betweenness-based centrality metrics to guide network reconfiguration and reinforcement through optimization frameworks, transitioning from vulnerability assessment to actionable network design for improved resilience.

References

[1] A. Mittal, J. Hazra, N. Jain, V. Goyal, D. P. Seetharam, and Y. Sabharwal, "Real time contingency analysis for power grids," in *Euro-Par 2011 Parallel Processing*, E. Jeannot, R. Namyst, and J. Roman, Eds. Berlin, Heidelberg: Springer Berlin Heidelberg, 2011, pp. 303–315.

[2] "Climate vulnerability considerations for the power sector: Transmission and distribution infrastructure," EPRI, Technical Report 3002026315, October 2023. [Online]. Available: https://www.epri.com/research/products/000000003002026315

[3] *IEEE Recommended Practice for Monitoring Electric Power Quality*, Institute of Electrical and Electronics Engineers Std. IEEE Std 1159-2019, 2019. [Online]. Available: https://standards.ieee.org/ieee/1159/6124/

[4] *Short-circuit currents in three-phase AC systems – Part 3: Currents during two separate simultaneous line-to-earth short circuits and partial short-circuit currents flowing through earth*, IEC Std. IEC 60 909-3:2009, 2009. [Online]. Available: https://webstore.iec.ch/en/publication/3890

[5] *Fault tree analysis (FTA)*, IEC Std. IEC 61 025:2006, 2006.

[6] R. Billinton and R. N. Allan, "Probabilistic assessment of power systems," *Proceedings of the IEEE*, vol. 88, no. 2, pp. 139–162, 2000. [Online]. Available: https://home.engineering.iastate.edu/ jdm/ee653/BasicConcept_Reliability Evaluation.pdf

[7] T. Gonen, *Electric Power Distribution Engineering*, 3rd ed. CRC Press, Taylor & Francis Group, 2014. [Online]. Available: http://students.aiu.edu/submissions/profiles/resources/onlineBook/Z7e5T7 _Electric_Power_Distribution_Engineering-_Third_Edition.pdf

[8] "Distribution annual performance report 2023/24," SP Energy Networks, Annual Performance Report, July 2024. [Online]. Available: https://www.spenergynetworks.co.uk/userfiles/file/SP__Energy_Networks _Distribution_Annual_Performance_Report_2023-24.pdf

[9] H. Farhangi, "The path of the smart grid," *IEEE Power and Energy Magazine*, vol. 8, no. 1, pp. 18–28, 2010. [Online]. Available: https://doi.org/10.1109/MPE.2009.934876

[10] D. Pudjianto, P. Djapic, G. Strbac, E. T. van Schalkwyk, and B. Stojkovska, "Der reactive services and distribution network losses," *CIRED*, vol. 2020, pp. 541–544, 2021. [Online]. Available: https://digital-library.theiet.org/doi/abs/10.1049/oap-cired.2021.0111

[11] I. B. Majeed and N. I. Nwulu, "Impact of reverse power flow on distributed transformers in a solar-photovoltaic-integrated low-voltage network," *Energies*, vol. 15, no. 23, 2022. [Online]. Available: https://www.mdpi.com/1996-1073/15/23/9238

[12] A. M. Howlader, S. Sadoyama, L. R. Roose, and Y. Chen, "Active power control to mitigate voltage and frequency deviations for the smart grid using smart pv inverters," *Applied Energy*, vol. 258, p. 114000, 2020. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S0306261919316873

[13] Y. Li and A. Jenn, "Impact of electric vehicle charging demand on power distribution grid congestion," *Proceedings of the National Academy of Sciences*, vol. 121, no. 18, p. e2317599121, 2024. [Online]. Available: https://www.pnas.org/doi/abs/10.1073/pnas.2317599121

[14] BloombergNEF, "Electric vehicle outlook 2025," June 2025, annual flagship report analyzing the future of road transport electrification, including market forecasts, battery trends, and policy impacts. [Online]. Available: https://about.bnef.com/insights/clean-transport/electric-vehicle-outlook/

[15] Trialog, "L'impact de l'intégration des véhicules électriques dans le réseau électrique," 2023. [Online]. Available: https://www.trialog.com/fr/impact-vehicules-reseau/

[16] K. Fenaughty, D. S. Parker, J. Butzbaugh, and C. Colon, "Detailed evaluation of electric demand load shifting potential of heat pump water heaters in a hot humid climate," *Science and Technology for the Built Environment*, vol. 29, no. 9, pp. 905–926, 2023. [Online]. Available: https://doi.org/10.1080/23744731.2023.2261808

[17] T. L. Lee, "Implications of heating electrification on distribution networks and distributed energy resources," S.M. Thesis, Massachusetts Institute of Technology, May 2022. [Online]. Available: https://dspace.mit.edu/handle/1721.1/144976

[18] J. Noh, S. Kang, J. Kim, and J.-W. Park, "A study on volt-watt mode of smart inverter to prevent voltage rise with high penetration of pv system," in *2019 IEEE Power Energy Society General Meeting (PESGM)*, 2019, pp. 1–5.

[19] W. Liu, T. Zhang, C. Lin, K. Li, and R. Wang, "Graph attention networks unleashed: A fast and explainable vulnerability assessment framework for microgrids," March 2025. [Online]. Available: https://arxiv.org/html/2503.00786v1

[20] S. Maity and P. Panigrahi, "Vulnerability and efficiency assessment of complex power grids using current-flow line centralities," *arXiv preprint arXiv:2404.10005*, April 2024. [Online]. Available: https://arxiv.org/pdf/2404.10005

[21] P. Crucitti, V. Latora, and S. Porta, "Centrality measures in spatial networks of urban streets," *Physical review. E, Statistical, nonlinear, and soft matter physics*, vol. 73, p. 036125, 04 2006.

[22] Y. Alrowaili, N. Saxena, and P. Burnap, "Towards developing an asset-criticality identification framework in smart grids," in *Proceedings of the IEEE International Conference on Cyber Security and Resilience (CSR)*. London, United Kingdom: IEEE, September 2024. [Online]. Available: https://orca.cardiff.ac.uk/id/eprint/171133/1/2024217970.pdf

[23] S. Porta, P. Crucitti, and V. Latora, "Multiple centrality assessment in parma: A network analysis of paths and open spaces," *Urban Design International*, vol. 13, no. 1, pp. 41–50, 2008. [Online]. Available: https://www.academia.edu/2818958/Multiple_centrality_assessment_in_Parma_a_network_analysis_of_paths_and_open_spaces

[24] J. Li, X. Wang, and T. Zhang, "Sequence-based centrality measures in maritime transportation networks," *IET Intelligent Transport Systems*, vol. 14, no. 14, pp. 2042–2051, 2020. [Online]. Available: https://ietresearch.onlinelibrary.wiley.com/doi/abs/10.1049/iet-its.2020.0301

[25] S. Forsberg, K. Thomas, and M. Bergkvist, "Power grid vulnerability analysis using complex network theory: A topological study of the nordic transmission grid," *Physica A: Statistical Mechanics and its Applications*, vol. 626, p. 129072, 2023. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S0378437123006271

[26] A. B. M. Nasiruzzaman, H. R. Pota, and F. R. Islam, "Method, impact and rank similarity of modified centrality measures of power grid to identify critical components," in *2012 11th International Conference on Environment and Electrical Engineering*, 2012, pp. 223–228.

[27] P. Espinoza-Arriagada, G. Ruz, and L. Gutierrez-Lagos, "Graph theory-based topological and nodal analysis for identifying critical points to enhance resilience in electrical distribution systems," 12 2024.

[28] N. Developers, "Networkx: Software for the creation, manipulation, and study of complex networks," 2025. [Online]. Available: https://networkx.org

[29] DSS-Extensions Developers, "Opendssdirect.py documentation," 2025. [Online]. Available: https://dss-extensions.org/OpenDSSDirect.py/index.html

[30] C. Martin and P. Niemeyer, "On the impact of network size and average degree on the robustness of centrality measures," *Network Science*, vol. 9, no. S1, p. S61–S82, 2021.