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Tight and compact MILP formulation for a high-resolution of start-up costs in the medium-term unit commitment

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Abstract

Nowadays, most modern power systems are evolving towards a considerable capacity expansion in their energy storage and interconnection facilities. However, these great developments are not being accomplished fast enough to accommodate the high penetration of variable renewable energy sources. This situation raises demand variability, requiring more flexibility from thermal generators, especially from their more frequent start-up and shut-down processes. Consequently, the unit commitment requires more accurate and detailed modeling while maintaining computational efficiency. This paper analyzes some of the best models to manage long-duration start-up costs according to the real fuel-consumption curves of a gas-fired generation portfolio. Moreover, we propose a tight and compact MILP piecewise formulation that enhances the resolution of start-up representations and achieves outstanding results compared to the literature benchmarks. The successful performance of this methodology is proven in several large-size case studies focusing on the medium term. Furthermore, conventional day-ahead problems are also run to demonstrate the overall competitiveness of the formulation.

Keywords: unit commitment, start-up costs, medium-term models, mixed-integer linear programming, piecewise linearization.

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Nomenclature

A. Sets

$g \in G$ Set of indexes of generating units.

$s \in S$ Set of indexes of start-up segments.

$t \in T$ Set of indexes of hourly periods of the time span.

B. Parameters

C_g^{F} Fuel cost of unit g [\$/MMBtu].

C_g^{P-LV} Linear variable production cost of unit g [\$/MWh].

C_g^{P-NL} Fixed production cost of unit g [\$/h].

C_g^{SD} Shut-down cost of unit g [\$].

$C_{g,s}^{SU-M}$ Linear variable cost for start-up s of unit g [\$/h].

$C_{g,s}^{SU-N}$ Fixed cost for the start-up type s of unit g [\$].

D_t Load demand in period t [MWh].

F_g^{P-LV} Linear variable fuel-consumption for producing electricity of unit g [MMBtu/MWh].

F_g^{P-NL} Fixed fuel-consumption for producing electricity of unit g [MMBtu/h].

F_g^{SD} Shut-down fuel-consumption of unit g [MMBtu].

$F_{g,s}^{SU-M}$ Linear variable fuel-consumption for the start-up type s of unit g [MMBtu/h].

$F_{g,s}^{SU-N}$ Fixed fuel-consumption for the start-up type s of unit g [MMBtu].

$\overline{H}$ High value to discerning the start-up type decisions or managing the coldest start-up properly (Big-M).

$\overline{O_g}$ Maximum offline hours that unit g could reach with respect to its initial status and horizon's duration [h].

$\overline{P_g}$ Maximum power output of unit g [MW].

$\underline{P}_g$ Minimum power output of unit g [MW].

P_g^0 Power output of unit g in the first period t [MW].

R_t Spinning reserve requirement in period t [MWh].

RD_g Ramp-down limit of unit g [MW/h].

RU_g Ramp-up limit of unit g [MW/h].

SD_g Shut-down capability of unit g [MW].

SU_g Start-up capability of unit g [MW].

$T_{g,s}^{SU}$ Minimum time period that unit g must be offline for the start-up type s [h].

TD_g Minimum down time of unit g [h].

TD_g^0 Offline hours of unit g in the first period t [h].

TD_g^R Number of hours that unit g must remain offline [h].

TU_g Minimum up time of unit g [h].

TU_g^0 Online hours of unit g in the first period t [h].

TU_g^R Number of hours that unit g must remain online [h].

U_g^0 Commitment status of unit g in the first period t.

C. Variables

1) Binary variables

$u_{g,t}$ Commitment decision of unit g in period t.

$v_{g,t,s}$ Start-up decision of unit g in period t and type s.

$w_{g,t}$ Shut-down decision of unit g in period t.

2) Positive and continuous/integer variables

$h_{g,t}^{SD}$ Number of hours that unit g has been offline in period t [h].

$h_{g,t,s}^{SU}$ Number of hours that unit g has been offline individualized to the segment s in which unit g starts-up in period t [h].

3) Positive and continuous variables

$c_{g,t}^{P}$ Production cost of unit g in period t [$].

$c_{g,t}^{SD}$ Shut-down cost of unit g in period t [$].

$c_{g,t}^{SU}$ Start-up cost of unit g in period t [$].

$p_{g,t}$ Power output above the minimum output of unit g in period t [MW].

$r_{g,t}$ Spinning reserve served by unit g in period t [MW].

1. Introduction

Most power systems currently experience an increasing penetration of variable renewable energy sources, like wind and solar. Therefore, the load profiles' variability faced by thermal technologies is being progressively intensified [1]. There is a growing interest in assuring power systems' stability to guarantee the security of supply [2] and accurately studying the possibilities of boosting operational flexibility [3].

In this way, fast-ramping thermal generators, like combined cycle gas turbines (CCGTs), offer a solid backup to deal with high-intermittent demand curves [4], increasing their starting-up and shutting-down frequency to satisfy this variability rise [5]. Consequently, it is imperative to represent these processes in detail, especially when thermal units manifest long-duration fuel-consumption curves in their start-up operations.

Regarding the optimal management of fuel-fired generators in electricity markets, the unit commitment (UC) problem provides optimum schedules while satisfying technical and market constraints [6]. The UC represents complex problems in which the modeling framework can be remarkably detailed if desired but also entails a high computational burden [7].

Although the UC computational efficiency has continuously been analyzed from different optimization perspectives [8, 9, 10], mixed integer linear programming (MILP) is frequently positioned as the best choice [11].

In that context, an ample catalog of different MILP models whose efficiency depends on good practices is available in the literature. The authors in [12] proposed to reduce the number of binary variables, stating that it could speed up solving times. With this purpose, they replaced the start-up and shut-down decisions with extra constraints and variables, leaving only the commitment decisions as unique binary variables.

Afterwards, the authors in [13] highlighted the concept of tightness, i.e., the proximity of the relaxed feasible region to the integer one, as the actual quality of the MIP formulations, where the ideal tightest possible formulation, i.e., convex hull, solves a MIP as an LP since all vertices of the relaxed feasible region satisfy all the integrality constraints [14]. Furthermore, the authors in [13] also claimed that the reduction of binary variables in [12] does not improve the model's performance, given that a tight UC formulation forces the start-up and shut-down variables to take binary values even if they are defined as continuous [12], [14]. Therefore, a tight UC formulation only needs the commitment decisions defined as binary variables, at the end presenting the same number of binary variables as the formulation in [12]. However, defining start-up and shut-down decisions as binary variables enhances the solver's capabilities to exploit their integrality properties [12], [15] (e.g., through preprocessing, creating cuts, and node exploration).

Authors in [15] remarked that there is a trade-off between tightness and compactness, where increasing model size brings computational advantages only if it significantly improves the tightness. This is actually one of the most powerful strategies of current MIP solvers: adding cuts to the model (increasing its size) to make it tighter. That is, although the LP iteration process is slower because of a larger model size, the problem is solved faster due to its reduced integrality complexity (tighter). Hence, using additional variables, even if binary, is desirable if they help to tighten and decrease the size of MIP problems, which inevitably leads to faster solving times.

Several methodologies have been presented in the literature to improve tightness in the unit commitment problem, focusing on minimum up and down times [16, 17], generation limits [13, 18], ramping processes [19, 20], turbine configurations [21, 22], hydro constraints [23], energy storage [24], or versatile formulating methods [25]. Besides, diverse approaches have been proposed for accurately modeling the start-up and shut-down processes, [18, 26, 27]. These are efficient stairwise formulations to handle start-up costs that achieve especially successful performances. Generally, the stairwise functions can closely approximate real start-up fuel-consumption curves with only a few start-up steps in short-term representations. Nonetheless, when medium-term horizons are evaluated, a significant amount of start-up steps are needed to reach high-detail linearizations [28]. This fact is illustrated in Fig. 1.

Piecewise functions offer a more efficient representation in medium-term horizons. In [28], the authors recently presented a compact piecewise MILP formulation compared to accurate stairwise models. This formulation introduces an offline-hour counter $h_{g,t}^{SD}$, a continuous variable with integer behavior that is linearly related to start-up decisions $v_{g,t,s}$ (binary variables) through the utilization of Big-M parameters. This methodology extends the use of piecewise linearizations in the unit commitment problem, whose application was limited to direct-linear relationships like production costs, emissions, hydro generation, or others, as described in [11].

Despite its successful results, Big-Ms considerably harm the formulations' tightness [21]. Therefore, this paper proposes a tighter and more compact piecewise MILP formulation for the start-up cost modeling that notably improves the computational performance. This piecewise methodology entails a disruptive modeling approach since the unit commitment literature only works with single-step or stairwise functions to linearly approximate start-up costs [8, 9, 10, 11, 29, 30, 31].

The main contributions are summarized below:

- This article proposes a tight and compact MILP formulation to accurately model high-resolution start-up costs in the medium-term unit commitment. The proposed model is thoroughly compared to high-performing formulations previously presented in the literature to prove its success (even when considering short-term horizons). Furthermore, the implications of using continuous/integer variable declarations in MILP optimization are addressed.
- This study compares the tightness and compactness of the formulations in both qualitative and quantitative ways. In addition, the resolution processes of these methodologies are also analyzed in detail to determine the best modeling practices. Besides, the improvements in the computational performance achieved with this proposal are exposed.
- This paper exposes the characterization of the technical features of a real gas-fired generation portfolio. Besides, the case studies represent the real operation of power systems with a high penetration of renewable generators and their corresponding demand variability. Therefore, the optimal schedules are obtained following modern market trends.

The reminder of this paper is organized as follows. Section 2 presents the tight and compact piecewise MILP formulation. It also shows the alternative efficient models with differences in

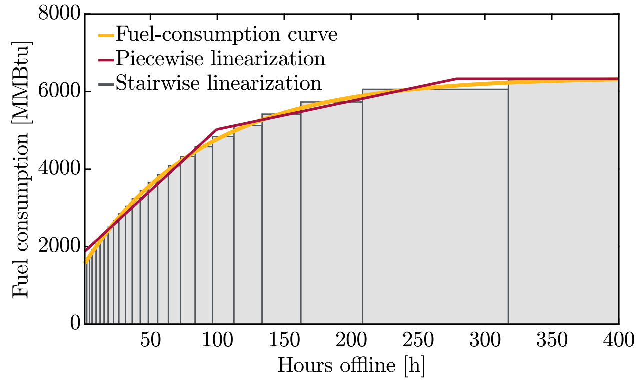

Figure 1: Representation of different linearization methodologies to approximate real long-duration fuel-consumption curves in the start-up process of a CCGT.

the integer/continuous declarations of variables that always manifest an integer behavior. Thereafter, Section 3 describes the case studies and analyzes the tightness, compactness, computational performance, and results of each formulation. To conclude, Section 4 gathers the conclusions.

2. Mathematical formulations

The start-up cost of a thermal unit, directly related to its fuel consumption, grows with the number of hours since it was shut down, flattening when the engines are completely cold. Hence, linearizations must be assumed to efficiently represent this cost in the unit commitment problem using MILP optimization.

Traditionally, stairwise linearizations have been employed to model start-up costs, where the offline hours in a thermal unit's operation are easily accounted for within their corresponding step intervals when the generator is started-up [18, 26, 27]. These approaches define a start-up type (s) per each desired time step in the stairwise linearization.

Figure 1 illustrates how the number of start-up costs (s) and model size grows proportional to the increasing resolutions when long-duration start-up cost curves are linearized through stairwise functions. On the contrary, piecewise approximations define a start-up type (s) per each segment cost, thus requiring fewer start-up types (s) to achieve the same modeling accuracy, significantly reducing problem sizes.

In [28], the authors employed Big-Ms to define the behavior of the offline-hour counters that enable the utilization of start-up costs' piecewise linearizations in MILP optimization. This formulation is remarkably more compact than stairwise models when the same accuracy is sought. However, Big-Ms harm the formulation's tightness, thus increasing run times.

This paper proposes a tight and compact MILP formulation that removes Big-Ms when modeling piecewise start-up costs. In turn, it analyzes the implications of declaring the offline-hour counters as continuous or integer variables. Later, these proposals are compared to the computationally efficient models presented in [18] and [28], which are described in the following subsections.

2.1. Tight & Compact Piecewise Formulation (TCPF)

The tight and compact piecewise formulation introduced in this paper uses an offline counter that indicates the number of hours that a thermal unit has been shut down in each time step, $h^{SD}_{g,t}$. This variable is linearly related to the time interval of the start-up segments $T^{SU}_{g,s}$, and the start-up decisions $v_{g,t,s}$ to determine another offline-hour counter which uniquely takes values in start-ups $h^{SU}_{g,t,s}$. Then, it multiplies the slope of the corresponding segment in the piecewise start-up cost function:

$$c^{SU}_{g,t} = \sum_{s \in S} \left(v_{g,t,s} C^{SU-N}_{g,s} + h^{SU}_{g,t,s} C^{SU-M}_{g,s} \right) \qquad \forall g, t \tag{1}$$

Equations (2)-(4) determine $h^{SD}_{g,t}$. When the unit is started-up, the offline-hour counter is set to zero. If the generator is still being committed, $h^{SD}_{g,t}$ remains zero. When the unit shut-downs, $h^{SD}_{g,t}$ starts increasing per time period in which the unit is not committed.

$$h^{SD}_{g,t} \leq (1 - \sum_{s \in S} v_{g,t,s}) \overline{O_g} \qquad \forall g, t \tag{2}$$

$$h^{SD}_{g,t} = TD^0_g + (1 - u_{g,t}) - \sum_{s \in S} h^{SU}_{g,t,s} \qquad \forall g, t = 1 \tag{3}$$

$$h^{SD}_{g,t} = h^{SD}_{g,t-1} + (1 - u_{g,t}) - \sum_{s \in S} h^{SU}_{g,t,s} \qquad \forall g, t \in [2, T] \tag{4}$$

Regarding $h^{SU}_{g,t,s}$, equations (2)-(6) guarantee that it is equal to $h^{SD}_{g,t-1}$ when the unit starts-up, where the equations (5) and (6) assure that the proper start-up segment is chosen according to its corresponding time interval $T^{SU}_{g,s}$.

$$h^{SU}_{g,t,s} \leq v_{g,t,s} T^{SU}_{g,s+1} \qquad \forall g, t, s \in [1, S_g) \tag{5}$$

$$h^{SU}_{g,t,s} \leq v_{g,t,s} \overline{O_g} \qquad \forall g, t, S_g \tag{6}$$

For the sake of simplicity, the remaining constraints of this formulation are shown in Appendix A. These tight equations employed in [28] are shared by every methodology described in the paper. (TCSF also uses them, making the corresponding changes when start-up decisions are involved).

It is important to mention that $h^{SD}_{g,t}$ and $h^{SU}_{g,t,s}$ are declared as continuous variables in this model. They can only take integer values because they are directly related to binary variables.

2.2. Tight & Compact Piecewise Formulation with an Integer-variable declaration (TCPFi)

In order to study the implications of the integer declaration of variables which have an implicit integer behavior, $h^{SD}_{g,t}$ and $h^{SU}_{g,t,s}$ from TCPF are defined integer for TCPFi, which also uses equations (1)-(6) and Appendix A.

2.3. Compact Piecewise Formulation (CPF)

The compact piecewise formulation presented in [28] is used to compare the efficiency of the proposed methodology. This approach also utilizes the offline counter $h^{SU}_{g,t,s}$ in equation (1) to determine the start-up cost. However, the relationships with $h^{SD}_{g,t}$ and $v_{g,t,s}$ are established by using Big-M parameters, $\overline{H}$. Equations (7)-(11) contribute to determining $h^{SD}_{g,t}$ at different circumstances:

$$h^{SD}_{g,t} \leq TD^0_g + 1 \qquad \forall g, t = 1 \tag{7}$$

$$h^{SD}_{g,t} \leq h^{SD}_{g,t-1} + 1 \qquad \forall g, t \in [2, T] \tag{8}$$

$$TD^0_g + 1 \leq h^{SD}_{g,t} + u_{g,t}\overline{H} \qquad \forall g, t = 1 \tag{9}$$

$$h^{SD}_{g,t-1} + 1 \leq h^{SD}_{g,t} + u_{g,t}\overline{H} \qquad \forall g, t \in [2, T] \tag{10}$$

$$h^{SD}_{g,t} - (1 - u_{g,t})\overline{H} \leq 0 \qquad \forall g, t \tag{11}$$

Equation (12) enables choosing the proper start-up type:

$$v_{g,t,s} \leq 1 + \frac{T^{SU}_{g,s+1} - h^{SD}_{g,t-1}}{\overline{H}} \qquad \forall g, t, s \in [1, S_g) \tag{12}$$

Equations (13)-(14) guarantee that the offline hours are only considered in the objective function when start-ups take place:

$$h^{SD}_{g,t-1} - (1 - v_{g,t,s})\overline{H} \leq h^{SU}_{g,t,s} \qquad \forall g, t, s \in [1, S_g) \tag{13}$$

$$h^{SU}_{g,t,s} \leq v_{g,t,s}\overline{H} \qquad \forall g, t, s \in [1, S_g) \tag{14}$$

It is important to note that, given the mathematical behavior of these constraints, it is not necessary to define them for the coldest segment, which achieves a more compact formulation.

2.4. Compact Piecewise Formulation with an Integer-variable declaration (CPFi)

The integer declaration of the offline-hour counters $h^{SU}_{g,t,s}$ and $h^{SD}_{g,t}$ is also addressed for the CPF approach proposed in [28]. Therefore, CPFi is introduced in this paper with the aim of acquiring a better perspective of the usefulness of declaring or not continuous variables when they have integer behaviors.

2.5. Tight & Compact Stairwise Formulation (TCSF)

The tight and compact model proposed in [18] is chosen as stairwise reference for comparing computational aspects in this article. This renowned formulation has demonstrated highly efficient performances in several unit commitment analyses [32, 33, 34], independently from the utilized commercial solver and software version.

Given that this approach implies using different set elements to model their start-up processes, the reader is referred to [18] for further detail. Moreover, [28] can be consulted for examining the stairwise linearization algorithm utilized to reach the same modeling accuracy as the piecewise approximations.

Table 1: Technical data of the thermal units.

UNIT	F_g^{P-LV} [MMBtu /MWh]	F_g^{P-NL} [MMBtu /h]	F_g^{SD} [MMBtu]	$F_{g,hot}^{SU-M}$ [MMBtu /h]	$F_{g,warm}^{SU-M}$ [MMBtu /h]	$F_{g,hot}^{SU-N}$ [MMBtu]	$F_{g,warm}^{SU-N}$ [MMBtu]	$F_{g,cold}^{SU-N}$ [MMBtu]	$\overline{P_g}$ [MW]	$\underline{P_g}$ [MW]	RD_g [MW /h]	RU_g [MW /h]	SD_g [MW]	SU_g [MW]	TD_g [h]	TU_g [h]	$T_{g,warm}^{SU}$ [h]	$T_{g,cold}^{SU}$ [h]
Unit A	6.6	300	1,100	392.3	77.9	1,517.4	3,545.3	4,899.2	412	157	215	215	157	157	7	7	7	18
Unit B	6.2	460	1,100	42.2	9.3	1,142.7	3,622.2	5,603.3	390	135	200	200	135	135	5	5	76	213
Unit C	5.4	820	1,900	326.3	9.0	768.6	5,280.0	8,696.9	856	285	425	425	285	285	11	11	18	50
Unit D	6.4	320	1,100	18.5	4.0	2,178.3	2,999.7	3,648.3	402	112	200	200	112	112	6	6	57	161
Unit E	6.8	280	1,200	12.6	2.8	2,691.1	3,505.9	4,169.3	413	157	215	215	157	157	7	7	84	236
Unit F	6.2	360	1,200	31.9	7.3	1,846.3	4,298.0	6,329.5	427	163	225	225	163	163	8	8	100	279
Unit G	5.6	740	1,900	25.9	5.8	4,821.5	6,578.8	7,995.8	796	225	385	385	225	225	10	10	88	246

3. Case studies and computational performance

The unit commitment formulations described in Section 2 are tested in large-size case studies that represent the operation of a modern power system. They have been run on a Intel Core i7-8700 @3.20 GHz with 12 logical processors and 32 GB of RAM and solved with Gurobi 10.0.1 [35] under GAMS 43.3.1 [36]. The optimality gap (OG) is used as the stopping criterion for the optimization processes. In minimization, is defined by (15):

$$OG\,(\%) = 100\,\frac{BI - BR}{BI} \tag{15}$$

Where BI is the best integer solution (upper bound) reached after the branch and bound (and cuts) process. BR is the best relaxed solution (lower bound). Hence, the OG is continuously updated in the node exploration with the values of BI and BR managed by the solver until the stopping criterion is attained.

3.1. Description of the case studies

The thermal portfolio used in every case study is composed by 7 CCGTs placed in the Iberian Electricity Market (MIBEL). The technical data of these real gas-fired generators are shown in Table 1. Their initial conditions are shown in Appendix B.

This paper uses three piecewise segments s for representing hot, warm, and cold start-up types. This distinction is usually made in the literature [37, 38, 39].

On the other hand, the step aggregation algorithm presented in [28] is employed to approximate the real start-up functions of these generation units to stairwise linearizations. Its application reveals that up to 36 start-up types (stairwise steps) are required to reach the same modeling accuracy achieved with three piecewise segments [28].

Concerning the construction of large-size case studies where the relevance of detailing real start-up costs can be manifested, 12 monthly case studies are considered on an hourly basis. Their hourly electricity demand profiles correspond to the real gas-fired generation in MIBEL in 2020, presented in [40]. Hence, it is possible to identify the influence of seasonality on demand variability. It is important to mention that the case studies run all the hours in their horizons simultaneously (they are not rolling horizon runs). Their lengths go from 696 hours (February 2020) to 744 hours, depending on the month. Therefore, a value of 744 is suitable when defining the $\overline{O_g}$ parameter. Meanwhile, $\overline{H}$ requires a more conservative value to avoid numerical-tolerance problems when handling $v_{g,t,s}$ decisions. Specifically, a value of 8760 is used as Big-M in these cases.

Equation (16) is applied to adjust the hourly demand of [40] to the maximum capacity of the thermal portfolio described in this paper, following the trends of a power system with a high penetration of renewable generation. In consequence, monthly peak demands correspond to the 95% of the portfolio capacity. Additionally, the demand series are subject to a data processing step. It replaces the hourly demands under the minimum power output of the smaller generator

Table 2: Hub trading, currency exchange rates and fuel prices.

	J	F	M	A	M	J	J	A	S	O	N	D
MIBGAS [€/MWht]	11.879	9.817	8.563	7.482	5.331	6.426	6.435	9.187	11.310	13.441	14.500	18.184
Exchange Rate [€/$]	0.901	0.917	0.904	0.921	0.917	0.889	0.873	0.845	0.848	0.849	0.845	0.822
Fuel Price [$/MMBtu]	3.864	3.138	2.776	2.381	1.704	2.118	2.160	3.186	3.909	4.639	5.029	6.483

by zero-demand hours to avoid infeasibilities. Later, a 5% of D_t is set as the spinning reserve requirements at each time period t.

$$D_t = \frac{Gas\ Fired\ Production_{\ t}}{max\{Gas\ Fired\ Production_{\ t}\}} 0.95 \sum_{g \in G} \overline{P_g} \tag{16}$$

Additionally, the monthly average gas prices of the Iberian Gas Market (MIBGAS) published in [41] are used to transform fuel consumptions into operational costs. These data are presented in Table 2, together with the monthly average currency exchange rates to obtain fuel prices in [$/MMBtu].

3.2. Tightness comparison

The tightness of a MIP problem is quantitatively measured by the integrality gap [42], which is defined as the proximity of the relaxed objective function (OF) to the integer one:

$$IG\ (\%) = 100\ \frac{OF_{Integer} - OF_{Relaxed}}{OF_{Integer}} \tag{17}$$

Where $OF_{Integer}$ is the integer solution and $OF_{Relaxed}$ is the relaxed one (here, we use the MIP root relaxation). As a clarification, the $OF_{Relaxed}$ values used in (17) are not modified as the MIP resolution processes evolve. (On the contrary, the $OF_{Relaxed}$ values employed in (15) are frequently updated by the solver with less restrictive objectives to meet the imposed OG, as described in [43].)

Given that this article analyzes different formulations and the implications of the continuous/integer variable declaration, the root relaxations are chosen as the $OF_{Relaxed}$. These values depend on utilizing presolve strategies. Therefore, the solver's heuristics introduce some differences between CPF and CPFi, and between TCPF and TCPFi, enabling a fairer comparison.

Moreover, it is important to highlight that TCSF will likely reach lower objective functions. Stairwise linearizations imply a group of offline hours having the same start-up cost. Hence, the solvers can take advantage of this fact to reduce OFs when managing alternative thermal generators' opportunities. On the contrary, piecewise linearizations manifest a different cost per offline hour. Therefore, utilizing these different linearization methods will unavoidably lead to different MIP solutions even if a 0% OG were defined.

In this Section, all the case studies are run until reaching a predefined 0.1% OG or until consuming a time limit of 15000 seconds. Table 3 presents the MIP objective functions and the calculated integrality gaps. TCSF is the tightest formulation. On the other hand, CPF and CPFi achieve practically identical integrality gaps. This fact also happens with TCPF and TCPFi.

Table 3: Integrality gap comparison.

#	MIP Objective Functions [$]					Final Optimality Gap [%]					Integrality Gap [%]				
	TCSF	CPF	CPFi	TCPF	TCPFi	TCSF	CPF	CPFi	TCPF	TCPFi	TCSF	CPF	CPFi	TCPF	TCPFi
J	**25,643,498**	25,654,729	25,649,145	25,647,524	25,647,524	0.28	0.37	0.36	**0.17**	**0.17**	**2.35**	3.20	3.20	2.85	2.85
F	**22,625,539**	22,626,437	22,626,437	22,626,437	22,626,437	**0.10**	0.16	0.15	0.13	**0.10**	**1.82**	2.75	2.74	2.35	2.35
M	**12,442,070**	12,444,577	12,444,577	12,444,577	12,444,577	0.17	0.28	0.26	0.16	**0.10**	**3.41**	4.45	4.44	4.06	4.06
A	**7,093,094**	7,096,645	7,096,461	7,095,447	7,095,260	**0.42**	0.74	0.79	0.54	0.56	**5.63**	7.17	7.15	6.82	6.82
M	**6,276,064**	6,277,911	6,277,912	6,277,912	6,277,912	0.16	0.21	0.24	0.17	**0.10**	**3.53**	4.53	4.53	4.21	4.21
J	**18,417,590**	18,418,742	18,418,742	18,418,742	18,418,394	0.25	0.14	0.16	0.21	**0.10**	**1.59**	2.08	2.08	1.85	1.85
J	23,965,340	**23,962,687**	**23,962,687**	**23,962,687**	**23,962,687**	0.11	0.15	0.15	0.12	**0.10**	**1.26**	1.54	1.54	1.38	1.38
A	**37,989,522**	37,992,159	37,992,155	37,992,155	37,995,032	**0.10**	**0.10**	**0.10**	**0.10**	**0.10**	**1.01**	1.35	1.35	1.18	1.18
S	39,120,747	**39,119,891**	**39,119,891**	**39,119,891**	39,120,023	0.14	0.13	0.22	0.15	**0.10**	**1.41**	1.77	1.77	1.64	1.64
O	**29,023,131**	29,032,698	29,036,945	29,032,526	29,032,526	0.20	0.36	0.32	0.33	**0.19**	**2.61**	3.55	3.55	3.21	3.21
N	**33,341,495**	33,348,229	33,356,060	33,348,229	33,348,229	0.17	0.19	0.23	0.29	**0.12**	**2.25**	3.24	3.24	2.71	2.71
D	**28,163,412**	28,174,323	28,176,373	28,168,154	28,173,120	**0.30**	0.45	0.58	0.40	0.32	**3.63**	5.18	5.18	4.72	4.72
$\overline{x}$	-	-	-	-	-	-	-	-	-	-	**2.54**	3.40	3.40	3.08	3.08

Nevertheless, the better root relaxations obtained in TCPF and TCPFi, compared to CPF and CPFi, quantitatively manifest the tightness improvements of the proposed formulations. Besides, in Appendix C, we conceptually show why the proposed model is tighter than the piecewise formulation of [28].

Furthermore, Table 3 presents the final optimality gaps of the case studies to provide insights into the accuracy of finding the optimal integer solutions. It can be appreciated that using 0.1% OGs imposes a very severe condition for these problems, being inaccessible for most of the models within the maximum time limit. In this regard, TCPFi shows the best performance.

3.3. Compactness comparison

As previously said, tightness represents a desirable quality whose improvement sometimes justifies the definition of extra variables and constraints in a formulation [13]. However, these additional elements can remarkably increase the model sizes, increasing their run times [15]. Table 4 presents the average problem sizes before and after presolve performance for each formulation listed in Section 2.

Here, it is important to mention that start-up types S differ for TCSF (with up to 36 steps s) and piecewise linearizations (3 segments s). Meanwhile, thermal units G are equal in every case study (7 generators g), and T depends on the month (from 696 hours in February 2020 to 744). Therefore, TCSF exposes a significantly larger problem size according to the constraints and

Table 4: Compactness comparison.

Form.	Constraints	Continuous Variables	Integer Variables	Binary Variables
TCSF	$\sim T \cdot (2 + G \cdot (6 + S)$	$T \cdot G \cdot 2$	-	$T \cdot G \cdot (3+S)$
CPFi	$\sim T \cdot (2 + G \cdot (6 + 3 \cdot S)$	$T \cdot G \cdot (2+S)$	-	$T \cdot G \cdot (2+S)$
CPFi	$\sim T \cdot (2 + G \cdot (6 + 3 \cdot S)$	$T \cdot G \cdot 2$	$T \cdot G \cdot S$	$T \cdot G \cdot (2+S)$
TCPF	$\sim T \cdot (2 + G \cdot (8 + S)$	$T \cdot G \cdot (3+S)$	-	$T \cdot G \cdot (2+S)$
TCPFi	$\sim T \cdot (2 + G \cdot (8 + S)$	$T \cdot G \cdot 2$	$T \cdot G \cdot (1+S)$	$T \cdot G \cdot (2+S)$
		Constraints & Variables Before Presolve [#]		
TCSF	177,629	10,248	-	210,084
CPF	78,230	25,620	-	25,620
CPFi	78,230	10,248	15,372	25,620
TCPF	57,734	30,744	-	25,620
TCPFi	57,734	10,248	20,496	25,620
		Constraints & Variables After Presolve [#]		
TCSF	132,856	8,803	-	120,240
CPF	56,396	22,461	-	22,926
CPFi	56,454	8,697	13,783	22,953
TCPF	50,701	26,427	-	23,256
TCPFi	50,704	8,803	17,642	23,240

binary variables.

On the other hand, CPFi is analogous to CPF, and TCPFi is similar to TCPF. They decrease S continuous variables per TG and increase integer variables in the same quantity. When comparing TCPF to CPF and TCPFi to CPFi, a constraint reduction is observed (they are equal for $S = 1$, and reduce $2S - 2$ per TG if $S > 1$). Besides, an increment of 1 variable per TG is also observed (continuous in TCPF and integer in TCPFi).

Despite the more thorough presolve performance in TCSF, their problem sizes are still notably larger. Furthermore, TCPF and TCPFi maintain the compactness improvements that CPF achieved in [28] while accomplishing a tightness enhancement as demonstrated in Section 3.2 and Appendix C. Therefore, it can be affirmed that the proposed formulations are tighter and more compact than those in [28].

The integer complexity of a MIP problem is defined by the distance between the relaxed and the integer feasible regions. That is, if the problem is the tightest possible (convex hull), the MIP problem can be solved as an LP. The larger this distance, the more trouble the branch and bound (and cuts) will face in finding the integer optimum. Meanwhile, the speed at which the solver explores this distance is defined by the size of the LPs that are repeatedly solved at each branch and bound node. Thus, establishing tight and compact trade-offs is mandatory.

Table 5: Resolution processes comparison - Optimization times.

#	MIP Run Time [s]					Generating MIP Model [s]					Presolve Time [s]					rMIP Run Time [s]				
	TCSF	CPF	CPFi	TCPF	TCPFi	TCSF	CPF	CPFi	TCPF	TCPFi	TCSF	CPF	CPFi	TCPF	TCPFi	TCSF	CPF	CPFi	TCPF	TCPFi
J	1958.9	858.5	721.7	223.3	**220.6**	269.9	8.4	8.3	8.1	8.2	52.5	8.1	8.5	6.0	5.7	221.3	10.8	10.8	12.0	12.3
F	441.9	275.8	522.8	**89.6**	157.5	155.6	7.4	7.5	7.1	7.2	54.3	104.6	113.7	5.9	5.8	200.2	9.0	9.2	10.7	10.7
M	1959.4	1393.2	884.5	379.8	**206.7**	204.4	8.3	8.4	8.1	8.0	50.8	7.1	7.2	3.9	3.9	245.2	10.5	10.4	12.2	12.1
A	4591.6	3084.8	3055.3	**1245.2**	1776.2	216.4	7.9	7.9	7.4	7.4	49.1	7.4	7.9	3.7	3.9	193.7	10.0	9.8	11.4	11.2
M	1868.2	1104.0	662.8	268.5	**154.9**	242.5	8.3	8.3	8.0	7.9	49.8	11.8	12.3	3.6	3.7	244.7	10.4	10.3	12.2	12.5
J	907.2	298.2	381.6	215.1	**200.5**	217.1	7.8	7.8	7.6	7.5	54.0	7.5	7.8	6.3	6.0	209.4	9.7	9.8	12.2	12.1
J	640.2	293.3	459.1	160.2	**154.4**	243.9	8.4	8.3	8.0	8.0	58.7	75.4	88.0	6.1	6.3	221.8	10.4	10.6	12.7	12.5
A	499.1	205.7	148.8	134.1	**120.2**	286.3	8.3	8.5	8.0	8.0	56.0	26.8	27.6	5.7	6.2	217.6	10.3	10.4	11.8	11.7
S	850.2	162.8	190.6	**142.2**	198.3	262.2	8.1	7.9	7.5	7.5	58.1	8.9	9.0	6.8	7.0	195.0	9.6	9.6	11.2	11.2
O	1032.3	1033.6	1240.2	491.3	**340.0**	218.4	8.7	8.3	7.9	8.0	53.7	8.1	8.0	5.3	5.6	212.0	10.5	10.6	12.4	12.3
N	847.7	561.9	514.6	297.2	**254.4**	199.8	7.9	7.8	7.5	7.6	53.6	8.0	8.2	3.8	3.8	198.1	9.8	9.9	12.2	12.1
D	1653.4	1140.7	1713.8	703.5	**449.7**	203.9	8.4	8.4	7.9	8.0	50.6	6.9	7.2	3.8	3.9	218.1	10.5	10.5	12.5	12.0
$\overline{x}$	1437.5	867.7	874.7	362.5	**352.8**	226.7	8.2	8.1	7.8	7.8	53.4	23.4	25.4	5.1	5.1	214.8	10.1	10.2	11.9	11.9
SUF	1.0	1.9	1.8	4.2	**4.6**	1.0	27.5	27.6	28.9	28.8	1.0	4.0	3.8	10.8	10.6	1.0	21.2	21.1	18.0	18.0

3.4. Resolution processes comparison

Although the tightness and compactness frequently predict the performance of MIP formulations, it is essential to evaluate computational performances to complete the analysis of these models. In accordance, the 60 case studies presented in Section 3.1 are carried out and shown in Table 5 and Table 6, using a 1% OG as the stopping criterion.

Table 5 shows the total run times of each case study. It also includes the required time to generate the model and presolve times (both metrics are part of the total run time). In turn, the run times obtained when running these 60 case studies as relaxed mixed-integer programming (rMIP) problems are also included to explain how the compactness of these problems is slowing down the node-resolution processes in the branch and bound.

With the aim of better comparing these formulations, Table 5 also shows average values $\overline{x}$. Additionally, speed-up factors (SUF) are used to globally contrast the run-time improvements performed with each formulation. They are calculated through Equation (18), a geometric mean of ratios of the n case studies in which TCSF is used as the benchmark formulation:

$$SUF = \sqrt[N]{\prod_{n=1}^{N} \frac{Formulation\ Run\ Time_{n}}{Benchmark\ Form.\ Run\ Time_{n}}} \tag{18}$$

Generally, the piecewise formulations solve faster than stairwise linearizations. CPF improves the performance of TCSF, obtaining similar results as in [28] independently from solver versions. Although CPFi also attains better performances than TCSF, the integer declaration of the offline-hour counters does not manifest a significant influence, accomplishing a slightly worse SUF and average run time.

On the other hand, TCPF and TCPFi achieve a remarkable performance improvement, raising their speed-up factors to 4.2 and 4.6, respectively, and also reducing the average run time to approximately a fourth part of the TCSF requirements. Here, the integer declaration of the offline-hour counters seems to be the right choice. Despite both formulations reaching a similar average run time, most case studies individually demand fewer run times to find the optimal solution in TCPFi. Therefore, its SUF is slightly better.

Regarding the presolve performance, CPF and CPFi employ around half time compared to TCSF. Also, TCPF and TCPFi uniquely spend the tenth part. Furthermore, problem sizes play a critical role in generating the MIP model. The bigger TCSF models require ∼28 more times than piecewise formulations. This effect can be observed in the rMIP run times, too. Model generation

Table 6: Resolution processes comparison - Branch & cut performances.

#	Nodes					Cuts					Heuristics					Simplex Iterations per second				
	TCSF	CPF	CPFi	TCPF	TCPFi	TCSF	CPF	CPFi	TCPF	TCPFi	TCSF	CPF	CPFi	TCPF	TCPFi	TCSF	CPF	CPFi	TCPF	TCPFi
J	814	4156	3003	861	1047	4234	10952	10138	6187	6040	39	51	52	41	41	433	899	1123	1456	1516
F	55	1863	2386	1	941	3161	9514	8437	4625	4384	15	26	37	35	38	375	1093	858	573	1582
M	550	3221	2666	1650	831	4053	10154	10656	5147	6830	30	33	50	47	32	443	975	1255	1417	1732
A	1212	11902	4721	2869	3523	4351	11317	10264	5348	5227	42	58	73	58	44	381	1666	1015	983	907
M	811	2864	1733	1204	373	2879	7644	7269	3408	5011	25	31	30	30	30	406	923	1067	1333	1496
J	449	3776	4279	721	509	3775	11055	11914	4943	5196	29	30	58	43	41	412	2138	2152	1351	1324
J	265	3025	4605	708	516	3411	10015	6134	4657	5213	26	36	37	30	28	408	1434	1231	1504	1504
A	1	1048	1443	1286	802	3463	7846	9013	4679	4794	15	12	22	23	28	272	1404	1852	1804	1486
S	489	851	1052	255	2185	3846	10518	8979	4856	5504	33	32	22	30	48	517	2162	1837	1177	1552
O	551	4744	3696	3124	3044	2991	9393	9189	4605	4671	29	50	73	56	52	439	802	837	1331	1333
N	515	3734	3358	2062	2919	3775	10460	9896	3420	2517	35	48	55	39	50	435	1389	1353	1501	1617
D	828	2357	7249	1310	2211	3392	9421	10163	4215	4818	25	44	84	51	53	351	668	1160	671	1116
$\overline{x}$	545	3628	3349	1338	1575	3611	9857	9338	4674	5017	29	38	49	40	40	406	1296	1312	1258	1430

complicates determining TCSF's relaxed solutions, which substantially slows down the branch and cut process of MIP optimization.

Respecting the solver's work in the branch and cut process, Table 6 shows the number of nodes explored in the enumeration tree, the amount of cutting planes introduced to improve the performance, the heuristics, and the simplex iterations per second. As expected, node exploration is significantly lower in TCSF, as well as the simplex iterations, cuts, and heuristics. It demonstrates that due to the larger model size, the stairwise formulation in the branch and cut process is more demanding than piecewise formulations.

On the other hand, the simplex iterations per second show similar average behaviors with CPF, CPFi, TCPF, and TCPFi. They present analogous efficiency, but TCPF and TCPFi need fewer iterations to reach the optimum. Thus, fewer nodes were explored, and fewer cuts were generated. Conversely, similar average heuristics are used despite their lower run times.

To sum up, TCPF and TCPFi not only manifest an enhanced tightness and compactness. The case studies demonstrate that they perform more efficient resolution processes. Additionally, the integer formulation of the offline-hour counters involves a stronger computational performance. TCPFi reaches a 4.6 SUF compared to TCSF. Moreover, the individual case studies often show better run times. Besides, TCPF is also efficiently solved, achieving a 4.2 SUF and an average run time close to TCPFi.

Table 7: Accuracy comparison - Operational costs with 1% optimality gap.

#	Production Costs [$]					Start-up & Shut-down Costs [$]				
	TCSF	CPF	CPFi	TCPF	TCPFi	TCSF	CPF	CPFi	TCPF	TCPFi
J	25,003,061	24,798,436	24,895,739	24,881,562	24,833,600	801,387	955,714	842,490	816,233	865,112
F	22,019,269	21,994,218	21,979,179	21,973,510	21,981,800	721,201	668,941	688,329	692,305	657,533
M	12,067,757	12,054,753	12,026,039	12,039,871	12,042,942	435,063	436,008	475,273	416,156	419,932
A	6,752,876	6,705,473	6,704,220	6,711,825	6,711,939	367,180	391,437	391,228	388,418	387,698
M	6,044,468	6,033,688	6,031,702	6,031,182	6,045,068	267,694	261,924	256,645	255,898	245,414
J	18,072,864	18,103,671	18,065,519	18,133,797	18,111,691	433,484	366,183	401,987	342,655	356,776
J	23,678,763	23,667,963	23,687,132	23,707,559	23,732,271	390,139	340,056	332,818	328,024	302,075
A	37,646,756	37,558,278	37,639,458	37,553,229	37,625,375	601,867	496,057	466,063	535,335	509,667
S	38,560,137	38,582,435	38,625,974	38,614,955	38,637,618	737,616	620,979	613,662	636,480	580,699
O	28,065,604	28,062,331	28,014,886	28,104,419	28,055,188	1,048,630	1,073,151	1,107,342	992,743	1,016,869
N	32,282,996	32,211,359	32,215,698	32,175,446	32,219,244	1,226,503	1,269,569	1,213,545	1,218,808	1,163,159
D	26,675,524	26,653,133	26,648,216	26,653,859	26,619,573	1,645,276	1,579,947	1,551,185	1,537,647	1,580,973

Table 8: Accuracy comparison - Operational costs with 0.1% optimality gap.

#	Production Costs [$]					Start-up & Shut-down Costs [$]				
	TCSF	CPF	CPFi	TCPF	TCPFi	TCSF	CPF	CPFi	TCPF	TCPFi
J	24,836,768	24,842,810	24,836,430	24,838,994	24,838,994	806,730	811,919	812,714	808,530	808,530
F	21,967,562	21,967,471	21,967,471	21,967,471	21,967,471	657,977	658,966	658,967	658,967	658,967
M	12,019,402	12,025,816	12,025,816	12,025,816	12,025,816	422,668	418,761	418,761	418,761	418,761
A	6,699,492	6,706,963	6,699,193	6,704,220	6,699,492	393,602	389,682	397,268	391,228	395,768
M	6,035,243	6,039,703	6,039,703	6,039,703	6,039,703	240,821	238,195	238,209	238,209	238,209
J	18,081,846	18,089,343	18,089,343	18,089,343	18,086,890	335,745	329,399	329,399	329,399	331,504
J	23,644,628	23,657,477	23,657,477	23,657,477	23,657,477	320,712	305,208	305,210	305,210	305,210
A	37,514,142	37,513,206	37,512,816	37,512,816	37,526,761	475,380	478,953	479,339	479,339	468,270
S	38,529,646	38,526,900	38,526,900	38,526,900	38,532,955	591,101	592,990	592,991	592,991	587,068
O	28,008,114	28,009,795	28,014,047	28,014,946	28,014,946	1,015,017	1,022,903	1,022,898	1,017,580	1,017,580
N	32,161,035	32,184,206	32,184,036	32,184,206	32,184,206	1,180,460	1,164,023	1,172,025	1,164,023	1,164,023
D	26,644,814	26,657,957	26,631,492	26,622,842	26,646,666	1,518,597	1,516,367	1,544,882	1,545,311	1,526,454

3.5. Results comparison

3.5.1. Influence of the optimality gap on the operational costs

This section compares the optimal solutions provided by the formulations analyzed in this paper under different optimality gaps. Table 7 and Table 8 present the production costs and the start-up and shut-down costs obtained in the 60 case studies introduced in Section 3.1 by imposing a 1% OG, like in Section 3.4, and those achieved after defining a 0.1% OG or a maximum time limit of 15000 seconds. In turn, the computational performance of the 0.1% OG cases is addressed in Table 10, exposing the run time or the final optimality gaps when the time limit is reached.

As previously said, 0.1% OG is a too strict convergence criterion given the nature of the case studies. It can be appreciated that most of the approaches reach one point where they slowly further improve the solution. Although these integer solutions are similar (final upper bounds), the TCPFi's problems starting lower bounds are greater than those of the piecewise models of [28] and perform a faster convergence towards the defined optimality gaps. This outstanding computational efficiency is in line with the conclusions of Section 3.4, where this model consumes less time in optimizing the large-size problems used in this article.

Regarding the quality of the results, the objective functions obtained with both optimality gaps are similar. The 0.1% OG imposition accomplishes less than a 0.3% average reduction. However, most of the case studies different from some TCPFi's cannot finish within the time limit. In turn, the contribution of start-up, shut-down, and production costs to the final objective function is relatively constant with these optimality gaps. If the average values of the 60 cases are compared, a ∼3.2% contribution of start-up and shut-down costs is obtained with 1% OG against ∼3.1% with 0.1% OG. Nevertheless, significantly more consistent results in the 0.1% OG cases are appreciated when distinguishing between the formulations. In that line, the coefficient of variation in the piecewise models (CPF, CPFi, TCPF, TCPFi) is reduced in one order of magnitude when the 0.1% OG is imposed.

Therefore, it can be concluded that the optimality gap does not preferentially boost any specific operational cost reduction. At the same time, its inherent reduction of the alternatives that satisfy the converge criteria makes the integer solutions more homogenous when working with different formulations.

3.5.2. Influence of the linearization methods on the start-up costs

Given the application of different linearization methods in the stairwise and piecewise formulations, their optimization processes will not reach the same optimal solution. However, the accuracy of these linearizations can be measured by comparing the start-up costs determined in each case study to those calculated with the real (exponential) start-up curves depending on the offline hours when start-up decisions are set. These real curves are obtained by multiplying the fuel-consumption curves of Appendix D by the monthly fuel prices gathered in Table 2.

Table 9: Start-up costs obtained with linearizations and with the real curves.

#	Start-up Costs with Linearizations [$]					Start-up Costs with Real Curves [$]				
	TCSF	CPF	CPFi	TCPF	TCPFi	TCSF	CPF	CPFi	TCPF	TCPFi
J	569,867	575,442	575,851	571,280	571,280	690,331	675,798	670,705	706,958	706,958
F	461,852	462,841	462,842	462,842	462,842	705,008	594,595	594,589	609,021	609,034
M	306,076	303,557	303,557	303,557	303,557	356,591	215,670	215,670	248,996	248,996
A	272,409	275,870	276,313	273,844	274,575	530,979	289,769	321,936	315,839	335,077
M	181,863	181,282	181,295	181,295	181,295	337,874	111,487	111,461	137,132	137,156
J	245,094	241,502	241,502	241,502	243,607	600,525	300,628	300,628	318,426	316,761
J	230,208	222,048	222,050	222,050	222,050	633,002	301,901	301,900	314,265	314,265
A	339,019	342,978	342,978	342,978	335,414	661,409	536,323	536,323	544,406	533,399
S	440,995	442,884	442,885	442,885	441,262	485,480	462,705	462,704	489,847	458,346
O	731,574	739,924	739,919	734,138	734,138	688,919	775,224	761,887	837,601	837,601
N	867,656	857,757	865,759	857,757	857,757	792,046	1,008,993	1,020,694	1,069,504	1,069,504
D	1,073,215	1,072,281	1,096,906	1,098,633	1,081,720	846,139	1,350,931	1,379,928	1,426,468	1,415,082

Table 9 shows the differences between the optimal-linearized costs and their corresponding real values. Most of the cases manifest that linearizations involve a reduction when modeled start-up costs are compared to their real values. This reduction is more significant when working with stairwise approaches. Besides, Table 10 also exposes the mean absolute percentage error (MAPE) committed in each case study's start-up cost representation. If the average MAPE of each formulation is compared, it can be appreciated that piecewise linearizations achieve better start-up costs characterizations (∼22% MAPE) than the stairwise linearization (∼32% MAPE).

Table 10: Run time with 0.1% optimality gap and MAPE committed with start-up costs' linearizations.

#	Run Time with 0.1% Optimality Gap [s]					MAPE: Linearized vs Real Start-up Costs [%]				
	TCSF	CPF	CPFi	TCPF	TCPFi	TCSF	CPF	CPFi	TCPF	TCPFi
J	(0.28%)	(0.37%)	(0.36%)	**(0.17%)**	**(0.17%)**	17.45	14.85	14.14	19.19	19.19
F	6726.7	(0.16%)	(0.15%)	(0.13%)	**2560.3**	34.49	22.16	22.16	24.00	24.00
M	(0.17%)	(0.28%)	(0.26%)	(0.16%)	**7265.7**	14.17	40.75	40.75	21.91	21.91
A	**(0.42%)**	(0.74%)	(0.79%)	(0.54%)	(0.56%)	48.70	4.80	14.17	13.30	18.06
M	(0.16%)	(0.21%)	(0.24%)	(0.17%)	**14821.2**	46.17	62.60	62.65	32.21	32.18
J	(0.25%)	(0.14%)	(0.16%)	(0.21%)	**6619.7**	59.19	19.67	19.67	24.16	23.09
J	(0.11%)	(0.15%)	(0.15%)	(0.12%)	**3130.2**	63.63	26.45	26.45	29.34	29.34
A	5476.8	1315.3	3854.2	2354.9	**370.3**	48.74	36.05	36.05	37.00	37.12
S	(0.14%)	(0.13%)	(0.22%)	(0.15%)	**1829.7**	9.16	4.28	4.28	9.59	3.73
O	(0.20%)	(0.36%)	(0.32%)	(0.33%)	**(0.19%)**	6.19	4.55	2.88	12.35	12.35
N	(0.17%)	(0.19%)	(0.23%)	(0.29%)	**(0.12%)**	9.55	14.99	15.18	19.80	19.80
D	**(0.30%)**	(0.45%)	(0.58%)	(0.40%)	(0.32%)	26.84	20.63	20.51	22.98	23.56
$\overline{x}$	-	-	-	-	-	32.02	22.65	23.24	22.15	22.03

3.6. Application to a large-size thermal portfolio in the short-term unit commitment problem

This section analyzes the models' performances when optimizing large power systems. Despite the proposed approaches focusing on the medium-term unit commitment, these horizons entail a high computational burden. Hence, the time spans of this section's case studies have been reduced to 24 hours. Five non-zero demand days per month exposed in Section 3.1 are chosen to construct this additional comparison benchmark.

Besides, a larger thermal portfolio that comprises 100 times the generators described in Table 1 is used. Consequently, 300 case studies with 700 generators operating in 24-hour horizons are run until reaching a 0.1% OG. The run times of each model are exposed in a box plot in Fig. 2. Here, it can be appreciated that although TCSF's distribution intervals are the closest, the median run time of TCPFi is the best, with its intervals placed below TCSF's. Regarding average run times, ∼300s, ∼765s, ∼550s, ∼270s, and ∼200s, are obtained for TCSF, CPF, CPFi, TCPF, and TCPFi, respectively, and their SUF are 1.0x, 0.7x, 0.9x, 1.5x, and 1.7x, manifesting the superiority of TCPFi.

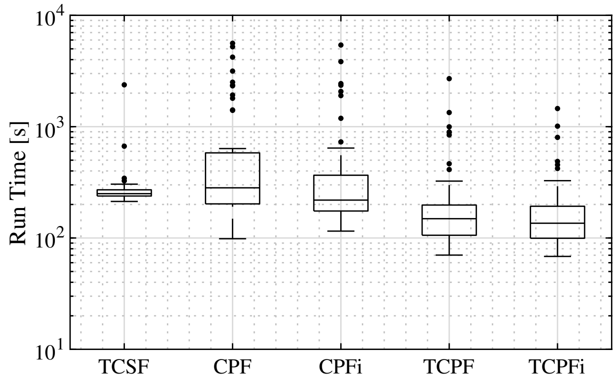

Figure 2: Box plot of run time required to solve the short-term problems with the large generation portfolio.

Note that these results involve symmetry effects because of the generator replication. In turn, this short-term representation is also appropriate for characterizing simple-cycle gas turbines, whose coolest steps are easily reached in just one day.

Finally, it is important to highlight that these case studies imply a remarkably lower computational burden. They end in reasonable run times despite using a severe converge criterion, entailing approximately three times larger problem sizes, and implying the presence of symmetry effects in the MIP resolution processes. This less arduous computational performance in the short-term unit commitment was also identified in [43]. Consequently, increasing modeling detail especially affects the medium-term representations, manifesting the importance of improving the efficiency of the formulations.

4. Conclusions

This paper presents a tight and compact formulation for accurately representing long-duration start-up costs in the unit commitment problem. This model manages start-up costs with piecewise linearizations in MILP optimization problems. Its effectiveness is proved by comparing its performance against current high-performing formulations in several large-size case studies. These cases also capture the intermittence in thermal generation due to the high penetration of renewable energy sources in modern power systems in medium-term horizons.

The proposed formulation is compared to a computationally efficient stairwise model, previously presented in the literature, and to a more recent MILP piecewise formulation. Moreover, the implications of declaring continuous or integer variables for implicitly integer optimization elements are studied in this article. Consequently, the tightness, compactness, and resolution processes of high-performing formulations are analyzed in depth. Despite the stairwise formulation being the tightest, its compactness is excessively far from the piecewise models. In this regard, the proposed formulation achieves a significantly improved tight and compact trade-off, reaching a 4.6 speed-up factor compared to the stairwise model and reducing the average run time to a fourth part of the stairwise formulation when the same modeling accuracy is sought in medium-term high-resolution case studies which are acutely computationally demanding.

The scope of this paper is limited to MILP formulations because they lead to global-optimal solutions. Moreover, the accuracy of their results is also demonstrated. The integer solutions obtained under diverse convergence criteria to finish the optimization processes are presented in this paper. They show that the contributions of the different operational costs to the final objective function do not significantly depend on the imposed optimality gaps. Therefore, it can be concluded that those decisions which involve more reduced costs are not disregarded in the resolution processes. On the other hand, non-linear mixed-integer programming approaches can be explored as future research to try to reduce run times even more by directly addressing the start-up cost functions (Appendix D), followed by in-depth analyses of the accuracy of their (non-global optimal) results.

Finally, this methodology is applied to case studies in which the operation of a large-size thermal portfolio is optimized in short-term horizons. This study concludes that short-term cases are less arduous to solve than the medium-term representations despite notably entailing larger problem sizes (involving more constraints and binary and continuous variables). Although the proposed formulation mainly focuses on the medium-term unit commitment, it also performs a more computationally efficient resolution process in these more easy-to-solve case studies. In this regard, the proposed formulation achieves a 1.7 speed-up factor compared to the stairwise model.

Appendix A. Objective function and technical constraints

The objective function and technical constraints used in this paper are presented below. They are tight and compact options to model generators' features in the unit commitment problem [18]:

- *Objective function:*

$$min \left(\sum_{g \in G} \sum_{t \in T} c^{P}_{g,t} + c^{SD}_{g,t} + c^{SU}_{g,t} \right) \tag{A.1}$$

- *Production cost:*

$$c^{P}_{g,t} = u_{g,t} C_g^{P-NL} + (u_{g,t} \underline{P_g} + p_{g,t}) C_g^{P-LV} \qquad \forall g, t \tag{A.2}$$

- *Generation limits:*

$$\begin{aligned} p_{g,t} + r_{g,t} \leq u_{g,t}(\overline{P_g} - \underline{P_g}) - \sum_{s \in S} v_{g,t,s}(\overline{P_g} - SU_g) \\ - w_{g,t+1}(\overline{P_g} - SD_g) \qquad \forall g \notin G^1, t \end{aligned} \tag{A.3}$$

$$\begin{aligned} p_{g,t} + r_{g,t} \leq u_{g,t}(\overline{P_g} - \underline{P_g}) \\ - \sum_{s \in S} v_{g,t,s}(\overline{P_g} - SU_g) \quad \forall g \in G^1, t \end{aligned} \tag{A.4}$$

$$\begin{aligned} p_{g,t} + r_{g,t} \leq u_{g,t}(\overline{P_g} - \underline{P_g}) \\ - w_{g,t+1}(\overline{P_g} - SD_g) \qquad \forall g \in G^1, t \end{aligned} \tag{A.5}$$

- *Ramping constraints:*

$$p_{g,t} + r_{g,t} - p_{g,t-1} \leq RU_g \qquad \forall g, t \in [2, T] \tag{A.6}$$

$$-p_{g,t} + p_{g,t-1} \leq RD_g \qquad \forall g, t \in [2, T] \tag{A.7}$$

- *Balance constraints:*

$$D_t \leq \sum_{g \in G} u_{g,t} \underline{P_g} + p_{g,t} \qquad \forall t \tag{A.8}$$

$$R_t \leq \sum_{g \in G} r_{g,t} \qquad \forall t \tag{A.9}$$

- *Shut-down cost:*

$$c^{SD}_{g,t} = w_{g,t} C^{SD}_g \qquad \forall g, t \tag{A.10}$$

- *Logic constraint:*

$$\sum_{s\in S} v_{g,t,s} - w_{g,t} = u_{g,t} - u_{g,t-1} \qquad \forall g, t \in [2, T] \qquad \text{(A.11)}$$

- *Minimum time up/down constraints:*

$$\sum_{i=t-TU_g+1}^{t} \sum_{s\in S} v_{g,i,s} \leq u_{g,t} \qquad \forall g, t \in [TU_g, T] \qquad \text{(A.12)}$$

$$\sum_{i=t-TD_g+1}^{t} w_{g,i} \leq 1 - u_{g,t} \qquad \forall g, t \in [TD_g, T] \qquad \text{(A.13)}$$

- *Initial condition constraints:*

$$\sum_{s\in S} v_{g,t,s} - w_{g,t} = u_{g,t} - U_g^0 \qquad \forall g, t = 1 \qquad \text{(A.14)}$$

$$p_{g,t} - (P_g^0 - U_g^0 \underline{P_g}) \leq RU_g \qquad \forall g, t = 1 \qquad \text{(A.15)}$$

$$-p_{g,t} + (P_g^0 - U_g^0 \underline{P_g}) \leq RD_g \qquad \forall g, t = 1 \qquad \text{(A.16)}$$

- *Operational-coherence constraint at the beginning:*

$$u_{g,t} = U_g^0 \qquad \forall g, t \in [1, TU_g^R + TD_g^R] \qquad \text{(A.17)}$$

– *Obtaining of pre-optimization parameters:*

$$\overline{O_g} = TD_g^0 + \sum_{t\in T} 1 \qquad \forall g \qquad \text{(A.18)}$$

$$C_g^{P-LV} = F_g^{P-LV} C_g^F \qquad \forall g \qquad \text{(A.19)}$$

$$C_g^{P-NL} = F_g^{P-NL} C_g^F \qquad \forall g \qquad \text{(A.20)}$$

$$C_g^{SD} = F_g^{SD} C_g^F \qquad \forall g \qquad \text{(A.21)}$$

$$C_{g,s}^{SU-M} = F_{g,s}^{SU-M} C_g^F \qquad \forall g, s \qquad \text{(A.22)}$$

$$C_{g,s}^{SU-N} = F_{g,s}^{SU-N} C_g^F \qquad \forall g, s \qquad \text{(A.23)}$$

$$TU_g^R = max\{0, (TU_g - TU_g^0) U_g^0\} \qquad \forall g \qquad \text{(A.24)}$$

$$TD_g^R = max\{0, (TD_g - TD_g^0)(1 - U_g^0)\} \qquad \forall g \qquad \text{(A.25)}$$

This model performs a conventional representation of the generators' technical operation, as it is frequently addressed when a computationally efficient formulation of the unit commitment problem is proposed [12, 18, 26, 27, 28]. However, implementing additional constraints, like advanced power equipment modeling, could be easily addressed.

Appendix B. Initial conditions of the thermal units

The technical information about the initial conditions of the thermal portfolio presented in this paper is shown in Table B.11.

Table B.11: Initial conditions of the thermal units.

Thermal Unit	P_g^0 [MW]	TD_g^0 [h]	TU_g^0 [h]	U_g^0
Unit A	314	0	7	1
Unit B	270	0	5	1
Unit C	570	0	11	1
Unit D	224	0	6	1
Unit E	314	0	7	1
Unit F	326	0	8	1
Unit G	450	0	10	1

Appendix C. Comparing feasible regions established by offline-hour counter constraints

This appendix compares the tightness of the constraints proposed in this paper (TCPF and TCPFi) for modeling the offline-hour counters' behavior to those introduced in [28] (used in CPF and CPFi). Note that the comparison between CPF and TCPF is the same as between CPFi and TCPFi due to the absence of differences in their constraints.

For the sake of simplicity, this analysis is presented for uniquely one thermal generator g and one start-up type s (for instance, the hottest start-up $s = 1$). Therefore, the notation of g and s disappears from the following equations.

It is important to mention that this study uses the assumption of considering a de-committed thermal unit at the beginning ($u_t = 0$). In turn, the time step t for performing the analysis is different from $t = 1$ given that the initial period depends on exogenous parameters to manage the initial status and handles fewer decision variables.

Hence, the logic constraint (A.11) that determines the behavior between commitment, start-up, and shut-down decisions becomes (C.1) under the previously described assumptions:

$$v_t - w_t = u_t - u_{t-1} \qquad \forall t \qquad \text{(C.1)}$$

Meanwhile, the constraint (A.12) that maintains a unit online for a certain amount of hours since it is started-up becomes:

$$\sum_{i=t-TU_g+1}^{t} v_i \leq u_t \qquad \forall t \in [TU_g, T] \qquad \text{(C.2)}$$

Then, the combination of (43) with (44) under the consideration of $u_{t-1} = 0$ provides the imposition of $u_t = v_t$.

- *Feasible region of the offline-hour counters' constraints in CPF and CPFi:* the terms of constraints (8), (10), and (11) can be reorganized as:

$$h_t^{SD} \leq h_{t-1}^{SD} + 1 \qquad \forall t \qquad \text{(C.3)}$$

$$h_t^{SD} \geq h_{t-1}^{SD} + 1 - u_t\overline{H} \qquad \forall t \qquad \text{(C.4)}$$

$$h_t^{SD} \leq (1 - u_t)\overline{H} \qquad \forall t \qquad \text{(C.5)}$$

The offline-hour counter h_t^{SD} is defined as a positive variable. Thus, its less-strong lower bound will always be 0 despite the problem's features. Moreover, constraint (C.4) links its lower bound with the decision of h_t^{SD} in the previous time step.

Besides, the more complicated bounding procedures in the optimization process are related to the upper bound. The typical upper bound for this positive variable is $+\infty$. Nonetheless, constraint (C.5) establishes a more restrictive upper bound independently from the decision for u_t. In this way, the upper bound imposed by (C.3), which depends on the level of h_t^{SD} in the previous time step, needs to be studied further to determine if it indirectly introduces a stronger bounding.

Therefore, the combination of (13) and (14) provides:

$$h_{t-1}^{SD} - (1 - v_t)\overline{H} \leq v_t\overline{H} \qquad \forall t \qquad \text{(C.6)}$$

This constraint imposes an upper bound for h_{t-1}^{SD} along the whole time span:

$$h_{t-1}^{SD} \leq \overline{H} \qquad \forall t \qquad \text{(C.7)}$$

Finally, the terms of constraint (12) can be reorganized as:

$$v_t\overline{H} \leq \overline{H} + T_{warm}^{SU} - h_{t-1}^{SD} \qquad \forall t \qquad \text{(C.8)}$$

Then, taking into account that $u_t = v_t$, the constraint (C.9) is obtained:

$$h^{SD}_{t-1} \leq \overline{H} - u_t\overline{H} + T^{SU}_{warm} \qquad \forall t \qquad \text{(C.9)}$$

Thus, constraint (C.9) determines that h^{SD}_{t-1} is still being bounded by $\overline{H}$ in case of remaining de-committed and by the minimum offline hours to incurring in the next start-up type s (warm in this example) if the generator is started-up. These circumstances conclude that (C.3) can not provide a stronger upper bound for h^{SD}_{t-1} than (C.5) does. In consequence, its upper bound in CPF is established by:

$$h^{SD}_{t} \leq \overline{H} - u_t\overline{H} \qquad \forall t \qquad \text{(C.10)}$$

- *Feasible region of the offline-hour counters' constraints in TCPF and TCPFi:* considering that uniquely one thermal unit g and one start-up type s (the hottest) are regarded in this comparison, together with the de-committed beginning $u_{t-1} = 0$ that imposes $u_t = v_t$ though constraints (C.1) and (C.2), the terms of (4) can be reorganized as:

$$h^{SU}_{t} = h^{SD}_{t-1} - h^{SD}_{t} + 1 - u_t \qquad \forall t \qquad \text{(C.11)}$$

Substituting h^{SU}_{t} from equation (C.11) into constraint (5):

$$h^{SD}_{t-1} + 1 + u_t - v_t T^{SU}_{warm} \leq h^{SD}_{t} \qquad \forall t \qquad \text{(C.12)}$$

and applying the imposition of $u_t = v_t$ under the analysis's conditions, the lower bound for h^{SD}_{t} is obtained:

$$h^{SD}_{t} \geq h^{SD}_{t-1} + 1 + u_t - u_t T^{SU}_{warm} \qquad \forall t \qquad \text{(C.13)}$$

As it can be appreciated, this lower bound manifests the same performance as in CPFi, being linked to the decision of h^{SD}_{t} in the previous time step and imposing 0 as the less-strong bound because of its positive definition.

Finally, the upper bound for h^{SD}_{t} is directly determined by constraint (2). Considering $u_t = v_t$, the following constraint provides an upper bound without any linkage to previous time steps:

$$h^{SD}_{t} \leq \overline{O} - u_t\overline{O} \qquad \forall t \qquad \text{(C.14)}$$

Comparing the CPFi formulation to TCPFi, it can be concluded that they entail a similar behavior in bounding the offline-hour counters. It can also be affirmed that the integer feasible regions generated by these models around h_t^{SD} are the same. However, if the problem is relaxed, it can be observed that the feasible region of TCPFi is tighter given that $\overline{O} << \overline{H}$, which offers stronger upper bounds with integer-relaxed programming.

Appendix D. Real fuel-consumption curves

This appendix shows the real fuel-consumption curves that characterize the start-up processes of the generation portfolio presented in Table 1. They are determined by equation (D.1):

$$Consumption_g^{SU} = A_g - B_g\, exp\left(\frac{\tau}{-C_g}\right) \tag{D.1}$$

where:

$Consumption_g^{SU}$	Real start-up fuel-consumption [MMBtu].
A_g	First parameter of the function [MMBtu].
B_g	Second parameter of the function [MMBtu].
C_g	Third parameter of the function [h].
τ	Offline hours before the start-up [h].

The start-up fuel-consumption parameters of this gas-fired generation portfolio are exposed in Table D.12.

Table D.12: Start-up fuel-consumption parameters.

Thermal Unit	A_g [MMBtu]	B_g [MMBtu]	C_g [h]
Unit A	4900	3820	5
Unit B	5630	4830	67
Unit C	8705	8640	15
Unit D	3654	1590	50
Unit E	4180	1600	75
Unit F	6374	4860	90
Unit G	8020	3440	78

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