ON THE SELECTION OF THE SLACK BUS IN MECHANISMS FOR TRANSMISSION NETWORK COST ALLOCATION THAT ARE BASED ON NETWORK UTILIZATION

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Abstract – There is no general consensus on the most suitable method for allocating the cost of a transmission network among its users. Most regulators are using some measure of network utilization as the basic criterion for transmission tariff design, being marginal participations (*areas of influence*) and average participations (*tracing flows*) among the most promising and employed alternatives when location signals are required. This paper offers some theoretical and practical results that help in clarifying the properties of these two approaches and their relationship. Some limitations of these two methods, when applied in the context of a regional or multinational market, may be overcome by a specific combination of them that is proposed in this paper. Application to a large multinational network model is shown.

Keywords: transmission pricing, nodal prices, slack bus, network utilization, marginal participation factors

1 INTRODUCTION

Transmission of electricity is almost universally considered, even in liberalized systems, as a regulated activity whose income has to be determined by the regulator and collected from the transmission users through a network tariff. However, there is no universal consensus on how to determine these tariffs or, more precisely, on which is the most suitable method for allocating the cost of a transmission network among its users.

Nodal prices, —i.e. short-term marginal prices with full geographical differentiation—, cannot provide a complete solution to this problem, since in most existing networks they are only able to recover a small fraction of the required income [1]. Although a cost allocation method that is based on the economic benefits that each transmission facility provides to every user [2] appears to be the most efficient one, it is in practice very difficult to implement. In consequence, most electricity markets are using as the basic element of their transmission pricing mechanism some measure of how much each agent is using the grid, which can be interpreted as an approximation of how much the agent is benefiting from the existence of the network.

Throughout the paper, we will refer indistinctly as network user, market participant or agent to any generator or load located at a generic network node *i* whose transmission charge is being calculated. We will also define a balanced subgroup of agents as a set of the previously defined agents, generally located at different network nodes, such that the sum of the generation for all of them equals the sum of their demand.

Under the simplest scheme, —the postage stamp method—, usage is just measured as the energy produced (or consumed) by each network user, or simply the connected or contracted capacity, or even a mix of both, with no locational differentiation. This may be suitable for well meshed grids that are designed to operate as a unified system, but whenever the network is not so meshed (Argentina, Chile or Norway are clear examples), or a regional market is created out of a sum of several systems that used to operate mostly on their own (Australia, the Internal Electricity Market of the European Union [3] or the USA), location signals are more necessary and some mechanisms of evaluation of the impact of any user on the power flow at any part of the network have to be adopted.

The method of marginal participations [2, 4, 5] (often named *areas of influence*, after Chile) has been used in countries such as Chile or Argentina since these systems were restructured. This procedure calculates how much would the flow in line *j* increase if the generation (or load) in node *i* increased by 1 MW, i.e., the method obtains the per unit measure of marginal participation on line *j* for any agent located at node *i*. This calculation is performed for every node, for all the lines in the grid and for a representative number of load flow scenarios. Then, the cost of each line is allocated to the different users according to their participations in the line. Several variations on this basic approach are possible.

The problem with this method is that, due to Kirchhoff's laws, any 1 MW increase in generation (or load) at node *i* has to be compensated by a corresponding 1 MW (ignoring losses) increase in load (or generation) at some other node or nodes. Thus, the calculation of how much an injection at a certain bus affects the flows in the network depends on the decision of which is the node that responds, and the basic answer that is demanded from the method is heavily conditioned by an assumption that it needs as an input. Different choices are possible for this "slack bus" (the responding node or nodes in power systems terminology): near the major load center (as in Argentina or Chile), a distributed virtual node so that all the demand responds pro rata, the marginal generator in the market or in the centralized economic dispatch, etc., but they may lead to widely different results. The selection of the slack bus becomes an important problem; the choice has much practical importance, but there is no clear criteria to decide.

The average participations method [2, 6, 7, 8] seems to be free from this dilemma. The method requires as its basic input data a complete power flow corresponding to the specific system conditions of interest. The algorithm is based on the assumption that electricity flows can be traced —or the responsibilities for causing them can be assigned— by supposing that, at any network node, the inflows are distributed proportionally between the outflows. Under these assumptions, the method *traces* the flow of electricity from individual sources to individual sinks; i.e., the model identifies, for each generator injecting power into the network, physical paths starting at the generator that extend into the grid until they reach certain loads where they end. Symmetrically, the paths from loads to generators are also found. Then, the cost of each line is allocated to the different users according to how much the flows starting at a certain agent have circulated along the corresponding line.

This makes several implicit hypotheses that may influence the final results heavily. In order to allow for a simple and intuitive calculation of these physical paths, a rule of distribution of power flows through an electricity network has been adopted that is not fully supported by engineering principles (and it cannot be, as power does not really flow in the networks as a fluid in a pipeline). Considering that different options could have been used that would have led to different results, then simplicity may not be the only reasonable design criterion.

This paper offers some results that help in clarifying the properties and the relationship between marginal and average participations. First, it explores the question of the arbitrary selection of the slack bus required by the marginal method and, building upon the mathematical properties of spot prices that were developed in [9], it explains how network charges to the network users vary with changes in the slack bus, and relates this property with a "global percentage" arbitrary decision that is often discussed by regulators. Second, the paper describes in depth some of the implicit hypotheses underlying the average participations method and presents some potentially conflictive cases of its application. Then, a new third method is proposed that may have some advantages in the context of a regional or multinational system. Finally, some numerical results are presented for the case of 16 European countries in the UCTE.

2 MARGINAL PARTICIPATIONS

2.1 Description

Any usage-based methodology tries to identify how much of the power that flows through each of the lines in the system is due to the existence of a certain network user, in order to charge it according to the adopted measure of utilization. To do so, the marginal participations method analyzes how the flows in the grid are modified when minor changes are introduced in the production (or consumption) of agent *i*, and it assumes that the relationship of the flow through line *j* with the behavior of agent *i* can be considered to be linear. For each one of the considered scenarios, the procedure can be described as follows:

1. Marginal participation sensitivities $A_{i,j}$ are obtained that represent how much the flow through line *j* increases when the injection in bus *i* is increased by 1 MW.
2. Total participations for each agent are calculated as the product of its net injection by its marginal participation. Net injection is positive for generators and negative for demands. So the total participation of a generic agent *i* in line *j* is $A_{i,j} \cdot (g_i - d_i)$.
3. The cost of each line is allocated pro rata to the different agents according to their total participation in the corresponding line.

The linearity assumption does not introduce significant errors. In fact, the DC model of the load flow [10] is perfectly linear, and it provides rather accurate results regarding the flows through the lines. The critical task is the computation of the sensitivity factors $A_{i,j}$.

It is a fundamental technical characteristic of power systems that generation and demand must be always balanced. Therefore, if the generation at node *i* is incremented in 1 MW so that one can compute its marginal sensitivities, then some other nodes in the grid must increment their demand or reduce their generation in order to keep the system in balance. When a DC load flow is used, it is implicitly assumed that the node that is defined in the model as the slack bus is the one absorbing any changes that may happen in generation or load. Hence, what the sensitivity $A_{i,j}$ is really expressing is how much the flow through line *j* increases when the generation at bus *i* is increased in 1 MW and the demand at the slack bus is increased in 1 MW (ignoring losses). In other words, $A_{i,j}$ is telling us how much of a hypothetical transaction starting at node *i* and ending at the slack bus would go through line *j*. The DC load flow model allows one to define any node or combination of nodes as the slack bus, with some easy numerical manipulation, but one must be aware that choosing the nodes that respond influences heavily the final results, so it is a decision not to be made arbitrarily. Note also that the method, as it is presently applied, always uses the same slack bus to respond to any increment in generation or load in the system. A common choice for the slack node is a major load center. Chile and other Latin-American countries use some network node that is as close as possible to the largest city, where a significant amount of the load concentrates. This makes the network users that are close to the slack bus (most of them consumers) pay reduced network charges, while distant market participants (most of them generators) tend to pay high transmission charges.

Another alternative is the use of a distributed virtual node, so all the generators (or all the loads, or both) respond homothetically to any unbalance, in proportion to their level of production (or consumption, in the case of loads). When calculating the marginal participations of an agent located at node *i*, this alternative implies considering a transaction that starts at *i* and reaches practically every node in the system, so the method generally results in participations for agent *i* in almost every line in the grid. Of course, participations are higher in the lines that are closer to the considered agent, but the area where they are not negligible is relatively wide. In fact, depending on the generation pattern, important participations may appear in lines that are very distant to node *i*.

2.2 Implications of the selection of the responding node

The previous example, where marginal factors are calculated using a slack bus at the load center, illustrates how the slack node can modify the allocation of network charges between supply and demand. Next, we will examine in detail the impact that a change of slack bus has on the transmission charge for any network user.

For any line *j*, the total participation $f_{i,j}$ for a generic agent *i* is

$$f_{i,j} = A_{i,j} \cdot (g_i - d_i)$$

Being T_i the per-MW transmission charge for a generic agent *i*, and considering that the cost C_j of each line *j* is allocated to all the users in the system in proportion to their total participation in that line, the network charges for agent *i* are

$$T_i \cdot (g_i - d_i) = \sum_j \left(C_j \cdot \frac{A_{i,j} \cdot (g_i - d_i)}{\sum_z A_{z,j} \cdot (g_z - d_z)} \right)$$

where *z* is a dummy index representing the different agents in the system.

As it is shown in [9], if the slack node is changed, the sensitivity factors for line *j* are only modified in a fixed additional term, which is constant for all the nodes. Using X_j to denominate these fixed terms, the total participation of agent *i* in line *j* with the new slack is

$$f'_{i,j} = \left(A_{i,j} + X_j \right) \cdot (g_i - d_i)$$

And the total network charge for agent *i* can be calculated as

$$T'_i \cdot (g_i - d_i) = \sum_j \left(C_j \cdot \frac{\left(A_{i,j} + X_j \right) \cdot (g_i - d_i)}{\sum_z \left(A_{z,j} + X_j \right) \cdot (g_z - d_z)} \right)$$

Ignoring losses, and assuming that the system is in balance, the sum of net injections $(g_z - d_z)$ for all the *z* agents is zero, so $\sum_z X_j \cdot (g_z - d_z) = 0$.

Then, the total payment for agent *i* can be re-written as

$$\sum_j \left(C_j \cdot \frac{A_{i,j} \cdot (g_i - d_i)}{\sum_z A_{z,j} \cdot (g_z - d_z)} \right) + (g_i - d_i) \cdot \sum_j \left(\frac{X_j \cdot C_j}{\sum_z A_{z,j} \cdot (g_z - d_z)} \right)$$

The first term in this equation is equal to the total network charge that was obtained when using the original slack bus. The second term is the same for all agents *i*, except for the $(g_i - d_i)$ factor, which is common to both terms. This means that, for every network user, a change in the slack node results in an additional term *K* in the per-MW transmission charge. This additional term is the same for all of the network users.

$$T'_i \cdot (g_i - d_i) = (T_i + K) \cdot (g_i - d_i)$$

The former is true if negative participations are fully taken into account. Some implementations of the marginal participations method have chosen to consider only positive contributions, ignoring the negative ones; in that case the property described above would not hold.

The fixed term that appears modifying the unitary transmission price after a change in the slack is affected by the net injection in each bus so, whenever it is additive for any generator, it is additive for all of the generators in the system and subtractive for all of the demands, and vice versa. A slack bus located near the major load centers would tend to increase the total network charges paid by the generators and to reduce the part of the transmission price that is born by the consumers, while a slack bus close to the generation areas would increase the share of the demand in total payments and reduce the charges for the generators. A change in the slack bus is, then, just a way of determining the global percentage of the network costs to be paid by all the producers and all the consumers. In other words, if for some reason the regulator has made an a priori decision regarding the global split of the total network costs into generators and consumers, then the slack bus can be selected with no arbitrariness. Both decisions are equivalent. This is a very attractive property of the marginal participations method, and it provides some meaning to the seemingly arbitrary decision of choosing the slack node. Note, however, that other usage methods are possible (see sections 3 and 4) where the split into global demand and generation charges and the selection of slack bus can be performed independently.

Another interesting feature of the marginal participations method is that the relative location signals between nodes are not affected by a change in the slack node. The decision of a generator about whether to install at one network node or another depends, —obviously among other reasons—, on the difference between the transmission tariffs that it will be charged at the two locations. Since the difference between tariffs does not depend on the choice of slack bus, the location signal is not influenced by this decision.

When the wholesale market is perfectly competitive, it can be easily shown that any payment or tax charged to all producers, —such as a uniform adder to the transmission tariff—, would be passed on to the consumers via market prices, sooner or later. The implication is that, regardless of the choice on the split of transmission charges between generators and consumers or, equivalently, regardless of the choice of slack bus, the final result would be the same. Then the choice of slack node would be immaterial. Note that the situation is different when the markets are not perfectly competitive, as it is the case in most existing electricity systems. Then the decision about the split of the transmission costs between generators and consumers becomes more relevant and, consequently, also the choice of the slack node.

2.3 *Dispersion of the participations*

An alternative way of analyzing the impact of the choice of the slack node on the transmission tariffs is the examination of the participations of the different agents in all the network lines. If a distributed slack node is adopted, —i.e. all of the generators (or demands) respond homothetically to any change—, it can be seen intuitively that each node participates in almost every line in the network. In the European case, for example, network users in Portugal would have non-negligible participations in systems as far as Poland, since part of the increments originated at the Portuguese nodes would be compensated by the Polish nodes, although it seems clear that the actions of the agents in Portugal will have very scarce actual influence on the Polish network. This dispersion is a direct consequence of an intrinsic feature of the method of marginal participations: a single node or combination of nodes responds to the increments of all the generators and loads in the system. Then the dispersion in the participation factors is unavoidable. This is particularly troublesome in the case of a multinational market where a single slack node has to be defined for the entire system. The usage methods to be presented later make use of a different slack node for each agent or group of agents, in order to overcome this difficulty.

A very extreme and intuitive example would consist of two well-meshed areas linked by a very weak interconnection, as in Figure 1. If a fully distributed slack node is chosen, for a generator located in one of the two areas the method of marginal participations would yield participations in lines at both halves of the system. Then, any network user would have to pay a relevant part of the network in the neighboring area. This would happen even though the power flow through the interconnection might be close to zero. Intuitively it seems that this generator should basically pay transmission charges for the network in the area where it is located and the allocation obtained from the method appears to be incorrect.

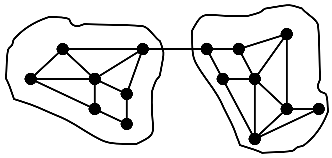

Figure 1: Two areas example

This is a relevant objection against the method of marginal participations. Note that in a regional market, the network users from any country that is far away from the slack node, —or if the slack node is a distributed one—, will generally have non zero and meaningless participations in a high number of distant lines. The same problem can be perceived when looking at the allocation of the transmission costs of that country between its generators and consumers. While the consumers (or generators) in that area will be paying a high transmission charge that represents a large and positive percentage of the costs of their national grid, the producers (or consumers) will be paying an also large but negative percentage, i.e., they will be receiving a net income. Although, theoretically, this is a possible way of allocating the totality (100%) of the transmission costs that are due to the agents located at the country, —as 300% to consumers and -200% to generators, for instance—, this allocation is hard to accept in practice.

2.4 *Balanced subgroups*

The method of marginal participations has some interesting features in case one is interested in considering subsets of network users with generation and demand that are completely balanced (we will refer to them as balanced subgroups). The net participations that are assigned by the method to a balanced group of users, —by adding up the participations of all the individuals forming the group—, are independent from the choice of slack node. In effect, by using the superposition principle, it can be seen that the flows originated from a 1 MW increment in the production of generator A and a 1 MW increment in the demand at the slack node, plus the flows resulting from a 1 MW increment in the generation of the slack node and a 1 MW increment in the demand at node B are equal to the flows resulting from a 1 MW transit between A and B, and they are the same for whatever slack node that might have been chosen.

Furthermore, it is possible to compute the flows that a balanced set of agents induce on the network in a clear an objective way, just by analyzing what happens when they are all removed from the system, without having to define any slack node. The total network usage calculated by the marginal participations method for a balanced group of agents coincides —assuming that the DC load flow is an acceptable model of the system— with the flows caused by this set of agents. For instance, a subgroup consisting of a generator and a demand of the same size and located at the same node will be assigned by the marginal method no network use. Whether this is a desirable feature or not is still an open issue.

This potential advantage is mostly conceptual, since it cannot be applied to design network tariffs for unbalanced sets of agents, which happen to be the majority, but it can be considered as an indicator of the soundness of the method. In any case, this feature should not be considered as a strong criterion to determine the value of a usage method, since there are several effects, such as congestions and other non-linearities in the performance of the network, —very much related to investment decisions—, that are not captured by the marginal participations method and which may weaken the reasoning above.

3 AVERAGE PARTICIPATIONS

3.1 Description

The basic intuition behind the average participations method is that the sources of the supply to loads and the destination of the power injected by generators can be assigned by employing very simple heuristic rules that only make use of the actual pattern of network flows. Although this procedure does not intend to capture the details of the physics of the problem, one could argue that in an electricity market that works reasonably well the power flows from nodes where it is less expensive to nodes where it is more expensive. Thus, using the actual network flow pattern may be a way of assigning sources and sinks to loads and generators, respectively, in a reasonable manner. It is not the only possible way, but it is intuitive, and simple to explain and to compute.

This is how the method works: for every individual generator *i*, a number of physical paths are constructed, starting at the node where the producer injects the power into the grid, following through the lines as the power spills over the network, and finally reaching several of the loads in the system. An analogous calculation is also performed for the demands, tracing upstream the energy consumed by a certain user, from the demand bus until some generators are reached. One such physical path (with as many branches as needed) is constructed for every producer, and for every demand.

In order to create these paths, a basic criteria is adopted: in each node of the network, the inflows are allocated proportionally to the outflows. A simple example is shown in Fig. 2.

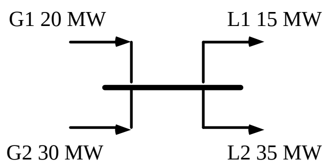

Figure 2: Proportionality principle in average participations

According to the proportional distribution, generator G1 would contribute 15x20/50 MW to the flow in line 1 and 35x20/50 MW to the flow in line 2.

The usage of line *j* by agent *i* is obtained as the part of the flows starting at agent *i* that pass through line *j*. The method implicitly results in a 50/50 global allocation of costs to generators and loads. However, if desired, an weighting factor could be used to modify this percentage. In the marginal participations method, this decision is linked to the choice of the slack node.

3.2 Dispersion of the participations

The marginal participations method calculates the contributions of a certain agent *i* to the flow through the different lines in the system by evaluating the impact of a simulated transit between the node *i* and the slack bus. The average participations method calculates the participations of agent *i* by tracking the influence in the network of a transit between node *i* and several ending nodes that result from the rules that conform the algorithm. These final nodes are different for each node *i* and they are implicitly defined by the procedure. As in the marginal participation method, global energy balance in the network requires that some buses must absorb the injection at node *i*. The difference here is that they are not defined by the user as in the marginal method, but they are a result of the model itself. The rules that are employed to obtain these nodes are completely pre-defined by the average model, although for some of them (the proportional allocation rule, for instance) other alternatives could have been adopted.

Anyway, the main difference between both methods is the use by the average method of a different set of final nodes for each bus *i* when evaluating its impact on the flows in the network, while the slack bus was always the same for marginal participations. As a consequence, the average participations method can construct a set of physical paths over the network linking generators to demands such that the sources or sinks of power for any agent located in an area where generation and load are more or less balanced tend to be found in the proximity of the agent, while agents located at a net exporting (or importing) area usually have their sources or sinks in more distant nodes, that are found by following the dominant flow patterns downstream (or upstream). This is a very interesting feature, since it avoids the dispersion problem described in 2.3 for the marginal participations method, which is clearly outperformed by the average method in this regard.

3.3 Balanced subgroups

However, using a different set of final nodes for each agent has other implications. For instance, let us consider the example described in Fig. 3.

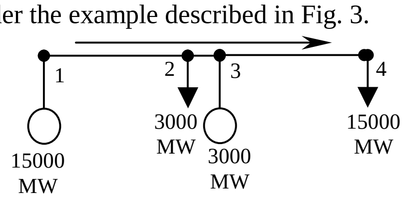

Figure 3: Four nodes example

The average participations method allocates the flow in line 3-4 to the flow coming from node 2 plus the production of the generator located at node 3. This means that the generator at 3 has a 3000 MW contribution to the flow of 15000 MW through line 3-4. Using a similar reasoning, the demand at node 2 has a 3000 MW contribution to the flow of 15000 MW through line 2-1. Thus, if we consider the load in node 2 and the generator in bus 3 as a balanced subgroup, the net charges to them include a fraction of the costs of lines 1-2 and 3-4 —the long and expensive lines in the system—, and no participation in the small line 2-3, which happens to connect both. However, if we analyze what would happen to the flows in the system if both the generator in 3 and the load in 2 were removed, the only change is a 3000 MW increment in line 2-3. Therefore, the aggregation of the network charges that are dictated by the method of average participations for the 3000 MW generator and the 3000 MW load, *when considered separately*, appears not to be related to the combined effect of the balanced group consisting of that generation and load on the pattern of flows in the network.

The example shows the large methodological difference between the marginal and average participations methods. In marginal participations the property of superposition of flows and charges for a balanced subgroup holds, i.e., regardless of the adopted slack, in the example in figure 3 the charges corresponding to the 3000 MW and the 3000 MW load when separately considered are exactly equal to the charges that should be applied to a flow of 3000 MW from the generator to the load. In average participations the sources of the flow feeding the 3000 MW load and the sinks of the flow originated at the 3000 MW generator will not coincide in general, therefore the combination of both flow patterns does not have any neat superposition property.

Although studies that are somehow based on the use of the superposition principle —dividing the use of the system into parts, obtaining results for each part separately, and summing up— are often performed in electrical engineering for other purposes, it is not clear that this is a desirable property in network cost allocation. One should be aware that the average participations method only provides meaningful results when applied to the actual flows in the network (i.e., those flows resulting from a complete network model, or from direct measurements, or from a combination of both) and not to other flows that may result from decomposition of the actual ones. Any analysis that implies using flows that are different from those actual ones may result in misleading conclusions.

4 HYBRID METHOD

While the marginal participations method fails to obtain reasonable measures of usage for individual agents, the average participations method results in counterintuitive results when several network users are jointly considered. A compromise between the two approaches may be attempted, whereby a) the entire population of market agents are aggregated into easily identifiable groups of generators and loads which will be treated as such and b) a suitable definition of the slack node for each one of these groups allows one to avoid the dispersion of participations in distant lines. The model that is presented hereafter groups generators and loads at country level and assigns a single slack bus per country. Countries seem to be a suitable grouping category when dealing with regional markets and multinational transmission tarification mechanisms.

One must be aware that such an approach is sympathetic with the claims of those that, because of historical reasons or the defense of national interests, argue that national networks are primary employed to allow their national demand to be met by their national generation, and that only the mismatches between their national generation and demand will result in export or import flows that will use other countries' networks. One can see that the final choice of a method of network cost allocation will not be exempt from this kind of regulatory considerations.

4.1 Description

The proposal is based on considering each country as a distinct entity for the purpose of calculation of transmission tariffs. A measure of the network usage of the generators and loads of a given country is obtained by considering the flows created by the generation and the demand in the country simultaneously, and then using some method to obtain a source (or a sink) for the net imports (or exports). In other words, national generators are primarily linked with national demands and only net imports or exports are linked with external nodes. Different alternatives could be used in this context, such as obtaining the responding nodes for each country using the average participations method and computing the flows through a marginal analysis, or using the average participations to define an area of responding nodes and allowing nodes in that area to respond proportionally to their size, etc. Here, a particular implementation of this idea is described.

The joint participation of all agents within country n in the different lines j in the entire system will be calculated as follows:

I. First, an external slack bus (source or sink) for each country n is found.

1. If the country is a net importer for the considered scenario of operation, then the average participations method is applied to each load i located at country n. If the country is a net exporter, then the method is applied to each generator i located within country n. The contributions in the different lines that result from these calculations are ignored; only the set of final nodes for every agent i and the amount of power injected or withdrawn by each of them are recorded.
2. Aggregating these data for all of the corresponding agents in country n and ignoring the final nodes that are located within country n, a global external slack node is obtained for country n. It consists of a set of buses, with a certain percentage of response for each one of them, which represents the generalized node that would respond to a unitary change in the net imports or exports of country n.
3. The per-unit response that is assigned to every bus included in this set is obtained by adjusting proportionally the quantities obtained at step 1 until they add up to 1 MW.

II. Now a marginal sensitivity analysis is performed.

4. A small incremental transaction is introduced in the system. Assuming that country n is a net exporter with a total generation G and a total consumption D, the transaction is defined as follows:
    - The generation of every production agent in n is increased by an amount that results from multiplying it by the factor 1/G.

- The demand of all the consumers in n is also increased by an amount that results from multiplying it by a 1/G factor.
- The external responding bus defined in steps 1-3 absorbs a total amount of (G-D)/G.

5. The flows originated by this transaction are computed and identified as the unitary participations of country *n* in the different lines *j*. Total participations are obtained by multiplying by the total production in the country (the former G factor).
6. The cost of each line is assigned to the different countries proportionally according to their contributions in the exercise that has just been described, when applied to each country. It is proposed here that negative contributions (i.e. in the opposite direction to the actual flow in the line) are set to zero, although other alternatives are possible.

4.2 Analysis

The participations that are identified for country *n* through this procedure tend to be reasonably distributed geographically, and they are consistent with the flows in the grid and the situation of country *n*. The use of a slack bus (sources or sinks) that is different for every country allows the model to avoid the dispersion problems that appeared in the marginal participations method. For example, the results of this approach for the case in Fig. 1 where each area corresponds to a different country would be the ones that are to be reasonably expected: network users will be mainly responsible for the cost of the lines within their own area.

See also that in the four nodes case, which is described in Fig. 3, if it is assumed that nodes 2 and 3 belong to the same country —while nodes 1 and 4 belong to other countries— their total participation in the network costs according to this new method only affects the line linking them. This is exactly what was intended by the proposed approach. Obviously, in a more realistic network, even if the generation and load in a country are completely balanced, the loop flows would imply some degree of external network use.

The approach that has been proposed here makes extensive use of political borders, which in principle should not influence transmission charges in a multinational market, and it is “transaction-based”, since it applies a specific treatment to the transaction of national generation meeting national demand, while the surpluses (exports) or deficits (imports) are treated in a different way. Although none of those features is theoretically desirable, the hybrid model allows to mitigate some of the difficulties that were detected with the average and marginal methods, and it may be adequate as the starting point in the implementation of multinational markets.

5 SAMPLE CASE

5.1 Data

In this example, 16 European countries that are members of the UCTE have been modeled to obtain some estimations of the behavior of the different models that have been analyzed in this paper. The data have been obtained from the real operation of the system on January 17, 2001, at 10.30. The model includes 3383 nodes and 3655 lines, and its basic elements are shown in Table 1 and Fig. 4. In order to deal with a single type of line throughout the network, 220 and 132 kV lines have been transformed into equivalent 400 kV lines, by applying a suitable factor.

country	400 kV equiv. lines	total demand (MW)	total production (MW)	net import (MW)
SPA	392.7	29093	28287	806
POR	80.5	6359	6891	-533
FRA	1123.2	64804	74125	-9322
ITA	450.8	31479	26508	4971
SWI	117.2	5720	5876	-156
GER	856.2	50260	49648	611
BEL	83.4	7059	6405	655
NED	69.1	10651	7134	3517
SVN	11.5	870	928	-59
AUS	54.7	2867	4295	-1428
CZE	62.5	8361	9705	-1345
POL	245.3	15179	16005	-826
BOS	8.4	426	437	-11
CRO	17.3	1057	1122	-65
HUN	41.2	3971	3655	317
SVK	40.4	2699	2723	-23

Table 1: Basic data for the case example

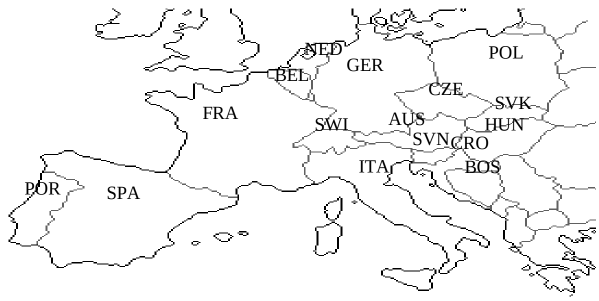

Figure 4: Countries included in the case example

Total results for the method of average participations and for the whole European network are presented in Table 2. The rows represent the participations of a certain country in the grids of every country in the system, including its own grid. The columns represent how the use of the grid of a given country, and thus its cost, is allocated among all the countries in the system, including the country itself. Numbers are expressed as a percentage of the total volume of the European network.

Regarding the marginal participations method, a distributed slack bus has been implemented such that, whenever there is an increase in the activity (production or consumption) of any agent, all of the remaining agents respond homothetically and 50% of the variation in the power injected or withdrawn from the system is provided by generators and 50% by the loads. On the other hand, when implementing the hybrid model, a simplified alternative has been used —although the procedure described in section 4.1 would have brought

	SPA	POR	FRA	ITA	SWI	GER	BEL	NED	SVN	AUS	CZE	POL	BOS	CRO	HUN	SVK
SPA	10,45	0,19	0,16	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
POR	0,11	2,01	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
FRA	0,19	0,00	29,26	0,34	0,15	0,04	0,15	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
ITA	0,00	0,00	0,40	11,32	0,24	0,00	0,00	0,00	0,01	0,04	0,00	0,00	0,00	0,03	0,01	0,00
SWI	0,00	0,00	0,33	0,49	2,61	0,22	0,00	0,00	0,00	0,06	0,00	0,00	0,00	0,00	0,00	0,00
GER	0,00	0,00	0,45	0,01	0,19	22,49	0,01	0,23	0,00	0,16	0,25	0,05	0,00	0,00	0,00	0,00
BEL	0,00	0,00	0,13	0,00	0,00	0,01	2,04	0,08	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
NED	0,00	0,00	0,00	0,00	0,00	0,30	0,08	1,58	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
SVN	0,00	0,00	0,00	0,02	0,00	0,00	0,00	0,00	0,22	0,03	0,00	0,00	0,00	0,05	0,01	0,00
AUS	0,00	0,00	0,00	0,05	0,02	0,20	0,00	0,00	0,06	1,15	0,07	0,00	0,00	0,01	0,01	0,01
CZE	0,00	0,00	0,00	0,00	0,00	0,08	0,00	0,00	0,00	0,02	1,28	0,07	0,00	0,00	0,01	0,03
POL	0,00	0,00	0,00	0,00	0,00	0,09	0,00	0,00	0,00	0,00	0,04	6,56	0,00	0,00	0,00	0,05
BOS	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,21	0,08	0,00	0,00
CRO	0,00	0,00	0,00	0,10	0,00	0,00	0,00	0,00	0,03	0,00	0,00	0,00	0,02	0,28	0,01	0,00
HUN	0,00	0,00	0,00	0,01	0,00	0,00	0,00	0,00	0,00	0,02	0,00	0,00	0,00	0,02	1,00	0,05
SVK	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,01	0,06	0,04	0,00	0,00	0,07	0,96
	10,75	2,20	30,75	12,34	3,21	23,44	2,28	1,89	0,31	1,50	1,71	6,71	0,23	0,47	1,10	1,10

Table 2: Global results for average participations

more accurate results— whereby the average participations method is employed to define an area of responding nodes and, then, a marginal sensitivity analysis is performed in which all of the nodes in that area respond in proportion to their sizes.

country	Average particip.		Marginal particip.	
	% of European grid	% of national grid	% of European grid	% of national grid
SPA	0	0	0,976	9,4
POR	0	0	0,094	4,4
FRA	0	0	3,642	11,1
ITA	0	0	1,234	10,8
SWI	0	0	0,506	15,2
GER	0,085	0,7	4,029	35,7
BEL	0	0	0,195	10,2
NED	0	0	0,349	12,4
SVN	0	0	0,107	33,7
AUS	0,016	1,1	0,769	40,4
CZE	1,278	74,6	0,766	55,1
POL	0,068	1,0	1,048	15,6
BOS	0	0	0,011	3,7
CRO	0	0	0,071	15,2
HUN	0,005	0,4	0,139	13,0
SVK	0,033	3,0	0,240	22,9

Table 3: Participations in the European grid for CZE

The marginal participations method, having a single slack bus for the whole system, is expected to yield participations in very distant lines for most agents, even though we should intuitively expect them to have little impact on those transmission systems. In order to illustrate this dispersion problem, Table 3 shows the participations of the Czech Republic (CZE) on the grids of the sixteen considered countries, both with the marginal participations method and with the average one. The first column in Table 3 shows how much of the grid of each one of the countries in the whole system is charged to the Czech Republic under average participations, where all number in the table are percentages of the total volume of lines in the entire grid. The second column presents the same data, but now expressed as a percentage of each country's national grid. Columns three and four show analogous results for the marginal participations method.

Under both approaches, the Czech Republic uses its own grid to a larger extent than any other, as one would have expected. However, while in the average participations method the participations of CZE in other countries are limited to neighboring countries and to small volumes of use, in the marginal participations approach the participations are remarkably large for most countries in the system.

country	Average particip.		New method	
	% of European grid	% of national grid	% of European grid	% of national grid
SPA	0,000	0,0	0,009	0,1
POR	0,000	0,0	0,007	0,3
FRA	0,334	1,1	0,164	0,5
ITA	0,493	4,0	0,066	0,5
SWI				
GER	0,222	0,9	0,205	0,9
BEL	0,000	0,0	0,003	0,1
NED	0,000	0,0	0,003	0,2
SVN	0,000	0,0	0,000	0,0
AUS	0,063	4,2	0,020	1,4
CZE	0,000	0,0	0,000	0,0
POL	0,000	0,0	0,001	0,0
BOS	0,000	0,0	0,000	0,0
CRO	0,000	0,0	0,000	0,0
HUN	0,000	0,0	0,000	0,0
SVK	0,000	0,0	0,000	0,0

Table 4: Participations in the European grid for SWI

On the other hand, as expected from the conceptual discussion on the four node example in Fig. 3, the average participations method may result in significant external use when applied to some small countries that are subject to strong transits, even though these countries may have generation and demand that are well balanced. The hybrid model suggested in the paper is designed to mitigate this effect. Table 4 presents the case of Switzerland, and compares the volumes of network usage assigned to this country in the remaining European countries both under average participations and under the alternative mechanism.

The results show that the relatively large contributions of Switzerland to the neighboring grids of France and Italy under the average participations method, are reduced with the new algorithm. There is a large dominant flow from France to Italy, which appears to be reflected in the results of the average model. Since that flow is basically a transit, and it is not caused by imports or exports of Switzerland itself, then the new method reduces the corresponding external participations. Participations in the German network, however, are not mitigated by the alternative procedure.

6 CONCLUSIONS

Electric network use has been adopted in this paper as a pragmatic proxy to the underlying cost function of a transmission network, which is believed to be more directly related to the economic benefits that the agents obtain from the network. Benefits are difficult to evaluate, and this is why electric utilization is frequently adopted. But electric use also presents implementation difficulties, since it cannot be defined unambiguously. Two well known alternative approaches for the evaluation of network utilization have been examined in depth: marginal participations and average participations. Their properties, —some of them presented here for the first time— have been evaluated and compared. None of the two methods has been found to be totally satisfactory, at least for all circumstances:

- Marginal participations uses the same slack bus in the computation of the impact of each generator or load on the flows in the network. This is the source of elegant mathematical properties of the resulting participations and charges, and it allows the aggregation of the network impact of balanced groups of generators and loads. However, it also results in an wide spread of the participations in the entire network and final charges that may contain an excessive locational content. The need to employ a single slack node appears to be both a blessing and a curse.
- Average participations uses different source and sink nodes (they can be seen as some sort of slack nodes) for each generator and load, in such a way that they take advantage of the actual flow patterns and result in reasonable distributions of participations and network charges. The method may result in non-negligible network charges for balanced groups of generators and loads that are closely located. This may be precisely the case of small countries in a regional market under specific circumstances of transit flows.

In the particular and complex case of a regional market, a new approach, which combines some characteristics of the other two methods, is proposed as an alternative that may be considered to improve the performance of the former approaches. This has been verified with a realistic example of the transmission network of 16 European countries included in the UCTE.

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