Illiquid Financial Markets and Monetary Policy*

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Abstract

This paper analyzes the role of money in asset markets characterized by search frictions. We develop a dynamic framework that brings together a model for illiquid financial assets *à la* Duffie, Gârleanu, and Pedersen, and a search-theoretic model of monetary exchange *à la* Lagos and Wright. The presence of decentralized financial markets generates an essential role for money, which helps investors re-balance their portfolios. We provide conditions that guarantee the existence of a monetary equilibrium. In this case, asset prices are always above their *fundamental value*, and this differential represents a *liquidity premium*. We are able to derive an asset pricing theory that delivers an explicit connection between monetary policy, asset prices, and welfare. We obtain a negative relationship between inflation and equilibrium asset prices. This key result stems from the complementarity between money and assets in our framework.

JEL classification: E44, E52, G11, G12.

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1 Introduction

Over the last two decades secondary financial markets have developed considerably, both in size and complexity. New financial products are generated through processes of securitization by which financial assets are derived from the value of an underlying asset. In their seminal paper, Duffie, Gârleanu, and Pedersen (2005) document that many of these financial products are traded in markets that are characterized by search frictions. Moreover, since the pioneering work of Kiyotaki and Wright (1989) search models have been used in order to study the role of monetary exchange in economies where trade is not centralized through some perfect and frictionless market. This paper attempts to bring these two strands of the literature together, by studying the role that money can play, through liquidity provision, in frictional financial markets.

Duffie et al. analyze over-the-counter markets where a financial asset trades without the explicit use of a liquid asset like fiat money. On the other hand, the literature stemming from Kiyotaki and Wright (1989) considers models where agents trade consumption goods for the liquid asset in decentralized frictional markets. Our objective is to bring the framework of Duffie et al into a dynamic monetary model, *à la* Lagos and Wright (2005), where financial markets with frictions may generate a role for money.[1] Within this framework, we address the following questions: Do equilibria in which fiat money (an intrinsically useless, unbacked asset) is valued exist? What is the effect of introducing money on welfare? Finally, how does monetary policy affect the volume of asset transactions and asset prices?

We develop a model in which agents can purchase a risk-free asset in a perfectly competitive market. The most interesting economic decision made by agents is whether to hold onto this *underlying* asset and receive the certain return or convert (part of) it into a *derivative* asset or *security*, which has a risky return and can only be traded in a frictional market. Following Duffie et al, we assume that the return on the derivative asset is idiosyncratic: the same asset has a higher return in the hands of certain agents (high-types) than in the hands of others (low-types). Hence, gains from trading these assets arise in the same spirit as Berentsen and Rocheteau (2003). Since these instruments are traded in frictional, non-Walrasian markets, there is a potential role for a liquid asset that can serve as a medium of exchange and facilitate the allocation of securities into the hands of the agents with higher valuation.

Like in Duffie et al, in our model, the driving force of the main results is the different valuation of assets among various agents and the consequent gains from trading these assets. However, their paper focuses on how intermediation and asset prices are affected by search and bargaining and not on the details of the exchange process. In

[1] It is well known that in models that feature Arrow-Debreu type of markets, it is very hard to support monetary equilibria other than those where agents are "forced" to hold money, e.g. money in the utility function or cash-in-advance models.

their model, whenever a low-type and a high-type agent meet, the low-type hands all her asset holdings to the high-type, who pays the low-type with some consumption good.[2] In our paper, the high-types can only acquire the risky securities by paying the low-types with another asset, namely fiat money, which is valued precisely for these liquidity services.

If trade of securities among the different types was not possible, agents would undertake the risky investment, only if its expected return exceeded the (safe) return of holding onto the underlying asset.[3] However, as long as high-types value the return on the derivative asset more than the safe yield, a benevolent Social Planner would choose to transform all the underlying asset into the risky investment, and *ex post* allocate the risky assets into the hands of high-types. In the absence of liquidity, the decentralized market cannot achieve this allocation as an equilibrium outcome. Money provides agents with liquidity and allows the economy to take full advantage of the benefits associated with the risky investment, thus reaching the first best.

Unlike the majority of search-theoretic models of monetary exchange, where money helps allocate *commodities* into the hands of agents with a higher valuation, in our model, money helps allocate *assets* into the hands of agents with a higher valuation.[4] We consider this set-up extremely interesting, since it allows us to study the welfare improving role of money in frictional financial markets, and to link monetary policy with asset prices. Since money helps agents re-balance their portfolios after the realization of the idiosyncratic shock, its introduction reduces the risk associated with investing in the derivative asset and induces agents to hold more securities. This is also reflected in the price of the underlying asset, which is higher than its so-called *fundamental value*, i.e. the discounted stream of dividends, in all monetary equilibria.

In monetary equilibrium, agents wish to hold money and the underlying asset concurrently. More precisely, in order to take advantage of the benefits associated with the risky investment (which is backed by the underlying asset), agents need to have sufficient liquidity (which is provided by fiat currency). Hence, money and the risk-free asset are complements. Consequently, inflationary monetary policy increases the cost of holding money, which decreases the demand and, hence, the equilibrium

[2] Agents are assumed to have access to a storage technology which replicates a risk-free bank account with interest rate equal to the rate of time preference. Hence, the bank account is interpreted as a liquid asset that can be traded instantly.

[3] Agents learn their valuation types only after they have transformed (part of) the safe asset into the risky investment. Since here agents are risk neutral, comparing the expected returns of various investment plans is sufficient in order to characterize the agents' optimal choices. However, all the results in this paper would go through in a model with risk averse agents.

[4] For example, in Kiyotaki and Wright (1993), it is assumed that agents can produce a good which they cannot consume (i.e. they have zero valuation for it). An agent who just produced such a good (seller) searches for an agent who gets positive utility by consuming that good (buyer). In any meeting between a buyer and a seller, the good is transferred to the agent with the higher valuation (the buyer) in exchange for cash.

price of the underlying asset. We prove that there exists a unique critical level of inflation above which any monetary equilibrium collapses. This level is shown to be strictly positive. Therefore, money improves welfare even under inflationary policies.

The negative relationship between inflation and asset prices is in contrast with findings in related literature, such as Geromichalos, Licari, and Suarez-Lledo (2007), Lester, Postlewaite, and Wright (2008), and Jacquet and Tan (2010). In these papers, agents meet in decentralized markets and purchase a special good using either money or claims to a *Lucas tree*. Since the two assets compete as media of exchange, they are substitutes.[5] An increase in inflation induces agents to move out of cash and into the alternative asset, leading to an increase in the price of the asset. In our framework, inflation has the opposite effect on asset prices, due to the complementarity of currency and the underlying asset. Our result is similar in spirit to Ferraris (2010), who considers a model with cash and credit. The author models credit as delayed cash payments and shows that inflation can harm credit more than it harms money.

The present paper is related to a growing literature of search-theotretic models of money, which introduce additional assets and study the effect of monetary policy on their equilibrium prices. Lagos and Rocheteau (2008) consider a model where money and capital compete as media of exchange. The authors show that there exist equilibria in which agents overaccumulate productive assets to use as media of exchange, and they discuss optimal monetary policy. Lagos (2010) presents a model where agents can use money and equity shares as means of payments and discusses the implications of changes in the nominal interest rate on equity prices, equity returns, and output. Finally, Ferraris and Watanabe (2011) introduce capital that can serve as collateral for monetary loans. They study the effects of fluctuations in the liquidation value of the collateral on the economy's allocation and its interaction with monetary policy.

Lastly, our paper is related to a number of papers which build upon Duffie et al in order to study financial markets with search frictions. Lagos and Rocheteau (2009) extend the framework of Duffie at al by allowing for unrestricted asset holdings. One of the novelties of their paper, is that market participants can accommodate trading frictions by adjusting their asset positions. Lagos, Rocheteau, and Weill (2011) consider crises and recoveries in over-the-counter markets. The authors study the efficiency of dealers liquidity provision and the desirability of policy intervention in these types of markets during crises. Although these papers consider financial markets which are very similar to ours, they do not model explicitly the welfare improving role that money can play in these markets through liquidity provision.

The rest of the paper is organized as follows. In section 2, we describe the physical environment. In Section 3, we discuss the optimal behavior of agents. In Section 4,

[5] In recent work, Geromichalos and Herrenbrueck (2016) consider an environment where assets cannot be used directly as means of payment but agents can sell them for cash in an Over-the-Counter secondary asset market. Hence, in this environment too, assets are effectively substitutes to money.

we define a monetary equilibrium, and we lay out the main results of the paper, most prominently, the relationship between asset prices and monetary policy. Section 5 contains the main conclusions.

2 The Model

The environment that we analyze takes the framework presented in Lagos and Wright (2005) as a starting point. Time is discrete, and goes on forever, $t \in \mathbb{N}$. There exists a unit measure of agents who live forever and discount future at rate $\beta \in (0,1)$. There are three assets in the model: a real asset that yields a deterministic return, R, at the end of every period, a risky financial asset to be discussed below in detail, and an intrinsically worthless object that we call money. Sometimes we refer to the risk-free asset as the underlying asset and to the risky security as the derivative asset. The supply of the real asset is fixed at A. Money supply is controlled by a monetary authority, and it evolves according to $M_{t+1} = (1+\mu)M_t$. Every period, t, is divided into three subperiods, in which agents engage in different economic activities.

In the first subperiod, agents with a certain amount of money holdings, m_t, and real asset holdings, a_t, decide how much of this asset to transform into a risky security, $s_t \in [0, a_t]$, thus giving up any claim on the safe return by the issued amount. Instead, investing in this new financial product will yield a high return, $y_{_H}$, or a low return, $y_{_L}$, at the end of the period with equal probability.[6] We assume that $0 \leq y_{_L} < y_{_H}$ and $y_{_H} > R$. These returns are idiosyncratic, in the sense that the same security yields $y_{_H}$ in the hands of a high-type and $y_{_L}$ in the hands of a low-type. Duffie et al provide a number of reasons that justify this assumption. For example, a low-type investor may have low liquidity (a need for cash), high financing costs, hedging reasons to sell, a relative tax disadvantage, or just a lower personal use of the asset. The idiosyncratic uncertainty is resolved after the investment decisions of agents have been made. The risky returns are i.i.d. distributed across periods and agents.

A few comments regarding the nature of the risky investment are in order. First, since $y_{_H} > R$, social optimality requires the transformation of the risk-free asset into the derivative asset, as long as the securities can find their way into the hands of the agents with high valuation, and the low-type agents are compensated for at least y_L per unit of asset. Second, we model the risky asset as a one-period security: after paying the yield $y_{_i}$ to an agent of type-i, $i = L, H$, the s_t units of securities held by the agent are physically identical to the $a_t - s_t$ units of the asset that were not transformed into the risky investment (but the agent does not receive the yield R on the s_t units). In this sense, the derivative asset is *backed* by the underlying asset.

[6] Clearly, all the results of the paper would go through under more complicated specifications of the random variable y, e.g. more than two states, uneven probabilities of state realization etc.

In the second subperiod, having learned their types, agents enter a *frictional financial market* (FFM). In this market, agents can re-balance their holdings of the risky asset (if they chose $s_t > 0$ in the first subperiod). Gains from trade are generated *à la* Berentsen and Rocheteau (2003) from low-return agents wanting to sell their securities to high-return agents in exchange for currency. We model the FFM as a market where all agents are price-takers.[7] Despite this competitive feature, anonymity and a double coincidence problem (due to the difference in assets' valuations), make the use of a medium of exchange essential. This point is also highlighted by Rocheteau and Wright (2005) and Temzelides and Yu (2003). Agents get access to the FFM with probability $\lambda \in (0, 1]$, which captures market frictions. Moreover, agents' types are public information.

In the third subperiod, agents enter a frictionless *centralized market* (CM) with their new holdings of the risky asset, the safe asset, and money, which depend on their actions in the previous two subperiods. At this stage, agents can acquire any quantity of money, m_{t+1}, and the real asset, a_{t+1}, for the next period, at prices ϕ_t and ψ_t, respectively. They also derive net utility, $U(X_t) - H_t$, from consuming X_t units of a general good and supplying H_t units of labor. We assume that agents have access to a technology that can transform one unit of labor into one unit of the general good. Notice that this is the only market in which agents consume and work. The function $U(X_t)$ is assumed to be twice continuously differentiable with $U' > 0$, $U'' \leq 0$. Finally, we assume that there exists X^* such that $U'(X^*) = 1$, with $U'(X^*) > X^*$.

3 Optimal Behavior

In this section, we analyze the optimal choices of agents, which will lead us to the discussion of equilibrium in the next section. We study agents' behavior in each subperiod separately. Since all value functions admit a recursive representation, we drop the time subscripts from now on whenever it does not lead to confusion.

[7] Rocheteau and Wright (2005) consider a money-search model where the terms of trade in the decentralized market are determined under three alternative specifications: bargaining, price taking, and price posting. The authors show that equilibrium and the effects of policy depend on market structure. However, in that paper, entry of sellers is modeled explicitly, and the objective is to study the effects of inflation on the extensive and intensive margin of decentralized trade. In our framework, the objective is to highlight the role of money in the FFM and to qualitatively link asset prices to monetary policy. Hence, the modeling choice of the market structure in the FFM does not affect the spirit of our exercise. We adopt price taking because it is the most simple out of these three specifications. Other papers that follow this approach include Berentsen, Camera, and Waller (2007), Ferraris and Watanabe (2010), and Ferraris and Watanabe (2011).

3.1 Centralized Market

For analytical purposes, it is convenient to solve the model backwards, starting from the third subperiod. For an agent of type $j = L, H$, the value function of entering the CM with money holdings m, real asset holdings b, and risky securities s, is

$$\begin{aligned} V^{3j}(m,b,s) &= \max_{\hat{m},\hat{a},X,H} \left\{ U(X) - H + \beta V^1(\hat{m},\hat{a}) \right\} \\ s.t.\ \phi\hat{m} + \psi\hat{a} + X &= H + \phi m + (\psi + R)b + (\psi + y_j)s, \end{aligned}$$

where $b = a - s$, and a represents the amount of the real asset chosen in the previous period (i.e. the last period's third subperiod). Variables with "hats" represent next period's choices. Notice that the value function in the third subperiod depends on the agent's type, while the value function in the first subperiod, V^1, does not. This follows from the fact that uncertainty has not been resolved in the first period and that all agents are ex ante identical.

Three observations are immediate regarding V^{3j}. First, in every period $X = X^*$, and we can write

$$\begin{aligned} V^{3j}(m,b,s) &= U(X^*) - X^* + \phi m + (\psi + R)b \\ &+ (\psi + y_j)s + \max_{\hat{m},\hat{a}} \left\{ -\phi\hat{m} - \psi\hat{a} + \beta V^1(\hat{m},\hat{a}) \right\}. \end{aligned} \tag{1}$$

Second, V^{3j} is linear in all its arguments,

$$V^{3j}(m,b,s) = \Lambda + \phi m + (\psi + R)b + (\psi + y_j)s, \tag{2}$$

where the definition of Λ is obvious. Third, it is easy to see from (1) that there are not any wealth effects: the agent's choices of $\hat{m}, \hat{a}$ do not depend on today's state variables m, b, s. This is a consequence of quasi-linearity of preferences.

3.2 Frictional Financial Market

Consider now the second subperiod. When agents enter the FFM, they know what return they will obtain on their investment in the risky asset, and they are allowed to re-balance their positions on this investment by trading securities. Since $y_L < y_H$, low-types will naturally arise as the sellers, while high-types will become the buyers. We assume that the FFM is a competitive market where all agents are price-takers. However, anonymity and imperfect credit make the use of a medium of exchange necessary.[8]

[8] As Rocheteau and Wright (2005) point out, price taking can be regarded as the monetary economics' analogue to the Lucas Jr and Prescott (1974) search model.

Let p be the dollar price of one unit of the risky securities. Conditional on being a low-type (a seller), an agent with state variables (m, b, s) solves the following problem

$$\max_{q_s \leq s} \quad V^{3L}\left(m + pq_s, b, s - q_s\right)$$

or alternatively

$$\max_{q_s \leq s} \quad \left\{\Lambda + \phi m + (\psi + R)b + (\psi + y_{_L})s + (\phi p - \psi - y_{_L})q_s\right\},$$

where q_s is the supply of securities. The seller's optimal behavior yields the individual supply function

$$q_s^* = \begin{cases} 0, \ if \ p < \frac{\psi + y_{_L}}{\phi}, \\ \in [0, s], if \ p = \frac{\psi + y_{_L}}{\phi}, \\ s, \ if \ p > \frac{\psi + y_{_L}}{\phi}. \end{cases}$$

The supply function of a low-type depends crucially on whether the FFM price of one unit of the securities, p, exceeds their value in the upcoming CM, namely $\psi + y_{_L}$, adjusted for a term that captures inflation. This is not surprising, given that low-types get paid in cash, and a low value of money (low ϕ) implies a low purchasing power in the third subperiod. The higher the ϕ, the more likely a low-type is to sell her assets. Finally, notice that the supply function is affected only by the agent's s holdings.

Next, consider the problem of a high-type with with state variables (m, b, s). This agent solves

$$\max_{q_b \leq m/p} \quad V^{3H}\left(m - pq_b, b, s + q_b\right)$$

or alternatively

$$\max_{q_b \leq m/p} \quad \left\{\Lambda + \phi m + (\psi + R)b + (\psi + y_{_H})s + (\psi + y_{_H} - \phi p)q_b\right\},$$

where q_b is the demand of securities. The demand function is given by

$$q_b^* = \begin{cases} 0, \ if \ p > \frac{\psi + y_{_H}}{\phi}, \\ \in [0, m/p], if \ p = \frac{\psi + y_{_H}}{\phi}, \\ m/p, \ if \ p < \frac{\psi + y_{_H}}{\phi}. \end{cases}$$

The demand for securities admits a similar interpretation as the supply, and it is affected only by the buyer's money holdings.

Figure 1 depicts the aggregate demand and supply curves. The equilibrium price

and quantity depend on the shape of the demand curve. In particular, if $\phi m < (\psi + y_L)s$ (represented by D_1 in Figure 1), we have $Q^* = (\lambda/2)(\phi m)/(\psi + y_L)$. If $\phi m \geq (\psi + y_L)s$ (represented by D_2 in Figure 1), we have $Q^* = \lambda s/2$. Therefore, $Q^* = (\lambda/2)\min\{s, (\phi m)/(\psi + y_L)\}$. The equilibrium quantity of securities that a representative high-type agent acquires in the FFM can be summarized as

$$q^* = \min\left\{s, \frac{\phi m}{\psi + y_L}\right\}, \tag{3}$$

and the prevailing price in the FFM is given by

$$p^* = \begin{cases} \frac{\psi + y_L}{\phi}, \text{ if } \phi m < (\psi + y_L)s, \\ \frac{m}{s}, \text{if } \phi m \in [(\psi + y_L)s, (\psi + y_H)s], \\ \frac{\psi + y_H}{\phi}, \text{ if } \phi m > (\psi + y_H)s. \end{cases} \tag{4}$$

The equilibrium in this market depends on real money balances, ϕm. Portfolio re-balancing generates a role for money and takes place at the equilibrium price, p^*, which depends on inflation through ϕ. Moreover, it is key to notice from equation (3), that trade in this market exists, $q^* > 0$, if and only if $\phi m > 0$. Therefore, the relationship between monetary equilibrium and financial trade is one-to-one. Let us emphasize on the fact that equilibrium in the FFM is described taking ϕ and ψ as given. However, in Section 4, these objects are treated as endogenous variables that will also be determined in equilibrium.

3.3 First Subperiod and Optimal Choice of s

At the beginning of every period, agents with money and real asset holdings m, a, choose what part of a to invest in the risky asset, in order to maximize their continuation value into the FFM. Recall that this choice is made before agents know their types. The first subperiod value function for the typical agent is given by[9]

$$V^1(m, a) = \max_{s \in [0,a]} \left\{ \frac{1}{2}\left[V^{2L}(m, b, s) + V^{2H}(m, b, s)\right] \right\}. \tag{5}$$

In order to examine the optimal choice of s for the agent, we need to replace the value functions V^{2j} with more useful expressions. To that end notice that

$$\begin{aligned} V^{2L}(m, b, s) &= \lambda V^{3L}(m + pq_s, b, s - q_s) + (1 - \lambda)V^{3L}(m, b, s), \\ V^{2H}(m, b, s) &= \lambda V^{3H}(m - pq_s, b, s + q_s) + (1 - \lambda)V^{3H}(m, b, s). \end{aligned}$$

[9] Since we have defined $b = a - s$, it does not matter whether the agent chooses s or b.

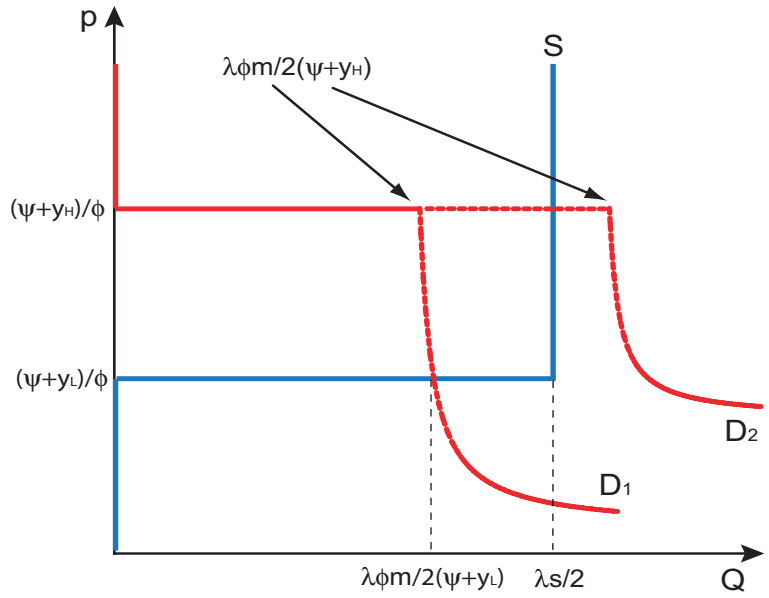

Figure 1: Equilibrium in the FFM.

Equivalently, using the results derived above regarding the terms of trade in the FFM, we can write

$$\begin{aligned} V^{2L}(m,b,s) &= \lambda V^{3L}(m+p^*q^*,b,s-q^*)+(1-\lambda)V^{3L}(m,b,s), \\ V^{2H}(m,b,s) &= \lambda V^{3H}(m-p^*q^*,b,s+q^*)+(1-\lambda)V^{3H}(m,b,s). \end{aligned}$$

Exploiting equations (2) (linearity of V^{3j}) and (3), implies that

$$\begin{aligned} &\frac{1}{2}\left[V^{2L}(m,b,s)+V^{2H}(m,b,s)\right]=\Lambda+\phi m+(\psi+R)a \\ &+\left[\frac{1}{2}(y_L+y_H)-R\right]s+\frac{\lambda}{2}(y_H-y_L)\min\left\{s,\frac{\phi m}{\psi+y_L}\right\}. \end{aligned} \tag{6}$$

Inserting (6) into (5) allows us to write the agent's objective function as

$$\begin{aligned} &\max_{s\in[0,a]}\left\{\left[\tfrac{1}{2}(y_L+y_H)-R\right]s+\tfrac{\lambda}{2}(y_H-y_L)\min\left\{s,\tfrac{\phi m}{\psi+y_L}\right\}\right\} \\ &\qquad=\max_{s\in[0,a]}\left\{\alpha_1 s+\alpha_2\min\{s,\alpha_3\}\right\}, \end{aligned}$$

where, for notational convenience, we have defined

$$\alpha_1 \equiv (y_L+y_H)/2-R,$$

$$\begin{aligned}\alpha_2 &\equiv (\lambda/2)\,(y_{_H} - y_{_L})\,,\\ \alpha_3 &\equiv \phi m/(\psi + y_{_L}).\end{aligned}$$

Notice that α_1, α_2 are constants (parameters). This is not true for α_3, which depends on the state variable m.

The agent's objective function is very intuitive. The term α_1 shows that for every unit of a that gets converted into the risky security, the agent forgoes the return R, and gains $y_{_L}$ or $y_{_H}$ with equal probability. The second term in the objective function is the expected gain from trade in the FFM, which is equal to the total number of units of s that trade in the FFM multiplied by the average gain from this transaction. For every unit of s that goes from the hands of a low-type to those of a high-type, a surplus equal to $y_{_H} - y_{_L}$ is generated.

Consider now the optimal choice of s, namely, s^*. To keep things interesting, we focus on the case where the expected return from the risky investment is less than the return on the safe asset, i.e. $\alpha_1 \leq 0$. Under this assumption, if agents are not able to trade in the FFM, they set $s^* = 0$.[10] In general, the key determinant of s^* is the term $\alpha_1 + \alpha_2 = [(1+\lambda)y_{_H} + (1-\lambda)y_{_L}]/2 - R$. When the frictions in the FFM are moderate, i.e. λ is big, more weight is put on $y_{_H}$ making $\alpha_1 + \alpha_2$ large. As $\lambda \to 0$, the gain from holding s coincides with the net expected return of the risky asset, which is assumed to be non-positive. If $\lambda = 1$, $\alpha_1 + \alpha_2 = y_{_H} - R > 0$. The following Lemma summarizes the optimal investment policy.

Lemma 1. *For a given state (m, a), the optimal choice of $s \in [0, a]$ is given by*

$$s^* = \begin{cases} 0, \textit{ if } \alpha_1 + \alpha_2 < 0, \\ \in [0, \min\{\alpha_3, a\}]\,, \textit{ if } \alpha_1 + \alpha_2 = 0, \\ \min\{\alpha_3, a\}, \textit{ if } \alpha_1 + \alpha_2 > 0 \textit{ and } \alpha_1 < 0, \\ \in [\min\{\alpha_3, a\}, a]\,, \textit{ if } \alpha_1 + \alpha_2 > 0 \textit{ and } \alpha_1 = 0. \end{cases} \tag{7}$$

Proof. The result follows immediately from inspection of the objective function. □

3.4 Optimal Choice of $\hat{m}, \hat{a}$

Having characterized the optimal choice of s, we can now analyze the optimal choice of $\hat{m}, \hat{a}$. Before we proceed with this task, we state a lemma that will be crucial for the forthcoming analysis. The lemma highlights the fact that the net gain of carrying assets across periods is non-positive. This implies that agents will only be willing

[10] Alternatively, the assumption that $\alpha_1 \leq 0$ shows the robustness of our results: if agents choose $s^* > 0$ when the net return (excluding the potential gains in the FFM) of the risky asset is non-positive, then they will definitely do so if $\alpha_1 > 0$.

to carry money and the real asset, if there are positive gains from re-balancing their investment positions in the FFM.

Lemma 2. *In any equilibrium*

$$\begin{aligned} \psi &\geq \beta\left(R+\hat{\psi}\right), \\ \phi &\geq \beta\hat{\phi}. \end{aligned}$$

Proof. A formal proof can be found in Geromichalos, Licari, and Suarez-Lledo (2007). Intuitively, if $\psi < \beta(\hat{\psi}+R)$, agents have an infinite demand for the asset, and equilibrium is not well defined. A similar argument applies for the price of money, ϕ. □

We now proceed to the characterization of the optimal choices of $\hat{m}, \hat{a}$ by examining the value function V^{3j}. This choice will depend crucially on the parameter values. Once again, the key term is $\alpha_1+\alpha_2$, because the sign of this term determines whether the agent chooses some $s^* > 0$. We shall see that when $s^* = 0$, there is no welfare improving role for money in the model.

Case 1: $\alpha_1 + \alpha_2 < 0$. Under these parameter values, the agent will choose to not invest anything in the risky asset in the next period, i.e. $\hat{s}^* = 0$, which implies $q^* = 0$ by (3). Rewrite equation (1) replacing V^1 with the expression given by (5). Notice that, since $\hat{s}^* = 0$, we have $\hat{b} = \hat{a}$. Also, the money holdings in the next period of an agent that chooses $\hat{m}$ in the current period's CM are given by $\hat{m} + \mu M$. These facts allow us to write the third subperiod value function as

$$\begin{aligned} V^{3j}(m,b,s) &= U(X^*) - X^* + \phi m + (\psi+R)b + (\psi+y_{_j})s \\ &+ \max_{\hat{m},\hat{a}}\left\{-\phi\hat{m} - \psi\hat{a} + \frac{\beta}{2}\left[V^{2L}(\hat{m}+\mu M,\hat{a},0) + V^{2H}(\hat{m}+\mu M,\hat{a},0)\right]\right\}. \end{aligned}$$

One can use (6) in order to substitute for the expressions V^{2L} and V^{2H} in the last equation. This implies that

$$\begin{aligned} V^{3j}(m,b,s) &= \Omega_1 + \max_{\hat{m},\hat{a}}\Big\{-\phi\hat{m} - \psi\hat{a} + \beta\Big[\hat{\Lambda} + \hat{\phi}(\hat{m}+\mu M) + (R+\hat{\psi})\hat{a} \\ &\quad +(\lambda/2)\left(y_{_H} - y_{_L}\right)\min\left\{0, \hat{\phi}(\hat{m}+\mu M)/(\hat{\psi}+y_{_L})\right\}\Big]\Big\} \\ &= \Omega_2 + \max_{\hat{m},\hat{a}}\left\{-\left(\phi - \beta\hat{\phi}\right)\hat{m} - \left[\psi - \beta\left(R+\hat{\psi}\right)\right]\hat{a}\right\}. \end{aligned} \tag{8}$$

The terms Ω_1, Ω_2, and $\hat{\Lambda}$ do not depend on $\hat{m}, \hat{a}$, and their definitions are obvious.[11]

We are now ready to describe the optimal choices of $\hat{m}, \hat{a}$ in Case 1. The following lemma provides the details.

[11] In particular, $\hat{\Lambda}$ is the term that we get if we lead Λ, defined in (2), by one period. Since Λ is independent of m, a, $\hat{\Lambda}$ is independent of $\hat{m}, \hat{a}$.

Lemma 3. *The optimal choices of $\hat{m}, \hat{a}$ satisfy*

$$\begin{aligned} \hat{m}^* &= \begin{cases} 0, \text{ if } \phi > \beta\hat{\phi}, \\ \in \mathbb{R}_+, \text{if } \phi = \beta\hat{\phi}, \end{cases} \\ \hat{a}^* &= \begin{cases} 0, \text{ if } \psi > \beta(\hat{\psi} + R), \\ \in \mathbb{R}_+, \text{if } \psi = \beta(\hat{\psi} + R). \end{cases} \end{aligned} \tag{9}$$

Proof. The proof follows immediately from inspection of (8) and Lemma 2. □

Case 2: $\alpha_1+\alpha_2 \geq 0$. Here $\hat{s}^* = \min\{\hat{a}, \hat{\alpha}_3\}$, where $\hat{\alpha}_3 = \hat{\phi}(\hat{m} + \mu M) / \left(\hat{\psi} + y_{_L}\right)$.[12] Following the same strategy as above, we can write

$$\begin{aligned} V^{3j}(m,b,s) = U(X^*) - X^* + \phi m + (\psi + R)b + (\psi + y_{_j})s \\ + \max_{\hat{m},\hat{a}} \Big\{ -\phi\hat{m} - \psi\hat{a} + \frac{\beta}{2}\left[V^{2L}(\hat{m} + \mu M, \hat{a} - \min\{\hat{a}, \hat{\alpha}_3\}, \min\{\hat{a}, \hat{\alpha}_3\}) \right. \\ \left. + V^{2H}(\hat{m} + \mu M, \hat{a} - \min\{\hat{a}, \hat{\alpha}_3\}, \min\{\hat{a}, \hat{\alpha}_3\})\right] \Big\}. \end{aligned}$$

Using (6), in order to substitute for the expressions V^{2L} and V^{2H} in the last equation, implies that

$$\begin{aligned} V^{3j}(m,b,s) = \Omega_1 + \max_{\hat{m},\hat{a}} \Big\{ -\phi\hat{m} - \psi\hat{a} + \beta\Big[\hat{\Lambda} + \hat{\phi}\hat{m} + \left(R + \hat{\psi}\right)(\hat{a} - \min\{\hat{a}, \hat{\alpha}_3\}) \\ + \left(\hat{\psi} + \frac{y_{_H} + y_{_L}}{2}\right)\min\{\hat{a}, \hat{\alpha}_3\} + \frac{\lambda}{2}(y_{_H} - y_{_L})\,\hat{q}^*\Big]\Big\}, \end{aligned} \tag{10}$$

where the term Ω_1 was defined previously.

Moreover, from (3), the quantity of s that changes hands in the FFM is given by

$$\begin{aligned} \hat{q}^* &= \min\left\{\hat{s}^*, \frac{\hat{\phi}(\hat{m} + \mu M)}{\hat{\psi} + y_{_L}}\right\} = \min\left\{\min\{\hat{a}, \hat{\alpha}_3\}, \frac{\hat{\phi}(\hat{m} + \mu M)}{\hat{\psi} + y_{_L}}\right\} = \\ &= \min\left\{\min\left\{\hat{a}, \frac{\hat{\phi}(\hat{m} + \mu M)}{\hat{\psi} + y_{_L}}\right\}, \frac{\hat{\phi}(\hat{m} + \mu M)}{\hat{\psi} + y_{_L}}\right\} = \min\left\{\hat{a}, \frac{\hat{\phi}(\hat{m} + \mu M)}{\hat{\psi} + y_{_L}}\right\}. \end{aligned}$$

Using this fact in (10), one can conclude that

$$V^{3j}(m,b,s) = \Omega_1 + \max_{\hat{m},\hat{a}} \Big\{ -\left(\phi - \beta\hat{\phi}\right)\hat{m} - \left[\psi - \beta\left(R + \hat{\psi}\right)\right]\hat{a}$$

[12] From Lemma 1, it follows that $\hat{s}^* = \min\{\hat{a}, \hat{\alpha}_3\}$ is always an optimal choice in the forthcoming period, although not the only one. This choice is the unique optimal, only if $\alpha_1 + \alpha_2 > 0$ and $\alpha_1 < 0$. Nevertheless, plugging $\hat{s}^* = \min\{\hat{a}, \hat{\alpha}_3\}$ into the value function V^{3j} is always without loss of generality.

$$+(\beta/2)\left[(1+\lambda)y_{_H}+(1-\lambda)y_{_L}-2R\right]\min\left\{\hat{a},\hat{\phi}\left(\hat{m}+\mu M\right)/(\hat{\psi}+y_{_L})\right\}\Big\}$$
$$=\Omega_1+\max_{\hat{m},\hat{a}}\left\{-\gamma_1\hat{m}-\gamma_2\hat{a}+\gamma_3\min\left\{\hat{a},\gamma_4\hat{m}+\gamma_5\right\}\right\},$$

where, for notational convenience, we have defined

$$\begin{aligned}
\gamma_1 &\equiv \phi-\beta\hat{\psi},\\
\gamma_2 &\equiv \psi-\beta(\hat{\psi}+R),\\
\gamma_3 &\equiv (\beta/2)\left[(1+\lambda)y_{_H}+(1-\lambda)y_{_L}-2R\right],\\
\gamma_4 &\equiv \hat{\phi}/(\psi+\hat{y}_{_L}),\\
\gamma_5 &\equiv \hat{\phi}\mu M/(\hat{\psi}+y_{_L}).
\end{aligned}$$

The agent's objective function has a very intuitive interpretation. The terms γ_1,γ_2 represent the cost of carrying money and real assets, respectively. The term $\gamma_3\min\{\hat{a},\gamma_4\hat{m}+\gamma_5\}$ stands for the expected benefit from carrying money and the asset. From Lemma 2, we know that $\gamma_1,\gamma_2\geq 0$. Also, $\gamma_3\geq 0$, $\gamma_4>0$, and the sign of γ_5 depends on whether the monetary authority is running inflation or deflation, i.e. the sign of μ. The fact that $\hat{m}$ and $\hat{a}$ appear inside a min operator in the benefit term, is related to a point that we have already made in the Introduction: money and the underlying asset are complements. Since the agent chooses $\hat{m},\hat{a}$ before knowing her type, she needs to set $\hat{a}>0$, in order to sell securities (assuming that she chose to convert some of the $\hat{a}$ units into the risky asset) if she turns out to be a low-type. At the same time, she needs to set $\hat{m}>0$, in order to have enough liquidity to buy securities, if she turns out to be a high-type.

The following lemma describes the agent's optimal behavior in Case 2.

Lemma 4. *The optimal choices of $\hat{m},\hat{a}$ satisfy*

$$(\hat{m}^*,\hat{a}^*)=\begin{cases}(0,0),\ \textit{if}\ \gamma_1+\gamma_2\gamma_4>\gamma_3\gamma_4,\\ (+\infty,+\infty),\ \textit{if}\ \gamma_1+\gamma_2\gamma_4<\gamma_3\gamma_4,\\ (z,\gamma_4 z+\gamma_5),\ \textit{for any}\ z\geq-\frac{\gamma_5}{\gamma_4},\ \textit{if}\ \gamma_1+\gamma_2\gamma_4=\gamma_3\gamma_4.\end{cases}\tag{11}$$

Proof. Since the objective function is linear in $\hat{m},\hat{a}$, the optimal solution depends on the magnitude of the various γ terms. If $\gamma_1+\gamma_2\gamma_4<\gamma_3\gamma_4$, the cost of carrying assets is relatively small to the benefit, inducing agents to want to carry unlimited amounts of both objects. The opposite is true if $\gamma_1+\gamma_2\gamma_4>\gamma_3\gamma_4$. In this case, the agent does not wish to carry any assets, and from the complementarity, there is no reason to carry any money either. When $\gamma_1+\gamma_2\gamma_4=\gamma_3\gamma_4$, optimality requires that the arguments inside the min operator, in the benefit term of the objective, should be equal to each other. Hence, any combination of $\hat{m}=z$ and $\hat{a}=\gamma_4 z+\gamma_5$ is optimal. The restriction $z\geq-\gamma_5/\gamma_4$ ensures that the optimal $\hat{a}$ satisfies non-negativity when $\gamma<0$. □

The intuition behind the agent's optimal choice is the following. For any given choice of $\hat{a}$, there is no reason to bring more than $\gamma_4 \hat{m} + \gamma_5$ units of cash, since carrying cash is costly, and this amount provides the agent with all the necessary liquidity. The complementarity between $\hat{m}$ and $\hat{a}$ highlighted above is, once again, crucial for this result. Having described the optimal behavior of agents, we are now ready to proceed to the analysis of equilibrium.

4 Equilibrium

4.1 Characterization of Equilibrium

We begin this section with the definition of equilibrium.

Definition 1. *A monetary equilibrium is a set of value functions V^{ij}, $i = 1, 2, 3$ and $j = L, H$ that satisfy the Bellman equations, a triplet (p^*, q^*, s^*) that satisfy (3), (4), and (7) in every period, and a pair of bounded sequences $\{\phi_t M_t\}_{t=0}^{\infty}$, $\{\psi_t\}_{t=0}^{\infty}$, such that agents behave optimally under the market clearing conditions $a_t = A$ and $m_t = M_t$, for all t.*

We analyze equilibrium separately for the two cases of parameters introduced above.

Case 1: $\alpha_1 + \alpha_2 < 0$. From Lemma 3, a necessary condition for equilibrium is $\psi = \beta(\hat{\psi} + R)$, which implies that the sequence of the asset price follows the difference equation $\psi_{t+1} = -R + (1/\beta)\psi_t$. Since $\beta < 1$, this can only be true if

$$\psi_t = \bar{\psi} \equiv \frac{\beta R}{1 - \beta}.$$

The asset price given by $\bar{\psi}$ is the fundamental value of the asset, i.e. the discounted stream of future dividends. The result, according to which $\psi_t = \bar{\psi}$ for all t, reflects the fact that, in the case under consideration, the asset is only valued for the dividend it yields: agents never hold it in order to invest in the risky asset and possibly trade in the FFM.

The role of money in this economy is not essential. From Lemma 2, agents have a positive demand for money only if $\phi = \beta\hat{\phi}$. In steady state this implies that $\mu = \beta - 1$, i.e. the monetary authority is following the Friedman rule.[13] Agents carry money because the holding cost is zero (equivalently, the nominal interest rate is zero). However, they never get to use that money in the FFM, since $s^* = 0$ and there is nothing to trade money with.

[13] In steady state $\phi M = \hat{\phi}\hat{M} \Rightarrow \phi M = \hat{\phi}(1 + \mu)M$. Therefore, $1 + \mu = \phi/\hat{\phi} = \beta$.

Case 2: $\alpha_1+\alpha_2 \geq 0$. Lemma 4 reveals that a necessary condition for the existence of equilibrium is

$$\gamma_1 + \gamma_2\gamma_4 = \gamma_3\gamma_4 \Leftrightarrow$$
$$\phi - \beta\hat{\phi} + \left[\psi - \beta\left(R + \hat{\psi}\right)\right]\frac{\hat{\phi}}{\hat{\psi}+y_L} = \beta(\alpha_1+\alpha_2)\frac{\hat{\phi}}{\hat{\psi}+y_L}. \tag{12}$$

To simplify equation (12), divide both sides by $\hat{\phi}$ and recall that, at the steady state, $\phi/\hat{\phi} = 1+\mu$. These operations yield

$$(1+\mu-\beta)\left(\hat{\psi}+y_L\right) = -\psi + \beta\left(R+\hat{\psi}\right) + \beta(\alpha_1+\alpha_2).$$

Using the definition of $\alpha_1+\alpha_2$ and solving with respect to $\hat{\psi}$ yields the following asset pricing difference equation

$$\hat{\psi} = \frac{\frac{\beta}{2}\left[(1+\lambda)y_H + (1-\lambda)y_L\right] - (1+\mu-\beta)y_L}{1+\mu-2\beta} - \frac{1}{1+\mu-2\beta}\psi. \tag{13}$$

Here we focus on steady state equilibria, that is, equilibria in which money balances and asset prices are constant in every period. In the appendix, we explore the possibility of some non-stationary equilibria. Let $\psi = \hat{\psi} = \psi^*$. Imposing this restriction in (13) and solving with respect to ψ^* yields

$$\psi^* = \psi^*(\mu) = \frac{\frac{\beta}{2}\left[(1+\lambda)y_H + (1-\lambda)y_L\right] - (1+\mu-\beta)y_L}{2(1-\beta)+\mu}. \tag{14}$$

Also, differentiating the equilibrium asset price with respect to μ yields

$$\frac{\partial\psi^*}{\partial\mu} = -\frac{\frac{\beta}{2}[(1+\lambda)y_H + (1-\lambda)y_L] + (1-\beta)y_L}{[2(1-\beta)+\mu]^2} < 0. \tag{15}$$

We discuss these results in the next sub-section.

4.2 Discussion

Equation (14) represents the steady state equilibrium price of the underlying asset and establishes a clear connection between the asset price and monetary policy. From (15), the price of the asset depends negatively on the growth rate of money. This result is in contrast with the predictions of related papers, like Geromichalos et al (2007), Lester et al (2008), and Jacquet and Tan (2010). In these papers, agents wish to purchase a special good, in a decentralized market, using either money or a real asset. Higher inflation makes carrying money more costly, so people turn to the

relatively cheaper asset, thus increasing its price. Unlike these papers, here money and the asset are complements, in the sense that agents need both objects in order to trade in the FFM. If money is costly to carry, the demand for the asset will decrease and so will its equilibrium price (given fixed supply).

A second important observation that follows from (14) is that the price of the asset is, in general, higher than its fundamental value. The steady state version of Lemma 2 implies that $\psi^* \geq \bar{\psi}$. Hence, our claim will be true as long as we can show that the maximum possible equilibrium asset price is greater than the fundamental value. Since ψ^* is decreasing in μ, it obtains its maximum value when $\mu = \beta - 1$.[14] The resulting expression is given by

$$\psi_{max} = \frac{\beta}{1-\beta}\frac{1}{2}\left[(1+\lambda)y_H + (1-\lambda)y_L\right].$$

Clearly, $\psi_{max} \geq \bar{\psi}$ if and only if $\alpha_1 + \alpha_2 \geq 0$, which is assumed to hold. If $\alpha_1 + \alpha_2 > 0$, then $\psi_{max} > \bar{\psi}$, which verifies our claim.

The asset pricing equation in this model reflects two dimensions. First, the asset is valued for its dividend R and, second, it is valued for its property to back the risky financial security, thus allowing agents to generate additional value by re-balancing their portfolios in the FFM. However, due to the frictional nature of this market, liquidity is absolutely crucial for the second property of the asset to be exploited. Hence, in monetary equilibria, and only in those, $\psi^* > \bar{\psi}$, and the distance $\psi^* - \bar{\psi}$ reflects the premium of the asset over its fundamental value. We refer to this term as the liquidity premium of the asset. The liquidity premium is maximized, and it is equal to $\alpha_1 + \alpha_2$, under the Friedman rule. Finally, it is not hard to show that the liquidity premium of the underlying asset increases as the frictions in the FFM become less severe. Formally,

$$\frac{\partial \psi^*}{\partial \lambda} = \frac{\beta}{2}\frac{y_H - y_L}{2(1-\beta)+\mu} > 0.$$

The next issue we wish to highlight, is that not all monetary policies are consistent with equilibrium. As μ becomes large, ψ^* eventually becomes negative.[15] Clearly, ψ^* cannot be negative but, moreover, it cannot be lower than $\bar{\psi}$, since this would violate Lemma 2. The upper bound of monetary policies consistent with equilibrium can be uniquely defined by $\bar{\mu} \equiv \left\{\mu : \psi^*(\mu) = \bar{\psi}\right\}$. A detailed characterization of $\bar{\mu}$ is provided in the Appendix. The existence of an upper bound of admissible monetary policies is very intuitive. There exists a critical level of inflation, above which the cost of

[14] As in any monetary model, in steady state, the so-called Friedman rule provides a lower bound for admissible monetary policies. In our model, this can be seen from the fact that $\mu < \beta - 1$ would lead to a straightforward violation of Lemma 2.

[15] More precisely, as $\mu \to \infty$, $\psi^* \to -y_L$.

holding money overweighs the benefit, so agents do not wish to hold any currency. As a consequence, the trade of securities in the FFM collapses due to the lack of liquidity. As intuition suggests, the upper bound of admissible policies is increasing in λ, since a higher λ increases the potential gains from trade in the FFM and makes agents more willing to tolerate some inflation.

In the Appendix, we show that $\bar{\mu}$ can be positive and, therefore, monetary equilibria with positive inflation can be supported. This is yet another result that stands in contrast with findings in Geromichalos et al (2007). In that paper, it is shown that monetary equilibrium collapses for any $\mu \geq 0$. The reason is as follows. The rate of return on the real asset is given by $(\psi_{t+1} + R - \psi_t)/\psi_t$, which, in steady state, equals $R/\psi > 0$. Since in that framework money and the real asset are perfect substitutes, no-arbitrage implies that the rate of return on money should also be positive. In steady state, the latter is given by $-\mu/(1+\mu)$, which can be positive only if $\mu < 0$. In this paper, money and assets are complements, and the argument above does not hold. In fact, all one needs in order to support monetary equilibria with inflation, is the assumption that money and assets are not perfect substitutes. Lester et al (2008) break down the perfect substitutability assumption, by allowing money to have superior liquidity properties compared to the asset. They show that, in this case, monetary equilibria with $\mu > 0$ can be supported.

Finally, consider welfare in the economy. In our model, there is no consumption good traded in a decentralized market. However, trade of assets in the FFM increases welfare, because it allows agents to achieve their desired CM consumption, X^*, with less work. Assuming that $\alpha_1 + \alpha_2 > 0$, in the first subperiod agents set $s^* = A$. In the FFM, the amount of assets that change hands is given by $Q^* = (\lambda/2)\min\{A, \phi M/(\psi + y_{_L})\} = (\lambda/2)A$. In other words, low-types sell all their securities to high-types, which is efficient from a social point of view. An interesting result is that, unlike asset prices, total welfare is unaffected by inflation, as long as $\mu \in [\beta - 1, \bar{\mu}]$.[16] The first best is achieved when $\lambda = 1$, since, in that case, the trade of financial assets (and, therefore, optimal re-allocation of resources) is maximized.

The following proposition summarizes the most important results of the paper.

Proposition 1. *The key variable for the determination of equilibrium is $\alpha_1 + \alpha_2$.* ***(a)*** *If $\alpha_1 + \alpha_2 \leq 0$, no trade takes place in the FFM, $q^* = s^* = 0$, and the risk-free asset is only valued for the dividend it yields, i.e. $\psi^* = \bar{\psi}$. A monetary equilibrium*

[16] This result is closely connected to the assumption of linear returns on the risky asset, which leads to corner solutions. In an alternative specification of the model, we consider the case where investing $s \in [0, a]$ units of the asset yields $\epsilon_j f(s)$, where $j = L, H$, and f is a strictly increasing and concave function with standard Inada conditions. Since $lim_{s\to 0} f'(s) = \infty$, it is never optimal for the low-type to give up all her assets. This creates a link between inflation and the amount of s that changes hands in the FFM. However, the analysis becomes very complex and we are only able to solve the model numerically. Since none of the major results of the model are altered, we prefer the more simple specification presented here, which also yields closed form solutions.

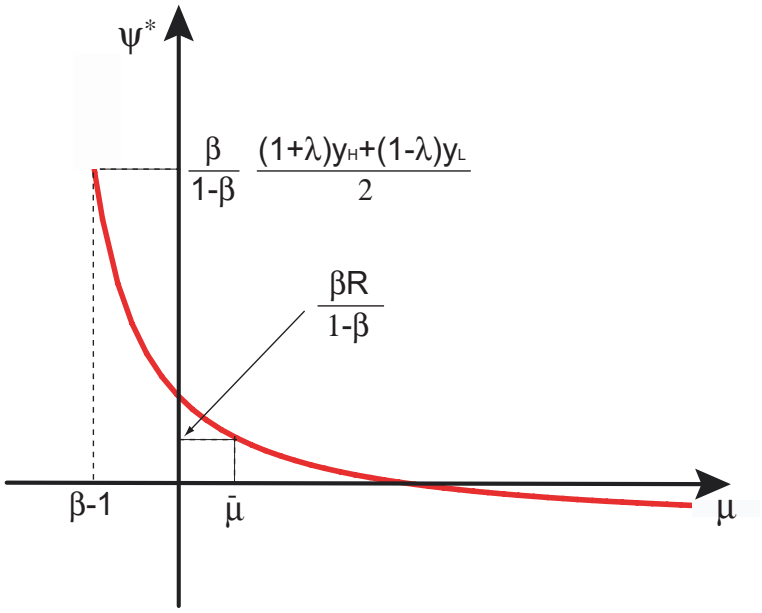

Figure 2: Equilibrium asset price and monetary policy.

can be supported by the Friedman rule, but even in that case there is no essential role for money.

(b) *If* $\alpha_1 + \alpha_2 > 0$, $\psi_t = \psi^*(\mu)$ *given by (14) for all* t. *The range of policies under which monetary equilibria can be supported is* $[\beta - 1, \bar{\mu}]$, *where* $\bar{\mu}$ *is defined by* $\bar{\mu} \equiv \{\mu : \psi^*(\mu) = \beta R/(1-\beta)\}$. *The upper bound of admissible monetary policies,* $\bar{\mu}$, *is increasing in* λ. *Moreover, for all* $\mu \in [\beta - 1, \bar{\mu}]$, $d\psi^*/d\mu < 0$. *The bigger the* λ *and the smaller the* μ, *the bigger the liquidity premium on the underlying asset,* $\psi^* - \bar{\psi}$. *These results are presented in Figure 2.*

Welfare depends on λ, *i.e. the probability with which agents get to trade in the FFM and re-balance their positions. In equilibrium, all agent set* $s^* = A$. *Hence, risky investment takes place even if the expected return on securities is worse than that on the safe asset (* $\alpha_1 \leq 0$*). In the FFM, low-types sell all their risky assets to high-types, so all equilibria are constrained efficient. The first best (the Social Planner's allocation) is achieved when* $\lambda = 1$.

5 Conclusion

We have presented a general equilibrium model that explicitly analyzes the role of money in financial markets characterized by frictions. We provide an innovative framework that models a risky financial asset that can be issued from the value of a safe-return real asset. Agents can re-balance their positions on these relatively illiquid assets using a more liquid instrument (fiat money) in a decentralized financial market. Money has an essential, welfare improving role, since it helps the economy overcome a double coincidence problem and allocate the risky financial securities into the hands of agents with high valuation. In equilibrium, this intrinsically useless object, is valued precisely for its liquidity services.

Our approach yields interesting results, some in contrast with the existing literature. In particular, since money and the underlying asset are complements, the asset price is negatively related to inflation. In monetary equilibrium, the price of the asset always exceeds its so-called fundamental value, reflecting a liquidity premium. However, not all monetary policy rules are consistent with a monetary equilibrium. We provide a closed form solution for an upper bound of admissible monetary policies. If the monetary authority allows inflation to exceed this bound, holding money becomes too expensive, and the trade of assets in the frictional market collapses, due to the lack of liquidity. The introduction of money can generate welfare gains even under inflationary money growth rates, as long as those rates are not too large.

References

Berentsen, A., G. Camera, and C. Waller (2007): "Money, credit and banking," *Journal of Economic Theory*, 135(1), 171–195.

Berentsen, A., and G. Rocheteau (2003): "Money and the gains from trade*," *International Economic Review*, 44(1), 263–297.

Duffie, D., N. Gârleanu, and L. Pedersen (2005): "Over-the-Counter Markets," *Econometrica*, 73(6), 1815–1847.

Ferraris, L. (2010): "On the complementarity of money and credit," *European Economic Review*, 54(5), 733–741.

Ferraris, L., and M. Watanabe (2010): "Liquidity constraints in a monetary economy," *Int. Econ. Rev. forthcoming*.

——— (2011): "Collateral fluctuations in a monetary economy," *Journal of Economic Theory*.

Geromichalos, A., and L. Herrenbrueck (2016): "Monetary Policy, Asset Prices, and Liquidity in Over-the-Counter Markets," *Journal of Money, Credit, and Banking*, 48(1), 35–79.

Geromichalos, A., J. M. Licari, and J. Suarez-Lledo (2007): "Monetary Policy and Asset Prices," *Review of Economic Dynamics*, 10(4), 761–779.

Jacquet, N., and S. Tan (2010): "The Liquidity Effects of Monetary Policy on Asset Prices," .

Kiyotaki, N., and R. Wright (1989): "On money as a medium of exchange," *The Journal of Political Economy*, pp. 927–954.

——— (1993): "A search-theoretic approach to monetary economics," *The American Economic Review*, pp. 63–77.

Lagos, R. (2010): "Asset prices, liquidity, and monetary policy in an exchange economy," *Journal of Money, Credit and Banking*.

Lagos, R., and G. Rocheteau (2008): "Money and capital as competing media of exchange," *Journal of Economic Theory*, 142(1), 247–258.

Lagos, R., and G. Rocheteau (2009): "Liquidity in asset markets with search frictions," *Econometrica*, 77(2), 403–426.

Lagos, R., G. Rocheteau, and P. Weill (2011): "Crises and liquidity in over-the-counter markets," *Journal of Economic Theory*, forthcoming.

Lagos, R., and R. Wright (2005): "A Unified Framework for Monetary Theory and Policy Analysis," *Journal of Political Economy*, 113(3), 463–484.

Lester, B., A. Postlewaite, and R. Wright (2008): "Information, Liquidity and Asset Prices," PIER Working Paper Archive 08-039, Penn Institute for Economic Research, Department of Economics, University of Pennsylvania.

Lucas Jr, R., and E. Prescott (1974): "Equilibrium search and unemployment," *Journal of Economic Theory*, 7(2), 188–209.

Rocheteau, G., and R. Wright (2005): "Money in search equilibrium, in competitive equilibrium, and in competitive search equilibrium," *Econometrica*, 73(1), 175–202.

Temzelides, T., and J. Yu (2003): "Sufficient conditions for the essentiality of money," *International Economic Review, Forthcoming*.

A Appendix

Non-steady state equilibria.
We prove here that equilibria other than the steady state described above, cannot arise in this model. More precisely, we cannot have sequences $\{\phi_t M_t\}_{t=0}^{\infty}$,$\{\psi_t\}_{t=0}^{\infty}$ that converge to a certain limit either monotonically or in an oscillating manner. Therefore, the only way to keep these sequences bounded is to have $\phi_t M_t$ and ψ_t constant in all periods (in order to make this statement one needs to exclude cycles, which is the case here). As part of an equilibrium, we are looking for bounded sequences of money balances and asset prices such that (12) holds and also

$$\phi_t M_t = (\psi_t + y_{_L})A, \ \ \forall t. \tag{16}$$

Define real balances as $z \equiv \phi M$. If we multiply (12) by $\hat{M}$ and use (16) to substitute for $\hat{\phi}/(\hat{\psi}+y_{_L})$ and ψ, we conclude that real money balances follow a first-order linear difference equation,

$$\hat{z} = -\frac{\beta\,(\alpha_1+\alpha_2+R)+(1-\beta)y_{_L}}{2\beta}A + \frac{2+\mu}{2\beta}z.$$

However, $\mu \geq \beta - 1$ always, which implies for the multiplier of z that $(2+\mu)/(2\beta) > 1$. Therefore, the sequence $\{\phi_t M_t\}_{t=0}^{\infty}$ will always be explosive, unless $(\phi M)_t = (\phi M)^*$ for all t. Since (16) has to hold in every period the same conclusion is true for $\{\psi_t\}_{t=0}^{\infty}$.

Optimal monetary policy range.
The equilibrium asset price ψ^* is decreasing in μ. Moreover, as $\mu \to \infty$, ψ^* becomes negative. Clearly, a negative asset price is impossible. However, Lemma 2 in steady state, places an even more binding restriction on ψ^*. In particular, ψ^* is bounded below by $\bar{\psi} > 0$. Hence, there exists a unique value of μ, such that $\psi^*(\mu) = \bar{\psi}$. If the monetary authority chooses a growth rate of money that is higher than this critical level, we would have $\psi^*(\mu) < \bar{\psi}$, and equilibrium would collapse. The upper bound for monetary policy consistent with a monetary equilibrium can be uniquely pinned down by $\bar{\mu} \equiv \{\mu : \psi^*(\mu) = \bar{\psi}\}$.

From the equilibrium asset price (14), we can set ψ^* equal to its fundamental value and then solve with respect to the money growth rate. We have

$$\psi^* = \frac{\frac{\beta}{2}[(1+\lambda)y_{_H} + (1-\lambda)y_{_L}] - (1-\mu-\beta)y_{_L}}{2(1-\beta)+\mu} = \frac{\beta R}{1-\beta}.$$

After some algebra one can solve with respect to μ, which yields the desirable expression for the upper bound of monetary policies. In particular,

$$\bar{\mu} = \beta - 1 + \frac{\beta(1-\beta)(\alpha_{_1}+\alpha_{_2})}{y_{_L}(1-\beta)+\beta R}.$$

Notice that this expression can be positive as long as $\alpha_1 + \alpha_2 > (1-\beta)y_L/\beta + R > 0$. Therefore, monetary equilibria exist even under inflationary policies.