RESEARCH ARTICLE

Combining economic and fluid dynamic models to determine the optimal spacing in very large wind farms

Richard J. A. M. Stevens[1,4], Benjamin F. Hobbs[2], Andrés Ramos[3] and Charles Meneveau[4]

[1] Department of Physics, Mesa+ Institute and J. M. Burgers Centre for Fluid Dynamics, University of Twente, 7500 AE Enschede, The Netherlands
[2] Department of Geography and Environmental Engineering and the Environment, Energy, Sustainability and Health Institute, Johns Hopkins University, Baltimore, Maryland 21218-2686, USA
[3] ICAI School of Engineering, Institute for Research in Technology, Comillas Pontifical University, C/ Santa Cruz de Marcenado 26, 28015 Madrid, Spain
[4] Department of Mechanical Engineering and Center for Environmental and Applied Fluid Mechanics, Johns Hopkins University, Baltimore, Maryland 21218, USA

ABSTRACT

Wind turbine spacing is an important design parameter for wind farms. Placing turbines too close together reduces their power extraction because of wake effects and increases maintenance costs because of unsteady loading. Conversely, placing them further apart increases land and cabling costs, as well as electrical resistance losses. The asymptotic limit of very large wind farms in which the flow conditions can be considered 'fully developed' provides a useful framework for studying general trends in optimal layouts as a function of dimensionless cost parameters. Earlier analytical work by Meyers and Meneveau (Wind Energy 15, 305–317 (2012)) revealed that in the limit of very large wind farms, the optimal turbine spacing accounting for the turbine and land costs is significantly larger than the value found in typical existing wind farms. Here, we generalize the analysis to include effects of cable and maintenance costs upon optimal wind turbine spacing in very large wind farms under various economic criteria. For marginally profitable wind farms, minimum cost and maximum profit turbine spacings coincide. Assuming linear-based and area-based costs that are representative of either offshore or onshore sites we obtain for very large wind farms spacings that tend to be appreciably greater than occurring in actual farms confirming earlier results but now including cabling costs. However, we show later that if wind farms are highly profitable then optimization of the profit per unit area leads to tighter optimal spacings than would be implied by cost minimization. In addition, we investigate the influence of the type of wind farm layout. © 2016 The Authors. *Wind Energy* Published by John Wiley & Sons, Ltd.

1. INTRODUCTION

Wind farms are becoming increasingly larger. For instance, the Alta and Roscoe wind farms in Texas have approximately 600 turbines. Therefore, it is important to better understand the influence of wake effects on the optimal turbine spacing in wind farms with hundreds or thousands of turbines.[1] For smaller wind farms, the majority of the turbines can be placed such that wake effects are rather limited. For the design of such wind farms, the industry uses site-specific, detailed optimization

calculations for wind turbine placement based on wake type models.[2–5] Such calculations aim to place the turbines such that wake effects are limited with respect to the main incoming wind direction at the site under consideration. There are also academic studies that use wake models to optimize the placement of a limited number of turbines on a given land area using Monte Carlo simulations,[6] genetic algorithms[7] or evolutionary algorithms.[8,9] For an overview on this, we refer to the review by Herbert–Acero *et al.*[10] The typical wind turbine spacing that is used in actual wind farms nowadays is $\sim 6 - 10D$, where D is the turbine diameter.

It is known that these wake type models do not explicitly capture the effect of the interaction between the atmosphere and very large wind farms.[11–15] These effects are better described by top-down type models, which are based on momentum analysis and horizontal averaging. These models give a vertical profile of the average velocity profile by assuming the existence of two logarithmic velocity regions, one above the turbine hub-height and the other below.[16–23] The mean velocity at hub-height is used to predict the wind farm performance as function of wind farm design parameters. The top-down model approach allows one to analytically calculate the wind farm performance for very large wind farms. Meyers and Meneveau[24] used this approach to analyze the optimal spacing in the limit of very large wind farms by accounting for the turbine and land costs and found an optimal spacing of $\sim 12 - 15D$, which is significantly larger than the value found in actual wind farms. Later work by Stevens[23] showed that the predicted optimal spacing is much closer to the values found in actual wind farms when the smaller number of turbines (and thus smaller contribution of wake effects) in these farms are taken into account.

The purpose of this paper is to generalize the Meyers and Meneveau[24] analysis for very large farms by including effects of cable and maintenance costs in addition to land and turbine costs upon optimal wind turbine spacing under various economic criteria. Figure 1 shows in schematic fashion the general trade-offs that we analyze. The horizontal axis is the spacing between turbines while the vertical axis expresses the levelized cost per MWh of energy production, defined as the present worth (at an assumed interest rate) over the wind farm lifetime of the farm's capital as well as operations and maintenance (O&M) costs, divided by the present worth of energy (MWh) produced over the farm's life. The cost is normalized so that 1.0 is the hypothetical levelized cost of a single wind turbine whose output is unaffected by wakes from other turbines and for which there are no capital costs beyond the turbine and its associated structure. A real wind farm with multiple turbines will have a levelized cost greater than this level because there will be some wake effects (decreasing the denominator) as well as additional costs for land acquisition (perhaps in the form of offshore leases), cables between turbines and associated electrical resistance losses, and roads (in the case of onshore wind farms). The downward sloping curve (turbine capital and O&M cost) shows how wakes affect the levelized turbine, structure and O&M costs; at a small spacing, wake effects significantly shrink the MWh production (and thus the denominator of levelized costs). That curve approaches the 1.0 asymptote for large spacings. Meanwhile, the upward sloping curve (quadratic and linear cost) represents the levelized cost of land, cables and roads, which approaches zero at the origin and is convex (bending upwards) if land costs are important, because they are a quadratic function of spacing. The sum of these two cost curves gives the overall levelized cost of power production, which will be U-shaped and will have a minimum at the cost-optimal spacing. The spacing that minimizes the cost of wind depends on shapes and levels of both the capital and O&M cost curve and the quadratic/linear cost curve.

In this paper, we use the coupled wake boundary layer (CWBL) model of Stevens *et al.*[13,14,25] to account for wake effects in the capital/O&M cost curve, and then combine it with an improved methodology for estimating the cost curve for very large farms. Because the CWBL model is an analytical model, it allows a very fast evaluation of the wind farms' performance. Important benefits of the CWBL model compared with the Calaf *et al.*[21] model are (i) that the CWBL model
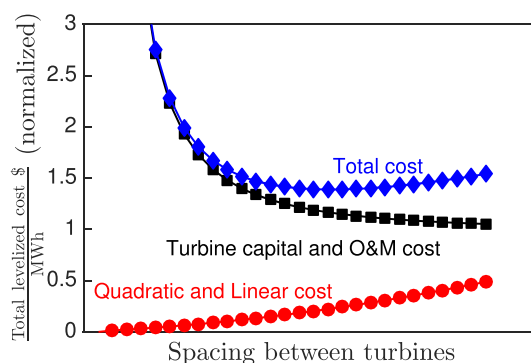
Figure 1. Schematic of analysis of optimal spacing in a large wind farms, showing trade-off between wake losses (reflected in higher capital and O&M costs for lower values of turbine spacing) and land, cable and other spacing-related expenditures (reflected in increased 'quadratic plus linear costs' as a function of spacing).

is able to predict the effect of the turbine layout on the performance of very large wind farms and (ii) that the CWBL model captures the entrance effects. Thus, the use of the CWBL model allows us to analyze the optimal spacing of wind farms in more detail than the Calaf *et al.*[21] model. This CWBL approach can be used for wind farms of varying size. As wake effects on the optimal spacing are most pronounced in very large fully developed wind farms, we focus on that case here. *A priori*, it is difficult to specify a very sharp definition of the 'fully developed' regime of large wind farms because it depends on case-by-case arrangements. For the purpose of mean power optimization as carried out in this paper, we take the view that fully developed means that the turbine power becomes nearly constant with downstream distance. Common experience such as in Horns Rev (80 turbines) shows that at the end of the wind farm turbines is exposed to fully developed wind conditions. Therefore, we expect fully developed wind conditions to become relevant for wind farms with several hundred turbines.

The improved costing methodology used in this paper considers cost categories that are 'quadratic' (e.g., area-related costs arising from leasing or occupying land) and 'linear' (e.g., the expense of inter-turbine cabling, electrical resistance losses and roads) related to the inter-turbine spacing. Based on an order of magnitude estimation of the quadratic and linear costs, we argue in this paper that linear costs are significant and are especially important for offshore wind farms. In addition, to account for the effect of both quadratic and linear cost components in this paper, we also present versions of the model that address other issues not considered by Meyers and Meneveau.[24] One version accounts for the effect of spacing-affected turbulence upon cost-optimal spacing, because closely spaced turbines are likely to experience increased turbulence-related fatigue and damage. Another version recognizes that areas that are particularly profitable for wind development can be limited, implying that the objective of design is not to minimize cost of production but to maximize profit per unit area.

In Section 2, we start with the definition of the quadratic and linear cost components and the dimensionless cost parameters that are important for very large wind farms. In Section 3, we introduce the CWBL model[13, 25] that will be used to estimate the power and turbulence intensity for different wind farm designs. In Section 4, the modeling approach introduced by Meyers and Meneveau[24] is extended to include linear costs (e.g., the costs of cables, roads and resistance losses that scale linearly with the distance between turbines) while in Section 5 the optimal turbine spacing is determined by optimizing the profit per unit area of the wind farms instead of the normalized power per unit cost. In Section 6, the effect of the maintenance costs, based on the predicted turbulence intensity by the CWBL model, is evaluated. Subsequently, we will use the CWBL model to estimate the effect of the wind farms layout on the optimal turbine spacing in very large wind farms in Section 7. Conclusions are given in Section 8.

2. DEFINITION OF DIMENSIONLESS PARAMETERS

In this paper, we will study the effect of different cost factors on the optimal turbine distance $S = sD$, where D is the turbine diameter, and s the dimensionless turbine distance that will be used in the remainder of this paper. For convenience, we introduce the following dimensionless parameters to analyze the effect of the main economic influences on optimal turbine spacing:

$$\theta = \frac{\text{Cost}_{\text{quadratic}}}{\text{Cost}_{\text{turbine}}/D^2}, \quad \beta = \frac{\text{Cost}_{\text{linear}}}{\text{Cost}_{\text{turbine}}/D}, \quad \gamma = \frac{\text{Rev}_1}{\text{Cost}_{\text{turbine}}}, \quad \epsilon = \frac{\text{Main}_1}{\text{Cost}_{\text{turbine}}} \tag{1}$$

where all the cost factors have been normalized with respect to the turbine costs and the turbine diameter when appropriate. $\text{Cost}_{\text{turbine}}$ is defined as the present worth (at some discount rate) of the capital and O&M cost of a single turbine, excluding all costs that depend on distance between turbines, such as cabling, land leasing and roads. $\text{Cost}_{\text{quadratic}}$ and $\text{Cost}_{\text{linear}}$ are, respectively, the present worth of area and length dependent capital and other costs that increase with turbine spacing. Quadratic costs include for example costs arising from leasing or occupying land while linear costs include items such as inter-turbine cabling, electrical resistance losses and roads. 'Rev$_1$' is the present worth of expected revenue of a single wind turbine that does not experience wake effects. 'Main$_1$' are the maintenance costs over lifetime, again for a single turbine without wake effects, the estimate should indicate for which turbulence intensity this estimate was obtained, such

Table I. Reference estimates of reasonable dimensionless cost parameters for wind farms, see details in the Appendix.

	θ (quadratic)	β (linear)	γ (revenue)	ϵ (maintenance)
Onshore	1×10^{-3}	5×10^{-3}	1.5	0.1
Offshore	2×10^{-5}	1×10^{-2}	1.5	0.1

The cost parameters θ (quadratic), β (linear), γ (revenue) and ϵ (maintenance) are normalized with the turbine costs, see equation (1).

that this can be accounted for in the calculations. Lifetime revenue is defined as being net of the expense of interconnecting the farm to the main power grid, because that cost is not included in $\text{Cost}_{\text{turbine}}$.

In Sections 4–7, we assess the optimal spacing and profitability for wide ranges of the linear (β) and quadratic (θ) turbine cost parameters, in acknowledgement of the large uncertainty in their values for particular locations. In the Appendix, we present some order of magnitude estimates for the dimensionless cost parameters for onshore and offshore wind farms, see Table I, to direct the reader's attention to more practically relevant regions of the parameter space. The parameter γ is meant to include all sources of revenues, including tax incentives and renewable energy credits. The precise value is not based on actual experience but upon an expectation that constructed wind farms will be profitable. Therefore, we assume the same γ for both onshore and offshore facilities under the assumption that public policy and market conditions are such that either are somewhat but not greatly profitable for development. Thus, although costs are higher for offshore facilities, we anticipate that revenues will also be higher, so that the ratio of revenues to costs for financially viable wind farms will somewhat but not greatly above 1.0. For instance, offshore farms are given more subsidies in the UK than onshore farms.[26] To assure the aforementioned assumption of wind farm profitability, we have selected $\gamma = 1.5$ for the onshore and offshore reference cases.

3. INPUT PREDICTIONS FROM THE CWBL MODEL

In the CWBL model, the top-down approach by Calaf *et al.*[21] is used to calculate the performance of the wind turbines in the fully developed region of the wind farm. In the CWBL model, the ratio of the mean velocity to the reference incoming velocity at hub-height in the fully develop region is

$$\frac{\langle \overline{u} \rangle (z_h)}{\langle \overline{u_0} \rangle (z_h)} = \frac{\ln \left(\delta_H / z_{0,\text{lo}} \right)}{\ln \left(\delta_H / z_{0,\text{hi}} \right)} \ln \left[\left(\frac{z_h}{z_{0,\text{hi}}} \right) \left(1 + \frac{D}{2z_h} \right)^b \right] \left[\ln \left(\frac{z_h}{z_{0,\text{lo}}} \right) \right]^{-1} \tag{2}$$

Here, δ_H indicates the height of the atmospheric boundary layer in the fully developed region of the wind farm, $b = v_w^*/(1 + v_w^*)$, $v_w^* \approx 28\sqrt{\pi C_T/(8 w_f s_x s_y)}$, $z_{0,\text{hi}}$ denotes the roughness length of the wind farm, which is evaluated in the model according to $z_{0,\text{hi}} = z_h (1 + R^*)^b \exp\left(-[q + \ln[(z_h/z_{0,\text{lo}})(1 - R^*)^b])^{-2}]^{-1/2}\right)$, where $R^* = D/(2z_h)$, $q = \pi C_T/(8 w_f s_x s_y \kappa^2)$, $\langle u_0 \rangle (z_h) = u_*/\kappa \ln \left(z_h/z_{0,lo} \right)$ and w_f indicates the effective wake area coverage, which is obtained from the two-way coupling with the wake model part of the CWBL model, and is equal to the ratio of wake area divided by total area.[25] Because of the two-way coupling between the wake and the top-down model, w_f depends on parameters such as the streamwise distance between the turbines s_x, the relative positioning of the turbines and the wake coefficient in the fully developed region of the wind farm $k_{w,\infty}$. As we focus on the optimal spacing for very large wind farms, we assume that the average power output of the wind turbine is the same as the turbine power output in the fully developed region of the wind farms. The power ratio P_∞/P_1 is given by the ratio of cubed mean velocity at hub-height with wind turbines compared with the reference case without wind farms:

$$\frac{P_\infty}{P_1}(s_x, s_y, \text{layout}, \ldots) = \left(\frac{\langle \overline{u} \rangle (z_h)}{\langle \overline{u_0} \rangle (z_h)} \right)^3 \tag{3}$$
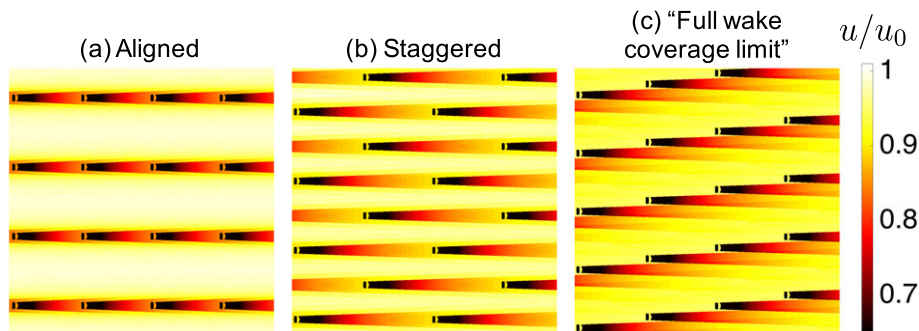
Figure 2. The velocity field, obtained with the CWBL model, in the fully developed part of a very large wind farm with a spacing of $s = 10.5$ with an (a) aligned, (b) staggered and (c) full wake coverage limit layout. The black lines indicate the turbine positions. The wake coverage area w_f is defined as the percentage of the area in the fully developed regime of the wind farm where $u < 0.95 u_0$.[25] Here, $w_f = 0.35$ (aligned), $w_f = 0.60$ (staggered) and $w_f = 1.00$ (full wake coverage limit).

In earlier work, we showed[13,25] that the CWBL model gives improved predictions for the power output in the fully developed region compared with the top-down model introduced by Calaf *et al.*[21] or stand-alone wake models.[3] In addition, the CWBL model is able to predict the difference between the performance of different wind farms geometries. Here, we specifically focus on aligned, staggered and 'full wake coverage limit' layouts. Figure 2 shows a visualization of these different wind farm layouts. The full wake coverage limit is defined as a wind farm configuration for which the wake coverage area $w_f = 1$ in the CWBL model. In the full wake coverage limit layout, the performance of the wind farm is better than for aligned or staggered wind farms, because the distance between directly upstream turbines is larger, by placing turbines such that turbines on the next downstream row are just outside the wake. For $s < 7$, the staggered wind farm layout still has a wake area coverage $w_f = 1$, and therefore, the full wake coverage limit only outperforms the staggered layout for $s > 7$. This effects is discussed in more detail in the work of Stevens *et al.*[27] We note that most calculations presented in this paper, except in Section 7 where the effect of the wind farms layout is discussed, are based on the power estimates for the full wake coverage limit layout.

4. OPTIMAL TURBINE SPACING OF VERY LARGE WIND FARMS WITH CABLE COST

In order to investigate the robustness of the results from Meyers and Meneveau,[24] we include, in addition to the quadratic (land) costs, a linear cost component defining the total cost per turbine as

$$\text{Cost} = \text{Cost}_{\text{turbine}} + (sD)\text{Cost}_{\text{linear}} + (sD)^2\text{Cost}_{\text{quadratic}} \tag{4}$$

Following the approach by Meyers and Meneveau,[24] this leads to the following normalized power per unit cost

$$
\begin{aligned}
P^*(s_x, s_y, \text{layout}, ...) = \frac{P_\infty}{\text{Cost}} &= \frac{P_\infty}{\text{Cost}_{\text{turbine}} + (sD)\text{Cost}_{\text{linear}} + (sD)^2\text{Cost}_{\text{quadratic}}} \\
&= \frac{P_1}{\text{Cost}_{\text{turbine}}} \frac{P_\infty}{P_1} \frac{1}{1 + \beta s + \theta s^2}
\end{aligned}
\tag{5}
$$

where $P_\infty = P_\infty(s_x, s_y, \text{layout}, ...)$ is the average power production over the life of a typical turbine in a farm with the assumed spacing and layout. P_1 is the average power production over the life of a turbine if it was lone-standing without a wind farm. The ratio (P_∞/P_1) thus contains the wind farm related power reductions because of wake and layout effects. Costs are levelized to present worth. Thus, the ratio $(P_1/\text{Cost}_{\text{turbine}})$ is the reciprocal levelized cost of power for an individual wind turbine, while $P^*(s_x, s_y, \text{layout})$ is the reciprocal levelized cost of power for a wind turbine in a wind farm.

First, in order to explore equation (5), we maximize it for the simple case of a very large fully developed wind farm. We use equation (3) to determine the ratio P_∞/P_1 and replace it in equation (5) and find s that maximizes P^*. Results are shown in Figure 3(b). In agreement with the results of Meyers and Meneveau,[24] Figure 3(b) shows that without linear costs (β approaching zero) the optimal turbine spacing strongly depends on the quadratic costs. In addition, Figure 3(b) reveals that with increasing linear costs, the predicted optimal spacing becomes smaller.

As can be seen, for offshore wind farms, it is the linear costs that determine the value of the optimal distance, while for onshore farms, quadratic costs become equally or more important. This is indicated in Figure 3(b), where the reference
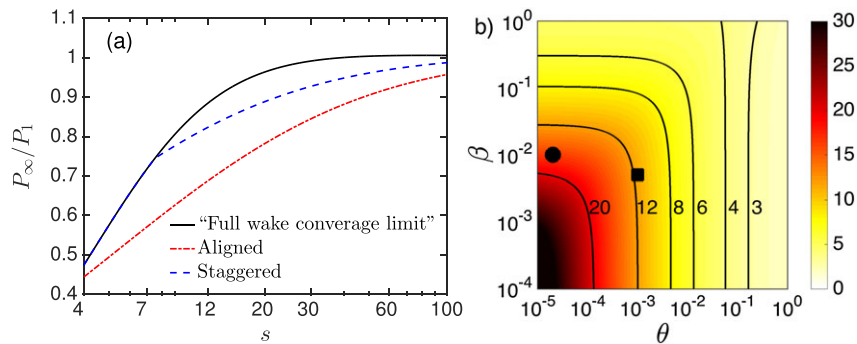
Figure 3. (a) The predicted power output in the fully developed region of a wind farms compared with a free standing wind turbine (P_∞/P_1) using equation (3) as function of the geometric mean turbine spacing $s = \sqrt{s_x s_y}$ for $s_x/s_y = 1$. (b) Optimal spacing as function of 'quadratic costs' θ and 'linear costs' β, see equation (1). The circle and square indicates the reference offshore and onshore case, respectively.

offshore farm (indicated by a dot) has a higher linear costs than the reference onshore farm (indicated by a square), while the onshore farm has a higher quadratic costs. Based on these reference values defined in the Appendix, the figure indicates that the best spacing for offshore farms is about 16 rotor diameters. Onshore farms, in contrast, have a smaller optimal spacing of about 12 diameters. Both values are appreciably greater than values typical of actual wind farms designs. Our calculations therefore show that in very large wind farms wake effects are large enough to justify significantly greater expenditures on land, cables, roads and other distance-dependent categories in order to lower the delivered cost of wind power. Perhaps, actual design practice chooses smaller distances on occasion because the cost of wakes is under appreciated in planning; however, as we show in the next section, smaller distances can in fact be justified if developable area is limited and wind development is highly profitable.

5. PROFIT OPTIMIZATION ON A FIXED AREA

Up to now, the optimal spacings we calculated were based on optimizing the normalized power per unit cost. In a competitive wind industry in which competitive pressures drive wind power revenues close to the cost of supply, a company maximizes its profit by minimizing its cost, or else, it will lose out to competitors. This is consistent with the neoclassical microeconomic theory of the firm, in which free entry by identical competitors will cause companies to produce in a way that minimizes total cost.[28] However, this textbook result makes a number of strong assumptions, including the assumption that the industry has achieved a long run equilibrium. This is manifestly not the case here, as the present wind industry is still growing rapidly in much of the world because of large subsidies while at the same time technology costs are falling. Furthermore, economic 'rents' (which is the economist's term for the excess of revenues over expenses, including the cost of capital) can persist if entry of new wind turbines is restricted. This can occur, for instance, because concerns over landscape degradation or wildlife might limit the areas that can be developed for wind, which is especially the case for large onshore farms. In this section, we consider the implications of a situation in which anticipated revenues from selling wind power exceed costs by a significant margin when the developable area is finite. In this situation, the profit-maximizing spacing needs to consider the fact that a wider spacing not only incurs more costs per unit output but will also reduce the number of turbines that can be installed in the limited land available, and so lower revenues. In a sense, there is what economists call an 'opportunity cost' associated with wider spacings, in the form of foregone revenues from turbines that wider spacings prevent from being installed. The implication is that a developer with a limited amount of area to develop will maximize profit per unit area developed and not cost per unit output, because total profit is equal to total area times that profit per unit area. We show later that this leads to tighter optimal spacings than would be implied by cost minimization if the ratio of revenues to costs is high.

To be consistent with the discussion earlier, we define profit per single turbine in a wind farm as

$$\text{Profit} = \text{Revenue over lifetime} - \left[\text{Cost}_{\text{turbine}} + \text{Cost}_{\text{linear}}(sD) + \text{Cost}_{\text{quadratic}}(sD)^2 \right] \tag{6}$$

The revenue over lifetime is given by the product of the lifetime revenue of a single turbine Rev_1 multiplied by the fraction of lifetime power because of the presence of the wind farm, i.e., 'Revenue over lifetime $= \text{Rev}_1 P_\infty(s_x, s_y, \text{layout})/P_1$'; equation (6) can now be rearranged as follows to give a profit function:

$$\text{Profit}^* = \frac{\text{Profit}}{\text{Cost}_{\text{turbine}}} = \frac{P_\infty(s_x, s_y, \text{layout}, ...)}{P_1} \gamma - \left[1 + \beta s + \theta s^2 \right] \tag{7}$$

The ratio of present worth of revenues to turbine cost (γ) varies considerably from market to market. As a profit is only possible for $\gamma > 1$, only this region of the parameter space will be considered. As a very rough approximation, we consider values of γ ranging from 1.25 to 2, with the latter value indicating that the revenues are double the turbine cost, including O&M costs but excluding the distance dependent linear and quadratic costs.

Because the area of the wind farms is fixed, the objective of the farm developer is not to maximize the profit per turbine but to maximize the profit over the given area. This is equivalent to maximizing the profit per unit area, because that can be multiplied by the constant area to obtain the total profit. Profit per unit area is, in turn, $\text{Cost}_{\text{turbine}}$ times the normalized profit (7) divided by the area per turbine (which is proportional to s^2), implying that total profit over the area is expression (equation (7)) divided by s^2 times a constant proportional to $\text{Cost}_{\text{turbine}}$ times the farm area. Thus, maximizing (7)/s^2, shown thereafter, is the same as maximizing total profit for the entire farm:

$$\frac{\text{Profit}^*}{s^2} = \frac{1}{s^2} \left[\frac{P_\infty(s_x, s_y, \text{layout}, ...)}{P_1} \gamma - \left(1 + \beta s + \theta s^2 \right) \right] \tag{8}$$

So our problem is to choose the spacing s (or, more particularly, to choose s_x, s_y, and the arrangement of turbines) to maximize equation (8) for each value of β, θ and γ within some specified domain.
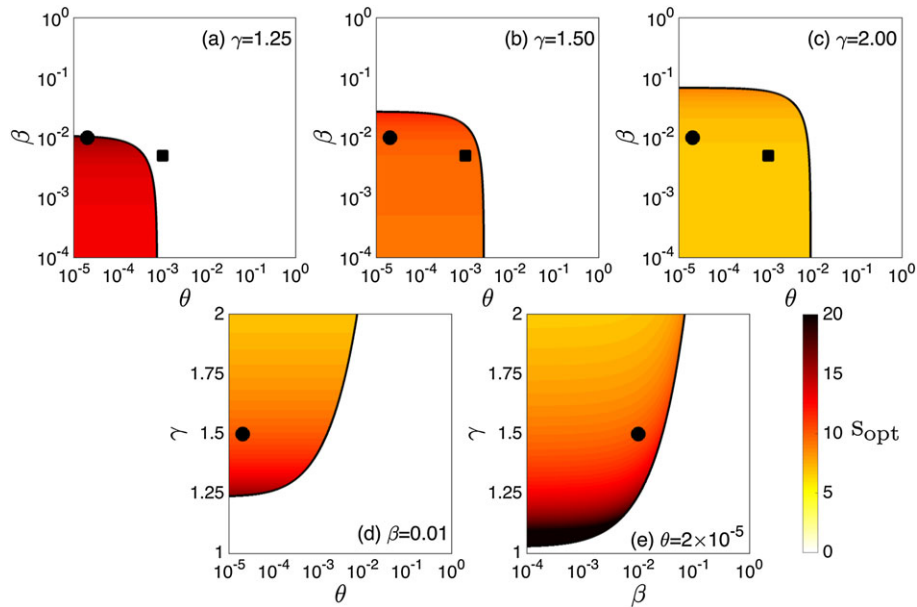
Figure 4. The top panels $((a) - (c))$ indicate the optimal spacing under profit maximization as function of θ and β for different γ. The lower panels ((d) and (e)) indicate the optimal spacing as function of γ and θ, and γ and β for the reference offshore case. The optimal spacing is only shown for $\theta - \beta - \gamma$ combinations that are profitable.

In Figure 4, the optimal spacing based on maximizing profit per unit area is shown as function of θ, β and γ. The bottom two panels show different cuts through the three-dimensional parameter space assuming our reference offshore case $\theta = 2 \times 10^{-5}$, $\beta = 0.01$, which is indicated by the dot. Note that in these figures the optimal spacing is only shown when there is a profit, i.e., the white regions indicate combinations of θ, β and γ for which the wind farms will not be profitable. Meanwhile, the top panels of Figure 4 show the optimal spacing for three different fixed γ values.

Overall, the results in Figure 4 show that for high γ, the profitable region in the $\theta - \beta$ plane is large, whereas for lower γ the profitable region in this plane is small. This is as expected, because a higher revenue will enable farms with higher costs to be profitable. However, the magnitude of the optimal spacing depends on γ, with smaller optimal spacings for higher γ, i.e., when the revenue obtained from the turbines is high compared with the turbine cost. This effect is also shown in the lower panels, which show the optimal spacing in the $\gamma - \theta$ and $\gamma - \beta$ planes.

This result can be explained as follows. As wind power becomes more profitable, the opportunity cost (in terms of foregone net revenue from turbines not built) of using a wide spacing to mitigate wake effects increases. This incents the developer to crowd the turbines more closely together. At $\gamma = 1.25$ (Figure 4(a)), under typical land and cable costs, both onshore and offshore farms are barely, if at all profitable, and the only way that the developer can make money is by minimizing cost. Consequently, for that γ, the optimal spacings are similar to the cost minimizing spacings of the previous section, above (16 diameters for offshore, but about 14 diameters for the onshore case instead of 12 to minimize the wake losses). But at higher revenues (Figure 4(b) and (c)), the optimal spacings fall to about 10 ($\gamma = 1.5$) and 7 ($\gamma = 2$). Thus, the $6 - 10$ diameter spacings seen in actual large wind farms could, at least in some cases, be due to limited land and a high profitability. Discount rate and amortization period will play a similar role to γ and that is also taken into account by the companies.

6. MAINTENANCE COSTS

Another important cost component is the maintenance costs. As noted in the Appendix, levelized maintenance costs amount to approximately a tenth to a fifth of the levelized capital costs for both offshore and onshore wind farms. Assuming that an appreciable portion of those costs depend on the turbulence intensity to which turbines are exposed, we derived in the Appendix an approximate value of $\epsilon (\approx 0.1)$, the ratio of turbulence-affected O&M costs to the turbine cost. The turbulence-related O&M costs will depend on the wind farms design as the turbines will experience higher loads when the turbines are placed close together. Here, for simplicity, we assume that the maintenance costs depend linearly on the turbulence intensity, and we use the CWBL model to get a first order approximation of how the turbulence intensity depends
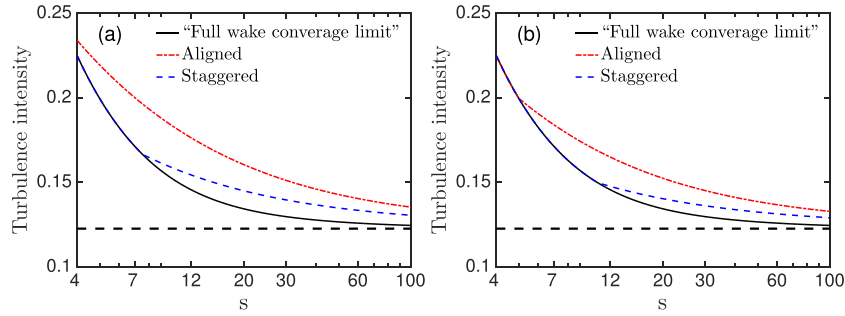
Figure 5. The predicted turbulence intensity at hub-height according to the CWBL model (equation (9)) as function of the geometric mean turbine spacing $s = \sqrt{s_x s_y}$ for (a) $s_x/s_y = 1$ and (b) $s_x/s_y = 2$.
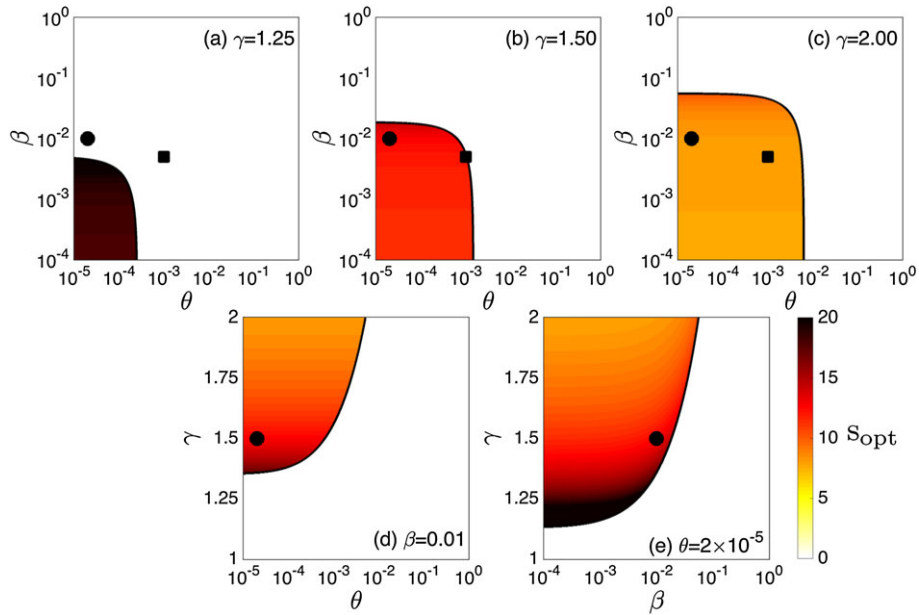
Figure 6. The top panels ($(a) - (c)$) indicate the optimal spacing under profit maximization per unit area with turbulence-related O&M costs as function of θ and β for different γ. The lower panels ((d) and (e)) indicate the optimal spacing as function of γ and θ, and γ and β for the reference offshore case. The optimal spacing is only shown for $\theta - \beta - \gamma$ combinations that are profitable.

on the wind farms design by modeling the turbulence intensity as

$$
\mathrm{TI}_{\mathrm{hub}} = \frac{\left[\langle \overline{(u'^+)^2} \rangle \right]^{1/2}}{\langle \overline{u} \rangle} = \frac{[B_1 - A_1 \log(z_{\mathrm{h}}/\delta_H)]^{1/2}}{\langle u \rangle (z_{\mathrm{h}})} \frac{u_{*,\mathrm{hi}} + u_{*,\mathrm{lo}}}{2 u_*}
\tag{9}
$$

where $\langle u \rangle (z_{\mathrm{h}})$ is the mean velocity at hub-height according to the CWBL model, see equation (2), '$B_1 - A_1 \log(z_{\mathrm{h}}/\delta_H)$' the logarithmic law for the variance evaluated at hub-height z_h, and $u_{*,\mathrm{hi}}/u_*$ and $u_{*,\mathrm{lo}}/u_*$ are evaluated from the CWBL model using (equations (9) and (16) in the work of Meneveau[22]). Figure 5 shows the predictions for the turbulence intensity at hub-height obtained from the CWBL model. According to experiments and high resolution LES,[29] $A = 1.25$, $B = 1.6$ and $\kappa = 0.4$ are reasonable values to estimate the turbulence intensity in an atmospheric boundary layer. For the data shown here, we assumed a roughness height $z_{\mathrm{lo},0} = 10^{-4} \delta_H$, a hub-height $z_h/\delta_H = 0.1$, where δ_H is the height of the boundary layer.

We could insert the wake turbulence-dependent O&M cost expression in the cost equation 4 and then minimize normalized cost. For the cost minimization case, by penalizing closer spacings for their greater O&M costs from wake turbulence, the outcome would be even wider spacings than found optimal in Figure 3(b) in Section 4 ($s \approx 16$ for offshore, $s \approx 12$ for

onshore). For brevity, we proceed directly to derivation and analysis of the case of profit maximization per unit area (as in Section 5), in which we assume that revenues are significantly in excess of costs while developable area is limited.

Following the procedure outlined in Section 5, we start by expressing profit per turbine, accounting for turbulence-induced O&M costs, as follows:

$$\text{Profit} = \text{Revenue over lifetime}$$
$$- \left[\text{Cost}_{\text{turbine}} + \text{Cost}_{\text{main}} \text{TI}'(s_x, s_y, \text{layout}, ...) + \text{Cost}_{\text{linear}} sD + \text{Cost}_{\text{quadratic}} (sD)^2 \right] \tag{10}$$

where $\text{Cost}_{\text{main}}$ is the present worth for a single turbine of turbulence-dependent O&M costs over the life of the turbine, calculated assuming no wake-induced turbulence. $\text{Cost}_{\text{turbine}}$ should then be interpreted, for the purposes of equation (8), as the present worth of capital and non-turbulence-related O&M for one turbine. $\text{TI}'(s_x, s_y, \text{layout}, ...)$ is defined as the ratio of total turbulence experienced by a turbine at a given spacing and arrangement to that experienced if there is zero wake-induced turbulence, i.e., $\text{TI}'(s_x, s_y, \text{layout}, ...) = \text{TI}(s_x, s_y, \text{layout}, ...)/\text{TI}(s_x \to \infty, s_y \to \infty, \text{layout}, ...)$. Dividing by $\text{Cost}_{\text{turbine}}$ and s^2 gives the relative profit per area (which generalizes equation (8)):

$$\frac{\text{Profit}^*}{s^2} = \frac{1}{s^2} \left[\frac{P_\infty(s_x, s_y, \text{layout}, ...)}{P_1} \gamma - \left[1 + \beta s + \theta s^2 + \epsilon \text{TI}'(s_x, s_y, \text{layout}, ...) \right] \right] \tag{11}$$

where ϵ should be interpreted as the ratio of turbulence-dependent O&M costs (for the case of no wake) to $\text{Cost}_{\text{turbine}}$, which we have estimated as about 0.1 in the Appendix. We now solve for the s (or, more particularly, to choose s_x, s_y, and the arrangement of turbines) that maximizes this normalized profit per unit area (equation (11)) for each value of β, θ, and γ within the ranges considered.

Figure 6 shows the optimal spacing including the maintenance costs for $\epsilon = 0.1$. The optimal spacings in that figure for a given combination of θ, β and γ suggest a larger turbine spacing than without incorporating the maintenance costs term. In particular, the s values that maximize profit per unit area in Figure 4 ($s = 7$ and $s = 10$ for $\gamma = 2$ and $\gamma = 1.5$, respectively) become about one-third larger when these wake effects on turbulence related maintenance costs are accounted for. The optimal s now falls in the range of $8-12$, see Figure 6(c) and (b).

7. WIND FARM LAYOUT

All the results presented earlier have been obtained for the full wake coverage limit layout for a given geometrical turbine spacing $s = \sqrt{s_x s_y}$. In this section, we will compare the predicted optimal spacing using the power curves shown in Figure 3(a) by considering the normalized power per unit cost as in Section 4 for the aligned, staggered and full wake coverage limit layout. Figure 7 shows the optimal spacing for aligned, staggered and full wake coverage limit layouts for $\beta = 0$. Interestingly, this figure shows that, depending on s_x/s_y and θ, the turbines in an aligned wind farms should be placed further apart or closer together than using the full wake coverage limit layout to get the optimal normalized power per unit cost. A larger geometrical mean turbine spacing in an aligned wind farm is necessary when the optimal spacing is relatively large using the full wake coverage limit layout. In that case, the full wake coverage limit layout does not benefit much more from placing the turbines even further apart although this can still be beneficial for aligned wind farms for which wake effects are stronger. The geometrical mean turbine spacing in aligned wind farms should be smaller than using
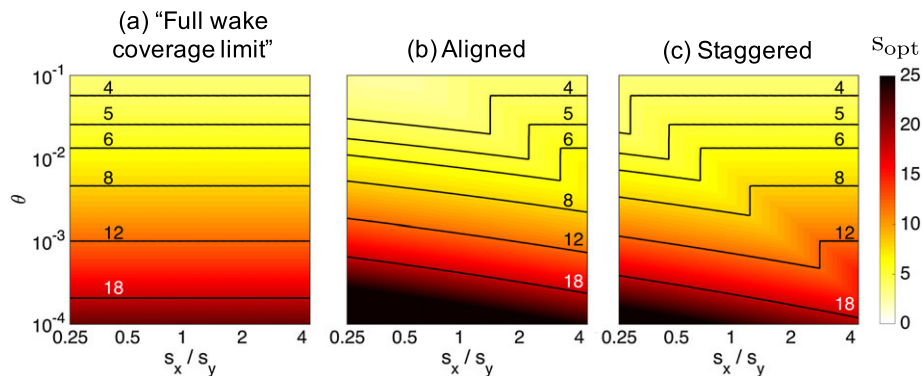
Figure 7. Optimal turbine spacing based on the normalized power per unit cost for the (a) full wake coverage limit, (b) aligned and (c) staggered layout as function of the ratio s_x/s_y and θ.

the full wake coverage limit layout in the regime of the parameter space where the turbines are placed relatively close together and when the spanwise spacing is sufficiently large. In that case, a wind farm using the full wake coverage limit layout will use the available land area more efficiently than an aligned one, which results in a smaller optimal spacing. The same differences compared with the full wake coverage limit layout can be observed for staggered wind farms, although in that case, the effects are less pronounced.

8. CONCLUSIONS

In this work, we have extended the work of Meyers and Meneveau[24] on the determination of the optimal spacing in very large wind farms using analytical models. In addition to the earlier work, which only included the incorporation of the land (quadratic) and turbine costs, we now include a linear cost component (such as roads and cabling), as well as turbulence-affected maintenance costs. Just as in previous work, the results are presented as function of dimensionless cost ratios, which allow one to look up the relevant numbers for a specific case of interest as the relevant parameters vary from site to site. In addition, we now presented an analysis that uses the expected profit per unit area instead of the normalized power per unit cost to find the optimal turbine spacing. Depending on the expected revenue of the turbines, this alters the optimal spacing that is found. We have shown that if wind farms are highly profitable then optimization of the profit per unit area leads to tighter optimal spacings than would be implied by cost minimization. This analysis also allows the determination of the 'profitable region' in the parameter space.

In summary, we find that optimal turbine spacings are approximately 10 turbine diameters or higher if cost per unit of energy production is minimized or if profit per unit is maximized while accounting for the effect of wake-related turbulence on maintenance costs. The optimal spacing under some reference dimensionless cost parameters can be as much as 15 turbine diameters or greater. This spacing is appreciably more than what is found in actual wind farms layouts, which tend to be on the order of 6–10 turbine diameters. We find that these smaller values are only optimal if wind farms are highly profitable (revenues approximately twice the cost), developable area is limited and wake turbulence effects on maintenance costs are disregarded.

The presented analysis has made a number of simplifying assumptions, such as flat terrain (in the case of onshore developments), uniform spacing, a single wind direction and the assumption of asymptoticly large fully developed wind farm. In the case of the terrain assumption, varying topography can significantly influence spacing. We do not expect that the uniform spacing and wind direction assumptions would change our qualitative conclusion that full consideration of wake effects would imply larger optimal spacings for large wind farms than seen in practice, but future research should consider, for instance, the effect of varying wind directions and magnitudes upon the precise numerical conclusions we have reached.

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APPENDIX

In this Appendix, we present some order of magnitude estimates for the dimensionless cost parameters to give an idea of the perhaps more interesting regions within the parameter space. The values of the cost parameters β, θ and ϵ, which we derive thereafter, are summarized in Table I. We emphasize that the dimensionless cost parameters will differ for each specific site/market and are likely to change over time.

We consider a 1.91 MW turbine with a diameter of 96.9 m for onshore and a 4.3 MW turbine with a diameter of 119.4 m for offshore case, see Moné *et al.*,[30] as reference. The capital cost for such a turbine is assumed to be \$1728 kW^{-1} for onshore turbines and \$5187 kW^{-1} for offshore turbines, based on US values reported in Moné *et al.*[30] Based on a ratio of levelized O&M costs per MWh to levelized capital costs per MWh of about 30% for onshore turbines,[30] we obtain onshore capital cost (including fixed O&M costs that do not depend on additional wake turbulence because of spacing effects) of Cost$_{\text{turbine}} \approx \$4,300,000$/turbine. A similar calculation based on a $\sim 22\%$ ratio[30] of levelized O&M to capital costs for offshore turbines results in an offshore turbine cost Cost$_{\text{turbine}} \approx \$27,000,000$/turbine.

Later, we derive our reference values for θ, β and ϵ, which are, respectively, the quadratic and linear cost components, and the turbulence-influenced O&M cost parameter. A summary of some of the main values that are used in the estimates

Table A.I. Summary of values used in the estimate of the reference onshore and offshore dimensionless cost parameters.

	Onshore	Offshore
Diameter	119.4 m	96.9 m
Power	4.3 MW	1.91 MW
Capital costs	$1728 kW^{-1}	$5187 kW^{-1}
Present worth of capital and O&M cost	$2246 kW^{-1}	$6328 kW^{-1}
Capital + O&M costs	$4,300,000$/turbine	$27,000,000$/turbine
Capital costs	$51 MWh^{-1}	$175 MWh^{-1}
Present worth of capital and O&M cost	$66 MWh^{-1}	$213 MWh^{-1}
Quadratic costs	$0.40 m^{-2}	$0.04 m^{-2}
Linear costs	$205 m^{-1}	$1485 m^{-1}
Maintenance costs	$9 MWh^{-1}	$24 MWh^{-1}

The dimensionless cost parameters can be found in Table I.

is given in Table A.I. The estimates indicate that quadratic cost component dominate spacing-related costs for onshore turbines, while linear costs are most important for offshore turbines, see Figure 3(b).

A.1. Onshore parameters

We start with an estimate of the quadratic costs. The cost of lease payments is $8 kW^{-1} year^{-1} based on Table 6 in the work of Moné,[30] which is broadly consistent with earlier estimates of $4–$6 kW^{-1} year^{-1}.[31] To convert this to a present worth cost per turbine, we first use a present worth factor (to convert annual costs to present worth) of 10.37, implied by Table 10 in the work of Moné[30] or $\sim$ $80 kW^{-1} or $\sim$ $300,000$/turbine. Not all of this cost will increase if turbine spacing increases, because the turbine foundation pad size is constant. So as a rough approximation, we lower this by 20% to $240,000. We convert this into $ m^{-2} by assuming that these leasing costs reflect current practice of $s \sim 8$ diameters spacing between turbines, yielding $0.40 m^{-2} for the present worth of land costs. This implies $\theta \approx 0.001$.

The estimate of β should account for all costs that are anticipated to be linearly related to turbine spacing, such as cabling, resistance losses in those cables and roads. Based on European experience, the cost of onshore farm cabling includes approximately $25 m^{-1} for trenching, $30 m^{-1} for cable ($10 m^{-1} per electrical phase) and $5 m^{-1} for cable installation for three phases or a total of $60 m^{-1}. Adding 20% for engineering overhead brings this value to approximately $70 m^{-1}. The length of cable needed between turbines might be appreciably longer than the inter-turbine distance if redundancy is built into the cable layout, but to be conservative, we assume a radial network with no such redundancy. However, we also need to include the expense of roads as well as electrical resistance losses. For instance, Windustry[32] says that $\sim$ $80 m^{-1} should be budgeted for roads in wind farms, which is about the same as the aforementioned cost of cable. Meanwhile, resistance losses occurring in farm cabling are on the order of 1%.[33] Assuming that this applies to a $s = 8$ farm, this implies a loss per metre of 25 W m^{-1} per turbine for a 1.91 MW turbine. Multiplied by the present worth of the onshore turbines cost, this implies an opportunity cost of losses per turbine of $\sim$ $55 m^{-1}. Altogether, the linear costs for our reference onshore turbines amount to roughly $205 m^{-1}, which gives $\beta \sim 0.005$.

Turning to ϵ, total O&M costs include some items that are not affected by wake-induced turbulence, such as property taxes, utilities and insurance. These are included in the Cost of turbine. Moné *et al.*[30] describe a subset of O&M costs (which they call 'MAIN') that are unplanned maintenance and other costs that may vary throughout the project life and depend on output. We assume that only that subset is affected by turbulence, which for onshore turbines is estimated to be $9 MWh^{-1} (levelized). When divided by the levelized capital and maintenance cost, this is about 0.14. Because some of this cost is related to the amount of MWh produced rather than turbulence, we suggest the use of a smaller value of 0.1 for ϵ.

A.2. Offshore parameters

In Maryland, area-related lease payments for offshore wind sites are over $100/acre.[34] In addition to state payments, there is an annual payment to the Federal government of $3 per acre per year[35] or about $30/acre in present worth terms. Thus, an approximate cost of offshore leasing in the USA is perhaps $150/acre or $\sim$ 0.04$ m^{-2}. So we obtain $\theta \approx 2 \times 10^{-5}$.

Turning to the linear costs, our calculation for offshore farms is based on Green *et al.*[36] That study quantifies the costs of cables and losses for offshore farms, and although the authors do not explicitly quantify how spacing affects those costs, we can make some reasonable inferences. The analysis reports a cost of about $240–$314 kW^{-1} for collector cables, based on 830 m of cable length between 3 MW turbines that are 630 m apart (about 6D). Using a value of $300 kW^{-1}, this is $(300/6328) = \sim 5\%$ of the capital cost of an offshore wind turbine. This appears high relative to Tables 4 and 5 in the work of Irena,[37] which report that about $\sim$ 2% of the capital cost of an offshore wind farms is the cost of cable. Therefore, we

recommend a value of $\sim$ \$200 kW^{-1} for cabling between turbines. We convert this value into a \$ m^{-1} value by dividing by the assumed 830 m distance to get \$0.24 kW^{-1} m^{-1} or about \$1030 m^{-1}, which gives $\beta \approx 0.005$ for our reference offshore turbine.

In the case of electrical losses, Green *et al.*[36] do a variety of analyses showing how losses depend on cable size and loading. A typical analysis shows that 12 turbines arrayed on a network with 630 mm^2 (cross-sectional area) cable would yield losses of 55 W m^{-1}. For a 630 m spacing that they assume between turbines (which yields 830 m of cabling according to their analysis, about 6D), this yields $\sim$ 50 kW resistance loss between adjacent turbines. For their 500 MW wind farm, with 167 turbines, that would be about an 8 MW loss, which is consistent with their total loss calculations of about 8–14 MW for a variety of cable designs and farm layouts. We can convert this loss cost into \$ m^{-1} as follows. We multiply 55 W m^{-1} by the ratio 830/630 and obtain $\sim$ 72 W m^{-1} of loss per metre distance between turbines. This loss, when multiplied by our estimated present worth offshore turbine capital and O&M cost, implies an economic cost of losses of \$455 m^{-1} thus an additional loss component of $\beta \approx 0.002$. Adding the cable cost and loss components together yields an order of magnitude estimate of $\beta \approx 0.01$ for offshore turbines.

Turning to value of ϵ for offshore turbines, we use the same procedure as for onshore turbines. We divide the levelized relevant portion of the O&M cost by the levelized capital cost of offshore wind energy. Moné *et al.*[30] (Table 17) suggest a maintenance cost of \$24 MWh^{-1}, which when divided by the present worth of capital and O&M cost of yields $\epsilon = 0.11$. However, because this will include some fixed components that would not depend on turbulence, we suggest an order of magnitude estimate of $\epsilon = 0.1$.

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