Full Active-Reactive DC Power Flow Model with Loss Compensation

F. M. Echavarren, E. Lobato and L. Rouco, *Senior Member, IEEE*

***Abstract*—This paper formulates a generic DC approximation of the full active-reactive AC power flow problem. This novel approximation includes a novel linear reactive DC model than can be used to estimate bus voltage magnitudes and MVAr flows (DC-Q model). In addition, the combination of the DC-P and DC-Q models also provides a better estimation of network power losses, improving the accuracy of the well-known DC-P model. This novel DC-Q model is based on a change of variables, where the bus voltage module is substituted by its natural logarithm, and presents the same advantages recognized in the DC-P model. Due to this fact, it can be easily integrated into any software application that is already using the DC-P model. Using a small size 2-bus system and actual real-time scenarios of the Spanish electric power system, the precision performance of the DC-Q model is assessed, and the accuracy of the DC-P and DC-Q models are also compared.**

***Index Terms*—DC power flow, contingency analysis, voltage security assessment.**

I. INTRODUCTION

The power flow (PF) of an electric power system is the basic computation tool of different security and market applications such as on-line and off-line security-constrained operation, optimal power flows, medium term and long-term unit commitment models, contingency screening and ranking, security-constrained market clearing tools, etc.

PF equations include both active and reactive balances in each bus of the power system. They are non-linear expressions formulated with modules and angles of bus voltages as variables. Under normal operation conditions, the behavior of AC electric power systems tends to be smooth, even linear for small disturbances. Nevertheless, if the system is highly loaded of some severe outage has occurred, then the behavior deteriorates to highly non-linear, or even unsolvable if the point of work is beyond the voltages collapse point. Voltages collapse point is a saddle-node bifurcation of the PF equations, thus is the boundary between solvable and unsolvable PF [1, 2].

Because of the non-linearity of PF equations, all the methods presented in the literature through decades are formulated as iterative processes [3]. Since the very first attempts during the 50s, researches have applied different algorithms (Jacobi, Gauss-Seidel, Newton-Raphson, etc.), formulations (adopting either polar or cartesian coordinates, using either admittance or impedance matrix, etc.) and strategies (Pθ and QV decoupling, invariant approximations for the Jacobian matrix, step control, etc.) to not only solve the PF problem, but also identify unsolvable scenarios. Moreover, classic algorithms tend to fail when the system is so close to the voltages collapse point [4]. As a consequence, special methods have been designed to be robust in case of ill-conditioned systems, such as step size optimization [5], continuation power flow and optimization techniques [6, 7], or even the more recently Holomorphic Embedding Load-flow Method (HELM) formulation [8].

One additional difficulty associated to power flow solution is the computation speed. For several applications such as security analysis or planning issues the problem is to compute the solution for large sets of power system scenarios as fast as possible. Taking a set of sensible simplifications on the active power equations of the PF, the active DC linear power flow model is built to obtain approximate values of the bus voltage angles and active power flows through lines and transformers. This well-known model will be referred as DC-P along the paper. Among the main features that make DC-P power flows very attractive are [9]: linearity, uniqueness of solution, simplicity of methods and software, minimization of required input data, suitability to security-constrained and economic tools such as optimal power flows and contingency analysis. Even though recent research has pointed that a DC-P model may not be very accurate under certain conditions [9-13], it is a widespread used model where the values of angles and MW line flows are accepted as reasonable by the industrial and research community for a large set of software applications and studies.

DC-P models have been classified into two different types: (a) hot-start or state-dependent models and (b) cold-start or independent-state models. The former is constructed on a known solution of the power flow that serves as base case and has better precision, where the latter is built when no reliable previous solution is available and thus is less accurate. Among the cold-start models two different subtypes are described in the literature (b1) classical DC-P model (ignoring line active power losses) and (b2) DC-P power flow model with loss compensation which includes the estimated value of active power losses distributed along the power system. In order to obtain an estimated value of system losses, an initial run of the classical DC-P model is needed. Precision and comparison among these DC-P models is tackled within the literature [9, 10, 14].

A DC-P model gives no information about bus voltage magnitudes and reactive flows of branches. In order to assess voltage and reactive magnitudes, full AC models are used. Integrating in different ways the DC-P model solution within the AC power flow is an option to improve the full AC model performance [12, 14, 15]. For specific applications such as voltage-related contingency assessment, different techniques such as the 1P-1Q iteration and zone related methods can be employed for ranking and screening [16-18].

Even though recent attempts considering reactive power within a DC-P power flow model can be found [13], no linear power flow DC model as such can be found in the literature for reactive problems. This paper formulates a generic full active-reactive DC linear power flow model and derives from this formulation a novel linear DC-Q model than can be used to make estimates of bus voltage magnitudes and MVAr flows. The novel DC-Q model here proposed has the same advantages listed for DC-P models. In this sense, the DC-Q model can be easily integrated in software applications for different purposes. As will be seen in the following sections, the key to formulate the full active-reactive power DC-model of an electric power system is to use the logarithm of the bus voltage as variable.

Similarly to the cold-start DC-P models (it is assumed that no previous reliable solution of the power system is available), both a DC-Q classical model ignoring reactive power losses, and a DC-Q model with reactive loss compensation will be formulated in order to improve the accuracy. Using a small size 2-bus system and actual real-time scenarios of the Spanish electric power system, the precision performance of the DC-Q model is assessed, and the accuracy of the DC-P and DC-Q models are also compared.

The paper is organized as follows. Section II. presents the DC modeling of the network using the log-voltage notation and certain network simplifications. Section III. explains how to solve the DC models. Results presented in section V. illustrate the performance of the DC-Q model. Finally, conclusions of the paper are presented in section VI.

II. DC MODELING OF THE NETWORK

The present section details how to formulate the approximated DC model of an electric power system considering not just the active power problem, but also the reactive power problem. To formulate the DC-Q model, bus voltages are substituted by their logarithms.

The devices considered to be modeled in the network are power lines, power transformers and shunt devices (reactors and capacitors). The bus active and reactive power balance equations are formulated using the admittance matrix and considering bus voltage angles and modules as state variables. After the equations are obtained and linearized, they are rearranged thus independent terms are summed at the right-hand-side, and terms with voltage angles or log-modules are placed as a linear application in the left-hand-side.

Since the system is linearized, power losses are not considered in first attempt. After solving the DC models, both active and reactive power losses can be estimated. Using power losses estimation, they can be introduced in the network as new independent energy sources. Therefore a second DC models solving is needed to consider network power losses and obtain a more accurate approximation to the actual solution.

A. Notation

V_i	Bus i voltage
$\lvert V_i \rvert$	Module of the bus i voltage
ω_i	Log-module of the bus i voltage ($\omega_i = \ln(\lvert V_i \rvert)$)
θ_i	Angle of the bus i voltage
PD_i	Active power generation absorbed from bus i
QD_i	Reactive power generation absorbed from bus i
PG_i	Active power generation injected into bus i
QG_i	Reactive power generation injected into bus i
$I_{i>j}$	Branch complex current leaving bus i to bus j
$S_{i>j}$	Branch complex power flow leaving bus i to bus j
$P_{i>j}$	Branch active power flow leaving bus i to bus j
$Q_{i>j}$	Branch reactive power flow leaving bus i to bus j
$S_{i>0}$	Shunt complex power flow leaving bus i to ground
$P_{i>0}$	Shunt active power flow leaving bus i to ground
$Q_{i>0}$	Shunt reactive power flow leaving bus i to ground
I	Series current of branch connecting buses i and j
$Ploss_{ij}$	Fraction associated to bus i of the branch ij active power losses estimation
$Qloss_{ij}$	Fraction associated to bus i of the branch ij reactive power losses estimation
r_{ij}	Series resistance of a branch connecting buses i and j
x_{ij}	Series reactance of a branch connecting buses i and j
b_{ij}	Shunt susceptance of a line connecting buses i and j
t_{ij}	Tap placed at bus i of a transformer connecting buses i and j
$\lvert t_{ij} \rvert$	Module of the transformer tap placed at bus i of a transformer connecting buses i and j
τ_{ij}	Log-module of the transformer tap placed at bus i of a transformer connecting buses i and j ($\tau_i = \ln(\lvert t_{ij} \rvert)$)
ϕ_{ij}	Angle of the transformer tap placed at bus i of a transformer connecting buses i and j
b_i	Susceptance of a shunt device connecting bus i to ground
$Qloss_{i0}$	Reactive power losses estimation for the shunt device connecting bus i to ground

B. Log-V notation for the DC-Q model formulation

One key aspect why the DC model has been applied to the active power problem and not for the reactive power problem is because angles and modules of the bus voltages present different behavior. The modules of bus voltages appear directly multiplying the equations, whereas the angles appear multiplied by the imaginary unit as the exponent of e. To overcome this difficulty, the idea of considering as variables the logarithm of the bus voltages instead of the module is proposed.

$$V_i = \lvert V_i \rvert e^{j\theta_i} = e^{\omega_i + j\theta_i} \rightarrow \omega_i = \ln\left(\lvert V_i \rvert\right) \quad (1)$$

In (1), the bus i voltage phasor is represented in both the classic module-angle notation and using the proposed log-voltage notation. In the latter, bus i voltage V_i is defined as the exponentiation of a complex number whose real part is the log-voltage ω_i and the imaginary part is the angle θ_i.

The log-voltage ω_i represents a novel contribution for AC circuit analysis, but it actually has always been there, masked inside the classic power flow solving process. To construct the Jacobian matrix of power flow equations, their derivatives with respect to system variables are needed. Concerning the derivatives with respect to the voltages modules, it is very common to multiply the derivative by the voltage module, i.e. $\lvert V_i \rvert \cdot \partial(\cdot)/\partial \lvert V_i \rvert$), and define the updating vector as a relative updating ($\Delta \lvert V_i \rvert / \lvert V_i \rvert$). All these manipulations lead to the same results obtained by considering the log-voltage ω_i as variable instead of the module $|V_i|$. On one hand, following the chain rule it can be observed that the derivatives are the same.

$$|V_i|\cdot\frac{\partial(\bullet)}{\partial|V_i|}=|V_i|\cdot\frac{\partial(\bullet)}{\partial\omega_i}\cdot\frac{\partial\omega_i}{\partial|V_i|}=\frac{\partial(\bullet)}{\partial\omega_i} \tag{2}$$

On the other hand, the relative updating of the module $|V_i|$ is equivalent to the log-voltage ω_i updating.

$$|V_i|=e^{\omega_i}\xrightarrow{\partial}\Delta|V_i|=e^{\omega_i}\cdot\Delta\omega_i=|V_i|\cdot\Delta\omega_i \tag{3}$$

But this change of variable not only can be applied to the bus voltages modules. Other network variables that can be redefined using log notation are the power transformer taps. Tap t_{ij} is considered as a complex number since taps may control either bus voltages with its module $|t_{ij}|$ or active power flows with its angle ϕ_{ij}. Therefore, also t_{ij} can be formulated using exponential notation:

$$t_{ij}=|t_{ij}|e^{j\phi_{ij}}=e^{\tau_{ij}+j\phi_{ij}} \tag{4}$$

Using the log-module τ_{ij} instead of $|t_{ij}|$ provides simpler derivatives for the Jacobian matrix, and also allows to consider transformer taps as control variables in the DC-Q model, in the same way that angles ϕ_{ij} appear in the DC-P model.

C. Generators, loads and equations system definition

In both the DC-P and the DC-Q models there are two kinds of buses: reference buses and variable buses. The former present their voltages set to a fixed value and their net current injection free, thus they have to be computed after solving the DC model. The latter are the opposite, i.e. they present specified net current injections and their voltages are the variables of the system to be solved.

There exist a lot of control strategies in the literature to control the power systems which manipulate the references and the variables to achieve different objectives, such as setting energy interchange between areas, or controlling the voltage of a HV bus using a generation unit, a shunt device or transformer taps. The formulation of both the DC-P and the DC-Q can be adapted to work considering different equations and variables rearrangements, and also can be both integrated into more complex tools where the classic DC-P model is already integrated. Nevertheless, in this paper the reference and variable buses definition used is the classic version of the power flow problem. As a consequence, the power demand will be always specified, but the power generated will depend on the kind of bus, and on the considered DC network. Fig. 1 shows the load and the generator models used in the DC networks.

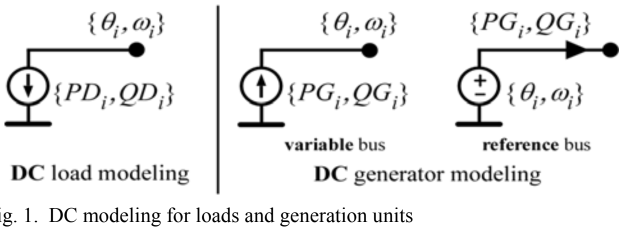

Fig. 1. DC modeling for loads and generation units

In the left-hand-side of Fig. 1, loads are modeled as current sources, absorbing the corresponding demand from the network. That model is valid for both the DC-P and the DC-Q models. However, the generator model (right-hand-side of Fig. 1) depends on what kind of is bus *i*. If it is a variable bus, then the model is a current source injecting the power production into the network, thus the voltage of the bus will be a variable of the system. If the bus is a reference bus, then the model is a voltage source setting the voltage of the bus, thus the power production will be unknown until the circuit is solved.

The sets of reference buses and variable buses for the DC-P and the DC-Q networks are different to each other. Concerning the DC-P model, one and only one reference bus is considered, also known as slack bus. For the DC-Q model, every bus with at least one operative generation unit is considered as a reference bus.

D. DC networks modeling

The devices considered to be modeled in the network are power lines, power transformers and shunt devices (reactors and capacitors). Those are the most relevant and numerous devices in actual distribution and transmission networks. Other electric power devices, such as FACTS or HVDC links, have been discarded, although their integration in the DC models proposed in this paper is feasible.

1) Power line

Network power lines have been represented using a Π-scheme, with a series impedance $r_{ij}+jx_{ij}$ from bus to bus, and half of the shunt susceptance b_{ij} at each bus. For the DC modeling of power lines, the series resistance r_{ij} is neglected, thus only series reactance x_{ij} and shunt susceptance b_{ij} are considered. Using this model, the current leaving bus *i* to bus *j* is:

$$I_{i>j}=\frac{V_i-V_j}{jx_{ij}}+V_i\cdot\frac{jb_{ij}}{2}=(-j)\cdot\left[-\frac{b_{ij}}{2}\cdot V_i+\frac{1}{x_{ij}}\cdot(V_i-V_j)\right] \tag{5}$$

Considering exponential notation (1) for bus voltages, the power injected in the line from bus *i*, computed as $S_{i>j}=V_i\cdot(I_{i>j})^*$, results to be:

$$\begin{aligned}S_{i>j}&=e^{\omega_i+j\theta_i}\cdot(+j)\cdot\left[-\frac{b_{ij}}{2}\cdot e^{\omega_i-j\theta_i}+\frac{1}{x_{ij}}\cdot\left(e^{\omega_i-j\theta_i}-e^{\omega_j-j\theta_j}\right)\right]\\&=j\cdot\left[-\frac{b_{ij}}{2}\cdot e^{2\omega_i}+\frac{1}{x_{ij}}\cdot\left(e^{2\omega_i}-e^{\omega_i+j\theta_i+\omega_j-j\theta_j}\right)\right]\end{aligned} \tag{6}$$

Considering that $e^z\sim 1+z$ when z tends to zero, (6) is expanded and simplified, thus the active and reactive power injected in the line from bus *i* are approximated as follows:

$$P_{i>j}+jQ_{i>j}\sim\left\{\frac{\theta_i-\theta_j}{x_{ij}}\right\}+j\left\{\frac{\omega_i-\omega_j}{x_{ij}}-\frac{b_{ij}}{2}(1+2\omega_i)\right\} \tag{7}$$

Equation (7) is analog for power flow injected in the line from bus *j*. From (7) it can be observed that the DC-P model represented as the active power flow is proportional to bus angle θ difference and inverse proportional to the series reactance. In the case of the DC-Q model, the reactive power flow is also proportional to bus log-module ω difference and inverse proportional to the series reactance, plus an extra term to represent reactive power flow through shunt susceptance. A complete representation of the line and their DC-P and DC-Q models is depicted in Fig. 2.

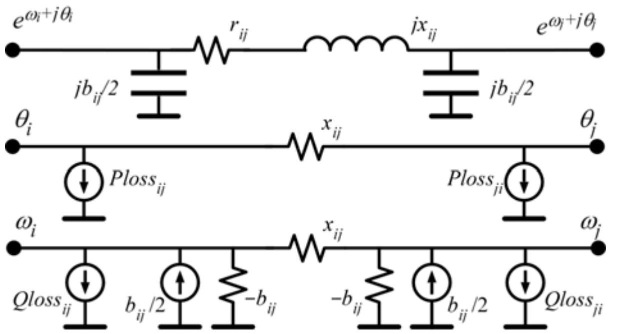

Fig. 2. Power line electric model, and corresponding DC-P and DC-Q models

Fig. 2 representation of the DC-P model includes x_{ij} as a resistance between the buses, and additional current sources representing the active power losses of the line. Representation of the DC-Q model includes x_{ij} as a series resistance and additional current sources for the reactive power losses as well, but b_{ij} is also represented as a negative conductance and a current source at each of both buses.

Concerning active power losses, current sources $Ploss_{ij}$ and $Ploss_{ji}$ are placed at corresponding bus in the DC-P circuit. Each source will absorb half of the active power losses in the series resistance r_{ij} of the line due to the series current I.

$$Ploss_{ij} = Ploss_{ji} = \frac{1}{2} r_{ij} \cdot |I|^2 \tag{8}$$

Series current I is the series current from bus to bus, i.e. the current due to the voltages drop applied to the series impedance of the line. The module of that series current is computes as follows:

$$|I|^2 = \left(\frac{\theta_i - \theta_j}{x_{ij}}\right)^2 + \left(\frac{\omega_i - \omega_j}{x_{ij}}\right)^2 \tag{9}$$

In the case of reactive power losses, current sources $Qloss_{ij}$ and $Qloss_{ji}$ are placed at corresponding bus in the DC-Q circuit. Each source will absorb half of the reactive power losses in the series reactance x_{ij} of the line due to the series current I, but also will inject the reactive power excess from the shunt susceptance b_{ij}. At bus i (analog for bus j), the reactive power absorbed by the shunt susceptance halved is approximated as follows:

$$-\frac{b_{ij}}{2}|V_i|^2 = -\frac{b_{ij}}{2} e^{2\omega_i} \sim -b_{ij} \cdot \left(\frac{1}{2} + \omega_i\right) - b_{ij} \cdot \omega_i^2 - \dots \tag{10}$$

In (10), the first two terms of the Taylor approximation are computed in the DC-Q circuit, whereas the third term is considered as the reactive power excess. As a consequence, current sources $Qloss_{ij}$ and $Qloss_{ji}$ are set as follows:

$$Qloss_{ij} = \frac{1}{2} x_{ij} \cdot |I|^2 - b_{ij} \cdot \omega_i^2 \tag{11}$$

$$Qloss_{ji} = \frac{1}{2} x_{ij} \cdot |I|^2 - b_{ij} \cdot \omega_j^2 \tag{12}$$

2) Power transformer

Network power transformers have been represented using a series impedance $r_{ij}+jx_{ij}$ from bus to bus, and taps with voltage and phase control at both sides of the transformer, t_i and t_j. For the DC modeling of power transformers, the series resistance r_{ij} is neglected, thus only series reactance x_{ij} is considered Using this model, the current leaving bus i to bus j will be:

$$I_{i>j} = \frac{1}{t_i^*} \cdot \left(\frac{V_i/t_i - V_j/t_j}{jx_{ij}}\right) = \frac{-j}{x_{ij}} \cdot \left(\frac{V_i}{t_i^* \cdot t_i} - \frac{V_j}{t_i^* \cdot t_j}\right) \tag{13}$$

Considering exponential notation (1) for the bus voltages and the transformer taps, the power injected in the line from bus i, computed as $S_{i>j}=V_i \cdot (I_{i>j})^*$, results to be:

$$\begin{aligned} S_{i>j} &= e^{\omega_i + j\theta_i} \cdot \frac{+j}{x_{ij}} \cdot \left(\frac{e^{\omega_i - j\theta_i}}{e^{\tau_i + j\phi_i} \cdot e^{\tau_i - j\phi_i}} - \frac{e^{\omega_j - j\theta_j}}{e^{\tau_i + j\phi_i} \cdot e^{\tau_j - j\phi_j}}\right) \\ &= \frac{j}{x_{ij}} \left(e^{2\omega_i - 2\tau_i} - e^{\omega_i + j\theta_i + \omega_j - j\theta_j - \tau_i - j\phi_i - \tau_j + j\phi_j}\right) \end{aligned} \tag{14}$$

Considering that $e^z \sim 1+z$ when z tends to zero, (14) is expanded and simplified, thus the active and reactive power injected in the line from bus i are approximated as follows:

$$P_{i>j} + jQ_{i>j} \sim \left\{\frac{\theta_i - \theta_j - \phi_i + \phi_j}{x_{ij}}\right\} + j\left\{\frac{\omega_i - \omega_j - \tau_i + \tau_j}{x_{ij}}\right\} \tag{15}$$

Equation (15) is analog for power flow injected in the transformer from bus j. From (15) it can be observed that the DC-P model represented as the active power flow is proportional to bus angle θ and tap angle ϕ differences, and inverse proportional to the series reactance. In the case of the DC-Q model, the reactive power flow is proportional to bus log-module ω and log-tap module τ differences, and inverse proportional to the series reactance. In both models, DC-P and DC-Q, transformer taps are included as independent current sources. A complete representation of the transformer and their DC-P and DC-Q models is depicted in Fig. 3.

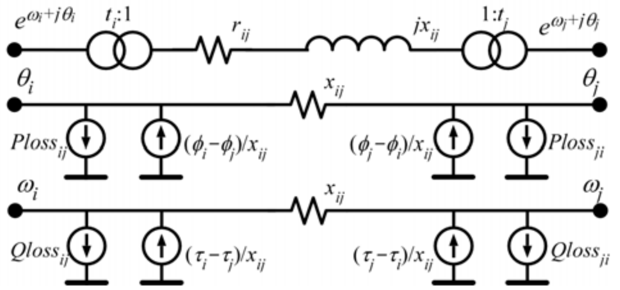

Fig. 3. Power transformer electric model, and corresponding DC-P and DC-Q models

Concerning power losses, current sources are placed at corresponding bus in both the DC-P ($Ploss_{ij}$ and $Ploss_{ji}$) and DC-Q ($Qloss_{ij}$ and $Qloss_{ji}$) circuits. Each source will absorb half of the power losses in the series impedance $r_{ij}+jx_{ij}$ of the line due to the series current I.

$$Ploss_{ij} = Ploss_{ji} = \frac{1}{2} r_{ij} \cdot |I|^2 \tag{16}$$

$$Qloss_{ij} = Qloss_{ji} = \frac{1}{2} x_{ij} \cdot |I|^2 \tag{17}$$

In this case, the series current I to be considered is the inter-taps transformer current, i.e. the current which pass through the series impedance of the transformer.

$$|I|^2 = \left(\frac{\theta_i - \theta_j - \phi_i + \phi_j}{x_{ij}}\right)^2 + \left(\frac{\omega_i - \omega_j - \tau_i + \tau_j}{x_{ij}}\right)^2 \tag{18}$$

3) Shunt device

Network shunt reactors and capacitors have been represented using their shunt susceptance b_i. Using this model, the current leaving bus i to ground through the shunt is:

$$I_{i>0} = jb_i \cdot V_i \tag{19}$$

Considering exponential notation (1) for bus voltages, the power injected in the shunt from bus i results:

$$S_{i>0} = V_i I_{i>0}^* = e^{\omega_i + j\theta_i} \cdot (-jb_i) \cdot e^{\omega_i - j\theta_i} \quad (20)$$

Assuming that $e^z \sim 1+z$ when z tends to zero, then the active and reactive power injected in the shunt from bus *i* is:

$$P_{i>0} + jQ_{i>0} \sim \{0\} + j\{-b_i(1+2\omega_i)\} \quad (21)$$

From (21) it can be observed that the DC-P model represented is blank since shunt devices are purely reactive. In the case of the DC-Q model, the reactive power flow represents reactive power flow through shunt susceptance as a constant plus a linear function of ω_i. A complete representation of the line and their DC-P and DC-Q models is depicted in Fig. 4.

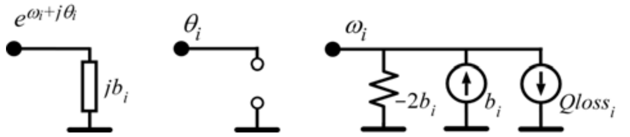

Fig. 4. Shunt reactor/capacitor electric model, and corresponding DC-P and DC-Q models

Concerning reactive power losses, current source $Qloss_i$ is placed at the bus in the DC-Q circuit. That source will inject the reactive power excess from the shunt susceptance b_i. At bus *i*, the reactive power absorbed by the shunt susceptance is approximated as follows:

$$-b_i |V_i|^2 = -b_i \cdot e^{2\omega_i} \sim -2b_i \cdot \left(\frac{1}{2} + \omega_i\right) - 2b_i \cdot \omega_i^2 - \ldots \quad (22)$$

In (22), the first two terms of the Taylor approximation are computed in the DC-Q circuit, whereas the third term is considered as the reactive power excess. As a consequence, current source $Qloss_{i0}$ is set as follows:

$$Qloss_{i0} = -2b_i \cdot \omega_i^2 \quad (23)$$

III. DC NETWORKS SOLVING

DC approximations presented in previous section provide linear circuits where power flows are represented by currents and voltage variables are bus voltages. As a consequence, resolution of both the DC-P and the DC-Q models can be worked out by solving linear equations systems. These equations are bus current balance, and are formulated using the network admittance matrix and the different current sources.

A. DC-P network solving

For the DC-P network modeling, the reference (*r*) buses are the swing bus, where the active power generation is free to compensate power losses, and the rest will be variable (*v*) buses, where active power generation and demand are specified thus active power mismatches can be formulated. Reference bus (*r*) will be also the reference for the voltages angles, therefore the angles of variable (*v*) buses will be the system variables. DC-P model equations will be:

$$\mathbf{PG} - \mathbf{PD} - \mathbf{LP} + \mathbf{TP} = \mathbf{BP} \cdot \boldsymbol{\theta} \quad (24)$$

In the LHS of (24) are the vectors included as independent energy sources: **PG** and **PD** as bus active power generation and demand; **LP** as active power losses; and **TP** as transformer phase compensation.

To build vector **LP**, each element *i* contains the sum of the active power losses of every device connected to the corresponding bus *i*.

$$LP_i = \sum_j Ploss_{ij} \quad (25)$$

Vector **TP** contains the correction ϕ introduced by transformers' angle taps. To build vector **TP**, each element *i* contains the sum of the differences between transformers' angle taps, divided by corresponding series reactance.

$$TP_i = \sum_j \left(\frac{\phi_i - \phi_j}{x_{ij}} \right) \quad (26)$$

Finally, admittance matrix **BP** is built considering the inverse of series reactances in off-diagonal (-) and diagonal (+) elements of the matrix:

$$BP_{ii} = \sum_j \frac{1}{x_{ij}} \quad ; \quad BP_{ij} = -\frac{1}{x_{ij}} \quad (27)$$

To obtain the solution for the angles of variable (*v*) buses, the next linear equations system has to be solved:

$$\mathbf{BP}_{vv} \cdot \boldsymbol{\theta}_v = \mathbf{PG}_v - \mathbf{PD}_v - \mathbf{LP}_v + \mathbf{TP}_v - \mathbf{BP}_{vr} \cdot \boldsymbol{\theta}_r \quad (28)$$

System (28) is obtained from system (24) removing equations corresponding with reference (*r*) buses. It is important to remark that, in the DC-P model, the reference bus angle θ_r is always equal to zero, but for generalization purposes it has not been removed from the equations.

Finally, by removing equations corresponding with variable (*v*) buses, active power generation of reference buses (*r*) is obtained:

$$\mathbf{PG}_r = \mathbf{PD}_r + \mathbf{LP}_r - \mathbf{TP}_r + \mathbf{BP}_{rv} \cdot \boldsymbol{\theta}_v + \mathbf{BP}_{rr} \cdot \boldsymbol{\theta}_r \quad (29)$$

B. DC-Q network solving

For the DC-Q network modeling, the reference (*r*) buses are those with operative generation units. The rest, the variable (*v*) buses, will be any bus with no generation unit connected. Reference buses (*r*) will have a constant voltage, thus the log-modules of variable (*v*) buses will be the system variables. DC-Q model equations will be:

$$\mathbf{QG} - \mathbf{QD} - \mathbf{LQ} + \mathbf{TQ} + \mathbf{B} = \mathbf{BQ} \cdot \boldsymbol{\omega} \quad (30)$$

In the LHS of (30) are the vectors included as independent energy sources: **QG** and **QD** as bus reactive power generation and demand; **LQ** as reactive power losses, included as independent energy sources; **TQ** as transformer tap compensation; and **B** as reactive injections from the shunt elements and the susceptances in power lines.

To build vector **LQ**, each element *i* contains the sum of the reactive power losses of every device connected to the corresponding bus *i*.

$$LQ_i = \sum_j Qloss_{ij} \quad (31)$$

Important remark for (31): index *j* also includes '0' (i.e. to ground) to take the shunt devices loss correction into account.

Vector **TQ** contains the correction τ introduced by transformers' log-module taps. To build vector **TQ**, each element *i* contains the sum of the differences between transformers' log-module taps, divided by corresponding series reactance.

$$TQ_i = \sum_j \left(\frac{\tau_i - \tau_j}{x_{ij}} \right) \quad (32)$$

Vector **B** contains the constant terms of shunt susceptances, including shunt elements and the susceptances in power lines, modeled as current sources. To build vector **B**, each element *i* contains the sum of any shunt susceptance connected to the bus.

$$B_i = b_i + \sum_j \frac{b_{ij}}{2} \quad (33)$$

Finally, admittance matrix **BQ** is built considering the inverse of series reactances in off-diagonal (-) and diagonal (+) elements of the matrix. Also, shunt susceptances from lines and shunt devices are placed in the diagonal elements doubled and with minus sign:

$$BQ_{ii} = -2b_i + \sum_j \left(\frac{1}{x_{ij}} - b_{ij} \right) \quad ; \quad BQ_{ij} = -\frac{1}{x_{ij}} \quad (34)$$

To obtain the solution for the log-module of variable (v) buses, the next linear equations system has to be solved:

$$\mathbf{BQ}_{vv} \cdot \boldsymbol{\omega}_v = \mathbf{QG}_v - \mathbf{QD}_v - \mathbf{LQ}_v + \mathbf{TQ}_v + \mathbf{B}_v - \mathbf{BQ}_{vr} \cdot \boldsymbol{\omega}_r \quad (35)$$

System (35) is derived from system (30) removing equations corresponding with reference (r) buses. In the DC-Q model, the log-voltages of the reference buses ω_r correspond with the scheduled voltages of the generation units. Therefore, ω_r must remain in the equations.

Finally, by removing equations corresponding with variable (v) buses, reactive power generation of reference buses (r) is obtained:

$$\mathbf{QG}_r = \mathbf{QD}_r + \mathbf{LQ}_r - \mathbf{TQ}_r - \mathbf{B}_r + \mathbf{BQ}_{rv} \cdot \boldsymbol{\omega}_v + \mathbf{BQ}_{rr} \cdot \boldsymbol{\omega}_r \quad (36)$$

IV. Fast contingency analysis using DC networks

The DC-P is also used to compute fast contingency analysis. Since the DC-P model is a linear circuit, combination of superposition and substitution principles allow changing the topology of the network without updating the corresponding admittance matrix. This application is also available for the DC-Q model.

Using a generic notation, let **S** be the vector of net currents injected from buses into network (**P** or **Q**), and **x** the vector of bus voltages (θ or ω). Then both are linearly related by admittance matrix **B**. This relationship works for both the base case (^{0}) and the variation due to the outage (Δ).

$$\mathbf{S^0} = \mathbf{B} \cdot \mathbf{x^0} \rightarrow \Delta\mathbf{S} = \mathbf{B} \cdot \Delta\mathbf{x} \quad (37)$$

Let **N** be the inverse of **B**, then:

$$\mathbf{N} = \mathbf{B}^{-1} \rightarrow \Delta\mathbf{x} = \mathbf{N} \cdot \Delta\mathbf{S} \quad (38)$$

The strategy to simulate the outage of the branch km is to include additional power injections ΔS_k and ΔS_m. To achieve no flux at branch km, ΔS_k and ΔS_m have to be equal to the corresponding flux after the outage, as Fig. 5 depicts.

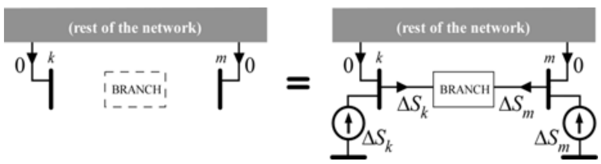

Fig. 5. Post-contingency simulation strategy

If only additional power injections ΔS_k and ΔS_m are considered, then the updating of any variable x_i can be expressed as follows.

$$\Delta x_i = N_{ik} \cdot \Delta S_k + N_{im} \cdot \Delta S_m \quad (39)$$

Power flow through branch km after the outage $S_{k>m}$ will be the base case power flow $S^0_{k>m}$ plus the 2×2 admittance matrix of the branch km. multiplied by voltages x_k and x_m update vectors, and thus equal to the additional power injections ΔS_k and ΔS_m.

$$\begin{vmatrix} S_{k>m} \\ S_{m>k} \end{vmatrix} = \begin{vmatrix} S^0_{k>m} \\ S^0_{m>k} \end{vmatrix} + \mathbf{Bbrn} \cdot \begin{vmatrix} \Delta x_k \\ \Delta x_m \end{vmatrix} = \begin{vmatrix} \Delta S_k \\ \Delta S_m \end{vmatrix} \quad (40)$$

A submatrix of N is defined corresponding with buses k and m.

$$\mathbf{N}\big|_{km} = \begin{vmatrix} N_{kk} & N_{km} \\ N_{mk} & N_{mm} \end{vmatrix} \rightarrow \begin{vmatrix} \Delta x_k \\ \Delta x_m \end{vmatrix} = \mathbf{N}\big|_{km} \cdot \begin{vmatrix} \Delta S_k \\ \Delta S_m \end{vmatrix} \quad (41)$$

Using (41) and (40), and defining $\mathbf{I}_2$ as a 2×2 identity matrix, the adequate magnitudes for the additional power injections ΔS_k and ΔS_m are computed.

$$\begin{vmatrix} \Delta S_k \\ \Delta S_m \end{vmatrix} = \left(\mathbf{I}_2 - \mathbf{Bbrn} \cdot \mathbf{N}\big|_{km} \right)^{-1} \cdot \begin{vmatrix} S^0_{k>m} \\ S^0_{m>k} \end{vmatrix} \quad (42)$$

From the results in (42), all the variables are updated using (39). The new point of work corresponds with the DC solution if the branch would be with the values they would have if the branch was indeed switched off. As a consequence, the solution obtained is exactly the DC approximation of the network after the outage, and can be used to estimate post-contingency power flows and bus voltages.

V. Results

To illustrate the performance of the DC-Q model presented in this paper, two different studies have been carried out. First part of results presents a generator-to-load connection through a standard power line. Those results presented compare the behavior of the three models considered: the full AC, the DC-Q and the DC-Q considering power losses. The second part of results illustrates the performance of the DC-Q modeling when it's applied to actual scenarios of large-scale networks, such as the Spanish electric power system.

A. Generator-to-load basis network

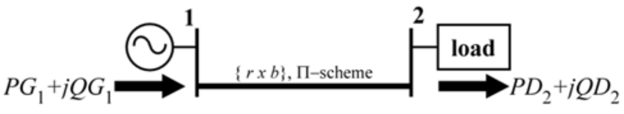

Fig. 6. Generator-to-load system

First step to check the performance of the DC-P and DC-Q models combination is to consider the simplest system, i.e. a generation unit feeding a constant power load through a power line, as Fig. 6 shows. The power line connecting both buses has been considered as a standard 400 kV single circuit line, whose parameters are presented in TABLE I and disposed using a Π-scheme (see Fig. 2). The line is 300 km long, approximately the 5% of its wavelength λ.

TABLE I
Electrical Parameters of the Generator-to-Load Line

R [Ω/m]	X [Ω/m]	B [S/m]	Zc [Ω]	Pc [W]	λ [km]
0.027	0.277	4.167	258.3	619.5	5838

r [pu]	x [pu]	b [pu]
0.005	0.052	2.000

The generation unit placed in bus 1 is the slack bus of the system, thus its angle will be used as reference (θ_1=0.0) and its active power generation will be unspecified to balance the active power losses of the network. Besides, the generation unit has its voltage module set to 1.0 pu (ω_1=0.0), and its reactive power generation unspecified to balance the reactive power losses of the network. The load placed in bus 2 is considered as a constant power demand with an invariant power factor.

For each point of work defined by the active power demand and its power factor, a full AC power flow is run. The solution is thus compared with the approximation provided by the combination of DC-P and DC-Q models, both neglecting and considering power losses correction, namely DC_0 and DC models, respectively. To illustrate the DC models processes, they are going to be exposed in detail for the generator-to-load example. Fig. 7 presents both of the DC circuits, the DC-P and the DC-Q.

In the DC-P circuit, since bus 1 is the slack of the system, it is considered as reference node and its voltage θ_1 is set to 0.0. The unknown variable of the bus is the net current injected into the network, i.e. the active power generated by the slack unit PG_1. Bus 2 corresponds with the load, thus the active power demand PD_2 is set and represented by a current source, and the voltage θ_2 is considered as variable of the system. Both buses are connected by a resistance equal to the series reactance x_{12} of the line. In addition, the line presents current sources placed at both extremes, absorbing the active power losses correction (25).

In the DC-Q circuit, since bus 1 has a generation unit with an scheduled voltage equal to 1.0 pu, it presents a voltage ω_1 set to 0.0. The unknown variable of the bus is the net current injected into the network, i.e. the reactive power generated by the generation unit QG_1. Bus 2 corresponds with the load, thus the reactive power demand QD_2 is set and represented by a current source, and the voltage ω_2 is considered as variable of the system. Both buses are connected by a resistance equal to the series reactance x_{12}, and present shunt conductances equal to minus the shunt susceptance b_{12} of the line. In addition, the line presents current sources placed at both extremes, one injecting the half of the shunt susceptance b_{12} of the line and the other absorbing the reactive power losses correction (31).

First step is to build the admittance matrices **BP** and **BQ**, following (27) and (34) respectively. The matrices obtained, showed in TABLE II, only differ in the diagonal elements, where the shunt susceptance b_{12} of the line appears substracted in the DC-Q model matrix **BQ**.

TABLE II
ADMITTANCE MATRICES FOR THE DC MODELING OF THE GENERATOR-TO-LOAD NETWORK

$$\mathbf{BP} = \begin{vmatrix} 19.268 & -19.268 \\ -19.268 & 19.268 \end{vmatrix} \qquad \mathbf{BQ} = \begin{vmatrix} 17.270 & -19.268 \\ -19.268 & 17.270 \end{vmatrix}$$

The current balance applied to bus 2 in both the DC-P and the DC-Q networks provides lineal equations to compute the voltages θ_2 and ω_2.

$$BP_{22}\cdot\theta_2 = \cancel{PG_2}^{0} - PD_2 - LP_2 + \cancel{TP_2}^{0} - BP_{21}\cdot\cancel{\theta_1}^{0}$$

$$BQ_{22}\cdot\omega_2 = \cancel{QG_2}^{0} - QD_2 - LQ_2 + \cancel{TQ_2}^{0} + B_2 - BQ_{21}\cdot\cancel{\omega_1}^{0}$$

In both equations, the voltages depend on the load and the losses compensation. As an example, three loads has been considered and solved. Those three points of work correspond with an active power demand equal to the characteristic load (see TABLE I), i.e. equal to 6.2 pu. The reactive power demand is defined by three power factors, the resistive one (1.00), and 0.98 both inductive and capacitive. Results of the full AC power flow are displayed in TABLE III, with power magnitudes in MW and MVAr, voltages in kV and angles in degrees.

TABLE III
RESULTS OF THE FULL AC MODEL APPLIED TO THE GENERATOR-TO-LOAD NETWORK

PF	PD_2	QD_2	PG_1	QG_1	V_2	θ_2
0.98c	620	-125.9	640.5	-119.3	413.4	-18.86
1.00	620	0	641.5	30.6	383.0	-19.97
0.98i	620	125.9	646.1	224.4	344.2	-21.81

Once the demands at bus 2 are defined, the DC models are solved and a first approximation of the power flow solution is obtained. This first approximation (DC_0) has been computed without considering power losses compensation. This DC_0 approach is used to estimate power losses, and a second and definitive approximation is computed considering power losses compensation (DC). Results on the DC models applied to the generator-to-load network are presented in TABLE IV (DC-P) and TABLE V (DC-Q), with angles and log-modules in radians, and power magnitudes in pu.

TABLE IV
RESULTS ON THE DC-P MODEL APPLIED TO THE GENERATOR-TO-LOAD NETWORK

PF	θ_2 (AC)	PD_2	θ_2 (DC_0)	LP_2	θ_2 (DC)
0.98c	-0.3276	6.20	-0.3218	0.108	-0.3274
1.00	-0.3471	6.20	-0.3218	0.096	-0.3268
0.98i	-0.3790	6.20	-0.3218	0.093	-0.3266

TABLE V
RESULTS ON THE DC-Q MODEL APPLIED TO THE GENERATOR-TO-LOAD NETWORK

PF	ω_2 (AC)	QD_2	ω_2 (DC_0)	LQ_2	ω_2 (DC)
0.98c	0.0342	-1.26	0.1307	1.128	0.0654
1.00	-0.0418	0.00	0.0578	1.023	-0.0014
0.98i	-0.1476	1.26	-0.0151	0.999	-0.0729

Once the solution is obtained, next step is to compute the active and reactive power production of generation units. More precisely, to compute the active power production of the slack bus and the reactive power production of each generation unit in the network. Applying current balance to bus 1 in both the DC-P and the DC-Q networks, it is obtained lineal equations to compute PG_1 and QG_1. Results are displayed in TABLE VI, with all power magnitudes in pu.

$$PG_1 = \cancel{PD_1}^{0} + LP_1 - \cancel{TP_1}^{0} + BP_{11}\cdot\cancel{\theta_1}^{0} + BP_{12}\cdot\theta_2$$

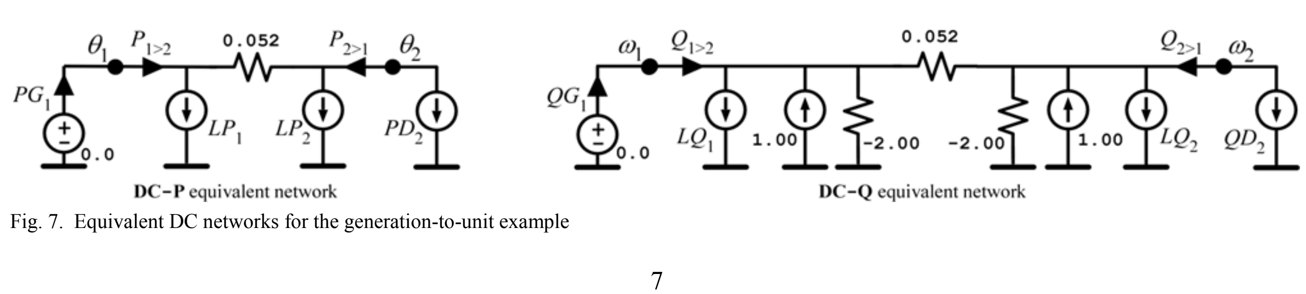

Fig. 7. Equivalent DC networks for the generation-to-unit example

$$QG_1 = \cancel{QD_1^0} + LQ_1 - \cancel{TQ_1^0} - B_1 + BQ_{11}\cdot\cancel{\omega_1^0} + BQ_{12}\cdot\omega_2$$

TABLE VI
RESULTS ON THE POWER PRODUCTION FOR BOTH THE DC-P AND THE DC-Q MODELS APPLIED TO THE GENERATOR-TO-LOAD NETWORK

	PG_1			QG_1		
PF	(AC)	(DC_0)	(DC)	(AC)	(DC_0)	(DC)
0.98c	6.40	6.20	6.42	-1.20	-3.52	-1.10
1.00	6.41	6.20	6.39	0.31	-2.11	0.06
0.98i	6.46	6.20	6.39	2.24	-0.71	1.41

Results on network variables of TABLE IV (DC-P) and TABLE V (DC-Q) show that the DC-P network is more precise than the DC-Q. However, DC-Q results after power losses present small deviations from the AC solution. In terms of bus voltages, only the '098i' scenario presents a deviation for ω greater than 0.05 pu. Concerning the power produced (or absorbed) by the generation unit (see TABLE VI), the DC-P network presents lower deviations from the AC solution than the DC-Q network, but the latter present deviations low enough to consider the utility of the information provided by the DC-Q network.

To analyze the performance of the model proposed for different loads, three vectors of active power demands for bus 2 have been defined, from zero to the voltages collapse point. Each vector represents one of the power factors defined, the resistive one (1.00), and 0.98 both inductive and capacitive. Since the DC-Q is proposed to compute an approximation of the system bus voltages, the curves depicted in Fig. 8 are the load bus voltage and its approximations evolution during active power demand growth, also known as nose curves. For each load power factor there is a set of three curves represented by a marker. For each set, the thickest curves correspond with the full AC power flow solutions, whereas the thinner are DC approximations, with (continuous line) and without (dashed line) considering power losses.

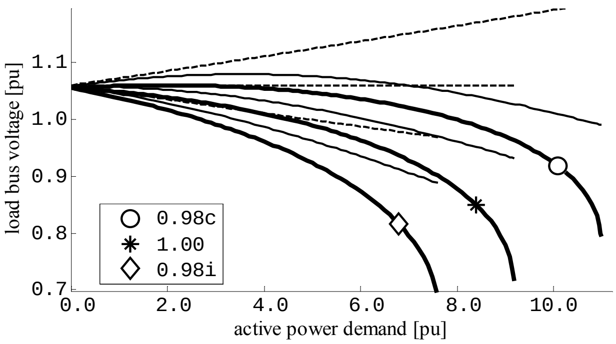

Fig. 8. Load bus voltage evolution during active power demand growth

As Fig. 8 shows, the DC-Q model without considering power losses behaves almost linearly with the active power demand because the reactive power demand is proportional. Therefore, the deviation from the full AC solution is unacceptable for reasonable load levels. However, the DC-Q considering power losses present manifolds much more similar to the AC ones. The deviation grows with the demand, but still the precision is acceptable. Considering a deviation of 0.05 pu in the load bus voltage, scenarios '098i', '1.00' and '098c' present lower deviations with active power demand magnitudes less than 5.6 pu, 6.8 pu and 8.3 pu, respectively.

The other key variable involved in the DC-Q modeling proposed is the reactive power flows. An accurate approximation of reactive power flows through the network branches provides useful information to optimize the reactive power balance in the network, and to compute more precise branch power flows during security analysis. The curves depicted in Fig. 9 are the reactive power produced by the generation unit and its approximations evolution during active power demand growth. For each load power factor there is a set of three curves represented by a marker. For each set, the thickest curve corresponds with the full AC power flow solution, whereas the thinner are DC approximations, with (continuous line) and without (dashed line) considering power losses.

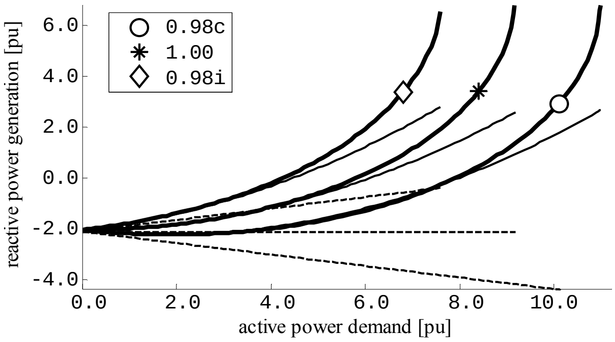

Fig. 9. Reactive power produced by the generation unit evolution during active power demand growth

The behavior of the reactive power manifolds is qualitatively equal to the bus voltages. In Fig. 9, the reactive power without considering power losses is almost linearly with the active power demand because the reactive power demand is proportional. Considering power losses, then the reactive power approximations follow the actual solutions until a point close to voltages collapse where trajectories diverge. Considering a deviation threshold of 0.5 pu in the reactive power, scenarios '098i', '1.00' and '098c' present lower deviations with active power demand magnitudes of less than 5.6 pu, 7.0 pu and 9.3 pu, respectively.

B. Transmission network

To illustrate the performance of the DC-Q model proposed in this paper, it has been applied to actual real-time scenarios of the Spanish electric power system. The system includes the HV and part of the MV network, and includes up to 2238 buses connected by 2331 lines, 979 transformers an 90 shunt devices.

First step was to select a 24 hours set of real-time 136 scenarios and thus compare the precision of the DC-Q model using the same network but with different demand levels. In Fig. 10, hourly evolution of system demand and precision measurements of the voltages estimated by the DC-Q model are presented.

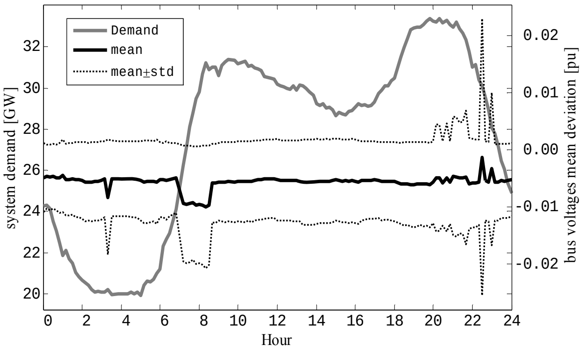

Fig. 10. System demand and bus voltages deviation statistics during 24 hours

Fig. 10 shows that both the mean and the standard deviation of the bus voltage DC-Q estimation error present an almost neutral behavior as demand varies, just a couple of perturbations when the demand slope is at its highest, around hours 8 and 22.

To get more detailed results on DC-Q model performance, the peak-load scenario at 19h50m has been analyzed, not only in the base case, but also considering the outage of each 400 kV line in the network. Fig. 11 depicts the cumulative distribution function of the bus voltage DC-Q estimation error among 400 and 220 kV buses in the network.

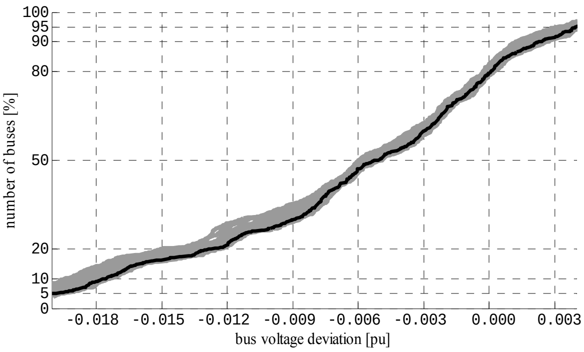

Fig. 11. Bus voltages deviation distribution functions

In Fig. 11, the base case error is represented with thick black line, whereas the error of each contingency analysis is plotted in thin grey line. All the error distributions are quite uniforms and present mean values around -0.005 pu. The 90% of the buses present an estimation error greater than -0.018 pu and shorter than 0.003 pu.

Bus voltages estimation has been proved as an important contribution of the DC-Q model proposed in this paper. But the other network magnitudes under surveillance in security analysis, i.e. branch power flows, can be estimated with more precision combining the DC-P and the DC-Q models. Fig. 12 illustrates the cumulative distribution function of the branch power flow estimation error among the HV branches of the network, i.e. the 400 and 220 kV lines, and 132/220, 132/220 and 220/400 kV transformers.

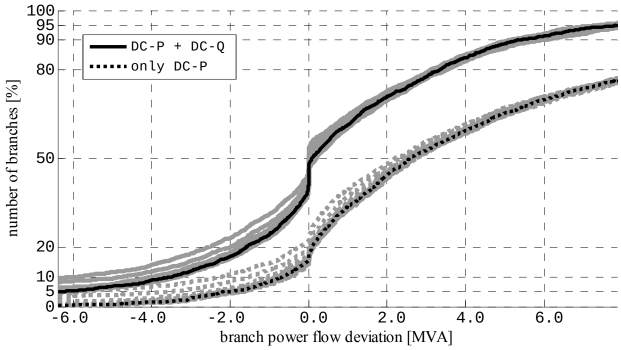

Fig. 12. Branch power flow distribution functions

In Fig. 12, the base case error is represented with thick black line, whereas the error of each contingency analysis is plotted in thin grey line. There are two sets of manifolds, corresponding with the DC-P estimation (dotted lines) and the combined DC-P and DC-Q estimation (continuous line). It can be observed that the distributions corresponding with the DC-P and DC-Q combination are centered on zero and present lower standard deviations with respect to the estimations based only in the DC-P model.

VI. Conclusions

This paper introduces a novel DC-Q model approximation for the power flow equations. The model is built using the logarithm of bus voltage as variable of the system instead of the module. With this change of variable, the simplifications applied result in two linear circuits where active and reactive power flows are currents, and bus angles and log-voltages are node voltages. Power losses are not considered in first attempt, thus after solving the DC models power losses are estimated and used as new independent energy sources and a second DC models solving is needed. Linearity of both circuits also allows computing fast contingency analysis using substitution and superposition theorems.

The DC-Q model presents a large number of possible applications. It provides a useful tool for each application where fast approximations of power flow solutions are needed and the QV information is relevant, such as optimal power flows, security analysis or voltage stability studies. The linearity of both models allows to easily optimizing QV control strategies, even including transformer taps as part of the control variables set. DC models can also be applied to obtain an appropriate starting point for power flow convergence.

Results obtained over different actual scenarios of the Spanish electric power system note that the DC-Q model is an adequate tool to compute estimations of power flow QV solutions. The DC-Q model by itself provides accurate bus voltage estimations, but also the DC-P and DC-Q combination allows to obtaining branch power flow estimations more precise than the obtained using only the DC-P model. Finally, it is important to remark that the accuracy of the DC models estimations are as precise as the system is away from voltages collapse point.

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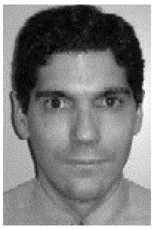
VIII. Biographies

Francisco M. Echavarren received his Ingeniero Industrial and Doctor Ingeniero Industrial degrees (2001 and 2006) from Universidad Pontificia Comillas, Madrid, Spain. Since 2001 he has been a researcher in the IIT. He is also a lecturer in the Department of Electrical Engineering. His areas of interest include voltage stability phenomena, analysis of power system constraints and power system protection.



Enrique Lobato Miguélez was born in Burgos, Spain, in 1974. He received the degree of Electrical Engineer in 1998 and the PhD degree in 2002, from *Universidad Pontificia Comillas*, Madrid, Spain. Since June 1998, he is a Researcher at the *Instituto de Investigación Tecnológica*, *Universidad Pontifica Comillas*. His areas of interest include analysis, planning, operation and economics in electric power systems. He has participated in several research projects for different firms related with the energy industry

Luis Rouco (S'89, M'91) obtained his Ingeniero Industrial and Doctor Ingeniero Industrial degrees from Universidad Politécnica de Madrid in 1985 and 1990. He is Professor of the School of Engineering of Universidad Pontificia Comillas and is attached to the Department of Electrical Engineering. He served as Department Head from 1999 to 2005. He develops his research activities in the IIT. His areas of interest are modeling, analysis, simulation, and identification of electric power systems.