



Master in Industrial Engineering (MII)

Assessment of dynamic load-altering attacks on power
system small signal stability

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Abstract

The development of information and communication technologies (ICT) combined with the implementation of smart grids has significantly enhanced the efficiency and reliability of power grid systems. However, without robust security measures, these technological innovations can introduce new vulnerabilities, making power grids susceptible to a wide range of cyberattacks.

In cyberattacks targeting the generation sector, an attacker may attempt to hack into large power plants to disrupt or take control of generation units. In attacks on the distribution and transmission sectors, the attacker might try to manipulate with energy sensors installed throughout the power grid. In the consumer sector, the focus may be on executing load-altering attacks (LAA) to disrupt normal operation.

The shared goal of these attacks is to compromise power system stability which is defined as the capability of the system to return to normal operation status after a disturbance.

Disturbances can be classified as large disturbances which are related to major events related to generator or transmission line outages whose equations cannot be linearized while small disturbances are related to minor events such as small generation or demand variations and whose equations can be linearized. Therefore, if a system is not stable under small disturbances, it will not be stable under large disturbances either so it is important to first evaluate small disturbance stability also known as small signal stability.

The small signal stability analysis is based on the calculation of the eigenvalues of the state matrix A of the system which is given by formulating the equations of the system in standard linearized form in which the derivatives of the state variables are related to the state variables with through the state matrix A . If eigenvalues have negative real part the system is stable but if at least one eigenvalue have positive real part the system becomes unstable.

Related to demand cyberattacks, LAA attempts to control and modify the demand of a group of remotely controllable and insecure loads in order to damage the grid. Several types of loads are potentially vulnerable to attacks of this type, e.g. remotely controllable loads, loads that automatically respond to price commands or direct load control signals, frequency-dependent loads etc.

LAA disruption attacks can be classified based on type, controller type and scope. In this thesis we focus on closed-loop multi-point dynamic load altering attacks (D-LAA) which are characterized of being demand multi-attacks in which the attacker has real-time monitoring of network conditions and focuses on coordinated attacks on multiple loads.

Previously described attack is modeled in frequency dependent loads (FDL), which are those demands that incorporate a frequency controller that modifies the demanded load as a function of the frequency variation in the system mainly for system stabilization purposes. The goal is to design the frequency controller in order to destabilize the system instead of stabilizing it.

Therefore, this master's final thesis has as its overall objective to analyze small signal stability to multi-point closed-loop FDL D-LAA on the IEEE 39-bus system using small signal stability analysis Matlab toolbox.

To this end, this master's final thesis aims to fulfill the following specific objectives:

- Development of a fundamental model to analyze FDL D-LAA small signal stability
- Implementation of FDL D-LAA into a small signal stability analysis Matlab toolbox, which allow more accurate and complex calculations than the fundamental model
- Evaluation of the impact of existing stabilization means such as Power System Stabilizers on the effectiveness of FDL D-LAA

From the development of the fundamental model to analyze FDL D-LAA small signal stability, a system of n generators was modeled with network simplifications such as DC power flow or simplified dynamic generator models, obtaining the following state matrix A for the subsequent eigenvalue calculation.

$$A^{sys} = \begin{bmatrix} 0 & \Omega_{base} & 0 \\ -\frac{1}{2 \cdot \omega_0} \cdot H^{-1} \cdot (B^{agg} + B^{agb} \cdot B^{\theta\delta}) & -\frac{1}{2 \cdot \omega_0} \cdot H^{-1} \cdot D & -\frac{1}{2 \cdot \omega_0} \cdot H^{-1} \cdot B^{agb} \cdot B^{\theta\varphi} \\ -\frac{1}{Tf} \cdot B^{\theta\delta} & 0 & \frac{1}{Tf} \cdot (B^{\theta\varphi} - I) \end{bmatrix} \quad (0.1)$$

In addition, the frequency controller was modeled with the following transfer function where K_j^L is the controller gain and $\Delta\theta_j(s)$ is the bus angle.

$$\Delta p_j^L(s) = K_j^L \cdot \frac{s}{1 + s \cdot Tf} \cdot \Delta\theta_j(s) \quad (0.2)$$

To carry out the FDL D-LAA small signal stability was analyzed by means of Matlab toolbox developed for this purpose. The toolbox needed to be updated to extend its capability to analyze the impact of FDL D-LAA and to enable to design of FDL D-LAA.

A destabilizing strategy was subsequently designed based on shifting the eigenvalues of the system towards the plane with positive real part by varying the controller gain having the following main steps:

- Calculation of the system eigenvalues and selection of the weakest eigenvalue
- Selection of nodes on which to perform the attack based on weakest eigenvalue sensitivities analysis
- Implementation of the attack by means of two control destabilization strategies, manual iterative design and coordinated eigenvalue design

Analyses were conducted in two scenarios within the IEEE 39-bus system: Scenario 1, where only some of the system's generators have stabilizers, and Scenario 2, all of the system's generators have stabilizers.

Some of the main results obtained include: In Scenario 1, using the manual iterative method, the system is destabilized at a controller gain value of $k = -10$. Conversely, in Scenario 2, the system becomes unstable at a controller gain value of $k = -80$.

These gain values indicate that the attacker must be able to modify the demand by a factor of 10 and 80 respectively in order to destabilize the system. That is, with a variation in demand of 1%, the attacker must be able to vary the demand by 10% and 80% in each of the corresponding scenarios.

Based on the results obtained, it was clearly observed that:

- The presence of stabilizers in the generators significantly hinders the destabilization of the system, requiring the attacker to manipulate large amounts of demand to induce system instabilities, which is often not feasible
- The weakest eigenvalue is not necessarily the easiest to destabilize, which makes it difficult to determine which nodes in the system are the most effective for a cyberattack

Additionally, several constraints were identified that must be met to enable a successful cyberattack:

- The attacker must have prior knowledge of which system loads are most vulnerable to causing system instability
- The attacker must have a foundational understanding of controllers design
- The targeted loads must have sufficient power capacity to be increased or decreased

Resumen

El desarrollo de las tecnologías de la información y la comunicación (TIC), combinado con la implantación de redes inteligentes, ha mejorado considerablemente la eficiencia y la fiabilidad de los sistemas eléctricos. Sin embargo, sin medidas de seguridad sólidas, estas innovaciones tecnológicas pueden introducir nuevas vulnerabilidades, haciendo que las redes eléctricas sean susceptibles de sufrir una amplia variedad de ciberataques.

En los ciberataques dirigidos al sector de la generación, un atacante puede intentar manipular grandes centrales eléctricas para interrumpir o tomar el control de las unidades de generación. En los ataques a los sectores de distribución y transmisión, el atacante podría intentar manipular con sensores instalados a lo largo de toda la red. En el sector de los consumidores, el objetivo puede ser ejecutar ataques de alteración de la demanda (LAA).

El objetivo común de estos ataques es comprometer la estabilidad del sistema eléctrico, que se define como la capacidad del sistema de volver a su estado normal de funcionamiento tras una perturbación.

Las perturbaciones se pueden clasificar en grandes perturbaciones, que están relacionadas con grandes eventos como caídas de generadores o líneas de transmisión, cuyas ecuaciones no se pueden linealizar, mientras que las pequeñas perturbaciones están relacionadas con eventos menores, como pequeñas variaciones de la generación o la demanda, y cuyas ecuaciones se pueden linealizar. Por lo tanto, si un sistema no es estable ante pequeñas perturbaciones, tampoco lo será ante grandes perturbaciones, por lo que es importante evaluar primero la estabilidad ante pequeñas perturbaciones, también conocida como estabilidad de pequeña señal.

El análisis de estabilidad pequeña señal se basa en el cálculo de los autovalores de la matriz de estado A del sistema, que se obtiene formulando las ecuaciones del sistema en forma linealizada estándar en la que las derivadas de las variables de estado se relacionan con las variables de estado a través de la matriz de estado A . Si todos los autovalores tienen parte real negativa, el sistema es estable, sin embargo, si al menos un autovalor tiene parte real positiva el sistema es inestable.

En relación con los ciberataques de la demanda, los LAA intenta controlar y modificar la demanda de un grupo de cargas controlables e inseguras con el fin de generar interrupciones en el servicio. Varios tipos de cargas son potencialmente vulnerables a ataques de este tipo, por ejemplo, cargas controlables a distancia, cargas que responden automáticamente a comandas de precios, cargas dependientes de la frecuencia, etc.

Los ataques de alteración de la demanda pueden clasificarse en función del tipo, el control y el alcance. En este trabajo nos centramos en los ataques dinámicos de alteración de la demanda (D-LAA) multipunto y en lazo cerrado que se caracterizan por ser ataques múltiples por demanda en los que el atacante dispone de monitorización en tiempo real de las condiciones de la red y se centra en ataques coordinados a múltiples cargas.

Este tipo de ataque se modelan en cargas dependientes de la frecuencia (FDL), que son aquellas que incorporan un controlador de frecuencia que modifica la carga demandada en función de la variación de frecuencia principalmente con fines de estabilización del sistema. El objetivo es diseñar el controlador de frecuencia para desestabilizar el sistema en lugar de estabilizarlo.

Por lo tanto, este trabajo de fin de máster tiene como objetivo general analizar la estabilidad de pequeña señal ante D-LAA en lazo cerrado y multipunto sobre FDL en el sistema IEEE 39-buses utilizando una toolbox de Matlab especializada en el análisis de estabilidad de pequeña señal.

Para ello, es necesario cumplir con los siguientes objetivos específicos:

- Desarrollar un modelo fundamental para analizar estabilidad de pequeña señal ante D-LAA en FDL
- Implementar D-LAA en FDL en una toolbox de Matlab especializada en el análisis de estabilidad de pequeña señal ya que ofrece mayor precisión y potencia de cálculo que el modelo fundamental
- Evaluar el impacto de la presencia de estabilizadores en los generadores de la red en la efectividad de los D-LAA en FDL

A partir del desarrollo del modelo fundamental para analizar la estabilidad de

pequeña señal ante D-LAA en FDL, se modeló un sistema de n generadores con simplificaciones de red como el uso del flujo de cargas DC o el modelo dinámico simplificado de generador, obteniendo la siguiente matriz de estado A para el posterior cálculo de autovalores.

$$A^{sys} = \begin{bmatrix} 0 & \Omega_{base} & 0 \\ -\frac{1}{2 \cdot \omega_0} \cdot H^{-1} \cdot (B^{agg} + B^{agb} \cdot B^{\theta\delta}) & -\frac{1}{2 \cdot \omega_0} \cdot H^{-1} \cdot D & -\frac{1}{2 \cdot \omega_0} \cdot H^{-1} \cdot B^{agb} \cdot B^{\theta\varphi} \\ -\frac{1}{T_f} \cdot B^{\theta\delta} & 0 & \frac{1}{T_f} \cdot (B^{\theta\varphi} - I) \end{bmatrix} \quad (0.3)$$

Además, el control de frecuencia se modeló con la siguiente función de transferencia donde K_j^L representa la ganancia del control y $\Delta\theta_j(s)$ el ángulo del nudo.

$$\Delta p_j^L(s) = K_j^L \cdot \frac{s}{1 + s \cdot T_f} \cdot \Delta\theta_j(s) \quad (0.4)$$

Para llevar a cabo D-LAA en FDL se analizó la estabilidad de pequeña señal mediante la toolbox de Matlab especializada. Para ellos fue necesario primero actualizar la herramienta para ampliar su capacidad de analizar el impacto de D-LAA en FDL y actualizarla para permitir el diseño del control de FDL.

Se diseñó posteriormente una estrategia de desestabilización basada en el desplazamiento de los autovalores del sistema hacia el plano con parte real positiva variando la ganancia del control teniendo los siguientes pasos:

- Cálculo de los autovalores del sistema y selección del autovalor más débil
- Selección de los nudos de demanda sobre los que realizar el ataque a partir de un análisis de las sensibilidades del autovalor más débil
- Implementación del ataque basado en dos estrategias de desestabilización del control, diseño iterativo manual y diseño coordinado de autovalores

Los análisis se llevaron a cabo en dos configuraciones distintas dentro del sistema IEEE de 39 buses: el Escenario 1, con estabilizadores solamente en algunos de los generadores del sistema; y el Escenario 2, con estabilizadores en todos los generadores del sistema.

Algunos de los resultados más destacados indican que, en el Escenario 1, utilizando el método iterativo manual, el sistema se desestabiliza con un valor de ganancia del controlador de $k = -10$. Por otro lado, en el Escenario 2, también mediante el método iterativo manual, se observa que el sistema se vuelve inestable con un valor de ganancia del controlador de $k = -80$.

Estos valores de ganancia indican que el atacante debe ser capaz de modificar la demanda en un factor de 10 y 80 respectivamente para desestabilizar el sistema. Es decir, con una variación de la demanda del 1%, el atacante debe ser capaz de variar la demanda un 10% y un 80% en cada uno de los escenarios correspondientes.

A partir de los resultados obtenidos en los distintos análisis, se pudo apreciar claramente que:

- La presencia de estabilizadores en los generadores dificulta considerablemente la desestabilización del sistema, ya que obliga al atacante a manipular grandes cantidades de demanda para inducir inestabilidades en el sistema, lo que a menudo no es factible
- El autovalor más débil no es necesariamente el más fácil de desestabilizar, lo que dificulta determinar qué demanda del sistema son los más eficaces para un ciberataque.

Además, se identificaron varias limitaciones que deben cumplirse para que un ciberataque tenga éxito:

- El atacante debe tener conocimiento previo de qué cargas del sistema son más vulnerables a causar inestabilidad en el sistema
- El atacante debe tener conocimientos básicos de diseño de controles
- Las cargas a atacar deben tener suficiente reserva de potencia a subir o a bajar

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1 Introduction

1.1 Problem statement

The development of information and communication technologies (ICT) combined with the implementation of smart grids has significantly enhanced the efficiency and reliability of power grid systems. However, without robust security measures, these technological innovations can introduce new vulnerabilities, making power grids susceptible to a wide range of cyberattacks.

In cyberattacks targeting the generation sector, an attacker may attempt to hack into large power plants to disrupt or take control of generation units. In attacks on the distribution and transmission sectors, the attacker might try to tamper with energy sensors installed throughout the power grid. In the consumer sector, the focus may be on executing load-altering attacks (LAA) to disrupt normal operation. Across all these scenarios, the common goal is typically to inject false data into the wide-area control system to induce network instability.

Related to demand cyberattacks, LAA attempts to control and modify the demand of a group of remotely controllable and insecure loads in order to damage the grid. Several types of loads are potentially vulnerable to attacks of this type, e.g. remotely controllable loads, loads that automatically respond to price commands or direct load control signals, frequency-dependent loads etc.

One possible way of LAA cyberattacks is the manipulation of demand via Internet of Things also known as MaDIoT. IoT devices typically possess lower security levels, and when compromised on a large scale, they can be exploited to diminish the overall security margins of the power system. The ultimate goal of these attacks is to manipulate power demand in a way that overloads or under-utilises the power system, thus causing service interruptions. The possible results of these disruptions can range from localised blackouts to major power outages affecting entire regions or even entire countries.

This master thesis aims to implement and analyze system's small signal stability under dynamic load altering attacks (D-LAAs), which are characterized by being multi-attacks per load, with focus on converting loads to destabilizing frequency-dependent loads (FDL).

Power system stability refers to the grid's ability to return to normal operating conditions after a disturbance. The term "normal" is emphasized because the post-perturbation state must be one where key variables (angles, voltages, and frequency) remain within acceptable ranges defined by system operators. Additionally, the system's topology should remain intact, meaning that protection devices and control actions triggered by the disturbance should not lead to significant system losses or grid separation into islands, which are protective mechanisms to prevent total blackouts.

Disturbances can be classified as "large" or "small." Large disturbances, or transient stability issues, involve significant events like short circuits or major transmission line outages. Small disturbances involve minor perturbations that can be analyzed through linearization of system model equations.

Power system stability is categorized based on key system variables: generator rotor angles, bus voltage magnitudes, and system frequency, as shown in the accompanying figure. The stability classifications are defined and characterized as follows:

- **Angle Stability.** Rotor angle stability refers to the ability of synchronous machines in the grid to remain in synchronism after disturbances.
- **Voltage Stability.** Voltage stability is the ability of the power system to maintain steady voltages at all buses following a disturbance.
- **Frequency Stability.** Frequency stability involves the recovery of system frequency after significant imbalances between generation and load due to disturbances.

1.2 State of the art

Nowadays, studies related to cyber-attack protection systems and the possible consequences of cyber-attacks are crucial to understand the possible risks and mitigation actions in

the face of an accelerated technological development driven by IoT and the artificial intelligence boom.

Several projects related to the need for reliable and secure electricity systems have been developed, such as the development of security systems to protect power grids against cyber-attacks, the study of the security attributes of power grid systems current cyber-security problem or the impact of cyber-attacks on electricity grid generation.

Firstly, the concept of an internet-based load-altering attack was defined, identifying direct and indirect loads that could potentially be compromised [8]. The MaDIoT attack was introduced as an attack that disrupts the normal operation of the power grid by altering power demand using IoT devices to which the attacker has access [11]. They studied these attacks on the Polish grid model, managing to cause local outages and large blackouts in the grid. However, studies suggest the possibility that the Polish grid model under analysis was not N-1 secure, which would lead to an overestimation of the impact of the attacks [3].

The previously mentioned studies show that causing a wide area blackout in a large North American regional system using evenly distributed MaDIoT attacks is extremely difficult. Even if the grid is in a vulnerable state before the attack, such attacks would only cause partial blackouts due to the partial disconnection of loads and generators. The system would quickly recover its stability after this [3].

Researchers examined MaDIoT attacks on the IEEE 39-Bus system by assuming that the attacker has advanced knowledge of the system, allowing them to carry out more sophisticated attacks targeting the most vulnerable nodes in the power system. Results show that these attacks have success rates between 67% and 91% in causing widespread blackouts. However, the likelihood of an attacker with the required system knowledge and resources is estimated to be low.[10]

To date, the LAA literature has mainly focused on static load disruption attacks (S-LAA), where the attack focuses mainly on the volume of vulnerable loads being altered in a single attack per load. In contrast, this project is concerned with D-LAA, which is characterized

by being multi-attacks per load rather than a single attack [9] [6] [5].

Related to D-LAA, we can find a paper which proposes protection schemes after D-LAA based on stability results obtained with a different method from the one that will be used in this project for the small signal stability analysis [1].

1.2.1 D-LAA classification

A Dynamic Load Altering Attack can operate in two different ways: open-loop or closed-loop.

In an open-loop D-LAA, the attacker manipulates unsecured loads without real-time monitoring of grid conditions or the attack's impact on the grid. This approach relies on historical data collected before the attack to determine a pre-programmed trajectory for the compromised loads [4].

In contrast, a closed-loop D-LAA involves continuous monitoring of grid conditions such as price, voltage magnitude or frequency among others depending on the typology of the load. The attacker uses sensors into the existing power system monitoring infrastructure to adjust the load trajectory based on real-time grid conditions. This method allows for precise control over the load changes at the victim load buses [4].

D-LAAs can also be categorized by scope: single-point or multi-point. Single-point D-LAAs target vulnerable loads at one victim load bus, while multi-point D-LAAs involve coordinated attacks on multiple load buses [7]. Figures (1a) and (1b) illustrate examples of single-point and multi-point closed-loop D-LAAs, respectively.

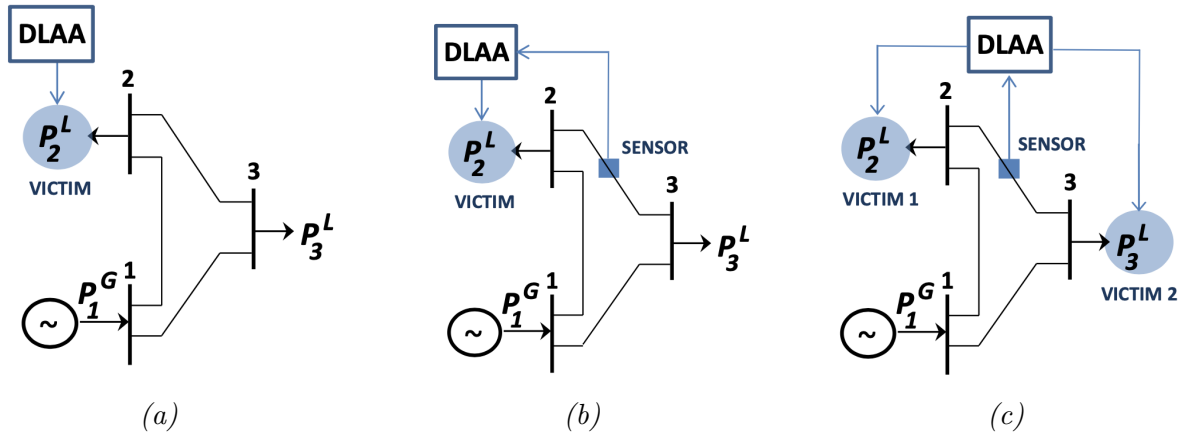


Figure 1: Classification of dynamic load altering attacks: a) open-loop D-LAA, b) single-point closed-loop D-LAA, c) multi-point closed-loop D-LAA

Finally, D-LAAs can be classified based on the type of controller used to manipulate load consumption at the victim buses. Open-loop attacks may use feed-forward controllers, while closed-loop attacks typically use feedback controllers. In closed-loop D-LAAs, attackers might employ controllers such as P, PI, or PID, or more complex feedback control mechanisms [1].

To execute a successful D-LAA, the attacker must compromise a sufficient amount of vulnerable loads, being more effective with a higher amount of unsecured and flexible loads to manipulate.

1.2.2 Closed-loop FDL D-LAA

Frequency-dependent loads are essential in modern power systems, enhancing grid stability, efficiency, and reliability by adjusting power consumption based on frequency changes having the possibility of reducing consumption when the frequency drops and increasing it when the frequency rises helping balance supply and demand.

However, through the internet of things, there is potential for cyber manipulation of frequency controllers to achieve the opposite effect, aiming to destabilize the system in response to frequency changes [1]. Therefore, attacks on FDL can be classified as D-LAA since frequency is a parameter that is continuously measured by the frequency controller, leading to continuous multi-attacks.

As discussed in [12], the impact of this type of attack can potentially force generators offline, causing major system disruptions. These disturbances can trigger cascading effects across the interconnected system, with small localized perturbations potentially causing disruptive impacts in distant areas [2].

To carry out a closed-loop FDL D-LAA, a cyber attacker may typically follow these three main steps:

- **Frequency Monitoring.** Measuring the power grid frequency is generally simple and can be done at any power outlet using an inexpensive commercial sensor. Such sensing mechanisms are already embedded in FDLs that help regulate power usage for frequency regulation.
- **Load Calculation.** Calculate the amount of vulnerable load that can be compromised at the victim bus(es) based on information about how much load is currently being consumed and how much load can potentially be manipulated.
- **Destabilizing Controller Design.** Adjust the victim load frequency controller to generate system instability based on the monitored frequency and the calculated load alteration capability.

1.3 Objectives and methodological approach

This master's final thesis has as its overall objective to analyze small signal stability to multi-point closed-loop FDL D-LAA on the IEEE 39-bus system using small signal stability analysis Matlab toolbox.

To this end, this master's final thesis aims at the following specific objectives:

- Development of a fundamental model to analyze FDL D-LAA small signal stability
- Implementation of FDL D-LAA into a small signal stability analysis Matlab toolbox, which allow more accurate and complex calculations than the fundamental model
- Evaluation of the impact of existing stabilization means such as Power System Stabilizers on the effectiveness of FDL D-LAA

In order to achieve the overall and specific objectives, the small signal disturbance is analyzed by means of Matlab toolbox developed for this purpose. The toolbox needs to be updated to extend its capability to analyze the impact of FDL D-LAA. Further, the toolbox needs to be updated to enable to design of FDL D-LAA.

1.4 Thesis structure

The thesis is organized into seven main sections to facilitate a clear and methodical understanding.

- **Section 1** provides a general introduction to the thesis, setting the context and objectives of the study
- **Section 2** describes the fundamentals of small signal stability, providing an essential theoretical foundation for the analyses
- **Section 3** introduces the FDL D-LAA model
- **Section 4** presents a methodology for designing effective FDL D-LAA systems, outlining the steps and considerations involved
- **Section 5** presents the analysis results, detailing the outcomes of implementing the FDL D-LAA in the IEEE 39-bus system.
- **Section 6** discusses how the work aligns with the Sustainable Development Goals (SDGs), highlighting the contributions and relevance of the study to broader sustainability objectives
- **Section 7** illustrates the conclusions drawn from the research, summarizing key findings and suggesting directions for future work

2 Small signal stability analysis

This section details the study of small signal stability. First, simplified small signal stability analysis for simple systems is described and then the study methodology is generalized for any system.

2.1 Simplified small signal stability analysis

In order to carry out explanation, we consider the classical eigenvalue of a single machine connected to an infinite bus. In this case, small signal stability analysis consists of determining whether the generator's equilibrium point comes back to the original stable equilibrium point, or reaches a new stable equilibrium point after a small-disturbance in the mechanical power supplied by the turbine. This study assumes that the initial equilibrium point is stable and that after the perturbation.

In the case of small disturbances, the nonlinear differential equations that describe the generator dynamic behavior can be linearized around the operating point to study the generator response as follows, based on a Taylor-series expansion:

$$\frac{d\Delta\delta}{dt} = \Delta\omega \quad (2.1)$$

$$\begin{aligned} \frac{d\Delta\omega}{dt} &= \frac{\omega_0}{2H} \left(\Delta P_m - \frac{E'V_\infty}{X_e} \cos \delta_0 \Delta\delta - \frac{D}{\omega_0} \Delta\omega \right) \\ &= \frac{\omega_0}{2H} \left(\Delta P_m - K\Delta\delta - \frac{D}{\omega_0} \Delta\omega \right) \end{aligned} \quad (2.2)$$

Observe in eq. (2.2) that, apart from the mechanical torque (represented by ΔP_m), there is a synchronizing torque (proportional to the rotor angle, that is, $K\Delta\delta$) and a damping torque (proportional to the rotor speed deviation, that is, $\frac{D}{\omega_0}\Delta\omega$) applied to the generator's rotor. The constant K is typically referred to as the synchronizing torque coefficient.

Equations eq. (2.1) and eq. (2.2) can be also written in matrix form as

$$\begin{bmatrix} \Delta\dot{\delta} \\ \Delta\dot{\omega} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{K\omega_0}{2H} & -\frac{D}{2H} \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta\omega \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{\omega_0}{2H} \end{bmatrix} \Delta P_m \quad (2.3)$$

which in “standard” linear system compact form are

$$\Delta \dot{x} = A\Delta x + b\Delta u \quad (2.4)$$

where A is the state matrix and b is the input vector.

The small-signal stability of a generator connected to an infinite bus can be analyzed by applying the Laplace transform to the set of linear differential equations. Assuming zero initial conditions, the result is:

$$\Delta x(s) = (sI - A)^{-1}b\Delta u(s) \quad (2.5)$$

$$\frac{\Delta x(s)}{\Delta u(s)} = \frac{b}{sI - A} \quad (2.6)$$

The small-signal stability of the generator can be therefore determined by calculating the roots of the characteristic equation:

$$\det(sI - A) = 0 \quad (2.7)$$

which results in

$$\det \begin{bmatrix} s & -1 \\ \frac{K\omega_0}{2H} & s + \frac{D}{2H} \end{bmatrix} = s^2 + \frac{D}{H}s + \frac{K\omega_0}{2H} = 0 \quad (2.8)$$

If damping D is positive, the oscillations are damped while if is negative, the oscillations are undamped.

This simple example of one generator can be extended to several generators. Equation (2.9) shows the state-space equations for n generators represented by their classical model¹.

$$\begin{bmatrix} \Delta \dot{\delta} \\ \Delta \dot{\omega} \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & \Omega_{base} \\ -\frac{1}{2\omega_0} \cdot H^{-1} \cdot K^s & -\frac{1}{2\omega_0} \cdot H^{-1} \cdot D \end{bmatrix}}_{=A^{sys}} \cdot \begin{bmatrix} \Delta \delta \\ \Delta \omega \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{2\omega_0} \cdot H^{-1} \end{bmatrix} \cdot \Delta P^m \quad (2.9)$$

¹For more information about state-space representation for n classical model generators see A

Small signal stability is determined by the eigenvalues of the system matrix, A^{sys} . If all eigenvalues have negative real part, the system is asymptotically stable. In other words, if Δp^m is disturbed, generator speeds start oscillating but these oscillations are damped out over time. The damping is mainly affected by the equivalent damping matrix, D , and the distribution of the inertia, whereas the oscillation frequency is affected by the synchronizing power matrix, K^s , and the distribution of the inertia.

2.2 Generalization of small signal stability analysis

Let us consider a dynamic system described by a set of non linear differential equations written in explicit form (the derivatives of the state variables depend only on the state variables x):

$$\dot{x} = f(x) \quad x \in \mathbb{R}^{N \times l} \quad (2.10)$$

If the set of non linear differential equations are linearized around an operating point $x = x_0$, it results in:

$$\Delta \dot{x} = \left. \frac{\partial f(x)}{\partial x} \right|_{x=x_0}, \Delta \dot{x} = A \Delta x, \quad A \in \mathbb{R}^{N \times N}, \quad \Delta x = x - x_0 \quad (2.11)$$

The small signal stability analysis of complex eigenvalues is not performed in practice based on computing the roots of the characteristic equation, given the difficulties of calculating the determinant of a matrix that can be of large dimension. Thus, this analysis is typically carried out by determining the analytical solution of the linear system expressed in terms of the exponential of the state matrix A .

$$\Delta x = e^{At} \Delta x(0) \quad (2.12)$$

The exponential of the state matrix A may be computed using the Taylor expansion:

$$e^{At} = I + \frac{A}{1!}t + \frac{A^2}{2!}t^2 + \dots \quad (2.13)$$

However, this method is not always numerically robust. A physically meaningful alternative is based on the eigenvalues and eigenvectors of the state matrix A . An eigenvalue λ_i of

the state matrix A and the associated right v_i and left w_i eigenvectors are defined as:

$$Av_i = v_i \lambda_i \quad (2.14)$$

$$w_i^T A = \lambda_i w_i^T \quad (2.15)$$

The study of eq. (2.14) and eq. (2.15) indicates that the right and left eigenvectors are not uniquely determined (they are computed as the solution of a linear system of N equations and $N + 1$ unknowns). An approach to eliminate that degree of freedom is to introduce a normalization such as:

$$w_i^T v_i = 1 \quad (2.16)$$

in case of N distinct eigenvalues, eq. (2.14) and eq. (2.15) and can be written together for all eigenvalues in matrix form as:

$$AV = V\Lambda \quad WA = \Lambda W \quad WV = I \quad (2.17)$$

where Λ , V y W are respectively the matrices of eigenvalues and right and left eigenvectors:

$$\Lambda = \begin{bmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_N \end{bmatrix}, \quad V = [v_1 \cdots v_N], \quad W = \begin{bmatrix} w_1^T \\ \vdots \\ w_N^T \end{bmatrix} \quad (2.18)$$

If the exponential of the state matrix e^{At} is expressed in terms of eigenvalues and right and left eigenvectors of the state matrix, it results in:

$$\begin{aligned} e^{At} &= VW + \frac{V\Lambda W}{1!}t + \frac{V\Lambda^2 W}{2!}t^2 + \dots \\ &= V \left(I + \frac{\Lambda}{1!}t + \frac{\Lambda^2}{2!}t^2 + \dots \right) W = Ve^{\Lambda t}W \end{aligned} \quad (2.19)$$

The solution of the set of linear differential equations eq. (2.11) can be expressed in terms of the eigenvalues and right and left eigenvectors of the state matrix A as:

$$\Delta x = Ve^{\Lambda t}W\Delta x(0) = \sum_{i=1}^N v_i e^{\lambda_i t} [w_i^T \Delta x(0)] \quad (2.20)$$

The study of eq. (2.20) allows to draw the following conclusions:

- The system response is expressed as the combination of the system response for N eigenvalues.
- The eigenvalues of the state matrix determine the system stability. A real negative (positive) eigenvalue indicates an exponentially decreasing (increasing) behaviour. A complex eigenvalue of negative (positive) real part indicates an oscillatory decreasing (increasing) behaviour.
- The components of the right eigenvector v_i indicate the relative activity of each variable in the i -th eigenvalue.
- The components of the left eigenvector w_i weight the initial conditions in the i -th eigenvalue.

2.2.1 Modal controllability and observability factors

Let us consider that in the linear dynamic system written in explicit form eq. (2.11) an input u and an output y have been selected:

$$\begin{aligned}\Delta\dot{x} &= A\Delta x + b\Delta u \\ \Delta y &= c^T \Delta x\end{aligned}\tag{2.21}$$

Let us apply to the previous equations a variable transformation defined by the matrix of right eigenvectors $\Delta x = V\Delta\tilde{x}$:

$$\begin{aligned}\Delta\dot{\tilde{x}} &= \Lambda\Delta\tilde{x} + Wb\Delta u \\ \Delta y &= c^T V\Delta\tilde{x}\end{aligned}\tag{2.22}$$

or:

$$\begin{cases} \Delta\dot{\tilde{x}}_i = \lambda_i\Delta\tilde{x}_i + w_i^T b\Delta u \\ \Delta y = c^T v_i\Delta\tilde{x}_i \end{cases} \quad i = 1, \dots, N\tag{2.23}$$

The study of eq. (2.23) allows to draw the following conclusions:

- $w_i^T b$ measures the controllability of the eigenvalues associated to the variable $\Delta \tilde{x}_i$ from the input Δu . In other words, it indicates if the eigenvalue λ_i can be controlled from the input Δu .
- $c v_i$ measures the observability of the eigenvalue associated to the variable $\Delta \tilde{x}_i$ from the input Δy . In other words, it indicates if the eigenvalue λ_i can be observed from the variable Δy .

These results can be summarized as follows: the effectiveness of a control action on an eigenvalue requires that both the eigenvalue is observable from the measured variable Δy and the eigenvalue is controllable from the control variable Δu .

2.2.2 Transfer function residues

The transfer function between Δu and Δy is obtained applying the Laplace transform to the equations (3.16) and eliminating the Laplace transform of the state variables $\Delta x(s)$:

$$\frac{\Delta y(s)}{\Delta u(s)} = c^T (sI - A)^{-1} b \quad (2.24)$$

The transfer function eq. (2.24) can also be written as a partial fraction expansion in terms of the poles p_i and the associated residues $R_{\Delta y/\Delta u, i}$:

$$\frac{\Delta y(s)}{\Delta u(s)} = \sum_{i=1}^N \frac{R_{\Delta y/\Delta u, i}}{(s - p_i)} \quad (2.25)$$

If equation eq. (2.25) is written in terms of the eigenvalues and eigenvectors of the state matrix, it becomes:

$$\frac{\Delta y(s)}{\Delta u(s)} = c^T V (sI - \Lambda)^{-1} W b = \sum_{i=1}^N \frac{c^T v_i w_i^T b}{(s - \lambda_i)} \quad (2.26)$$

The comparison of eq. (2.25) and eq. (2.26) confirms that the eigenvalues of the state matrix are the poles of the open loop transfer function and that the transfer function residues can be computed as the product of the modal controllability and observability factors.

$$R_{\Delta y/\Delta u, i} = c v_i w_i^T b = v_{\Delta y, i} w_{\Delta u, i} \quad (2.27)$$

2.2.3 Eigenvalue sensitivities in feedback systems written in transfer function form

The state space representation of linear dynamic systems is appropriate for the analysis of large scale systems. In this context, the expressions obtained in the previous sections are very useful. However, the transfer function representation of linear dynamic systems is more useful when the design of control systems is considered.

Let us consider the feedback system of Figure 2. The plant to be controlled is eigenvalued by the transfer function $H(s)$ and the controller is represented by the transfer function $F(s, q)$.

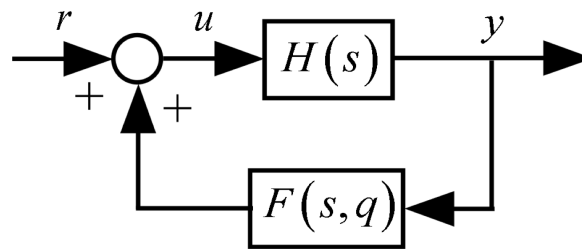


Figure 2: Feedback system in transfer function form

The sensitivity of a pole (eigenvalue) λ_i of the close loop transfer function $\Delta y(s)/\Delta r(s)$ with respect to a parameter q of the controller transfer function $F(s, q)$ is product of the residue of closed loop transfer function $\Delta y(s)/\Delta r(s)$ corresponding to the pole λ_i and the partial derivative of the controller transfer function with respect to the parameter q for $s = \lambda_i$:

$$\frac{\partial \lambda_i}{\partial q} = R_{\Delta y/\Delta r, i} \cdot \left. \frac{\partial F(s, q)}{\partial q} \right|_{s=\lambda_i} \quad (2.28)$$

Therefore, the residue plays a crucial role in the design of controllers, as the variation of an eigenvalue λ_i with respect to a certain controller parameter q for a specific input and output $\Delta y(s)/\Delta r(s)$ is proportional to the residue $R_{\Delta y/\Delta r, i}$ associated with that eigenvalue, input, and output.

3 FDL D-LAA small signal stability model

3.1 Fundamental model

Equation (2.9) can be extended to include frequency-dependent loads. This will show how frequency-dependent loads can affect state matrix and thus the eigenvalues of the system.

A frequency-dependent load at bus j varies its active power consumption according to the bus frequency deviation.

$$\Delta p_j^L = K_j^L \cdot \Delta \dot{\theta}_j \quad (3.1)$$

where K_j^L is the load-frequency damping factor. Since the pure derivative of the bus voltage angle is not causal, the following approximation is used:

$$\Delta \dot{\varphi}_j = \frac{1}{T^f} \cdot (-\Delta \varphi_j + \Delta \theta_j) \quad (3.2a)$$

$$\Delta p_j^L = \frac{K_j^L}{T^f} \cdot (-\Delta \varphi_j + \Delta \theta_j) \quad (3.2b)$$

where $\Delta \varphi_j$ is a new state variable that represents the bus voltage frequency at bus j and T^f is a small filter time constant. Equation (3.2) is the state-space formulation of the following transfer function:

$$\Delta p_j^L(s) = K_j^L \cdot \frac{s}{1 + s \cdot T^f} \cdot \Delta \theta_j(s) \quad (3.3)$$

In matrix form,

$$\Delta \dot{\varphi}^b = \frac{I}{T^f} \cdot (-\Delta \varphi^b + \Delta \theta^b) \quad (3.4a)$$

$$\Delta P^L = \frac{K^L}{T^f} \cdot (-\Delta \varphi^b + \Delta \theta^b) \quad (3.4b)$$

where $K^L = \text{diag}([\dots, K_j^L, \dots])$ is diagonal matrix with the load-frequency damping factors on its diagonal.

Considering small perturbations, the bus voltage angle variations can be expressed in

terms of the nodal load variations and the internal voltage angle variations (see eq. (A.9)):

$$-\Delta P^L = B^{abg} \cdot \Delta\delta + B^{abb} \cdot \Delta\theta^b \quad (3.5)$$

Further, if all loads were frequency dependent, merging eq. (3.4b) and eq. (3.5) leads to:

$$\begin{aligned} \Delta\theta^b &= \left(\frac{K^L}{Tf} + B^{abb} \right)^{-1} \cdot \left(\frac{K^L}{Tf} \cdot \Delta\varphi^b - B^{abg} \cdot \Delta\delta \right) \\ &= B^{\theta\varphi} \cdot \Delta\varphi^b - B^{\theta\delta} \cdot \Delta\delta \end{aligned} \quad (3.6)$$

$$B^{\theta\delta} = \left(\frac{K^L}{Tf} \cdot \Delta\varphi^b - B^{abg} \cdot \Delta\delta \right) \quad (3.7)$$

Equation (3.4a) becomes then:

$$\Delta P^L = \frac{K^L}{Tf} \cdot (B^{\theta\varphi} - I) \cdot \Delta\varphi^b - \frac{K^L}{Tf} \cdot B^{\theta\delta} \cdot \Delta\delta \quad (3.8a)$$

$$\Delta\dot{\varphi}^b = \frac{1}{Tf} \cdot (B^{\theta\varphi} - I) \cdot \Delta\varphi^b - \frac{1}{Tf} \cdot B^{\theta\delta} \cdot \Delta\delta \quad (3.8b)$$

By using eq. (3.7), active power generation injection can now be computed in terms of the state variable as follows:

$$\begin{aligned} \Delta P^{Gg} &= B^{agg} \cdot \Delta\delta + B^{agb} \cdot \Delta\theta^b \\ &= (B^{agg} - B^{agb} \cdot B^{\theta\delta}) \cdot \Delta\delta + B^{agb} \cdot B^{\theta\varphi} \cdot \Delta\varphi^b \end{aligned} \quad (3.9)$$

Equation (A.12) finally becomes by including the load-frequency dynamics:

$$\begin{bmatrix} \Delta\dot{\delta} \\ \Delta\dot{\omega} \\ \Delta\dot{\varphi} \end{bmatrix} = A^{sys} \cdot \begin{bmatrix} \Delta\delta \\ \Delta\omega \\ \Delta\varphi \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{2 \cdot \omega_0} \cdot H^{-1} \\ 0 \end{bmatrix} \cdot \Delta P^m \quad (3.10)$$

where

$$A^{sys} = \begin{bmatrix} 0 & \Omega_{base} & 0 \\ -\frac{1}{2 \cdot \omega_0} \cdot H^{-1} \cdot (B^{agg} + B^{agb} \cdot B^{\theta\delta}) & -\frac{1}{2 \cdot \omega_0} \cdot H^{-1} \cdot D & -\frac{1}{2 \cdot \omega_0} \cdot H^{-1} \cdot B^{agb} \cdot B^{\theta\varphi} \\ -\frac{1}{Tf} \cdot B^{\theta\delta} & 0 & \frac{1}{Tf} \cdot (B^{\theta\varphi} - I) \end{bmatrix} \quad (3.11)$$

Given the state matrix eq. (3.11), small signal stability can be analyzed using the simplified method by directly calculating the roots of the characteristic equation, as described in the example in Section 2.1, or by applying the generalized methodology outlined in Section 2.2.

3.2 Detailed model

Although the fundamental model allows us to simplify the analysis of small signal stability for FDL D-LAA, this model employs simplifications both in terms of calculations and representation of network elements, which does not clearly reflect reality.

So, to analyze FDL D-LAA small signal stability with greater accuracy, frequency-dependent load model was implemented in a Matlab small signal stability toolbox. In this tool, the various network elements are represented in more detail, and employs more accurate calculation methods, such as AC-PF instead of DC-PF, or the generalized methodology for small signal stability calculation described in Section 2.2. Elemental network elements are modeled as follows:

- Generator units are modeled taking into account the rotor equations, the turbine governor, the excitation system, and the stabilizing units
- Loads can be modeled as constant admittances, constant current, or constant power
- The network is represented by its admittance matrix expanded into its real and imaginary parts

FDLs were modeled in Matlab toolbox using the following dynamic controller system which corresponds with previously described eq. (3.3):

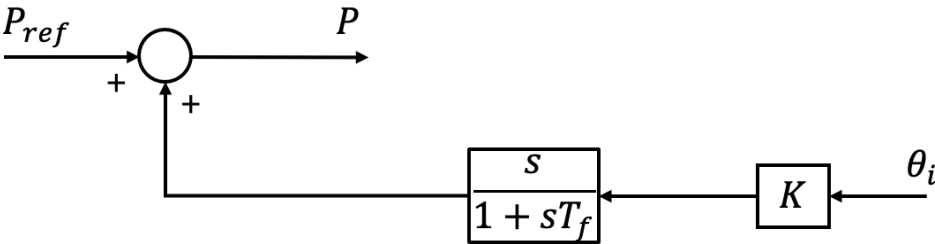


Figure 3: FDL controller

4 FDL D-LAA destabilizing methodology

Small signal stability analysis for FDL D-LAA were carried out using the Matlab toolbox previously described in Section 3 with the aim of determining how easy or difficult it was to destabilize the IEEE 39-bus system ².

The methodology used is as follows:

- First, eigenvalues of the system are calculated, and the weakest eigenvalue selected
- Secondly, the demand buses on which to carry out the cyberattacks are selected. For the weakest eigenvalue, sensitivity analysis was conducted for a transfer function with frequency as the input and power as the output. This analysis aimed to identify the demand buses with the highest residues related to this eigenvalue and transfer function, as attacks on these nodes would have a greater impact on the eigenvalue and consequently to system stability
- Finally, the attack was carried out on the three demand buses with the highest residues by designing a destabilizing FDL controllers with the aim of destabilizing the system. Two different destabilizing controller designs were implemented: manual iterative design and coordinated eigenvalue sensitivity design

4.1 Manual iterative design

This destabilizing controller design consists of iteratively modifying the value of the selected FDL controller gain K shifting eigenvalues to the half-plane with positive real part until one of the system's eigenvalues, usually the one that was initially the weakest, begins to have positive real part, thereby destabilizing the system.

It is important to consider the sign of the controller gain to shift eigenvalues in the right direction and help destabilize the system. For the demand to contribute to destabilizing the system, it should be reduced when the frequency rises to increase excess generation and increased when the frequency drops to worsen the generation deficit. These actions

²For more information about the IEEE 39-bus system see Appendix B

amplify the disparity between generation and demand, leading to greater instability in the system's frequency. Therefore, the sign of the FDL controller gain k must be negative.

4.2 Coordinated eigenvalue sensitivity design

The coordinated eigenvalue sensitivity approach to design a destabilizing controller to destabilize an eigenvalue comprises two steps, the design of the phase compensation network of the controller and the computation of the controller gain.

- The phase compensation network of the controller need to be designed so that the phase of the eigenvalue sensitivity becomes 0 degrees at the eigenvalue natural frequency
- The controller gain needs to be determined such that the eigenvalue moves to the desired position, which in this case is to the point where the eigenvalue begins to have a positive real part

The following Figure 4 shows a geometric interpretation of the eigenvalue sensitivity approach to design a destabilizing controller.

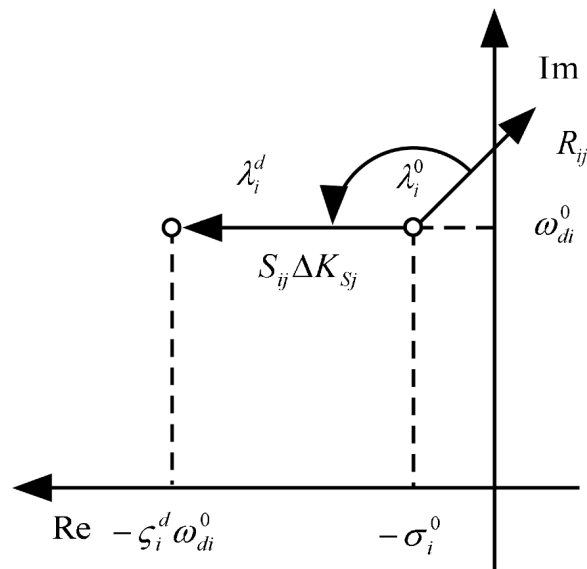


Figure 4: geometric interpretation of the eigenvalue sensitivity approach

Assuming the filtering ratio α_j and the number of stages N_{sj} of the phase compensation networks of the j -th controller, the design of the phase compensation network consists of

determining the time constant T_{sj} of the transfer function

$$\left(\frac{1 + sT_{sj}}{1 + sT_{sj}/\alpha_j} \right)^{N_{sj}} \quad (4.1)$$

so that the phase of the eigenvalue sensitivities with respect to the j -th controller becomes as close as possible to 0 degrees. In other words:

$$\max_{T_{sj}} G(T_{sj}) = \max_{T_{sj}} \sum_{i=1}^{N_E} \beta_{ij} \cos(\arg[S_i(T_{sj})]) \quad (4.2)$$

where N_E is the total number of eigenvalues and:

$$\beta_{ij} = \frac{|R_j|}{\sum_{k=1}^{N_E} |R_{kj}|} \quad (4.3)$$

The filtering ratio of the controllers is determined from the average phase of the sensitivities corresponding to the nodes of interest and assuming the number of stages of the phase compensation networks. It should be noted that φ_j is the average phase of the eigenvalue sensitivities:

$$\varphi_j = \arg \left(\sum_{i=1}^{N_k} S_i(T_{sj} = 0) \right) \quad (4.4)$$

Once the phase of the sensitivities is close to 0, the gains of the controllers are determined to move the eigenvalues to the desired position. The gains of the controllers are determined by solving a linear programming problem. The objective function is to minimize the control action. The control action is expressed as the sum of the gains weighted by the sensitivities:

$$\min \sum_{j=1}^{N_C} \gamma_j \Delta K_{sj} \quad (4.5)$$

where N_C is the total number of controllers being designed and:

$$\Delta K_{sj} = K_{sj} - K_{sj}^0 \quad (4.6)$$

$$\gamma_j = \sum_{i=1}^{N_E} \left| \frac{\partial \lambda_i}{\partial K_{sj}} \right| \quad (4.7)$$

The constraints are the maximum values of the real part of the eigenvalues and the lower and upper bounds of the gains:

$$\sum_{j=1}^{N_C} \operatorname{Re} \left(\frac{\partial \lambda_i}{\partial K_{sj}} \right) \Delta K_{sj} \geq \operatorname{Re} (\lambda_i^d - \lambda_i^0), \quad i = 1, \dots, N_E \quad (4.8)$$

$$K_{sj}^{\min} \leq K_{sj}^0 + \Delta K_{sj} \leq K_{sj}^{\max}, \quad j = 1, \dots, N_C \quad (4.9)$$

λ_i^0 and λ_i^d are respectively the original and the desired eigenvalues. Assuming that the phase of the eigenvalue sensitivity is 0 degrees, the imaginary part of the desired eigenvalue remains constant and the real part is defined by the desired eigenvalue.

The estimated eigenvalue λ_i^c after incorporating the destabilizing controllers can also be determined using the first order eigenvalue sensitivity:

$$\lambda_i^c = \lambda_i^0 + \Delta \lambda_i = \lambda_i^0 + \sum_{j=1}^{N_C} \frac{\partial \lambda_i}{\partial K_{sj}} \Delta K_{sj} = \lambda_i + \sum_{j=1}^{N_C} S_i(T_{sj}) \cdot K_{sj} \quad (4.10)$$

5 Results analysis

Small signal stability analysis for FDL D-LAA were conducted using the detailed model described in Section 3 and following the destabilizing methodology outlined in Section 4. Two different scenarios were considered:

- Scenario 1: Stabilizers on some of the IEEE 39-bus system generators
- Scenario 2: Stabilizers on all of the IEEE 39-bus system generators

5.1 Scenario 1 results analysis

First, system eigenvalues illustrated in the Figure 5 were calculated.

Complex eigenvalues:

Nº.	Real	Imag	Damp	Freq	Variable	Dev	Bus
23	-0.4378	8.8392	4.9472	1.4085	omega	ROT	36
24	-0.3736	8.8146	4.2342	1.4041	omega	ROT	37
25	-0.3904	8.5916	4.5396	1.3688	omega	ROT	33
26	-0.1766	7.4765	2.3610	1.1903	omega	ROT	32
27	-0.2978	7.1872	4.1404	1.1449	omega	ROT	30
28	-0.2590	6.8731	3.7653	1.0947	omega	ROT	35
29	-0.4857	6.2436	7.7564	0.9967	omega	ROT	38
30	-0.2279	6.1878	3.6798	0.9855	omega	ROT	34
31	-0.0866	3.9811	2.1760	0.6338	omega	ROT	39
32	-3.0741	1.5101	89.7556	0.5451	psifd	GEN	38
33	-3.5817	1.4180	92.9785	0.6131	psifd	GEN	38
34	-1.1793	0.9034	79.3833	0.2364	psifd	GEN	36
35	-0.8049	0.8130	70.3584	0.1821	exc2	EXC	36
36	-0.6928	0.8093	65.0319	0.1696	psifd	GEN	30
37	-0.8477	0.7785	73.6539	0.1832	exc2	EXC	32
38	-0.4565	0.6739	56.0812	0.1295	exc2	EXC	37
39	-0.5901	0.6691	66.1441	0.1420	exc2	EXC	31
40	-0.4923	0.6417	60.8743	0.1287	exc2	EXC	35
41	-1.5048	0.6175	92.5126	0.2589	exc2	EXC	38
42	-0.4874	0.6075	62.5779	0.1240	psifd	GEN	34
43	-1.0394	0.2318	97.6020	0.1695	exc2	EXC	39
44	-0.0908	0.0912	70.5328	0.0205	omega	ROT	39

Figure 5: Scenario 1 eigenvalues

It can be observed that the weakest eigenvalue is associated with the generator rotor speed of bus 39, as it has the real part closest to positive values (-0.0866) and the lowest damping ratio (2.176).

Sensitivity analysis was then performed on the previously identified eigenvalue. The residues associated with the weakest eigenvalue for each node of the system, with a frequency input and power output transfer function, are shown in the following Figure 6.

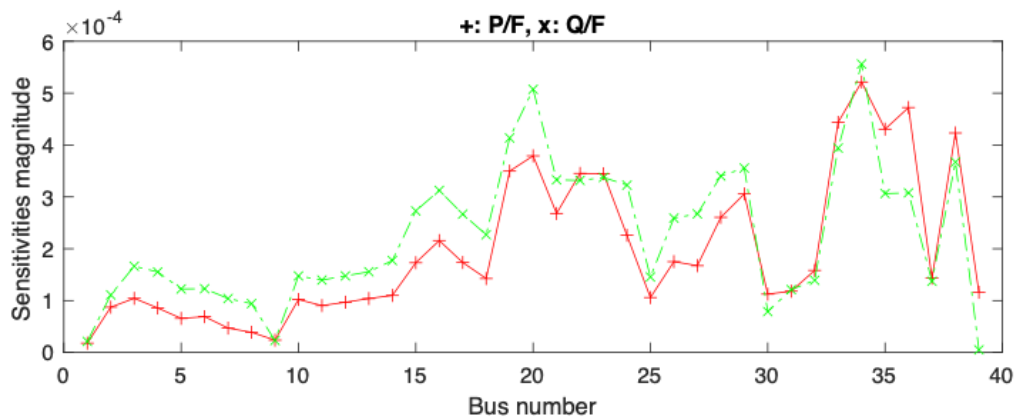


Figure 6: Scenario 1 weakest eigenvalue residues

The demand nodes where the attack would have the greatest impact were selected, being buses 20, 23 and 29 due to their higher residues.

First, manual iterative design of destabilizing FDL controller was implemented in these three nodes. After iterating the control gain value K for each of the selected FDLs, from a value of $K = -10$, the system became unstable. The recalculated eigenvalues of the system for this gain value are shown below in Figure 7.

Complex eigenvalues:

Nº.	Real	Imag	Damp	Freq	Variable	Dev	Bus
22	-0.4354	8.8373	4.9207	1.4082	omega	R0T	36
23	-0.3737	8.8148	4.2359	1.4042	omega	R0T	37
24	-0.3756	8.5914	4.3682	1.3687	omega	R0T	33
25	-0.1762	7.4767	2.3566	1.1903	omega	R0T	32
26	-0.2956	7.1905	4.1076	1.1454	omega	R0T	30
27	-0.2175	6.8925	3.1543	1.0975	omega	R0T	35
28	-0.3590	6.3129	5.6770	1.0064	omega	R0T	38
29	-0.2281	6.1236	3.7229	0.9753	omega	R0T	34
30	0.0143	3.9585	-0.3608	0.6300	omega	R0T	39
31	-2.9660	1.6764	87.0566	0.5422	psifd	GEN	38
32	-3.6075	1.1459	95.3075	0.6024	psifd	GEN	38
33	-1.1856	0.9022	79.5796	0.2371	psifd	GEN	36
34	-0.8050	0.8131	70.3533	0.1821	exc2	EXC	36
35	-0.6927	0.8091	65.0339	0.1695	psifd	GEN	30
36	-0.8494	0.7785	73.7190	0.1834	exc2	EXC	32
37	-0.4568	0.6738	56.1126	0.1296	exc2	EXC	37
38	-0.5902	0.6690	66.1533	0.1420	exc2	EXC	31
39	-0.4923	0.6417	60.8684	0.1287	exc2	EXC	35
40	-1.5105	0.6205	92.5003	0.2599	exc2	EXC	38
41	-0.4874	0.6076	62.5781	0.1240	psifd	GEN	34
42	-1.0581	0.2471	97.3791	0.1729	exc2	EXC	39

Figure 7: Scenario 1 eigenvalues with FDLs at buses 20, 23, and 29, with a gain value of $K = -10$

It can be observed that the eigenvalue associated with the generator rotor speed of bus 39 now has a positive real part (0.0143) and consequently a negative damping ratio (-0.3608), destabilizing the system.

This result indicates that the demand should be changed by a factor of 10 times the frequency in pu in each of the three demand nodes to destabilize the system. For example, if there is a 1% deviation in frequency, demand must be modulated by 10% to destabilize the system.

Secondly, coordinated eigenvalue sensitivity design of FDL destabilizing controller was implemented in the previous three demand nodes, resulting in a value of $k = -6$ for each of the nodes to destabilize the selected eigenvalue and consequently the system, accompanied by a phase compensation of 91° .

This result indicates that if a phase compensation of 91° is added to the controller, the demand should be changed by a factor of 6 times the frequency in pu in each of the three demand nodes to destabilize the system. For example, if there is a 1% deviation in frequency, demand must be modulated by 6% to destabilize the system.

5.2 Scenario 2 results analysis

First, system eigenvalues illustrated in the Figure 8 were calculated.

Complex eigenvalues:

Nº.	Real	Imag	Damp	Freq	Variable	Dev	Bus
33	-2.3846	10.5029	22.1410	1.7141	omega	ROT	37
34	-1.9387	10.0405	18.9589	1.6275	omega	ROT	33
35	-2.6459	8.6579	29.2257	1.4409	omega	ROT	36
36	-1.6227	8.5984	18.5444	1.3926	omega	ROT	32
37	-0.4835	7.2669	6.6381	1.1591	omega	ROT	30
38	-1.4039	7.0828	19.4435	1.1492	omega	ROT	31
39	-2.3425	7.0543	31.5147	1.1830	omega	ROT	38
40	-12.6146	6.6732	88.3935	2.2713	sta 2	STA	35
41	-1.5459	6.4062	23.4577	1.0488	omega	ROT	34
42	-4.5118	4.2878	72.4869	0.9906	sta 2	STA	38
43	-0.5414	3.6862	14.5315	0.5930	omega	ROT	39
44	-5.6714	3.6496	84.0926	1.0734	sta 2	STA	34
45	-4.3339	2.9682	82.5056	0.8360	sta 2	STA	37
46	-2.9155	2.4868	76.0825	0.6099	psifd	GEN	38
47	-2.0441	1.6335	78.1206	0.4164	psifd	GEN	38
48	-4.3475	1.5282	94.3415	0.7334	sta 1	STA	33
49	-1.1864	0.9172	79.1121	0.2387	psifd	GEN	33
50	-0.7911	0.8114	69.8090	0.1804	exc2	EXC	36
51	-0.6900	0.8114	64.7829	0.1695	psifd	GEN	30
52	-0.8688	0.7800	74.4123	0.1858	exc2	EXC	32
53	-0.5907	0.6792	65.6257	0.1433	exc2	EXC	31
54	-0.4503	0.6749	55.4985	0.1291	exc2	EXC	37
55	-0.4853	0.6273	61.1903	0.1262	exc2	EXC	35
56	-0.4808	0.6078	62.0464	0.1233	exc2	EXC	33
57	-1.4727	0.5185	94.3251	0.2485	exc2	EXC	38
58	-1.0351	0.2331	97.5564	0.1689	exc2	EXC	39
59	-0.0970	0.0028	99.9580	0.0154	sta 3	STA	36
60	-3.3181	0.0004	100.0000	0.5281	gov 2	GOV	34
61	-13.3400	0.0001	100.0000	2.1231	gov 1	GOV	38
62	-0.1000	0.0000	100.0000	0.0159	sta 3	STA	34
63	-0.1000	0.0000	100.0000	0.0159	sta 3	STA	32
64	-0.1000	0.0000	100.0000	0.0159	sta 3	STA	39

Figure 8: Scenario 2 eigenvalues

It can be observed that the weakest eigenvalue is now associated with the generator rotor speed of bus 30, as it has the real part closest to positive values (-0.4835) and the lowest damping ratio (6.6381).

Sensitivity analysis was then performed on the previously identified eigenvalue. The residues associated with the weakest eigenvalue for each node of the system, with a frequency input and power output transfer function, are shown in the following Figure 9.

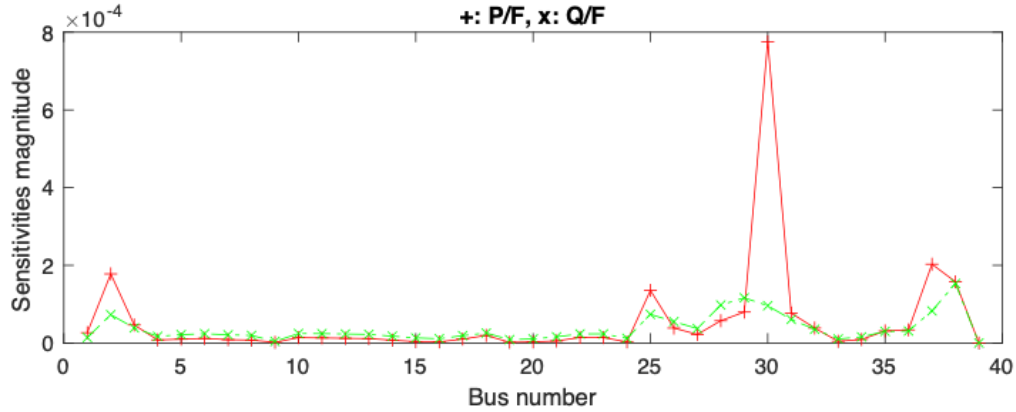


Figure 9: Scenario 2 weakest eigenvalue residues

The demand nodes where the attack would have the greatest effect were selected, being buses 28, 29 and 31 due to their higher residues.

First, manual iterative design of destabilizing FDL controller was implemented in these three nodes. After iterating the control gain value K for each of the selected FDLs, from a value of $K = -80$, the system became unstable. The recalculated eigenvalues for this gain value of the system are shown below in Figure 10.

Complex eigenvalues:

Nº.	Real	Imag	Damp	Freq	Variable	Dev	Bus
35	-2.3943	10.5077	22.2166	1.7152	omega	ROT	37
36	-1.9414	10.0421	18.9808	1.6278	omega	ROT	33
37	-2.6452	8.6576	29.2202	1.4408	omega	ROT	36
38	-0.9442	8.5438	10.9849	1.3681	omega	ROT	38
39	-1.6710	8.5070	19.2747	1.3798	omega	ROT	32
40	0.0236	7.5566	-0.3128	1.2027	omega	ROT	31
41	-0.5089	6.9386	7.3153	1.1073	omega	ROT	30
42	-12.5855	6.6917	88.2953	2.2686	sta 2	STA	35
43	-1.4487	6.6201	21.3780	1.0786	omega	ROT	34
44	-0.3898	3.8513	10.0688	0.6161	omega	ROT	39
45	-4.2273	3.7260	75.0187	0.8968	sta 2	STA	37
46	-5.6903	3.6596	84.1074	1.0768	sta 2	STA	34
47	-4.3062	3.1412	80.7897	0.8483	sta 2	STA	37
48	-3.0505	2.3754	78.9000	0.6153	psifd	GEN	38
49	3.4576	1.9118	-87.5136	0.6288	smesp1	SMES	31
50	-2.1378	1.6102	79.8781	0.4260	psifd	GEN	38
51	-4.3363	1.5246	94.3389	0.7316	sta 1	STA	33
52	-1.2515	1.1206	74.5005	0.2674	psifd	GEN	39
53	-0.7873	0.8146	69.4952	0.1803	exc2	EXC	36
54	-0.6903	0.8091	64.9066	0.1693	psifd	GEN	30
55	-0.9210	0.7451	77.7435	0.1885	exc2	EXC	31
56	-0.4509	0.6743	55.5886	0.1291	exc2	EXC	37
57	-0.6438	0.6311	71.4140	0.1435	exc2	EXC	32
58	-0.4845	0.6269	61.1496	0.1261	exc2	EXC	35
59	-0.4806	0.6086	61.9753	0.1234	exc2	EXC	33
60	-1.4417	0.5514	93.4017	0.2457	exc2	EXC	38
61	-1.1574	0.4587	92.9638	0.1981	exc2	EXC	39
62	-1.5173	0.0201	99.9912	0.2415	psikq1	GEN	39
63	-3.3189	0.0003	100.0000	0.5282	gov 2	GOV	34
64	-0.1001	0.0002	99.9998	0.0159	sta 3	STA	35
65	-13.3427	0.0001	100.0000	2.1236	gov 1	GOV	32
66	-1.6673	0.0000	100.0000	0.2654	gov 4	GOV	30
67	-0.1000	0.0000	100.0000	0.0159	gov 3	GOV	34
68	-0.1000	0.0000	100.0000	0.0159	gov 3	GOV	37

Figure 10: Scenario 2 eigenvalues with FDLs at buses 28, 29, and 31, with a gain value of $K = -80$

It can be observed that the eigenvalue that destabilizes the system had changed. The eigenvalue associated with the generator rotor speed of bus 31 now has a positive real part (0.0236) and consequently a negative damping ratio (-0.3128), destabilizing the system.

This result indicates that the demand should be changed by a factor of 80 times the frequency in pu in each of the three demand nodes to destabilize the system. For example, if there is a 1% deviation in frequency, demand must be modulated by 80% to destabilize the system.

It can be seen that the PSS makes it much more difficult to destabilize the system compared to scenario 1. The difficulty is due to the fact that a larger amount of demand has to be modified, which indicates that MaDIoT must infiltrate much more demand.

Secondly, coordinated eigenvalue sensitivity design of FDL destabilizing controller was implemented in the previous three demand nodes, resulting in a value of $k = -10000$ for each of the nodes to destabilize the selected eigenvalue and consequently the system, accompanied by a phase compensation of 105° .

This high result for the control gain k is due to the fact that the coordinated eigenvalue sensitivity design focuses on destabilizing the selected eigenvalue without considering the values of the other eigenvalues in the system. In this case, the eigenvalue that begins to destabilize the system is not the weakest eigenvalue, but the eigenvalue associated to generator rotor speed of bus 31 being this eigenvalue the most sensitive to an attack on the selected nodes. The weakest eigenvalue would start to destabilize at much higher values of k , by which the system would already be destabilized.

5.3 Sensitivity analysis

After conducting the analyses for both scenarios, sensitivity studies were developed, which involved extrapolating the results obtained in Scenario 1 to Scenario 2.

First, the nodes associated with the weakest eigenvalue in Scenario 1—the generator rotor speed of bus 39 eigenvalue—where the attack would have the greatest impact were selected. Nodes 20, 23, and 29 were targeted in Scenario 2 with a controller gain value of $k = 10$, which successfully destabilized the system in Scenario 1.

Secondly, the eigenvalues for Scenario 2 were calculated. As shown in the Figure 11, attacking the previously mentioned FDL demand nodes with a control gain value of $k = 10$ does not succeed in destabilizing the system having all eigenvalues negative real part.

Complex eigenvalues:

Nº.	Real	Imag	Damp	Freq	Variable	Dev	Bus
31	-2.3858	10.5022	22.1525	1.7141	omega	ROT	37
32	-1.9065	10.0461	18.6450	1.6274	omega	ROT	33
33	-2.6451	8.6612	29.2084	1.4413	omega	ROT	36
34	-1.6223	8.5989	18.5396	1.3927	omega	ROT	32
35	-0.4906	7.2703	6.7324	1.1597	omega	ROT	30
36	-2.1314	7.1368	28.6159	1.1854	omega	ROT	38
37	-1.3957	7.0938	19.3050	1.1507	omega	ROT	31
38	-12.5597	6.6857	88.2727	2.2645	sta 2	STA	35
39	-1.5000	6.3950	22.8358	1.0454	omega	ROT	34
40	-4.4267	4.2160	72.4133	0.9729	sta 2	STA	38
41	-0.4828	3.7088	12.9087	0.5953	omega	ROT	39
42	-5.6760	3.6104	84.3771	1.0706	sta 2	STA	34
43	-4.3290	2.9895	82.2860	0.8373	sta 2	STA	37
44	-2.9302	2.4793	76.3405	0.6109	psifd	GEN	38
45	-2.0358	1.6404	77.8669	0.4161	psifd	GEN	39
46	-4.3563	1.5169	94.4383	0.7342	sta 1	STA	33
47	-1.1929	0.9157	79.3231	0.2393	psifd	GEN	33
48	-0.7909	0.8117	69.7860	0.1804	exc2	EXC	36
49	-0.6900	0.8112	64.7885	0.1695	psifd	GEN	30
50	-0.8698	0.7805	74.4282	0.1860	exc2	EXC	32
51	-0.5907	0.6791	65.6296	0.1432	exc2	EXC	31
52	-0.4504	0.6747	55.5191	0.1291	exc2	EXC	37
53	-0.4851	0.6274	61.1679	0.1262	exc2	EXC	35
54	-0.4809	0.6079	62.0435	0.1234	exc2	EXC	33
55	-1.4767	0.5203	94.3181	0.2492	exc2	EXC	38
56	-1.0548	0.2483	97.3391	0.1725	exc2	EXC	39
57	-3.3182	0.0004	100.0000	0.5281	gov 2	GOV	34
58	-13.3399	0.0001	100.0000	2.1231	gov 1	GOV	38
59	-0.1000	0.0000	100.0000	0.0159	gov 3	GOV	34
60	-0.1000	0.0000	100.0000	0.0159	gov 3	GOV	35

Figure 11: Scenario 2 eigenvalues with FDLs at buses 20, 23, and 29, with a gain value of $K = -10$

Thirdly, the gain value K of the FDL controller for demand nodes 20, 23, and 29, at which the Scenario 2 system begins to destabilize, was determined. Manual iterative design of the destabilizing FDL controller was implemented, resulting in a gain value of $k = 70$, from which the system in Scenario 2 becomes unstable, as shown in Figure 12, with the generator rotor speed of node 39 eigenvalue having a positive real part.

Complex eigenvalues:

N0.	Real	Imag	Damp	Freq	Variable	Dev	Bus
31	-2.3920	10.4985	22.2151	1.7137	omega	ROT	37
32	-1.7328	10.1389	16.8468	1.6371	omega	ROT	33
33	-2.6421	8.6822	29.1131	1.4444	omega	ROT	36
34	-1.6175	8.6009	18.4824	1.3929	omega	ROT	32
35	-1.1452	7.8110	14.5066	1.2564	omega	ROT	38
36	-0.5703	7.2127	7.8818	1.1515	omega	ROT	30
37	-1.3040	7.1650	17.9053	1.1591	omega	ROT	31
38	-12.3622	6.7850	87.6643	2.2444	sta 2	STA	35
39	-1.2077	6.5346	18.1739	1.0576	omega	ROT	34
40	-3.9679	3.9008	71.3105	0.8856	sta 2	STA	38
41	0.0496	3.8676	-1.2833	0.6156	omega	ROT	39
42	-5.6564	3.4667	85.2606	1.0559	sta 2	STA	34
43	-4.3693	3.1176	81.4024	0.8543	sta 2	STA	37
44	-3.0060	2.4188	77.9100	0.6141	psifd	GEN	38
45	-1.9927	1.6824	76.4101	0.4151	psifd	GEN	39
46	-4.3923	1.4628	94.8770	0.7368	sta 1	STA	33
47	-1.2214	0.9043	80.3687	0.2419	psifd	GEN	33
48	-0.7902	0.8127	69.7096	0.1804	exc2	EXC	36
49	-0.6899	0.8106	64.8136	0.1694	psifd	GEN	30
50	-0.8730	0.7823	74.4718	0.1866	exc2	EXC	32
51	-0.5906	0.6786	65.6496	0.1432	exc2	EXC	31
52	-0.4510	0.6745	55.5814	0.1291	exc2	EXC	37
53	-0.4845	0.6276	61.1084	0.1262	exc2	EXC	35
54	-0.4810	0.6084	62.0172	0.1234	exc2	EXC	33
55	-1.4990	0.5282	94.3155	0.2529	exc2	EXC	38
56	-1.1157	0.2853	96.8828	0.1833	exc2	EXC	39
57	-3.3189	0.0003	100.0000	0.5282	gov 2	GOV	34
58	-0.1000	0.0000	100.0000	0.0159	sta 3	STA	34
59	-0.1000	0.0000	100.0000	0.0159	sta 3	STA	34
60	-0.1000	0.0000	100.0000	0.0159	sta 3	STA	39

Figure 12: Scenario 2 eigenvalues with FDLs at buses 20, 23, and 29, with a gain value of $K = -70$

These results indicate that the eigenvalue selected to determine which nodes would be most effective for a cyberattack may not necessarily be directly related to the weakest eigenvalue.

In this case, targeting nodes 20, 23, and 29—identified from the sensitivity analysis of the generator rotor speed eigenvalue of node 39—proved to be more effective than attacking

nodes 28, 29, and 31, which were identified from the sensitivity analysis of the weakest eigenvalue in Scenario 2, associated with the generator rotor speed of bus 30, requiring FDL controller gain values of 70 and 80 to destabilize the system, respectively.

This ultimately reaffirms the conclusion that the weakest eigenvalue is not necessarily the easiest to destabilize, complicating the task of identifying which nodes in the system are the most effective targets for a cyberattack.

6 Alignment with Sustainable Development Goals (SDGs)

The development and implementation of smart power grids are intrinsically linked to SDG 9 and SDG 11, which emphasize building resilient infrastructure, sustainable cities, promoting inclusive and sustainable industrialization, and fostering innovation.

Smart grids integrate power systems with information and communication technologies (ICT), enhancing reliability and flexibility but also opening the door to cyberattacks.

6.1 SDG 9: Industry, Innovation, and Infrastructure

Power grid infrastructure is a vital component for economic development and job creation. Cyberattacks can severely damage power grid infrastructure, disrupting industries and hindering the ability to innovate. Resilient infrastructure against cyberattacks is critical to maintaining the integrity of these systems and promoting sustainable economic growth.

This project contributes to SDG 9 by providing tools to analyze and understand cyber threats to power grid systems through both a fundamental model and a more precise Matlab implementation. These tools enhance the ability to detect vulnerabilities and improve the resilience of power grid infrastructure.

By advancing cybersecurity in the power grid, the project stimulates innovation in cybersecurity technologies and more efficient energy management systems. This innovation not only protects critical infrastructure but also contributes to the development of advanced technological solutions that can be applied in other industries and sectors, thus fostering resilience and innovation.

6.2 SDG 11: Sustainable Cities and Communities

Urban areas rely heavily on electric power to ensure a high quality of life. Protecting the power grid from cyberattacks is essential for creating safer and more sustainable cities by preventing power supply disruptions that could affect daily life and the operation of

essential services.

This project supports SDG 11 by enhancing the capability to study and mitigate the impacts of cyberattacks on smart grids, thereby improving the resilience of urban power infrastructures. The availability of reliable power is critical to the development of both urban and rural communities.

By providing tools for comprehensive analysis and protection against frequency dependent loads dynamic load altering attacks, this research aims to ensure uninterrupted power supply, which promotes the continuous operation of essential services in cities.

Furthermore, studying cyberattacks is essential for the smooth integration of renewable energy sources within smart grids, contributing to sustainable energy management and reducing the environmental impact of urban energy consumption. This holistic approach not only makes cities more resilient but also fosters a cleaner and more sustainable urban environment.

7 Conclusions

It's crucial to understand the significant impact that cyber threats can have on the evolving landscape of smart power grids. The integration of electrical power systems with IoT and smart grids enhances reliability and flexibility, but also opens the door to cyberattacks.

This master's thesis aimed to analyze small signal stability in the IEEE 39-bus system using a Matlab toolbox for small signal stability analysis.

To achieve this, the thesis focused on several specific goals. Firstly, the development of a fundamental model to assess small signal stability of FDL D-LAA. Secondly, the implementation FDL D-LAA into an advanced Matlab toolbox, enhancing its capability for more accurate and complex calculations compared to the fundamental model. Lastly, the impact evaluation of existing stabilization measures, such as Power System Stabilizers, on the effectiveness of FDL D-LAA.

To meet these objectives, the analysis of small signal disturbances were conducted using the specialized Matlab toolbox after extending its functionality to both analyze and design FDL D-LAA effectively.

Based on the results obtained, it was clearly observed that:

- The presence of stabilizers in the generators significantly hinders the destabilization of the system, requiring the attacker to manipulate large amounts of demand to induce system instabilities, which is often not feasible
- The weakest eigenvalue is not necessarily the easiest to destabilize, which makes it difficult to determine which nodes in the system are the most effective for a cyberattack

Additionally, several constraints were identified that must be met to enable a successful cyberattack:

- The attacker must have prior knowledge of which system loads are most vulnerable to causing system instability

- The attacker must have a foundational understanding of controllers design
- The targeted loads must have sufficient power generation capacity to be increased or decreased

For future work, the small signal stability results presented in this work must be confirmed by non-linear time-domain simulations taking into account the limited amount of load available. This helps understanding to what extent FDL D-LAA could be an actual threat. Further, other type of input signals instead of frequency should be analyzed, to further understand the impact of the input signal. In general terms and independently of the cyberattack framework, the selection of appropriate eigenvalues a control should act upon without affecting others should be addressed.

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A State-space representation for n classical model generators

A.1 DC power flow (DC-PF)

The DC-PF assumes that (i) voltage magnitudes are around nominal (1 pu), (ii) branch resistances can be neglected (which is commonly an acceptable assumption in transmission networks), (iii) the angle difference between two adjacent buses is typically small. Equation (A.1a) shows the active power flow of a branch between buses j and k . The active power balance at bus j is shown in (A.1b) according to Kirchhoff's law.

$$p_{jk} = \frac{\theta_j - \theta_k}{x_{jk}} \quad (\text{A.1a})$$

$$p_j = \sum_{k \in \mathcal{B}_j} p_{jk} = p_j^G - p_j^L \quad (\text{A.1b})$$

where p_j and θ_j are the power injection and the voltage angle at bus j and p_{jk} and x_{jk} are the branch flow and the branch reactance of branch from bus j to bus k . The nodal power injection is the difference between the nodal generation, p_j^G , and load, p_j^L . Equation (A.1b) can be generalized for all branches and buses in the system as follows:

$$P^l = X^{-1} \cdot A \cdot \theta^b \quad (\text{A.2a})$$

$$P^b = A^T \cdot P^l = P^G - P^L \quad (\text{A.2b})$$

where $X = \text{diag}([\dots, x_{jk}, \dots])$ is a diagonal matrix with the branch reactances on its diagonal, and A is the incidence matrix relating branches and buses in the same order as in X . P^l and P^b are vectors of branch flow and bus injections, and P^G and P^L are vectors of nodal generation and load. From (A.2) one can obtain:

$$P^G - P^L = A^T \cdot X^{-1} \cdot A \cdot \theta^b = B' \cdot \theta^b \quad (\text{A.3})$$

Note that B' is singular (not invertible) given the linear dependency of the A . One way to deal with the singularity is to define a reference bus, where the voltage angle is arbitrarily

fixed (e.g., 0 rad/s). The corresponding row and column of this bus are then eliminated for matrix inversion. Further, eq. (A.2b) can be expanded to explicitly represent generator (g) and non-generator buses (ng) in the following way:

$$\begin{bmatrix} P^{Gg} - P^{Lg} \\ -P^{Lng} \end{bmatrix} = B' \cdot \theta^b \quad (\text{A.4})$$

Note that the first buses are generator buses, followed by non-generator buses. Nodal generation at non-generator buses is 0.

A.2 Augmented DC-PF by explicitly representing generators

Matrix B' can be augmented to include the transient reactances of generators. Generators are modelled by a simplified electrical circuit representing an internal voltage, e_g , behind a transient reactance, x_{gk}' , of the generator at bus k as follows:

$$e_g = u_k + j \cdot x_{gk}' \cdot i_{gk} \quad (\text{A.5})$$

In this case, the active power injected by the generator g at bus k can be approximated as shown in (A.6) by making use of similar hypotheses as for the DC-PF:

$$p_{gk} = \frac{\delta_g - \theta_k}{x_{gk}'} \quad (\text{A.6})$$

The expression in eq. (A.6) very much resembles the one in eq. (A.1a) and the active power injection can be handled as an active power inflow from the internal voltage of generator g to the bus k . Equation (A.6) can be generalized as follows:

$$P^{lG} = (X')^{-1} \cdot \begin{bmatrix} A^g & A^{ng} \end{bmatrix} \cdot \begin{bmatrix} \delta \\ \theta^b \end{bmatrix} \quad (\text{A.7a})$$

$$P^{Gg} = \begin{bmatrix} (A^g)^T \\ (A^{ng})^T \end{bmatrix} \cdot (X')^{-1} \cdot \begin{bmatrix} A^g & A^{ng} \end{bmatrix} \cdot \begin{bmatrix} \delta \\ \theta^b \end{bmatrix} \quad (\text{A.7b})$$

where $X' = \text{diag}([\dots, x_{gk}', \dots])$ is a diagonal matrix with the transient reactances on its diagonal and $\begin{bmatrix} A^g & A^{ng} \end{bmatrix}$ is the incidence matrix of the active power inflows. P^{lG} and P^{Gg} are

the vector of active power inflow and the vector of nodal generation, respectively. Note that A^g is a diagonal matrix if the explicit generator representation follows the same order as in eq. (A.4). In that case, the first columns corresponding to the generators in A^{ng} also form a diagonal matrix.

The explicit generator representation can then be included by augmenting B' appropriately. By using eq. (A.4) and eq. (A.7) the augmented DC-PF becomes:

$$\begin{bmatrix} P^{Gg} \\ -P^L \end{bmatrix} = \begin{bmatrix} (A^g)^T \cdot (X')^{-1} \cdot A^g & (A^g)^T \cdot (X')^{-1} \cdot A^{ng} \\ (A^{ng})^T \cdot (X')^{-1} \cdot A^g & B' + (A^{ng})^T \cdot (X')^{-1} \cdot A^{ng} \end{bmatrix} \cdot \begin{bmatrix} \delta \\ \theta^b \end{bmatrix} \quad (\text{A.8})$$

or

$$\begin{bmatrix} P^{Gg} \\ -P^L \end{bmatrix} = \begin{bmatrix} B^{agg} & B^{agb} \\ B^{abg} & B^{abb} \end{bmatrix} \cdot \begin{bmatrix} \delta \\ \theta^b \end{bmatrix} = B^a \cdot \begin{bmatrix} \delta \\ \theta^b \end{bmatrix} \quad (\text{A.9})$$

Further, the matrix B^a in eq. (A.9) is still singular. Again, this can be handled by defining a reference generator (e.g., the first one) and eliminating the corresponding row and column. Note that B^{abb} is non-singular.

A.3 Fundamental dynamic model

The fundamental dynamic model of the power system couples the dynamics of the generators by means of the network. The network is represented by the augmented DC-PF. The dynamics of the generator g at bus k are modelled by means of the classical generator model.

$$\dot{\delta}_g = \Omega_{base} \cdot (\omega_g - \omega_0) \quad (\text{A.10a})$$

$$2 \cdot H_g \cdot \omega_0 \cdot \dot{\omega}_g = p_g^m - p_g^e - D_g \cdot (\omega_g - \omega_0) \quad (\text{A.10b})$$

where H_g and D_g are the inertia constant and the equivalent damping factor (representing damper windings, PSS, etc.) of the generator. Ω_{base} and ω_0 are the base angular speed in rad/s and the nominal angular speed per unit (i.e., 1 pu). p_g^m and p_g^e are the mechanical and electrical power of the generator g at bus k . Note that $p_g^e = p_{gk}$ (see also eq. (A.6)). Equation (A.10) can be expressed in matrix form as follows:

$$\dot{\delta} = \Omega_{base} \cdot (\omega - \omega_0 \cdot I) \quad (\text{A.11a})$$

$$2 \cdot H \cdot \omega_0 \cdot \dot{\omega} = P^m - P^{Gg} - D \cdot (\omega - \omega_0 \cdot I) \quad (\text{A.11b})$$

where H and D are diagonal matrices with the inertia constants and equivalent damping factors on their diagonals. I is the identity matrix. If the perturbations are sufficiently small, eq. (A.11) can be linearized.

$$\Delta \dot{\delta} = \Omega_{base} \cdot \Delta \omega \quad (\text{A.12a})$$

$$2 \cdot H \cdot \omega_0 \cdot \Delta \dot{\omega} = \Delta P^m - \Delta P^{Gg} - D \cdot \Delta \omega \quad (\text{A.12b})$$

Similarly and if the load does not vary (e.g., constant power loads), eq. (A.9) becomes for small variations:

$$\begin{bmatrix} \Delta P^{Gg} \\ 0 \end{bmatrix} = \begin{bmatrix} B^{agg} & B^{agb} \\ B^{abg} & B^{abb} \end{bmatrix} \cdot \begin{bmatrix} \Delta \delta \\ \Delta \theta^b \end{bmatrix} \quad (\text{A.13})$$

From eq. (A.13) it becomes clear that first, under no (or neglectable) load variations, there exists a direct relation between the bus voltage angles, $\Delta \theta$, and the angles of the internal voltages, $\Delta \delta$. Second, generation variations and variations of the angles of the internal voltages are related, too. Finally, bus frequencies (derivative of bus voltage angles) depend on generator speeds (derivative of internal voltage angles)³.

$$\Delta \theta^b = -(B^{abb})^{-1} \cdot B^{abg} \cdot \Delta \delta \quad (\text{A.14a})$$

$$\Delta P^{Gg} = (B^{agg} - B^{agb} \cdot (B^{abb})^{-1} \cdot B^{abg}) \cdot \Delta \delta = K^s \cdot \Delta \delta \quad (\text{A.14b})$$

$$\Delta \omega^b = -(B^{abb})^{-1} \cdot B^{abg} \cdot \Delta \omega \quad (\text{A.14c})$$

If eq. (A.14b) is substituted in eq. (A.12), then

³This relationship has been denoted *frequency divider* in the literature since it highlights that bus frequencies are mainly a result of generator speeds. Indeed, in power systems with synchronous generation, the terminal voltage frequency is due to the rotating field. The frequency divider highlights that bus frequencies are weighted generator speeds, where the weights depend on the electrical distances.

$$\begin{bmatrix} \Delta \dot{\delta} \\ \Delta \dot{\omega} \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & \Omega_{base} \\ -\frac{1}{2 \cdot \omega_0} \cdot H^{-1} \cdot K^s & -\frac{1}{2 \cdot \omega_0} \cdot H^{-1} \cdot D \end{bmatrix}}_{=A^{sys}} \cdot \begin{bmatrix} \Delta \delta \\ \Delta \omega \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{2 \cdot \omega_0} \cdot H^{-1} \end{bmatrix} \cdot \Delta P^m \quad (\text{A.15})$$

which is the matrix form of a n-coupled oscillators. Small signal stability is determined by the eigenvalues of the system matrix, A^{sys} . If all eigenvalues have negative real part, the system is asymptotically stable. In other words, if Δp^m is disturbed, generator speeds start oscillating but these oscillations are damped out over time. The damping is mainly affected by the equivalent damping matrix, D , and the distribution of the inertia, whereas the oscillation frequency is affected by the synchronizing power matrix, K^s , and the distribution of the inertia.

B IEEE 39-bus system

The IEEE 39-bus system, commonly known as the New England Test System, has been widely employed in various studies with different objectives, most of which are related to small signal stability analysis and control. There are multiple versions of the New England Test System, including those with different system technologies, FACTS integration, among other modifications. For this study, we chose to adhere closely to the original data source.

Network elements data are shown on Tables (1), (3), (2) while generators dynamic parameters are provides in Tables (4), (5), (6).

From Bus	To Bus	R (p.u.)	X (p.u.)	B (p.u.)
1	2	0.0035	0.0411	0.6987
1	39	0.001	0.025	0.75
2	3	0.0013	0.0151	0.2572
2	25	0.007	0.0086	0.146
3	4	0.0013	0.0213	0.2214
3	18	0.0011	0.0133	0.2138
4	5	0.0008	0.0128	0.1342
4	14	0.0008	0.0129	0.1382
5	6	0.0002	0.0026	0.0434
5	8	0.0008	0.0112	0.1476
6	7	0.0006	0.0092	0.113
6	11	0.0007	0.0082	0.1389
7	8	0.0004	0.0046	0.078
8	9	0.0023	0.0363	0.3804
9	39	0.001	0.025	1.2
10	11	0.0004	0.0043	0.0729
10	13	0.0004	0.0043	0.0729
13	14	0.0009	0.0101	0.1723
14	15	0.0018	0.0217	0.366
15	16	0.0009	0.0094	0.171
16	17	0.0007	0.0089	0.1342
16	19	0.0016	0.0195	0.304
16	21	0.0008	0.0135	0.2548
16	24	0.0003	0.0059	0.068
17	18	0.0007	0.0082	0.1319
17	27	0.0013	0.0173	0.3216
21	22	0.0008	0.014	0.2565
22	23	0.0006	0.0096	0.1846
23	24	0.0022	0.035	0.361
25	26	0.0032	0.0323	0.513
26	27	0.0014	0.0147	0.2396
26	28	0.0043	0.0474	0.7802
26	29	0.0057	0.0625	1.029
28	29	0.0014	0.0151	0.249

Table 1: Transmission Line Data

From Bus	To Bus	R (p.u.)	X (p.u.)	Tap (p.u.)
6	31	0	0.025	1.007
10	32	0	0.02	1.007
19	33	0.0007	0.0142	1.007
20	34	0.0009	0.018	1.009
22	35	0	0.0143	1.025
23	36	0.0005	0.0272	1
25	37	0.0006	0.0232	1.025
2	30	0	0.0181	1.025
29	38	0.0008	0.0156	1.025

Table 2: Transformer Data

From Bus	To Bus	R (p.u.)	X (p.u.)	Tap (p.u.)
12	11	0.0016	0.0435	1.006
12	13	0.0016	0.0435	1.006
19	20	0.0007	0.0138	1.006

Table 3: Generator Step-Up Transformer Data

Unit No.	T_R	K_A	T_A	T_B	T_C	V_{setpoint}	$E_{\text{fd,Max}}$	$E_{\text{fd,Min}}$
1	0.01	200.0	0.015	10.0	1.0	10.300	5.0	-5.0
2	0.01	200.0	0.015	10.0	1.0	0.9820	5.0	-5.0
3	0.01	200.0	0.015	10.0	1.0	0.9831	5.0	-5.0
4	0.01	200.0	0.015	10.0	1.0	0.9972	5.0	-5.0
5	0.01	200.0	0.015	10.0	1.0	10.123	5.0	-5.0
6	0.01	200.0	0.015	10.0	1.0	10.493	5.0	-5.0
7	0.01	200.0	0.015	10.0	1.0	10.635	5.0	-5.0
8	0.01	200.0	0.015	10.0	1.0	10.278	5.0	-5.0
9	0.01	200.0	0.015	10.0	1.0	10.265	5.0	-5.0
10	0.01	200.0	0.015	10.0	1.0	10.475	5.0	-5.0

Table 4: AVR data for the New England Test System

Unit No.	H	R_a	x'_d	x'_q	x_d	x_q	T'_{do}	T'_{qo}	x_l
1	500	0	0.006	0.008	0.02	0.019	7.0	0.7	0.003
2	30.3	0	0.0697	0.17	0.295	0.282	6.56	1.5	0.035
3	35.8	0	0.0531	0.0876	0.2495	0.237	5.7	1.5	0.0304
4	28.6	0	0.0436	0.166	0.262	0.258	5.69	1.5	0.0295
5	26	0	0.132	0.166	0.67	0.62	5.4	0.44	0.054
6	34.8	0	0.05	0.0814	0.254	0.241	7.3	0.4	0.0224
7	26.4	0	0.049	0.186	0.295	0.292	5.66	1.5	0.0322
8	24.3	0	0.057	0.0911	0.29	0.28	6.7	0.41	0.028
9	34.5	0	0.057	0.0587	0.2106	0.205	4.79	1.96	0.0298
10	42	0	0.031	0.008	0.1	0.069	10.2	0.0	0.0125

Table 5: Generator Data for the New England Test System

Unit No.	K	T_w	T_1	T_2	T_3	T_4	$V_{PSS,Max}$	$V_{PSS,Min}$
1	$1.0/(120\pi)$	10.0	5.0	0.60	3.0	0.50	0.2	-0.2
2	$0.5/(120\pi)$	10.0	5.0	0.40	1.0	0.10	0.2	-0.2
3	$0.5/(120\pi)$	10.0	3.0	0.20	2.0	0.20	0.2	-0.2
4	$2.0/(120\pi)$	10.0	1.0	0.10	1.0	0.30	0.2	-0.2
5	$1.0/(120\pi)$	10.0	1.5	0.20	1.0	0.10	0.2	-0.2
6	$4.0/(120\pi)$	10.0	0.5	0.10	0.5	0.05	0.2	-0.2
7	$7.5/(120\pi)$	10.0	0.2	0.02	0.5	0.10	0.2	-0.2
8	$2.0/(120\pi)$	10.0	1.0	0.20	1.0	0.10	0.2	-0.2
9	$2.0/(120\pi)$	10.0	1.0	0.50	2.0	0.10	0.2	-0.2
10	$1.0/(120\pi)$	10.0	1.0	0.05	3.0	0.50	0.2	-0.2

Table 6: PSS data for the New England Test System