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Temporal aggregation for large-scale multi-area power system models

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Abstract

The study of integrated electricity systems that consist of several interconnected areas in the long term often results in large-scale complex models, that are difficult to solve. The already large spatial size of these systems, combined with a fine-grained time representation, necessary to capture the short-term variability arising from the high penetration of renewable generation, increases the complexity of the problem, and thus, its computational cost. To overcome this issue, temporal reduction techniques are generally applied. However, the application of time aggregation in interconnected systems represents a challenge. The goal is to select the best possible time aggregation that considers at the same time the particularities of each of the areas that make up the whole system. To do so, the authors propose a new methodology for temporal aggregation in multi-area energy system models. By implementing a multi-dimensional clustering algorithm, the original hourly data is transformed into system states, or group of hours that share similar characteristics, reducing significantly the computational burden required to solve it. Together, an accurate representation of the variability of the system is achieved. The main conclusions are derived from a real-size case study based on the electricity markets of three European countries. The sensitivity analysis performed shows the degree of accuracy of the results obtained, as well as the computing cost incurred for different temporal configurations. Ultimately, the results show the benefits of using this methodology over a more conventional approach.

1 | INTRODUCTION

In the forthcoming years, the power sector is set to undergo a profound development towards integrated, low-carbon and more efficient and sustainable energy systems. In this transition, carbon-emitting technologies are being replaced by different forms of renewable generation. This generation, however, is highly intermittent. Its inherent variability requires the use of flexibility resources to counterbalance the potential lack or excess of supply [1, 2]. Hence, a significant share of renewable penetration in the years to come may have a strong impact in the operation of power systems.

In this context, interconnected systems provide a great solution to manage this variability at a large scale. In general, market integration aims at optimizing welfare [3]. This brings important benefits, in socioeconomic and environmental terms. The increase of interconnections also leads to an enhanced use of

the excess of generation as well as an increase in reliability and security of supply [4]. For this reason, it constitutes a key objective in current energy policies throughout the world. Nevertheless, integrated multi-area markets rely on cross-border capacities. The limited capacity in the interconnections between the regions may lead to congestion, reducing competitiveness and creating challenging situations.

In this context, policy makers, regulators, market agents and system planners need tools to evaluate the best path to face this transition, analysing the possible outcomes and addressing emerging difficulties. Traditionally, energy system models have been the preferred tool for medium- to long-term energy system planning, to analyse and forecast scenarios for the evaluation of the future development of energy systems in the coming months, years or decades. For this reason, they are very important for providing decision support in the operation of power systems.

These models usually differ in terms of the objectives for what they are developed for: operational planning, capacity expansion planning, integrated assessment, and so forth [5]. This paper focuses on medium- to long-term operational planning models for multi-area systems. The combination of a long-time horizon and large spatial resolution makes the size of the problem larger and, consequently, it becomes difficult to solve. To avoid running into intractable problems, generally, simplifications are carried out. In this regard, given the detailed technical representation required in operational planning models, the temporal resolution is commonly reduced into different forms of time-period aggregations. Selecting a suitable temporal resolution is crucial for the performance and outcomes of the model.

In particular, with the increasing penetration of renewable generation, this approach can be difficult and challenging [6]. Renewable generation introduces significant variability in the system, especially in the short term. This makes it increasingly necessary to have a good short-term representation of the operation of power systems in these models, normally intended for medium- to long-term studies. Furthermore, operational constraints at a unit level, such as ramping rates or unit commitments, require chronological information.

In this situation, a failure to capture the short-term variability would result in unrealistic outcomes, not suitable for the decision making. For this reason, it is essential to ensure the intra-period and intra-regional characteristics of the intermittent supply and the operation of flexibility resources, such as storage [7].

1.1 | Literature review

Temporal aggregation in the context of power systems has been widely investigated in the literature in the last decade.

Initially, the most common temporal aggregation technique consisted of load duration blocks, that is, blocks of time with a fixed demand and duration [8–10]. Although the aggregation achieved by this method is considerable, this technique was soon replaced by new methodologies due to the lack of chronology between blocks, combined with the growth of renewable generation, critical in power systems nowadays. System states, introduced in [11], represents a step forward in this direction. The advantage of this methodology lies in the use of a transition matrix, that enables to retain chronological information.

Further work has been conducted in this field of investigation. For instance, Ref. [12] proposes a new formulation to optimize the operation of storage technologies in power systems. Through the use of a system state approximation of the balance equation in storage units, this work is able to achieve an accurate representation of the system maintaining the problem computationally tractable. Furthermore, the authors in [13] integrate this methodology in a capacity investment model, and enhance the formulation for a better co-optimization of both short- and long-term energy storage systems. With the same purpose, Ref. [14] introduces a new temporal aggregation methodology to obtain chronological clusters. The preserved chronology between periods allows to obtain a more efficient long-term capacity investment plan with similar usage of computational resources to other methods.

Another frequent approach recently developed to aggregate the time steps is by selecting representative periods, also known as typical periods or time slices. Representative periods are groups of consecutive time steps that characterize operational cycles, such as days, weeks or, in general, time intervals with a pre-defined duration. The use of consecutive time steps allows this methodology to retain the chronological information between them. This feature is one of the main reasons why this approach has been extensively investigated in the literature, where many research contributions can be found on this problem [15–17].

However, a new challenge arises as to how to withhold the inter-period relationships and variability. In this sense, numerous recent studies have explored the impact of representative periods in the modelling of the operation of power systems, to achieve a more accurate representation of the short-term variability due to the increase in renewable penetration. A novel formulation is introduced in [18] to address the coupling of typical days. This allows for capturing seasonal variability. However, this approach is applied at a regional level, raising concerns about its extensibility to large-scale more complex problems. Additionally, Ref. [13] introduces an enhanced version of the representative periods methodology to account for the chronology between days. The results show an overall better representation of short- and long-term dynamics. Finally, a comprehensive review of clustering methods for temporal representation in power system model is given in [19].

All the works above consider only a single-area system. A multi-area system approach adds further complexity to energy system models. The investigation of power systems consisting of several areas not only entails the use of larger models, which are already more difficult to solve, but also makes the temporal transformation considerably more challenging [20].

As aforementioned, the definition of a temporal aggregation methodology is very much connected to the particularities of the system under study. In this sense, demand and, more recently, net demand, have been the most common inputs on which to support this aggregation. Net demand, calculated by subtracting the non-dispatchable generation to the demand, is probably the parameter that best explains the price as it encapsulates the operation of dispatchable technologies. This definition becomes more complex in multi-area systems. Although the demand in different areas may be somewhat correlated, renewable generation has a significant regional component. The intrinsic geographic characteristics of each region determine to a great extent the impact of this kind of generation on the operation of the system. In this circumstance, it is rather unclear on the basis of which variable hours should be grouped.

Furthermore, the presence of cross-border constraints has a substantial impact on the distribution of energy across regions, as well as on the coordination of the sources of flexibility through the whole system. For instance, when not constrained, interconnections enable energy exports from areas with high shares of renewable generation to areas with high demand. Otherwise, congestion causes bottlenecks that may result in an

increment in the use of conventional generation or storage units. Therefore, market operation in multi-area power systems will be subject to changes in the status of the interconnections. In this context, the temporal representation becomes burdensome in terms of how to consider at the same time the specific characteristics intrinsic to each system, as well as the interconnections that link them together.

The authors in [21] propose a hierarchical clustering approach to derive the temporal structure of a long-term power system model through the use of representative days. They apply this methodology into a system consisting of multiple European countries and analyse the accuracy of the representation of renewable variability with different number of representative days. Nevertheless, this approach relies on the assumption of projecting demand and variable renewable energy (VRE) time series based solely on historical information. Not making ex ante considerations of market expectations may cause the model to overlook important changing relationships in the fundamentals of the system. Additionally, their temporal aggregation solution does not account with inter-day storage operation. Meanwhile, in Ref. [22], the authors study the storage operation in systems considering the transmission network. For this purpose, they extend the system states methodology in [11] to systems with several nodes. However, this framework is just examined on a rather simplified bus system, and its application to larger scale systems consisting of several areas remains to be tested.

Other works have applied similar methodologies to reduce the operational states of the system considering network constraints. The authors in [23] introduce the information of network congestion as relevant criteria for the selection of clusters. Likewise, Ref. [24] proposes an ex post reduction method for representative periods selection considering the reinforcement lines as inputs to the clustering process. Nevertheless, these studies are contextualized outside the scope of this paper. They focus on the transmission expansion planning problems, and therefore, lack the necessary detail for analysing the system from an operational perspective in the medium term.

Hitherto, to the best knowledge of the authors, no temporal reduction approaches have been proposed for their application in comprehensive real-size multi-area operational planning models for the medium to long term.

1.2 | Objectives and contributions

Against this backdrop, this paper introduces a comprehensive approach to reduce the temporal scope of planning models applied in systems with several areas in the medium to long term. The proposed methodology does so by considering a multi-dimensional clustering procedure which results in fewer representatives time slices that preserve the necessary short-term variability of the individual regions that conform the whole system.

In order to validate the effectiveness of this methodology, we measure the ability of different time aggregation configurations to replicate the solution of the computationally intensive hourly resolution model. For this matter, several indicators are defined, that help to determine which method achieves an overall accurate output.

In this light, the main contributions of this paper are outlined below:

- The development of a multi-dimensional clustering methodology to reduce the temporal dimension of multi-area planning models, making them tractable and adequate for the decision-making process in the medium and long term. This methodology is flexible, configurable and can be adapted to any electricity system with multiple areas.
- The application of the proposed approach to a real-size case study based on the electricity system formed by Portugal, Spain and France.

The remainder of this paper is organized as follows: in Section 2, the methodology is presented, where the proposed clustering process and the model employed are described. Moreover, its application to a real-size system is provided in Section 3, coupled with an analysis of the main findings and takeaways. Finally, Section 4 outlines the final conclusions and suggests lines of future research.

2 | METHODOLOGY

When modelling large-scale power systems for operational planning in the medium to long term, it is common to result in massive models, that generally turn computationally intractable. For these models to be suitable for the decision-making process, it is necessary to reach adequate solutions in reasonable times. In this situation, temporal aggregation becomes a critical part of the problem.

2.1 | Multi-dimensional temporal aggregation

Temporal aggregation consists of reducing the time framework of the model. The ultimate goal is to sub-sample the time steps at which variables are defined, so that the problem becomes smaller, and therefore, less demanding computing-wise, and easier to solve. The accuracy of the results obtained from these models, though, strongly relies on the number of time steps represented. Hence, there is a trade-off between the accuracy of the drawn solutions and the tractability of the resulting model. Depending on how to carry out this reduction, the solution will be more or less close to the real operation of the system.

Programmatically, time aggregation methods group the hourly observations of an input variable, often referred as control variable. In this sense, to accurately capture the functioning of the market it is required to select, as control variable, input data that retains the higher degree of information on the variability of the system, both in the short and the long term. For this reason, the estimated demand combined with renewable generation profiles are usually
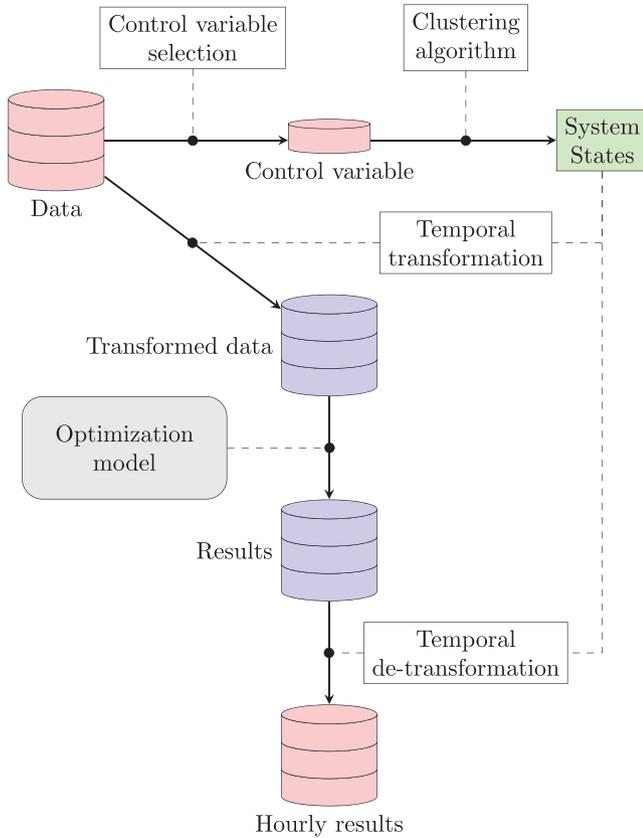
FIGURE 1 Conceptual diagram of the proposed methodology. The time dimension of the input data is first transformed into system states following the multi-dimensional clustering algorithm described in Section 2.1. Then, the optimization model (Section 2.2) is run over the transformed data to obtain the results. These results are ultimately converted to an hourly resolution with the information of the correspondence hour-state found in the clustering process

adopted as control variable, although, in theory, any time varying information that carries within itself a good representation of the market operation might be employed for this purpose.

The methodology of this paper is built from the selection of a control variable that reflects the patterns present in the electricity system. The time series of this control variable is first divided into periods, usually weeks or months. The hours corresponding to each period are subsequently grouped in the so-called system states, following a similar approach to the temporal framework proposed in [11]. Once this time structure has been defined on the control variable, the rest of the parameters of the fundamental model are transformed according to this scheme.

A conceptual illustration of the methodology is represented in Figure 1.

Delving deeper into the details, the methodology proposed here can be divided in the following steps (Figure 2).

Step 1: **Define the control variable**

Initially, let $X = \{X^1, \ldots, X^a, \ldots, X^A\}$ be the set of time series that represents the multi-dimensional control variable in which the clustering process is applied,

where A is the total number of areas in the system, and $X^a = \{x_1^a, \ldots, x_{hr}^a, \ldots, x_{HR}^a\}$ the times series of the control variable in area a. Meanwhile, x_{hr}^a stands for the observation at hour hr in area a, and HR is the total number of hours throughout the full horizon of the problem.

The so-called net demand is probably the most common selection as input to the clustering process. Net demand is defined as the difference between the electricity load and the non-dispatchable generation, that includes renewable power outputs as well as other non-controllable generation, such as the run-off-the-river in hydro units. In this sense, the net demand equals the dispatchable generation. In the current situation of electricity systems, where the share of renewable generation is rapidly accelerating, this parameter provides a useful indicator of the dynamics and variability present in power systems, since it accurately captures the relevant patterns that characterize their operation.

The variability of the net demand is naturally contingent to the individual characteristics of each electricity system. Hence, it is different from one location to another. In the context of a multi-area system, however, the territorial distribution of wind farms or solar plants has a smoothing effect on the net demand, reducing its overall variability. Therefore, the use of an aggregated system-wise net demand would result in an important loss of the particular features that drive the market outcomes in each system separately. For this reason, it is essential to use a vector parameter as control variable with the net demand by area, to preserve its value and to ensure a good representation of the local variations in each market. Instructions on how to accomplish this will be addressed in Step 4.

Step 2: **Divide the time series into periods**

For each area a and period p, let Y_p^a be the p-partition of X^a with cardinality P_p, containing the time series observations $y_n^a \in X$, $n = 1, \ldots, P_p$ corresponding to the hours of period p in area a.

Further along the procedure, the time series of the control variable is split into periods. Because specific aspects of the resource management are done on a weekly or monthly basis, as, for instance, the hydro operation, it is worth dividing the control variable observations to better model and represent these features. Hence, periods will be preset to reflect the dynamics of the system.

Step 3: **Normalize the weekly time series**

Normalize the time series of area a at period p, Y_p^a, into $\hat{Y}_p^a$, such that each data point of the original time series y_n^a is transformed in $\hat{y}_n^a$ according to $\hat{y}_n^a = \frac{y_n^a}{max(y_j^a : j=1,\ldots P_p)}$.

Through the normalization of the control variable, it is assured that the weighted contribution of each area in the clustering process is the same. This prevents an over-representation of the larger areas.
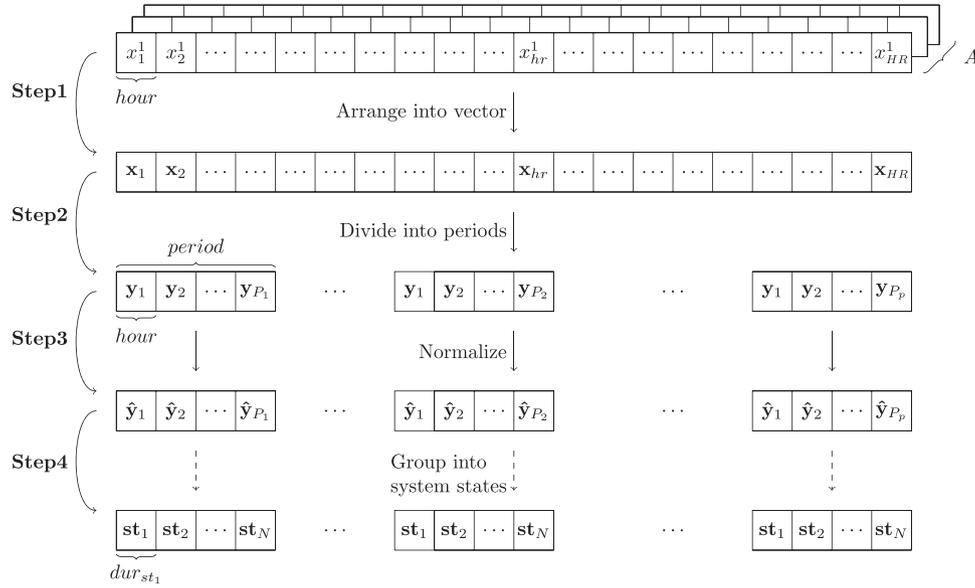
FIGURE 2 Diagram of the multi-dimensional temporal aggregation of the control variable to extract the system states that characterize the temporal representation of the parameters and variables of the model. The original hourly time series from the control variable of each area are first arranged into a vector form (Step 1). Then, the vectorized control variable is divided into periods (Step 2) and subsequently normalized (Step 3), to finally get the system states in each period (Step 4)

Step 4: Aggregate period-level data into system states

Given the vector $\mathbf{y} = (\hat{Y}^a, \ldots, \hat{Y}^{\mathcal{A}})$, resulted from rearranging the set of points containing the observations in period p for the normalized time series of all areas in the whole system, for each period p, find N clusters $ST = \{ST_1, \ldots, ST_N\}$ and its corresponding cluster centres $\{\mathbf{st}_1, \ldots, \mathbf{st}_N\} \in \mathbb{R}^a$ (hereinafter referred as centroids or 'system states') by means of a multi-dimensional k-means clustering algorithm, so that that the Euclidean distance from each vector realization $\mathbf{y_n} = (\hat{Y}_n^a, \ldots, \hat{Y}_n^{\mathcal{A}})$ to its respective cluster centroid $\mathbf{st}$ is minimized.

A clustering algorithm is implemented to group the hourly observations contained in each pre-defined period of the control variable into system states. Therefore, each system state constitutes a group of hours of the period not necessarily consecutive.

When implementing the proposed methodology over a unique region, this arrangement is relatively straightforward. In this case, the net demand of that region is selected as control variable and the clusters are calculated according to the values of this parameter.

In the case of considering multi-area systems, however, the geographical situation makes clustering more challenging. Renewable sources or electricity demand can be strongly correlated depending on the location analysed, as they are clearly driven by meteorological conditions. These correlations are more significant within a geographical region or inside the boundaries of individual markets. In large spatial systems, though, where areas with different climatic conditions and market regulations are considered, other factors, such as network constraints or firms' strategic behaviours, can also limit or strengthen these correlations. In this context, the consideration of a multi-area system influences to a great extent the definition of this temporal aggregation.

A blunt attempt to implement this aggregation would be by individually grouping the hourly data of net demand of each area into system states. Henceforth, this procedure will be refereed as *independent clustering*, to be distinguished from the *multi-dimensional clustering* proposed here. By taking this approach, different sets of system states for each period and for each area would be obtained. Because the model is solved once for the entire electricity system, covering all areas at the same time, these sets of system states must be necessarily combined together to create a definitive unique temporal structure for the model. In this way, the total resulting system states would be the combination of all the system states defined at a region level. To illustrate this process, a short example is provided hereunder.

Given a system with two areas A and B, for instance, consider there are three operational states defined in each area, characterized by a low, medium and high value of its net demand, respectively. In this situation, the resulting system-wise states would be the combination of the three operational states of each area, that is, nine total final states. A representation of this example is portrayed in Figure 3(a). In general, according to this approach, considering n areas with m_i possible operational states in each area, independent from each other, the total number of system states across the multi-area system is equal to $\prod_{i=1}^{n} m_i$.
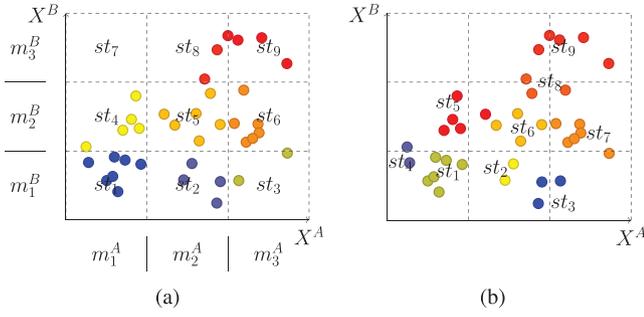
FIGURE 3 Representation of the distribution of the system states (st_1 to st_9) resulted from the implementation of independent ((a)) and multi-dimensional clustering ((b)) on a two-area (A and B) system with three operational states modelled in each area

As can be appreciated, this process is highly combinatorial. If this kind of clustering procedure is implemented in larger electricity systems with more areas, or when a higher granularity is desired in the representation of the control variable of each area, the resulting total number of system states will be significantly high. By contrast, in the reality, these combinations are not necessarily present in the operation of the power systems. Certain situations, like a low value of net demand in region A and a high value of net demand in region B might not make sense, since they hardly ever occur, or maybe are present only in a few hours of the entire horizon. Therefore, considering all these possible combinations will needlessly add complexity to the problem by enlarging the time sample of the model.

The proposed solution to address this issue is the implementation of a *multi-dimensional clustering* process (Figure 3(b)). Since the most representative combinations of net demand in each area are selected intrinsically within the clustering process, multi-dimensional system states clearly constitute an upgrade, and prevents over-representing sections of the combinatorial grid with non-significant occurrence. By entering the net demand in each area as a vectorized parameter to the clustering algorithm, a more accurate characterization of the spatio-temporal correlation of net demand among areas is guaranteed with a lower number of system states. A comparison of the two methods is performed in Section 3 to show this effect.

2.2 | The optimization model

The temporal structure resulting from the clustering procedure is applied to the formulation of a fundamental optimization model. For this purpose, a similar model to the one presented in [25] is used. This section is intended to provide a brief description of the formulation of the model as well as its integration together with the clustering process presented above.

The model used here calculates the market equilibrium using the conjectural variations approach. The selection of this particular type of model goes in line with the ultimate objective for which the methodology is developed, that is, the operational planning of power systems in the medium to long term. Market equilibrium models, based on the principle of Nash equilibrium [26], are suitable for the analysis of different types of competition within power markets [27]. In this case, through the use of conjectures, it is possible to have a more realistic representation of the outcome of the system, when strategic behaviours are present among market participants. As discussed in [25], considering these strategies is particularly important in multi-area systems, since the exercise of market power in certain regions may be determined to a great extent by the status of the interconnections.

As stated in [8], and later extended in [25] to account for several areas, the market equilibrium is found with the following quadratic optimization problem (1)–(4):

$$\min_{Q_{e,a,p}, f_{l,p}} \sum_{e,a,p} \left(C_{e,a,p}(Q_{e,a,p}) \right.$$
$$\left. + \sum_{d'} \frac{\theta_{e,a,d',p}}{2} Q_{e,a,p} Q_{e,a,d',p} + S_{e,a,p} Q_{e,a,p} \right) \tag{1}$$

s.t.

$$\sum_e Q_{e,a,p} - \sum_l H_{a,l} f_{l,p} = D_{a,p} : \lambda_{a,p} \qquad \forall a, p \tag{2}$$

$$\underline{F_l} \leq f_{l,p} \leq \overline{F_l} \qquad \forall l, p \tag{3}$$

$$\mathcal{H}(Q_{e,a,p}) \geq 0 \qquad \forall e, a, p. \tag{4}$$

Market power in this problem is modelled through parameters $\theta_{e,a,d',p}$ and $S_{e,a,p}$. Both represent the reaction of the competitor's strategies: Conjecture $\theta_{e,a,d',p}$ is formulated as the change in the electricity price $\lambda_{a,p}$ in area a with respect to the agent's production $P_{e,d',p}$ in area d' (5), whereas $S_{e,a,p}$ denotes the price spread increment assigned to each market agent depending on the state of the interconnections.

$$\theta_{e,a,d',p} = -\frac{\partial \lambda_{a,p}}{\partial Q_{e,d',p}} \qquad \forall e, a, d', p. \tag{5}$$

In this regard, the electricity price $\lambda_{a,p}$ can be obtained as the dual variable of Equation (2), which defines the demand balance for every area, taking into account the energy exchange between areas.

According to the temporal aggregation defined in the previous section, the total production of each market agent can be broken down into thermal, hydro and renewable energy

production, by means of Equation (6):

$$
Q_{e,a,p} = \sum_{st} \left(\sum_{t \in a : o_t = e} pt_{t,p,st} + \sum_{g \in a : o_g = e} pres_{g,p,st} \right.
$$

$$
\left. + \sum_{h \in a : o_h = e} (ph_{h,p,st} - be_{h,p,st}) \right) dur_{st} \quad \forall e, a, p. \tag{6}
$$

Finally, Equation (4) summarizes the rest of the constraints, such as power limits, ramps, maintenance, efficiency and other technical and economic considerations covered within the model. Note that, although not described in this section, these technical and economic characteristics are taken into account in the simulations of the model performed in Section 3.

In this sense, to include some technical features, such as ramping constraints or start-up and shut-down decisions, chronology must be preserved between system states. This is made possible through the use of a transition matrix, as proposed in [11]. This matrix, obtained in the clustering algorithm, is defined with the number of transitions between the different system states. Overall, this formulation enables the integration of the temporal aggregation methodology consisting of periods and system states with the planning model.

Furthermore, as far as the topology of the network is concerned, this model represents a simplified system where the different areas considered are mapped as a network of single nodes connected by the existing capacity between the interconnected countries. Therefore, the topology within each area is ignored. This modelling approach allows us to have a good representation of the interconnection, so congestion and stability issues between countries are considered. In this way, the model is prepared to consider limitations and variations in the exchange capacity present in the grid for each hour of the simulation time horizon. However, since the internal network of each area is not represented, the model does not take into account unavailability or congestion of the internal transport network nor network losses or other circumstances. At the same time, discarding the network within each area prevents us from running into intractable problems, computationally intensive and difficult to solve. This model is aimed at the planning of the electricity market in the medium and long term, and stability and congestion in the internal network are not generally a big concern in this time scope. In this sense, the additional computational effort required to compute the internal network within each system is generally not justified to obtain more detail on operational signals that are not so relevant in the medium and long term.

3 | CASE STUDY AND RESULTS

This section covers the application of the temporal framework proposed here to a real-size case study with the intention of illustrating its strengths and performance. Initially, a description of the considered system is presented in Section 3.1, where the inputs to the model are briefly outlined. The implementation of the methodology is explained in Section 3.2. Finally, Section 3.3 contains a discussion of the obtained results.

Note that the data used in this section are solely meant to provide a real-size analysis of a multi-area electricity system with the ultimate purpose of demonstrating the potential benefits of this methodology. In no case should the results of the model be interpreted as an accurate representation of reality.

3.1 | System description

In order to have a realistic measure of the performance of the proposed methodology, a real-size model of the electricity markets existing in the regions of Portugal, Spain and France is implemented. Portugal and Spain, which make up the Iberian peninsular system (MIBEL), and France, which in turn is part of the Central Western European (CWE) system, are all geared, together with other European countries, under the same unified market, commonly referred as the European Internal electricity market. This integrated system is structured as a zonal market, where each area is characterized by a single price country-wide. For this reason, when modelling this system, a reduced network of one node per region is considered, accounting only for the net transfer capacity available in the interconnections between areas, rather than the physical capacity of individual transmission lines within each area. Still, it is worth noting that in the particular case of the Iberian system, the model includes technical constraints for specific generating units that guarantee operation procedures, demand coverage and network availability in certain regions, according to the regulation of this market.

Having defined this structure, the fundamental model implemented in this case study comprises all the technical and economic characteristics of every area. In this sense, a high technical representation of the operation of the individual thermal and hydro units is considered, taking also into account pumping capabilities within reservoirs. Additionally, non-dispatchable generation is considered using different time series aggregated by technology and region. Furthermore, this model also includes other aspects of the market operation and regulatory policies present in each of these areas.

3.2 | Temporal aggregation implementation

The fine-grained technical representation of the model makes the problem big and complex. Considering this size, and the computational resources available, the targeted system in this case study is analysed for the year 2020. This temporal sample is sufficient to evaluate the effectiveness of the model under different circumstances, in terms of climatic conditions, market changes or technical variations, that may affect significantly the outcome of the system, from an operational point of view. Hence, as far as the model is concerned, the time horizon considered in each simulation is 1 year.

With this definition, an hourly resolution representation of the model is calculated. The results of this simulation establish the benchmark against which the rest of the simulations

are measured. These additional simulations are computed over the same model with different degrees of temporal aggregation, according to the proposed methodology in Section 2.

Following this methodology, the time horizon is first divided into time periods. In this case, months are selected as periods, as some features of this electricity system are driven by monthly signals, as it is the case of the hydro operation of the reservoirs, the negotiation of energy bilateral contracts by market players or other contracting decisions, such as take or pay clauses or third-party access to the gas network (TPA), for instance. Anyway, a different period selection could have been considered. All comes down to the particularities of the operation of the system under study, the degree of detail desired, and the computational resources available.

Having this monthly segmentation, the hours of each month are subsequently grouped into system states. For this clustering process, two different procedures for obtaining the clusters are analysed:

- *Independent clustering*. Clusters are found independently for each of the three areas that constitute the system. Once the clusters are obtained, the resulting system states are the combinations of these clusters (Figure 3(a)).
- *Multi-dimensional clustering*. A three-dimensional vector, consisting of the control variable of Portugal, Spain and France, is considered. A unique clustering algorithm is run, and the results are the final system states (Figure 3(b)).

According to these procedures, two different parameters are considered as the control variable used to obtain the clusters: demand and net demand. As mentioned in Section 2.1, net demand, understand as the dispatchable generation, is the control variable chosen for this methodology. However, traditionally, demand has been the basis for time blocks aggregation. In fact, as previously commented in the literature review, load duration blocks were initially the most extended method for the synthesis of the temporal representation of medium- and long-term models. In this case study, both variables will be included in the analysis to have a clear view of the differences in the results associated with the choice of demand as a control variable against the consideration of net demand, which theoretically accounts for more information of the variability present in each electricity system.

Therefore, for the *independent clustering* procedure, demand and net demand of each area constitute the control variable used in each clustering process. Likewise, a three-dimensional vector of demand and net demand of each area will be the basis for clustering generation in the *multi-dimensional* procedure.

The resulting system states under this parametrization of the clustering process will be entered in the model, simulated and finally compared in Section 3.3.

In this sense, the choice of the number of system states is again decisive for the accuracy of the solutions. In exchange, it is also crucial for the computational tractability of the model. There is no systematic formula for deciding which is the most appropriate temporal aggregation configuration in terms of accuracy and computational performance. This decision natu-

rally relies on the particularities of the operation of the system under study, the degree of detail desired and the computational resources available. As expected, the more system states are modelled, the better results are obtained. However, with several areas represented, CPU times and RAM rapidly increase with the system states.

To study the sensitivity of the results to the number of system states, different configurations of temporal aggregations are analysed with 2, 3, 4, 5, 6, 7, 8 and 9 system states for the independent clustering approach, and 8, 27, 64, 125, 216, 343, 512 and 729 system states for the multi-dimensional algorithm. In any case, the resulting number of the final system states is the same under each parametrization. This ensures a fair comparison.

Ultimately, apart from the hourly simulations mentioned above, 32 different executions are carried out: one for each system state configuration, control variable selection and clustering process.

As designed, in each simulation, a generation dispatch of the units of all the systems considered is carried out for all the modelled hours (or groups of hours). This dispatch is performed respecting all the constraints of the units in such a way that a very detailed representation of the system operation is achieved. Predictably, the accuracy of the model results will depend on the time aggregation configuration selected.

The data pre-processing, as well as the clustering process and the post-processing, were executed in MATLAB R2020b, whereas the different realizations of the model were computed in GAMS 32.2.0, using the commercial solver CPLEX 12.10.0.0. The equipment used for these tests was a PC with Intel (R) Xeon (R) Silver 4116 CPU @2.10 GHz with 40 logical processors and 128 GB of installed RAM memory running 64-bit Windows Server 2019.

3.3 | Results and discussion

Temporal aggregation is implemented with the main objective of generating simplified, computationally tractable models, that will be, therefore, easier to solve. For this reason, it is important to begin this discussion by analysing the resources used in the tests performed, in terms of CPU time and RAM memory.

To have a clear reference, the hourly model consists of 18,347,757 variables and 9,337,382 equations, and took 69,535 s to solve, requiring 201 GB of RAM memory. This execution time is not suitable for an agent in his decision-making process, since in the medium term, it is usual to make probabilistic forecasts, for example, by means of Monte Carlo simulations. Monte Carlo simulations require executing deterministic realizations of the model hundreds of times under different scenarios of uncertain variables. In this context, the computational effort required plays a key role in this decision. In the end, users need to find a compromise between accuracy in the results and the computational power required according to their needs. The results of the rest of the simulations are shown in Figure 4. In this figure, it can be observed that the results present slight differences depending on the control variable chosen for the simulation.
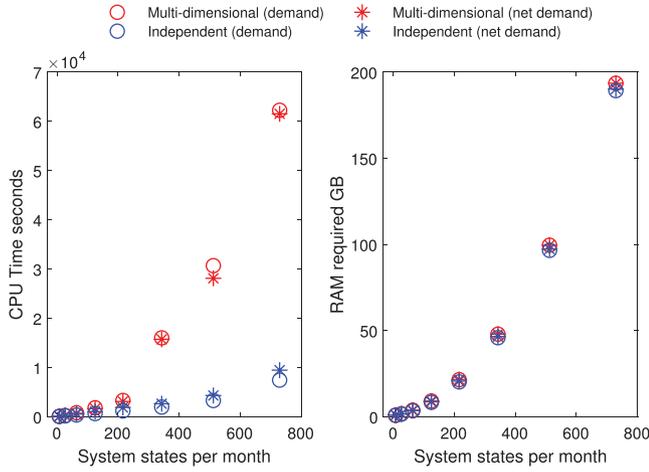
FIGURE 4 Illustration of CPU times and RAM memory values obtained in every simulation for the correspondent number of system states modelled under both clustering procedures: multi-dimensional and independent. Note that for each procedure, each point in the plot represents the corresponding result for the simulation considering demand and net demand as the control variable in the clustering process

However, these differences are not significant. The choice of demand or net demand has little impact on the number of system states considered. Therefore, the size of the model, in terms of variables and equations remains pretty much the same, and the run times are equivalent for both selections.

When comparing with the hourly model, a significant improvement is observed, in both the execution times and the amount of memory required for each test. This reduction highlights the huge degree of simplification achieved with this methodology. As expected, the more system states employed in the representation of the temporal framework of the model, the more memory is required, and the longer it takes to solve it.

In this sense, it is also worth noting the differences between the simulations with independent and multi-dimensional system states. As can be appreciated in Figure 4, the tests performed with independent system states required considerably less CPU time and memory than those with multi-dimensional system states. This difference is greater as more states are modelled. This result is not obvious. Intuitively, the use of the resources should be similar, considering the same number of system states are being pre-defined for both approaches. However, the construction of these system states is different between the two procedures. On the one hand, multi-dimensional system states are calculated jointly for all areas, thus taking into account the existing correlations between areas. In contrast, by the independent clustering approach, system states are calculated as the possible combinations between the operational states independently computed in every area.

Figure 5 illustrates this difference for a specific period with eight total system states (or two independent ones for each area). When the control variable, the net demand in this case, shows some kind of correlation between areas, some of these combinations do not necessarily occur. This means that some system states have no representation, and are, therefore, not included in the model. Hence, the number of non-zero variables decreases, with respect to the alternative approach, and the computation time gets reduced. This effect is better shown in Figure 6. In this case, a graphical plot of the clustering results over the same period with 27 system states reveals that st_7 and st_{16} has no representation in the independent clustering approach. This circumstance is indeed the source of the previous results.

Although CPU times are favourable to the independent clustering procedure, the outcomes of the model have yet to be evaluated. Before discussing these results, another reliable ex ante indicator of the performance of each clustering procedure is the clustering error. When comparing the evolution of the net demand over a specific period of time (as depicted in Figure 7) against the hourly curve approximated by system states (27 system states in the case of Figure 7), a significant difference can be appreciated. The better distribution of the clusters over the three-dimensional space achieved by the multi-dimensional clustering procedure results in a more accurate representation of the variability of the net demand for every area. This impact becomes greater as the number of system states increases. This result already shows the limitations of the independent clustering as compared to the multi-dimensional one. In the case of considering demand as control variable, equivalent results are obtained. These results are not shown in the paper due to simplification purposes.

Turning now to the outputs of the model, to evaluate the accuracy of the results it is essential to define metrics that provide a reliable indicator of the performance of this methodology.

The validation parameter selected as the main criteria for this analysis is the relative mean absolute error MAE% (7). This performance indicator describes the relative deviation of the prediction residuals.

$$MAE\% = \frac{\sum_{br} |f_b - y_b|}{\sum_{br} |f_b|}. \qquad (7)$$

In Equation (7), f_b and y_b are the hourly values of an output from the benchmarked model and the approximated one, respectively. In this sense, although the model is formulated with a time resolution of system states, it is possible to obtain the respective hourly results. Since the methodology presented here creates an exact correspondence between the pair 'system state' – hour, an ex post calculation derives these hourly values from the solutions of the model.

The following analysis is carried out for the main outputs of the model: productions and prices. These variables are critical in the decision-making process of market agents in this time horizon.

Firstly, system-aggregated productions over the whole year are analysed in Table 1. As evidenced by the results, the error obtained for both clustering procedures decreases exponentially with the number of system states. However, it is worthy to note that multi-dimensional system states always result in lower errors than the ones obtained with independent clustering. The difference between these measures is more significant as the
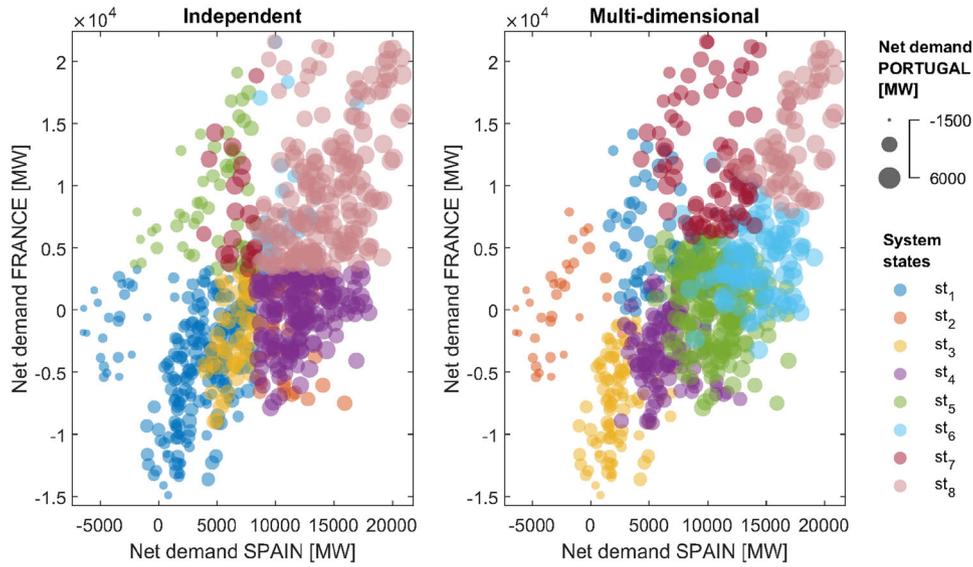
FIGURE 5 Representation of the eight clusters obtained under each procedure according to the values of net demand in each area for a specific month of the annual time series
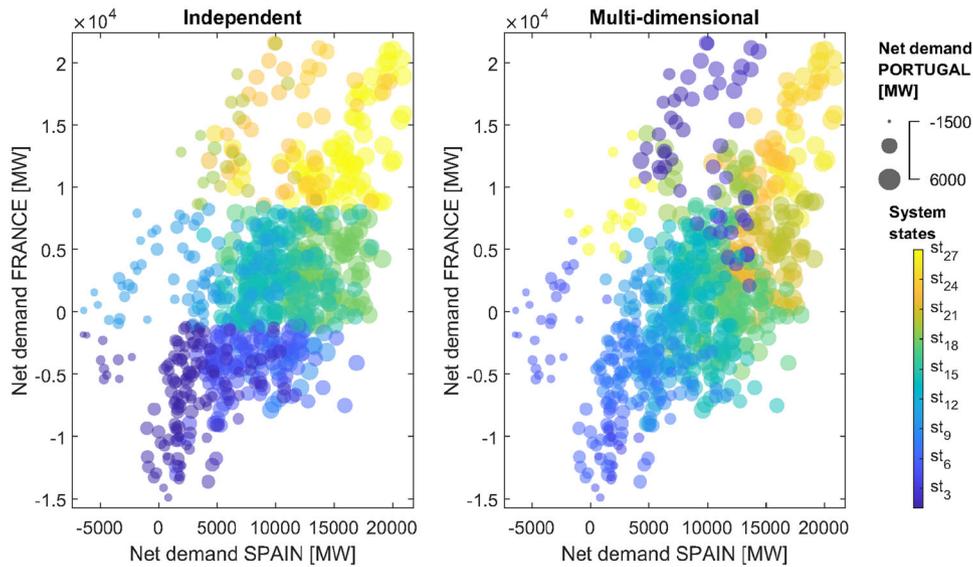
FIGURE 6 Representation of the 27 clusters obtained under each procedure according to the values of net demand in each area for the same month as in Figure 5. Clusters st_7 and st_{16} are not found in the independent approach because of the existent correlation between net demand in each area

number of system states increases. This performance further confirms the expected behaviour according to the clusters distribution achieved in each procedure.

Furthermore, regarding the differences derived from the control variable chosen in the simulations, results show that, as expected, the use of net demand gives a better accuracy in the results. In fact, in all simulations, the error obtained for simulations with net demand is always lower than the one obtained using demand as control variable. As a matter of fact, in some cases, the difference is quite significant. This demonstrates that, for the selected areas, net demand achieves a better capture of

the variability of the system. This leads to an accurate representation of the main outputs of the model.

To have a clearer view of the sensitivity of these results for each area to different market conditions, a Box and Whisker plot of the monthly prediction error is represented in Figure 8. Following the multi-dimensional clustering algorithm proposed here, the error is reduced significantly as the number of system states increase. This is true for every area and every month. In contrast, a meaningfully worse representation is achieved by using the other procedure, which fails to reach low errors, even at a high number of system states.

TABLE 1 Annual error prediction (MAE%) in the production of different technologies (aggregated for the whole system) to the number of system states per month represented, obtained under each clustering implementation

| | Multi-dimensional | | | | | | Independent | | | | | |
| | Thermal | | Hydro | | Non-dispatchable | | Thermal | | Hydro | | Non-dispatchable | |
System States	Demand	Net Demand	Demand	Net Demand	Demand	Net Demand	Demand	Net Demand	Demand	Net Demand	Demand	Net Demand
8	5.1%	4.3%	23.5%	17.9%	19.9%	20.0%	5.2%	4.6%	23.6%	19.6%	20.7%	20.6%
27	4.7%	3.7%	21.8%	15.6%	18.7%	17.8%	5.1%	4.1%	23.0%	16.8%	19.5%	18.6%
64	4.4%	3.4%	20.4%	13.9%	17.4%	15.7%	4.9%	3.8%	21.9%	15.2%	18.7%	17.2%
125	4.1%	3.2%	19.0%	12.8%	16.0%	14.1%	4.7%	3.5%	21.2%	14.0%	18.1%	16.0%
216	3.8%	2.9%	17.2%	11.6%	14.1%	12.2%	4.5%	3.3%	20.5%	13.2%	17.3%	14.7%
343	3.3%	2.5%	14.9%	10.2%	11.2%	9.6%	4.4%	3.2%	19.7%	12.5%	16.8%	13.7%
512	2.6%	2.1%	11.4%	8.5%	6.8%	5.9%	4.3%	3.1%	19.4%	12.1%	16.0%	12.7%
729	0.5%	0.5%	2.4%	2.3%	0.7%	0.6%	4.2%	2.9%	18.7%	11.5%	15.6%	11.8%
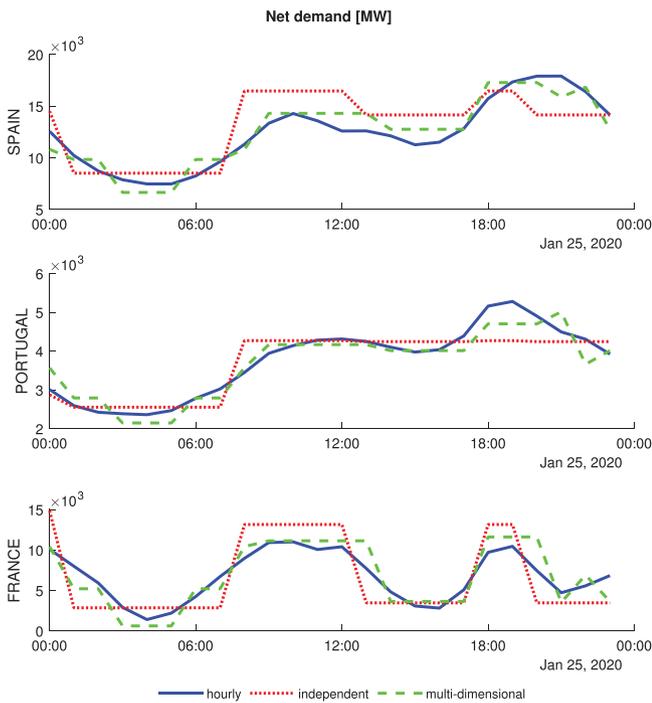
FIGURE 7 Hourly representation of the net demand of 25 January 2020 in each area under different temporal aggregation schemes. The hourly curves resulting from both the multi-dimensional and independent clustering approaches with 27 system states clearly follow the solid line that illustrates the benchmarked hourly simulation. However, the approximation is better achieved with the multi-dimensional procedure

On the whole, the results show that monthly values of MAE% follow the same trend as the annual ones. Furthermore, after further examination, it is possible to observe that the errors obtained in the thermal production are considerably lower than the rest. This is largely due to the fact that most of the thermal generation is accounted for technologies that act as a baseload. This generally translates into a lower variability than the one present in hydro or non-dispatchable units, for instance. In turn, a lower variability results in a lower error. On the other hand,

taking a look at the error distribution for each type of generation, it is discernible that the same pattern is observed for each area. In this context, Portugal profiles represent a special case. The high metrics found in thermal and hydro productions for specific months of the year are result of situations with many hours with production close to zero. This special circumstances have a great impact on the error indicator used, and strongly affect the error distribution in this area.

Moving on with the assessment of the results, electricity price is another variable of major concern in power markets. Figure 9 illustrates the distribution of the monthly values of MAE% for the electricity price obtained in every area under each temporal aggregation configuration.

Again, a similar performance to productions is observed in the electricity price. For every area, the multi-dimensional clustering approach is able to obtain errors 3–4% lower than by using independent clusters when a small number of system states is represented. Meanwhile, this error difference increases to more than 5% for a larger number of states. The same is valid in relation to the choice of the control variable. Lower errors are always found in the case of selecting net demand as the basis for the clustering process

Finally, the gap between both approaches becomes even more evident when analysing the energy flows in the interconnections (Table 2). While the average MAE% in the interconnection between Spain and France following the multi-dimensional approach decreases exponentially with the number of system states until it reaches values close to 0 with 729 states per month, the error found using independent clusters does not drop below 43%.

This highlights the significant improvement of the multi-dimensional approach to properly capture the dynamics between the different areas.

4 | CONCLUSIONS

This paper proposes a new methodology for temporal aggregation in multi-area energy system models. The main objective
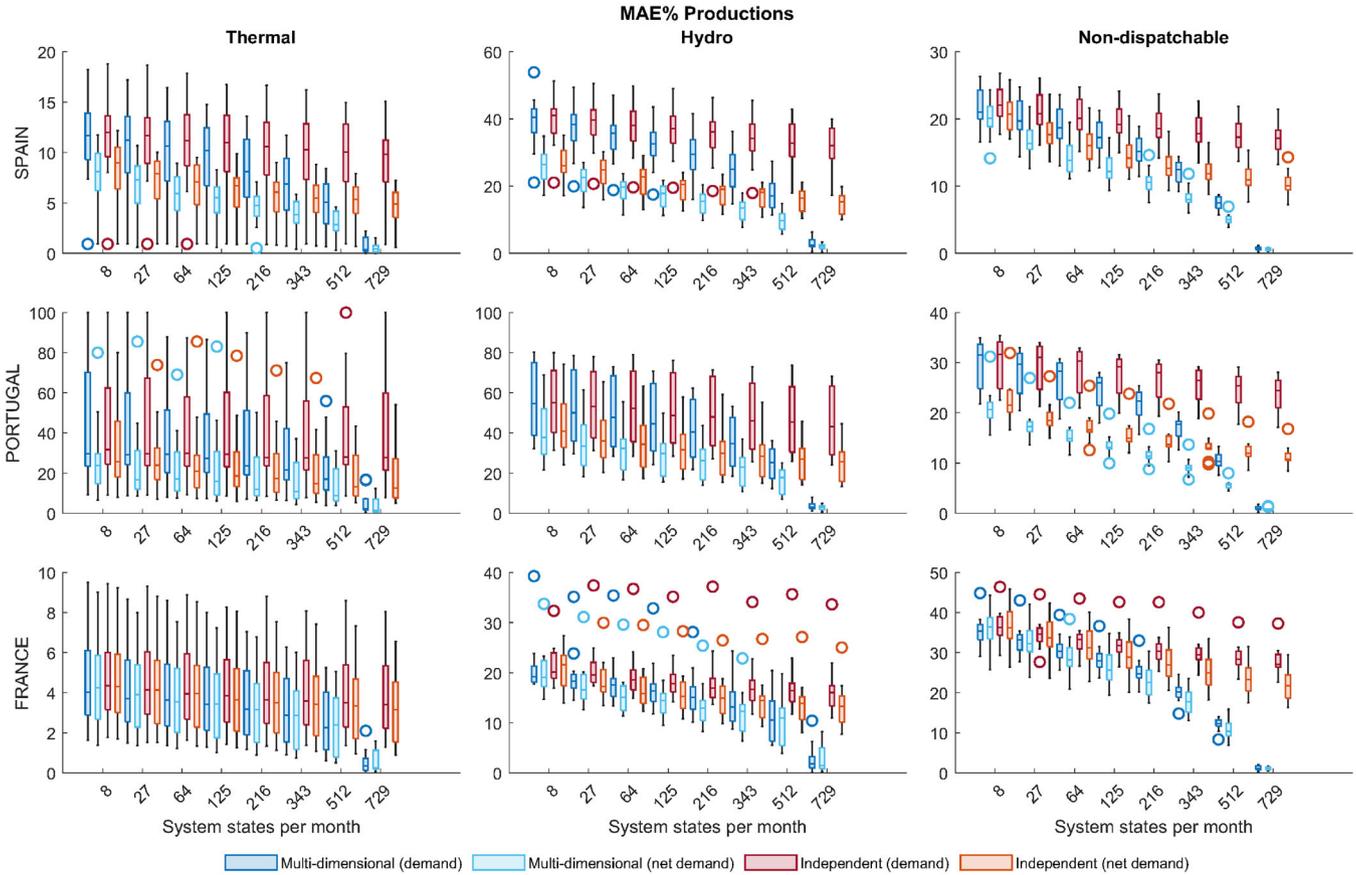
FIGURE 8   Box and Whisker plot of the monthly error prediction (MAE%) in productions of different technologies for each area. Note that the x-axis does not retain a uniform spacing between labels

TABLE 2   Sensitivity of annual error prediction (MAE%) in the energy flows of existing interconnections to the number of system states per month represented, obtained under each clustering implementation

System states	ES-FR				ES-PO			
	Multi-dimensional		Independent		Multi-dimensional		Independent	
	Demand	Net demand	Demand	Net demand	Demand	Net demand	Demand	Net demand
8	64.31%	53.17%	65.38%	58.29%	90.25%	80.45%	89.52%	79.69%
27	58.37%	45.58%	62.30%	49.68%	85.87%	66.91%	88.49%	71.65%
64	53.80%	40.58%	59.65%	45.44%	80.47%	59.71%	84.79%	64.86%
125	49.26%	35.95%	56.91%	40.84%	74.55%	52.32%	82.66%	59.09%
216	43.10%	31.02%	54.21%	38.10%	66.60%	45.06%	80.21%	54.89%
343	34.47%	24.40%	52.16%	35.02%	55.68%	37.07%	77.47%	51.27%
512	22.73%	15.91%	50.58%	33.03%	36.73%	25.30%	75.45%	47.26%
729	2.68%	2.04%	48.33%	29.62%	5.77%	4.12%	71.59%	43.89%

is to obtain computationally tractable models that achieve an accurate representation of the variability present in the system, making them suitable for the medium- and long-term planning of the market.

Nowadays, recent policies and developments in the electricity sector are causing power systems to increase the levels of interconnection between regions while migrating to more sustainable generation technologies. The increasing penetration of
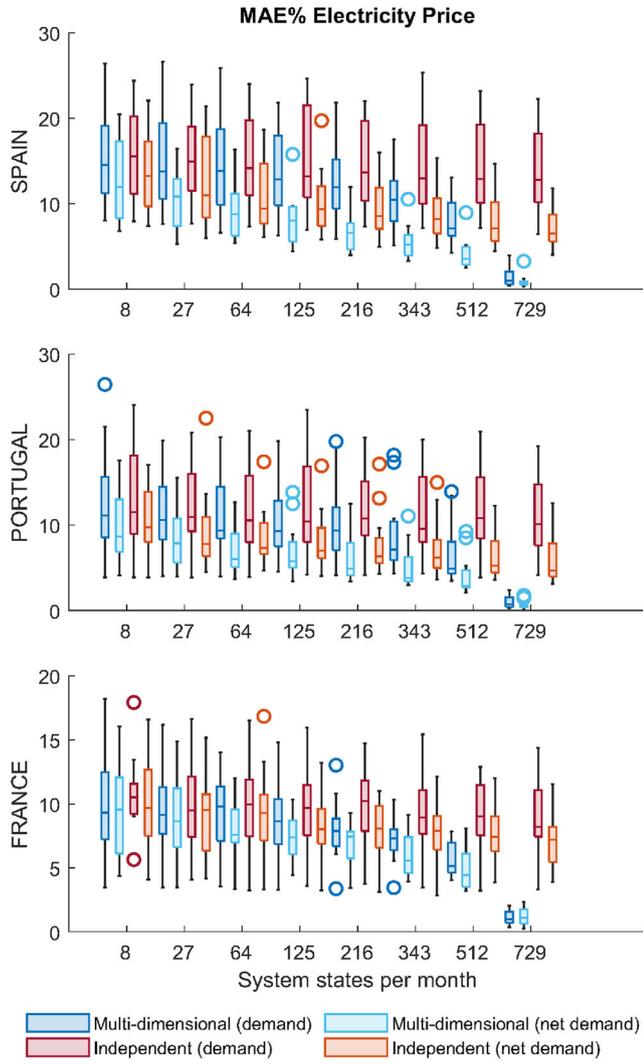
FIGURE 9 Box and Whisker plot of the monthly error prediction (MAE%) in electricity prices for each area. Note that x-axis does not retain a uniform spacing between labels

renewable generation brings along an increment in the short-term variability. In this transition to integrated decarbonized power systems, energy system models play a relevant role. They constitute the main tool used for market agents, regulators and system analysts for the medium- to long-term planning of the system, since they provide a good representation of the market operation. However, to be incorporated in the decision-making process, these tools need to provide realistic and accurate outcomes while remaining computationally tractable.

On the one hand, to provide a realistic representation of the system, energy models need to account for the aforementioned short-term variability, since it plays a determinant role in the operation of the market. On the other hand, to lighten the computational burden, it is common to resort to time reduction techniques. In this case, to avoid losing the necessary spatial detail from an operational point of view, the temporal dimension can be reduced into different forms of time aggregation. As a consequence, multi-area systems modelling represents a challenge because of its complexity and computational burden.

To this end, this paper presents a multi-dimensional clustering algorithm, based on a control variable that drives the market operation in each area. By means of this procedure, it is possible to transform the hourly resolution of the input data into system states, reducing significantly the computational burden required to solve it. This temporal arrangement is highly configurable, according to the desired level of detail and the computational resources available.

To evaluate the effectiveness of this methodology, a real-size case study of the system integrated by Portugal, Spain and France has been implemented. Comparing with the outcomes of a benchmarked hourly simulation, the results show the prediction accuracy provided for this methodology for various temporal aggregation configurations. As presented, the error indicator follows the same trend for monthly and annual values, decreasing exponentially as more system states are represented in the model. Moreover, results also demonstrate the benefits of this methodology against a more conventional approach based on independent clusters per region. In contrast to this approach, the clusters obtained employing the proposed methodology better represent the control variable defined, even when it presents a high correlation between certain areas. In addition, the multi-dimensional clusters offer a wide range of system states selection, while the independent clusters definition is very much constrained by the number of regions modelled.

NOMENCLATURE

Indices and Sets

$a, d' \in A$	Areas
$e \in E$	Market agents
$g \in G$	Renewable energy sources (RES)
$h \in H$	Hydro generation units
$hr \in HR$	Hours
$l \in L$	Interconnections between areas
$p \in P$	Periods (e.g., weeks, months)
$st \in ST$	System states
$t \in T$	Thermal generation units

Parameters

$\overline{F_l}$	Max. power flow in interconnection l [MWh]	
$\theta_{e,a,d',p}$	Conjectured-price response [€/MWh^2]	
$\underline{F_l}$	Min. power flow in interconnection l [MWh]	
$D_{a,p}$	Demand in area a and period p [MWh]	
dur_{st}	Duration of system state st [h]	
$H_{a,l}$	Matrix where a correspondence between areas and interconnections is defined as follows:• 1 if interconnection l starts in area a • −1 if interconnection l ends in area a• 0 if interconnection l does not correspond with area a	
o_g	Ownership of RES unit g [p.u.]	
o_h	Ownership of hydro unit h [p.u.]	
o_t	Ownership of thermal unit t [p.u.]	
$S_{e,a,p}$	Price spread increment [€]	

Variables

$\lambda_{a,p}$ Electricity price in period p [€/MWh]

$be_{h,p,st}$ Pumped power [MW]

$C_{e,a,p}$ Cost function for agent e [€]

$f_{l,p}$ Power flow in interconnection l [MWh]

$ph_{h,p,st}$ Hydro production [MW]

$pres_{g,p,st}$ Renewable energy production [MW]

$pt_{t,p,st}$ Thermal production [MW]

$Q_{e,a,p}$ Production of agent e [MWh]

CONFLICT OF INTEREST

The authors declare no conflict of interest.

DATA AVAILABILITY STATEMENT

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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