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IMPACT OF GEOPOLITICAL CONFLICTS ON U.S. DEFENSE INDUSTRY MARKET EFFICIENCY: A MULTIFRACTAL DETRENDED FLUCTUATION ANALYSIS (MF-DFA)

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Abstract.

Amidst mounting global political uncertainty, the global defense industry has seen a surge in investor interest, leading to a shift in capital towards military and security-related equities. This study analyses how geopolitical events such as the Russia–Ukraine war and the Israel–Hamas conflict have impacted the market efficiency across different subsectors of the U.S. aerospace and defense industry, given the United States' role as a dominant force in this industry. By applying Multifractal Detrended Fluctuation Analysis (MF-DFA) and the Magnitude of Long Memory (MLM) index to daily returns, we uncover and quantify scaling behavior, long-range correlations, and market inefficiencies, offering actionable insights for investors and laying the groundwork for future research.

Our findings reveal pronounced multifractality and heterogeneous responses across the four subsectors. The Services & Support segment exhibits the widest multifractal spectrum and the greatest long-memory effects, followed by the Technology/C4ISR segment shows sharp multifractal peaks during conflict periods. Prime System Contractors experienced elevated persistence during the war in Ukraine but regained efficiency amid the Israel conflict, reflecting shifting investor expectations. In contrast, the manufacturing sector remains comparatively stable, with moderate multifractality and minimal long-memory variation.

Keywords.

War conflicts, arms, geopolitical, Ukraine, Israel, Hamas, industry, United States, Aerospace & Defense, multifractality, MFDFA, subsector, investor, geopolitics, persistence, long-memory.

Resumen.

En medio de la creciente incertidumbre política global, la industria armamentística ha experimentado un aumento del interés de los inversores, lo que ha generado un movimiento de capitales hacia acciones relacionadas con el ámbito militar y la seguridad. Este estudio analiza cómo ciertos acontecimientos geopolíticos como la guerra entre Rusia y Ucrania y el conflicto entre Israel y Hamas han afectado a la eficiencia del mercado en diferentes subsectores de la industria armamentística de los Estados Unidos, dado el papel dominante de este país en dicha industria. Mediante la aplicación del análisis multifractal de fluctuaciones sin tendencia (MF-DFA) y el índice de magnitud de memoria larga (MLM) de los rendimientos diarios, descubrimos y cuantificamos el comportamiento escalar, las correlaciones de largo alcance y las ineficiencias de este mercado, lo que ofrece información útil para los inversores y sienta las bases para futuras investigaciones sobre el comportamiento de esta industria en estos escenarios.

Nuestros resultados revelan una multifractalidad pronunciada y reacciones heterogéneas en los diferentes subsectores. El segmento de servicios y soporte presenta el espectro multifractal más amplio y los mayores efectos de memoria larga, seguido por el segmento de tecnología/C4ISR, que muestra picos multifractales pronunciados durante los periodos de estudio. Los principales contratistas experimentaron una elevada persistencia durante la guerra en Ucrania, pero recuperaron la eficiencia en medio del conflicto de Israel, lo que refleja un cambio en las expectativas de los inversores y un ajuste de la eficiencia de este. Por el contrario, el sector manufacturero se mantiene relativamente estable, con una multifractalidad moderada y una variación mínima de la memoria larga.

Palabras clave.

Conflictos bélicos, armamento, geopolítica, Ucrania, Israel, Hamás, industria, Estados Unidos, Aeroespacial y Defensa, multifractalidad, MFDFA, subsector, inversor, geopolítica, persistencia, memoria larga.

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I. INTRODUCTION

An understanding of the defense industry from an investment perspective is imperative, considering contemporary geopolitical instability and the inherent characteristics of these securities. These securities are often regarded as a means of risk management and diversification during periods of uncertainty. However, it is noteworthy that sudden shocks, such as the Russian invasion of Ukraine and the emergence of the Israel-Hamas conflict, have the potential to disrupt the defense industry, thereby influencing market efficiency and volatility. Many studies analyze the effects of the pandemic on the US stock market (Choi, 2021). Nevertheless, there is a lack of analysis regarding the changes in the defense industry, which is responsible for 5% of the U.S. GDP. The Russia–Ukraine war has reshaped the global economy, causing significant disruptions across financial markets. While financial markets were still recovering from the pandemic's impact, on February 24th of 2022, the Russian invasion of Ukraine triggered another wave of economic uncertainty. This geopolitical conflict marked the most significant war in Central Europe since World War II, causing unprecedented shifts in global trade, energy supply, and investor sentiment, especially regarding the defense sector.

Both events can be considered "black swan" events, as they were largely unforeseen and had substantial economic consequences. According to Assaf et al. (2023), the war's impact on financial markets was evident through abnormal stock price reactions, increased risk aversion, and heightened volatility, particularly in Europe and emerging economies. The financial sector responded negatively, yet certain industries, such as aerospace and defense, experienced a surge in investor interest due to increased military spending by governments. Similarly, Singh et al. (2022) found that the Russia-Ukraine conflict led to a recalibration of investor preferences, with energy and defense stocks emerging as net receivers of return spillover effects, highlighting a shift towards sectors perceived as more resilient in times of geopolitical instability. Moreover, Bouri et al. (2024), this surge can be attributed to a "flight-to-arms," analogous to the more familiar "flight-to-quality." In this scenario, portfolio managers perceive weapon contractors as being less economically sensitive and more reliant on robust, long-term government procurement cycles, particularly during periods of intense geopolitical uncertainty.

Recent studies have also emphasized the necessity of re-evaluating market behavior and efficiency during periods of crisis. For instance, Memon et al. (2022) applied the Multifractal Detrended Fluctuation Analysis (MF-DFA) method to assess the impact of global shocks on commodity markets, revealing significant inefficiencies and herding behavior during both the pandemic of 2020 and the Global Financial Crisis. Their findings confirm that such extreme events can disrupt market efficiency and alter investor dynamics.

The defense industry, which includes companies involved in weapons manufacturing, aerospace systems, and military technology, has historically been influenced by government contracts and geopolitical developments. The outbreak of war has intensified defense spending, particularly in Europe and the United States, as nations reassess their security strategies. As Singh et al. (2022) highlight, many investors previously avoided defense stocks due to ESG (Environmental, Social, and Governance) considerations, but the war has shifted this narrative, reinforcing the sector's role in national security and economic stability.

The Efficient Market Hypothesis (EMH) posits that stock prices fully reflect all available information, making it difficult for investors to achieve consistently above-market returns. Furthermore, policymakers need to investigate weak-form efficiency in the stock market because it is used to allocate resources (Choi, 2021). However, extreme events such as pandemics and wars may create inefficiencies as markets struggle to price geopolitical risk and economic uncertainty (Assaf et al., 2023). Analyzing the impact of these events on financial markets, particularly within the defense industry, provides valuable insight into how markets respond to crises.

Due to the lack of research on market efficiency in the U.S. defense industry, this paper examines how two significant geopolitical events, the Russia–Ukraine war and the Israel–Hamas conflict, have affected the efficiency and pricing of the industry's subsectors. Using multifractal detrended fluctuation analysis (MF-DFA), we evaluate whether these events altered return correlations and long-memory properties in the subsectors comprising the SPADE Defense Index. By comparing our findings with broader benchmarks, such as the S&P500, and a fractional Gaussian noise (*fGn*) baseline, we can identify the subsectors that experienced the greatest efficiency shifts and uncover potential arbitrage opportunities for asset managers and traders.

a. Data and methodology

The present study utilizes data from the SPADE Defense Index to evaluate the efficiency of the U.S. defense industry and its subsectors. The SPADE Defense Index is a metric that tracks the performance of publicly traded U.S. companies involved in defense, homeland security, and aerospace. To gain more granular insights, the index is decomposed by identifying and extracting its main constituent companies. These firms are then categorized by subsectors such as aerospace systems, cybersecurity, weapons manufacturing, and military technology, based on their core business operations and classification in financial databases.

The logarithmic returns of the index and its subsector components are calculated and used as inputs for the Multifractal Detrended Fluctuation Analysis (MF-DFA), the primary technique applied in this study to assess market efficiency and further compared against the S&P500 for a broader perspective, and with the fractional Gaussian Noise (*fGn*) for a fully efficient market perspective. MF-DFA is a powerful numerical algorithm that identifies scaling behavior and long-range correlations in non-stationary time series. Initially developed for the analysis of natural phenomena and biomedical data, it has witnessed a surge in adoption in econophysics and finance, largely due to its ability to detect multifractal properties and inefficiencies in complex markets (Rydin Gorjão et al. (2022) ; Kantelhardt et al. (2002)). We then apply the Magnitude of Long Memory (MLM), a method that ranks defense industry subsectors by their market efficiency (Memon et al., 2022; Wang et al., 2009), to identify and compare the changes in subsector market efficiency during the identified periods. The MF-DFA method analyzes the fluctuation patterns of a time series across multiple time scales. It reveals the presence of self-similarity, persistence, or anti-persistence. This provides a quantitative characterization of market efficiency. It is deemed efficient when the series exhibits characteristics of a purely random process (i.e., a Hurst exponent = of 0.5). Conversely, deviations from this indicate the presence of memory and potential inefficiencies.

While a growing body of research has applied MF-DFA to traditional financial markets, such as stock indices or commodities, its application to sector-specific indices, particularly the defense sector, remains limited. A select number of studies, including those by Aslam et al. (2020), have explored frontier and European markets during periods of heightened uncertainty, such as the outbreak of COVID-19, illuminating the multifractal nature of market dynamics during global crises. However, the intersection between multifractal analysis and defense finance remains unexplored.

Furthermore, their applications to the defense industry are scarce despite their geopolitical and macroeconomic significance. This paper aims to contribute to and expand this subject by offering a systematic analysis of the market efficiency of the U.S. defense industry. The analysis focuses on the impact of major global shocks, specifically the Russia-Ukraine war and the Israel-Hamas conflict, on the price dynamics and fractal structure of the subsectors, opening the door to arbitrage opportunities in certain subsectors of the industry during periods of heightened war, conflict, and geopolitical instability.

II. EFFICIENT MARKET HYPOTHESIS AND FRACTAL THEORY: A COMPREHENSIVE ANALYSIS

The efficient market hypothesis (EMH) and fractal theory provide contrasting but complementary frameworks for understanding financial markets. EMH, developed by Fama (1970) posits that markets process information efficiently, meaning that asset prices always reflect all available information. In contrast, the fractal theory, developed by Benoît Mandelbrot (Mandelbrot, 2005), challenges the traditional assumptions of market randomness and normality in price distributions, suggesting that financial markets exhibit self-similarity and fractal characteristics. This paper provides an in-depth exploration of both theories, focusing on their implications for market efficiency and the application of Multifractal Detrended Fluctuation Analysis (MFDFA) in assessing market behavior.

Understanding market efficiency is crucial for investors, policymakers, and financial researchers. If markets are efficient, then no investor can consistently earn abnormal returns, and all known information is already incorporated into asset prices (Fama, 1970). However, studies have challenged this assumption, showing that financial markets are prone to inefficiencies, especially during times of economic crisis (Jarrow & Larsson, 2012). The testing of these theories not only contributes to academic discussions but also has practical applications in risk management, trading strategies, and financial modeling (Erer et al., 2023). The efficient market hypothesis is one of the fundamental concepts of modern financial economics. It asserts that asset prices fully reflect all available information, preventing investors from consistently outperforming the market through either technical or fundamental analysis (Fama, 1970). The EMH is categorized into three different forms, each defining a different level of market efficiency.

In the weak form of EMH, past price movements and trading volumes are considered irrelevant for predicting future prices (Fama, 1970). This implies that technical analysis, which relies on historical price patterns to predict future trends, is ineffective. Weak-form efficiency suggests that stock prices follow a random walk, meaning that future movements are independent of past trends. Empirical studies have tested this hypothesis using statistical methods in different markets, often rejecting this hypothesis (Gupta & Basu, 2011).

The semi-strong form of EMH extends the weak form by claiming that all publicly available information, including financial statements, economic indicators, and news announcements, is already incorporated into stock prices (Fama, 1970). This means that investors cannot gain an advantage through fundamental analysis, which involves evaluating financial reports and market conditions to estimate the intrinsic value of stocks. Event studies, which examine how quickly stock prices adjust to new public information, have been used to test semi-strong efficiency, with results showing that markets generally react quickly but not always perfectly (Jarrow & Larsson, 2012).

The strong form of EMH takes market efficiency to its highest level, arguing that all information, including insider knowledge, is fully reflected in stock prices (Fama, 1970). Under this assumption, no investor, even one with privileged access to corporate information, can consistently earn abnormal returns. This version of EMH is more controversial and has been widely criticized, as empirical evidence has shown that insider trading can indeed lead to significant profits (Chau & Vayanos, 2008).

Under the Efficient Market Hypothesis, any empirical test simultaneously evaluates market efficiency and the chosen equilibrium model for price setting. As Blackledge & Lamphiere (2021) note, this is the 'joint hypothesis problem: if observed returns diverge from theoretical predictions, one cannot distinguish whether the discrepancy arises from model misspecification or true market inefficiency.

Despite its theoretical appeal, the EMH has been subject to extensive criticism. Behavioral finance research has highlighted the role of cognitive biases, irrational investor behavior, and market anomalies that challenge the notion of perfectly efficient markets Erer et al., 2023). Behavioral finance research has shown that investors are prone to various cognitive biases, such as overconfidence, loss aversion, and herd behavior, which cause the stock prices to deviate from their fundamental value, leading to market inefficiencies (Lin, 2023). Similarly, value investing strategies, which involve buying undervalued stocks and selling overvalued ones, have historically generated excess returns, challenging the semi-strong form of EMH (Jarrow & Larsson, 2012). Even from a physical perspective, the idea of market efficiency is not reasonable since market information is not obtained simultaneously (Blackledge & Lamphiere, 2021). Nevertheless, Malkiel (2003) further affirms that despite apparent irregularities and patterns in stock returns that can even persist over the short term due to the irrationality of some market participants, in the long run, there are no sufficiently recognizable and exploitable patterns to generate excess returns.

On the other hand, Fractal theory offers an alternative perspective on financial market behavior (Mandelbrot, 2005). Unlike EMH, which assumes that price movements follow a random walk with normally distributed returns, fractal theory suggests that markets exhibit self-similarity and heavy-tailed distributions. Peters (1989) studies the Fractal Structure in capital Markets, stating investor sentiment reflects how investors interpret events affecting capital markets, which do not immediately reflect market prices. Instead, they gradually emerge as persistent return biases, sometimes persisting for decades. This stands in contrast to the Efficient Market Hypothesis. These biases significantly impact overall market returns.

A key concept in fractal theory is multifractality, which implies that markets exhibit different scaling behaviors at different time horizons (Mandelbrot, 2005). This feature challenges the traditional notion of market efficiency by showing that asset prices exhibit long-run dependence, meaning that past price movements can influence future trends over long periods, which contradicts the weak form of EMH.

One of the main tools for analyzing multifractal properties in financial markets is Multifractal Detrended Fluctuation Analysis, used to distinguish the underlying cause of multifractality between long-range correlations and a broad probability density function (Salat et al., 2017). This method allows researchers to quantify the degree of multifractality in asset returns and to identify patterns that deviate from the assumptions of the EMH. Studies have shown that financial markets are characterized by power laws, where extreme events such as financial crises and market crashes occur more frequently than predicted by normal distribution models (Mandelbrot, 2005).

III. DATA OVERVIEW

When determining the subsectors we are analyzing, we have followed the SPADE Defense Index criteria and, as we consider to be the most accurate US Defense Index, with more data available. The index covers all aspects of the sector, including infrastructure, services, IT and cyber activities, and support. This index is calculated using a modified market capitalization weighting methodology (TrueCap) that accurately reflects the weightings assigned to highly diversified firms (i.e., Lockheed Martin or General Electric) by focusing on the relevant business activity within a sector/theme (i.e., defense, aerospace, homeland security, space) as if it were a standalone entity. This way, these firms can be measured according to a particular market segment where they operate, eliminating disparities between the big, highly diversified, and the more niche firms (i.e., Kratos Defense).

The SPADE Defense Index is calculated using a modified market capitalization weighting methodology, with component weights modified to conform to asset diversification rules applied in conjunction with the scheduled quarterly updates to the Index (Sacknoff, 2023). However, its modified market methodology caps the total weight of the largest companies to reduce their influence on the overall index. This approach seeks to ensure a balanced representation of both large contractors and the smaller companies that support them (TrueCap Methodology).

To follow the Index accuracy in this study, the components are grouped in four subsectors according to SPADE Indexes LLC criteria:

i) Prime System Contractors: Companies that are awarded a contract or program that requires a significant amount of funding. For example, Lockheed Martin is the prime contractor for the F-35 fighter jet, meaning it oversees the development of the jet and its subcontractors. Since the Cold War, the major prime contractors have been Lockheed Martin (LMT), Northrop Grumman (NOC), Raytheon Technologies (RTX), Boeing (BA), and General Dynamics (GD). They are referred to as the military complex.

ii) Manufacturing: Systems, subsystems, components, and hardware: Companies that focus on building the parts for a program or contract. They are usually referred to as subcontractors.

iii) Services and Support: Specialize in providing support to the military during operations.

iv) Technology/C4ISR: Companies that integrate command, control, communications, computers, intelligence, surveillance, and reconnaissance systems into the military and the battlefield. It has recently been on the watch list of many investors due to its strong link with AI and machine learning, as developments in the Ukrainian War.

Consequently, Table 1 shows an overview of the firms and subsector constituents for this sample, their closing price, and market capitalization as of December 31st, 2024. Because of the Index changes in size and composition, we kept the same firms as in the last Index report. (see attached January 2025 Investor Report for full review). The predominant sector in terms of the number of firms is the Manufacturing Systems, subsystems, components, and hardware; nevertheless, the biggest in terms of market capitalization per firm is the Prime System Contractors. It gives a picture of how structured each subsector is and how difficult it is for companies to get in.

Table 1
Overview of the company selection and subsectors. Data is obtained from Refinitiv Eikon with the corresponding ticker (data as of December 31st, 2024)

#	Company	Ticker	Subsector	Price	Market Cap ($Bn.)
1	AAR	AIR	Manufacturing: Systems, Subsystems, Components, and Hardware	61.28	2.21
2	AeroVironment	AVAV	Manufacturing: Systems, Subsystems, Components, and Hardware	153.89	4.34
3	Amentum	AMTM	Services and Support	21.03	5.12
4	Amphenol	APH	Manufacturing: Systems, Subsystems, Components, and Hardware	69.45	84.01
5	ATI	ATI	Manufacturing: Systems, Subsystems, Components, and Hardware	55.04	7.76
6	AXON International Inc.	AXON	Manufacturing: Systems, Subsystems, Components, and Hardware	594.32	46.27
7	Barnes	B	Manufacturing: Systems, Subsystems, Components, and Hardware	47.26	2.42
8	Boeing	BA	Prime System Contractors	177.00	133.46
9	Booz Allen	BAH	Services and Support	128.70	16.30
10	BWX Technologies	BWXT	Manufacturing: Systems, Subsystems, Components, and Hardware	111.39	10.18
11	C3.ai	AI	Technology/C4ISR	34.43	4.57
12	CACI Intl	CACI	Technology/C4ISR	404.06	8.89
13	Cadre Holdings	CDRE	Services and Support	32.30	1.31
14	CAE	CAE	Manufacturing: Systems, Subsystems, Components, and Hardware	25.38	8.11
15	Curtiss Wright	CW	Manufacturing: Systems, Subsystems, Components, and Hardware	354.87	13.38
16	Ducommun	DCO	Manufacturing: Systems, Subsystems, Components, and Hardware	63.66	0.95
17	Eaton	ETN	Manufacturing: Systems, Subsystems, Components, and Hardware	331.87	130.02
18	Elbit Systems	ESLT	Manufacturing: Systems, Subsystems, Components, and Hardware	258.07	11.52
19	General Dynamics	GD	Prime System Contractors	263.49	70.72
20	General Electric	GE	Manufacturing: Systems, Subsystems, Components, and Hardware	166.79	177.86
21	Heico	HEI	Manufacturing: Systems, Subsystems, Components, and Hardware	237.74	33.03
22	Hexcel	HXL	Manufacturing: Systems, Subsystems, Components, and Hardware	62.70	5.04
23	Honeywell	HON	Manufacturing: Systems, Subsystems, Components, and Hardware	225.89	145.18
24	Howmet	HWM	Manufacturing: Systems, Subsystems, Components, and Hardware	109.37	44.24
25	Huntington Ingalls	HII	Manufacturing: Systems, Subsystems, Components, and Hardware	188.97	7.41
26	Iridium	IRDM	Technology/C4ISR	29.02	3.14
27	Jacobs Engineering	J	Services and Support	133.62	16.37
28	KBR	KBR	Services and Support	57.93	7.52
29	Keysight	KEYS	Technology/C4ISR	160.63	27.76
30	Kratos Defense & Security	KTOS	Manufacturing: Systems, Subsystems, Components, and Hardware	26.38	4.04
31	L3 Harris	LHX	Manufacturing: Systems, Subsystems, Components, and Hardware	210.28	39.31
32	Leidos	LDOS	Technology/C4ISR	144.06	18.47
33	Leonardo DRS	DRS	Manufacturing: Systems, Subsystems, Components, and Hardware	32.31	8.60
34	LOAR	LOAR	Manufacturing: Systems, Subsystems, Components, and Hardware	73.91	6.91
35	Lockheed Martin	LMT	Prime System Contractors	485.94	113.85
36	Mercury Computer Sys	MRCY	Manufacturing: Systems, Subsystems, Components, and Hardware	42.00	2.51
37	Moog Inc.	MOG.A	Manufacturing: Systems, Subsystems, Components, and Hardware	196.84	6.21
38	Northrop Grumman	NOC	Prime System Contractors	469.29	67.54
39	Oshkosh	OSK	Manufacturing: Systems, Subsystems, Components, and Hardware	95.07	6.14
40	OSI Systems	OSIS	Manufacturing: Systems, Subsystems, Components, and Hardware	167.43	2.81
41	Palantir	PLTR	Technology/C4ISR	75.63	178.46
42	Parker Hannifan	PH	Manufacturing: Systems, Subsystems, Components, and Hardware	636.03	81.90
43	Parsons	PSN	Technology/C4ISR	92.25	9.85
44	Raytheon Technologies	RTX	Prime System Contractors	115.72	154.60
45	Rocketlab	RKLB	Manufacturing: Systems, Subsystems, Components, and Hardware	25.47	11.55
46	SAIC	SAIC	Technology/C4ISR	111.78	5.27
47	Spirit	SPR	Manufacturing: Systems, Subsystems, Components, and Hardware	34.08	4.00
48	Teledyne	TDY	Manufacturing: Systems, Subsystems, Components, and Hardware	464.13	21.75
49	Textron	TXT	Manufacturing: Systems, Subsystems, Components, and Hardware	76.49	13.81
50	Transdigm	TDG	Manufacturing: Systems, Subsystems, Components, and Hardware	1267.28	71.07
51	Triumph	TGI	Manufacturing: Systems, Subsystems, Components, and Hardware	18.66	1.44
52	TTM	TTMI	Manufacturing: Systems, Subsystems, Components, and Hardware	24.75	2.52
53	V2X	VVX	Manufacturing: Systems, Subsystems, Components, and Hardware	47.83	1.51
54	Viasat	VSAT	Manufacturing: Systems, Subsystems, Components, and Hardware	8.51	1.10
55	Woodward	WWD	Manufacturing: Systems, Subsystems, Components, and Hardware	166.42	9.88

Source: Own Elaboration using data from FactSet

Our sample period ranges from January 2nd, 2020, to December 31st, 2024, so the Ukraine and Israel conflict outbreaks are covered. We gather daily data for each company that takes part in the SPADE Defense Index Aerospace & Defense subsector according to its Index Last Report (The SPADE Investor, Jan 25). Afterwards, we transform the daily returns into log-returns, plotted in Appendix Figure a1, where we can see big fluctuations, especially during the war period (without taking into consideration those from COVID time). We can already see that one of the most volatile subsectors is Technology/C4ISR, with firms like Palantir (PLTR) or Keysight (KEYS). A summary of key statistics is reported in Appendix Table a1 (January 2020 – January 2022 period) and Appendix Table a2 (Mid-February 2022 – December 2024 period).

Apart from the previously mentioned observations regarding the Index, exchange-traded funds (ETFs) represent a compelling option for investors who are less risk-averse or have other investment objectives. Considering each of the following ETFs focuses on the defense sector, they exhibit notable disparities. The Invesco ETF (PPA), which replicates the SPADE Defense Index, is noteworthy for its systematic inclusion of all firms deemed relevant to the Pentagon, encompassing non-manufacturers involved in information technology, cybersecurity, surveillance, reconnaissance, and command and control software (Invesco Aerospace & Defense ETF, 2025). In contrast, ITA (iShares) and XAR (State Street) focus exclusively on infrastructure and have one-third fewer firms than PPA (iShares by BlackRock,2024). The ITA's primary holding typically accounts for approximately 20% of its total weight, while XAR stands out as the sole fund that equals its holdings (*SPDR ® S&P ®*, 2025). This approach implies that a tier-three supplier would have the same rebalancing weight as a prime contractor, such as RTX or Lockheed Martin. The Invesco Aerospace & Defense ETF is the only one to be set as a Sector Benchmark, has easily accessible historical data, is diversified by market cap and activity, and has publicly available rules.

IV. MF-DFA

a. Origins and Development of MF-DFA

Multifractal Detrended Fluctuation Analysis (MF-DFA) is an extension of the traditional Detrended Fluctuation Analysis (DFA), which was originally developed to study long-range correlations in time series DFA itself was originally developed by Peng et al. (1994) to analyze long-range correlations in non-stationary time series, particularly in biological and physiological signals such as heart rate variability (Ihlen, 2012). While DFA is effective for detecting monofractal properties, it assumes that a single Hurst exponent can describe the system. However, many real-world systems exhibit multifractality, meaning that different parts of the time series scale differently (Kantelhardt et al., 2002). MF-DFA overcomes this limitation by introducing a spectrum of scaling exponents, allowing for a more detailed examination of the complexity in time series data. This extension is significant in financial markets, climatology, neuroscience, and physics, where systems often display heterogeneous scaling behavior due to varying degrees of persistence and non-linearity (Thompson & Wilson, 2016). Other studies support the fractal nature of financial markets, especially in emerging markets tend to show positive long-range correlation (Menezes et al., 2018) and higher multifractality degree (Zunino et al., 2008).

MF-DFA is built on several fundamental principles from fractal and multifractal analysis:

Scaling and Self-Similarity: Many complex systems exhibit self-similar properties, meaning that patterns observed at small scales reappear at larger scales. In monofractal systems, this scaling behavior is characterized by a single Hurst exponent HHH, whereas in multifractal systems, different segments may exhibit different scaling exponents (Kantelhardt et al., 2002).

Long-Range Dependence: Some time series exhibit persistent correlations over long periods, which cannot be captured using conventional statistical tools. MF-DFA allows for detecting these dependencies by analyzing fluctuations over multiple scales (Rydin Gorjão et al., 2022).

Multifractality: Unlike monofractal series, which can be described by a single power-law exponent, multifractal series exhibit a broad spectrum of scaling exponents. This spectrum is described by the generalized Hurst exponent $h(q)$, the scaling exponents $\tau(q)$, and the singularity spectrum $D(\alpha)$ (Bacry et al., 2010).

MF-DFA is particularly useful for analyzing the financial stability of defense companies, which are often influenced by geopolitical tensions, military conflicts, and government spending on national security. Unlike conventional financial assets, we find that defense stocks tend to display long-range dependencies and volatility clustering that are amplified by political and economic uncertainty.

b. Applications of MF-DFA to the U.S. Defense Industry Market Efficiency

MF-DFA is particularly valuable when applied to the evaluation of the efficiency and complexity of the U.S. defense industry. Defense companies' stocks and market behaviors are notably influenced by diverse, complex, and interconnected factors such as geopolitical tensions, international conflicts, and fluctuations in government spending on national defense (Thompson & Wilson, 2016). Unlike standard financial assets, the stocks of defense companies often exhibit pronounced long-range dependencies, strong nonlinear behaviors, and volatility clustering that arise directly from political decisions, defense budget announcements, or geopolitical events (Klein, 2024)

By leveraging MF-DFA, researchers and analysts can:

Quantify the Multifractal Spectrum: Through MF-DFA, the multifractal spectrum of returns on defense companies' stocks can be estimated, revealing subtle and intricate differences in scaling behavior across different temporal segments. This spectrum provides insights into market efficiency, detecting periods when stocks might not follow a purely random walk, potentially unveiling hidden memory effects and herding behaviors.

Assess Market Efficiency: Market efficiency implies random-walk behaviors with little to no memory. However, the multifractal spectrum, as indicated by the width ($\Delta\alpha$) or the difference between minimum and maximum Hurst exponents, provides a direct measure of market inefficiency. A wider multifractal spectrum suggests strong multifractality and more pronounced inefficiency, as the market does not fully and quickly integrate available information (Ihlen, 2012; Rydin Gorjão et al., 2022).

Detect Periods of Stress and Herding Behavior: MF-DFA allows detailed examination of the market's reaction to specific geopolitical events (e.g., conflicts, military interventions, major defense spending announcements). By comparing periods of geopolitical tensions or crises (such as the Russian invasion of Ukraine) with more stable periods, MF-DFA can identify critical changes in market behavior, detecting whether the market’s response is more persistent or anti-persistent, and thereby providing insights into investor psychology and behavior under stress.

Evaluate the Impact of Policy Changes: Defense companies are significantly affected by government decisions, spending priorities, and regulatory changes. MF-DFA provides a powerful statistical tool to assess how these policy decisions influence the market dynamics, volatility clustering, and overall risk behavior of the defense stocks. This capability makes MF-DFA a vital tool for policymakers aiming to understand the economic implications of their actions.

c. Steps of MF-DFA Implementation

As presented by Kandelhart et al. (2002), the procedure consists of five steps, the first three being the same as those for DFA (Detrended Fluctuation Analysis). For series x_k where k = (1, 2, N) and N is the length of x_k, the MF-DFA method is as follows:

Convert the original time series (x_k) into a cumulative sum (profile), transforming noise-like data into random-walk-like data suitable for multifractal analysis (Ihlen, 2012).

$$Y(i) \equiv \sum_{k=1}^{i} [x_k - (\overline{x})] , \mathrm{i} = 1{,}2 \ldots , \mathrm{N}$$

The time series is to be divided into Ns segments of equal size that do not overlap. The last points of the data must be discarded, since the total length of the data is not always a multiple of the segment length, s. To avoid discarding data points, the process is repeated starting at the opposite end of the series, resulting in a total of $2N_s$ segments. (Ihlen, 2012).

$$N\mathrm{s} = int(N/s)$$

To eliminate local non-stationarities and trends (which typically follow linear or quadratic forms), it is necessary to remove local polynomial trends from each segmented sub-series. For each segment v and an order m of the polynomial previously determined, a polynomial $yv(i)$ of degree m is fitted to the aggregated observations within the segment of length s via least squares (Rydin Gorjão et al., 2022). Then, the local polynomial trend is subtracted from each data point, resulting in detrended fluctuations:

$$F^2(v,s) = \frac{1}{s}\sum_{i=1}^{s} \{Y[(v-1)s+i] - yv(i)\}^2$$

This step, as initially proposed by Kantelhardt et al. (2002), guarantees that the analysis focuses exclusively on the intrinsic variability of the data, thereby ensuring the absence of local polynomial trends. This methodological approach enables precise multifractal characterization.

Compute the generalized fluctuation function by averaging fluctuations across all segments for each order q. The fluctuation function, denoted $Fq(s)$ is contingent upon two parameters: the segment size s and the *q-(th)* power. This allows emphasis on small *(q<0)* or large *(q>0)* fluctuations (Rydin Gorjão et al., 2022).

$$F_q(s) = \{\frac{1}{2N_s}\sum_{v=1}^{2N_s}[F^2(v,s)]^{q/2}\}^{1/q} \quad , q \neq 0$$

The objective of this study is to ascertain the dependence of generalized q-dependentuation functions, denoted $Fq(s)$, on the time scale s for distinct values of q. To this end, steps 2 to 4 must be repeated for multiple time scales s. $Fq(s)$will increase with rising s (Englehart, et al., 2002)

Analyze the scaling relationship of the data by examining the log–log plot of the fluctuation function $Fq(s)$ against scale *s*, in case the data shows power-law correlations.

$$Fq(s) \sim S^{h(q)}$$

The generalized Hurst exponent h(q) is determined by the slope of a log-log plot of Fq (s) versus s. A constant $h(q)$ indicates monofractal behavior while varying $h(q)$ values for different q indicate multifractality. Specifically:

If *h(q)* is independent of q, the series is monofractal

If *h(q)* varies significantly with q, the series is multifractal

The Hurst exponent *h(q)* is a critical measure for evaluating market efficiency, as it captures long-range correlations in financial time series. The value of *h(q) > 0.5* indicates positive autocorrelation and persistence, suggesting that past movements tend to influence future trends. This persistent behavior, frequently associated with herding phenomena, signifies inefficiency due to the gradual integration of information into market prices (Memon et al., 2022; Peters, 1989). Conversely, a value of *h(q) < 0.5* reflects negative autocorrelation or anti-persistent behavior, indicative of frequent reversals and instability, typically implying overreactions and corrective actions in investor behavior (Salat et al.,2017).

Finally, a Hurst exponent *h(q)* = 0.5 describes a purely stochastic process, often referred to as Brownian motion or a classical random walk, where price changes are entirely independent and the market efficiently incorporates information without systematic memory (Peters, 1989;

Memon et al., 2022). In the multifractal context, the variation of *h(q)* across different orders q reveals the presence of multifractality, with broader variations highlighting more complex market dynamics and heterogeneous investor behaviors (Salat et al., 2017).

If the data under consideration exhibits multifractal characteristics, the multifractal scaling exponent, denoted by *τ(q),* which serves to quantify the variation in scaling behavior as a function of the order q, can be subjected to rigorous examination. This exponent is mathematically related to the generalized Hurst exponent *h(q)* by:

$$\tau(q) = qh(q) - 1$$

where the generalized Hurst exponent *h(q)* measures scaling properties across different moments q.

In order to gain further insight into multifractality, it is possible to construct the singularity spectrum *D(α)* through the Legendre transform. Under the assumption that *τ (q)* is sufficiently smooth, the singularity strength α can be calculated as follows:

$$\alpha = \tau'(q) = h(q) + qh'(q)$$

From where the singularity spectrum can be calculated as:

$$D(\alpha) = q\alpha - \tau(q)$$

The singularity spectrum *D(α)* describes the dimension of the subset of the time series that is characterized by the singularity strength α (Salat et al., 2017). The width of the singularity spectrum, defined as the range of α, provides a measure of the degree of multifractality within the time series. A broader spectrum is indicative of stronger multifractality, while a narrower spectrum is indicative of weaker multifractality, with the maximum point typically centered around the most prominent scaling exponent, h (Rydin Gorjão et al., 2022).

The implementation of multifractal detrended fluctuation analysis (MF-DFA) in the context of stock markets has led to the substantiation of market inefficiencies, a phenomenon that is particularly pronounced in emerging markets, where the presence of substantial multifractal structures has been observed (Zunino et al., 2008). Conversely, developed markets demonstrate reduced multifractality, indicating a heightened degree of efficiency.

Furthermore, commodity markets, including gold, oil, and agricultural goods, have demonstrated notable multifractality due to volatility clustering and speculative trading, especially during crisis (Memon et al., 2022). In addition, Choi (2021) further analyzes the impact of the Great Financial Crisis and COVID-19 on different sectors of the U.S. economy, showing that each crisis event affects the efficiency of each subsector differently. Moreover, Bentes (2016) find that gold volatility exhibits long memory with different patterns across geopolitical periods. Nevertheless, no study addresses it in the Aerospace and Defense Industry (A&D).

Conversely, the application of this concept to the domain of the defense industry and military stocks represents a burgeoning area of research, particularly in light of the geopolitical risks and structural shifts in military expenditure.

d. Magnitude of Long Memory (MLM)

To better analyze and quantify the changes in the subsector market efficiency across our sample period, we apply the Magnitude of Long Memory (MLM) index based on the generalized Hurst exponent, denoted by (Wang et al., 2009) as:

$$D = \frac{1}{2}\ (|h(-5) - 0.5| + |h(5) - 0.5|$$

As stated in the previous section, and following a range of q [-5,5], fluctuations between these ranges follow a random walk process, meaning that MLM < 0, the volatility of subsector returns is efficient with no longer-term memory. Therefore, the lower the value of MLM, the lower the level of long memory and the lower the herding behavior in the U.S Aerospace and Defense subsectors (Mnif & Jarboui, 2021).

V. RESULTS

As previously mentioned in the Sample Section, the objective is to assess the study's capacity to evaluate the market efficiency of the U.S. Aerospace & Defense Industry during geopolitical events. To this end, the constituents of the SPADE Defense Index are grouped into four subsectors. For each subsector, a multifractal detrended fluctuation analysis is performed over three primary geopolitical macro events that have occurred over the past six years. Firstly, we carried out a preliminary descriptive analysis of the sample, as seen in Table 2, which shows annualized main financial descriptives from the beginning of the sample period (January 2020) to December 2024. It is noticeable how the SPADE Defense Index Sharpe Ratio, which measures the risk-adjusted returns, has been outperforming our broader index benchmark since the year of the Ukraine conflict outbreak. Moreover, from the Skewness, which indicates the asymmetry of a distribution, and Kurtosis levels, which measure the tails of a distribution relative to a normal, we can tell the Services and Support subsector has had drastic moves compared to its comparable.

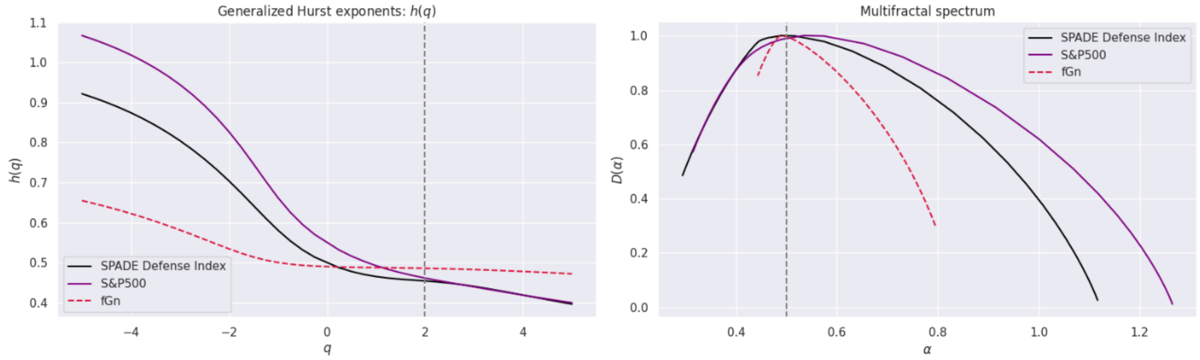
Table 2 – Key Statistics of the SPADE Defense Index and subsectors (2020-2024)

#	Year	CategoryName	Annual Return (%)	Annualized Volatility (%)	Sharpe	Skewness	Kurtosis
0	2020	SPADE Defense	-2.359	41.623	-0.066	-0.641	5.594
1	2020	Manufacturing: Systems, Subsystems, Components, and Hardware	-3.643	46.421	-0.087	-0.86	5.481
2	2020	Prime System Contractors	-19.383	42.664	-0.513	-0.524	5.961
3	2020	Services and Support	17.464	36.842	0.427	-0.859	7.457
4	2020	Technology/C4ISR	0.654	42.704	0.007	-1.382	8.347
5	2021	SPADE Defense	6.613	17.105	0.372	-0.146	1.052
6	2021	Manufacturing: Systems, Subsystems, Components, and Hardware	7.67	19.469	0.377	-0.196	1.371
7	2021	Prime System Contractors	13.184	17.29	0.714	-0.473	1.811
8	2021	Services and Support	-1.693	29.49	-0.059	-6.385	71.833
9	2021	Technology/C4ISR	-5.689	19.813	-0.298	-0.557	2.162
10	2022	SPADE Defense	8.592	22.489	0.276	-0.014	0.337
11	2022	Manufacturing: Systems, Subsystems, Components, and Hardware	-2.111	24.094	-0.173	0.008	0.261
12	2022	Prime System Contractors	27.035	24.061	0.91	-0.211	1.719
13	2022	Services and Support	1.331	23.708	-0.03	0.015	0.372
14	2022	Technology/C4ISR	3.955	23.23	0.08	-0.064	0.571
15	2023	SPADE Defense	17.851	14	0.802	0.099	0.658
16	2023	Manufacturing: Systems, Subsystems, Components, and Hardware	32.655	15.989	1.443	-0.19	0.204
17	2023	Prime System Contractors	-2.935	16.573	-0.493	1.378	12.556
18	2023	Services and Support	17.025	16.964	0.621	-0.45	1.043
19	2023	Technology/C4ISR	7.812	17.316	0.135	-0.15	0.856
20	2024	SPADE Defense	24.879	14.581	1.175	-0.418	2.307
21	2024	Manufacturing: Systems, Subsystems, Components, and Hardware	20.055	17.491	0.754	-0.18	2.068
22	2024	Prime System Contractors	-0.944	13.681	-0.441	-0.106	3.387
23	2024	Services and Support	-7.925	25.132	-0.531	-3.703	30.715
24	2024	Technology/C4ISR	21.566	20.196	0.715	-1.894	12.631
25	(2020-2024)	SPADE Defense	10.707	24.266	0.315	-0.729	12.912
26	(2020-2024)	Manufacturing: Systems, Subsystems, Components, and Hardware	10.072	27.142	0.26	-0.948	12.546
27	(2020-2024)	Prime System Contractors	2.182	25.17	-0.015	-0.574	13.648
28	(2020-2024)	Services and Support	4.734	27.255	0.077	-2.638	30.019
29	(2020-2024)	Technology/C4ISR	5.276	26.339	0.099	-1.431	14.742

Source: Own Elaboration using data from FactSet

Before looking in more detail at each subsector, we examined the overall efficiency of the SPADE Defense Index concerning a market benchmark index (the S&P 500) and fractional Gaussian noise (*fGn*) as a baseline for an efficient series. The MF-DFA results, illustrated in Figure 2, suggest multifractality and differences in efficiency between these indices. Figure 2(a) plots the generalized Hurst exponent *(h(q))* against the moment order *(q)*. The SPADE Defense Index and the S&P500 display pronounced multifractal behavior with significant variations in *h(q)*. For negative *q* values, which emphasize small fluctuations, both indices show significantly higher *h(q)* values, indicating persistence in minor market movements. For positive *q* values, which highlight larger fluctuations, *h(q)* drops significantly below 0.5. This reflects pronounced anti-persistence and strong mean reversion following major market shifts. Compared to *fGn*, both indices deviate from market efficiency, as is particularly evident in their anti-persistent behavior. Figure 2(b) shows the singularity spectrum *D(α)* plotted against the Hölder exponent *(α).* While the fGn exhibits a narrow, symmetrical spectrum, the SPADE Defense Index and the S&P 500 both demonstrate broader, asymmetrical spectrum characteristics. The S&P 500's spectrum is notably broader than that of the SPADE Defense Index, indicating greater complexity and heterogeneity in its local scaling behaviors. This wider range of spectrum values *(Δα)* suggests that the S&P500 is more impacted by diverse economic and geopolitical events as COVID-19, exhibiting a more pronounced response to market shocks, especially given its weighting towards the technology sector. Its wide spectrum indicates a large variety of local scaling behaviors and a highly heterogeneous time series (Ihlen, 2012). While the SPADE Defense Index is sensitive to sector-specific geopolitical developments, it exhibits comparatively lower multifractality, reflecting more moderate variability relative to the broader market, as it is primarily made up of manufacturing firms.

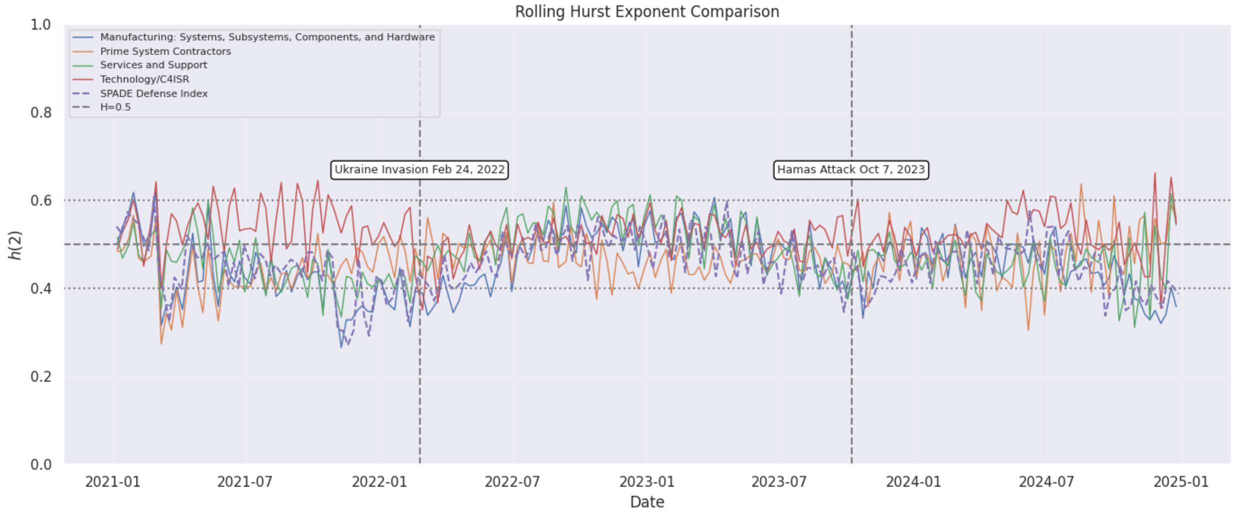
Figure 1 – The MFDFA results of the daily SPADE Defense Index log returns time series and the S&P500 for the period (2020-2024)



Source: Own Elaboration using data from FactSet

To gain a better understanding of the evolution of market efficiency within the US defense sector, we calculate the rolling Hurst exponent *h (2)* for each industry subsector and the SPADE Defense Index using a 252-day window to simulate the working year. This method captures long-range dependence in return dynamics: values above *0.5* indicate persistence (i.e., trending behavior), while values below *0.5* suggest anti-persistence (i.e., mean reversion). We establish a confidence interval of *0.4* and *0.6* and create a neutral zone to help identify long memory effects from statistical noise. Although the Ukraine–Russia war overlaps with the Israel-Hamas conflict, as both are still ongoing as of December 31st 2024, we opted for reducing the conflict periods to 1.5 years for each to be as rigorous as possible when comparing their effects on the industry. As shown in Figure 2, the SPADE Defense Index (plotted as a dashed black line) tends to fluctuate within the neutral zone, acting as a smoothed average of the underlying subsectors. Although this chart does not show abrupt structural breaks that align exactly with the dates of geopolitical shocks, such as the Russian invasion of Ukraine on 24 February 2022 and the Hamas attack on Israel on 7 October 2023, this is to be expected due to the smoothing effect of the rolling window. Nevertheless, notable divergences between the index and specific subsectors emerge during key periods. At the start of the sample period, the Technology/C4ISR subsector has a higher Hurst exponent than the overall index, indicating persistent behavior and greater market inefficiency compared to the broader sector. Additionally, Prime System Contractors show a significant increase in persistence around mid-2022, diverging from the index, suggesting investor confidence or consistent momentum in large-cap contractors, that are being used as hedges against ongoing uncertainty which resonates with Gurdgiev et al. (2022) findings about using these contractors as a hedge against the general market reaction to war conflicts where the U.S. becomes publicly involved.

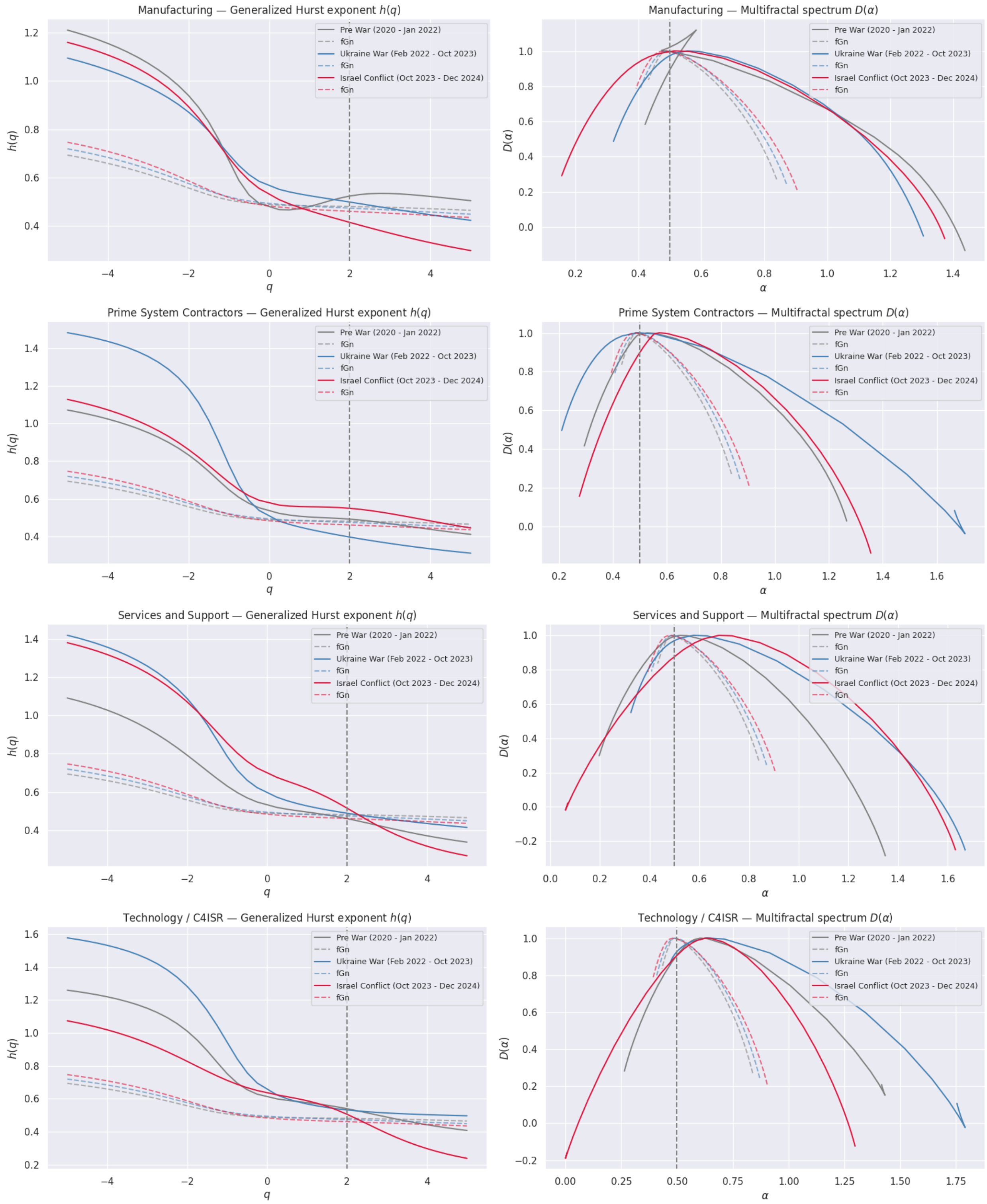
Figure 2 – Rolling Hurst of SPADE Defense Index and its Subsectors for the period (2020-2024)



Source: Own Elaboration using data from FactSet

As seen in Figure 3, wider singularity spectrum and non-constant Hurst exponents over q denote strong multifractality over the four subsectors across different periods. Nevertheless, there are some particularities. The Manufacturing subsector displays a narrow and slow shifting Δh compared to the rest of the subsector, indicating that the scaling behavior of small and large fluctuations remains more uniform over time, suggesting that stocks of different magnitudes are absorbed more evenly. This stability implies that the underlying process behind manufacturing returns has a more homogeneous set of temporal correlations: neither small-scale persistence nor large-scale anti-persistence changes dramatically across geopolitical stress periods which can be attributed to its business diversification as firms with broad product lines, multiple end markets, or geographically extended operations effectively mitigate idiosyncratic shocks so that neither the tails nor the center of the return distribution dominates as much during geopolitical conflicts. On the other hand, regarding the Ukraine conflict period, the Prime System contractors show the widest singularity spectrum during the period of the Ukraine War, indicating greater complexity and heterogeneity in market behavior. While small fluctuations were persistent, suggesting stable investor confidence, larger shocks triggered diverse responses. The broader spectrum shows that the market did not treat the subsector uniformly but differentiated firms based on contract exposure and perceived geopolitical sensitivity. Compared to the narrower, more symmetric spectrum of the pre-war period, this indicates an environment of amplified investor selectivity and asymmetric reactions under geopolitical stress. On the other hand, there is minimal variation in services and support across conflict periods $h(q)$ curves, and a stable multifractal spectrum suggests that geopolitical events had a minimal impact on return dynamics. Technology/C4ISR exhibits a wide, asymmetric spectrum, indicating a highly reactive and heterogeneous structure driven by speculative behavior of its constituents' profile (i.e., Palantir). During the Israel–Hamas conflict, there was a noticeable increase in anti-persistence for $q>0$.is observed across most subsectors. Unlike the war in Ukraine, which triggered sustained structural shifts, the Israel conflict appears to have generated intense but localized tension, prompting tactical investor behavior and less directional persistence, especially in U.S. aerospace and Defense stocks due to their strong relationships with Israel. This may reflect a limited perceived long-term impact despite short-term geopolitical stress.

Figure 3 – Generalized Hurst exponent *h(q)* and Singularity Spectrum *D(α)* per subsector across three periods: Pre-War (2020–Feb 2022), Ukraine War (Feb 2022–Oct 2023), and Israel Conflict (Oct 2023–Dec 2024).



Source: Own Elaboration using data from FactSet

Examining the spectral width, in Table 3, across the three conflict-defined windows reveals highly heterogeneous responses among subsectors and indexes. Over the pre-war period, the Technology / C4ISR sector exhibits the largest multifractal spectrum ($\Delta\alpha$ = 0.8515) across the subsectors, reflecting the highest local scaling heterogeneity. Nevertheless, the SPADE Defense Index had the highest multifractal width during the pre-war period, exceeding both the S&P 500 and any individual subsector. This is because it aggregates a mix of highly volatile manufacturing, services, and technology stocks at a time when the effects of the pandemic and supply chain disruptions were unevenly distributed across these sectors. Despite their high levels of multifractality, the different indexes and subsectors recorded the highest levels across different conflict periods. Notably, during the Ukraine War, the SPADE Defense Index ($\Delta\alpha$ = 0.3724), which became the least inefficient during that period, reflected its diversified profile, predominantly composed of manufacturing companies, the subsector that exhibited the lowest inefficiency behavior throughout the sample period. The Prime System Contractors' spectrum widens dramatically to $\Delta\alpha$ = 1.1712, surpassing all other subsectors. This peak multifractality can be attributed to heavy capital flows into large, well-known defense contract firms whose market capitalization and government-backed order books make them the main beneficiaries of a sector-wide rally (Chordia et al., 2005). Moreover, Technology / C4ISR also displays pronounced multifractality peaks during the Ukraine War, which can be attributed to the conflict's acceleration of digital and autonomous warfare capabilities and is further adjusted by the market during the second conflict. In contrast, the SPADE Index spectrum narrows, demonstrating how an aggregate benchmark can smooth out idiosyncratic volatility, even amid severe geopolitical stress. In the Israel Conflict window, the SPADE Index spectrum expands sharply to $\Delta\alpha$ = 1.1167, its widest of all three periods, suggesting that this "black swan" event induced more uneven, large-scale fluctuations in the benchmark than the Ukraine War, still lower compared to the index benchmark which recorded the highest spectral width of the period. Similarly, the Services & Support spectrum reaches its maximum width ($\Delta\alpha$ = 1.1153), while the Manufacturing subsector levels slightly increase compared to its peers. Regarding Market Long Memory (MLM) measure, where lower values indicate greater efficiency (less herding), the SPADE index registered the lowest value (MLM = 0.1862) in line with its spectral width during the Ukrainian invasion. On the other hand, Prime System Contractors were the least efficient (MLM = 0.5856), reflecting acute long-memory effects on their returns. Conversely, by the Israel-Hamas conflict period and having a defined possible outcome of the Ukrainian war, investors had presumably adjusted their expectations regarding prime contractors, recognizing their robustness amid prolonged geopolitical conflicts. Capital inflows likely stabilized around these large-caps firms, consistent with Chordia et al. (2005) findings, reducing volatility and inefficiency, and thus lowering their MLM score. Finally, manufacturing maintains moderate MLM scores throughout, displaying stable but robust inefficiency that barely fluctuates under geopolitical stress. This resilience likely reaffirms the previous statement that it is due to its broad product portfolios and geographically diverse operations that balance idiosyncratic shocks and prevent dramatic shifts in long-memory dynamics. This finding contrasts with the study by Choi (2021), where they find strong market efficiency changes across their sample period, suggesting heightened sensitivity and decreased efficiency in manufacturing during crises.

Table 3 – Multifractal Metrics of A&D Industry, its subsectors, and the S&P500 across the periods of study.

	Pre War (2020-Jan 2022)			Ukraine War (Feb 2022-Oct 2023)			Israel Conflict (Oct 2023-Dec 2024)		
Index / Subsector	$\Delta\alpha$	α^*	MLM	$\Delta\alpha$	α^*	MLM	$\Delta\alpha$	α^*	MLM
fGn	0.1321	0.4776	0.06607	0.1331	0.4749	0.06656	0.1621	0.4781	0.08106
S&P500	0.6278	0.5937	0.3118	0.8668	0.5913	0.4334	**1.4059**	**0.5724**	**0.7029**
SPADE Defense Index	1.0120	0.6471	0.5287	0.3724	0.5135	0.1862	**1.1167**	**0.5711**	**0.5584**
Manufacturing	0.7441	0.5841	0.3579	0.6712	0.5532	0.3356	**0.8613**	**0.5103**	**0.4306**
Prime System Contractors	0.6604	0.5241	0.3302	**1.1712**	**0.4840**	**0.5856**	0.6830	0.5686	0.3415
Services & Support	0.7546	0.5196	0.3773	1.0058	0.5733	0.5029	**1.1153**	**0.6766**	**0.5577**
Technology / C4ISR	0.8515	0.6005	0.4258	**1.0801**	**0.6257**	**0.5401**	0.8352	0.6241	0.4176

Source: Own Elaboration using data from FactSet

VI. CONCLUSION

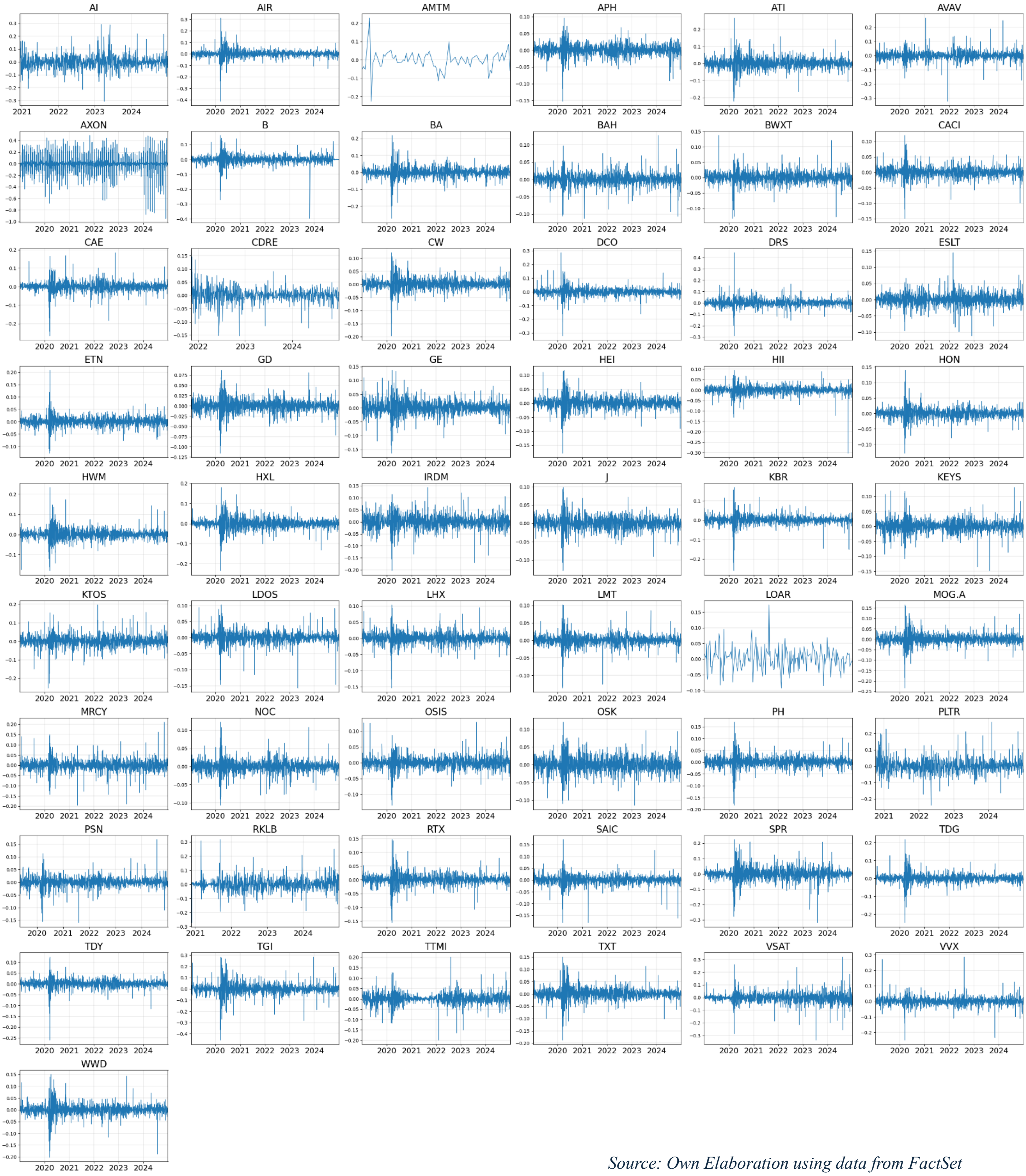
The global defense sector has undergone significant growth and transformation over the past five years, a scale of change not observed since the Cold War, driven by escalating geopolitical tensions and conflicts. This period has marked a shift in warfare, transitioning from traditional methods to more autonomous and digital forms, with financial markets also reflecting these changes and attracting large capital inflows. This study aims to extend and deepen the limited research on the impact of recent major conflicts on the defense industry by evaluating the efficiency of the U.S. aerospace and defense subsectors over the last five years. It specifically examines how the efficiency levels shifted during the Russian invasion of Ukraine, which commenced in February 2022, and the Israel–Hamas conflict, which began in October 2023, the analysis employs multifractal detrended fluctuation analysis and the generalized Hurst exponent, followed by the application of the Magnitude of Long Memory (MLM) index to rank efficiency levels throughout the overall period and during specific conflict periods compared to the S&P500 as U.S. stock Index benchmark, and the Fractional Gaussian Noise as the optimal market efficiency level.

The Aerospace and Defense (A&D) industry is highly affected by geopolitical conflicts and has experienced heightened anti-persistent behavior, reflecting the overreactions of the market regarding geopolitical conflicts. However, the geopolitical conflict effects are not equal across subsectors. We can denote that the Services and Support subsector exhibited the highest level of inefficiency and multifractality throughout the sample period, with significant efficiency fluctuations during crisis periods. This trend suggests that the prices of recurring service stocks are becoming increasingly mispriced under successive shocks, making this subsector particularly appealing for both short and long-term arbitrage strategies. Additionally, Prime System Contractors showed notable volatility in efficiency rankings, particularly between the two major conflicts studied. However, during the Israel–Hamas conflict, a notable recovery in efficiency ranking was observed, likely due to investors adjusting expectations regarding the long-term stability and government reliance of these prime contractors. Such significant shifts in investor perception highlight short-term arbitrage opportunities, especially in the early stages of geopolitical crises, where price reactions often over or under-estimate the actual long-term impact of geopolitical events on firm fundamentals. Similarly, the pronounced multifractal peaks in Technology/C4ISR during geopolitical stress suggest time-limited arbitrage opportunities, in which portfolio managers or informed traders can exploit mispricing before long-memory effects diminish. Meanwhile, the Manufacturing subsector maintained a relatively stable position in terms of multifractality and efficiency. Its moderate MLM rankings throughout the period, combined with less pronounced volatility during geopolitical events, indicated a smaller window for arbitrage opportunities during geopolitical events, making it a more defensive core holding for mid-long-term strategies.

This study has several important limitations. First, the two conflict periods analyzed are still ongoing, so the data may not fully reflect their complete market impact. As a result, the findings observed in the latter part of the sample, particularly for the Israel–Hamas conflict and the ongoing Ukraine war, cannot be attributed with complete confidence. Second, the Ukraine conflict remains active in the most recent sub-period and continues to influence market dynamics, further complicating causal interpretation. Third, due to the unavailability of daily market capitalization data for each subsector constituent, we computed subsector returns using an unweighted arithmetic average rather than market value weighting. This approach may introduce measurement bias, as a properly weighted index would likely yield more precise and representative results. In this regard, the study opens avenues for future and deeper research. Firstly, exploring a broader range of geopolitical conflicts could further validate and expand the current findings, providing a deeper understanding of how varying geopolitical shocks influence arbitrage opportunities based on the relationship between the nation's sector and the conflict's location, and expanding to global players. Future research could explore the role of investor sentiment, media influence (meme stocks), and psychological factors driving sector inefficiencies and further enhance the ability to identify and exploit these arbitrage opportunities when markets react irrationally and prices deviate from fundamental values, for instance, using the Black-Litterman theory regarding portfolio optimization. Additionally, incorporating high-frequency trading could offer richer insights regarding short-term arbitrage, particularly around the immediate onset of geopolitical events, for example, using the Moving Hurst as a technical indicator.

VII. APPENDIX

Figure a1 - SPADE Defense Index Log Returns (2020-2024)



Source: Own Elaboration using data from FactSet

Table a1- SPADE Defense Index constituents and their statistics (January 2020 – January 2022)

#	Company	Ticker	Subsector	Price	Mean	Max	Min	Market Cap ($bn.)	Total Return	Std. Dev.	Skewness	Kurtosis
1	Boeing	BA	Prime System Contractors	217.73	260.11	440.62	95.01	164.170	-39.7%	0.0137	-0.4041	12.8675
2	Raytheon Technologies	RTX	Prime System Contractors	94.04	76.50	96.00	44.21	125.633	38.7%	0.0304	-0.2511	10.8801
3	Honeywell	HON	Manufacturing: Systems, Subsystems, Components, and Hardware	189.53	183.11	234.18	103.86	121.808	36.3%	0.0103	-0.2228	10.8456
4	Lockheed Martin	LMT	Prime System Contractors	382.22	362.82	439.85	258.08	89.553	36.7%	0.0049	-1.0370	15.9826
5	General Electric	GE	Manufacturing: Systems, Subsystems, Components, and Hardware	62.81	51.46	71.45	27.33	66.977	48.9%	0.0572	-0.0901	4.3280
6	Eaton	ETN	Manufacturing: Systems, Subsystems, Components, and Hardware	155.53	111.35	174.66	57.77	60.932	81.8%	0.0194	0.3963	16.8240
7	General Dynamics	GD	Prime System Contractors	212.77	173.03	215.44	106.60	57.107	30.2%	0.0100	-0.5734	6.9632
8	Northrop Grumman	NOC	Prime System Contractors	384.77	334.26	406.62	239.92	55.379	44.6%	0.0054	-0.0422	8.3300
9	Amphenol	APH	Manufacturing: Systems, Subsystems, Components, and Hardware	38.40	28.95	43.97	17.09	46.449	65.5%	0.0640	-1.0502	9.9415
10	L3 Harris	LHX	Manufacturing: Systems, Subsystems, Components, and Hardware	217.04	196.32	244.73	130.05	40.575	48.9%	0.0096	-0.5755	11.1832
11	Parker Hannifan	PH	Manufacturing: Systems, Subsystems, Components, and Hardware	306.34	228.17	334.00	97.56	39.446	71.1%	0.0112	-0.6114	11.2974
12	Transdigm	TDG	Manufacturing: Systems, Subsystems, Components, and Hardware	656.21	533.18	685.00	245.79	36.803	65.9%	0.0055	-0.5708	17.5057
13	Palantir	PLTR	Technology/C4ISR	14.17	22.19	39.00	9.03	33.437	40.4%	0.2113	0.9846	3.3715
14	Keysight	KEYS	Technology/C4ISR	169.77	119.65	207.93	58.32	29.338	101.7%	0.0185	0.0761	5.6227
15	Heico	HEI	Manufacturing: Systems, Subsystems, Components, and Hardware	143.62	119.76	152.37	61.66	19.956	63.7%	0.0201	-0.3448	8.0240
16	Teledyne	TDY	Manufacturing: Systems, Subsystems, Components, and Hardware	425.44	348.72	463.38	200.06	19.936	72.6%	0.0063	-2.2339	29.5122
17	Howmet	HWM	Manufacturing: Systems, Subsystems, Components, and Hardware	34.99	23.04	35.73	9.19	14.152	90.0%	0.1363	-0.0226	10.0485
18	Textron	TXT	Manufacturing: Systems, Subsystems, Components, and Hardware	70.54	51.34	78.25	21.66	12.735	42.6%	0.0553	-0.4423	7.7789
19	Jacobs Engineering	J	Services and Support	103.37	85.43	123.95	47.84	12.667	75.0%	0.0227	-0.4559	6.5216
20	AXON International Inc.	AXON	Manufacturing: Systems, Subsystems, Components, and Hardware	143.25	105.66	203.51	43.29	11.152	104.0%	0.1246	-0.1560	3.4857
21	Leidos	LDOS	Technology/C4ISR	84.50	90.66	123.22	51.33	10.834	48.5%	0.0222	-0.7608	10.8020
22	Booz Allen	BAH	Services and Support	73.71	75.99	97.84	43.97	9.335	50.8%	0.0231	-0.9767	8.9936
23	CAE	CAE	Manufacturing: Systems, Subsystems, Components, and Hardware	26.71	24.17	34.05	9.94	8.538	36.3%	0.1277	-1.1897	17.6939
24	Elbit Systems	ESLT	Manufacturing: Systems, Subsystems, Components, and Hardware	172.81	142.17	180.25	111.88	7.711	40.5%	0.0120	-0.2248	3.2731
25	Oshkosh	OSK	Manufacturing: Systems, Subsystems, Components, and Hardware	115.42	90.50	136.92	48.78	7.459	62.8%	0.0263	-0.2193	3.1875
26	Huntington Ingalls	HII	Manufacturing: Systems, Subsystems, Components, and Hardware	180.13	199.56	278.57	137.83	7.068	-6.7%	0.0101	-0.5885	6.0480
27	Woodward	WWD	Manufacturing: Systems, Subsystems, Components, and Hardware	118.81	103.92	129.93	50.24	7.053	47.5%	0.0273	-0.5035	10.1094
28	Spirit	SPR	Manufacturing: Systems, Subsystems, Components, and Hardware	52.22	52.66	99.35	17.16	6.126	-32.7%	0.0829	-0.2267	8.0342
29	KBR	KBR	Services and Support	44.77	29.50	49.17	13.08	5.808	104.7%	0.0965	-1.5475	21.0359
30	Viasat	VSAT	Manufacturing: Systems, Subsystems, Components, and Hardware	44.12	56.15	94.25	27.46	5.697	-27.4%	0.0562	-0.3146	16.8797
31	CACI Intl	CACI	Technology/C4ISR	252.47	233.08	289.00	140.81	5.552	57.6%	0.0083	-0.7468	11.5547
32	Curtiss Wright	CW	Manufacturing: Systems, Subsystems, Components, and Hardware	138.03	118.06	149.25	76.52	5.203	29.3%	0.0202	-0.7523	10.8178
33	Rocketlab	RKLB	Manufacturing: Systems, Subsystems, Components, and Hardware	10.16	11.52	20.72	8.10	4.608	-23.1%	0.4064	1.1807	16.5551
34	Hexcel	HXL	Manufacturing: Systems, Subsystems, Components, and Hardware	56.25	58.73	85.33	26.75	4.523	-2.6%	0.0551	-0.5645	9.2311
35	BWX Technologies	BWXT	Manufacturing: Systems, Subsystems, Components, and Hardware	44.79	55.83	70.43	38.10	4.094	14.1%	0.0342	-0.5137	10.7061
36	Iridium	IRDM	Technology/C4ISR	35.75	30.84	54.37	17.91	3.867	63.3%	0.1013	-0.4662	4.3993
37	SAIC	SAIC	Technology/C4ISR	81.73	83.82	103.10	51.92	3.855	25.5%	0.0275	-1.3077	17.1704
38	Parsons	PSN	Technology/C4ISR	32.70	36.35	45.03	25.31	3.492	8.3%	0.0645	-1.0168	8.7484
39	C3.ai	AI	Technology/C4ISR	25.71	66.24	177.47	22.54	3.413	-162.1%	0.0822	0.9400	6.1525
40	ATI	ATI	Manufacturing: Systems, Subsystems, Components, and Hardware	23.93	18.18	29.26	5.23	3.376	6.8%	0.2181	-0.0848	7.2749
41	Mercury Computer Sys	MRCY	Manufacturing: Systems, Subsystems, Components, and Hardware	52.98	69.05	92.80	43.11	3.162	14.8%	0.0403	-0.1838	9.0598
42	Leonardo DRS	DRS	Manufacturing: Systems, Subsystems, Components, and Hardware	10.80	7.23	14.44	2.20	2.874	141.6%	0.5247	1.4549	28.6516
43	Kratos Defense & Security	KTOS	Manufacturing: Systems, Subsystems, Components, and Hardware	17.08	20.54	33.24	10.20	2.618	20.8%	0.1556	-1.2124	11.4042
44	Moog Inc.	MOG.A	Manufacturing: Systems, Subsystems, Components, and Hardware	77.50	77.41	97.55	35.00	2.444	0.2%	0.0382	-0.4085	10.8498
45	Barnes Group	B	Manufacturing: Systems, Subsystems, Components, and Hardware	47.01	49.20	66.92	32.66	2.412	-14.4%	0.0624	-1.0072	14.6356
46	Triumph	TGI	Manufacturing: Systems, Subsystems, Components, and Hardware	23.40	17.04	29.20	3.43	1.812	66.8%	0.3459	-0.7400	10.8346
47	AeroVironment	AVAV	Manufacturing: Systems, Subsystems, Components, and Hardware	60.76	77.68	137.94	46.72	1.715	-12.0%	0.0408	-0.7567	21.1860
48	AAR	AIR	Manufacturing: Systems, Subsystems, Components, and Hardware	43.05	34.09	51.88	9.44	1.554	13.9%	0.1180	-0.9990	23.3703
49	V2X	VVX	Manufacturing: Systems, Subsystems, Components, and Hardware	43.67	45.08	59.87	21.90	1.379	68.3%	0.0606	0.2145	22.7024
50	OSI Systems	OSIS	Manufacturing: Systems, Subsystems, Components, and Hardware	81.95	90.94	117.03	54.26	1.377	13.1%	0.0215	-0.0815	10.4323
51	TTM	TTMI	Manufacturing: Systems, Subsystems, Components, and Hardware	12.30	12.72	15.81	8.53	1.250	22.8%	0.1990	-0.9020	10.1205
52	Cadre Holdings	CDRE	Services and Support	22.99	21.11	25.42	15.30	0.935	40.7%	0.2344	0.7412	1.8598
53	Ducommun	DCO	Manufacturing: Systems, Subsystems, Components, and Hardware	45.14	44.58	64.43	16.61	0.671	21.4%	0.0800	-0.3801	17.6008
54	Amentum	AMTM	Services and Support	-	-	-	-	-	-	-	-	-
55	LOAR	LOAR	Manufacturing: Systems, Subsystems, Components, and Hardware	-	-	-	-	-	-	-	-	-

Source: Own Elaboration using data from FactSet

Table a2- SPADE Defense Index constituents and their statistics (February 2022 - December 2024)

#	Company	Ticker	Subsector	Price	Mean	Max	Min	Market Cap ($Bn.)	Total Return	Std. Dev.	Skewness	Kurtosis
1	Palantir	PLTR	Technology/C4ISR	75.63	19.05	82.38	6.00	178.46	167.47%	0.225	0.5148	6.0224
2	General Electric	GE	Manufacturing: Systems, Subsystems, Components, and Hardware	166.79	98.10	194.23	38.02	177.86	97.67%	0.019	-0.4251	3.6035
3	Raytheon Technologies	RTX	Prime System Contractors	115.72	97.42	127.21	69.38	154.60	20.75%	0.015	-0.3965	7.8818
4	Honeywell	HON	Manufacturing: Systems, Subsystems, Components, and Hardware	225.89	198.55	236.00	166.97	145.18	17.55%	0.006	-0.1613	1.8554
5	Boeing	BA	Prime System Contractors	177.00	184.23	264.27	115.86	133.46	-20.71%	0.013	-0.4348	2.4075
6	Eaton	ETN	Manufacturing: Systems, Subsystems, Components, and Hardware	331.87	221.17	377.52	125.04	130.02	75.79%	0.008	-0.3199	2.1956
7	Lockheed Martin	LMT	Prime System Contractors	485.94	464.48	614.61	382.22	113.85	24.01%	0.003	0.3679	7.5059
8	Amphenol	APH	Manufacturing: Systems, Subsystems, Components, and Hardware	69.45	47.33	75.35	31.26	84.01	59.25%	0.034	-0.5715	3.8160
9	Parker Hannifan	PH	Manufacturing: Systems, Subsystems, Components, and Hardware	636.03	411.84	709.46	236.37	81.90	73.06%	0.004	0.2992	3.7004
10	Transdigm	TDG	Manufacturing: Systems, Subsystems, Components, and Hardware	1267.28	907.97	1442.53	509.83	71.07	65.81%	0.002	-0.1441	2.8424
11	General Dynamics	GD	Prime System Contractors	263.49	249.72	314.03	204.18	70.72	21.38%	0.005	-0.1133	4.9299
12	Northrop Grumman	NOC	Prime System Contractors	469.29	470.56	549.01	384.77	67.54	19.86%	0.003	0.1658	7.5950
13	AXON International Inc.	AXON	Manufacturing: Systems, Subsystems, Components, and Hardware	594.32	238.81	689.78	84.37	46.27	142.28%	0.063	-1.7240	10.5545
14	Howmet	HWM	Manufacturing: Systems, Subsystems, Components, and Hardware	109.37	56.10	120.09	30.70	44.24	113.97%	0.034	0.9803	8.6616
15	L3 Harris	LHX	Manufacturing: Systems, Subsystems, Components, and Hardware	210.28	216.16	270.74	161.28	39.31	-3.16%	0.007	0.3652	5.1796
16	Heico	HEI	Manufacturing: Systems, Subsystems, Components, and Hardware	237.74	182.29	279.02	127.59	33.03	50.40%	0.009	-0.4181	3.3509
17	Keysight	KEYS	Technology/C4ISR	160.63	153.33	188.51	119.31	27.76	-5.53%	0.013	-0.5895	10.5835
18	Teledyne	TDY	Manufacturing: Systems, Subsystems, Components, and Hardware	464.13	413.54	489.24	331.10	21.75	8.70%	0.004	-0.7771	5.9181
19	Leidos	LDOS	Technology/C4ISR	144.06	114.00	201.39	76.92	18.47	53.35%	0.014	-1.8760	24.3712
20	Jacobs Engineering	J	Services and Support	133.62	111.67	149.25	90.58	16.37	25.67%	0.014	-0.5583	3.4132
21	Booz Allen	BAH	Services and Support	128.70	118.14	186.00	70.48	16.30	55.73%	0.015	0.1317	9.2565
22	Textron	TXT	Manufacturing: Systems, Subsystems, Components, and Hardware	76.49	75.65	96.98	57.93	13.81	8.10%	0.023	0.1632	5.8242
23	Curtiss Wright	CW	Manufacturing: Systems, Subsystems, Components, and Hardware	354.87	211.42	389.49	125.45	13.38	94.43%	0.007	-0.1810	2.4579
24	Rocketlab	RKLB	Manufacturing: Systems, Subsystems, Components, and Hardware	25.47	6.26	28.44	3.53	11.55	91.90%	0.709	0.3649	3.3597
25	Elbit Systems	ESLT	Manufacturing: Systems, Subsystems, Components, and Hardware	258.07	201.44	263.50	163.03	11.52	40.10%	0.009	0.0485	8.1951
26	BWX Technologies	BWXT	Manufacturing: Systems, Subsystems, Components, and Hardware	111.39	75.54	133.28	42.78	10.18	91.11%	0.022	0.3077	9.1133
27	Woodward	WWD	Manufacturing: Systems, Subsystems, Components, and Hardware	166.42	127.29	187.29	80.26	9.88	33.70%	0.015	-1.0120	18.2537
28	Parsons	PSN	Technology/C4ISR	92.25	60.08	113.31	30.76	9.85	103.71%	0.029	1.2615	16.6058
29	CACI Intl	CACI	Technology/C4ISR	404.06	345.93	572.44	246.28	8.89	47.03%	0.004	-0.3211	4.2871
30	Leonardo DRS	DRS	Manufacturing: Systems, Subsystems, Components, and Hardware	32.31	18.12	37.05	7.76	8.60	109.58%	0.155	-0.0778	6.7819
31	CAE	CAE	Manufacturing: Systems, Subsystems, Components, and Hardware	25.38	21.30	27.74	15.33	8.11	-5.11%	0.110	-0.0407	11.8289
32	ATI	ATI	Manufacturing: Systems, Subsystems, Components, and Hardware	55.04	41.85	67.71	20.96	7.76	83.29%	0.060	-0.2091	4.5625
33	KBR	KBR	Services and Support	57.93	56.82	72.02	42.97	7.52	25.77%	0.031	-1.4648	15.9957
34	Huntington Ingalls	HII	Manufacturing: Systems, Subsystems, Components, and Hardware	188.97	231.68	296.43	180.13	7.41	4.79%	0.008	-5.9147	92.7807
35	LOAR	LOAR	Manufacturing: Systems, Subsystems, Components, and Hardware	73.91	69.08	94.34	46.00	6.91	39.66%	0.054	0.4094	2.1524
36	Moog Inc.	MOG.A	Manufacturing: Systems, Subsystems, Components, and Hardware	196.84	124.40	226.58	70.35	6.21	93.21%	0.015	0.2863	6.6214
37	Oshkosh	OSK	Manufacturing: Systems, Subsystems, Components, and Hardware	95.07	96.71	127.15	70.29	6.14	-19.40%	0.020	-0.1561	3.1937
38	SAIC	SAIC	Technology/C4ISR	111.78	111.99	154.10	79.10	5.27	31.31%	0.015	-1.5418	22.2950
39	Amentum	AMTM	Services and Support	21.03	25.99	33.40	18.93	5.12	-28.63%	0.216	0.0697	6.6171
40	Hexcel	HXL	Manufacturing: Systems, Subsystems, Components, and Hardware	62.70	64.30	78.04	48.77	5.04	10.86%	0.030	-0.8625	4.7965
41	C3.ai	AI	Technology/C4ISR	34.43	24.29	46.37	10.26	4.57	29.20%	0.218	0.5147	5.6926
42	AeroVironment	AVAV	Manufacturing: Systems, Subsystems, Components, and Hardware	153.89	122.32	235.17	56.93	4.34	92.93%	0.026	0.8162	10.0522
43	Kratos Defense & Security	KTOS	Manufacturing: Systems, Subsystems, Components, and Hardware	26.38	16.74	28.29	9.06	4.04	43.47%	0.173	1.0442	5.9813
44	Spirit	SPR	Manufacturing: Systems, Subsystems, Components, and Hardware	34.08	30.82	53.11	14.84	4.00	-42.68%	0.116	-1.1093	13.5955
45	Iridium	IRDM	Technology/C4ISR	29.02	41.78	67.26	24.65	3.14	-20.86%	0.057	-0.3321	7.1099
46	OSI Systems	OSIS	Manufacturing: Systems, Subsystems, Components, and Hardware	167.43	114.03	187.75	70.83	2.81	71.45%	0.016	0.5294	4.9604
47	TTM	TTMI	Manufacturing: Systems, Subsystems, Components, and Hardware	24.75	15.57	26.66	11.21	2.52	69.92%	0.167	0.4317	10.4895
48	Mercury Computer Sys	MRCY	Manufacturing: Systems, Subsystems, Components, and Hardware	42.00	42.59	69.81	26.23	2.51	-23.22%	0.065	0.0937	11.5517
49	Barnes	B	Manufacturing: Systems, Subsystems, Components, and Hardware	47.26	37.75	47.91	19.83	2.42	1%	0.065	-5.60	95.09
50	AAR	AIR	Manufacturing: Systems, Subsystems, Components, and Hardware	61.28	56.11	75.54	34.97	2.21	35.31%	0.036	-0.6770	4.2255
51	V2X	VVX	Manufacturing: Systems, Subsystems, Components, and Hardware	47.83	44.55	68.82	29.81	1.51	9.10%	0.063	0.5929	23.0980
52	Triumph	TGI	Manufacturing: Systems, Subsystems, Components, and Hardware	18.66	13.71	27.31	7.17	1.44	-22.64%	0.277	-0.3407	13.5964
53	Cadre Holdings	CDRE	Services and Support	32.30	28.02	40.22	18.15	1.31	34.00%	0.092	-1.2477	6.5165
54	Viasat	VSAT	Manufacturing: Systems, Subsystems, Components, and Hardware	8.51	27.44	51.38	6.83	1.10	-164.57%	0.169	-0.2474	10.0432
55	Ducommun	DCO	Manufacturing: Systems, Subsystems, Components, and Hardware	63.66	51.65	69.18	39.66	0.95	34.38%	0.041	-0.4424	8.0907

Source: Own Elaboration using data from FactSet

Python Code

```
!pip install MFDFA

%config InlineBackend.figure_format = 'png' # ‘png’, ‘retina’, ‘jpeg’, ‘svg’, ‘pdf’

import pandas as pd

from pathlib import Path

import numpy as np

import matplotlib.pyplot as plt

from MFDFA import MFDFA

from numpy.polynomial.polynomial import polyfit

import plotly.graph_objects as go
```

```
from plotly.subplots import make_subplots
import pandas as pd
import seaborn as sns
!pip install fbm
from fbm import FBM # Corrected import statement
sns.set()



import numpy as np
import pandas as pd
from fbm import FBM
from MFDFA import MFDFA
from numpy.polynomial.polynomial import polyfit
from joblib import Parallel, delayed
import matplotlib.pyplot as plt

# 1) Función MFDFA

def get_mfdfa(data,
           q=None,
           order=1,
           integrate=True,
           lag_start=None,
           lag_end=None,
           lag_steps=None,
           window=None,
           tipo='real'):

    n = len(data)
    if lag_start is None:
        lag_start, lag_end, lag_steps = select_lag_params(n, tipo=tipo)
    lags = np.unique(
```

```
        np.logspace(np.log10(lag_start), np.log10(lag_end), lag_steps).astype(int)
    )
    if q is None:
        q = np.linspace(-5, 5, 41)
        q = q[q != 0]
    series = (data - data.mean()).cumsum() if integrate else data
    ext = {'window': window} if window else {}
    lag, dfa = MFDFA(series, lag=lags, q=q, order=order, extensions=ext)
    start, end = int(0.0 * len(lag)), int(0.6 * len(lag))
    slope = polyfit(np.log10(lag[start:end]), np.log10(dfa[start:end]), 1)[1]
    hq = slope - 1
    tau = q * hq - 1
    alpha = np.gradient(tau, q)
    Dalpha = q * alpha - tau
    metrics = pd.DataFrame({
        'slope': slope,
        'hq': hq,
        'tau': tau,
        'alpha': alpha,
        'Dalpha': Dalpha
    }, index=q)
    spectral_width = metrics['Dalpha'].max() - metrics['Dalpha'].min()
    return q, lag, dfa, slope, hq, tau, alpha, Dalpha, metrics, spectral_width


# metemos los lags en una función para establecer unos u otros en función del tipo, ya sea para el fGn o los índices/subsectore

def select_lag_params(length, tipo='real'):
    if tipo == 'fGn':
        return 5, min(100, length // 4), length // 4
    return 3, min(100, length // 6), length // 6


# Función para generar el Fractional Gaussian Noise segun el tamaño de la muestra

def generate_fgn_series(length, hurst=0.5, method='daviesharte'):
```

```
    f = FBM(n=length-1, hurst=hurst, length=1, method=method)
    return f.fgn()

# Rolling Hurst

def hurst_rolling(log_rets, window=252, step=1):

    def _calculate_hq(x):
        # compute only h(q=2)
        q, _, _, _, hq, _, _, _, metrics, _ = get_mfdfa(
            x.values,
            q=[-2, 2],
            window=False,
            tipo='real'
        )
        # metrics indexed by q, pick h at q=2
        return metrics.loc[2, 'hq']

    # rolling window with step
    rolled = log_rets.rolling(window=window, min_periods=window).apply(_calculate_hq)
    return rolled.iloc[::step]

# Plotting Rolling Hurst

def plot_rolling_hurst(hurst_dict, confidence_interval=(0.4, 0.6), events=None, figsize=(14,6)):

    plt.figure(figsize=figsize)
    for name, series in hurst_dict.items():
        if name == 'SPADE Defense Index':
            plt.plot(series.index, series.values, '--', label=name, linewidth=1.5)
        else:
            plt.plot(series.index, series.values, label=name, linewidth=1.0)
```

```
    # intervalor de confianza
    plt.axhline(0.5, color='gray', linestyle='--', label='H=0.5')
    for ci in confidence_interval:
        plt.axhline(ci, color='gray', linestyle=':')

    # Para mostrar en el gráfico la etiqueta de los conflictos
    if events:
        y_top = plt.ylim()[1]
        for label, date in events.items():
            dt = pd.to_datetime(date)
            plt.axvline(dt, color='gray', linestyle='--')
            plt.text(dt, y_top, label, ha='center', va='top',
                     bbox=dict(facecolor='white', edgecolor='black', boxstyle='round'), fontsize=9)
    plt.title('Rolling Hurst Exponent Comparison')
    plt.xlabel('Date')
    plt.ylabel(r'$h(2)$')
    plt.ylim(0,1)
    plt.legend(loc='upper left', fontsize=8)
    plt.grid(alpha=0.3)
    plt.tight_layout()
    plt.show()

#Función boodtraping para el fGn generado
# Iteramos 1000  veces ya que aunque n_jobs = -1, tarda bastante
def bootstrap_mfdfa(series, q_vals, n_iter=1000, n_jobs=-1):

    def one_it(_):
        sample = np.random.choice(series, size=len(series), replace=True)
        lag_s, lag_e, lag_st = select_lag_params(len(sample), tipo='fGn')
        _, _, _, _, hq, _, alpha, Dalpha, _, _ = get_mfdfa(
            sample,
```

```
            q=q_vals,
            lag_start=lag_s,
            lag_end=lag_e,
            lag_steps=lag_st,
            tipo='fGn'
        )
        return hq, alpha, Dalpha
    out = Parallel(n_jobs=n_jobs)(delayed(one_it)(i) for i in range(n_iter))
    hq_arr, alpha_arr, Dalpha_arr = map(np.array, zip(*out))
    return {
        'hq_mean': hq_arr.mean(0),
        'hq_ci': np.percentile(hq_arr, [2.5, 97.5], axis=0),
        'alpha_mean': alpha_arr.mean(0),
        'alpha_ci': np.percentile(alpha_arr, [2.5, 97.5], axis=0),
        'Dalpha_mean': Dalpha_arr.mean(0)
    }

# Calculamos Δα, α*, MLM para cada periodo tnato para el fGn como de los índices

def compute_metrics_over_periods(series_dict, periods, qs, n_iter=1000, n_jobs=-1):

    records = []
    for name, series in series_dict.items():
        for period_name, (start, end) in periods.items():
            seg = series.loc[start:end] if hasattr(series, 'loc') else series
            vals = seg.values if hasattr(seg, 'values') else seg
            if name == 'fGn':
                stats = bootstrap_mfdfa(vals, qs, n_iter, n_jobs)
                h_vals = stats['hq_mean']
                D_vals = stats['Dalpha_mean']
                alpha_vals = stats['alpha_mean']
            else:
```

```
            _, _, _, _, h_vals, _, alpha_vals, D_vals, _, _ = get_mfdfa(
                vals,
                q=qs,
                tipo='real'
            )
        delta_h = np.max(alpha_vals) - np.min(alpha_vals)
        alpha_star = alpha_vals[np.argmax(D_vals)]
        try:
            h_m = h_vals[qs == qs.min()][0]
            h_p = h_vals[qs == qs.max()][0]
            mlm = 0.5 * (abs(h_m - 0.5) + abs(h_p - 0.5))
        except IndexError:
            mlm = np.nan
        records.append({
            'Index': name,
            'Period': period_name,
            'Δα': delta_h,
            'α*': alpha_star,
            'MLM': mlm
        })
    return pd.DataFrame(records)

# Calculamos Δα, α*, MLM de cada subsector en cada periodo

def compute_subsector_summary(subsector_df, periods, qs):

    summary = []
    for subsector, series in subsector_df.items():
        for period_name, (start, end) in periods.items():
            vals = series.loc[start:end].dropna().values
            _, _, _, _, h_vals, _, alpha_vals, D_vals, _, _ = get_mfdfa(
                vals,
```

```
            q=qs,
            tipo='real'
        )
        width = np.max(alpha_vals) - np.min(alpha_vals)
        alpha_star = alpha_vals[np.argmax(D_vals)]
        try:
            h_m = h_vals[qs == qs.min()][0]
            h_p = h_vals[qs == qs.max()][0]
            mlm = 0.5 * (abs(h_m - 0.5) + abs(h_p - 0.5))
        except IndexError:
            mlm = np.nan
        summary.append({
            'Subsector': subsector,
            'Period': period_name,
            'Width(Δh)': width,
            'α*': alpha_star,
            'MLM': mlm
        })
    return pd.DataFrame(summary)

# Graficamos el Hurst exponent y el Multifractal Spectrum de los indices y fGn durante todo el periodo

def plot_hq_and_spectrum(results, boot_stats, qs, colors):
    fig, ax = plt.subplots(1, 2, figsize=(14, 5))
    # h(q)
    for name, res in results.items():
        if name != 'fGn':
            ax[0].plot(res['q'], res['hq'], label=name, color=colors[name])
    ax[0].plot(qs, boot_stats['hq_mean'], '--', color=colors['fGn'], label='fGn')
    ax[0].set(xlabel='q', ylabel='h(q)', title='Generalized Hurst exponent')
    ax[0].axvline(qs.mean(), linestyle='--', color='gray')
    ax[0].legend(); ax[0].grid(True)
```

```
    # spectrum
    for name, res in results.items():
        if name != 'fGn':
            ax[1].plot(res['alpha'], res['Dalpha'], label=name, color=colors[name])
    ax[1].plot(boot_stats['alpha_mean'], boot_stats['Dalpha_mean'], '--', color='crimson', label='fGn')
    ax[1].set(xlabel='α', ylabel='D(α)', title='Multifractal spectrum')
    ax[1].axvline(0.5, linestyle='--', color='gray')
    ax[1].legend(); ax[1].grid(True)
    plt.tight_layout()
    plt.show()

#Graficamos el Hurst exponent y el Multifractal Spectrum de los subsector y fGn durante en cada periodo

def compute_subsector_curves(subsector_df, periods, qs):
    curves = {}
    for subsector, series in subsector_df.items():
        curves[subsector] = {}
        for pname, (start, end) in periods.items():
            vals = series.loc[start:end].dropna().values
            _, _, _, _, h_vals, _, alpha_vals, D_vals, _, _ = get_mfdfa(vals, q=qs)
            curves[subsector][pname] = {'hq': h_vals, 'alpha': alpha_vals, 'Dalpha': D_vals}
    return curves

#sacamos el fGn boostrapped en cada periodo
def compute_fgn_period_bootstrap(fgn_series, periods, qs, n_iter=1000, n_jobs=-1):
    fgn_curves = {}
    for pname, (start, end) in periods.items():
        vals = fgn_series.loc[start:end].dropna().values
        stats = bootstrap_mfdfa(vals, qs, n_iter, n_jobs)
        fgn_curves[pname] = {
            'hq_mean': stats['hq_mean'],
            'alpha_mean': stats['alpha_mean'],
```

```
        'Dalpha_mean': stats['Dalpha_mean']
    }
  return fgn_curves

#Graficamos los metricas de cada subsector en cada periodo junto con el fGn por periodo

def plot_subsector_periods(subsector, curves, fgn_curves, qs, period_colors):
  fig, ax = plt.subplots(1, 2, figsize=(12, 5))
  # h(q)
  for pname, col in period_colors.items():
    # subsector
    sub_h = curves[subsector][pname]['hq']
    ax[0].plot(qs, sub_h, label=pname, color=col)
    # fGn
    fgn_h = fgn_curves[pname]['hq_mean']
    ax[0].plot(qs, fgn_h, linestyle='--', color=col, label=f'fGn {pname}')
  ax[0].set(title=f"{subsector} — h(q)", xlabel='q', ylabel='h(q)')
  ax[0].legend(); ax[0].grid(True)

  # D(α)
  for pname, col in period_colors.items():
    # trazado del subsector
    sub_a = curves[subsector][pname]['alpha']
    sub_D = curves[subsector][pname]['Dalpha']
    ax[1].plot(sub_a, sub_D, label=pname, color=col)
    # fGn D(α) con mismo color de periodo
    fgn_a = fgn_curves[pname]['alpha_mean']
    fgn_D = fgn_curves[pname]['Dalpha_mean']
    ax[1].plot(fgn_a, fgn_D, linestyle='--', color=col, label=f'fGn {pname}')
  ax[1].set(title=f"{subsector} — D(α)", xlabel='α', ylabel='D(α)')
  ax[1].legend(); ax[1].grid(True)
```

```
    plt.tight_layout()
    plt.show()


#PROCEDEMOS A EJECUTAR EL CODIGO
#Cargamos y calulamos los daily log rets de los indices
df = pd.read_excel('/content/Datos_SPADE_SP500_.xlsx', parse_dates=['Date']).set_index('Date')
log_rets_spade = df['SPADE Defense Index'].loc['2020-01-01':]
log_rets_spade = np.log(log_rets_spade / log_rets_spade.shift(1)).dropna()


# Generamos y añadimos el fGn
df['fGn'] = generate_fgn_series(len(df) + 1, hurst=0.5)


qs = np.linspace(-5, 5, 40)
qs = qs[qs != 0]


# Ejecutamos el MFDFA para los indices
results = {}
for name in df.columns:
    q, lag, dfa, slope, h_vals, tau, alpha_vals, D_vals, metrics, sw = get_mfdfa(df[name].dropna().values,
q=qs, tipo = 'real')
    results[name] = {'q': q, 'hq': metrics['hq'], 'alpha': metrics['alpha'], 'Dalpha': metrics['Dalpha']}


# Bootstrap fGn
boot_stats = bootstrap_mfdfa(df['fGn'].values, qs, n_iter=1000, n_jobs=-1)


# Graficos de los indices vs fGn
colors = {'SPADE Defense Index':'black','S&P500':'purple','fGn':'crimson'}
plot_hq_and_spectrum(results, boot_stats, qs, colors)


# Definimos los periodos
periods = {
    'Pre War': ('2020-01-01','2022-02-23'),
```

```
    'Ukraine War': ('2022-02-24','2023-10-06'),
    'Israel Conflict': ('2023-10-07','2024-12-31')
}

# Calculamos las metricas por periodo para los indices y el fGn
series_dict = {name: df[name] for name in df.columns}
index_summary = compute_metrics_over_periods(series_dict, periods, qs, n_jobs=-1)
print(index_summary)

# Procedemos con los subsectores
#Cargamos datos y calculamos los log rets por subsector
sub_data = pd.read_excel('/content/Merged_Cleaned_Defense_Data - copia (2019-2024).xlsx', parse_dates=['Date'])
sub_data = sub_data[(sub_data['Date'] >= '2020-01-01') & (sub_data['Date'] <= '2024-12-31')]

sub_idx = (
    sub_data
    .groupby(['Date', 'Subsector'])['Close Price']
    .mean()
    .unstack('Subsector')
    .sort_index()
)
sub_log_rets = np.log(sub_idx / sub_idx.shift(1)).dropna()

#Rolling Hurst del SPADE Defense Index vs los subsectores
hurst_dict = {}
for name, series in sub_log_rets.items():
    hurst_dict[name] = hurst_rolling(series, window=252, step=5).clip(0,1)
hurst_dict['SPADE Defense Index'] = hurst_rolling(log_rets_spade, window=252, step=5).clip(0,1)

# Añadimos la fecha de los conflictos
events = {
    'Ukraine Invasion Feb 24, 2022': '2022-02-24',
```

```
    'Hamas Attack Oct 7, 2023': '2023-10-07'
}
plot_rolling_hurst(hurst_dict, confidence_interval=(0.4,0.6), events=events)

# Sacamos las metricas por subsector y periodo
subsector_summary = compute_subsector_summary(sub_log_rets, periods, qs)
print("Subsector summary metrics:")
pivot_sub = subsector_summary.pivot(index='Subsector', columns='Period', values=['Width(Δh)', 'α*',
'MLM'])
display(pivot_sub)
subsector_curves = compute_subsector_curves(sub_log_rets, periods, qs)

# Graficamos los G. Hurst Exponents y Multifractal Spectrum de cada subsector en cada period + el fGn
por periodo
fgn_curves = compute_fgn_period_bootstrap(df['fGn'], periods, qs,
                                   n_iter=1000, n_jobs=-1)
# Colors for periods
period_colors = {'Pre War':'gray','Ukraine War':'blue','Israel Conflict':'red'}
for subsector in sub_log_rets.columns:
    plot_subsector_periods(subsector, subsector_curves, fgn_curves, qs, period_colors)
```

VIII. BIBLIOGRAPHY

Aslam, F., Mohti, W., & Ferreira, P. (2020). Evidence of Intraday Multifractality in European Stock Markets during the recent Coronavirus (COVID-19) Outbreak. *International Journal of Financial Studies*, *8*(2), 31. https://doi.org/10.3390/ijfs8020031

Assaf, R., Gupta, D., & Kumar, R. (2023). The price of war: Effect of the Russia–Ukraine war on the global financial market. *The Journal of Economic Asymmetries, 28*, e00328. https://doi.org/10.1016/j.jeca.2023.e00328

Bacry, E., Gloter, A., Hoffmann, M., & Muzy, J. F. (2010). Multifractal analysis in a mixed asymptotic framework. *The Annals of Applied Probability, 20*(5), 1729–1760. https://doi.org/10.1214/09-AAP670

Bentes, S. R. (2016). Long memory volatility of gold price returns: How strong is the evidence from distinct economic cycles? *Physica A: Statistical Mechanics and Its Applications*, *443*, 149–160. https://doi.org/10.1016/j.physa.2015.09.065

Blackledge, J., & Lamphiere, M. (2022). A Review of the Fractal Market Hypothesis for Trading and Market Price Prediction. *Mathematics*, *10*(1), 117. https://doi.org/10.3390/math10010117

Chau, M., & Vayanos, D. (2008). Strong-Form Efficiency with Monopolistic Insiders. *The Review of Financial Studies*, *21*(5), 2275–2306. https://doi.org/10.1093/rfs/hhl029

Choi, S.-Y. (2021). Analysis of stock market efficiency during crisis periods in the US stock market: Differences between the global financial crisis and COVID-19 pandemic. *Physica A: Statistical Mechanics and Its Applications*, *574*, 125988. https://doi.org/10.1016/j.physa.2021.125988

Erer, D., Erer, E. & Güngör, S. The aggregate and sectoral time-varying market efficiency during crisis periods in Turkey: a comparative analysis with COVID-19 outbreak and the global financial crisis. *Financ Innov* **9**, 80 (2023). https://doi.org/10.1186/s40854-023-00484-4

Fama, E. F. (1970). Efficient capital markets: A review of theory and empirical work. *The Journal of Finance, 25*(2), 383–417. https://doi.org/10.2307/2325486

Gurdgiev, C., Henrichsen, A., & Mulhair, A. (2022). The budgets of wars: Analysis of the U.S. defense stocks in the post-Cold War era. *International Review of Economics & Finance, 82*, 335–346. Retrieved from: https://doi.org/10.1016/j.iref.2022.06.023

Gupta, R., & Basu, P. K. (2011). Weak Form Efficiency In Indian Stock Markets. *International Business & Economics Research Journal (IBER)*, *6*(3). https://doi.org/10.19030/iber.v6i3.3353

Ihlen, E. A. F. (2012). Introduction to multifractal detrended fluctuation analysis in Matlab. *Frontiers in Physiology, 3*, 141. https://doi.org/10.3389/fphys.2012.00141

Invesco | Product Detail | Invesco Aerospace & Defense ETF. (2025). Invesco.com. https://www.invesco.com/us/financial-products/etfs/product-detail?audienceType=Investor&ticker=PPA

Jarrow, R. A., & Larsson, M. (2012). The meaning of market efficiency. *Mathematical Finance, 22*(1), 1–30. https://doi.org/10.1111/j.1467-9965.2011.00497.x

Kantelhardt, J. W., Zschiegner, S. A., Koscielny-Bunde, E., Havlin, S., Bunde, A., & Stanley, H. E. (2002). Multifractal detrended fluctuation analysis of nonstationary time series. *Physica A: Statistical Mechanics and its Applications, 316*(1–4), 87–114. https://doi.org/10.1016/S0378-4371(02)01383-3

Klein, T. (2024). Investor behavior in times of conflict: A natural experiment on the interplay of geopolitical risk and defense stocks. Journal of Economic Behavior and Organization, 222, 294–313. Sourced from: https://doi.org/10.1016/j.jebo.2024.04.020

Lin, X. (2023). The Limitations of the Efficient Market Hypothesis. *Highlights in Business, Economics and Management*, *20*, 37–41. https://doi.org/10.54097/hbem.v20i.12311

Malkiel, B. G. (2003). The Efficient Market Hypothesis and Its Critics. *Journal of Economic Perspectives*, *17*(1), 59–82. https://www.aeaweb.org/articles?id=10.1257/089533003321164958

Mandelbrot, B. The inescapable need for fractal tools in finance. *Annals of Finance* **1**, 193–195 (2005). https://doi.org/10.1007/s10436-004-0008-1

Memon, B. A., Yao, H., & Naveed, H. M. (2022). Examining the efficiency and herding behavior of commodity markets using multifractal detrended fluctuation analysis. *Resources Policy, 77*, 102715. https://doi.org/10.1016/j.resourpol.2022.102715

Menezes, R., Oliveira, Á., & Portela, S. (2018). Investigating detrended fluctuation analysis with structural breaks. *Physica a Statistical Mechanics and Its Applications*, *518*, 331–342. https://doi.org/10.1016/j.physa.2018.12.006

Mnif, E., & Jarboui, A. (2021). COVID-19, bitcoin market efficiency, herd behaviour. *Review of Behavioral Finance*, *13*(1), 69–84. https://doi.org/10.1108/rbf-09-2020-0233

Peters, E. E. (1989). Fractal structure in the capital markets. *Financial Analysts Journal, 45*(4), 32–37. https://doi.org/10.2469/faj.v45.n4.32

Rydin Gorjão, L., Hassan, G., Kurths, J., & Witthaut, D. (2022). MFDFA: Efficient multifractal detrended fluctuation analysis in Python. *Computer Physics Communications, 273*, 108254. https://doi.org/10.1016/j.cpc.2021.108254

Salat, H., Murcio, R., & Arcaute, E. (2017). Multifractal methodology. *Physica A: Statistical Mechanics and Its Applications*, *473*, 467–487. https://doi.org/10.1016/j.physa.2017.01.041

Sacknoff, S. (2023). Investing in the defense sector: Strategic growth, technological innovation, and policy risk. SPADE Indexes, 28(1), 4–15. Retrieved from:

https://spadeindex.com/Files/Investing_Defense_Sector_Guide.pdf

Singh, A., Patel, R., & Singh, H. (2022). Recalibration of priorities: Investor preference and Russia–Ukraine conflict. Finance Research Letters, 50, 103294. Sourced from: https://doi.org/10.1016/j.frl.2022.103294

SPDR ® S&P ®. Retrieved June 17, 2025, from https://www.ssga.com/library-content/products/factsheets/etfs/us/factsheet-us-en-xar.pdf

Thompson, J. R., & Wilson, J. R. (2016). Multifractal detrended fluctuation analysis: Practical applications to financial time series. *Mathematics and Computers in Simulation, 126*, 63–88. https://doi.org/10.1016/j.matcom.2016.03.003

iShares by BlackRock. (2024). *iShares U.S. Aerospace & Defense ETF (ITA): Fund profile*. https://www.ishares.com/us/literature/fact-sheet/ita-ishares-u-s-aerospace-defense-etf-fund-fact-sheet-en-us.pdf

Wang, Y., Liu, L., & Gu, R. (2009). Analysis of efficiency for Shenzhen stock market based on multifractal detrended fluctuation analysis. *International Review of Financial Analysis*, *18*(5), 271–276. https://doi.org/10.1016/j.irfa.2009.09.005

Zunino, L., Tabak, B. M., Pérez, D. G., Garavaglia, M., & Rosso, O. A. (2008). Inefficiency in Latin American market indices. *The European Physical Journal B, 60*(1), 111–121. https://doi.org/10.1016/j.physa.2008.08.028

Declaración de Uso de Herramientas de Inteligencia Artificial Generativa en Trabajos Fin de Grado

ADVERTENCIA: Desde la Universidad consideramos que ChatGPT u otras herramientas similares son herramientas muy útiles en la vida académica, aunque su uso queda siempre bajo la responsabilidad del alumno, puesto que las respuestas que proporciona pueden no ser veraces. En este sentido, NO está permitido su uso en la elaboración del Trabajo fin de Grado para generar código porque estas herramientas no son fiables en esa tarea. Aunque el código funcione, no hay garantías de que metodológicamente sea correcto, y es altamente probable que no lo sea.

Por la presente, yo, Mario Laureano Simón Alonso, estudiante de E2 + Business Analytics de la Universidad Pontificia Comillas al presentar mi Trabajo Fin de Grado titulado "THE MODERN INVESTOR’S DILEMMA: INVESTING IN DEFENSE COMPANIES AN ANALYSIS OF THE IMPACT OF

WAR CONFLICTS ON THE RISK-RETURN RELATIONSHIP IN THE U.S. DEFENSE INDUSTRY", declaro que he utilizado la herramienta de Inteligencia Artificial Generativa ChatGPT u otras similares de IAG de código sólo en el contexto de las actividades descritas a continuación [el alumno debe mantener solo aquellas en las que se ha usado ChatGPT o similares y borrar el resto. Si no se ha usado ninguna, borrar todas y escribir “no he usado ninguna”]:

1. **Brainstorming** de ideas de investigación: Utilizado para idear y esbozar posibles áreas de investigación.

2. **Referencias**: Usado conjuntamente con otras herramientas, como Zotero, para identificar referencias preliminares que luego he contrastado y validado.

4. **Interpretador de código**: Para realizar análisis de datos preliminares.

6. **Corrector de estilo literario y de lenguaje**: Para mejorar la calidad lingüística y estilística del texto.

7. **Sintetizador y divulgador de libros complicados:** Para resumir y comprender literatura compleja.

8. **Revisor**: Para recibir sugerencias sobre cómo mejorar y perfeccionar el trabajo con diferentes niveles de exigencia.

Afirmo que toda la información y contenido presentados en este trabajo son producto de mi investigación y esfuerzo individual, excepto donde se ha indicado lo contrario y se han dado los créditos correspondientes (he incluido las referencias adecuadas en el TFG y he explicitado para que se ha usado ChatGPT u otras herramientas similares). Soy consciente de las implicaciones académicas y éticas de presentar un trabajo no original y acepto las consecuencias de cualquier violación a esta declaración.

Fecha: 18/06/2025

Firma: ___Mario Laureano Simón Alonso_________________________