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OFFICIAL MASTER'S DEGREE IN THE
ELECTRIC POWER INDUSTRY

Market Equilibrium in Natural Gas Systems: Analysis of the Implementation of a Hub

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Abstract

The introduction of competition in the natural gas market, increases the interaction among shippers, changing the scenario they have to face and therefore their behavior. Gas demand is expected to be more flexible, mainly because of the use of natural gas fired power plants (NGGFPPs) to back up intermittent generation. Conversely, most of the contracts are long-term contracts, which are not suitable for this flexible demand scenario and oblige shippers to balance their position in the OTC market. With the introduction of hubs, transactions costs are reduced and additional flexibility is achieved.

The general objective of this master thesis is to represent the strategic behavior of agents in a market environment, in which they will try to maximize their profit. Each agent maximizes its profit facing a captive demand where it behaves as a monopoly, and interacting with the rest of the agents in the hub, in the electricity market and in foreign markets, holding different market behaviors. The equilibrium is stated as a Mixed Complementarity Problem and the three methodologies which are used for solving the problem are described.

Finally, in order to overcome the limitations of the Mixed Complementary Problem (MCP) formulation, an iterative optimization problem is proposed.

***Index Terms*—Natural Gas Market Equilibrium, Gas Hub, Natural Gas Systems**

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Notation

The main notation used in this master thesis is stated below for quick reference.

Indexes

a	Index of agent
i	Captive demand
e	Electric demand
x	Foreign market
p	Index of periods

Constants

$P^{c_o}_{ap}$	Intercept of agents´ cost function per period
α^{c}_{ap}	Slope of agents´ cost function per period
$P^{i_o}_{ap}$	Intercept of agents´ captive demand function per period
α^{i}_{ap}	Slope of agents´ captive demand per period p
$P^{e_o}_{p}$	Intercept of the electric demand function per period
α^{e}_{ap}	Slope of agents´ electric demand per period p
$P^{x_o}_{p}$	Intercept of foreign demand function per period
α^{x}_{p}	Slope of foreign demand per period p
$\bar{Q}^{c}_{ap}$	Maximum gas volume contracted per agent
k	Scalar used to solve the problem as a MILP

Variables:

q^{i}_{ap}	Gas demanded by agent for its captive demand
q^{e}_{ap}	Gas demanded by agent for its electric demand
q^{x}_{p}	Gas demanded by agent for foreign market
q^{c}_{ap}	Gas contracted by agent from long term contracts
Δq_{ap}	Agents´ gas purchase in the hub per period
∇q_{ap}	Agents´ gas sales in the hub per period
λ	Price in the Hub
ε_{ap}	Dual variable of the upper bound on gas demanded by an agent
$\mu^{q_i}_{ap}$	Dual variable of the lower bound on gas demanded by an agent for its captive demand
$\mu^{q_e}_{ap}$	Dual variable of the lower bound on gas demanded by an agent for its electric demand
$\mu^{q_x}_{ap}$	Dual variable of the lower bound on gas demanded by an agent in foreign markets
$\mu^{\Delta q}_{ap}$	Dual variable of the lower bound on gas purchases by an agent in the hub
$\mu^{\nabla q}_{ap}$	Dual variable of the lower bound on gas sales by an agent in the hub
b_ε_{ap}	Binary variable corresponding to the complementarity between ε_{ap} and q^{c}_{ap}
$b\_\mu^{q_i}_{ap}$	Binary variable corresponding to the complementarity between $\mu^{q_i}_{ap}$ and q^{i}_{ap}
$b\_\mu^{q_e}_{ap}$	Binary variable corresponding to the complementarity between $\mu^{q_e}_{ap}$ and q^{e}_{ap}
$b\_\mu^{q_x}_{ap}$	Binary variable corresponding to the complementarity between $\mu^{q_x}_{ap}$ and q^{x}_{ap}

$b\_\mu_{ap}^{\Delta q}$	Binary variable corresponding to the complementarity between $\mu_{ap}^{\Delta q}$ and Δq_{ap}
$b\_\mu_{ap}^{\nabla q}$	Binary variable corresponding to the complementarity between $\mu_{ap}^{\nabla q}$ and ∇q_{ap}

Chapter 1

Introduction

Dreaming, after all, is a form of planning. –Gloria Steinem

1.1. World´s Natural gas outlook

The Medium-Term Gas Market Report [1], forecasts that global gas demand will reaccelerate following a marked slowdown in both 2013 and 2014. The global natural gas demand is expected to grow by 1.9% p.a. [2], by 2035. Growth is driven by non-OECD demand, which grows 2.5% p.a. while OECD demand grows more modestly at 1.1% p.a. [2].

The natural gas sector is facing huge uncertainties, related to technological advances, geopolitical changes, and strategic policy shifts, that can reshape unexpectedly the gas markets. For instance, large-scale shale gas developments in Mexico and Argentina, a rapid uptake of gas in the transportation sector, the emergence of the Islamic Republic of Iran as an LNG exporter, the Russian Federation’s shift to the East or the large quantities of flexible supplies from the United States that might be on the way.

The most remarkable events and a picture of the current gas market is given by the IEA, in [1]. A quick outlook of the world´s gas sector is given below based in [1].

In OECD countries, the crisis and the continued deployment of renewables damped gas demand increases while very high import prices in 2013 and 2014 have undermined gas consumption growth, especially in the power sector. On the other hand, strong environmental policies may play a role in enhancing the position of gas, which offers flexibility and low carbon emissions.

In Asia, the expansion on the use of coal in some countries, the deceleration in China´s gas demand growth or Japan nuclear power policy uncertainty slow Asian demand growth and OECD Asia LNG imports fell for the first time since 2009.

US gas production shoot up. US gas production increased by 5.7% in 2014, the fastest growth since 2011 [1].

Demand growth in Latin America, Africa, and the Middle East is constrained by supply availability being that supply shortages a chronic problem.

In January 2015, oil prices were below USD 50/bbl. While prices have recovered from their lows, they remain locked in a USD 55-70 range. As gas is directly and indirectly linked to oil, this fall in oil prices has result in slower gas production, because of reductions in capital expenditure programs.

Another consequence of lower oil prices and the fact that the bulk of LNG trade is still oil-indexed is that regional price differentials have narrowed substantially and has brought Asian prices more in line with European benchmarks, changing the current gas flows directions.

Figure 1-1 represents the global gas context and how some of the remarks commented before affect prices, pushing them up or down.

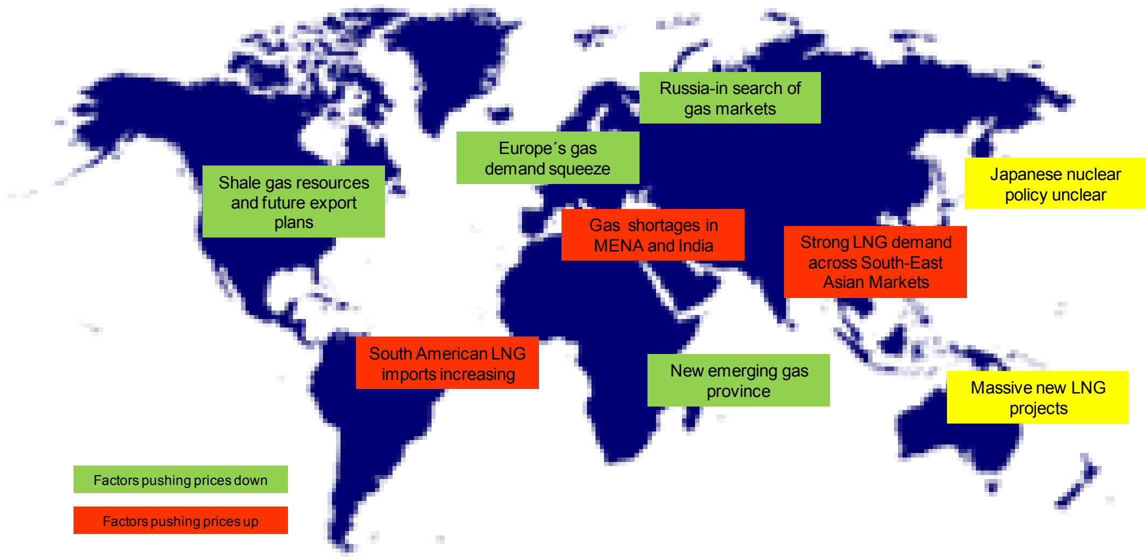

Figure 1-1 –Global gas context[1].

1.2. Natural gas outlook in Europe

The Natural gas sector has become a focus in Europe in the last decade and even more in the last years, as the supply and **demand picture has become increasingly uncertain**. A combination of factors lie behind: 1) the ongoing **liberalization process** started in 1998 with the first package (Directive 98/30/CE); 2) the **security of supply** concerns due to the concentration of suppliers most of them coming from countries considered risky and Europe's dependency in these imports; 3) the globalization of the natural gas markets through the **LNG**, its recent convergence in global gas prices and its sustainability over time; 4) the evolving framework of **long-term contracts** based on the cost of alternative fuels (oil price) and big take-or-pay commitments to liquid markets with transparent gas index prices; 5) the **production of shale gas** in North America, its

[1]Based in the one from Prof. Manfred Hafner, International and European natural gas markets (supply and demand) and geopolitics: Developments and Outlook

global present and future output and possible shale gas developments in Europe; and 6) **the role of natural gas in a low carbon Europe**. In particular, the shale gas revolution in America has put gas-intensive European industrial companies at a competitive disadvantage. At the same time, the displaced coal from the American generation mix has lowered coal prices in Europe, and worldwide, such that coal-fired generation is now more profitable than running gas-fired power stations. The low emission allowance price has also intensified this phenomenon.

Furthermore, for a long period, gas demand had been rising, whilst on the supply side, European Union (EU) production, which is located largely in the UK and the Netherlands, is declining.

1.2.1. Liberalization process

The European gas industry was historically managed by national, vertically integrated companies. Liberalization, introduced by the First Gas Directive (European Union, 1998), which sought to create competitive markets by ensuring access to the network to third parties was refined in the Second Gas Directive, (European Union, 2003), named the Acceleration Directive, which mandated regulated third party access (rTPA) for all existing infrastructures[2], commanded legal unbundling as the minimum level of unbundling, and reinforced the importance of the regulator being the result of a push toward a single European competitive gas market by the European institutions and the member states.

This series of legislative measures, followed in 2009 by the set of directives and regulations known as the Third Gas Package (European Union, 2009), led to the unbundling of the transmission assets, to the implementation of an entry-exit transport gas system and the creation of virtual hubs. This succession of changes and adjustments has not come to an end, and the European legislation will continue to adapt, in order to complete the vision enclosed in the regularly updated Gas Target Model[3].

This liberalization process has changed the legal and economic framework of the gas industry and how agents behave in this new environment is the main motivation of this

2 Negotiated third party access is not omitted on purpose as it is indeed an exception to regulated third party access. Instead of being imposed by regulatory authorities, the cost recovery framework will be decided by the infrastructure owner. Nevertheless, the owner is not totally free to set access rules, which are normally subject to regulatory authorities' approval.

3 In September 2010, the European energy regulators initiated a process to establish a target model for European gas markets. CEER published the Vision for a European Gas Target Model and Conclusions Paper on 1 December 2011. The review of this paper was undertaken by ACER in 2014

master thesis, more precisely the impact on agents´ behavior due to the incorporation of virtual hubs.

1.2.2. Security of supply

The EU imports more than half of all the energy it consumes. Its import dependency is particularly high for crude oil (more than 90%) and natural gas (66%)[4]. The total import bill is more than €1 billion[5] per day.

Security of supply is in the spotlight of the European Commission, because of:

1. The concentration of suppliers, many countries are heavily reliant on a single supplier.
2. To a large extent, the supplies come from countries that are perceived as “risky”, for instance, Central and Eastern Europe currently suffers from a strong dependency on Russian natural gas exports.

This dependence leaves Europe vulnerable to supply disruptions, as for instance, the 2009 gas dispute between Russia and the transit-country Ukraine, left many EU countries with severe shortages or in Algeria where the state is constantly undermined by islamist rebels (even if it has never been interrupted due to this reason).

Aiming at improving supply security and in response to these concerns, the European Commission released its Energy Security Strategy in May 2014 addressing short- and long-term security of supply. The Strategy aims to ensure a stable and abundant supply of energy for Europe.

1.2.3. Globalization of gas markets

The European market is increasingly becoming part of a globalized natural gas market with the interconnection of the world regions via liquefied natural gas (LNG). Even though, two thirds of gas imports are still done through pipelines. The flow by pipeline in 2014 accounted for 663.9 billion cubic meters whereas total trade liquefied natural gas in 2014 accounted for 333.3 billion cubic meters according to [3].

[4] European Comission http://ec.europa.eu/energy/en/topics/imports-and-secure-supplies

[5] European Comission http://ec.europa.eu/energy/en/topics/imports-and-secure-supplies

Billion cubic metres	2013 Pipeline imports	2013 LNG imports	2013 Pipeline exports	2013 LNG exports	2014 Pipeline imports	2014 LNG imports	2014 Pipeline exports	2014 LNG exports
US	78.9	2.7	44.4	0.1	74.6	1.7	42.3	0.4
Canada	25.8	1.1	78.9	–	21.8	0.6	74.6	–
Mexico	18.6	7.8	†	–	20.5	9.3	†	–
Trinidad and Tobago	–	–	–	19.8	–	–	–	19.3
Other S. & Cent. America	18.6	19.6	18.6	5.7	17.8	21.4	17.8	5.8
France	30.5	8.7	1.1	0.6	27.4	7.1	1.1	0.6
Germany	98.4	–	15.1	–	85.0	–	10.1	–
Italy	51.6	5.5	0.2	–	46.9	4.5	0.2	–
Netherlands	21.5	0.8	51.3	0.2	23.2	1.1	44.1	0.6
Norway	†	–	102.4	3.8	†	–	101.1	5.3
Spain	15.3	14.9	0.9	2.6	15.4	15.5	0.6	5.1
Turkey	38.2	6.1	0.6	–	41.1	7.3	0.6	–
United Kingdom	40.0	9.4	9.0	–	32.9	11.3	10.6	–
Other Europe	99.5	6.0	11.8	1.6	90.0	5.3	10.8	2.0
Russian Federation	27.0	–	212.0	14.2	24.2	–	187.4	14.5
Ukraine	25.0	–	–	–	17.5	–	–	–
Other Former Soviet Union	32.2	–	67.1	–	32.1	–	69.3	–
Qatar	–	–	19.9	105.6	–	–	20.1	103.4
Other Middle East	25.5	4.5	9.4	28.5	27.2	5.4	9.6	27.5
Algeria	–	–	28.8	14.9	–	–	23.5	17.3
Other Africa	7.2	–	9.3	31.6	8.5	–	10.8	31.2
China	27.3	24.5	–	–	31.3	27.1	–	–
Japan	–	119.0	–	–	–	120.6	–	–
Indonesia	–	–	10.0	22.4	–	–	9.5	21.7
South Korea	–	54.2	–	–	–	51.1	–	0.2
Other Asia Pacific	26.4	40.4	16.7	73.5	26.5	43.9	20.0	78.6
Total World	**707.5**	**325.3**	**707.5**	**325.3**	**663.9**	**333.3**	**663.9**	**333.3**

†Less than 0.05.

Source: Includes data from CISStat, FGE MENAgas service, GIIGNL, IHS Waterborne, PIRA Energy Group, Poten, Wood Mackenzie.

Table 1-2: Gas trade in 2013 and 2014 (billion cubic meters). Source: BP Statistical Review of World Energy June 2015

The total LNG regasification capacity in OECD Europe represents about 45% of the region’s consumption [1], with Spain leading in number with six regasification plants.

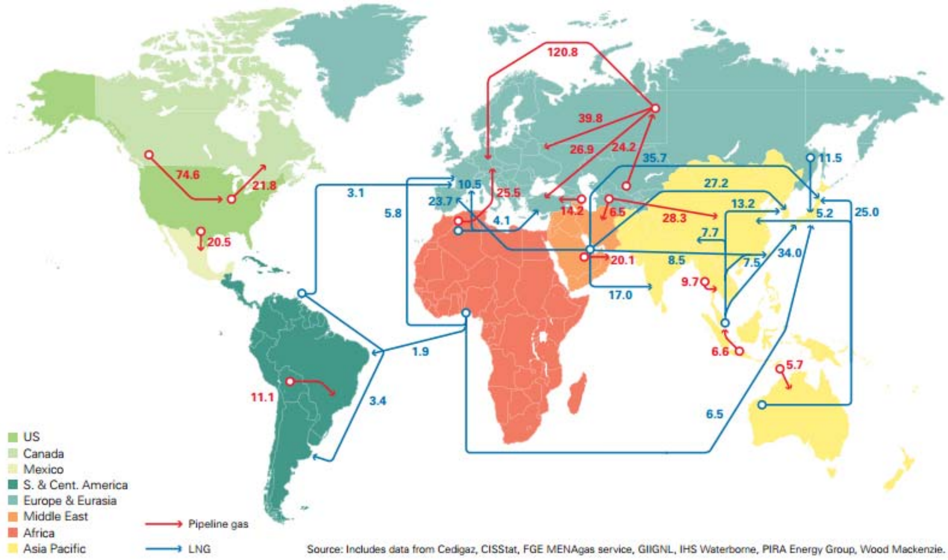

Figure 1-2 –Major trade movements 2014 Trade flows worldwide (billion cubic meters). Source: BP Statistical Review of World Energy June 2015

Inter-regional gas trade will expand by 40% between 2014 and 2020, surpassing 780 billion cubic meters (bcm) by 2020. Liquefied natural gas (LNG) will account for 65% of the increase [1].

1.2.4. Long-term contracts

Gas markets have traditionally relied on long-term bilateral contracts for covering gas demand. Producers signed long-term supply contracts with shippers, in which producers guarantee the recovery of their huge investments in capital-intensive facilities and shippers guarantee a firm supply at prices well-known in advance. While long-term contracts may have slowed down the natural gas market liberalization process, they have favored the development of long-term, capital-intensive supply projects such as pipelines, and LNG terminals. Realization of these projects would not be possible without the insurance provided by long-term contracts. Also consumers, like industries or local authorities on behalf of households, signed long-term contracts with shippers for similar reasons. Nowadays, producers, shippers and consumers still sign these long-term contracts. Conversely, gas demand is expected to be more flexible (as, for example, is the case of natural gas fired power plants) in the future and yet current pricing and market structures are not amenable to that outcome.

The price of gas was based on the cost of alternative fuels (oil price) and contracts rely on price review clauses. The framework is changing and even if long-term contracts still play a key role, they are becoming more flexible and there is an ongoing transition from long-term oil-indexed contracts to hub based contracts.

1.2.5. Shale gas developments

Shale gas has been in production for several decades, but started to grow rapidly only in the mid- 2000s, growing at more than 45% per year between 2005 and 2010. Unconventional gas production was nearly 60% of total gas production in the United States in 2010 [4].The reasons for this shale gas boom, as stated in [5], are related to government policy, private entrepreneurship, technology innovations, private land and mineral rights ownership, high natural gas prices in the 2000s, and a number of other factors.

This remarkable growth of shale gas production in the United States leads to interest in exploring shale resources in other areas of the world. A number of countries, including China, Mexico, Argentina, Poland, India, and Australia considered developing their own shale gas resources.

Currently, globally the outlook is less positive than it was a couple of years ago. Development in China, in particular, has proved slower than expected. In Europe, companies' interest in shale gas is evaporating fast, even in those countries where governments have proven supportive, such as Poland and Romania. Disappointing test wells, regulatory constraints, and continued public hostility have added to deteriorating economics as a result of lower oil and gas prices that have all contributed to dimming the outlook for shale gas.[1]. In the United Kingdom, although the government is supportive of the technology, the debate remains due to public hostility against fracking. In Spain, exploration permits are unlikely to be granted before 2020. Germany approved a draft law for the commercial exploitation of shale gas and oil.

Political indecision and public hostility against European shale gas development leads us to conclude a minor role for European shale gas production in the near future.

1.2.6. Low carbon framework

Strong environmental policies can play a role in enhancing the position of gas. Natural gas is advertised as a clean fossil fuel able to cope with the climate change and is often seen as an important energy carrier on the way to low carbon economy. Compared to other fossils fuels, natural gas has the lowest carbon content per unit of energy and due to the high flexibility of natural gas fired power plants (NGGFPPs), they are used as a backup for intermittent renewable power generation.

The role of natural gas as a transitional fuel in a Climate and Energy framework was highlighted in the EU Energy Roadmap 2050[6].

"... gas will play an important role, in the short to medium term, in the transformation of the energy system, since it represents a relatively quick and cost-efficient way of reducing reliance on other more polluting fossil fuels;..."

"… recognises natural gas potential as a flexible back-up for balancing variable renewable energy supply alongside electricity storage, interconnection and demand-response; considers affording greater importance to gas, particularly if technologies for carbon capture and storage become more widely available; believes that the objective of reducing greenhouse gas emissions must be the core consideration here, and the prevailing objective in the energy mix;..."

On the other hand, the deployment of renewables leaves limited space for gas demand increases, which currently could not compete with increasingly competitive renewables.

[6] http://www.roadmap2050.eu/

High LNG prices in recent years have influenced the viability of gas. Consumption growth is been slowed due to competition from coal and renewables.

1.3. The context of this project research in the natural gas sector

The 3rd Gas Directive (2009/73/EC) proposes the unbundling of activities, the implementation of entry-exit access systems and the constitution of national or supra-national virtual hubs in order to enlarge the market, reduce the barriers to entry and encourage the degree of competition. The liberalization process has changed the legal and economic framework of the gas industry, from big monopolies to oligopolies.

The introduction of competition in the gas market due to the ongoing liberalization in Europe, increases the interaction among shippers in downstream gas systems. As the entries and exits from the balancing zones may be uncertain, shippers buy and sell gas to balance their position. Shippers usually perform Over the Counter (OTC) bilateral operations in the search of balancing their entries, exits and inventory variations daily. With the introduction of virtual hubs (balancing electronic platforms, linked to balancing zones), transactions costs are reduced.

The appearance of continental European gas hubs encouraged by the 3rd Gas Directive has brought up the following question: How do agents behave in the hub since in many cases there are few firms competing? We are interested in the equilibrium solution of such game, looking for the simultaneous solution of all players (Nash equilibrium) [6] instead of each individual profit.

The motivation of this master thesis is twofold. First, to represent the strategic behavior of agents in a market environment, in which they will try to maximize their profit. Each agent maximizes their profit facing a captive demand where it behaves as a monopoly, interacting with the rest of the agents in the hub, in the electricity market and in foreign markets. With this aim, the decision-taking process of the different shippers will be modeled. Second, the paper models and assesses how the introduction of a gas hub modifies the behavior of the different agents.

1.4. Nash equilibrium

Economic agents can interact strategically in a variety of ways, and many of these have been studied by using game theory. In game theory, the **Nash equilibrium** [6] is a solution concept of a non-cooperative game involving two or more players, in which each player is assumed to know the equilibrium strategies of the other players, and no player has anything to gain by changing unilaterally strategy[7].

Let *(S,f)* be a game with *n* players, where S_i is the set of strategies for each player *i,* $S = S_1 \times S_2 .... \times S_n$ is the set of strategies profiles and $f = (f_1(x),...,f_n(x))$ is the payoff function of $x \in S$. Let x_i be a strategy profile of player *i,* and x_{-i} a strategy profile of all players except for player *i*. When a player $i \in \{1,...,n\}$ chooses strategy x_i resulting in strategy profile $x = (x_1,...,x_n)$ then player *i* obtains a payoff $f_i(x)$. Then, a strategy profile $x^* \in S$ is a Nash equilibrium if $\forall i, x_i \in S_i : f_i(x_i^*, x_{-i}^*) \geq f_i(x_i, x_{-i}^*)$, that is, no player has incentive to deviate from his strategy given that the other players do not deviate.

We therefore have a Nash equilibrium when each agent is making the optimal choice, given the other agents' choice. It may sometimes appear non-rational in a third-person perspective. This is because it may happen that a Nash equilibrium is not Pareto Optimal.

The Nash equilibrium can be defined either as "strict" or "weak", according if the best-response means strict best response (strictly greater pay-off) or weak best response (as good as any other alternative).

1.5. Modeling approaches

The following subsections summarize the two used techniques in this thesis for modeling the natural gas sector markets: optimization and equilibrium modeling.

1.5.1. Optimization techniques

Linear programming was developed during the 40s under the leadership of G. B. Dantzig. Today, optimization is applicable to a large set of problems and plays an important role in planning and forecasting in nearly all types of industries. It is frequently used for applications in production processes but many other applications exist.

[7]Osborne, Martin J., and Ariel Rubinstein. *A Course in Game Theory*. Cambridge, MA: MIT, 1994.

Linear and non-linear programming (LP and NLP) consist of the maximization or minimization of linear and non-linear functions of one or several variables under some linear equality and/or inequality constraints. The complexity of the function and the constraints can vary from few variables and a linear structure, to numerous variables and nonlinear problems.

The basic setup can be expressed in the following form:

$$\underset{x}{Max}\, f(x) \text{ s.t. } gi(x) \leq 0 \;\forall i \in I \tag{1.1}$$

Where x denotes the decision variable, f(x) the objective function and gi(x) the i'th constraint.

The optimization technique has been widely used for modeling operational problems in the natural gas sector. For example, [7] solve the gas distribution problem as a cost minimization subject to nonlinear flow-pressure relations, material balances, and pressure bounds or GASCOOP model [8] that optimizes the system operation, minimizing agent´s costs in an entry-exit access system.

There exists a lot of market models using an optimization approach for representing gas markets. However, optimization models fails when market power is included in the model, as imperfect market games (e.g., Cournot) are difficult to model with optimization techniques.

1.5.2. Equilibrium modeling. Complementarity approach.

The European natural gas market is not a perfect market; therefore, the cost minimization approach may not be representative for the European natural gas market and its imperfect market structure.

For the representation of the equilibria, complementarity structures are used in a general equilibrium framework and in non-cooperative game setting. By simultaneously solving the optimization problems of several players within the complementarity system, this model type gives the equilibrium solution to the entire market game.

Hence, the equilibrium solution goes beyond the solution of the individual optimization problem of each player, by giving the simultaneous solution to all agents in the game. In many situations the individualistic interests of each player causes the equilibrium solution not to be Pareto optimal, like the known example of the Prisoner's Dilemma.

As described in [9], having a function $F: \Re^n \rightarrow \Re^n$ the pure nonlinear complementarity problem denoted *NCP(F)* is to find $x \in \Re^n$ such that for all i :

$$\begin{aligned} &F_i(x) \geq 0 \\ &x_i \geq 0 \\ &x_i \cdot F_i(x) = 0 \end{aligned} \tag{1.2}$$

The mixed version of the complementarity problem (MCP) is closely related but also allows for both equations with corresponding free variables and inequalities with associated nonnegative variables. The general form of the MCP is stated as follows [10]:

Finding a vector x, assuming $f(x) \geq 0$, satisfying the complementarity condition $f(x)^T \cdot x = 0$, for each element I of the vector x, either x_i or $f_i(x)$ must equal zero:

$$0 \leq x_i \perp f_i(x) \geq 0, \quad \forall i \in I \tag{1.3}$$

Accordingly the variables x and $f(x)$ are called *complementary.*

A general maximization problem becomes:

$$\max_x f(x) \tag{1.4}$$

s.t.

$$g_i(x) \leq 0, \quad (\lambda_i), \quad \forall i \in I \tag{1.5}$$

$$h_j(x) = 0, \quad (\mu_j), \quad \forall j \in J \tag{1.6}$$

$$x \geq 0 \tag{1.7}$$

Where *i* and *j* are indexing the inequalities and equalities respectively.

The corresponding Karush-Kuhn-Tucker conditions (KKT) are the necessary and sufficient conditions for optimality of the problem if we have a convex objective function and a convex solution space feasible region.

$$\nabla f(x) + \sum_i \lambda_i \cdot \nabla g_i(x)^T + \sum_j \mu_j \cdot \nabla h_j(x)^T = 0 \tag{1.8}$$

$$0 \geq g_i(x) \quad \perp \quad \lambda_i \geq 0 \quad \forall i \in I \tag{1.9}$$

$$h_j(x) = 0, \qquad \mu_i \text{ free}, \quad \forall j \in J \tag{1.10}$$

Equation (1.8) makes sure the solution is stationary, (1.9) guarantees complementarity and (1.10) feasibility. Note that the dual variable λ_i of the inequality has to be greater than or equal to zero, while the dual μ_j of the equality can take any real number.

Depending on the character of the constraints of the optimization problem, $g_i(x)$ or $h_j(x)$, different types of complementarity problems can be distinguished:

If the constraints are exogenous parameters, the linear or non-linear complementarity problem can be expressed as a **MCP.** Their common characteristic is the simultaneous solution to all optimization problems in the model. There can be several linked optimization problems in such a model, either in a game context (linked via reaction functions) or in any other setup where the link is done via physical balance or market clearing conditions. Market games such as Cournot games can be modeled in the MCP format. More generally, MCP models allow to represent Nash games in pure strategies.

If the constraint is itself the result of another equilibrium problem, two types can be distinguish:

MPEC (mathematical program with equilibrium constraints) if the objective function of the program is the optimization problem of a single player (e.g., Stackelberg game, welfare optimization).

EPEC (equilibrium program with equilibrium constraints) if the objective function gives the solution of another equilibrium problem.

In general, solving the resulting system of equations and proving uniqueness and existence of the solution happens to be mathematically challenging.

This master thesis has used some of the above-mentioned modeling methods, specifically MCP, for solving the equilibria described in the following sections and linear programming (LP) for solving an iterative optimization problem that leads to the same solution of the equilibria.

1.6. State of the art and motivation

In this section, the state of the art is described, paying special attention to those models that have been focused on solving the gas market equilibrium. A gap of improvement has been detected in the representation of the demands in the downstream market as well as the upcoming necessity of representing and study the impact of virtual hubs in this environment.

1.6.1. State of the art

Because European natural gas sector has been described as a Cournot oligopoly [11], most of the models proposed in the literature for solving the equilibrium are Mixed Complementary Problems (MCP), Mathematical Program with Equilibrium Constraints (MPEC) or Equilibrium Program with Equilibrium Constraint (EPEC) [9]. Some of the models that represent gas markets with detail are described below.

The evaluated models are focused on the European natural gas market. MCP models have been the preferred format of the European natural gas sector, because of the imperfect character of the market. Another common characteristic of the current literature is that, most of the models are focused in the long to medium term.

GASTALE [13] is used to analyze the European natural gas market and focuses primarily on the role of the downstream trading companies and their interaction with gas producers. Producers of natural gas are assumed to form an oligopoly meanwhile, downstream within-country traders of gas are represented in different versions of the model as local oligopolies or perfect competitors. The model therefore has a two-level structure, in which producers engage in competition a la Cournot, and each producer is a Stackelberg leader with respect to traders, who may be Cournot oligopolies or perfect competitors. The model is formulated as a complementarity problem, and is solved by nonlinear programming.

NATGAS: The NATural GAS [14] model is an integrated model of the European wholesale gas market providing long-run projections of supply, transport, storage and consumption patterns in the model region, aggregated in 5-year periods, distinguishing two seasons (winter and summer).

GASMOD [15] is a model of the European natural gas supply, which is structured as a two-stage game of successive natural gas exports to Europe (upstream market) and wholesale trade within Europe (downstream market), and which explicitly includes infrastructure capacities. It allows the representation of different markets scenarios in both markets (Cournot competition or Perfect competition) concluding that Cournot competition on both markets is the most realistic representation of today's European natural gas market.

GASCOOP [8] is a market model which capture accurately the performance of a gas market based on an entry-exit access system through cost minimization, providing reliable outcomes, not only for academic purposes, but also for any stakeholder, such as a market participant, a regulatory authority or a facility operator. It contains a detailed representation of infrastructure operation, optimizing operation and capacity contracting decisions considering the influence of long-term supply contracts and LNG carriers movements.

1.6.2. Contribution of this research

While large-scale gas models have been developed and used extensively, GASTALE model by [13], NATGAS model by [14], and GASMOD model by [15]; EUGAS model by [16], the model presented in this master thesis differs from earlier models in its representation of demands and the representation of the hub.

			Market Representation	Operation details	Demand representation
(Hobbs et al., 2003)	GASTALE	LT	●	◔	◔
(Zwart et al., 2006)	NATGAS	LT	●	◐	◐
(Holz et al., 2009)	GASMOD	LT	●	◐	◐
(Dueñas et al., 2013)	OMEGA	LT/MT	◔	●	◐
OBJECTIVE	➡	ST	◕	○	●

Table 1-3 – Main models comparative

Table 1-3 contains the results of the evaluation, in which a white circle points out that a shortcoming is not addressed at all and a black circle indicates that a shortcoming is fully addressed.

We have detected a relevant and generalized lack of models regarding the operation in the short term with a detailed analysis of the different downstream gas demands. This gap is studied and modeled in our model.

The model also addresses establishing an organized market (Hub), as a balancing market, where shippers trade to balance their position. This market is represented as a perfect market. One limitation of our model is that we do not capture anticompetitive behavior in the hub, which may actually occur in immature and imperfect gas markets.

1.7. Master thesis objectives

The general objective of this master thesis is to represent the strategic behavior of gas agents in a market environment, in which they will try to maximize their profit. Each agent maximizes their profit facing a captive demand where it behaves as a monopoly, and interacting with the rest of the agents in the hub, in the electricity market and in foreign markets, holding different market behaviors. With this aim, the decision-taking process of the different shippers will be simulated.

Along the lines of the previous objective, the EU Third Energy Package includes the so-called Gas Target Model, which defines the constitution of national or supra-national virtual hubs. In this new environment, we want to answer: How do agents act in the hub since in many cases there are few firms competing? For this purpose, we model and assess how the introduction of a gas hub modifies the behavior of the different agents.

Aware of the intrinsic limitations of MCP formulation, we propose an iterative optimization problem. The strengths and weaknesses of each formulation will be explained as well as the limitations of the approach are noted, as the complexity of the model is not enough yet to draw conclusions.

1.8. Structure of the document

In Chapter 1, an overview of the natural gas sector was given for introducing afterward the problem context. Next, the Nash equilibrium and the different modeling approaches used in this master thesis has been explained. Finally, the master thesis objectives are set.

The rest of the document is organized as follows:

- In Chapter 2, the different regulatory frameworks (entry-exit and point-to-point) and the modeling assumptions are described. Special attention has been paid to the demand segmentation.
- In Chapter 3, the equilibrium model, in which the strategic behavior of agents in a market environment is represented, is explained. Two cases have been set out: First, the behavior of the agents have been represented without considering the hub. Afterwards, a virtual hub is introduced in the problem and how agents behave under this new framework is studied. A small case study is presented, where the obtained results considering and not considering the hub are explained. Finally, conclusions are drawn.

- In Chapter 4, an iterative optimization problem is proposed to solve the equilibrium problem. A small case study is presented and conclusions are drawn.
- Chapter 5 gathers the conclusions reached in the previous two chapters and proposes next steps and future research.

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Chapter 2

Model description

Teach thy tongue to say, “I do not know,” and thous shalt progress. –Maimonides

2.1. Introduction

In this section, all necessary assumptions to build the proposed model are introduced.

2.2. Regulatory framework: Point-to-point vs. Entry-exit systems

The liberalization process of the gas sector has end up with the definition of two main different regulation frameworks for the network services like contracting and operating rules and a cost recovery framework for the regulated infrastructure[8].

However, the way to coordinate such network services is still under debate.

On the one hand, point-to-point systems establish two prices at both pipeline extremes. The difference between both prices reflects transportation costs and scarce capacity valuation when transportation constraints appear. On the other hand, entry-exit systems fragment the market by defining balancing zones where the network is embedded and establish entry and exit tariffs. Balancing zones disregard transportation and distribution network characteristics, except at entry and exit points [16].

Figure 2-1 contains a graphical representation of the entry exist systems and the point-to-point.

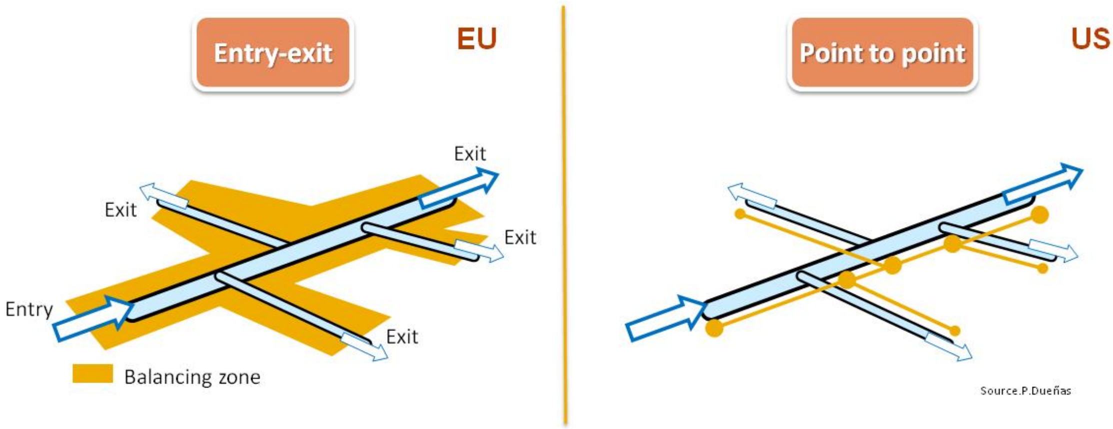

Figure 2-1 – Graphical representation of entry-exit systems and point-to-point system

[8] Negotiated third party access is not omitted on purpose as it is indeed an exception to regulated third party access. Instead of being imposed by regulatory authorities, the cost recovery framework will be decided by the infrastructure owner. Nevertheless, the owner is not totally free to set access rules, which are normally subject to regulatory authorities' approval.

Entry-Exit	Point-to-Point
Primary allocation of capacity for introducing and removing gas in the point defined as "Entry Points" or "Exit Points". - Associated to **Virtual Hubs** ✓ Flexibility in the allocation of capacity ✓ Promotes liquidity ✗ Imbalances need penalties ✗ Cost associated with flexibility	Capacity allocation associated with an exit point and a path or route of transport determined. You cannot hire separate input and output. - Associated to **Physical Hub** ✓ Efficient use of transport infrastructure ✓ Tariffs are cost reflective ✗ High barriers to entry ✗ It requires a mature market
Europe	**United States**

Table 2-1 – Characteristics of Entry - exit vs. Point-to-point systems

This master thesis is focused on the entry-exit access systems which are being implemented in the EU in line with the Third Energy Package to constitute an internal gas market.

2.3. Modeling Assumptions

2.3.1. Demand Segmentation

The different types of demands supplied by a gas company have been categorized into: households, electricity markets and foreign markets.

2.3.1.1. Households

The Second legislative package in 2003, (Directive 2003/55 / EC) allowed the entry of new suppliers of gas in the Member States, and opened the possibility that consumers could choose freely their gas supplier. Nonetheless, the downstream gas market still relies on monopolistic structures at some point and, for example, households changing from their suppliers, is even today not so common[9].

For this reason, **households have been represented as a captive demand of each agent**, not considering the consumers' switching rate among companies. As gas has substitute goods, like oil or electricity, consumers will look for other alternatives, being the demand elastic, although not much due to the complexity of changing from one fuel to another in the short term (except for the industrial demand, which has been omitted in this research). **It is assumed that each gas agent supplies its own market, acting as a monopoly, in which the demand is a linear function of the price.**

A monopoly would recognize its influence over the market price and choose that level of price and output which maximizes its overall profits. Of course, it cannot choose price and output independently; for any given price, the monopoly will be able to sell only what the market will bear. It can be seen as the monopoly choosing the quantity and letting the consumers decide what price they will pay for the quantity.

The demand behavior of the consumers will constrain the monopolist's choice of price and quantity [18].

The monopolist´s profit-maximization problem then takes the following form:

Let p(y) denote the market inverse demand curve and c(y) to denote the cost function, then $r(y) = p(y) \cdot y$ is the revenue function of the monopolist.

$$\max_{y} \; r(y) - c(y) \tag{2.1}$$

The optimality condition for this problem is where the marginal revenues (MR) equals the marginal cost (MC)

$$\frac{\Delta r}{\Delta y} = \frac{\Delta c}{\Delta y} \tag{2.2}$$

If marginal revenues were less than marginal costs it would force the firm to decrease outputs, since the savings in costs would be more than the resulting loss in revenues. If the marginal revenues were greater than the marginal costs, it would force the firm to

9 In Spain, the switching rate in 2014 was of 10% according to "Informe anual de supervision de los cambios de comercializador – Año 2014". CNMC

increase outputs. The only point where the firm has no incentive to change outputs is where marginal revenues equal marginal costs.

Expressing the marginal revenue in terms of the elasticity $\varepsilon(y)$, and rewriting the optimality condition "marginal revenue equals marginal cost":

$$MR(y) = p(y)\left[1 - \frac{1}{|\varepsilon(y)|}\right] = MC(y) \tag{2.3}$$

If the monopolist faces a household´s linear demand curve $p(y) = a - b \cdot y$, the revenue function is

$$r(y) = p(y) \cdot y = a \cdot y - b \cdot y^2 \tag{2.4}$$

And the marginal revenue function is

$$MR(y) = a - 2 \cdot b \cdot y \tag{2.5}$$

The marginal revenue function has the same vertical intercept *a*, as the demand curve, but it is twice as steep.

Figure 2-2 represents a monopoly with a linear demand curve and shows that the monopolist´s profit-maximizing output occurs where marginal revenue equals marginal cost.

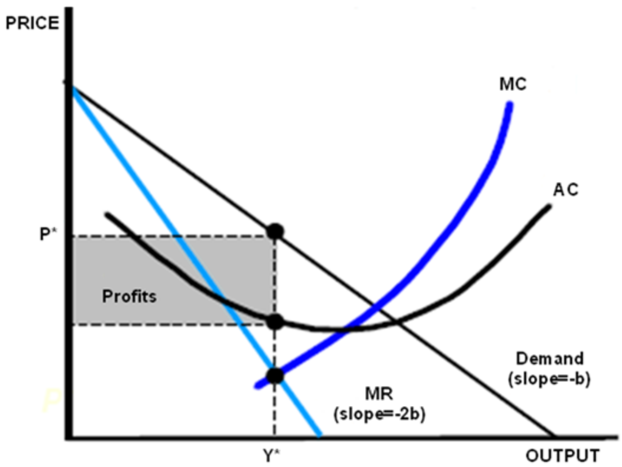

Figure 2-2 – Monopoly with a linear demand curve.

2.3.1.2. Electricity sector demand

The share of gas demand used for power generation over the total gas consumption has augmented during these years, reaching globally around 40% in 2012 [19]. Besides, the worldwide share of gas used in the electric power sector, which is produced by natural gas fired power plants NGFPPs, has doubled (from 10% to nearly 22.2% [20].

In this master thesis, **agents participate in the electricity market with their own natural gas fired power plants (NGFPPs), but there are not so many agents as to regard each of them as having a negligible effect on price.** This is the situation known as **oligopoly.**

The electricity sector is not represented in this article, assuming each agent owns some gas-fired power plants which request a gas demand. This gas demand, is represented by a price-quantity affine function for all agents, where the influence of each agent in the market is represented by the value of the slope for each agent.

This market has been represented as a Cournot, where each agent chooses a profit-maximizing output for itself.

Assume that there are only two agents and agent 1 expects that agent 2 will produce y_2^e units of output. (The *e* stands for expected output.) If firm 1 decides to produce y_1 units of output, it expects that the total output produced will be $Y = y_1 + y_2^e$ and the output will yield a market price of $p(Y) = p(y_1 + y_2^e)$. The profit-maximization problem of agent 1 is then [18]:

$$\max_{y_1} \ \mathrm{p}\cdot\left(y_1 + y_2^e\right)\cdot y_1 - c\left(y_1\right) \qquad (2.6)$$

For any given belief about the output of agent 2, y_2^e , there will be some optimal choice of output for agent 1, y_1 . The reaction function gives one firm's optimal choice as a function of its beliefs about the other agent's choice.

$$\begin{aligned} y_1 &= f_1(y_2^e) \\ y_2 &= f_2(y_1^e) \end{aligned} \qquad (2.7)$$

Then, the optimal output combination is such that the output of choices $\left(y_1^*, y_2^*\right)$ satisfies:

$$\begin{aligned} y_1^* &= f_1(y_2^*) \\ y_2^* &= f_2(y_1^*) \end{aligned} \qquad (2.8)$$

Such a combination of output levels is known as **Cournot equilibrium.**

For example, in case of the linear demand function and zero marginal costs the reaction function for both agents takes the form

$$y_2 = \frac{a - b \cdot y_1^e}{2 \cdot b};\ y_1 = \frac{a - b \cdot y_2^e}{2 \cdot b} \qquad (2.9)$$

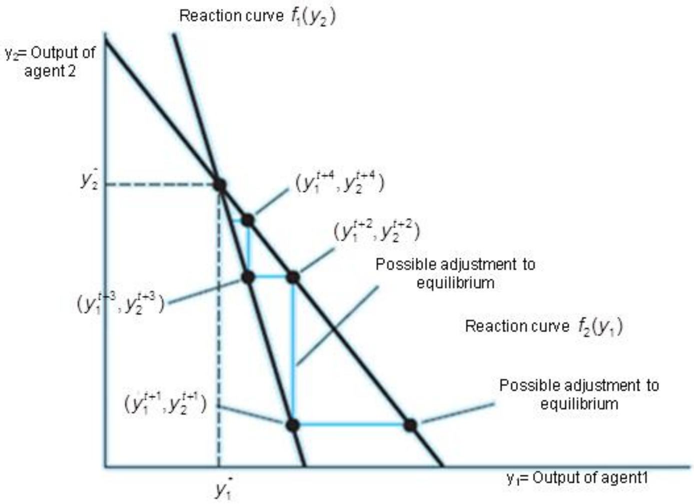

Figure 2-3 –Cournot equilibrium.

2.3.1.3. Deliveries to foreign markets

The world is interconnected via liquefied natural gas (LNG), favoring the globalization of the natural gas sector and creating an international gas market. In this scenario, it becomes necessary to account for the foreign gas market. As a global market, the influence of each agent on its price is reduced; hence, the market is quite elastic, i.e., the price maintains almost constant while gas is delivered.

This market could be represented as a **perfectly competitive market, assuming that the goods being offered for sale are all the same and that the buyers and sellers are so numerous that no single buyer or seller can influence in the market price.** Each agent assumes that the market price is independent of its own level of output and that its decision is how much output it wants to produce, being price takers.

By definition a competitive agent ignores its influence on the market price. Thus the maximization problem facing a competitive agent is maximizing its profits (revenue minus cost) [18].

$$\max_{y} \; p \cdot y - c(y) \qquad (2.10)$$

The agent will operate in the point where marginal revenue equals marginal cost, that is, when the extra revenue gained by one more unit of output just equals the extra cost, of producing another unit.

$$p = MC(y) \qquad (2.11)$$

Thus, a competitive firm will choose a level of output y where the marginal cost (MC(y)) that it faces is equal to the market price (p).

In this master thesis, foreign markets have been represented through an elastic demand function as sales are considered to have slight influence on prices in any case.

2.3.2. Including a Hub

A wholesale gas market is a market where the participants such as producers, regulated and unregulated utilities, and traders buy and sell natural gas. A gas hub is a place where gas wholesale trading is facilitated. Organized markets have been often seen as a prerequisite for gas pricing through gas-to-gas competition.

One key question in the design of a wholesale gas market is the way in which the hub design deals with the gas network.

Two types of organized gas markets depending on the aforementioned regulatory framework (point-to-point transportation in the US or entry-exit access in the EU) can be found: physical hubs and virtual hubs [8]. **Physical hubs** are linked to a specific gas facility where the shipper trade with gas at a price at the location, usually pipelines junctions where a significant amount of gas sales and purchases takes place, and where storage services can be also traded. On the other hand, **virtual hubs** are balancing electronic platforms, associated with a standard set of delivery points. Therefore, virtual hubs are not linked to a specific gas facility or any physical junction of pipelines, but to the gas facilities embedded in a balancing zone.

US wholesale gas markets are fundamentally based on bilateral contracts among producers and marketers (referred as shippers in this document), without the need for any mandatory organized market to trade. Nonetheless, gas supply and demand patterns are highly volatile, so the agents solve their imbalances in the short term at the physical hub, where the delivery of the commodity takes place. The Henry Hub, located in Louisiana, is the intersection of more than a dozen interstate pipelines and it is the most liquid trading hub in North America and the most important physical hub worldwide.

The price coordination between long- and short-term decisions is done through financial contracts in organized financial markets New York Mercantile Exchange (NYMEX), where the underlying asset is usually the delivered gas in the hub [16].

EU gas markets did not go so far into the network details and favor organizing gas transactions around a virtual hub, which is not a physical representation of pipelines, but instead a regulated set of delivery points with a very simplified representation of the actual physical characteristics of the network [21]. The fundamental logic for virtual hubs is to increase the market liquidity associated with the simplification of the network. Several national virtual hubs have been constituted since the mid 90`s. The most liquid are NBP in the UK (1996) and TTF (2003) in the Netherlands. The NBP works as the pricing point for the Intercontinental Exchange (ICE) market, and it is the most liquid trading point in Europe.

Physical and virtual hubs may offer similar services. The most important difference is the operation of the network assets. In a virtual hub, whose main characteristic is embedding transmission and distribution networks in balancing zones, the gas transmission system operator offers a set of network services calculated from the operation of the network and is commonly in charge of monitoring shippers' entries, exits, and inventory variations within the balancing zone. In a physical hub, it exists the possibility of tracking the flow of gas within the pipelines and network services are offered by a company (which may be a pipeline owner or not), using a specific set of physical assets.

Virtual Hub	Physical Hub
- Not linked to a specific **gas facility** - **Balancing electronic platforms** - Gas delivery point: **Virtual location (Balancing zone)** - Services: Network´s short-term balancing Transportation between the entry and exits points ✓ Simplification of the network ✓ Promotes liquidity ✘ Necessary to have some kind of penalties to penalize imbalances	- **Linked to a specific gas facility** - Gas delivery point: **Physical location** - Services: Physical coverage of short-term receipt/delivery balancing needs Transportation between interconnections with other pipelines ✓ More detailed representation of the network ✓ No necessity of penalties or balances procedure ✘ It requires a mature market
Europe: NBP, TTF, GPL	**United States: Henry Hub; Europe: Zeebruge**

Table 2-2– Characteristics of virtual hubs vs. physical hubs

2.3.3. Cost function

The European gas markets previous to the liberalization rely on long term contracts, with big take-or-pay commitments.

Shippers signed long-term import contracts with different suppliers, with take-or-pay commitments, restrictions of use, etc. Figure 2-4 represents an example of a shipper gas portfolio made up of different long-term contracts, with its price quantity relationship, depending on the contract.

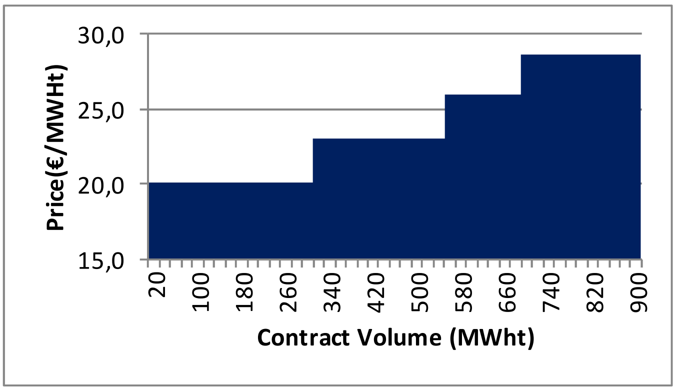

Figure 2-4 –Example of shippers´ gas portfolio contracted through LT contracts

In the model proposed, the portfolio of long-term contracts has been simplified and is represented as an increasing linear cost function, considering that each agent has signed different contracts with different prices. This difference in prices between contracts can be related, mainly, to the flexibility of the contract. For the sake of simplicity, the contracts clauses have not been modeled even though we are conscious of their importance in the current gas sector and their impact in the operation decisions made by shippers.

In this master thesis context, neither contract clauses nor inter-temporarily relationships between the gas consumed in all the periods considered has been modeled, only a maximum amount of gas for each shipper for each period.

Chapter 3

Downstream market representation and impact of agents´ behavior in a gas hub environment

Everything has beauty, but not everyone can see. –Confucius

3.1. Introduction

In the liberalization process, individual incentives may be misaligned with the system's total welfare. Within a market environment, gas companies try to maximize their profits facing a different scenario than the one they were used to with a centralized planner and under a regulated environment. Under this approach, we model the strategic behavior of agents within a market environment, within which they try to maximize their profit. Each agent maximizes their profit facing a captive demand where it behaves as a monopoly, and interacting with the rest of the agents in the hub, in the electricity market and in foreign markets. In order to study the impact of the introduction of a gas hub and how it modifies the behavior of the different agents, two different cases are modeled and compared. First, the decision-taking process of the different shippers will be simulated, without the hub. Second, the hub will be modeled and simulated. Finally, both cases will be compared, to analyze the impact of the hub in agents´ behavior. We are interested in the equilibrium solution of such game, looking for the simultaneous solution of all players (Nash equilibrium) [6] instead of each individual profit.

It is expected that with the introduction of the virtual hub, the marginal cost of all shippers reach a unique value, which coincide with the gas hub price. The previous statement does not hold when supply constraints appear for the shipper, in which case the difference between the hub price and the marginal cost is equal to the willingness to pay of the shipper for an additional unit of gas.

For this purpose, different formulations of the problem are used.

The first resolution approach, named as MCP, is formulated as a Mixed Complementarity Problem, wherein each shipper decides its sales and purchases in the hub subject to a complementarity problem that defines the market equilibrium. The MCP enables the formulation of equilibrium problems but does not guarantee either the existence or the uniqueness of the solution, unless the problem is convex.

The second resolution approach named as MILP reformulates the problem, replacing the complementarities of the problem by inequalities, linearizing them by using binary variables, which permit its formulation as a Mixed Integer Linear Problem (MILP). Hence, the MCP becomes a MILP, guaranteeing the existence and uniqueness of the solution.

The third resolution approach named as KKT, solve the KKT (Karush-Kuhn-Tucker) conditions of the equilibrium problem using a NLP solver.

In this chapter, the model formulation is described in detail, the methods to efficiently solve the problem are presented, a small case then illustrates the previous formulation and, finally, conclusions are drawn.

3.2. Market representation

This subsection describes how agents make their operation decisions, in a deregulated context, competing in quantity to maximize their profits.

An assumption in this model is that firms´ operation decision-making processes occur simultaneously. Therefore, modeling this type of market equilibrium requires the simultaneous consideration of each agents´ profit maximization problem, linked through the hub price resulting from the interaction of all of them, also through the electricity market and through the foreign market. Figure 3-1 represents this market equilibrium subject to the set of constraints *h* and *g*.

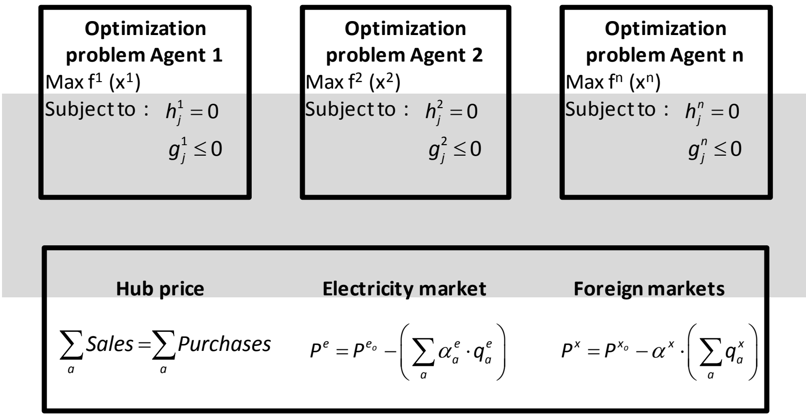

Figure 3-1 – Market equilibrium subject to a set of constraints and all agents´ profit maximization problem linked through the electricity market, foreign market and the hub.

Figure 3-2 shows the scheme of the market equilibrium, where each shipper *a*, maximizes its profits subject to the set of constraints h and g linked through the electricity market and through the foreign markets, without considering the hub.

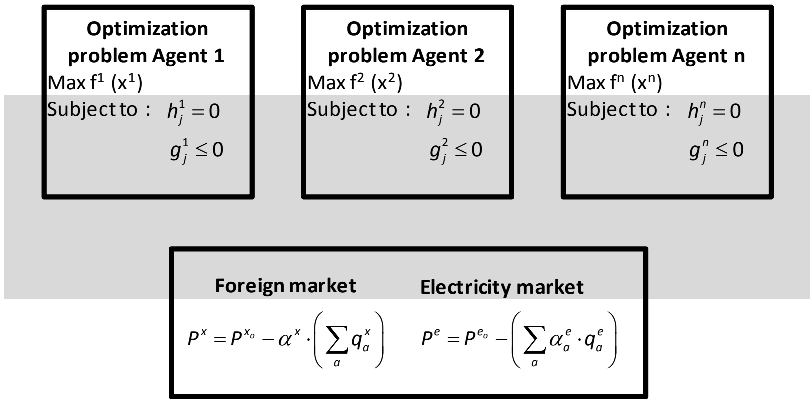

Figure 3-2 – Market equilibrium subject to a set of constraints and all agents´ profit maximization problem linked through the electricity market and foreign markets.

Both markets equilibria are solved and compared, to analyze the influence of the hub in agents´ behavior.

3.3. Model Description

As it has been mentioned before, each agent has its own gas contract portfolio, generally made up of long-term contracts. This market is represented by the following function:

$$C_{ap}(q_{ap}^c) = (P_{ap}^{c_0} + \alpha_{ap}^c \cdot q_{ap}^c) \quad \forall a,p \tag{3.1}$$

Being C_{ap} agents´ cost function, which consist of the intercept of agents´ cost function $P_{ap}^{C_0}$ and the slope of agents´ cost function α_{ap}^c. The volume of gas contracted is represented by q_{ap}^c.

Each agent can use its contracted gas to supply its captive demand q_{ap}^i, to supply its electric demand q_{ap}^e, to send it to the foreign market q_{ap}^x or to sell it in the hub ∇q_{ap}. Each agent can also buy gas in the hub Δq_{ap} to supply its demands.

$$q_{ap}^c = q_{ap}^i + q_{ap}^e + q_{ap}^x - \Delta q_{ap} + \nabla q_{ap} \tag{3.2}$$

The agent behaves as a monopoly when selling the gas to its captive demand. This behavior is represented by the following linear function P_{ap}^i, where $P_{ap}^{i_0}$ is the intercept of agents´ captive demand function per period and α_{ap}^i the slope of agent´s captive demand per period p. The volume of gas supplied to the captive demand is represented by q_{ap}^i.

$$P_{ap}^i = P_{ap}^{i_0} - \alpha_{ap}^i \cdot q_{ap}^i \quad \forall a,p \tag{3.3}$$

Each agent interacts with the rest of the agents in the electricity market. This behavior is represented by the following linear function P_p^e considering an individual agent's influence in the market through the value of the slope α_{ap}^e. $P_p^{e_0}$ is the intercept of the electric demand function per period and q_{ap}^e the gas supplied to the natural gas electric power plants

$$P_p^e = P_p^{e_0} - \left(\sum_a \alpha_{ap}^e \cdot q_{ap}^e \right) \quad \forall p \tag{3.4}$$

Each agent competes in the foreign market with the rest of the agents being represented by the following elastic demand function P_p^x. $P_p^{x_0}$ is the intercept of the electric demand function per period, α_p^x the slope and q_{ap}^x the gas supplied to foreign markets.

$$P_p^x = P_p^{x_0} - \alpha_p^x \cdot \left(\sum_a q_{ap}^x \right) \quad \forall p \tag{3.5}$$

In the proposed formulation, the following market equilibrium has been adopted for representing the gas market. Agents compete in quantity maximizing their profits and choosing their output. The optimization problem for every agent is given by (3.6) subject to (3.7) and (3.8), where P_p^{Hub} represents the price of the hub and $\bar{Q}_{ap}^c$ the maximum amount of gas available from each agents´ long term contracts.

$$\begin{aligned} Max\ & P_{ap}^i(q_{ap}^i) \cdot q_{ap}^i + P_p^e(\sum_a q_{ap}^e) \cdot q_{ap}^e + P_p^x(\sum_a q_{ap}^x) \cdot q_{ap}^x \\ & - C_{ap}(q_{ap}^c) \cdot (q_{ap}^c) - P_p^{Hub} \cdot \Delta q_{ap} + P_p^{Hub} \cdot \nabla q_{ap} \end{aligned} \quad \forall a,p \tag{3.6}$$

s.t.

$$\bar{Q}_{ap}^c \geq q_{ap}^i + q_{ap}^e + q_{ap}^x - \Delta q_{ap} + \nabla q_{ap} \quad \forall a,p \quad : \varepsilon_{ap} \tag{3.7}$$

$$q_{ap}^i, q_{ap}^e, q_{ap}^x, \Delta q_{ap}, \nabla q_{ap} \geq 0 : \mu_{ap}^{q_i}, \mu_{ap}^{q_e}, \mu_{ap}^{q_x}, \mu_{ap}^{\Delta q}, \mu_{ap}^{\nabla q} \quad \forall a,p \tag{3.8}$$

Figure 3-3 represents each agent maximization problem:

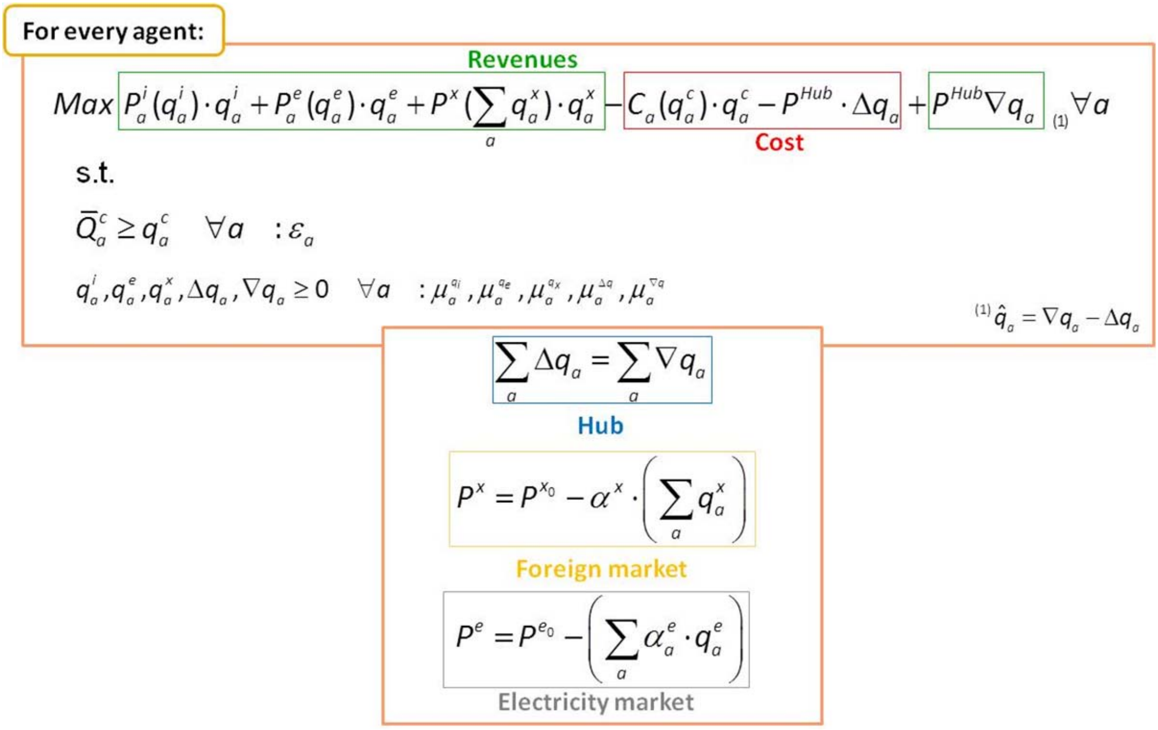

Figure 3-3 – Mathematical model representation of the optimization problem for every agent

The problem is represented as a complementary problem because the complementary slackness conditions have the structure of a complementary problem. This set of equations consists of: the inequality constraints multiplied by their corresponding dual variables, $\varepsilon_{ap}, \mu_{ap}^{qi}, \mu_{ap}^{qe}, \mu_{ap}^{qx}, \mu_{ap}^{\Delta q}, \mu_{ap}^{\nabla q}$ and equal to zero; the inequality constraints themselves; the explicit statement of dual variables, $\varepsilon_{ap}, \mu_{ap}^{qi}, \mu_{ap}^{qe}, \mu_{ap}^{qx}, \mu_{ap}^{\Delta q}, \mu_{ap}^{\nabla q}$ as negative ones.

$$\varepsilon_{ap}\cdot(-\bar{Q}_{ap}^c + q_{ap}^i + q_{ap}^e + q_{ap}^x - \Delta q_{ap} + \nabla q_{ap}) = 0 \quad \forall a,p \tag{3.9}$$

$$\begin{aligned} &\mu_{ap}^{q_i}\cdot(-q_{ap}^i) = 0 \\ &\mu_{ap}^{q_e}\cdot(-q_{ap}^e) = 0 \\ &\mu_{ap}^{q_x}\cdot(-q_{ap}^x) = 0 \qquad \forall a,p \\ &\mu_{ap}^{\Delta q}\cdot(-\Delta q_{ap}) = 0 \\ &\mu_{ap}^{\nabla q}\cdot(-\nabla q_{ap}) = 0 \end{aligned} \tag{3.10}$$

And finally, the gradient of the Lagrangian function with respect to the decision variables: gas demanded by each agent for its captive market q_{ap}^i, gas demanded by each agent for its electric demand q_{ap}^e, gas sent by each agent to its foreign market q_{ap}^e, and the purchases Δq_{ap} and sales ∇q_{ap} of each agent in the hub:

$$\nabla_{q_i} L() = \frac{\partial L}{\partial q^i_{ap}} = -\frac{\partial P^i_{ap}}{\partial q^i_{ap}} \cdot q^i_{ap} - P^i_{ap} + \frac{\partial C_{ap}(q^c_{ap})}{\partial q^i_{ap}} + \varepsilon_{ap} - \mu^{q_i}_{ap} \tag{3.11}$$

$$\nabla_{q_e} L() = \frac{\partial L}{\partial q^e_{ap}} = -\frac{\partial P^e_{p}}{\partial q^e_{ap}} \cdot q^e_{ap} - P^e_{p} + \frac{\partial C_{ap}(q^c_{ap})}{\partial q^e_{ap}} + \varepsilon_{ap} - \mu^{q_e}_{ap} \tag{3.12}$$

$$\nabla_{q_x} L() = \frac{\partial L}{\partial q^x_{ap}} = -\frac{\partial P^x_{p}}{\partial q^x_{ap}} \cdot q^x_{ap} - P^x_{p} + \frac{\partial C_{ap}(q^c_{ap})}{\partial q^x_{ap}} + \varepsilon_{ap} - \mu^{q_x}_{ap} \tag{3.13}$$

$$\nabla_{\Delta q} L() = \frac{\partial L}{\partial \Delta q_{ap}} = P^{Hub}_{p} + \frac{\partial C_{ap}(q^c_{ap})}{\partial \Delta q_{ap}} - \varepsilon_{ap} - \mu^{\Delta q}_{ap} \tag{3.14}$$

$$\nabla_{\nabla q} L() = \frac{\partial L}{\partial \nabla q_{ap}} = -P^{Hub}_{p} + \frac{\partial C_{ap}(q^c_{ap})}{\partial \nabla q_{ap}} + \varepsilon_{ap} - \mu^{\nabla q}_{ap} \tag{3.15}$$

Grouping together all agent´s optimality conditions, the gradient of the Lagrangian function with respect to the decision variables $q^i_{ap}, q^e_{ap}, q^x_{ap}, \Delta q_{ap}, \nabla q_{ap}$ and the complementary slackness conditions leads to a Mixed Complementary Problem. All the optimization problems are linked together through the market price resulting from the interaction of all of them in the hub (3.16), the electricity market (3.17) and the foreign market (3.18.

$$\sum_a \Delta q_{ap} = \sum_a \nabla q_{ap} \quad \forall p \tag{3.16}$$

$$P^e_p = P^{e_0}_{ap} - \left(\alpha^e_p \cdot \sum_a q^e_{ap} \right) \quad \forall p \tag{3.17}$$

$$P^x_p = P^{x_0}_{ap} - \alpha^x_p \cdot \left(\sum_a q^x_{ap} \right) \quad \forall p \tag{3.18}$$

3.4. Solution methods

A standard way to solve these kinds of market equilibrium is by stating the problem in terms of a MCP, as in [22] , [23] mainly due to computational advantages such as the existence of available MCP solving software, such as the PATH solver (algorithm) in the software GAMS [24] , [25]. Besides, with a MCP approach, all the decisions variables of all the firms are considered to be made at the same time, as it should happen in such an equilibrium game. Furthermore, this optimization problem is a convex problem, so a local solution of this optimization problem is also a global one and moreover, considering convexity of the cost functions, it can be said that this solution will be unique. Even though the previously mentioned approach seems very appropriate for small numerical cases, it can be hardly tractable for real-size systems.

It can also be solved very efficiently, by reformulating the problem, replacing the complementarities of the problem by linear inequalities, which permits its formulation as a Mixed Integer Linear Problem (MILP). The complementarities are linearized by using binary variables $b\_\mu_{ap}^{q_i}, b\_\mu_{ap}^{q_e}, b\_\mu_{ap}^{q_x}, b\_\mu_{ap}^{\Delta q}, b\_\mu_{ap}^{\nabla q}$ as follows:

$$\begin{aligned} &Max\ P_{ap}^{i}(q_{ap}^{i}) \cdot q_{ap}^{i} + P_{p}^{e}(q_{ap}^{e}) \cdot q_{ap}^{e} + P_{p}^{x}(\sum_{a} q_{ap}^{x}) \cdot q_{ap}^{x} \\ &-C_{ap}(q_{ap}^{c}) \cdot (q_{ap}^{c}) - P_{p}^{Hub} \cdot \Delta q_{ap} + P_{p}^{Hub} \nabla q_{ap} \end{aligned} \quad \forall a,p \tag{3.19}$$

$$\begin{aligned} &k \cdot b\_\varepsilon_{ap} \geq \varepsilon_{ap} \\ &k \cdot \left(1 - b\_\varepsilon_{ap}\right) \cdot \left(\bar{Q}_{ap}^{c} - q_{ap}^{i} - q_{ap}^{e} - q_{ap}^{x} + \Delta q_{ap} - \nabla q_{ap}\right) = 0 \end{aligned} \quad \forall a,p \tag{3.20}$$

$$\begin{aligned} &k \cdot b\_\mu_{ap}^{q_i} \geq \mu_{ap}^{q_i} \\ &k \cdot \left(1 - b\_\mu_{ap}^{q_i}\right) \geq q_{ap}^{i} \end{aligned} \quad \forall a,p \tag{3.21}$$

$$\begin{aligned} &k \cdot b\_\mu_{ap}^{q_e} \geq \mu_{ap}^{q_e} \\ &k \cdot \left(1 - b\_\mu_{ap}^{q_e}\right) \geq q_{ap}^{e} \end{aligned} \quad \forall a,p \tag{3.22}$$

$$\begin{aligned} &k \cdot b\_\mu_{ap}^{q_x} \geq \mu_{ap}^{q_x} \\ &k \cdot \left(1 - b\_\mu_{ap}^{q_x}\right) \geq q_{ap}^{x} \end{aligned} \quad \forall a,p \tag{3.23}$$

$$\begin{aligned} &k \cdot b\_\mu_{ap}^{\Delta q} \geq \mu_{ap}^{\Delta q} \\ &k \cdot \left(1 - b\_\mu_{ap}^{\Delta q}\right) \geq q_{ap}^{\Delta q} \end{aligned} \quad \forall a,p \tag{3.24}$$

$$\begin{aligned} &k \cdot b\_\mu_{ap}^{\nabla q} \geq \mu_{ap}^{\nabla q} \\ &k \cdot \left(1 - b\_\mu_{ap}^{\nabla q}\right) \geq q_{ap}^{\nabla q} \end{aligned} \quad \forall a,p \tag{3.25}$$

The MCP problem becomes a MILP and the existence and uniqueness of the solution can be guaranteed in most practical situations.

As mentioned before the equilibrium is also solved as a nonlinear problem (NLP), by solving the Karush–Kuhn–Tucker (KKT) optimality conditions.

Therefore, any of the methodologies proposed above can be used for solving the optimization problem.

3.5. Equilibrium solution

Setting up the problem with and without considering the hub and solving the system of equations leads to the following equilibria, the former for the problem without hub and the later with hub.

$$C'(q_{ap}^{c}) = \frac{\partial P_{ap}^{i}}{\partial q_{ap}^{i}} \cdot q_{ap}^{i} + P_{ap}^{i} = \frac{\partial P_{p}^{e}}{\partial q_{ap}^{e}} \cdot q_{ap}^{e} + P_{p}^{e} = \frac{\partial P_{p}^{x}}{\partial q_{ap}^{x}} \cdot q_{ap}^{x} + P_{p}^{x} \quad \forall a,p \tag{3.26}$$

$$P_{p}^{Hub} = C'(q_{ap}^{c}) = \frac{\partial P_{ap}^{i}}{\partial q_{ap}^{i}} \cdot q_{ap}^{i} + P_{ap}^{i} = \frac{\partial P_{p}^{e}}{\partial q_{ap}^{e}} \cdot q_{ap}^{e} + P_{p}^{e} = \frac{\partial P_{p}^{x}}{\partial q_{ap}^{x}} \cdot q_{ap}^{x} + P_{p}^{x} \quad \forall a,p \tag{3.27}$$

The marginal cost of all shippers reach a unique value, which coincide with the gas hub price. The previous statement does not hold when supply constraints appear for the shipper, in which case the difference between the hub price and the marginal cost is equal to the willingness to the pay of the shipper for an additional unit of gas ε_{ap}.

$$C'(q_{ap}^{c}) + \varepsilon_{ap} = \frac{\partial P_{ap}^{i}}{\partial q_{ap}^{i}} \cdot q_{ap}^{i} + P_{ap}^{i} = \frac{\partial P_{p}^{e}}{\partial q_{ap}^{e}} \cdot q_{ap}^{e} + P_{p}^{e} = \frac{\partial P_{p}^{x}}{\partial q_{ap}^{x}} \cdot q_{ap}^{x} + P_{p}^{x} \quad \forall p \tag{3.28}$$

$$P_{p}^{Hub} = C'(q_{ap}^{c}) + \varepsilon_{ap} = \frac{\partial P_{ap}^{i}}{\partial q_{ap}^{i}} \cdot q_{ap}^{i} + P_{ap}^{i} = \frac{\partial P_{p}^{e}}{\partial q_{ap}^{e}} \cdot q_{ap}^{e} + P_{p}^{e} = \frac{\partial P_{p}^{x}}{\partial q_{ap}^{x}} \cdot q_{ap}^{x} + P_{p}^{x} \quad \forall p \tag{3.29}$$

3.6. Case Study

The problem has been implemented in the GAMS language and solved by using PATH for the MCP formulation and CPLEX for the MILP, linearizing the complementarities using binary variables, using an *epgap* of 0% tolerance.

Two different cases have been studied and compared. Both of them consider the captive demand, the electric demand and the foreign market. The first case, named as case (a), optimizes each agents' benefit considering the three markets mentioned above without the hub. The second case is named as case (b) and considers the three previous markets of case (a) plus the interactions of the agents in the hub. The results of both cases are compared and both cases are solved using the three methods (MCP, MILP and NLP) mentioned above.

3.6.1. Case description

The case study represents a hypothetical gas system with three agents, of different sizes. The scope is one period. The cost functions for each agent are presented in Table 3-1.

COST[1]	**Agent 1**	**Agent 2**	**Agent 3**
Intercept	18,0	17,0	18,5
Slope	0,01	0,015	0,0123

Table 3-1 – Cost functions from each agent

And the maximum amount of gas available from the long-term contracts per agent in Table 3-2

AGENT	**qc**
1	660,00
2	500,00
3	700,00

Table 3-2 – Maximum amount of gas from the L.T. contracts

The demand functions faced by each agent are shown in Table 3-3.

Demand[2]		**Agent 1**	**Agent 2**	**Agent 3**
Captive	Intercept	60,00	59,00	65,00
	Slope	0,060	0,070	0,083
Electric	Intercept	90,00	90,00	90,00
	Slope	**0,03**	**0,02**	**0,05**

Table 3-3 – Demand functions faced by each agent

The foreign market is represented by the elastic demand function shown in Table 3-4.

Foreign market	
Intercept	100
Slope	0,04

Table 3-4 – Foreign markets demand function

3.6.2. Results

The obtained results in both cases are shown below. Those results are consistent with theoretical predictions since the agents behave as a monopoly with their captive market and as an oligopoly in the electricity market, and in perfect competition in the hub; hence, the equilibrium is fulfilled. In the equilibrium with the hub, the global profit increases, although individually two agents increase their profits while another decreases its profits.

[1] The units used for the cost curve of captive markets, are MWht for the intercept and €/MWht for the slope.

[2] The units used for the demand functions of captive markets, electricity market and foreign market are MWht for the intercept and €/MWht for the slope.

3.6.2.1. Case (a)

In this case, agents do not interact in the hub, but interact in the electricity market and in the foreign market.

The price of the electricity market is 62,778 €/MWht and of the foreign market is 65,278€/MWht. Most of the gas is used in the electricity market or diverted to foreign markets, while the captive demand receives less. Agent 1 has the higher profit and agent 2 the lower.

Agent	q^{i}_{ap}	q^{e}_{ap}	q^{x}_{ap}	Δq_{ap}	∇q_{ap}
1	52,888	311,207	295,905	-	-
2	13,456	282,696	203,848	-	-
3	87,070	244,635	368,294	-	-

Table 3-5 Gas consumption by conventional demand, used in electricity sector, sent to foreign market, per agent in MWht.

Agent	Income	Cost	Profit
1	41852,9	16236	25616,9
2	31835,16	12250	19585,16
3	44429,5	18976,96	25452,54
		Total	**70654,6**

Table 3-6 Profits by agent (€)

3.6.2.2. Case (b)

In this case, as mentioned before, agents interact in the hub, fulfilling the equilibrium condition that the volume of sold gas in the hub by all agents should be the same as the bought one. They also interact in the electricity market and in the foreign market. The price of the electricity market is 63,946 €/MWht, the price of the foreign market is 66,446 €/MWht and the price in the hub is 55,261 €/MWht. Most of the gas is used in the electricity market or is diverted to foreign markets, while conventional demand receives less. Agent 3 has the higher profit and agent 2 the lower. Agents 1 and 3 sell gas and agent 2 purchases gas in the hub. The volume of diverted gas to foreign markets with the hub is the same for all agents, because they have the same influence in the market as the equilibrium conditions indicate.

Agent	q^{i}_{ap}	q^{e}_{ap}	q^{x}_{ap}	Δq_{ap}	∇q_{ap}
1	38,219	289,493	279,62		52,668
2	26,823	434,24	279,62	240,683	
3	58,67	173,696	279,62		188,015

Table 3-7 Gas consumption by conventional demand, used in electricity sector, sent to foreign market, purchase and sales in the hub per agent in MWht.

Agent	Income	Cost	Profit
1	42204,61	16236,00	25968,61
2	47879,95	25550,38	22329,57
3	43604,54	18977,04	24627,51
		Total	**72925,69**

Table 3-8 Profits by agent (€)

3.6.3. Conclusions

From the case study, we obtained that most of the gas is used in the electricity market or is diverted to foreign markets, while conventional demand receives less. These conclusions are input-data-dependent and related to the used price functions for each market, but show in which markets agents would obtain larger profits since the elasticity values are close to reality.

With the incorporation of the hub the global profit increases, although not all the agents increase their profits with their participation in the hub. Furthermore, some agents can have their profits reduced for participating in the hub.

Regarding the different formulations used for solving the problem, all of them lead to the same solution and in similar times (0,015-0,016 Sec).

3.7. General conclusions

This study presents the behavior of agents in the hub. The marginal costs of all shippers reach a unique value, which coincide with the gas hub price. The previous statement does not hold when supply constraints appear for the shipper, in which case the difference between the hub price and the marginal cost is equal to the willingness to pay of the shipper for an additional unit of gas ε_{ap} .

$$P_p^{Hub} = C'(q_{ap}^c) + \varepsilon_{ap} \quad \forall p \tag{3.30}$$

The aggregated profit of the agents is increasing even when anticompetitive behavior is not explicitly represented. Constituting a hub might therefore be a necessary, but not sufficient, solution to introduce competition.

The volume of diverted gas to foreign markets with the hub is the same for all agents, because they have the same influence in the market. This conclusion is based on the equilibrium solution:

$$P_p^{Hub} = \frac{\partial P_p^x}{\partial q_{ap}^x} \cdot q_{ap}^x + P_p^x \quad \forall p \tag{3.31}$$

As it has been mentioned above, the three formulations (MCP, MILP and NLP (KKT)) which have been used are adequate for solving the problem. However, each formulation has its own advantages and disadvantages.

- **MCP** formulation, using PATH solver, is the most appropriate one, as this solver has been specially developed to solve this type of non-linear-problems. If the problem is not convex, it can lead to different equilibrium solutions and the global optimum cannot be ensured.
- **MILP** formulation, using CPLEX solver, can be more time consuming for bigger problems, due to the introduction of binary variables in order to linearize the complementarities. Another drawback of this formulation might be the selection of the constant value. On the other hand, with this formulation, it can be ensured, that the obtained solution is the global optimum.
- **NLP** formulation, using the KKT and solved with CONOPT solver, works properly for small cases as the one presented. However, this type of solvers are prepared to solve any type of non-linearity, so it might be more appropriate to use MCP formulation and PATH solver, that are specific for this kind of problems and non-linearities.

Chapter 4

Iterative optimization problem

A person who never made a mistake never tried anything new. – Albert Einstein

4.1. Introduction

Even if the most common way for solving equilibrium problems is the Mixed Complementary Problem (MCP) approach, this formulation have some drawbacks. First, the size of the involved problem as it cannot be used to solve large problems. Second, binary variables cannot be used and the problem should be convex in order to ensure that the solution found in this optimization problem is also a global one. For solving these shortcomings, we propose an iterative optimization problem that leads to the same solution.

Therefore, the objective is to propose an iterative optimization problem which represents the behavior of the agents in the hub. As a first step in the development of this iterative optimization problem the former MCP model has been simplified and only a captive demand has been considered. This model is later compared with the proposed iterative optimization model.

4.2. Model Description: MCP Formulation

The former model has been simplified and only one demand has been considered in this case, the conventional demand. For avoiding duplicity due to the synergies with the former model, only the maximization problem is formulated, where P_{ap}^{i} is the captive demand price function for each agent, q_{ap}^{i}, the volume of gas used to supply each agents´ captive demand, C_{ap} each agents´ cost function, q_{ap}^{c} the volume of gas contracted by agent from long-term contracts, P^{Hub} is the price in the hub and $\Delta q_{ap}, \nabla q_{ap}$ purchases and sales in the hub by agent and period.

$$Max\ P_{ap}^{i}(q_{ap}^{i}) \cdot q_{ap}^{i} - C_{ap}(q_{ap}^{c}) \cdot (q_{ap}^{c}) - P_{p}^{Hub} \cdot \Delta q_{ap} + P_{p}^{Hub} \nabla q_{ap} \quad \forall a,p \tag{4.1}$$

s.t.

$$\bar{Q}_{ap}^{c} \geq q_{ap}^{i} - \Delta q_{ap} + \nabla q_{ap} \quad \forall a,p \quad : \varepsilon_{ap} \tag{4.2}$$

$$q_{ap}^{i}, \Delta q_{ap}, \nabla q_{ap} \geq 0 : \mu_{ap}^{q_i}, \mu_{ap}^{\Delta q}, \mu_{ap}^{\nabla q} \quad \forall a,p \tag{4.3}$$

Each agent can use its contracted gas to supply its captive demand q_{ap}^{i} or to sell it in the hub ∇q_{ap}. The volume of gas available by each agent from long term contracts is limited by a maximum $\bar{Q}_{ap}^{c}$. Each agent can also buy gas in the hub Δq_{ap} to supply its captive demand.

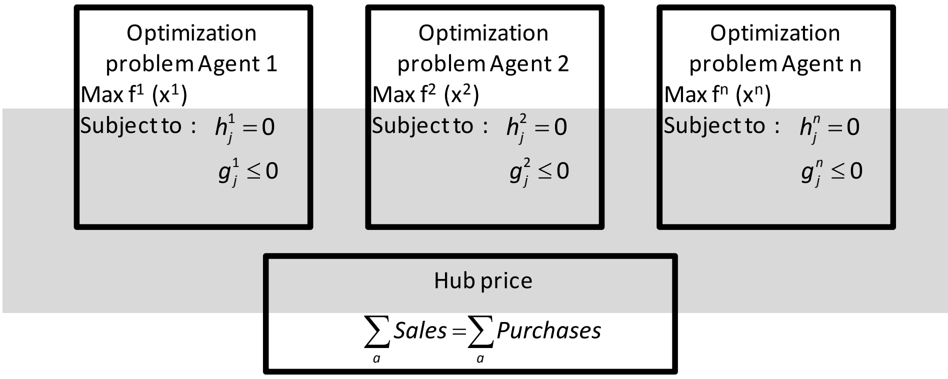

Figure 4-1 – Market equilibrium subject to a set of constraints and all agents´ profit maximization problem linked through the hub.

4.3. Model Description: Iterative optimization problem Formulation

The aim of this section is to formulate an iterative optimization problem to solve the market equilibrium as an optimization problem, in order to overcome the limitations of the MCP formulation as it has been done in [26].

The proposed structure is an iterative optimization problem which is presented in Figure 4-3:

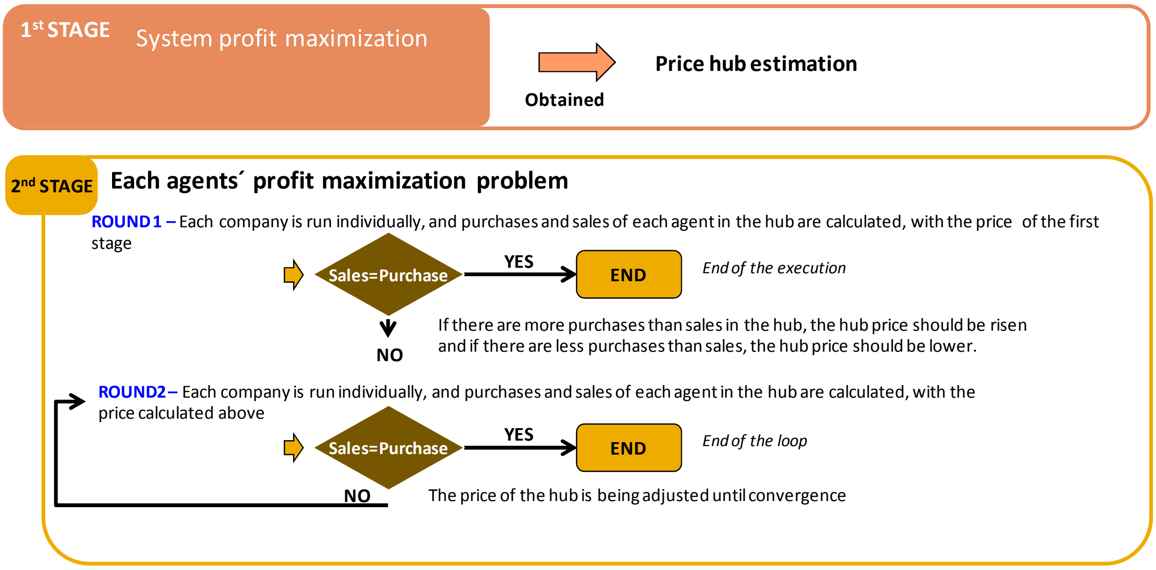

Figure 4-3 – Structure of the proposed iterative optimization problem

In the first stage the system profit maximization is calculated, maximizing revenues minus costs. Each agent sells a volume of gas q_{ap}^{i} to its captive demand at $P_{ap}^{i}\left(q_{ap}^{i}\right)$.

Costs are represented by the convex cost function $C_{ap}^{c}\left(q_{ap}^{c}\right)$, where q_{ap}^{c} is the volume of gas contracted by agent form long term contracts.

$$Max\sum_{a} P_{ap}^{i}(q_{ap}^{i})\cdot q_{ap}^{i} - \sum_{a} C_{ap}(q_{ap}^{c})\cdot(q_{ap}^{c}) \quad \forall a,p \tag{4.4}$$

s.t.

$$\bar{Q}_{ap}^{c} \geq q_{ap}^{i} - \Delta q_{ap} + \nabla q_{ap} \quad \forall a,p \tag{4.5}$$

$$q_{ap}^{i}, \Delta q_{ap}, \nabla q_{ap} \geq 0 \quad \forall a,p \tag{4.6}$$

$$\sum_{a} \Delta q_{ap} = \sum_{a} \nabla q_{ap} \quad \forall p \quad \rightarrow \quad \lambda_0 : \text{Price Hub} \tag{4.7}$$

The price of the hub is the dual variable λ_0 of the hub constraint which states that the summation of the purchase of all agents must be equal to the summation of the sales of all agents in the hub.

The second stage is split into two rounds. In the first round, each agents' profit maximization is calculated using the price hub P_{p}^{Hub} obtained in the systems' profit maximization in the first stage.

$$Max\ P_{ap}^{i}(q_{ap}^{i})\cdot q_{ap}^{i} - C_{ap}(q_{ap}^{c})\cdot(q_{ap}^{c}) - P_{p}^{Hub}\cdot\Delta q_{ap} + P_{p}^{Hub}\cdot\nabla q_{ap} \quad \forall a,p \tag{4.8}$$

s.t.

$$\bar{Q}_{ap}^{c} \geq q_{ap}^{i} - \Delta q_{ap} + \nabla q_{ap} \quad \forall a,p \tag{4.9}$$

$$q_{ap}^{i}, \Delta q_{ap}, \nabla q_{ap} \geq 0 \quad \forall a,p \tag{4.10}$$

Each agents' purchases Δq_{ap} and sales ∇q_{ap} in the hub are obtained for that hub price. Next, it is checked if the balance (4.11) of the hub is fulfilled.

$$\sum_{a} \Delta q_{ap} = \sum_{a} \nabla q_{ap} \quad \forall p \tag{4.11}$$

If the hub balance (4.11) is fulfilled, then the solution has been found and the price corresponds to the obtained price by solving the equilibrium.

If the sum of the purchase of all the agents in the hub is greater than the sum of the sales of all agents, it means that the obtained hub price in the first stage from the system profit maximization is lower than the sought price corresponding to the price in the equilibrium. In this case, the price hub must be risen for the second round. On the contrary, if the sum of the purchase of all the agents in the hub is lower than the sum of the sales of all agents, means that the obtained hub price in the first stage from the system profit maximization is higher than the sought price. In this case, the price hub must be lowered for the second round.

In the second round, for the first iteration, each agents' profit maximization is calculated by using the new price hub, modified in the first round. The price for the rest of the iterations in the loop is calculated using linear functions to find the next price until the problem converges. Convergence may be assure by the Cobwed theorem [27].

If the supply curve is steeper than the demand curve, then the fluctuations decrease in magnitude with each cycle, as plot in Figure 4-4. This is called the stable or convergent case.

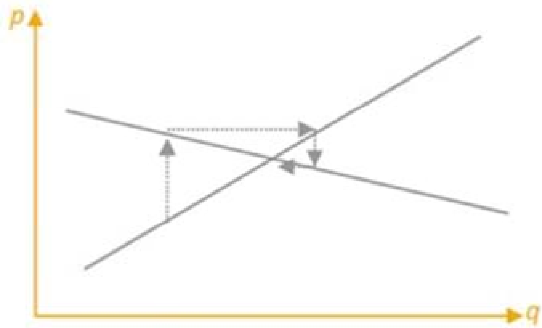

Figure 4-4 – Plot of the prices and quantities over time would look like an inward spiral

If the slope of the supply curve is less than the absolute value of the slope of the demand curve, then the fluctuations increase in magnitude with each cycle, so that prices and quantities spiral outwards, Figure 4-5. This is called the unstable or divergent case.

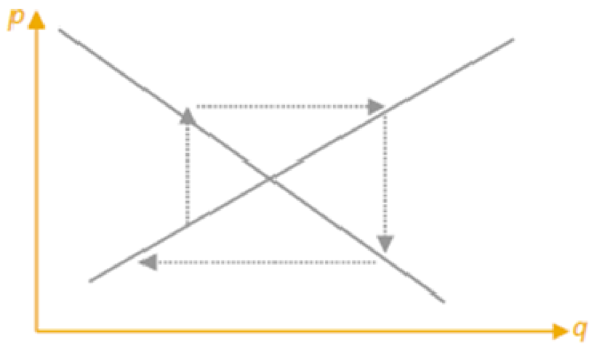

Figure 4-5 – Plot of the prices and quantities in a unstable case

4.4. Solution methods

Two different ways to solve the problem are proposed and afterwards compared.

The first one is by stating the problem in term of a MCP as it has been done in Chapter 3and by using the PATH solver in the software GAMS [24] , [25].The second way is reformulating the problem as an iterative optimization problem, which permits the formulation of the problem as a quadratic constrained programming (QCP), in GAMS software.

4.5. Case Study

The problem has been implemented in the GAMS language and solved by using PATH for the MCP formulation and CPLEX for the QCP formulation.

Each agent optimizes its benefits by considering just the captive demand and the interaction of the agents in the hub. The problem is solved using the above mentioned methodologies. Both methodologies to solve the problem are studied and compared.

The first case, named as case (a), is solved by using MCP formulation while the second case is named as case (b) and is solved using the iterative optimization problem.

4.5.1. Case description

The case study represents a hypothetical gas system with three agents, of different sizes. The scope is one period. The cost functions for each agent are presented in Table 4-1.

COST[3]	**Agent 1**	**Agent 2**	**Agent 3**
Intercept	18	17	18,5
Slope	0,01	0,015	0,0123

Table 4-1 – Cost functions for each agent

And the maximum amount of gas available from the long term contracts per agent in Table 4-2.

[3]The units used for the cost curve of captive markets, are MWht for the intercept and €/MWht for the slope.

AGENT	Quantity
1	660,00
2	500,00
3	700,00

Table 4-2 – Maximum amount of gas from the L.T. contracts

The demand functions faced by each agent are shown in Table 4-3

Demand[4]		Agent 1	Agent 2	Agent 3
Captive	Intercept	60,00	59,00	65,00
	Slope	0,06	0,07	0,083

Table 4-3 – Demand functions faced by each agent

4.5.2. Results

Both models lead to the same results (quantity used to supply gas for conventional demand, exchanges between agents in the hub and the hub price.

Agent	q^i_{ap}	Δq_{ap}	∇q_{ap}
1	288,575		22,260
2	249,521	8,965	
3	245,682	13,296	

Table 4-4 – Gas used to supply conventional demand and purchase and sales in the hub per agent in MWht.

The price obtained in the hub is 24.217 €/MWht.

In this case, due to the simplicity of the presented case study, is not possible to draw general conclusions. In the described case study, the obtained price of the hub by using the MCP formulation is the same as the one obtained in the first stage of the iterative problem, with the system profit maximization. This occurs, because each agents' profit maximization problem, is just linked with the rest of the agents through the hub.

[4]The units used for the demand functions of captive markets, electricity market and foreign market are MWht for the intercept and €/MWht for the slope.

4.6. Conclusions

Firstly, the equilibrium where each agent supplies it captive demand and interacts with the rest of the agents in the hub maximizing its profits is modeled as an MCP.

Secondly, an alternative formulation for this equilibrium problem is proposed to overcome the limitations of the MCP formulation, solving the equilibrium through an iterative optimization problem that leads to the same solution as the MCP formulation.

Thirdly, a bigger and more complicated case is necessary to draw conclusions about the effectiveness of the methodology for solving the equilibrium as an iterative optimization problem.

Finally, this iterative problem will have some limitations when trying to solve the equilibrium problem. We can lead to non-converge, as in this case, the iterative problem does not have the same equilibrium problem conditions and, as it has been defined, it cannot lead to a Nash equilibrium that is not Pareto optimal. This means, that in the Nash equilibrium, each agent is making the optimal choice, given the other agent's choice. The Nash equilibrium, can lead to situations, which might appear non-rational in a third-person perspective. In the iterative optimization problem, as in the second stage, each agent is optimized isolated, without taking into account other agent's decisions; hence, the agent would never choose any decision that is not its optimal decision.

Chapter 5

Conclusions and future research

I would rather die of passion than of boredom. –Vincent van Gogh

5.1. Conclusions

The modeling contribution is one of the main contributions of this master thesis. This master thesis filled a gap in the representation of the different demands and in the representation of a hub in an entry-exit system model. Furthermore, the proposed iterative optimization problem methodology, for solving the equilibrium, can be implemented in bigger optimization problems, for solving the interaction of the agents in the hub. However, there is still much work to do.

The conclusions drawn in the previous chapters (Chapter 3 and Chapter 4) are summarized below:

In Chapter 3 the **behavior of agents in the hub is presented**. The demand each agent faces has been segmented in three different categories: Each agent can use its contracted gas to: 1) supply its **captive demand**, which has been represented as a monopoly, 2) supply its **electric demand**, represented as a linear function considering individual agent's influence in the market through the value of the slope, 3) send it to the **foreign market**, represented by an elastic function, 4) sell it in the hub. Each agent can also buy gas in the hub to supply its demands.

Agents compete in quantity maximizing their profits and choosing their outputs. The problem is represented as a complementary problem. Three different formulations are used to solve the problem. Firstly the problem is stated as a Mixed Complementary Problem **(MCP)**. Secondly, the complementarities are linearized using binary variables and the problem is solved as an Mixed Integer Linear Programming **(MILP).** Finally, the third resolution approach, solve the KKT (Karush-Kuhn-Tucker) conditions of the equilibrium problem using a Non-Linear Programming **(NLP)** solver.

Solving the equilibrium problem, the following conclusions are drawn:

- The marginal costs of all shippers reach a unique value, which coincide with the gas hub price. The previous statement does not hold when supply constraints appear for the shipper, in which case the difference between the hub price and the marginal cost is equal to the willingness to pay of the shipper for an additional unit of gas.
- The agents prefer to use their gas to generate electricity or to divert their gas to foreign markets because greater profits are obtained through these activities.
- The volume of diverted gas to foreign markets with the hub is the same for all agents, because they have the same influence in the market.

- With the incorporation of the hub the global profit increases, although not all the agents increase their profits with their participation in the hub. Furthermore, some agents can have their profits reduced for participating in the hub.
- The mere constitution of a hub might not improve the competitiveness of the market, even when anticompetitive behavior in the hub has not been considered.
- The three formulations (MCP, MILP and NLP (KKT)) which have been used are adequate for solving the problem. Even each formulation has its own advantages and disadvantages **MCP** formulation, using PATH solver, is the most appropriate one, as this solver has been specially developed to solve this type of non-linear-problems. The problem is convex, so in this case, the obtained solution of this optimization problem is also a global one. The **MILP** formulation, using CPLEX solver, can be more time consuming for bigger problems, due to the introduction of binary variables in order to linearize the complementarities, but it can be used to ensure, that the obtained solution is the global optimum and as a starting point for the MCP formulation if necessary.

In Chapter 4, the first steps for an alternative formulation for this equilibrium problem are proposed, in order to overcome the limitations of the MCP formulation, solving the equilibrium through an **iterative optimization problem** that leads to the same solution as the MCP formulation.

The proposed formulation has been modeled for a simple case and compared to the MCP, reaching both the same solution. A bigger and more complicated case is necessary to draw conclusions about the effectiveness of the methodology for solving the equilibrium as an iterative optimization problem.

Even if this formulation might overcome some limitations of the MCP formulation (the size of the problem and binary variables), this iterative problem may have some limitations when trying to solve the equilibrium problem. We can lead to non-converge and, as it has been defined, it cannot lead to a Nash equilibrium that is not Pareto optimal.

5.2. Future research

As future research guidelines:

Although one of the main contributions of this master thesis is the segmentation of the different demands, further research considering different elasticity, trying to represent each market closer to reality, cross-price elasticity of demand, which measures the responsiveness of gas demand to a change in a substitute good price or the effect of consumers' switching behavior could be incorporated.

As supply contracts representation has been simplified, it might be interesting to improve how supply contracts are modeled, by including minimum volumes and take-or-pay clauses. Moreover, supply contracts could be modeled over periods.

The iteration optimization problem methodology needs to be tested in more complex cases, to be able to draw general conclusions.

Finally, both proposed models could be included in a bigger or more complex model.

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