Profit-based optimal scheduling of a hydro chain in the day-ahead electricity market

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Abstract:

This paper presents a profit-based model for short-term hydro scheduling adapted to pool-based electricity markets. The objective is to determine a feasible operation of a set of coupled hydro units belonging to a small or medium-size hydroelectric company. The company is assumed to be price-taker, and therefore, market prices are considered exogenous variables and modeled via scenarios. In order to be protected against the worse prices scenarios, a minimum profit constraint is introduced in the model as a risk-aversion criterion. In order to ensure a feasible operation, the model takes into account a very detailed representation of the generating units. In particular, the non-linear relationship among the electrical power, the net-head and the turbine water discharge is treated by means of an under-relaxed iterative procedure where net-heads are successively updated. During each algorithm stage, previous iterations' information is used to build the input-output curves and the resulting hydro scheduling problem can be formulated as a MILP optimization problem, where unit-commitment decisions are modeled with {0/1} variables. The model has been successfully applied to a real-size example case, which is also presented in this paper.

Keywords: Hydroelectric power generation, mixed integer linear programming, short term hydro scheduling, day-ahead energy markets, profit maximization.

1. Introduction

This paper presents an optimization model to help a hydro-generation company to schedule its hydroelectric units in the very short-term (up to 24 hours) under a competitive environment. The model is formulated as a stochastic profit-based hydro scheduling and the pool is supposed to be organized as day-ahead market.

The basic functioning of a day-ahead market is the following one. The market operator is the coordinating authority who receives the offers to buy and to sell electricity and performs the auction model to obtain the hourly marginal prices (used to remunerate all the generation), and the set of accepted and rejected bids. Generation companies are the power producers who try to sell electricity in the market. Energy service companies represent the load requirement of electricity customers and therefore they submit offers to buy electricity. Each hour, marginal price is found as the intersection between the hourly aggregated supply and demand functions.

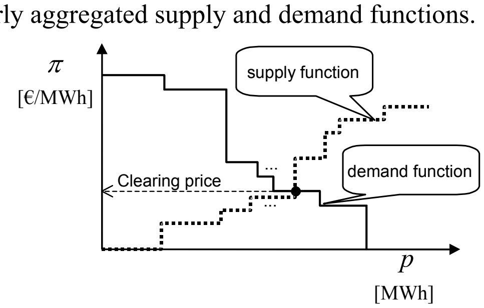


Figure 1: Market clearing

Some international experiences of this kind of markets can be found in Spain, Netherlands, PJM, Ontario, New England, Australia, etc.

In this framework, a pure hydro-generation company has to:

1. Elaborate a daily operation plan of its hydro resources in order to asses the available energy that could be offered in the day-ahead market.
2. Build the competitive hourly bids to sell that energy, and submit them to the market operator.
3. Throughout the operating day, operate its hydro-units trying to fulfill the market clearing schedule, and modify the program in the intra-day energy markets if necessary as real-time operation is getting closer.

This paper is focused on the first step, and therefore, the strategic bidding problem related to the second one, and the real-time reservoir management related to the third one, are out of the scope of this work.

In an electricity markets it is possible to discriminate two kinds of selling agents depending on their capacity to alter market prices: oligopolistic agents and price-taker agents. In both cases, as in any other business, the utilities' criterion should be the maximization of their expected profit, defined as the difference between market revenues and operation costs. However, the difference between them is that oligopolistic agents cannot consider market prices as exogenous variables, as their own decisions can affect market results. Therefore, an oligopolistic agent has to estimate its competitors' behavior and to consider explicitly its influence on clearing prices. This can be achieved by the estimation of the hourly residual demand functions, (García-González and Barquín 2000; Baíllo, Ventosa et al. 2004).

In case of a price-taker, its schedule cannot modify the hourly market clearing prices. Thus, market uncertainty is limited to the expected prices for the next 24 hours, (Conejo, Arroyo et al. 2002). It is important to note that during the first stages of the deregulation process, many electricity markets can be considered oligopolies, i.e. with a relatively small number of suppliers. However, as the market evolves, the demand grows and the system gets mature, new entrants might decide to participate as selling agents, normally as price takers. Therefore, there exists a real need for tools adapted to these new entrants. This paper is aimed at providing an optimization model to help an small or medium-size hydro-generation company that can be considered as a price-taker in the spot market. The main features of the proposed model are the following ones:

- The model is stochastic and considers simultaneously a predetermined number of price scenarios weighted with their corresponding probabilities.
- A risk aversion criterion is introduced in order to ensure at least a minimum profit.
- A very detailed representation of hydro equipment is taken into account. An special emphasis has been done in the modeling of the head-dependant units. This way, the non-linear relationship among water discharge, power generation and net-heads is considered accurately in the optimization problem.

The remaining paper is organized as follows. First of all, section 2 presents the state of the art of the short-term hydro scheduling problem. Section 3 shows the proposed model overview. Then, the mathematical formulation of the optimization problem solved each iteration is presented in section 4. After that, a real application is presented in section 5. Finally, concluding remarks are given in section 6.

2. Short-term hydro schedulling: state of the art

The generation company has to ensure a feasible operation of its available generation resources. Therefore, all the physical constraints of the hydro chain have to be accomplished: initial and final reservoir level conditions, water rights, detraction flows for water consumption, spillage management, hydro plants space-temporal relationships, etc. Among these technical constraints, the nonlinear relationship among the hydroelectric generation, the turbine discharge, and the net hydraulic head of the corresponding reservoir (Wood and Wollenberg 1984) is one of the main difficulties to cope with. Depending on the particular structure of the hydro sub-system, this dependence might be neglected, but in this case the head dependency has been considered within the scheduling algorithm in order to obtain feasible and realistic results.

Basically, the main strategies that can be found in the literature to deal with this problem are the following ones: to consider a discrete family of curves (up to 3 ó 5) (Conejo, Arroyo et al. 2002), to build an approximation of the truthful input-output surface by meshing and triangulation (García-González and Castro 2001), to implement an iterative procedure where each iteration considers a fixed head, which is successively updated (Pereira and Pinto 1983; Medina 1997) or to make a very realistic and detailed modeling of the problem and apply computer simulation techniques to find a near-optimum solution.

In this paper, an iterative procedure based on the under-relaxed nonlinear programming technique (Ortega and Rheinboldt 1970) has been implemented. This method was firstly introduced by the authors to solve the input-output curves problem under a traditional cost-minimization scheme in (García-González, Parrilla et al. 2003). Its main advantages are:

- The under-relaxed updating strategy avoid the oscillating solutions that can arise using other updating strategies as in (Pereira and Pinto 1983; Medina 1997).
- The company does not have to decide a priori the candidate input-output curves (Conejo, Arroyo et al. 2002), because the model selects iteratively the most accurate ones.
- The method presented here overcomes the size limitations of (García-González and Castro 2001), being suitable for larger hydro systems or, like in this case, for including additional features such as considering the maximum efficiency points, the start-up costs related to the loss of water, wear and tear o the equipment, etc. (Nilsson and Sjelvgren 1997).

Regarding the optimization techniques applied to solve this problem, a natural approach is to model the system as a network flow (N.Nabona, González et al. 1992), but this approach fails when it is necessary to consider particular characteristics of the hydroelectric equipment that cannot be simplified such us discrete unit-commitment decisions. Other techniques that can be found in the related literature are dynamic programming (Arce, Ohishi et al. 2002), Lagrangian Relaxation (Ni, Xing et al. 1999), Linear Programming (Medina, Conejo et al. 1994), Mixed Integer Linear Programming (MILP) (Chang, Waight et al. 2001; García-González and Castro 2001; Conejo, Arroyo et al. 2002), optimal feedback control, artificial neural networks, etc.

In this paper, the problem solved each iteration has been stated as a MILP problem (solved with the standard *Branch&Bound*), because of its modeling flexibility, modularity and quality of the solutions obtained.

3. Model overview

Price takers can split the optimal short-term problem into two separate phases (see Fig. 2):

1. The forecasting of the day-ahead market prices.
2. The optimization of its expected profit subject to the forecasted prices, and to a feasible schedule of the generating units. This problem will be denoted hereafter as the *Profit-Based hydro Scheduling problem* (PBS).

Regarding the first point, subsection 3.1 presents a discussion about the most appropriate methods available in the literature to forecast these prices. The PSB description, and the resolution method proposed in this paper is presented in subsection 3.2.

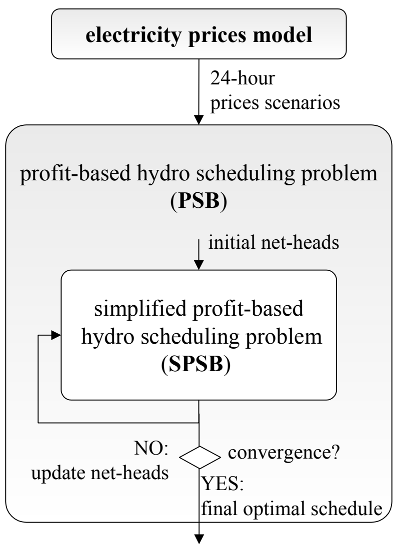


Figure 2: Model overview

3.1 Electricity prices models for the generation of price scenarios for the day-ahead spot market

Several models for electricity prices have been developed in the literature for the short term (Bunn 2004). In (Mateo, Muñoz et al. 2004) electricity prices models based on time series are reviewed and a novel classification is proposed. First of all, authors discriminate between stationary and non-stationary models. Although electricity prices exhibit a well known non-stationary component due to its multiple seasonalities (related to daily, weekly and monthly periodicity), several authors have proposed different stationary models for electricity prices series. For example, Dynamic Regression Models (Nogales, Contreras et al. 2002), Linear Transfer Function Models (Nogales, Contreras et al. 2002) or ARIMA models (Contreras, Espinola et al. 2003). In order to apply this kind of models the non-stationarity component has to be firstly removed by applying different techniques provided by classical statistics. However, as it is pointed out in Mateo et. al. this could not be enough in electricity markets as spot prices reflect a switching nature related to discrete changes in participants' strategies.

A more suitable alternative for modeling electricity prices is to apply non-stationary models. Econometric and Financial world have given rise the most of the non-stationary electricity prices models: Mean- Reversion models (Knittel and Roberts 2001), GARCH models (Batlle and Barquín 2002), two factor models (Schwartz and Smith 2000), or jump diffusion models (Deng 2000). Besides this, different authors have proved that neural networks can properly be used for modeling the evolution of the electricity prices series, (Szkuta, Sanabria et al. 1999). However the switching nature of spot prices require to apply switching models. In this group the most important econometric model is proposed in (Hamilton 1990) and (Fabra and Toro 2002) where price series is modeled through a Markovian switching process among autoregressive regimes, adapting to occasional discrete shifts in the level, variance and autoregressive dynamics of the series.

Finally in (Mateo, Muñoz et al. 2004) authors introduce a novel approach for modeling and forecasting electricity prices by the Input/Output Hidden Markov Model (IOHMM) originally proposed in (Bengio and Frasconi 1996). The switching nature, related to discrete changes in competitors´ strategies, can be represented by a set of dynamic models sequenced together by a Markov chain. In the IOHMM different market states are firstly identified and characterized by their more relevant explanatory variables. Moreover, a conditional probability transition matrix governs the probabilities of remaining in the same state, or changing to another. Finally, and at each time step the IOHMM model provides the probability density function conditioned to a set of input variables. This feature can be used for generating prices scenarios as proposed in (Mateo, 2005).

3.2 The problem PSB

In order to compute the global production of the hydro chain in the mentioned PBS, it is necessary to express accurately the hydroelectric generation functions of its hydro units. This can be attained by introducing in the PBS the following equation:

$$p_{ik} = \Phi_i(q_{ik}, h_{ik}), \ \forall i, \forall k \tag{1}$$

where p_{ik} is the power produced by unit i in hour k of the following day. This production depends on the turbine discharge q_{ik} and on the net-head of the associated reservoir, h_{ik}. When h_{ik} is constant $p_{ik} = \Phi_i(q_{ik})$ and only one curve is needed to characterize the generating unit. However, when the variation in the storage pond is a fairly large percentage of the overall net hydraulic head, it cannot be considered constant. Therefore, each hydro unit is characterized by its own function $\Phi_i(\cdot)$, which besides the efficiency curve of the turbine, the gravity constant and the water density, it might allow to model forbidden operating areas, multi-group effects, etc. The direct consideration of the whole real input-output curves in PBS could require excessive computational efforts and long execution times, (García-González and Castro 2001).

Suppose the case where the net head values for each reservoir along the temporal horizon were known, h_{ik}^{υ}. In that case, we could build the following time-varying functions where the net head dependence has been removed by substituting in $\Phi_i(\cdot)$ those known net head values:

$$\phi_{ik}^{\upsilon}(q_{ik}) = \Phi_i(q_{ik}, h_{ik}^{\upsilon}),\ \forall i, \forall k \qquad (2)$$

The hydro scheduling problem can be rewritten replacing the equations in by the following one,

$$p_{ik} = \phi_{ik}^{\upsilon}(q_{ik}),\ \forall i, \forall k \qquad (3)$$

The original problem is simplified, and the new will be denoted as SPBS (*Simplified Profit-Based hydro Scheduling problem*). The SPBS requires less computational effort to be solved than the PBS, as all the units are treated as non-head dependent. However, their solutions will remain different unless the functions $\phi_{ik}^{\upsilon}(\cdot)$ are the correct ones. For that reason, assume that the optimal solution of the PBS were known and denoted with an asterisk. In this case, the optimal power generation p_{ik}^{*} must satisfy:

$$p_{ik}^{*} = \Phi_i(q_{ik}^{*}, h_{ik}^{*}),\ \forall i, \forall k \qquad (4)$$

Thus, functions $\phi_{ik}^{\upsilon}(\cdot)$ leading to an equivalence between PBS and SPBS are the following ones:

$$\phi_{ik}^{*}(q_{ik}) = \Phi_i(q_{ik}, h_{ik}^{*}),\ \forall i, \forall k \qquad (5)$$

As the net heads are variables of the problem, univocally related to the reservoir levels at each time period, it is impossible to determine *a priori* the optimal evolution of net head values (h_{ik}^{*}), so it is necessary to introduce an iterative procedure to reach the optimal value (h_{ik}^{*}). In the appendix, there is a description of the under-relaxed iterative procedure implemented to solve this problem.

4. SPBS Mathematical formulation

The SPBS problem is formulated as a MILP optimization problem, which is solve at each iteration of the mentioned under-relaxed algorithm.

The following subsections include the mathematical formulation of objective function and the considered constraints.

4.1 Objective function

The main objective is to maximize the expected profits of the hydro chain in the day-ahead market, avoiding unnecessary spillages and considering possible start-up costs. The objective function takes into account all the price scenarios at once and weighed by its occurrence probability.

Maximize:

$$\sum_{n=1}^{N}\sum_{k=1}^{K}\rho_n \cdot \pi_{k,n} \cdot \sum_{i=1}^{I}\left(p_{ik}\right) - \sum_{i=1}^{I}\sum_{k=1}^{K}\left(ps \cdot s_{ik} + c_i \cdot y_{ik}\right) \qquad (17)$$

where ρ_n is the probability associated to the scenario n; $\pi_{k,n}$ is the price is the scenario n at the hour k; p_{ik} is the hourly electricity production of the hydro unit i; s_{ik} represents the water spillages of the units and ps its penalty factor; and $c_i \cdot y_{ik}$ are the start-up costs.

4.2 Constraints

4.2.1 Minimum profit constraint

In order to be protected against the worse prices scenarios, the risk aversion of the hydro utility can be modeled by the following set of constraints where a minimum profit $B_{\min}$ is required for every scenario n.

$$\left[\sum_{k=1}^{K}\pi_{k,n} \cdot \sum_{i=1}^{I}p_{ik}\right] - \left[\sum_{i=1}^{I}\sum_{k=1}^{K}c_i \cdot y_{ik}\right] \geq B_{\min},\ \forall n \qquad (19)$$

4.2.2 Reservoir water balance

For each reservoir i at each time period k, the following constraint establishes the water balance equation. The reservoir level at the beginning of a period $v_{i(k+1)}$ is the reservoir level at the beginning of the previous period v_{ik} minus the released volume (turbine discharged q_{ik} or spilled s_{ik}) plus the volume coming from direct upstream reservoirs (noted as the set Ω_i) and natural inflows w_{ik}.

$$v_{i(k+1)} = v_{ik} + \ell_k \cdot \begin{bmatrix} -q_{ik} - s_{ik} + w_{ik} \\ +\sum_{i\in\Omega_i}^{I}\left(q_{ik} + s_{ik}\right) \end{bmatrix},\ \forall i, \forall k \neq K \qquad (20)$$

4.2.3 Initial and final reservoir level

The reservoir level conditions, initial level v_i^o and final one v_i^f, are stated as follows:

$$v_{ik} = v_i^o \qquad \forall i, k = 1 \qquad (22)$$

$$v_{ik} = v_i^f \qquad \forall i, k = K+1 \qquad (23)$$

4.2.4 Reservoir capacity limits

The reservoirs management has to take into account their upper $\overline{v}_{ik}$ and lower $\underline{v}_{ik}$ dynamic capacity limits,

$$\underline{v}_{ik} \leq v_{ik} \leq \overline{v}_{ik} \qquad \forall i, \forall k \qquad (24)$$

4.2.5 Water Rights

It is usual in hydro systems the existence of water rights derived from ecological flows, irrigation requirements, etc. This water discharged limits ($\underline{\theta}_{ik}, \overline{\theta}_{ik}$) are applied to both the turbine discharges and the spillages:

$$\underline{\theta}_{ik} \le q_{ik} + s_{ik} \le \overline{\theta}_{ik} \qquad \forall i, \forall k \tag{25}$$

4.2.6 *Input-output curve modeling*

In order to model the truthful input-output curve for a given net head, a piece-wise linear approximation has been implemented taking into account the minimum, the maximum, the maximum efficiency discharge points(see Figure 3). Equations (26), (27) and (28) state that when the unit is off ($u_{ik} = 0$), the outflow is zero and when it is on, the outflow must be within the interval [$\underline{q}_{ik}, \overline{q}_{ik}$],

$$q_{ik} = u_{ik} \cdot \underline{q}_{ik} + q^a_{ik} + q^b_{ik} \quad \forall i, \forall k \tag{26}$$

$$q^a_{ik} \le u_{ik} \cdot \left(q^{mxef}_{ik} - \underline{q}_{ik} \right) \; \forall i, \forall k \tag{27}$$

$$q^b_{ik} \le u_{ik} \cdot \left(\overline{q}_{ik} - q^{mxef}_{ik} \right) \; \forall i, \forall k \tag{28}$$

where q^a_{ik} is the turbine discharged over the minimum one ($\underline{q}_{ik}$) and q^b_{ik} is the turbine discharged over the maximum efficiency point q^{mxef}_{ik}.

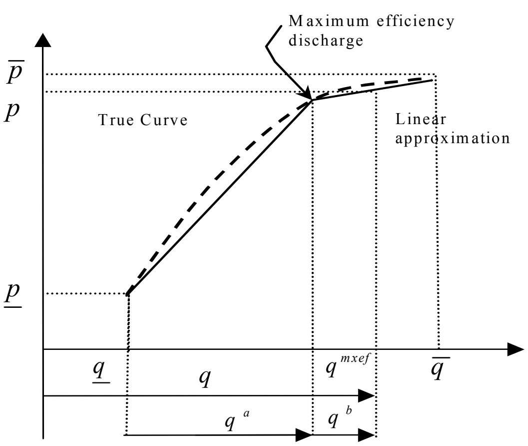


Figure 3: Input-Output curve approximation

After defining the water discharge physic limits, it is possible to express the power generation function p_{ik}. In this case this function has been modeled by a linear approximation presented in Figure 3 and in equation (29).

$$p_{ik} = u_{ik} \cdot \underline{p}_{ik} + q^a_{ik} \cdot \frac{(p^{mxef}_{ik} - \underline{p}_{ik})}{(q^{mxef}_{ik} - \underline{q}_{ik})} + q^b_{ik} \cdot \frac{(\overline{p}_{ik} - p^{mxef}_{ik})}{(\overline{q}_{ik} - q^{mxef}_{ik})} \qquad \forall i, \forall k \tag{29}$$

4.2.7 *Logic constraint*

This constraint ensures the coherence among the binary variables related to the discrete decisions, commitment u_{ik}, start up y_{ik} and stop z_{ik}. For instance, it does not allow to start-up a unit which is already on.

$$y_{ik} + u_{ik-1} - u_{ik} - z_{ik} = 0 \;\; \forall i, \forall k \tag{30}$$

This formulation let include easily additional logic constraints, such us the common daily maneuvers limitation.

5. **Study case**

The presented model has been implemented in GAMS, using the commercial solver CPLEX 7.1 to solve the MILP problems of each iteration. This section presents its application to a real size hydro chain.

5.1 *Input Data*

This hydro chain consists of ten cascaded reservoirs and seven hydraulic generating units. Figure 4 shows the complete topology and table I summarizes the most relevant characteristic parameters of each unit.

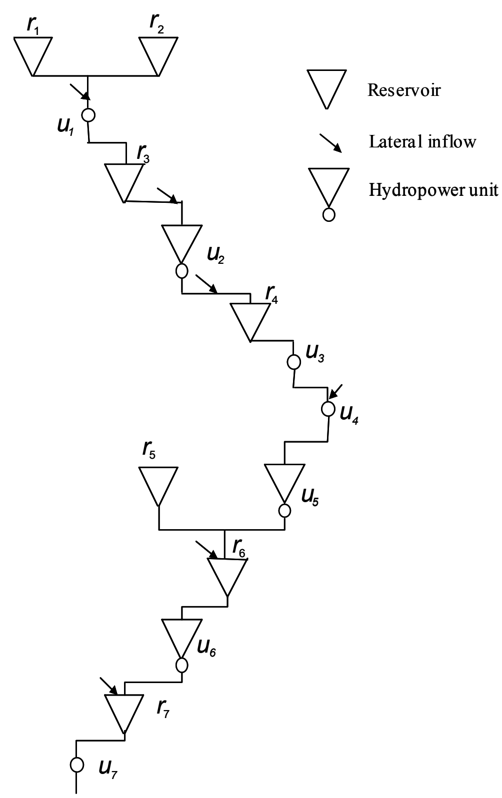


Figure 4: Hydro chain topology

TABLE I
CHARACTERISTICS PARAMETERS

	$\overline{v}$	$\underline{v}$	v^0	v^f	$\overline{p}_{c,k}$	$\overline{q}_{c,k}$	w	c_i
u1	-	-	-		14.2	11	5	35.5
u2	64.9	5.95	35.44	35.42	80	62	1	200
u3	-	-	-		37	35		92.5
u4	-	-	-		32	45		80

u5	313.1	60.4	164.5	163.5	72	100		180
u6	12.7	6	9.4	8.74	120	124		300
u7	-	-	-		30.6	120		76.5
r1	50	7	20.1	20.1	-	9		-
r2	20	3	10.1	10.1	-	11		-
r3	28.3	.5	14.4	14.12	-	31		-
r4	0.49	0.11	0.3	0.3	-	35	1	-
r5	1.66	07	1.18	1.1	-	48		-
r6	0.83	0.6	0.72	0.67	-	56.5	1	-
r7	12.55	7.1	3.0	2.79	-	120	12	-
	Hm^3	MW	m^3/s	€				

The hydropower units start-up costs have been estimated as a function of the nominal output power, $c_i = \overline{p} \cdot 2.5€/MW$ (Nilsson and Sjelvgren 1997). Besides these technical characteristics, unit u_2 has to fulfill a maximum outflow of 38 m^3/s during the whole day.

The temporal scope considered is 24 hourly periods, corresponding to the 24 auctions of the day-ahead market and five prices scenarios have been considered simultaneously. These price scenarios, see Figure 5 and table II, are weighed in the optimization problem with the same occurrence probability.

TABLE II: PRICE SCENARIOS

	h1	h2	h3	h4	h5	h6	h7	h8	h9	h10	h11	h12	h13	h14	h15	h16	h17	h18	h19	h20	h21	h22	h23	h24
n1	3.9	3.4	2.5	2.4	2.4	2.4	2.6	3.4	3.5	4.0	4.5	4.5	3.9	3.9	3.5	3.2	3.3	3.9	4.7	5.6	4.6	4.0	3.5	3.1
n2	3.6	3.6	2.7	2.6	2.4	2.6	3.6	3.9	3.7	4.0	4.1	4.3	4.0	4.0	3.5	3.5	3.6	4.0	5.0	5.5	4.5	4.0	3.6	3.9
n3	3.8	3.0	2.3	2.3	2.3	2.4	2.8	3.9	4.0	4.3	4.6	4.9	4.5	4.5	3.6	4.0	4.4	4.6	5.5	5.7	5.2	5.0	4.0	4.0
n4	3.8	3.6	2.7	2.7	2.5	2.5	3.6	4.0	4.1	4.1	4.2	4.2	4.1	4.1	3.6	3.7	3.8	4.1	4.8	5.3	4.1	4.1	3.9	4.1
n5	4.0	3.7	2.9	2.8	2.5	2.5	2.8	3.8	3.5	3.5	4.0	4.0	3.6	3.2	2.7	2.7	3.0	4.0	5.4	4.8	4.0	3.6	2.8	3.2

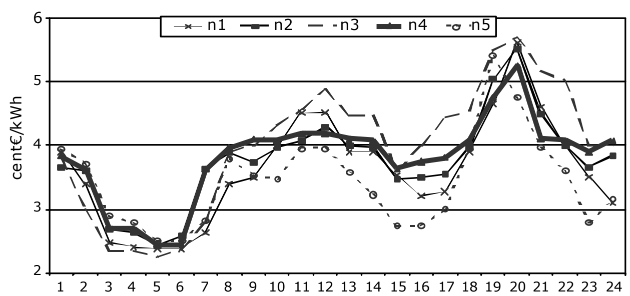


Figure 5: Price scenarios

Finally, the minimum daily profit required (market incomes minus start-up costs) was set to 56.500 €.

5.2 *Results analysis*

The example case was solved for a relaxation parameter $\alpha = 0.95$, on a Pentium IV, 2.4 GHz with 512 MB. Figure 6 and Table III present the resulting hydro scheduling. The production of u_1 is a good example of a run-of-river unit, which is forced to produce its own natural inflow during the whole day. The other units distribute their production during the higher expected prices hours.

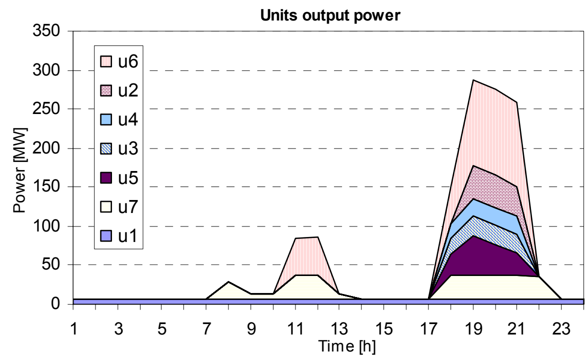


Figure 6: Hydro units productions

TABLE III: OBTAINED SCHEDULE

	h1	h2	h3	h4	h5	h6	h7	h8	h9	h10	h11	h12	h13	h14	h15	h16	h17	h18	h19	h20	h21	h22	h23	h24
u1	6.4	6.4	6.4	6.4	6.4	6.4	6.4	6.4	6.4	6.4	6.4	6.4	6.4	6.4	6.4	6.4	6.4	6.4	6.4	6.4	6.4	6.4	6.4	6.4
u2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	41.9	41.9	36.7	0.0	0.0	0.0
u3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	20.4	25.1	25.1	25.1	0.0	0.0	0.0
u4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	18.5	22.7	22.7	22.7	0.0	0.0	0.0
u5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	27.6	51.0	39.2	28.4	0.0	0.0	0.0
u6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	47.2	50.2	0.0	0.0	0.0	0.0	0.0	47.2	109.6	109.1	108.7	0.0	0.0	0.0
u7	0.0	0.0	0.0	0.0	0.0	0.0	0.0	21.7	7.5	7.5	30.5	30.5	7.5	0.0	0.0	0.0	0.0	30.5	30.5	30.5	30.5	29.1	0.0	0.0

Regarding the minimum profit constraint, table IV presents the profit obtained for each scenario in case the constraint is considered or not in the optimization. Note that except the fifth scenario, all the others accomplish this requirement in the situation of non considering this constraint. As it can be seen in figure 5, prices for this scenario are slightly lower than the others. Thus, when the constraint is active, the profits of all the other scenarios are reduced in favor of this one. This could be interpreted as if that scenario's probability were increased.

TABLE IV: EFFECT OF CONSIDERING THE MINIMUM PROFIT CONSTRAINT

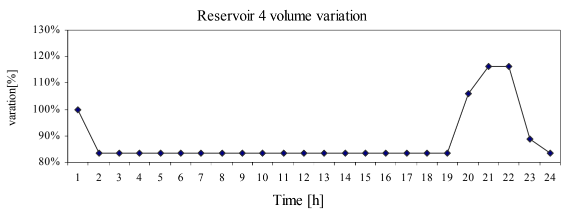
	n1	n2	n3	n4	n5
without:	59443	60048	66673	57942	56076
with:	59410	59903	65756	57742	56500

Table V shows the self-scheduling obtained without the minimum profit constraints active, and the differences with the previous one are market with dotted lines. The prices of the 5th scenario are the main cause of such differences. Units u3 and u4 start-up one hour before in case of no minimum profit required. This happens because the maximum price in the 5th scenario occurs during the interval 18h-21h, while in the other scenarios it takes place in the interval 19h-21h. Another example is the start-up of unit u6 in hours 11h-12h. In this case, the reason is that the price in these hours in the fith scenario are similar to the prices during evening hours.

TABLE V
HYDRO OBTAINED SCHEDULE WITHOUT MINIMUM PROFIT CONSTRAINTS

	h1	h2	h3	h4	h5	h6	h7	h8	h9	h10	h11	h12	h13	h14	h15	h16	h17	h18	h19	h20	h21	h22	h23	h24
u1	6.4	6.4	6.4	6.4	6.4	6.4	6.4	6.4	6.4	6.4	6.4	6.4	6.4	6.4	6.4	6.4	6.4	6.4	6.4	6.4	6.4	6.4	6.4	6.4
u2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	41.9	41.9	36.7	0.0	0.0	0.0
u3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	25.1	25.1	25.1	20.4	0.0	0.0
u4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	22.7	22.7	22.7	18.5	0.0	0.0
u5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	12.1	39.2	51.0	28.4	15.7	0.0	0.0
u6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	47.2	110.1	109.6	109.2	93.8	0.0	0.0
u7	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	12.4	30.5	30.5	30.5	0.0	0.0	0.0	0.0	30.5	30.5	30.5	30.5	30.5	0.0	0.0

Finally, another interesting result is the reservoir management. Figure 7 shows the evolution of the storage water in r_4 and r_6 along the 24 hours. During the last 4 hours, reservoir r_4 releases the water stored previously in order to satisfy its final volume target. On the other hand, reservoir r_6 behaves in a different way. In the last hours it stores the water previously discharged fulfilling the final level condition.



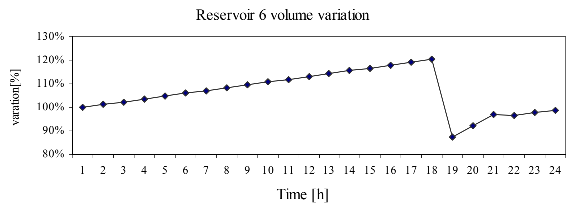


Figure 7: Hydro units productions

In the simulations done, the convergence has been reached in four iterations for the minimum profit case and in six without considering it. The execution time was less than 15 seconds in both cases.

6. Conclusions

This paper presents an optimization model to help a hydro-generation company to schedule its hydroelectric units in a pool-based organized as a day-ahead market. The objective function is the maximization of the expected profit, defined as the difference between expected market revenues and the start-up costs. The hydro-generation company is supposed to be price-taker, and therefore, market price is considered as an exogenous variable. A discussion about the most appropriate methods available in the literature to forecast these prices is also presented. Regarding the hydro units modeling, the net-head effect has been considered by means of an under-relaxed iterative procedure, and its application to an example case has been satisfactory.

7. Disclaimer and acknowledgment

This paper reports the research undertaken by the authors, but do not necessarily correspond to the opinions of the organizations they are representing.

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8. Appendix: Under-relaxed iterative procedure

The under relaxed method is a technique to update the variables involved in an iterative process. It was firstly introduced by the authors to the resolution of a traditional short-term hydro scheduling in (García-González, Parrilla et. al, 2003), so further information could be found in this paper. The application of this method to the presented problem could be summarized in four steps:

Step 1) Initialize the net-heads assuming an *a priori* reservoir management. The net-head h_i [m] of a hydro plant i measures the difference between the forebay elevation and the tailrace elevation. Therefore, it can be expressed as a function of its reservoir storage v_i [Hm^3] and the immediate downstream reservoir storage, v_j [Hm^3].

$$h_i = \rho_i(v_i, v_j) \tag{32}$$

In the Spanish system, the tailrace elevation can be considered constant in most of the reservoirs. Therefore, this relationship can be simplified:

$$h_i = \rho_i(v_i) \tag{33}$$

Step 2) Functions $\phi_{ik}^{\upsilon}(q_{ik})$ are built by applying to the current values h_{ik}^{υ}.

Step 3) Once the hydro units have been characterized by their time-varying input-output functions, the SPBS solution is obtained by solving the MILP optimization problem presented in section 4.

Step 4) The aim of this step is to check whether the convergence has been reached or not, and in this case, to prepare the input-data for the next iteration $\upsilon+1$. Firstly, it is necessary to define a convergence measure which in this case is $\varepsilon = (v_{jk} - v_{jk}^{\upsilon}) / v_{jk}^{\upsilon}$, where j is the index of the reservoir in which the stored water in period k has the highest mismatch between two consecutive iterations. If value of ε is smaller than a given tolerance (e.g. 0.1 %) the iterative process finishes, but if not, the last solution of the SPBS provides new values for the reservoir levels v_{ik}, that could be used directly to update $h_{ik}^{\upsilon+1}$ and the algorithm would continue in step 2:

$$h_{ik}^{\upsilon+1} = \rho_i\left(v_{ik}\right) \tag{34}$$

However, in order to avoid undesirable diverging oscillations, we propose to update the net heads using also previous iterations information. Let define the relaxation parameter $\alpha > 0$. The updated net heads can be obtained by the following equation:

$$h_{ik}^{\upsilon+1} = \rho_i\left(v_{ik}^{\upsilon+1}\right) = \rho_i\left(v_{ik}^{\upsilon} + \alpha \cdot \left[v_{ik} - v_{ik}^{\upsilon}\right]\right) \tag{35}$$

Note that is just a particular case of when $\alpha = 1$. The selection of the best under-relaxation factor is empiric and unfortunately, it can be case-dependent. However, a general rule can be stated: for the early stages of iterations lower values of the under-relaxation factor will help to avoid divergence, and as the iterations get closer to the converged state, values very close to 1 help to speed up the progress.

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