Efficient automatic operation of a metro line: eco-driving design with optimised use of regenerative energy and rolling stock consideration

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Abstract

Metropolitan railways are major consumers of energy and are potential targets for the application of energy efficiency techniques. In this paper, eco-driving, timetable design and regenerative braking are integrated to optimise operation while minimising energy consumption. In addition, the rolling stock required to provide the service is considered, as it has an important impact on operational costs. First, the design of efficient ATO (Automatic Train Operation) driving is carried out using a MOPSO (Multi-Objective Particle Swarm Optimization). Then the timetable is optimised including a regenerated energy model and the number of trains required to satisfy the periodic service. For the timetable design, several algorithms have been compared, proving that GLMO-NSGA-III (Grouped and Linked Mutation Operator – Non-dominated Sorting Genetic Algorithm III) is the best for the case study. The complete model has been applied to the Madrid Underground line, achieving energy savings of 24.79% compared to the typical operator's design criteria.

Keywords: energy-efficient train timetable, eco-driving, regenerative braking energy, Automatic Train Operation (ATO), rolling stock.

1. Introduction

In the current context of optimising energy efficiency and supporting the use of renewable energy, it is important to develop technologies that enable the rational and efficient use of resources. Europe is leading this change through emission reduction policies, creating a framework for action in this area. Among one of the major energy consumers is the transport sector. The rail sector is gaining attention given that it is one of the most efficient transport modes, and it is considered a keystone to reducing pollutant emissions in metropolitan areas. Metropolitan rail traffic is an electrified mode of transport and, despite its efficiency, is susceptible to the application of techniques to reduce energy consumption (Feng et al. 2013). Some of these methods are related to optimal operation, and they can be implemented in the short term, like the design of efficient driving or eco-driving, whether manual or automatic, and the design of the timetable. Automatic Train Operation (ATO) systems are also the desirable trend for rail systems because of the regularity in the train operation that provides capacity and safety improvements and the potential energy savings given by its ability to perform accurately the efficient driving commands, also known as eco-driving.

Eco-driving consists of executing a speed profile in an interstation to achieve the minimum energy savings for a specific running time (Yuan and Frey 2020). The most common strategies for the application of eco-driving in metro lines are speed regulation and the execution of cut-off traction and re-motor cycles with upper and lower speed limits.

The benefits of eco-driving implementation in mass transit lines have been demonstrated in several studies under different scenarios. Efficient driving achieved by GA (Genetic Algorithm) and PSO (Particle Swarm Optimisation) was validated in the case of the Istanbul metro achieving energy savings with manual operation between 20% and 30% (Yildiz, Arikan, and Keskin 2023b). In (Domínguez et al. 2012), the authors consider the regenerated energy and storage devices in the design of Automatic Train Operation (ATO) driving commands, obtaining a result of 11% energy savings. The ATO speed profiles search with MOPSO (Multi-Objective Particle Swarm Optimization) in (Domínguez et al. 2014) provides energy savings of 15% and it can be combined with robustness conditions to obtain driving commands considering mass variation (Fernandez-Rodriguez et al. 2015) or extended to include multiple time targets (Fernández et al. 2020). The authors in (Yildiz, Arikan, and Keskin 2023a) present a speed trajectory optimization using nature-inspired algorithms for the Istanbul M3 subway line considering a variable number of passengers. The algorithms used are GA, Simulated Annealing algorithm (SA), PSO and the Marine Predators Algorithm. The solution was tested in the real line achieving energy savings of 21.27%. In (S. Su et al. 2023), the speed curve of heavy-haul trains is optimized considering the cyclic air braking on long, steep downward slopes.

The first study related to the calculation of efficient driving applied to railways was carried out (Ichikawa 1968), applying Pontryagin's Maximum Principle. Ichikawa considers the analysis as a bounded state variable problem taking into account the speed limits and obtained for the first time the optimal regimes: maximum acceleration, holding speed, coasting and maximum deceleration. Applying the same principle, Howlett in (Howlett 1990) considers energy minimisation to find the optimal driving strategy. Different simulation methods and optimisation techniques are used applying the results of the Maximum Principle to solve different cases proposed in the literature.

Solutions from a constructive algorithm are achieved in (Howlett 1996) where coast and maximum power phases are alternate to obtain the optimal driving strategy. Variable speed restrictions, grade profile, traction and brake force and running time are constraints to optimal driving pattern calculation in (Khmelnitsky 2000) with the maximum principle analysis. In (Liu and Golovitcher 2003) the analytical solution offers the sequence of optimal train control. An optimisation and control of speed and dwell time are given in (Thomas Albrecht, Binder, and Gassel 2013).

Apart from constructive algorithms, different mathematical models have been applied to obtain the optimal solution for efficient driving such as Dynamic Programming (DP) (Thomas Albrecht, Binder, and Gassel 2013), Mixed-Integer programming (Zhou, You, and Fan 2020), Sequential Quadratic Programming (Miyatake and Ko 2010) or Lagrange Multipliers (Rodrigo et al. 2013). A mixed-integer non-linear programming problem is solved in (Y. Wang et al. 2014) minimising passengers travel time and traction energy consumption in urban rail transit system.

Previous methods obtain the analytical eco-driving solution but, when the model includes details such as automatic driving logic, the nonlinearities introduced require the application of different techniques. In contrast to mainline, mass transit lines are highly automated exploitations where the trains are automatically driven by ATO equipment. Therefore, it is important to develop simulation models that include ATO characteristics in eco-driving models for mass transit lines and to combine them with optimisation algorithms (Figueira and Almada-Lobo 2014) to obtain applicable solutions.

Artificial Neural Networks (ANN) are used in (Chuang et al. 2009) for optimising coasting speed in Mass Rapid Transit (MRT) systems while in (Açıkbaş and Söylemez 2008) are found the optimal coasting positions. GA have been frequently used for eco-driving as in (Bocharnikov et al. 2007) for a Direct Current (DC) metropolitan railway line, in (Chang and Sim 1997) to find the optimal coasting points and in (Lu et al. 2013) to calculate the efficient speed. In (T. Albrecht 2004), the GA is used to minimise energy consumption considering the headway, running time and dwell times. A multi-population GA is used in (Huang et al. 2015) to minimise traction energy taking into account driving strategy and trip time. Fuzzy logic (Bellman and Zadeh 1970) and GA can be combined to consider uncertainty in any of the parameters that affect railway operation. An example is presented in (Blanco-Castillo et al. 2022) where eco-driving is calculated and adapted to operational conditions considering variability in climatological conditions for a high-speed railway with a GA. NSGA-II (K. Deb et al. 2002) (Non-Dominated Sorting Genetic Algorithm II) is applied in (Dullinger, Struckl, and Kozek 2017) to optimise traction systems while, in (Carvajal-Carreño, Cucala, and Fernández-Cardador 2014), is combined with fuzzy logic to calculate ATO driving in metro lines considering variability in the load of passengers. In (Fernández, Fernández, and Cucala 2018) a dynamic version of NSGA-II was applied to re-calculate the train speed profiles during its journey. In (Sicre, Cucala, and Fernández-Cardador 2014) a fuzzy manual driving model and a GA are combined to solve eco-driving. Other alternatives to GA have been proposed in the literature to solve the energy minimisation problem in train driving such as Brute Force (Zhao et al. 2017), Monte Carlo (Metropolis and Ulam 1949) simulation in (Z. Tian et al. 2017), Differential Evolution in (Kim et al. 2013), Genetic Simulated Annealing in (Keskin and Karamancioglu 2017), Indicator Based Evolutionary Algorithm (Zitzler and Künzli 2004) (IBEA) in (Chevrier, Pellegrini, and Rodriguez 2013) and Ant Colony Optimisation (ACO) (Dorigo, Birattari, and Stutzle 2006) in (Lu et al. 2013).

In contrast to previous studies, approaches that combine timetabling and eco-driving problems can be found in literature to minimise the global traction energy consumption of traffic operations. Thus, the focus is on the complete line instead of minimising the consumption of each interstation separately. The running times between stations and dwell times can be optimised jointly with the eco-driving to spend the time where it is more beneficial reducing the traction energy for operation.

Driving and timetable are optimised with a combined PSO-GA (Ran et al. 2020) for energy efficiency purposes. Eco-driving and timetable level are solved in (S. Su et al. 2020) by DP and the SA algorithm, respectively, adding a substation-based energy consumption model. The timetable is also calculated in (Sánchez-Contreras et al. 2023) by applying ATO efficient driving when the mass transported is considered a fuzzy variable. Optimal design (Meng, Jia, and Xiang, 2018) focuses on the capacity of the timetable to remain robust when possible disruptions arise during operation.

Other studies address disruptions integrating the rescheduling problem of train timetables and rolling stock circulation (B. Su et al. 2024).

On the other hand, it is important to take into account not only the traction energy but also the use of regenerative energy to reduce the energy consumption in a railway line as much as possible. This is even more important in mass transit line where the power system is a DC system where returning energy from the railway grid to the utility grid is difficult (Bae 2009). Regenerative braking is a technology that allows the trains equipped with it to produce energy during the braking process. This energy is, firstly, used to feed the auxiliary equipment of the train that produces it, and the rest can be sent back to the catenary. This way, other trains can use it reducing the energy demanded from substations. The regenerated energy produced by a train can be used by another train that requires traction in DC systems when the trains share the same electrical section. Suppose there is not enough energy demand in the section. In that case, the regenerated energy must be wasted in on-board rheostats of the braking train to prevent an undesired increase of the catenary voltage.

Taking into account the previous concepts, there are authors who seek to reduce the total substation demand of the train traffic by synchronising accelerating and braking trains (Peña-Alcaraz et al. 2012) taking into account the power-saving factor between each pair of trains. In this case, an energy saving of 7% is achieved in a Madrid Underground case study just by the increase in regenerated energy use. The timetable is optimised in (Xin Yang et al. 2012), maximising the overlapping time between the accelerating and braking trains by means of an integer programming model and solving it with a GA.

The application of eco-driving decreases consumption but the use of regenerated energy decreases as well. The higher the speed, the higher the use of regenerated brake energy when trains arrive to the stopping points. So, there is a balance between efficient driving design and the ability to maximise the use of regenerated energy during braking, considering also the density of rail traffic (Bocharnikov, Tobias, and Roberts 2010). In (Huang et al. 2018), the driving is designed, and the timetable is then optimised considering regenerative braking exchange only when trains arrive and depart from stations with no consideration of motor efficiency or transmission losses.

In (P. Wang, Zhu, and Corman 2022) a resolution model is used to synchronise trains where the overlapping time of accelerating and braking trains (at the start-up and arrival at stations) is maximised and the travel time of a user and of all passengers globally is minimised, taking into account passenger transfers. In order to reuse the regenerated energy, this work takes into account the start-up and arrival of trains at stations without considering possible traction and braking that may occur during driving between stations. This study does not consider the use of regenerated energy but the overlapping time does not add the losses suffered during energy transmission and does not carry out an eco-driving design included in the model. Furthermore, the timetable is designed in such a way as to be periodic because, although flexibility is lost in adjusting starts and braking to maximise the use of regenerated energy, comfort and the perception of good service on the line are gained.

In the study carried out by (Xin Yang et al. 2015), the timetabling and coordination of trains during starting and braking, controlling dwell times, are proposed to take advantage of regenerated energy on the Beijing Metro Yizhuang Line. The results obtained show annual energy savings of 6.97% compared to the current planning. In this article, the speed profiles used are the real ones of the trains on the line analyzed, and the transmission loss coefficient is considered as constant.

The analysis in (Xin Yang et al. 2014) aims to maximize the use of regenerated energy while at the same time shortening passenger waiting time. It is formulated as a two-objective integer programming model with headway time and dwell time control, and then, it is designed a genetic algorithm to find the optimal solution. Transmission loss data are estimated using historical data.

An integrated method to maximize regenerative energy is presented in (Yildiz, Arikan, and Cakiroglu 2024) by optimizing train speed and timetables. Additionally, it minimises simultaneous acceleration or braking within the same electric zone. Simulations of Istanbul's M3 subway using genetic and simulated annealing algorithms show traction energy savings.

In (P. Wang et al. 2022) the design of the timetable is carried out considering the synchronisation between trains as in the previous work but now minimising power peaks. Possible delays and how they affect the timetable are analysed by means of the Monte Carlo method.

In (Qu et al. 2020) a two-step approach to timetable design is proposed. The first phase aims to reduce passenger waiting time and the second phase to reduce total consumption. The speed profile of the trains is divided into six zones for traction, coasting and braking. Regenerated energy is only taken into account during the overlap between acceleration and braking. In (He et al. 2019) a Chongqing metro line is studied for which the objective is to minimise the total energy consumed by taking into account the energy regenerated by accelerating and braking trains in the same power supply section. They use a matrix resolution model for searching the global solution, although with a higher computational cost, by taking the speed profiles of the trains. A bi-objective algorithm is proposed by (Als, Madsen, and Jensen 2023) to obtain efficient timetables where the consumption and passenger travel time are considered using real historical data of a railway network taking simplifications.

The literature found real-time timetable optimisation by means of train control as in (Sun et al. 2023). The synchronisation of the trains is taken into account to take advantage of the regenerated energy when the whole DC power supply system is connected in all the sections although the scenario where sections are disconnected is also considered. The response of the timetable is analysed against delays, and the speed profiles to be executed by the trains are adjusted using the strategy presented in (Sun et al. 2014).

(Chen et al. 2023) proposed a timetable optimisation and train control method to minimise substation energy consumption on a metro line with a DC electrical system equipped with reversible substations. In this case, the limitations of regenerative energy exchange are relaxed because reversible substations allow the flow of regenerative energy from the catenary to the utility grid. The train speed profile is included along with the timetable optimisation.

In the literature, different studies on energy-efficient train timetabling can be found but have rarely been taken into account train circulation plan, that is, the use of rolling stock (trains) required to provide the rail services defined in the timetable. However, the timetable design has an important impact on the use of the available rolling stock. When the timetable is optimised in order to save energy, it may be very costly to match the train circulation plan to the timetable due to an inadequate connection between train services (Zhou et al. 2023). Authors in (Liebchen and Möhring 2007) suggested that the train circulation plan should be taken into account in the optimization of the train schedule, but they did not focus on the energy-efficient timetabling problem.

In particular, depending on how the timetable is designed, the number of trains (rolling stock) needed to provide the service may be different, significantly impacting investment and operation.

In (Zhou et al. 2023) a synchronisation between trains is carried out for timetabling taking into account different electrical sections. The authors consider the possible synchronisation of energy regenerated/consumed along the interstation, and not only at arrival/departure. They optimize the timetable considering the energy saving and the train circulation plan for an urban rail line, by means of a mono-objective PSO optimization algorithm. However, they do not design the eco-driving at each interstation, taking into account the characteristics of the ATO equipment, the number of trains associated with the timetable is not calculated, and some simplifications are assumed in the train model.

The main objective and contribution of the present work is a multi-objective method based on detailed simulation to design the optimal ATO driving commands of a metro line and, as a result, define the traffic operation that minimises the energy consumption considering the rolling stock required to provide the service. This is achieved by integrating the optimisation of the speed profiles and the optimisation of the timetable, adding the contribution of the energy regenerated by braking trains during the train journey. The number of trains required to provide the service associated with the timetable is also considered to quantify costs related to rolling stock. In particular, it can be highlighted the following contributions:

- An integrated multi-objective approach to design the ATO driving commands and timetable of a metro line including: the minimization of energy consumption, the maximization of regenerative energy usage and the minimization of the rolling stock required to operate. This method can assist train operators to decide the timetable taking into account the trade-offs between commercial speed and the costs of energy and rolling stock.
- The integrated optimization model is a two-level optimization procedure where the local eco-driving level finds the set of efficient speed profiles at each interstation and the global timetable level finds the Pareto front of timetables. The local level is solved using a MOPSO algorithm which has demonstrated its performance in literature. The global timetable level is solved comparing four different algorithms, proving that GLMO-NSGA-III is the most effective for this optimization model.
- The model includes constraints associated with realistic ATO equipment at the local level and constraints associated with the reverse manoeuvre in terminal stations in order to avoid solutions that cannot be applied in real installations or that can require a greater number of trains.
- A Madrid Underground case study is carried out to prove the benefits of the optimization model proposed. The results illustrate that the integrated optimization achieves energy savings of 24.8% compared to the typical criteria used by operators to design the timetable.

References and previous work related to eco-driving and timetable design are listed in Table 1. As a summary, many studies in the literature ((Xin Yang et al. 2012), (P. Wang, Zhu, and Corman 2022), (Xin Yang et al. 2015), (Liebchen and Möhring 2007)) compared with the proposed model do not include an integrated optimization method with timetable and eco-driving. Literature models do not take into account constraints associated with ATO driving logic as the model proposed, except for (Sánchez-Contreras et al. 2023) and (He et al. 2019). However, in reference (Sánchez-Contreras et al. 2023) the regenerated energy is not considered and in (Sánchez-Contreras et al. 2023) (He et al.

2019) the rolling stock is not included as a goal. Regarding the regenerated energy, in (Yildiz, Arikan, and Keskin 2023a), (Ran et al. 2020), (S. Su et al. 2020), (Sánchez-Contreras et al. 2023), (Als, Madsen, and Jensen 2023) and (Liebchen and Möhring 2007) the regenerated energy is not modelled. In (Xin Yang et al. 2012), (Bocharnikov, Tobias, and Roberts 2010), (Huang et al. 2018), (P. Wang, Zhu, and Corman 2022), (Xin Yang et al. 2015), (Xin Yang et al. 2014), (Yildiz, Arikan, and Cakiroglu 2024) and (Qu et al. 2020) the use of regenerated energy is improved indirectly by maximizing the coincidences of train departures and arrivals. In (P. Wang et al. 2022), (He et al. 2019), (Sun et al. 2023), (Chen et al. 2023) and (Zhou et al. 2023) a similar approach to the model proposed in this paper is used to calculate the total regenerated energy transferred among trains. However, except for (Zhou et al. 2023) the rolling stock is not taken into account. The cost associated with the rolling stock is only included in the model in (Zhou et al. 2023) and (Liebchen and Möhring 2007). In both cases, the approach is to include the rolling stock cost as the total running time and the connection time. In the present paper, the approach is to directly calculate the number of trains needed to provide the service with the designed timetable and include it as a penalty in the objective function.

This paper is organised as follows: Section 2 describes the design objective for the optimal efficient operation of a metro line. In Section 3 the timetabling model is presented. Section 4 defines the eco-driving ATO design model for each interstation and the train simulator it uses to evaluate solutions. In Section 5 the method is applied to a case study of Madrid Underground. Section 6 presents the conclusions and a discussion of the proposed method.

Reference	Eco-driving consideration	ATO consideration	Regenerated energy consideration	Rolling stock consideration
(Yildiz, Arikan, and Keskin 2023a)	Yes	No	No	No
(Ran et al. 2020) , (S. Su et al. 2020)	Yes	No	No	No
(Sánchez-Contreras et al. 2023)	Yes	Yes	No	No
(Xin Yang et al. 2012)	No	No	Traction/Braking synchronization	No
(Bocharnikov, Tobias, and Roberts 2010)	Yes	No	Traction/Braking synchronization	No
(Huang et al. 2018)	Yes	No	Traction/Braking synchronization	No
(P. Wang, Zhu, and Corman 2022)	No	No	Traction/Braking synchronization	No
(Xin Yang et al. 2015)	No	No	Traction/Braking synchronization	No
(Xin Yang et al. 2014)	Yes	No	Traction/Braking synchronization	No
(Yildiz, Arikan, and Cakiroglu 2024)	Yes	No	Traction/Braking synchronization	No
(P. Wang et al. 2022)	Yes	No	Fully considered	No
(Qu et al. 2020)	Yes	No	Traction/Braking synchronization	No
(He et al. 2019)	Yes	Yes	Fully considered	No
(Als, Madsen, and Jensen 2023)	Yes	No	No	No
(Sun et al. 2023)	Yes	No	Fully considered	No
(Chen et al. 2023)	Yes	No	Fully considered	No

(Zhou et al. 2023)	Yes	No	Fully considered	Yes
(Liebchen and Möhring 2007)	No	No	No	Yes

Table 1. Literature review.

2. Design of the optimal ATO operation of a railway line

The model presented in this paper considers a typical topology of a mass transit railway line, a two-track line between terminal stations. The line is composed of Ns stations and equipped with ATO systems. All the electrical sections are supposed to be connected although regeneration will be more efficiently used by trains running close to the regenerating one, as electrical losses are modelled. Energy regenerated by a train running upwards can be consumed by a nearby train running in the same direction or in the downward direction.

Sections 2.1., 2.2., and 2.3. list the parameters, intermediate variables and decision variables used in the paper in Table 2, Table 3 and Table 4, respectively.

2.1. Parameters

$\boldsymbol{h}$	Headway
$\boldsymbol{N_s}$	Number of stations
$\boldsymbol{t_{min}}$	Minimum dwell time
$\boldsymbol{P}$	Penalty for the use of an additional train
$\boldsymbol{\sigma(s)}$	Transmission loss model dependent on train position
$\boldsymbol{M_{train}}$	Train mass at each interstation
$\boldsymbol{M_{eq}}$	Equivalent mass of the train
$\boldsymbol{F_{max}}$	Maximum traction effort according to the motor characteristics
$\boldsymbol{j_{max}}$	Maximum variation of traction/braking effort
$\boldsymbol{A}$	Constant related to the mechanical resistance to the motion
$\boldsymbol{B}$	Constant related to the resistance due to the air inlet in the train
$\boldsymbol{C}$	Constant related to the aerodynamic resistance
$\boldsymbol{g}$	Gravitational acceleration
$\boldsymbol{P_{track}(s)}$	Equivalent gradient of the track in the train position
$\boldsymbol{I_{max}}$	Maximum current
$\boldsymbol{U_{cat}}$	Nominal catenary voltage
$\boldsymbol{pf}$	Motors' efficiency

Table 2. Parameters.

2.2. Intermediate variables

$\boldsymbol{RT(x_g, t_g)}$	Global running time of the timetable
$\boldsymbol{C(x_g, t_g)}$	Cost function
$\boldsymbol{N_t}$	Number of trains
$\boldsymbol{t_1}$	Running time on track 1

t_2	Running time on track 2
T_1	Time difference between the arrival and departure of consecutive services in station 1
$f_1(x_n)$	Running time evaluation function for interstation n
$f_2(x_n)$	Energy consumption evaluation function for interstation n
EDC_n	Set of the efficient ATO driving commands of interstation n
s_i	Position of train *i*
σ_{ji}	Value of transmission losses for the regenerating train *j* and the accelerating train *i*
n_c	Number of trains consuming energy
n_r	Number of trains regenerating energy
E_i	Energy consumption used for traction by train *i*
R_j	Energy regenerated by train *j*
R_{ji}	Energy regenerated by train *j* and transmitted to train *i*
t	Time
a_{train}	Train acceleration
F_{track}	Resistance due to the track grades
F_{res}	Running resistance
a_{ref}	Traction/braking demand for the train
$K_{traction}$	Constant of the proportional controller for the traction mode
v_{target}	Minimum value between the maximum speed at the position of the train and the driving command *hs* (holding speed)
v_{train}	Speed of the train
$a_{gradient}$	Acceleration due to the gradients at the position of the train
$K_{braking}$	Constant of the proportional controller for the braking mode
F_{train}	Traction/braking effort of the train
P_{trac}	Traction electric power
P_{reg}	Regenerative electric power

Table 3. Intermediate variables.

2.3. Decision variables

x_g	ATO driving commands for all the interstations of the entire metro line
t_g	Dwell time at every station in the railway line
x_n	Vector with the ATO driving parameters for the interstation *n* (component of x_g)
t_n	Dwell time at station n (component of t_g)
x_{Ns}	ATO driving commands for interstation *Ns*
t_{Ns}	Dwell time at station *Ns*
c	Speed at which coasting is applied (component of x_n)
r	Speed at which traction is applied (component of x_n)
hs	Holding speed (component of x_n)

$\boldsymbol{b}$	Braking rate (component of $\boldsymbol{x_n}$)

Table 4. Decision variables.

2.4. Optimal ATO problem

The problem to solve is the design of the optimal ATO driving commands of a metropolitan railway line for a specific operational period. With this objective, the model designs jointly the efficient driving at each interstation and a periodic efficient timetable for the operational period.

The model includes a penalty associated to the number of trains (rolling stock) required to provide the rail services of the timetable. This way, timetables with a lower number of trains required will be preferred.

A multi-objective model is defined to propose the most efficient solutions for different total travel times of the line. As the travel time and energy consumption are conflicting objectives, the result will be the set of non-dominated solutions; i.e., those that cannot be improved in energy consumption and travel time simultaneously. This way, the operator can select the commercial running time of the line (i.e., the travel time to run the line completely) in view of the trade-off with the energy consumption and considering the number of trains required. The travel time is the sum of the running time of the speed profiles produced by the designed ATO driving commands for each interstation stretch, and the selected dwell time at each station. To optimise the energy consumption of the line, for each total travel time the proposed model takes into account the set of optimal ATO driving commands in each interstation (that produces eco-driving speed profiles), the distribution of time margins along the route to make it efficient and also the energy transferred between trains in the braking processes which, depending on the design of the timetables of all trains, makes it possible to reduce the overall energy consumption. In this paper, a two-level resolution model is proposed (Figure 1). First, at the eco-driving level, a local Pareto front of driving commands is obtained for each interstation, providing for each running time the eco-driving (ATO driving with the minimum energy consumption).

Then, at the timetable level, a global Pareto front of optimal timetable solutions (i.e., a driving command for each interstation and a dwell time for each station) is obtained taking into account the exchange of regenerative energy among trains on the entire metro line in order to maximise its use, as well as the time distribution along the line to minimise the total energy consumption of the system.

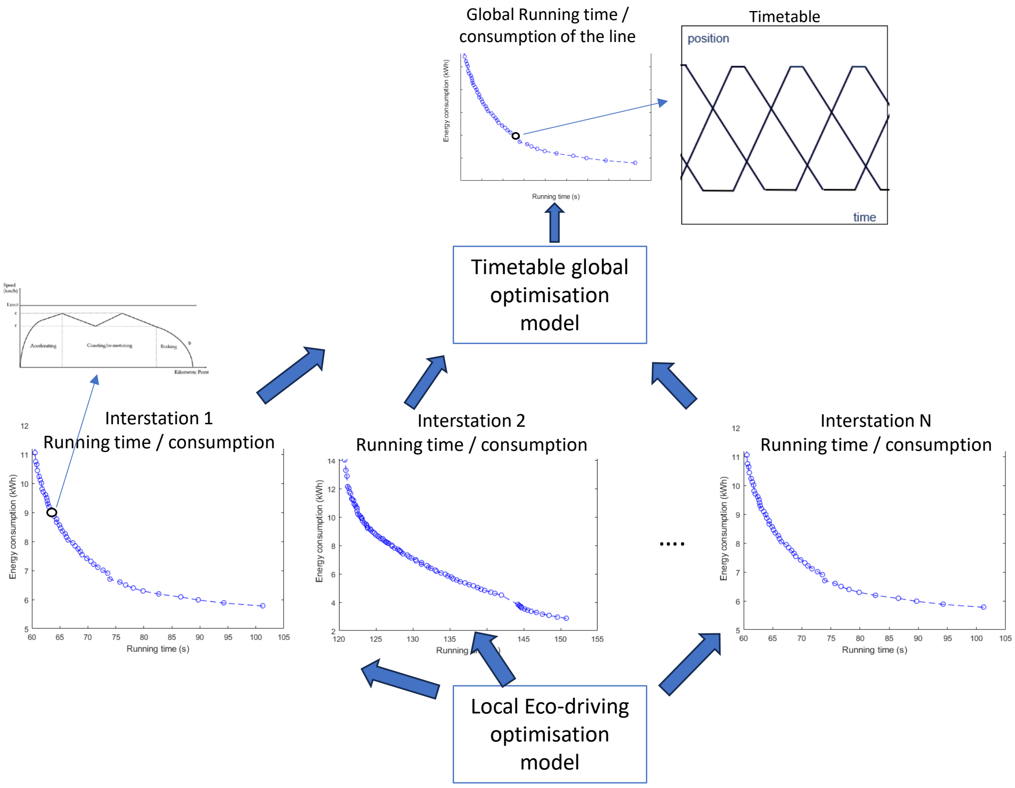

Figure 1. Proposed model for the efficient operation of a metro line.

3. Timetable design level

This section presents the optimisation of the timetable at the global level. The inputs received at this level are the eco-driving profiles designed by the Local eco-driving multi-objective optimisation model (Section 4).

A series of preconditions have been established to design the timetable for a specific operational period. A periodic timetable has been considered with all trains performing the same running time for the same route and the same stopping times for the same stations considering that the interval between trains is constant. Therefore, by calculating the timetable for each train in each period of time, the entire timetable is calculated.

When the timetable design problem is solved, a new global Pareto front is obtained where each solution has associated the total energy consumption and running time of the cycle for the entire line, considering that the trains are using the efficient driving calculated in the eco-driving level. Thus, each solution represents a specific set of efficient ATO driving parameters at each interstation (which provide the running times at interstations) and the dwell times at the stations of the line. For each solution, the total operating cycle time of the entire line is thus defined. In this way, the optimal timetable is established by defining the time of arrival and departure at stations and thus, the associated stopping times.

3.1. Global optimisation model

At the global level, the objective is to maximise the use of energy regenerated by trains during braking avoiding waste through rheostats and to distribute the time margins by reducing the running time at the interstations where greater traction energy reduction can be obtained.

The energy regenerated by a train is firstly used for auxiliary systems. The remaining regenerated energy is transmitted to other trains that are accelerating nearby the train or transmitted to rheostats if there is no demand to avoid voltage surges in the catenary.

To design the efficient timetable, the input parameters are the headway and the set of efficient eco-driving parameters obtained for each interstation at local level. The proposed optimisation model will modify the arrival/departures in order to synchronize trains improving the use of regenerated energy in braking. In this way, a braking train transmits as much energy as possible to nearby accelerating trains.

In the proposed model, the consumption profile on the route of each interstation is used to synchronise not only the braking on arrival at the station but also at any other moment that may occur, such as, for example due to speed limits along the route.

In addition, every timetable on a railway line has an associated number of trains required to provide that service. The model proposed imposes a penalty for timetables with similar running time and energy consumption that require an extra train to provide the service.

Thus, the objective function of the multi-objective optimisation problem of designing the line operation can be expressed as:

$$min\, f_g(x_g, t_g) = \left(RT(x_g, t_g)\,, C(x_g, t_g)\right) \quad (1)$$

where, RT is the total running time (s) for the mass transit line, C is the cost function that includes the total energy consumption (kWh) and the penalty in the number of trains (kWh), x_g is the ATO driving commands for all the interstations of the entire metro line with $x_g = (x_1, x_2, \dots, x_{Ns})$ being Ns the number of stations along the metro line and t_g are the values of dwell time (s) at every station in the line with $t_g = (t_1, t_2, \dots, t_{Ns})$.

The driving commands contained in x_g will be restricted by the following condition:

$$x_n \in EDC_n \quad (2)$$

where x_n is the component n of x_g (i.e., the ATO driving command of the interstation n) and EDC_n is the set of the efficient ATO driving commands of interstation n (i.e., the Pareto front obtained in the local eco-driving level for interstation n).

Moreover, the dwell times will be restricted by a minimum value that ensures that the passengers have enough time to get on board the train:

$$t_n \geq t_{min} \quad (3)$$

where t_n es the dwell time (s) at station n, and t_{min} is the minimum dwell time (s). The global running time of the timetable model $RT(x_g, t_g)$, presented in Equation 4, is calculated as the sum of the running time and the stopping time at each station n:

$$RT(x_g, t_g) = \sum_{n=1}^{Ns} (f_1(x_n) + t_n) \tag{4}$$

where, $f_1(x_n)$ is the running time (s) of interstation n.

The cost function $C(x_g, t_g)$, shown in Equation 5, is calculated knowing at each moment t the consumption of the traction trains, the energy returned by braking trains and the distance between trains to take into account the electrical losses in the energy transmission between regenerating trains and accelerating trains. In addition, this function includes a penalty for each extra train required in the solution to satisfy the timetable:

$$C(x_g, t_g) = \sum_{t=0}^{h} \left(\sum_{i=1}^{n_c} E_i - \sum_{i=1}^{n_c} \sum_{\substack{j=1 \\ j \neq i}}^{n_r} R_{ji} \cdot \sigma_{ji} \right) + P \cdot N_t \tag{5}$$

where, j is the train that transmits regenerated energy, i is the train that receives regenerated energy, n_c is the number of trains consuming energy, n_r is the number of trains regenerating energy, h is the headway (s), E_i is the energy consumption (kWh) used for traction by train i, R_{ji} is the energy regenerated (kWh) by train j and transmitted to train i at a specific instant, σ models the electrical losses and represents the percentage of regenerated energy that is transmitted from a braking train and utilised by a traction train. This function is dependent on the distance (m) between the regenerating train j and the train i receiving the energy as shown in Equation 6. An example is shown in Figure 2 although different shapes can be obtained depending on the characteristics of the specific railway line. P is the penalty (kWh) whereby the solution with the lowest number of trains satisfying the timetable is prioritised and N_t is the number of trains required to fulfil the timetable.

$$\sigma_{ji} = \sigma \cdot (|s_i - s_j|) \tag{6}$$

Where s_i and s_j are the kilometric position (m) of train i and j respectively.

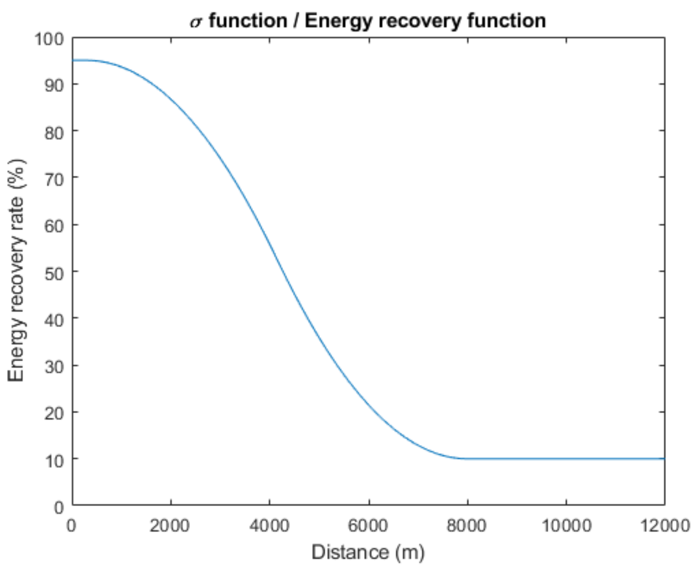

Figure 2. σ function.

Cost function C (Equation 5) is calculated as the sum of consumption at each instant of time t, over an interval equal to the headway. It is sufficient to calculate it in a headway since, as the timetable is periodic, it is repeated every headway h.

In Equation 5, the value of the energy consumption (E) of a train cannot be a negative value. A negative value would mean that the train would be braking, regenerating and transmitting energy, so instead, this value would be included in R. Similarly, the energy regeneration R of a train cannot be a negative value (in that case, the train would be consumed and should be modelled by means of E).

In addition, there are a series of restrictions to complete the model for the use of regenerated energy. The first expression (Equation 7) indicates that the reuse of the regenerated energy is maximised offering the greatest possible amount of energy to the nearest trains with the highest utilisation percentage (Figure 2):

$$max \sum_{i=1}^{n_c} R_{ji} \cdot \sigma_{ji} \tag{7}$$

The energy regenerated will always be higher than the energy received by the other trains for reuse due to the losses introduced in the calculation:

$$R_j > \sum_{i=1}^{n_c} R_{ji} \tag{8}$$

The total regenerated energy received by trains cannot exceed the energy required for their operation so energy is only transmitted to trains that are accelerating:

$$\sum_{i=1}^{n_c} \sigma_{ji} \cdot R_{ji} \leq E_i \tag{9}$$

In addition, in the cost function (Equation 5), a penalty is introduced associated to the number of trains (rolling stock) required to provide the service of the designed timetable. In this way, the optimal solution can be obtained with the lowest number of associated trains while reducing related costs.

The calculation of the number of trains is solved by means of Equations 10 and 11. T_1 is the result of the timetable optimisation and represents the time difference between the arrival and departure of consecutive services in terminal station 1. If this time is equal to or greater than minimum dwell time, t_{min}, that means the same rolling stock can perform both services. In this case, the number of trains is calculated by Equation 10, by considering the dwell time at terminal station 2 equal to the minimum and rounding the result to the greater closest integer value. That means that the dwell time at terminal station 2 is the needed value to obtain an integer value of trains subject to the minimum stopping time:

$$If\ T_1 \geq t_{min} \rightarrow N_t = ceil\left(\frac{t_1 + t_2 + T_1 + t_{min}}{h}\right) \tag{10}$$

If this time is lower than minimum dwell time (t_{min}), that means the same rolling stock cannot perform both services. That means that when a train arrives to terminal station 1, there is other train ready to start the service in the opposite direction. Therefore, the train that arrives to terminal station 1 has to wait T_1 plus an interval h to start the service in the opposite direction (Figure 3). The impact of this extra-time is that an additional train is needed to provide the service. In this case, the number of trains is calculated by Equation 11:

$$If\ T_1 < t_{min} \rightarrow N_t = ceil\left(\frac{t_1 + t_2 + T_1 + h + t_{min}}{h}\right) \tag{11}$$

where, T_1 is the dwell time (s) calculated in terminal station 1, t_{min} is the minimum dwell time (s) in terminal station, t_1 is the running time (s) on track 1 and t_2 is the running time (s) on track 2.

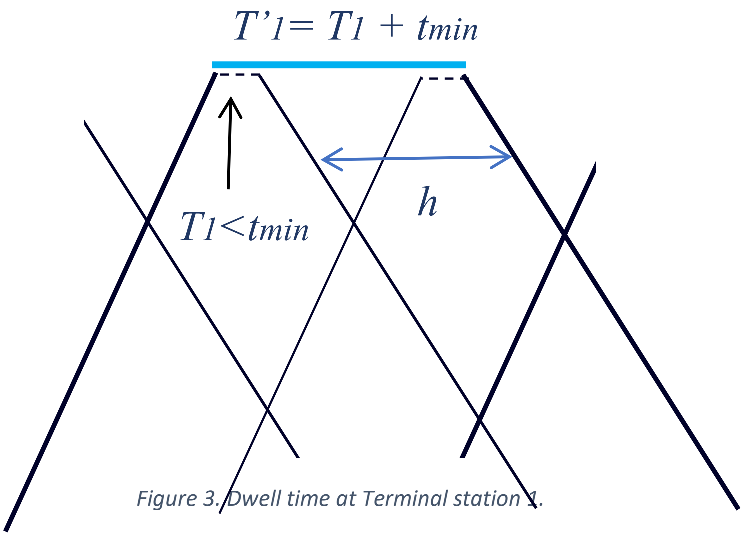

Figure 3. Dwell time at Terminal station 1.

The number of trains necessary for the operation is calculated as the complete cycle running time associated with a specific timetable divided by the headway between trains. For a given timetable it is analysed whether the time at the terminal station is greater than the minimum time necessary for the manoeuvre (or if more margin time is desired for delay recovery). Otherwise, if the result is less than the minimum time, it means that the train arriving at that terminal station must connect with the next timetable which starts a time equals to one headway later. As a consequence, in this case an extra train is needed to provide the service.

A Non-Sorted Genetic Algorithm III (Kalyanmoy Deb and Jain 2014) that uses the Grouped Linked Polynomial Mutation Operator (Zille et al. 2016a) (GLMO-NSGA-III) algorithm is proposed in this paper to solve the multi-objective optimization problem defined in the timetable level due to its performance in terms of diversity and optimality of the solutions obtained in complex problems.

3.2. GLMO-NSGA-III algorithm

The parameters related to the GLMO-NSGA-III algorithm are presented in Table 5.

$\vec{s}$	Set of solutions
s_i	Solution i of $\vec{s}$

$\boldsymbol{\mu}$	Distribution Index
$\vec{\boldsymbol{m}}$	Mutated Solution
$\boldsymbol{m_i}$	Mutated solution i of $\vec{m}$
$\vec{\boldsymbol{r}}$	Variables that are to be mutated chosen based on variable grouping
$\boldsymbol{n_g}$	Group index
$\boldsymbol{r_{n_g}}$	Variables with group index n_g
$\boldsymbol{j}$	Random group index
$\boldsymbol{h}$	Random value between (0,1)
$\boldsymbol{r_j}$	Group r with index $\boldsymbol{j}$
$\boldsymbol{s_{i,min}}$	Minimum value of $\vec{s}$
$\boldsymbol{s_{i,max}}$	Maximum value of $\vec{s}$
$\boldsymbol{\tau_1, \tau_q, \tau_2}$	Intermediate variable calculation

Table 5.GLMO-NSGA-III parameters.

GLMO-NSGA-III algorithm is the proposed method for solving the efficient timetable calculation model. The Grouped Linked Polynomial Mutation Operator (GLMO) (Zille et al. 2016a) is based on the following mutation techniques: Linked Polynomial Mutation and Grouped Polynomial Mutation. These two procedures are based on Polynomial Mutation and are explained below.

- Polynomial Mutation: This mutation operator is designed to modify the value of a variable based on a distribution around the value at that time. Two parameters are used to perform the mutation: the mutation probability and the distribution index. The probability is used to decide for each variable whether or not it undergoes mutation. The index determines the distribution from which the new value will be calculated, the probability of the new value being more similar to the current value when this index takes large values. When deciding whether a variable will mutate, another parameter h is taken from a uniform distribution indicating how the mutated variable will change. If the result of the mutation exceeds the limits of the distribution around which the original value is modified, this new value is placed at the limit of the distribution.

- Linked Polynomial Mutation: This operator follows a similar idea to the Polynomial Mutation but the parameter u remains constant for all variables. The Linked Polynomial Mutation is a version of the Polynomial Mutation in which a connection is established between variables that are susceptible to change. The procedure by which the connection is established is by keeping the relative amount of change of a variable fixed for each mutation and the same for each solution. With this change, all variables that undergo mutation considering a specific value of probability are mutated by the same amount.

- Grouped Polynomial Mutation: In this modification the variables are separated into different groups and the mutation is applied to these groups, varying each variable in the same group equally. The decision of which variables mutate changes from a randomised procedure to a procedure designed for that purpose assuming that the clustering method makes the appropriate decision of which variables interact and should be modified simultaneously. In this case the mutation probability is not necessary. In this operator, the variables are arbitrarily divided into groups and the variables in any of these groups will be susceptible to polynomial mutation. For each of the variables in the group, a value h is taken defining the mutation. The variables of the rest of the groups do not change.

- Grouped Linked Polynomial Mutation Operator (GLMO): This case is a combination of the two previous ones. The distribution parameter together with the grouping mechanism are the input information for this procedure. The selection of the group to mutate is done in a similar way to the Grouped Polynomial Mutation operator. In this case the value of the parameter h is previously selected as in the Linked Polynomial Mutation. In comparison to the Linked Polynomial Mutation, it is now assumed that the interacting variables are modified simultaneously and equally when they are in the same group. GLMO operator is used within the NSGA-III optimisation algorithm (Kalyanmoy Deb and Jain 2014) to improve the adaptability, the diversity and the convergence of the algorithm.

GLMO pseudocode is as follows:

Input: Solution $\vec{s}$, the Grouping Mechanism and Distribution Index μ

Output: Mutated Solution $\vec{m}$

$\{r_1, \ldots, r_{n_g}\} \leftarrow Apply\ Grouping\ Mechanism\ to\ \vec{r}\ producing\ n_g\ groups$
$j \leftarrow Pick\ a\ group\ index\ at\ random\ from\ \{1, \ldots, n_g\}$
$h \leftarrow takes\ random\ values\ between\ (0,1)$
$loop\ s_i\ values\ belonging\ to\ r_j\ do$
$if\ h\ \leq 0.5\ then$
$\quad \tau_1 = \frac{s_i - s_{i,min}}{s_{i,max} - s_{i,min}}$
$\quad \tau_q = (2 \cdot h + (1 - 2 \cdot h) \cdot (1 - \tau_1)^{\mu+1})^{\frac{1}{\mu+1}} - 1$
$else$
$\quad \tau_2 = \frac{s_{i,max} - s_i}{s_{i,max} - s_{i,min}}$
$\quad \tau_q = 1 - (2 \cdot (1 - h) + 2 \cdot (h - 0.5) \cdot (1 - \tau_2)^{\mu+1})^{\frac{1}{\mu+1}}$
$end\ if$
$\quad m_i = s_i + \tau_q \cdot (s_{i,max} - s_{i,min})$
$\quad repair(m_i)$
$end\ loop$
$loop\ s_i\ values\ not\ belonging\ to\ r_j\ do$
$\quad m_i = s_i$
$end\ loop$
$return\ \vec{m}$

Finally, NSGA-III optimisation algorithm flowchart is shown in Figure 4. The NSGA-III is an evolutionary algorithm where individuals evolve during iterations at the end of which the best fitted survives. The initial step of the algorithm is the random generation of the parent population. The global travel time and cost of the members of the parent population are evaluated using Equations 4 and 5. Then, the offspring population is generated by applying the GLMO operator to individuals in the parent population. This offspring population is evaluated, and a result population is generated, joining the parent and offspring populations. The dominance of the solutions in the result population is calculated. As a result, the selection criterion chooses the members of the result population that will survive depending on the domination level. Thus, firstly the group of non-dominated solutions are saved. After that, the group of solutions that are dominated by just one solution is saved, and the process is repeated with the following groups of solutions. If a group cannot be saved completely because the size of the population is achieved, the crowding distance operator is applied to select the solutions of the last group that will survive.

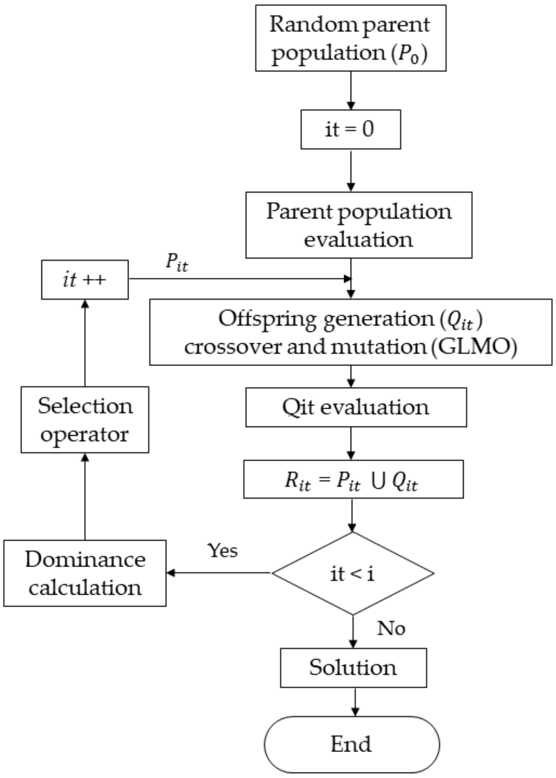

Figure 4. NSGA-III flowchart.

4. Local eco-driving multi-objective optimisation model

As previously described, the local optimisation model provides a Pareto front of solutions for each interstation. Each solution of a Pareto front represents the set of optimal ATO driving parameters for the interstation with the running time and energy consumption associated.

At this level of model resolution, which is the eco-driving level executed for each interstation of the line, the search of optimal solutions is modelled as a multi-objective problem where the aim is to minimise running time and energy consumption at the interstation. The evaluation function for interstation n is:

$$min\, f(x_n) = (f_1(x_n)\, , f_2(x_n)) \tag{12}$$

where, $f_1(x_n)$ is the objective function that defines the running time (s) between stations when a specific set of ATO driving parameters is executed, $f_2(x_n)$ is the objective function that defines the energy consumption (kWh) associated to each driving and x_n is the vector that contains the ATO driving parameters for the interstation n. The driving parameters are defined following the driving model presented in (Domínguez et al. 2014).

$$x_n = (c, r, hs, b) \tag{13}$$

where, c is the speed at which coasting is applied (km/h), r is the speed at which traction is applied (km/h) (re-motor), hs is the holding speed (km/h) and b is the braking rate (m/s^2).

Figure 5 shows an example of ATO speed profile where the eco-driving strategy is the regulation speed. In this case, the main driving parameter is hs and the coasting speed and re-motoring speed are equal to 0. When a regulation speed is greater than 0, the train aims to maintain the speed defined by c. On the other hand, Figure 6 shows a coasting/re-motoring case. Coasting/re-motoring strategies are characterized by two main parameters c and r while the holding speed hs is 0. In this case, the train tractions until its speed achieves the value of c and, then, it coasts until it reduces its speed r km/h that it is the moment when the train tractions again and the cycle is repeated. In both cases (regulation and coasting/re-motoring) the speed limitations are observed and the deceleration up to the stop in the station is defined by b.

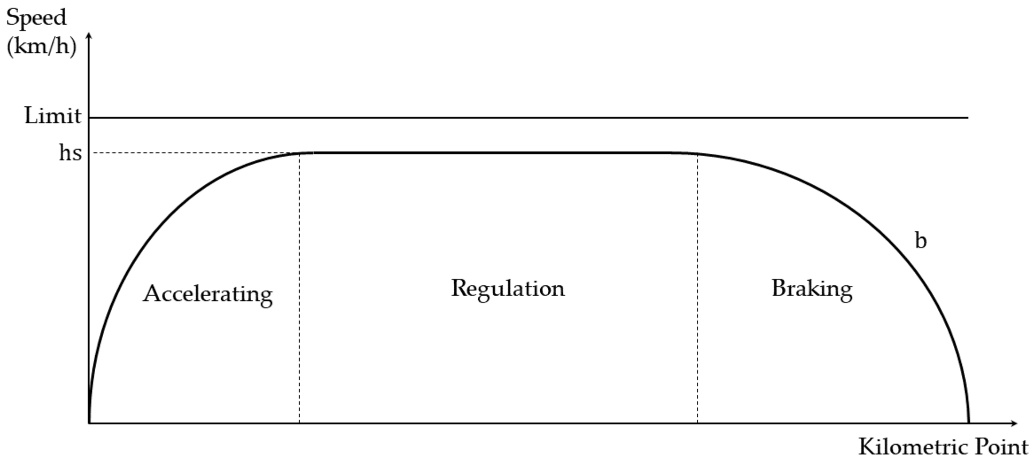

Figure 5. Regulation speed profile.

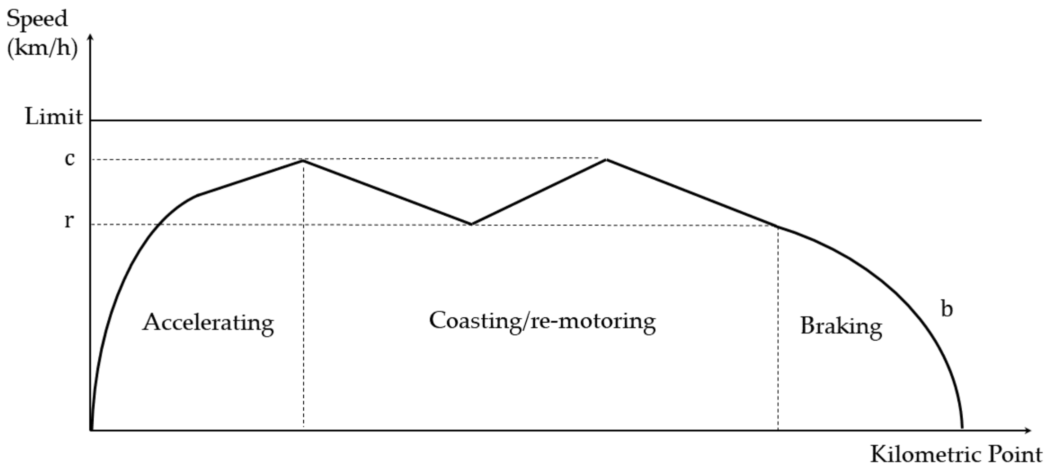

Figure 6. Coasting/re-motoring speed profile.

Efficient ATO driving must meet comfort requirements, otherwise they are discarded. The requirements to be fulfilled are: avoid traction cut-off on a ramp, avoid too many cut-off and re-motor cycles between interstations, and avoid low operating speeds and maintaining traction for a minimum time. In addition, accelerations and decelerations must be kept within a certain comfort range by controlling the rate of change of acceleration. Another desirable requirement is that the driving can be kept stable facing changes in the mass of passengers avoiding large variations in running time and consumption (Lin and Sheu 2008). For instance, the efficient ATO speed profiles generated in (Fernandez-Rodriguez et al. 2015) considers load variations.

It is important to note that the passenger load can vary during the journey and, in the proposed model, it is possible to configure a different passenger mass for each interstation. This way, the model takes into account the impact of the mass variation in the design of the eco-driving and in the use of regenerated energy. On the other hand, the objective is to provide a periodic timetable with a constant headway and with the same stopping regime for a specific time period (for instance, peak hours). In this kind of operation, the model considers that trains run with similar passenger load at the same stations. Therefore, the passenger mass used for the design of the ATO should be an average mass value for each interstation.

4.1. MOPSO algorithm

The parameters involved in the MOPSO algorithm section are presented in Table 6.

$\boldsymbol{x_i}$	Position of the particle
$\boldsymbol{n}$	Iteration number
$\boldsymbol{v_i}$	Velocity of the particle
$\boldsymbol{w}$	Inertial weight of the particle
$\boldsymbol{c_1, c_2}$	Parameters that determine individual and collective influence
$\boldsymbol{r_1, r_2}$	Parameters that take a random value chosen between 0 and 1
$\boldsymbol{p_i}$	Vector where the previously local best result of the $\boldsymbol{i}$ particle is stored
$\boldsymbol{p_g}$	Vector where the previously global best result is stored

Table 6. MOPSO parameters.

For the search of these possible solutions, a MOPSO algorithm has been chosen because it has been proven to provide better results for the eco-driving problem compared to other widely used algorithms (Domínguez et al. 2014). MOPSO algorithm is based on the PSO method, resembling the behaviour of a multitude of individuals present in nature (Kennedy and Eberhart 1995). MOPSO algorithm is a particle swarm optimisation technique for solving multi-objective optimisation problems. In MOPSO, each particle represents a potential solution to the optimisation problem and moves in a multidimensional search space. The goal of the algorithm is to find a set of optimal solutions that are as close as possible to the Pareto front. To achieve this, MOPSO uses a technique called "Pareto dominance", which allows solutions to be compared in terms of their quality across multiple goals. The algorithm adjusts the speed and direction of each particle based on its own previous experience and that of neighbouring particles, in an iterative process that seeks to continuously improve solutions.

The MOPSO algorithm presented in this paper uses the train simulation model presented in Section 4.2 to evaluate the fitness function in Equation (12) and the comfort criteria.

The PSO algorithm searches for an optimal solution by allowing an initial population to move through the search space according to mathematical rules. The movement of the particles is conditioned by the best local solutions, at the level of each particle, and at the global level, by the population as a whole. The PSO method is a relatively simple and effective algorithm, which makes use of the characteristic movement of particles through the search hyperspace by rapidly converging on better solutions. It makes use of a process similar to crossover, used in Genetic Algorithms, and the concept of fitness. MOPSO allows dealing with multi-objective optimisation problems. In this case, it is used the Pareto front concept (Goldberg 1989). The history of the best solutions found by a particle can be used to store previously generated non-dominated solutions. The use of attraction mechanisms combined with non-dominated solution vectors would trigger convergence to globally non-dominated solutions. Therefore, in each cycle, the past experience of each particle is saved. In addition, it is used the technique inspired by the external archive used in PAES (Knowles and Corne 2000) (Pareto Archive Evolution Strategy). Here, the information stored by the particles is updated based on the values of the objective function for each of the particles. The previously stored information is used by each individual to choose the leader to guide the search.

MOPSO algorithm is described by the following steps:

1. Initialise randomly the position and velocity of the particles.
2. The particles are evaluated in running time and energy consumption and those that do not fulfil comfort criteria are directly marked as dominated solutions.
3. Local best is initialised as the current position of each particle.
4. Non-dominated solutions are stored in the archive.
5. Global best is initialised selecting randomly a non-dominated solution.
6. Update the position and velocity values for each of the particles by means of the following equations:

$$x_i(n) = x_i \cdot (n-1) + v_i(n) \tag{14}$$

$$v_i(n) = w \cdot v_i \cdot (n-1) + c_1 \cdot r_1 \cdot (p_i - x_i \cdot (n-1)) + c_2 \cdot r_2 \cdot \left(p_g - x_i \cdot (n-1)\right) \tag{15}$$

where, x_i is the position of the particle, v_i is the velocity of the particle, n is the number of the iteration, w is the inertial weight of the particle and allows particles to find local optimal solutions based on global solutions at the outset, c_1 and c_2 are parameters that determine individual and collective influence, r_1 and r_2 are parameters that take a random value chosen between 0 and 1, p_g is the vector where the previously global best result is stored and it is the best position found by the swarm during all the iterations of the algorithm, p_i is the vector where the previously local best result of the i particle is stored and it is the best position found by the specific particle during all the iterations of the algorithm.

7. Calculate the consumption and travel time by evaluating the particles. Subsequently, comfort criteria can be applied to mark as dominated those driving that do not meet them.
8. The archive is updated including the new non-dominated solutions found and discarding those that are dominated by the new ones.
9. Update local best for each solution. If the local best and the current particle position are mutually non-dominated, then the local best will be maintained or updated by the new particle position randomly.
10. Calculate the crowding distance (CD) of each solution in the archive.
11. Update the global best result from the solutions of the archive. Solutions with higher value of CD will have higher probability of being selected as global best to promote diversity. Thus, the archive will be divided into two groups: the group with higher values of CD and the rest. The first group is composed by the fraction of the archive with higher values of CD according

to the "Top select" parameter. Then, a solution is randomly selected from the top group or from the rest with a probability defined by the "Top select probability" parameter.

12. Repeat steps 6 to 11 until the maximum number of iterations is achieved.

4.2. Train simulation model

The train simulation model is used to evaluate the solutions generated by the MOPSO algorithm. A detailed description of this simulation model and its validation can be found in (Domínguez et al. 2011). It is composed by several modules that represent the ATO logic, the traction/braking system, the train dynamics and the energy consumption calculation. The line characterization used by the simulation model includes location of stations (kilometric point), grades, grade transitions curves, track curvatures and speed limits. Moreover, the train characterization includes mass of the train, the mass of the load, length of the train, maximum speed, rotatory inertia, adhesion traction, traction effort curve, braking effort curve, running resistance coefficients, auxiliary systems consumptions and the ATO equipment configuration.

The simulation model works by calculating, at constant time steps, the state variables of the train motion. At each step, the process starts with the ATO module to obtain the traction/braking demand (a_{ref}) for the train. This demand is the ratio between the effort that the traction/braking system has to apply and the maximum traction/braking effort.

The ATO module receives as inputs the speed and position of the train, the maximum speed profile, the gradient profile and the position of the arrival station. With this information, the ATO modules differentiate 3 scenarios:

- The train is in traction mode and does not need to brake to observe maximum speed reduction or to the station arrival. In this case, the ATO calculates its output by means of a proportional controller using as a speed reference (v_{target}) the minimum value between the maximum speed at the position of the train and the driving command hs (in case the train is driven in holding speed mode). The result of the control is corrected with the gradient acceleration as shown in Equation (16).

$$a_{ref} = K_{traction} \cdot \left(v_{target} - v_{train}\right) + K_{acc} \cdot a_{gradient} \tag{16}$$

  where $K_{traction}$ is the constant of the proportional controller $\left(\frac{s}{m}\right)$ for the traction mode, v_{train} is the current train speed $\left(\frac{m}{s}\right)$, K_{acc} is the constant for the pre-fed acceleration $\left(\frac{s^2}{m}\right)$ and $a_{gradient}$ is the acceleration $\left(\frac{m}{s^2}\right)$ due to the gradients at the position of the train.

- The train is in braking mode to observe a speed restriction. At each simulation step, the ATO logic evaluates by means of a braking detection curve (Carvajal, Cucala, and Fernández 2016) if it is necessary to reduce the speed because of a maximum speed reduction. When the braking detection curve intersects the maximum speed profile, the ATO braking logic is started. In the braking mode, the output of the ATO is calculated by the proportional controller represented in Equation (17). This controller is corrected not only with the gradient acceleration (as the previous case) but also with the target acceleration a_{target} $\left(\frac{m}{s^2}\right)$. The target acceleration (a_{target}) is calculated as a braking curve from the current train position and speed to the position where the maximum speed is reduced and the new maximum speed value. The speed reference v_{target} $\left(\frac{m}{s}\right)$ in this case is the speed of the braking curve in the next time step.

$$a_{ref} = K_{braking} \cdot \left(v_{target} - v_{train}\right) + K_{acc} \cdot \left(-a_{target} + a_{gradient}\right) \tag{17}$$

where $K_{braking}$ is the constant of the proportional controller $\left(\frac{s}{m}\right)$ for the braking mode that depends on the train speed.

- The train is in final braking mode to brake up to the arrival station. At the beginning of the simulation, the braking curve up to the stop in the arrival station is calculated between the arrival point and the initial point using the driving command deceleration (b). When the train speed and position intersect this braking curve, the ATO final braking is started. In the final braking mode, the proportional ccontrolrepresented in Equation (17) is applied where the target acceleration is b and the speed reference (v_{target}) is the speed of the final braking curve in the next time step

The ATO traction/braking demand a_{ref} previously calculated is bounded by [-1,1] and hysteresis is applied to minimise that the train changes for traction to braking continuously. Besides, if the train is driven with coasting/re-motoring driving commands, a_{ref} will be bounded by a maximum value of 0 when the train is in traction mode and the speed has previously reached the coasting speed (c) and it is over the re-motoring speed (r).

Once the ATO output is calculated, the effort provided by traction/braking system is calculated using Equation (18).

$$F_{train} = a_{ref} \cdot F_{max} \tag{18}$$

where F_{train} is the traction/braking effort of the train (kN) and F_{max} is the maximum traction effort (kN) according to the motors characteristics, if $a_{ref} \geq 0$, or the maximum braking effort according to the braking system characteristics, if $a_{ref} < 0$. Both, maximum traction and maximum braking effort curves are dependent on the train speed.

The traction/braking system limits abrupt changes in the train effort applied. Therefore, the previously calculated traction/braking effort will be constrained applying a limitation as show in Equation (19).

$$-j_{max} < \frac{dF_{train}}{dt} < j_{max} \tag{19}$$

where j_{max} is the maximum variation of traction/braking effort $\left(\frac{kN}{s}\right)$ value configured.

The traction/braking effort will be the input for the dynamics module that will calculate firstly the train acceleration (a_{train}) using the Newton's second motion law as shown in Equation (20).

$$a_{train} = \frac{F_{train} - F_{track} - F_{res}}{M_{eq}} \quad (20)$$

where F_{track} is the resistance due to the track grades (kN), F_{res} (kN) is the running resistance and M_{eq} is the equivalent mass of the train (tons) (including the rotatory inertia).

The running resistance and the resistance due to track grades can be calculated using Equations (21) and (22).

$$F_{res} = A + B \cdot v_{train} + C \cdot (v_{train})^2 \quad (21)$$

$$F_{track} = g \cdot M_{train} \cdot P_{track} \quad (22)$$

where A (kN), $B \left(\frac{kN}{\frac{m}{s}}\right)$ and $C \left(\frac{kN}{\left(\frac{m}{s}\right)^2}\right)$ are coefficients that depend on the train characteristics, g is the gravitational acceleration $\left(\frac{m}{s^2}\right)$, M_{train} is the train mass (tons) and P_{track} is the equivalent gradient of the track $\left(\frac{m}{m}\right)$ in the train position. The equivalent gradient is calculated adding to the average gradient affecting the complete length of the train, the equivalent gradient that represents the resistance produce by the track curves.

The position s_{train} (m) and speed of the train $\left(\frac{m}{s}\right)$ can be updated using the train acceleration and Equations (23) and (24).

$$\frac{dv_{train}}{dt} = a_{train} \quad (23)$$

$$\frac{ds_{train}}{dt} = v_{train} \quad (24)$$

Finally, the traction electric power P_{trac} (kW) and regenerative electric power P_{reg} (kW) consumptions are calculated using Equations (25) and (26). With the power, the traction energy consumption can be calculated integrating the traction electric power.

$$P_{trac} = I_{max} \cdot \frac{F_{train}}{F_{max}} \cdot U_{cat} \cdot \frac{1}{pf} \qquad if\ F_{train} \geq 0 \quad (25)$$

$$P_{reg} = I_{max} \cdot \frac{F_{train}}{F_{max}} \cdot U_{cat} \cdot pf \qquad if\ F_{train} < 0 \quad (26)$$

where I_{max} is the maximum current (A) according to the motor characteristics that is associated to the maximum effort of traction or braking, F_{max} (kN), U_{cat} is the nominal catenary voltage (V) and pf is the motors' efficiency (%) that depends on the working load level (i.e., depends on the ratio between F_{train} and F_{max}).

5. Case study

This section presents a case study where the proposed model is applied to a line of Madrid Underground. The metro line consists of 16 interstations and two terminal stations spread over 14 kilometres and its infrastructure is modelled by means of gradients and curves. The characteristics of the train are: 160 tonnes, 78 tonnes of passenger load, a length of 90 metres and the maximum power offered is 1500 kW. The electric system is a 1.5 kV DC. Speed limits set by the railway operator are respected at all times. The dynamic simulation of the train taking into account the above parameters is carried out by means of a detailed model of the train movement and execution of the on-board ATO driving parameters. The minimum dwell time (t_{min}) takes a value of 20 seconds.

The variations of the possible values of the ATO driving parameters and their maximum and minimum limits are defined according to Table 7.

	Deceleration rate (m/s²)	Regulation speed (km/h)	Coasting speed (km/h)	Re-motoring speed (km/h)
Minimum	0.60	30	30	5
Maximum	0.80	80	80	50
Increase	0.05	0.25	0.50	1

Table 7. ATO Driving parameter settings.

The headway of the timetable to be designed is 420 seconds (7 min).

For the analysis of the case study, the process described in Section 4 is carried out searching for the optimal driving (Equation 12) for each interstation with MOPSO optimisation algorithm.

MOPSO parameters used in to generate the local eco-driving Pareto curves are listed in Table 8.

Swarm size	Iterations	c1	c2	w	Top select (%)	Top select probability (%)
40	1000	1	1	0.9	6%	98

Table 8. MOPSO parameters.

The result of this process is a Pareto front for each interstation and is the input information used for the following optimisation procedure. Figure 7 shows an example of solutions after the local eco-driving calculation process. Each point of a Pareto curve represents a specific ATO parametrisation that produces the most efficient driving for the corresponding running time at that interstation.

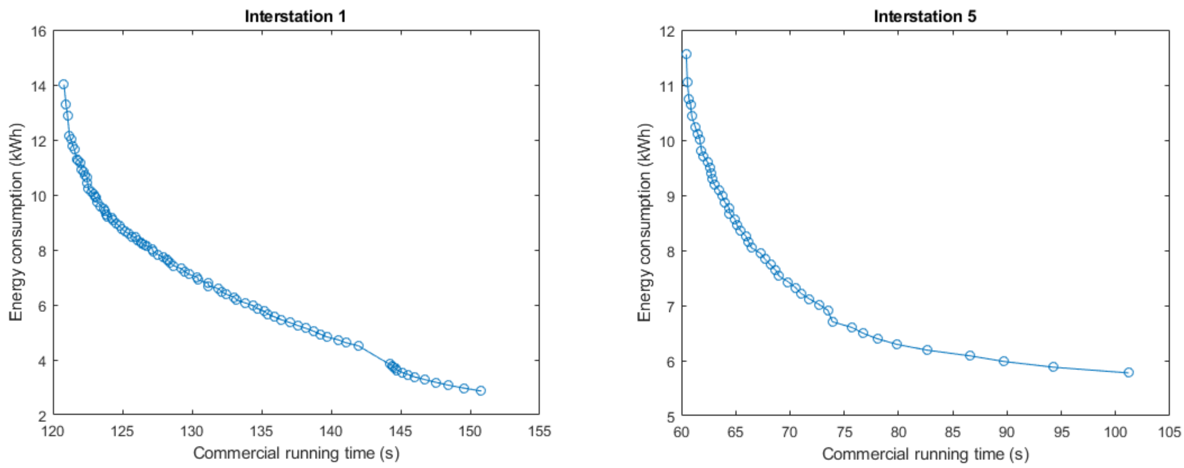

Figure 7. Examples of Pareto front of interstations 1 and 5: designed eco-driving, running time / energy consumption.

As previously described, the procedure continues with the timetable optimisation (Equation 1) and the global Pareto front is obtained. Each solution of this global front represents the set of efficient ATO driving parameters at each interstation of the entire railway line and stopping times. The obtained selection of running times and stopping times leads to synchronised braking and tractioning trains to maximise the use of regenerative energy.

The timetable optimisation problem is solved by the proposed algorithm GLMO-NSGA-III. In the following, this solution is compared as well with other algorithms: WOF (Zille et al. 2016b) (Weighted Optimization Framework), LCSA (Zille and Mostaghim 2019) (Linear Combination-based Search Algorithm), FDV (Xu Yang et al. 2021) (Fuzzy Decision Variable) and NSGA-III.

The optimisation process using the FDV algorithm is divided into two distinct parts, fuzzy evolution and precise evolution. In the first one, fuzzy evolution substages are added and the fuzzy operation is performed once the offspring has been generated at the beginning with the MOEA (Multi-Objective Evolutionary Algorithm) that has been chosen, being in this case the LMOCSO (Y. Tian et al. 2020) (Large-Scale Multi-Objective Optimization Competitive Swarm Optimizer). In the second phase, offspring generation operations are carried out. The next population generation is examined by means of LMOCSO. The fuzzy evolution phase in turn consists of: fuzzy evolution sub-stages division and fuzzy operation. The purpose of the first of the above phases is to make the algorithm's solution more accurate by dividing the fuzzy evolution into multiple phases in which the level of fuzzification decreases. In the second phase, the degree of fuzzification of the solution is determined.

LCSA algorithm is a procedure in which solutions of the optimisation problem are formed by linear combination of previous results to improve the quality of solutions in multi-objective problems. A population of coefficient vectors is formed and optimised by a metaheuristic procedure to improve the search for solutions. The algorithm starts with the optimisation of the population with any multi-objective method. The first non-dominated solution front of the existing population is used to form a solution matrix. Then, a random population of the linear combination of solution vectors is formed. These vectors are optimised through an arbitrary procedure, storing this result. Finally, the previous result is combined with the initial population, and the process is repeated. In this case study, the multi-objective optimisation algorithm applied has been the NSGA-III.

WOF algorithm has been designed as a metaheuristic procedure to be incorporated in a population-based optimisation mechanism. This method is based on the grouping and optimisation of the weighting variables to be applied, and the weighting variables are applied to cases where the objectives are multiple. In the WOF algorithm, the decision vector is divided into smaller vectors. The corresponding weighting factor is applied to each of these vectors. The whole process is divided into two different phases. Initially, an optimisation is performed with any of the following randomly chosen algorithms: SMPSO (Speed-constrained Multi-objective Particle Swarm Optimisation), MOEA/D (Multi-Objective Evolutionary Algorithm based on Decomposition), NSGA-II, NSGA-III. After this, a certain number of solutions are chosen from the current population and a new optimisation is performed with the NSGA-III algorithm and taking into account the corresponding reduced decision vector.

Table 9 presents the parameters of the algorithms used for the optimization. All algorithms have been executed with 220 generations and a population size of 100. Additionally, the random seed is initialized for the generation of the initial random population in all cases, and for all executed and compared algorithms. This uniformity in the initialization process and parameters further ensures a fair comparison of the algorithms' performance.

FDV	**Fuzzy evolution rate**	**Step acceleration**	**Optimiser**	**Aggregation function**	-
	0.8	0.4	LMOCSO	Penalty based Boundar Intersection	-
GLMO-NSGAIII	**Grouping method**	**Variable groups**	**Optimiser**	-	-
	Ordered	4	NSGA-III	-	-
LCSA	-	-	**Optimiser**	-	-
	-	-	NSGA-III	-	-
NSGAIII	**Simulated binary crossover probability**	**Crossover distribution index (η_c)**	**Mutation distribution index (η_m)**	**Polynomial mutation probability**	-
	1	30	20	$\frac{1}{number\ of\ variables}$	-
WOF	**Grouping method**	**Variable groups**	**Transformation function**	**Evaluations (original/transformed problem)**	**Fraction of function evaluations for the alternating weight-optimisation phase**
	Ordered	4	Interval	1000/500	0.5

Table 9. Algorithm configuration parameters.

Figure 8 presents the behaviour of the algorithms after the global optimisation process considering the electrical loss function.

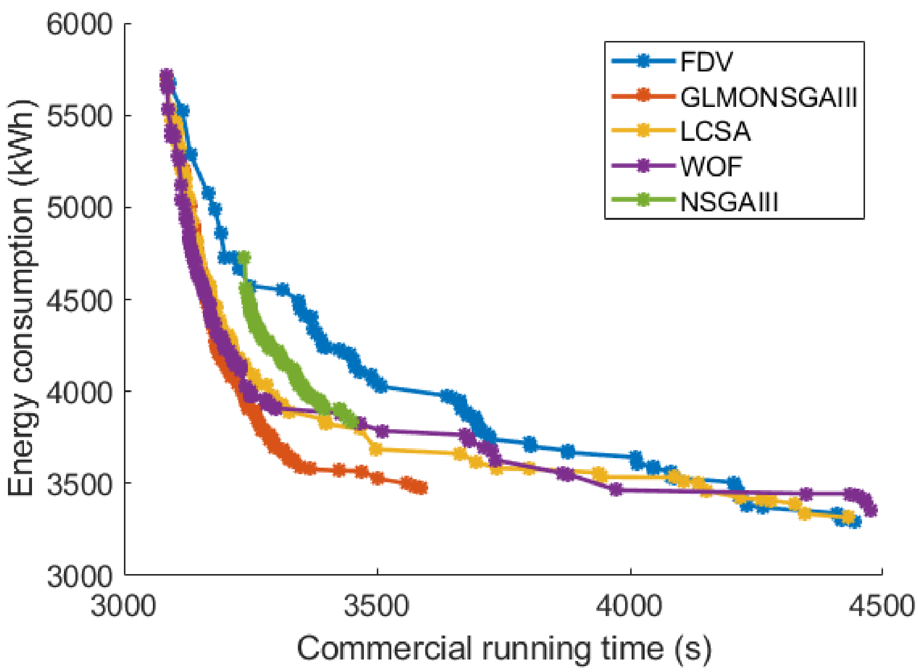

Figure 8. Pareto fronts comparison.

For the resolution and optimisation of the timetable as the second part of the problem, the GLMO-NSGA-III has been chosen due to its good results after the comparison with the rest of the algorithms. When NSGA-III is combined with the GLMO operator, the algorithm provides better solutions. It can be observed that GLMO-NSGA-III obtains better solutions in the range between 3100 seconds and 3600 seconds. After that, the LCSA algorithm provides more solutions. However, in the design of railway timetables the time margin that is usually given above the minimum time is between 3.5% and 10%, so the range for the running time is up to 500 seconds. Therefore, solutions provided by GLMO-NSGA-III are adequate from the operational point of view.

In the following, the proposed model is firstly executed without introducing the penalty on the number of trains to analyse the obtained results. The design of the timetable is carried out in two different cases depending on whether regenerated energy is considered or not in the optimisation model. The first case will refer to the optimal design of the timetable when the energy regenerated by the trains is ignored in the analysis. The second case will consider the energy regenerated during braking being more faithful to reality by adding the losses given during transmission. These losses are modelled by means of a function dependent on the distance that separates the train that is regenerating energy and the train that receives and uses the energy (Figure 2).

Figure 9 plots the optimisation scenarios representing each of them as the corresponding Pareto front. As expected, the driving with the highest associated consumption corresponds to the scenario where the regenerated energy is not reused.

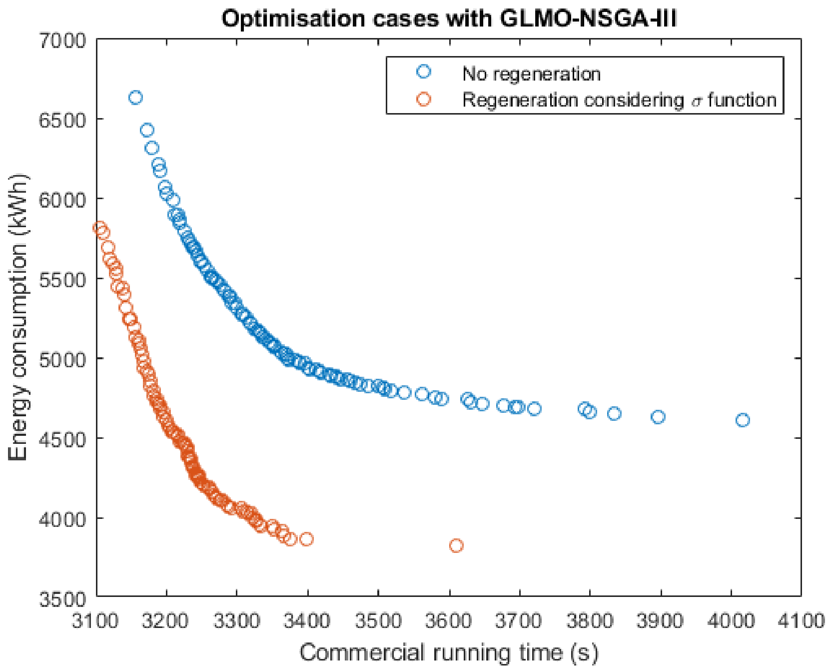

Figure 9. GLMO-NSGA-III optimisation cases.

Results in Figure 9 are the Pareto curves obtained by the optimisation algorithm in these cases. However, in reality, when the solutions obtained by the algorithm are tested on the line (by simulation) there is regenerated energy and the transmission of this energy between trains has associated losses that depend on the distance between trains. Therefore, the result of simulating the above Pareto fronts on a line with regenerated power transmission with losses according to the defined loss model is presented below in Figure 10. It can be seen in Figure 10 that the curve that was optimised taking into account the regenerated energy and σ function has a better behaviour in terms of consumption as it has been tried to preferentially synchronise accelerating and braking trains.

Figure 10 shows the optimisation result when regeneration is considered with the loss function and the result when regeneration is not considered in the optimisation, both simulated in the case where regeneration with losses during transmission is added.

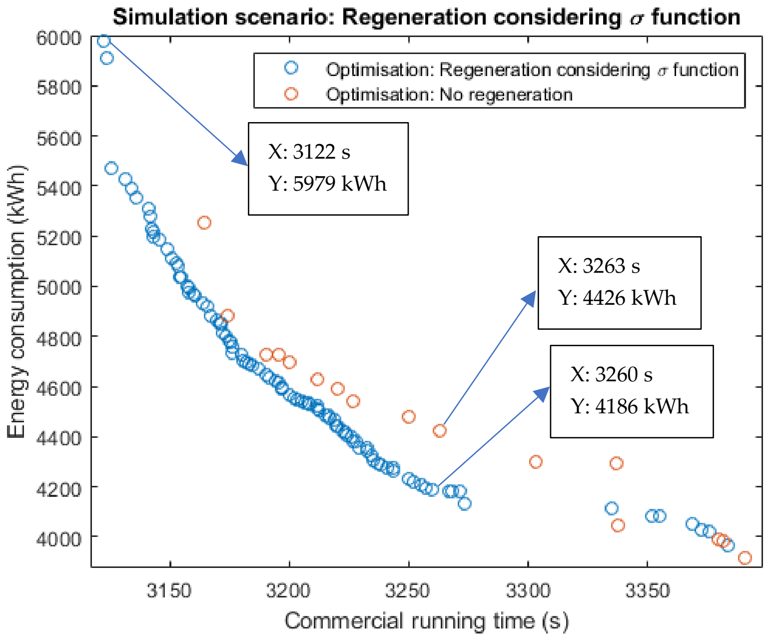

Figure 10. Optimisation results simulated considering σ function.

As can be seen in Figure 10 a better result is obtained for the optimised case where regeneration and the loss function are considered. The graph highlights the points corresponding to the fastest possible set of driving for the whole metro line and the same for a set of 4.52% slower driving with respect to the optimised case in the two optimised cases. Comparing the result of both optimisations for what could be the nominal set of driving shows an energy saving of 5.42% with the optimisation when considering regeneration and σ function.

Previous results do not take into consideration the number of trains required to meet the demand. Firstly, the number of trains is calculated as a further step to the optimisation (Equation 10 and 11), and the results are shown in Figure 11. In this case, no penalty has been introduced for the objective function. This is why the solutions between the 8-train and 9-train cases overlap in some areas of the graph. As shown in the figure, the Pareto front contains similar solutions that are optimal in terms of energy and running time but with different numbers of trains required to give the associated service. With the same consumption and running times some solutions require 8 trains and others require 9 trains, which impact critically on the investments and operation. The results of Figure 11 present the energy consumption vs the commercial running time. The commercial running time is the commercial speed experienced by the passengers, so the time spent by the trains at stops in terminal stations is excluded. This time is used by the trains for the reverse manoeuvres, to unload and load passengers, and as a buffer to absorb delays. However, it is also crucial to determine the synchronization of ascending and descending trains to use regenerated energy as was studied in (Roch-Dupré et al. 2018). Therefore, solutions with the same commercial running time could present different cycle time in the line and different number of trains to operate the timetable.

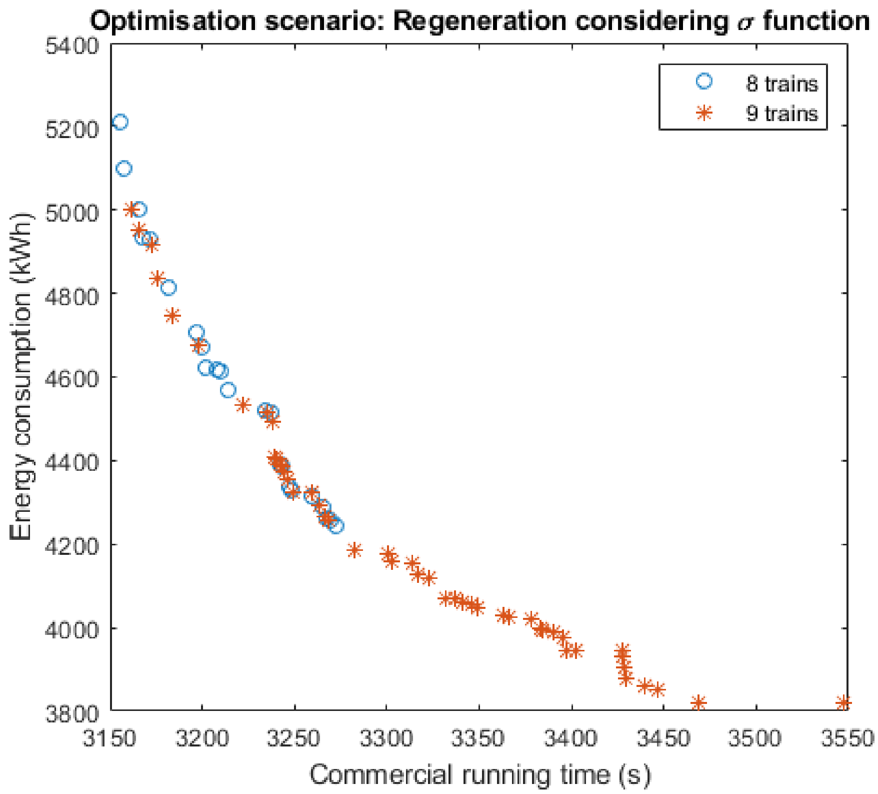

Figure 11. Timetable optimisation without penalty for the number of trains.

Figure 12 shows the optimisation after including in the objective function (Equation 1) a penalty or additional cost of 25 kWh for each extra train required in the achieved solution. This penalty is expressed in the term $P \cdot N_t$ of the Equation 5. Hence, the areas of the 8-train and 9-train cases are clearly differentiated.

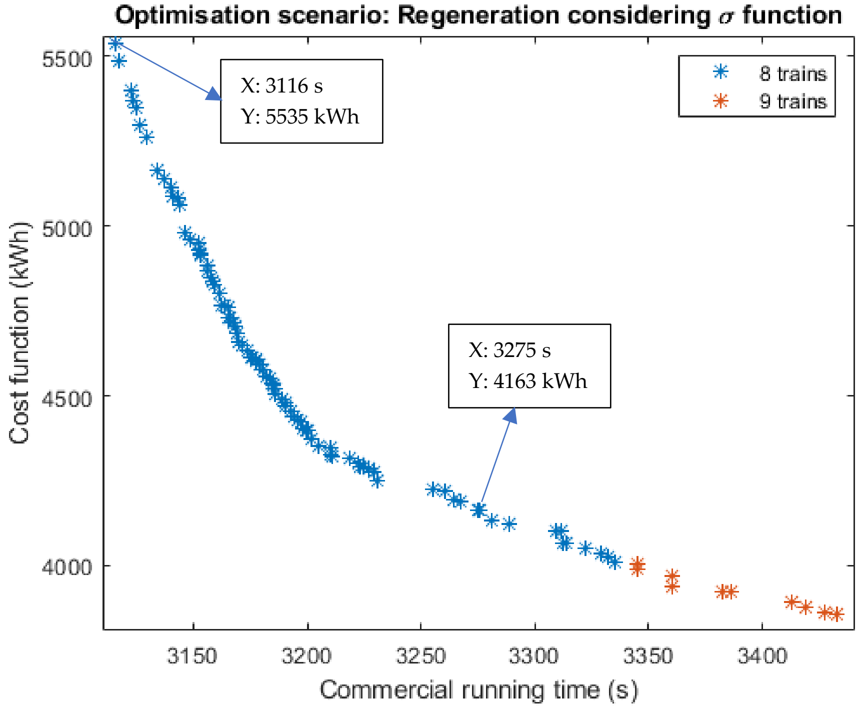

Figure 12. Timetable optimisation introducing penalty for the number of trains.

In this case, a timetable can be chosen with a running time around 5% slower than that associated with the flat-out driving between stations following the railway operator's criteria of setting a time margin for the design of the timetable. In this way, an energy saving in relation to the cost function of 1372 kWh is achieved, which represents a saving of 24.79%, not being necessary to include more trains on the line ensuring the fulfilment of the service.

Figure 13 compares the energy that would be consumed in rheostats in the results obtained in Figure 10. Both cases were simulated by considering the regenerated energy and the losses associated with its transmission. Orange bars show the result when the optimisation is carried out without considering the energy regenerated during braking and the blue bars show the result when the energy regenerated during braking is considered with the associated transmission losses.

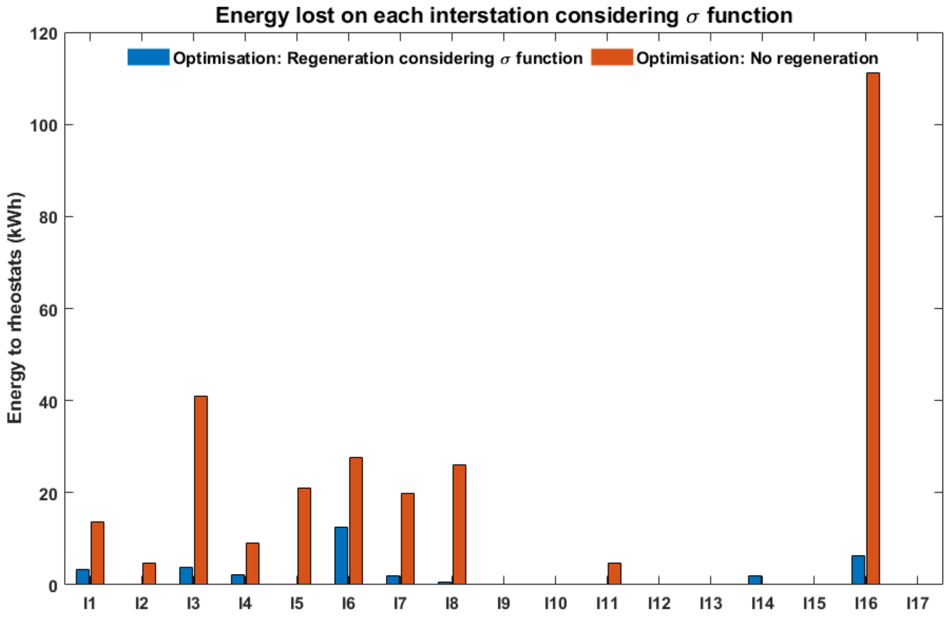

Figure 13. Energy dissipation in rheostats at interstations.

It can be seen from the result that synchronisation improves the use of regenerated energy in every interstation. The energy that would be wasted in the case where regeneration was not taken into account would be 279 kWh while this wasted energy drops to 33 kWh when the driving and synchronisation are designed taking regeneration into account.

Figure 14 and Figure 15 show the two optimised speed profiles for the cases compared for track 1 and track 2, respectively:

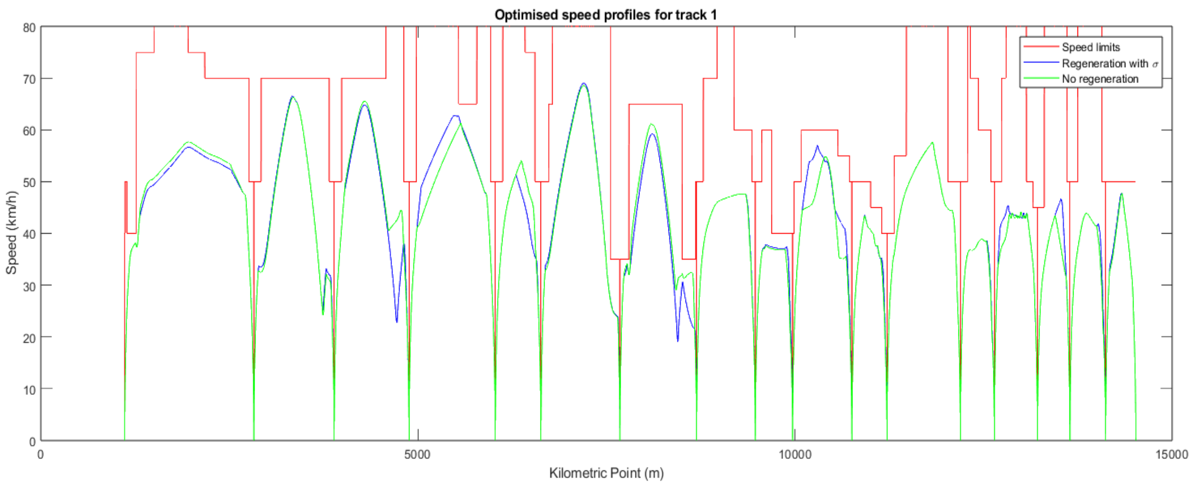

Figure 14. Speed profiles for track 1.

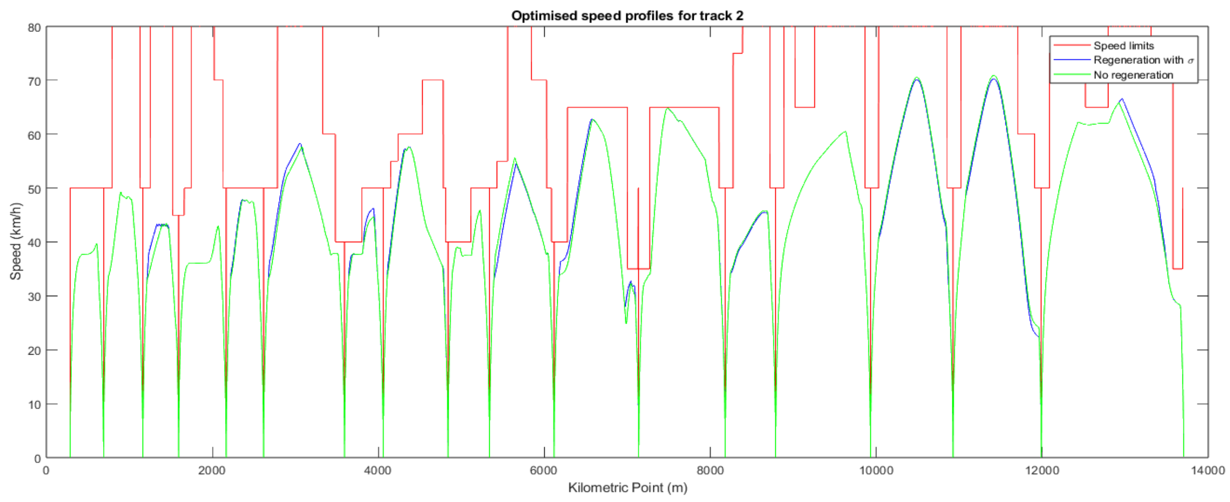

Figure 15. Speed profiles for track 2.

Table 10 and Table 11 show the arrival and dwell times for the compared cases, in track 1 and track 2. These figures have been obtained for a train that arrives to the first station in time equals to 0. As can be seen, when no regeneration is considered all the dwell times are the minimum value except for the reverse time in the first station that is 287s. However, when considering regeneration with σ function, there are cases where the dwell time is above the 20s (station S8 for instance) and both reverse times are greater than the minimum value (45s in S1 and 38s in S18). This is because the algorithm is synchronizing accelerating and braking trains to increase the use of regenerated energy.

	Optimisation: considering σ		**Optimisation: no regeneration**	
Station	**Arrival time (s)**	**Departure time (s)**	**Arrival time (s)**	**Departure time (s)**
S1	0	45	0	287
S2	183	203	423	443
S3	303	323	544	564
S4	415	435	649	669
S5	528	548	766	786
S6	614	634	848	868
S7	731	751	966	986
S8	864	885	1087	1107
S9	959	979	1181	1201
S10	1039	1059	1264	1284
S11	1133	1153	1363	1383
S12	1210	1230	1441	1461
S13	1322	1342	1553	1573
S14	1397	1417	1629	1649
S15	1478	1498	1712	1732
S16	1550	1570	1789	1809
S17	1624	1644	1865	1885

Table 10. Arrival and dwell times for track 1.

	Optimisation: considering σ		Optimisation: no regeneration	
Station	Arrival time (s)	Departure time (s)	Arrival time (s)	Departure time (s)
S18	1693	1731	1934	1954
S17	1780	1800	2004	2024
S16	1854	1874	2078	2098
S15	1926	1946	2150	2170
S14	2010	2030	2235	2255
S13	2082	2102	2307	2327
S12	2193	2213	2418	2438
S11	2269	2289	2495	2515
S10	2365	2385	2591	2611
S9	2443	2463	2670	2690
S8	2541	2561	2765	2785
S7	2657	2677	2884	2904
S6	2765	2785	2992	3012
S5	2852	2872	3079	3099
S4	2972	2992	3200	3220
S3	3074	3094	3301	3321
S2	3188	3208	3414	3434

Table 11. Arrival and dwell times for track 2.

6. Conclusion

This paper presents a procedure for optimal timetable design considering efficient driving for a metropolitan railway line based on detailed simulation. The objective is to design the optimal operation, minimising the energy consumption and considering the rolling stock required to provide the service. This is achieved by integrating the optimisation of the ATO driving parameters and the optimisation of the timetable, adding the contribution of the energy regenerated by braking trains. The number of trains required to fulfil the timetable is included in the model to consider costs associated with rolling stock. The model starts with designing the ATO eco-driving parameters for the whole line where the energy consumption is minimised as a function of running time. The search for efficient driving has been carried out by means of the MOPSO algorithm, and the solutions form a Pareto front where running time and consumption are conflicting objectives. These ATO parameters satisfy realistic operating conditions easing its implementation in real conditions. The timetable is then designed by optimising the time margin distribution, the stopping times at each station and synchronising traction and braking trains during their journey on the whole line to maximise the use of regenerated energy. Several optimisation algorithms have been tested for the efficient timetable design, including WOF, LCSA, NSGA-III, GLMO-NSGA-III and FDV. In this study, GLMO-NSGA-III has been found to be the best performing algorithm.

The procedure described has been applied to a line of Madrid Underground. The optimal design has been carried out in two different scenarios: ignoring regenerated energy and considering regenerated braking with transmission losses to show the importance of including regenerated energy use calculation in the optimisation model. It has been proved energy savings up to 5.42% when the proposed model considers the regenerative braking for the optimisation and it permits to take into account the number of trains required to provide the service when deciding the commercial speed of the line. Likewise, the calculated driving meets the railway operator's design criteria, giving a margin time for the nominal operation, offering also an accurate reference for metro railway operators.

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