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TÍTULO: Efficient Railway Traffic Operation

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Eco-driving and operation design, including energy recovery systems such as regenerative braking, are key techniques for improving efficiency in railway systems. These strategies, combined with advanced optimisation models, have demonstrated significant potential in several railway scenarios, as highlighted in the following studies.

The first part of the research study positions eco-driving as a cornerstone of energy reduction in the railway sector and a vital tool for advancing the Sustainable Development Goals in transportation. However, its application in real-world scenarios is often impacted by uncertainties arising from climatological factors, which are frequently disregarded in train driving optimisation models. This work introduces an eco-driving model developed to generate efficient driving commands while accounting for climatological uncertainties, such as temperature, pressure and wind. These uncertainties are modelled using fuzzy numbers, and the optimisation problem is solved with a Genetic Algorithm employing fuzzy parameters, utilising an accurate railway simulator. Applied to a realistic high-speed railway scenario in Spain, the model demonstrates that energy savings can increase from 29.76% to 34.70% when climatological factors are considered for the summer scenario. Moreover, an energy variation of 5.31% is observed between summer and winter scenarios, while punctuality constraints are fully satisfied. This model provides operators with an effective tool to better estimate and optimise energy consumption by adapting driving commands to prevailing climate conditions.

The following part of the thesis addresses metropolitan railway operation. This type of system, so widely used in large cities, is a significant energy consumer and a promising target for the implementation of energy efficiency techniques. This work integrates three main strategies: eco-driving, timetable design and regenerative braking, into a simulation-based model to optimise operation while minimising energy consumption. Additionally, the model incorporates the rolling stock required to meet service demand, given its substantial impact on operational costs. Efficient driving commands are developed using a MOPSO (Multi-Objective Particle Swarm Optimisation) algorithm and timetable optimisation is achieved by considering the proposed energy regeneration model and the number of trains required for periodic service. Applied to the Madrid Underground, the model achieves energy savings of up to 24.79% compared to the fastest cycle time, while adhering to operational criteria such as time margins for delay recovery. The study further demonstrates that the GLMO-NSGA-III (Grouped and Linked Mutation Operator – Nondominated Sorting Genetic Algorithm III) algorithm is the most effective for timetable design in this context.

Together, these studies illustrate the significant benefits of combining eco-driving, operational planning, and energy regeneration techniques in railway systems, ranging from high-speed networks to metropolitan lines, maximizing energy efficiency and enhancing sustainability.

Resumen

La conducción eficiente (eco-driving) y el diseño de la operación, incluyendo sistemas de recuperación de energía como el frenado regenerativo, son técnicas clave para mejorar la eficiencia en los sistemas ferroviarios. Estas estrategias, combinadas con modelos avanzados de optimización, han demostrado un gran potencial en diversos escenarios ferroviarios, como se destaca en los estudios presentados a continuación.

La primera parte del estudio de investigación posiciona la conducción eficiente como un pilar fundamental para la reducción del consumo energético en el sector ferroviario y como una herramienta vital para avanzar en los Objetivos de Desarrollo Sostenible en el ámbito del transporte. Sin embargo, su aplicación en escenarios reales se ve frecuentemente afectada por incertidumbres derivadas de factores climatológicos, que a menudo son ignoradas en los modelos de optimización de la conducción de trenes. Este trabajo presenta un modelo de eco-driving desarrollado para generar consignas de conducción eficientes teniendo en cuenta dichas incertidumbres climatológicas, como la temperatura, presión y viento. Estas incertidumbres se modelan mediante números borrosos (lógica borrosa), y el problema de optimización se resuelve utilizando un Algoritmo Genético con parámetros borrosos, junto con un simulador ferroviario. Aplicado a un escenario realista de Alta Velocidad en España, el modelo demuestra que los ahorros energéticos pueden aumentar del 29.76 % al 34.70 % cuando se consideran los factores climatológicos en condiciones de verano. Además, se observa una variación del 5.31 % en el consumo energético entre los escenarios de verano e invierno, cumpliendo en ambos casos con las restricciones de puntualidad. Este modelo proporciona a los operadores una herramienta eficaz para estimar y optimizar mejor el consumo energético, adaptando las consignas de conducción a las condiciones climáticas predominantes.

La segunda parte de la tesis aborda la operación ferroviaria metropolitana. Este tipo de sistema, ampliamente utilizado en grandes ciudades, representa un alto consumo de energía y es un objetivo prometedor para la implementación de técnicas de eficiencia energética. Este trabajo integra tres estrategias principales: eco-driving, diseño de horarios y frenado regenerativo, en un modelo basado en simulación para optimizar la operación minimizando el consumo de energía. Además, el modelo incorpora el material rodante necesario para cubrir la demanda del servicio, dado su impacto significativo en los costes operativos. Las consignas de conducción eficientes se desarrollan utilizando el algoritmo MOPSO (Multi-Objective Particle Swarm Optimisation), y la optimización del horario se logra considerando el modelo propuesto de regeneración de energía y el número de trenes requeridos para el servicio periódico. Aplicado a la red de Metro de Madrid, el modelo alcanza ahorros energéticos de hasta el 24.79 % en comparación con el tiempo de ciclo más rápido, cumpliendo además con criterios operativos como los márgenes de tiempo para la recuperación de retrasos. El estudio demuestra además que el algoritmo GLMO-NSGA-III (Grouped and Linked Mutation Operator – Nondominated Sorting Genetic Algorithm III) es el más eficaz para el diseño de horarios en este contexto.

En conjunto, estos estudios ilustran los beneficios de combinar técnicas de conducción eficiente, planificación del tráfico y regeneración de energía en los sistemas ferroviarios, tanto en Alta Velocidad como en líneas metropolitanas, maximizando la eficiencia energética y fomentando la sostenibilidad.

CHAPTER 1

INTRODUCTION AND MOTIVATION OF THE THESIS

The growing need for sustainable economic development is prompting developed countries to consider climate-related aspects and possible future scenarios resulting from previous industrial development. Similarly, the population trend in all countries is to concentrate in large metropolitan areas. This demographic shift is particularly evident in developed countries, where societies are increasingly concentrated in huge metropolis. In the current context of optimising energy efficiency and supporting the use of renewable energy it is important to develop technologies that enable the rational and efficient use of the resources. This paradigm, combined with increased environmental awareness, positions the transportation sector as one of the main strategies for sustainable development. The efficient generation and utilisation of energy are central to the effort focused on sustainable development.

Specifically, the transportation sector is one of the largest energy consumers globally, alongside the industry, commercial and public services and the residential sector [1]. On a global scale, this sector accounts for a large share of energy demand, making energy efficiency improvements crucial. The rational use of energy is particularly relevant for the transport sector because of the massive amount of energy required in developed countries [2], especially when the trend is towards unification into an electrified system.

Also, the transportation sector is responsible for a significant share of emissions, accounting for around 21% of global carbon dioxide (CO_2) emissions, rising to 24% when considering only CO_2 emissions from energy use. Road transport is the dominant source, generating approximately 75% of transport-related emissions. Within this, passenger vehicles−including cars and buses−contribute 45.1%, while freight trucks account for 29.4%. As a result, road transport alone represents about 15% of total global CO_2 emissions. In contrast, rail transport contributes a comparatively small share, with rail travel and freight accounting for only 1% of transport emissions, making it the most energy-efficient mode of land transport. Aviation is responsible for 11.6% of transport emissions (around 2.5% of total global CO_2 emissions), while international shipping accounts for 10.6%. Other transport methods, such as pipelines used to move materials like water, oil and gas, contribute 2.2% [3].

Rail transport has significantly improved its energy efficiency, reducing energy consumption per transported unit by 23.9% in 2021. It remains the most energy-efficient mode, consuming less than a third of the energy required by other transport modes. Specifically, it is 2.4 times more efficient than road transport for passengers and 4.6 times more efficient for freight, reinforcing its role as the most sustainable land transport alternative [4].

This highlights the importance of rationalising and making energy use more efficient across all sectors and transportation modes to minimise pollutant emissions. Within the transport sector, rail transport is characterised by high energy efficiency, as it consumes less energy per transported load compared to other modes of transport, and this efficiency is even more pronounced due to the increase in electrified traffic [4]. Even so, they still present significant opportunities for improvement [5]. Railways play a vital role in connecting large metropolitan areas, facilitating mobility between urban centres and promoting sustainable transportation. As the demand for efficient and low-emission transport solutions increases, rail networks are expected to play an increasingly important role in the future of urban and inter-urban mobility [6].

Urban and intercity railway mobility systems provide a valuable opportunity to transition toward more efficient and sustainable transportation modes. They offer enhanced comfort and speed for passengers while delivering significant energy savings and environmental benefits [7]. Metropolitan rail transport plays a crucial role in ensuring mobility within large cities, while long-distance railway connections are essential for maintaining strong intercity links. In this context, high-speed rail transport becomes highly relevant, positioning itself as a fast, efficient, safe and comfortable mode of transportation with potential environmental benefits [8]. High-speed railway has significantly contributed to enhancing long-distance mobility while reducing dependence on private vehicles. This type of transport system has emerged as a significant competitor to other mobility options between major urban centres, aligning with sustainable development goals and helping mitigate the negative effects of road and air transportation [9]. The expansion of railway capacity to meet the growing demand for passenger services, and to a lesser extent, freight operations, has resulted in an equivalent increase in traction energy consumption for trains and other services related to railway operations. Therefore, it is crucial to implement measures for the efficient use of energy in railway systems to ensure the development of rail transport while progressing towards sustainability goals [10].

Europe has taken the lead in the decarbonisation of the transport sector, energy generation and industry. This leadership is driven by emission reduction policies establishing a framework for action in this area [11]. Within this framework, the rail sector focuses on the development and application of technology aimed at improving the efficiency of the systems involved. Innovation and development have the potential to improve energy efficiency reducing energy consumption without reducing the quality of service, offering multiple benefits, such as lower energy generation demand, lower operating costs and reducing negative environmental impact [5].

In addition to technological advances, it is vital to increase the attractiveness of rail transport for both passengers and freight. Increased rail traffic can relieve road congestion and reduce the pollution associated with road transport. High-speed railways, in particular, responds effectively to long-distance mobility demands while minimising greenhouse gas emissions, making it a key element of sustainable transport networks. The transition to a more efficient and sustainable rail system depends on collaborative efforts between stakeholders, including policy makers, technology innovators, research institutions, infrastructure managers and rail operators. These actors are key to implementing technologies that reduce energy consumption, improve operational efficiency and ensure passenger satisfaction [12].

Despite its inherent efficiency, the railway sector remains a major energy consumer. Therefore, the continuous exploration of innovative solutions is essential to further optimise its operations. As electrification becomes the dominant paradigm in transport, rail is in a unique position to lead the transition to a sustainable and energy efficient future. This aligns with global sustainability goals, particularly in the European context, where railway development is a strategic component of decarbonisation policies. By integrating cutting-edge technology with coordinated policy and research, the rail sector can serve as a global model for addressing mobility challenges within the framework of sustainable development [13].

Technological and innovation solutions for improving energy efficiency in railway systems can be categorised into three key areas: infrastructure, rolling stock and operation. In the case of infrastructure, there are different areas that can play an important role enabling some improvements in the operation. Some of them are the electrification of lines without electric power supply, the use of materials with better mechanical properties, an optimal track design, the integration of renewable energy sources [14] and the utilisation of energy storage systems [15]. For rolling stock, the reduction of train weight, aerodynamic optimisation [16], the use of regenerative braking [17] and the integration of on-board systems with new signalling systems contribute to reducing the energy needs without compromising the performance of the operation ([18], [19]). With this last improvement possibility, significant energy savings can be achieved by adapting the efficient driving of the train in real-time to the operational conditions. Operational improvement strategies, such as driving optimisation techniques, timetable optimisation, automation and traffic regulation systems, mean that energy consumption is reduced, ensure passenger comfort, punctuality of service and safety during operation. Also, the use of new technologies applied to traction power generation, such as hydrogen generation [20], is an option for improving the efficiency of the railway system reducing total gas emission.

1.1. PROBLEM STATEMENT AND APPROACH

The railway sector includes three main areas where research efforts can be directed to improve efficiency and sustainability: infrastructure, rolling stock and operation. Although significant progress has been made in all three areas, this thesis specifically focuses on improving railway operation by applying techniques to improve the efficiency of the system. Among the various potential improvements to increase the energy efficiency of railway systems, two key operational optimisations stand out: eco-driving design and timetable optimisation.

Economic driving, or eco-driving [21], is the driving that involves identifying the most energy-efficient driving strategy for a train by generating an optimal speed profile for a specific route that minimises energy consumption over a predetermined running time [22]. This process takes into account constraints related to passenger comfort, the established schedule and operational limitations. The theory applied to efficient driving indicates that the best pattern for designing the speed profile consists of an initial start with maximum acceleration, followed by speed regulation, and ending the travel profile with a coasting phase, that is, without applying traction or braking, and a final braking phase with maximum deceleration until the stopping point, where the train completes the speed profile [21]. On the other hand, timetable optimisation consists of determining the optimal running times between stations and the appropriate dwell times at each stop, with the goal of minimising overall energy consumption while adhering to comfort and operational constraints [23].

Operational improvements, particularly in eco-driving techniques and timetabling, offer cost-effective solutions that do not require substantial financial investments in new infrastructure or replacing existing trains with more efficient models. Instead, these strategies rely on the application of advanced optimisation methods and computational techniques. Once these solutions are developed, their implementation typically involves minimal costs, as they mainly depend on the integration of software programs and operator training rather than large-scale physical upgrades.

A key element of these solutions is their generation through simulation models, algorithms and computational techniques, which allow efficient driving commands and schedules to be designed and tested in a virtual environment before deployment. These tools enable the exploration of numerous scenarios and the identification of optimal solutions that can then be quickly and effectively applied to real railway systems. This rapid applicability is a significant advantage, as it minimises downtime and resource allocation while achieving immediate benefits in energy efficiency and operational performance. In this context, rail transport also benefits from emerging technologies, including artificial intelligence, which can enhance efficiency, safety and sustainability [24].

In this context, fuzzy logic [25] has been introduced to handle uncertainty in railway systems, such as variations in track conditions, weather, or measurement errors in stopping times. Fuzzy logic allows these uncertain parameters to be modelled more realistically, which improves decision making under variable conditions. This methodology assigns degrees of membership to values instead of assigning only a true or false value, allowing vague or uncertain aspects in operational data to be dealt with in a quantified manner.

The integration of fuzzy logic into optimization models allows for more robust and adaptable solutions to be created, as a wider range of uncertain variables affecting railway operation can be considered. This improves the flexibility of decisions made, especially in unpredictable or changing situations. By using fuzzy logic, models become more capable of adjusting to real operating conditions, optimizing both energy efficiency and operational reliability.

The implementation of eco-driving techniques provides tangible benefits in reducing energy consumption for a given running time without compromising passenger comfort or the quality of railway services. Similarly, the application of efficient timetabling strategies ensures reduced energy consumption during operations while maintaining compliance with predefined schedules. These results highlight the ability of operational improvements to balance energy efficiency with service reliability and customer satisfaction.

Eco-driving leverages computational models and real-time data to adapt driving behaviours to specific conditions such as gradients, speed limits, and energy recovery systems. Likewise, train service management, incorporating timetable optimisation and regenerative braking coordination, aims to enhance operational efficiency and energy utilisation by ensuring synchronised operations across the network.

The choice to focus on operational solutions is due to their scalability, adaptability and immediate impact. Unlike infrastructure improvements, which often require long-term planning and substantial financial resources, or rolling stock upgrades, which are constrained by manufacturing timelines and acquisition budgets, operational strategies can be rapidly implemented within existing systems. Moreover, operational optimisation is compatible with advancements in other areas, making it a complementary approach that maximises the overall efficiency of the railway network. This approach stands out for its unique advantages, particularly in contrast to the high costs associated with infrastructure or rolling stock improvements, as well as its rapid applicability to real systems.

By addressing railway efficiency from an operational perspective, this thesis contributes to the broader goal of sustainable transportation. It provides a pathway to achieving energy savings and operational improvements without requiring large economic investments, aligning with the overarching objectives of reducing greenhouse gas emissions and supporting global sustainable development initiatives.

1.1.1. ECO-DRIVING IN LONG-DISTANCE SCENARIOS

Long-distance or high-speed rail operation has a series of particular characteristics, differentiating it from other types of operation, such as metropolitan rail operation.

One key distinction lies in the level of automation currently being implemented in high-speed rail systems [26]. Automatic driving systems are under development and implementation in high-speed trains, although in general, driving continues to be carried out by the driver, with this driving being adjusted during the operation according to the needs and specific conditions of the service [27]. Automation not only makes it possible to optimise energy consumption in real time, but also introduces a new level of complexity, as the driving design must adapt dynamically in case of delays or deviations from the timetable.

The basic efficient driving strategy for long-distance or high-speed trains is speed regulation. Speed regulation consists of the application of traction to keep the train speed constant to the driving command speed. This approach minimises excessive acceleration and braking, preserving passenger comfort. The route can be divided into several sections where the driving instructions are varied to achieve a certain regulation speed. Even if the route can be divided into several regulation sections, there should not be too many of them, as this would impair the driver's driving performance. A refinement and more efficient approach is speed regulation without the use of brakes, which prevents braking when the train naturally accelerates beyond the regulation speed due to favourable gradients ([28], [29]). In this case, the established speed limit is respected, but it is not necessary to respect the regulation speed, thus achieving additional energy savings in sections where the train does not need to consume more traction energy to obtain a higher speed.

Figure 1 shows the application of an efficient driving command. The train follows this speed command by allowing the speed to be exceeded when there is a gradient that allows the speed to be increased, while respecting the speed limits.

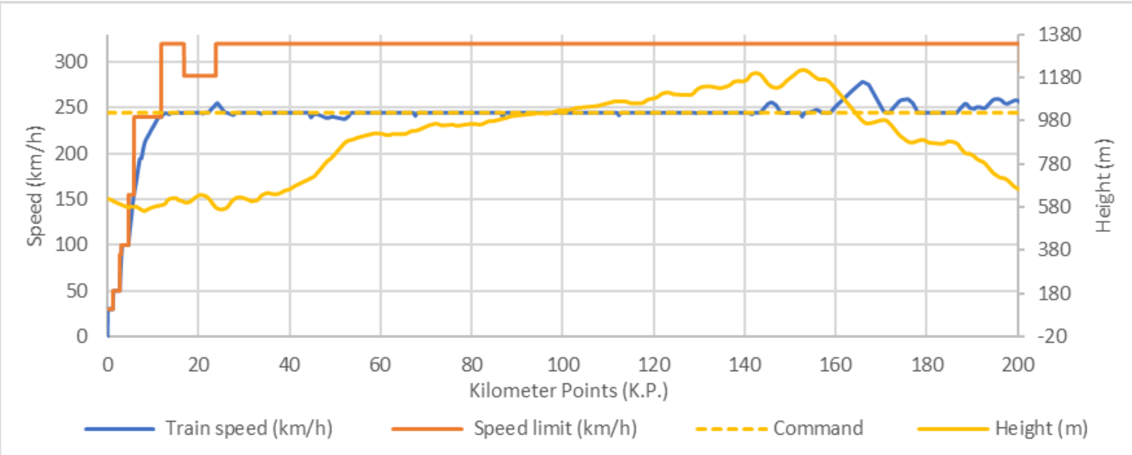

Figure 1. Regulation speed without braking.

Additionally, mixed traffic operation is a common characteristic, where different passenger services and freight trains coexist on the same tracks. It may be common for different passenger services to share the same track on different trains and for freight trains with different commercial speeds and restrictions [30]. This peculiarity implies an added effort in the design of the driving and for the regulation of traffic that tries to minimise the impact and delays caused during the service by conflicts between trains [31].

In this context, the application of efficient driving to long-distance or high-speed lines becomes a key element in energy savings both in the operation planning phase and in real-time traffic regulation. During the planning phase, efficient driving strategies are designed with consideration for running times, scheduled stops and punctuality criteria. During operation, eco-driving strategies should be applied dynamically when deviations occur or when conditions differ from the initial plan.

In order to achieve accurate results, simulation and optimisation models must consider all parameters influencing railway operations. En general, para el diseño de la conducción no se incluye la influencia de parámetros externos a la propia operación. However, previous studies incorporate the impact of some external factors and their associated uncertainty. In [32] the variation in passenger mass is included, in [33] the uncertainty in manual driving is considered and in [34] the uncertainty in delays is taken into account for the eco-driving design. Among these factors, weather conditions significantly affect the energy consumption of trains and, to a lesser extent, the running times. Studies that have addressed climatic aspects typically focus on winter and snowy scenarios, aiming to predict delays or traffic disruptions [35]. Nevertheless, railway operators tend to exclude climatic factors from their models, considering them highly random and unpredictable variables.

In the context of high-speed railway services, it is crucial to assess the influence of wind and aerodynamic resistance [36]. Regarding wind effects, both its direction relative to train movement and its velocity must be analysed, as it can either assist or hinder train operation by altering its relative speed. Additionally, aerodynamic resistance is strongly influenced by the density of the surrounding air, which, in turn, depends on atmospheric pressure and temperature. This component becomes increasingly relevant at higher operating speeds since it is proportional to the square of the velocity.

Given the inherent variability of climatic parameters, their analysis requires methods that account for the associated uncertainty. While uncertainty in certain parameters has been addressed using fuzzy logic [25], the literature has not specifically examined the uncertainty of climatic factors. Chapter 2 considers the influence of meteorological variables and their uncertainty, modelled using fuzzy numbers, integrating them into the efficient driving optimization model. The results highlight the significance of these factors in driving optimization.

1.1.2. ECO-DRIVING IN METROPOLITAN SCENARIOS

Just like in long-distance or high-speed railway operations, metropolitan railway operation has its own characteristics that determine the energy-saving techniques that can be applied.

Metropolitan operation is mostly carried out automatically through Automatic Train Operation (ATO) systems although manual driving is also used in certain lines [37]. The journeys between stations in metro-type systems are numerous and much shorter compared to long-distance travel and the ATO driving patterns applied are pre-programmed in the onboard systems. Since trains share the same line and follow the same type of routes and stops, the possible driving patterns designed for the trains are shared by all trains on the line.

Typically, railway operators usually consider four possible ATO driving patterns to be executed by the train. The first is the fastest possible run, taking into account the characteristics of the train and the track, used for delay recovery. The second is the nominal run, which is used under normal conditions while following the timetable. The remaining two patterns are progressively slower and are used when the train needs to be delayed for traffic regulation purposes.

Therefore, efficient driving can be applied to automatic operation through the speed regulation technique or the coasting/re-motoring technique considering both time and energy-saving criteria [38]. Coasting/re-motoring technique consists of the application of traction by the train until the speed reaches the established upper limit. Once the speed reaches this limit, the coasting phase is applied until the speed reaches the lower limit. When the speed reaches the lower limit, the traction phase starts again. Coasting/re-motoring technique becomes more relevant in metro-type systems, where higher accelerations are allowed compared to long-distance or high-speed trains, taking advantage of the optimal full-throttle regime and the application of coasting, where energy consumption is zero.

Figure 2 shows an example of coasting/re-motoring driving technique.

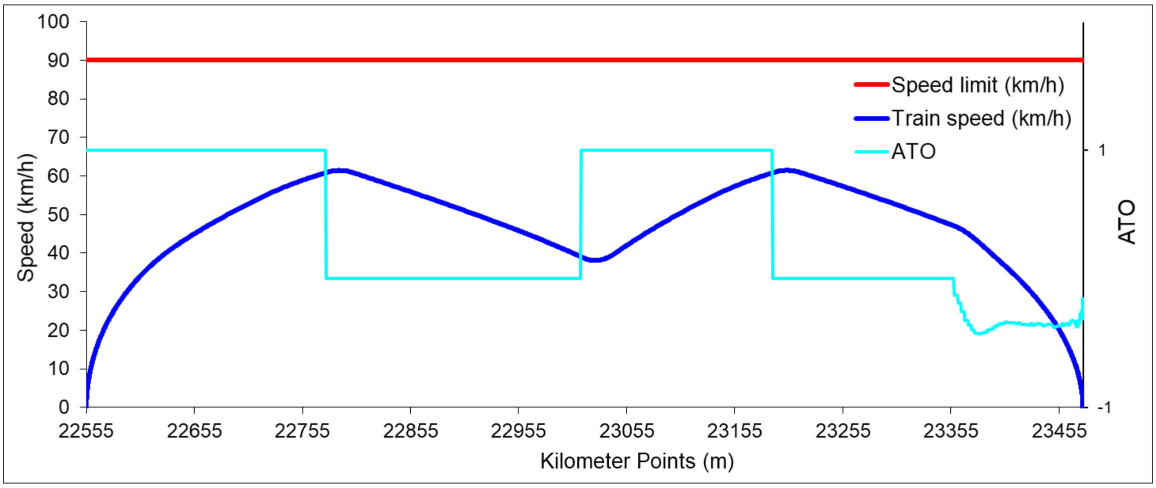

Figure 2. Coasting/remotoring speed profile.

Eco-driving is applied similarly in the timetable phase of long-distance or high-speed services. However, metropolitan railway operations are frequently subject to deviations from the planned schedule, making the application of efficient driving during the traffic regulation phase particularly relevant. In the regulation phase, the operator aims to correct deviations from nominal conditions by assigning one of the possible driving patterns to each train based on delays, dwell times and train headways [39]. This communication from the control centre to the trains can be done intermittently through trackside beacons and signalling systems or continuously via radio, as with the CBTC (Communications-Based Train Control) system [40]. When communication with trains is continuous, the complexity of decision-making regarding the action's trains should take increases significantly, making computing techniques and processing speed highly relevant.

1.1.3. TIMETABLE DESIGN

The efficient planning and management of railway timetables is a key aspect of optimising operations and improving system performance. Timetable design in metropolitan and long-distance railway systems is conducted as a preliminary step before the commercial operation of a line.

Considering energy-saving criteria, timetabling enables the implementation of energy efficiency techniques such as eco-driving [41], the utilization of regenerative braking [42] and the optimisation of dwell times at stations [43]. In long-distance, high-speed and metro-type railways, timetabling may incorporate eco-driving strategies to minimise total energy consumption. In metro-type systems, timetables can be designed to synchronise headways and integrate technologies such as regenerative braking or energy storage systems to enhance system efficiency. Additionally, commercial running times between stations include time margins to account for contingencies.

Previous studies have addressed timetabling in metropolitan scenarios, considering running and dwell times to optimise synchronisation between trains [44]. Some have incorporated efficient driving strategies, while others have analysed the number of trains required to fulfil the service [45], treating this variable as an optimisation factor alongside operational cost. However, no study has yet integrated all these variables simultaneously.

For this reason, Chapter 3 integrates a two-level optimisation model for train operation on a Madrid metro line. On the first level, driving is optimised for each route of the line. On the second level, the timetable is optimised by distributing station stopping times to synchronise accelerating and braking trains, maximising the use of regenerated energy. Additionally, different algorithms have been compared and rolling stock constraints have been considered by penalising the use of a greater number of trains to balance operational efficiency and energy savings.

1.1.4. REGENERATIVE BRAKING

Regenerative braking is a key technology for improving the energy efficiency of railway systems, particularly in metropolitan and high-frequency networks [17]. It works by harnessing the train's kinetic energy during braking, converting it into electricity through electric motors that function as generators. This energy can be used to power the train’s auxiliary systems, be transferred to other trains within the same electrical sector, or, in some cases, be fed back into the power grid.

However, regenerative braking is not a stand-alone solution; it must be complemented by efficient driving strategies and proper traffic planning [46]. Its effectiveness depends on operational conditions and the responsiveness of the infrastructure. While it helps save a portion of the energy that would otherwise be lost, the extent of this energy recovery varies depending on circumstances. When properly coordinated with other strategies, such as eco-driving and timetable optimisation, regenerative braking maximises its impact, particularly during station approaches and sections where high speeds need to be reduced.

This combination of energy recovery and optimization strategies not only enhances operational efficiency but also contributes to the overall sustainability of the railway system. In metro-type networks, timetabling can take advantage of energy recovery by synchronizing trains that regenerate energy during braking with those accelerating at the same time. This coordination optimizes energy flows across the network, minimizing waste and increasing the effectiveness of regenerative braking.

1.1.5. ALGORITHMS AND OPTIMISATION MODELS

Improving the efficiency of railway systems by improving train driving and timetables is based on solving optimisation models through algorithms.

- **Eco-driving**

For solving the eco-driving problem different methods and algorithms have been applied. Initially, the work carried out in this sense was done by applying the Pontryagin Maximum Principle by Ichikawa in 1968 [21] to determine the optimal driving regimes. This approach has been the basis from which subsequent research has been carried out. The methods developed can be classified into analytical methods and numerical methods.

In analytical methods, the resolution of the problem is based on the mathematical and detailed formulation of the problem. In many cases, Pontryagin's Maximum Principle is used to find the optimality conditions necessary to subsequently apply the resolution algorithm considering minimizing or maximizing the variable of interest. These methods are characterized by their ability to obtain exact solutions under ideal mathematical conditions. However, their application is limited to relatively simple systems or to scenarios in which certain simplifications can be made, such as the linearisation of the equations or the simplification of the operating restrictions.

Some examples are constructive algorithms [47], dynamic programming [48] or sequential quadratic programming [49]. Constructive algorithms allow precise and fast solutions to be found in relation to the computing speed. However, the application of this type of algorithm is restricted due to the simplifications made in the models of both the train and the operating conditions.

On the other hand, numerical methods allow working with much more detailed models, without the need to simplify the problem conditions. This resolution approach is based on the search for approximate solutions in an iterative manner until it meets the search requirements.

Genetic Algorithms (GA) are a prominent example of numerical methods. These algorithms are inspired by natural selection and evolution processes, where a population of possible solutions evolves through selection, crossover and mutation processes. GAs efficiently explore large solution spaces and are particularly useful in complex optimization problems, such as speed profile optimization in railway environments, where possible input values can be numerous and non-linear [50]. GAs allow finding high-quality approximate solutions without the need to know the explicit formulation of the objective function, which gives them great flexibility to adapt to different train models and operating conditions. Within the numerical methods, we also find Particle Swarm Optimization (PSO), which is inspired by the collective behaviour of natural systems, such as swarms of birds or schools of fish. PSO adjusts solutions through collaboration between particles, seeking the best solution based on their evaluations. Like GA, PSO is effective in handling complex problems without the need for specific formulations, making it suitable for highly complex railway systems [51].

The combination of these methods, both analytical and numerical, together with the integration of fuzzy logic [34], allows for the creation of detailed models that can be implemented in advanced simulators. These simulators allow for accurate representation of train characteristics, track conditions and signalling systems, facilitating the evaluation of different driving strategies and their impact on energy consumption and service punctuality. By integrating these models into a simulation environment, various solutions can be tested before being implemented in real operation, improving the efficiency and reliability of the railway system in real time.

- **Timetabling**

Different methods and algorithms have been used to solve the timetabling problem. These methods can be classified into different main categories. First, there are heuristic and metaheuristic algorithms, such as the Genetic Algorithm (GA) and the Allocation Algorithm ([52], [53]), the Nested Heuristic Genetic Algorithm [54] and the Particle Swarm Algorithm (PSO)

[45], which are commonly used for resource optimisation and solving complex optimisation problems.

Secondly, mathematical and programming methods, including Mixed Integer Linear Programming (MILP) ([55], [56]), are used to address combinatorial optimisation problems, providing exact solutions in contexts where the variables are integers and the relationships between them are linear.

In addition, control and optimisation methods have been employed, such as the Matrix Control Algorithm [46], applied to the optimisation of speed and energy efficiency in railway systems, and Pontryagin's Maximum Principle [42], used in the optimisation of dynamic systems to determine optimal control strategies.

On the other hand, multi-step and spectral optimisation methods are applied, where the Pseudospectral Method (PSM) is compared with Dynamic Programming (DP) [57] to assess the validity of the proposed solutions in optimal control problems.

Finally, some studies resort to hybrid approaches, such as data-driven bi-objective metaheuristic [58], which combines exact mathematical techniques with heuristic strategies to solve complex optimisation problems by considering multiple objectives simultaneously.

1.2. OBJECTIVES

1.2.1. MAIN OBJECTIVE

The main objective of the thesis is to develop optimisation models for train driving and timetabling in both metropolitan and long-distance railway systems. These models are based on detailed simulations of train kinematics and dynamics, incorporating all operational parameters, constraints related to passenger comfort and uncertainties inherent in railway operations. Additionally, the research considers factors that influence the overall efficiency of railway systems, including timetable constraints, the utilisation of regenerated energy by trains during braking and the characteristics of the rolling stock. By integrating these elements, the proposed models aim to optimise operational strategies while maintaining system reliability and service quality. Through the application of advanced computational techniques and simulation frameworks, this thesis seeks to address critical challenges in railway operations, contributing to the broader goals of energy efficiency and sustainable mobility.

1.2.2. SPECIFIC OBJECTIVE

The specific objectives of the development of this doctoral thesis are described in detail. The main goal is the application of the eco-driving technique to reduce energy consumption during operation, taking into account the particularities of each case.

- **Objective 1: Design of efficient driving strategies for high-speed trains considering the uncertainty associated with climatic conditions.**

The design of efficient railway driving strategies is a problem extensively addressed in the literature; however, the impact of climatic conditions on railway operations and driving strategies has not been considered until now. The effects of wind and variations in air density can significantly influence the resistance that the train must overcome. The methodologies for identifying optimal solutions are diverse, and among them are nature-inspired algorithms.

To address this problem, a Genetic Algorithm is used alongside a detailed train movement simulator to optimise the proposed eco-driving model. Travel time and energy consumption are evaluated using a fitness function, with each parameter weighted by a specific factor. The resolution model incorporates pressure, temperature, air density, wind direction and speed as fuzzy parameters (fuzzy logic), thereby accounting for their associated uncertainty. In this way, the driving commands that must be executed by the high-speed train are determined to minimise energy consumption while meeting a specific travel time under given climatic conditions.

The proposed model and its results provide a reference framework for railway operators to evaluate the impact of external factors that were previously difficult to quantify.

The tasks to be completed to achieve Objective 1 are as follows:

- **Task 1**: Review of previous literature related to efficient driving strategies and the algorithms that can be employed for optimisation. Examination of prior studies addressing the role of climatic conditions in railway operations.
- **Task 2**: Development of the driving simulation model and the climatic model with fuzzy parameters. The climatic model includes the effects of wind (direction and speed) and variations in air density due to changes in temperature and pressure along the route. Development of the optimisation model for efficient driving commands, integrating the Genetic Algorithm and the climatic model with fuzzy parameters.
- **Task 3**: Application of the simulation and optimisation model to a real-world case study of a high-speed railway line in Spain, obtaining conclusions, and proposing future research directions.

- **Objective 2: Optimisation of ATO Driving Commands and Operation for Metropolitan Railway Lines Considering Rolling Stock and Regenerated Energy During Braking.**

The design of automatic driving commands (ATO) and proper operational planning are the primary challenges to address in the operation of metropolitan railway lines. To this end, an optimisation model is proposed that integrates the design of automatic driving commands and operational planning, considering the synchronisation of trains to leverage the energy flow generated by regenerative braking.

The first level of ATO driving design obtains Pareto frontiers for each interstation section using a MOPSO algorithm, where each solution represents the driving profile that achieves minimum energy consumption for a given travel time. The second optimisation level uses the solutions from the first level to maximise the use of regenerated braking energy and optimally distribute existing time margins to recover potential delays. At this level, the performance of various algorithms is compared: FDV (Fuzzy Decision Variable) [59], GLMO-NSGA-III (Grouped and Linked Mutation Operator – Non-dominated Sorting Genetic Algorithm III) ([60], [61]), LCSA (Linear Combination-based Search Algorithm) [62] and WOF (Weighted Optimisation Framework) [63]. The algorithm yielding the best results for the case study is selected.

The results obtained through the new model can serve as a tool for timetable design on metropolitan lines, meeting the nominal operational requirements typically imposed by railway operators.

- **Task 1.** Review of previous literature related to ATO driving commands and operational planning for metropolitan lines, as well as prior studies on the computational techniques used [38].
- **Task 2.** Development of the optimisation model integrating the ATO driving calculation level using a multi-objective algorithm and the global timetable optimisation level, where several algorithms will be compared. The model will account for regenerated energy during braking, including an energy loss function during transmission. Additionally, the rolling stock required to meet the designed operation will be considered.
- **Task 3.** Application of the model to a real case study of Metro de Madrid, taking into account the minimum number of trains required to satisfy the designed timetable and avoiding costs associated with rolling stock. Conclusions will be drawn based on the results.

The development and achievement of the objectives outlined will be described in the following chapters.

1.3. STRUCTURE OF THE THESIS

The thesis document is structured as follows:

- **Chapter 1** has introduced the thesis work, presented the motivation for the present work, the approach and focus of the problem and objectives to be completed and its contribution.
- **Chapter 2** presents the problem of efficient driving design when considering climatic parameters and their uncertainty. The problem is also solved by means of a genetic algorithm modelling the uncertainty using fuzzy logic.
- In **Chapter 3** a two-level timetable planning model for metropolitan services is developed. In a first stage, the efficient driving is designed for each route of a metro line. Then, the planning problem is solved by considering the energy regenerated during braking and the number of trains needed to satisfy the rail service.

- **Chapter 4** lists the conclusions obtained, the contributions of the thesis, the articles published in high impact journals and, finally, comments on the possible lines of work and future development.

The document ends with the bibliographical citations that have served for the correct development of the thesis.

CHAPTER 2

ECO-DRIVING CONSIDERING CLIMATIC UNCERTAINTY

2.1. INTRODUCTION

Eco-driving is one of the most popular methods for optimising energy consumption in railway operations because it can be applied in the short term, providing important energy reduction results [22]. Eco-driving is the obtention of the speed profile with the minimum energy consumption associated. The eco-driving speed profile is calculated for a specific target running time considering the previously designed timetable ([5], [44], [64], [65], [66], [67], [68], [69]).

Different eco-driving strategies have been analysed in the academic literature. In some simple scenarios, driving through speed regulation has been simulated ([2], [70], [71], [72]). Other studies have proposed design by coasting/re-motoring cycles, having upper and lower speed limits ([73], [74], [75]) or considering coasting points along the railway line ([21], [47], [50], [76], [77]). Coast-remotoring strategy is appropriate for short routes on metro lines but are not very efficient for longer railway lines as the HSR lines. The speed regulation without braking has been demonstrated as an efficient and easily executed eco-driving strategy for high-speed railway lines ([28], [29]).

The production of speed profiles with the minimum energy consumption requires an optimisation model that can return the best combination of eco-driving strategies. In study [21], the initial study on eco-driving was developed. This work applies Pontryagin's Maximum Principle to a simplified train dynamics model, obtaining optimal driving modes as maximum acceleration, cruising, coasting and maximum braking. The studies developed subsequently to [21] have improved the dynamic model of simulation, and the different optimisation techniques implemented are classified in study [78] as analytical and numerical methods.

Most of the analytical methods are developed based on the optimal control theory. Pontryagin's Maximum Principle is used in these methods to find the best energy efficiency solutions and apply algorithms to obtain the optimal switching points between the efficient regimes. These algorithms are based on Dynamic Programming ([48], [79], [80], [81]), constructive algorithms ([2], [47], [82], [83]), Sequential Quadratic Programming [49], and the method of Lagrange Multipliers over the discretised problem [84]. Other methods transform the situation into a non-linear problem [85]. The analytical methods are used to provide the optimal solution but, considering the complexity of the problem to be solved, a series of simplifications have to be made in the train model. Due to the simplifications, inaccuracies can be obtained in the results generated, and may require recalculations in real applications [71].

On the other hand, numerical methods have been used for solving the problem of efficient driving in railway operations as well. These methods are flexible enough to be used without reducing the complexity of the train model by using detailed simulation models, adjusting the comfort and driving requirements. Thus, these methods can be used to fulfil any need or constraint in real situations.

Among these numerical methods are Artificial Neural Networks, being used for the optimisation of the coasting points [86], the optimisation of the coasting speed for a Mass Rapid Transit System considering energy cost and travelling time cost [87], and for calculating a new indicator to characterise a railway traffic driving smoothness [88]. Nature-inspired optimisation algorithms such as Genetic Algorithm (GA) have been widely applied for determining the speed profile which minimise the energy consumption [89], the optimal coasting points [50], and the optimum train speed along the journey [79]. A single optimise train trajectory considering net energy between adjacent DC substations is described in [90]. In study [91], it is jointly optimised by means of a GA for the timetable and the speed profile. A coasting point control method is developed in ([92], [93], [94]) based on

GA to determine its number and position. Multi-population Genetic Algorithm (GA) is used in [95] to reduce energy consumption in a subway system with multiple stations, and in [96] in order to determine the acceleration, cruising and coasting points. GA combined with fuzzy logic is used in [74] to optimise energy traction by trading-off reductions in energy against time, in study [28] to find the economical pattern for an HSR balancing energy and running time, and used in studies ([27], [34]) to calculate optimal speed profiles considering uncertainty in manual driving. Direct search algorithms have been applied to minimise energy traction in [97] while Brute Force algorithm is used in [98] to minimise energy consumption in substations. A Monte Carlo Simulation model was proposed to determine the system energy flow considering regenerative braking trains [99]. Differential Evolution has been presented in [100] to optimise railway operation. Optimisation by minimising energy traction is carried out by Simulated Annealing in [101] and searching optimal coasting points in [102]. On the other hand, multi-objective models have been developed using numerical methods. Among them, Indicator Based Evolutionary Algorithm (IBEA) was proposed to optimise both the running time and energy consumption [103]. Non-dominated Sorting Genetic Algorithm II (NSGA-II) is used in [104] to design ATO CBTC speed profiles to minimise energy traction and in [38], its performance is compared with MOPSO generating the Pareto curve for real cases of the Madrid Underground. Ant Colony Optimisation can be seen in [79], proposing a distance-based train trajectory optimising the train trajectory, and in [105] to achieve the optimal driving strategy. Finally, a MOPSO algorithm is used in [51] to design efficient speed profiles for the ATO of the trains of a metro line.

With the purpose to obtain accurate results, it is important not only to use detailed simulation models but also to take into account the possible uncertainties that emerge from the HSR operation. These uncertainties can be classified in three groups: limitations in the movement authority produced by signals or preceding trains ([106], [33]), variability in the manual operation of the train [27], and the climate conditions. While the first two uncertainty sources have been widely studied, climate conditions have not been deeply considered in the eco-driving problem.

Most academic works focus on winter scenarios in Northern Europe to forecast delays or disruptions in traffic operation. Some studies have reviewed the issues during winter railway operations caused by snow and ice in Nordic countries [35]. In study [107], impact of humidity, temperature, ice, and snow depth on the occurrence of delays in high-speed operation in Sweden has been analysed through a Cox model and Markov chain model. A train delay prediction model with weather data has been designed in [108] and tested on two high-speed railway lines in China. The Nordland railway line has been assessed in [109], where extreme cold weather is a crucial factor related to delays and low punctuality. Other studies have focused on service delay time and its exposure time to bad weather [110]. A combination of fuzzy theory and rough sets under adverse weather conditions has been used to contribute to the forewarning method for train operation in [111]. Causal relationships between extreme weather patterns and derailments on railway turnouts have been investigated by means of Fuzzy Bayesian Network model [112]. The impact on freight railways in winter weather in Northern Europe has been discussed [113]. In study [114], an estimation of the effects of weather conditions on railway operator performance of passenger train services has been carried out, focusing on infrastructure disruptions. In study [115], a model to capture the cascade dynamics of delay propagation on railway networks under inclement weather has been presented.

As a general rule, climatological factors are considered random variables and instantly excluded from the models considered by railway operators. However, in HSR, the running resistance is the major factor determining the trains' energy consumption. Climatic conditions such as wind or air density can produce significant variations in running resistance value. Therefore, climatology plays an important role in the energy that high-speed trains demand. Previous reasons motivate this work to generate a new model to quantify the climatological affection in railway operation, both in energy consumption and in running time. Climatology has a great variability due to the impossibility of having complete knowledge of all the variables that model its behaviour. Therefore, climatic conditions such as pressure, temperature, and wind have been modelled by means of fuzzy logic [116]. This way, this affection is included in the running resistance, given the vagueness in the knowledge of these variables.

The main advantage of fuzzy logic modelling is the ability to handle imprecise or in-complete information. Additionally, a positive feature is its flexibility and simplicity to be developed and implemented to obtain efficient calculations. That is why many studies have applied fuzzy logic in train operation models. In study [117], the train service provider and the infrastructure provider have been modelled as software agents, and the negotiation for a train schedule and the associated track access charge as a prioritised fuzzy constrain satisfaction problem. In study [118], predictive fuzzy control is used for carrying out the train dwell-time control in an event-driven framework. A methodology for predictive fuzzy control in an event-driven environment for multi-objective decision making on DC railway systems has been presented in [119]. In study [120], an approach within the fuzzy framework for tackling the complexity of multidimensional service evaluations and the judgment on the goodness of the schedule has been proposed. In study [121], the authors investigated a passenger train timetable problem with fuzzy passenger demand. The paper presented by [122] exposed a framework to evaluate the logistics performance of inter-modal freight transportation by applying fuzzy set techniques. A multileveled distributed railway traffic fuzzy control system is proposed in [123]. An assistant support system for railway traffic control is described in [124] making use of fuzzy knowledge. A fuzzy optimisation model for rescheduling high-speed timetables is proposed in [69], focusing this work on punctuality and passengers' affection without considering energy minimisation. In studies ([125], [126]), a fuzzy model is proposed for an Automatic Train Control system. In studies ([27], [33]), uncertainty in the manual driving application of the speed regulation command is modelled using fuzzy numbers. In study [74], the optimal speed profile in urban DC railways is found with a GA, where the energy consumption and the running time are the fuzzy results. A fuzzy model of delays and punctuality constraints is proposed in [34] for the offline design of timetables when efficient driving is applied. The algorithm proposed in [104] is used for urban ATO operation including uncertainty in the mass, generating optimal energy–time Pareto curves of speed profiles. Fuzzy logic is applied in [127] to detect the railway wheelset conicity as a measure of the deterioration. In study [128], a fuzzy model is used to evaluate the potential risks of railway crossings. As can be seen, climatological parameters and their associated uncertainty have not been modelled previously; therefore, this possible affection to the railway operation has not been considered.

The main contribution of this chapter is the introduction of climatological parameters in the eco-driving optimisation model. The proposed model allows an efficient driving design considering the uncertainty due to climate conditions by means of fuzzy modelling. Climatological parameters are not typically considered in the design of speed profiles, and they could affect the running time and energy consumption.

The optimisation model is based on a Genetic Algorithm combined with fuzzy parameters. This algorithm applies a search process that optimises a driving holding speed without a braking commands set that can be easily applied by human drivers. Applying these commands generates the efficient driving with punctuality requirements at the destination. Using a detailed simulation platform makes solving all possible scenarios possible when the climatological parameters vary according to the proposed model. This methodology can also be applied to quantify the error obtained in the expected energy saving if climate conditions are not initially considered in the optimisation of the railway operation. The model has been applied in a simulation test based on real data of a Spanish HSR, to compare scenarios where different wind and air pressure conditions are considered.

This chapter is organised as follows: Section 2.2 describes the simulation model in which the weather parameters are included. The uncertainty associated with the climatological parameters is modelled in Section 2.3. Section 2.4 is a description of the optimisation model developed to generate efficient driving. Section 2.5 analyses the case study and presents the results obtained for a real high-speed railway line using the models proposed in previous sections. A discussion of the proposed procedure and the results are presented in Section 2.6. At the end of the document, the bibliographical references and previous academic studies from which information for the present analysis has been obtained are presented.

References and previous work related to eco-driving and the treatment of climatic parameters in railway operations are listed in Table 1.

Table 1. Literature review.

Reference	Algorithm or method applied	Optimisation and simulation target	Uncertainty treatment	Fuzzy logic	Railway line	Objective
[2]	Pontryagin's Principle	Energy consumption	No	No	Conventional	Eco-driving
[5]	Simulation	Speed profile	No	No	Conventional	Eco-driving
[21]	Pontryagin's Maximum Principle	Driving regimes	No	No	Conventional	Dynamic model for optimal driving
[22]	-	-	No	No	-	Overview of energy-saving technologies
[27]	Genetic Algorithm	Speed profile	Yes (manual driving variability)	Yes	Conventional	Speed profile under driving variability
[28]	Fuzzy c-Means and Genetic Algorithm	Multi-objective: energy and running time	Yes (fuzzy rule-based control)	Yes	High-Speed	Optimise energy-time trade-off control
[29]	Simulation	Speed profile	No	No	High-Speed	Eco-driving without braking
[33]	Multi-Objective Optimisation Algorithm	Energy consumption and delay	Yes (manual driving)	Yes	High-Speed	Eco-driving and delay recovery
[34]	Genetic Algorithm	Timetable and energy	Yes (delays and manual driving variability)	Yes	High-Speed	Eco-driving and timetable design under delay uncertainty
[35]	Statistical Analysis	Impact of weather on operation	Yes (climatic conditions)	No	Conventional / High-Speed	Delay/disruption prediction under weather conditions
[38]	NSGA-II and MOPSO	Optimisation of Pareto curves	No	No	Mass Rapid Transit	Algorithm comparison for metro operations
[44]	Genetic Algorithm	Regenerative energy	No	No	Mass Rapid Transit	Synchronize trains to maximise regenerative energy
[47]	Pontryagin's Maximum Principle	Energy consumption	No	No	Conventional	Explore energy savings in parts of train journeys
[48]	Dynamic Programming	Speed profile	No	No	Mass Rapid Transit	Speed optimisation for traffic management
[49]	Sequential Quadratic Programming	Speed profile	No	No	Conventional	Speed profile optimisation
[50]	Genetic Algorithm	Coasting strategy	No	No	Conventional	Coasting point optimisation
[51]	MOPSO	Speed profile	No	No	Mass Rapid Transit	ATO speed profile design

[64]	Variable Neighbourhood Search	Timetable and passenger demand	No	No	Mass Rapid Transit	Timetable optimisation
[65]	Mixed Integer Linear Programming	Profits for high-speed railway operators	No	No	High-Speed	Timetable optimisation
[66]	Mixed Integer Linear Programming with VNS	Passenger travel time	No	No	Mass Rapid Transit	Timetabling to enhance passenger experience
[67]	Distributed Search and GRASP	Total train running time	No	No	Conventional	Efficient train timetabling
[68]	Variable Neighborhood Search	Energy consumption	Yes (variability in acceleration and braking times)	No	Mass Rapid Transit	Energy-efficient timetable design
[69]	Fuzzy Mixed Integer Programming	Mismatch between timetable and passenger demand	Yes (passenger demand)	Yes	High-Speed	Timetable optimisation considering uncertainty
[70]	Parametric Evaluation and Optimisation Modelling	Energy consumption	No	No	Conventional	Formation scales and traction capacities study
[71]	Pontryagin's Maximum Principle	Energy consumption	No	No	Conventional	Energy-optimal driving design
[72]	Heuristic and rule-based algorithms	Speed profile considering constraints	No	No	Conventional	Develop algorithms to automatically generate speed profiles
[73]	Pontryagin's Maximum Principle	Energy consumption	No	No	Conventional	Energy-optimal driving strategies
[74]	Dynamic Programming	Energy consumption	No	No	Mass Rapid Transit	Energy-efficient driving strategies for DC systems
[75]	Optimisation Algorithms	Energy consumption in ATO metro system	No	No	Mass Rapid Transit	Energy-efficient ATO speed profile design
[76]	Nonlinear Train Dynamics	Energy consumption	No	No	Conventional	Trajectory optimisation
[77]	Coasting Board Strategies / Optimal Control Approaches	Energy efficiency and operational performance	No	No	Conventional	Coasting board and optimal control for minimising energy use
[78]	Review	-	No	No	Conventional	Classification of analytical and numerical methods
[79]	Dynamic Programming / GA / ACO	Train trajectory	No	No	Conventional	Eco-driving
[80]	Dynamic	Energy	No	No	Conventional	Eco-driving

	Programming / Gradient Method / Sequential Quadratic Programming	consumption				
[81]	Sequential Quadratic Programming	Energy consumption considering energy storage device	No	No	Conventional	Eco-driving with energy storage system
[82]	Optimal Control Theory	Energy consumption minimisation	No	No	Conventional	Eco-driving
[83]	Statistical Analysis	Urban development potential near metro stations	No	No	Mass Rapid Transit	Evaluate urban rail densification
[84]	Lagrange Multipliers	Optimisation of discretised problem	No	No	Conventional	Lagrangian proach to energy optimisati
[85]	Non-linear optimisation	Speed profile	No	No	Conventional	Optimization with non-linear model
[86]	Artificial Neural Network	Coasting strategy	No	No	Mass Rapid Transit	Coasting points optimisation
[87]	Artificial Neural Network	Coasting speed	No	No	Mass Rapid Transit	Coasting speed optimisation
[88]	Artificial Neural Network	Driving smoothness indicator	No	No	Mass Rapid Transit	Driving behaviour characterisation
[89]	Genetic Algorithm	Speed profile	No	No	Conventional	Minimise energy consumption
[90]	Genetic Algorithm	Net energy	No	No	Conventional	Optimisation of energy between substations
[91]	Genetic Algorithm	Timetable + speed profile	No	No	Conventional	Timetable and speed profile optimisation
[92]	Genetic Algorithm	Coasting points	No	No	Mass Rapid Transit	Energy minimisation optimising coasting points
[93]	Comparative Search Methods	Coasting points	No	No	Mass Rapid Transit	Traction energy reduction via coasting control
[94]	Mixed Integer Programming and Heuristic Search	Running time and efficiency	No	No	Conventional	Minimisation of total running time
[95]	Multi-pop Genetic Algorithm	Speed and coasting points	No	No	Mass Rapid Transit	Energy reduction in subway networks
[96]	Multi-population Genetic Algorithm	Energy consumption	No	No	Mass Rapid Transit	Train energy-saving control with fixed running time
[97]	Direct Search	Traction energy	No	No	Conventional	Eco-driving
[98]	Brute Force	Substation energy	No	No	Conventional	Substation energy consumption
[99]	Monte Carlo Simulation	Energy flow estimation	No	No	Conventional	Simulation with regenerative

						braking
[100]	Differential Evolution Algorithm	Trade-off optimisation: time and energy	No	No	Conventional	Energy-time trade-off under different track profiles
[101]	Simulated Annealing	Traction energy	No	No	Mass Rapid Transit	Eco-driving under punctuality constraints
[102]	Simulated Annealing	Coasting strategy	No	No	Conventional	Coasting points optimisation
[103]	IBEA	Multi-objective: energy and time	No	No	Conventional	Energy-time optimisation
[104]	NSGA-II	Multi-objective: energy and time	Yes (train mass variability)	Yes	Mass Rapid Transit	ATO speed profile optimisation
[105]	MAX-MIN Ant System Algorithm	Energy consumption between stations	No	No	Mass Rapid Transit	Online optimisation of train operation modes
[106]	Multi-Phase Optimal Control Formulation	Energy consumption and delay	No	No	Conventional	Eco-driving
[107]	Statistical Analysis	Delay patterns	Yes (delay prediction)	No	High-Speed	Investigate how winter climate affects punctuality
[108]	Recurrent Neural Networks	Delays under weather conditions	Yes (data variability)	No	Conventional / Mass Rapid Transit	Develop data-driven models for delay prediction
[109]	Statistical Regression Analysis	Delays from weather variables	Yes (punctuality impact)	No	Conventional	Quantify the impact of weather conditions on punctuality
[110]	Correlation Analysis	Delays and weather exposure	Yes (weather exposure)	No	Conventional	Quantify correlation between weather and delays
[111]	Fuzzy C-Means + Rough Sets	Early warnings for delays	Yes (disruption forecast)	Yes	High-Speed	Improve forecasting of delay risk under weather conditions
[112]	Bayesian Networks	Derailment probability	Yes (derailment risk)	No	Conventional	Analyse the probability of derailments under weather conditions
[113]	Empirical Analysis	Vulnerability of freight railways to weather	Yes (freight disruption)	No	Conventional	Examine the operational impact and risk of weather on freight rail services
[114]	Econometric Modelling	Weather disturbances	Yes (infrastructure failures)	No	Conventional	Weather impact on operation and infrastructure
[115]	Network Modelling	Cascading failures	Yes (cascade	No	Conventional	Weather disruption

	and Dynamic Simulation	simulation	effects)			propagation modelling
[116]	Fuzzy Modelling	-	-	-	-	Foundational theory of fuzzy decision-making
[117]	Prioritised Fuzzy Constraint Satisfaction Approach	Timetable and track access charge	Yes (service provider and infrastructure provider negotiation)	Yes	Conventional	Timetabling through fuzzy constraint negotiation
[118]	Predictive Fuzzy Control	Dwell time and scheduling decisions	Yes (dwell time and decision-making)	Yes	Mass Rapid Transit	Optimising dwell time to improve operation
[119]	Predictive Fuzzy Control	Energy, regularity and dwell time	Yes (decision-making and control)	Yes	Conventional	Optimise DC operation using fuzzy control
[120]	Fuzzy Analytical Hierarchy Process	Waiting time variables	Yes (timetable evaluation and decision-making)	Yes	Conventional	Timetable selection optimisation
[121]	Branch-and-Bound Algorithm	Passenger time and Delay	Yes (passenger demand)	Yes	Conventional	Timetable design under uncertainty
[122]	Fuzzy Analytical Hierarchy Process + Fuzzy Multi-Criteria Decision Making	Performance scores and criteria weights	Yes (subjective evaluation)	Yes	Conventional	Evaluate intermodal logistics performance
[123]	Fuzzy Distributed Decision-Making Control	Real-time train dispatching and schedule recovery	Yes (priority rating and delay)	Yes	Conventional	Real-time traffic optimisation
[124]	Fuzzy Petri Nets	Traffic quality and response to disturbances	Yes (expert dispatching knowledge)	Yes	Conventional	Support train dispatching with expert knowledge
[125]	Fuzzy Logic Control	ATC system control	Yes (system variability)	Yes	Mass Rapid Transit	Automatic train control under uncertainty
[126]	Pareto-based Differential Evolution	Energy, punctuality and comfort	Yes (Fuzzy memberships functions)	Yes	Mass Rapid Transit	Tune fuzzy ATO for optimal train operation
[127]	Fuzzy Diagnostic Model	Conicity detection	No	Yes	Conventional	Wheelset deterioration detection
[128]	Fuzzy Risk Evaluation	Risk assessment at crossings	No	Yes	Conventional	Railway crossings risk analysis
[123]	Fuzzy Multiattribute Decision-Making	Traffic control strategy	Yes (decision-making attributes)	Yes	Conventional	Develop a distributed traffic control system

2.2. SIMULATION

2.2.1. SIMULATION AND CLIMATOLOGICAL PARAMETERS

Table 2. Simulation model parameters.

$\boldsymbol{F_m}$	Motor force (kN)
$\boldsymbol{F_r}$	Running resistance (kN)
$\boldsymbol{F_g}$	Force due to the railway grades (kN)
$\boldsymbol{F_c}$	Force due to track curvature (kN)
$\boldsymbol{m_{eq}}$	Equivalent mass (tons)
$\boldsymbol{t}$	Time (s)
$\boldsymbol{a}$	Acceleration (m/s^2)
$\boldsymbol{v_{train}}$	Speed of the train (m/s)
$\boldsymbol{m_0}$	Empty train mass (tons)
$\boldsymbol{\lambda}$	Rotating mass factor
$\boldsymbol{m_l}$	Load mass (tons)
$\boldsymbol{g}$	Gravity acceleration (m/s^2)
$\boldsymbol{\theta}$	Slope angle
$\boldsymbol{p}$	Gradient of the track (mm/m or ‰)
$\boldsymbol{K_r}$	Constant defined by track gauge (m)
$\boldsymbol{r_c}$	Radius of track curvature (m)
$\boldsymbol{A}$	Constant related to the mechanical resistance to the motion (kN)
$\boldsymbol{B}$	Constant related to the resistance due to the air inlet in the train (kN/(m·s))
$\boldsymbol{C}$	Constant related to the aerodynamic resistance (kN/(m·s)^2)
$\boldsymbol{k}$	Tunnel factor
$\boldsymbol{E_{pantograph}(t)}$	Energy consumed by the train measured at the pantograph (W·s) at instant t (W)
$\boldsymbol{P_{pantograph}(t)}$	Electrical power consumed at instant t by the train measured at the pantograph (W)
$\boldsymbol{E_{substation}(t)}$	Energy consumption estimated at the electrical substation at instant t (Ws)
$\boldsymbol{P_{substation}(t)}$	Power consumed at the substation at instant t (W)
$\boldsymbol{P_{mec}(t)}$	Mechanical power at instant t (W)
$\boldsymbol{\eta_T}$	Electrical chain efficiency for the traction case (%)
$\boldsymbol{P_{aux}}$	Power consumed by the auxiliary systems (W)
$\boldsymbol{nz_{start}}$	Initial position of every neutral electrical zone (m)

nz_{end}	Final position of every neutral electrical zone (m)
$s(t)$	Position of the train at every moment (m)
η_B	Electrical chain efficiency for the braking case (%)
$r(s)$	Electrical line resistance in the position s (Ω)
V	Nominal line voltage (V)
$\cos\varphi$	Power factor

Table 3. Climatological model parameters.

F_d	Drag force (N)
v	Speed (m/s)
C_D	Drag coefficient
ρ_{ISA}	Density of the air under standard conditions (kg/m^3)
ρ	Density of the air ρ (kg/m^3)
S	Cross-sectional area (m^2)
p	Pressure (N/m^2)
R	Ideal gas constant (J/(kg·K))
T	Temperature (K)
ϕ , β	Angles defined by the velocities (Figure 4)
φ_{rail} , φ_{wind}	Track and wind angles with respect to the north (Figure 4)
v_{rel}	Train speed relative to the wind (m/s)
v_{wind}	Velocity of the wind (m/s)
$C(\rho, \beta)$	Value of C considering air density variation and wind effects (kN/(m·s)^2)
F_{tunnel}	Running resistance when the train travels through a tunnel (kN)

2.2.2. SIMULATION MODEL

The simulation model is described in this section. This model is detailed in [27] and the results provided by the simulation were compared with real data of a Spanish HSR to validate the simulator. Differences of 1.2% for the running time and differences of 0.4% when compared with the energy consumption were shown. The model was used from [27] to design eco-driving in a high-speed line, and measurements were recorded. As a result, energy savings between 20% and 34% were measured in commercial services. It was a detailed discrete event deterministic simulation model, that combines next-event and fixed-discrete time steps for advancing the simulation clock ([129], [130]). The simulation model takes several input data related to the rolling stock involved in the operation and related to the railway line:

- Rolling stock.
    1. Physical parameters: mass of the train, the mass of the load, length, maximum speed, rotatory inertia, and adhesion traction.
    2. Traction effort curve, braking effort curve, running resistance coefficients, energetic and power systems and auxiliary systems consumptions.
- Railway line. Includes Kilometric Points of relevant information as stations (K.P.), grades, track curvatures, speed limits, tunnels, and grade transitions considering the length of the train and the electric neutral zones.

The equation that models the motion of the train at each simulation step considering the forces acting on the train is:

$$F_m - \left(F_r + F_g + F_c\right) = m_{eq} \cdot a \tag{1}$$

where F_m is the motor force (kN), F_r is the running resistance of the train (kN) defined by the Davis formula (Equation (7)), F_g is the force due to the railway grades (kN), F_c is the force due to track curvature (kN), m_{eq} is the equivalent mass (tons) and a is the acceleration of the train (m/s^2).

By definition, the acceleration of the train in every step of the simulation is:

$$a = \frac{d}{dt}(v_{train}) \tag{2}$$

where v_{train} is the speed of the train (m/s).

The equivalent mass, effective mass or inertial mass can be calculated by:

$$m_{eq} = m_0 \cdot (1 + \lambda) + m_l \tag{3}$$

where m_0 is the empty train mass (tons), m_l is the load mass (tons) and λ is the dimensionless rotating mass factor.

Force due to gradient (Equation (4)) shows acceleration due to gravity effect, considering uphill or downhill motion. Therefore, the force due to gradient profile can be written as:

$$F_g = (m_0 + m_l) \cdot g \cdot sin\ (\theta) \tag{4}$$

where g is the gravity acceleration (m/s²) and θ is the slope angle.

Given the small values of slope angle in typical railway tracks, Equation (4) can be written as:

$$F_g = (m_0 + m_l) \cdot g \cdot \frac{p}{1000} \tag{5}$$

where p is the gradient of the track (mm/m or ‰).

Likewise, the force due to the track curvature can be expressed as follows:

$$F_c = \ (m_0 + m_l) \cdot g \cdot \frac{K_r}{r} \tag{6}$$

where K_r is an empirical constant defined by track gauge (m) and r_c is the radius of track curvature (m).

The traction effort or motor force (F_m) is bounded by a maximum value of the electrical traction effort curve and a maximum electrical braking effort curve both dependent on the train speed. The simulation model considers at every moment the speed limits established along the rail track by the railway administration. Speed limits are defined by stretches along the railway where the train must drive respecting the maximum speed limit considering the total length of the train. When a train passes through a neutral zone, auxiliary systems cannot be fed from a catenary, and motors apply at least a constant braking effort to maintain the charge of the batteries that feed auxiliary systems along neutral zones.

Running resistance is the model that defines and groups a series of resistances opposing the train's movement. These include mechanical resistances (rolling and internal friction), air intake resistances (for engine cooling and passenger air renewal), and aero-dynamic resistance [85]. The running resistance is modelled with a second-degree equation dependent on the instantaneous velocity and defined by the Davis expression [86]:

$$F_r = A + B \cdot v_{train} + C \cdot k \cdot v_{train}^2 \quad (7)$$

where A is a constant related to the mechanical resistance to the motion (kN), B is a constant related to the resistance due to the air inlet in the train $\left(\frac{kN}{\frac{m}{s}}\right)$, C is a constant related to the aerodynamic resistance $\left(\frac{kN}{\left(\frac{m}{s}\right)^2}\right)$, k is the tunnel factor and v_{train} is the velocity of the train (m/s^2).

The Davis formulae and similar expressions of a polynomial nature are models of reality. Each term of this equation refers to different physical components of the total running resistance. However, it must be considered that these polynomials are fitted models of highly complex phenomena that interact themselves. Therefore, although for general purposes certain approximations can be estimated, in reality, they are not completely accurate and do not represent reality in its entirety. Likewise, within the running resistance model, there are parameterised coefficients (A, B and C) that represent different oppositions to the movement of the train. These values are experimental, complex to define, and depend on the physical characteristics of the rolling stock. In order to obtain them, manufacturers test their trains and their relationships with the resistances generated. These coefficients are the nominal values, so they have been evaluated under standard conditions of pressure and temperature given by the International Standard Atmosphere, ISA (ISO, 1975: $T = 15\ ^{\circ}\mathrm{C}$; $\rho = 1.225\ \mathrm{kg/m^3}$).

The term of the running resistance model (Equation (7)) that is not related to the effect of the air outside the train is called mechanical resistance. In this term, the coefficient A is involved. In the most general case, it is derived from the frictional resistance between mechanical components, rail-to-wheel contact, track irregularities, and energy losses in the traction and suspension equipment of the vehicles due to the oscillating or parasitic movements of the suspended mass.

The term of the train speed-dependent running resistance corresponds, mainly, to the resistance caused by the air intake of the train. In trains, there is a constant and noticeable air flow, necessary for cooling engines and other equipment as well as that required for air renewal for passengers. This term is related to the B coefficient of the running resistance model.

The so-called aerodynamic running resistance is the longitudinal force that opposes the train's movement as a consequence of the interaction between the train and the surrounding air with which it collides and envelops it. The coefficient C is responsible for modelling this resistance.

The tunnel factor k increases the quadratic term of the running resistance model when the train runs through a tunnel due to the increased air friction against the outer surface of the train. When a tunnel is not involved in the simulation, the factor takes the value 1.

Equations (1) – (7) describe the motion of the train in next-event and fixed-discrete simulation steps, limited by speed limits, calculating braking curves to stop at the arrival point or station point with comfort and safety requirements.

The model related to the energy consumed by the train measured at the pantograph at instant t (Ws) ($E_{pantograph}$) is expressed as shown in Equation (8) while the train energy consumption estimated at the electrical substation at instant t (Ws) ($E_{substation}$) is defined by means of Equation (9):

$$E_{pantograph} = \int_0^t P_{pantograph}(t) \cdot dt \qquad (8)$$

$$E_{substation} = \int_0^t P_{substation}(t) \cdot dt \qquad (9)$$

where $P_{pantograph}$ is the electrical power consumed at instant t by the train measured at the pantograph (W) and $P_{substation}$ is the estimation of the power consumed at the substation at instant t (W).

Previous calculus requires some other equations to complete the model:

$$P_{mec} = F_m \cdot v_{train} \qquad (10)$$

$$P_{pantograph} = \frac{P_{mec}}{\eta_T} + P_{aux} \;\; if \;\; P_{mec} \geq 0 \; and \; not \; (nz_{start} \leq s \leq nz_{end}) \qquad (11)$$

$$P_{pantograph} = P_{mec} \cdot \eta_B + P_{aux} \;\; if \;\; P_{mec} < 0 \; and \; not \; (nz_{start} \leq s \leq nz_{end}) \qquad (12)$$

$$P_{pantograph} = 0 \;\; if \; nz_{start} \leq s \leq nz_{end} \qquad (13)$$

$$P_{substation} = P_{pantograph} + r \cdot \left(\frac{P_{pantograph}}{V \cdot cos\, \varphi}\right)^2 \;\; if \; not \; (nz_{start} \leq s \leq nz_{end}) \qquad (14)$$

where P_{mec} is the mechanical power at instant t (W), P_{aux} is the power consumed by the auxiliary systems (W), η_T is the electrical chain efficiency for the traction case (%), η_B is the electrical chain efficiency for the braking case (%), V is the nominal line voltage (V), $\cos\varphi$ is the power factor, r is the electrical line resistance in the position s (Ω), nz_{start} and nz_{end} are the initial and final position of every neutral electrical zone (m) and s is the position of the train (m).

2.2.3. CLIMATOLOGICAL MODEL

The following section focuses on the running resistance variation when the railway operation is developed under real climatological circumstances. Initially, the influence of the variation of density along the railway line due to altitude changes has been modelled. Then, the wind's influence on the running resistance model is incorporated. When motion equations are applied (Equations (1) – (7)) for simulating the train's driving in a railway line, it is supposed that pressure and temperature values are constant along the whole line under standard conditions, given by the International Standard Atmosphere [131]. Thus, Equation (7) is just modified by the velocity of the train and it incorporates coefficients B and C given by the manufacturer. For that reason, previous coefficients are also constant along the railway operation in absence of wind and they are always the same for each line, no matter the geographical location and the season of the year considered for the railway operation. The purpose of this section is to include in the simulation model the influences on the running resistance with climatological considerations of pressure, temperature and wind.

The railway operation can occur between points of different altitudes, and it is, therefore, to be expected that the model must deal with this variation along the railway line. In summary, throughout the journey, the air density will vary due to the change in altitude and this variation will modify the running resistance equation according to the model described below. When the train runs on a line without wind, the running resistance term can be defined by Equation (7). In the running resistance model, lineal and quadratic terms vary as a function of train speed.

The following expression relates the drag force (N) (F_d) to the parameter C in Equation (7) [132]:

$$F_d = C \cdot v^2 = \frac{1}{2} \cdot C_D \cdot \rho_{ISA} \cdot S \cdot v^2 \quad (15)$$

where v is the speed (m/s), C_D is the drag coefficient, ρ_{ISA} is the density of the air under standard conditions (kg/m^3) [89] and S is the cross-sectional area (m^2).

From Equation (15), it is deduced that factor C of the Davis formula is linearly dependent on the density of the air ρ (kg/m^3) and, therefore, the coefficient C is parameterised as a function of density as follows considering standard weather conditions as a reference [89]:

$$C(\rho) = C \cdot \frac{\rho}{\rho_{ISA}} \quad (16)$$

Finally, when all information related is considered, the running resistance model can be written as follows. However, this equation models the situation where the train is running without being influenced by the wind:

$$F_r = A + B \cdot v_{train} + C(\rho) \cdot k \cdot v_{train}^2 \quad (17)$$

In this manner, a main and essential task is to know the air density at every point or analyse the stretch along the railway line. The density is a function of the temperature and pressure of the air, and its relationship can be expressed by the equation of state defined in the ideal gas law as follows:

$$\rho = \frac{p}{R \cdot T} \quad (18)$$

where ρ is the density (kg/m^3), p is the pressure (N/m^2), R is the ideal gas constant $\left(\frac{J}{kg \cdot K}\right)$ and T is the temperature (K).

Pressure and temperature values strongly depend on the meteorological conditions given a specific area due to its geographical characteristics and the year's season. Therefore, considering the climatological model, they can be significantly different from one area to another. This work analyses these differences, which are present in the variations along the railway line.

On the other side, the running resistance force is also significantly affected by the wind and its variation along the route. The influence of the wind is modelled by the combination of its intensity, or wind speed, and the angle of incidence in each stretch relative to the train (ϕ) ([132], [133], [134]). The train speed relative to the wind is the train speed minus the wind speed vectors, as it can be seen in Figure 3. Figure 4 shows the reference frame considered in this work where angle ϕ (Equation (19)) is the principal parameter to model the wind influence in the railway operation.

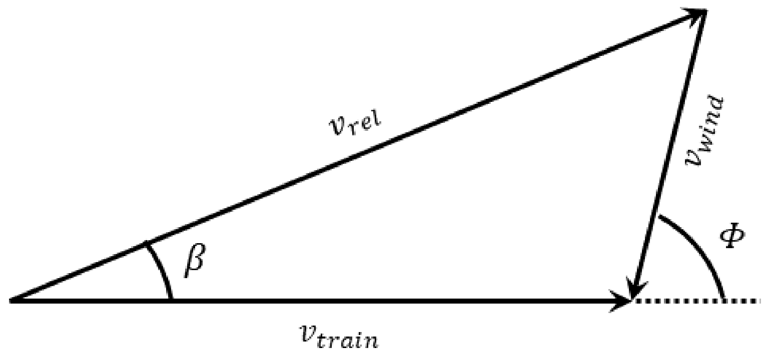

Figure 3. Velocities diagram.

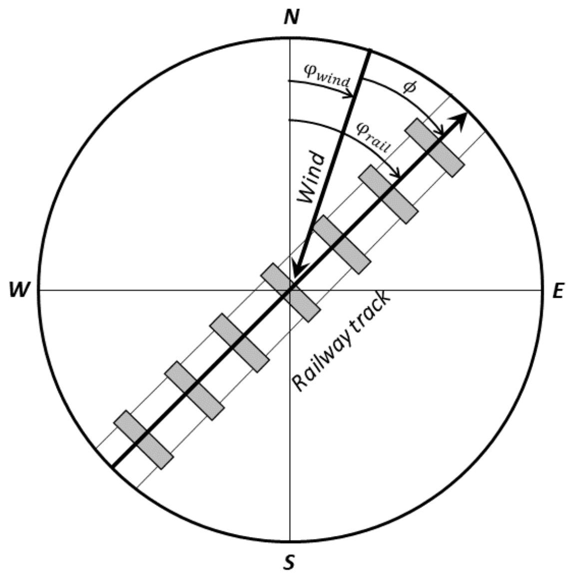

Figure 4. Reference frame.

$$\phi = \varphi_{rail} - \varphi_{wind} \tag{19}$$

where β and ϕ are the angles defined by the velocities, v_{wind} is the velocity of the wind (m/s) and v_{rel} is the relative speed of the train with respect to the wind (m/s), graphically defined in Figure 3.

Running resistance model, given by Equation (7), is valid just when the wind has not been considered, so the model has to be modified, now influenced by the speed of the train and wind vectors, as shown in Figure 3. The modification consists of replacing the train speed v_{train} in Equation (7) by the train speed relative to the wind v_{rel}, which is equivalent to considering zero wind in the original equation:

$$F_r = A + B \cdot v_{rel} + C \cdot k \cdot v_{rel}^2 \tag{20}$$

Besides and in addition to the previous model, it has to be considered that factor C also depends on the angle β as shown in Figure 3. Yaw angle can be calculated using Equation (21):

$$\beta = arctan\left(\frac{v_{wind} \cdot sin\,\phi}{v_{train} + v_{wind} \cdot cos\,\phi}\right) \tag{21}$$

To get the final model for the running resistance of a train, the relative train speed must be calculated, determining the simulation model as follows:

$$v_{rel} = \sqrt{v_{train}^2 + v_{wind}^2 + 2 \cdot v_{train} \cdot \ v_{wind} \cdot cos\,\phi} \tag{22}$$

Equation (23) is proposed in [135]. This expression establishes the variation of the constant C according to the angle β. This expression becomes meaningless when β > 30°. An angle with such magnitude for typical speed values in operation would imply extreme winds, which would affect the train operation or could lead to the total suspension of the service.

$$C(\rho, \beta) = C(\rho) \cdot (1 + 0.02 \cdot \beta) \tag{23}$$

where $C(\rho)$ is the value of C when density variation is considered according to Equation (16).

Finally, the variable running resistance considering climatological factors is modelled as:

$$F_r = a + b \cdot v_{rel} + c(\rho, \beta) \cdot v_{rel}^2 \tag{24}$$

When the train runs through a tunnel, the force exerted by the influence of the wind shall not be considered. Therefore, for the sections simulated in a tunnel, the expression for the running resistance must be modelled with an expression that considers the train speed as input data, including the tunnel factor in the quadratic term. As applicable, it is used a k parameter as the tunnel factor:

$$F_{tunnel} = A + B \cdot v_{train} + k \cdot C \cdot v_{train}^2 \qquad (25)$$

Pressure, temperature and wind models have been discretised along the railway line. If the altitude suffers any variation along the journey, the model can deal with the change by measuring the impact of the air density variance. Two different reasons have justified the division or discretisation along the railway line. Firstly, the variability of the wind makes necessary a partition along the line where the climatological and geographical characteristics determine the main direction in which the wind blows. And secondly, the change of directions taken by the train along the railway line has been dealt implementing stretches where it can be supposed that the relative angle between the wind and the train movement is approximately constant along a straight line. Hence, the proposed wind model considers both the railway track angle and the incidence wind angle to be parametrised. In conclusion, previous considerations allow the simulation model of the train movement to deal with the wind behaviour's complexity without compromising the final results' accuracy.

2.3. FUZZY CLIMATOLOGICAL MODEL

2.3.1. FUZZY PARAMETERS

Table 4. Fuzzy model parameters.

$\hat{p}$	Fuzzy pressure (N/m^2)
$\hat{T}$	Fuzzy temperature (K)
$\hat{v}_{wind}$	Fuzzy wind speed (m/s)
$\hat{\phi}$	Fuzzy angle of incidence (º)
$\hat{T}_r$	Punctuality constraint (s)
$t_{objective}$	Objective running time (s)
α_p	Measure of the possibility
n_p	Required necessity
N	Possibility measure

This study quantifies the relevance of the climatological factors and their variation by modelling them with their associated uncertainty during railway operation by means of a climate fuzzy model.

The parameters modelled by fuzzy numbers [92] are defined in Section 2.2.2: the pressure and temperature defining, consequently, the density, and the intensity and the angle of incidence of the wind.

Among the many possibilities for the membership function of the fuzzy numbers: piece-wise linear [93], linear, hyperbolic, exponential and S-shaped ([138], [139]), climatological factors have been defined as triangular fuzzy numbers. But the model proposed in this chapter would be applicable if any other shape is utilised to model climatological parameters' uncertainty. Each of the fuzzy numbers, the temperature, barometric pressure, air density, wind intensity (or speed) and the angle of incidence of wind are defined by the shape shown in Figure 5. Therefore, a fuzzy number is defined by its core, its lower limit and its upper limit of the support.

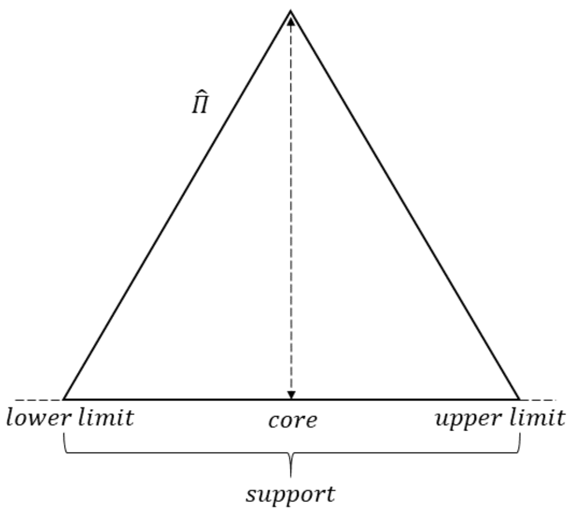

Figure 5. Triangular fuzzy number.

Fuzzy pressure ($\hat{p}$), fuzzy temperature ($\hat{T}$), fuzzy wind speed ($\hat{v}_{wind}$) and fuzzy angle of incidence ($\hat{\phi}$) have been defined for every division realised along the railway line in order to get a more detailed climatological model on railway track sections as explained in Section 2.2.2.

The running time is calculated from the indicated fuzzy parameters and the model input data (train and railway line, Section 2.2.1). This result will be a fuzzy parameter due to the uncertainty associated with some of the calculation parameters. In the proposed fuzzy model, the punctuality constraint is imposed on the fuzzy running time, which must be less or equal to the objective running time:

$$\hat{T}_r \leq t_{objective} \tag{26}$$

The punctuality constraint can be expressed either as a measure of the possibility of arriving at the scheduled time or as a measure of the necessity of fulfilling that objective. Thus, if it is expressed as a measure of the possibility α_p, the expression is:

$$\Pi\ (\hat{T}_r \leq t_{objective}) = \alpha_p \tag{27}$$

The corresponding necessity measure, in this case, is 0: $N\ (\hat{T}_r \leq t_{obj}) = 0$.

If the punctuality constraint is expressed as a measure of the required necessity n_p, (Figure 6) the expression is:

$$N\ (\hat{T}_r \leq t_{objective}) = n_p = 1 - \alpha \tag{28}$$

The corresponding possibility measure, in this case, is 1:

$$N\ (\hat{T}_r \leq t_{objective}) = 1 \tag{29}$$

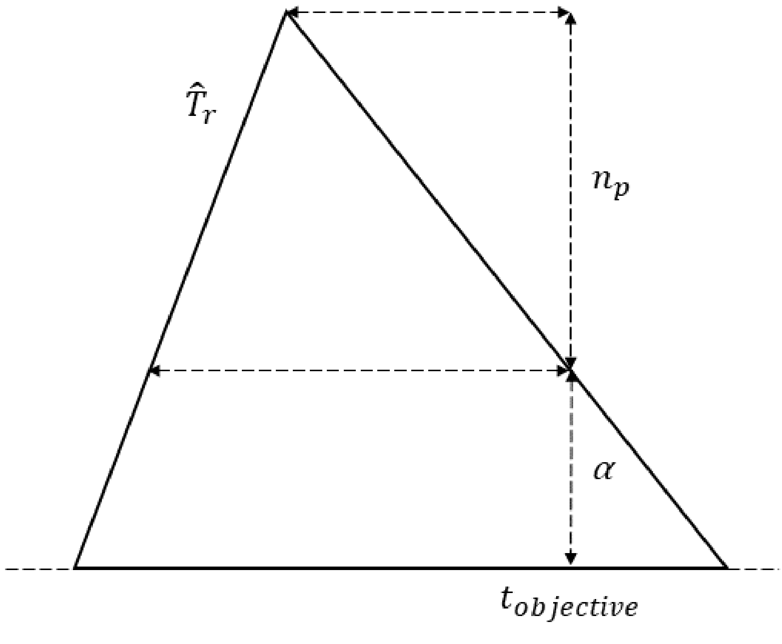

Figure 6. Fuzzy running time.

2.4. ECO-DRIVING OPTIMISATION WITH FUZZY PARAMETERS

Having introduced the climatological fuzzy model and how it affects the train motion simulation model, the objective is now to present the driving optimisation model. This optimal driving will be constructed as a combination of efficient commands considering coasting points known as Command Matrix (C_m). Then, the problem and proposed algorithm for the problem resolution will be presented: Genetic Algorithm with Fuzzy Parameters (GA-F).

2.4.1. EFFICIENT DRIVING COMMANDS

In order to design the eco-driving, it is necessary to make use of efficient commands. In this chapter the driving commands are based on the coasting application and the speed regulation without braking strategy. This driving strategy is the efficient version of the speed regulation strategy and it is easily applied by the drivers on long distance and high-speed railway lines. To execute this strategy, the constant regulation speed is applied as long as it is needed for the traction. It occurs in horizontal or non-steep track alignment. Moreover, braking is not applied if needed to maintain the cruising speed and the maximum speed limits allow it [23, 24]. It occurs in steep downhill alignments. This way, the train uses the track grades to increase speed without consuming energy. It allows gaining time that can be used to drive the train at a lower speed at other track sections where energy is needed to maintain the cruising speed. Braking is applied just to observe maximum speed limitations and braking curves up to reductions of maximum speed profile or up to stopping points.

Commands are modelled in the form of Command Matrix (C_m) [29]. The matrix is the result of the optimisation process and means the division of the railway line where the driver is required to apply certain driving commands. The C_m is defined as a matrix formed with the values of the points where every stretch end and the value of the regulation speed without braking are applied in every section. The end (Kilometric Point) of the divisions forms the first column, and the regulation speed value without braking forms the second column. Thus, the initial point of a division is the end of the previous section except for the first division, where the initial point is the journey starting position. Furthermore, the end of the last division will be the coasting section's starting point until the braking curve is reached up to the station. The numbers of rows of the matrix plus 1 are the number of the sections defined.

Table 5 shows the Command Matrix that can be applied by the train driver or the automatic train operation system.

Table 5. Command matrix.

Command Matrix, C_m		
Initial K.P.	**Final K.P.**	**Speed (km/h)**
$x_{initial}$	x_1	$v_{initial}$
x_1	x_2	v_1
…	…	…
x_{n-2}	x_{n-1}	v_{n-1}
x_{n-1}	x_n	v_n
x_n	x_{final}	$Coasting\ and\ braking$

Figure 7 shows the application of the driving previously design in Table 1 by the divisions along the railway line where the speed regulation without braking is applied.

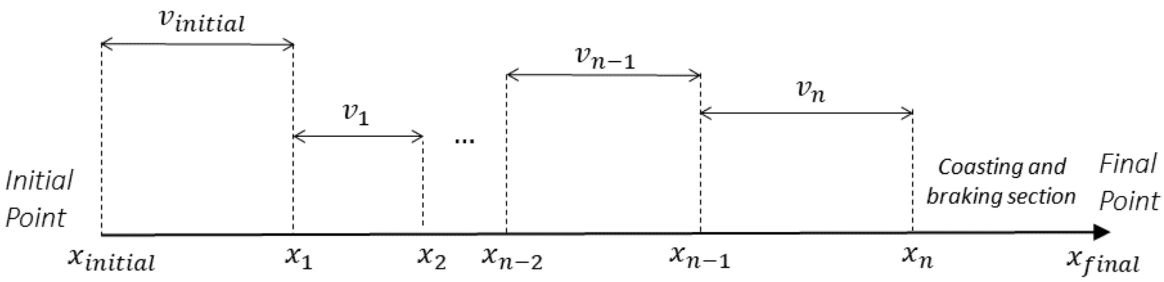

Figure 7. Command sections.

The application of the Command Matrix calculated considers the operational requirements as the speed limits and the stopping points at the stations. Moreover, the optimisation model includes comfort demands for the passengers and punctuality constraints. Hence, the Command Matrix (C_m) obtained as a solution to the problem will be efficient, safe and comfortable driving.

The solutions given by the algorithm are feasible and easily understood, and executed by the driver because they represent realistic driving. The number of driving commands to be executed by the driver is a choice to be made by the railway operator according to the desired complexity during the railway operation. The definition of driving must consider that the more divisions are taken, the more complicated it will be to execute the eco-driving strategy.

2.4.2. GENETIC ALGORITHM WITH FUZZY PARAMETERS: GA-F

Once the fuzzy numbers of the climatological model are given, the driving simulated will have an associated fuzzy energy consumption ($\hat{E}$) and a fuzzy running time ($\hat{T}_r$). When the efficient driving is designed, the objective is to find the set of commands that will produce the minimum value of energy consumption for the required running time (t_{obj}).

The fitness function $\hat{F}(\hat{T}_r, \hat{E})$ is shown in Equation (29) and defines the evaluation of both the running time and energy consumption affected by the weighting factors considered (w_e, w_t) to minimise the energy consumption considering a design or target running time. Energy consumption associated with the fastest possible driving along the railway line with operational constraints is called $E_{flat-out}$.

$$\hat{F}(\hat{T}_r, \hat{E}) = \begin{cases} w_e \cdot \dfrac{\hat{E}}{E_{flat-out}} + w_t \cdot \dfrac{\hat{T}_r}{t_{obj}} & if \quad \hat{T}_r > t_{obj} \\ w_e \cdot \dfrac{\hat{E}}{E_{flat-out}} + w_t \cdot \dfrac{t_{obj}}{\hat{T}_r} & if \quad \hat{T}_r \leq t_{obj} \end{cases} \tag{29}$$

where, w_e and w_t are weighting factors, $E_{flat-out}$ is the energy consumption when the flat-out driving is applied and t_{obj} is the target running time.

The Genetic Algorithm (GA) used to solve the problem evolves a population of possible solutions (individuals) by means of iterations. An elite group, with the fittest solutions, survives from one generation to the next while the process is moving forward using C_m as the genome of the individuals. Additionally, crossover and mutation operators are applied to generate and offspring of individuals from the elite group at each iteration. The fuzzy optimisation algorithm is the procedure for the calculation of the driving commands when the fuzzy parameters are included. The GA-F process is graphically described in Figure 8 and it is explained as follows:

1. First random generation of individuals (C_m) equal to the population size N_{pop}.
2. Simulation of command matrix of the population. This step generates a running time $\hat{T}_r$ and an energy consumption $\hat{E}$ associated to each individual.
3. Evaluation of each individual by the fitness function (Equation (29)).
4. Sorting of each individual in increasing order by means of their fitness value.
5. Elite group selection. The first individuals N_e survive in the next iteration population and the rest are eliminated.
6. Offspring generation. Mutation and crossover operators are applied to the elite group to generate new individuals until the population reaches its size N_{pop}. In this step, a number of mutations N_m and a number of crossovers N_c are applied.
7. The process is repeated from step 2 until the iteration number (it) is equal to the maximum numbers of iterations defined (it_{max}), giving the best solution to the fittest in the last population.

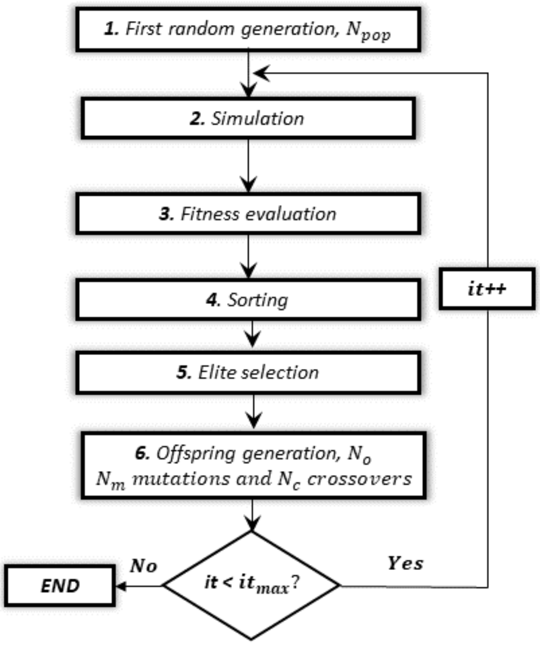

Figure 8. Flowchart of the GA.

The proposed algorithm can be expressed in terms of the α-cuts of the fuzzy numbers [96]. As shown in Figure 6, if the punctuality requirement is expressed as a necessity measure n_p, the upper limit of the α-cuts of the fuzzy running time (equivalent to the upper limit of the α-cuts of the fuzzy running time) is necessary for the evaluation of the punctuality constraint. For this upper limit to be obtained, given the fuzzy numbers associated with the climatological parameters, it has to be analysed whether the running time is an increasing or a decreasing function in each of them.

As discussed in the model, the running time and energy consumption are dependent on the running resistance model. The running resistance force is a function of the fuzzy parameters introduced in this model: the air density and the wind during operation. In turn, the density is a function of the pressure and the temperature. The wind has been modelled by its intensity or speed and their angle of incidence related to the train's movement.

With equal C_m, train and railway track data, the running time calculated is a function of the uncertainties considered: pressure ($\hat{p}$), temperature ($\hat{T}$), velocity of the wind ($\hat{v}_{wind}$), and angle of incidence of the wind ($\hat{\phi}$). All climatological factors, running time, and energy consumption are defined as fuzzy parameters.

$$\hat{T}_r = F_1(\hat{p}, \hat{T}, \hat{v}_{wind}, \hat{\phi}) \tag{30}$$

Function F_1 (Equation (30)) is monotonous with all climatological parameters. The greater the running resistance, the lower the acceleration and the greater the running time as a result. In that sense, function F_1 is increasing with the air density, and thus, according to Equation (18), it is monotonically increasing with the pressure ($\hat{p}$) and monotonically decreasing with the temperature ($\hat{T}$).

Regarding the angle of incidence of the wind ($\hat{\phi}$), it is necessary to analyse every division of the railway line (explained at the end of Section 2.2). In this case, and taking Figure 3 as a reference, F_1 is defined differently depending on the situation. Moreover, the presence of the angle of incidence in Equation (22) has to be considered in order to analyse the behaviour of the function F_1. In this way, the variation of the angle and, therefore, the change of $\cos\phi$ in the term of Equation (22) determines the different situations in the high-speed railway scenario. If the angle of incidence lies between:

(1) $0° < \hat{\phi} < 180°$ then F_1 is monotonically decreasing with the angle ϕ.

(2) $180° < \hat{\phi} < 360°$ then F_1 is monotonically increasing with the angle ϕ.

After considering the term of the angle of incidence, the speed is another factor related to the wind and its contribution to the railway operation. Considering Figure 3, the behaviour is as follows:

(1) $0° < \hat{\phi} < 90°$ or $270° < \hat{\phi} < 360°$ then F_1 is monotonically increasing with the velocity of the wind.

(2) $90° < \hat{\phi} < 270°$ then F_1 is monotonically decreasing with the velocity of the wind.

Likewise, the energy consumption associated to the railway operation is also a function of the climatological parameters:

$$\hat{E} = F_2\left(\hat{p}, \hat{T}, \hat{v}_{wind}, \hat{\phi}\right), \tag{31}$$

Function F_2 (Equation (31)) is monotonous with all climatological parameters as F_1 which permits the application of α-cut calculations. Moreover, the dependence of F_2 with these parameters is the same as F_1 because the greater the running resistance, the greater the traction effort and energy consumption.

In conclusion, the upper and lower limits of $\hat{T}_r$ α-cuts, and the upper and lower limits of $\hat{E}$ α-cuts can be calculated by means of the alfa-cut of the parameters. Equations (32) – (39) show the corresponding formulas for the 4 quadrants of the angle of incidence as follows, considering the situations presented previously:

- For $0° < \hat{\phi} < 90°$:

$$T_r^{\overline{\alpha}}, E^{\overline{\alpha}} = F_1, F_2\left(p^{\overline{\alpha}}, T^{\underline{\alpha}}, v_{wind}^{\overline{\alpha}}, \phi^{\underline{\alpha}}\right) \tag{32}$$

$$T_r^{\underline{\alpha}}, E^{\underline{\alpha}} = F_1, F_2\left(p^{\underline{\alpha}}, T^{\overline{\alpha}}, v_{wind}^{\underline{\alpha}}, \phi^{\overline{\alpha}}\right) \tag{33}$$

- For $90° < \hat{\phi} < 180°$:

$$T_r^{\overline{\alpha}}, E^{\overline{\alpha}} = F_1, F_2\left(p^{\overline{\alpha}}, T^{\underline{\alpha}}, v_{wind}^{\underline{\alpha}}, \phi^{\underline{\alpha}}\right) \tag{34}$$

$$T_r^{\underline{\alpha}}, E^{\underline{\alpha}} = F_1, F_2\left(p^{\underline{\alpha}}, T^{\overline{\alpha}}, v_{wind}^{\overline{\alpha}}, \phi^{\overline{\alpha}}\right) \tag{35}$$

- For $180° < \hat{\phi} < 270°$:

$$T_r^{\overline{\alpha}}, E^{\overline{\alpha}} = F_1, F_2\left(p^{\overline{\alpha}}, T^{\underline{\alpha}}, v_{wind}^{\underline{\alpha}}, \phi^{\overline{\alpha}}\right) \tag{36}$$

$$T_r^{\underline{\alpha}}, E^{\underline{\alpha}} = F_1, F_2\left(p^{\underline{\alpha}}, T^{\overline{\alpha}}, v_{wind}^{\underline{\alpha}}, \phi^{\underline{\alpha}}\right) \quad (37)$$

- For $270° < \hat{\phi} < 360°$:

$$T_r^{\overline{\alpha}}, E^{\overline{\alpha}} = F_1, F_2\left(p^{\overline{\alpha}}, T^{\underline{\alpha}}, v_{wind}^{\overline{\alpha}}, \phi^{\overline{\alpha}}\right) \quad (38)$$

$$T_r^{\underline{\alpha}}, E^{\underline{\alpha}} = F_1, F_2\left(p^{\underline{\alpha}}, T^{\overline{\alpha}}, v_{wind}^{\underline{\alpha}}, \phi^{\underline{\alpha}}\right) \quad (39)$$

Figure 9 shows an example corresponding to the case $180° < \hat{\phi} < 270°$ of how the climatological fuzzy parameters are considered and the results associated with a specific necessity level of punctuality.

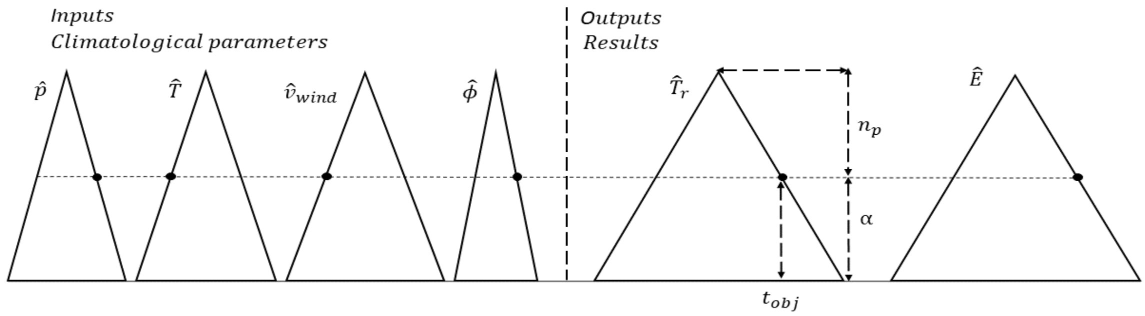

Figure 9. Climatological fuzzy parameters and fuzzy results.

Therefore, the proposed problem can be solved with the application of the GA-F by means of α-cuts with $\alpha = 1 - n_p$. The parameter n_p is the imposed punctuality constraint. The algorithm will search for the solution in the form of a Command Matrix with a fuzzy running time associated. Its upper α-cut gives the objective running time as a result, considering punctuality requirements and minimising the energy consumption.

2.5. CASE STUDY

2.5.1. INTRODUCTION

Real commercial service and infrastructure data of a Spanish high-speed railway line have been analysed in this section. The railway line is operated by a train with an unladen mass of 322 t, a length of 200 m, a maximum traction effort of 200 kN and a maximum speed of 300 km/h.

This line has been divided as shown in the following figures to elaborate an appropriate data analysis as explained at the end of Section 2.2.2. The railway line has been divided into zones where the wind is considered constant as can be seen in Figure 10a, and into sections where the angle of the track φ_{rail} (Table 1) is considered constant as shown in Figure 10b.

(a)

(b)

Figure 10. (a) Wind zones; (b) Constant trip directions zones.

Table 6. Angle of the railway track.

Initial K.P.–Final K.P.	Angle of the Rail Track (º)
0 – 45	65
45 – 135	48
135 – 200	59
200 – 245	57
245 – 275	41
275 – 335	114
335 – 435	82
435 – 495	117
495 – 515	161
515 – 580	62
580 – 621	145

2.5.2. FUZZY CLIMATOLOGICAL PARAMETERS

The climatological parameters have been modelled taking into account the temperature, the wind intensity, and its angle of incidence collected by the State Meteorological Agency of the Spanish Government [97] in every one of the closest weather stations to the railway line for this case study, and the pressure is a function of the track altitude according to the International Standard Atmosphere [131].

The following tables collect the fuzzy climatological numbers modelled: temperature, pressure, intensity, and angle of incidence of the wind (Table 7, Table 8, Table 9 and Table 10, respectively).

Table 7. Fuzzy temperature, $\hat{T}$.

Fuzzy Temperature			
Initial K.P.	Summer (°C)		
	Lower Limit	Core	Upper Limit
0	28.25	34.31	40.37
95	27.86	33.30	38.74
170	24.14	32.38	40.61
240	25.10	33.11	41.12
330	25.21	32.07	38.93
400	26.89	33.63	40.37
475	27.03	31.39	35.75
Initial K.P.	Winter (°C)		
	Lower Limit	Core	Upper Limit
0	5.46	11.77	18.09
95	4.90	11.35	17.80
170	5.00	11.47	17.93
240	3.54	11.71	19.87
330	3.85	11.22	18.59
400	1.10	10.69	20.27
475	9.55	15.35	21.14

Table 8. Fuzzy pressure, $\hat{p}$.

Fuzzy Pressure			
Initial K.P.	Pressure (mbar)		
	Lower Limit	Core	Upper Limit
0	927.91	941.17	954.44
45	929.15	942.42	955.68
95	916.55	929.58	942.60
170	931.78	945.25	958.72
240	968.48	983.10	997.72
330	956.92	971.09	985.25
400	976.49	991.22	1005.94
475	990.38	1004.74	1019.11
500	995.28	1009.65	1024.01

Table 9. Fuzzy intensity of the wind, $\hat{v}_{wind}$.

Fuzzy Intensity of the Wind / Summer and Winter (m/s)			
Initial K.P.–Final K.P.	**Lower Limit**	**Core**	**Upper Limit**
0–45	0.6	3.2	5.8
45–200	1.4	4	6.6
200–240	0	2.8	5.6
240–330	0	3.6	7.9
330–400	0	3.9	8.8
400–475	0	2.5	5.4
475–500	1.6	4.5	7.4
500–550	0.4	3.3	6.2
550–580	0	3.7	7.4
580–621	0	3	6.7

Table 10. Fuzzy angle of incidence of the wind, $\hat{\phi}$.

Fuzzy Angle of Incidence of the Wind (°) / Summer				
Initial K.P.–Final K.P.	**Lower Limit**	**Core**	**Upper Limit**	**Type of Wind**
0–45	215	230	245	Unfavourable
45–135	55	70	85	Favourable
135–200	59	70	85	
200–240	25	40	55	
240–330	295	310	325	Unfavourable
330–335	245	260	275	
335–400	245	260	262	
400–475	245	260	275	
475–495	103	110	125	Favourable
495–500	95	110	125	
500–515	125	140	155	
515–550	125	140	152	
550–580	62	70	85	
580–621	145	160	175	

Fuzzy Angle of Incidence of the Wind (°) / Winter				
Initial K.P.–Final K.P.	**Lower Limit**	**Core**	**Upper Limit**	**Type of Wind**
0–45	5	20	35	Unfavourable
45–135	55	70	85	Favourable
135–200	59	70	85	
200–240	25	40	55	
240–330	295	310	325	Unfavourable
330–335	245	260	275	
335–400	245	260	262	
400–475	245	260	275	
475–495	103	110	125	Favourable
495–500	95	110	125	
500–515	265	280	295	
515–550	265	280	295	
550–580	62	70	85	
580–621	145	160	175	

2.5.3. RESULTS

2.5.3.1. FLAT-OUT DRIVING

Initially, the flat-out driving simulations are presented for the Initial Case scenario, with no climatological parameters, and also for the scenarios when the climatological parameters are considered (both in winter and summer). The flat-out driving is the fastest possible in the journey fulfilling the speed limitations of the line. This type of driving has the fastest running time and the associated highest energy consumption.

The running time in the simulated Initial Case (no climatological parameters) is 2:18:49, and the associated energy consumption is 10.804 MWh. Table 11 presents the results.

Table 11. Flat-out driving results.

Scenario	Energy Consumption (MWh)	Running Time (hh:mm:ss)
Initial Case	10.804	2:18:49
Winter	10.603	2:18:48
Summer	10.061	2:18:40

Figure 11 shows the speed profile of the train for the Initial Case, when the flat-out driving is applied and climatological parameters are not impacting the operation.

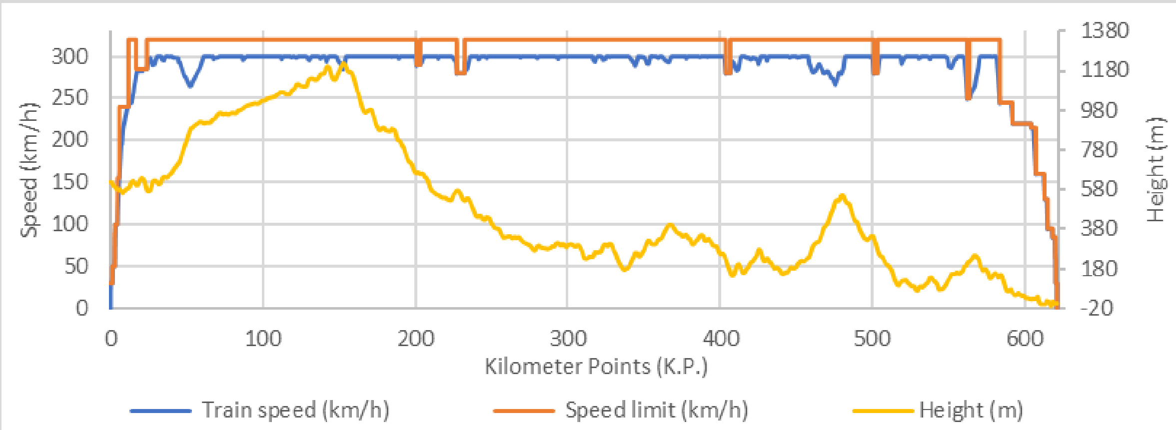

Figure 11. Flat-out speed profile.

The running time and the energy consumption shown in Table 11 for winter and summer climatological conditions, correspond to the value calculated for the core of the fuzzy sets of temperature, pressure, wind intensity, and angle of incidence.

Figures 12 and 13 show the energy consumption of the flat-out driving and the associated running time when the uncertainty in the climatological parameters are included.

The impact of climatological parameters on energy consumption is more important than on the running time. In the summer scenario shown in Figure 12, the variation of energy consumption for the minimum and maximum values (support of the fuzzy numbers) is 14.8% in summer and 13.2% in winter, relative to the minimum value. To analyse the impact of winter and summer conditions on the energy consumption, the core of the fuzzy energy consumption in summer and winter can be compared; this comparison shows that the energy consumption is 5.4% less in summer than in winter.

On the other hand, the impact on the running times is less significant (Figure 13). In summer, the variation is 0.18%, and 0.88% in winter, both relative to the minimum value in each case. The variation between summer and winter conditions (core values) shows a difference of 0.10%, approximately.

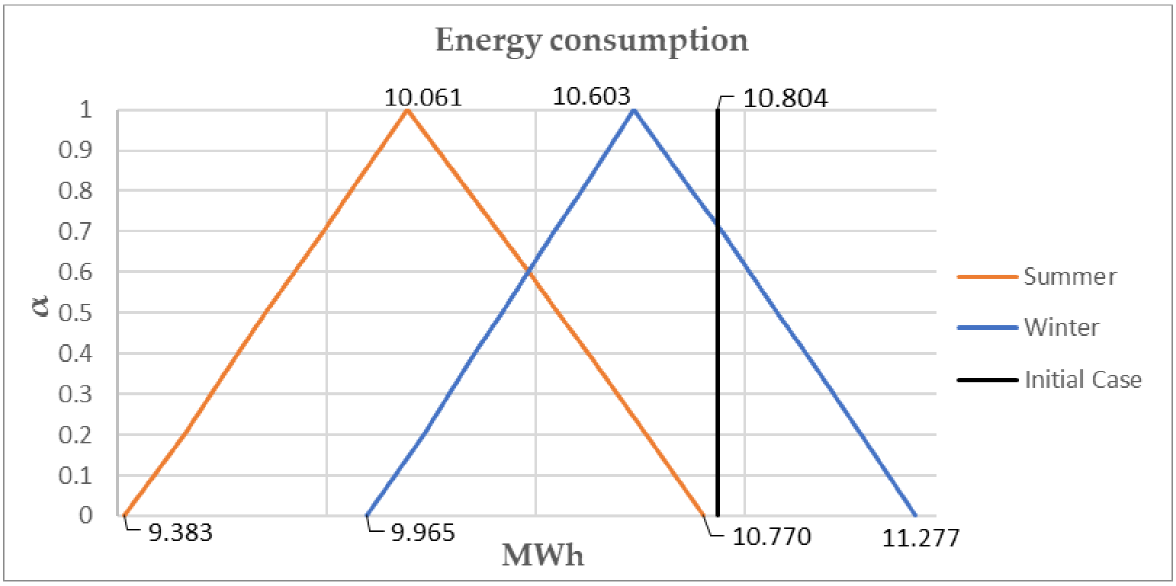

Figure 12. Fuzzy energy consumption applying flat-out driving.

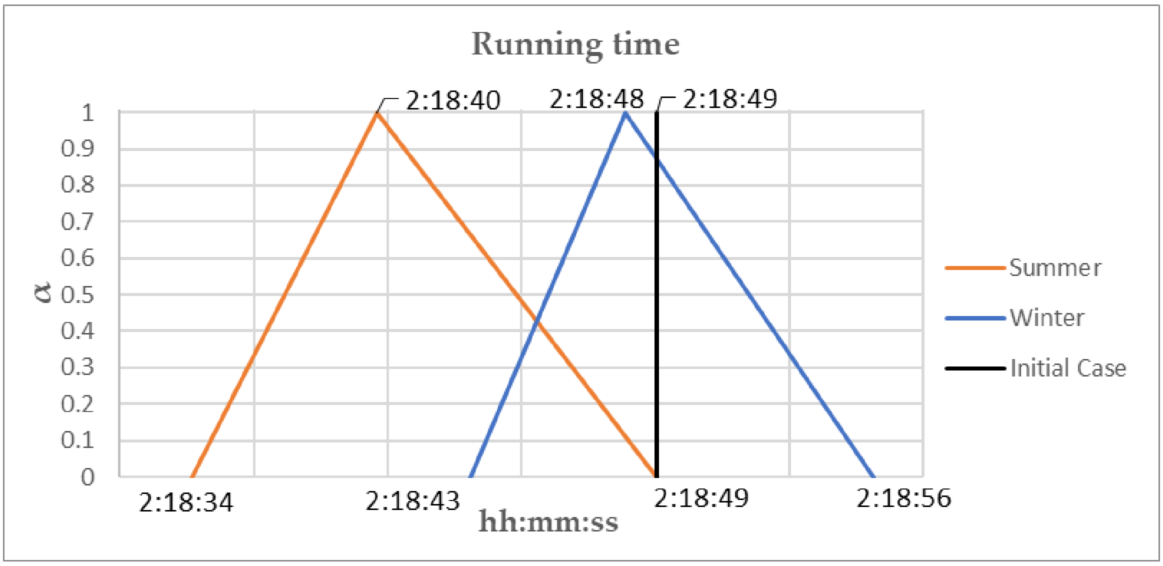

Figure 13. Fuzzy running time applying flat-out driving.

2.5.3.2. ECO-DRIVING DESIGN WITHOUT CLIMATOLOGICAL PARAMETERS

After analysing the impact of weather conditions on the flat-out driving, the driving optimisation model is run to design the eco-driving without taking into account the weather conditions, which is solved by means of a GA. This would be the typical approach to solving the problem and will serve as a basis for comparison with the model proposed in this article. The objective travel time for the journey is 2 h and 45 min.

The result of the optimisation is the Command Matrix to apply to obtain the optimal driving, shown in Table 12.

Table 12. Command Matrix.

Command Matrix		
Initial K.P.	**Final K.P.**	**Speed (km/h)**
0	271.08	244.73
271.08	422.03	230.16
422.03	621	240.17

Figure 14 shows the speed profile for the eco-driving obtained from the optimisation model. The driving commands produce the nominal driving and define the commercial eco-driving designed in a specific railway operation. This driving produces an energy consumption of 7.589 MWh.

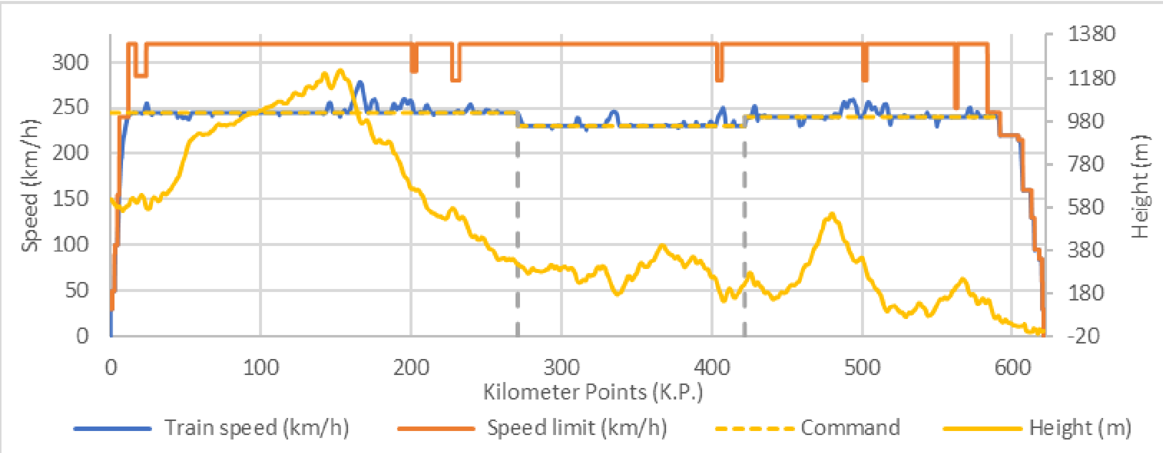

Figure 14. Eco-driving speed profile.

In Table 13, the energy consumption obtained by the eco-driving design and the associated energy saving compared to the flat-out driving can be seen. Thus, following this typical procedure, the energy saving that the railway operator would expect would be 29.76%. The substantial and significant energy saving is achieved due to the slack time included in the timetable (necessary to satisfy punctuality when a delay arises), and due to the eco-driving design to use the slack time optimally to minimise the energy consumption. Other studies show similar energy savings, 31.27% in [74] and 35.54% in [86].

Table 13. Energy savings with no weather conditions.

Energy Consumption	Flat-Out (MWh)	Eco-Driving (MWh)	Saving
Initial Case	10.804	7.589	29.76%

2.5.3.3. ECO-DRIVING WITH FUZZY CLIMATOLOGICAL PARAMETERS APPLYING GA-F

The next step is the execution of the proposed GA-F model to optimise the driving considering the fuzzy climatological parameters (Tables 7–10) with an objective running time of 2:45:00. The punctuality constraint is expressed as a necessity measure $np = 0.5$. The fuzzy optimisation model has been applied for the summer and winter scenarios.

Table 14 collects the results (energy consumption and running time) obtained by simulating for different α-cut with the eco-driving commands obtained with the optimisation model. Specifically, the lower and upper limits of the α-cuts with $\alpha = 0$ and with $\alpha = 0.5$ are shown, as well as the core ($\alpha = 1$).

Table 14. Fuzzy results obtained with the eco-driving design applying GA-F with np = 0.5 in winter and in summer.

Season	**Winter**			**Summer**		
Value	α	**Energy (MWh)**	**Running Time (hh:mm:ss)**	α	**Energy (MWh)**	**Running Time (hh:mm:ss)**
Lower	0	6.989	2:44:06	0	6.550	2:44:13
	0.5	7.221	2:44:25	0.5	6.801	2:44:31
Core	1	7.451	2:44:43	1	7.055	2:44:46
Upper	0.5	7.710	2:45:00	0.5	7.335	2:45:00
	0	7.957	2:45:14	0	7.603	2:45:12

For the fuzzy energy consumption results, the difference between the core values in summer and winter is 5.31%. The variation between the lower limit of α-cut = 0 and the upper limit of α-cut = 0 in winter is 12.16% and 13.85% in summer. On the other hand, the variation related to the running time is 00:01:08 hh:mm:ss for the winter scenario (0.69%) and 0:00:59 for the summer scenario (0.60%).

Figures 15 and 16 are the fuzzy energy consumption and the fuzzy running times obtained by applying the proposed GA-F model with fuzzy climatological parameters. Figure 16 shows that the objective running time is achieved when $\alpha = 0.5$ as the necessity measure for punctuality imposed was np = 0.5 = 1 − α.

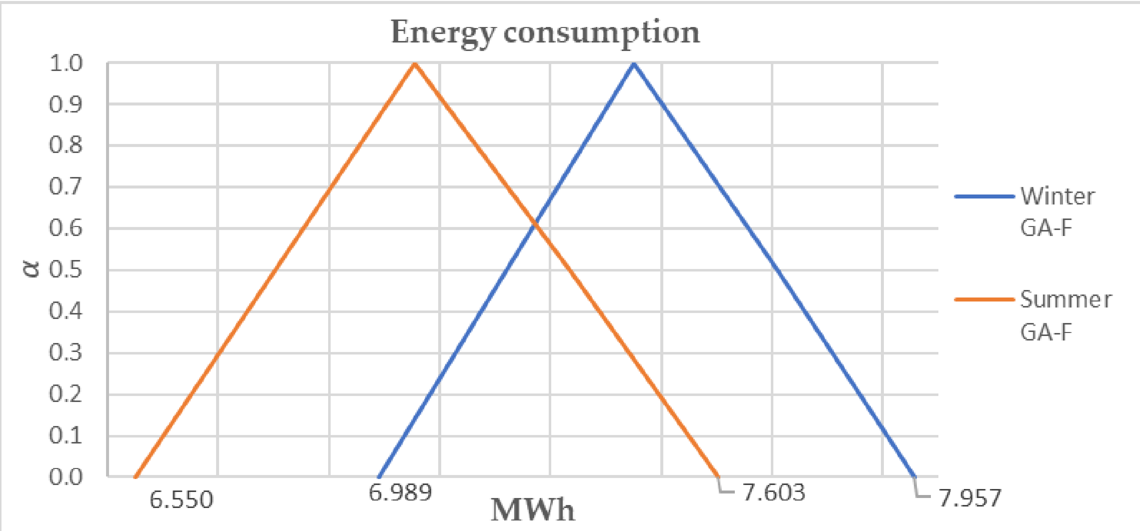

Figure 15. Fuzzy energy consumption applying GA-F with np = 0.5 in winter and in summer.

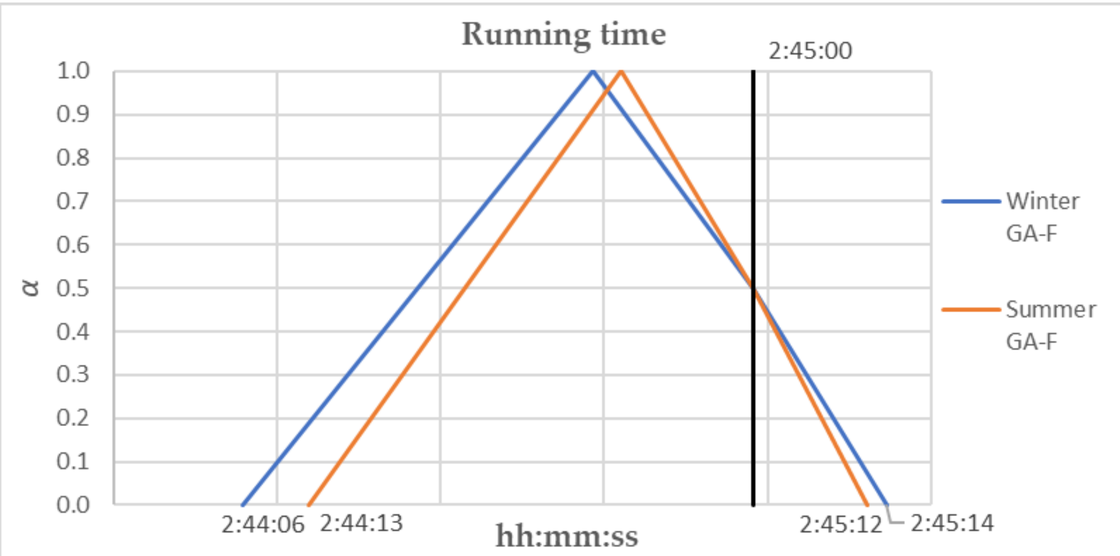

Figure 16. Fuzzy running time applying GA-F with np = 0.5 in winter and in summer.

To conclude, the typical eco-driving design procedure where weather conditions are not considered would provide expected energy savings of 29.76%, as previously indicated. With the application of the proposed GA-F model with fuzzy climatological parameters, the energy consumption of the new eco-driving designs is lower, and thus, the expected energy savings are higher (34.70% in summer and 31.03% in winter). Weather conditions have a significant impact on energy consumption and to a lesser extent, on journey time, and it is worth incorporating this information into the model in order to adapt driving to these pressure, temperature and wind conditions on the journey under analysis.

2.6. CONCLUSIONS AND CONTRIBUTIONS

This chapter proposes a new model to design eco-driving in railway lines taking into account climatological conditions. The climatological parameters affecting the model are the temperature, pressure, the wind intensity and its angle of incidence. The uncertainty associated with these climatological parameters is modelled by fuzzy numbers, and the optimisation problem is solved by means of a GA-F, Genetic Algorithm with fuzzy parameters. The analysis of the energy consumption and running time functions depending on these parameters permits an efficient calculation and resolution of the problem applying α-cut simulations.

The proposed model has been applied to a real Spanish high-speed railway line. First, the flat-out driving has been simulated and the important impact that the climatological parameters have on the energy consumption of the train and to a lesser extent on the running times have been shown. Then, the efficient driving is designed without and with climatological parameters (applying the proposed GA-F). The eco-driving design is based on applying coasting commands and speed regulation without braking commands. In this case study, it has been shown that the expected energy savings obtained in this process are underestimated when eco-driving is designed without considering climatological parameters.

Of course, depending on the specific characteristics of the railway line under study, the climatological conditions could be less or more favourable. However, it can be concluded that weather conditions have a significant impact on energy consumption and to a lesser extent, on journey time, and it is worth incorporating this information into the model in order to adapt driving to the pressure, temperature, and wind conditions on the journey under analysis. The final aim is to maximise energy savings considering as much as possible parameterised data involved in the rail traffic operations.

In conclusion, the results obtained show that the expected energy savings without considering climatological factors is 29.76%, while, if these are considered, the savings can increase up to 34.7% in summer conditions. With the proposed model, a variation in energy consumption of 5.31% is obtained between the summer and winter scenarios (core values), in both cases complying with the punctuality restrictions. On the other hand, although weather conditions have a significant impact on energy consumption, their effect on travel time is practically negligible, with a difference of only 0.03% between the two scenarios (core values). This demonstrates that the model allows the operator to more accurately estimate energy consumption, offering optimised driving adapted to real weather conditions. Future developments will be oriented to the application of the model in real-time to adapt the optimal driving to the current climatological conditions and include other sources of uncertainty related to the traffic conditions.

The main contribution of this chapter is the proposal of an innovative model of efficient driving for High-Speed trains, which incorporates weather conditions through fuzzy logic. Unlike previous studies, parameters such as pressure, temperature, air density and wind are explicitly considered as fuzzy variables, allowing their uncertainty to be represented and their impact on energy consumption and travel time to be analysed. The model combines a genetic algorithm with a detailed simulation of the train's movement to optimise driving profiles in different weather scenarios, ensuring that journey times are met. It has been validated by simulation on a real High-Speed line in Spain, demonstrating relevant improvements in the estimation of energy consumption and providing a practical approach for railway operators seeking more efficient strategies adapted to real conditions.

CHAPTER 3

EFFICIENT MASS TRANSIT OPERATION: ECO-DRIVING, REGENERATIVE ENERGY AND ROLLING STOCK OPTIMISATION

3.1. INTRODUCTION

The benefits of eco-driving implementation in mass transit lines have been demonstrated in several studies under different scenarios. Efficient driving achieved by GA (Genetic Algorithm) and PSO (Particle Swarm Optimization) was validated in the case of the Istanbul metro achieving energy savings with manual operation between 20% and 30% [37]. In [17], the authors consider the regenerated energy and storage devices in the design of ATO driving commands obtaining a result of 11% of energy savings. The ATO speed profiles search with MOPSO in [38] provide energy savings of 15% and it can be combined with robustness conditions to obtain driving commands considering mass variation [51].

The authors in [142] present a speed trajectory optimisation using nature-inspired algorithms for the Istanbul M3 subway line considering a variable number of passengers. The algorithms used are a GA, Simulated Annealing algorithm (SA), PSO and the Marine Predators Algorithm. The solution was tested in the real line achieving energy savings of 21.27%.

The first study related to the calculation of efficient driving applied to railways was carried out in [21] applying Pontryagin's Maximum Principle. Ichikawa considers the analysis as a bounded state variable problem taking into account the speed limits and obtained for the first time the optimal regimes: maximum acceleration, holding speed, coasting and maximum deceleration. Applying the same principle, Howlett in [143] considers energy minimisation to find the optimal driving strategy. Different simulation methods and optimisation techniques are used applying the results of the Maximum Principle to solve different cases proposed in the literature.

Solutions from constructive algorithms are achieved in [73] where coast and maximum power phases are alternated to obtain the optimal driving strategy. Variable speed restrictions, grade profile, traction and brake force and running time are constraints to optimal driving pattern calculation in [82] with the Maximum Principle analysis. In [2] the analytical solution offers the sequence of optimal train control. An optimisation and control of speed and dwell time are given in [48].

Apart from constructive algorithms, different mathematical models have been applied to obtain the optimal solution for efficient driving such as Dynamic Programming (DP) [48], Mixed-Integer programming [65], Sequential Quadratic Programming [80] or Lagrange Multipliers [84]. A mixed-integer non-linear programming problem is solved in [17] minimising passengers travel time and traction energy consumption in urban rail transit systems.

Previous methods obtain the analytical eco-driving solution but, when the model includes details such as automatic driving logic, the nonlinearities introduced require the application of different techniques. In contrast to mainlines, mass transit lines are highly automated exploitations where the trains are automatically driven by ATO equipment. Therefore, it is important to develop simulation models that include ATO characteristics in eco-driving models for mass transit lines and to combine them with optimisation algorithms [144] to obtain applicable solutions.

Artificial Neural Networks (ANN) are used in [87] for optimising coasting speed in Mass Rapid Transit (MRT) systems while in [86] are found the optimal coasting positions. GA have been frequently used for eco-driving as in [74] for a Direct Current (DC) metropolitan railway line, in [50] to find the optimal coasting points and in [79] to calculate the efficient speed. In [43] is used the GA to minimise energy consumption considering the headway, running time and dwell times. A multi-population GA is used in [95] to minimise traction energy taking into account driving strategy and trip time.

Fuzzy logic [116] and GA can be combined to consider uncertainty in any of the parameters that affect railway operation. An example is presented in [145] where eco-driving is calculated and adapted to operational conditions considering variability in climatological conditions for a high-speed railway with a GA. NSGA-II [146] (Non-Dominated Sorting Genetic Algorithm II) is applied in [147] to optimise traction systems while, in [104], is combined with fuzzy logic to calculate ATO driving in metro lines considering variability in the load of passengers. In [148] a dynamic version of NSGA-II was applied to re-calculate the train speed profiles during its journey. In [27] a fuzzy manual driving model and a GA are combined to solve eco-driving. Other alternatives to GA have been proposed in the literature to solve the energy minimisation problem in train driving such as Brute Force in [98], Monte Carlo [149] simulation in [99], Differential Evolution in [100], Genetic Simulated Annealing in [101], Indicator Based Evolutionary Algorithm [150] (IBEA) in [103] and Ant Colony Optimisation (ACO) [151] in [79].

In contrast to previous studies, approaches that combine timetabling and eco-driving problems can be found in the literature to minimise the global traction energy consumption of traffic operation. Thus, the focused is in the complete line instead of minimising the consumption of each interstation separately. The running times between stations and dwell times can be optimised jointly with the eco-driving to spend the time where it is more beneficial reducing the traction energy for operation.

Driving and timetable are optimised with a combined PSO-GA in [41] for energy efficiency purposes. Eco-driving and timetable levels are solved in [152] by DP and the SA algorithm, respectively, adding a substation-based energy consumption model. Timetable is also calculated in [32] by applying ATO efficient driving when the mass transported is considered as a fuzzy variable. Optimal design in [153] focuses on the capacity of the timetable to remain robust when possible disruptions arise during operation.

On the other hand, it is important to take into account not only the traction energy, but also the use of regenerative energy to reduce as much as possible the energy consumption in a railway line. This is even more important in mass transit line where the power system is a DC system where returning energy from the railway grid to the utility grid is difficult [154]. Regenerative braking is a technology that allows the trains equipped with it to produce energy during the braking processes. This energy is, firstly used to feed the auxiliary equipment of the train that produces it and, the rest, can be sent back to the catenary. This way, other trains can use it reducing the energy demanded from substations. The regenerated energy produced by a train can be used by another train that requires traction in DC systems when the trains share the same electrical section. If there is not enough energy demand in the section, the regenerated energy must be wasted in on-board rheostats of the braking train to prevent an undesired increase of the catenary voltage.

Taking into account the previous concepts, there are authors who seek to reduce the total substation demand of the train traffic by synchronising accelerating and braking trains [44] taking into account the power-saving factor between each pair of trains. In this case, an energy saving of 7% is achieved in a Madrid Underground case study just by the increase in regenerated energy use. Timetable is optimised in [155] maximising the overlapping time between the accelerating and braking trains by means of an integer programming model and solving it with a GA.

The application of eco-driving decreases consumption but the use of regenerated energy decreases as well. The higher the speed, the higher the use of regenerated brake energy when trains arrive at the stopping points. So, there is a balance between efficient driving design and the ability to maximise the use of regenerated energy during braking considering also the density of rail traffic [90]. In [156] the driving is designed and timetable is then optimised considering regenerative braking exchange only when trains arrive and depart from stations with no consideration of motors efficiency or transmission losses.

In [157] a resolution model is used to synchronise trains where the overlapping time of accelerating and braking trains (at the start-up and arrival at stations) is maximised and the travel time of a user and of all passengers globally is minimised, taking into account passenger transfers. In order to reuse the regenerated energy, this work takes into account the start-up and arrival of trains at stations without considering possible traction and braking that may occur during driving between stations. This study does not consider the use of regenerated energy but the overlapping time, does not add the losses suffered during energy transmission and does not carry out an eco-driving design included in the model. Furthermore, the timetable is designed in such a way as to be periodic because, although flexibility is lost in adjusting starts and braking to maximise the use of regenerated energy, comfort and the perception of good service on the line are gained.

In the study carried out in [52] the timetabling and coordination of trains during starting and braking, controlling dwell times, are proposed to take advantage of regenerated energy on the Beijing Metro Yizhuang Line. The results obtained show annual energy savings of 6.97% compared to the current planning. In this article, the speed profiles used are the real ones of the trains on the line analysed and the transmission loss coefficient is considered as constant.

The analysis in [53] aims to maximise the use of regenerated energy while at the same time shortening passenger waiting time. It is formulated a two-objective integer programming model with headway time and dwell time control and then, it is designed a genetic algorithm to find the optimal solution. Transmission loss data are estimated using historical data.

An integrated method to maximise regenerative energy is presented in [158] by optimizing train speed and timetables. Additionally, it minimises simultaneous acceleration or braking within the same electric zone. Simulations of Istanbul's M3 subway using genetic and simulated annealing algorithms show traction energy savings.

In [55] the design of the timetable is carried out considering the synchronisation between trains as in the previous work but now minimising power peaks. Possible delays and how they affect the timetable are analysed by means of the Monte Carlo method.

In [54] a two-step approach to timetable design is proposed. The first phase aims to reduce passenger waiting time and the second phase to reduce total consumption. The speed profile of the trains is divided into six zones for traction, coasting and braking. Regenerated energy is only taken into account during the overlap between acceleration and braking. In [46] a Chongqing metro line is studied for which the objective is to minimise the total energy consumed by taking into account the energy regenerated by accelerating and braking trains in the same power supply section. They use a matrix resolution model for searching the global solution although with a higher computational cost by taking the speed profiles of the trains. A bi-objective algorithm is proposed in [58] to obtain efficient timetables where the consumption and passenger travel time are considered using real historical data of a railway network taking simplifications.

In the literature it is found real-time timetable optimisation by means of train control as in [159]. The synchronisation of the trains is taken into account to take advantage of the regenerated energy when the whole DC power supply system is connected in all the sections although the scenario where sections are disconnected is also considered. The response of the timetable is analysed against delays and the speed profiles to be executed by the trains are adjusted by means of the strategy presented in [160].

In [57] it is proposed a timetable optimisation and train control method to minimise substation energy consumption on a metro line with a DC electrical system equipped with reversible substations. In this case the limitations of regenerative energy exchange are relaxed because reversible substations allow the flow of regenerative energy from catenary to the utility grid. The train speed profile is included along with the timetable optimisation.

In the literature, different studies on energy efficient train timetabling can be found, but has rarely been taken into account train circulation plan, that is, the use of rolling stock (trains) required to provide the rail services defined in the timetable. However, the timetable design has an important impact on the use of the available rolling stock. When the timetable is optimised to save energy, it may be very costly to match the train circulation plan to the timetable due to an inadequate connection between train services [45]. Authors in [56] suggested that the train circulation plan should be taken into account in the optimisation of the train schedule, but they did not focus on the energy-efficient timetabling problem.

In particular, depending on how the timetable is designed, the number of trains (rolling stock) needed to provide the service may be different, which has a significant impact on investment and operation.

In [45] a synchronisation between trains is carried out for timetabling taking into account different electrical sections. The authors consider the possible synchronisation of energy regenerated/consumed along the interstation, and not only at arrival / departure. They optimize the timetable considering the energy saving and the train circulation plan for an urban rail line, by means of a mono-objective PSO optimisation algorithm. However, they do not design the eco-driving at each interstation taking into account the characteristics of the ATO equipment, the number of trains associated to the timetable is not calculated, and some simplifications are assumed in the train model.

The main objective and contribution of the present work is a multi-objective method based on detailed simulation to design the optimal ATO driving commands of a metro line and, as a result, define the traffic operation that minimises the energy consumption considering the rolling stock required to provide the service. This is achieved by integrating the optimisation of the speed profiles and the optimisation of the timetable adding the contribution of the energy regenerated by braking trains during all the train journey. The number of trains required to provide the service associated to the timetable are also considered to quantify costs related to rolling stock. In particular, it can be highlighted the following contributions:

- An integrated multi-objective approach to design the ATO driving commands and timetable of a metro line including: the minimisation of energy consumption, the maximisation of regenerative energy usage and the minimisation of the rolling stock required to operate. This method can assist train operators to decide the timetable taking into account the trade-offs between commercial speed and the costs of energy and rolling stock.
- The integrated optimisation model is a two-level optimisation procedure where the local eco-driving level finds the set of efficient speed profiles at each interstation and the global timetable level finds the Pareto front of timetables. The local level is solved using a MOPSO algorithm which has demonstrated its performance in literature. The global timetable level is solved comparing four different algorithms, proving that GLMO-NSGA-III is the most effective for this optimisation model.
- The model includes constraints associated to realistic ATO equipment at the local level and constraints associated to the reverse manoeuvre in terminal stations in order to avoid solutions that cannot be applied in real installations or that can require a greater number of trains.
- A Madrid Underground case study is carried out to prove the benefits of the optimisation model proposed. The results illustrate that the

integrated optimisation achieves energy savings of 24.8% compared to the typical criteria used by operators to design the timetable.

References and previous work related to eco-driving and timetable design are listed in Table 15. As a summary, many studies in the literature ([52], [56], [155], [157]) compared with the proposed model do not include an integrated optimisation method with timetable and eco-driving. Literature models do not take into account constraints associated to ATO driving logic as the model proposed except for [32] and [46]. However, in reference [32] the regenerated energy is not considered and in [32] and [46] the rolling stock is not included as a goal. Regarding the regenerated energy, in [32], [41], [56], [142], [152], [161] and the regenerated energy is not modelled. In ([52], [53], [54], [90], [155], [156], [157] and [158]) the use of regenerated energy is improved indirectly by maximizing the coincidences of trains' departures and arrivals. In [45], [46], [55], [57] and [159] and a similar approach to the model proposed in this chapter is used to calculate the total regenerated energy transferred among trains. However, except for [45] the rolling stock is not taken into account. The cost associated to the rolling stock is only included in the model in [45] and [56]. In both cases, the approach is to include the rolling stock cost as the total running time and the connection time. In the present chapter, the approach is to directly calculate the number of trains needed to provide the service with the designed timetable and include it as a penalty in the objective function.

This chapter is organised as follows: Section 3.2 describes the design objective for the optimal efficient operation of a metro line. In Section 3.3 the timetabling model is presented. Section 3.4 defines the eco-driving ATO design model for each interstation and the train simulator it uses to evaluate solutions. In Section 3.5 the method is applied to a case study of Madrid Underground. Section 3.6 presents the conclusions and a discussion of the proposed method.

Table 15. Literature review.

Reference	Eco-driving consideration	ATO consideration	Regenerated energy consideration	Rolling stock consideration
[23]	Yes	No	No	No
[42]	Yes	No	Fully considered	No
[52]	No	No	Traction/Braking synchronisation	No
[53]	Yes	No	Traction/Braking synchronisation	No
[54]	Yes	No	Traction/Braking synchronisation	No
[55]	Yes	No	Fully considered	No
[57]	Yes	No	Fully considered	No
[58]	Yes	No	No	No
[142]	Yes	No	No	No
[152]	Yes	No	No	No
[155]	No	No	Traction/Braking synchronisation	No
[156]	Yes	No	Traction/Braking synchronisation	No
[157]	No	No	Traction/Braking synchronisation	No
[158]	Yes	No	Traction/Braking synchronisation	No
[162]	Yes	Yes	No	No
[163]	Yes	No	Traction/Braking synchronisation	No
[164]	Yes	Yes	Fully considered	No
[165]	Yes	No	Fully considered	Yes
[166]	No	No	No	Yes

3.2. DESIGN OF THE OPTIMAL ATO OPERATION OF A RAILWAY LINE

The model presented in this chapter considers a typical topology of a mass transit railway line, a two-track line between terminal stations. The line is composed of N_S stations and equipped with ATO systems. All the electrical sections are supposed to be connected although regeneration will be more efficiently used by trains running close to the regenerating one, as electrical losses are modelled. Energy regenerated by a train running upwards can be consumed by a nearby train running in the same direction or in downwards direction.

Sections 3.2.1, 3.2.2, and 3.2.3 list the parameters, intermediate variables and functions and decision variables used in the chapter in Table 13, Table 14 and Table 15, respectively.

3.2.1. PARAMETERS

Table 16. Parameters.

$\boldsymbol{h}$	Headway (s)
$\boldsymbol{N_s}$	Number of stations
$\boldsymbol{t_{min}}$	Minimum dwell time (s)
$\boldsymbol{P}$	Penalty for the use of an additional train (kW)
$\boldsymbol{\sigma}$	Transmission loss model dependent on train position
$\boldsymbol{m_0}$	Empty train mass (tons)
$\boldsymbol{m_l}$	Load mass (tons)
$\boldsymbol{m_{eq}}$	Equivalent mass of the train
$\boldsymbol{F_{max}}$	Maximum traction effort according to the motor characteristics
$\boldsymbol{j_{max}}$	Maximum variation of traction/braking effort
$\boldsymbol{A}$	Constant related to the mechanical resistance to the motion
$\boldsymbol{B}$	Constant related to the resistance due to the air inlet in the train
$\boldsymbol{C}$	Constant related to the aerodynamic resistance
$\boldsymbol{g}$	Gravitational acceleration (m/s^2)
$\boldsymbol{P_{track}}$	Equivalent gradient of the track in the train position
$\boldsymbol{I_{max}}$	Maximum current
$\boldsymbol{U_{cat}}$	Nominal catenary voltage
$\boldsymbol{\eta}$	Motor efficiency (%)

3.2.2. INTERMEDIATE VARIABLES

Table 17. Intermediate variables.

$RT(x_g, t_g)$	Global running time of the timetable (s)
$C(x_g, t_g)$	Cost function (kWh)
N_t	Number of trains
t_1	Running time on track 1 (s)
t_2	Running time on track 2 (s)
T_1	Time difference between the arrival and departure of consecutive services in station 1 (s)
$f_1(x_n)$	Running time evaluation function for interstation $\boldsymbol{n}$ (s)
$f_2(x_n)$	Energy consumption evaluation function for interstation $\boldsymbol{n}$ (kWh)
EDC_n	Set of the efficient ATO driving commands of interstation $\boldsymbol{n}$
s_i	Position of train *i* (m)
σ_{ji}	Value of transmission losses for the regenerating train *j* and the accelerating train *i* (kWh)
n_c	Number of trains consuming energy
n_r	Number of trains regenerating energy
E_i	Energy consumption used for traction by train *i* (kWh)
R_j	Energy regenerated by train *j* (kWh)
R_{ji}	Energy regenerated by train *j* and transmitted to train *i* (kWh)
t	Time (s)
a	Acceleration (m/s^2)
F_g	Force due to the railway grades (kN)
F_c	Force due to track curvature (kN)
F_r	Running resistance (kN)
a_{ref}	Traction/braking demand for the train (m/s^2)
$K_{traction}$	Constant of the proportional controller for the traction mode (s/m)
K_{acc}	Constant for the pre-fed acceleration (s^2/m)
v_{target}	Minimum value between the maximum speed at the position of the train and the driving command *hs* (holding speed) (m/s)
v_{train}	Speed of the train (m/s)
$a_{gradient}$	Acceleration due to the gradients at the position of the train (m/s^2)
$K_{braking}$	Constant of the proportional controller for the braking mode (s/m)
F_m	Motor force (kN)

$\boldsymbol{P_{trac}}$	Traction electric power (kW)
$\boldsymbol{P_{reg}}$	Regenerative electric power (kW)

3.2.3. DECISION VARIABLES

Table 18. Decision variables.

$\boldsymbol{x_g}$	ATO driving commands for all the interstations of the entire metro line
$\boldsymbol{t_g}$	Dwell time at every station in the railway line (s)
$\boldsymbol{x_n}$	Vector with the ATO driving parameters for the interstation n (component of $\boldsymbol{x_g}$)
$\boldsymbol{t_n}$	Dwell time at station $\boldsymbol{n}$ (component of $\boldsymbol{t_g}$) (s)
$\boldsymbol{x_{Ns}}$	ATO driving commands for interstation *Ns*
$\boldsymbol{t_{Ns}}$	Dwell time at station *Ns* (s)
$\boldsymbol{c}$	Speed at which coasting is applied (component of $\boldsymbol{x_n}$) (m/s)
$\boldsymbol{r}$	Speed at which traction is applied (component of $\boldsymbol{x_n}$) (m/s)
$\boldsymbol{hs}$	Holding speed (component of $\boldsymbol{x_n}$) (m/s)
$\boldsymbol{b}$	Braking rate (component of $\boldsymbol{x_n}$) (m/s^2)

3.2.4. OPTIMAL ATO PROBLEM

The problem to solve is the design of the optimal ATO driving commands of a metropolitan railway line for a specific operational period. With this objective, the model designs jointly the efficient driving at each interstation and a periodic efficient timetable for the operational period.

The model includes a penalty associated to the number of trains (rolling stock) required to provide the rail services of the timetable. This way, timetables with a lower number of trains required will be preferred.

A multi-objective model is defined to propose the most efficient solutions for different total travel times of the line. As the travel time and energy consumption are conflicting objectives, the result will be the set of non-dominated solutions; i.e., those that cannot be improved in energy consumption and travel time simultaneously. This way, the operator can select the commercial running time of the line (i.e., the travel time to run the line completely) in view of the trade-off with the energy consumption and considering the number of trains required. The travel time is the sum of the running time of the speed profiles produced by the designed ATO driving commands for each interstation stretch, and the selected dwell time at each station. To optimise the energy consumption of the line, for each total travel time the proposed model takes into account the set of optimal ATO driving commands in each interstation (that produces eco-driving speed profiles), the distribution of time margins along the route to make it efficient and also the energy transferred between trains in the braking processes which, depending on the design of the timetables of all trains, makes it possible to reduce the overall energy consumption. In this study, a two-level resolution model is proposed (Figure 17). First, at the eco-driving level, a local Pareto front of driving commands is obtained for each interstation, providing for each running time the eco-driving (ATO driving with the minimum energy consumption).

Then, at the timetable level, a global Pareto front of optimal timetable solutions (i.e., a driving command for each interstation and a dwell time for each station) is obtained taking into account the exchange of regenerative energy among trains on the entire metro line in order to maximise its use, as well as the time distribution along the line to minimise the total energy consumption of the system.

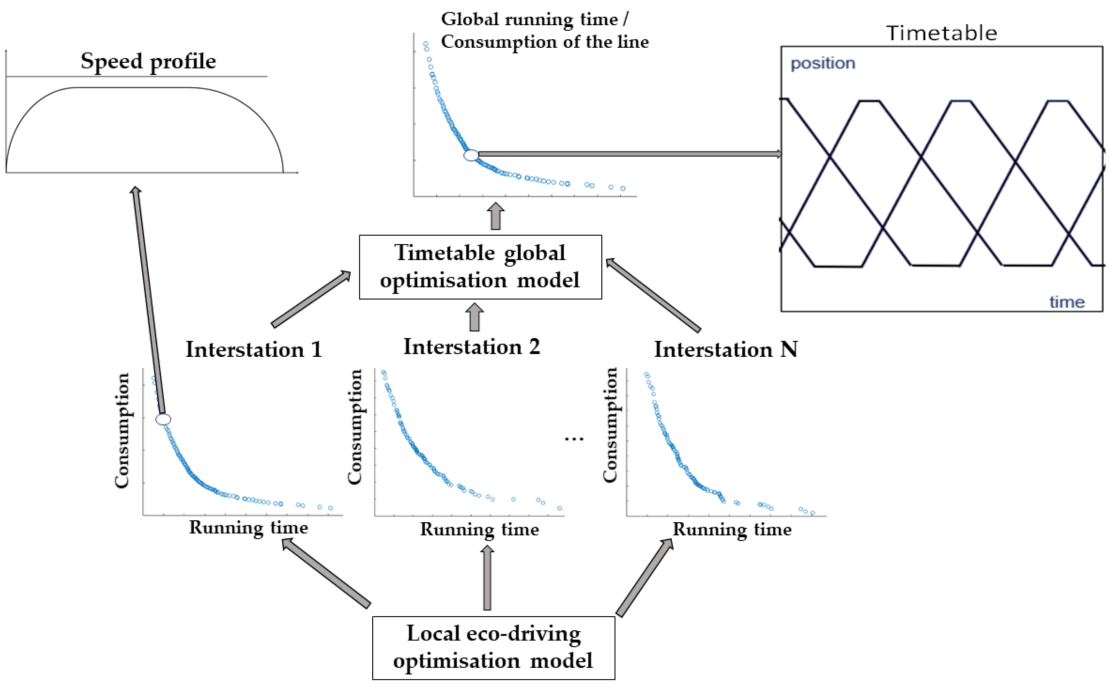

Figure 17. Proposed model for the efficient operation of a metro line.

3.3. TIMETABLE DESIGN LEVEL

This section presents the optimisation of the timetable at the global level. The inputs received at this level are the eco-driving profiles designed by the Local eco-driving multi-objective optimisation model (Section 4). A series of preconditions have been established to design the timetable for a specific operational period. A periodic timetable has been considered with all trains performing the same running time for the same route and the same stopping times for the same stations considering that the interval between trains is constant. Therefore, by calculating the timetable for each train in each period of time, the entire timetable is calculated.

When the timetable design problem is solved, a new global Pareto front is obtained where each solution has associated the total energy consumption and running time of the cycle for the entire line, considering that the trains are using the efficient driving calculated in the eco-driving level. Thus, each solution represents a specific set of efficient ATO driving parameters at each interstation (which provide the running times at interstations) and the dwell times at the stations of the line. For each solution, the total operating cycle time of the entire line is thus defined. In this way, the optimal timetable is established by defining the time of arrival and departure at stations and thus, the associated stopping times.

3.3.1. GLOBAL OPTIMISATION MODEL

At the global level, the objective is to maximise the use of energy regenerated by trains during braking avoiding waste through rheostats and to distribute the time margins by reducing the running time at the interstations where greater traction energy reduction can be obtained. The energy regenerated by a train is firstly used for auxiliary systems. The remaining regenerated energy is transmitted to other trains that are accelerating nearby the train or transmitted to rheostats if there is no demand to avoid voltage surges in the catenary. To design the efficient timetable, the input parameters are the headway and the set of efficient eco-driving parameters obtained for each interstation at local level. The proposed optimisation model will modify the arrival / departures in order to synchronize trains improving the use of regenerated energy in braking. In this way, a braking train transmits as much energy as possible to nearby accelerating trains.

In the proposed model, the consumption profile on the route of each interstation is used to synchronise not only the braking on arrival at the station but also at any other moment that may occur, such as, for example due to speed limits along the route. In addition, every timetable on a railway line has an associated number of trains required to provide that service. The model proposed imposes a penalty for timetables with similar running time and energy consumption that require an extra train to provide the service.

Thus, the objective function of the multi-objective optimisation problem of designing the line operation that can be expressed as:

$$min\, f_g(x_g, t_g) = \left(RT(x_g, t_g)\, , C(x_g, t_g)\right) \tag{40}$$

Where, RT is the total running time (s) for the mass transit line, C is the cost function that includes the total energy consumption (kWh) and the penalty in the number of trains (kWh), x_g is the ATO driving commands for all the interstations of the entire metro line with $x_g = (x_1, x_2, \dots, x_{Ns})$ being Ns the number of stations along the metro line and t_g are the values of dwell time (s) at every station in the line with $t_g = (t_1, t_2, \dots, t_{Ns})$.

The driving commands contained in x_g will be restricted by the following condition:

$$x_n \in EDC_n \tag{41}$$

where x_n is the component n of x_g (i.e., the ATO driving command of the interstation n) and EDC_n is the set of the efficient ATO driving commands of interstation n (i.e., the Pareto front obtained in the local eco-driving level for interstation n).

Moreover, the dwell times will be restricted by a minimum value that ensures that the passengers have enough time to get on board the train:

$$t_n \geq t_{min} \tag{42}$$

where t_n es the dwell time (s) at station n, and t_{min} is the minimum dwell time (s). The global running time of the timetable model $RT(x_g, t_g)$, presented in Equation 43, is calculated as the sum of the running time and the stopping time at each station n:

$$RT(x_g, t_g) = \sum_{n=1}^{Ns} (f_1(x_n) + t_n) \tag{43}$$

where $f_1(x_n)$ is the running time (s) of interstation n.

The cost function $C(x_g, t_g)$, shown in Equation 44, is calculated knowing at each moment t the consumption of the traction trains, the energy returned by braking trains and the distance between trains to take into account the electrical losses in the energy transmission between regenerating trains and accelerating trains. In addition, this function includes a penalty for each extra train required in the solution to satisfy the timetable:

$$C(x_g, t_g) = \sum_{t=0}^{h} \left(\sum_{i=1}^{n_c} E_i - \sum_{i=1}^{n_c} \sum_{\substack{j=1 \\ j \neq i}}^{n_r} R_{ji} \cdot \sigma_{ji} \right) + P \cdot N_t \tag{44}$$

where j is the train that transmits regenerated energy, i is the train that receives regenerated energy, n_c is the number of trains consuming energy, n_r is the number of trains regenerating energy, h is the headway (s), E_i is the energy consumption (kWh) used for traction by train i, R_{ji} is the energy regenerated (kWh) by train j and transmitted to train i at a specific instant, σ models the electrical losses and represents the percentage of regenerated energy that is transmitted from a braking train and utilised by a traction train. This function is dependent on the distance (m) between the regenerating train j and the train i receiving the energy as shown in Equation 45. An example is shown in Figure 18 although different shapes can be obtained depending on the characteristics of the specific railway line. P is the penalty (kWh) whereby the solution with the lowest number of trains satisfying the timetable is prioritised and N_t is the number of trains required to fulfil the timetable.

$$\sigma_{ji} = \sigma \cdot (|s_i - s_j|) \tag{45}$$

where s_i and s_j are the kilometric position (m) of train i and j respectively.

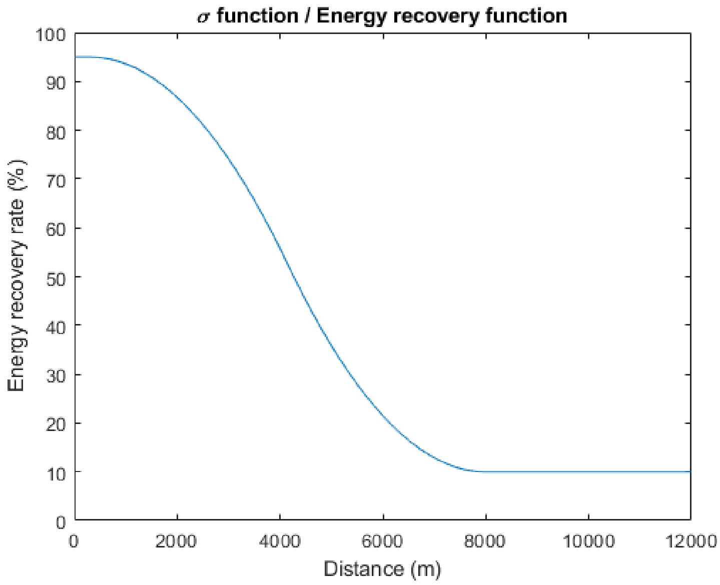

Figure 18. σ function.

Cost function C (Equation 44) is calculated as the sum of consumption at each instant of time t, over an interval equal to the headway. It is sufficient to calculate it in a headway since, as the timetable is periodic, it is repeated every headway h.

In Equation 44, the value of the energy consumption (E) of a train cannot be a negative value. A negative value would mean that the train would be braking, regenerating and transmitting energy, so instead, this value would be included in R. Similarly, the energy regeneration R of a train cannot be a negative value (in that case, the train would be consumed and should be modelled by means of E).

In addition, there are a series of restrictions to complete the model for the use of regenerated energy. The first expression (Equation 46) indicates that the reuse of the regenerated energy is maximised offering the greatest possible amount of energy to the nearest trains with the highest utilisation percentage (Figure 18):

$$max \sum_{i=1}^{n_c} R_{ji} \cdot \sigma_{ji} \tag{46}$$

The energy regenerated will always be higher than the energy received by the other trains for reuse due to the losses introduced in the calculation:

$$R_j > \sum_{i=1}^{n_c} R_{ji} \tag{47}$$

The total regenerated energy received by trains cannot exceed the energy required for their operation so energy is only transmitted to trains that are accelerating:

$$\sum_{i=1}^{n_c} \sigma_{ji} \cdot R_{ji} \leq E_i \tag{48}$$

In addition, in the cost function (Equation 44), a penalty is introduced associated to the number of trains (rolling stock) required to provide the service of the designed timetable. In this way, the optimal solution can be obtained with the lowest number of associated trains while reducing related costs.

The calculation of the number of trains is solved by means of Equations 49 and 50. T_1 is the result of the timetable optimisation and represents the time difference between the arrival and departure of consecutive services in terminal station 1. If this time is equal to or greater than minimum dwell time, t_{min}, that means the same rolling stock can perform both services. In this case, the number of trains is calculated by Equation 49, by considering the dwell time at terminal station 2 equal to the minimum and rounding the result to the greater closest integer value. That means that the dwell time at terminal station 2 is the needed value to obtain an integer value of trains subject to the minimum stopping time:

$$If\ T_1 \geq\ t_{min}\ \rightarrow N_t = ceil\left(\frac{t_1 + t_2 + T_1 + t_{min}}{h}\right) \tag{49}$$

If this time is lower than minimum dwell time (t_{min}), that means the same rolling stock cannot perform both services. That means that when a train arrives to terminal station 1, there is other train ready to start the service in the opposite direction. Therefore, the train that arrives to terminal station 1 has to wait T_1 plus an interval h to start the service in the opposite direction (Figure 19). The impact of this extra-time is that an additional train is needed to provide the service. In this case, the number of trains is calculated by Equation 50:

$$If\ T_1 < t_{min} \rightarrow N_t = ceil\left(\frac{t_1 + t_2 + T_1 + h + t_{min}}{h}\right) \qquad (50)$$

where T_1 is the dwell time (s) calculated in terminal station 1, t_{min} is the minimum dwell time (s) in terminal station, t_1 is the running time (s) on track 1 and t_2 is the running time (s) on track 2.

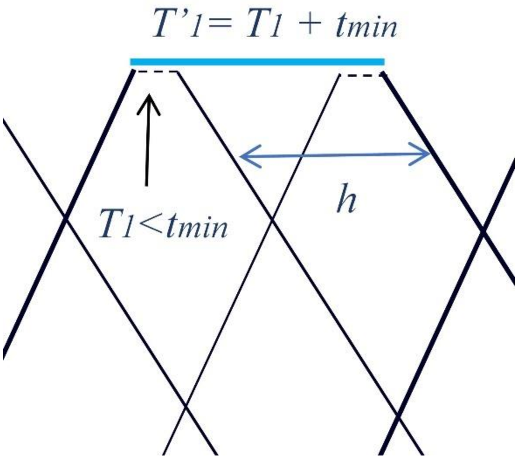

Figure 19. Dwell time at Terminal station 1.

The number of trains necessary for the operation is calculated as the complete cycle running time associated with a specific timetable divided by the headway between trains. For a given timetable it is analysed whether the time at the terminal station is greater than the minimum time necessary for the manoeuvre (or if more margin time is desired for delay recovery). Otherwise, if the result is less than the minimum time, it means that the train arriving at that terminal station must connect with the next timetable which starts a time equals to one headway later. As a consequence, in this case an extra train is needed to provide the service.

A Non-Sorted Genetic Algorithm III [167] that uses the Grouped Linked Polynomial Mutation Operator [60] (GLMO-NSGA-III) algorithm is proposed in this work to solve the multi-objective optimisation problem defined in the timetable level due to its performance in terms of diversity and optimality of the solutions obtained in complex problems.

3.3.2. GLMO-NSGA-III ALGORITHM

The parameters related to the GLMO-NSGA-III algorithm are presented in Table 19.

Table 19. GLMO-NSGA-III parameters.

$\vec{s}$	Set of solutions
s_i	Solution i of $\vec{s}$
μ	Distribution Index
$\vec{m}$	Mutated Solution
m_i	Mutated solution $\boldsymbol{i}$ of $\vec{m}$
$\vec{r}$	Variables that are to be mutated chosen based on variable grouping
n_g	Group index
r_{n_g}	Variables with group index n_g
j	Random group index
h	Random value between (0, 1)
r_j	Group r with index j
$s_{i,min}$	Minimum value of $\vec{s}$
$s_{i,max}$	Maximum value of $\vec{s}$
$\tau_1, \tau_q, \tau_{:}$	Intermediate variable calculation

GLMO-NSGA-III algorithm is the proposed method for solving the efficient timetable calculation model. The Grouped Linked Polynomial Mutation Operator (GLMO) [60] is based on the following mutation techniques: Linked Polynomial Mutation and Grouped Polynomial Mutation. These two procedures are based on Polynomial Mutation and are explained below.

- Polynomial Mutation: This mutation operator is designed to modify the value of a variable based on a distribution around the value at that time. Two parameters are used to perform the mutation: the mutation probability and the distribution index. The probability is used to decide for each variable whether or not it undergoes mutation. The index determines the distribution from which the new value will be calculated, the probability of the new value being more similar to the current value when this index takes large values. When deciding whether a variable will mutate, another parameter h is taken from a uniform distribution indicating how the mutated variable will change. If the result of the mutation exceeds the limits of the distribution around which the original value is modified, this new value is placed at the limit of the distribution.

- Linked Polynomial Mutation: This operator follows a similar idea to the Polynomial Mutation but the parameter u remains constant for all variables. The Linked Polynomial Mutation is a version of the Polynomial Mutation in which a connection is established between variables that are susceptible to change. The procedure by which the connection is established is by keeping the relative amount of change of a variable fixed for each mutation and the same for each solution. With this change, all variables that undergo mutation considering a specific value of probability are mutated by the same amount.

- Grouped Polynomial Mutation: In this modification the variables are separated into different groups and the mutation is applied to these groups, varying each variable in the same group equally. The decision of which variables mutate changes from a randomised procedure to a procedure designed for that purpose assuming that the clustering method makes the appropriate decision of which variables interact and should be modified simultaneously. In this case the mutation probability is not necessary. In this operator, the variables are arbitrarily divided into groups and the variables in any of these groups will be susceptible to polynomial mutation. For each of the variables in the group, a value h is taken defining the mutation. The variables of the rest of the groups do not change.

- Grouped Linked Polynomial Mutation Operator (GLMO): This case is a combination of the two previous ones. The distribution parameter together with the grouping mechanism are the input information for this procedure. The selection of the group to mutate is done in a similar way to the Grouped Polynomial Mutation operator. In this case the value of the parameter h is previously selected as in the Linked Polynomial Mutation. In comparison to the Linked Polynomial Mutation, it is now assumed that the interacting variables are modified simultaneously and equally when they are in the same group. GLMO operator is used within the NSGA-III optimisation algorithm [167] to improve the adaptability, the diversity and the convergence of the algorithm.

GLMO pseudocode is as follows:

Input: Solution $\vec{s}$, the Grouping Mechanism and Distribution Index μ
Output: Mutated Solution $\vec{m}$

$\left\{r_1, \dots, r_{n_g}\right\} \leftarrow Apply\ Grouping\ Mechanism\ to\ \vec{r}\ producing\ n_g\ groups$
$j \leftarrow Pick\ a\ group\ index\ at\ random\ from\ \{1, \dots, n_g\}$
$h \leftarrow takes\ random\ values\ between\ (0,1)$
$loop\ s_i\ values\ belonging\ to\ r_j\ do$
$if\ h\ \leq 0.5\ then$
$\quad \tau_1 = \frac{s_i - s_{i,min}}{s_{i,max} - s_{i,min}}$
$\quad \tau_q = (2 \cdot h + (1 - 2 \cdot h) \cdot (1 - \tau_1)^{\mu+1})^{\frac{1}{\mu+1}} - 1$
$else$
$\quad \tau_2 = \frac{s_{i,max} - s_i}{s_{i,max} - s_{i,min}}$
$\quad \tau_q = 1 - (2 \cdot (1 - h) + 2 \cdot (h - 0.5) \cdot (1 - \tau_2)^{\mu+1})^{\frac{1}{\mu+1}}$
$end\ if$
$\quad m_i = s_i + \tau_q \cdot (s_{i,max} - s_{i,min})$
$\quad repair(m_i)$
$end\ loop$
$loop\ s_i\ values\ not\ belonging\ to\ r_j\ do$
$\quad m_i = s_i$
$end\ loop$
$return\ \vec{m}$

Finally, NSGA-III optimisation algorithm flowchart is shown in Figure 20. The NSGA-III is an evolutionary algorithm where individuals evolve during iterations at the end of which the best fitted survives. The initial step of the algorithm is the randomly generation of the parent population. The global travel time and cost of the members of parent population are evaluated using Equations 43 and 44. Then, offspring population is generated applying the GLMO operator to individuals in the parent population. This offspring population is evaluated and a result population is generated joining parent and offspring population. The dominance of the solutions in the result population is calculated. As a result, the selection criterion chooses the members of the result population that will survive depending on the domination level. Thus, firstly the group of non-dominated solutions are saved. After that, the group of solutions that are dominated just by one solution is saved and the process is repeated with the following groups of solutions. If a group cannot be saved completely because the size of the population is achieved, the crowding distance operator is applied to select the solutions of the last group that will survive.

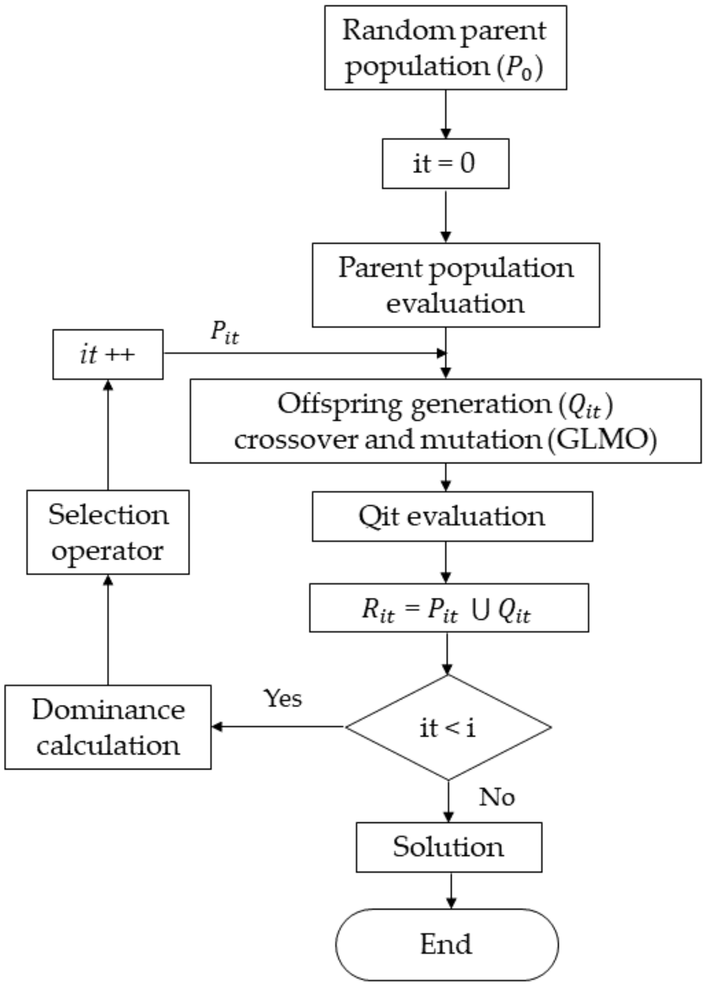

Figure 20. NSGA-III flowchart.

3.4. LOCAL ECO-DRIVING MULTI-OBJECTIVE OPTIMISATION MODEL

As previously described, the local optimisation model provides a Pareto front of solutions for each interstation. Each solution of a Pareto front represents the set of optimal ATO driving parameters for the interstation with the running time and energy consumption associated.

At this level of model resolution, which is the eco-driving level executed for each interstation of the line, the search of optimal solutions is modelled as a multi-objective problem where the aim is to minimise running time and energy consumption at the interstation. The evaluation function for interstation n is:

$$min\, f(x_n) = (f_1(x_n)\,, f_2(x_n)) \tag{51}$$

where $f_1(x_n)$ is the objective function that defines the running time (s) between stations when a specific set of ATO driving parameters is executed, $f_2(x_n)$ is the objective function that defines the energy consumption (kWh) associated to each driving and x_n is the vector that contains the ATO driving parameters for the interstation n. The driving parameters are defined following the driving model presented in [168].

$$x_n = (c, r, hs, b) \tag{52}$$

where c is the speed at which coasting is applied (km/h), r is the speed at which traction is applied (km/h) (re-motor), hs is the holding speed (km/h) and b is the braking rate (m/s^2).

Figure 21 shows an example of ATO speed profile where the eco-driving strategy is the regulation speed. In this case, the main driving parameter is hs and the coasting speed and re-motoring speed are equal to 0. When a regulation speed is greater than 0, the train aims to maintain the speed defined by c. On the other hand, Figure 22 shows a coasting/re-motoring case. Coasting/re-motoring strategies are characterized by two main parameters c and r while the holding speed hs is 0. In this case, the train tractions until its speed achieves the value of c and, then, it coasts until it reduces its speed r km/h that it is the moment when the train tractions again and the cycle is repeated. In both cases (regulation and coasting/re-motoring) the speed limitations are observed and the deceleration up to the stop in the station is defined by b.

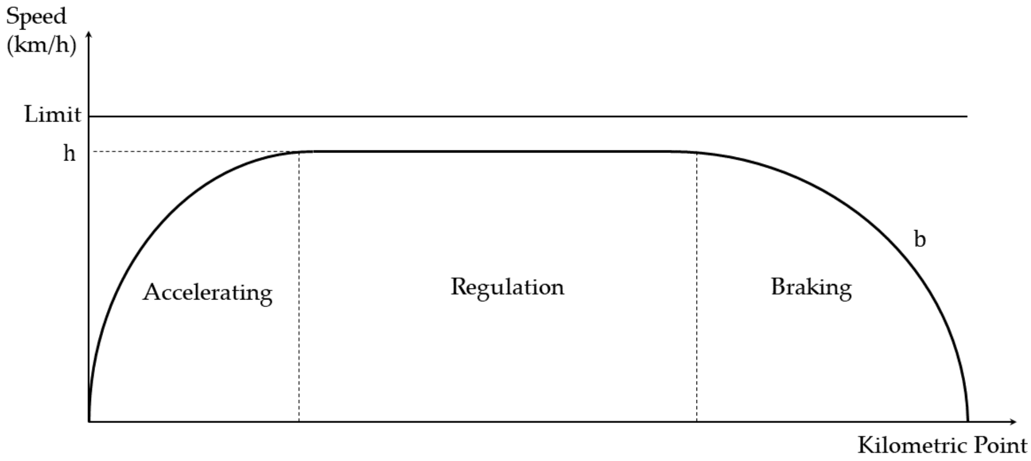

Figure 21. Regulation speed profile.

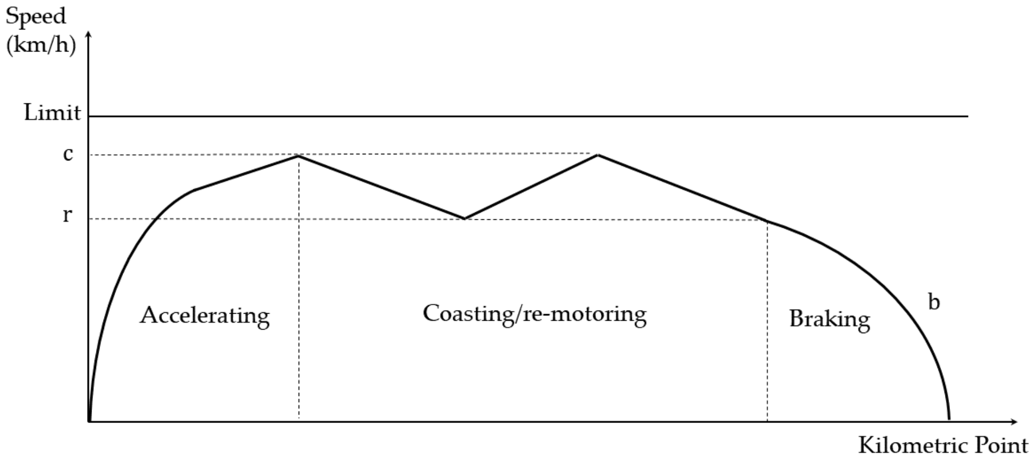

Figure 22. Coasting / re-motoring speed profile.

Efficient ATO driving must meet comfort requirements, otherwise are discarded. The requirements to be fulfilled are: avoid traction cut-off on a ramp, avoid too many cut-off and re-motor cycles between interstations, and avoid low operating speeds and maintaining traction for a minimum time. In addition, accelerations and decelerations must be kept within a certain comfort range by controlling the rate of change of acceleration. Another desirable requirement is that the driving can be kept stable facing changes in the mass of passengers avoiding large variations in running time and consumption [169]. For instance, the efficient ATO speed profiles generated in [51] considers load variations.

It is important to note that the passenger load can vary during the journey and, in the proposed model, it is possible to configure a different passenger mass for each interstation. This way, the model takes into account the impact of the mass variation in the design of the eco-driving and in the use of regenerated energy. On the other hand, the objective is to provide a periodic timetable with a constant headway and with the same stopping regime for a specific time period (for instance, peak-hours). In this kind of operation, the model considers that trains run with similar passenger load at the same stations. Therefore, the passenger mass used for the design of the ATO should be an average mass value for each interstation.

3.4.1. MOPSO ALGORITHM

The parameters involved in the MOPSO algorithm section are presented in Table 15.

Table 20. MOPSO parameters.

$\boldsymbol{x_i}$	Position of the particle
$\boldsymbol{n}$	Iteration number
$\boldsymbol{v_i}$	Velocity of the particle
$\boldsymbol{w}$	Inertial weight of the particle
$\boldsymbol{c_1, c_2}$	Parameters that determine individual and collective influence
$\boldsymbol{r_1, r_2}$	Parameters that take a random value chosen between 0 and 1
$\boldsymbol{p_i}$	Vector where the previously local best result of the $\boldsymbol{i}$ particle is stored
$\boldsymbol{p_g}$	Vector where the previously global best result is stored

For the search of these possible solutions, a MOPSO algorithm has been chosen because it has been proven that provides better results for the eco-driving problem compared to other widely used algorithms [168]. MOPSO algorithm is based on the PSO method, resembling the behaviour of a multitude of individuals present in nature [170]. MOPSO algorithm is a particle swarm optimisation technique for solving multi-objective optimisation problems. In MOPSO, each particle represents a potential solution to the optimisation problem and moves in a multidimensional search space. The goal of the algorithm is to find a set of optimal solutions that are as close as possible to the Pareto front. To achieve this, MOPSO uses a technique called "Pareto dominance", which allows solutions to be compared in terms of their quality across multiple goals. The algorithm adjusts the speed and direction of each particle based on its own previous experience and that of neighbouring particles, in an iterative process that seeks to continuously improve solutions.

The MOPSO algorithm presented in this chapter uses the train simulation model presented in Section 3.4.2 to evaluate the fitness function in Equation (51) and the comfort criteria.

The PSO algorithm searches for an optimal solution by allowing an initial population to move through the search space according to mathematical rules. The movement of the particles is conditioned by the best local solutions, at the level of each particle, and at the global level, by the population as a whole. The PSO method is a relatively simple and effective algorithm, which makes use of the characteristic movement of particles through the search hyperspace by rapidly converging on better solutions. It makes use of a process similar to crossover, used in Genetic Algorithms, and the concept of fitness. MOPSO allows dealing with multi-objective optimisation problems. In this case, it is used the Pareto front concept [171]. The history of the best solutions found by a particle can be used to store previously generated non-dominated solutions. The use of attraction mechanisms combined with non-dominated solution vectors would trigger convergence to globally non-dominated solutions. Therefore, in each cycle, the past experience of each particle is saved. In addition, it is used the technique inspired by the external archive used in PAES [172] (Pareto Archive Evolution Strategy). Here, the information stored by the particles is updated based on the values of the objective function for each of the particles. The previously stored information is used by each individual to choose the leader to guide the search.

MOPSO algorithm is described by the following steps:

1. Initialise randomly the position and velocity of the particles.
2. The particles are evaluated in running time and energy consumption and those that do not fulfil comfort criteria are directly marked as dominated solutions.
3. Local best is initialised as the current position of each particle.
4. Non-dominated solutions are stored in the archive.
5. Global best is initialised selecting randomly a non-dominated solution.
6. Update the position and velocity values for each of the particles by means of the following equations:

$$x_i(n) = x_i \cdot (n-1) + v_i(n) \tag{53}$$

$$v_i(n) = w \cdot v_i \cdot (n-1) + c_1 \cdot r_1 \cdot (p_i - x_i \cdot (n-1)) + c_2 \cdot r_2 \cdot \left(p_g - x_i \cdot (n-1)\right) \quad (54)$$

where x_i is the position of the particle, v_i is the velocity of the particle, n is the number of the iteration, w is the inertial weight of the particle and allows particles to find local optimal solutions based on global solutions at the outset, c_1 and c_2 are parameters that determine individual and collective influence, r_1 and r_2 are parameters that take a random value chosen between 0 and 1, p_g is the vector where the previously global best result is stored and it is the best position found by the swarm during all the iterations of the algorithm, p_i is the vector where the previously local best result of the i particle is stored and it is the best position found by the specific particle during all the iterations of the algorithm.

7. Calculate the consumption and travel time by evaluating the particles. Subsequently, comfort criteria can be applied to mark as dominated those driving that do not meet them.
8. The archive is updated including the new non-dominated solutions found and discarding those particles that are dominated by the new ones.
9. Update local best for each solution. If the local best and the current particle position are mutually non-dominated, then the local best will be maintained or updated by the new particle position randomly.
10. Calculate the crowding distance (CD) of each solution in the archive.
11. Update the global best result from the solutions of the archive. Solutions with higher value of CD will have higher probability of being selected as global best to promote diversity. Thus, the archive will be divided into two groups: the group with higher values of CD and the rest. The first group is composed by the fraction of the archive with higher values of CD according to the "Top select" parameter. Then, a solution is randomly selected from the top group or from the rest with a probability defined by the "Top select probability" parameter.
12. Repeat steps 6 to 11 until the maximum number of iterations is achieved.

3.4.2. TRAIN SIMULATION MODEL

The train simulation model is used to evaluate the solutions generated by MOPSO algorithm. A detailed description of this simulation model and its validation can be found in [75]. It is composed by several modules that represent the ATO logic, the traction/braking system, the train dynamics and the energy consumption calculation. The line characterisation used by the simulation model includes location of stations (kilometric point), grades, grade transitions curves, track curvatures and speed limits. Moreover, the train characterisation includes mass of the train, the mass of the load, length of the train, maximum speed, rotatory inertia, adhesion traction, traction effort curve, braking effort curve, running resistance coefficients, auxiliary systems consumptions and the ATO equipment configuration.

The simulation model works calculating, at constant time steps, the state variables of the train motion. At each step, the process starts with the ATO module to obtain the traction/braking demand (a_{ref}) for the train. This demand, is the ratio between the effort that the traction/braking system has to apply and the maximum traction/braking effort.

The ATO module receives as inputs the speed and position of the train, the maximum speed profile, the gradient profile and the position of the arrival station. With this information, the ATO modules differentiate 3 scenarios:

- The train is in traction mode and does not need to brake to observe maximum speed reduction or to the station arrival. In this case, the ATO calculates its output by means of a proportional controller using as a speed reference (v_{target}) the minimum value between the maximum speed at the position of the train and the driving command hs (in case the train is driven in holding speed mode). The result of the control is corrected with the gradient acceleration as shown in Equation (55).

$$a_{ref} = K_{traction} \cdot \left(v_{target} - v_{train}\right) + K_{acc} \cdot a_{gradient} \qquad (55)$$

where $K_{traction}$ is the constant of the proportional controller (s/m) for the traction mode, v_{train} is the current train speed (m/s), K_{acc} is the constant for the pre-fed acceleration (s^2/m) and $a_{gradient}$ is the acceleration (m/s^2) due to the gradients at the position of the train.

- The train is in braking mode to observe a speed restriction. At each simulation step, the ATO logic evaluates by means of a braking detection curve [173] if it is necessary to reduce the speed because of a maximum speed reduction. When the braking detection curve intersects the maximum speed profile, the ATO braking logic is started. In the braking mode, the output of the ATO is calculated by the proportional controller represented in Equation (56). This controller is corrected not only with the gradient acceleration (as the previous case) but also with the target acceleration a_{target} (m/s²). The target acceleration (a_{target}) is calculated as a braking curve from the current train position, the speed of the position where the maximum speed is reduced and the new maximum speed value. The speed reference v_{target} (m/s) in this case is the speed of the braking curve in the next time step.

$$a_{ref} = K_{braking} \cdot (v_{target} - v_{train}) + K_{acc} \cdot (-a_{target} + a_{gradient}) \quad (56)$$

  where $K_{braking}$ is the constant of the proportional controller (s/m) for the braking mode that depends on the train speed.

- The train is in final braking mode to brake up to the arrival station. At the beginning of the simulation, the braking curve up to the stop in the arrival station is calculated between the arrival point up to the initial point using the driving commands deceleration (b). When the train speed and position intersect this braking curve, the ATO final braking is started. In the final braking mode, the proportional controlled represented in Equation (56) is applied where the target acceleration (m/s²) is b and the speed reference (v_{target}) is the speed (km/h) of the final braking curve in the next time step.

The ATO traction/braking demand a_{ref} previously calculated is bounded by [-1,1] and hysteresis is applied to minimise that the train changes for traction to braking continuously. Besides, if the train is driven with coasting/re-motoring driving commands, a_{ref} will be bounded by a maximum value of 0 when the train is in traction mode and the speed has previously reached the coasting speed (c) and it is over the re-motoring speed (r).

Once the ATO output is calculated, the effort provided by traction/braking system is calculated using Equation (57).

$$F_m = a_{ref} \cdot F_{max} \tag{57}$$

where F_m is the traction/braking effort of the train (kN) and F_{max} is the maximum traction effort (kN) according to the motors characteristics, if $a_{ref} \geq 0$, or the maximum braking effort according to the braking system characteristics, if $a_{ref} < 0$. Both, maximum traction and maximum braking effort curves are dependent on the train speed.

The traction/braking system limits abrupt changes in the train effort applied. Therefore, the previously calculated traction/braking effort will be constrained applying a limitation as show in Equation (58).

$$-j_{max} < \frac{dF_m}{dt} < j_{max} \tag{58}$$

where j_{max} is the maximum variation of traction/braking effort $\left(\frac{kN}{s}\right)$ value configured.

The traction/braking effort will be the input for the dynamics module that will calculate firstly the train acceleration (a) using the Newton's second motion law as shown in Equation (59).

$$a = \frac{F_m - F_g - F_c - F_r}{m_{eq}} \tag{59}$$

where F_g (kN) and F_c (kN) are defined in Equation (5) and Equation (6), respectively, F_r is the running resistance (kN) and m_{eq} is the equivalent mass of the train (tons), including the rotatory inertia (Equation (3)).

The running resistance F_r can be calculated using Equation (60):

$$F_r = A + B \cdot v_{train} + C \cdot v_{train}^2 \tag{60}$$

where A (kN), $B \left(\frac{kN}{\frac{m}{s}}\right)$ and $C \left(\frac{kN}{\left(\frac{m}{s}\right)^2}\right)$ are coefficients that depend on the train characteristics.

The position s_{train} (m) and speed of the train $\left(\frac{m}{s}\right)$ can be updated using the train acceleration and Equations (62) and (63).

$$\frac{dv_{train}}{dt} = a \tag{62}$$

$$\frac{ds_{train}}{dt} = v_{train} \tag{63}$$

Finally, the traction electric power P_{trac} (kW) and regenerative electric power P_{reg} (kW) consumptions are calculated using Equations (64) and (65). With the power, the traction energy consumption can be calculated integrating the traction electric power.

$$P_{trac} = I_{max} \cdot \frac{F_{train}}{F_{max}} \cdot U_{cat} \cdot \frac{1}{\eta} \qquad if \quad F_m \geq 0 \tag{64}$$

$$P_{reg} = I_{max} \cdot \frac{F_{train}}{F_{max}} \cdot U_{cat} \cdot \eta \qquad if \quad F_m < 0 \tag{65}$$

where I_{max} is the maximum current (A) according to the motor characteristics that is associated to the maximum effort of traction or braking, F_{max} (kN), U_{cat} is the nominal catenary voltage (V) and η is the motor efficiency (%) that depends on the working load level (i.e., depends on the ratio between F_m and F_{max}).

3.5. CASE STUDY

This section presents a case study where the proposed model is applied to a line of Madrid Underground. The metro line consists of 16 interstations and two terminal stations spread over 14 kilometres and its infrastructure is modelled by means of gradients and curves. The characteristics of the train are: 160 t, 78 t of passenger load, a length of 90 m and the maximum power offered is 1500 kW. The electric system is a 1.5 kV DC. Speed limits set by the railway operator are respected at all times. The dynamic simulation of the train taking into account the above parameters is carried out by means of a detailed model of the train movement and execution of the on-board ATO driving parameters. The minimum dwell time (t_{min}) takes a value of 20 seconds.

The variations of the possible values of the ATO driving parameters and their maximum and minimum limits are defined according to Table 21.

Table 21. ATO driving parameters settings.

	Deceleration rate (m/s²)	Regulation speed (km/h)	Coasting speed (km/h)	Re-motoring speed (km/h)
Minimum	0.60	30	30	5
Maximum	0.80	80	80	50
Increase	0.05	0.25	0.50	1

The headway of the timetable to be designed is 420 seconds (7 min).

For the analysis of the case study, the process described in Section 3.4 is carried out searching for the optimal driving (Equation 51) for each interstation with MOPSO optimisation algorithm.

MOPSO parameters used in to generate the local eco-driving Pareto curves are listed in Table 22.

Table 22. MOPSO parameters.

Swarm size	Iterations	c1	c2	w	Top select (%)	Top select probability (%)
40	1000	1	1	0.9	6%	98

The result of this process is a Pareto front for each interstation and is the input information used for the following optimisation procedure. Figure 23 shows an example of solutions after the local eco-driving calculation process. Each point of a Pareto curve represents a specific ATO parametrisation that produces the most efficient driving for the corresponding running time at that interstation.

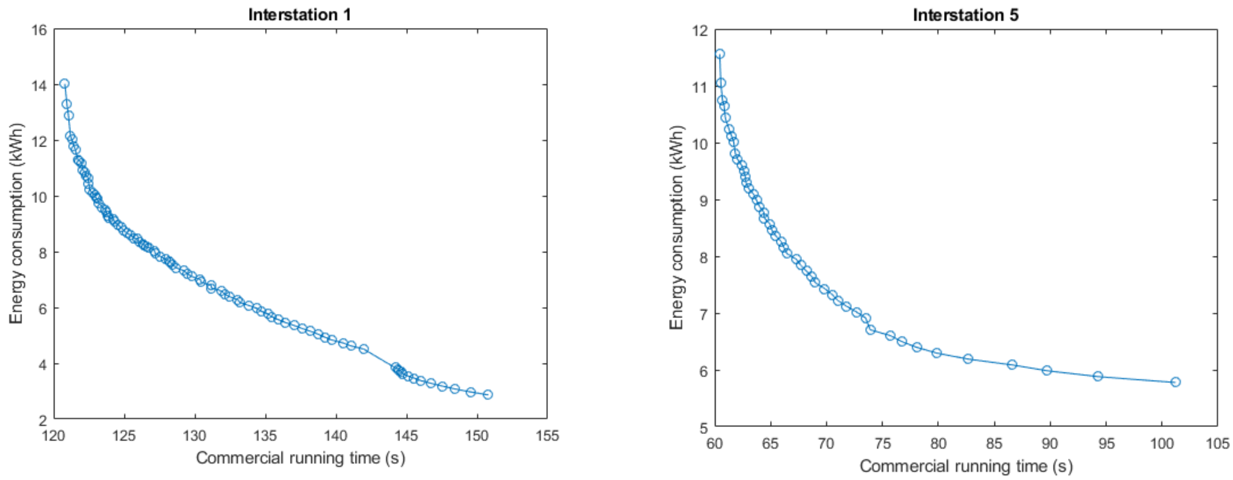

Figure 23. Examples of Pareto front of interstations 1 and 5: designed eco-driving, running time / energy consumption.

As previously described, the procedure continues with the timetable optimisation (Equation 40) and the global Pareto front is obtained. Each solution of this global front represents the set of efficient ATO driving parameters at each interstation of the entire railway line and stopping times. The obtained selection of running times and stopping times leads to synchronised braking and tractioning trains to maximise the use of regenerative energy.

The timetable optimisation problem is solved by the proposed algorithm GLMO-NSGA-III. In the following, this solution is compared as well with other algorithms: WOF [174] (Weighted Optimization Framework), LCSA [175] (Linear Combination-based Search Algorithm), FDV [59] (Fuzzy Decision Variable) and NSGA-III.

The optimisation process using the FDV algorithm is divided into two distinct parts, fuzzy evolution and precise evolution. In the first one, fuzzy evolution substages are added and the fuzzy operation is performed once the offspring has been generated at the beginning with the MOEA (Multi-Objective Evolutionary Algorithm) that has been chosen, being in this case the LMOCSO [176] (Large-Scale Multi-Objective Optimization Competitive Swarm Optimizer). In the second phase, offspring generation operations are carried out. The next population generation is examined by means of LMOCSO. The fuzzy evolution phase in turn consists of: fuzzy evolution sub-stages division and fuzzy operation. The purpose of the first of the above phases is to make the algorithm's solution more accurate by dividing the fuzzy evolution into multiple phases in which the level of fuzzification decreases. In the second phase, the degree of fuzzification of the solution is determined.

LCSA algorithm is a procedure in which solutions of the optimisation problem are formed by linear combination of previous results to improve the quality of solutions in multi-objective problems. A population of coefficient vectors is formed and optimised by a metaheuristic procedure to improve the search for solutions. The algorithm starts with the optimisation of the population with any multi-objective method. The first non-dominated solution front of the existing population is used to form a solution matrix. Then, a random population of the linear combination of solution vectors is formed. These vectors are optimised through an arbitrary procedure, storing this result. Finally, the previous result is combined with the initial population and the process is started again. In this case study, the multi-objective optimisation algorithm applied has been the NSGA-III.

WOF algorithm has been designed as a metaheuristic procedure to be incorporated in a population-based optimisation mechanism. This method is based on the grouping and optimisation of the weighting variables to be applied, and applies the weighting variables for cases where the objectives are multiple. In WOF algorithm the decision vector is divided into smaller vectors. The corresponding weighting factor is applied to each of these vectors. The whole process is divided into two different phases. Initially, an optimisation is performed with any of the following randomly chosen algorithms: SMPSO (Speed-constrained Multi-objective Particle Swarm Optimisation), MOEA/D (Multi-Objective Evolutionary Algorithm based on Decomposition), NSGA-II, NSGA-III. After this, a certain number of solutions are chosen from the current population and a new optimisation is performed with the NSGA-III algorithm and taking into account the corresponding reduced decision vector.

Table 23 presents the parameters of the algorithms used for the optimisation. All algorithms have been executed with 220 generations and a population size of 100. Additionally, the random seed is initialized for the generation of the initial random population in all cases, and for all executed and compared algorithms. This uniformity in the initialisation process and parameters further ensures a fair comparison of the algorithms' performance.

Table 23. Algorithm configuration parameters.

FDV	**Fuzzy evolution rate**	**Step acceleration**	**Optimiser**	**Aggregation function**	-
	0.8	0.4	LMOCSO	Penalty based Boundary Intersection	-
GLMO-NSGAIII	**Grouping method**	**Variable groups**	**Optimiser**	-	-
	Ordered	4	NSGA-III	-	-
LCSA	-	-	**Optimiser**	-	-
	-	-	NSGA-III	-	-
NSGAIII	**Simulated binary crossover probability**	**Crossover distribution index (η_c)**	**Mutation distribution index (η_m)**	**Polynomial mutation probability**	-
	1	30	20	$\frac{1}{number\ of\ variables}$	-
WOF	**Grouping method**	**Variable groups**	**Transformation function**	**Evaluations (original/transformed problem)**	**Fraction of function evaluations for the alternating weight-optimisation phase**
	Ordered	4	Interval	1000/500	0.5

Figure 24 presents the behaviour of the algorithms after the global optimisation process considering the electrical loss function.

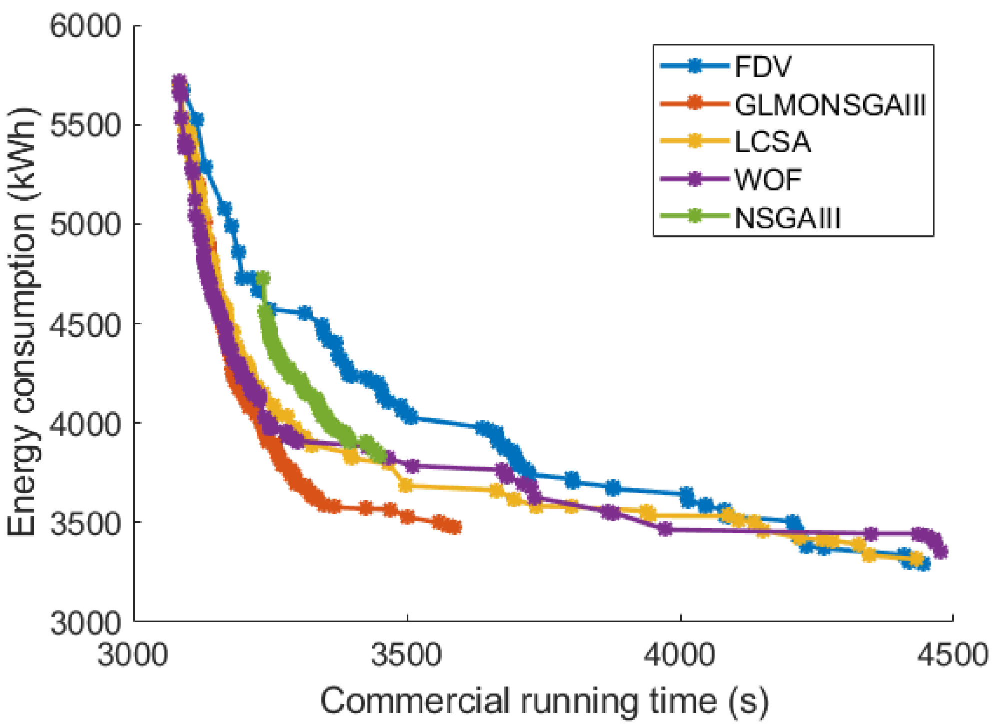

Figure 24. Pareto fronts comparison.

For the resolution and optimisation of the timetable as the second part of the problem, the GLMO-NSGA-III has been chosen due to its good results after the comparison with the rest of the algorithms. When NSGA-III is combined with the GLMO operator, the algorithm provides better solutions. It can be observed that GLMO-NSGA-III obtains better solutions in the range between 3100 seconds and 3600 seconds. After that, the LCSA algorithm provides more solutions. However, in the design of railway timetables the time margin that is usually given above the minimum time is between 3.5% and 10%, so the range for the running time is up to 500 seconds. Therefore, solutions provided by GLMO-NSGA-III are adequate from the operational point of view.

In the following, the proposed model is firstly executed without introducing the penalty on the number of trains to analyse the obtained results. The design of the timetable is carried out in two different cases depending on whether regenerated energy is considered or not in the optimisation model. The first case will refer to the optimal design of the timetable when the energy regenerated by the trains is ignored in the analysis. The second case will consider the energy regenerated during braking being more faithful to reality by adding the losses given during transmission. These losses are modelled by means of a function dependent on the distance that separates the train that is regenerating energy and the train that receives and uses the energy (Figure 18).

Figure 25 plots the optimisation scenarios representing each of them as the corresponding Pareto front. As expected, the driving with the highest associated consumption correspond to the scenario where the regenerated energy is not reused.

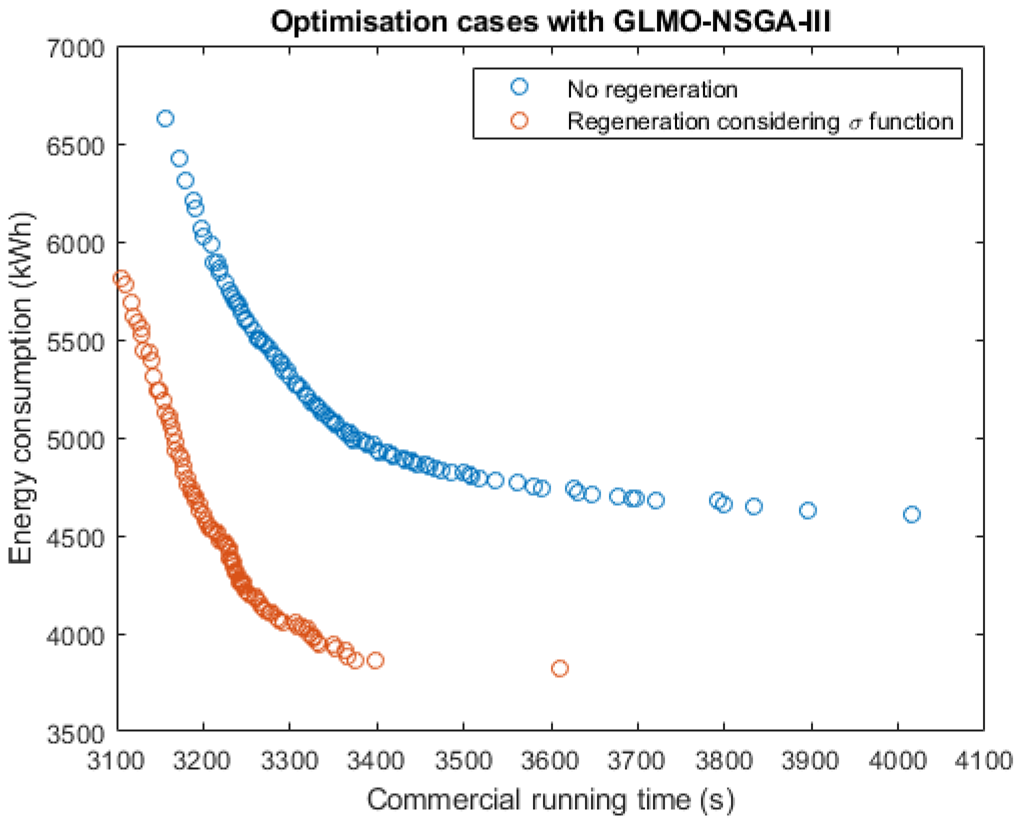

Figure 25. GLMO-NSGA-III optimisation cases.

Results in Figure 25 are the Pareto curves obtained by the optimisation algorithm in these cases. However, in reality, when the solutions obtained by the algorithm are tested on the line (by simulation) there is regenerated energy and the transmission of this energy between trains has associated losses that depend on the distance between trains. Therefore, the result of simulating the above Pareto fronts on a line with regenerated power transmission with losses according to the defined loss model is presented below in Figure 26. It can be seen in Figure 26 that the curve that was optimised taking into account the regenerated energy and σ function has a better behaviour in terms of consumption as it has been tried to preferentially synchronise accelerating and braking trains.

Figure 26 shows the optimisation result when regeneration is considered with the loss function and the result when regeneration is not considered in the optimisation, both simulated in the case where regeneration with losses during transmission is added.

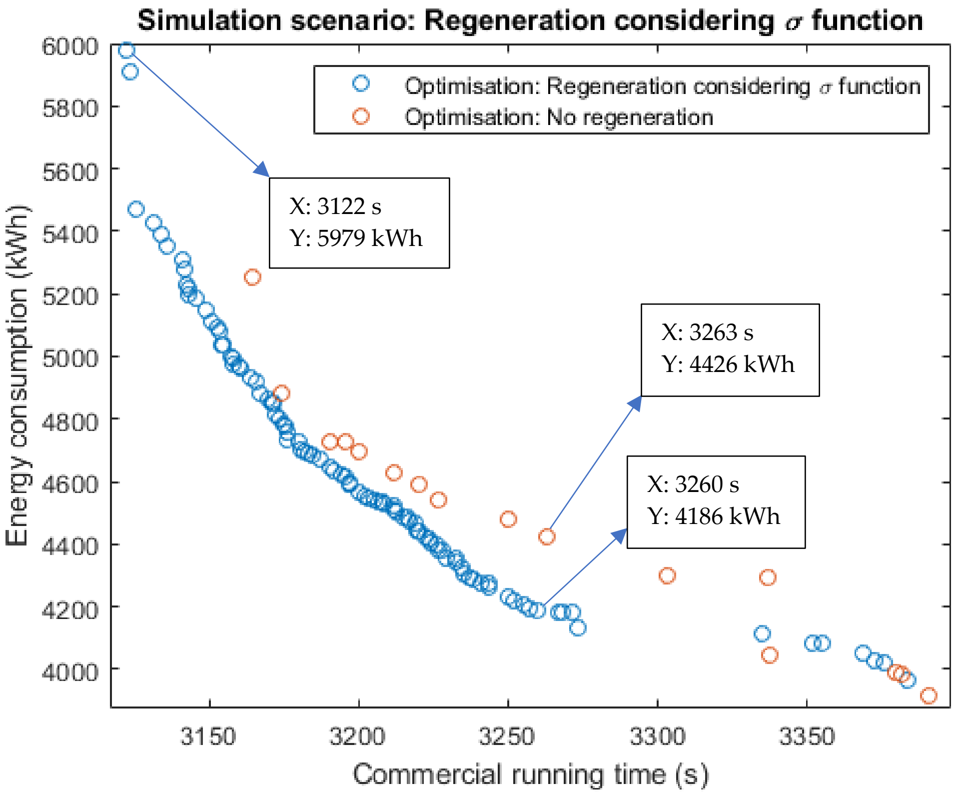

Figure 26. Optimisation results simulated considering σ function.

As can be seen in Figure 26 a better result is obtained for the optimised case where regeneration and the loss function are considered. The graph highlights the points corresponding to the fastest possible set of driving for the whole metro line and the same for a set of 4.52% slower driving with respect to the optimised case in the two optimised cases. Comparing the result of both optimisations for what could be the nominal set of driving shows an energy saving of 5.42% with the optimisation when regeneration and σ function are considered.

Previous results do not take into consideration the number of trains required to meet the demand. Firstly, the number of trains are calculated as a further step to the optimisation (Equation 49 and 50) and the results are shown in Figure 27. In this case no penalty has been introduced in the objective function. This is why the solutions between the 8-train and 9-train case overlap in some areas of the graph. As shown in the figure, the Pareto front contains similar solutions that are optimal in terms of energy and running time, but with different number of trains required to give the associated service. With the same consumption and running times some solutions require 8 trains and others require 9 trains, which impact critically on the investments and operation. The results of Figure 27 present the energy consumption vs the commercial running time. The commercial running time is the commercial speed experienced by the passengers, so the time spent by the trains at stops in terminal stations is excluded. This time is used by the trains for the reverse manoeuvres, to unload and load passengers, and as a buffer to absorb delays. However, it is also crucial to determine the synchronisation of ascending and descending trains to use regenerated energy as was studied in [177]. Therefore, solutions with the same commercial running time could present different cycle time in the line and different number of trains to operate the timetable.

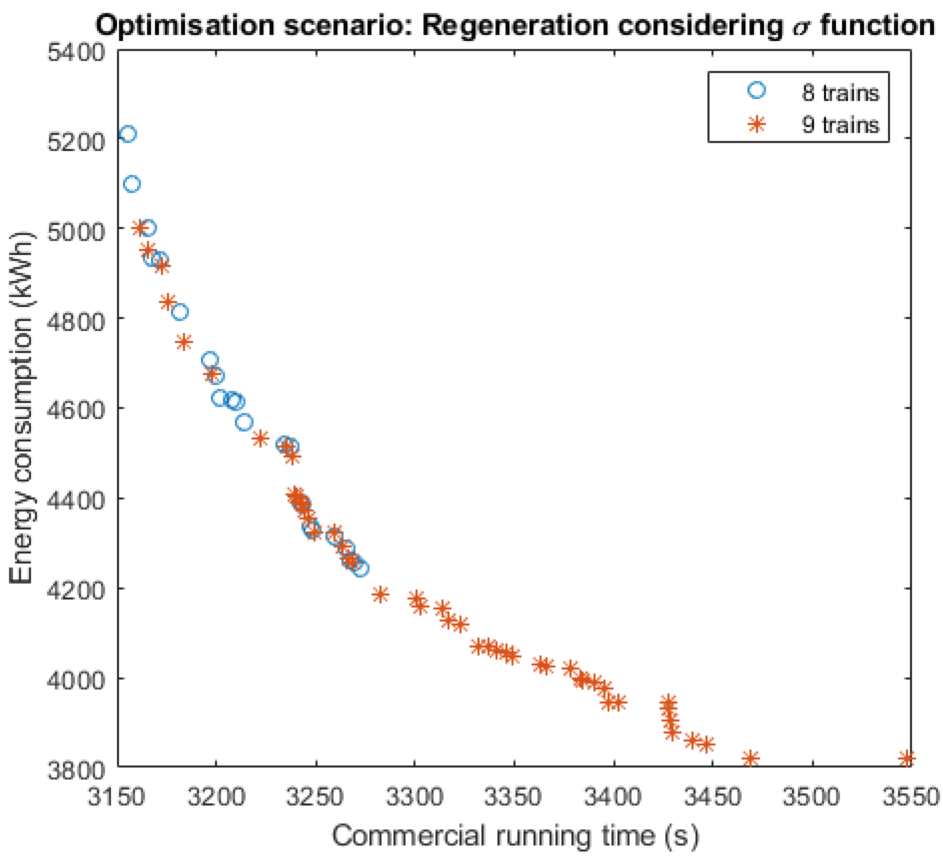

Figure 27. Timetable optimisation without penalty for the number of trains.

Figure 28 shows the optimisation after including in the objective function (Equation 40) a penalty or additional cost of 25 kWh for each extra train required in the achieved solution. This penalty is expressed in the term $P \cdot N_t$ of the Equation 44. Hence, the areas of the 8-train and 9-train cases are clearly differentiated.

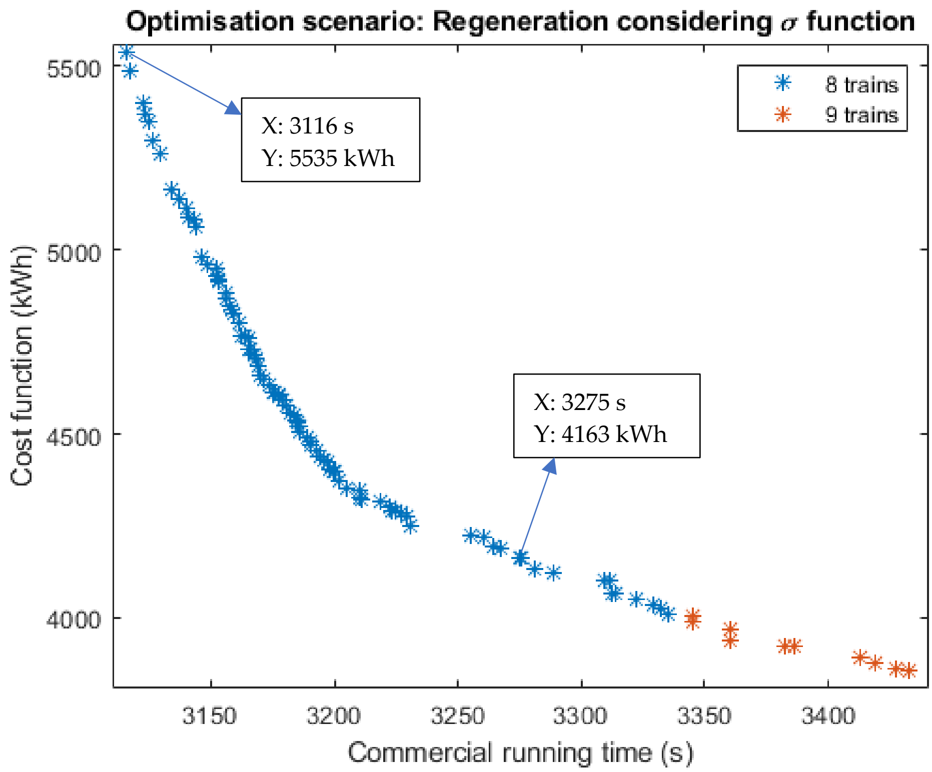

Figure 28. Timetable optimisation introducing penalty for the number of trains.

In this case, a timetable can be chosen with a running time around 5% slower than that associated with the flat-out driving between stations following the railway operator's criteria of setting a time margin for the design of the timetable. In this way, an energy saving in relation to the cost function of 1372 kWh is achieved, which represents a saving of 24.79%, not being necessary to include more trains on the line ensuring the fulfilment of the service.

Figure 29 compares the energy that would be consumed in rheostats in the results obtained in Figure 26. Both cases have been simulated when considering the regenerated energy and the losses associated with its transmission. Orange bars show the result when the optimisation is carried out without considering the energy regenerated during braking and the blue bars show the result when the energy regenerated during braking is considered with the associated transmission losses.

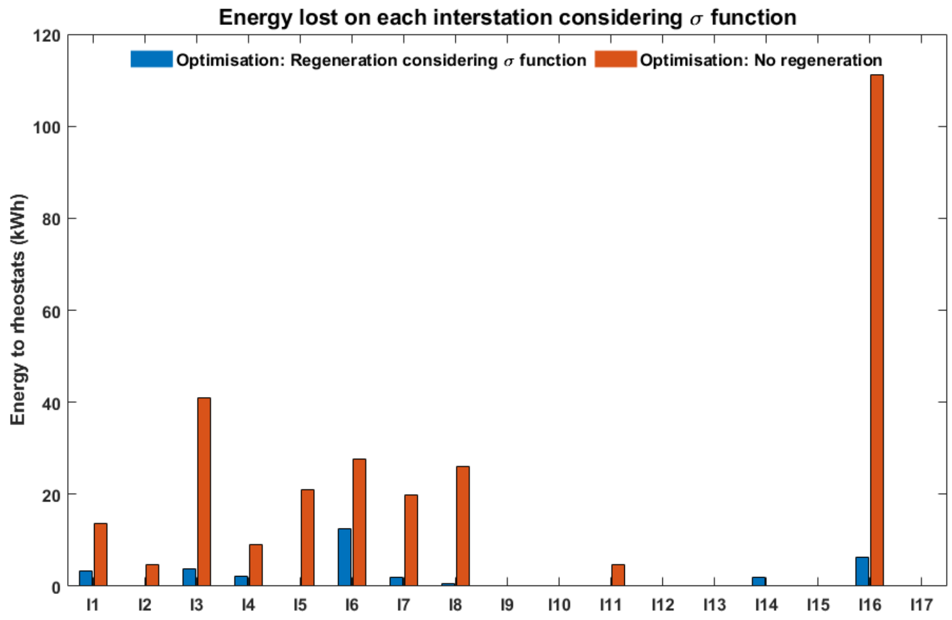

Figure 29. Energy dissipation in rheostats at interstations.

It can be seen from the result that synchronisation improves the use of regenerated energy in every interstation. The energy that would be wasted in the case where regeneration was not taken into account would be 279 kWh while this wasted energy drops to 33 kWh when the driving and synchronisation are designed taking regeneration into account.

Figure 30 and Figure 31 show the two optimised speed profiles for the cases compared for track 1 and track 2, respectively:

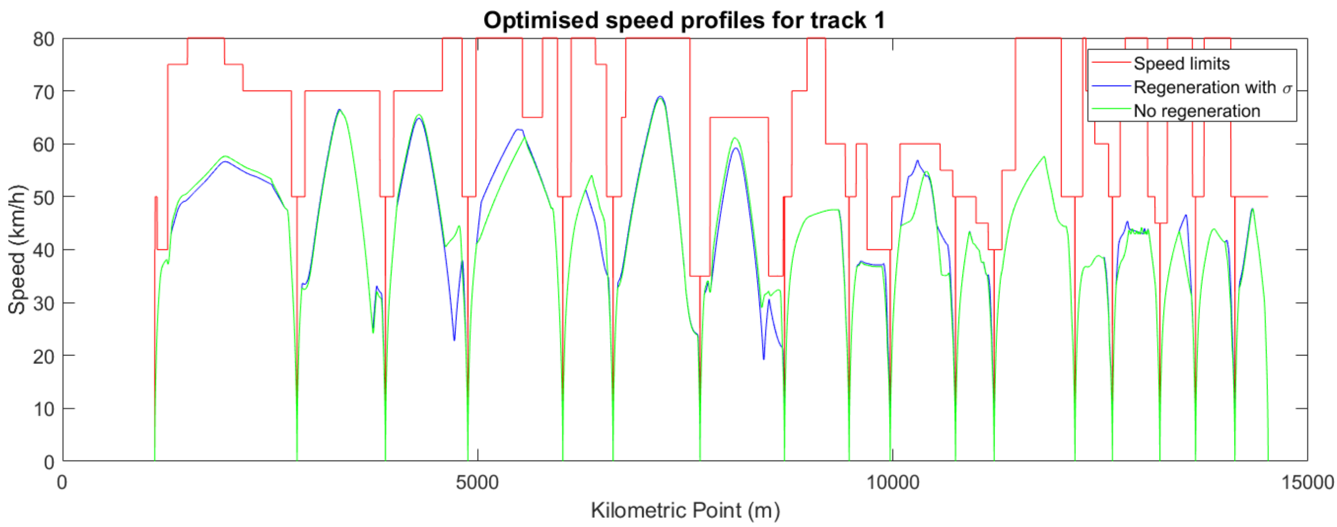

Figure 30. Speed profiles for track 1.

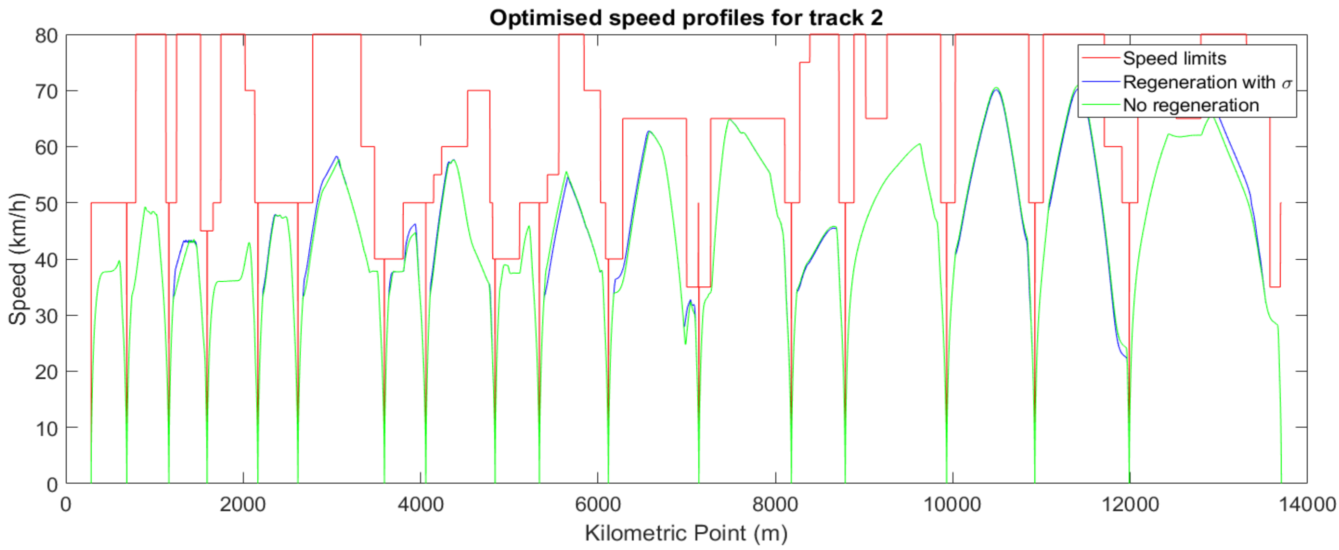

Figure 31. Speed profiles for track 2.

Table 24 and Table 25 show the arrival and dwell times for the compared cases, in track 1 and track 2. These figures have been obtained for a train that arrives to the first station in time equals to 0. As can be seen, when no regeneration is considered all the dwell times are the minimum value except for the reverse time in the first station that is 287s. However, when considering regeneration with σ function, there are cases where the dwell time is above the 20s (station S8 for instance) and both reverse times are greater than the minimum value (45s in S1 and 38s in S18). This is because the algorithm is synchronizing accelerating and braking trains to increase the use of regenerated energy.

Table 24. Arrival and dwell times for track 1.

	Optimisation: considering σ		**Optimisation: no regeneration**	
Station	**Arrival time (s)**	**Departure time (s)**	**Arrival time (s)**	**Departure time (s)**
S1	0	45	0	287
S2	183	203	423	443
S3	303	323	544	564
S4	415	435	649	669
S5	528	548	766	786
S6	614	634	848	868

S7	731	751	966	986
S8	864	885	1087	1107
S9	959	979	1181	1201
S10	1039	1059	1264	1284
S11	1133	1153	1363	1383
S12	1210	1230	1441	1461
S13	1322	1342	1553	1573
S14	1397	1417	1629	1649
S15	1478	1498	1712	1732
S16	1550	1570	1789	1809
S17	1624	1644	1865	1885

Table 25. Arrival and dwell times for track 2.

	Optimisation: considering σ		**Optimisation: no regeneration**	
Station	**Arrival time (s)**	**Departure time (s)**	**Arrival time (s)**	**Departure time (s)**
S18	1693	1731	1934	1954
S17	1780	1800	2004	2024
S16	1854	1874	2078	2098
S15	1926	1946	2150	2170
S14	2010	2030	2235	2255
S13	2082	2102	2307	2327
S12	2193	2213	2418	2438
S11	2269	2289	2495	2515
S10	2365	2385	2591	2611
S9	2443	2463	2670	2690
S8	2541	2561	2765	2785
S7	2657	2677	2884	2904
S6	2765	2785	2992	3012
S5	2852	2872	3079	3099
S4	2972	2992	3200	3220
S3	3074	3094	3301	3321
S2	3188	3208	3414	3434

3.6. CONCLUSIONS AND CONTRIBUTIONS

Chapter 3 presents a procedure for optimal timetable design considering efficient driving for a metropolitan railway line based on detailed simulation. The objective is to design the optimal operation, minimising the energy consumption and considering the rolling stock required to provide the service. This is achieved by integrating the optimisation of the ATO driving parameters and the optimisation of the timetable adding the contribution of the energy regenerated by braking trains. The procedure described has been applied to a line of Madrid Underground. The optimal design has been carried out in two different scenarios: ignoring regenerated energy and considering regenerated braking with transmission losses to show the importance of including regenerated energy use calculation in the optimisation model.

The presented analysis supports the conclusion that the proposed model is effective in reducing energy consumption on metropolitan railway lines by integrating efficient driving design, timetable optimisation, and the use of regenerated energy. Its application to a real case on the Madrid Metro network has validated the approach, achieving a total energy saving of 24.79% compared to the fastest possible driving strategy. Furthermore, it has been demonstrated that including regeneration in the timetable optimisation yields an additional 5.42% improvement in efficiency. The new timetable design meets punctuality requirements while providing sufficient time margins to recover from potential delays. In addition, the model optimises the number of trains required to operate the service, considering their associated operating costs. Finally, it has been shown that synchronisation between trains significantly improves the use of regenerated energy across all interstation segments: while 279 kWh would be wasted if regeneration were not considered, this value is reduced to just 33 kWh when the new model is applied—representing a reduction in energy waste to 11.83% compared to the optimisation without energy regeneration. Likewise, the calculated driving meets the railway operator's design criteria, giving a margin time for the nominal operation and offering an accurate reference for metro railway operators.

A key contribution of this chapter is the integration of a two-level optimisation model: first, the design of eco-driving parameters for all interstation runs of the line, and second, the global timetable optimisation, which includes station dwell times and synchronisation of traction and braking trains to maximise regenerative energy transfer. The number of trains required to fulfil the timetable are included in the model to consider costs associated with rolling stock. This allows the model to select the solution that satisfies the timetable with the minimum number of active trains, achieving energy efficiency while considering operational cost. The model starts with the design of the ATO eco-driving parameters for the whole line where the energy consumption is minimised as function of running time. The search for efficient driving has been carried out by means of the MOPSO algorithm and the solutions form a Pareto front where running time and consumption are conflicting objectives. These ATO parameters satisfy realistic operating conditions easing its implementation in real conditions. The timetable is then designed by optimising the time margin distribution, the stopping times at each station and synchronising traction and braking trains during their journey on the whole line to maximise the use of regenerated energy. Several optimisation algorithms have been tested for the efficient timetable design, including WOF, LCSA, NSGA-III, GLMO-NSGA-III and FDV. In this study, GLMO-NSGA-III has been found to be the best performing algorithm. Furthermore, it allows evaluating the impact of commercial speed decisions on fleet size and energy usage, providing a solution that simultaneously minimises the number of trains in use and total energy consumed.

Future developments will focus on real-time energy and traffic optimisation in metro systems, integrating rolling stock management and improving the use of regenerated energy, supported by advanced communications and high-performance computing.

CHAPTER 4

CONCLUSIONS, CONTRIBUTIONS, PUBLICATIONS AND FUTURE DEVELOPMENTS

4.1. CONCLUSIONS AND CONTRIBUTIONS

The importance of increasing efficiency in railway operations lies in the growing use of services of high-speed trains and mass transit transport systems. These systems consume a significant amount of energy, and it is necessary to provide technological solutions that enable better energy use, which could potentially result in benefits for the passengers. For this reason, this thesis is based on the application of energy efficiency techniques in railway systems from the perspective of train operation. The main techniques for achieving efficient operation include driving design and timetable optimisation.

Efficient driving design in railway systems has been the subject of study for decades, with research focusing on the design of driving strategies, planning and regulation of rail traffic. Several computational techniques and optimisation algorithms have been used to solve the efficient driving models proposed in the literature. The application of these techniques covers long-distance, high-speed railway systems and metropolitan transport networks. However, the effectiveness of these strategies depends not only on their design but also on the accurate consideration of external factors influencing train operation.

In railway operation, various factors directly affect energy consumption and journey times, which are generally not considered in designs due to the difficulty of addressing them in simulations. The relevance of these factors lies in the uncertainty inherent in their nature, which complicates their quantification and, consequently, their integration into models without appropriate treatment. To address this complexity, fuzzy logic can be used to model and quantify these uncertainties, providing a flexible representation of variations and unforeseen factors affecting railway operation. This ensures that the application of the model and the validity of the results remain consistent with actual operating conditions.

Previous studies have considered the uncertainty of certain driving-related parameters, such as the variability in driving styles and the drive's behaviour during manual driving in high-speed rail services. However, climatological factors have often been overlooked. Driving strategies are conventionally designed based on nominal conditions, omitting the consideration of external environmental factors, including fluctuations in air properties along the route and the influence of wind, which can critically affect train performance.

Therefore, in Chapter 2, the impact of climatological uncertainty on railway operations—and consequently, on the design of efficient driving for high-speed services—has been analysed. The parameters subject to uncertainty, modelled by means of fuzzy logic, have been air density, which is influenced by pressure and temperature, wind intensity and the wind incidence angle relative to the train's direction of travel. These fuzzy parameters influence the aerodynamic resistance of the train, which acts as an opposing force to its motion. Thus, climatological conditions directly affect the train's traction energy consumption and the total running time required to complete the journey.

To solve the efficient driving problem, a genetic algorithm (NSGA-II) has been used, integrating the parameters modelled through fuzzy logic. The efficient driving model also incorporates punctuality requirements. This punctuality constraint has been modelled as a necessity level, representing the requirement to meet the scheduled arrival time under given climatic conditions.

In this way, efficient driving has been designed for high-speed rail service on a Spanish railway line, considering uncertainty associated with climatological conditions and punctuality requirements. The case where the driving is carried out under nominal conditions, i.e., without taking climatic factors into account, has been compared with the case where the driving has been designed considering these factors. Finally, the solutions of the model have been compared for a typical summer case and a typical winter case.

The conclusions derived from the research focused on the development of eco-driving optimisation for high-speed trains under climate uncertainty - where climate parameters are modelled using fuzzy logic - are presented below:

- Taking climatological conditions into account when optimising eco-driving strategies has a relevant impact on the expected energy savings for a railway line. In the case study, the difference reached approximately 5%, increasing the expected energy savings from 29.76% to 34.70% under summer conditions. This highlights the significant potential of incorporating weather variability into energy-saving models.

- The model reveals a seasonal variation in energy consumption of 5.31% between summer and winter scenarios, despite maintaining compliance with punctuality constraints in both cases. This underlines the importance of adapting strategies to different weather conditions. This point underlines the importance of tailoring eco-driving strategies to seasonal changes to achieve optimal performance year-round.

- Although climatic conditions influence energy consumption, their impact on running time is practically negligible. The results show a variation of only 0.03% between the optimised summer and winter scenarios, confirming that energy optimisation does not compromise punctuality. The balance between efficiency and service reliability is critical for practical implementation.

- Adapting train driving strategies to real climatological conditions leads to a more accurate estimation of energy consumption. The proposed model was successfully simulated in a real high-speed railway line in Spain, demonstrating its effectiveness and practical applicability.

- The optimisation case that does not consider climatological parameters shows a behaviour more similar to the winter scenario than to the summer one. Specifically, the expected energy savings without weather consideration (29.76%) are much closer to those obtained under winter conditions (31.03%) than to those achieved in summer (34.70%). This suggests that traditional eco-driving strategies might unintentionally approximate winter conditions, as the simulation results of the initial case and the winter scenario are more similar, reinforcing the value of including weather data to maximise efficiency under varying climates. Therefore, ignoring climatological variability could lead to suboptimal energy savings during more favourable conditions, such as in summer.

The main contributions of this work on eco-driving for high-speed trains under climatic uncertainty are listed below:

- A new eco-driving model for railway lines, incorporating climatological conditions such as temperature, pressure, wind intensity and its angle of incidence. By integrating these environmental factors, the model captures the interaction between climate and train dynamics, allowing for an adaptative optimisation. The incorporation of climatological data into the optimisation model allows for maximizing energy savings, achieving additional traction energy reductions.

- The incorporation of the uncertainty of the climatological parameters through the application of fuzzy logic, facilitating the treatment of their variability. This approach provides a flexible and robust framework that can handle inaccurate or incomplete meteorological data, reflecting the unpredictability in a real case.

- The optimisation problem is solved using a Genetic Algorithm with fuzzy parameters (GA-F), integrating fuzzy logic to account for the impact of climatological uncertainty on railway operations. This approach enables the derivation of driving commands that are better adapted to real-world conditions and consider punctuality constraints. As a result, the method achieves a balance between energy efficiency and operational reliability, essential aspects for its practical implementation.

- The application of the model to a high-speed railway line in Spain demonstrating that climatological parameters have a significant impact on energy consumption and a lesser effect on running times. These findings underscore the importance of incorporating meteorological data into eco-driving strategies to optimise performance under diverse climate scenarios.

- A comparison has been carried out between the case where it is not considered climatological conditions and the cases where they are considered, showing that neglecting these factors underestimates potential energy savings. Additionally, summer and winter scenarios are compared. This comparison highlights the risk of relying on traditional models that do not account for climate variability, which can lead to missed opportunities for efficiency improvements during more favourable weather conditions.

Metropolitan railways, metro-type systems or mass rapid transit (MRT) are characterized as passenger rail transport with the capacity to move a large number of people with a high frequency service. Additionally, this system can connect a large part of the city with its metropolitan areas, with each line having multiple stops along its route. In these reasons, along with the potential benefits or drawbacks for passengers, lies the importance of an effective timetable design for these type of railway services.

An essential distinction between this type of railway transport and long-distance transport is its high degree of automation. While in long-distance trains the driver is responsible for the manual operation of the train, metropolitan systems are increasingly automated, with most trains equipped with ATO systems. Just like high-speed rail operations, metropolitan systems are major energy consumers and are therefore suitable for the application of technology and techniques to achieve energy savings.

The high level of automation in these types of railway lines allows for the application of efficient driving techniques where the driver loses significance. In this case, the train's movements are programmed in the onboard and track-side systems. These automatic driving can be designed considering energy-saving and comfort criteria, taking into account running times between stations, dwell times at stations and headways.

One of the most relevant technologies for optimizing energy use in metro-type systems is the regenerative braking. The energy produced during the braking of trains can be used by other trains that are being powered, returned to the grid through electrical substations or stored in accumulators located along the track.

Thus, Chapter 3 presents a procedure for optimal timetable design in metropolitan railway lines, focusing on minimising energy consumption while considering the rolling stock required for the service. The model integrates the optimisation of ATO driving parameters based on real data and the design of the timetable, taking into account the energy regenerated by braking trains. The optimisation process starts with the design of the efficient ATO parameters for the entire line, where energy consumption is minimised as a function of running time. The search for efficient driving is carried out using MOPSO algorithm, which generates one Pareto front for each interstation that balances the conflicting objectives of running time and energy consumption.

The timetable optimisation process determines the dwell times at each station considering the previous eco-driving design and synchronising traction and braking trains along the entire route to maximise the use of regenerated energy. Several optimisation algorithms were tested, including WOF, LCSA, NSGA-III, GLMO-NSGA-III and FDV. The procedure was applied to the Madrid Underground, where two scenarios were considered: the first one that ignored regenerated energy and the second one that accounted for regenerative braking considering transmission losses.

The integration of eco-driving, timetable optimisation, and the use of regenerated energy into a single model has enabled a coordinated and effective approach to reducing energy consumption in metropolitan railway systems. Based on this proposal, the following conclusions have been reached:

- The application of the model to a real case on the Madrid Metro railway network has validated its effectiveness, achieving an energy saving of 24.79% compared to the fastest possible driving scenario. This result demonstrates the practical applicability of the methodology in real operational environments.

- The inclusion of the rolling stock required to provide the service and its operational costs within the model has allowed the selection of solutions that require a lower number of trains, thereby optimising not only energy consumption but also material and economic resources. Specifically, for the case study presented, it has been observed that, for the same energy consumption and commercial running time, there are solutions that allow the operation of the line with 8 trains instead of 9, representing a significant reduction in investment and operating costs. This difference becomes clearly visible when a penalty associated with the number of trains is introduced in the objective function, reinforcing the model's usefulness in supporting efficient operational decision-making.

- The inclusion of regenerative braking in timetable optimisation has led to an additional energy saving of 5.42% compared to the case in which regenerated energy is not considered, confirming that coordination between braking and traction phases is key to maximising energy efficiency.

- The model has succeeded in significantly reducing energy losses: from 279 kWh to only 33 kWh, which represents just 11.83% of the initially wasted energy. This result highlights the importance of train synchronisation to enhance the use of regenerated energy and increase overall system efficiency.

- Finally, as a more general conclusion, it is evident that integrating multiple strategies into the planning and optimisation of railway operations —including eco-driving, timetable design, energy regeneration management and rolling stock allocation— enables more comprehensive, sustainable, and realistic solutions. The more relevant factors are incorporated into the optimisation model, the greater the potential for improvement in energy, operational, and economic efficiency, making this integrated approach a key tool for better design of metropolitan railway systems.

The main contributions of this study are summarised as follows:

- The design of an optimal timetable procedure for a metropolitan railway line, integrating the optimization of ATO driving parameters and the use of energy regenerated by braking trains. This integration ensures an approach that seeks to balance energy efficiency and operational performance.

- The simultaneous optimization of energy consumption and running time through the incorporation of ATO driving parameters, using the MOPSO algorithm to obtain the Pareto front between these two objectives. This multi-objective method allows for evaluating and selecting solutions that optimize both energy efficiency and punctuality.

- The incorporation of rolling stock costs into the model, by considering the number of trains required to meet the timetable and the associated costs, enabling a more comprehensive and realistic analysis of the operation. This economic dimension strengthens the model’s applicability for sustainable and cost-effective railway planning.

- The detailed optimization of the timetable through the proper distribution of time margins, dwell times, and the synchronization of traction and braking trains to maximize the use of regenerated energy. This fine coordination between acceleration and braking phases is key to reducing energy losses.

- The evaluation of several optimization algorithms, highlighting that GLMO-NSGA-III showed the best performance for efficient timetable design. The rigorous comparison of methods ensures the selection of the most suitable technique for the problem at hand.

- The application of the model to a real line scenario, demonstrating energy savings considering energy regeneration and confirming that the design meets the railway operator’s criteria, providing a margin for nominal operation. This validates the practical feasibility of the approach and its alignment with real operational requirements.

4.2. PUBLICATIONS

The main content and development the doctoral thesis work and its results have been collected in the next articles and published in academic journals.

Initially, the outcome of the work in Chapter 2 is reflected in an article published in the journal Sustainability:

Blanco-Castillo, M.; Fernández-Rodríguez, A.; Fernández-Cardador, A.; Cucala, A.P. Eco-Driving in Railway Lines Considering the Uncertainty Associated with Climatological Conditions. ***Sustainability*** **2022,** ***14*****, 8645. https://doi.org/10.3390/su14148645**

The result of the work in Chapter 3 is then published in an article in the journal Engineering Optimisation:

Blanco-Castillo, M., Fernández-Rodríguez, A., Su, S., Fernández-Cardador, A., & Cucala, A. P. (2024). Efficient automatic operation of a metro line: eco-driving design with optimized use of regenerative energy and rolling stock consideration. ***Engineering Optimization*****, 1–33. https://doi.org/10.1080/0305215X.2024.2385583**

4.3. FUTURE DEVELOPMENTS

At this point, possible ideas for future work on the topic discussed during this work are discussed. The development of previous works and the current thesis led to the consideration of possible lines of research and future work.

4.3.1. ENHANCEMENTS ECO-DRIVING FOR HIGH-SPEED OPERATIONS UNDER CLIMATIC INFLUENCE

Future research could explore how high-speed train eco-driving strategies can be further optimized by incorporating real-time weather data and environmental variability. Current models could be extended to include uncertain or fluctuating meteorological parameters. By integrating on-board sensors and improved data acquisition, ATO systems could dynamically adjust speed profiles to minimise energy usage without compromising punctuality. Furthermore, computational advances will support faster recalculations of speed commands in response to weather changes, allowing the control strategy to evolve continuously during the journey.

4.3.2. METROPOLITAN OPERATION AND REGENERATIVE ENERGY OPTIMISATION

For metro and suburban railway systems, future research could focus on refining energy-saving strategies through regenerative braking optimisation. This involves not only optimising driving profiles or maximising regenerated energy between trains, but also enhancing power flows and overall energy efficiency through the integration with other technologies. In particular, research could also expand into system-level energy management, where energy flows are directed not only between trains but also toward stationary storage systems or auxiliary station services. The increasing deployment of communications systems as CBTC allows more flexible and responsive driving strategies, which can be further leveraged by multi-objective optimisation to balance energy, punctuality and headway constraints in real-time.

4.3.3. ENERGY-EFFICIENCY, CAPACITY OPTIMISATION AND VIRTUAL COUPLING

While the optimisation of railway capacity has traditionally focused on increasing train frequency and minimising conflicts, future developments may increasingly consider energy-efficiency constraints in the formulation of operational capacity models. Given the limited infrastructure in many metropolitan and intercity lines, a promising line of research is the joint optimization of capacity and energy consumption, especially under high-density traffic scenarios.

Modern capacity assessment techniques based on timetable compression (UIC 406) allow for a detailed representation of track occupancy, signalling constraints, and rolling stock performance. These approaches can be extended to explicitly incorporate energy-related parameters, such as braking and traction power demands, regeneration opportunities and dynamic headways dependent on train energy profiles.

In this context, the concept of virtual coupling (VC) emerges as a potentially disruptive technology. By enabling reduced headways through continuous communication between trains, VC can unlock additional capacity without the need for major infrastructure expansion. However, the dynamic coordination required by VC also presents a unique opportunity to synchronise traction and braking phases of neighbouring trains to maximise the transfer or storage of regenerative energy.

Thus, future work could explore multi-objective optimisation models that simultaneously aim to:

- Maximise capacity utilisation.
- Minimise total energy consumption.
- Enhance regenerative energy use through coordinated control of virtually coupled trains.

These models could build upon existing simulation tools and real-time traffic management algorithms, offering a path forward for sustainable and high-capacity railway operations.

4.3.4. REAL-TIME SIMULATION, DECISION SUPPORT AND ALGORITHMIC ENHANCEMENT

Further advancements can be made in the speed, robustness and adaptability of simulation tools and real-time optimisation algorithms used in railway operations. The integration of real-time data input, including train diagnostics, weather information and network traffic, would allow dynamic re-optimisation of train trajectories during commercial service. Moreover, on-board computing systems could benefit from algorithmic improvements, such as metaheuristic acceleration, parallel processing or reduced-order modelling, to produce rapid solutions that are deployable in operational settings. These improvements would be critical for real-time ATO decision-making, dispatching support systems, and adaptive energy-efficient control across varying traffic scenarios.

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