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Redesigning Probabilistic Production Costing models and reliability measures in the presence of market demand elasticity

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Abstract—. **The ability to analyze the market performance, set proper objectives and to anticipate and analyze the market response to potential new mechanisms is essential for regulators. In this context, simulation models represent powerful tools to assist the regulator in his duties.**

The objective of this paper is to illustrate and deal with the proper definition of metrics to evaluate whether the market is reaching efficient outcomes. To do so, we first extend the classic formulation of the classic Probabilistic Production Cost models (PPC) by proposing a novel algorithm that allows easily and efficiently to introduce demand elasticity and then on this basis we illustrate how traditional reliability measures are not suitable metrics to be used when a non-negligible part of the demand is elastic.

***Index Terms*— Electricity markets, Security of electricity supply, Probabilistic Production Cost models, demand elasticity**

I. Introduction

The ability to analyze the electricity market performance, in order to set proper objectives and to anticipate and analyze the market response to potential new mechanisms is essential for regulators. In this context, simulation models represent powerful tools to assist them in their duties.

In particular, the first necessary step in the complex task of achieving an optimal level of security of generation supply is to detect whether or not (and assess to what extent) there is a problem. In order to carry out such an assessment it is necessary to define metrics to evaluate the electric power system and market performance. These metrics, when referred to the firmness and adequacy dimensions, have been usually expressed in terms of "pure" reliability (for instance, based on the Lost of Load Probability (LOLP), Lost of Load Expectancy (LOLE) and Expected Non-Served Energy (ENSE) values), assigning a secondary role to the actual cost of enjoying the level required

One of the simulation tools which has been traditionally used to carry out electric power generation system reliability assessments is the so-called Probabilistic Production Cost (PPC) model.

In this paper, our purpose is to contribute to the development of this approach:

- We first face one of the challenges in this context: we propose to use an algorithm that allows extending the classic PPC models design to explicitly represent demand elasticity. The methodology proposed is a simple, robust, closed and consistent solution which clears up the task without complicating the original approach.
- Then, this redesign will serve us to illustrate how, in the presence of demand elasticity, the aforementioned traditional reliability measures are no longer a consistent proxy to estimate the system performance, since as we discuss, they leave aside relevant information. Indeed, in a fully elastic demand context, the LOLP or the NSE value as traditionally defined, would take a zero value, no matter which the generation availability would be.

Next in the remainder of this introduction, we describe the basic PPC model methodology. Then in section II the algorithm to model demand elasticity is developed. Finally, in section III, we discuss the reasons that lead us to state that reliability measures are not suitable metrics in a context in which demand elasticity is significant. This will lead to the discussion on the ways to better define a proper metric to estimate to what extent the market outcomes are adequate.

A. Probabilistic production cost models: a classic tool to measure the reliability of a power system

Probabilistic Production Cost models (PPC models) have been traditionally used as a support tool in the centralized long-term decision-making process in electric power systems. These models are characterized for centering all the computational efforts in representing the random nature of some of the most relevant variables involved in the long-term planning problem (typically the demand values and the forced outage rates of each generating unit). They allow for reliability assessments of real-size electric power systems with little computational effort. However, this is achieved at the cost of making strong simplifications regarding short- and medium-term operational and planning constraints of the generation plants.

This approach has attracted considerable efforts from academia since the late 60's. The basic model corresponds to its application to a non-constrained thermal system; see the pioneering works of (Baleriaux et al, 1967) and (Booth, 1972).

The major outputs that were first calculated using these models were:

- Reliability measures: the loss of load probability, the loss of load expectancy, the expected value of the non served energy, etc.

- Expected production schedules, that is, the expected energy generated by each generating unit.
- Expected production costs.

Although for the purposes of the present chapter, this basic approach will serve as the starting point to illustrate the effect of demand elasticity, is it important to note that there are dozens of papers where several developments have been introduced to the classic approach. For instance, there are remarkable works focused on introducing simplified alternatives to include hydraulic units (Finger, 1978), (Ramos et al., 1991) or (Malik, 2004), storage units (Conejo, 1987) or (Invernizzi et al., 1988), time dependent units (Conejo et al., 1985), etc.

One of the most popular extensions to the basic model is the so-called frequency and duration method. This extension introduces information on frequency and durations of the different states (demand interval, outage rates, etc.). This allows calculating additional information, as for instance the mean time existing between two consecutive events (typically scarcities). There are different approaches to introduce this frequency and duration data, see for instance (Ayoub & Patton, 1976) or (Finger, 1979).

There are also different approaches to compute some additional results, for instance in (Leite da Silva et al., 1988) or in (Lee et al., 1990) a means to calculate the underlying variance of the results is provided. A description on how to estimate derivatives (e.g. marginal values), can be found in (Ramos et al, 1994) and also in (Maceira & Pereira, 1996).

These PPC models have been applied to determine the marginal contribution of each generating unit to the regulator's reliability objectives. One of the first works in this respect is the one developed in (Garver, 1966). A more recent work trying to determine this contribution to reliability objectives (in this case, the contribution of wind energy) by means of a PPC model can be found in (Kahn, 2004). This sort of calculations have served for instance to set the remuneration for each generating unit in some real systems in which a capacity payment mechanism had been implemented (this was for example the case of the former Chilean mechanism or the Panamanian case[1]).

However, in the literature the way to address electricity demand elasticity in PPC models is still a pending issue, since no close solutions to deal with this issue can be found. Inon & Hobbs (2004) approach the problem by resorting to a stylized model in which they represent demand elasticity in a very simplified way by considering two different realizations of the demand curve and comparing the impact on the expected values of LOLP in the long run.

Malik (2001) aimed at representing the impact of demand-side programs, but the chief objective of his approach was to introduce and assess the effect of load shifting programs, which are demand side-management programs that seek to move the load consumption from peak to off-peak hours. This is carried out by breaking down the load shifting operation into two operations that can be separately modeled within the PPC context by means of limited energy generators. This way the model separates peak clipping (reduction of the consumption on the peak) and valley filling (increasing it on off-peak hours), and models the first operation as an equivalent hydro unit and the second as the process of loading a pump storage unit (for more details, see any of the references provided before on how to perform such operations). However, in this model, the energy to be shifted from peak load to valley load is introduced as an exogenous parameter, thus, in rigor no explicit response to prices is modeled.

As stated, our objective is not to model load shifting programs, but introducing explicitly demand response to prices. Here we propose a methodology that solves the problem in a simple and compact way. In order to ease and clarify the description, we will present the proposed new formulation on the basis of the simplest PPC model design (representing just non-energy limited thermal generating plants, modeled through their maximum output and their forced outage rate). As it is latter shown, thanks to the consistency and straightforwardness of the algorithm proposed, the methodology allows further complication of the system representation on exactly the same basis as the traditional approach itself.

B. Description of the Basic PPC Model

The basic PPC model is built upon the assumption that all generation plants can produce at full capacity at any time unless when they are out-of-order due to a forced outage. Hourly demand is considered to be inelastic and stochastic.

These models were conceived to check a basic reliability condition: whenever the system's (inelastic) demand exceeds the available generating capacity a loss of load takes place. The probability of such an event happening (the Loss of Load Probability or LOLP) and the corresponding expected non-served energy (ENSE), have been the main reliability results obtained from these PPC models.

In such a context, the loss of load probability distribution can be evaluated by means of the distribution of the difference between two random variables: the demand and the total generation available[2]. This difference is usually evaluated in a generic random hour. Longer-term results (e.g. the ENSE in a whole year) are calculated by directly extending the results obtained when computing this generic hour.

If all variables (demand and failure rates in the most simple case) are statistically independent, then the computation of the former difference considerably simplifies, since the sum (or difference) of two independent random variables is equal to

[1] The *Centro Nacional de Despacho* (CND) of the *Empresa de Transmisión Eléctrica S.A.* (ETESA) in Panama uses the FLOP model (see www.iit.upcomillas.es/aramos/flop.htm), a PPC model to calculate the so-called Firm Capacity according to which generating plants are paid in the context of the capacity mechanism in force.

[2] If only loss of load is being evaluated, just positive values of such difference would be of interest.

the convolution of their probability distribution functions.

We next present how the demand and the thermal generating units are modeled and the order followed in the convolution operation to simulate the generating units' scheduling. Then we explain how to interpret the results obtained when performing this operation.

1) Hourly load probability distribution

The probability curve for the electricity demand in a generic random hour should ideally be calculated by means of probabilistic forecasting techniques. However, it is commonly accepted as a well-suited proxy to take a large set of historical data and then assign the same probability to each one of the historical realizations of the demand, that is, each realization is supposed to have a probability of $1/n$, being n the number of hourly data considered. This way, the percent of time that a given load level (or a greater than a given load level) occurs in the set of data considered will be interpreted as a probability. Thus, at any given time (hour) there will be a probability of 1 that the load will be higher than the minimum load being considered.

Under the latter assumption, and as illustrated in **¡Error! No se encuentra el origen de la referencia.**, we can calculate the Load Complementary Distribution Function (LCDF) just by rotating the axes of the load duration curve corresponding to the historical horizon considered, and then normalizing the time period so that the vertical axis gives the percent of time (the probability) that a certain value of demand level is exceeded. This is the reason why the hourly Load Complementary Distribution Function (LCDF) is sometimes referred to as the Inverted Load Duration Curve (ILDC) or just Load Duration Curve (LDC).

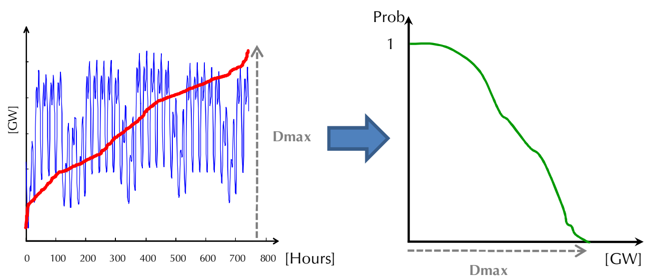


Fig. 1. Chronologic demand, corresponding load duration curve and estimated load complementary distribution function.

However, it is important to bear in mind that the LDCF curve does not represent anymore a demand monotone, but a complementary distribution function of the demand in a generic hour.

2) Thermal plants modeling

Each generator's available capacity is modeled as an independent discrete random variable. The simplest representation would be the two-state model, where the plant either is able to produce at maximum capacity (probability p) or it cannot produce because of a forced outage (probability $q = 1 - p$).

3) Dispatch criteria: the merit order

When operating constraints are not considered, the dispatch that results in the minimum operating cost is the one in which generators are dispatched in order of increasing marginal cost[3]. This ranking of the generators is usually known as the *merit order* or the *loading order*. This way, the convolution operation is performed following this merit order.

4) The equivalent load and the results provided by the basic model

As described in (Booth, 1972), let us introduce the concept of the "equivalent load after dispatching the first n units", denoted by EqL_n. This EqL_n represents the distribution function of the non-served load after having dispatched the first n generating groups in the merit order. This equivalent load can be computed by carrying out the convolution of the variables involved, as expressed next:

$$EqL_n = L - \sum_n C_n, \qquad n = 0,1,2,3,... \tag{1}$$

The first equivalent curve (n=0) represents the complementary distribution function of the load consumption (L) of the system, when no generator has been dispatched yet. The successive equivalent loads represent the load yet to be covered after dispatching each generator in the system. The last curve, EqL_N, represents the complementary distribution function of the demand left uncovered once all the system generators have been dispatched. **¡Error! No se encuentra el origen de la referencia.** illustrates this procedure, where the successive equivalent loads are calculated as a result of the successive probabilistic dispatch of the units.

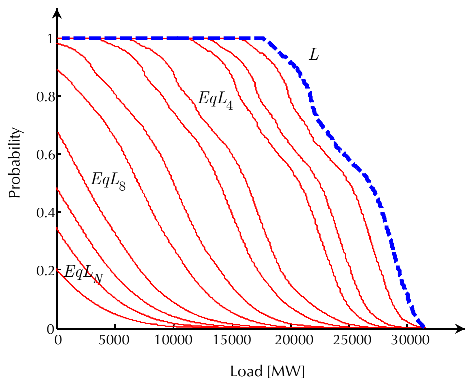


Fig. 2. Equivalent load curve

The last equivalent load curve represents the distribution function of the non-served energy. From this last equivalent curve we can extract the two following valuable pieces of information (see **¡Error! No se encuentra el origen de la referencia.**):

- The loss of load probability (LOLP) is the point where this last curve intercepts the y-axis (probability). It represents

[3] In practice, based on heuristic algorithms, this merit order can be modified in order to consider approximately some operating constraints (high start-up, on-line or shutdown costs, network transmission constraints, etc.).

the probability that there will still be non-served demand left to be satisfied after all the generators have been dispatched.

- The non-served energy expectation (ENSE) is the area beneath this last curve. It represents the expected amount of energy in MWh that is left unsupplied (in the generic random hour being represented) in the system after all the generators have been dispatched.

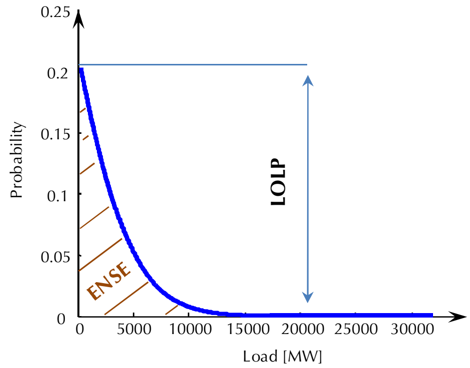


Fig. 3. The last equivalent curve: the non-served energy distribution function

II. Introducing Demand Elasticity in PPC Models

To introduce explicitly demand response to prices in PPC models we opt for modeling elastic demand bids by means of a set of equivalent thermal units, which as we show next, allows us to solve one of the pending issues in the literature regarding PPC modeling, introducing demand price elasticity in an extremely simple and robust way[4]. Our proposal allows clearing up the task without complicating the original approach.

A. *Redefining the merit order: modeling demand offer bids as equivalent generators*

For the sake of clarity we have opted for illustrating the basic idea of the algorithm proposed making use of both a deterministic demand and a deterministic set of the thermal units being available to produce. Once presented the main idea, it will be straightforward to introduce an analogous reasoning in the PPC methodology.

Let us consider that demand marginal utility (the demand offer curve) is given by the red step-wise curve presented in Figure 1. The available generators' capacities (MW) and their corresponding marginal costs (€/MWh) have been represented as an aggregated step-wise curve (dotted in blue) in the same chart.

[4] This solution has been previously used in other modelling approaches, as for instance in the context of a long-term deterministic simulation tool representing market agents' strategic behavior, see (Batlle & Barquín, 2005), as well as in former market equilibrium models designed by Prof. Barquín.

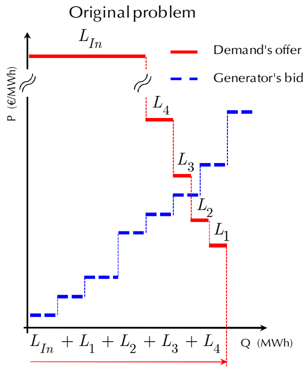


Figure 1. Demand and generation offer curves

Note that in the demand representation there are two differentiated parts:

The inelastic load consumption, denoted here by L_{In}. The associated offer price representing this portion of the consumption is assumed to be much higher than the variable cost of any of the generators. This price should ideally represent the value of loss of load (VOLL), also known as the Non-Served Energy Cost (NSEC).

The offers corresponding to elastic demand consumption; each step of the elastic fragment has been denoted by L_i, where i represents the elastic offer index, offered at a price, p_i^L.

The algorithm proposed consists in solving an equivalent problem (see Figure 2), in which the elastic demand is substituted by:

A new fictitious inelastic demand which is equal to the sum of quantities of all the consumption of the original demand curve (i.e. including both the inelastic and elastic consumption). This new fictitious inelastic demand, has been denoted in Figure 2 by L_{In}^F.

A set of fictitious generators, each one representing an offer step from the original demand curve. This way, each L_i, becomes a fictitious generator G_{Li}^F The corresponding quantity (MWh) and price (€/MWh) are those defining the original demand offer.

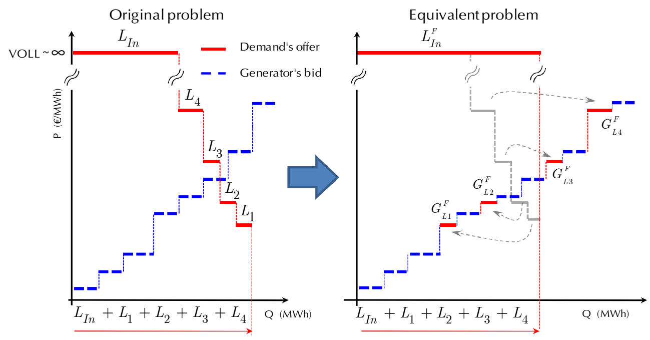


Figure 2. The equivalent model formulation

In the figure it can also be checked how the market outcome remains unchanged, since the resulting price, and the committed generators are exactly the same in both cases.

Note that in the equivalent problem, the production of the fictitious generators corresponds to those blocks of the demand which were not committed, that is, it corresponds to the energy that was not purchased because the offer price was below the resulting market price[5].

B. The PPC model with an elastic demand

The aforementioned methodology allows including demand elasticity in the PPC framework by performing well-known operations (the dispatch of thermal units). As described, the algorithm consists in solving an equivalent problem, where the elastic demand is substituted by an inelastic demand and a set of fictitious thermal units.

In the following, for the sake of simplicity we consider a unitary value for all the availability rates of the fictitious thermal generators. However, note that by means of the availability failure rate of the fictitious thermal units we can represent stochastic demand elasticity (exactly in the same way that the thermal units are modeled).

As it has just been pointed out, the production of the fictitious generators corresponds to demand consumption not committed at the corresponding price. This production can be calculated by means of the resulting equivalent load before dispatching each one of the fictitious unit.

In Figure 3 it is shown how this production can be calculated using the equivalent load after dispatching the previous units in the merit order (i.e. EqL_{n-1}), and the capacity of the fictitious generator. Assuming that the failure rate is zero, the dotted line (in red) represents the complementary distribution function of the production of the fictitious unit. The dahed area (in each of the charts included in the figure), represents the expected production (i.e. the expected non-purchased energy at that price). In the figures it has also been represented the probability of the corresponding demand block not being committed, what will be termed here as non-purchased energy probability (NPEP)[6].

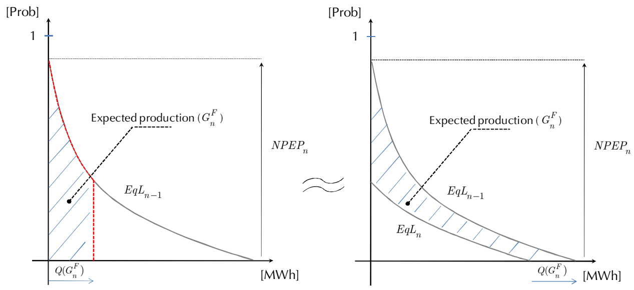


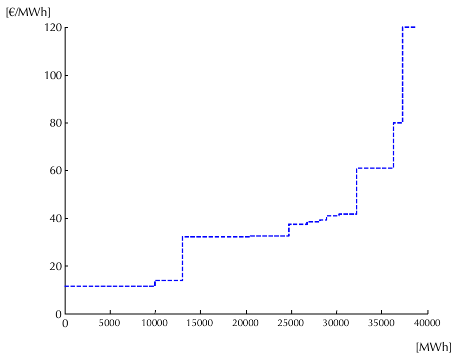
Figure 3. Expected production of the fictitious generator

[5] In other words, this is a way to represent that when the market price rises up to the level of an elastic segment of demand, this energy is retired. This is modeled by artificially committing a virtual generator that covers with its fictitious production this "hole".

[6] Note that this non-purchased energy probability is nothing but what is usually known as the $LOLP_{n-1}$, that is, the resulting LOLP after having dispatched al the preceding units in the merit order.

C. Numerical case example

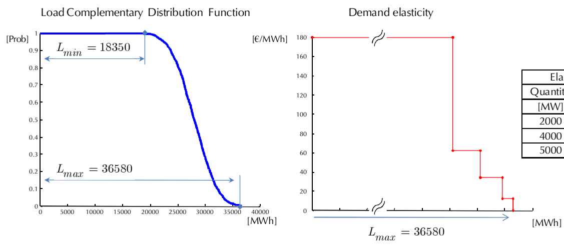
In order to better illustrate the procedure, we resort to a case example. In Figure 4 we have represented the thermal plants that are considered in the analysis as well as its characteristics (installed capacity, availability and variable costs).



Installed capacity	Availability	Variable costs
[MW]	[p.u.]	[€/MWh]
5000	0,95	11,50
5000	0,95	11,50
3000	0,93	14,00
2500	0,90	32,00
2500	0,90	32,00
2500	0,50	32,00
1500	0,90	32,29
1500	0,93	32,29
1250	0,93	32,29
1000	0,92	37,41
1000	0,92	37,41
700	0,92	38,45
700	0,92	38,45
700	0,92	39,05
700	0,92	41,00
700	0,92	41,00
2000	0,92	41,42
1000	0,78	60,85
1000	0,78	60,85
1000	0,78	61,00
1000	0,78	61,00
1000	0,78	80,00
1500	0,78	120,00

Figure 4. Thermal plants considered in the case study

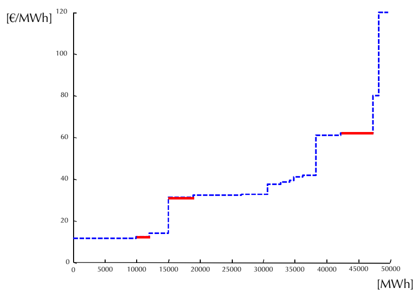
Next, in Figure 5 we present the Load Complementary Distribution Function as well as the demand elasticity[7]. The minimum demand value (including both the inelastic and elastic consumption) is equal to 18350 MWh, and the maximum 36580 MWh.



Elastic offers	
Quantity	Price
[MW]	[€/MWh]
2000	12
4000	31
5000	62

Figure 5. LCDF and demand elasticity function

We introduce the demand elastic offers in the model by means of the fictitious generators. The resulting generating mix considered, including these fictitious units (in red), have been represented in Figure 6.



Installed capacity	Availability	Variable costs
[MW]	p.u.	[€/MWh]
5000	0,95	11,50
5000	0,95	11,50
2000	**1,00**	**12,00**
3000	0,93	14,00
2500	0,90	32,00
2500	0,90	32,00
2500	0,50	32,00
1500	0,90	32,29
1500	0,93	32,29
1250	0,93	32,29
4000	**1,00**	**31,00**
1000	0,92	37,41
1000	0,92	37,41
700	0,92	38,45
700	0,92	38,45
700	0,92	39,05
700	0,92	41,00
700	0,92	41,00
2000	0,92	41,42
5000	**1,00**	**62,00**
1000	0,78	60,85
1000	0,78	60,85
1000	0,78	61,00
1000	0,78	61,00
1000	0,78	80,00
1500	0,78	120,00

Figure 6. The equivalent thermal unit problem

[7] Note that in order to simplify the algorithm understanding, implicitly we have assumed that demand elasticity remains unchanged during all the yearly blocks, i.e. roughly speaking it is the same in the off-peak hours than in the peak ones.

1) Results

In Figure 7 it has been represented the successive results obtained from dispatching probabilistically each one of generators considered in the equivalent problem. The areas dashed in red correspond to the dispatch of the fictitious generators.

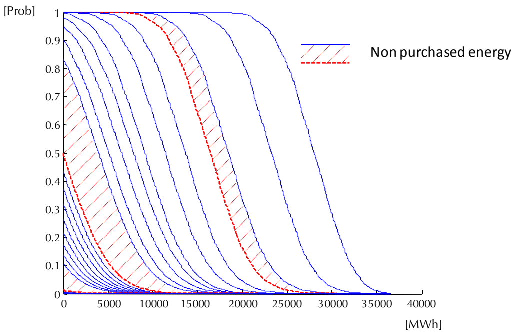


Figure 7. Case example results

Following the procedure described in Figure 3, we can calculate the probability that each one of the offers does not result accepted for falling below the market price. We have termed this probability associated to each offer as the Non-Purchased Energy Probability (NPEP). It is also possible to calculate the Expected Non-Purchased Energy (ENPE) corresponding to each one of the demand offers.

In the following table we have gathered these results.

Elastic offers		Results	
Quantity	Price	NPEP	ENPE
[MWh]	[€/MWh]	[p.u.]	[MWh]
2000	12	1.00	2000
4000	31	0.83	2718
5000	62	0.10	102

This way, the first block is never dispatched, since the NPEP value is 1, and the expected value of the non purchased energy (ENPE) is equal the quantity being offered. On the other extreme, the block offered at a highest price, is not dispatched 10% of the time, and the expected energy not being committed is 102 MWh.

The classic reliability measures take the following values:

LOLP	1,8E-04
ENSE	0,2325

III. NON-SERVED ENERGY AND NON-PURCHASED ENERGY

In real markets, the regulator is particularly concerned about guaranteeing the electricity supply for the inelastic demand. This happens for several reasons, among which we can highlight the following:

the inelastic consumption represents a large percentage of overall energy consumption,

the underlying marginal demand utility (ideally, the demand's offer price) is considered to be much higher than the one corresponding to the elastic demand,

In Figure 8 we have a real example of both the system demand and generator curves corresponding to the Spanish spot market. It can be clearly observed the two types of consumption already mentioned (i.e. the inelastic L_{In} and the elastic consumption L_{El})[8].

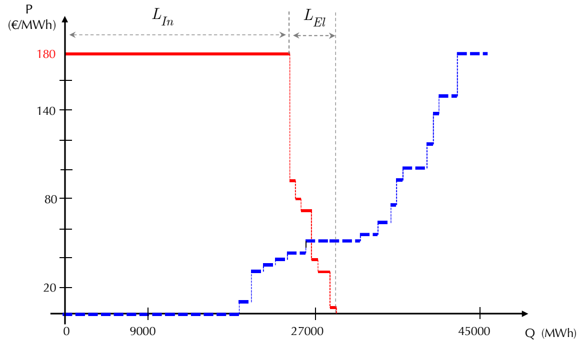


Figure 8. Demand and offer curves in the Spanish spot market

For the three reasons just mentioned, traditionally the regulator has defined its security of supply objectives in terms of the system capability to supply the inelastic consumption. This capability to provide this "critical" fraction of the demand is what is usually analyzed by the different so-called reliability measures. This way, the LOLP represents the probability of not being able to supply this inelastic consumption and the NSE the non-served energy corresponding to this inelastic consumption.

Therefore, the non-served consumption corresponding to the elastic demand blocks not being committed (which will be denoted in the following as non-purchased energy or NPE), does not intervene in the reliability measures.

To illustrate this, we have taken the previous example representing a tighter generation availability scenario (just three thermal groups are now available). In this new case we have represented the non-served energy (NSE) and the non-purchased energy (NPE) in both the real market situation and in the equivalent problem formulation.

[8] As pointed out in **¡Error! No se encuentra el origen de la referencia.**, section **¡Error! No se encuentra el origen de la referencia.**, in many systems the regulator has imposed price limits avoiding the system to reflect the true marginal demand utility. This is the case in Spain, where a limit of 180€/MWh has been in force since the electricity market was introduced.

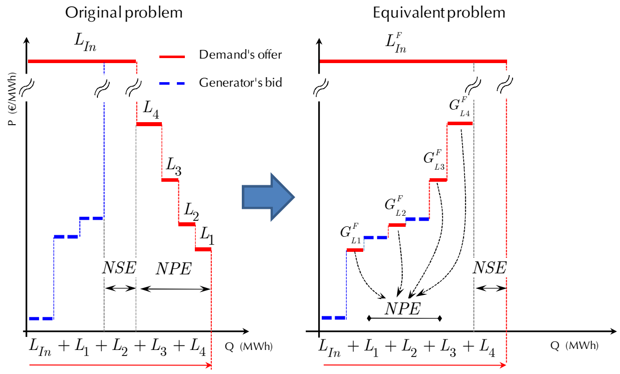


Figure 9. The Non Served Energy (NSE) and the Non Purchased Energy (NPE)

A. *Reliability measures in the presence of demand elasticity*

We next show the fact that, when assessing the system performance by means of the reliability measures, since we are just taking into account the inelastic consumption, we are leaving aside relevant information. This has been illustrated in the following figures, which serve us as the basis to give rise to the discussion about the weakness of these reliability measures in the presence of an elastic demand.

Three different scenarios of demand have been considered: a fully inelastic scenario, a partially elastic scenario and a fully elastic scenario. Each of these demand scenarios have been confronted with three deterministic scenarios of generation availability, one presenting a sufficient reserve margin, one in which several groups are unavailable and also one representing a severe scarcity (just a few groups are available). This is what we can observe:

In the case of the inelastic demand, see Figure 10, there are two generation availability scenarios which lead to non-served energy.

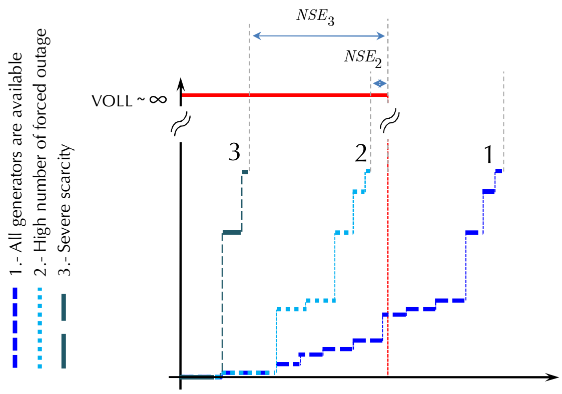


Figure 10. Fully inelastic demand scenario

In the case of the partial elastic demand, see Figure 11, there is only non-served energy in the scenario presenting the tightest reserve margin.

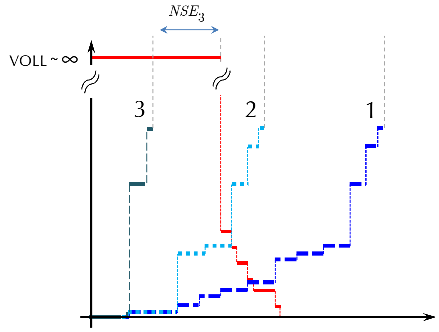


Figure 11. Partially elastic demand scenario

In the case with a fully elastic demand, see Figure 12, resorting to the classic definition of the aforementioned reliability measures, the non-served energy is equal to zero.

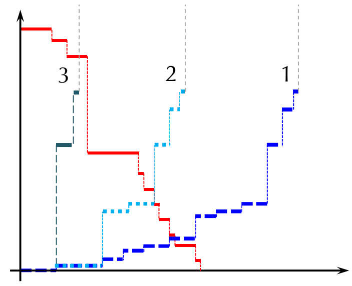


Figure 12. Fully elastic demand scenario

Does this mean that in a context where demand is fully elastic, reliability is not longer a problem? Resorting to the definition provided previously the answer is that effectively, reliability as classically defined is not longer a problem.

However, this does not mean that security of supply as defined in the present work is not longer a problem. Note that in this fully elastic demand case, a permanent scenario of severe scarcity will clearly be a problem from the regulator perspective. As previously discussed in **¡Error! No se encuentra el origen de la referencia.**, the objective of the regulator is to maximize the net social benefit, and this implies evaluating how far the outcomes provided by the market are from an ideal benchmark (the most efficient investments, resource management, scheduling, etc.).

Indeed, comparing the market results with a benchmark is exactly what is done when the regulator seeks to achieve a reliability standard (like the one day in ten years). These standards are supposed to represent the optimal benchmark, and this benchmark is supposed to take into account investment costs, the value of loss of load, etc.

B. *The metric to evaluate market performance*

In order to take into account the elastic consumption in the performance measure, we propose to evaluate the distribution function of the total value of what we have termed the Non-Purchased Energy.

This Non-Purchased Energy Distribution Function would have to be compared with that of resulting from the benchmark. This comparison would provide a more precise idea of how well the market is performing its “job” than the mere reliability criteria. The comparison can be performed in terms of the expected value, the VaR, the CVaR, or in general in terms of any measure of a distribution function. In Figure 13 in has been illustrated how this comparison could be performed in a generic case (the measure represented in the figure is the CVaR).

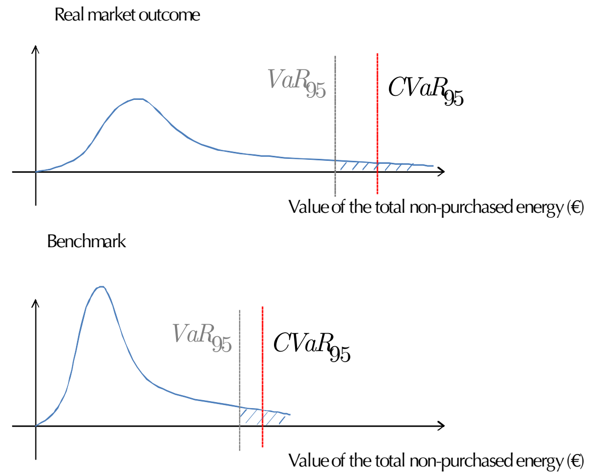


Figure 13. Measuring the market performance comparing with the benchmark solution

Note that the LOLP would no longer make sense in the fully elastic demand scenario. In this new context it can rather be calculated the probability of not supplying a certain demand block.

C. *Numerical case example*

The expected value[9] of the previous case example can be computed taking into account for each offer, the expected non-purchased energy and the corresponding price (we have considered a value of loss of load (VOLL) equal to 10000 €/MWh).

The expected value of the total non-purchased energy (ENPE):

$$ENSE{\cdot}VOLL + \sum_{i} ENPE_i{\cdot}p_i^L =$$

$$= 0,2325 \cdot 10000 + 2000 \cdot 12 + 2718 \cdot 31 + 102 \cdot 62 = \quad 116907 \text{ €}$$

This value should be compared with that of resulting from the benchmark scenario.

IV. Conclusion

We have developed the Probabilistic Production Cost (PPC) model approach by extending the classic formulation on the basis a novel algorithm that allows introducing demand elasticity in a simple and closed way.

Then, we have taken advantage of this model formulation to show how the traditional reliability measures, such as the LOLP or the ENSE, as a measure to assess the level of security of supply should be reconsidered in the presence of significant demand elasticity. This has led us to propose a better way to define a proper metric to estimate if the market results comply with the regulator's standards.

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[9] In our case example, the distribution function of the non-purchased energy value could also be easily computed taking into account that if any energy offer is not committed, then any energy offer presenting a lower price will also be left uncommitted. This is true since we are considering that demand elastic offers have a failure rate equal to zero.

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