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# A multi-objective algorithm for train driving energy reduction with multiple time targets

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**Abstract:** Eco-driving is one of the most promising methods to reduce the energy consumption of existing railways. Considering the practical situation in complex railway lines, this paper proposes a new multi-objective searching algorithm to obtain the set of most efficient speed profiles in train journey for each combination of arrival and intermediate times. This algorithm makes use of the Particle Swarm optimization principles. However, a criterion of minimum energy consumption for a combination of objective arrival and passing times is applied to avoid the gaps that could appear in Pareto fronts. The multi-dimensional set of speed profiles obtained by means of the algorithm proposed can help railway operators to make better decisions when designing timetables. In the simulation, variations of 25 % can be observed in the energy consumption of two speed profiles with the same arrival time but with different passing times.

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**Keywords:** Energy efficiency, train simulation, eco-driving, multi-objective, Particle Swarm optimization, speed profile, operational restrictions

## 1. Introduction

Despite of being one of the most energy efficient transport modes, railways are subject nowadays to processes of energy reduction worldwide. These changes in the railway sector are motivated by the rising energy prices and the climate crisis. The latter is

reflected in the Paris agreement of 2015 where energy reductions goals and specific CO<sub>2</sub> emissions reduction goals were established for the railway sector (IEA and UIC, 2016).

The problem of energy reduction in railways has been widely study in literature. The methods proposed could be divided in three general groups (X. Yang et al., 2016): methods related to the rolling stock (Beusen et al., 2013; Matsuoka and Kondo, 2014), methods related to infrastructure (Liu et al., 2018; Sousa et al., 2019) and methods related to the traffic operation (Abril et al., 2008; Fay, 2000; Huang and Li, 2017; Peña-Alcaraz et al., 2011). The methods related to the traffic operation is more popular since it allows providing great energy reduction results in the short-term, being applied in railways lines already built, and using low investments.

Among the traffic operation methods, speed profile optimization, also known as eco-driving, stands out as one of the most important and most studied methods. A variety of studies and real life applications consistently show that important energy reduction figures can be obtained by means of the efficient driving in railways (Douglas et al., 2015). The objective of the speed profile optimization problem is to find the driving commands that a train must perform for a safe, punctual, comfortable, and energy-efficient train operation.

The first speed profile optimization work was developed by Ichikawa (Ichikawa, 1968). In this work, the Pontryagin's Maximum Principle was applied to a very simplified train model to obtain the optimal regimes of the train control. Later, the research developed by Howlett (Howlett, 1996) demonstrated that the solution of the eco-driving problem is unique and that in a flat track, it consists in a sequence of a maximum acceleration period, a cruising period, a coasting period and maximum braking period.

Since then, many researchers have continued this research line proposing more detailed train models taking into account regenerated energy (Khmelnitsky, 2000) or

variable motor efficiency (Franke et al., 2000). Track details have also been taken into account by means of variable grades and speed limits (Liu and Golovitcher, 2003). The optimal way to negotiate steep grades was obtained in (Howlett et al., 2009). As details were incorporated in the model, the complexity of the optimal solution increased.

Many techniques have been applied to solve the speed profile optimization problem. Among them, it can be found constructive algorithms (A. R. Albrecht et al., 2013; Howlett et al., 2009; Khmelnitsky, 2000; Liu and Golovitcher, 2003; Su et al., 2016, 2013; J. Yang et al., 2016), Dynamic Programming (T. Albrecht et al., 2013; Lu et al., 2013; Miyatake and Ko, 2010; Miyatake and Matsuda, 2009), Sequential Quadratic Programming (Gu et al., 2014; Miyatake and Ko, 2010), Lagrange multiplier method over the discretised problem (Rodrigo et al., 2013) and pseudo-spectral methods (Wang et al., 2014, 2013). Furthermore, it can be found artificial intelligence techniques to solve this problem such as: Artificial Neural Networks (Acikbas and Soylemez, 2008; Chuang et al., 2009), genetic algorithms (GA) (Bocharnikov et al., 2010; Chang and Sim, 1997; Jia et al., 2018; Lechelle and Mouneimne, 2010; Li and Lo, 2014; Sicre et al., 2012; Wong and Ho, 2004, 2003; Yang et al., 2012), Simulated Annealing (SA) (Keskin and Karamancioglu, 2017; Xie et al., 2013), Ant Colony Optimisation (ACO) (Ke et al., 2012; Yan et al., 2016) or Differential Evolution (DE) (Kim et al., 2013).

This paper proposes a new searching algorithm for obtaining the set of most efficient speed profiles to be performed by trains in stretches between stations by taking into account the effect of intermediate target times. The objective of this algorithm is to calculate the set of solutions that perform the minimum energy consumption for the possible combination of arrival and intermediate passing times. This algorithm improves existing multi-objective approaches taking into account not only energy consumption and running time but also the passing time at intermediate target positions. Furthermore, it

includes mechanisms to allow the inclusion of energy efficient solutions that are dominated, making possible the selection of speed profiles that perform the exact intermediate and arrival times. This algorithm has been developed in the basis of Multi-Objective Particle Swarm Optimization (MOPSO) algorithm. The main contribution of this paper is described as the following.

- The eco-driving problem is proposed formulating the speed profile optimization problem as a multi-objective optimization problem such that the decision-maker can consider the trade-offs in energy consumption and running time by using the two-dimensional Pareto front (Carvajal-Carreño et al., 2014; Chevrier et al., 2013; Domínguez et al., 2014; Fernández-Rodríguez et al., 2018b, 2015).
- The intermediate passing target times in complex railway lines which are used to ensure the passing order of crossing trains or the separation of trains in critical positions of the infrastructure (T. Albrecht et al., 2013) are considered and taken as objectives in the optimization model.
- A new algorithm called “Eco-driving Particle Swarm Searching Algorithm” (EPSSA) is proposed that is combined with a detailed simulation model. The Pareto selection criterion has been substituted by a criterion of minimum energy consumption for a combination of objective arrival and passing times. And a smooth set of the most efficient speed profiles is obtained avoiding the gaps that could appear in Pareto front.

The paper is structured as follows: Section 2 presents the driving model used as the basis of the speed profile solutions, Section 3 explains the algorithm proposed to obtain the set of efficient speed profiles, Section 4 introduces the simulation model used to evaluate the possible solutions during the algorithm execution, Section 5 presents the results obtained in this piece of research and Section 6 details the main conclusions.

## 2. Driving model

The solutions obtained by the proposed algorithm are speed profiles characterized by a set of driving commands which are presented in the algorithm by means of the Command Matrix ( $C_m$ ) scheme proposed by Sicre et al. (Sicre et al., 2012). This scheme results in a speed profile where the driving commands of the train are maximum motoring, cruising, coasting, and maximum service deceleration. This follows the optimal driving modes derived from the optimal control theory as shown in (Su et al., 2013).

$C_m$  divides the journey into  $n_s$  sections where the train applies a certain driving command. The first  $n_s - 1$  sections are defined by a value of holding speed without braking command. In other words, in these sections the train has as objective to reach and maintain the target speed given by the commands but only motoring. However, if braking is demanded to maintain the target speed (for instance during steep downhills), the train will apply coast (null traction). This way, the train takes advantage of the gravitational force to increase its speed reducing the running time without increasing the energy consumed. The last section ( $n_s$ ) is defined by a coasting command that must be fulfilled up to the braking curve to stop at the station. The orders collected in  $C_m$  are limited by the braking curves. If the train needs to brake to observe a speed limitation and the driving command demands coast or traction, the train will apply the corresponding service brake (see Figure 1).

This figure shows in solid line the speed profile of the train, in dot-dashed line the traction force (positive values) or braking force (negative values) provided by the train motors, in dotted line the holding speed without braking command at each position, in dashed line the speed limitations and the grey areas present the track profile.

As can be seen, between 0 and 35 km the train needs to apply traction to achieve and maintain the speed command of 250 km/h. Between 35 and 55 km there is a downhill

and the train would need to apply braking to maintain 250 km/h. However, the train applies coasting to speed-up without energy consumption as can be seen in the null value of the motor force. Later, between 55 and 70 km the train applies a certain amount of braking force not to overcome the maximum speed limitation. From 70 km, the track is almost flat so the train applies coasting to reduce its speed until the 250km/h command and, in that moment, it applies traction to hold its speed. After that, it coasts when the driving command is 0 and brakes when crosses the braking curve up to the station.

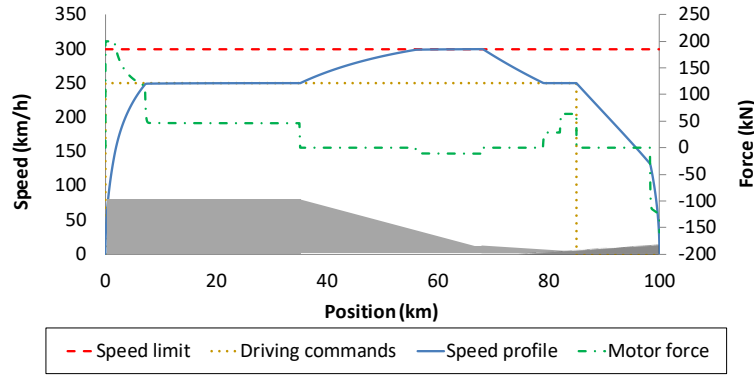


Figure 1. Train journey using a 250 km/h holding speed without braking command and a coasting command before the final braking

The command matrix  $C_m$  is as a matrix of  $n_s - 1$  rows and 2 columns (see Figure 2). The first column ( $sc_k$  values) contains the points of the track (position) where the sections end. The second column represents the values of the holding speed without braking command ( $vc_k$ ) that will be applied in that section.

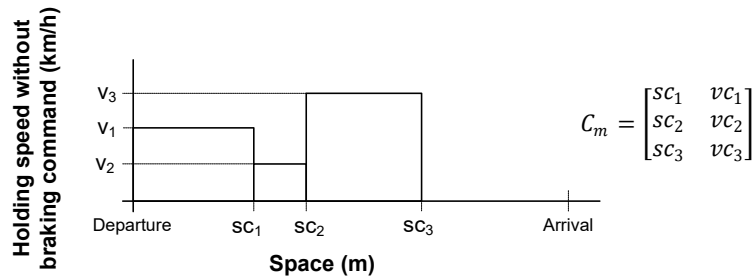


Figure 2. Command matrix of four sections

The value of  $n_s$  must be established by the railway operator depending on the journey length, the speed limitation profile and the number of intermediate target points. The railway operator must take into account also comfort criteria to reduce the frequency of changes in the driving commands that can be perceived unpleasant for passenger or stressful for drivers.

### 3. Eco-driving Particle Swarm Searching Algorithm

The algorithm proposed has the objective of obtaining the set of speed profiles with the lower energy consumption for each combination of arrival and intermediate passing times. This algorithm follows the framework of the MOPSO algorithm proposed in (Coello et al., 2004). However, the domination criteria is substituted by an energy efficiency criterion in order to not discard dominated solutions. These dominated solutions are included in the optimal set to provide a smooth and a complete set of eco-driving speed profiles.

In the EPSSA, as well as the MOPSO, the velocity of the particles (*pbest*) and its position (*gbest*) are updated at each iteration. Particularising the notation for the problem of speed profile optimization, the notation is as follows: The  $j^{th}$  particle of the swarm in an iteration is a specific set of driving commands and, therefore, a specific speed profile with a result of arrival time, intermediate passing times and energy consumption. It is represented by a matrix  $X_j(it)$  in iteration  $it$  and contains the elements of a specific  $C_m$ . The velocity of the  $j^{th}$  particle in iteration  $it$  is  $V_j(it)$  and has the same dimensions as  $X_j(it)$ . The best previous position of the  $j^{th}$  particle is  $pbest_j$  and the best combination of commands in the swarm is *gbest*.

In successive iterations, the particle's new velocity is calculated using its previous velocity (weighted by an inertia factor), and the distances from its current position to its



$pbest$  and to the  $gbest$  positions (weighted by social factors) as shown in Eq. 1. The new position of the particle is calculated with Eq. (2).

$$v_j(it) = w \cdot v_j(it - 1) + c_1 \cdot r_1 \cdot (pbest_j - x_j(it - 1)) + c_2 \cdot r_2 \cdot (gbest - x_j(it - 1)) \quad (1)$$

$$X_j(it) = X_j(it - 1) + V_j(it) \quad (2)$$

where,  $i = 1, 2, \dots, n_{swarm}$ ;  $it = 1, 2, \dots, it_{max}$ . The parameter  $n_{swarm}$  is the size of the swarm, and  $it_{max}$  is the iteration limit;  $c_1$  and  $c_2$  are positive constants (called “social factors”), and  $r_1$  and  $r_2$  are random numbers between 0 and 1;  $w$  is inertia weight that controls the impact of the previous history of the velocities on the current velocity, influencing the trade-off between the global and local experiences.

The selection of  $pbest$  and  $gbest$  are based on the particle’s results in the  $no$  dimensions of the search space which are the passing times at the intermediate target points, the arrival time and the energy consumption in this paper. To improve the typical MOPSO algorithm, the proposed EPSSA discards the dominance principle to rank the solutions found. An energy consumption principle combined with a mechanism based on reference points is used instead. This principle prioritizes the solutions with the minimum energy consumption for each combination of target times. EPSSA creates a set of reference points evenly distributed in the intermediate and arrival time dimensions of the searching space to compare solutions and rank them. The reference points are created within the practical bounds of the journey considered. That is, the values of these points will be greater or equal to the times obtained by the flat-out driving (i.e. the fastest possible speed profile) and lower than or equal to the times performed by the slowest possible speed profile (i.e. the speed profile that performs the driving commands with the lowest value of holding speed). Furthermore, the separation of the reference points at each

time dimensions will be a constant value  $\Delta\sigma$ . This parameter determines the desired separation between speed profiles.

EPSSA ranks the positions found by particles and includes the best in the optimal set using the reference points explained before. First of all, each solution is associated to a reference point calculating its Euclidean distance to each reference point and selecting the closest one. Once each solution is associated to a reference point, it can be calculated the fitness of all solutions comparing all the solutions by pairs. The fitness of a solution in the proposed algorithm is calculated as the number of solutions that are associated to the same reference point and has a lower energy consumption. Thus, if two solutions are associated to different reference points it cannot be said what solution is better. However, if a solution is compared with another solution associated to same reference point, the best solution will be the one with the lowest energy consumption. For a given reference point, the solution with the lowest energy consumption will have a fitness value of 0 as no other solution associated to that reference point has lower energy consumption.

EPSSA maintains an external archive  $A$  where the solutions with zero fitness value are contained (i.e. the solutions with the minimum energy consumption per each reference point). The archive is updated at each iteration calculating the fitness of the swarm positions and the positions stored in  $A$  by comparison of all of them by pairs.

Besides, a niche count mechanism is applied to ensure the diversity in the optimal set. Niche count mechanism allows to know how crowded the region around a solution of the archive  $A$  is. This mechanism consists in calculating the number of solutions that falls into a hypersphere with a fixed radius ( $\theta$ ) determined by the user after normalising the results of the solution. The normalization of the results is carried out using Eq. (3).

$$Vnorm_j = \frac{Var_j + Vmin}{Vmax - Vmin} \quad (3)$$

where  $Vnorm_j$  is the normalized variable and  $Var_j$  is the value of the variable before the normalization of solution  $j$ .  $Vmax$  and  $Vmin$  which are taken from the speed profile with the minimum and maximum possible running times are the maximum and the minimum possible values for the objective.

EPSSA selects the *gbest* for the swarm with the Top Select probability from among the best positions in  $A$  with the lowest niche count values. This way, the swarm is incentivized to move towards the least crowded regions in the objective space.

On the other hand, the update of *pbest* is carried out at each iteration by comparison of the current position of each particle with its own *pbest*. If the particle's *pbest* and current position are associated to the same reference point, the new *pbest* will be the one with the lowest energy consumption. On the contrary, if the particle's *pbest* and current position are associated to a different reference point, the new *pbest* will be selected randomly between them with equal probability.

### 3.1 EPSSA flow diagram

1. The algorithm begins with the initialization process. An empty external archive  $A$  and the set of the reference points are generated. Then, a swarm of  $n_{swarm}$  combinations of commands is created. The particle positions ( $X_j$ ) and velocities ( $V_j$ ) are initialised randomly. The driving commands of the particles are always maintained within the minimum and maximum holding speed values ( $vc_{min}$  and  $vc_{max}$  respectively) after each update.
2. After that, the result of each particle position in the objective space (energy consumption, arrival and passing times) are evaluated using the simulator explained in Section 4 and the driving commands of each particle as input data.

3. The particles obtained are associated to a reference point. Particles' position and positions stored in  $A$  are compared by pairs to calculate the fitness of all of them. With the new value of fitness, the archive  $A$  is updated adding the particles' positions with fitness equal to 0 and removing the positions stored with fitness greater to 0.
4. The  $pbest$  is updated comparing the current position of each particle with its own  $pbest$ . In the first iteration, as there is no  $pbest$ , it is set to the current position of each particle.
5. The niche count is calculated and the positions stored in  $A$  are sorted in increasing order of niche count.
6. The  $gbest$  is selected randomly from the Top Select portion with a Top Select Probability. Otherwise, it is randomly selected from the remaining portion of  $A$ .
7. At each iteration  $it$ , the velocities and the positions of the particles are updated using equations (1) and (2), respectively.
8. The process is repeated from point 2 until the maximum number of iterations is reached.

The EPSSA algorithm described is represented in Figure 3.

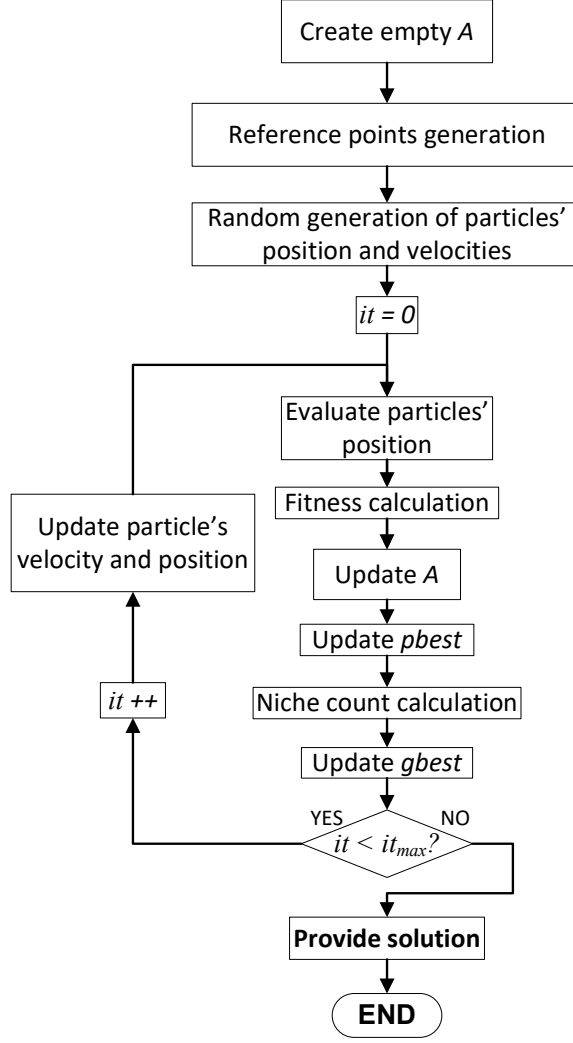


Figure 3. EPSSA diagram flow

#### 4. Train simulation model

The simulation model is used in combination with EPSSA to evaluate the results of the particles' position. The simulator receives as input the driving commands contained in the position of the particle, obtains the corresponding speed profile defined by the input  $C_m$  and returns the results in the energy and time dimensions.

This section briefly presents the simulation model which consists of the train module, the line module and the driving module. The detailed description of the simulation model and its validation can be found in (Fernández-Rodríguez et al., 2018a;

Sicre et al., 2012). The simulator was validated with real data measured from high-speed trains on commercial services and nocturnal tests on the Madrid–Barcelona high-speed corridor. An average error of 1.2% in running time and 0.4% in energy consumption was obtained comparing simulation results and the measurements. This accurate simulator was used in a project developed in collaboration with Renfe and Adif for the design of efficient speed profiles for this high-speed line and important energy consumption reduction figures were measured (Sicre et al., 2012).

A closed-loop proportional-integral speed regulator performs the control of the train. The variable  $U_{reg}(t)$  is the tractive/braking effort demanded by the control and it is obtained at each simulation step using the following expressions:

$$e(t) = v_{obj}(t) - v(t) \quad (4)$$

$$U_{prop}(t) = K_p \cdot e(t) \quad (5)$$

$$U_{int}(t) = \frac{K_p}{K_I} \cdot \frac{(e(t) + e(t - ts))}{2} + U_{int}(t - ts) \quad (6)$$

$$\begin{aligned} U_{reg}(t) &= U_{prop}(t) + U_{int}(t) && \text{if } v(t) \geq v_{max}(t) \text{ or } U_{prop}(t) + U_{int}(t) \geq 0 \\ U_{reg}(t) &= 0 && \text{if } v(t) < v_{max}(t) \text{ and } U_{prop}(t) + U_{int}(t) < 0 \end{aligned} \quad (7)$$

where  $U_{prop}(t)$  is the proportional contribution to  $U_{reg}(t)$  and  $U_{int}(t)$  is the integral contribution to  $U_{reg}(t)$ .  $K_p$  and  $K_I$  are the proportional and the integral constants. The error of regulation  $e(t)$  is calculated as the difference between the objective speed  $v_{obj}(t)$  and the current speed of the train  $v(t)$ . The constant  $ts$  is the simulation time-step and  $e(t - ts)$  is the error of regulation in the previous simulation step. The minimum value for the total tractive/braking effort demanded is set to 0 when the train speed is below the maximum  $v_{max}(t)$  to perform the holding speed without braking driving

command (Eq. (7)).

Several limitations are applied to the value of  $U_{reg}(t)$  to no surpass the train motors characteristics and comfort limitations introduced by the maximum jerk allowed. Thus,  $F_m$  is obtained using the maximum traction/braking curves as boundaries for  $U_{reg}(t)$  and limiting its variations by taking into account the maximum jerk limitation. The objective speed  $v_{obj}(t)$  is derived from the driving commands contained in the input  $C_m$  and the maximum speed  $v_{max}(t)$  at each simulation step.

Once the demand for the motors is obtained, the simulation model calculates the train acceleration at each time step using Eq. (8).

$$M_{eq} \cdot a(t) = F_m - (F_r(v) + F_g(s)) \quad (8)$$

where  $M_{eq}$  is the train mass including the effect of rotary inertia,  $t$  is the time,  $v$  is the speed of the train,  $s$  is the position of the train,  $a(t)$  is the train acceleration,  $F_m$  is the motors force, and  $F_r(v)$  is the running resistance.  $F_g(s)$  includes the gravitational force caused by the track grades and the resistance caused by the track bends.

The running resistance is calculated with the Davis' formula at each simulation step as shown in Eq. (10):

$$F_r(v) = A + B \cdot v + C \cdot v^2 \quad (9)$$

The resistance force produced by the track profile is obtained using Eq. (10).

$$F_g(s) = g \cdot m \cdot p(s) \quad (10)$$

where  $g$  is the gravity acceleration,  $m$  is the mass of train and  $p(s)$  is the average equivalent track grade affecting the complete length of the train.

The simulator derives the train speed and position at the next simulation step using the acceleration value calculated and the uniformly accelerated motion equations (11) and (12).

$$\frac{ds}{dt} = v(t) \quad (11)$$

$$\frac{dv}{dt} = a(t) \quad (12)$$

After solving the train dynamics, the simulation model calculates the train energy consumption by using the following equations:

$$P_{mec} = F_m \cdot v \quad (13)$$

$$P_p = \frac{P_{mec}}{\eta} + P_{aux} \quad \text{If } P_{mec} \geq 0 \quad (14)$$

$$P_p = P_{mec} \cdot \eta + P_{aux} \quad \text{If } P_{mec} < 0$$

where  $P_{mec}$  is the mechanical power,  $P_p$  is the electrical power at pantograph,  $P_{aux}$  is the power consumed by auxiliary equipment and  $\eta$  is the efficiency of the electrical chain that is a function of the train speed and the ratio of the motor force divided to the maximum motor force.

Finally, the simulation model calculates the increment of energy consumed by the train measured at pantograph as a time integral of  $P_p$ . Furthermore, the estimation of the effect of the energy consumed by the train at substation is calculated adding the line power losses to  $P_p$ .

## 5. Case study

The proposed EPSSA algorithm combined with the train simulation model have been



applied to a realistic case study using data from a Spanish high-speed line. The stretch analysed runs between two consecutive stations (Calatayud to Zaragoza) and has a length of 85.4 km. The train data used is taken from the Talgo-Bombardier class 102 high-speed train. This train has two motors of 8 MW and 200 kN of maximum traction effort. The train length is 200 m and the train mass is 324 t. The operational restrictions for EPSSA to generate possible sets of driving commands are set to  $v_{c_{min}} = 150$  km/h to avoid too low speed phases in the middle of the journey, and  $v_{c_{max}} = 300$  km/h that is the maximum speed allowed in the journey.

The case study includes a train travel with one intermediate timing point besides the arrival timing point. The intermediate timing point is located at the kilometric point 264 km, between the departure point (221.3 km) and the arrival point (306.7 km). Therefore, the solution space will have 3 dimensions: The arrival hour at 306.7 km, the passing hour at 264 km and the energy consumed by the train at the end of the journey. Figure 4 shows the track profile of the journey as well as the speed limitation profile and the neutral zones where the train has no power.

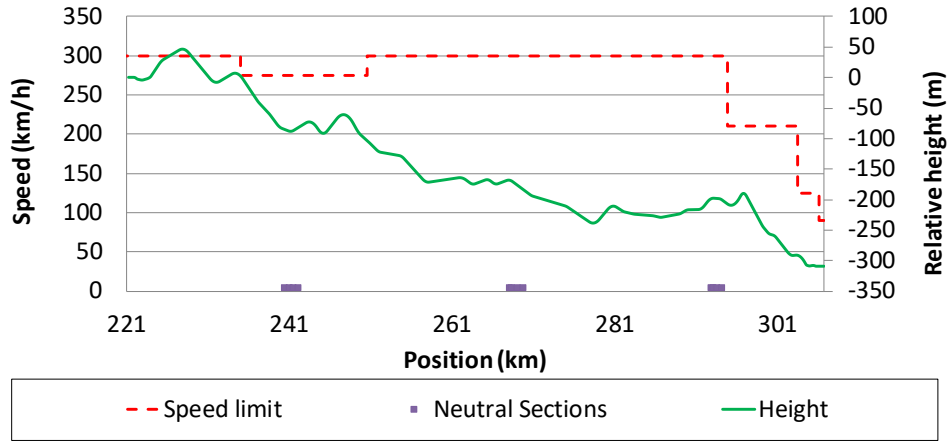


Figure 4. Track profile, speed limitation profile and neutral zones of the case study

Table 1 presents the tuned parameters of the algorithm. These parameters have been tuned by trial and error as usual in population-based algorithms.

Iterations ( $it_{max}$ )	Swarm size ( $n_{swarm}$ )	Social factor ( $c_1$ )	Social factor ( $c_2$ )	Inertia weight ( $w$ )	Niche count radius ( $\theta$ )	A Top select	A Top select probability
400	250	2	2	0.3	0.1	3 %	99 %

Table 1. Tuned parameters of EPSSA

The distance between reference points ( $\Delta\sigma$ ) is set to 10 s. This distance will result in the approximate distance between the solutions obtained in the time dimension so it seems that 10 s is an adequate separation given that the minimum possible running time for the journey is 1320 s and the maximum is 2476 s.

### 5.1. Convergence analysis

Several metrics have been proposed in literature to measure the convergence of multi-objective optimization algorithms results with the known optimal solution (Jiménez et al., 2013) or with the designed discard dominated solutions such as the hyper-volume (While et al., 2012), which are not valid to measure the results of EPSSA. Therefore, a new metric has been proposed to compare the results of a set of efficient speed profiles against others to study the convergence of EPSSA results. This metric takes advantage of the set of reference points created for the algorithm execution to calculate the proximity of the solutions to the reference points.

The convergence metric associates to each reference point the energy consumption of the nearest solution contained in the archive  $A$ . If no solution is associated to the reference point, the energy consumption of this reference point will be equal to the energy consumption of the flat-out driving. Finally, the value of the convergence metric is calculated by the summation of the energy consumption associated to all the reference

points. Therefore, the lower the value of the metric the closer to the best set of speed profiles.

Figure 5 presents the evolution of the metric during the iterations of the algorithm for the case study presented. As can be seen, the value of the proposed convergence metric decreases at each iteration of the algorithm execution. That means that the solution is improved through the EPSSA iterations. The value decreases dramatically during the first 100 iterations. After that, between 100 and 300 iterations the convergence metric reduction is slower. Finally, between 300 and 400 iterations the reduction ratio is minimum, which shows the algorithm has converged to the best solution.

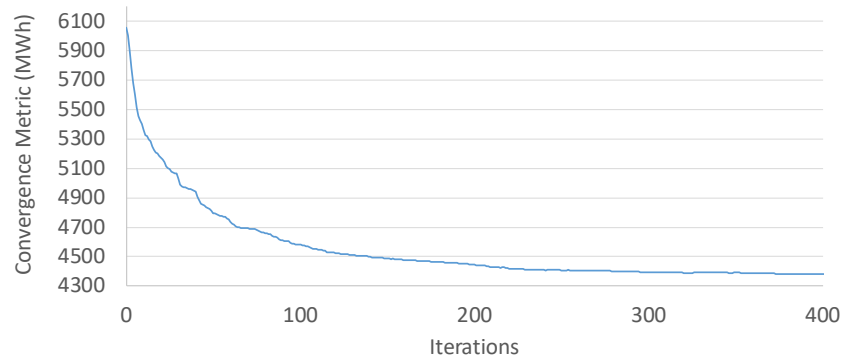


Figure 5. Evolution of convergence metric for EPSSA

Figure 6 presents the set of efficient speed profiles obtained in the three dimensions considered (intermediate passing time, arrival time and energy consumption) as a result of EPSSA execution at the end of the 400<sup>th</sup> iteration.

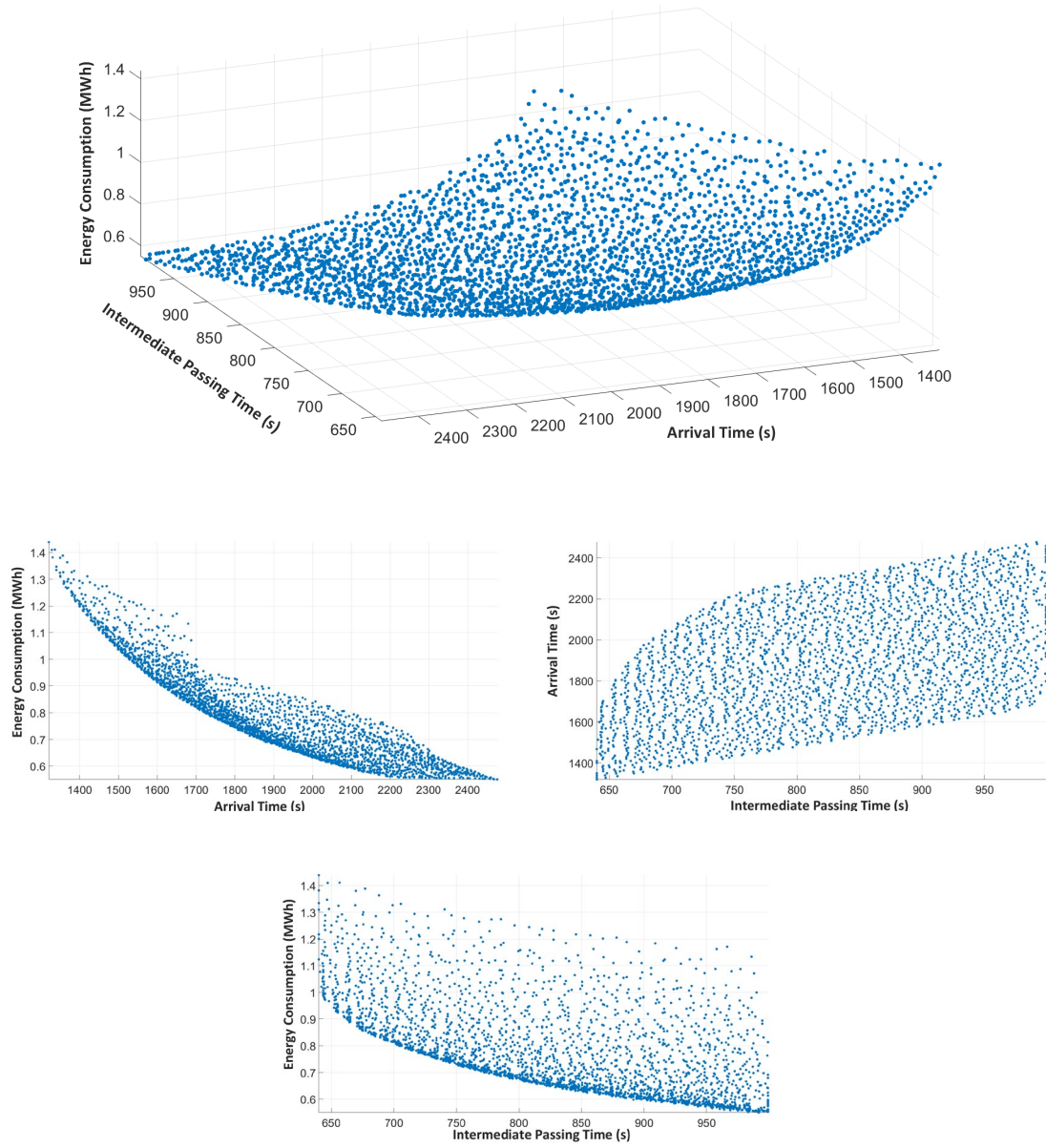


Figure 6. Results of the algorithm

The set of speed profiles obtained by the algorithm are well distributed in the time space made of arrival and intermediate passing time dimensions. Furthermore, they form a front that smoothly changes the energy consumption when moving through the time dimensions.

It can be observed that the energy consumption tends to increase as lower is the arrival time. For the same value of intermediate passing time, the results show important differences in energy consumption. This is caused because, for the same intermediate passing time, it can be found a wide variety of solutions with different values of arrival time and the running time for the whole journey has a capital influence in the energy consumption determination. However, if we consider the intermediate passing time, the results also show important differences in the energy consumption obtained for the same arrival time. These variations in energy consumption are lower because the possible combinations of arrival and intermediate passing time are reduced when the running time is close to the minimum.

The optimization computation cost depends on the number of particle position evaluations and the number of iterations. Each particle evaluation takes 0.15 s on average because of the detailed simulation performed. The computational cost of the whole swarm evaluation is 37 s on average at each iteration. Compared to this, the computational cost of the remaining optimization tasks of the algorithm can be neglected.

## ***5.2. Speed profile analysis***

The results obtained by EPSSA can be used to design the set of driving commands that the train must perform in a journey to fulfil a target arrival and intermediate passing times.

The nominal trip time for the complete journey of the case study is 26 minutes (1560 s). There are 20 solutions in the efficient set of speed profiles obtained by EPSSA which arrival time is lower or equal to 1560 s with a maximum difference of 5 s. From these solutions close to the nominal running time, it can be extracted three main representatives: the speed profile with the minimum energy consumption, the speed

profile with the minimum intermediate passing time and the speed profile with the maximum intermediate passing time (see Table 2).

	Intermediate passing time (s)	Arrival time (s)	Energy consumption (MWs)	Difference with the minimum energy consumption
Minimum energy consumption speed profile	725.0	1559.5	0.962	0 %
Minimum intermediate passing time speed profile	653.5	1559.0	1.0174	5.8 %
Maximum intermediate passing time speed profile	878.0	1558.0	1.2055	25.3 %

Table 2. Results of representative speed profiles for 1560 s running time

All the speed profiles allow arriving on time with the nominal running time. However, depending of the traffic conditions a speed profile can be chosen instead of another to ensure a determined passing time. This will lead to a different energy consumption result as can be seen in Table 2. It is necessary to pass through the intermediate target at 725 s to achieve the minimum energy consumption. However, if it is demanded to pass early through the intermediate target, the train could advance the passing up to 72 s early, which would lead to increase the energy consumption by 5.8%. Similarly, if it is necessary to delay the passing at the intermediate target, the train could pass up to 225 s of delay but with a 25.3 % growth in energy consumption. Figure 7 presents the three speed profiles explained before:

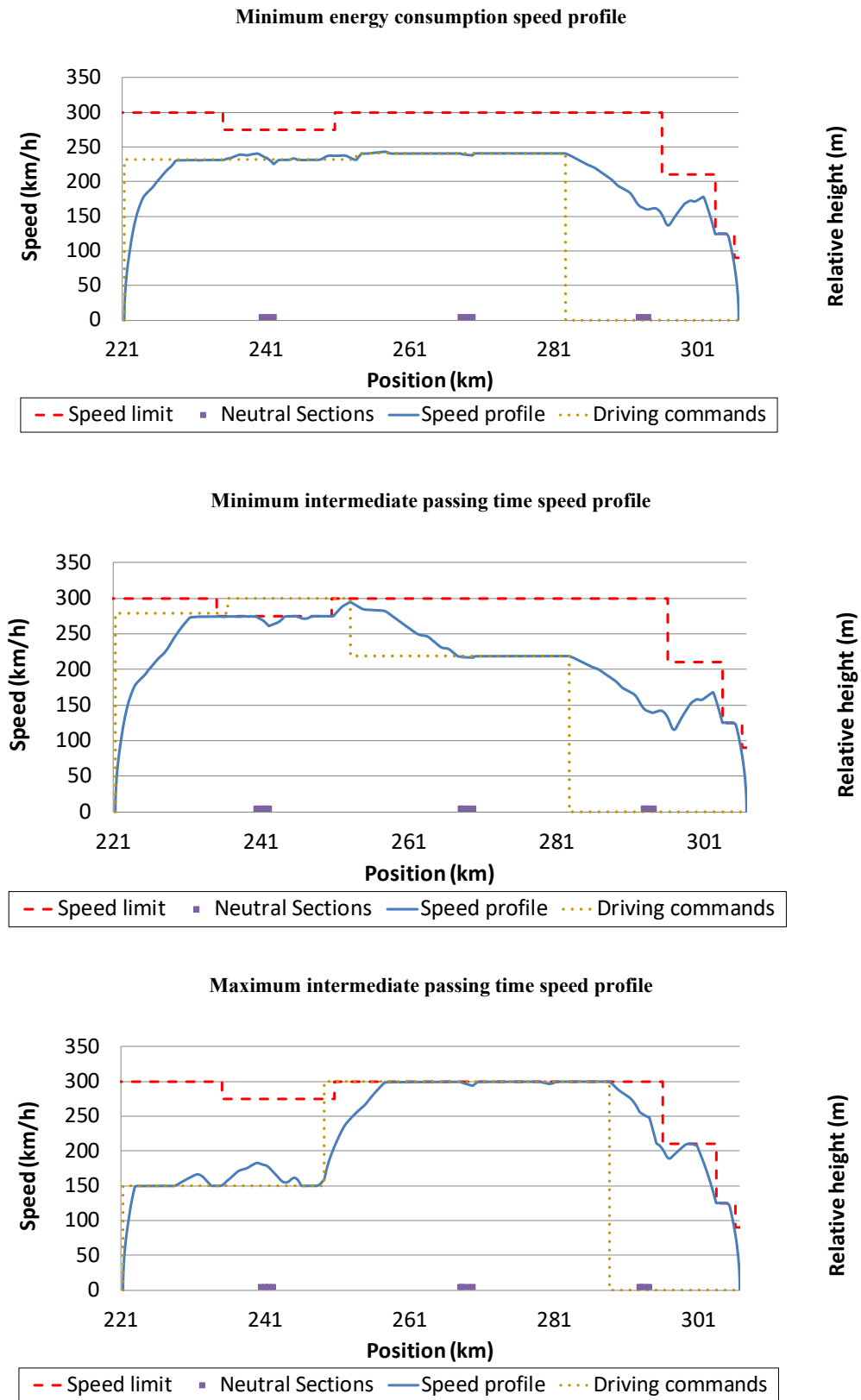


Figure 7. Representative speed profiles for 1560 s running time

The minimum energy consumption speed profile presents stable driving commands between 230 and 240 km/h up to the coasting period in 282 km. However, the speed profile presents more important speed variations in the driving commands of the minimum intermediate passing time example. The train drives practically at maximum speed until it starts coasting to reach a command of 220 km/h in this example. Later, it starts to coast in a similar position than in the minimum energy consumption example. By this way, it arrives to the intermediate target in a short time and reduces the speed of the train in the second half of the journey to maintain the global running time. The relevant speed variations that experiment the train during the minimum intermediate passing time example makes that the energy consumption grows respect to the most efficient speed profile.

On the other hand, the variations of the driving commands are more important in the maximum intermediate passing time example. This speed profile starts the journey with the minimum driving command of 150 km/h. However, in the 250 km position, it starts to accelerate to drive the train at the maximum speed of 300 km/h in order to perform the target arrival time. The journey finish starting a coasting period at 289 km position up to the final braking stage. The bigger speed variations in the driving commands of this example explains why this is the most energy consuming case for the running time of 1560 s.

## **6. Conclusions**

Taking the intermediate target times into consideration, a new multi-objective searching algorithm, called EPSSA, has been proposed in this paper to obtain the set of most efficient speed profiles in train journey with intermediate target times to meet. This algorithm contributes to the existing multi-objective speed profile optimization models including passing times at intermediate target positions as additional objectives besides



energy consumption and running time. Railway operators can select the best combination of passing times at target positions using the results from the algorithm proposed taking into account possible trade-offs among solutions.

This algorithm is built in the basis of particle swarm optimization algorithm. However, as a difference of other multi-objective algorithms, EPSSA does not ignore dominated solutions. A criterion of minimum energy consumption for a combination of objective arrival and passing times is applied, instead of the typical Pareto selection criterion, with the objective of obtaining the most efficient speed profile for each possible combination of arrival and intermediate passing times. The result obtained by this algorithm is a smooth and complete set of most efficient speed profiles avoiding the gaps that could appear in Pareto fronts because of the presence of dominated solutions.

The proposed EPSSA has been applied to a case study using real data from a Spanish high-speed corridor. The result obtained by the algorithm is a well distributed set of speed profiles in the time dimensions of the problem. Furthermore, the set obtained does not present gaps and the variations in energy consumption are smooth when moving in the time dimensions. A new metric has been proposed to evaluate the convergence of the algorithm. This metric has allowed to determine the convergence speed of EPSSA.

The analysis of the efficient set of solutions allows to evaluate the cost in energy consumption when selecting a specific combination of target times. Variations of 25 % can be observed in the energy consumption of two speed profiles with the same arrival time but with different passing times. These differences are reduced in low running time regions because the possibilities of different speed profiles are reduced.

The multi-dimensional set of efficient speed profiles obtained by means of EPSSA can also help operators to make better decisions when designing efficient timetables.

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