UNIVERSIDAD PONTIFICIA COMILLAS DE MADRID
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ANALYSIS OF THE IMPACT OF SUBSEQUENT MARKETS AND MECHANISMS USED TO RESOLVE TECHNICAL CONSTRAINTS ON WHOLESALE ELECTRICITY MARKETS

Análisis del impacto de mercados y mecanismos posteriores usados para resolver las restricciones técnicas en los mercados de electricidad

Tesis para la obtención del grado de Doctor

Director: Dr. D. Javier Reneses Guillén

Autor: MSc. D. Andrés Ramiro Delgadillo Vega

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AUTOR: Andrés Ramiro Delgadillo Vega

DIRECTOR: Javier Reneses Guillén

TUTOR-PONENTE:

DEPARTAMENTO: INSTITUTO DE INVESTIGACIÓN TECNOLÓGICA

FACULTAD O ESCUELA: ESCUELA TÉCNICA SUPERIOR DE INGENIERÍA

Miembros del Tribunal Calificador:

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VOCAL: **Firma:**

VOCAL: **Firma:**

SECRETARIO: **Firma:**

Fecha de lectura:

Calificación:

A mi familia

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Dissertation

This doctoral thesis includes an analysis of the effects of network constraints in the strategic behavior of generation companies. This thesis is based on the work of 7 articles (2 in journals with JCR, 2 under review and 3 in conferences). These are:

Journal Articles

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Delgadillo, A., Reneses, J., Barquín, J., Feb. 2013. Effect of network congestions between areas on single-price electricity markets. IEEE Trans. Power Systems 28 (1), 93–101.

Under review

Delgadillo, A., Reneses, J., 2014a. Analysis of the effect of voltage level requirements on an electricity market equilibrium model.

Delgadillo, A., Reneses, J., 2014b. Counter-trading mechanisms in Europe. The cases of Spain, the Netherlands, Portugal and Germany.

Conference Papers

Delgadillo, A., Reneses, J., Jul. 2013. Analysis of the Spanish congestion management mechanism. In: 2013 IEEE Power and Energy Society General Meeting. pp. 1–5.

Delgadillo, A., Reneses, J., May 2014. Electricity market equilibrium model with voltage constraints. In: 11th International Conference on the European Energy Market - EEM 14. pp. 1–4.

Delgadillo, A., Reneses, J., Barquín, J., Aug. 2011. Effect of technical network constraints on single-node electricity markets. In: 17th Power Systems Computation Conference - PSCC'11. pp. 1–6.

Chapter 1

Introduction

Contents

This Chapter introduces the context of this thesis and defines the main objectives. It presents a literature review of the different studies related to the relevant research topics.

1.1 Context

Until recently, electric power systems were natural monopolies subject to government control. Decisions about managing and operating power systems were centralized and the ability of private companies to act independently was limited by public authorities. However, in the past decades several countries have liberalized and deregulated their electric power systems. These processes tend toward disintegration, and the development of market economies and competition in the different activities of the sector with the aim of seeking greater efficiency, as well as more safety, quality, reliability and sustainability of electricity service.

These liberalization processes in the generation activity have resulted in the development of wholesale electricity markets in which most of the energy is traded, and causing the decentralization of decision-making processes. Therefore, generation companies have to make their own decisions, taking into account the behavior of other companies, meteorological variables, fuel prices, etc.

On the other hand, in recent years, the markets and mechanisms used to clear technical and security constraints are becoming more important due to the increase in the number and severity of network constraints caused by the emergence of distributed generation, the penetration of renewable energies and the flexibility of demand with response capability.

This thesis studies the effects of different markets and mechanisms used to clear technical and security constraints, such as network congestion, voltage level and reserve requirements, on the behavior of the generation companies in single-price electricity markets in the medium-term horizon. The thesis explores the use and generalization of industrial organization models to this problem. Specifically, the thesis analyzes the application of oligopoly models based on conjectural variations initially developed for the analysis of electricity markets without technical constraints.

1.2 Background

This section describes the problem addressed, including a critical review of existing research on the thesis research topics. Section 1.2.1 provides a general description of the different game theory models used in electricity markets. A more detailed analysis and classification of the different models taking the technical and security constraints into account is presented in 1.2.2.

Studies carried out in recent years to analyze different designs and operation of electricity markets have required the use of appropriate analysis and modeling techniques to understand companies' strategic behavior in those markets. The study of this strategic behavior is a research area in which a variety of fields converge. In addition to electrical engineering, this research depends on methods from microeconomic theory, game theory, mathematical programming, computational simulation models and operations research.

An important factor in the companies' decision-making processes is the determination of the companies' behavior in the different markets because the decisions made in one market can directly affect the decisions made in other markets. The companies' strategic behavior can be analyzed in different time frames, from short term to long term. Short-term (defined as real time operation up to a few weeks) decisions include bids in the power market and power system operation. Medium-term (defined as few months up to 2-3 years) decisions are related with water resources management, fuel purchases, bilateral contracts, as well as analysis of regular situations that agents can predict such as usual network congestion. Finally, long-term (defined as more than 3 years) decisions involve expansion of the power system, the building of new plants and research into new technologies.

This thesis focuses on the medium-term horizon. The majority of the medium-term models developed to study companies' strategic behavior in electricity markets have focused on the analysis of the day-ahead electricity market and usually do not assess the impact of subsequent markets and mechanisms used to clear technical and security constraints.

1.2.1 Game theory

In the medium-term, companies' strategic behavior is usually studied using game theory models. These models try to determine the outcome of the interaction between different companies under the assumption of rational behavior. Rational companies are those that make their decisions with the goal of maximizing their own welfare. Each generation company develops strategies based on the characteristics of its generation units, the constraints of the power system, the predicted behavior of the different variables subject to uncertainty (meteorological variables, fuel prices, etc.) and predictions of the past, present and future behavior of its competitors.

In game theory there are different formulations and representations of the companies' behavior. There are cooperative and noncooperative games. The theory of the cooperative games develops axioms in order to model the idea of fairness while in the noncooperative games the decisions of the companies are based only on their self-interest and made may depend on their forecast of the decisions of the rest of the companies. However, companies in noncooperative games can exhibit cooperative behaviors under certain circumstances (Fudenberg and Tirole, 1991).

The different results of the interaction of the companies are known as equilibrium points of the game. One of the most-used definitions of equilibrium in noncooperative games is the Nash Equilibrium (Nash, 1950). At the Nash Equilibrium, the companies' behavior is modeled using a strategic game where companies take an action knowing that the rest of companies play in the same way. Thus, the result of this interaction is reached when each company maximizes its own profit, taking into account that the rest of the companies also maximize their profits. In this solution, companies have no incentive to modify their strategic behavior unilaterally because any deviation entails a decrease in benefits. Hence, game theory models are useful for analyzing the different variables that affect the companies' strategic behavior and to determine the different equilibrium states of the system. Equilibrium models based on game theory are based on a formal definition of the problem which is expressed in systems of algebraic or differential equations.

Therefore, the goal in equilibrium models is to determine the best strategy for the companies based on assumptions about the best strategies for the other companies. The solution to this problem can be found by enumerating all the possible combinations of the different strategies of all companies or determining the optimality conditions for each company to represent their decision process.

The most widely used equilibrium models assume that companies decide their strategies simultaneously, without the possibility to react against the decisions of its rivals. Among them are perfect competition models; Bertrand models where companies compete in prices; Cournot models where agents compete in quantities; supply function equilibrium models where strategic behavior is modeled by means of supply functions that combine price and quantity competition; and conjectural-variation-based equilibrium models where the supply functions are restricted to a smaller set of parameterized curves.

1.2.1.1 Perfect competition models

Perfect competition describes markets in which companies do not have market power to set the price. One of the characteristics of these markets is that there are a large number of buyers and sellers with complete and symmetric information, and therefore none of them may influence the price of the market and all companies are price-takers. The result of the perfect competition corresponds to the maximization of the total social welfare and it is useful to contrast the results of other market situations.

In the literature there are some perfect competition models that analyze the influence of the network constraints on the outcome of the electricity market. Among them are the models proposed in Holmberg and Lazarczyk (2012) and Oggioni and Smeers (2013) as presented in Section 1.2.2.

1.2.1.2 Bertrand models

In Bertrand models, companies determine the price of a homogenous product and the buyers decide the quantity to buy at that price. The Nash equilibrium in this market situation corresponds to the non-cooperative result in which the companies charge the marginal cost of the product, and therefore this is equivalent to the perfect competition result when it is assumed that each company can supply the whole demand. However, the Bertrand solution is not necessarily the perfect competition solution when the power network is taken into account, and prices can be above marginal cost as presented in Hobbs (1986) and Younes and Ilic (1999).

1.2.1.3 Cournot models

In Cournot models, companies produce an identical product and their strategies are to decide the quantity to be produced. The market price is determined by the aggregate supply function and the market demand function. Typically, the market price in this situation is considerably higher than the price in a perfect competition model.

These models have been widely used in studies of market power in electricity markets (Younes and Ilic, 1999; Day et al., 2002; Yu et al., 2002; Peng and Tomsovic, 2003; Wang et al., 2007; Bautista et al., 2007a,b; Barquín and Vazquez, 2008; Yao et al., 2008; Dijk and Willems, 2011) with and without network constraints as shown in Section 1.2.2. The market power of a company in an oligopoly market refers to the influence of this company on the market price and the distance between the equilibrium point and the equilibrium reached in a perfect competition model.

1.2.1.4 Supply function equilibrium models

Unlike Cournot models where strategies are only given in terms of quantities, supply function equilibrium models developed by Klemperer and Meyer (1989) are useful to analyze markets in which companies determine their strategies in terms of both price and quantities. Although supply function equilibrium models extend the set of strategies with respect to Cournot models, their disadvantage is that a set of differential equations or a set of non-linear equations have to be solved with a high computational cost, and therefore, these models are intractable in real-size electricity systems.

There are several works that study the strategic behavior of generation companies in electricity markets using supply function equilibrium models as presented in Section 1.2.2. Among them there are the models proposed in Green and Newbery (1992); Younes and Ilic (1999); Berry et al. (1999); Day et al. (2002); Xian et al. (2004); Bompard et al. (2006); Bautista et al. (2007c); Wang et al. (2007); Liu and Wu (2007); Bompard et al. (2010); Dijk and Willems (2011); Soleymani (2013); Petoussis et al. (2013); Langary et al. (2014).

1.2.1.5 Conjectural-variation-based equilibrium models

In conjectural-variation-based equilibrium models, supply functions are restricted to a smaller set of parametrized curves. In this situation, the strategic behavior is modeled by means of a parameter known as the company conjecture. This parameter reflects the reaction of a company when it decides its optimal offer, and it could represent the reaction of the competitors, the change in market price when the company changes the quantity produced or the demand elasticity with respect to the company's production. Usually, this parameter is defined exogenously, and it is determined using historical data. An important feature of conjectural-variation-based equilibrium models is that it is possible to study different kinds of competition, from perfect competition to Cournot oligopoly.

In recent years, different works have presented conjectural-variation-based models to analyze the outcome of electricity markets (Day et al., 2002; Hobbs and Rijkers, 2004; Hobbs et al., 2004; Barquín et al., 2004; Song et al., 2004; Fernández-Menéndez et al., 2005; Nam et al., 2006; Centeno et al., 2007; Liu et al., 2007; Barquín et al., 2009; Chitkara et al., 2009; Kurzidem, 2010; Díaz et al., 2012; Oggioni et al., 2012; Petoussis et al., 2013; Díaz et al., 2014) as explained in Section 1.2.2.

1.2.2 Technical and security system constraints

Technical and security system constraints can play a fundamental role in modeling electricity markets. Varied and contradictory results can arise depending on the fundamental assumptions made in the electricity market model (Neuhoff et al., 2005). Disregarding the effect of the network can result in an inadequate representation of the strategic behavior of market participants. This is primarily because the existence of the network can increase the opportunities for exercising market power (Cardell et al., 1997; Hogan, 1997; Younes and Ilic, 1999; Borenstein et al., 2000; David and Wen, 2001; Yu et al., 2002; Peng and Tomsovic, 2003; Xiao and Wang, 2004; Xian et al., 2004; Barquín and Vazquez, 2008). For example, congestion in one or more lines can isolate certain areas of the power system, and therefore only a small number of plants can supply the demand in those areas.

1.2.2.1 Network congestion

Network congestion is one of the system constraints that may alter the outcome of the electricity market. Congestion occurs when the operational or policy constraints of the transmission network are violated and it is therefore not possible to deliver all the energy from one node to another. Different mechanisms can be used to clear the congestion. The choice of a particular mechanism depends on the market design and the system size, i.e., it is highly dependent on regulatory models, and may undergo significant changes in a short time period. A comprehensive review of the literature on congestion management in different deregulated electricity markets can be found in Kumar et al. (2005).

The most efficient way to determine electricity prices is the nodal-pricing system (Green, 2007; Neuhoff et al., 2011; Dijk and Willems, 2011; Holmberg and Lazarczyk, 2012; Oggioni and Smeers, 2013). This market design explicitly takes the different technical constraints imposed by the power network into account and efficiently remunerates the costs of producing and transporting energy through the different nodes, which may lead to different electricity prices in each node of the system (Schweppe et al., 1988). Nevertheless, many countries have chosen to use a single-price system. In this market design, the electricity network is not taken into account in the market-clearing process. It is assumed that the energy is traded on a single node, and that the electricity price is the same for all areas in the system. The single-price electricity market design may be suitable when the transmission network is sufficiently robust and when there are no constraints to cause significant congestion in the system. However, internal congestion is a problem that has arisen in several markets with a single-price system. Therefore, in order to clear this congestion, an additional mechanism, usually known as counter-trading, is implemented subsequent to the market-clearing process. In this mechanism, necessary adjustments are made to remove congestion that has appeared in the system, and to reward the companies involved.

In addition to the nodal-pricing system and single-pricing system with counter-trading use to deal with the internal congestion, explicit and implicit auctions have also been used to deal with congestion at the interconnections of multiple systems, which are usually interconnections between several countries (Pérez-Arriaga and Olmos, 2005). A comparison of the different congestion management methods can be found in Knops et al. (2001).

In the literature, there are different models to analyze the effect of network congestion on the companies' strategic behavior and the market equilibrium in the different types of electricity markets. Table 1.1 classifies those models taking the market structure and the equilibrium model used into account. In the red cells this thesis seeks to provide models in order to study the equilibrium.

Table 1.1: Equilibrium Models with Network Congestion

	Market splitting	**Market coupling**
Conjectural-variation-based equilibrium models	Fernández-Menéndez et al. (2005); Barquín et al. (2009)	Kurzidem (2010); Oggioni et al. (2012); Oggioni and Smeers (2013)

	Nodal-price markets
Perfect competition	Holmberg and Lazarczyk (2012)
Cournot models	Younes and Ilic (1999); Berry et al. (1999); Day et al. (2002); Yu et al. (2002); Barquín and Vazquez (2008); Yao et al. (2008)
Supply function equilibrium models	Younes and Ilic (1999); Xian et al. (2004); Bompard et al. (2006); Nam et al. (2006); Liu and Wu (2007); Bompard et al. (2010)
Conjectural-variation-based equilibrium models	Day et al. (2002); Hobbs and Rijkers (2004); Liu et al. (2007); Díaz et al. (2012); Nappu et al. (2013)

	Single-price markets	
	Companies internalize congestion	Companies **do not** internalize congestion
Cournot models	Peng and Tomsovic (2003); Dijk and Willems (2011)	Wang et al. (2007)
Supply function equilibrium models		Green and Newbery (1992); Wang et al. (2007)
Conjectural-variation-based equilibrium models		Barquín et al. (2004); Song et al. (2004); Centeno et al. (2007)

- **Market splitting and market coupling**
  Regarding the models of the mechanisms used for management congestion between different areas, Fernández-Menéndez et al. (2005) and Barquín et al. (2009) proposed a conjectural-variation-based equilibrium model to compute zonal prices in an electricity market with market splitting. These models internalize the effect of transmission constraints making the conjectured price responses a function of the state of congestion. The integration of European electricity markets through a market coupling

design is analyzed in Kurzidem (2010), Oggioni et al. (2012) and Oggioni and Smeers (2013).

- **Nodal-price markets**

  Younes and Ilic (1999); Berry et al. (1999); Hobbs and Rijkers (2004); Xian et al. (2004); Nam et al. (2006); Bompard et al. (2006); Liu and Wu (2007); Barquín and Vazquez (2008); Holmberg and Lazarczyk (2012); Díaz et al. (2012) studied the strategic behavior of generation companies in nodal-price electricity markets.

  Younes and Ilic (1999) argued that a transmission line could become a source of inefficiency and could lead to higher prices if generators realize that they can make profits by strategically constraining it. Berry et al. (1999) showed that the transmission constraints can increase the profit of the generators at the expense of the owner of the transmission rights. Hobbs and Rijkers (2004) presented a conjectural-variation-based equilibrium model to represent the strategic behavior of large producers. The underlying assumption is that large producers may affect the prices of transmission services. Nam et al. (2006) analyzes the effect of long-term contracts on the market equilibrium in the electricity market with transmission constraints. The analysis is only suitable in two-area systems. Xian et al. (2004); Bompard et al. (2006) and Liu and Wu (2007) developed a supply function equilibrium model to analyze the impact of the transmission network on the equilibrium of an electricity market. Barquín and Vazquez (2008) developed a Cournot model for describing the effect of transmission constraints on the generation companies' strategic behavior and opportunities to exercise market power by influencing the set of constrained lines. Yao et al. (2008) modeled the forward and spot markets as a two-period Cournot Game that takes the network congestion into account using a nodal-pricing system. Holmberg and Lazarczyk (2012) concluded that under perfect competition, an electricity market with zonal pricing and a counter-trading mechanism result in the same dispatch of a nodal-price market. However, the zonal system is inefficient because there are additional payments to generation companies in the exporting nodes. Díaz et al. (2012) proposed an iterative algorithm to compute the conjectured supply function equilibrium with DC transmission network constraints. Nappu et al. (2013) established a methodology for the identification of the generator most likely to behave strategically and to congest the system.

- **Single-price markets**

  The models that analyze the single-price markets often obviate the subsequent market mechanism to solve the network constraints (Green and Newbery, 1992; Barquín et al., 2004; Song et al., 2004; Wang et al., 2007; Centeno et al., 2007). Therefore, the basic assumption made in those models is that the generation companies do not internalize the effect of network congestion on their strategic behavior. Green and Newbery (1992) analyzed strategic behavior in duopolies in an electricity spot market using a supply function equilibrium model. Song et al. (2004) proposed a learning method to determine the best value of the conjecture parameter use by each generation company. Wang et al. (2007) presented Cournot and supply function equilibrium models for representing strategic behavior in electricity markets considering load and supply side uncertainties. A conjectured price-response market equilibrium is proposed in Barquín et al. (2004) and Centeno et al. (2007) to study the medium-term strategic behavior of generation companies. Langary et al. (2014) and Díaz et al. (2014) studied the existence of the market equilibrium in electricity markets using a supply equilibrium model and a conjectural-variation-based equilibrium model, respectively.

  On the other hand, Peng and Tomsovic (2003) and Dijk and Willems (2011) analyze the effect of the transmission network on a single-price market. Peng and Tomsovic (2003) studied the influence of network congestion on the process of determining generation companies' bidding strategies in an electricity market. Additionally, in the Cournot model developed, the best response of each company is evaluated. However, the theoretical approach is only suitable for simple networks with a small number of companies. Dijk and Willems (2011) compared the efficiency between the use of nodal pricing and counter-trading system as mechanisms to manage congestion in a power system. This work assessed the effects of these mechanisms on the entry of new companies and on the competitiveness of the market. Their model is only suitable in a two-area system with a congested transmission line. In the imported area there are several incumbent symmetric Cournot companies with the same marginal cost and the entrant is competitive and located in the exporting area. The most important result is that with a counter-trading mechanism, the potential benefits of additional competition (more competitive prices and lower production cost) do not outweigh the distortions (additional investment cost for the entrant and socialization of the congestion cost to consumers).

It can be concluded from the literature review and classification of the the different equilibrium models proposed to study the electricity markets that there are a vast number of models for nodal-pricing electricity markets. There are fewer models that analyze single-pricing electricity markets. However, these models usually disregard the effect of the network constraints on the strategic behavior of generation companies, and the works that analyze the effect of network congestion are only suitable for small-scale systems with few companies or several symmetric companies. Therefore, there is a lack of research on market equilibrium models of single-pricing electricity markets as shown in Table 1.1.

1.2.2.2 Voltage level requirements

The models presented in the previous section only study the congestion caused by the thermal limits of the transmission lines. They use a DC approximation of the power flow equations, and are unable to analyze other technical constraints such as voltage constraints or reactive power requirements.

Few models study the effect of voltage constraints on the companies' strategic behavior. Among them there are the models proposed in Bautista et al. (2007a,b,c); Chitkara et al. (2009); Almeida and Senna (2011); Petoussis et al. (2013); Soleymani (2013). Bautista et al. (2007a,b) presented a Cournot model to study the influence of the reactive power requirements on the active power dispatch. These works argue that the DC approximation of the power flow is not accurate enough because it does not take into account the capability curve of the generation units that models the tradeoff between active and reactive power. Bautista et al. (2007c) was an extension of the previous approaches using a supply function equilibrium model. Chitkara et al. (2009) proposed a model to analyze the companies' strategic behavior in a reactive power market. This model assumes that the active power is already scheduled, thereby there is no feedback between the reactive and active power markets, i.e., reactive power requirements do not modify strategic behavior in the active power market. Almeida and Senna (2011) proposed a bilevel optimization problem that models the active and reactive power dispatch under competence. The first level corresponds to the active power market and the second level minimizes the opportunity cost of the reactive power which is defined in terms of the marginal price of the power active market. Petoussis et al. (2013) assessed different parametrization methods of the companies' supply functions in an active power market taking into account an AC representation of the network. Finally, Soleymani (2013) developed a supply function equilibrium model for optimal bidding strategy of generation companies in active and reactive power markets, where the companies have incomplete information about their rivals.

Similar to the analysis of network congestion on electricity markets, there are some models that study the effect of voltage requirements. However, all of them are focused on nodal-price electricity markets, and none assess the effect on single-price electricity markets as shown in Table 1.2. This thesis proposes a model in order to fill these research gaps marked in red.

Table 1.2: Equilibrium Models with Voltage Constraints

	Single-price markets	**Nodal-price markets**
Perfect competition		Almeida and Senna (2011)
Cournot models		Bautista et al. (2007a,b)
Supply function equilibrium models		Bautista et al. (2007c); Soleymani (2013); Petoussis et al. (2013)
Conjectural-variation-based equilibrium models		Chitkara et al. (2009); Petoussis et al. (2013)

1.2.2.3 Reserve markets

The reserve markets and mechanisms are one of the ancillary services used to guarantee the security and reliability of the power systems. However, the classification of the different reserve services and the regulation of this markets may vary considerably among countries (Raineri et al., 2006; Milligan et al., 2010).

In the literature there are several approaches for analyzing the relation and interdependence between the energy and reserve markets. These models may differ in the dispatch method (joint or sequential dispatch), the reserve classification (primary, secondary, tertiary, spinning or non-spinning reserve), the electricity market design (single-price or nodal-price electricity market), the temporal constraints (single-period or multi-period), and in the strategic behavior of generation companies (perfect competition, Cournot models, supply function equilibrium models or conjectural-variation-based equilibrium models). A detailed literature review of the different reserve markets models and implementations can be found in González et al. (2014).

Several models have proposed different market mechanisms to determine the reserve level of the generation units and to remunerate the reserve services. These models do not analyze the strategic behavior of the generation companies and assume that there is a centralized market or a perfect competition market. Gan and Litvinov (2003); Galiana et al. (2005); Stacke and Cuervo (2008); Morales et al. (2009) and Amjady et al. (2013) studied a market model in which the energy and reserve are cleared jointly. Galiana et al. (2005) proposed that all the different types of reserve were paid at a same price denominated security price. Stacke and Cuervo (2008) presented a pricing model that takes the bilateral contracts, electricity pool and reserve markets in a joint market of services. Morales et al. (2009) proposed a methodology to determine the required level of spinning and non-spinning reserves in a power system with high penetration of wind power. Amjady et al. (2013) analyzed a stochastic market-clearing model for joint energy and reserve. The participation of wind generators in reserve markets is studied in Liang et al. (2011); Saiz-Marin et al. (2012).

Regarding the strategic behavior of generation companies, Bautista et al. (2006); Haghighat et al. (2007); Jianlin et al. (2010) and Nazir and Galiana (2011) studied the integration between the electricity and reserve markets. They argued that even a competitive reserve market have an effect on the electricity market efficiency. In Bautista et al. (2006) and Haghighat et al. (2007), electricity is traded in a nodal-price system and there is only one reserve price for the whole system. The difference between both models is that Bautista et al. (2006) used a conjectural-variation-based equilibrium model while Haghighat et al. (2007) used a supply function equilibrium model. Jianlin et al. (2010) proposed a conjectural-variation-based equilibrium model considering locational spinning reserve requirements. Wieschhaus and Weigt (2008) and Nazir and Galiana (2011) presented a Cournot model in which there is one electricity price and one reserve price for the whole power system. In Nazir and Galiana (2011) the electricity and reserve markets are dispatched jointly while in Wieschhaus and Weigt (2008) modeled the Germany mechanism where the reserve market is cleared before the electricity market. Zhao et al. (2011) studied the reserve price cap as a measurement to mitigate market power in joint energy and reserve markets. Jia et al. (2006) and Hongxing et al. (2010) studied the strategic behavior of generation companies in the reserve market under a sequential dispatch. Both models assumed that the electricity market is already cleared, and therefore, there is no interdependence between both markets. Jia et al. (2006) used a conjectural-variation-based equilibrium model, and Hongxing et al. (2010) a Cournot model.

Table 1.3 classifies the different models proposed to analyze electricity and reserve markets. There are several models that study both nodal-price and single-price electricity markets. The majority of these models assume that electricity and reserve are dispatched jointly. Although a joint dispatch gives the solution with higher social welfare (Galiana et al., 2005), there are several countries like Spain where the electricity and reserve are dispatched sequentially. Moreover, the few models that study the strategic behavior of generation companies in the reserve markets assume that the result of the reserve market does not affect the result in the electricity market.

Table 1.3: Equilibrium Models with Reserve markets

	Joint dispatch	
	Single-price markets	**Nodal-price markets**
Perfect competition	Stacke and Cuervo (2008); Morales et al. (2009); Amjady et al. (2013)	Gan and Litvinov (2003); Galiana et al. (2005);
Cournot models	Nazir and Galiana (2011);	Zhao et al. (2011)
Supply function equilibrium models		Haghighat et al. (2007)
Conjectural-variation-based equilibrium models		Bautista et al. (2006); Jianlin et al. (2010)

	Sequential dispatch	
	Single-price markets	**Nodal-price markets**
Perfect competition		
Cournot models	Wieschhaus and Weigt (2008); Hongxing et al. (2010);	
Supply function equilibrium models		
Conjectural-variation-based equilibrium models	Jia et al. (2006)	

1.2.3 Thesis research topics

As shown in the previous section, in the literature there are several models to study the effect of system congestion and voltage level requirements on the strategic behavior of generation companies in electricity markets. However, the majority of these models only study electricity markets with a nodal-pricing mechanism and they do not analyze the effect on electricity markets with a single-pricing mechanism. This is a major gap in the literature because several countries have implemented a single-pricing mechanism.

The models that analyze the interdependence between electricity and reserve markets assume that these markets are dispatched jointly. Although there are some models for the sequential dispatch, these models do not correctly study how the reserve markets affect the result of the electricity market.

Therefore, the thesis seeks to fill these research gaps. To do this, this thesis proposes a new market equilibrium model based on conjectural variations. The equilibrium model takes into account the effect of network constraints on the strategic behavior of generation companies in a single-price electricity market. The thesis also proposes a model to effectively analyze the interdependence between the electricity and reserve markets under a sequential dispatch like the one implemented in Spain.

The general idea of the proposed models is that the mechanisms used to clear technical and security constraints modify the cost of the generation units perceived by the companies in the day-ahead electricity market. Hence, the generation companies modify their strategies, which leads in changes in the dispatch of the day-ahead electricity market.

1.3 Objectives

1.3.1 Main objective

The main objective of this thesis is to analyze how the markets and mechanisms used to resolve the technical and security system constraints may affect the strategic behavior of generation companies involved in a single-price electricity market in the medium term. This analysis will be both qualitative and quantitative.

1.3.2 Specific objectives

More specifically, the objectives that has been covered within the context of the thesis are:

- To analyze the effect of the market and mechanisms (counter-trading mechanisms, voltage level requirements, reserve markets, etc.) that solve the network and security constraints (network congestion, voltage regulation, etc.) on the strategic behavior of generation companies in the medium term in single-price electricity markets.
- To develop an equilibrium model of a single-price electricity market. This model will represent the effect of markets and mechanisms to clear network constraints which are not taken into account in current models. These markets and mechanisms are relevant to analyze the strategic behavior of generation companies.
- To apply the proposed model in real-size power systems, assessing the economic and physical implications of the markets and mechanisms used to clear network constraints in single-price electricity markets in the medium-term horizon.

1.4 Thesis outline

The remainder of the thesis is organized as follows:

Chapter 2.
This chapter shows how the congestion management mechanisms implemented in several European countries (Spain, Portugal, Germany, and the Netherlands) are inefficient. These mechanisms allow generation companies to exercise market power between the day-ahead electricity market and the counter-trading mechanism. This analysis is made using a market equilibrium model that incorporates the strategic behavior between markets.

Chapter 3.
This chapter proposes a conjectural-variation-based equilibrium model to study the effect of system congestion on the strategic behavior of generation companies. The model includes two kinds of strategic behavior. The first is the ability to

modify the day-ahead electricity price; the second is the ability to behave strategically between markets. This equilibrium model is implemented in a two-area system.

Chapter 4.
In this chapter, the equilibrium model presented in the previous chapter is extended to study real-size power systems. A two-stage iterative algorithm to compute the market equilibrium is also proposed. The day-ahead market clearing process is computed in the first stage. Network congestion is cleared using a DC optimal power flow in the second stage. The proposed model is implemented in a real-size power system. The convergence of the iterative algorithm is also studied.

Chapter 5.
The effect of network congestion caused by the thermal limits of the transmission lines is studied in the previous chapter. However, the DC approximation of the power flow is not suitable to analyze the voltage level requirements. Therefore, this chapter presents an AC power flow in order to solve the network constraints.

Chapter 6.
This chapter proposes a model to study the mechanism implemented in Spain to contract and manage additional upward reserve in the power system.

Chapter 7.
This last chapter presents the conclusions of this thesis, and guidelines for future work are outlined.

Appendix A.
Nomenclature.

Appendix B-H.
Articles developed in the thesis.

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Chapter 2

Inefficiencies of counter-trading mechanisms

Contents

In electricity markets, different mechanisms are used to deal with congestion in the system. In Spain, the Netherlands, Portugal and Germany, the day-ahead electricity market is cleared without taking technical constraints into account, and subsequently a counter-trading mechanism is used to deal with congestion in the system. The counter-trading mechanism allows generation companies to behave strategically between markets since they may modify their bids to avoid them being dispatched in the day-ahead market, and to enable them to be dispatched in the counter-trading mechanism. In contrast, such behavior does not occur in a nodal-pricing system.

This chapter presents a simple case in order to analyze the inefficiencies of the congestion management mechanisms implemented in Spain, the Netherlands, Portugal, and Germany, comparing them with a nodal-pricing system. The identification of these inefficiencies motivates the development of appropriate models to study in more detail the effect of subsequent markets on the result of the electricity market and on the companies' strategic behavior.

2.1 Introduction

The objective of this chapter is to identify the technical and economic inefficiencies that appear in the electricity market due to the counter-trading mechanism. To achieve this objective, this chapter introduces a market equilibrium model. In the model, the generation companies behave strategically between the day-ahead electricity market and the congestion management mechanism. Thus, generation companies may have incentive to congest the system and not be dispatched in the day-ahead electricity market, and be dispatched in the counter-trading mechanism. The results of this chapter motivate the development of more complex models to study the effect of subsequent mechanisms on the strategic behavior of generation companies in the day-ahead electricity market. The development of these models as well as the critical analysis of the obtained results is the main objective of this thesis.

2.1.1 Counter-trading mechanisms

Several European countries like Spain, Portugal, Germany and the Netherlands use congestion management mechanisms to deal with internal congestion in the power system. In general terms, these mechanisms work as follows: generation companies make bids in the day-ahead electricity market and the market operator then performs the market-clearing process taking into account these bids but without considering technical network constraints. This process does not guarantee a technically feasible method of transmitting electricity because it may breach maximum flow constraints, resulting in system congestion. In such cases, the counter-trading mechanism is implemented in order to clear system congestion. The generation companies bid on the production that they can increase or reduce with respect to the day-ahead market. The Transmission System Operator (TSO) receives these bids and determines a solution that meets the network constraints. The increments are paid, and the reductions are charged at the prices determined in the counter-trading mechanism. Thus, in a single-price electricity market with a counter-trading mechanism, generation companies have to decide on their bids in both the day-ahead market and the counter-trading mechanism.

The idea behind the implementation of a counter-trading mechanism is:

1. To try to obtain the same final technical solution as that which exists in the nodal-pricing scheme
2. To do this in a way that lowers consumer payments relative to the nodal-pricing scheme, by discriminating among units in an importing area so as to pay greater remuneration only to generation units which participate in the counter-trading mechanism.

Assuming that generation companies know which generation units are necessary to clear congestion, they place more or less value on their generation units when there is system congestion and may behave strategically, which will lead to a different outcome in the day-ahead market. Generation companies may reduce their production in the exporting areas in order not to be penalized in the counter-trading mechanism. However, this effect does not guarantee an increase in production in the importing area in the day-ahead market. This is mainly because, as generation companies know that their production in the importing area is more valuable, there is an incentive for them to enter in the counter-trading mechanism, increasing their offer price in the day-ahead market.

Several studies have analyzed the inefficiencies of the counter-trading mechanisms used in different electricity markets. Bompard et al. (2003) performed a comparative analysis of different schemes (England and Wales, Norway, Sweden, PJM and California) implemented to clear network congestion. Another comparative analysis between a single-pricing system and a nodal-pricing system for England and Wales was carried out by Green (2007), who concluded that a nodal-pricing system may increase the total social welfare since the electricity market becomes less vulnerable to the market power of generation companies and gives the right investments signal. Neuhoff et al. (2011) and Oggioni and Smeers (2013) focused on the integration of the European electricity markets. Neuhoff et al. (2011) analyzed the criteria to be met by the congestion management mechanism used in the integration of European electricity markets, and concluded that a nodal-pricing system is the optimal solution. Oggioni and Smeers (2013) assessed different counter-trading mechanisms in different versions of the market coupling scheme. Their main conclusion was that the integration of European markets may work well or may be inefficient depending on the zonal decomposition, and the degree of coordination in the counter-trading between Transmission System Operators. Holmberg and Lazarczyk (2012) argued that a counter-trading mechanism is inefficient because it results in additional payments to producers in exporting areas. Dijk and Willems (2011) and van Blijswijk and de Vries (2012) assessed the counter-trading mechanism implemented in the Netherlands in 2011. Both studies gave different and contradictory results. Dijk and Willems (2011) studied the entry and exit of power plants in the Dutch system. Their analysis showed that the counter-trading that takes place gives the wrong long-term signals, causing an over-entry in the exporting areas and an under-entry in the importing areas. On the other hand, van Blijswijk and de Vries (2012) argued that the potential for companies to exercise market power by using the new mechanism is limited. However, this conclusion is based on the assumption that network congestion is not structural, and will only be temporary since it is expected that the Dutch TSO will make the necessary reinforcements to the system to mitigate any congestion.

To summarize, all these studies conclude that counter-trading mechanisms are inefficient because:

- Not all generation units can take part in the redispatch process because only some units can solve the congestion. In some importing areas the units must increase their generation, and in some exporting areas the units must reduce their generation.

- Generation companies have strong incentives to behave strategically between markets. In exporting areas, they may receive a payment even if they are charged for reducing the production in the counter-trading mechanism. In importing areas, they may prefer not to be dispatched in the day-ahead market and expect to be required to participate in the counter-trading mechanism.
- The counter-trading mechanism gives the wrong investment signals for locating new plants.
- The day-ahead market price does not reflect the fact that some areas are more expensive than others.

Taking into account this scenario, this chapter analyzes the counter-trading mechanisms used in Spain, the Netherlands, Portugal and Germany. Furthermore, this work compares the market outcome of those mechanisms with that of a nodal-pricing system. The results show that these counter-trading mechanisms are technically or economic inefficient. In order to properly analyze these inefficiencies is necessary to develop appropriate models which will be addressed in the following chapters. Section 2.2 describes the mechanisms used in each country to clear system congestion. Section 2.3 describes the characteristics of the market equilibrium model used to assess the strategic behaviors of generation companies. Section 2.4 presents a case study which compares the solution provided by these counter-trading mechanisms with a nodal-pricing system. Finally, section 2.5 draws appropriate conclusions.

2.2 Congestion management mechanisms

This chapter focuses on the congestion management mechanism used in Spain (Furió and Lucia, 2009; SEE, 2012), the Netherlands (Dijk and Willems, 2011; van Blijswijk and de Vries, 2012), Portugal (ERSE, 2009), and Germany (BNetzA, 2012; Burstedde, 2013; Nüßler, 2012).

2.2.1 Spain

A congestion management mechanism is implemented subsequent to the outcome of the day-ahead market. The TSO performs a number of security analysis to identify congestion that may appear in the system, taking into account the total amount of electricity produced in the day-ahead market and the security constraints. Furthermore, the TSO receives price and quantity bids made by the units that are able to increase or reduce their production with respect to the day-ahead market. When congestion is identified, some units have to increase or reduce their production. Increased production is determined by the TSO as the lower cost solution evaluating the bids made by generation companies. The increased quantity of electricity is paid by using the bid price made by the unit in the counter-trading mechanism. On the other hand, the quantity reduced depends on the Generator Shift Factor (Dobson et al., 2001). This factor quantifies the change in the flow at the interconnection when the generation unit changes its production. First, the unit with the highest factor has to reduce its production, and this reduction then continues in the order given by the factor until the congestion disappears. When several units have the same factor, the reduction is proportional to their production. The units that reduce their production are charged at the day-ahead market price. This means that generation units in exporting areas with congestion are only paid for the quantity produced, while receiving nothing for the quantity withdrawn.

2.2.2 The Netherlands

In May 2011, a counter-trading mechanism was implemented to deal with system congestion. As in Spain, this mechanism is a corrective method in the sense that it is implemented after the electricity market has closed. In the electricity market, the generation companies are paid for their scheduled production. When congestion occurs, production must be reduced in the exporting areas and increased in the importing areas. To achieve this, there are two additional markets: a market to increase production and a market to reduce production. In the exporting areas, generation companies make bids of the price at which they are willing to reduce their production, and the TSO accepts the bids of the generation companies that are willing to pay more. In the importing areas, generation companies make bids of the price at which they are willing to increase their production, and TSO accepts the cheapest bids. In both cases, the TSO charges/pays generation companies on a pay-as-bid basis.

2.2.3 Portugal

As in Spain, generation companies make price and quantity bids for increasing or reducing their production with respect to the day-ahead market. The TSO performs a security analysis to identify network congestion that may appear in the system, taking into account the outcome of the day-ahead market. When congestion is identified, it determines the lowest cost solution that will resolve the problem. When a generation company has increased its production, the increased quantity is paid for at the minimum value between its bids made in the day-ahead market and the congestion management mechanism. On the other hand, when a generation company has reduced its production, the reduced quantity is charged at the maximum value between its bids made in the day-ahead market, the congestion management mechanism and 0.85 times the day-ahead market price.

2.2.4 Germany

Unlike in Spain, the Netherlands or Portugal, in Germany there is no real market to clear system congestion, i.e., generation companies do not make bids for increasing or reducing their production with respect to the day-ahead market. However, when there is a technical constraint, the TSOs are obliged and empowered to intervene in the electricity market to ensure the safe operation of the power system. Specifically, TSOs implement a cost-based redispatch when congestion occurs. This means that the units used to clear congestion are selected according to their marginal costs. The units that increase their production are paid their generation costs while the units that reduce their production receive the difference between the day-ahead market price and their marginal costs. This is equivalent to charge the unit's marginal cost on the reduced quantity. In October 2012, the German regulator established a new procedure for calculating the marginal costs of units in order to establish a common and consistent mechanism (BNetzA, 2012).

2.3 Description of the model used for the study

In order to assess the different congestion management mechanisms used in Spain, the Netherlands, Portugal and Germany, this chapter presents a market equilibrium model which is explained with more detail in chapter 3[1]. As discussed above, the objective of this chapter is to identify the inefficiencies of the counter-trading mechanisms in order to motivate the development of more detailed models as presented in chapters 3 and 4. The main feature of this model is that the market equilibrium equations take into account the effect of the counter-trading mechanism on the generation companies' behavior. In the model, different degrees of competition can be analyzed, from perfect competition to extreme oligopoly markets (such as Cournot Equilibrium). Moreover, the model includes two kinds of strategic behavior. The first is the ability to modify the electricity price; the second is the ability to behave strategically between markets.

Since the market equilibrium equations presented in chapter 3 simulate the mechanism used in Spain, some slight changes are carried out to adjust them to the mechanisms used in the Netherlands, Portugal and Germany, as shown below. Furthermore, unlike those in chapter 3, the market equilibrium equations do not take into account the market power in the day-ahead market, and only simulate strategic behavior between markets. The other difference between the model in this chapter and the one in chapter 3 is the methodology used to determine the solution of the market equilibrium equations. An equivalent quadratic minimization problem is formulated in chapter 3 while in this chapter the equilibrium equations are solved using a Mixed Complementarity Problem (MCP). Although both approaches are equivalent.

2.3.1 The model's structural assumptions

For the sake of clarity and simplicity, the formulation is based on the following modeling assumptions:

1. There are two areas, one exporting area (EX) and one importing area (IM), interconnected by a flowgate with limited transfer capacity ($\overline{F_l}$), as shown in Fig. 2.1.

2. Each company i can own generation units in both areas. Thus, P_i^{EX} and P_i^{IM} are the productions of company i in the exporting and importing areas, respectively.

[1]Delgadillo et al. (2013) presents the market equilibrium model for the Spanish electricity market

3. In both areas, the demand (D^{EX} and D^{IM}) is inelastic.

4. Both areas belong to the same electricity system, in which the day-ahead market-clearing process does not take network constraints into account. Therefore, the day-ahead electricity market price λ is the same for both areas.

5. The result of the day-ahead electricity market is that the total production in the importing area is lower than the demand in that area. Therefore, there is an energy flow from EX to IM and the flow reaches the maximum value causing the connection between EX and IM to be congested.

6. There is a counter-trading mechanism that clears congestion between the two areas. In order to eliminate overflows, the total generation of the exporting area has to be reduced while the total generation of the importing area has to be incremented. Thus, ΔP_i^{EX} and ΔP_i^{IM} are the changes in the generation of company i in the exporting and importing areas, respectively.

7. The difference between the real production and the result of the day-ahead market will be paid or charged at a certain price. These payments can be viewed as an income or a cost depending on whether the generation unit increases or reduces its production. Increases in unit production are paid at the price γ, and reductions are charged at the price χ. These prices depend on the regulatory framework of each country, as detailed in section 2.2.

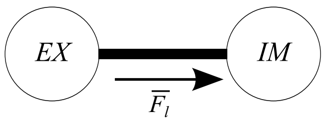

Figure 2.1: Two-area system

2.3.2 Market clearing conditions

In the electricity market, total generation and demand have to be balanced. In the day-ahead market, the sum of the generation of all units equals the total demand ($D^{IM} + D^{EX}$). In the counter-trading mechanism, the increased power in the importing area and the reduced power in the exporting area must be equal in order to maintain the power system balance. The total change (ΔP) is the change in the production of the units that clear system congestion in the counter-trading mechanism. The value of ΔP is determined using an iterative procedure as described in Section 2.3.5. I is the set of generation companies. Thus, the power balance constraints are:

$$\sum_{i \in I} \left(P_i^{IM} + P_i^{EX} \right) = D^{IM} + D^{EX} \tag{2.1}$$

$$\sum_{i \in I} \Delta P_i^{IM} = \Delta P \tag{2.2}$$

$$\sum_{i \in I} \Delta P_i^{EX} = \Delta P \tag{2.3}$$

2.3.3 The generation company's problem

The behavior of the generation company i is modeled by the maximization problem (2.4)-(2.8). The generation company i determines the production of its generation units in the day-ahead market (P_i^{IM} and P_i^{EX}) and counter-trading mechanism (ΔP_i^{IM} and ΔP_i^{EX}) that is required to maximize profits taking into account the market prices (λ, γ and χ) and the cost functions ($C_i^{IM}\left(P_i^{IM} + \Delta P_i^{IM}\right)$ and $C_i^{EX}\left(P_i^{EX} - \Delta P_i^{EX}\right)$) .

Equation (2.4) is the company's profit that is equal to the revenue in the day-ahead market, plus the income due to the increment in the generation at area IM, minus the charge due to the reduction in the generation at area EX, minus the production costs in each area. Constraints (2.5) and (2.6) are the maximum production in the importing and exporting areas, respectively. Constraint (2.7) indicates that the maximum reduction in the exporting area in the counter-trading mechanism is less than or equal to the production in the exporting area in the day-ahead market. Constraints (2.8) model that the decision variables are positive. μ_i, ν_i and ξ_i represent the dual variables associated to constraints (2.5), (2.6) and (2.7), respectively.

$$\max_{P_i^{IM}, P_i^{EX}, \Delta P_i^{IM}, \Delta P_i^{EX}} \quad \lambda \cdot \left(P_i^{IM} + P_i^{EX} \right) + \gamma \cdot \Delta P_i^{IM} - \chi \cdot \Delta P_i^{EX} - C_i^{IM}\left(P_i^{IM} + \Delta P_i^{IM} \right) - C_i^{EX}\left(P_i^{EX} - \Delta P_i^{EX} \right) \tag{2.4}$$

s.t.

$$\overline{P}_i^{IM} - P_i^{IM} - \Delta P_i^{IM} \geq 0 \qquad : (\mu_i) \tag{2.5}$$

$$\overline{P}_i^{EX} - P_i^{EX} \geq 0 \qquad : (\nu_i) \tag{2.6}$$

$$P_i^{EX} - \Delta P_i^{EX} \geq 0 \qquad : (\xi_i) \tag{2.7}$$

$$P_i^{IM} \geq 0, \; P_i^{EX} \geq 0, \; \Delta P_i^{IM} \geq 0, \; \Delta P_i^{EX} \geq 0 \tag{2.8}$$

2.3.4 Market equilibrium

The market equilibrium point corresponds to the solution of the Mixed Complementary Problem (2.9)-(2.18). Equations (2.9)-(2.11) are the market clearing constraints. The generation companies behavior is modeled by means of equations (2.12)-(2.18). These equations are the Karush–Kuhn–Tucker (KKT) conditions of the problem (2.4)-(2.8) for each company i. The operator $\perp$ denotes the inner product of two vectors equal to zero, i.e., $0 \le x \perp f(x) \ge 0$ corresponds to the system equations $x \ge 0$, $f(x) \ge 0$ and $x \cdot f(x) = 0$.

$$(\lambda \text{ unrestricted}) \quad \sum_i \left(P_i^{IM} + P_i^{EX}\right) = D^{IM} + D^{EX} \tag{2.9}$$

$$(\gamma \text{ unrestricted}) \quad \sum_i \Delta P_i^{IM} = \Delta P \tag{2.10}$$

$$(\chi \text{ unrestricted}) \quad \sum_i \Delta P_i^{EX} = \Delta P \tag{2.11}$$

$$0 \le \mu_i \perp \overline{P}_i^{IM} - P_i^{IM} - \Delta P_i^{IM} \ge 0 \qquad \forall i \in I \tag{2.12}$$

$$0 \le \nu_i \perp \overline{P}_i^{EX} - P_i^{EX} \ge 0 \qquad \forall i \in I \tag{2.13}$$

$$0 \le \xi_i \perp P_i^{EX} - \Delta P_i^{EX} \ge 0 \qquad \forall i \in I \tag{2.14}$$

$$0 \le P_i^{IM} \perp -\lambda + \frac{\partial C_i^{IM}\left(P_i^{IM} + \Delta P_i^{IM}\right)}{\partial P_i^{IM}} + \mu_i \ge 0 \qquad \forall i \in I \tag{2.15}$$

$$0 \le P_i^{EX} \perp -\lambda + \frac{\partial C_i^{EX}\left(P_i^{EX} - \Delta P_i^{EX}\right)}{\partial P_i^{EX}} + \nu_i - \xi_i \ge 0 \qquad \forall i \in I \tag{2.16}$$

$$0 \le \Delta P_i^{IM} \perp -\gamma + \frac{\partial C_i^{IM}\left(P_i^{IM} + \Delta P_i^{IM}\right)}{\partial \Delta P_i^{IM}} + \mu_i \ge 0 \qquad \forall i \in I \tag{2.17}$$

$$0 \le \Delta P_i^{EX} \perp \chi + \frac{\partial C_i^{EX}\left(P_i^{EX} - \Delta P_i^{EX}\right)}{\partial \Delta P_i^{EX}} + \xi_i \ge 0 \qquad \forall i \in I \tag{2.18}$$

2.3.5 Solution methodology

It is important to note that the total power change (ΔP) in (2.10) and (2.11) is the result of the counter-trading mechanism. An iterative procedure is used to determine the value of ΔP. This procedure works as follows:

1. Initialize $\Delta P = 0$. This case corresponds to the case without congestion.

2. Solve the problem (2.9)-(2.18).

3. Update the value of ΔP that clears congestion, $\Delta P = \sum_{i \in I} P_i^{EX} - D^{EX} - \overline{F_l}$. If the change in ΔP with the previous iteration is less than ϵ value, the algorithm stops; otherwise it goes to 2.

2.4 Illustrative case study

This section presents a simple example that permits an analysis of the performance of the different counter-trading mechanisms. The results are compared with those obtained if the electricity market uses a nodal-pricing system in which a counter-trading mechanism is not necessary to clear system congestion.

The case considers two areas: an exporting area *EX* and an importing area *IM* with demands $D^{EX} = 100$ MW and $D^{IM} = 300$ MW, respectively. Both areas are connected by a flowgate with a maximum transmission capacity $\overline{F_l} = 150$ MW. There are four generation companies with generation units in both areas. Table 2.1 presents the total installed capacity and the generation cost of the units in each area. In this system, it can be observed that the generation company 2 owns the units with the lowest costs, and the units located in area *EX* are cheaper than those in area *IM*.

Table 2.1: Generation Units

	Area			
	EX		***IM***	
Generation Company	$\overline{P}$ (MW)	Variable Cost (€/MWh)	$\overline{P}$ (MW)	Variable Cost (€/MWh)
1	80	55.6		
2	300	52.9	100	53.8
3	50	55.7	40	57
4			60	57.5

Different cases are modeled in order to represent different policy mechanisms. Case A is a preliminary case without a limit on the interconnection capacity; thus, there is no system congestion. Case B represents a nodal-pricing system. Case C represents the solution expected by the policy makers in the different countries in which generation companies behave competitively and bid their power units at a price equal to their marginal costs. Case D, E ,F and G model the policy mechanisms used in Spain, the Netherlands, Portugal and Germany, respectively.

In these cases, generation companies may behave strategically between markets, i.e., they may bid their power units differently than their marginal costs because they know that their units will be redispatched in the counter-trading mechanism in order to clear congestion.

2.4.1 Case without congestion (A)

In a perfect competitive market and without congestion between areas, the electricity market price is the same in both areas. Generation company 2 produces 300 and 100 MW in areas *EX* and *IM*, respectively. The electricity price is 53.8 €/MWh which corresponds to the variable cost of the marginal unit, and the power flow from area *EX* to *IM* would be equal to 200 MW (Table 2.2).

Table 2.2: Results of case A

Generation Company	**Power (MW)** *EX*	*IM*	**Income (€)**	**Cost (€)**	**Profit (€)**
1	-	-	-	-	-
2	300	100	21520$^{(*)}$	21250$^{(**)}$	270
3	-	-	-	-	-
4	-	-	-	-	-
Total	300	100	21520	21250	270
Power Exchanges	-200	200			
Price (€/MWh)	53.8				

$(*)\ (300+100)\cdot 53.8 = 21520$

$(**)\ 300\cdot 52.9 + 100\cdot 53.8 = 21520$

2.4.2 Nodal-pricing mechanism (B)

With a nodal-pricing mechanism, there is a market separation between areas when congestion occurs. In this case, electricity prices are different in each area, and they correspond to the cost of the marginal unit in each area.

The unit production in the exporting area of generation company 2 is reduced to 250 MW while the production of units in the importing area of generation companies 3 and 4 is increased to 40 MW and 10 MW, respectively. This result occurs because the maximum transmission capacity is explicitly taken into account in the nodal-pricing mechanism, and therefore, the power flow cannot exceed the maximum capacity of 150 MW. The electricity price in the exporting area *EX* is reduced to 52.9 €/MWh, which is equal to the costs of the marginal unit in that area. Meanwhile, the price in area *IM* is increased to 57.5 €/MWh because more expensive units have to meet the demand in that area. Thus, when system congestion is taken into account in the day-ahead market, total consumer cost is increased by $21850 - 21520 = 330$ €, and generation companies receive an additional profit which is equal to $390 - 270 = 120$ €(Table 2.3).

Table 2.3: Results of case B

Generation Company	**Power (MW)** *EX*	*IM*	**Income (€)**	**Cost (€)**	**Profit (€)**
1	-	-	-	-	-
2	250	100	18975$^{(*)}$	18605$^{(**)}$	370
3	-	40	2300	2280	20
4	-	10	575	575	0
Total Power	250	150	21850	21460	390
Power Exchanges	-150	150			
Price (€/MWh)	52.9	57.5			

$(*)\, 250 \cdot 52.9 + 100 \cdot 57.5 = 18975$

$(**)\, 250 \cdot 52.9 + 100 \cdot 53.8 = 18605$

2.4.3 Consumer payments minimization (C)

Within the different regulatory frameworks analyzed (Spain, the Netherlands, Portugal, Germany), the day-ahead electricity market does not explicitly take into account the transmission network, as in case B, and system congestion that appears in the system is cleared using a counter-trading mechanism. In this scenario, the market operator expects that companies will behave competitively and bid their marginal costs, i.e., the generation companies will not change their behavior in the day-ahead market, and the day-ahead market outcome will be the same as in case A. When congestion occurs, the necessary adjustments will be made by the counter-trading mechanism, and the total amount of electricity produced would be the same as the production in the nodal-pricing mechanism (case B).

In this case, the unit in the exporting area of generation company 2 decreases its production by 50 MW in the counter-trading mechanism. Meanwhile, the production of units in the importing area of generation companies 3 and 4 increases by 40 MW and 10 MW, respectively (Table 2.4). This result gives a total consumer cost equal to 21685 €, which is lower than the consumer cost of the nodal-pricing mechanism (Table 2.9). However, although this would be the optimal outcome from the point of view of the regulator, this result is not a market equilibrium because the generation companies can behave strategically to increase their profits.

Table 2.4: Results of case C

Generation Company	Power (MW) DA EX	DA IM	CT EX	CT IM	Final EX	Final IM	Income (€)	Cost (€)	Profit (€)
1	-	-	-	-	-	-	-	-	-
2	300	100	-50	-	250	100	18830$^{(*)}$	18605$^{(**)}$	225
3	-	-	-	40	-	40	2280	2280	0
4	-	-	-	10	-	10	575	575	0
Total Power	300	100	-50	50	250	150	21685	21460	225
Power Exchanges	-200	200			-150	150			
Price (€/MWh)	53.8								

$(*)\ (300+100)\cdot 53.8 - 50\cdot 53.8 = 18830$

$(**)\ (300-50)\cdot 52.9 + 100\cdot 53.8 = 18605$

2.4.4 Spanish case (D)

When congestion occurs, generation companies value their production in each area differently, giving more importance to generation in the importing area because they know that, while units in the exporting area may be penalized, units in the importing area are necessary for the removal of congestion. Since nothing is paid for reducing production in the counter-trading mechanism, the companies may prefer to bid the units in the importing area above their marginal cost because the day-ahead market price will rise, and, in any case, the production in the importing area will be dispatched to deal with the congestion.

In this case, generation company 2 knows that its generation has to be reduced in the exporting area and increased in the importing area in order to clear system congestion. Therefore, there is a strong incentive for this company to congest the interconnection, producing nothing in the importing area. This means that other agents' more expensive units have to be dispatched in the day-ahead market, thereby increasing the price of electricity and the amount of system congestion. This causes an increase in the amount of electricity traded in the counter-trading mechanism. Thus, the final outcome is that generation company 2 can exercise market power by bidding its unit in the importing area at a price above the marginal cost of this unit. In this way, this unit is not dispatched in the day-ahead market, but has to be dispatched in the counter-trading mechanism to remove the congestion. Thus the outcome of the counter-trading mechanism is not optimal from a technical perspective because the final production of electricity is not the same as in case B (Table 2.5), and neither is it optimal from an economic perspective because the counter-trading mechanism allows generation companies to exercise market power, which they do not have under a nodal-pricing system.

Table 2.5: Results of case D

	Power (MW)								
Generation Company	**DA**		**CT**		**Final**		**Income (€)**	**Cost (€)**	**Profit (€)**
	EX	***IM***	***EX***	***IM***	***EX***	***IM***			
1	80	-	-30	-	50	-	2785	2780	5
2	300	-	-112.5	100	187.5	100	15823.75$^{(*)}$	15298.75$^{(**)}$	525
3	20	-	-7.5	40	12.5	40	2976.25	2976.25	0
4	-	-	-	10	-	10	575	575	0
Total Power	400	-	-150	150	250	150	22160	21630	530
Power Exchanges	-300	300			-150	150			
Price (€/MWh)	55.7								

$(*)\, 300 \cdot 55.7 - 112.5 \cdot 55.7 + 100 \cdot 53.8 = 15823.75$

$(**)\, (300 - 112.5) \cdot 52.9 + 100 \cdot 53.8 = 15298.75$

2.4.5 Dutch case (E)

As in Spain, the generation companies may behave strategically in the day-ahead market. They may have an incentive to congest the network by increasing the price bid of their units in the importing area so that they will be dispatched in the counter-trading mechanism.

As regards the counter-trading mechanism, the pay-as-bid system is used to remunerate generation companies (section 2.2.2). The objective is that the changes in production be paid/charged at a minimum cost for the system. However, generation companies will try to obtain the same profits that they could obtain in a marginal-pricing system (Ren and Galiana, 2004). Thus, in the importing area, generation companies may bid prices above their cost until the bid of the most expensive unit necessary to clear congestion is made. In the exporting area, generation companies may behave in a similar fashion, making bids at a price below that of the cheapest unit in order to be able to return the least amount of money possible. If they behave in this fashion, changes in the production of the units in the exporting area are charged at a price of 52.9 €/MWh, while changes in the production of the units in the importing area are paid at a price of 57.5 €/MWh. These values are equal to the prices corresponding to the nodal-pricing case (Table 2.3). However, in this case, these prices only remunerate the changes in production in the counter-trading mechanism, as shown in Table 2.6. Although the final production of units corresponds to the technically optimal solution, i.e., the total amount of electricity produced is the same as that found in case B, the companies' total profit and consumers' total costs are much higher than they are in case B.

Table 2.6: Results of case E

	Power (MW)								
Generation Company	**DA**		**CT**		**Final**		**Income (€)**	**Cost (€)**	**Profit (€)**
	EX	***IM***	***EX***	***IM***	***EX***	***IM***			
1	80	-	-80	-	0	-	224	0	224
2	300	-	-50	100	250	100	19815$^{(*)}$	18605$^{(**)}$	1210
3	20	-	-20	40	0	40	2356	2280	76
4	-	-	-	10	-	10	575	575	0
Total Power	400	-	-150	150	250	150	22970	21460	1510
Power Exchanges	-300	300			-150	150			
Price (€/MWh)	55.7								

$(*)\ 300 \cdot 55.7 - 50 \cdot 52.9 + 100 \cdot 57.5 = 19815$

$(**)\ (300 - 50) \cdot 52.9 + 100 \cdot 53.8 = 18605$

2.4.6 Portuguese case (F)

In the counter-trading mechanism implemented in Portugal, a generation unit that increases its production is paid at the minimum value between its bids in the day-ahead market and the counter-trading mechanism. Thus, the unit does not have any incentive to make a bid higher than its bid in the day-ahead market. Meanwhile a unit that reduces its production is charged at the maximum between its bids in the day-ahead market, the counter-trading mechanism and 0.85 times the day-ahead market price. Therefore, the unit does not have any incentive to make a bid lower than its bid in the day-ahead market. Consequently, the policy implemented in Portugal reduces the possibility of generation companies exercising market power in the counter-trading mechanism because the bids made by the generation companies are limited by the bids made in the day-ahead market.

However, this policy does not prevent generation companies from exercising market power in the day-ahead market in order to congest the system: generation companies may behave strategically by increasing the bid prices of the units in the importing area. Thus the day-ahead market price may increase, and the units in the importing area will not be dispatched in the day-ahead market, but will be necessary to clear congestion in the counter-trading mechanism.

As in the Dutch case, it achieves the optimal production costs, but consumer payments are higher. Comparing this solution with the case B, the final amount of electricity produced is the same, but the total companies profit and consumer costs are higher (Table 2.7).

Table 2.7: Results of case F

	Power (MW)								
Generation Company	**DA**		**CT**		**Final**		**Income (€)**	**Cost (€)**	**Profit (€)**
	EX	***IM***	***EX***	***IM***	***EX***	***IM***			
1	80	-	-80	-	0	-	8	0	8
2	300	-	-50	100	250	100	19445$^{(*)}$	18605$^{(**)}$	840
3	20	-	-20	40	0	40	2280	2280	0
4	-	-	-	10	-	10	575	575	0
Total Power	400	-	-150	150	250	150	22308	21460	848
Power Exchanges	-300	300			-150	150			
Price (€/MWh)	55.7								

$(*)\ 300 \cdot 55.7 - 50 \cdot 52.9 + 100 \cdot 53.8 = 19445$

$(**)\ (300-50) \cdot 52.9 + 100 \cdot 53.8 = 18605$

2.4.7 German case (G)

In Germany, the generation companies only make bids in the day-ahead market because there is no additional market for the resolution of system congestion. The units involved in the congestion clearing procedure are remunerated at their marginal costs. From a policy point of view, the main disadvantage of this mechanism is that the determination of the marginal costs is not a simple task for regulators to perform.

Although the congestion management mechanism in Germany uses only the marginal costs of the units, this does not guarantee that generation companies do not exercise market power in the day-ahead market with the aim of congesting the system. Under this mechanism, the total amount of electricity produced correspond to the optimum production cost. However, the companies' total profit and the consumers' total costs increase, in contrast to to case B (Table 2.8).

Table 2.8: Results of case G

Generation Company	Power (MW) DA EX	DA IM	CT EX	CT IM	Final EX	Final IM	Income (€)	Cost (€)	Profit (€)
A1	80	-	-80	-	0	-	8	0	8
A2	300	-	-50	100	250	100	19445$^{(*)}$	18605$^{(**)}$	840
A3	20	-	-20	40	0	40	2280	2280	0
A4	-	-	-	10	-	10	575	575	0
Total Power	400	-	-150	150	250	150	22308	21460	848
Power Exchanges	-300	300			-150	150			
Price (€/MWh)	55.7								

$(*)\ 300 \cdot 55.7 - 50 \cdot 52.9 + 100 \cdot 53.8 = 19445$

$(**)\ (300-50) \cdot 52.9 + 100 \cdot 53.8 = 18605$

2.4.8 Comparison

The technical solutions found in cases B and C are the same: the optimal solution to clear congestion between the areas. However, the consumers' cost and companies' profit are different in these cases since the changes in production made by the units in the counter-trading mechanism are not properly remunerated in case C (Table 2.9). It is naive to think that companies are not going to behave strategically and to think that the market outcome will be the solution found in case C. When generation companies take into account the effect of the counter-trading mechanism in their strategic behavior, the market solution is that found in cases D, E, F and G. In the Spanish case (D), the technical solution is worse than the optimal solution. Meanwhile, in the Dutch, Portuguese and German cases (E, F and G), the amount of electricity produced by the units is the same as the optimal solution. However, in all four cases, companies' total profit and consumers' total cost increase significantly with respect to case B.

Table 2.9: Case comparison

Case	**Total Consumers' Cost (€)**	**Total Production Cost (€)**	**Total Generators' Profit (€)**
A - Case without congestion	21520	21250	270
B - Nodal-pricing mechanism	21850	21460	390
C - Consumer payments minimization	21685	21460	225
D - Spain	22160	21630	530
E - The Netherlands	22970	21460	1510
F - Portugal	22308	21460	848
G - Germany	22308	21460	848

2.5 Conclusion

This chapter has studied the congestion management mechanisms implemented in Spain, the Netherlands, Portugal and Germany, comparing them with a nodal-pricing system. The idea behind is to motivate and to show how subsequent mechanism may have a very significant impact on the strategic behavior of the generation companies, and therefore on the results of the electricity market. The results have shown that the counter-trading mechanisms in these countries are inefficient because they allow generation companies to exercise market power. When generation companies detect that system congestion will occur, they vary the bid prices of their generation depending on the area, giving more importance to importing area production because the production in this area is necessary to clear congestion. Thus, generation companies may behave strategically between markets, bidding their units in the importing area at prices higher than their marginal costs in order to increase the price of electricity, and ensure that units are not dispatched in the day-ahead market but in the counter-trading mechanism.

In the four cases analyzed, the Spanish one was the only case in which the final the amount of electricity produced by the units do not correspond to the technically optimal solution, while in the Netherlands, Portugal and Germany the final quantities are the same as those found in the nodal-pricing system. However, in all four cases, the total consumer cost is higher than the cost in the optimal solution. The reason is that the electricity price in the four cases does not properly reflect the real production cost in the different zones of the power system. Therefore, in an electricity market with a counter-trading mechanism, the final outcome may not be technically or economically optimal and may increase the prices paid by consumers.

This chapter shows that single-price electricity markets with a subsequent mechanism used to clear technical constraints can be inefficient. These mechanisms allow generation companies to behave strategically and exercise market power. The identification of such inefficiencies and the effect on the companies' strategic behavior is the main goal of this thesis and the models presented in the following chapters. Chapters 3 and 4 generalize the model presented in this chapter to study the effect of congestion management mechanisms. Chapter 5 analyzes the effect of voltage requirements. Finally, chapter 6 study the mechanism implemented in Spain to contract and manage additional upward reserve.

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Chapter 3

The effect of system congestion on the strategic behavior of generation companies - Two-area system

Contents

This chapter presents a conjectural-variation-based equilibrium model of a single-price electricity market. The distinctive modeling feature introduced in this chapter is the formalization of the equilibrium equations taking into account the effect of congestion between areas on the generation companies' behavior. The results show that, when there is congestion between two areas, generation companies value differently the production of each area, giving more importance to the importing area.

3.1 Introduction

Chapter 2 has introduced an equilibrium model to analyze the inefficiencies of the counter-trading mechanisms implemented in some European countries. The existence of the counter-trading mechanism to clear system congestion allows generation companies to be able to exercise market power. Therefore, the generation companies may behave strategically between markets, bidding some units at prices higher than their marginal costs in order to ensure that these units are not dispatched in the day-ahead electricity market but in the counter-trading mechanism. Generation companies also may behave strategically by modifying the day-ahead electricity price.

This chapter proposes a market equilibrium model for assessing the effect of system congestion on the bidding strategies of generation companies in a single-price electricity market[1]. The model includes two kinds of strategic behavior. The first is the ability to modify the day-ahead market electricity price; the second is the ability to behave strategically between markets. The model is suitable in cases in which congestion can isolate certain areas of the power system, and therefore, a generation redispatch is necessary because only a small number of plants can supply the demand in those areas. The model generalizes a previous model (Barquín et al., 2004; Centeno et al., 2007) used to find the equilibrium in a single-price electricity market by means of an optimization procedure. Both models can be classified as conjectural-variation-based equilibrium models. Furthermore, due to the non-convexity and non-linearity of the proposed model, this chapter provides an iterative methodology in order to find the optimal solution. The performance of the proposed approach is successfully validated with numerical simulations.

[1]This market equilibrium model is presented in Delgadillo et al. (2013)

The major contributions of this chapter are:

1. The conjectural-variation-based equilibrium model of a single-price electricity market (Barquín et al., 2004; Centeno et al., 2007) is extended by adding the effect of congestion between areas in the mathematical formulation of the market equilibrium equations.

2. The model is suitable for power systems with two-areas, one exporting area and one importing area. Although this is the simplest representation of congestion in a power system, it allows to understand clearly the effect of system congestion. Moreover, this representation is adequate for real systems in which only one flowgate is congested splitting the power system in two areas. Chapter 4 extends the model to multiple areas.

3. A solution method is proposed to solve the equations of the formulated equilibrium problem by using an iterative procedure that solves an optimization problem in each iteration.

The remainder of this chapter is organized as follows. Section 3.2 presents an overview of the model described in (Barquín et al., 2004) and (Centeno et al., 2007). Section 3.3 adds the effect of congestion in the model and describes the proposed solution algorithm. Section 3.4 provides and analyzes a numerical example. Finally, Section 3.5 draws relevant conclusions.

3.2 Market equilibrium model without network constraints

This section describes the basic model presented in Barquín et al. (2004) and Centeno et al. (2007) which disregards the network constraints. The market equilibrium under deterministic conditions is studied in Barquín et al. (2004), while the uncertainty and stochasticity of the different variables such as total demand and cost function is analyzed in Centeno et al. (2007). More details about the model complexity, intertemporal constraints and hydro management can be found in the mentioned references.

Under the framework of game theory, the market equilibrium is reached at the point where each generation company maximizes its own profit, taking into account that the rest of the companies also maximize their profits. This equilibrium point is known as the Nash Equilibrium (Nash, 1950), in which the companies do not have an incentive to unilaterally modify their strategic behavior because any deviation entails a decrease in benefits.

The equilibrium model allows different strategic behaviors to be represented by means of a parameter called the conjectured-price response. It can be assumed that the conjectured-price response is a known non-negative constant because major structural or regulatory changes are not expected in the medium-term (Díaz et al., 2010). In the model, this parameter is considered an exogenous variable and the value can be estimated from historical data by means of different methods, among which are the methodologies presented in García-Alcalde et al. (2002); Bunn (2004); de Haro et al. (2007) and Díaz et al. (2010). The conjectured-price response θ_i of company i is the negative of the derivative of the electricity market price λ with respect to the production quantity P_i of the generation company, that is:

$$\theta_i = -\frac{\partial \lambda}{\partial P_i} \geq 0 \qquad \forall i \in I \tag{3.1}$$

where I is the set of generation companies.

In the simplest situation, the profit π_i of the generation company i at the clearing price λ is equal to the revenues minus the costs of the company:

$$\pi_i = \lambda \cdot P_i - C_i\left(P_i\right) \qquad \forall i \in I \tag{3.2}$$

The equilibrium point is then calculated by expressing the first-order profit-maximization condition for each generation company, which yields:

$$\frac{\partial \pi_i}{\partial P_i} = \lambda + P_i \cdot \frac{\partial \lambda}{\partial P_i} - \frac{\partial C_i\left(P_i\right)}{\partial P_i} = 0 \qquad \forall i \in I \tag{3.3}$$

Substituting the conjectural variation (3.1) into (3.3):

$$\lambda - \theta_i \cdot P_i = \frac{\partial C_i\left(P_i\right)}{\partial P_i} \qquad \forall i \in I \tag{3.4}$$

In this manner, the market equilibrium is reached when the marginal revenue MR_i (the left hand side on (3.4)) equals the marginal cost MC_i (the right hand side on (3.4)) for each company i:

$$MR_i = MC_i \qquad \forall i \in I \tag{3.5}$$

Furthermore, in electric power systems, the generation and demand must be balanced:

$$\sum_{i \in I} P_i = D \tag{3.6}$$

The inverse demand curve is the relationship between market price and demand (3.7). To ensure the existence of the equilibrium, the inverse demand curve must satisfy certain properties. This function has to be continuous, differentiable, monotone and strictly decreasing which are reasonable assumptions.

$$\lambda = \lambda (D) \tag{3.7}$$

The market equilibrium is therefore defined by (3.4), (3.6) and (3.7). This equilibrium can be calculated as the solution of the minimization problem:

$$\begin{aligned} \min_{P_i, D} \quad & \sum_{i \in I} \overline{C_i} (P_i) - U(D) \\ \text{s.t} \quad & \\ & \sum_{i \in I} P_i = D \qquad : (\lambda) \end{aligned} \tag{3.8}$$

where $\overline{C_i} (P_i)$ is the so-called effective cost function of company i, and $U(D)$ is the utility demand function. It is important to note that the clearing price λ is the dual variable associated to the power balance constraint (see Barquín et al. (2004) for further details).

The effective cost function is defined as:

$$\overline{C_i} (P_i) = C_i (P_i) + \theta_i \cdot \frac{{P_i}^2}{2} \qquad \forall i \in I \tag{3.9}$$

And the utility demand function is defined as:

$$U(D) = \int_0^D \lambda (D) \, dD \tag{3.10}$$

In some electricity markets, it is common to assume that the demand is inelastic, i.e., the demand is a known constant. In this situation, the optimization problem does not include the term $U(D)$, and the market equilibrium is solved in the same manner.

3.3 Effect of congestion on a single-price electricity market

This section studies the effect of congested transmission flowgates in the single-price model presented previously. The formulation is based on the same structural assumptions presented in section 2.3.1. In the model, there are two areas, one exporting area (EX) and one importing area (IM), interconnected by a flowgate with limited transfer capacity ($\overline{F_l}$) as shown in Fig. 3.1. Chapter 4 extends the model to multiple areas. In both areas, the demands (D^{EX} and D^{IM}) are inelastic. Both areas belong to the same electricity system, and therefore, the day-ahead electricity market price is the same in both areas. The day-ahead market-clearing process determines the productions P_i^{EX} and P_i^{IM} for each company i. If the power network is not taken into account in the day-ahead market, the resulting flows between the areas *EX* and *IM* may be not feasible since flows may exceed the maximum capacity. In that case, in order to eliminate overflows, the total generation of the exporting area has to be reduced ΔP_i^{EX} while the total generation of the importing area has to be incremented ΔP_i^{IM}. The difference between the real production and the result of the day-ahead market will be paid or charged at a certain price. These payments can be viewed as an income or a cost depending on whether the unit increases or reduces its production.

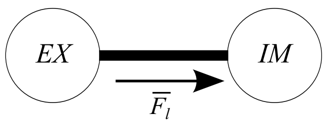

Figure 3.1: Two-area system

In different systems, there are different mechanisms to clear system congestion as presented in chapter 2. The model described in this chapter focuses on the congestion management mechanism used in Spain (SEE, 2012), although the proposed methodology can be adapted to other systems with different congestion management mechanisms. A more detailed description of the management of power system constraints in the Spanish electricity market can be found in Miguelez et al. (2004).

A brief description of the mechanism used in Spain is as follows. The congestion management mechanism is subsequent to the day-ahead market clearing process. The system operator receives price and quantity bids from the generation companies for the production that the generation units can increase or reduce with respect to the result of the day-ahead market. After that, the system operator performs different security analyses taking into account the unit productions established in the day-ahead market with the aim of identifying system congestion. Therefore, some units will reduce or increase their production in order to clear congestion. For those units that have to increase their production, the difference between the real production and the result of the day-ahead market will be paid at the company's offer price in the congestion management mechanism. For those units that have to reduce their production, the difference between the result of the day-ahead market and the real production will be charged at the day-ahead market price which means that there is no loss of profit.

The quantity increased by the units will be established at the lowest cost solution. The minimum cost solution is determined by the bids submitted by the generation companies in the congestion management mechanism. The quantity reduced by the units depends on the so-called contribution factor to the congestion, commonly referred in the literature as Generator Shift Factor (Dobson et al., 2001). This factor expresses the change in the flow over the interconnection line when the generation of each unit at the exporting area changes. First, the production of the unit with the highest contribution factor is reduced and the following reductions will continue in the order of the contributing factors until the congestion disappears. When many units have the same contribution factor the reduction will be proportional to their production.

3.3.1 Market equilibrium equations

In order to determine the market equilibrium, one possible solution is that the market equilibrium model explicitly includes a constraint that limits the production in the exporting area. In that case, the result of the market clearing process does not exceed the maximum flow. Although this is the correct approach in a centralized cost-minimization operation, such an approach would incorrectly model the congestion management mechanism and its influence on the day-ahead market, because: i) with that constraint, an additional dual variable would be necessary. This new dual variable would appear on the optimality conditions of the units in the exporting area, i.e., optimality conditions of problem (3.8) would be modified and they would not be equivalent to the market equilibrium conditions, and ii) that constraint would be equivalent to exporting area units bidding at lower prices.

Let λ be the day-ahead market price, and γ the price in the congestion management mechanism. Then, the profit π_i of the generation company i is equal to the revenue in the day-ahead market, plus the income of the increased generation in *IM*, minus the charge of the reduced generation in *EX*, minus the production costs in each area:

$$\begin{aligned}\pi_i = \lambda \cdot \left(P_i^{IM} + P_i^{EX}\right) + \gamma \cdot \Delta P_i^{IM} - \lambda \cdot \Delta P_i^{EX} \\ - C_i^{IM}\left(P_i^{IM} + \Delta P_i^{IM}\right) - C_i^{EX}\left(P_i^{EX} - \Delta P_i^{EX}\right) \qquad \forall i \in I\end{aligned} \tag{3.11}$$

In the two-area system studied, increasing the generation of any unit at the exporting area will result in the same flow change in the interconnection line. Hence, all units in area *EX* have the same contribution factor and the generation reduced by any unit in area *EX* will be proportional to its own production. Thus, there is a relationship between ΔP_i^{EX} and P_i^{EX}:

$$\Delta P_i^{EX} = \frac{f_l - \overline{F_l}}{\sum_{h \in I} P_h^{EX}} \cdot P_i^{EX} = m \cdot P_i^{EX} \qquad \forall i \in I \tag{3.12}$$

where h is the index of generation companies and m is the so-called reduction factor. The m value used must be consistent with the physical demand and productions, as shown in (3.13). f_l is the flow that would result if the final productions were the solution of the day-ahead market, as shown in (3.14). The value of f_l is greater than $\overline{F_l}$ because the model is describing the case with congestion between the two areas.

$$m = \frac{f_l - \overline{F_l}}{\sum_{h \in I} P_h^{EX}} \tag{3.13}$$

$$f_l = \sum_{h \in I} P_h^{EX} - D^{EX} \tag{3.14}$$

Therefore, the profit π_i is a function of P_i^{IM}, P_i^{EX} and ΔP_i^{IM}:

$$\begin{aligned}\pi_i = \lambda \cdot \left(P_i^{IM} + (1-m) \cdot P_i^{EX}\right) + \gamma \cdot \Delta P_i^{IM} \\ - C_i^{IM}\left(P_i^{IM} + \Delta P_i^{IM}\right) - C_i^{EX}\left((1-m) \cdot P_i^{EX}\right) \qquad \forall i \in I\end{aligned} \tag{3.15}$$

The equilibrium point is then calculated by expressing the first-order profit-maximization condition for the production in each area and the increasing production in area *IM* for each generation company. This yields (3.16) for the production at area *IM*, (3.17) for the production at area *EX* and (3.18) for the increasing production at area *IM*.

$$\begin{aligned}\frac{\partial \pi_i}{\partial P_i^{IM}} =& \lambda + \frac{\partial \lambda}{\partial P_i^{IM}} \cdot \left(P_i^{IM} + (1-m) \cdot P_i^{EX}\right) \\ & - \frac{\partial C_i^{IM}\left(P_i^{IM} + \Delta P_i^{IM}\right)}{\partial P_i^{IM}} = 0 \quad \forall i \in I \end{aligned} \tag{3.16}$$

$$\begin{aligned}\frac{\partial \pi_i}{\partial P_i^{EX}} =& \lambda \cdot (1-m) + \frac{\partial \lambda}{\partial P_i^{EX}} \cdot \left(P_i^{IM} + (1-m) \cdot P_i^{EX}\right) \\ & - \frac{\partial C_i^{EX}\left((1-m) \cdot P_i^{EX}\right)}{\partial P_i^{EX}} = 0 \quad \forall i \in I \end{aligned} \tag{3.17}$$

$$\begin{aligned}\frac{\partial \pi_i}{\partial \Delta P_i^{IM}} =& \gamma + \frac{\partial \gamma}{\partial \Delta P_i^{IM}} \cdot \Delta P_i^{IM} \\ & - \frac{\partial C_i^{IM}\left(P_i^{IM} + \Delta P_i^{IM}\right)}{\partial \Delta P_i^{IM}} = 0 \quad \forall i \in I \end{aligned} \tag{3.18}$$

Equations (3.16) and (3.17) show that the day-ahead market price depends on the marginal cost of the units, the strategic behavior of companies in the day-ahead market, and the system congestion valued by the factor m. Equation (3.18) indicates that the price in the congestion management mechanism depends on the marginal cost of the units in the importing area since these are the units that have to increase the production in order to clear the system congestion.

A generation company can affect the day-ahead market price by modifying its production in either area because both areas are in the same electricity system, and, furthermore, the power network is not taken into account in the day-ahead market clearing process and there is only one day-ahead market price for both areas. Hence, the company's conjectured-price response is the same for both areas (3.19). The conjectured price response of the generation company in the congestion management mechanism is defined in (3.20). The values of these parameters can be estimated using historical data as proposed in García-Alcalde et al. (2002); Bunn (2004); de Haro et al. (2007) and Díaz et al. (2010).

$$\theta_i = -\frac{\partial \lambda}{\partial P_i^{IM}} = -\frac{\partial \lambda}{\partial P_i^{EX}} \geq 0 \qquad \forall i \in I \tag{3.19}$$

$$\beta_i = -\frac{\partial \gamma}{\partial \Delta P_i^{IM}} \geq 0 \qquad \forall i \in I \tag{3.20}$$

Let MC_i^{IM} and MC_i^{EX} the marginal costs in the importing area *IM* and exporting area *EX*, respectively. Then, the market equilibrium equation (3.16) for area *IM* can be written as shown in (3.21) substituting $\theta_i = -{\partial\lambda}/{\partial P_i^{IM}}$ and isolating the market clearing price λ. The market equilibrium equation (3.17) for area *EX* can be written as shown in (3.22) substituting $\theta_i = -{\partial\lambda}/{\partial P_i^{EX}}$, isolating the market clearing price λ and dividing by $(1-m)$. Similarly, the market equilibrium equation (3.18) can be written as shown in (3.23) substituting $\beta_i = -{\partial\gamma}/{\partial\Delta P_i^{IM}}$ and isolating the market clearing price γ:

$$\lambda = \theta_i \cdot P_i^{IM} + \theta_i \cdot (1-m) \cdot P_i^{EX} + MC_i^{IM}\left(P_i^{IM} + \Delta P_i^{IM}\right) \qquad \forall i \in I \tag{3.21}$$

$$\lambda = \frac{\theta_i}{1-m} \cdot P_i^{IM} + \theta_i \cdot P_i^{EX} + MC_i^{EX}\left((1-m) \cdot P_i^{EX}\right) \qquad \forall i \in I \tag{3.22}$$

$$\gamma = \beta_i \cdot \Delta P_i^{IM} + MC_i^{IM}\left(P_i^{IM} + \Delta P_i^{IM}\right) \qquad \forall i \in I \tag{3.23}$$

Note that the case without congestion, $m = 0$, equations (3.21) and (3.22) are the same that equation (3.4) which corresponds to the market equilibrium without network constraints. In the equilibrium market equation for the production at the importing area (3.21), the conjectural variation is modified by the factor $(1-m)$ for the production in *EX*. In the same way, in the equilibrium market equation for the production at the exporting area (3.22), the conjectural variation is modified by the factor ${1}/{(1-m)}$ for the production in *IM*. Hence, in both equilibrium market equations, the weight of the production at the importing area is greater than the weight of the production at the exporting area, $\theta_i > \theta_i \cdot (1-m)$ in (3.21) and ${\theta_i}/{(1-m)} > \theta_i$ in (3.22). That is to say, in a two-area system with congestion in the interconnection between areas, a generation company values the production in each area differently, giving a greater weight to generation in the importing area. This is an intuitive result, because a generation company can predict the state of congestion and therefore it knows that its production in *EX* will be reduced while production in *IM* will be incremented. However, the important point is that the proposed model is able to quantify the impact of congestion on the market taking into account the company's strategic behavior.

Finally, in an electricity market, the total generation and demand have to be balanced. Equations (3.24) and (3.25) are the power balance constraints in the day-ahead market and the congestion management mechanism, respectively:

$$\sum_{i\in I} P_i^{IM} + \sum_{i\in I} P_i^{EX} = D^{IM} + D^{EX} \tag{3.24}$$

$$\sum_{i\in I} \Delta P_i^{IM} = \sum_{i\in I} m\cdot P_i^{EX} \tag{3.25}$$

3.3.2 Equivalent minimization problem

The market equilibrium defined by (3.21), (3.22), (3.23), (3.24) and (3.25) can be calculated as the solution of the following quadratic minimization problem:

$$\min_{P_i^{IM}, P_i^{EX}, \Delta P_i^{IM}} \sum_{i\in I} \overline{C_i}\left(P_i^{IM}, P_i^{EX}, \Delta P_i^{IM}\right) \tag{3.26}$$

s.t.

$$\sum_{i\in I} P_i^{IM} + (1-m)^2 \cdot \sum_{i\in I} P_i^{EX} = D^* \qquad :(\lambda) \tag{3.27}$$

$$\sum_{i\in I} \Delta P_i^{IM} = f_l - \overline{F_l} \qquad :(\gamma) \tag{3.28}$$

where, the modified effective cost function $\overline{C_i}\left(P_i^{IM}, P_i^{EX}, \Delta P_i^{IM}\right)$ is defined as:

$$\begin{aligned}\overline{C_i}\left(P_i^{IM}, P_i^{EX}, \Delta P_i^{IM}\right) = & C_i^{IM}\left(P_i^{IM} + \Delta P_i^{IM}\right) \\ & + (1-m)\cdot C_i^{EX}\left((1-m)\cdot P_i^{EX}\right) \\ & + \frac{\theta_i}{2}\cdot\left(P_i^{IM} + (1-m)\cdot P_i^{EX}\right)^2 \\ & + \frac{\beta_i}{2}\cdot\left(\Delta P_i^{IM}\right)^2 \quad \forall i\in I\end{aligned} \tag{3.29}$$

and D^* is defined as:

$$D^* = D^{IM} + D^{EX} + \left(f_l - \overline{F_l}\right)\cdot(m-2) \tag{3.30}$$

Constraint (3.27) is the power balance constraint. However, the original constraint (3.24) has been modified so that the solution of the minimization problem is equivalent to the market equilibrium equations as shown in Section 3.3.2.2. Constraint (3.28) determines the quantity incremented in the importing area. Note that the dual variable of (3.27) is again the price in the day-ahead market, while the dual variable of (3.28) corresponds to the price in the congestion management mechanism.

3.3.2.1 Equivalence between the minimization problem and the equilibrium equations

This section presents the proof of the equivalence between the minimization problem and the equilibrium market equations.

The solution to the minimization problem can be calculated by introducing the constraints in the objective function using the Lagrangian function $\mathscr{L}\left(P_i^{IM}, P_i^{EX}, \Delta P_i^{IM}\ \forall i \in I, \lambda, \gamma\right)$. In this case, the objective is to find a saddle point that minimizes the primal variables and maximizes the dual variables.

$$\min_{P_i^{IM}, P_i^{EX}, \Delta P_i^{IM}\ \forall i \in I} \quad \max_{\lambda, \gamma} \mathscr{L}\left(P_i^{IM}, P_i^{EX}, \Delta P_i^{IM}\ \forall i \in I, \lambda, \gamma\right)$$

where $\mathscr{L}\left(P_i^{IM}, P_i^{EX}, \Delta P_i^{IM}\ \forall i \in I, \lambda, \gamma\right)$ is defined as:

$$\begin{aligned}\mathscr{L} = &\sum_{i\in I} \overline{C_i}\left(P_i^{IM}, P_i^{EX}, \Delta P_i^{IM}\right) \\ &- \lambda \cdot \left(\sum_{i\in I} P_i^{IM} + (1-m)^2 \cdot \sum_{i\in I} P_i^{EX} - D^*\right) \\ &- \gamma \cdot \left(\sum_{i\in I} \Delta P_i^{IM} - \left(f_l - \overline{F_l}\right)\right) \qquad (3.31)\end{aligned}$$

the first-order optimization conditions are:

$$\frac{\partial \mathscr{L}}{\partial P_i^{IM}} = MC_i^{IM}\left(P_i^{IM} + \Delta P_i^{IM}\right) + \theta_i \cdot \left(P_i^{IM} + (1-m)\cdot P_i^{EX}\right) - \lambda = 0 \quad \forall i \in I \qquad (3.32)$$

$$\begin{aligned}\frac{\partial \mathscr{L}}{\partial P_i^{EX}} = &(1-m)^2 \cdot MC_i^{EX}\left((1-m)\cdot P_i^{EX}\right) \\ &+ \theta_i \cdot (1-m) \cdot \left(P_i^{IM} + (1-m)\cdot P_i^{EX}\right) - \lambda \cdot (1-m)^2 = 0 \quad \forall i \in I \qquad (3.33)\end{aligned}$$

$$\frac{\partial \mathscr{L}}{\partial \Delta P_i^{IM}} = MC_i^{IM}\left(P_i^{IM} + \Delta P_i^{IM}\right) + \beta_i \cdot \Delta P_i^{IM} - \gamma = 0 \quad \forall i \in I \qquad (3.34)$$

Isolating the market clearing price λ in (3.32) and (3.33), the resulting equations are equivalent to the equilibrium conditions (3.21) and (3.22). Equation (3.34) is equivalent to (3.23).

3.3.2.2 Power balance constraints

This section presents the mathematical derivation of the power balance constraints (3.27) and (3.28).

1. **Day-ahead market**
   Substituting $f_l - \overline{F_l} = m \cdot \sum_i P_i^{EX}$ in (3.30), gives:

$$D^* = D^{IM} + D^{EX} + m \cdot (m-2) \cdot \sum_{i \in I} P_i^{EX} \tag{3.35}$$

   substituting (3.35) in (3.27), gives:

$$\sum_{i \in I} P_i^{IM} + (1-m)^2 \cdot \sum_{i \in I} P_i^{EX} = D^{IM} + D^{EX} + m \cdot (m-2) \cdot \sum_{i \in I} P_i^{EX} \tag{3.36}$$

   rewriting the equation:

$$\sum_{i \in I} P_i^{IM} + \left(1 - 2m + m^2 - m^2 + 2m\right) \cdot \sum_{i \in I} P_i^{EX} = D^{IM} + D^{EX} \tag{3.37}$$

   thus, the balance between generation and demand is reached in the day-ahead market:

$$\sum_{i \in I} P_i^{IM} + \sum_{i \in I} P_i^{EX} \quad = \quad D^{IM} + D^{EX} \tag{3.38}$$

2. **Congestion management mechanism**
   Substituting $f_l - \overline{F_l} = m \cdot \sum_i P_i^{EX}$ in (3.28), gives:

$$\sum_{i \in I} \Delta P_i^{IM} = m \cdot \sum_{i \in I} P_i^{EX} \tag{3.39}$$

   thus, generation incremented in area *IM* is equal to generation reduced in area *EX:*

$$\sum_{i \in I} \Delta P_i^{IM} = \sum_{i \in I} \Delta P_i^{EX} \tag{3.40}$$

3.3.3 Solution methodology

Assuming that the m value is constant, the problem (3.26)-(3.28) is a quadratic optimization problem with linear constraints, and that available commercial software can effectively solve. However, in order to find an optimal solution that is consistent with the real value of m, an iterative methodology is proposed and it works as follows:

1. Initialize $m = 0$ and $f_l = \overline{F_l}$. This case corresponds to a case without congestion.

2. Solve the minimization problem (3.26)-(3.28). This gives a solution for P_i^{IM}, P_i^{EX}, ΔP_i^{IM}, λ and γ.

3. Update the values m and f_l using (3.13) and (3.14). If the change of m with respect to the previous iteration is less than an ϵ value, the algorithm stops; otherwise it goes to 2.

3.4 Numerical example

In order to analyze the effect of congestion on the model presented previously, a simple example will be used. The model considers 7 generation units owned by 4 generation companies as shown in Table 3.1. In this example, the generation companies do not exercise market power in the counter-trading mechanism, $\beta_i = 0$, because the aim is to analyze the effect on the strategic behavior in the day-ahead electricity market. Demand in area *IM* is $D^{IM} = 300$ MW and in area *EX* is $D^{EX} = 100$ MW. The proposed algorithm and the optimization problem are solved with CPLEX 12.1 (CPLEX, 2014) under GAMS (GAMS, 2014). The average computing time required to achieved the optimal solution of the problem (3.26)-(3.28) was 0.08 s, and with $\epsilon = 3 \times 10^{-6}$ average 70 iterations were necessary. The number of iterations required to solve the problem increases when the maximum flow is lower.

Table 3.1: Characteristics of the units.

Generation company	θ_i $[\frac{€/MWh}{MW}]$	β_i $[\frac{€/MWh}{MW}]$	**Generation unit**	**Area**	**Variable cost [€/MWh]**	**Maximum production [MW]**
G1	0.01	0	U1_1E	*EX*	42.0	100
			U1_2E	*EX*	42.5	70
G2	0.02	0	U2_E	*EX*	42.5	90
			U2_I	*IM*	42.9	70
G3	0.05	0	U3_E	*EX*	37.0	110
			U3_I	*IM*	38.8	70
G4	0.05	0	U4_I	*IM*	42.5	60

Table 3.2 presents the results of the reference case. In this case, there is no congestion between areas. The day-ahead market clearing price is $\lambda = 44.21$ €/MWh and the flow between areas *EX* and *IM* is $f_l = 265.71$ MW.

Table 3.2: Results of the reference case.

Generation company	**Generation unit**	P **[MW]**
G1	U1_1E	100.0
	U1_2E	70.0
G2	U2_E	85.7
	U2_I	0.0
G3	U3_E	110.0
	U3_I	0.0
G4	U4_I	34.3

In the following simulations, the flow between areas *EX* and *IM* is limited to be between 266 MW (no congestion) and 100 MW (the most extreme case). Figures 3.2-3.5 show the results of these simulations in terms of production, price, and flow. In these figures, there are five specific intervals determining the generation companies' behavior. The first interval corresponds to a maximum flow, $\overline{F_l}$, between 266 MW and 198 MW. The second interval is between 198 MW and 181 MW. The third interval is between 181 MW and 155 MW. The fourth interval is between 155 MW and 123 MW. Finally, the fifth interval is between 123 MW and 100 MW.

Fig. 3.2 presents the final production (the day-ahead market production plus the production changes set in the congestion management mechanism) for each generation unit. As the maximum flow is lower, the production of the units located at the exporting area (U1_1E, U2_E, U3_E) decreases. On the other hand, the production of units U2_I and U3_I increases with respect to the case without maximum flow limit until their maximum production is reached.

The existence of congestion between areas not only affects the final behavior of the generation companies, but also affects their behavior in the day-ahead market. For example, in the third and fourth intervals, it is interesting to analyze the behavior of generation company G2 shown in Fig. 3.3. In the day-ahead market, generation company G2 gives more importance to the production of its unit in the importing area, even though U2_I is more expensive than U2_E. Thus, the production of unit U2_I increases while the production of unit U2_E decreases.

When $\overline{F_l} = 152$ MW, it can be noted how the modification of the conjectural variation due to the presence of congestion (Equations (3.21) and (3.22)) affects the cost perceived by the generation company G2. In that case, the production of unit U2_I is equal to production of unit U2_E, i.e. generation company G2 perceives the same *apparent cost*[2] for both units.

Therefore, the results obtained for the production of the units confirm the behavior of the equilibrium presented in equations (3.21) and (3.22). These results show that, when the congestion between areas is considered, generation companies give more value to the production in the importing area than the production in the exporting area.

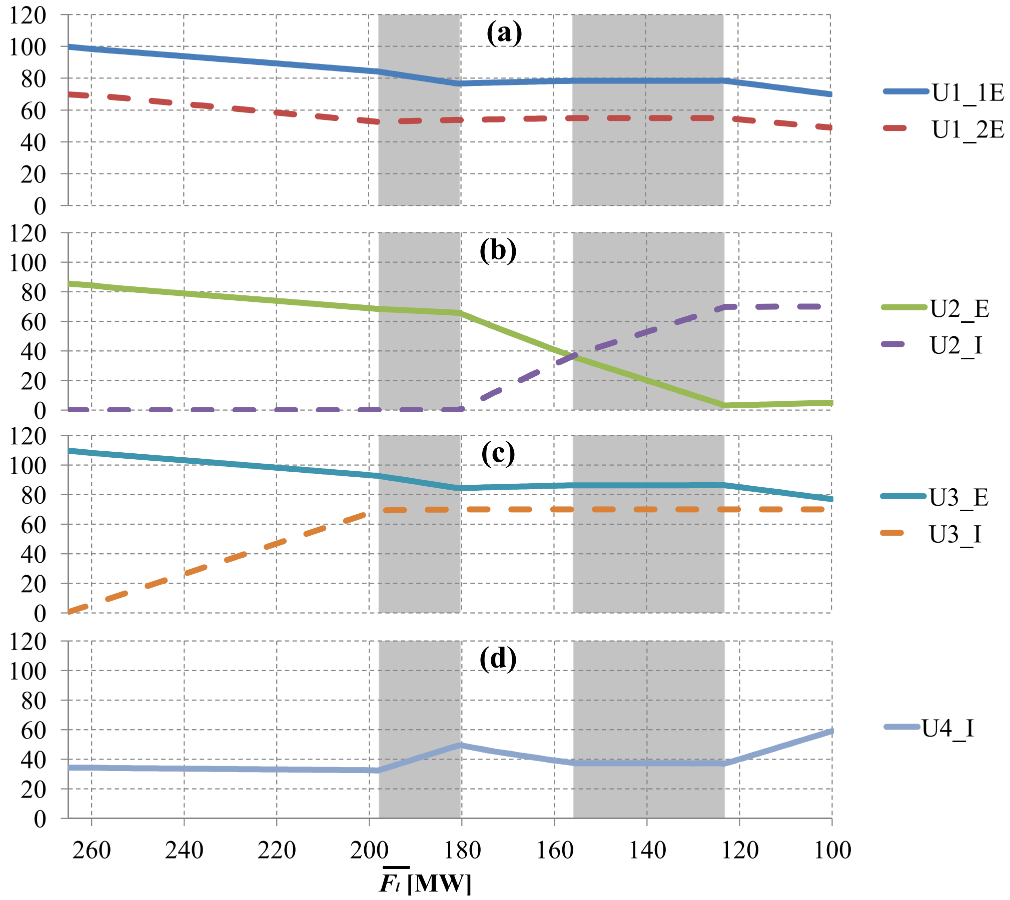

Figure 3.2: Unit production [MW]. (a) Generation company G1. (b) Generation company G2. (c) Generation company G3. (d) Generation company G4.

[2]The apparent cost corresponds to the cost at which the generation company should offer the production of the unit in order to maximize its profit. It can only be determined when the solution of market equilibrium is found because it depends on the unit's production and the total generation of the company in the day-ahead market. The value can be calculated as an equivalent marginal cost that is perceived by the system when the unit produces a determined quantity in the day-ahead market (Reneses et al., 2004).

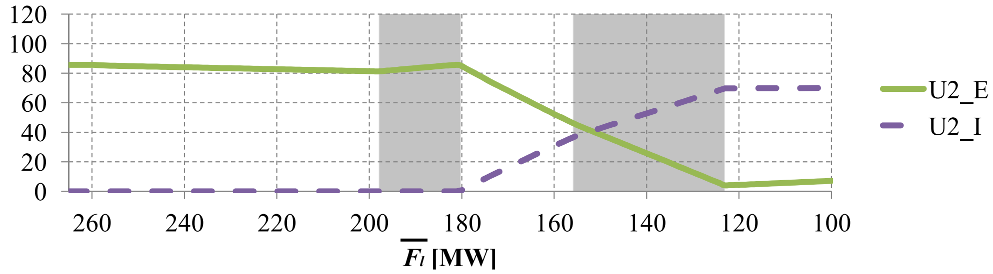

Figure 3.3: Production of the units [MW] of generation company G2 in the day-ahead market.

Fig. 3.4 presents the market clearing price for the day-ahead market and the congestion management mechanism. In the first interval, the day-ahead market price decreases from 44.21 €/MWh to 44.12 €/MWh. The price in the congestion management mechanism is 38.8 €/MWh because the increased production required in area *IM* is produced entirely by unit U3_I. In the second and third intervals, the day-ahead market price increases progressively from 44.12 €/MWh to 44.36 €/MWh. The price in the congestion management mechanism is 42.5 €/MWh because the increased production required in area *IM* is produced by units U3_I and U4_I. In the fourth interval, the day-ahead market price is 44.36 €/MWh and the price in the congestion management mechanism is 38.8 €/MWh. In this interval, the production of generation companies G2 and G4 in the area *IM* is fully negotiated in the day-ahead market. As the unit U2_I increases its production, the required production in area *IM* decreases and it can be fully supplied by the unit U3_I. Finally, in the fifth interval the day-ahead market price increases progressively to 44.64 €/MWh, and once again the increased production required in area *IM* is produced by units U3_I and U4_I, causing the price in the congestion management mechanism to be equal to 42.5 €/MWh.

Although production at the two areas are valued differently when there is congestion between them, there is not a constraint that explicitly limits production in the exporting area and therefore the resulting flow in the day-ahead market may be greater than the maximum flow. However, the flow calculated with the final productions, $P_i^{IM} + \Delta P_i^{IM}$ and $P_i^{EX} - \Delta P_i^{EX}$, meets the maximum flow constraint, as shown in Fig. 3.5 (a). In this figure, it can be inferred that the flow decreases because the production at the importing area increases when there is congestion. Finally, Fig. 3.5 (b) illustrates the value of the reduction factor m. This value starts at zero because there is no congestion and the resulting flow does not exceed the maximum flow. The value of m increases gradually while the maximum flow decreases.

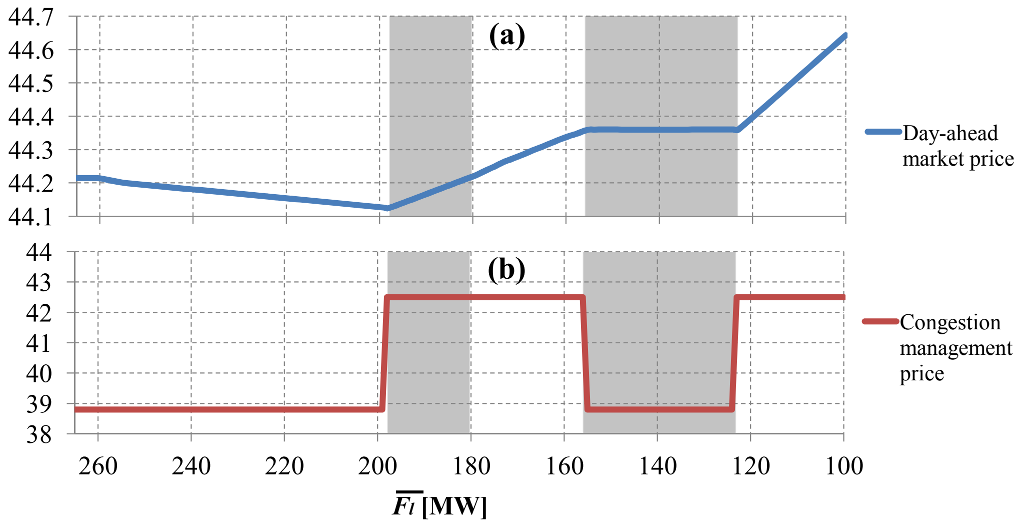

Figure 3.4: Market clearing price [€/MWh]. (a) Day-ahead market. (b) Congestion management mechanism.

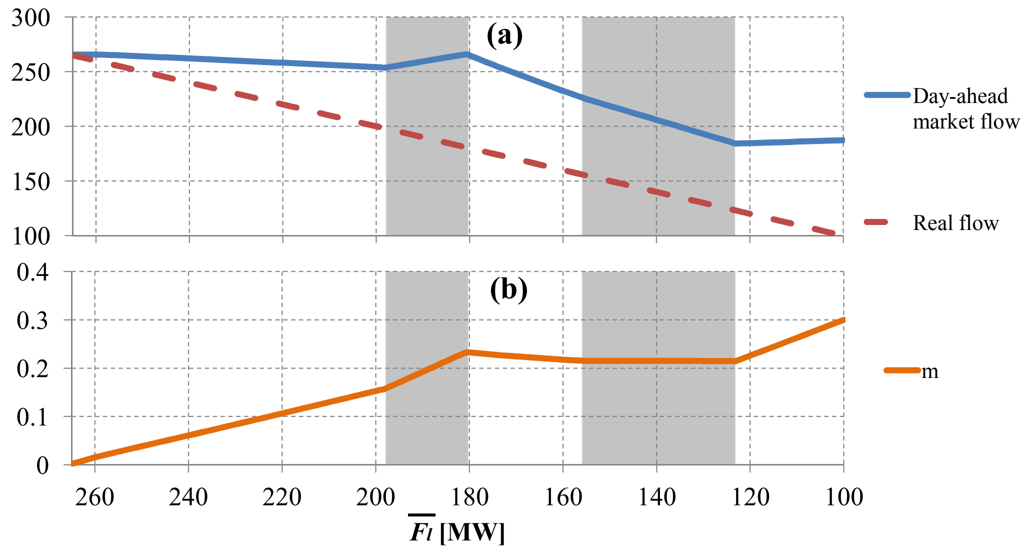

Figure 3.5: Flow and factor m. (a) Resulting flow [MW]. (b) Reduction factor m.

3.5 Conclusion

This chapter has studied the effect of congestion between areas on the generation companies' strategic behavior in a single-price electricity market by means of a conjectural-variation-based equilibrium model. The main feature of this model is that the market equilibrium equations can be formulated taking network congestion into account. Furthermore, the model and the methodology of the solution have been validated through a simple numerical example which quantifies the effect of network congestion.

The model and the results have shown that if there is congestion between two areas, the generation companies have to modify their behavior in order to maximize their profit. The most noteworthy result is that, even if the conjectural variation is the same for each area (such as the case of a single-price market), the value of the conjectural variation perceived by the generation company is modified due to the presence of congestion, i.e., the conjectured-price response is multiple by a term that depends on the reduction factor, and this term is different for the importing and exporting areas as shown in equations (3.21) and (3.22). Therefore, the companies' strategic behavior in the day-ahead electricity price depends on the system congestion, i.e., the congestion management mechanism modifies the solution of the day-ahead electricity market. As a consequence, the production of the importing area is incremented while the production of the exporting area is reduced.

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Chapter 4

The effect of system congestion on the strategic behavior of generation companies - Multi-area system

Contents

This chapter presents a conjectural-variation-based equilibrium model of a single-price electricity market. In the electricity market, firstly, the market clearing process is performed in the day ahead market and after that, a counter-trading mechanism is used to clear network congestion. The system may have any configuration, either radial or meshed, and there is no restriction on the size of the system. The main contribution of the model is that the market equilibrium equations incorporate the effect of congestion between multiple areas on the generation companies' strategic behavior. Furthermore, the market equilibrium equations are solved using an equivalent optimization problem. The optimization problem has two stages: the first stage corresponds to the day-ahead market and the second stage is a DC optimal power flow that clears network congestion. Numerical results are provided to illustrate the performance of the proposed approach in a real-size power system. The convergence of the iterative algorithm is also studied.

4.1 Introduction

Chapter 3 shows the strategic behavior of generation companies in a single-price electricity market with a counter-trading mechanism. In the exporting areas, generation companies have incentives to reduce their production to ensure that they will not be penalized in the counter-trading mechanism. However, this behavior does not ensure that congestion will not occur. In the importing areas, generation companies know that their production becomes more valuable because they have a chance to participate in the counter-trading mechanism. Thus, the opportunity cost of the units in the importing areas has increased. The main feature of the model presented in chapter 3 is that these effects can be easily quantified by modifying the conjectured-price response of the generation company in a medium-term conjectural-variation-based equilibrium model. However, the model is only suitable in power systems with two areas, one exporting area and one importing area connected by a flowgate with limited transfer capacity.

This chapter proposes a model[1] that generalizes the one presented in chapter 3. Both models analyze the effect of system congestion on the generation companies' strategic behavior. This effect can be calculated using the amount that units must reduce in order to resolve congestion. In chapter 3, those values can

[1]This model is presented in Delgadillo and Reneses (2013)

be easily calculated from the resulting flow over the interconnection because it is a two-area system. However, in more complex systems, it is necessary to carry out a power flow. Therefore, the major contribution of this chapter is that the power system is not limited to simple networks. The power system can have any configuration, both radial and meshed networks, and there is no restriction on the maximum number of areas. Thus, the formulation presented in chapter 3 is only a particular example of the formulation proposed in this chapter.

By including the power flow solution, the market equilibrium equations have to be solved using an equivalent two-stage optimization problem while the model in chapter 3 is a single-level optimization problem. The first stage models the day-ahead market clearing process. In the second stage, a DC optimal power flow (DC-OPF) clears network congestion. The decision variables of the day-ahead market clearing process affect the solution of the DC-OPF, and the solution of the DC-OPF must be taken into account in the day-ahead market clearing process. The equivalent two-stage optimization problem is nonconvex and nonlinear. Thus, the objective of this chapter is to provide an iterative procedure to solve this noncovex and nonlinear two-stage optimization problem.

The remainder of this chapter is organized as follows: Section 4.2 describes the effect of network congestion on the market equilibrium equations as well as the equivalent optimization problem. In Section 4.3, numerical results illustrate the performance of the model. Finally, relevant conclusions are drawn in section 4.4.

4.2 Model with multiple areas

4.2.1 Market equilibrium equations

This section generalizes the models presented in chapter 3 in order to incorporate the effect of congestion between multiple areas on the generation companies' strategic behavior in a single-price electricity market. In the electricity market, the day-ahead market clearing process is performed first and then a countertrading mechanism is used to clear network congestion.

When the connection between areas is congested, the final production of the generation units may be different from those assigned in the market clearing of the day-ahead market. Hence, some generation units increase their production while other units reduce their production to meet the power system constraints. The quantities increased in the final production are paid at a price γ, while the reductions are charged at the same price than the day-ahead market λ. In this case, the profit π_i of company i is equal to the revenue in the day-ahead market, plus the income of the increased generation in the importing areas, minus the charge of the reduced generation in the exporting areas, minus the production costs of the generation units:

$$\pi_i = \lambda \cdot \sum_{j \in J_i} P_j + \gamma \cdot \sum_{j \in J_i} X_j - \lambda \cdot \sum_{j \in J_i} W_j - \sum_{j \in J_i} C_j \left(P_j + X_j - W_j\right) \quad \forall i \in I \tag{4.1}$$

where J_i is the set of units which belongs to company i, I is the set of generation companies, P_j is the day-ahead market production of unit j, and X_j and W_j are the quantities incremented and reduced in the counter-trading mechanism, respectively. Since the reductions are charged at the day-ahead market price, it is possible to represent the quantity reduced W_j as a ratio of the day-ahead market production P_j, i.e., $W_j = m_j \cdot P_j$, where m_j represents the proportion of the generation that unit j has to reduce in order to meet the network constraints. Thus, the value of m_j has to be computed taking into account the power network. Thus, the company's profit is:

$$\pi_i = \lambda \cdot \sum_{j \in J_i} \left(1 - m_j\right) \cdot P_j + \gamma \cdot \sum_{j \in J_i} X_j - \sum_{j \in J_i} C_j \left(\left(1 - m_j\right) \cdot P_j + X_j\right) \quad \forall i \in I \tag{4.2}$$

The equilibrium point can be calculated by expressing the first-order profit-maximization condition for the production of each unit and the increasing production of each unit for each generation company, This yields to (4.3) for the unit production P_j, and to (4.4) for the increasing production X_j:

$$\begin{aligned}\frac{\partial \pi_i}{\partial P_j} =& \left(1 - m_j\right) \cdot \lambda + \frac{\partial \lambda}{\partial P_j} \cdot \sum_{k \in J_i} (1 - m_k) \cdot P_k \\ & - \left(1 - m_j\right) \cdot \frac{\partial C_j \left(\left(1 - m_j\right) \cdot P_j + X_j\right)}{\partial \left(\left(1 - m_j\right) \cdot P_j + X_j\right)} = 0 \quad \forall i \in I,\ \forall j \in J_i \end{aligned} \tag{4.3}$$

$$\frac{\partial \pi_i}{\partial X_j} = \gamma + \frac{\partial \gamma}{\partial X_j} \cdot \sum_{k \in J_i} X_k - \frac{\partial C_j \left(\left(1 - m_j\right) \cdot P_j + X_j\right)}{\partial \left(\left(1 - m_j\right) \cdot P_j + X_j\right)} = 0 \quad \forall i \in I,\ \forall j \in J_i \tag{4.4}$$

A generation company may exercise market power by modifying the production of its units, and therefore, affecting the market price. However, in the day-ahead market, the generation company is indifferent to the location of its units because when it modifies the production of any of its units, the market price is affected in the same way. This is because, in a single-price electricity market, the market price is the same for all areas since the market clearing process does not take into account the power network. Therefore, the company's conjectured-price response is the same for all the units of the generation company. The conjectured-price response of the generation company in the day-ahead market and the congestion management market are defined in (4.5) and (4.6), respectively.

$$\theta_i = -\frac{\partial \lambda}{\partial P_j} \qquad \forall i \in I,\ \forall j \in J_i \tag{4.5}$$

$$\beta_i = -\frac{\partial \gamma}{\partial X_j} \qquad \forall i \in I,\ \forall j \in J_i \tag{4.6}$$

Let MC_j be the marginal cost function of the unit j. The market equilibrium equations (4.3) and (4.4) can be written as shown in (4.7) and (4.8):

$$\lambda = MC_j\left(\left(1-m_j\right)\cdot P_j + X_j\right) + \frac{\theta_i}{\left(1-m_j\right)} \cdot \sum_{k\in J_i} (1-m_k)\cdot P_k \quad \forall i \in I,\ \forall j \in J_i \tag{4.7}$$

$$\gamma = MC_j\left(\left(1-m_j\right)\cdot P_j + X_j\right) + \beta_i \cdot \sum_{k\in J_i} X_k \quad \forall i \in I,\ \forall j \in J_i \tag{4.8}$$

In (4.7) it is clear that the congestion of the lines modifies the conjectured-price response of the company. The factor m_j is equal to 0 for those units that do not affect system congestion while the factor m_j is greater than 0 for those units required to reduce their production in the counter-trading mechanism. Thus, the conjectured-price response is modified by the factor $1/(1-m_j) \geq 1$, leading to an increase in the unit's *apparent cost*. Thereby, the units in the exporting areas are more expensive because the company knows that they will have to reduce their production in order to clear congestion. For the two-area system, constraints (4.7) and (4.8) are the same as the market equilibrium equations (3.21), (3.22) and (3.23) presented in chapter 3.

Finally, in an electricity market, the total generation and demand have to be balanced. Equations (4.9) and (4.10) are the power balance constraints in the day-ahead market and the counter-trading mechanism, respectively:

$$\sum_{j\in J} P_j = \sum_{a\in A} D_a \tag{4.9}$$

$$\sum_{j\in J} X_j = \sum_{j\in J} m_j \cdot P_j \tag{4.10}$$

where a is the index of the nodes or areas of the system, D_a is the demand in each area a, J is the set of all generation units, and A is the set of areas.

4.2.2 Equivalent minimization problem

The equations (4.7)-(4.10) are the market equilibrium equations considering the companies' strategic behavior in both the day-ahead market and in the counter-trading mechanism. Following the procedure presented in Barquín et al. (2004), it can be shown that these market equilibrium equations are identical to the optimality conditions of the minimization problem (4.11)-(4.17).

$$\min_{P_j, X_j, m_j, w_j} \sum_{i \in I} \overline{C_i}\left(P_j, X_j\ \forall j \in J_i\right) \tag{4.11}$$

$$\text{s.t.}$$

$$\sum_{j \in J} \left(1 - m_j\right)^2 \cdot P_j = D^* \qquad : (\lambda) \tag{4.12}$$

$$\sum_{j \in J} \left(1 - m_j\right) \cdot X_j = Y^* \qquad : (\gamma) \tag{4.13}$$

$$0 \le P_j \le \overline{P_j} \qquad \forall j \in J \tag{4.14}$$

$$0 \le X_j \le \overline{P_j} \cdot w_j \qquad \forall j \in J \tag{4.15}$$

$$P_j + X_j \le \overline{P_j} \qquad \forall j \in J \tag{4.16}$$

$$\left\{m_j, w_j\right\} \in \arg\Omega \tag{4.17}$$

where $\overline{C_i}\left(P_j, X_j\ \forall j \in J_i\right)$ is the modified effective cost function which is a quadratic cost function defined in (4.18).

Equations (4.12) and (4.13) represent the power balance constraints in the day-ahead market and the counter-trading mechanism, respectively. As in the two-area system, these constraints have been modified with respect to the original constraints (4.9) and (4.10), so that the solution of the minimization problem is equivalent to the market equilibrium equations. D^* and Y^* are defined in (4.19) and (4.20), respectively.

Constraints (4.14)-(4.16) define the limits of the decision variables. The binary variable w_j is equal to 1 if the unit j increments its production in the counter-trading mechanism and 0 otherwise. Thus, according to constraint (4.15), the increased quantity X_j is equal to 0 when the unit has to reduce its production

($w_j = 0$), i.e., the unit j cannot increase and reduce its production simultaneously.

Finally, constraint (4.17) indicates that the variables m_j and w_j are in the solution of the problem Ω which is a DC optimal power flow defined in (4.21)-(4.34). This makes the problem (4.11)-(4.17) nonconvex and nonlinear. The way proposed to deal with this issue is to use an iterative process as shown in section 4.2.3. Thus, the problem (4.11)-(4.17) is solved by fixing the values of the variables m_j and w_j, which are updated in each iteration, solving the problem Ω.

$$\begin{aligned}
\overline{C_i}\left(P_j, X_j\ \forall j \in J_i\right) = & \sum_{j \in J_i} \left(1 - m_j\right) \cdot C_j\left(\left(1 - m_j\right) \cdot P_j + X_j\right) \\
& + \frac{\theta_i}{2} \cdot \left\{ \sum_{j \in J_i} \left(\left(1 - m_j\right) \cdot P_j\right) \right\}^2 \\
& + \frac{\beta_i}{2} \cdot \left\{ \sum_{j \in J_i} \left(\left(1 - m_j\right) \cdot X_j\right) \right\}^2 \quad \forall i \in I \qquad (4.18)
\end{aligned}$$

$$D^* = \sum_{a \in A} D_a + \sum_{j \in J} m_j \cdot \left(m_j - 2\right) \cdot P_j \qquad (4.19)$$

$$Y^* = \sum_{j \in J} m_j \cdot P_j \qquad (4.20)$$

As mentioned, the problem Ω represents a DC optimal power flow (DC-OPF) that minimizes the cost of the changes in the productions of the units:

$$\min_{\Xi} \sum_{j \in J} \left\{ AC_j \cdot \left(X_j^{\Omega} + W_j^{\Omega} \right) \right\} \qquad (4.21)$$

s.t.

$$\sum_{j \in J_a} P_j^{\Omega} - \sum_{l \in L} H_{al} \cdot f_l = D_a \qquad \forall a \in A \qquad (4.22)$$

$$f_l = e_l \cdot \left(\delta_{FR(l)} - \delta_{TO(l)}\right) \qquad \forall l \in L \qquad (4.23)$$

$$P_j^{\Omega} = P_j + X_j^{\Omega} - W_j^{\Omega} \qquad \forall j \in J \qquad (4.24)$$

$$W_j^{\Omega} = m_j \cdot P_j \qquad \forall j \in J \qquad (4.25)$$

$$m_j = m_k \qquad \forall j, k \in J_a \qquad (4.26)$$

$$w_j = w_k \qquad \forall j, k \in J_a \qquad (4.27)$$

$$0 \le P_j^{\Omega} \le \overline{P_j} \qquad \forall j \in J \qquad (4.28)$$

$$0 \le X_j^{\Omega} \le \overline{P_j} \cdot w_j \qquad \forall j \in J \qquad (4.29)$$

$$0 \le W_j^{\Omega} \le \overline{P_j} \cdot \left(1 - w_j\right) \qquad \forall j \in J \qquad (4.30)$$

$$-\overline{F_l} \leq f_l \leq \overline{F_l} \qquad \forall l \in L \tag{4.31}$$

$$-\overline{\delta} \leq \delta_a \leq \overline{\delta} \qquad \forall a \in A \tag{4.32}$$

$$m_j \in [0,1] \qquad \forall j \in J \tag{4.33}$$

$$w_j \in \{0,1\} \qquad \forall j \in J \tag{4.34}$$

where the decision variables are $\Xi = \{P_j^\Omega, X_j^\Omega, W_j^\Omega, m_j, w_j \; \forall j \in J, f_l \; \forall l \in L, \delta_a \; \forall a \in A\}$. P_j^Ω is the final production of unit j. X_j^Ω and W_j^Ω are the necessary changes in the production of the unit j that clear system congestion in the counter-trading mechanism. f_l is the power flow of flowgate or transmission line l. δ_a is the phase angle at area a. L is the set of transmission lines.

In the objective function (4.21), the costs of the changes, both positive and negative, in production are evaluated using the *apparent cost* AC_j of the units. As explained in the previous chapter, the apparent cost is used because it corresponds to the cost at which the generation company must offer the production of the unit in order to maximize its profit. This definition of the objective function minimizes the changes in the production of the units with respect to the solution of the day-ahead market. However, any other definition of cost and objective function could be used, e.g., seek to increase the production of the units with lower apparent cost and reduce the production of the units with higher apparent cost as proposed in the model of chapter 5.

Constraints (4.22) express the power balance in each area of the system. H_{al} is the element of the network incidence matrix that is equal to 1 if area a is the sending area of flowgate l, -1 if area a is the receiving bus of flowgate l,and 0 otherwise.

Constraints (4.23) represent the power flows in each flowgate. e_l is the inverse of the reactance of flowgate l in p.u., and $FR(l)$ and $TO(l)$ are the sending and receiving areas of flowgate l, respectively.

Constraints (4.24) define the final production of the unit P_j^Ω as the day-ahead market production P_j plus the increment X_j^Ω minus the reduction W_j^Ω. The reduction factor of production m_j is defined in (4.25) as the ratio between the reduction W_j^Ω and the day-ahead market production P_j which is considered a data in problem Ω.

Constraints (4.26) and (4.27) represent the counter-trading mechanism implemented in Spain (SEE, 2012; Miguelez et al., 2004) where J_a is the set of generation units located at area a. In this mechanism, the quantity reduced by the units depends on the so-called contribution factor to the congestion which expresses the change in the flow over the flowgate that results from increasing the unit's generation. The reduction factor of production m_j will be the same for all units with the same contribution factor. Given the characteristics of the power networks and disregarding the local network constraints within the areas, all units belonging to the same area affect congestion equally between areas. Therefore, they have the same contribution factor, which implies that the value of the reduction factor m_j is the same for all units in the same area a. If a different countertrading mechanism is under consideration, constraints (4.26) and (4.27) would be different, without affecting the proposed methodology.

The boundaries of the decision variables are set in constraints (4.28)-(4.34). Constraints (4.29) and (4.30) ensure that the increased production, X_j^{Ω}, and the decrease in production, W_j^{Ω}, are not simultaneously positive. Although these constraints are not completely necessary taking the definition of the objective function used into account, for other definitions may be necessary to include these constraints.

4.2.2.1 Equivalence between the market equilibrium equations and the minimization problem

This section presents the proof of the equivalence between the minimization problem and the market equilibrium equations assuming that the problem Ω is already solved. Thus, m_j and w_j are fixed and they are not used in the derivatives of the Lagrangian function $\mathscr{L}\left(P_j, X_j\ \forall j \in J, \lambda, \gamma\right)$.

The solution of the minimization problem can be calculated by introducing the constraints in the objective function using the Lagrangian function $\mathscr{L}\left(P_j, X_j\ \forall j \in J, \lambda, \gamma\right)$. In this case, the objective is to find a saddle point that minimizes the primal variables and maximizes the dual variables.

$$\min_{P_j, X_j} \max_{\lambda, \gamma} \mathscr{L}\left(P_j, X_j\ \forall j \in J, \lambda, \gamma\right)$$

where $\mathscr{L}\left(P_j, X_j\ \forall j \in J, \lambda, \gamma\right)$ is defined as:

$$\begin{aligned}\mathscr{L} = &\sum_{i\in I} \overline{C_i}\left(P_j, X_j\ \forall j\in J_i\right)\\ &-\lambda\cdot\left(\sum_{j\in J}\left(1-m_j\right)^2\cdot P_j - D^*\right)\\ &-\gamma\cdot\left(\sum_{j\in J}\left(1-m_j\right)\cdot X_j - Y^*\right)\end{aligned} \tag{4.35}$$

The first-order optimization conditions are:

$$\begin{aligned}\frac{\partial\mathscr{L}}{\partial P_j} = &\left(1-m_j\right)^2\cdot MC_j\left(\left(1-m_j\right)\cdot P_j + X_j\right)\\ &+\theta_i\cdot\left(1-m_j\right)\cdot\sum_{k\in J_i}(1-m_k)\cdot P_k - \lambda\cdot\left(1-m_j\right)^2 = 0 \quad \forall i\in I,\ \forall j\in J_i\end{aligned} \tag{4.36}$$

$$\begin{aligned}\frac{\partial\mathscr{L}}{\partial X_j} = &\left(1-m_j\right)\cdot MC_j\left(\left(1-m_j\right)\cdot P_j + X_j\right)\\ &+\beta_i\cdot\left(1-m_j\right)\cdot\sum_{k\in J_i}(1-m_k)\cdot X_k - \gamma\cdot\left(1-m_j\right) = 0 \quad \forall i\in I,\ \forall j\in J_i\end{aligned} \tag{4.37}$$

In (4.37), there is the term $\sum_{k\in J_i}(1-m_k)\cdot X_k$. However, by definition the unit cannot both increase and reduce its production simultaneously, i.e. $m_k = 0 \Leftrightarrow X_k \neq 0$. Thus, that term can be replaced by the term $\sum_{k\in J_i} X_k$. Isolating the day-ahead market price λ in (4.36) and the counter-trading mechanism price γ in (4.37), the resulting equations are equivalent to the market equilibrium conditions (4.7) and (4.8).

4.2.2.2 Power balance constraints

This section presents the mathematical derivation of the power balance constraints (4.12) and (4.13).

1. **Day-ahead market**

   Substituting (4.19) in (4.12), gives:

$$\sum_{j\in J}\left(1-m_j\right)^2\cdot P_j = \sum_{a\in A} D_a + \sum_{j\in J} m_j\cdot\left(m_j-2\right)\cdot P_j \tag{4.38}$$

   Rewriting the equation:

$$\sum_{j\in J}\left(1-2m_j+m_j^2-m_j^2+2m_j\right)\cdot P_j=\sum_{a\in A}D_a \tag{4.39}$$

Thus, the balance between generation and demand is reached in the day-ahead market:

$$\sum_{j\in J}P_j=\sum_{a\in A}D_a \tag{4.40}$$

2. **Congestion management mechanism**
   Substituting $W_j=m_j\cdot P_j$ in (4.20), gives:

$$Y^*=\sum_{j\in J}W_j \tag{4.41}$$

   Substituting (4.41) in (4.13), gives:

$$\sum_{j\in J}\left(1-m_j\right)\cdot X_j=\sum_{j\in J}W_j \tag{4.42}$$

   By definition, the unit cannot both increase and reduce its production simultaneously, i.e. $m_j=0\Leftrightarrow X_j\neq 0$. Thus, the term $\sum_{j\in J}\left(1-m_j\right)\cdot X_j$ can be replaced by the term $\sum_{j\in J}X_j$. Thus, the total increased generation is equal to the total reduced generation:

$$\sum_{j\in J}X_j=\sum_{j\in J}W_j \tag{4.43}$$

4.2.3 Solution methodology

The problem (4.11)-(4.17) is a convex quadratic problem with linear constraints if the variables m_j and w_j are constants and the cost functions are linear or quadratic. The optimal solution in such a case can be found using available commercial software. However, the variables m_j and w_j are decision variables in problem (4.21)-(4.34). Therefore, an iterative method is used to find the optimal solution, and it works as follows:

1. Initialize the iteration counter $\kappa=1$, and the variables $m_j^{(1)}=0$, $w_j^{(1)}=0$, $D^{*^{(1)}}=\sum_a D_a$ and $Y^{*^{(1)}}=0$. This case corresponds to the case without limits in the interconnection capacity.

2. Solve the minimization problem (4.11)-(4.17). This gives a solution for P_j, X_j and λ.

3. Update the values $P_j^{(\kappa)}$, $X_j^{(\kappa)}$, $\lambda^{(\kappa)}$, $AC_j^{(\kappa)}$:

$$P_j^{(\kappa)} = \begin{cases} P_j & \text{if } \kappa = 1 \\ \alpha \cdot P_j + (1-\alpha) \cdot P_j^{(\kappa-1)} & \text{if } \kappa > 1 \end{cases} \tag{4.44}$$

$$X_j^{(\kappa)} = \begin{cases} 0 & \text{if } \kappa = 1 \\ \alpha \cdot X_j + (1-\alpha) \cdot X_j^{(\kappa-1)} & \text{if } \kappa > 1 \end{cases} \tag{4.45}$$

$$\lambda^{(\kappa)} = \begin{cases} \lambda & \text{if } \kappa = 1 \\ \alpha \cdot \lambda + (1-\alpha) \cdot \lambda^{(\kappa-1)} & \text{if } \kappa > 1 \end{cases} \tag{4.46}$$

$$\begin{aligned} AC_j^{(\kappa)} =& MC_j \left(\left(1 - m_j^{(\kappa)}\right) \cdot P_j^{(\kappa)} + X_j^{(\kappa)} \right) \\ &+ \frac{\theta_i}{\left(1 - m_j^{(\kappa)}\right)} \cdot \sum_{k \in J_i} \left(1 - m_k^{(\kappa)}\right) \cdot P_k^{(\kappa)}, \forall j \in J_i \end{aligned} \tag{4.47}$$

The learning rate α is used to achieve a smooth convergence in the value of the variables, and to prevent the solution from jumping between different values. A value of $\alpha = 1$ means that the variables are updated using only the information given in the last iteration while a value of $\alpha = 0$ represents the case in which only the information given in the first iteration is used[2].

4. Solve the minimization problem (4.21)-(4.34). This gives a solution of P_j^{Ω}, m_j and w_j.

5. Update the values $P_j^{\Omega^{(\kappa+1)}}$, $m_j^{(\kappa+1)}$, $w_j^{(\kappa+1)}$, ${D^*}^{(\kappa+1)}$ and ${Y^*}^{(\kappa+1)}$ and :

$$P_j^{\Omega^{(\kappa+1)}} = \alpha \cdot P_j^{\Omega} + (1-\alpha) \cdot P_j^{\Omega^{(\kappa)}} \tag{4.48}$$

$$m_j^{(\kappa+1)} = \alpha \cdot m_j + (1-\alpha) \cdot m_j^{(\kappa)} \tag{4.49}$$

$$w_j^{(\kappa+1)} = \alpha \cdot w_j + (1-\alpha) \cdot w_j^{(\kappa)} \tag{4.50}$$

$${D^*}^{(\kappa+1)} = \sum_a D_a + \sum_{j \in J} m_j^{(\kappa)} \cdot \left(m_j^{(\kappa)} - 2\right) \cdot P_j^{(\kappa)} \tag{4.51}$$

$${Y^*}^{(\kappa+1)} = \sum_{j \in J} m_j^{(\kappa)} \cdot P_j^{(\kappa)} \tag{4.52}$$

6. If the change of $P_j^{(\kappa)}$, $X_j^{(\kappa)}$ and $m_j^{(\kappa)}$ with respect to the previous iteration is less than an ϵ value, the algorithm stops; otherwise increase the iteration counter κ and go to 2.

[2] Different ways of setting α could be used. For example, this value could remain constant in all iterations, or it can have a lower value in the first iterations and increase progressively in the following iterations.

4.3 Numerical example

This section presents two illustrative examples. The first case corresponds to a small-size three-area system to analyze the companies' strategic behavior when there are different transfer capacities between areas. The second case is a more complex system to assess the capabilities of the model in large systems, as well as the convergence properties of the solution methodology. The proposed algorithm and optimization problems were solved with CPLEX 12.1 (CPLEX, 2014) under GAMS (GAMS, 2014).

4.3.1 Three-area system

This case is a three-area system as shown in Fig. 4.1. In the market, there are 5 generation companies owning 9 generation units as shown in Table 4.1. The day-ahead market is an oligopoly where generation companies can exercise market power, i.e. $\theta_i \neq 0$, while in the counter-trading mechanism, generation companies act in a perfectly competitive market, i.e. $\beta_i = 0$. For the sake of clarity and simplicity, this example disregards the power flow constraints which relate the flow in each flowgate with the phase angles at areas similar to a model of a pipeline network, and the cost functions used are linear. Demand is concentrated only in area 3 and is equal to 100MW.

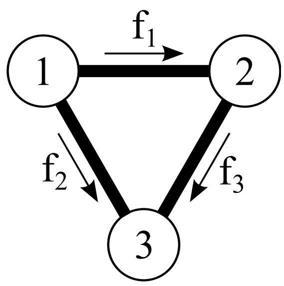

Figure 4.1: Three-area system

Table 4.2 shows the case studies used to analyze the effect of network congestion on the power market equilibrium. In case A, the maximum capacities of all lines is infinite, and therefore no line is congested. This case is considered as the base case. In the other 4 cases, the maximum transfer capacity of the flowgates is limited differently. In cases B and C, the entire generation of area 1 cannot be exported. In case D, the entire generation of area 2 cannot be exported. Finally, in case E, the generation that areas 1 and 2 can export is limited.

Table 4.1: Characteristics of generation companies and units

Generation company i	θ_i [$\frac{€/MWh}{MW}$]	β_i [$\frac{€/MWh}{MW}$]	**Unit** j	**Area** a	**Variable cost** [€/MWh]	$\overline{P_j}$ [MW]
1	0.1	0	1	1	40.3	70
			2	2	41.0	70
			3	3	42.0	70
2	0.1	0	4	1	41.5	70
			5	2	40.7	70
			6	3	42.0	70
3	0.1	0	7	1	40.0	70
4	0.1	0	8	2	40.3	70
5	0.1	0	9	3	42.0	70

Table 4.2: Maximum transfer capacity of each flowgate for each case study

Flowgate	$\overline{F_l}$ [MW]				
l	**Case A**	**Case B**	**Case C**	**Case D**	**Case E**
1	∞	10	20	10	100
2	∞	10	0	100	20
3	∞	100	100	10	20

Tables 4.3-4.7 show the results of the case studies. The production of units in the day-ahead market and the final results are presented in table 4.3. The power flows of each flowgate are found in table 4.4. The reduction factor and the apparent cost of the units are provided in tables 4.5 and 4.6, respectively. Finally, table 4.7 shows the electricity market prices.

Table 4.3: Power generation

Unit	P_j [MW]					P_j^{Ω} [MW]				
j	**Case A**	**Case B**	**Case C**	**Case D**	**Case E**	**Case A**	**Case B**	**Case C**	**Case D**	**Case E**
1	23.6	-	-	24.6	23.6	23.6	-	-	24.6	9.4
2	-	18.0	18.0	-	-	-	18.0	18.0	-	-
3	-	-	-	-	-	-	-	-	-	18.0
4	-	-	-	7.6	-	-	-	-	7.6	-
5	19.6	21.0	21.0	8.2	19.6	19.6	21.0	21.0	5.0	9.1
6	-	-	-	-	-	-	-	-	-	18.0
7	26.6	28.0	28.0	27.6	26.6	26.6	20.0	20.0	40.3	10.6
8	23.6	25.0	25.0	24.6	23.6	23.6	33.0	33.0	15.0	10.9
9	6.6	8.0	8.0	7.6	6.6	6.6	8.0	8.0	7.6	24.0

Table 4.4: Power flows

Flowgate	**Day-ahead result - f_l [MW]**					**Final result - f_l [MW]**				
l	**Case A**	**Case B**	**Case C**	**Case D**	**Case E**	**Case A**	**Case B**	**Case C**	**Case D**	**Case E**
1	-	-	-	-	-	-	10.0	20.0	-10.0	-
2	50.2	28.0	28.0	59.7	50.2	50.2	10.0	-	69.7	20.0
3	43.2	64.0	64.0	32.7	43.2	43.2	82.0	92.0	10.0	20.0

Table 4.5: Reduction factor

Unit	m_j				
j	**Case A**	**Case B**	**Case C**	**Case D**	**Case E**
1	-	0.28583	0.28583	-	0.60145
2	-	-	-	0.38890	0.53691
3	-	-	-	-	-
4	-	0.28583	0.28583	-	0.60145
5	-	-	-	0.38890	0.53691
6	-	-	-	-	-
7	-	0.28583	0.28583	-	0.60145
8	-	-	-	0.38890	0.53691
9	-	-	-	-	-

Table 4.6: Apparent cost

Unit	**Apparent cost [€/MWh]**				
j	**Case A**	**Case B**	**Case C**	**Case D**	**Case E**
1	42.66	42.82	42.82	42.76	42.66
2	43.36	42.80	42.80	45.02	43.03
3	44.36	43.80	43.80	44.46	42.94
4	43.46	44.44	44.44	42.76	43.78
5	42.66	42.80	42.80	42.76	42.66
6	43.96	44.10	44.10	43.26	42.91
7	42.66	42.80	42.80	42.76	42.66
8	42.66	42.80	42.80	42.76	42.66
9	42.66	42.80	42.80	42.76	42.66

Table 4.7: Market Prices

	Prices [€/MWh]				
	Case A	**Case B**	**Case C**	**Case D**	**Case E**
λ	42.66	42.80	42.80	42.76	42.66
γ	-	40.30	40.30	40.00	42.00

In case A, no flowgate is congested, so the final production of units is not modified with respect to the solution found in the day-ahead market (Table 4.3). The optimal solution occurs when the apparent cost of the marginal units (1, 5, 7, 8 and 9) is equal to 42.66 €/MWh (Table 4.6). Thus, the day-ahead market price is equal to 42.66 €/MWh (Table 4.7). This result occurs because the market equilibrium corresponds to an oligopolistic market, in which generation companies can increase the market price by modifying the productions of their units. If the electricity market was perfectly competitive, the units dispatched would be the most economic units (1,7 and 8) and the market price would be equal to 40.3 €/MWh.

In cases B and C, the maximum transfer capacity of flowgates 1 and 2 is limited. This causes the entire generation produced in area 1 to not be able to be exported. When such a situation occurs, the companies with generation units located in several areas give more importance to their production in areas 2 and 3 because they anticipate that the generation in area 1 will be reduced in order to meet the maximum flow constraints. In this example, companies 1 and 2 are the only companies that respond in this way because they own units in all the areas. The apparent cost of units 1 and 4 is increased (Table 4.6) because they belong to area 1. Thus, companies prefer to produce in the day-ahead market with units 2 and 5 from area 2 and to produce nothing with units 1 and 4 from area 1, as shown in the third and fourth columns of Table 4.3. Although with this behavior the power flow in flowgate 2 is reduced to 28 MW (Table 4.4), the maximum power flow is not met. Thus, it is necessary to reduce the production of unit 7 and to increase the production of unit 8. In cases B and C, the day-ahead market price is 42.8 €/MWh which is equal to the apparent cost of the marginal units. The price of the increased production is 40.3 €/MWh which is equal to the marginal cost of unit 8 because companies do not exercise market power in the counter-trading mechanism.

In case D, the maximum transfer capacity of flowgates 1 and 3 is limited. Therefore, the entire generation produced in area 2 cannot be exported. In this case generation companies 1 and 2 perceive a higher apparent cost of the units 2 and 5 which are in area 2 (Table 4.6). This makes the production of unit 5 decrease and the production of units 1 and 4 increase in contrast to case A. In this case, the power flow in flowgate 3 is 32.7 MW. However, this solution does not meet maximum power flow constraints, so it is necessary to increase the production of unit 7 and to reduce the production of units 5 and 8 with respect to the day-ahead solution (Table 4.3). In this case, the apparent cost of the marginal units is 42.76 €/MWh, and the price of the increasing production is 40 €/MWh, which is equal to the marginal cost of unit 7.

In case E, the maximum power flows in flowgates 2 and 3 are limited. Thus, the power production that areas 1 and 2 can export is limited. In this case, generation companies 1 and 2 give more importance to production in area 3, by making the apparent cost of units 3 and 6 decrease in contrast to case A. However, the reduction in costs is not sufficient and the apparent cost of units 1 and 5 is even smaller (Table 4.6). Therefore, even if the companies know that the productions in areas 1 and 2 will be reduced, the optimal production in the day-ahead market does not change with respect to case A (Table 4.3). However, in order to meet the maximum power flow constraints, it is necessary to reduce the productions of units 1, 5, 7 and 8, and to increase the productions of units 3, 6 and 9. In this case, the day-ahead market price is the same as the price in case A, 42.66 €/MWh, and the price of increasing the production is the highest of all cases, because the units that increase their production are in area 3, which is the most expensive area.

4.3.2 RTS-96

The objective of this section is to analyze the convergence properties of the proposed methodology when it is applied to a large-size system. The numerical example is based on the IEEE One Area Reliability Test System-1996 (RTS-96) (Grigg et al., 1999). This system comprises 38 interconnection lines, 24 nodes and 32 generation units as shown in Fig. 4.2. Table 4.8 presents the maximum transfer capacity of the interconnection lines.

In this example, there are 6 generation companies that behave strategically in the day-ahead market and in the counter-trading mechanism. For all generation companies, the conjectured-price responses are $\theta_i = 0.02\frac{€/\text{MWh}}{\text{MW}}$ and $\beta_i = 0.04\frac{€/\text{MWh}}{\text{MW}}$. The maximum capacity and the variable cost of the generation units of each company and in each node appear in tables 4.9 and 4.10, respectively.

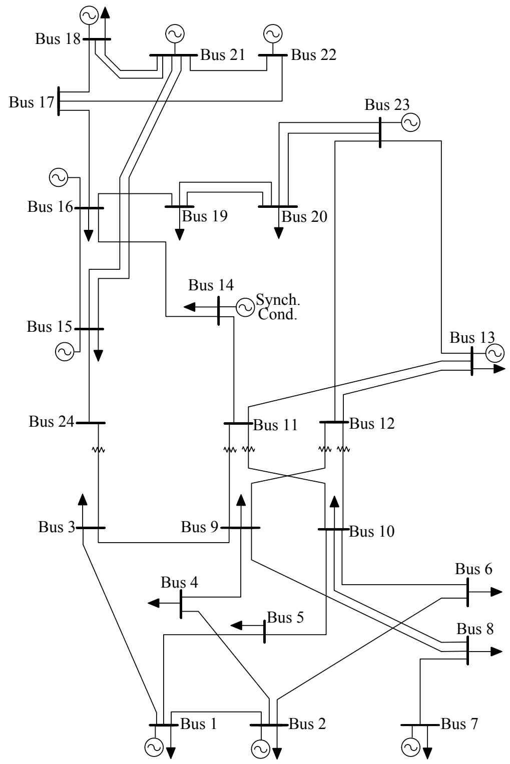

Figure 4.2: IEEE One Area RTS-96

Table 4.8: Interconnection lines

Line l	**From node**	**To node**	$\overline{F_l}$ **[MW]**	**Line** l	**From node**	**To node**	$\overline{F_l}$ **[MW]**
1	1	2	87.5	20	12	13	250
2	1	3	87.5	21	12	23	250
3	1	5	87.5	22	13	23	100
4	2	4	87.5	23	14	16	200
5	2	6	87.5	24	15	16	50
6	3	9	87.5	25	15	21	250
7	3	24	170	26	15	21	250
8	4	9	100	27	15	24	155
9	5	10	100	28	16	17	250
10	6	10	90	29	16	19	50
11	7	8	87.5	30	17	18	250
12	8	9	50	31	17	22	250
13	8	10	87.5	32	18	21	250
14	9	11	150	33	18	21	250
15	9	12	200	34	19	20	250
16	10	11	150	35	19	20	250
17	10	12	210	36	20	23	250
18	11	13	250	37	20	23	250
19	11	14	150	38	21	22	250

Table 4.9: Maximum capacity

	Generation company i						
	1	**2**	**3**	**4**	**5**	**6**	
Node a	**Maximum capacity [MW]**						**Total**
1	20	20	76	76			192
2	20	20	76	76			192
7	100	100	100				300
13	197	197	197				591
15	12	12	12	12	12	155	215
16	155						155
18	400						400
21	400						400
22	50	50	50	50	50	50	300
23	155	155	350				660
Total	1509	554	861	214	62	205	3405

Table 4.10: Variable cost

	Generation company i					
	1	**2**	**3**	**4**	**5**	**6**
Node a	**Variable cost [€/MWh]**					
1	43.4	48.7	34.1	38.0		
2	40.2	48.9	34.7	31.4		
7	40.9	44.2	43.6			
13	42.2	39.4	43.3			
15	43.3	43.3	44.7	42.3	43.9	35.8
16	28.6					
18	27.6					
21	28.4					
22	24.8	30.0	31.1	26.3	30.9	27.4
23	32.3	33.3	31.0			

4.3.2.1 Convergence

This section studies the convergence properties of the proposed methodology. As shown in section 4.2.3, the variables are updated using a learning rate α. In this numerical example, different values of α have been used in order to assess the convergence of the methodology and the number of iterations required. The average time taken by each iteration is equal to 0.441 seconds. Figures 4.3 and 4.4 present the values obtained in each iteration for the day-ahead market price

λ, the congestion market price γ, and the reduction factor m of the units located at node 22. When high values of α are used, e.g. $\alpha = 0.7$, it is possible that no convergence is reached and the algorithm continues to oscillate between several solutions. On the contrary, when very low values of α are used, e.g. $\alpha = 0.02$, the number of iterations required may increase significantly. It is important to note that for a given learning rate, some variables may converge while others may be oscillating. For the cases of $\alpha = 0.3$ and $\alpha = 0.1$, the day-ahead market price and the congestion market price reach their values in 70 and 200 iterations, respectively. However, the reduction factor m of the units at node 22 continues to oscillate between several values. In this example, in order to achieve the convergence of the value of the reduction factor m of the units at node 22 is necessary to use a very low value of $\alpha = 0.02$ and a large number of iterations as shown in Fig. 4.4d.

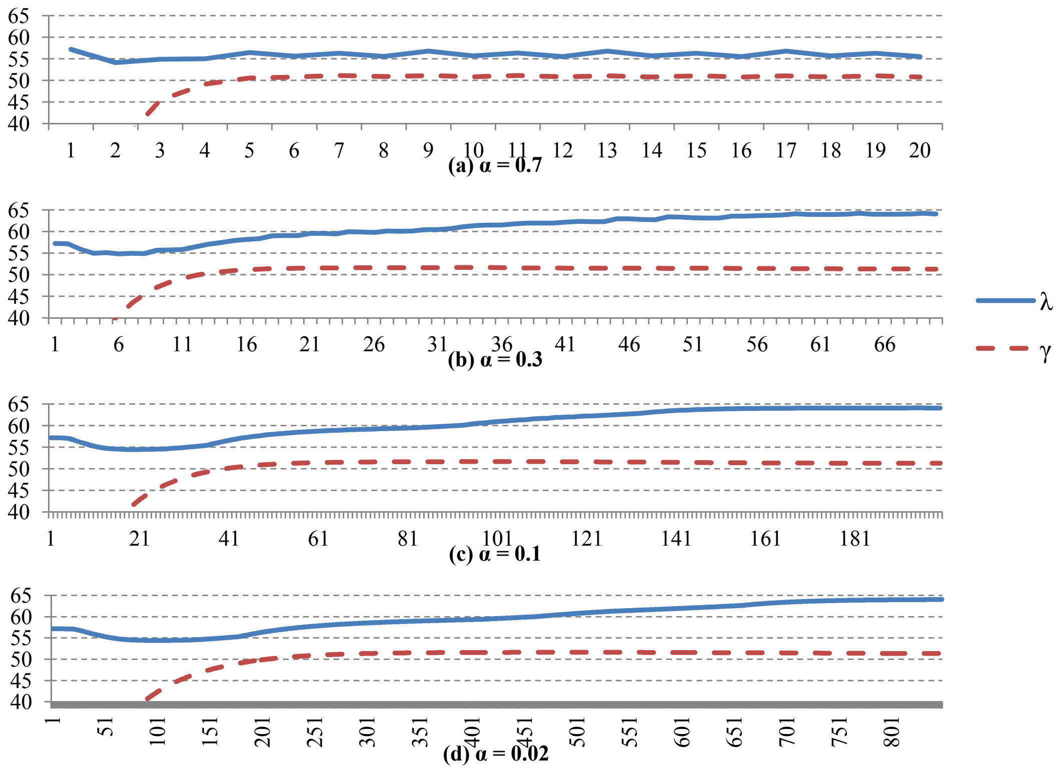

Figure 4.3: Day-ahead market price λ and congestion market price γ in each iteration using different learning rates α

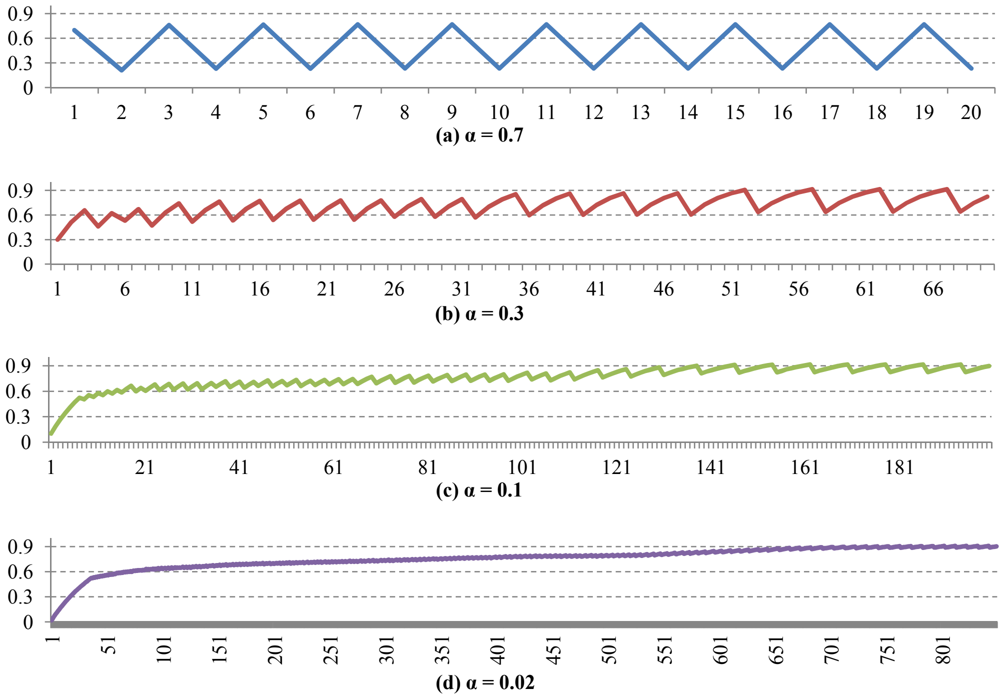

Figure 4.4: Reduction factor m of units at node 22 in each iteration using different learning rates α

4.3.2.2 Numerical results

Tables 4.11 and 4.12 show the total power generation on the nodes and the power flow over the congested lines. The units at node 22 are those that most affect congestion, and therefore, they have a high value of m. In order to clear congestion, it is necessary to reduce the generation in that node and increase the generation in other nodes. Comparing the results between the first and the last iterations, it can be noted that in the day-ahead market, the generation at node 22 decreases while the generation at nodes 1, 2, 7 and 13 increases, and therefore the flow over the lines decreases. This variation means that the congestion management mechanism affects the companies' strategic behavior in the day-ahead market. Although this does not guarantee that the day-ahead market solution meets the maximum flow constraints as shown in Table 4.12. Thus, the counter-trading mechanism is necessary to clear system congestion.

Table 4.11: Power Generation

	First Iteration	Last Iteration		
Node	**Day-ahead**	**Day-ahead**	**Final**	
a	**[MW]**	**[MW]**	**[MW]**	**m**
1	152.0	160.8	192.0	-
2	152.0	162.2	192.0	-
7	100.0	188.9	212.5	-
13	340.0	382.0	573.1	-
15	191.0	191.0	95.6	0.4997
16	155.0	155.0	139.7	0.0990
18	400.0	400.0	400.0	-
21	400.0	400.0	400.0	-
22	300.0	150.0	14.5	0.9031
23	660.0	660.0	630.7	0.0444

Table 4.12: Power Flow

		First Iteration	Last Iteration	
Line	$\overline{F}_l$	**Day-ahead**	**Day-ahead**	**Final**
l	**[MW]**	**[MW]**	**[MW]**	**[MW]**
7	170	231.2	202.1	149.7
10	90	101.4	99.4	89.5
12	50	99.7	55.0	42.8
16	150	195.1	162.1	130.9
17	210	224.4	208.7	205.0
19	150	178.0	112.3	6.0
22	100	197.3	167.7	92.7
23	200	372.0	306.3	200.0
24	50	86.9	44.8	50.0
27	155	231.2	202.1	149.7
28	250	322.9	244.1	160.3
29	50	92.8	37.7	50.0

Table 4.13 presents the total generation and profits of the companies. The companies with a higher number of units and high generation capacity (companies 1, 2 and 3 as shown in Table 4.9) have more opportunities to exercise market power. The reason is that these companies have more options for increasing their production in some areas when they know that they have to reduce production in another area. Thus, the counter-trading mechanism allows companies with high generation capacity to behave strategically and exercise market power, which leads to the corresponding increase in their profits and a decrease in efficiency in the system.

Table 4.13: Total results

	First Iteration		Last Iteration		
Generation company i	$\sum_{j\in J_i} q_j$ **[MW]**	**Profit [€]**	$\sum_{j\in J_i} q_j$ **[MW]**	$\sum_{j\in J_i} q_j^{\Omega}$ **[MW]**	**Profit [€]**
1	1160	33292	1114	1319	40763
2	514	10018	504	491	12135
3	695	15926	750	783	21104
4	214	5141	214	163	4779
5	62	1471	62	11	282
6	205	4804	205	82	2371
Total	2850	70651	2850	2850	81434

4.4 Conclusion

This chapter has studied the effect of congestion between multiple areas on the market equilibrium of a single-price electricity market. The electricity market uses a counter-trading mechanism to solve network congestion that appears after the day-ahead market clearing process. The main feature of the model is that network congestion modifies the generation companies' strategic behavior in the day-ahead market. In the model, the generation companies know which units must modify their productions in order to solve congestion in the counter-trading mechanism. Therefore, the generation companies anticipate this result by modifying their strategies in the day-ahead market. This results in an increment in the apparent cost perceived by the generation companies of the units that affect congestion while the units that solve congestion become more valuable to generation companies. Hence, there is a change in the dispatch of the day-ahead market with respect to the situation where network congestion is not taken into account.

The results of the numerical examples show that the generation companies most likely to exercise market power are those with more generation units in the different areas or nodes of the power system because these companies have more possibilities to increase the production in some importing areas when they have to reduce the production in the exporting areas in order to solve the system congestion.

Another important aspect presented in this chapter is the analysis of the convergence of the proposed methodology in large-size power systems. The methodology shows satisfactory results depending on the value of the learning parameter.

However, some variables may oscillate between several values for a high value of the learning parameter. In such cases, the convergence can be achieve decreasing the value of the learning parameter and increasing the number of iterations. However, this could result in a significant increase on the computational time in a large-size power systems.

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Chapter 5

The effect of voltage level requirements on the strategic behavior of generation companies

Contents

This chapter presents a conjectural-variation-based equilibrium model of a single-price electricity market. The main characteristic of the model is that the market equilibrium equations incorporate the effect of the voltage constraints on the companies' strategic behavior. A two-stage optimization model is used to solve the market equilibrium. In the first stage, the day-ahead market clearing process is solved using a mixed complementary problem. In the second stage, some generation units have to modify their active and reactive power levels in order to meet the technical constraints of the transmission network. These generation changes are determined by computing an AC optimal power flow.

5.1 Introduction

The models presented in chapters 3 and 4 analyze the effect of network congestion on the strategic behavior of generation companies. Theses models are only suitable for studying network congestion caused by the thermal limits of the transmission lines. Thus, the model in chapter 4 uses a DC approximation of the power flow which assumes that there is enough reactive power compensation in all nodes to maintain voltage at the desired level, so the terms related to reactive power are discarded and the voltage levels are equal to 1 p.u. in all nodes. However, this DC approximation is not suitable for studying the effect of the voltage level requirements because it is not possible to assume that voltage levels are constant in all nodes. Moreover, depending on the characteristics of the power system it may be necessary to dispatch certain generation units in some areas of the power system in order to maintain the voltage levels in an appropriate range. Therefore, the generation companies may anticipate this event and modify their behavior in the day-ahead electricity market. Although in the model presented in chapter 4 the generation companies anticipate the effect of some network constraints, that model is not valid to represent the effect of the voltage level requirements.

This chapter presents a conjectural-variation-based model of a single-price electricity market. The main characteristic of this model is that the companies' strategic behavior takes the effect of the voltage constraints into account. The market equilibrium equations are solved by means of a two-stage optimization problem. In the first stage, a mixed complementary problem models the market clearing process. In the second stage, an AC optimal power flow (AC-OPF) is solved to determine the changes in active and reactive power needed to meet the voltage system requirements. This chapter also presents an iterative algorithm to resolve the two-stage optimization problem.

The remainder of this chapter is organized as follows: Section 5.2 presents the market equilibrium model that includes the effect of the voltage constraints on the companies' strategic behavior. Section 5.3 provides and analyzes a numerical example. Finally, section 5.4 draws the most relevant conclusions.

5.2 Market Equilibrium Model

This section generalizes the model presented in chapter 4 in order to study the effect of voltage constraints on the companies' strategic behavior in a single-price electricity market. In the electricity market, the scheduled day-ahead generation is usually determined first. Then, a subsequent procedure is carried out if the day-ahead market solution does not meet the technical requirements necessary to maintain system stability. Different technical constraints are assessed and the power produced by units may change with respect to the scheduled day-ahead generation.

5.2.1 Market clearing conditions

The day-ahead market clearing process determines the active power P_j of each generation unit j as well as the market price λ. Since it is a single-price electricity market, the total generation and demand have to be balanced (5.1) and the market price λ is equal to the bid of the marginal unit:

$$\sum_{j \in J} P_j = \sum_{a \in A} DP_a + Z \tag{5.1}$$

where DP_a is the active power demand in the node a, Z are the losses in the power system, J is the set of generation units, and A is the set of nodes.

Subsequently, the changes in production necessary to maintain system stability are determined using a mechanism to solve the technical constraints. This chapter models the Spanish mechanism (SEE, 2012) in which the active power increments X_j are paid at the price γ while the reductions W_j are charged at the day-ahead market price λ. However, the proposed method can be extended in order to simulate different schemes used to remunerate the changes in active power as the mechanisms presented in chapter 2. In order to maintain the active power balance in the system, the total active power increment is equal to the total active power reduction:

$$\sum_{j \in J} X_j = \sum_{j \in J} W_j \tag{5.2}$$

5.2.2 The generation company's problem

A generation company i will try to maximize its profit by determining the production of its generation units, P_j, as well as the production changes, X_j and W_j, required to meet the system's technical constraints. Moreover, since the generation company behaves strategically, it can change the electricity prices when the production of its units changes. This strategic behavior can be modeled by means of the parameters θ_i and β_i. θ_i corresponds to the conjectured-price response in the day-ahead market (Barquín et al., 2004) and β_i to the conjectured-price response in the subsequent mechanism.

Since in the Spanish mechanism the reductions are charged at the day-ahead market price, it is possible to represent the quantity reduced W_j as a ratio of the day-ahead market production P_j, i.e., $W_j = m_j \cdot P_j$, where m_j represents the proportion of the active power generation that unit j has to reduce in order to meet the network constraints. Thus, the value of m_j has to be computed taking the power flow constraints into account. Therefore, the profit maximization problem of company i is:

$$\max_{P_j, X_j \ \forall j \in J_i} \quad \lambda \cdot \sum_{j \in J_i} (1 - m_j) \cdot P_j + \gamma \cdot \sum_{j \in J_i} X_j - \sum_{j \in J_i} C_j \left((1 - m_j) \cdot P_j + X_j \right) \tag{5.3}$$

s.t.

$$\left. \frac{\partial \lambda}{\partial \tilde{P}_j} \right|_{\tilde{P}_j = P_j} = \theta_i \tag{5.4}$$

$$\left. \frac{\partial \gamma}{\partial \tilde{X}_j} \right|_{\tilde{X}_j = X_j} = \beta_i \tag{5.5}$$

$$\overline{P_j} - P_j \geq 0 \qquad : (\mu_j) \qquad \forall j \in J_i \tag{5.6}$$

$$\overline{P_j} \cdot w_j - X_j \geq 0 \qquad : (\nu_j) \qquad \forall j \in J_i \tag{5.7}$$

$$\overline{P_j} - P_j - X_j \geq 0 \qquad : (\xi_j) \qquad \forall j \in J_i \tag{5.8}$$

$$P_j \geq 0, \quad X_j \geq 0 \qquad \forall j \in J_i \tag{5.9}$$

where J_i is the set of generation units owned by the generation company.

The equation (5.3) is the profit of the company i which is equal to the revenue in the day-ahead market, plus the income of the increased generation in the subsequent mechanism, minus the charge of the reduced generation in the subsequent mechanism, minus the production costs of the generation units.

Equations (5.4) and (5.5) represent how the company conjectures that electricity prices will change if the company changes its production. Equation (5.4) is the conjecture for the day-ahead market price, and equation (5.5) is the conjecture for the subsequent mechanism. This representation of the conjectured-price response is equivalent to the equations (4.5) and (4.6) presented in chapter 4. Another representation of the conjectured-price response is the one presented in Hobbs and Rijkers (2004). However both representations are mathematically equivalent.

Constraints (5.6)-(5.9) are the boundaries of the variables. The binary variables w_j indicate the units that have to increase their generation in the subsequent mechanism in order to meet the power system's constraints. Thus, $X_j = 0$ when $w_j = 0$.

In the event that the scheduled active power determined in the day-ahead market does not meet the system's technical constraints, the units' generation has to be modified in the subsequent mechanism. Assuming that these modifications happen on a regular basis, the companies can predict them, and may use this information to behave strategically. Thus, in the company's optimization problem, this information is modeled using the reduction factors, m_j, and the binary variables, w_j. Both are determined in the subsequent mechanism as shown in section 5.2.4.

5.2.3 Market equilibrium

By gathering together the first-order conditions for all companies and then adding the market-clearing conditions, the mixed complementarity model MCP (5.10)-(5.17) can be defined, and the market equilibrium corresponds to the solution of this MCP. An alternative way to compute this market equilibrium is by means of an equivalent quadratic optimization problem as shown in chapter 4. However, that methodology may not be successful in solving this problem because the power balance constraints have to be modified in each iteration and the convergence of the procedure is not guaranteed.

$$(\lambda \text{ unrestricted}) \quad \sum_{j\in J} P_j = DP \tag{5.10}$$

$$(\gamma \text{ unrestricted}) \quad \sum_{j\in J} X_j = Y \tag{5.11}$$

$$0 \leq \mu_j \perp \overline{P_j} - P_j \geq 0 \qquad \forall j \in J_i,\ \forall i \in I \tag{5.12}$$

$$0 \leq \nu_j \perp \overline{P_j} \cdot w_j - X_j \geq 0 \qquad \forall j \in J_i,\ \forall i \in I \tag{5.13}$$

$$0 \leq \xi_j \perp \overline{P_j} - P_j - X_j \geq 0 \qquad \forall j \in J_i,\ \forall i \in I \tag{5.14}$$

$$\begin{aligned} 0 \leq P_j \perp -\left(1-m_j\right)\cdot\lambda + \theta_i \cdot \sum_{k\in J_i}(1-m_k)\cdot P_k + \\ \left(1-m_j\right)\cdot MC_j\left(\left(1-m_j\right)\cdot P_j + X_j\right) + \\ \mu_j + \xi_j \geq 0 \qquad \forall j \in J_i,\ \forall i \in I \end{aligned} \tag{5.15}$$

$$\begin{aligned} 0 \leq X_j \perp -\gamma + \beta_i \cdot \sum_{k\in J_i} X_k + MC_j\left(\left(1-m_j\right)\cdot P_j + X_j\right) + \\ \nu_j + \xi_j \geq 0 \qquad \forall j \in J_i,\ \forall i \in I \end{aligned} \tag{5.16}$$

$$\left\{m_j, w_j\right\} \in \arg\Omega \tag{5.17}$$

Equations (5.10)-(5.11) are the market-clearing constraints. The values of DP and Y are the total active power demand and the total active power increments, and they are computed using an iterative procedure as presented in section 5.2.5.

The generation company's behavior is modeled by means of equations (5.12)-(5.16). These equations are the Karush-Kuhn-Tucker (KKT) conditions of the problem (5.3)-(5.9) for each company i. The operator $\perp$ denotes the inner product of two vectors equal to zero, i.e., $0 \leq x \perp f(x) \geq 0$ corresponds to the system equations $x \geq 0$, $f(x) \geq 0$ and $x \cdot f(x) = 0$.

Equation (5.17) indicates that the variables m_j and w_j are in the solution of the problem Ω which is a AC optimal power flow defined in (5.20)-(5.31).

For units whose productions P_j and X_j are between the minimum and maximum values, constraints (5.6), (5.7) and (5.8) are not binding and the dual variables μ_j, ν_j and ξ_j are equal to zero, the equations (5.15) and (5.16) could be written as:

$$\lambda = MC_j\left(\left(1-m_j\right)\cdot P_j + X_j\right) + \frac{\theta_i}{\left(1-m_j\right)} \cdot \sum_{k\in J_i}(1-m_k)\cdot P_k \tag{5.18}$$

$$\gamma = MC_j\left(\left(1-m_j\right)\cdot P_j + X_j\right) + \beta_i \cdot \sum_{k\in J_i} X_k. \tag{5.19}$$

The right-hand side on (5.18) and (5.19) corresponds to the *apparent cost* of the unit in the day-ahead market and in the subsequent mechanism, respectively. The *apparent cost* is defined as the cost at which the generation company must offer the production of the unit in order to maximize its profit (Reneses et al., 2004). In the apparent cost perceived by the company in the day-ahead market, the conjectured-price response is modified by factor $1/(1-m_j)$ which is greater than 1 when $m_j > 0$. This means that the company perceives that the unit j is more expensive in the day-ahead market because it knows that the active power of the unit has to be reduced in the subsequent mechanism in order to meet the system's technical constraints.

5.2.4 Subsequent mechanism

In single-price electricity markets, a procedure is used to clear the system's technical constraints when the day-ahead market solution is not technically feasible. This procedure is subsequent to the day-ahead market and determines the changes in active power as well as reactive power needed to maintain system stability. With those results, the companies can determine the reduction factors m_j and which units increase active power generation ($w_j = 1$). The optimal power flow Ω (5.20)-(5.31) models this procedure. In chapter 4, the OPF is solved using a DC approximation in which the voltage levels are fixed to 1 p.u. and the reactive power and system losses are disregarded. The DC approximation is valid to analyze the effect of congestion due to thermal limits of the lines. However, in order to study the effect of voltage requirements it is necessary to use an AC-OPF where the voltage levels are not fixed and the active and reactive power levels are taken into account.

In the AC-OPF (5.20)-(5.31) the decision variables are $\Xi = \{P_j^\Omega, Q_j^\Omega, X_j^\Omega, W_j^\Omega, u_j \ \forall j \in J, V_a, \delta_a \ \forall a \in A\}$. P_j^Ω is the final active production of unit j. Q_j^Ω is the final reactive production of unit j. X_j^Ω and W_j^Ω are the necessary changes in the production of the unit j that clear system congestion in the counter-trading mechanism. u_j is the commitment variable of unit j. V_a is the voltage magnitude at node a. δ_a is the phase angle at area a.

$$\min_{\Xi} \sum_{j \in J} ACX_j \cdot X_j^{\Omega} + \left(K - ACW_j\right) \cdot W_j^{\Omega} \tag{5.20}$$

s.t.

$$\sum_{j \in J_a} P_j^{\Omega} - DP_a = \sum_{b \in A} V_a \cdot V_b \cdot \left(G_{ab} \cos\left(\delta_a - \delta_b\right) + B_{ab} \sin\left(\delta_a - \delta_b\right)\right) \; \forall a \in A \tag{5.21}$$

$$\sum_{j \in J_a} Q_j^{\Omega} - DQ_a = \sum_{b \in A} V_a \cdot V_b \cdot \left(G_{ab} \sin\left(\delta_a - \delta_b\right) - B_{ab} \cos\left(\delta_a - \delta_b\right)\right) \; \forall a \in A \tag{5.22}$$

$$P_j^{\Omega} = P_j + X_j^{\Omega} - W_j^{\Omega} \qquad \forall j \in J \tag{5.23}$$

$$\underline{V_a} \leq V_a \leq \overline{V_a} \qquad \forall a \in A \tag{5.24}$$

$$0 \leq X_j^{\Omega} \leq \overline{P_j} \qquad \forall j \in J \tag{5.25}$$

$$0 \leq W_j^{\Omega} \leq \overline{P_j} \qquad \forall j \in J \tag{5.26}$$

$$\underline{P_j} \cdot u_j \leq P_j^{\Omega} \leq \overline{P_j} \cdot u_j \qquad \forall j \in J \tag{5.27}$$

$$\underline{Q_j} \cdot u_j \leq Q_j^{\Omega} \leq \overline{Q_j} \cdot u_j \qquad \forall j \in J \tag{5.28}$$

$$Q_j^{\Omega} \leq Q_j^{0,max} \cdot u_j + n_j^{max} \cdot P_j^{\Omega} \qquad \forall j \in J \tag{5.29}$$

$$Q_j^{\Omega} \geq Q_j^{0,min} \cdot u_j + n_j^{min} \cdot P_j^{\Omega} \qquad \forall j \in J \tag{5.30}$$

$$u_j \in \{0, 1\} \qquad \forall j \in J \tag{5.31}$$

The objective function (5.20) minimizes the *total apparent cost* of the changes in active power with respect to the day-ahead market solution. As in the models presented in chapters 3 and 4, the apparent cost is used because it corresponds to the cost at which the generation company must offer the production of the unit in order to maximize its profit. Unlike the model presented in chapter 4 which minimizes the positive and negative changes in the production, the model in this chapter seeks to increase the production of units with lower cost and reduce the production of units with higher cost, although other objective functions can be used. In the model, two different apparent costs ACX_j and ACW_j are considered. The apparent cost ACX_j corresponds to the cost when the generation unit j has to increase its active power while ACW_j corresponds to the cost when it has to reduce its active power. The values of these apparent costs are determined using the equations (5.36) and (5.37) as explained below. The solution of the minimization problem is that the units with lower apparent cost ACX_j increase their active power while the units with higher apparent cost ACW_j reduce their active power production. The term K is a constant higher than the maximum value of ACW_j, $K \geq \max_{j \in J}\{ACW_j\}$. This constant is necessary to properly model that the objective function seeks to reduce the active power production of units with higher apparent cost.

Constraints (5.21) and (5.22) are the power flow equations for active and reactive power, respectively. G_{ab} and B_{ab} are the elements of the conductance and the susceptance matrices in p.u., respectively.

Constraints (5.23) define the active power in the OPF P_j^{Ω} as the day-ahead market production P_j plus the increment X_j^{Ω} minus the reduction W_j^{Ω}.

Constraints (5.24)-(5.30) establish the minimum and maximum bounds of the variables. A linear approximation of the P-Q capability curve of the generation units, known as *D-curve*, is modeled with constraints (5.27)-(5.30) where $Q_j^{0,max}$, $Q_j^{0,min}$, n_j^{max}, n_j^{min} are parameters of the lineal approximation of this curve as illustrated in Fig. 5.1, where the shaded portion represents the feasible operating region for the unit. This curve models the trade-off between active and reactive power of the generation units, and therefore it determines the feasible operation region where it is not possible to produce the maximum active power and maximum reactive power at the same time. It is important to note that the binary variables u_j are necessary to meet the minimum and maximum requirements of the generation units, and to avoid solutions in which the active power of a unit is below the minimum to generate more reactive power. Finally, constraints (5.31) indicates that variables u_j are binary.

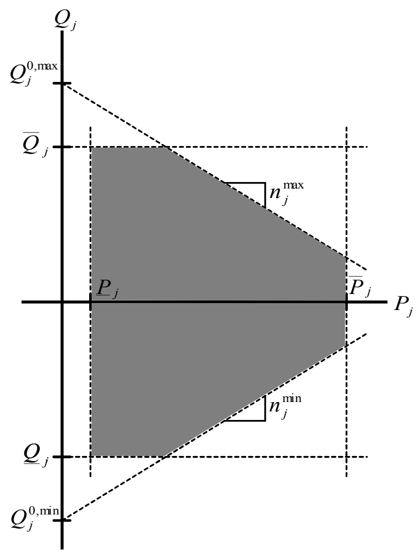

Figure 5.1: P-Q Capability curve

5.2.5 Solution methodology

An iterative algorithm similar to the one presented in chapter 4 is used to determine the market equilibrium taking into account the power changes required to meet the technical constraints:

1. Initialize the iteration counter $\kappa = 1$, and the variables $m_j^{(1)} = 0$, $w_j^{(1)} = 0$, $DP^{(1)} = \sum_a DP_a$, $Y^{(1)} = 0$. These values correspond to the case without network constraints.

2. Solve the MCP (5.10)-(5.16). This gives a solution for P_j, X_j, λ, γ.

3. Update the active power, prices and apparent cost values:

$$P_j^{(\kappa)} = \begin{cases} P_j & \text{if } \kappa = 1 \\ \alpha \cdot P_j + (1-\alpha) \cdot P_j^{(\kappa-1)} & \text{if } \kappa > 1 \end{cases} \tag{5.32}$$

$$X_j^{(\kappa)} = \begin{cases} 0 & \text{if } \kappa = 1 \\ \alpha \cdot X_j + (1-\alpha) \cdot X_j^{(\kappa-1)} & \text{if } \kappa > 1 \end{cases} \tag{5.33}$$

$$\lambda^{(\kappa)} = \begin{cases} \lambda & \text{if } \kappa = 1 \\ \alpha \cdot \lambda + (1-\alpha) \cdot \lambda^{(\kappa-1)} & \text{if } \kappa > 1 \end{cases} \tag{5.34}$$

$$\gamma^{(\kappa)} = \begin{cases} 0 & \text{if } \kappa = 1 \\ \alpha \cdot \gamma + (1-\alpha) \cdot \gamma^{(\kappa-1)} & \text{if } \kappa > 1 \end{cases} \tag{5.35}$$

$$ACX_j^{(\kappa)} = MC\left(\left(1-m_j^{(\kappa)}\right) \cdot P_j^{(\kappa)} + X_j^{(\kappa)}\right) + \beta_i \cdot \sum_{k \in J_i} X_k^{(\kappa)} \tag{5.36}$$

$$ACW_j^{(\kappa)} = MC\left(\left(1-m_j^{(\kappa)}\right) \cdot P_j^{(\kappa)} + X_j^{(\kappa)}\right) + \frac{\theta_i}{\left(1-m_j^{(\kappa)}\right)} \cdot \sum_{k \in J_i} \left(1-m_k^{(\kappa)}\right) \cdot P_k^{(\kappa)} \tag{5.37}$$

The learning rate α is used to achieve a smooth convergence in the value of the variables, and to prevent the solution from jumping between different values. A value of $\alpha = 1$ means that the variables are updated using only the information given in the last iteration while a value of $\alpha = 0$ means that only the information given in the first iteration is used.

4. Solve the AC-OPF (5.20)-(5.30). This gives a solution for P_j^{Ω}, X_j^{Ω}, W_j^{Ω}, V_a, δ_a, u_j.

5. Update the reduction factor, the units that increase their generation and the demand values:

$$P_j^{\Omega(\kappa+1)} = \alpha \cdot P_j^{\Omega} + (1-\alpha) \cdot P_j^{\Omega(\kappa)} \tag{5.38}$$

$$X_j^{\Omega(\kappa+1)} = \alpha \cdot X_j^{\Omega} + (1-\alpha) \cdot X_j^{\Omega(\kappa)} \tag{5.39}$$

$$W_j^{\Omega(\kappa+1)} = \alpha \cdot W_j^{\Omega} + (1-\alpha) \cdot W_j^{\Omega(\kappa)} \tag{5.40}$$

$$m_j^{(\kappa+1)} = \alpha \cdot \frac{W_j^{\Omega}}{P_j^{\Omega}} + (1-\alpha) \cdot m_j^{(\kappa)} \tag{5.41}$$

$$w_j^{(\kappa+1)} = \begin{cases} 1 \text{ if } X_j^{\Omega(\kappa)} > 0 \\ 0 \text{ if } X_j^{\Omega(\kappa)} = 0 \end{cases} \tag{5.42}$$

$$DP^{(\kappa+1)} = \sum_{j\in J} P_j^{\Omega(\kappa+1)} \tag{5.43}$$

$$Y^{(\kappa+1)} = \sum_{j\in J} W_j^{\Omega(\kappa+1)} \tag{5.44}$$

6. If the change of the variables is lower than an ϵ value, the algorithm stops; otherwise increase the iteration counter κ and go to 2.

5.3 Numerical Example

This section presents a simple example to study the effect of voltage constraints on the companies' strategic behavior. The market equilibrium is solved using PATH (Ferris and Munson, 2000) in GAMS (GAMS, 2014) and the AC-OPF is solved using MATPOWER (Zimmerman et al., 2011) in Matlab (MATLAB, 2014).

In the AC-OPF (5.20)-(5.30), the binary variables u_j are the commitment variables of the generation units. Since MATPOWER cannot compute binary variables in the solution of the OPF, the approach taken is therefore to evaluate all of the possible combinations of these binary variables and select the case with the lowest value in the objective function. This methodology reduces the size of the models that can be studied. Another inconvenience is that the algorithm used to find the optimal solution could converge to a local optimum depending on the initial values of the variables, so in the iterative procedure used in this model different solutions could be found and there is no certainty about the convergence of the model. In practice, the solution methodology has achieved satisfactory results in terms of convergence as shown in the previous chapter.

The power network has 3 nodes connected by 3 transmission lines as shown in Fig. 5.2 and Table 5.1. The values of the parameters of the transmission lines are significantly higher than the actual parameters, in order to highlight the effect of voltage requirements. The demand is equal to 100 MW and 35 MVAR and it is concentrated at node 3. The three nodes have generation units; however, the units located at node 3 are the most expensive. Thus, the day-ahead market solution is that units at nodes 1 and 2 supply the demand at node 3. If this was the final solution, there would be a significant voltage drop in the lines 1-3 and 2-3 caused by the impedance of these lines. In that case, the voltage level at node 3 would be lower than the specified minimum (0.95 p.u.).

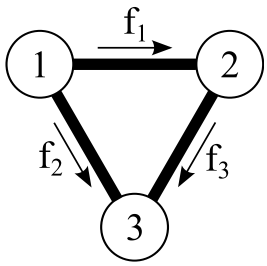

Figure 5.2: Three-node system

Table 5.1: Parameters of the lines

From Node	**To Node**	**Resistance [p.u.]**	**Reactance [p.u.]**	**Susceptance [p.u.]**
1	2	0.12	0.35	0.01
1	3	0.24	0.70	0.01
2	3	0.24	0.70	0.01

The parameters are in the base of 100 MVA

Three different cases are analyzed. In case A, companies 1 and 2 own generation units at nodes 1 and 2, and there is only one unit at node 3 owned by company 3. Therefore, this unit is the only one that can solve the voltage drop at node 3. In case B, the generation units are the same as in case A, but company 1 also owns one generation unit at node 3, so there are now 2 units that can solve the voltage requirements. Finally in case C, the three companies own generation units at node 3. In the three cases, the strategic behavior of company 3 is studied modifying its conjectured-price response in the subsequent mechanism, β, from the case in which the company does not exercise market power, i.e, $\beta_3 = 0$, and increasing the market power to $\beta_3 = 0.1$ and $\beta_3 = 1$. The data of generation units and the conjectured-price response of the companies are shown in Tables 5.2, 5.3 and 5.4.

Table 5.2: Conjectured-price responses

Company i	θ **[(€/MWh)/MW]**	β **[(€/MWh)/MW]**
1	0.05	0.1
2	0.05	0
3	0	0 - 0.1 - 1

Table 5.3: Generation Units

Unit j	Case	**Node** a	**Company** i	**Variable Cost** **[€/MWh]**	$\underline{P}$ **[MW]**	$\overline{P}$ **[MW]**	$\underline{Q}$ **[Mvar]**	$\overline{Q}$ **[Mvar]**
1	A, B, C	1	1	40.5	7	70	-40	40
2	A, B, C	2	1	42.0	7	70	-40	40
3	A, B, C	1	2	42.0	7	70	-40	40
4	A, B, C	2	2	40.0	7	70	-40	40
5	A	3	3	43.5	14	70	-58	58
5	B, C	3	3	43.5	7	35	-29	29
6	B, C	3	1	40.0	7	35	-29	29
7	C	3	2	44.0	7	35	-29	29

Table 5.4: Parameters of the D-curve of the generation units

Unit j	Case	$Q^{0,max}$ **[Mvar]**	$Q^{0,min}$ **[Mvar]**	n^{max}	n^{min}
1	A, B, C	56.4	-56.4	-0.643	0.643
2	A, B, C	56.4	-56.4	-0.643	0.643
3	A, B, C	56.4	-56.4	-0.643	0.643
4	A, B, C	56.4	-56.4	-0.643	0.643
5	A	72	-72	-1	1
5	B, C	36	-36	-1	1
6	B, C	36	-36	-1	1
7	C	36	-36	-1	1

5.3.1 Case A

If the generation companies do not take the voltage level requirements into account in their bids to the day-ahead market then generation units 1 and 4 at areas 1 and 2 are dispatched. However, in that solution, the voltage level at area 3

Table 5.5: Voltage levels

Node	First iteration	Last iteration
a	V [p.u]	V [p.u]
1	1.11	1.02
2	1.11	1.04
3	0.82	0.95

is only 0.82 p.u. and this value is below the required minimum of 0.95 as shown in Table 5.5.

Companies 1 and 2 do not modify their strategic behavior in the day-ahead market because they do not own any unit at area 3 to meet the voltage requirements. Hence, the final solution in the day-ahead market is not modified. On the other hand, unit 5 owned by company 3 is the only unit able to resolve the voltage constraint at area 3. This unit has to generate the maximum reactive power in order to reach the voltage level of 0.95 at area 3, and its active power generation is equal to the minimum given its P-Q capability curve. The active power increased by this unit in the subsequent mechanism is compensated by a reduction in the active power of unit 1 as shown in Table 5.6.

Table 5.6: Power Solution

	Day-ahead market		Subsequent mechanism	
Unit	First iteration	Last iteration	Last iteration	
j	P [MW]	P [MW]	P [MW]	Q [Mvar]
1	50.3	50.3	36.3	1.1
2	-	-	-	-
3	-	-	-	-
4	60.4	60.4	60.4	4.1
5	-	-	14	58

5.3.2 Case B

In this case, unit 6 at area 3 is dispatched to the maximum of its active power in the initial day-ahead market. However, this unit cannot generate the reactive power necessary to maintain the voltage level at area 3 due to its capability curve (Table 5.7), and therefore, as in the previous case, unit 5 is necessary in the subsequent mechanism.

Table 5.7: Voltage levels

Node a	First iteration V [p.u]	Last iteration V [p.u]
1	1.05	1.01
2	1.04	1.05
3	0.77	0.95

Unlike case A in which company 1 does not modify its strategic behavior in the day-ahead market, now this company foresees that the active power generation of unit 6 has to be at the minimum while the reactive power generation has to be at the maximum for maintaining the voltage level at area 3. This makes the reduction factor $m_6 > 0$, and therefore its *apparent cost* increases in the day-ahead market as explained in section (5.2.3). Thus, a higher *apparent cost* of this unit results in a change in the strategic behavior of company 1 in the day-ahead market generating only 9.3 MW with unit 6 (Table 5.8). This result shows how the voltage level requirements modify the strategic behavior of the generation companies in the day-ahead electricity market. The company 1 anticipates to the reduction of unit 6 in the subsequent mechanism modifying its strategy in the day-ahead market. On the other hand, unit 5 is dispatched in the subsequent mechanism to the minimum active power and the maximum reactive power to reach a voltage level equal to 0.95 p.u. at area 3.

Table 5.8: Power Solution

	Day-ahead market		Subsequent mechanism	
Unit	First iteration	Last iteration	Last iteration	
j	P [MW]	P [MW]	P [MW]	Q [Mvar]
1	15.3	41.8	37.1	-8.1
2	-	-	-	-
3	-	-	-	-
4	60.4	59.6	59.6	13.5
5	-	-	7	29
6	35	9.3	7	29

5.3.3 Case C

The initial day-ahead market solution of this case is the same as the initial solution to case B. Thus, another generation unit at area 3 is required to maintain the voltage level (Table 5.9).

Table 5.9: Voltage levels

Node	**First iteration**	**Last iteration**
a	V **[p.u]**	V **[p.u]**
1	1.05	1.01
2	1.04	1.05
3	0.77	0.95

The final result in the day-ahead market is exactly the same as in case B. This means that the strategic behavior of company 1 in the day-ahead market is not altered by the new power unit at area 3. Nevertheless, the outcome of the subsequent mechanism is modified depending on the strategic behavior of company 3. In cases A and B, company 3 could exercise market power because its unit was the only one that could resolve the voltage constraint. However, in case C, company 2 also has a unit at area 3. Thus, the market power of company 3 is mitigated, and the value of its conjectured-price response in the subsequent mechanism cannot be higher than 0.071 (€/MWh)/MW because a higher value would cause the apparent cost of unit 5 be greater than the apparent cost of unit 7. Table 5.10 shows how unit 5 is dispatched in the subsequent mechanism when $\beta_3 = 0$ while unit 7 is dispatched when $\beta_3 = 0.1$ and $\beta_3 = 1$.

Table 5.10: Power Solution

		Day-ahead market		**Subsequent mechanism**	
	Unit	**First iteration**	**Last iteration**	**Last iteration**	
	j	P **[MW]**	P **[MW]**	P **[MW]**	Q **[Mvar]**
	1	15.3	41.8	37.1	-8.1
	2	-	-	-	-
	3	-	-	-	-
	4	60.4	59.6	59.6	13.5
	6	35	9.3	7	29
(a)	5	-	-	7	29
	7	-	-	-	-
(b)	5	-	-	-	-
	7	-	-	7	29

(a) Solution for $\beta_3 = 0$

(b) Solution for $\beta_3 = 0.1$ and $\beta_3 = 1$

5.3.4 Prices

In the results above, the day-ahead market generation is affected by the voltage constraints at area 3 in cases B and C. These changes occur because the *apparent cost* of the units is modified by the reduction factor m_j. However, the changes in the day-ahead market price, λ, are not significant, and they are only equal to 0.04 €/MWh between the initial and the final solution.

On the other hand, the price in the subsequent mechanism γ may be modified by the market power of the companies which have the generation units necessary to maintain the voltage levels. In cases A and B, unit 5 of company 3 is indispensable to resolve the voltage constraints, and therefore this company has significant market power in the subsequent mechanism. On the contrary, in case C, the market power of company 3 is limited by unit 7 of company 2. Hence, company 3 cannot make bid prices of unit 5 above the bids of unit 7 in order to be dispatched. Moreover, the price γ in case C when β_3 is higher than 0.071 (€/MWh)/MW is equal to the variable cost of unit 7 as shown in Table 5.11.

Table 5.11: Prices

	λ [€/MWh]						**γ [€/MWh]**		
	First iteration			**Last iteration**			**Last iteration**		
β_3	**Case A**	**Case B**	**Case C**	**Case A**	**Case B**	**Case C**	**Case A**	**Case B**	**Case C**
0	43.02	43.02	43.02	43.02	42.98	42.98	43.5	43.5	43.5
0.1	43.02	43.02	43.02	43.02	42.98	42.98	44.9	44.2	44.0
1	43.02	43.02	43.02	43.02	42.98	42.98	57.5	50.5	44.0

5.4 Conclusion

This chapter has studied the effect of voltage requirements on companies' strategic behavior in a single-price electricity market. The market equilibrium equations take the solution of the mechanism used to clear the technical constraints into account. This mechanism is modeled by means of an AC optimal power flow. One of the contributions of this chapter is that the technical constraints are not limited to congestion due to the thermal constraints of the transmission lines, but the model can also analyze other technical constraints such as voltage levels or reactive power requirements.

An important difference with the effect of the system congestion studied in the previous chapters is that it is necessary to dispatch units that generate reactive power in order to maintain the voltage levels in an adequate range. Thus, there may be the case that units with active power generation in the day-ahead market must continue being dispatched in the subsequent mechanism but reducing their active power and increasing their reactive power. This issue makes necessary the use of binary variables in the modeling of the subsequent mechanism which is non-linear because the AC representation of the power flow. Therefore, this is a hard resolution problem. In this chapter, the model and methodology have been applied in a small-size power system. However, the resolution of this problem in large-size power systems becomes an important research point that could be addressed with more detail in future work.

With respect to the market power in the mechanism used to clear the network constraints, the generation units that can maintain the voltage levels in an area of the power system are precisely the units of this area. This issue may be used by the generation companies in order to behave strategically, and therefore to exercise market power. The results of the numerical example show that this market power is mitigated as more companies are able to resolve this technical constraint.

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Chapter 6

Analysis of the additional upward reserve mechanism implemented in Spain

Contents

In addition to the mechanisms used to clear network congestion and meet voltage requirements, there are other mechanisms subsequent to the day-ahead market used to guarantee the security of power systems, such as the reserve markets. This chapter presents a market equilibrium model to analyze the Spanish mechanism used to contract and manage additional upward reserve. The purpose of this mechanism is to guarantee the security of the power system in situations with low reserve margins which may occur depending on the forecast of intermittent generation and demand. In the model two different decision-making processes are assessed. In the joint decision approach, the generation companies determine their decision variables in the day-ahead electricity market, the intraday electricity markets, and the additional upward reserve mechanism simultaneously. In the sequential decision approach, the generation companies first make their decisions in the day-ahead electricity market, and after this market is cleared they make their decisions in the intraday electricity market and the additional upward reserve mechanism. These two decision-making processes are compared to an ideal situation in which the electricity and reserve markets are cleared jointly. The results show that the generation companies incur in an additional cost in the intraday markets for providing the necessary system reserve.

6.1 Introduction

The previous chapters have analyzed the effect of network congestion and voltage level requirements on the strategic behavior of generation companies in the electricity market. Another mechanisms that may modify the companies' strategic behavior in the electricity market are the reserve markets. As presented in section 1.2.2.3, there are several models that analyze the interdependence between the electricity and reserve markets in systems in which both markets are dispatched jointly. However, there are several countries like Spain where the electricity and reserve markets are dispatched sequentially[1]. Although there are a few

[1] Galiana et al. (2005) argue that the sequential dispatch is inefficient because:

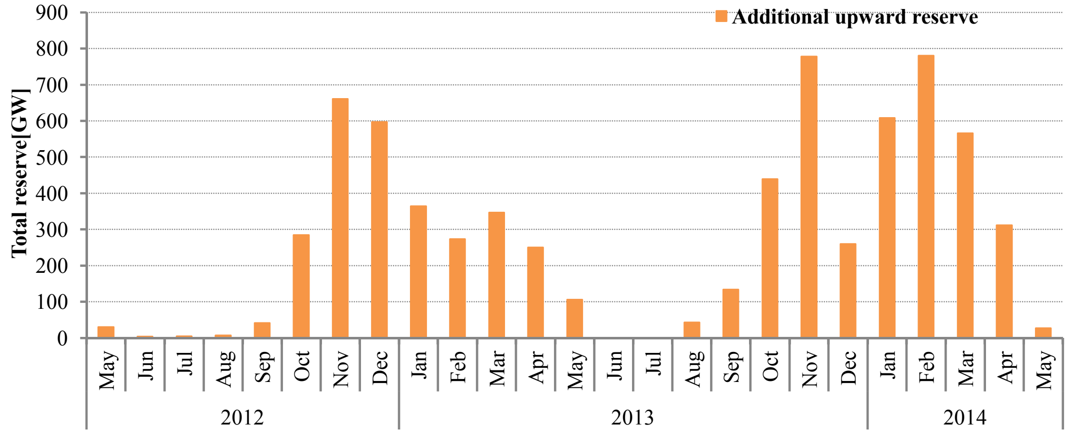
- If a complete redispatch of the generation units is permitted in the sequential steps, then these intermediate problems are as difficult to solve as a joint dispatch.
- If the generation units dispatched in the electricity market must remain committed, then the subsequent market clearing process may be unfeasible.
- The final solution of the sequential dispatch results in a lower social welfare than the joint dispatch.

models that study the strategic behavior of generation companies in the reserve markets in a sequential dispatch, these models assume that the result of the reserve market does not affect the result in the electricity market.

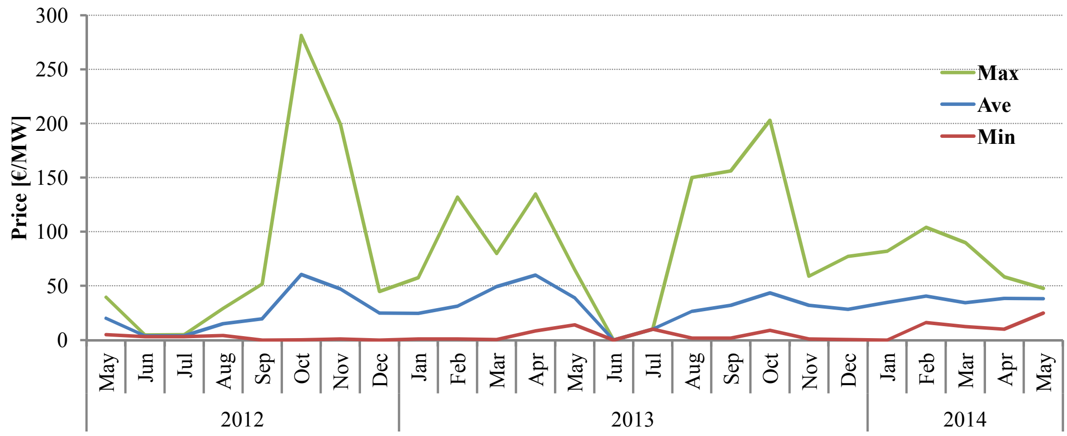
This chapter is a first approach to study the interdependence between the electricity market and the reserve markets. This chapter proposes a model to study the mechanism implemented in Spain to contract and manage the additional upward thermal reserve in the power system (SEE, 2013), and how this mechanism affects the results of the electricity market. However, the Spanish electricity market comprises other reserve markets such as the secondary reserve market and tertiary reserve market that have not been treated in this thesis.

The additional upward reserve mechanism is subsequent to the day-ahead electricity market, and it has been in operation since 2012. It can be seen in Figure 6.1 that this mechanism is not negligible. The average price is around 30 €/MW; however there have been months with prices above 100 €/MW (Fig. 6.1b). Moreover, the average total monthly cost paid by consumers is equal to 10.5 million euros, and there are months with total cost above 20 million euros (Fig. 6.1c). This mechanism is implemented to handle situations with low reserve margins, which may occur depending on the demand forecast, intermittent generation forecast, and conventional generation units dispatched. The generation units that can bid on the mechanism are only the thermal units that were not dispatched in the day-ahead electricity market. If the units dispatched in the day-ahead electricity market reduce their generation in order to provide more upward reserve, that reserve is not traded in the mechanism. When a thermal generation unit is dispatched in the additional upward reserve mechanism, this unit must bid in the different sessions of the intraday markets in order to be committed in the power system. Participation in the intraday markets is considered an effective provision of the upward reserve. Moreover, if the system operator carries out the imbalance management market, the generation units dispatched in the additional upward reserve mechanism are required to bid with at least the difference between the allocated reserve and the power dispatched in the intraday markets. All thermal generation units that provide additional upward reserve are paid at the marginal price (€/MW) of the mechanism.

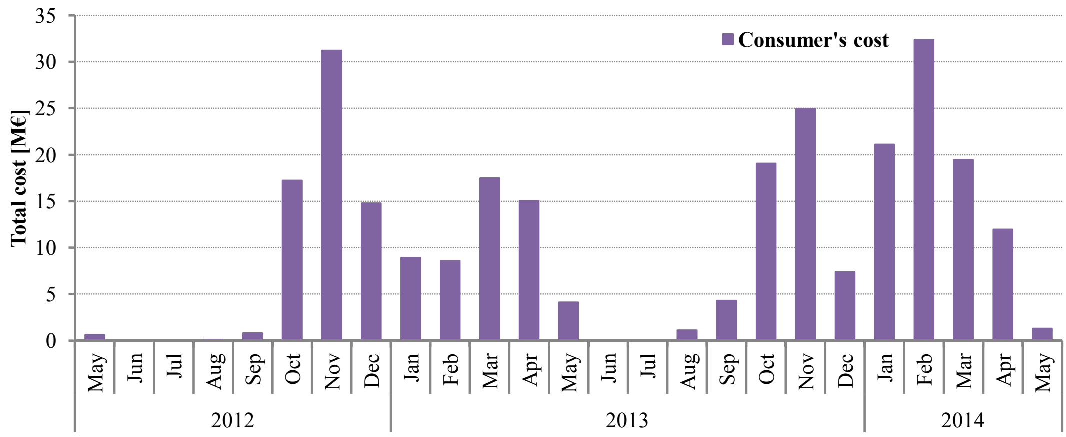
The remainder of this chapter is organized as follows: Section 6.2 presents the market equilibrium model proposed to study the additional upward reserve mechanism implemented in Spain. In Section 6.3, numerical results illustrate the performance of the model. Finally, relevant conclusions are drawn in section 6.4.



(a) Total reserve requirements [GW]



(b) Reserve price [€/MW]



(c) Total consumers' cost [M€]

Figure 6.1: Additional upward reserve. Source: Red Eléctrica de España (http://www.esios.ree.es/web-publica/)

6.2 Market Equilibrium Model

This section proposes a medium-term model to study the mechanism implemented in Spain to contract and manage the additional upward thermal reserve that may be necessary with respect to the available thermal reserve that results from the day-ahead electricity market dispatch. The model uses a temporal representation based on load levels as used in Barquín et al. (2004), although it is possible to use a more general representation based on system states as proposed in Wogrin et al. (2014). In the model, the periods t represent different types of days such as workdays and holidays, and the index s represents the hours with different load levels such as peak, plateau and off-peak. The constant d_{ts} is the duration in hours of the load level s in period t.

6.2.1 Market clearing conditions

The day-ahead market clearing process determines the power generation P_{jts} of each generation unit j as well as the market price λ_{ts} for each period t and load level s. In order to maintain power balance in the system, the sum of the power generated by all units must be equal to the system demand D_{ts} in each period t and load level s:

$$\sum_{j \in J} P_{jts} = D_{ts} \qquad \forall t, s \tag{6.1}$$

where J is the set of all generation units.

The mechanism to contract and manage additional upward reserve in the power system is subsequent to the day-ahead market. This mechanism is carried out when the thermal reserve available as a result of the day-ahead market dispatch is not enough to meet the reserve requirement DR_{ts} that guarantees the security of the power system. Only the generation units that were not dispatched in the day-ahead market can participate in this mechanism. The system operator determines the additional upward reserve R_{jts} of each generation unit, and the upward reserve price ϕ_{ts} in each period t and load level s that corresponds to the offer of the marginal unit in this mechanism. Equation (6.2) indicates that the sum of the additional upward reserve must be greater or equal to the reserve requirement minus the reserve available from the day-ahead electricity market dispatch. The dispatch variable u_{jt} is equal to 1 when the generation j is dispatched in the period t in the day-ahead market, and 0 otherwise.

$$\sum_{j\in J} R_{jts} \geq DR_{ts} - \sum_{j\in J} \left(\overline{P_j} \cdot u_{jt} - P_{jts}\right) \qquad \forall t, s \tag{6.2}$$

A requirement of the Spanish mechanism is that the units that have been assigned to provide additional upward reserve must participate in the intraday electricity markets in order to be committed in the power system. Therefore, these units are programmed with a generation level below their maximum output and can provide the reserve when it is required. Thus, these units increase their power generation in the intraday electricity markets. The proposed model assumes that there is not additional demand in the intraday markets. This means that the increments X_{jts} in the power generation of this units have to be compensated with power reductions W_{jts} of other generation units as shown in (6.3). This assumption is made because the model seeks to analyze the effect of the additional upward reserve mechanism in the solution of the electricity market, but not the effect of the intraday markets by themselves. Both the increments and reductions are traded at the intraday market price ψ_{ts}.

$$\sum_{j\in J} \left(X_{jts} - W_{jts}\right) = 0 \qquad \forall t, s \tag{6.3}$$

6.2.2 The generation company's problem

A generation company i maximizes its profit by determining the production of its generation units P_{jts} in the day-ahead electricity market, the increments X_{jts} and reductions W_{jts} in the intraday electricity markets, the additional upward reserve R_{jts}, the dispatch decision variables u_{jt}, as well as the start-up y_{jt} and shut-down z_{jt} variables in the day-ahead market, and the start-up x_{jt} and shut-down w_{jt} variables in the intraday market.

The equation (6.4) states the profit of company i which is equal to the revenue in the day-ahead market, plus the income or charge in the intraday market, plus the income in the additional upward reserve market, minus the production, dispatch, start-up and shut-down costs of its generation units. The production costs are assessed in the final production of the generation units $(P_{jts} + X_{jts} - W_{jts})$. In the same way, the dispatch costs are assessed taking the start-up and shut-down of the generation units in the intraday market into account $(u_{jt} + x_{jt} - w_{jt})$.

The start-up costs take the start-up of the generation units in the day-ahead and intraday markets into account $(y_{jt} + x_{jt})$. The shut-down costs takes the shut-down of the generation units in the day-ahead and intraday markets into account $(z_{jt} + w_{jt})$. d_{ts} is the duration of the load level s in period t, and J_i is the set of generation units owned by the company.

$$\sum_{j\in J_i}\Bigg\{\sum_{t}\Bigg\{\sum_{s}\Big\{\lambda_{ts}\cdot d_{ts}\cdot P_{jts}+\psi_{ts}\cdot d_{ts}\cdot\left(X_{jts}-W_{jts}\right)+\phi_{ts}\cdot d_{ts}\cdot R_{jts}$$
$$-CV_j\cdot d_{ts}\cdot\left(P_{jts}+X_{jts}-W_{jts}\right)-CU_j\cdot d_{ts}\cdot\left(u_{jt}+x_{jt}-w_{jt}\right)\Big\}$$
$$-CY_j\cdot\left(y_{jt}+x_{jt}\right)-CZ_j\cdot\left(z_{jt}+w_{jt}\right)\Bigg\}\Bigg\} \tag{6.4}$$

The profit of the company is maximized subject to constraints (6.5)-(6.17):

$$P_{jts}-\underline{P_j}\cdot u_{jt}\geq 0 \qquad :\left(\underline{\tau_{P_{jts}}}\right) \qquad \forall j\in J_i,t,s \tag{6.5}$$
$$\overline{P_j}\cdot u_{jt}-P_{jts}\geq 0 \qquad :\left(\overline{\tau_{P_{jts}}}\right) \qquad \forall j\in J_i,t,s \tag{6.6}$$
$$u_{jt}-\left(u_{j\,t-1}+x_{j\,t-1}-w_{j\,t-1}\right)-y_{jt}+z_{jt}=0 \qquad :\left(\nu_{jt}\right) \qquad \forall j\in J_i,t \tag{6.7}$$
$$X_{jts}-\underline{P_j}\cdot x_{jt}\geq 0 \qquad :\left(\underline{\tau_{X_{jts}}}\right) \qquad \forall j\in J_i,t,s \tag{6.8}$$
$$\overline{P_j}\cdot x_{jt}-X_{jts}-R_{jts}\geq 0 \qquad :\left(\overline{\tau_{X_{jts}}}\right) \qquad \forall j\in J_i,t,s \tag{6.9}$$
$$1-\left(u_{jt}+x_{jt}\right)\geq 0 \qquad :\left(\mu_{x_{jt}}\right) \qquad \forall j\in J_i,t \tag{6.10}$$
$$P_{jts}-W_{jts}-\underline{P_j}\cdot\left(u_{jt}-w_{jt}\right)\geq 0 \qquad :\left(\underline{\tau_{W_{jts}}}\right) \qquad \forall j\in J_i,t,s \tag{6.11}$$
$$\overline{P_j}\cdot\left(u_{jt}-w_{jt}\right)-P_{jts}+W_{jts}\geq 0 \qquad :\left(\overline{\tau_{W_{jts}}}\right) \qquad \forall j\in J_i,t,s \tag{6.12}$$
$$u_{jt}-w_{jt}\geq 0 \qquad :\left(\mu_{w_{jt}}\right) \qquad \forall j\in J_i,t \tag{6.13}$$
$$1-\left(x_{jt}+w_{jt}\right)\geq 0 \qquad :\left(\xi_{jt}\right) \qquad \forall j\in J_i,t \tag{6.14}$$
$$P_{jts}\geq 0,\quad X_{jts}\geq 0,\quad W_{jts}\geq 0,\quad R_{jts}\geq 0 \qquad \forall j\in J_i,t,s \tag{6.15}$$
$$u_{jt}\in\{0,1\},\quad x_{jt}\in\{0,1\},\quad w_{jt}\in\{0,1\} \qquad \forall j\in J_i,t \tag{6.16}$$
$$y_{jt}\in\{0,1\},\quad z_{jt}\in\{0,1\} \qquad \forall j\in J_i,t \tag{6.17}$$

Constraints (6.5)-(6.7) model the decisions in the day-ahead electricity market while constraints (6.8)-(6.14) model the decisions in the additional upward reserve mechanism and the intraday electricity market.

Constraints (6.5) and (6.6) model minimum and maximum values of the power generation in the day-ahead electricity market. Constraint (6.7) relates the dispatch decisions in the day-ahead market with the start-up and shut-down decisions in both the day-ahead and intraday markets. Thus, the decision variables of the intraday market affect the decision variables in the day-ahead market. If the model was a single-period model, this constraint would not be taken into account, and the decision variables in the day-ahead market would not be affected by the start-up and shut-down variables in the intraday market. Figure 6.2 shows the start-up and shut-down decisions.

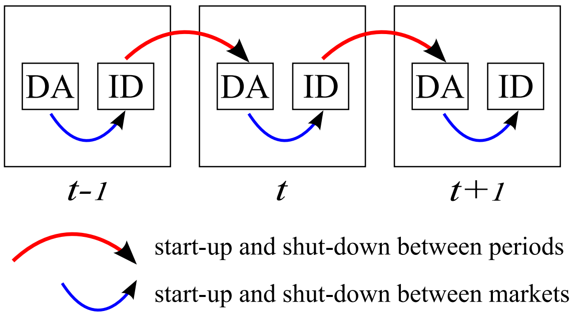


Figure 6.2: Start-up and shut-down decisions

Constraints (6.8)-(6.10) model the generation units that are assigned to provide additional upward reserve, and therefore, these units have to be dispatched in the intraday markets. Constraints (6.8) and (6.9) are the minimum and maximum values of the power generation X_{jts} and the additional upward reserve R_{jts}. Constraint (6.10) guarantees that the units that are dispatched in the day-ahead market cannot be dispatched in the intraday markets in order to provide additional upward reserve, i.e., if $u_{jt} = 1$ then $x_{jt} = 0$, $X_{jts} = 0$, and $R_{jts} = 0$.

Constraints (6.11)-(6.13) allow the generation units to shut down or reduce their production in the day-ahead market. Constraints (6.11) and (6.12) are the minimum and maximum values of the power reductions W_{jts} in the intraday markets. Constraint (6.13) indicates that only the units dispatched in the day-ahead market can be shut down in the intraday markets. Constraint (6.14) assures that units cannot start up or shut down simultaneously in the intraday markets.

Finally, constraints (6.15) model that the power variables and additional upward reserve are positive, and constraints (6.16) and (6.17) model that the dispatch, start up and shut down variables are binary.

6.2.3 Market equilibrium

By gathering together the first-order maximization conditions for all companies and then adding the market-clearing conditions, a mixed complementarity model MCP is defined, and the market equilibrium corresponds to the solution of the MCP. It is assumed that the companies behave perfect competitively in the different markets. Thus, there are no parameters that model the strategic behavior of generation companies like in the previous chapters. The market equilibrium is calculated by relaxing the binary decision variables. The reason is that the market equilibrium may not exist due to the non-convexity of the model with binary variables. Moreover, the electricity and reserve prices determined in a model with binary variables may not be enough to compensate for the total costs that include the dispatch, start-up and shut-down costs (O'Neill et al., 2005; Garciá-Bertrand et al., 2006; Delgadillo et al., 2012). A possible interpretation in the medium-term of the relaxed dispatch variables, $u_{jt} \in [0,1]$, is that the generation unit j is fully dispatched only during certain hours of the day t, e.g., $u_{jt} = 0.7$ means that the generation unit j is dispatched in the 70% of the time.

In this section, two different decision-making processes by the generation companies are analyzed in the market equilibrium. In the first case, called joint decision, each generation company determines the power generation of their units in the day-ahead and intraday electricity markets and the additional upward reserve simultaneously. In the second case, called sequential decision, each generation company first determines the power generation of their units in the day-ahead electricity market, and after the solution of this market is computed, each generation company determines the power generation in the intraday electricity market and the additional upward reserve.

6.2.3.1 Joint decision approach

In the joint decision case, it is not possible to use an equivalent minimization problem in which the dual variables of the power balance and reserve requirements constraints corresponds to the market prices because it would assume that both the available reserve that results from the day-ahead electricity market and the additional upward reserve are paid the reserve price. That approach would not work to model the Spanish mechanism in which only the additional upward reserve assigned in the mechanism is paid at the reserve price while the available reserve that results from the day-ahead electricity market and the additional reserve of units that reduce their generation in the intradays markets is not paid.

Therefore, the market equilibrium is determined using the MCP (6.18)-(6.39) in which only the additional upward reserve is paid. The operator $\perp$ denotes the inner product of two vectors equal to zero, i.e., $0 \le x \perp f(x) \ge 0$ corresponds to the system equations $x \ge 0$, $f(x) \ge 0$ and $x \cdot f(x) = 0$.

Equations (6.18)-(6.20) are the market clearing constraints (6.1)-(6.3):

$$(\lambda_{ts} \text{ unrestricted}) \quad \sum_{j \in J} P_{jts} = D_{ts} \qquad \forall t, s \tag{6.18}$$

$$0 \le \phi_{ts} \perp \sum_{j \in J} R_{jts} \ge DR_{ts} - \sum_{j \in J} \left(\overline{P_j} \cdot u_{jt} - P_{jts} \right) \qquad \forall t, s \tag{6.19}$$

$$\left(\psi_{ts} \text{ unrestricted}\right) \quad \sum_{j \in J} \left(X_{jts} - W_{jts} \right) = 0 \qquad \forall t, s \tag{6.20}$$

The generation company's behavior is modeled by means of equations (6.21)-(6.39). These equations are the Karush-Kuhn-Tucker (KKT) conditions of the maximization problem (6.4)-(6.17) for each company i.

Equations (6.21)-(6.30) are the primal constraints of the maximization problem (6.4)-(6.17):

$$0 \le \underline{\tau_{P_{jts}}} \perp P_{jts} - \underline{P_j} \cdot u_{jt} \ge 0 \qquad \forall j \in J, t, s \tag{6.21}$$

$$0 \le \overline{\tau_{P_{jts}}} \perp \overline{P_j} \cdot u_{jt} - P_{jts} \ge 0 \qquad \forall j \in J, t, s \tag{6.22}$$

$$0 \le \underline{\tau_{X_{jts}}} \perp X_{jts} - \underline{P_j} \cdot x_{jt} \ge 0 \qquad \forall j \in J, t, s \tag{6.23}$$

$$0 \le \overline{\tau_{X_{jts}}} \perp \overline{P_j} \cdot x_{jt} - X_{jts} - R_{jts} \ge 0 \qquad \forall j \in J, t, s \tag{6.24}$$

$$0 \le \mu_{x_{jt}} \perp 1 - \left(u_{jt} + x_{jt} \right) \ge 0 \qquad \forall j \in J, t \tag{6.25}$$

$$0 \le \underline{\tau_{W_{jts}}} \perp P_{jts} - W_{jts} - \underline{P_j} \cdot \left(u_{jt} - w_{jt} \right) \ge 0 \qquad \forall j \in J, t, s \tag{6.26}$$

$$0 \le \overline{\tau_{W_{jts}}} \perp \overline{P_j} \cdot \left(u_{jt} - w_{jt} \right) - P_{jts} + W_{jts} \ge 0 \qquad \forall j \in J, t, s \tag{6.27}$$

$$0 \le \mu_{w_{jt}} \perp u_{jt} - w_{jt} \ge 0 \qquad \forall j \in J, t \tag{6.28}$$

$$0 \le \xi_{jt} \perp 1 - \left(x_{jt} + w_{jt} \right) \ge 0 \qquad \forall j \in J, t \tag{6.29}$$

$$\left(v_{jt} \text{ unrestricted} \right) u_{jt} - \left(u_{j\,t-1} + x_{j\,t-1} - w_{j\,t-1} \right) - y_{jt} + z_{jt} = 0 \qquad \forall j \in J, t \tag{6.30}$$

Equations (6.31)-(6.39) are the dual constraints of the maximization problem

(6.4)-(6.17):

$$0 \le P_{jts} \perp -\lambda_{ts} \cdot d_{ts} + CV_j \cdot d_{ts} - \underline{\tau_{P_{jts}}} + \overline{\tau_{P_{jts}}} - \underline{\tau_{W_{jts}}} + \overline{\tau_{W_{jts}}} \ge 0 \qquad \forall j,t,s \tag{6.31}$$

$$0 \le u_{jt} \perp CU_j \cdot \sum_s d_{ts} + \underline{P_j} \cdot \sum_s \underline{\tau_{P_{jts}}} - \overline{P_j} \cdot \sum_s \overline{\tau_{P_{jts}}} + \mu_{x_{jt}} - \mu_{w_{jt}} + \underline{P_j} \cdot \sum_s \underline{\tau_{W_{jts}}} - \overline{P_j} \cdot \sum_s \overline{\tau_{W_{jts}}} - \nu_{jt} + \nu_{j\,t+1} \ge 0 \qquad \forall j,t \tag{6.32}$$

$$0 \le X_{jts} \perp -\psi_{ts} \cdot d_{ts} + CV_j - \underline{\tau_{X_{jts}}} + \overline{\tau_{X_{jts}}} \ge 0 \tag{6.33}$$

$$0 \le W_{jts} \perp \psi_{ts} \cdot d_{ts} - CV_j + \underline{\tau_{W_{jts}}} - \overline{\tau_{W_{jts}}} \ge 0 \qquad \forall j,t,s \tag{6.34}$$

$$0 \le R_{jts} \perp -\phi_{ts} \cdot d_{ts} + \overline{\tau_{X_{jts}}} \ge 0 \qquad \forall j,t,s \tag{6.35}$$

$$0 \le x_{jt} \perp CU_j \cdot \sum_s d_{ts} + CY_j + \underline{P_j} \cdot \sum_s \underline{\tau_{X_{jts}}} - \overline{P_j} \cdot \sum_s \overline{\tau_{X_{jts}}} + \mu_{x_{jt}} + \xi_{jt} + \nu_{j\,t+1} \ge 0 \qquad \forall j,t \tag{6.36}$$

$$0 \le w_{jt} \perp -CU_j \cdot \sum_s d_{ts} + CZ_j - \underline{P_j} \cdot \sum_s \underline{\tau_{W_{jts}}} + \overline{P_j} \cdot \sum_s \overline{\tau_{W_{jts}}} + \mu_{w_{jt}} + \xi_{jt} - \nu_{j\,t+1} \ge 0 \qquad \forall j,t \tag{6.37}$$

$$0 \le y_{jt} \perp CY_j + \nu_{jt} \ge 0 \qquad \forall j,t \tag{6.38}$$

$$0 \le z_{jt} \perp CZ_j - \nu_{jt} \ge 0 \qquad \forall j,t \tag{6.39}$$

6.2.3.2 Sequential decision approach

In the sequential decision case, a Mixed Complementary Problem is used in order to determine the market equilibrium as in the joint decision case. Assuming that the start-up and shut-down costs are the same in the day-ahead and intraday markets, the constraint that relates the dispatch decision variables with the start-up and shut-down variables is the same as presented in (6.7).

An alternative approach is to use a bilevel optimization problem in which the upper-level corresponds to the day-ahead electricity market clearing process and the lower-level models the additional upward reserve mechanism and the intraday electricity market. In that approach, constraint (6.7) is in the upper-level problem. In a single-period model, this constraint is not taken into account, and therefore, the upper-level problem is not affected by the lower-level decision variables, i.e., there is no feedback between the two levels. In that case, both problems can be solved separately, by first determining the solution in the day-ahead electricity market and using that solution to determine the solution in the additional upward reserve mechanism and the intraday electricity market, as in the models presented in Jia et al. (2006) and Hongxing et al. (2010).

One way to solve the bilevel problem among multiple periods is to transform the bilevel problem into an equivalent single-level problem. This transformation is made possible by changing the lower-level problem by its KKT conditions. However, the equivalent single-level problem is non-linear and non-convex due to the complementary constraints. These constraints can be converted to mixed-integer lineal constraints using the formulation proposed in Fortuny-Amat and McCarl (1981). The main inconvenience is that the transformation of each complementary constraint requires a binary variable, and the number of binary variables increases with the number of periods, load levels and generation companies. Thus, the bilevel problem approach is not suitable to large-scale problems.

Instead of using the bilevel problem as explained above, the market equilibrium is computed using a Mixed Complementary Problem in which the the day-ahead electricity market decisions P_{jts} and u_{jt} are constants in constraints (6.8)-(6.13) that corresponds to the intraday and reserve markets in the maximization problem of the generation company. The solution to this MCP is the same as the solution to the bilevel problem. The constraints that model the market equilibrium in this MCP are the same as the MCP in the joint decision (6.18)-(6.39) except the constraints (6.31) and (6.32) which become as presented in (6.40) and (6.41):

$$0 \leq P_{jts} \perp -\lambda_{ts} \cdot d_{ts} + CV_j \cdot d_{ts} - \underline{\tau_{P_{jts}}} + \overline{\tau_{P_{jts}}} \geq 0 \qquad \forall j,t,s \quad (6.40)$$

$$0 \leq u_{jt} \perp CU_j \cdot \sum_s d_{ts} + \underline{P_j} \cdot \sum_s \underline{\tau_{P_{jts}}} - \overline{P_j} \cdot \sum_s \overline{\tau_{P_{jts}}} - \nu_{jt} + \nu_{jt+1} \geq 0 \qquad \forall j,t \quad (6.41)$$

6.3 Numerical Example

This section presents a simple example to study the additional upward reserve mechanism implemented in Spain. The market equilibrium is solved using PATH (Ferris and Munson, 2000) in GAMS (GAMS, 2014).

The power system model of this case study consists of 13 thermal power plants. The characteristics of these units, i.e., their technology, the number of units installed, the minimum and maximum power, and the start-up, shut-down, variable and fixed costs can be found in Table 6.1.

Table 6.1: Generation units

Unit	**Technology**	**Power**		**Cost**			
		Minimum [MW]	**Maximum [MW]**	**Start-up [k€]**	**Shut-down [k€]**	**Variable [€/MWh]**	**Fixed [k€/h]**
1	Nuclear	900	1 000	54.00	5.40	4.00	0
2	Coal	170	350	30.04	3.00	26.76	0.81
3	Coal	170	350	30.38	3.04	26.76	0.81
4	Coal	210	350	30.06	3.01	32.98	0.67
5	Coal	160	330	30.90	3.09	32.93	0.80
6	Coal	160	350	30.30	3.03	39.01	0.78
7	Combined cycle	180	440	20.87	2.09	52.10	3.24
8	Combined cycle	180	440	20.71	2.07	54.15	3.24
9	Combined cycle	180	440	20.50	2.05	49.87	3.24
10	Combined cycle	180	440	20.44	2.04	49.15	3.24
11	Combined cycle	180	440	20.87	2.09	45.16	3.24
12	Fuel oil	60	250	5.16	0.52	75.05	2.08
13	Fuel oil	60	250	4.77	0.48	69.73	2.08

Three different cases are considered in this example. In the first case (A), the electricity and upward reserve are determined simultaneously, i.e., there is no subsequent mechanism for determining the additional upward reserve necessary in the system, and therefore the intraday markets are not taken into account. This approach can be seen as an ideal case. The second case (B) corresponds to the joint decision-making process presented in section 6.2.3.1, and the third case (C) corresponds to the sequential decision-making process presented in section 6.2.3.2.

The temporal structure is modeled using 7 periods and 3 load levels of demand (off-peak, plateau, and peak). The total demand D_{ts} and the total reserve requirement DR_{ts} for each period and load level appear in Table 6.3. The daily average duration in hours of these load levels is shown in Table 6.2.

Table 6.2: Average duration

	Duration [hours]
off-peak	6.6
plateau	11.4
peak	6.0

Table 6.3: Demand and Reserve

Period	Demand off-peak [MW]	Demand plateau [MW]	Demand peak [MW]	Reserve off-peak [MW]	Reserve plateau [MW]	Reserve peak [MW]
1	2 255	2 462	2 709	656	858	1 021
2	2 356	2 585	2 804	619	819	858
3	2 478	2 691	2 892	570	843	1 030
4	2 649	2 808	2 963	620	852	1 074
5	2 419	2 601	2 812	583	883	1 005
6	2 163	2 357	2 566	726	775	949
7	2 089	2 153	2 377	796	894	956
Average	2 325	2 536	2 732	662	847	985

6.3.1 Total system cost

Table 6.4 compares the overall system cost yielded in each case. In case A, the day-ahead electricity market and the reserve mechanism are cleared jointly, and the intraday electricity markets are not needed to modify the power production of the generation units. Therefore, this case represents the model with a lower overall system cost.

In case B, the overall system cost is greater than in case A. In the intraday market, some generation units must start up in order to provide the necessary upward reserve in the system while other generation units reduce their power production or are shut down in order to maintain the power system balance. This trade-off between start up and shut down/reduction may cause a reduction in the total variable and fixed cost. In case B, this variable and fixed cost reduction is equal to 338 k€. However, the start-up and shut-down costs increase by 757 k€. In a hypothetical case in which the start-up and shut-down costs are zero, the final dispatch of the generation companies in cases A and B would be exactly the same, and therefore, both cases would have the same overall system cost, i.e., the reduction in the variable and fixed costs in case B would be enough so that the overall system costs in both cases would be the same. Moreover, this would mean that the generation companies are indifferent to producing in the day-ahead market and the intraday markets, and the final dispatch would be the same. However, in real cases the start-up and shut-down costs maybe significant, which would result in a different dispatch in both cases, and the overall cost solution in case B would be greater than the overall cost solution in case A.

In case C, the overall system cost is the greatest of the three cases. Although the system cost in the day-ahead electricity market is lower than in case B, the system cost in the intraday markets is much greater. The reason is that in case C, the generation companies first made their decisions in the day-ahead electricity market, and these decisions are fixed in the intraday markets in which the companies make the necessary changes and incur higher costs. Therefore, this results in a different dispatch between cases B and C.

Table 6.4: Total cost

	case A	case B			case C		
		Day-ahead	Intraday	Total	Day-ahead	Intraday	Total
	[k€]	[k€]	[k€]	[k€]	[k€]	[k€]	[k€]
Start-up and shut-down	272	217	757	974	409	460	869
Variable and fixed	11 575	11 996	-338	11 658	9 407	3 220	12 627
Total	11 847	12 213	419	12 632	9 816	3 680	13 496

6.3.2 Power productions and reserve

Table 6.5 presents the total generation for each technology in each case. It can be seen that the total energy traded in the intraday markets is lower in case B than in case C.

In case B, the generation companies make their decisions in the day-ahead electricity market, intraday markets, and reserve mechanism simultaneously. Thus, the companies decide to start up combined cycle and fuel-oil generations units that are more expensive (higher variable cost), and therefore, the system cost in the day-ahead market increases (Table 6.4). However, these expensive generation units reduce their power production in the intraday markets, and the reserve in the system is provided by starting up coal units with lower variable and fixed costs but higher start-up costs.

Something different happens in case C. The units with the lower variable and fixed costs are dispatched in the day-ahead electricity market, and therefore, the system cost in the day-ahead market is the lowest of the three cases (Table 6.4). However, in the intraday markets, the generation companies incur higher costs because they have to reduce the power production of coal units with low variable and fixed costs, and start-up combined cycle and fuel-oil units with high variable costs in order to provide the reserve in the system.

Table 6.5: Total production

	case A	case B		case C	
		Day-ahead	Intraday	Day-ahead	Intraday
	[GWh]	[GWh]	[GWh]	[GWh]	[GWh]
Nuclear	167.5	168.0	0	168.0	0
Coal	189.6	162.6	30 .8	249.2	-78.4
Combined cycle	67.1	66.0	-9.3	7.4	58.2
Fuel oil	0.4	27.9	-21.5	-	20.2
Total	424.6	424.6	0	424.6	0

Table 6.6: Average production and reserve

(a) case A

	Final power production			Reserve		
	off-peak	plateau	peak	off-peak	plateau	peak
	[MW]	[MW]	[MW]	[MW]	[MW]	[MW]
Nuclear	989	1 000	1 000	11	0	0
Coal	945	1 128	1 330	785	602	400
Combined cycle	390	405	400	563	586	577
Fuel oil	2	2	2	7	8	7
Total	2 325	2 536	2 732	1 366	1 195	985

(b) case B

	Final power production			Reserve from day-ahead			Additional reserve		
	off-peak	plateau	peak	off-peak	plateau	peak	off-peak	plateau	peak
	[MW]	[MW]	[MW]	[MW]	[MW]	[MW]	[MW]	[MW]	[MW]
Nuclear	998	1 000	1 000	0	0	0	0	0	0
Coal	961	1 153	1 355	130	28	0	155	319	375
Combined cycle	327	344	338	161	129	0	63	237	488
Fuel oil	39	39	39	92	52	0	60	82	122
Total	2 325	2 536	2 732	384	209	0	278	638	985

(c) case C

	Final power production			Reserve from day-ahead			Additional reserve		
	off-peak	plateau	peak	off-peak	plateau	peak	off-peak	plateau	peak
	[MW]	[MW]	[MW]	[MW]	[MW]	[MW]	[MW]	[MW]	[MW]
Nuclear	1 000	1 000	1 000	0	0	0	0	0	0
Coal	823	1 021	1 222	342	160	0	35	35	41
Combined cycle	382	395	390	42	49	0	97	226	564
Fuel oil	120	120	120	0	0	0	146	377	380
Total	2 325	2 536	2 732	384	209	0	278	638	985

The average of the final power production and reserve assigned to each technology appears in Table 6.6. In case A, Table 6.6a shows that during peak hours the allocated reserve equals the system reserve requirement. During plateau and off-peak hours, the generation units are dispatched significantly below their maximum power, and therefore, the allocated reserve in those hours is much greater than the system reserve requirement.

Cases B and C are similar to case A (Tables 6.6b and 6.6c) in that during peak hours, the reserve allocated in the subsequent mechanism equals the system reserve requirement. On the other hand, during plateau and off-peak hours, only a proportion of the system reserve requirement is allocated in the subsequent mechanism because the generation units are dispatched significantly below their maximum power in the day-ahead market, and they can naturally provide upward reserve.

6.3.3 Prices

Table 6.7 shows the average prices of the electricity in day-ahead and intraday markets, and the average prices of the upward reserve in the subsequent mechanism for each load level. In the three cases, the reserve price is equal to zero in the plateau and off-peak hours while the reserve price is greater than zero during peak hours.

The generation units that are allocated in the subsequent reserve mechanism have to be dispatched in the intraday markets. In case B, during plateau and off-peak hours, it is not necessary to dispatch generation units to provide additional upward reserve and the day-ahead and intraday electricity prices are exactly the same during those hours. In case C, since generation companies make their decisions first in the day-ahead market and subsequently make the necessary changes in the intraday markets, the prices are different in all the load levels in the day-ahead and intraday markets. Moreover, the reserve price in case C is greater than the reserve price in cases A and B.

Table 6.7: Average prices

		off-peak	plateau	peak
case A	**Day-ahead [€/MWh]**	15.9	27.7	102.1
	Reserve [€/MW]	0	0	70.0
case B	**Day-ahead [€/MWh]**	18.4	27.9	88.6
	Intraday [€/MWh]	18.4	27.9	84.2
	Reserve [€/MW]	0	0	90.1
case C	**Day-ahead [€/MWh]**	33.7	36.6	74.2
	Intraday [€/MWh]	26.8	29.9	36.3
	Reserve [€/MW]	0	0	117.7

6.3.4 Comparison and discussion

In the comparison results of Table 6.8, case A reaches the lowest total production cost while cases B and C have a higher total production cost, as explained in section 6.3.1.

The day-ahead electricity price decreases and the reserve price increases during peak hours in case B compared to case A (Table 6.7). This means that the cost paid by consumers in the day-ahead market decreases and the reserve consumers' cost increases in case B. However, the increment in the reserve cost is lower than the reduction in the day-ahead cost. Thus, consumers pay less in case B than in case A, which makes the total generators' profit lower in case B.

The day-ahead electricity prices during off-peak and plateau hours in case C are greater than the prices in case A which results in an increment in the total production cost. In the same way, the reserve price during peak hours is also greater in case C, and therefore, the reserve consumers' cost also increases. Thus, case C is the most inefficient solution because it has the highest total production and consumers' costs.

These results may be analogous to results of chapter 2 which analyzes the effect of the counter-trading mechanism in the electricity market. Three different situations are assessed in chapter 2. The first situation corresponds to the ideal solution that is a nodal-pricing mechanism. The second situation corresponds to the expected solution by the regulator in which the generation companies do not behave strategically and the total cost paid by consumers is minimized. Finally, a more realistic case is modeled in the third situation in which the generation companies behave strategically.

Similarly, the results of Table 6.8 present three different situations. Case A corresponds with an ideal solution in which the total production cost is minimized. In case B, the total cost paid by consumers decreases although the total production cost increases. Case C is the least efficient solution in which both production and consumer costs increase. Case B assumes that generation companies can completely internalize the effect of the reserve and intraday markets on their behavior in the day-ahead market. However, that assumption may not be applicable to real situations. For example, in the Spanish market generation companies may not completely internalize those effects, and therefore, case C may be more appropriate to represent the actual outcome of the market.

Table 6.8: Comparison

	case A	**case B**	**case C**
Day-ahead production cost	11 847	12 213	9 816
Intraday production cost	-	419	3 680
Total production cost	11 847	12 632	13 496
Day-ahead consumers' cost	19 105	17 865	19 671
Intraday consumers' cost	-	0	0
Reserve consumers' cost	2 904	3 729	4 873
Total consumers' cost	22 009	21 594	24 544
Total generators' profit	10 162	8 962	11 048

6.4 Conclusion

This chapter made a first attempt to analyze the interdependence of the reserve and electricity markets under a sequential dispatch. In order to do this, a model to study the mechanism implemented in Spain for the allocation of additional upward reserve is presented. This model makes it possible to study two types of decision-making processes by generation companies: joint and sequential decisions. In both cases, the results show that the overall system cost is greater than an ideal market in which the electricity and reserve markets are cleared simultaneously. The reason is that the generation companies incur additional costs in the intraday markets for starting up generation units to provide upward reserve and for shutting down generation units to maintain the power system balance.

The joint-decision model may be more appropriate in a situation in which generation companies can completely internalize the effects of the reserve and intraday markets into their behavior in the day-ahead electricity market. Therefore, this situation tends to be closer to an ideal market in which the electricity and reserve markets are cleared simultaneously. However, to internalize these effects could cause a decrease in the day-ahead electricity prices, being counterproductive for generation companies because their profit would decrease.

On the other hand, in a real situation as in the Spanish market, it is not clear that companies completely internalize the effects of the reserve and intraday markets into their behavior in the day-ahead electricity market. Thus, generation companies decide their strategies in the day-ahead market and, considering the result on this market, they make the necessary changes in the reserve and intraday markets. In this situation, the sequential-decision model is more appropriate although it is the most inefficient solution for both the system and the consumers.

Therefore, these two decision-making processes may simulate two different situations. The joint-decision approach would correspond to the expected result by the regulator in which the total consumer cost is minimized while the sequential-decision approach would model more realistic cases.

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Chapter 7

Conclusions, contributions and future research

Contents

This Chapter presents the conclusions and most relevant contributions. It also discusses possible directions for future research.

7.1 Conclusions

In recent years, there has been an increase in technical and security constraints in power systems. This have been caused by several factors, such as the emergence of distributed generation, the penetration of renewable energies and the active demand response. This new context has augmented the importance of the markets and mechanisms used to resolve those constraints. One of the main assumptions made in the previous models developed in the literature is that since most of the energy is traded in the day-ahead electricity market, this market is sufficiently important and the models can disregard the effects of other markets and mechanisms used to resolve technical and security constraints. Although that assumption could be quite reasonable in the models of the former electricity markets, this thesis has shown that an adequate model of the electricity markets must include all the different markets and mechanisms in which the generation companies are involved. Thus, the models that simulate the electricity market cannot disregard the effect of these mechanisms because the behavior of generation companies cannot be accurately modeled without it.

In this context, several models have been developed in this thesis in order to analyze the effect of the mechanisms used to clear technical and security constraints on the strategic behavior of generation companies. A conjectural-variation-based equilibrium model has been proposed to study the effect of system congestion. This model has been first applied in a two-area power system and subsequently it has been generalized in power systems with several areas. A simplified version of this model has been used to assess the counter-trading mechanisms implemented in several European countries. A more complex version of this model that incorporates an AC power flow has been proposed to study the effect of other network constraints such as voltage level requirements. Finally, this thesis has also analyzed the effect of the additional upward reserve mechanism implemented in Spain.

Several European countries have single-price electricity markets that use a counter-trading mechanism in order to clear system congestion. This thesis has shown that these mechanisms are technically or economically inefficient because they allow generation companies to exercise market power. This market power is caused by the fact that generation companies have incentives to bid above the marginal cost of the units that clear system congestion in the day-ahead electricity market. Therefore, these units are not be dispatched in the day-ahead market but are necessary in the counter-trading mechanism. The inefficiencies of single-price electricity markets with counter-trading mechanisms can be counteracted by implementing nodal-price electricity markets as has occurred in the electricity markets in some countries, such as the PJM or CAISO markets in the United States.

One of the fundamental assumptions made in models of single-price electricity markets is that the strategic behavior of a generation company in the day-ahead market is the same in the whole power system because this company can affect the electricity price by modifying the generation in any area of the power system. However, this thesis has demonstrated that the congestion management mechanism may affect the strategic behavior of generation companies in the day-ahead market by means of a conjectural-variation-based equilibrium model. In the model, the conjectured-price response of the company is modified by a factor that models the power reduction of the units necessary to clear the congestion in the congestion management mechanism. Hence, the market equilibrium equations take system congestion into account. This model has been first developed for the two-area power system case, and subsequently it has been generalized in power systems with multiple areas by including a DC representation of the power flow. The market equilibrium is computed by means of an equivalent optimization problem. Since this problem is non-linear and non-convex, an iterative methodology has been proposed in order to solve this optimization problem with satisfactory results. In general terms, the results show that the generation companies value each area of the power system differently, and therefore their strategic behavior varies by area. When the generation companies know that the units located at the importing areas must increase their generation while the units located at the exporting area must decrease their generation in the congestion management mechanism, then the generation companies anticipate this result by modifying their behavior in the day-ahead electricity market and give more importance to the units at the importing areas.

Regarding the effect of the voltage level requirements, the conjectural-variation-based equilibrium model has been generalized by including an AC representation of the power flow and binary commitment variables. The main drawback is that the optimal power flow is a hard-resolution problem. A similar iterative approach to the methodology used in the model that analyzes the congestion management has been applied. This approach is satisfactory in a small-size power system. The resolution of this problem in large-size power systems is a key point that could be addressed with more detail in future work. However, even in a small-size power system, this model provides significant results showing that the voltage level is a local problem that can only be solved by the generation units located in the areas where the voltage levels are not at the allowable levels. As a result, the generation companies may use this information in order to exercise market power in those areas in the subsequent mechanism. As in the congestion management case, one possible solution is to consider the network constraints in the day-ahead market clearing process and to have different prices in the nodes of the power system. However, that solution might reduce the transparency of the market because it would be necessary to use an AC power flow.

In addition to the mechanisms used to clear network congestion and meet voltage requirements, there are other mechanisms subsequent to the day-ahead market used to guarantee the security of power systems, such as the reserve markets. This thesis has made a first attempt to study the effect of the reserve markets. Specifically, the additional upward reserve mechanism implemented in Spain has been analyzed by means of a model in which the generation companies behave perfectly competitively in the day-ahead electricity market. In the model, two different decision-making processes by generation companies have been studied: joint and sequential decisions. In both cases, the results show that the additional upward reserve mechanism results in a final dispatch that is neither technically nor economically optimal. Specifically, generation companies may incur in an additional start-up or shut-down cost in the intraday markets in order to provide the necessary reserve in the system. Moreover, in cases in which the generation companies cannot fully internalize the effect of the reserve and intraday markets on their decision in the day-ahead market, the final market solution becomes more inefficient. This problem could be solved with a market design in which the energy and reserve are dispatched jointly.

7.2 Contributions

The major contributions of this thesis are as follows:

1. A market equilibrium model that shows that the counter-trading mechanisms implemented in several European countries (Spain, Portugal, Germany, and the Netherlands) to solve congestion are inefficient because these mechanisms allow generation companies to exercise market power between markets even if there is not market power in each one separately.

2. A formalization of the market equilibrium equations taking into account the effect of congestion between areas on the strategic behavior of generation companies. This market equilibrium model is a conjectural-variation-based model. Initially, the model has been used to study the congestion between two areas. Subsequently, the model has been extended to analyze congestion in large-size power systems with multiple areas.

3. A solution method for solving the market equilibrium equations which is based on an iterative process. Two stages are solved in the iterative process. The first stage corresponds to the day-ahead market clearing process and the second stage is a DC optimal power flow that solves network congestion.

4. A market equilibrium model that takes into account the effect of voltage level requirements on the strategic behavior of generation companies.

5. A solution method for solving the market equilibrium equations which are solved by means of a two-stage optimization problem. In the first stage, a mixed complementary problem models the day-ahead market clearing process. In the second stage, an AC optimal power flow is solved to determine the changes in active and reactive power needed to meet the voltage system requirements.

6. A market equilibrium model for analyzing the effect of the additional upward reserve mechanism implemented in Spain.

7. A literature review and classification of the different equilibrium models used to study the effect of network constraints, such as system congestion and voltage requirements, and reserve markets on the outcome of electricity markets.

7.3 Future research

This section briefly outlines possible future research lines taking as reference the work presented in this thesis.

1. The model that analyzes the effect of system congestion on the result of the electricity market could be represented using a bilevel optimization problem in which the upper level represents the market equilibrium and the lower level simulates the congestion management mechanism. A drawback to this representation is that in the upper level the variable that models the reduction in power generation in the congestion management mechanism multiplies the power generation in both the power balance constraints and objective function. Therefore, even transforming the bilevel problem into an equivalent single-level optimization problem, this equivalent problem would still be nonlinear and nonconvex. Different alternative resolution techniques such as heuristic approaches could be explored in order to solve this equivalent problem.

2. A proper model of the mechanism that resolves the voltage level requirements need the use of an AC representation of the power flow equations and binary dispatch variables. In the thesis, this problem was tackled taking all the possible solutions of the binary dispatch variables into account, and selecting the solution with the best value in the objective function. Although that approach could be valid in small-size power systems, it would be intractable in large-size power systems. Therefore, it is necessary to study effective and efficient solution algorithms for nonlinear and nonconvex problems with binary variables.

3. With respect to the markets and mechanisms used to contract and manage reserve, this thesis has studied the effect of the additional upward reserve mechanism. The results have shown that even with perfectly competitive behavior in the day-ahead market, the final dispatch may be inefficient when the reserve and intraday markets are taken into account. Therefore, it is necessary to study other mechanisms used to contract other kinds of reserve such as secondary and tertiary reserves, and to analyze the effect of these mechanisms in oligopoly environments.

4. Other factors that may affect the strategic behavior of generation companies are related to policy or regulatory constraints, such as the remuner-

ation of non-convexities. A semi-complex offer with minimum income is used in Spain to dispatch only those generation units that can recover their production, start-up, shut-down, and commitment costs. These mechanisms have not been explored in this thesis but they should be studied because they may modify the strategies of the generation companies in the electricity market.

Appendices

Appendix A

Nomenclature

A.1 Indices

a	Area (node) index
b	Area (node) index
h	Generation company index
i	Generation company index
j	Production unit index
k	Production unit index
l	Flowgate (transmission line) index
s	Load level index
t	Time period index
EX	Exporting area index
IM	Importing area index
κ	Iteration counter index
Ω	Optimal power flow OPF

A.2 Sets

A	Set of indices of nodes or areas
I	Set of indices of generation companies
J	Set of indices of production units
J_a	Set of indices of production units connected to area a
J_i	Set of indices of production units owned by company i
L	Set of indices of flowgates
Ξ	Set of decision variables in the OPF

A.3 Constants

d_{ts}	Duration of period t and load level s [hour]
e_l	Constant equal to the base power of the system [MW] divided by the reactance of flowgate l [p.u.]
n_j^{max}	Parameter of the capability curve of unit j
n_j^{min}	Parameter of the capability curve of unit j
B_{ab}	Element of the susceptance matrix [p.u.]
CU_j	Commitment cost of unit j [MW]
CV_j	Variable cost of unit j [MW]
CY_j	Start-up cost of unit j [MW]
CZ_j	Shut-down cost of unit j [MW]
D	Total demand [MW]
D_a	Demand at area a [MW]
D^{IM}	Demand at importing area [MW]
D^{EX}	Demand at exporting area [MW]
D_{ts}	Total demand in period t and load level s [MW]
DR_{ts}	Up-reserve requirement in period t and load level s [MW]
$\overline{F_l}$	Power flow capacity of flowgate l [MW]
$FR(l)$	Sending area of flowgate l
G_{ab}	Element of the conductance matrix [p.u.]
H_{al}	Element of the network incidence matrix that is equal to 1 if area a is the sending area of flowgate l, -1 if area a is the receiving bus of flowgate l,and 0 otherwise
$\underline{P_j}$	Minimum active power generation of unit j [MW]
$\overline{P}_j$	Maximum active power generation of unit j [MW]
$\overline{P}_i^{IM}$	Maximum production of company i in the importing area [MW]
$\overline{P}_i^{EX}$	Maximum production of company i in the exporting area [MW]

$\underline{Q}_j$	Minimum reactive power generation of unit j [Mvar]
$\overline{Q}_j$	Maximum reactive power generation of unit j [Mvar]
$Q_j^{0,max}$	Parameter of the capability curve of unit j [Mvar]
$Q_j^{0,min}$	Parameter of the capability curve of unit j [Mvar]
$TO(l)$	Receiving area of flowgate l
α	Learning rate. It could take values in the interval [0,1]
β_i	Conjectured-price response of generation company i in the congestion management market [(€/MWh)/MW]
$\overline{\delta}$	Bound of the nodal phase angles [rad]
ϵ	Level of solution accuracy
θ_i	Conjectured-price response of generation company i in the day-ahead market [(€/MWh)/MW]

A.4 Variables

f_l	Power flow of flowgate l [MW]
m	Reduction factor
m_j	Reduction factor of production unit j
u_j	Commitment variable of unit j
u_{jt}	Commitment variable of unit j in period t
w_j	Binary variable that is equal to 1 if the unit j increments its production and 0 otherwise
w_{jt}	Shut-down variable of unit j in period t in the intraday market
x_{jt}	Start-up variable of unit j in period t in the intraday market
y_{jt}	Start-up variable of unit j in period t in the day-ahead market
z_{jt}	Shut-down variable of unit j in period t in the day-ahead market
AC_j	Apparent cost of unit j [€/MWh]
ACW_j	Apparent cost of unit j (Reduction) [€/MWh]

ACX_j	Apparent cost of unit j (Increment) [€/MWh]
C_j	Production cost of unit j [€]
$\overline{C}_j$	Effective cost of unit j [€]
D^*	Modified demand [MW]
DP_a	Active power demand at area a [MW]
DQ_a	Reactive power demand at area a [Mvar]
MC_i	Marginal cost of generation company i [€/MWh]
MC_j	Marginal cost of unit j [€/MWh]
MR_i	Marginal revenue of generation company i [€/MWh]
MR_j	Marginal revenue of unit j [€/MWh]
P_i	Generation of company i in the day-ahead market [MW]
P_i^{IM}	Generation of company i in the importing area in the day-ahead market [MW]
P_i^{EX}	Generation of company i in the exporting area in the day-ahead market [MW]
P_j	Active power generation of production unit j in the day-ahead market [MW] or [p.u]
P_j^{Ω}	Active power generation of production unit j in the optimal power flow [MW] or [p.u]
P_{jts}	Active power generation of production unit j in period t and load level s in the day-ahead market [MW]
Q_j	Reactive power generation of production unit j in the day-ahead market [Mvar]
Q_j^{Ω}	Reactive power generation of production unit j in the optimal power flow [Mvar]
R_{jts}	Additional up-reserve of unit j in period t and load level s [MW]
V_a	Voltage magnitude at node a [p.u.]
W_j	Decrement in the production of unit j [MW]
W_j^{Ω}	Decrement in the production of unit j in the optimal power flow [MW]
W_{jts}	Decrement in the production of unit j in period t and load level s in the intraday market [MW]

X_j	Increment in the production of unit j [MW]
X_j^{Ω}	Increment in the production of unit j in the optimal power flow [MW]
X_{jts}	Increment in the production of unit j in period t and load level s in the intraday market [MW]
Y^*	Modified increment [MW]
Z	Losses in the power system
γ	Congestion market price (Increments) [€/MWh]
δ_a	Phase angle at area a [rad]
λ	Day-ahead market price [€/MWh]
λ_{ts}	Day-ahead market price in period t and load level s [€/MWh]
μ	Dual variable
ν	Dual variable
ξ	Dual variable
τ	Dual variable
π_i	Profit of agent i [€]
ϕ_{ts}	Up-reserve market price in period t and load level s [€/MW]
ψ_{ts}	Intraday market price in period t and load level s [€/MWh]
χ	Congestion market price (Reductions) [€/MWh]
ΔP_i^{IM}	Change in the production of company i in the importing area in the counter-trading mechanism
ΔP_i^{EX}	Change in the production of company i in the exporting area in the counter-trading mechanism
ΔP	Total change in the production of the units that clear system congestion

A.5 Functions

$C_i(P_i)$	Cost function of company i [€]
$\overline{C_i}(P_i)$	Effective cost function of company i, [€]

$C_i^{IM}\left(P_i^{IM}\right)$	Cost function of company i in the importing area [€]
$C_i^{EX}\left(P_i^{EX}\right)$	Cost function of company i in the exporting area [€]
$U(D)$	Utility demand function [€]
$\lambda(D)$	Inverse demand function

Appendix B

Article 1 - Effect of Network Congestions Between Areas on Single-Price Electricity Markets

Delgadillo, A., Reneses, J., Barquín, J., Feb. 2013. Effect of network congestions between areas on single-price electricity markets. IEEE Trans. Power Systems 28 (1), 93–101.

Abstract: This paper presents a conjectural-variation-based equilibrium model of a single-price electricity market. The distinctive modeling feature introduced in this paper is the formalization of the equilibrium equations taking into account the effect of congestion between areas on generators' behavior. The results show that, when there is a congestion between two areas, generators valued differently the production of each area, giving more importance to the importing area.

Index Terms: Conjectural variation, market equilibrium, network constraints, single-price electricity markets.

Appendix C

Article 2 - Conjectural-Variation-Based Equilibrium Model of a Single-Price Electricity Market With a Counter-Trading Mechanism

Delgadillo, A., Reneses, J., Nov. 2013. Conjectural-variation-based equilibrium model of a single-price electricity market with a counter-trading mechanism. IEEE Trans. Power Systems 28 (4), 4181–4191.

Abstract: This paper presents a new conjectural-variation-based equilibrium model of a single-price electricity market. In the electricity market, firstly, the market clearing process is performed in the day-ahead market and after that, a counter-trading mechanism is used to clear the network congestion. The system may have any configuration, either radial or meshed, and there is not restriction on the size of the system. The main contribution of the model is that the market equilibrium equations incorporates the effect of congestion between multiple areas on the agents' strategic behavior. Furthermore, the market equilibrium equations are solved using an equivalent optimization problem. The optimization problem has two levels. The first level corresponds to the day-ahead market and the second level is a DC optimal power flow that solves the network congestion. Numerical results are provided to illustrate the performance of the proposed approach.

Index Terms: Conjectured-price response, counter-trading, electricity market, market equilibrium, network congestion.

Appendix D

Article 3 - Analysis of the effect of voltage level requirements on an electricity market equilibrium model

Analysis of the effect of voltage level requirements on an electricity market equilibrium model

Andrés Delgadillo[a,*], Javier Reneses[a]

[a] *Institute for Research in Technology (IIT), Technical School of Engineering (ICAI). Universidad Pontificia Comillas, Madrid E-28015, Spain*

Abstract

This paper presents a conjectural-variation-based equilibrium model of a single-price electricity market. The main characteristic of the model is that the market equilibrium equations incorporate the effect of the voltage constraints on the companies' strategic behavior. A two-stage optimization model is used to solve the market equilibrium. In the first stage, an equivalent optimization problem is used to compute the day-ahead market clearing process. In the second stage, some generation units have to modify their active and reactive power in order to meet the technical constraints of the transmission network. These generation changes are determined by computing an AC optimal power flow.

Keywords: Voltage constraints, equilibrium model, conjectural variations, electricity market

1. Introduction

Deregulation in electric power systems has been conducted using different processes in the past decades in several countries. Electric power systems have gone from being centralized and vertically integrated to systems with different degrees of competition in their different activities. In the generation activity, electricity markets were created to determine the amount of energy scheduled of the generation units, as well as the ancillary services that they should provide in order to maintain system stability.

*Corresponding author

Email addresses: `andres.delgadillo@iit.upcomillas.es` (Andrés Delgadillo), `javier.reneses@iit.upcomillas.es` (Javier Reneses)

Several models have been developed to study electricity markets. Usually, these models are based on game theory and they try to determine the outcome of the interaction between different companies under the hypothesis of rational behavior. The companies' behavior is modeled using a strategic game where companies take an action knowing that the rest of companies play in the same way. Among the game theory models are Perfect Competitive models [1], [2], Cournot models where companies compete in quantities [3]-[8], Bertrand models where companies compete in prices [3], Supply Function Equilibrium models where strategic behavior is modeled by means of supply functions that combine price and quantity competition [9]-[16], and Conjetural Variation Based Equilibrium models where the supply functions are parametrized with a parameter known as the company's conjecture [11], [17]-[25].

Most of these models have focused on solving the day-ahead electricity market and they disregard ancillary service markets and mechanisms used to clear the different technical constraints that may appear on the electric power system. Some models include the effect of network congestion on the companies' strategic behavior [1], [3]-[6], [12]-[16], [19]-[23]. However, they only study the congestion caused by the thermal limits of the transmission lines. Therefore, they use a DC approximation of the power flow equations, and it is not possible to analyze other technical constraints such as voltage constraints or reactive power requirements.

Few models [2], [7]-[11], [17] study the effect of voltage constraints on the companies' strategic behavior. However, all of them are focused on nodal-price electricity markets, and none of them assess the effect on single-price electricity markets. Almeida and Senna [2] proposed a bilevel optimization problem that models the active and reactive power dispatch under competence. The first level corresponds to the active power market and the second level minimizes the opportunity cost of the reactive power which is defined in terms of the marginal price of the power active market. Bautista et al. [7, 8] presented a Cournot model to study the influence of the reactive power requirements on the active power dispatch. These works argue that the DC approximation of the power flow is not accurate enough because it does not take into account the capability curve of the generation units that models the tradeoff between active and reactive power. Bautista et al. [9] was an extension of the previous approaches using a supply function equilibrium model. Soleymani [10] developed a supply function equilibrium model for optimal bidding strategy of generation companies in active and reactive power markets, where the companies have incomplete information about their rivals. Petoussis et al. [11] assessed different parametrization methods of the companies' supply functions in an active power market taking into account an AC representation of the network. Chitkara et al. [17] proposed a model to analyze the companies' strategic behavior in a reactive power market. This model assumes that the active power is already scheduled, thereby there is no feedback between the reactive and active power markets, i.e., reactive power requirements do not modify strategic behavior in the active power market.

This paper presents a conjectural-variation-based model of a single-price electricity market. The main characteristic of this model is that the companies' strategic behavior takes into account the effect of the voltage constraints. The market equilibrium equations are solved by means of a two-stage optimization problem. In the first stage, a minimization problem models the day-ahead market clearing process. In the second stage, an optimal power flow is solved to determine the changes in active and reactive power needed to meet the voltage system requirements. Moreover, this paper presents an iterative algorithm to resolve the two-stage optimization problem. This model is based on the model proposed in [23]. The main difference between the two models is that the model in [23] only analyzes the effect of network congestion caused by the thermal limits of the transmission lines. Thus, the model in [23] uses a DC-OPF which assumes that there is enough reactive power compensation in all nodes to maintain voltage at the desired level, so the terms related to reactive power are discarded and the voltage levels are equal to 1 p.u. in all nodes. However, this DC approximation is not suitable to study the effect of the voltage level requirements because it is not possible to assume that voltage levels are constant in all nodes. Hence, the model presented in this paper uses an AC-OPF to properly model the voltage requirements.

The remainder of this paper is organized as follows: Section 2 presents the market equilibrium model that includes the effect of the voltage constraints on the companies' strategic behavior. Section 3 provides and analyzes a numerical example. Finally, Section 4 draws the most relevant conclusions.

2. Market Equilibrium Model

This section generalizes the model presented in [23] in order to study the effect of voltage constraints on the companies' strategic behavior in a single-price electricity market. In the electricity market, the scheduled day-ahead generation is usually determined first. Then, a subsequent procedure is carried on if the day-ahead market solution does not meet the technical requirements necessary to maintain system stability. Different technical constraints are assessed and the power produced by units may change with respect to the scheduled day-ahead generation.

2.1. Market clearing conditions

The day-ahead market clearing process determines the active power P_j of each generation unit j as well as the market price λ. Since it is a single-price electricity market, the total generation and demand have to be balanced (1) and the market price λ is equal to the bid of the marginal unit:

$$\sum_{j \in J} P_j = \sum_{a \in A} DP_a + losses. \tag{1}$$

Subsequently, the changes in production necessary to maintain system stability are determined using a mechanism to solve the technical constraints. There are different schemes to remunerate the power active changes as presented in [26]-[29]. In this paper, the Spanish mechanism [26] is modeled in which the power active increments X_j are paid at the price γ while the reductions W_j are charged at the day-ahead market price λ. In order to maintain the system active power balance, the total active power increment is equal to the total active power reduction:

$$\sum_{j \in J} X_j = \sum_{j \in J} W_j. \tag{2}$$

2.2. The company's problem

A generation company i will try to maximize its profit by determining the production of its generation units, P_j, as well as the production changes, X_j and W_j, required to meet the technical system constraints. Moreover, since the generation company behaves strategically, it can change the electricity prices when the production of its units changes. This strategic behavior could be modeled by means of the parameters θ_i and β_i. θ_i corresponds to the conjectured-price response in the day-ahead market [18] and β_i to the conjectured-price response in the subsequent mechanism.

Since the reductions are charged at the day-ahead market price, it is possible to represent the quantity reduced W_j as a ratio of the day-ahead market production P_j, i.e., $W_j = m_j \cdot P_j$, where m_j represents the proportion of the active power generation that unit j has to reduce in order to meet the network constraints. Thus, the value of m_j has to be computed taking into account the power flow constraints.

Therefore, the profit maximization problem of company i is:

$$\max_{\lambda_i, \gamma_i, P_j, X_j} \lambda_i \cdot \sum_{j \in J_i} (1 - m_j) \cdot P_j + \gamma_i \cdot \sum_{j \in J_i} X_j - \sum_{j \in J_i} C\left((1 - m_j) \cdot P_j + X_j\right) \tag{3}$$

s.t.

$$\lambda_i = \lambda^* - \theta_i \cdot \left(\sum_{j \in J_i} P_j - \sum_{j \in J_i} P_j^* \right) \tag{4}$$

$$\gamma_i = \gamma^* - \beta_i \cdot \left(\sum_{j \in J_i} X_j - \sum_{j \in J_i} X_j^* \right) \tag{5}$$

$$\overline{P_j} - P_j \geq 0 \qquad : (\overline{\mu_j}) \qquad \forall j \tag{6}$$

$$\overline{P_j} \cdot w_j - X_j \geq 0 \qquad : (\overline{\nu_j}) \qquad \forall j \tag{7}$$

$$\overline{P_j} - P_j - X_j \geq 0 \qquad : (\overline{\xi_j}) \qquad \forall j \tag{8}$$

$$P_j \geq 0, \quad X_j \geq 0 \qquad \forall j \tag{9}$$

In the event that the scheduled active power determined in the day-ahead market does not meet the technical system constraints, the units' generation has to be modified in the subsequent mechanism. Assuming that these modifications happen on a regular basis, the companies can predict them, and may use this information to behave strategically. Thus, in the company's optimization problem, this information is modeled using the reduction factors, m_j, and the binary variables, w_j, that indicate which units have to increase generation. Both are determined in the subsequent mechanism as shown in section 2.4. The equation (3) is the profit of the company i. Constraints (4) and (5) represent how the company conjectures that electricity prices will change if the company changes its production. Each company i has an estimation of the prices λ_i and γ_i. However, in the equilibrium these prices are equal to the day-ahead market price, λ^*, and the price of the active power increments, γ^*, respectively. Constraint (4) is the conjecture for the day-ahead market price, in which the company assumes that λ_i deviates from λ^* if the company's active power generations P_j change from their equilibrium values P_j^*. Constraint (5) is the conjecture for the subsequent mechanism, showing how γ_i changes from γ^* when the increments of active power of the generation units, X_j, are shifted from X_j^*. Constraints (6)-(9) are the boundaries of the variables.

2.3. Market equilibrium

By gathering together the first-order conditions for all companies and then adding the market-clearing conditions, the mixed complementarity model MCP (10)-(16) can be defined, and the market equilibrium corresponds to the solution of this MCP. An alternative way to compute this market equilibrium is by means of an equivalent quadratic optimization problem as shown in [23]. However, that methodology was not successful in solving this problem because the power balance constraints have to be modified in each iteration and the convergence of the procedure is not guaranteed.

$$\sum_{j \in J} P_j = DP \qquad \lambda \text{ unrestricted} \tag{10}$$

$$\sum_{j \in J} X_j = Y \qquad (\gamma \text{ unrestricted}) \tag{11}$$

$$0 \leq \overline{\mu_j} \perp \overline{P_j} - P_j^* \geq 0 \qquad \forall j \in J_i,\ \forall i \in I \tag{12}$$

$$0 \leq \overline{\nu_j} \perp \overline{P_j} \cdot w_j - X_j^* \geq 0 \qquad \forall j \in J_i,\ \forall i \in I \tag{13}$$

$$0 \leq \overline{\xi_j} \perp \overline{P_j} - P_j^* - X_j^* \geq 0 \qquad \forall j \in J_i,\ \forall i \in I \tag{14}$$

$$\begin{aligned} 0 \leq P_j^* \perp & -(1-m_j) \cdot \lambda^* + \theta_i \cdot \sum_{k \in J_i} (1-m_k) \cdot P_k^* + \\ & (1-m_j) \cdot MC_j\left((1-m_j) \cdot P_j^* + X_j^*\right) + \overline{\mu_j} + \overline{\xi_j} \geq 0 \qquad \forall j \in J_i,\ \forall i \in I \end{aligned} \tag{15}$$

$$\begin{aligned} 0 \leq X_j^* \perp & -\gamma^* + \beta_i \cdot \sum_{k \in J_i} X_k^* + MC_j\left((1-m_j) \cdot P_j^* + X_j^*\right) + \\ & \overline{\nu_j} + \overline{\xi_j} \geq 0 \qquad \forall j \in J_i,\ \forall i \in I \end{aligned} \tag{16}$$

Equations (10)-(11) are the market-clearing constraints. The values of DP and Y are the total active power demand and the total active power increments, and they are computed using an iterative procedure as presented in section 2.5. The generation company's behavior is modeled by means of equations (12)-(16). These equations are the Karush-Kuhn-Tucker (KKT) conditions of the problem (3)-(9) for each company i. It is important to note that the variables λ_i and γ_i are substituted by constraints (4) and (5), respectively. Moreover, since the solution of the MCP corresponds to the equilibrium, the production variables P_j and X_j are replaced by the equilibrium variables P_j^* and X_j^*, respectively. The operator $\perp$ denotes the inner product of two vectors equal to zero, i.e., $0 \leq x \perp f(x) \geq 0$ corresponds to the system equations $x \geq 0$, $f(x) \geq 0$ and $x \cdot f(x) = 0$.

For units whose productions P_j^* and X_j^* are less than the maximum values, i.e., constraints (6), (7) and (8) are not binding and the dual variables $\overline{\mu_j}$, $\overline{\nu_j}$ and $\overline{\xi_j}$ are equal to zero, the equations (15) and (16) could be written as:

$$\lambda^* = MC_j\left(\left(1-m_j\right)\cdot P_j^* + X_j^*\right) + \frac{\theta_i}{\left(1-m_j\right)}\cdot\sum_{k\in J_i}\left(1-m_k\right)\cdot P_k^* \tag{17}$$

$$\gamma^* = MC_j\left(\left(1-m_j\right)\cdot P_j^* + X_j^*\right) + \beta_i\cdot\sum_{k\in J_i} X_k^*. \tag{18}$$

The right-hand side on (17) and (18) corresponds to the *apparent cost* of the unit in the day-ahead market and in the subsequent mechanism, respectively. The *apparent cost* is defined as the equivalent marginal cost perceived by the system when the unit produces a determined quantity in the market [30]. In the apparent cost perceived by the company in the day-ahead market, the conjectured-price response is modified by factor $1/\left(1-m_j\right)$ which is greater than 1 when $m_j > 0$. This means that the company perceives that this unit is more expensive in the day-ahead market because it knows that the active power of the unit has to be reduced in the subsequent mechanism in order to meet the technical system constraints.

2.4. Subsequent mechanism

In single-price electricity markets, a procedure is used to clear the technical system constraints when the day-ahead market solution is not technically feasible. This procedure is subsequent to the day-ahead market and determines the changes in active power as well as reactive power needed to maintain system stability. With those results, the companies can determine the reduction factors m_j and which units increase active power generation ($w_j = 1$). The optimal power flow (19)-(29) models this procedure. In [23], the OPF is solved using a DC approximation in which the voltage levels are fixed to 1 p.u. and the reactive power and system losses are disregarded. The DC approximation is valid to analyze the effect of congestion due to thermal limits of the lines. However, in order to study the effect of voltage requirements it is necessary to use an AC-OPF where the voltage levels are not fixed and the active and reactive power levels are taken into account.

$$\min_{\Xi} \sum_{j \in J} ACX_j \cdot X_j^{\Omega} + (K - ACW_j) \cdot W_j^{\Omega} \tag{19}$$

s.t.

$$\sum_{j \in J_a} P_j^{\Omega} - DP_a = \sum_{b \in A} V_a \cdot V_b \cdot \Big(G_{ab} \cos(\delta_a - \delta_b) + B_{ab} \sin(\delta_a - \delta_b) \Big) \ \forall a \in A \tag{20}$$

$$\sum_{j \in J_a} Q_j^{\Omega} - DQ_a = \sum_{b \in A} V_a \cdot V_b \cdot \Big(G_{ab} \sin(\delta_a - \delta_b) - B_{ab} \cos(\delta_a - \delta_b) \Big) \ \forall a \in A \tag{21}$$

$$P_j^{\Omega} = P_j + X_j^{\Omega} - W_j^{\Omega} \qquad \forall j \in J \tag{22}$$

$$\underline{V_a} \leq V_a \leq \overline{V_a} \qquad \forall a \in A \tag{23}$$

$$0 \leq X_j^{\Omega} \leq \overline{P_j} \qquad \forall j \in J \tag{24}$$

$$0 \leq W_j^{\Omega} \leq \overline{P_j} \qquad \forall j \in J \tag{25}$$

$$\underline{P_j} \cdot u_j \leq P_j^{\Omega} \leq \overline{P_j} \cdot u_j \qquad \forall j \in J \tag{26}$$

$$\underline{Q_j} \cdot u_j \leq Q_j^{\Omega} \leq \overline{Q_j} \cdot u_j \qquad \forall j \in J \tag{27}$$

$$Q_j^{\Omega} \leq Q_j^{0,max} \cdot u_j + n_j^{max} \cdot P_j^{\Omega} \qquad \forall j \in J \tag{28}$$

$$Q_j^{\Omega} \geq Q_j^{0,min} \cdot u_j + n_j^{min} \cdot P_j^{\Omega} \qquad \forall j \in J \tag{29}$$

where the decision variables are $\Xi = \left\{ P_j^{\Omega}, X_j^{\Omega}, W_j^{\Omega}, V_a, \delta_a, u_j \right\}$. The objective function (19) minimizes the *apparent cost* of the changes in active power with respect to the day-ahead market solution. The apparent cost is used because it corresponds to an equivalent marginal cost perceived by the system. In the model, two different apparent costs ACX_j and ACW_j have been considered. The apparent cost ACX_j corresponds to the cost when the generation unit j has to increase its active power while ACW_j corresponds to the cost when it has to reduce its active power. Therefore, in the minimization problem, the units with lower apparent cost ACX_j increase their active power while the units with higher apparent cost ACW_j reduce their active power production. The term K is a constant higher than the maximum value of ACW_j. Constraints (20) and (21) are the power flow equations for active and reactive power, respectively. Constraints (22) relate the active power in the OPF with the active power in the day-ahead market.

Constraints (23)-(29) establish the minimum and maximum bounds of the variables. A linear approximation of the P-Q capability curve of the generation units, known as *D-curve*, is modeled with constraints (26)-(29). This curve models the trade-off between active and reactive power of the generation units, and therefore it determines the feasible operation region where it is not possible to produce the maximum active power and maximum reactive power at the same time as illustrated in Fig. 1, where the shaded portion represents the feasible operating region for the unit. It is important to note that the binary variables u_j are necessary to meet the minimum and maximum requirements of the generation units, and to avoid solutions in which the active power of a unit is below the minimum to generate more reactive power. The problem with those binary variables is that the software used, Matpower, cannot compute binary variables in the solution of the OPF. The approach taken was therefore to evaluate all of the possible combinations of these binary variables and select the case with the lowest value in the objective function. However, that methodology reduces the size of the models that can be studied. Another inconvenience is that the algorithm used to find the optimal solution could converge to a local optimum depending on the initial values of the variables, so in the iterative procedure used in this model different solutions could be found and there is no certainty about the convergence of the model. In practice, the solution methodology has achieved satisfactory results in terms of convergence as shown in [23].

Figure 1: P-Q Capability curve

2.5. Solution methodology

An iterative algorithm similar to the one presented in [23] is used to determine the market equilibrium taking into account the power changes required to meet the technical constraints:

1. Initialize the variables $\kappa = 1$, $m_j^{(\kappa)} = 0$, $w_j^{(\kappa)} = 0$, $DP^{(\kappa)} = \sum_a DP_a$, $Y^{(\kappa)} = 0$. These values correspond to the case without network constraints.
2. Solve the MCP (10)-(16). This gives a solution for P_j^*, X_j^*, λ^*, γ^*.

3. Update the active power, prices and apparent cost values:

$$P_j^{(\kappa)} = \alpha \cdot P_j^* + (1-\alpha) \cdot P_j^{(\kappa-1)} \tag{30}$$

$$X_j^{(\kappa)} = \alpha \cdot X_j^* + (1-\alpha) \cdot X_j^{(\kappa-1)} \tag{31}$$

$$\lambda^{(\kappa)} = \alpha \cdot \lambda^* + (1-\alpha) \cdot \lambda^{(\kappa-1)} \tag{32}$$

$$\gamma^{(\kappa)} = \alpha \cdot \gamma^* + (1-\alpha) \cdot \gamma^{(\kappa-1)} \tag{33}$$

$$ACX_j^{(\kappa)} = MC\left(\left(1 - m_j^{(\kappa)}\right) \cdot P_j^{(\kappa)} + X_j^{(\kappa)}\right) + \beta_i \cdot \sum_{k \in J_i} X_k^{(\kappa)} \tag{34}$$

$$ACW_j^{(\kappa)} = MC\left(\left(1 - m_j^{(\kappa)}\right) \cdot P_j^{(\kappa)} + X_j^{(\kappa)}\right) + \frac{\theta_i}{\left(1 - m_j^{(\kappa)}\right)} \cdot \sum_{k \in J_i} \left(1 - m_k^{(\kappa)}\right) \cdot P_k^{(\kappa)} \tag{35}$$

the learning rate α is used to achieve a smooth convergence in the value of the variables, and to prevent the solution from jumping between different values. A value of $\alpha = 1$ means that the variables are updated using only the information given in the last iteration while a value of $\alpha = 0$ means that only the information given in the first iteration is used.

4. Solve the AC-OPF (19)-(29). This gives a solution for P_j^{Ω}, X_j^{Ω}, W_j^{Ω}, V_a, δ_a, u_j.

5. Update the reduction factor, the units that increase their generation and the

demand values:

$$P_j^{\Omega(\kappa)} = \alpha \cdot P_j^{\Omega} + (1-\alpha) \cdot P_j^{\Omega(\kappa-1)} \tag{36}$$

$$X_j^{\Omega(\kappa)} = \alpha \cdot X_j^{\Omega} + (1-\alpha) \cdot X_j^{\Omega(\kappa-1)} \tag{37}$$

$$W_j^{\Omega(\kappa)} = \alpha \cdot W_j^{\Omega} + (1-\alpha) \cdot W_j^{\Omega(\kappa-1)} \tag{38}$$

$$m_j^{(\kappa)} = \alpha \cdot \frac{W_j^{\Omega}}{P_j^{\Omega}} + (1-\alpha) \cdot m_j^{(\kappa-1)} \tag{39}$$

$$w_j^{(\kappa)} = \begin{cases} 1 \text{ if } X_j^{\Omega(\kappa)} > 0 \\ 0 \text{ if } X_j^{\Omega(\kappa)} = 0 \end{cases} \tag{40}$$

$$DP^{(\kappa)} = \sum_{j \in J} P_j^{\Omega(\kappa)} \tag{41}$$

$$Y^{(\kappa)} = \sum_{j \in J} W_j^{\Omega(\kappa)} \tag{42}$$

6. If the change of the variables is lower than an ϵ value, the algorithm stops; otherwise increase the iteration counter κ and go to 2.

3. Numerical Example

This section presents a simple example to study the effect of voltage constraints on the companies' strategic behavior. The market equilibrium is solved using PATH [31] in GAMS [32] and the AC-OPF is solved using MATPOWER [33] in Matlab [34].

The power network has 3 nodes connected by 3 transmission lines as shown in Fig. 2 and Table 1. It is important to note that the values of the parameters of the transmission lines are significantly higher than the actual parameters, in order to highlight the effect of voltage requirements. The demand is equal to 100 MW and 35 Mvar and it is concentrated at node 3. The three nodes have generation units; however, the units located at node 3 are the most expensive. Thus, the day-ahead market solution is that units at nodes 1 and 2 have to supply the demand at node 3. If this was the final solution, there would be a significant voltage drop in the lines 1-3 and 2-3 caused by the impedance of these lines. In that case, the voltage level at node 3 would be lower than the specified minimum (0.95 p.u.).

Figure 2: Three-node system

Table 1: Parameters of the lines

Three different cases are analyzed. In case A, companies 1 and 2 own generation units at nodes 1 and 2, and there is only one unit at node 3 owned by company 3. Therefore, this unit is the only one that can solve the voltage drop at node 3. In case B, there are the same units as case A, but company 1 also has one generation unit at node 3, so there are now 2 units that can solve the voltage requirements. Finally in case C, the three companies have generation units at node 3. In the three cases, the strategic behavior of company 3 is studied modifying its conjectured-price response in the subsequent mechanism, β, from the case in which the company does not exercise market power, i.e, $\beta_3 = 0$, and increasing the market power to $\beta_3 = 0.1$ and $\beta_3 = 1$. The data of generation units and the conjectured-price response of the companies are shown in Tables 2 and 3, respectively.

Table 2: Generation Units

Table 3: Conjectured-price response

3.1. case A

If the generation companies do not take into account the voltage level requirements in their bids to the day-ahead market then the generation units 1 and 4 at areas 1 and 2 are dispatched. However, in that solution, the voltage level at area 3 is only 0.82 p.u. and this value is below the required minimum of 0.95 as shown in Table 4.

Table 4: Voltage levels

Companies 1 and 2 do not modify their strategic behavior in the day-ahead market because they do not own any unit at area 3 to meet the voltage requirements. Hence, the final solution in the day-ahead market is not modified. On the other hand, unit 5 owned by company 3 is the only unit that can resolve the voltage constraint at area 3. This unit has to generate the maximum reactive power in order to reach the voltage level of 0.95 at area 3, and its active power generation is equal to the minimum given its P-Q capability curve. The active power increased by this unit in the subsequent mechanism is compensated by a reduction in the active power of unit 1 as shown in Table 5.

Table 5: Power Solution

3.2. case B

In this case, unit 6 at area 3 is dispatched to the maximum of its active power in the initial day-ahead market. However, this unit cannot generate the reactive power necessary to maintain the voltage level at area 3 due to its capability curve (Table 6), and therefore, as in the previous case, unit 5 is necessary in the subsequent mechanism.

Table 6: Voltage levels

Unlike case A, in which company 1 does not modify its strategic behavior in the day-ahead market, company 1 foresees that the active power generation of unit 6 has to be at the minimum while the reactive power generation has to be at the maximum for maintaining the voltage level at area 3. This makes the reduction factor $m_6 > 0$, and therefore its *apparent cost* increases in the day-ahead market as explained in section (2.3). Thus, a higher *apparent cost* of this unit results in a change in the strategic behavior of company 1 in the day-ahead market generating only 9.3 MW with unit 6 (Table 7). If the company does not change the active power generation of its units, the apparent cost of this units decreases, and therefore the day-ahead market price also decreases. However, that is not a good strategy because the company knows that the active power generation of unit 6 has to be reduced in the subsequent mechanism. On the other hand, unit 5 is dispatched in the subsequent mechanism to the minimum active power and the maximum reactive power to reach a voltage level equal to 0.95 p.u. at area 3.

Table 7: Power Solution

3.3. case C

The initial day-ahead market solution of this case is the same as the initial solution to case B. Thus, another generation unit at area 3 is required to maintain the voltage level (Table 8).

Table 8: Voltage levels

The final result in the day-ahead market is exactly the same as in case B. This means that the strategic behavior of company 1 in the day-ahead market is not altered by the new power unit at area 3. Nevertheless, the outcome of the subsequent mechanism is modified depending on the strategic behavior of company 3. In cases A and B, company 3 could exercise market power because its unit was the only one that could resolve the voltage constraint. However, in case C, company 2 also has a unit at area 3. Thus, the market power of company 3 is mitigated, and the value of its conjectured-price response in the subsequent mechanism cannot be higher than 0.071 (€/MWh)/MW because a higher value would cause the apparent cost of unit 5 be greater than the apparent cost of unit 7. Table 9 shows how unit 5 is dispatched in the subsequent mechanism when $\beta_3 = 0$ while unit 7 is dispatched when $\beta_3 = 0.1$ and $\beta_3 = 1$.

Table 9: Power Solution

3.4. Prices

In the results above, the day-ahead market generation is affected by the voltage constraints at area 3 in cases B and C. These changes occur because the *apparent cost* of the units is modified by the reduction factor m_j. However, the changes in the day-ahead market price, λ, are not significant, and they are only equal to 0.04 €/MWh between the initial and the final solution.

On the other hand, the price in the subsequent mechanism γ may be modified by the market power of the companies which have the generation units necessary to maintain the voltage levels. In cases A and B, unit 5 of company 3 is indispensable to resolve the voltage constraints, and therefore this company has a high market power in the subsequent mechanism. On the contrary, in case C, the market power of company 3 is limited by unit 7 of company 2. Hence, company 3 cannot make bid prices of unit 5 above the bids of unit 7 in order to be dispatched. Moreover, the price γ in case C when β_3 is higher than 0.071 (€/MWh)/MW is equal to the variable cost of unit 7 as shown in Table 10.

Table 10: Prices

4. Conclusions

This paper has studied the effect of voltage requirements on companies' strategic behavior in a single-price electricity market. The market equilibrium equations take into account the solution of the mechanism used to clear the technical constraints which is modeled by means of an AC optimal power flow. One of the contributions of this paper is that the technical constraints are not limited only to congestion due to the thermal constraints of the transmission lines, but the model can also analyze other technical constraints such as voltage levels or reactive power requirements.

The results of the numerical example show how one company may exercise market power in the mechanism used to clear the technical requirements if it is the only company that can resolve the voltage constraints. Also, it has been shown how this market power is mitigated as more companies are able to resolve this technical constraint.

5. List of symbols

5.1. Indices

a	Node index
b	Node index
i	Company index
j	Production unit index

k	Production unit index
κ	Iteration counter index
Ω	Optimal power flow

5.2. Sets

A	Set of indices of nodes
I	Set of indices of companies
J	Set of indices of production units
J_a	Set of indices of production units connected to node a
J_i	Set of indices of production units owned by company i
Ξ	Set of optimization variables in the OPF

5.3. Constants

n_j^{max}	Parameter of the capability curve of unit j
n_j^{min}	Parameter of the capability curve of unit j
DP_a	Active power demand at area a [MW]
DQ_a	Reactive power demand at area a [Mvar]
G_{ab}	Element of the conductance matrix [p.u.]
$\underline{P_j}$	Minimum active power generation of unit j [MW]
$\overline{P}_j$	Maximum active power generation of unit j [MW]
$\underline{Q_j}$	Minimum reactive power generation of unit j [Mvar]
$\overline{Q}_j$	Maximum reactive power generation of unit j [Mvar]
$Q_j^{0,max}$	Parameter of the capability curve of unit j [Mvar]
$Q_j^{0,min}$	Parameter of the capability curve of unit j [Mvar]
α	Learning rate (can take values in the interval [0,1])

β_i	Conjectured-price response of company i in the subsequent market [(€/MWh)/MW]
ϵ	Level of solution accuracy
θ_i	Conjectured-price response of company i in the day-ahead market [(€/MWh)/MW]

5.4. Variables

m_j	Reduction factor of production unit j
u_j	Commitment variable of unit j
w_j	Binary variable that is equal to 1 if the unit j increments its production and 0 otherwise
ACW_j	Apparent cost of unit j [€/MWh]
ACX_j	Apparent cost of unit j [€/MWh]
C_j	Production cost of unit j [€]
DP	Total active power demand [MW]
MC_j	Marginal cost of unit j [€/MWh]
P_j^*	Equilibrium value of the active power generation of unit j [MW]
P_j^Ω	Active power generation of unit j in the optimal power flow [MW]
Q_j^Ω	Reactive power generation of unit j in the optimal power flow [MW]
V_a	Voltage magnitude at node a [p.u.]
W_j^Ω	Decrement in the production of unit j in the optimal power flow [MW]
X_j^*	Equilibrium value of the increment in the production of unit j [MW]
X_j^Ω	Increment in the production of unit j in the optimal power flow [MW]
Y	Total active power increment [MW]
γ^*	Equilibrium price of the active power increments [€/MWh]
γ_i	Estimation of the price of the active power increments made by agent i [€/MWh]
δ_a	Phase angle at area a [rad]
λ^*	Day-ahead market equilibrium price [€/MWh]
λ_i	Estimation of the day-ahead market price made by agent i [€/MWh]

μ Dual variable

ν Dual variable

ξ Dual variable

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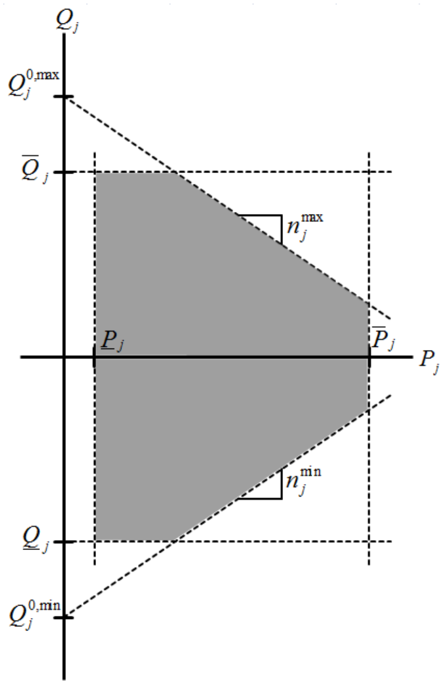

Figure 1: P-Q Capability curve

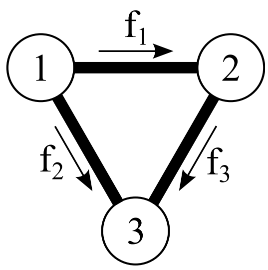

Figure 2: Three-node system

Table 1: Parameters of the lines

From Node	To Node	Resistance [p.u.]	Reactance [p.u.]	Susceptance [p.u.]
1	2	0.12	0.35	0.01
1	3	0.24	0.70	0.01
2	3	0.24	0.70	0.01

The parameters are in the base of 100 MVA

Table 2: Generation Units

Unit j	Case	Node a	Company i	Variable Cost [€/MWh]	$\underline{P}$ [MW]	$\overline{P}$ [MW]	$\underline{Q}$ [Mvar]	$\overline{Q}$ [Mvar]	$Q^{0,max}$ [Mvar]	$Q^{0,min}$ [Mvar]	n^{max}	n^{min}
1	A, B, C	1	1	40.5	7	70	-40	40	56.4	-56.4	-0.643	0.643
2	A, B, C	2	1	42.0	7	70	-40	40	56.4	-56.4	-0.643	0.643
3	A, B, C	1	2	42.0	7	70	-40	40	56.4	-56.4	-0.643	0.643
4	A, B, C	2	2	40.0	7	70	-40	40	56.4	-56.4	-0.643	0.643
5	A	3	3	43.5	14	70	-58	58	72	-72	-1	1
5	B, C	3	3	43.5	7	35	-29	29	36	-36	-1	1
6	B, C	3	1	40.0	7	35	-29	29	36	-36	-1	1
7	C	3	2	44.0	7	35	-29	29	36	-36	-1	1

Table 3: Conjectured-price response

Company i	θ [(€/MWh)/MW]	β [(€/MWh)/MW]
1	0.05	0.1
2	0.05	0
3	0	0 - 0.1 - 1

Table 4: Voltage levels

Node	First iteration	Last iteration
a	V [p.u]	V [p.u]
1	1.11	1.02
2	1.11	1.04
3	0.82	0.95

Table 5: Power Solution

	Day-ahead market		Subsequent mechanism	
Unit	First iteration	Last iteration	Last iteration	
j	P [MW]	P [MW]	P [MW]	Q [Mvar]
1	50.3	50.3	36.3	1.1
2	-	-	-	-
3	-	-	-	-
4	60.4	60.4	60.4	4.1
5	-	-	14	58

Table 6: Voltage levels

Node	First iteration	Last iteration
a	V [p.u]	V [p.u]
1	1.05	1.01
2	1.04	1.05
3	0.77	0.95

Table 7: Power Solution

	Day-ahead market		Subsequent mechanism	
Unit	First iteration	Last iteration	Last iteration	
j	P [MW]	P [MW]	P [MW]	Q [Mvar]
1	15.3	41.8	37.1	-8.1
2	-	-	-	-
3	-	-	-	-
4	60.4	59.6	59.6	13.5
5	-	-	7	29
6	35	9.3	7	29

Table 8: Voltage levels

Node	First iteration	Last iteration
a	V [p.u]	V [p.u]
1	1.05	1.01
2	1.04	1.05
3	0.77	0.95

Table 9: Power Solution

		Day-ahead market		Subsequent mechanism	
	Unit	First iteration	Last iteration	Last iteration	
	j	P [MW]	P [MW]	P [MW]	Q [Mvar]
	1	15.3	41.8	37.1	-8.1
	2	-	-	-	-
	3	-	-	-	-
	4	60.4	59.6	59.6	13.5
	6	35	9.3	7	29
(a)	5	-	-	7	29
	7	-	-	-	-
(b)	5	-	-	-	-
	7	-	-	7	29

(a) Solution for $\beta_3 = 0$

(b) Solution for $\beta_3 = 0.1$ and $\beta_3 = 1$

Table 10: Prices

	λ [€/MWh]						γ [€/MWh]		
	First iteration			Last iteration			Last iteration		
β_3	Case A	Case B	Case C	Case A	Case B	Case C	Case A	Case B	Case C
0	43.02	43.02	43.02	43.02	42.98	42.98	43.5	43.5	43.5
0.1	43.02	43.02	43.02	43.02	42.98	42.98	44.9	44.2	44.0
1	43.02	43.02	43.02	43.02	43.98	42.98	57.5	50.5	44.0

Appendix E

Article 4 - Counter-trading mechanisms in Europe. The cases of Spain, the Netherlands, Portugal and Germany

Counter-trading mechanisms in Europe. The cases of Spain, the Netherlands, Portugal and Germany

Andrés Delgadillo[a,*], Javier Reneses[a]

[a]*Institute for Research in Technology (IIT), Technical School of Engineering (ICAI). Universidad Pontificia Comillas, Madrid E-28015, Spain*

Abstract

In electricity markets, different mechanisms are used to deal with congestion in the system. In Spain, the Netherlands, Portugal and Germany, the day-ahead electricity market is cleared without taking technical constraints into account, and subsequently a counter-trading mechanism is used to deal with congestion in the system. The counter-trading mechanism allows generation companies to behave strategically between markets since they may modify their bids to avoid them being dispatched in the day-ahead market, and to enable them to be dispatched in the counter-trading mechanism. In contrast, such behavior does not occur in a nodal-pricing system.

This paper presents a simple case in order to analyze the inefficiencies of the congestion management mechanisms implemented in Spain, the Netherlands, Portugal, and Germany, comparing them with a nodal-pricing system.

Keywords: Congestion management, Counter-trading mechanism, Spain, the Netherlands, Portugal, Germany.

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1. Introduction

In a number of European states, the electricity sector has been liberalized and deregulated. As a result, the electricity sector has changed from a centralized industry with government-controlled entities to a liberalized sector in which most of the

*Corresponding author

Email addresses: andres.delgadillo@iit.upcomillas.es (Andrés Delgadillo), javier.reneses@iit.upcomillas.es (Javier Reneses)

energy is traded in the wholesale electricity markets. In order to maintain the power system stability there are subsequent markets and mechanisms which are becoming more important due to the increasing influence of system constraints resulting from the emergence of distributed generation, the high penetration of renewable energy and the flexibility of demand with response capacity.

Network congestion is one of the system constraints that may alter the outcome of the electricity market. Congestion occurs when the operational or policy constraints of the transmission network are violated and it is therefore not possible to deliver all the energy from one node to another. Different mechanisms can be used to clear the congestion. The choice of a particular mechanism depends on the market design and the system size, i.e., it is highly dependent on regulatory models, and could thus be exposed to significant changes in a short time period. A comprehensive review of the literature on congestion management in different deregulated electricity markets can be found in Kumar et al. [1].

The most efficient way to determine electricity prices is the so called nodal-pricing system [2, 3, 4, 5, 6]. This market design explicitly takes into account the different technical constraints imposed by the power network and efficiently remunerates the costs of producing and transporting energy through the different nodes, which may lead to different electricity prices in each node of the system. Nevertheless, many countries have chosen to use a single-pricing system. In this market design, the electricity network is not taken into account in the market-clearing process. It is assumed that the energy is traded on a single node, and that the electricity price is the same for all areas in the system. The single-price electricity market design may be suitable when the transmission network is sufficiently robust and when there are no constraints to cause significant congestion in the system. However, internal congestion is a problem that has arisen in several markets with a single-pricing system. Therefore, in order to clear this congestion, an additional mechanism, usually known as counter-trading, is implemented subsequent to the market-clearing process. In this mechanism, necessary adjustments are made to remove congestion that has appeared in the system, and to reward the agents involved. In addition, explicit and implicit auctions have also been used to deal with congestion at the interconnections of multiple systems, which are usually interconnections between several countries [7]. A comparison of the different congestion management methods can be found in Knops et al. [8].

A power network may favor strategic behavior on the part of generators [9, 10, 11, 12]. For example, the generation companies may be able to benefit from deliberately congesting and isolating certain areas of the system, thus exercising market power. Pérez-Arriaga and Olmos [7] proposed different congestion management schemes for the integration of electricity markets in Europe. The methods are a simplified version of a nodal-pricing system and explicit capacity auctions. Glachant and Pignon [13] presented an analysis of the congestion management mechanisms used in the Nordic electricity market - NordPool. Nappu et al. [12] established a methodology for the identification of the generator most likely to behave strategically and so congest the system.

1.1. Counter-trading mechanism

Several European countries like Spain, Portugal, Germany, and the Netherlands use congestion management mechanisms to deal with internal congestion in the power system. In general terms, these mechanisms work as follows: generation companies make bids in the day-ahead electricity market and the market operator then performs the market-clearing process taking into account these bids but without considering technical network constraints. This process does not guarantee a technically feasible method of transmitting electricity because it may breach maximum flow constraints, resulting in system congestion. In such cases, the counter-trading mechanism is implemented in order to clear system congestion. The generation companies bid on the production that they can increase or reduce with respect to the day-ahead market. The Transmission System Operator (TSO) receives these bids and determines a solution that meets the network constraints. The increments are paid, and the reductions are charged at the prices determined in the counter-trading mechanism. Thus, in a single-price electricity market with a counter-trading mechanism, generation companies have to decide on their bids in both the day-ahead market and the counter-trading mechanism.

The idea behind the implementation of a counter-trading mechanism is:

1. To try to obtain the same final technical solution as that which exists in the nodal-pricing scheme
2. To do this in a less costly way than in the nodal-pricing scheme, since the generation located in the importing area does not receive a greater remuneration (only those generation units which participate in the counter-trading mechanism).

Assuming that generation companies know which generation units are necessary to clear congestion, they place more or less value on their generation units when there is system congestion and may behave strategically, which will lead to a different outcome in the day-ahead market. Generation companies may reduce their production in the exporting areas in order not to be penalized in the counter-trading mechanism. However, this effect does not guarantee an increase in production in the importing area in the day-ahead market. This is mainly because, as generation companies know that their production in the importing area is more valuable, there is an incentive for them to enter in the counter-trading mechanism, increasing their offer price in the day-ahead market. These effects are easily quantified in the model presented in Delgadillo et al. [14] and Delgadillo and Reneses [15]. This model is a medium-term conjectural variation-based equilibrium model, its main feature being that the market equilibrium equations include the effect of the counter-trading mechanism. This model therefore makes possible an analysis of how the strategic behavior of generation companies is modified when they foresee the state of system congestion.

Several studies have analyzed the inefficiencies of the counter-trading mechanisms used in different electricity markets. Bompard et al. [16] performed a comparative analysis of different schemes (England and Wales, Norway, Sweden, PJM and California) implemented to clear network congestion. Another comparative analysis between a single-pricing system and a nodal-pricing system for England and Wales was carried out by Green [2], who concluded that a nodal-pricing system may increase the total social welfare since the electricity market becomes less vulnerable to the market power of generation companies and gives the right investments signal. Neuhoff et al. [3] and Oggioni and Smeers [4] focused on the integration of the European electricity markets. Neuhoff et al. [3] analyzed the criteria to be met by the congestion management mechanism used in the integration of European electricity markets, and concluded that a nodal-pricing system is the optimal solution. Oggioni and Smeers [4] assessed different counter-trading mechanisms in different versions of the market coupling scheme. Their principle conclusion was that the integration of European markets may work well or may be inefficient depending on the zonal decomposition, and the degree of coordination in the counter-trading between Transmission System Operators. Holmberg and Lazarczyk [5] argued that a counter-trading mechanism is inefficient because it results in additional payments to producers in exporting areas. Dijk and Willems [6] and van Blijswijk and de Vries [17] assessed the counter-trading mechanism implemented in the Netherlands in 2011. Both studies gave different and contradictory results. Dijk and Willems [6] studied the entry and exit of power plants in the Dutch system. Their analysis showed that the counter-trading that takes place gives the wrong long-term signals, causing an over-entry in the exporting areas and an under-entry in the importing areas. On the other hand, van Blijswijk and de Vries [17] argued that the potential for companies to exercise market power by using the new mechanism is limited. However, this conclusion is based on the assumption that network congestion is not structural, and will only be temporary since it is expected that the Dutch TSO will make the necessary reinforcements to the system to mitigate any congestion.

To summarize, all these studies conclude that counter-trading mechanisms are inefficient because:

- Not all generation units can take part in the redispatch process.
- Generation companies have strong incentives to behave strategically between mar-

kets. In exporting areas, they may receive a payment even if they are charged for reducing the production in the counter-trading mechanism. In importing areas, they prefer not to be dispatched in the day-ahead market and expect to be required to participate in the counter-trading mechanism.

- The counter-trading mechanism gives the wrong investment signals for locating new plants.
- The day-ahead market price does not reflect the fact that some areas are more expensive than others.

Taking into account this scenario, the objective of this paper is to analyze the counter-trading mechanisms used in Spain, the Netherlands, Portugal and Germany. Furthermore, this work compares the market outcome of those mechanisms with a nodal-pricing system. Section 2 describes the mechanisms used in each country to clear system congestion. Section 3 describes the characteristics of the market equilibrium model used to assess the strategic behaviors of generation companies. Section 4 presents a case study which compares the solution provided by these counter-trading mechanisms with a nodal-pricing system. Finally, Section 5 draws appropriate conclusions.

2. Congestion management mechanisms

This paper focuses on the congestion management mechanism used in Spain [18, 19], the Netherlands [6, 17], Portugal [20], and Germany [21, 22, 23].

2.1. Spain

A congestion management mechanism is implemented subsequent to the outcome of the day-ahead market. The TSO performs a number of security analysis to identify congestion that may appear in the system, taking into account the total amount of electricity produced in the day-ahead market and security constraints. Furthermore, the TSO receives price and quantity bids made by the units that are able to increase or reduce their production with respect to the day-ahead market. When congestion is identified, some units have to increase or reduce their production. Increased production is determined by the TSO as the lower cost solution evaluating the bids made by generation companies. The increased quantity of electricity is paid by using the bid price made by the unit in the counter-trading mechanism. On the other hand, the quantity reduced depends on the Generator Shift Factor Dobson et al. [24]. This factor quantifies the change in the flow at the interconnection when the generation unit increases its production. First, the unit with the highest factor has to reduce its production, and this reduction then continues in the order given by the factor until the congestion disappears. When several units have the same factor, the reduction is proportional to their production. The units that reduce their production are charged at the day-ahead market price. This means that generation units in exporting areas with congestion are only paid for the quantity produced, while receiving nothing for the quantity withdrawn.

2.2. The Netherlands

In May 2011, a counter-trading mechanism was implemented to deal with system congestion. As in Spain, this mechanism is a corrective method in the sense that it is implemented after the electricity market has closed. In the electricity market, the generation companies are paid for their scheduled production. When congestion occurs, production must be reduced in the exporting areas and increased in the importing areas. To achieve this, there are two additional markets: a market to increase production and a market to reduce production. In the exporting areas, generation companies make bids of the price at which they are willing to reduce their production, and the TSO accepts the bids of the generation companies that are willing to pay more. In the importing areas, generation companies make bids of the price at which they are willing to increase their production, and TSO accepts the cheapest bids. In both cases, the TSO charges/pays generation companies on a pay-as-bid basis.

2.3. Portugal

As in Spain, generation companies make price and quantity bids for increasing or reducing their production with respect to the day-ahead market. The TSO performs a security analysis to identify network congestion that may appear in the system, taking into account the outcome of the day-ahead market. When congestion is identified, it determines the lowest cost solution that will resolve the problem. When a generation company has increased its production, the increased quantity is paid for at the minimum value between its bids made in the day-ahead market and the congestion management mechanism. On the other hand, when a generation company has reduced its production, the reduced quantity is charged at the maximum value between its bids made in the day-ahead market, the congestion management mechanism and 0.85 times the day-ahead market price.

2.4. Germany

Unlike in Spain, the Netherlands or Portugal, in Germany there is no real market to clear system congestion, i.e., generation companies do not make bids for increasing or reducing their production with respect to the day-ahead market. However, when there is a technical constraint, the TSOs are obliged and empowered to intervene in the electricity market to ensure the safe operation of the power system. Specifically, TSOs implement a cost-based redispatch when congestion occurs. This means that the units used to clear congestion are selected according to their marginal costs. The units that increase their production are paid their generation costs while the units that reduce their production receive the difference between the day-ahead market price and their marginal costs. In October 2012, the German regulator established a new procedure for calculating the marginal costs of units in order to establish a common and consistent mechanism [21].

3. Description of the model used for the study

In order to assess the different congestion management mechanisms used in Spain, the Netherlands, Portugal and Germany, this paper presents a model based on that proposed in Delgadillo et al. [14] and Delgadillo and Reneses [15]. The main feature of this model [14, 15] is that the market equilibrium equations take into account the effect of the counter-trading mechanism on the generation companies' behavior. In the model, different degrees of competition can be analyzed, from perfect competition to extreme oligopoly markets (such as Cournot Equilibrium). Moreover, the model includes two kinds of strategic behavior. The first is the ability to modify the electricity price; the second is the ability to behave strategically between markets.

The solution to the market equilibrium problem described above is found by using an iterative process. The iterative process works as follows: First, an equivalent optimization problem performs the clearing process of the day-ahead market. This problem computes the market equilibrium without taking into account network constraints [25, 26]. This solution does not guarantee that the flow over the interconnection is between the maximum values. The counter-trading mechanism is therefore simulated in order to determine the amount of electricity produced that clear congestion. A factor measures the difference between the productions found in the day-ahead market and the counter-trading mechanism. This factor explicitly modifies the strategic behavior of the generation units in the day-ahead market and it is therefore necessary to perform a new iteration that finds a new solution. The iterative process stops when the value of this factor converges.

Since the market equilibrium equations presented in [14, 15] simulate the mechanism used in Spain, some slight changes were carried out to adjust them to the mechanisms used in the Netherlands, Portugal and Germany, as shown below. Furthermore, unlike those in [14, 15], the market equilibrium equations do not take into account the market power in the day-ahead market, and only simulate strategic behavior between markets. The other difference between the model in this paper and the one in [14, 15] is the methodology used to determine the solution of the market equilibrium equations. An equivalent quadratic minimization problem is formulated in [14, 15] while in this paper the equilibrium equations are solved using a Mixed Complementarity Problem (MCP).

3.1. The model's structural assumptions

For the sake of clarity and simplicity, the formulation is based on the following modeling assumptions:

1. There are two areas, one exporting area (EX) and one importing area (IM),

interconnected by a flowgate with limited transfer capacity ($\overline{F_l}$), as shown in Fig. 1.

2. Each company i can own generation units in each area. Thus, P_i^{EX} and P_i^{IM} are the productions of company i in the exporting and importing areas, respectively.
3. In both areas, the demands (D^{EX} and D^{IM}) are inelastic.
4. Both areas belong to the same electricity system, in which the day-ahead market-clearing process does not take network constraints into account. Therefore, the day-ahead electricity market price λ is the same for both areas.
5. The result of the day-ahead electricity market is that the total production in the importing area is less than the demand in that area. Therefore, there is an energy flow from EX to IM and the flow reaches the maximum value causing the connection between EX to IM to be congested.
6. There is a counter-trading mechanism that clears congestion between the two areas. In order to eliminate overflows, the total generation of the exporting area has to be reduced while the total generation of the importing area has to be incremented. Thus, ΔP_i^{EX} and ΔP_i^{IM} are the changes in the generation of company i in the exporting and importing areas, respectively.
7. The difference between the real production and the result of the day-ahead market will be paid or charged at a certain price. These payments can be viewed as an income or a cost depending on whether the generation unit increases or reduces its production. Increases in unit production are paid at the price γ, and reductions are charged at the price χ. These prices depend on the regulatory framework of each country.

Figure 1: Two-area system

3.2. Market clearing conditions

In the electricity market, total generation and demand have to be balanced. In the day-ahead market, the sum of the generation of all units equals the total demand

$(D^{IM} + D^{EX})$. In the counter-trading mechanism, the increased power in the importing area and the reduced power in the exporting area must be equal in order to maintain the power system balance. The total change (ΔP) is the change in the production of the units that clear system congestion in the counter-trading mechanism. Thus, the power balance constraints are:

$$\sum_i \left(P_i^{IM} + P_i^{EX}\right) = D^{IM} + D^{EX} \tag{1}$$

$$\sum_i \Delta P_i^{IM} = \Delta P \tag{2}$$

$$\sum_i \Delta P_i^{EX} = \Delta P \tag{3}$$

3.3. The generation company's problem

The behavior of the generation company i is modeled by the maximization problem (4)-(8). The generation company i determines the production of its generation units in the day-ahead market (P_i^{IM} and P_i^{EX}) and counter-trading mechanism (ΔP_i^{IM} and ΔP_i^{EX}) that is required to maximize profits taking into account the market prices (λ, γ and χ) and the cost functions $\left(C_i^{IM}\left(P_i^{IM} + \Delta P_i^{IM}\right) \text{ and } C_i^{EX}\left(P_i^{EX} - \Delta P_i^{EX}\right)\right)$.

Equation (4) is the company's profit that is equal to the revenue in the day-ahead market, plus the income due to the increment in the generation at area IM, minus the charge due to the reduction in the generation at area EX, minus the production costs in each area. Constraints (5) and (6) are the maximum production in the importing and exporting areas, respectively. Constraint (7) indicates that the maximum reduction in the exporting area in the counter-trading mechanism is less than or equal to the production in the exporting area in the day-ahead market. Constraints (8) model that the decision variables are positive. μ, ν and ξ are the dual variables associated to constraints (5), (6) and (7), respectively.

$$\max_{P_i^{IM}, P_i^{EX}, \Delta P_i^{IM}, \Delta P_i^{EX}} \quad \lambda \cdot \left(P_i^{IM} + P_i^{EX}\right) + \gamma \cdot \Delta P_i^{IM} - \chi \cdot \Delta P_i^{EX}$$
$$- C_i^{IM}\left(P_i^{IM} + \Delta P_i^{IM}\right) - C_i^{EX}\left(P_i^{EX} - \Delta P_i^{EX}\right) \tag{4}$$

s.t.

$$\overline{P}_i^{IM} - P_i^{IM} - \Delta P_i^{IM} \geq 0 \qquad : (\mu) \tag{5}$$

$$\overline{P}_i^{EX} - P_i^{EX} \geq 0 \qquad : (\nu) \tag{6}$$

$$P_i^{EX} - \Delta P_i^{EX} \geq 0 \qquad : (\xi) \tag{7}$$

$$P_i^{IM} \geq 0, \; P_i^{EX} \geq 0, \; \Delta P_i^{IM} \geq 0, \; \Delta P_i^{EX} \geq 0 \tag{8}$$

3.4. Market equilibrium

The market equilibrium point corresponds to the solution of the Mixed Complementary Problem (9)-(18). Equations (9)-(11) are the market clearing constraints. The generation companies behavior is modeled by means of equations (12)-(18). These equations are the Karush–Kuhn–Tucker (KKT) conditions of the problem (4)-(8) for each company i. The operator $\perp$ denotes the inner product of two vectors equal to zero, i.e., $0 \leq x \perp f(x) \geq 0$ corresponds to the system equations $x \geq 0$, $f(x) \geq 0$ and $x \cdot f(x) = 0$.

$$\sum_i \left(P_i^{IM} + P_i^{EX}\right) = D^{IM} + D^{EX} \qquad (\lambda \text{ unrestricted}) \tag{9}$$

$$\sum_i \Delta P_i^{IM} = \Delta P \qquad (\gamma \text{ unrestricted}) \tag{10}$$

$$\sum_i \Delta P_i^{EX} = \Delta P \qquad (\chi \text{ unrestricted}) \tag{11}$$

$$0 \leq \mu \perp \overline{P}_i^{IM} - P_i^{IM} - \Delta P_i^{IM} \geq 0 \qquad \forall i \tag{12}$$

$$0 \leq \nu \perp \overline{P}_i^{EX} - P_i^{EX} \geq 0 \qquad \forall i \tag{13}$$

$$0 \leq \xi \perp P_i^{EX} - \Delta P_i^{EX} \geq 0 \qquad \forall i \tag{14}$$

$$0 \leq P_i^{IM} \perp -\lambda + \frac{\partial C_i^{IM}\left(P_i^{IM} + \Delta P_i^{IM}\right)}{\partial P_i^{IM}} + \mu \geq 0 \qquad \forall i \tag{15}$$

$$0 \leq P_i^{EX} \perp -\lambda + \frac{\partial C_i^{EX}\left(P_i^{EX} - \Delta P_i^{EX}\right)}{\partial P_i^{EX}} + \nu - \xi \geq 0 \qquad \forall i \tag{16}$$

$$0 \leq \Delta P_i^{IM} \perp -\gamma + \frac{\partial C_i^{IM}\left(P_i^{IM} + \Delta P_i^{IM}\right)}{\partial \Delta P_i^{IM}} + \mu \geq 0 \qquad \forall i \tag{17}$$

$$0 \leq \Delta P_i^{EX} \perp \chi + \frac{\partial C_i^{EX}\left(P_i^{EX} - \Delta P_i^{EX}\right)}{\partial \Delta P_i^{EX}} + \xi \geq 0 \qquad \forall i \tag{18}$$

3.5. Solution methodology

It is important to note that the total power change (ΔP) in (10) and (11) is the result of the counter-trading mechanism. Following the method proposed in [14], an iterative procedure is used to determine the value of ΔP. This procedure works as follows:

1. Initialize $\Delta P = 0$. This case corresponds to the case without congestion.
2. Solve the problem (9)-(18).
3. Update the value of ΔP that clears congestion,$\Delta P = \sum_i P_i^{EX} - D^{EX} - \overline{F_l}$. If the change in ΔP with the previous iteration is less than ϵ value, the algorithm stops; otherwise it goes to 2.

4. Case study

This section presents a simple example that permits an analysis of the performance of the different counter-trading mechanisms. The results are compared with those obtained if the electricity market uses a nodal-pricing system in which a counter-trading mechanism is not necessary to clear system congestion.

The case considers two areas: an exporting area *EX* and an importing area *IM* with demands $D^{EX} = 100$ MW and $D^{IM} = 300$ MW, respectively. Both areas are connected by a flowgate with a maximum transmission capacity $\overline{F_l} = 150$ MW. There are four generation companies with generation units in both areas. Table 1 presents the total installed capacity and the generation cost of the units in each area. In this system, it can be observed that the generation company 2 has units with lower costs, and the units located in area *EX* are cheaper than those in area *IM*.

Table 1: Generation Units

Different cases are modeled in order to represent different policy mechanisms. Case A is a preliminary case without a limit on the interconnection capacity; thus, there is no system congestion. Case B represents a nodal-pricing system. Case C represents the solution expected by the policy makers in the different countries in which generation companies behave competitively and bid their power units at a price which will cover their marginal costs. Case D, E ,F and G model the policy mechanisms used in Spain, the Netherlands, Portugal and Germany, respectively. In these cases, generation companies may behave strategically between markets, i.e., they may bid their power units differently than their marginal costs because they know that their units will be redispatched in the counter-trading mechanism in order to clear congestion.

4.1. Case without congestion (A)

In a perfect competitive market and without congestion between areas, the electricity market price is the same in both areas. Generation company 2 produces 300 and 100 MW in areas *EX* and *IM*, respectively. The electricity price is 53.8 €/MWh which corresponds to the variable cost of the marginal unit, and the power flow from area *EX* to *IM* would be equal to 200 MW (Table 2).

Table 2: Results of case A

4.2. Nodal-pricing mechanism (B)

With a nodal-pricing mechanism, there is a market separation between areas when congestion occurs. In this case, electricity prices are different in each area, and they correspond to the cost of the marginal unit in each area.

The unit production in the exporting area of generation company 2 is reduced to 250 MW while the production of units in the importing area of generation companies 3 and 4 is increased to 40 MW and 10 MW, respectively. This result occurs because the maximum transmission capacity is explicitly taken into account in the nodal-pricing mechanism, and therefore, the power flow cannot exceed the maximum capacity of 150

MW. The electricity price in the exporting area *EX* is reduced to 52.9 €/MWh, which is equal to the costs of the marginal unit in that area. Meanwhile, the price in area *IM* is increased to 57.5 €/MWh because more expensive units have to meet the demand in that area. Thus, when system congestion is taken into account in the day-ahead market, total consumer cost is increased by $21850 - 21520 = 330$ €, and generation companies receive an additional profit which is equal to $390 - 270 = 120$ €(Table 3).

Table 3: Results of case B

4.3. Minimum cost solution (C)

Within the different regulatory frameworks analyzed (Spain, the Netherlands, Portugal, Germany), the day-ahead electricity market does not explicitly take into account the transmission network, as in case B, and system congestion that appears in the system is cleared using a counter-trading mechanism. In this scenario, the market operator expects that companies will behave competitively and bid their marginal costs, i.e., the generation companies will not change their behavior in the day-ahead market, and the day-ahead market outcome will be the same as in case A. When congestion occurs, the necessary adjustments will be made by the counter-trading mechanism, and the total amount of electricity produced would be the same as the production in the nodal-pricing mechanism (case B).

In this case, the unit in the exporting area of generation company 2 decreases its production by 50 MW in the counter-trading mechanism. Meanwhile, the production of units in the importing area of generation companies 3 and 4 increases by 40 MW and 10 MW, respectively (Table 4). This result gives a total consumer cost equal to 21685 €, which is lower than the consumer cost of the nodal-pricing mechanism (Table 9). However, although this would be the optimal outcome from the point of view of the regulator, this result is not a market equilibrium because the generation companies can behave strategically to increase their profits.

Table 4: Results of case C

4.4. *Spanish case (D)*

When congestion occurs, generation companies value their production in each area differently, giving more importance to generation in the importing area because they know that, while units in the exporting area may be penalized, units in the importing area are necessary for the removal of congestion. Since nothing is paid for reducing production in the counter-trading mechanism, the companies may prefer to bid the units in the importing area above their marginal cost because the day-ahead market price will rise, and, in any case, the production in the importing area will be dispatched to deal with the congestion.

In this case, generation company 2 knows that its generation has to be reduced in the exporting area and increased in the importing area in order to clear system congestion. Therefore, there is a strong incentive for this company to congest the interconnection, producing nothing in the importing area. This means that other agents' more expensive units have to be dispatched in the day-ahead market, thereby increasing the price of electricity and the amount of system congestion. This causes an increase in the amount of electricity traded in the counter-trading mechanism. Thus, the final outcome is that generation company 2 can exercise market power by bidding its unit in the importing area at a price above the marginal cost of this unit. In this way, this unit is not dispatched in the day-ahead market, but has to be dispatched in the counter-trading mechanism to remove the congestion. Thus the outcome of the counter-trading mechanism is not optimal from a technical perspective because the final production of electricity is not the same as in case B (Table 5), and neither is it optimal from an economic perspective because the counter-trading mechanism allows generation companies to exercise market power, which they do not have under a nodal-pricing system.

Table 5: Results of case D

4.5. *Dutch case (E)*

As in Spain, the generation companies may behave strategically in the day-ahead market. They may have an incentive to congest the network by increasing the price bid of their units in the importing area so that they will be dispatched in the counter-trading mechanism.

As regards the counter-trading mechanism, the pay-as-bid system is used to remunerate generation companies (Section 2.2). The objective is that the changes in production be paid/charged at a minimum cost. However, generation companies may try to obtain the same profits that they could obtain in a marginal-pricing system Ren and Galiana [27]. Thus, in the importing area, generation companies may bid prices above their cost until the bid of the most expensive unit necessary to clear congestion is made. In the exporting area, generation companies may behave in a similar fashion, making bids at a price below that of the cheapest unit in order to be able to return the least amount of money possible. If they behave in this fashion, changes in the production of the units in the exporting area are charged at a price of 52.9 €/MWh, while changes in the production of the units in the importing area are paid at a price of 57.5 €/MWh, prices which are equal to the prices in the nodal-pricing case (Table 3). However, in this case, these prices only remunerate the changes in production in the counter-trading mechanism, as shown in Table 6. It is important to note that the final production of units corresponds to the technically optimal solution, i.e., the total amount of electricity produced is the same as that found in case B. However, the companies' total profit and consumers' total costs are higher than they are in case B.

Table 6: Results of case E

4.6. Portuguese case (F)

In the counter-trading mechanism implemented in Portugal, a generation unit that increases its production is paid at the minimum between its bids in the day-ahead market and the counter-trading mechanism. Thus, the unit does not have any incentive to make a bid higher than its bid in the day-ahead market. Meanwhile a unit that reduces its production is charged at the maximum between its bids in the day-ahead market, the counter-trading mechanism and 0.85 times the day-ahead market price. Thus, the unit does not have any incentive to make a bid lower than its bid in the day-ahead market. Consequently, the policy implemented in Portugal reduces the possibility of generation companies exercising market power in the counter-trading mechanism because the bids made by the generation companies are limited by the bids made in the day-ahead market.

However, this policy does not prevent generation companies from exercising market power in the day-ahead market in order to congest the system: generation companies may behave strategically by increasing the bid prices of the units in the importing area. Thus the day-ahead market price may increase, and the units in the importing area will not be dispatched in the day-ahead market, but will be necessary to clear congestion in the counter-trading mechanism.

As in the Dutch case, the final solution is technically optimal but economically inefficient. Comparing this solution with the case B, the final amount of electricity produced is the same, but the total companies profit and consumer costs are higher (Table 7).

Table 7: Results of case F

4.7. German case (G)

In Germany, the generation companies only make bids in the day-ahead market because there is no additional market for the resolution of system congestion. The units involved in the congestion clearing procedure are remunerated at their marginal costs. From a policy point of view, the main disadvantage of this mechanism is that the determination of the marginal costs is not a simple task for regulators to perform.

Although the congestion management mechanism in Germany uses only the marginal costs of the units, this does not guarantee that generation companies do not exercise market power in the day-ahead market with the aim of congesting the system. Under this mechanism, total amount of electricity produced correspond to the technically optimal solution. However, the companies' total profit and the consumers' total costs increase, in contrast to to case B (Table 8).

Table 8: Results of case G

4.8. Comparison

The technical solutions found in cases B and C are the same: the optimal solution to clear congestion between the areas. However, the consumers' cost and companies' profit are different in these cases since the changes in production made by the units in the counter-trading mechanism are not properly remunerated in case C (Table 9). It is naive to think that companies are not going to behave strategically and to think that the market outcome will be the solution found in case C. When generation companies take into account the effect of the counter-trading mechanism in their strategic behavior, the market solution is that found in cases D, E, F and G. In the Spanish case (D), the technical solution is worse than the optimal solution. Meanwhile, in the Dutch, Portuguese and German cases (E, F and G), the amount of electricity produced by the units is the same as the optimal solution. However, in all four cases, companies' total profit and consumers' total cost increase with respect to case B. Thus the four counter-trading mechanisms are inefficient because the market outcomes are not optimal in either technical or economic terms.

Table 9: Case comparison

5. Conclusion

This paper has studied the congestion management mechanisms implemented in Spain, the Netherlands, Portugal and Germany, comparing them with a nodal-pricing system. The results have shown that the counter-trading mechanisms in these countries are inefficient because they allow generation companies to exercise market power. When generation companies detect that system congestion will occur, they vary the bid prices of their electricity depending on the area, giving more importance to importing area production because the production in this area is necessary to clear congestion. Thus, generation companies may behave strategically between markets, bidding their units in the importing area at prices higher than their marginal costs in order to increase the price of electricity, and ensure that units are not dispatched in the day-ahead market but in the counter-trading mechanism.

In the four cases analyzed, the Spanish one was the only case in which the final the amount of electricity produced by the units do not correspond to the technically optimal solution, while in the Netherlands, Portugal and Germany the final quantities are the same as those found in the nodal-pricing system. However, in all four cases, the total consumer cost is higher than the cost in the optimal solution. The reason is that the electricity price in the four cases does not properly reflect the real production cost in the different zones of the power system. Therefore, in an electricity market with a counter-trading mechanism, the final outcome may not be technically or economically optimal and may increase the prices paid by consumers.

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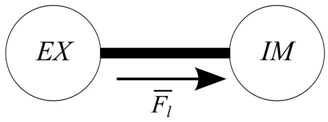

Figure 1: Two-area system

Table 1: Generation Units

	Area			
	E		I	
Company	Pmax (MW)	Cost (€/MWh)	Pmax (MW)	Cost (€/MWh)
A1	80	55.6		
A2	300	52.9	100	53.8
A3	50	55.7	40	57
A4			60	57.5

Table 2: Results of case A

	Power (MW)		Income	Cost	Profit
	E	I	(€)	(€)	(€)
A1	-	-	-	-	-
A2	300	100	21520$^{(*)}$	21250$^{(**)}$	270
A3	-	-	-	-	-
A4	-	-	-	-	-
Total	300	100	21520	21250	270
Power Exchanges	-200	200			
Price (€/MWh)	53.8				

$(*)\ (300 + 100) \cdot 53.8 = 21520$

$(**)\ 300 \cdot 52.9 + 100 \cdot 53.8 = 21520$

Table 3: Results of case B

	Power (MW)		Income	Cost	Profit
	E	I	(€)	(€)	(€)
A1	-	-	-	-	-
A2	250	100	18975$^{(*)}$	18605$^{(**)}$	370
A3	-	40	2300	2280	20
A4	-	10	575	575	0
Total Power	250	150	21850	21460	390
Power Exchanges	-150	150			
Price (€/MWh)	52.9	57.5			

$(*)\ 250 \cdot 52.9 + 100 \cdot 57.5 = 18975$

$(**)\ 250 \cdot 52.9 + 100 \cdot 53.8 = 18605$

Table 4: Results of case C

	Power (MW)						Income	Cost	Profit
	DA		CT		Final				
	E	I	E	I	E	I	(€)	(€)	(€)
A1	-	-	-	-	-	-	-	-	-
A2	300	100	-50	-	250	100	18830(*)	18605(**)	225
A3	-	-	-	40	-	40	2280	2280	0
A4	-	-	-	10	-	10	575	575	0
Total Power	300	100	-50	50	250	150	21685	21460	
Power Exchanges	-200	200			-150	150			
Price (€/MWh)	53.8								

(*) $(300 + 100) \cdot 53.8 - 50 \cdot 53.8 = 18830$

(**) $(300 - 50) \cdot 52.9 + 100 \cdot 53.8 = 18605$

Table 5: Results of case D

	Power (MW)						Income	Cost	Profit
	DA		CT		Final				
	E	I	E	I	E	I	(€)	(€)	(€)
A1	80	-	-30	-	50	-	2785	2780	5
A2	300	-	-112.5	100	187.5	100	15823.75(*)	15298.75(**)	525
A3	20	-	-7.5	40	12.5	40	2976.25	2976.25	0
A4	-	-	-	10	-	10	575	575	0
Total Power	400	-	-150	150	250	150	22160	21630	530
Power Exchanges	-300	300			-150	150			
Price (€/MWh)	55.7								

(*) $300 \cdot 55.7 - 112.5 \cdot 55.7 + 100 \cdot 53.8 = 15823.75$

(**) $(300 - 112.5) \cdot 52.9 + 100 \cdot 53.8 = 15298.75$

Table 6: Results of case E

	Power (MW)						Income	Cost	Profit
	DA		CT		Final				
	E	I	E	I	E	I	(€)	(€)	(€)
A1	80	-	-80	-	0	-	224	0	224
A2	300	-	-50	100	250	100	19815(*)	18605(**)	1210
A3	20	-	-20	40	0	40	2356	2280	76
A4	-	-	-	10	-	10	575	575	0
Total Power	400	-	-150	150	250	150	22970	21460	1510
Power Exchanges	-300	300			-150	150			
Price (€/MWh)	55.7								

(*) $300 \cdot 55.7 - 50 \cdot 52.9 + 100 \cdot 57.5 = 19815$

(**) $(300 - 50) \cdot 52.9 + 100 \cdot 53.8 = 18605$

Table 7: Results of case F

	Power (MW)						Income	Cost	Profit
	DA		CT		Final				
	E	I	E	I	E	I	(€)	(€)	(€)
A1	80	-	-80	-	0	-	8	0	8
A2	300	-	-50	100	250	100	19445$^{(*)}$	18605$^{(**)}$	840
A3	20	-	-20	40	0	40	2280	2280	0
A4	-	-	-	10	-	10	575	575	0
Total Power	400	-	-150	150	250	150	22308	21460	848
Power Exchanges	-300	300			-150	150			
Price (€/MWh)	55.7								

$(*)\ 300 \cdot 55.7 - 50 \cdot 52.9 + 100 \cdot 53.8 = 19445$

$(**)\ (300 - 50) \cdot 52.9 + 100 \cdot 53.8 = 18605$

Table 8: Results of case G

	Power (MW)						Income	Cost	Profit
	DA		CT		Final				
	E	I	E	I	E	I	(€)	(€)	(€)
A1	80	-	-80	-	0	-	8	0	8
A2	300	-	-50	100	250	100	19445$^{(*)}$	18605$^{(**)}$	840
A3	20	-	-20	40	0	40	2280	2280	0
A4	-	-	-	10	-	10	575	575	0
Total Power	400	-	-150	150	250	150	22308	21460	848
Power Exchanges	-300	300			-150	150			
Price (€/MWh)	55.7								

$(*)\ 300 \cdot 55.7 - 50 \cdot 52.9 + 100 \cdot 53.8 = 19445$

$(**)\ (300 - 50) \cdot 52.9 + 100 \cdot 53.8 = 18605$

Table 9: Case comparison

Case	Total Consumers' Cost (€)	Total Production Cost (€)	Total Generators' Profit (€)
A	21520	21250	270
B	21850	21460	390
C	21685	21460	225
D	22160	21630	530
E	22970	21460	1510
F	22308	21460	848
G	22308	21460	848

Appendix F

Article 5 - Effect of technical network constraints on single-node electricity markets

Delgadillo, A., Reneses, J., Barquín, J., Aug. 2011. Effect of technical network constraints on single-node electricity markets. In: 17th Power Systems Computation Conference - PSCC'11. pp. 1–6.

Abstract: This paper presents a conjectural-variation-based equilibrium model of a single-node electricity market. The distinctive modeling feature introduced in this paper is the effect of congestion between areas on generators' behavior. The results show that if there is a congestion between two areas, generators valued differently the production of each area, and give more importance to the importing area.

Index Terms: Conjectural Variation, Market Equilibrium, Network Constraints, Single-node electricity markets.

Appendix G

Article 6 - Analysis of the Spanish congestion management mechanism

Delgadillo, A., Reneses, J., Jul. 2013. Analysis of the Spanish congestion management mechanism. In: 2013 IEEE Power and Energy Society General Meeting. pp. 1–5.

Abstract: In electricity markets, different mechanisms are used to solve congestion on the system. In the Spanish electricity market, the day-ahead is cleared without taking into account the technical constraints, and subsequently a counter-trading mechanism is used to solve the congestion that appear in the system. Even assuming that generation companies cannot modify the electricity price, they may behave strategically. They may modify their bids to avoid being dispatched in the day-ahead market, but being dispatched in the counter-trading mechanism. Therefore, the counter-trading mechanism allows generation companies to behave strategically in the electricity market. Meanwhile, that behavior does not exist in a zonal price system. This paper presents a simple case to analyze the inefficiencies of the congestion management mechanism used in Spain, comparing it with a zonal price system.

Index Terms: Congestion management, Counter-trading, Spanish electricity market.

Appendix H

Article 7 - Electricity market equilibrium model with voltage constraints

Delgadillo, A., Reneses, J., May 2014. Electricity market equilibrium model with voltage constraints. In: 11th International Conference on the European Energy Market - EEM 14. pp. 1–4.

Abstract: A conjectural-variation-based equilibrium model of a single-price electricity market is used to analyze the effect of voltage constraints on the generators' strategic behavior. The solution of the model is computed by means of a two-stage optimization procedure. In the first stage, the day-ahead market clearing process is modeled. In the second stage, an AC optimal power flow determines the active and reactive production changes of the generation units in order to meet the technical network constraints.

Index Terms: Voltage constraints, equilibrium model, electricity market