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Analyzing the computational performance of balance constraints in the medium-term unit commitment problem: Tightness, compactness, and arduousness

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ABSTRACT

Since its beginning, the computational performance of numerical optimization techniques has depended on utilizing efficient mathematical formulations to deal with large-size problems successfully. This fact is manifested in the unit commitment literature. Several approaches have been proposed to handle the complexity of accurately modeling real power systems. However, most of these methodologies focus on strengthening the technical features' representations by reducing the number of constraints and variables of the associated optimization problem or approximating its relaxed feasible region to the integer one to improve resolution processes. Hence, the state-of-art of these effective procedures is periodically studied under operational research and commercial solvers developments. Nevertheless, the formulation comparisons frequently obviate analyzing the impact of the balance equations on the computational burden of the unit commitment problem. This constraint links every single technical restriction along the time span and sometimes provides an ample optimization space, sometimes a narrow one, directly affecting resolution proceedings. It can impose an electricity generation equal to demand, allow production excesses, include non-served energy, or establish profit-based relationships. This paper presents a computational analysis of the most popular balance equations, detailing solver performances and determining these methodologies' tightness, compactness, and arduousness. Therefore, 1010 case studies were run utilizing different input profiles and optimality-convergence criteria.

1. Introduction

The efficient management of power systems requires the optimization of thermal generation. Accordingly, the unit commitment problem has been studied in-depth in operational research as a powerful tool to find the most profitable schedule [1]. Several approaches have been applied in the literature, and it is possible to discern that the more rigorous the methodology is, the higher the quality of the obtained solution.

The unit commitment problem is frequently addressed as an optimization problem [2]. The available resolution techniques' state-of-art highlights the convenience of using numerical optimization resources since evolutionary optimization algorithms cannot categorically guarantee the quality of the solution [3]. Moreover, the great advances in commercial solvers during the last decades allow their convergence towards a global-optimal solution in reasonable run times when utilizing mixed integer linear programming (MILP). However, the high computational burden associated with this methodology demands a trade-off between detail modeling, run time, and solution accuracy [4].

In this way, several MILP formulations have been proposed to improve the performance of commercial solvers and exploit their features [5–9]. These methodologies merge theoretical concepts of numerical optimization and solvers' mathematical backgrounds to enhance the resolution process. Thus, different alternatives to model the same technical issues concerning the thermal units have been proposed, trying to reach the leading representation. Nevertheless, the literature usually focuses on ramp constraints [10], time-up and time-down constraints [11], start-up and shut-down representation [12–15], or their inter-relationships [15–17], ignoring the impact of the selection of a specific balance equation, or its substitution by a profit-based optimization. Then, a lack of information about how this choice affects the resolution performance of the unit commitment methodologies is identified.

Furthermore, the implications of utilizing stable or high intermittency demand profiles should also be considered when analyzing modeling efficiency, as exposed in [18]. Besides, unit commitment formulations are frequently tested with large generation portfolios that entail

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Nomenclature

A. Sets

$g \in G$	Set of indexes of generating units.
$s \in S$	Set of indexes of start-up segments.
$t \in T$	Set of indexes of hourly periods of the time span.

B. Parameters

C_g^F	Fuel cost of unit g [\$/MMBtu].
C_g^{LV}	Linear variable production cost of unit g [\$/MWh].
C_g^{NL}	Fixed production cost of unit g [\$/h].
C^{NSE}	Cost of non-served energy [\$/MWh].
C_g^{SD}	Shut-down cost of unit g [\$].
$C_{g,s}^{SU}$	Start-up cost for the start-up type s of unit g [\$].
D_t	Load demand in period t [MWh].
F_g^{LV}	Linear variable production fuel-consumption of unit g [MMBtu/MWh].
F_g^{NL}	Fixed prod. fuel-consumption of unit g [MMBtu/h].
F_g^{SD}	Shut-down fuel-consumption of unit g [MMBtu].
$F_{g,s}^{SU}$	Start-up fuel-consumption for the start-up type s of unit g [MMBtu].
L_t	Electricity price in period t [\$/MWh].
$\overline{P}_g$	Maximum power output of unit g [MW].
$\underline{P}_g$	Minimum power output of unit g [MW].
P_g^0	Power output of unit g in the first period t [MW].
RD_g	Ramp-down limit of unit g [MW/h].
RU_g	Ramp-up limit of unit g [MW/h].
SD_g	Shut-down capability of unit g [MW].
SU_g	Start-up capability of unit g [MW].
$T_{g,s}^{SU}$	Minimum time period that unit g must be offline for the start-up type s [h].
TD_g	Minimum down time of unit g [h].
TD_g^0	Offline hours of unit g in the first period t [h].
TD_g^R	Number of hours that unit g must remain offline [h].
TU_g	Minimum up time of unit g [h].
TU_g^0	Online hours of unit g in the first period t [h].
TU_g^R	Number of hours that unit g must remain online [h].
U_g^0	Commitment status of unit g in the first period t.

C. Variables

(1) Positive and continuous variables

$c_{g,t}^P$	Production cost of unit g in period t [\$].
$c_{g,t}^{SD}$	Shut-down cost of unit g in period t [\$].
$c_{g,t}^{SU}$	Start-up cost of unit g in period t [\$].
nse_t	Non-served energy in period t [MW].
$p_{g,t}$	Power output above the minimum output of unit g in period t [MW].

(2) Binary variables

$\delta_{g,s,t}$	Start-up type s of unit g in period t
$u_{g,t}$	Commitment decision of unit g in period t.
$v_{g,t}$	Start-up decision of unit g in period t.
$w_{g,t}$	Shut-down decision of unit g in period t.

large-size optimization problems. However, some base thermal units are often replicated to construct these portfolios. This fact introduces symmetry effects in the resolution processes, whose efficient treatment is still under study [19–21], and categorical conclusions have not been reached yet. For that reason, it would also be desirable to test methodologies with large-size unsymmetrical input data, which can be achieved through a horizon expansion instead of a thermal unit replication.

Therefore, this article intends to fulfill these research gaps by analyzing the unit commitment formulations' most popular balance equations and establishing a comparison benchmark to determine their corresponding computational implications. The main contributions of this paper are summarized below:

- Different balance equations are evaluated under the same technical representation of the unit commitment problem to study their effect on the computational performance. Consequently, their feasible regions are uniquely modified by utilizing one constraint or another, allowing the isolation of their implications.
- Each approach's tightness and compactness (T&C) are also analyzed. These criteria are widely utilized in the unit commitment literature to characterize expectable model behaviors. However, the inherent uncertainty of dealing with mixed integer programming (MIP) problems makes T&C sometimes fail when predicting success. For that reason, the concept of arduousness is introduced in this paper to provide information about how a formulation's T&C is related to the computational performance in numerical optimization. Thus, this additional metric contributes to better intuiting a methodology resolution process.
- Different generation portfolios are represented under real market conditions, manifesting the solver's capabilities to manage stable load curves and high intermittence demand profiles in medium-term horizons. The portfolios do not replicate generation units in these cases, so symmetry effects in MILP resolutions are avoided. Moreover, the models presented in this paper are also tested in large-size case studies with traditional short-term horizons and large portfolios with unit replication to compare their computational performances.
- The operation at each formulation, portfolio, and input profile is optimized to various convergence criteria, running a total of 1010 real-size case studies to establish a theoretical framework of the unit commitment's response to specific input parameters and modeling options.

Section 2 presents the mathematical formulations compared in this report. Later, the case studies are exposed in Section 3, followed by the illustration of the resolution performance and a result discussion. Finally, conclusions are shown in Section 4.

2. Methodology

This section presents a common framework for representing the technical aspects involved in the unit commitment problem. Accordingly, the renowned mathematical formulation exposed in [7] is chosen to fulfill this task given that its computational efficiency has been widely demonstrated [15–17]. Thereafter, the different balance equations and the corresponding objective function (OF) are described from Sections 2.1 to 2.5, establishing a benchmark to perform the modeling comparison.

The technical constraints utilized in this common framework are gathered below. The feasibility of the operational schedules is guaranteed by constraints (1)–(18) at the optimization step. Meanwhile, Eqs. (19)–(24) are used before the resolution process to calculate some needed input parameters. Note that the subset G^1 comprises thermal units with TU_g equal 1, which means that the thermal unit can start up and shut down in the same hourly period.

- *Production cost:* a linear equation is employed to simplify the formulation. Piecewise or quadratic functions can make the resolution processes more difficult, and this is an aspect that is out of the paper's scope.

$$c_{g,t}^P = u_{g,t} C_g^{NL} + (u_{g,t} \underline{P}_g + p_{g,t}) C_g^{LV} \qquad \forall g, t \tag{1}$$

- *Generation limits:* these tight and compact constraints need to be separated for those thermal units that are not able to start-up and shut-down in the same hourly period (Eq. (2)) and those that can (Eqs. (3), (4)).

$$p_{g,t} \le u_{g,t}(\overline{P_g} - \underline{P_g}) - v_{g,t}(\overline{P_g} - SU_g) - w_{g,t+1}(\overline{P_g} - SD_g) \quad \forall g \notin G^1, t \tag{2}$$

$$p_{g,t} \le u_{g,t}(\overline{P_g} - \underline{P_g}) - v_{g,t}(\overline{P_g} - SU_g) \quad \forall g \in G^1, t \tag{3}$$

$$p_{g,t} \le u_{g,t}(\overline{P_g} - \underline{P_g}) - w_{g,t+1}(\overline{P_g} - SD_g) \quad \forall g \in G^1, t \tag{4}$$

- *Ramping constraints:* up- and down-ramps are formulated per $t \in [2, T]$ because the ramping capacity of each thermal unit at the initial hourly period ($t = 1$) depends on the power output parameter for $t = 0$. The ramping constraints for $t \in [1, 2)$ are presented with the initial condition constraints.

$$p_{g,t} - p_{g,t-1} \le RU_g \quad \forall g, t \in [2, T] \tag{5}$$

$$-p_{g,t} + p_{g,t-1} \le RD_g \quad \forall g, t \in [2, T] \tag{6}$$

- *Shut-down cost:* this cost, frequently obviated in many unit commitment formulations, is modeled by a single-step cost for each thermal unit.

$$c^{SD}_{g,t} = w_{g,t} C^{SD}_g \quad \forall g, t \tag{7}$$

- *Start-up cost:* the start-up cost depends on the time that a generator has been offline. For that reason, different start-up types ‘s’ are defined in the following stairwise function:

$$c^{SU}_{g,t} = \sum_{s \in S} \delta_{g,s,t} C^{SU}_{g,s} \quad \forall g, t \tag{8}$$

- *Start-up constraints:* (Eqs. (9), (10)) constitute one of the most efficient ways to model the correspondence between an offline time and its start-up segment determination in MILP formulations.

$$\delta_{g,s,t} \le \sum_{i=T^{SU}_{g,s}}^{T^{SU}_{g,s+1}-1} w_{g,t-i} \quad \forall g, s \in [1, S_g), t \in [T^{SU}_{g,s+1}, T] \tag{9}$$

$$v_{g,t} = \sum_{s \in S_g} \delta_{g,s,t} \quad \forall g, t \tag{10}$$

- *Logic constraint:* defines the chronological behavior of commitments, start-up, and shut-down processes. It also needs an initial parameter for $t = 1$.

$$v_{g,t} - w_{g,t} = u_{g,t} - u_{g,t-1} \quad \forall g, t \in [2, T] \tag{11}$$

- *Minimum time up/down constraints:* these operational constraints manifest an efficient performance dealing with the minimum times that thermal units should be online/offline after starting-up or shutting-down.

$$\sum_{i=t-TU_g+1}^{t} v_{g,i} \le u_{g,t} \quad \forall g, t \in [TU_g, T] \tag{12}$$

$$\sum_{i=t-TD_g+1}^{t} w_{g,i} \le 1 - u_{g,t} \quad \forall g, t \in [TD_g, T] \tag{13}$$

- *Initial condition constraints:* the optimization needs initial parameters to work with the operational decisions at $t = 1$ properly.

$$v_{g,t} - w_{g,t} = u_{g,t} - U^0_g \quad \forall g, t \in [1, 2) \tag{14}$$

$$p_{g,t} - (P^0_g - U^0_g \underline{P_g}) \le RU_g \quad \forall g, t \in [1, 2) \tag{15}$$

$$-p_{g,t} + (P^0_g - U^0_g \underline{P_g}) \le RD_g \quad \forall g, t \in [1, 2) \tag{16}$$

- *Operational-coherence constraints at the beginning:* according to the initial status of the thermal units, certain commitment and start-up-type conditions have to be accomplished at the beginning of the time span to guarantee operational coherence.

$$\delta_{g,s,t} = 0 \quad \forall g, s \in [1, S_g), t \in (T^{SU}_{g,s+1} - TD^0_g, T^{SU}_{g,s+1}) \tag{17}$$

$$u_{g,t} = U^0_g \quad \forall g, t \in [1, TU^R_g + TD^R_g] \tag{18}$$

– *Determination of the pre-optimization parameters:*

In order to keep the coherence between the magnitudes used in the mathematical formulations and the technical information provided for thermal generators, the fuel consumption of their operations has to be transformed into costs:

$$C^{LV}_g = F^{LV}_g C^F_g \quad \forall g \tag{19}$$

$$C^{NL}_g = F^{NL}_g C^F_g \quad \forall g \tag{20}$$

$$C^{SD}_g = F^{SD}_g C^F_g \quad \forall g \tag{21}$$

$$C^{SU}_{g,s} = F^{SU}_{g,s} C^F_g \quad \forall g, s \tag{22}$$

In turn, the hours that the units must remain online or offline at the beginning of the problem have to be determined from the initial-condition information:

$$TU^R_g = max\{0, (TU_g - TU^0_g)U^0_g\} \quad \forall g \tag{23}$$

$$TD^R_g = max\{0, (TD_g - TD^0_g)(1 - U^0_g)\} \quad \forall g \tag{24}$$

– *Determination of the resolution accuracy:*

This paper analyzes the convergence of these methodologies toward an optimal solution with different numerical tolerances. The optimality gap (OG) is chosen as the stopping criterion for these MILP optimizations [22]. Therefore, up to three different OGs are selected for each base case to accurately study the solver’s performances with their respective settings.

2.1. Equal to Demand Unit Commitment (EDUC)

The EDUC formulation imposes a total power output equal to demand at every time period of the horizon. This obligation can cause infeasible situations due to the problem inflexibility, like the unavailability to satisfy the exact demand as a result of ramping limitations. These approaches are valuable for power systems with a high thermal generation in which the demand profiles are remarkably homogeneous. The EDUC approach comprises the constraints (1)–(24), the balance Eq. (25) and the objective function (26), as employed in [5]:

$$D_t = \sum_{g \in G} u_{g,t} \underline{P_g} + p_{g,t} \quad \forall t \tag{25}$$

$$min \left(\sum_{g \in G} \sum_{t \in T} c^P_{g,t} + c^{SD}_{g,t} + c^{SU}_{g,t} \right) \tag{26}$$

2.2. Equal to Demand Unit Commitment with Non-Served Energy (EDUC-N)

The EDUC-N formulation is more flexible than the EDUC. It avoids infeasible situations through the introduction of non-served energy (NSE) terms, penalized in the OF. Nevertheless, there are circumstances where it is more profitable not to meet demand. That entails a risk to the security of supply. Moreover, restrained values have to be assigned to the non-served energy term. Too low costs can distort the input profiles compared to the final power output. Extremely high values can lead to underestimating the rest of the involved costs because of numerical difficulties in the solver’s performance. The EDUC-N methodology, utilized in [7], comprises constraints (1)–(24), the balance Eq. (27) and the objective function (28):

$$D_t - nse_t = \sum_{g\in G} u_{g,t}\underline{P_g} + p_{g,t} \qquad \forall t \tag{27}$$

$$min\left(\sum_{g\in G}\sum_{t\in T}(c^P_{g,t} + c^{SD}_{g,t} + c^{SU}_{g,t}) + \sum_{t\in T} nse_t C^{NSE}\right) \tag{28}$$

2.3. Greater than Demand Unit Commitment (GDUC)

The GDUC formulation manifests a more flexible and actual behavior. It avoids infeasibilities by the allowance of a power output surplus. Accordingly, the thermal units assume an extra production cost instead of falling into more expensive start-up and shut-down processes to meet the instantly exact demand. This approach assures the security of supply and allows a more efficient power system operation. In turn, storage technologies like batteries or pumping facilities can leverage the generation surplus. This modeling is valuable for representing electricity markets with a high penetration of non-dispatchable resources, given its strengthened robustness against sudden demand variations [18]. The GDUC method comprises constraints (1)–(24), the balance Eq. (29) and the objective function (30). It is important to note that this formulation assumes that the generation surpluses can be unlimitedly taken by storage facilities or compensated through renewable curtailment (at zero cost) to guarantee the security of the system's operation. For the sake of clarity, these facilities are not represented in this paper.

$$D_t \le \sum_{g\in G} u_{g,t}\underline{P_g} + p_{g,t} \qquad \forall t \tag{29}$$

$$min\left(\sum_{g\in G}\sum_{t\in T} c^P_{g,t} + c^{SD}_{g,t} + c^{SU}_{g,t}\right) \tag{30}$$

2.4. Greater than Demand Unit Commitment with Non-Served Energy (GDUC-N)

The GDUC-N formulation allows a generation surplus when it is profitable for the system and non-served energy situations too. It is the most flexible approach analyzed in this paper. The GDUC-N provides ample space for optimization, bringing the advantages of high-flexible modeling with their corresponding difficulties in adjusting the problem. The GDUC-N comprises constraints (1)–(24), the balance Eq. (31) and the OF (32):

$$D_t - nse_t \le \sum_{g\in G} u_{g,t}\underline{P_g} + p_{g,t} \qquad \forall t \tag{31}$$

$$min\left(\sum_{g\in G}\sum_{t\in T}(c^P_{g,t} + c^{SD}_{g,t} + c^{SU}_{g,t}) + \sum_{t\in T} nse_t C^{NSE}\right) \tag{32}$$

2.5. Profit Based Unit Commitment (PBUC)

The PBUC formulation has been frequently used in the unit commitment problem. It differs from the previously described methodologies by not considering demand-satisfaction profiles in the approach. Instead, electricity price inputs are utilized to guide the optimization of operational decisions. Therefore, the generation is addressed from a profitability perspective along the time span without any balance constraint. This formulation is quite valuable for situations where a market player with a thermal portfolio (small enough not to alter the market trends) desires to maximize the benefits of their asset management. It comprises constraints (1)–(24) and the objective function (33), like in [13]:

$$max\left(\sum_{g\in G}\sum_{t\in T} L_t(u_{g,t}\underline{P_g} + p_{g,t}) - c^P_{g,t} - c^{SD}_{g,t} - c^{SU}_{g,t}\right) \tag{33}$$

Fig. 1 illustrates this article's methodology. It focuses on analyzing the most common alternatives to model the balance constraints when the unit commitment is addressed as an optimization problem. The tightness and compactness of the different approaches are studied together with the resolution processes' performance. Furthermore, a new metric is introduced to improve predictions about the computational efficiency in numerical optimization.

3. Case studies and computational performance

This article focuses on analyzing the computational burden associated with the election of the balance equation in the unit commitment problem. For that reason, several case studies are run using the previously described methodologies. With the aim of performing an accurate and thorough comparison, different generation portfolios are employed in each base case, as shown in Section 3.1. In turn, different demand and electricity-price profiles are evaluated in the case studies. Section 3.2 gathers high-renewable-intermittence (HRI) profiles with the gas-fired demand and the prices of a real power system. Besides, stable thermal generation (STG) profiles are also considered in order to contrast the resolution processes. The constitution of medium-term and short-term horizons is described in this section. The numerical results and the computational performances are discussed in Section 3.3 (medium-term cases) and in Section 3.4 (short-term against medium-term).

3.1. Description of the thermal portfolios

Four different generation portfolios are utilized in the case studies to evaluate the possible influence of their system sizes or characteristics. Table 1 exposes the technical information of the thermal units. Some of these operational data are provided in fuel-consumption magnitudes to facilitate a later adjustment to the casuistry of each case, given the fluctuation of electricity prices and thermal demand with the high volatility of fuel cost. It is important to note that two start-up types s (hot and cold) are used. Therefore, a hot or cold fuel consumption ($F^{SU}_{g,hot}$ or $F^{SU}_{g,cold}$) will be assumed in start-ups, depending on whether the offline hours are greater or lower than $T^{SU}_{g,cold}$.

- *Portfolio 1 (P1):* comprises Unit A to G. It was presented in [23], where operational costs were calculated assuming a natural gas price of 5 $/MMBtu, as it is manifested in the fuel consumption exposed in [18]. These generation units correspond to real combined cycle gas turbines (CCGTs) that are placed in the Iberian Power System. They constitute a thermal portfolio of large-size power plants ($\overline{P_g}$ > 400 MW). Generally, large thermal units have lower operational costs than small- and medium-size. However, they often show higher fuel consumption for start-up and shut-down processes.
- *Portfolio 2 (P2):* comprises Unit 1 to 8. It has been widely employed in the literature. This paper takes the data from [7]. According to the portfolio's power capacities and technical features, a fuel price of 2.5 $/MMBtu is considered to obtain their operations' fuel consumption. This portfolio includes large-, medium- ($\overline{P_g}$ = 100–400 MW) and small-size gas-fired generators ($\overline{P_g}$ < 100 MW). One of these units can start-up and shut-down in the same hourly period, producing at its maximum capacity if desired. It brings a high operational flexibility to the problem. Nevertheless, this unit also entails more significant fuel consumption.
- *Portfolio 3 (P3):* gathers Portfolio 1 and Portfolio 2. Despite that the number of thermal units is not high enough to represent a complex power system, each generator is unique, and symmetry effects are avoided in the resolution processes. Furthermore, it would be easy to replicate power plants if the representation of greater electricity markets is preferred.
- *Portfolio 4 (P4):* comprises 35 times Portfolio 3. It represents a complex system with 525 thermal units. Hence, their corresponding case studies will manifest remarkable generation flexibility when optimizing operational decisions in the unit commitment problem. On the other hand, resolution processes could be affected by symmetry effects.

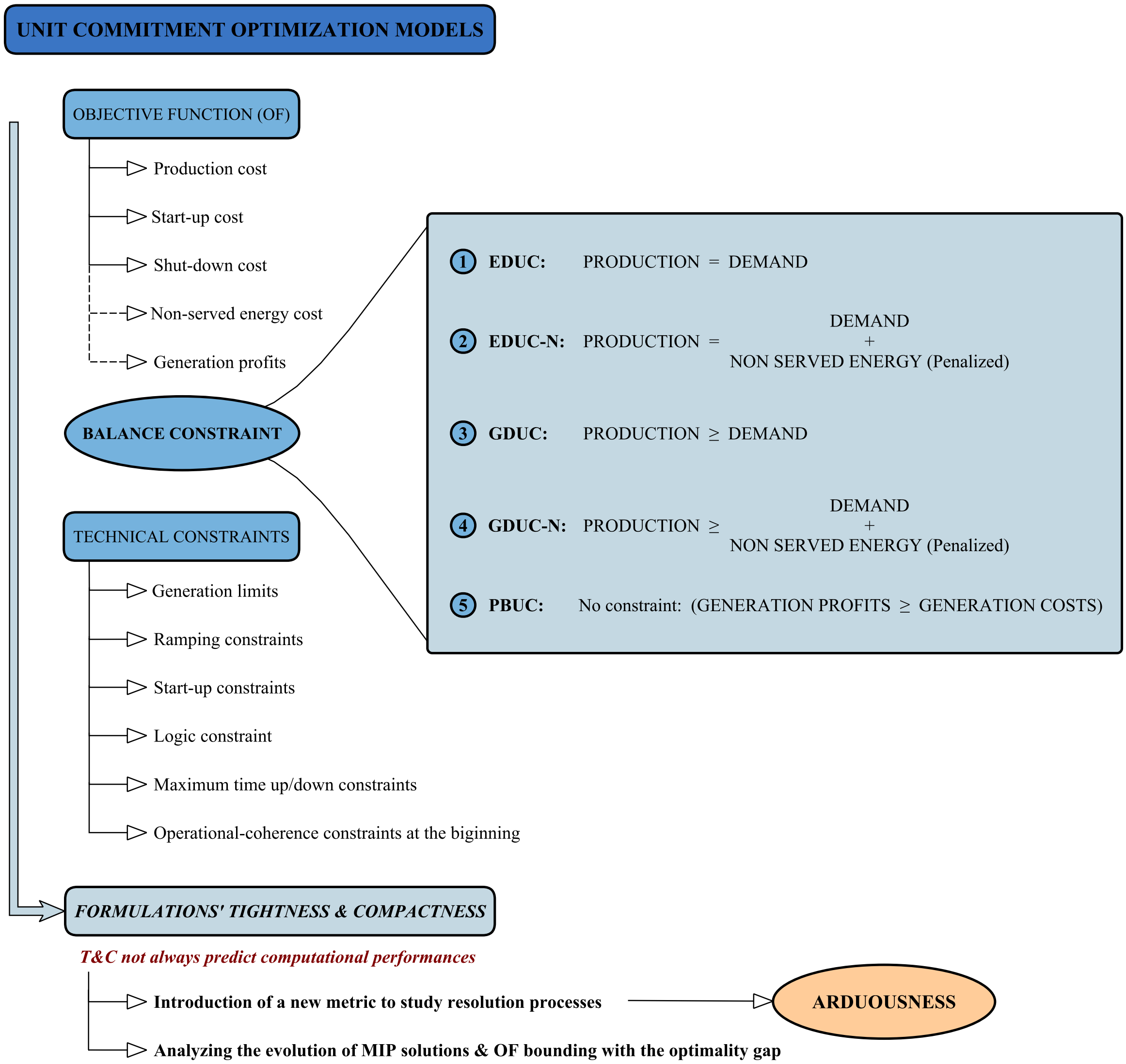

Fig. 1. Illustrative description of the paper's methodology and main contributions.

Table 1
Technical data of the thermal units.

Thermal unit	F_g^{LV} [MMBtu /MWh]	F_g^{NV} [MMBtu /h]	$F_{g,hot}^{SU}$ [MMBtu]	$F_{g,cold}^{SU}$ [MMBtu]	F_g^{SD} [MMBtu]	$\overline{P}_g$ [MW]	$\underline{P}_g$ [MW]	RD_g [MW/h]	RU_g [MW/h]	SD_g [MW]	SU_g [MW]	TD_g [h]	TU_g [h]	$T_{g,cold}^{SU}$ [h]	P_g^0 [MW]	TD_g^0 [h]	TU_g^0 [h]	U_g^0 [ad.]
Unit A	6.6	300	2891	4893	1100	412	157	215	215	157	157	7	7	12	314	0	7	1
Unit B	6.2	460	2798	5474	1100	390	135	200	200	135	135	5	5	79	270	0	5	1
Unit C	5.4	820	3996	8654	1900	856	285	425	425	285	285	11	11	20	570	0	11	1
Unit D	6.4	320	2737	3618	1100	402	112	200	200	112	112	6	6	61	224	0	6	1
Unit E	6.8	280	3239	4121	1200	413	157	215	215	157	157	7	7	88	314	0	7	1
Unit F	6.2	360	3483	6148	1200	427	163	225	225	163	163	8	8	103	326	0	8	1
Unit G	5.6	740	5992	7887	1900	796	225	385	385	225	225	10	10	91	450	0	10	1
Unit 1	6.5	400	1800	3600	0	455	150	225	225	150	150	8	8	14	300	0	8	1
Unit 2	6.9	388	2000	4000	0	455	150	225	225	150	150	8	8	14	300	0	8	1
Unit 3	6.6	280	220	440	0	130	20	50	50	20	20	5	5	10	40	0	5	1
Unit 4	6.6	272	224	448	0	130	20	50	50	20	20	5	5	10	40	0	5	1
Unit 5	7.9	180	360	720	0	162	25	60	60	25	25	6	6	11	50	0	6	1
Unit 6	8.9	148	68	136	0	80	20	60	60	20	20	3	3	8	40	0	3	1
Unit 7	11.1	192	104	208	0	85	25	60	60	25	25	3	3	6	50	0	3	1
Unit 8	10.4	264	12	24	0	55	10	135	135	10	10	1	1	2	20	0	1	1

Table 2
Hub trading, currency exchange rates and fuel prices.

	J	F	M	A	M	J	J	A	S	O	N	D
MIBGAS [€/MWht]	11.879	9.817	8.563	7.482	5.331	6.426	6.435	9.187	11.310	13.441	14.500	18.184
Exchange Rate [€/$]	0.901	0.917	0.904	0.921	0.917	0.889	0.873	0.845	0.848	0.849	0.845	0.822
Fuel Price [$/MMBtu]	3.864	3.138	2.776	2.381	1.704	2.118	2.160	3.186	3.909	4.639	5.029	6.483

3.2. Description of the demand and electricity-price profiles

3.2.1. Medium-term horizons

Every time span evaluated in the medium-term case studies comprises one month on an hourly basis. Consequently, large-size problems are generated to study the unit commitment formulations' balance equations properly. Furthermore, this detailed representation of technical and economic features in medium-term horizons can be used to illustrate the ongoing trends in real power systems. For that reason, considering longer time spans was prioritized over the duplication of thermal units in order to prepare large-size problems. In addition, this decision avoids the appearance of symmetry effects during the solver performance [4]. Finally, fifteen base cases are established in this analysis benchmark. Their characteristics are following described:

- The first twelve cases employ input profiles taken from a real power system with a high penetration of non-dispatchable generators. Each case represents a month of 2020 in the Iberian Electricity Market (MIBEL), capturing thermal intermittence and seasonality trends, an increasingly important matter in unit commitment [24–26]. In accordance, the gas-fired generation in the MIBEL [27] is scaled through Eq. (34), employed in [18], and utilized as demand profiles in EDUC, EDUC-N, GDUC, and GDUC-N formulations. Moreover, the corresponding electricity-price information [28] is used at PBUC. These are the so-called HRI cases.

$$D_t = \frac{Gas\ Fired\ Production_t}{max\{Gas\ Fired\ Production_t\}}\ 0.95 \sum_{g \in G} \overline{P_g} \tag{34}$$

- The last three cases take profiles from the literature about systems with stable thermal generation. These are the so-called STG cases. Some papers repeat daily curves on an hourly basis to extend the time span. The same procedure is assumed in this report when necessary. Case X utilizes profile [6] for demand and [12] for prices. Case Y takes [7,13] respectively. Case Z employs [8,14].
- The fuel costs of the HRI cases are taken from the Iberian Gas Market (MIBGAS) to assure operational coherence. The monthly average prices negotiated in the hub [29] are used in each case. Table 2 gathers them and the currency exchange rates to calculate fuel costs in $/MMBtu and the hourly electricity prices in $/MWh. Regarding the STGs, a fuel cost of 2.5 $/MMBtu is employed in these cases.

These proceedings establish fifteen medium-term base cases to test the approaches employing different generation portfolios and optimization options. Fig. 2 reports four monthly HRI demand curves (one per season of the year) and the STG load profiles. It can be appreciated how HRI internalizes the high renewable penetration in modern power systems, diminishing the thermal demand and making it significantly more intermittent when compared to conventional STGs.

3.2.2. Short-term horizons

Besides, 71 short-term base cases are also established to compare the computational performance of the described methodologies with a different time span duration:

- The short-term HRI cases are prepared by randomly choosing five daily profiles from each medium-term HRI case. The unique condition is that the load profile cannot be zero to avoid valueless cases. Then, the corresponding electricity price series are chosen

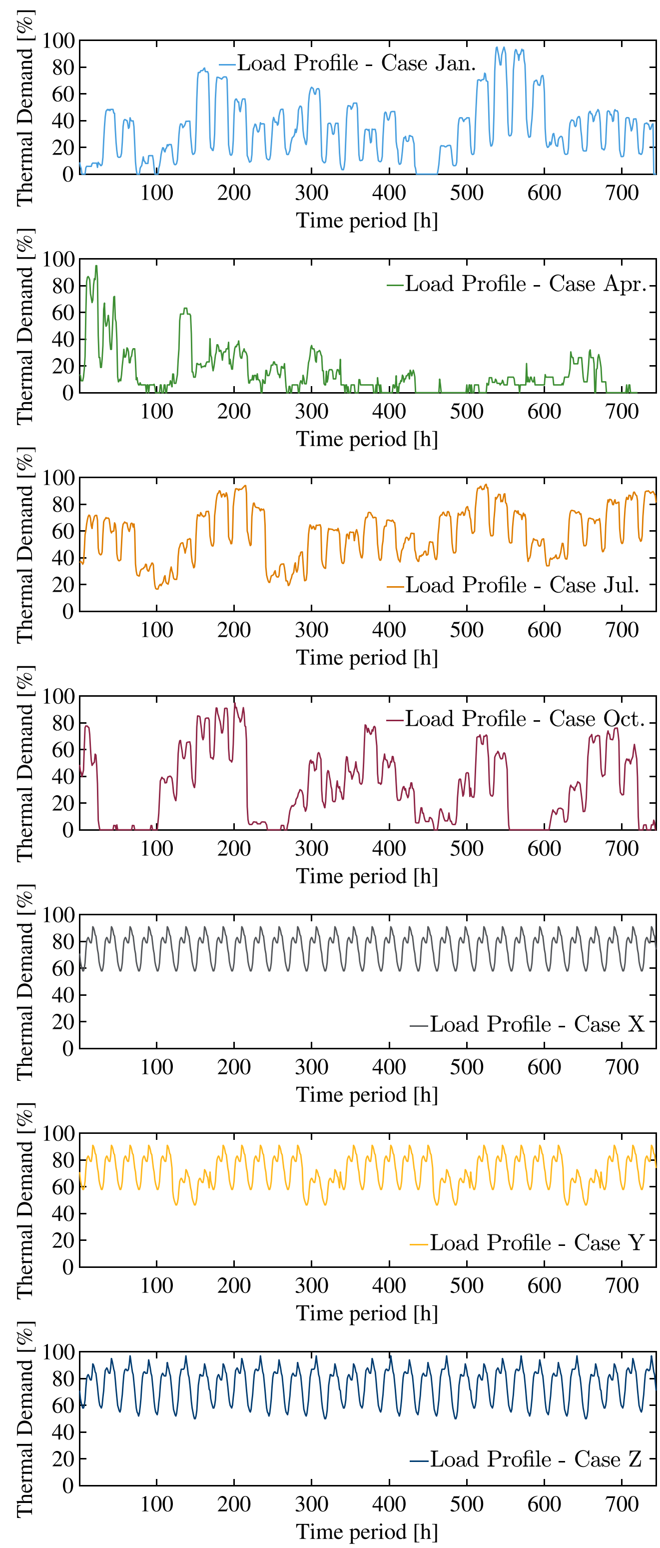

Fig. 2. Monthly load profiles for EDUC, EDUC-N, GDUC, and GDUC-N methodologies on an hourly basis.

Table 3
Run time comparison.

#	EDUC			EDUC-N			GDUC			GDUC-N			PBUC		
	Optimality gap			Optimality gap			Optimality gap			Optimality gap			Optimality gap		
	10^{-2}	10^{-4}	10^{-6}	10^{-2}	10^{-4}	10^{-6}	10^{-2}	10^{-4}	10^{-6}	10^{-2}	10^{-4}	10^{-6}	10^{-2}	10^{-4}	10^{-6}
							Portfolio 1								
J	*20.9*	*446.5*	*510.6*	38.8	1407.0	1526.7	26.1	150.7	157.9	45.8	201.1	224.6	9.8	9.8	9.8
F	*12.7*	*24.6*	*30.6*	18.5	63.4	66.4	19.2	41.4	41.4	28.5	53.4	53.9	10.1	10.1	10.1
M	*17.1*	*333.2*	*333.2*	42.2	754.9	754.9	27.8	596.5	599.0	48.0	572.6	579.2	10.2	10.2	10.2
A	*21.7*	*21.7*	*21.7*	19.1	79.6	79.6	35.9	674.3	674.3	63.0	608.9	608.9	10.6	10.7	10.7
M	*15.8*	*162.0*	*162.0*	27.1	218.1	218.5	25.0	180.7	180.8	27.6	319.8	326.5	10.2	10.2	10.3
J	*16.5*	*145.5*	*145.5*	37.2	218.7	223.8	23.0	187.8	193.0	42.8	247.6	265.7	9.9	9.9	9.9
J	16.0	190.1	200.7	23.3	267.2	322.5	15.9	206.6	296.4	26.3	275.1	312.8	9.8	9.8	9.8
A	12.6	51.8	86.0	24.9	57.4	59.9	12.9	102.2	107.0	20.7	44.1	48.3	9.8	9.8	9.8
S	*13.1*	*112.0*	*119.2*	26.4	143.8	166.7	16.9	234.0	269.3	31.4	398.1	481.9	10.1	10.1	10.1
O	*13.6*	*115.9*	*121.7*	28.8	324.7	363.7	25.5	132.8	134.4	31.8	151.1	165.2	10.1	10.1	10.2
N	*16.2*	*308.4*	*317.9*	27.0	2055.5	2074.0	25.9	6905.0	(0.0080%)	36.7	3850.1	4232.6	10.1	10.1	10.2
D	*10.2*	*18.0*	*22.9*	20.1	98.4	102.2	26.0	481.1	500.2	36.6	1114.7	1145.6	10.4	10.4	10.4
X	10.6	13.1	14.0	19.6	20.8	20.0	10.5	12.8	14.8	19.5	19.4	21.2	9.8	9.8	9.8
Y	11.4	15.6	16.4	21.8	21.7	23.0	11.2	15.2	16.5	20.7	20.6	22.2	9.9	9.9	9.9
Z	10.2	14.9	16.4	22.0	30.0	31.2	9.8	14.3	15.8	21.8	30.0	31.6	10.1	10.4	10.4
							Portfolio 2								
J	*62.6*	*(0.1454%)*	*(0.1454%)*	94.0	(0.1653%)	(0.1653%)	49.7	(0.0166%)	(0.0166%)	66.9	(0.0124%)	(0.0124%)	7.3	7.3	7.3
F	*29.9*	*224.9*	*319.3*	43.9	1624.6	2453.2	21.6	2340.5	2530.1	27.0	3200.7	3548.0	7.4	7.4	7.4
M	*45.2*	*(0.0801%)*	*(0.0801%)*	50.5	(0.0888%)	(0.0888%)	70.5	3449.5	3537.4	59.5	(0.0320%)	(0.0320%)	7.8	7.9	8.0
A	*24.2*	*1317.6*	*1317.6*	31.5	(0.0106%)	(0.0106%)	92.6	3565.7	3561.0	91.0	5080.6	5014.9	7.4	7.4	7.4
M	*19.5*	*65.7*	*67.1*	29.0	157.3	162.0	21.7	459.2	468.2	19.1	333.7	333.7	7.7	7.7	7.7
J	*18.2*	*2403.8*	*2829.1*	42.8	3070.0	3969.7	24.7	1037.1	1205.9	50.4	847.8	1195.0	7.3	7.3	7.3
J	14.6	615.7	3026.7	36.1	1376.2	2207.1	14.3	577.1	3182.2	31.5	1504.7	2910.8	7.2	7.2	7.2
A	10.8	60.2	77.8	26.9	75.3	94.0	12.8	50.1	59.6	26.2	85.3	102.3	7.1	7.1	7.1
S	*27.4*	*409.3*	*801.2*	40.6	2123.0	2506.2	32.0	3619.3	4218.7	48.5	3651.6	5648.9	7.3	7.4	7.4
O	*28.9*	*1196.9*	*1403.1*	40.1	2428.5	2667.6	27.7	3154.5	3415.9	34.6	5418.8	7179.5	7.8	7.8	7.8
N	*35.2*	*1792.0*	*2074.3*	43.2	2544.4	2677.7	15.3	730.4	853.7	19.9	2977.0	3099.4	7.5	7.6	7.6
D	*12.6*	*45.9*	*46.3*	26.0	58.0	58.7	27.5	1469.2	1580.3	30.0	374.4	399.3	7.6	7.6	7.6
X	11.2	11.7	15.9	18.2	18.2	21.5	9.7	15.0	15.0	10.2	21.8	25.8	7.2	7.2	7.2
Y	12.1	58.0	68.0	25.7	62.2	75.0	9.9	32.7	97.7	14.7	61.5	86.3	7.2	7.2	7.2
Z	9.0	78.0	113.4	27.1	340.5	390.7	8.9	87.1	200.6	19.0	350.2	385.4	8.2	8.2	8.2
							Portfolio 3								
J	*241.7*	*(0.1145%)*	*(0.1145%)*	228.9	(0.1727%)	(0.1727%)	86.4	(0.0794%)	(0.0794%)	145.7	(0.0866%)	(0.0866%)	24.9	24.9	24.9
F	*77.7*	*(0.0295%)*	*(0.0295%)*	74.0	(0.0444%)	(0.0444%)	44.6	(0.0557%)	(0.0557%)	56.7	(0.0559%)	(0.0559%)	25.4	25.4	25.4
M	*194.9*	*(0.1954%)*	*(0.1954%)*	224.6	(0.2401%)	(0.2401%)	192.2	(0.1354%)	(0.1354%)	147.4	(0.1435%)	(0.1435%)	25.7	25.7	25.7
A	*343.0*	*680.8*	*740.3*	71.0	5055.9	6044.5	124.2	5996.1	(0.0047%)	181.7	3669.7	5276.6	25.8	25.9	25.9
M	*110.4*	*(0.0428%)*	*(0.0428%)*	153.9	(0.0695%)	(0.0695%)	97.1	(0.0757%)	(0.0757%)	167.8	(0.1009%)	(0.1009%)	25.7	25.7	25.7
J	*168.2*	*(0.0655%)*	*(0.0655%)*	224.7	(0.0641%)	(0.0641%)	39.3	(0.0426%)	(0.0426%)	65.9	(0.0469%)	(0.0469%)	25.3	25.3	25.3
J	129.5	(0.0565%)	(0.0565%)	143.6	(0.0499%)	(0.0499%)	38.5	(0.0469%)	(0.0469%)	113.6	(0.0508%)	(0.0508%)	24.7	24.7	24.7
A	35.7	5866.3	(0.0050%)	123.9	4384.0	(0.0082%)	40.2	2364.8	(0.0036%)	136.3	4181.3	(0.0084%)	24.9	24.9	24.9
S	*115.9*	*(0.0436%)*	*(0.0436%)*	167.5	(0.0482%)	(0.0482%)	39.7	(0.0436%)	(0.0436%)	161.6	(0.0481%)	(0.0481%)	25.0	25.0	25.1
O	*120.7*	*(0.0560%)*	*(0.0560%)*	83.9	(0.0992%)	(0.0992%)	78.6	(0.0810%)	(0.0810%)	91.5	(0.0855%)	(0.0855%)	25.2	25.2	25.2
N	*130.0*	*(0.0547%)*	*(0.0547%)*	142.8	(0.1166%)	(0.1166%)	80.2	(0.1091%)	(0.1091%)	67.9	(0.1041%)	(0.1041%)	25.6	25.6	25.7
D	*426.8*	*(0.1415%)*	*(0.1415%)*	83.4	(0.2632%)	(0.2632%)	124.2	(0.2388%)	(0.2385%)	120.9	(0.2015%)	(0.2015%)	25.5	26.0	26.0
X	34.1	209.8	1397.4	78.6	332.1	5043.8	34.0	226.5	1462.4	83.3	422.7	(0.0008%)	26.0	26.0	26.0
Y	33.1	283.3	1882.9	94.8	291.4	(0.0006%)	33.4	282.8	1696.1	106.5	323.0	3949.7	25.5	25.5	25.5
Z	32.0	190.4	1240.8	69.3	381.2	1952.7	30.1	181.2	1062.7	66.2	357.0	3060.8	25.0	25.8	25.8

for the PBUC approach. Hence, 60 base cases with a 24-hour horizon are generated.

- The short-term STG cases are prepared by taking the eight different daily demand profiles and the three daily electricity prices from the monthly STG cases. Thus, eleven additional base cases with a 24-hour horizon are generated.

3.3. Numerical results of the medium-term case studies

Fifteen base cases are established to compare the aforementioned balance equations in medium-term horizons. Each base case is run using five unit commitment formulations with three different optimality gaps. These setups are tested with the technical data of three thermal portfolios (P1, P2, and P3). Hence, 675 case studies are analyzed in this section.

They have been run in a computer Intel Core i7-8700 @3.20 GHz with 12 logical processors and 32 GB of installed RAM memory running 64-bit Windows 10 Pro, and solved with the commercial solver Gurobi 9.5 under GAMS. Optimality gaps of 10^{-2}, 10^{-4}, and 10^{-6} are imposed as numerical tolerances to finish the resolution processes. In addition, a maximum run time of 7200 s is established to end the optimization of those excessively computationally-demanding case studies.

It is also important to mention that a value of 1000 $/MWh was set as non-served energy cost according to the input data. This value is frequently employed in the unit commitment literature because it represents undesired situations (according to market conditions like those considered in this paper) but also provides some flexibility to the thermal operation.

3.3.1. Objective function & run time trade-offs

Despite each methodology entails a different optimization problem, and their corresponding feasible regions cannot be directly compared, a trade-off between the quality of the solution (influenced by the imposed OG) and the run time employed to achieve it can be established for each approach and parametrized case study. For the sake of clarity, this analysis is addressed in Appendix. On the other hand, Table 3 shows the run times in seconds or their final OG in % when the maximum CPU time is accomplished.

It is important to highlight that some EDUC cases constitute infeasible problems due to the aggressive thermal intermittence of some HRI cases and their inflexibility to meet the exact demand dealing with the portfolios' ramping rates, minimum up and down times, or their initial conditions. However, the cases become feasible when flexible resources such as generation surpluses or NSE options are available.

Besides, when feasible regions are being qualitatively analyzed, some metrics like run times, integrality gaps, or arduousness can be evaluated in spite of utilizing a different demand profile. For that reason, the performance of the EDUC methodology has been compared to the others by modifying its infeasible demand profiles. These feasible-made EDUC cases are included in italics in Table 3 and further.

Regarding the run times, some differences can be appreciated between 10^{-4} and 10^{-6} OGs despite both performing a one-order magnitude increase from 10^{-2}. These variations are more significant at P2 and P3 in STG cases, which means that portfolios with small- and medium-generation units are more difficult to optimize with strict OG conditions than only large units. Furthermore, greater run times are observed for P2 (whose number of units is similar to P1) and P3.

Comparing HRI and STG cases, Table 3 shows that HRI profiles, which represent the ongoing trends in many modern power systems, require more computational resources than STG curves. Moreover, they frequently reach the maximum time limit in P3 with 10^{-4} and 10^{-6} OGs, a quite less common situation with STGs.

Finally, examining PBUC formulation, significantly lower run times are required to find the optimal solution. As expected, the solver identifies that each thermal unit can be treated individually without any shared target (the system's demand) to interrelate them. This simplifies the resolution process, and tiny OGs are quickly achieved. Consequently, there are not many differences between 10^{-4} and 10^{-6} OGs, neither between real market profiles (PBUC - HRI) and literature curves (PBUC - STG). However, portfolios' sizes and configurations introduce a distinction. Larger portfolios (P3), of course, require more run time, but surprisingly, optimizing large generation units (P1) in this approach is more challenging to manage than a mix of small-, medium- and large ones (P2).

After this preliminar analysis in which many results could be considered, a priori, expectable, an in-depth study and comparison between these methodologies is shown in the following sections. The tightness and compactness of each formulation are evaluated. Moreover, the concept of arduousness is introduced to describe computational behaviors that cannot be uniquely explained through T&C, and an analysis of the solver performance is exposed to clarify the obtained results and support the conclusions.

3.3.2. Tightness

The tightness of a mixed integer programming problem is frequently defined as the closeness of the relaxed solution to the integer. It is a desirable characteristic in MILP formulations because the obtention of a more limited feasible region in the relaxed problems' polytopes helps the solver in the branch & cut processes. With the aim of comparing the tightness of different approaches, the integrality gap (IG) was presented in [6]:

$$IG\,(\%) = 100\,\frac{OF_{Integer} - OF_{Relaxed}}{OF_{Integer}} \tag{35}$$

Eq. (35) determines the integrality gap of a MIP minimization problem. Meanwhile, in maximization problems, the IG is calculated through:

$$IG\,(\%) = 100\,\frac{OF_{Relaxed} - OF_{Integer}}{OF_{Relaxed}} \tag{36}$$

As previously mentioned in [17], integrality gaps can utilize the linear programming (LP) solution or the root relaxation as the relaxed OF. In this paper, the root relaxation is employed in the IG calculations because it offers a better idea about the solvers' performances and their abilities to handle these complex problems, manifesting more realistic relaxed targets to whom approximate the integer solution despite taking advantage of heuristics strategies to tight them.

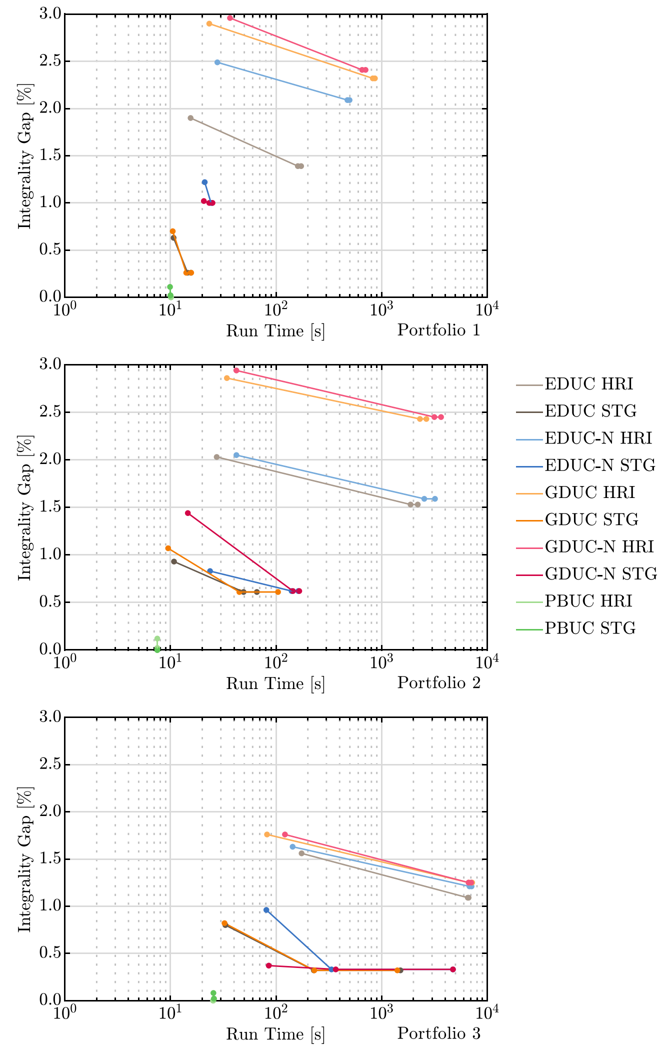

Fig. 3. Integrality gaps obtained with each optimality-gap definition and run times to reach them.

Table 4 specifies the particular integrality gap of each case in %. Additionally, mean IG values and the average run times to reach them are plotted in Fig. 3 for every methodology, portfolio, and numerical convergence criterion (10^{-2}, 10^{-4} and 10^{-6} OGs), distinguishing between the HRI and STG cases. It is important to remember that EDUC - HRI cases are considered in spite of using modified profiles because the tightness is a qualitative feature that measures the relaxed-integer proximity.

It can be appreciated that STG cases are not only generally faster than HRI (as exposed in Table 3), but they also achieve better integrality gaps. Therefore, the optimal relaxed solution in the HRI cases is further to the integer one than in STGs. It could be explained by the greater number of start-up and shut-down processes that entail higher intermittency [30], which can be leveraged in the relaxed formulation to adopt partial decisions in the corresponding binary variables. These behaviors distance to the real operation of a thermal portfolio and would be reduced with stable load profiles. Similarly, cheap electricity prices in PBUC's case Z boost the relaxed solution.

Table 4
Tightness comparison: integrality gap of each case study.

#	EDUC			EDUC-N			GDUC			GDUC-N			PBUC		
	Optimality gap			Optimality gap			Optimality gap			Optimality gap			Optimality gap		
	10^{-2}	10^{-4}	10^{-6}	10^{-2}	10^{-4}	10^{-6}	10^{-2}	10^{-4}	10^{-6}	10^{-2}	10^{-4}	10^{-6}	10^{-2}	10^{-4}	10^{-6}
							Portfolio 1								
J	*2.27*	*1.98*	*1.98*	2.91	2.35	2.35	3.07	2.27	2.27	2.55	2.39	2.39	0.00	0.00	0.00
F	*2.20*	*1.38*	*1.38*	2.59	2.12	2.12	2.07	1.55	1.55	2.24	1.67	1.67	0.00	0.00	0.00
M	*2.90*	*2.41*	*2.41*	2.96	2.52	2.52	3.92	3.52	3.52	4.26	3.58	3.58	0.00	0.00	0.00
A	*0.18*	*0.18*	*0.18*	5.56	5.18	5.18	5.70	5.16	5.16	5.73	5.23	5.23	0.01	0.00	0.00
M	*2.78*	*2.11*	*2.11*	3.02	2.46	2.46	3.55	3.34	3.34	4.13	3.47	3.47	0.00	0.00	0.00
J	*1.88*	*1.24*	*1.24*	1.42	1.25	1.25	2.41	1.54	1.54	1.72	1.60	1.60	0.00	0.00	0.00
J	1.73	0.98	0.98	2.01	1.17	1.17	1.77	0.98	0.98	2.01	1.18	1.18	0.00	0.00	0.00
A	1.48	0.84	0.84	1.24	0.98	0.98	1.60	0.84	0.84	1.88	0.98	0.98	0.00	0.00	0.00
S	*1.43*	*1.00*	*1.00*	1.22	1.09	1.09	1.95	1.20	1.20	1.62	1.30	1.30	0.01	0.01	0.00
O	*2.14*	*1.45*	*1.45*	2.26	1.81	1.81	2.75	2.09	2.09	2.98	2.23	2.23	0.00	0.00	0.00
N	*1.75*	*1.49*	*1.49*	2.50	2.26	2.26	2.44	1.94	1.93	2.30	1.91	1.91	0.01	0.01	0.00
D	*2.02*	*1.57*	*1.57*	2.19	1.84	1.83	3.64	3.40	3.40	4.06	3.37	3.37	0.01	0.00	0.00
X	0.66	0.24	0.24	1.22	1.14	1.14	0.66	0.24	0.24	1.14	1.14	1.14	0.00	0.00	0.00
Y	0.88	0.36	0.36	1.06	1.06	1.06	0.81	0.36	0.36	1.06	1.06	1.06	0.00	0.00	0.00
Z	0.35	0.18	0.18	1.39	0.80	0.80	0.63	0.18	0.18	0.87	0.80	0.80	0.32	0.07	0.07
							Portfolio 2								
J	*3.15*	*2.71*	*2.71*	3.16	2.78	2.78	3.55	3.08	3.08	3.50	3.13	3.13	0.00	0.00	0.00
F	*1.10*	*0.87*	*0.87*	2.36	1.79	1.79	1.97	1.62	1.62	2.20	1.64	1.64	0.00	0.00	0.00
M	*2.55*	*2.16*	*2.16*	2.84	2.39	2.39	4.10	3.55	3.55	4.14	3.56	3.56	0.16	0.02	0.02
A	*2.75*	*2.53*	*2.53*	2.23	1.80	1.80	5.12	5.00	5.00	5.25	4.99	4.99	0.21	0.08	0.08
M	*2.59*	*1.90*	*1.90*	2.31	1.72	1.72	3.61	3.53	3.53	3.84	3.58	3.58	0.02	0.01	0.01
J	*2.27*	*1.47*	*1.47*	2.02	1.37	1.37	2.46	1.80	1.80	1.92	1.82	1.82	0.00	0.00	0.00
J	1.83	1.15	1.15	1.65	1.17	1.17	1.97	1.18	1.18	1.76	1.19	1.19	0.00	0.00	0.00
A	1.75	1.00	1.00	1.03	1.01	1.01	1.54	1.03	1.03	1.93	1.04	1.04	0.00	0.00	0.00
S	*2.03*	*1.30*	*1.30*	1.82	1.30	1.30	1.80	1.48	1.48	2.08	1.50	1.50	0.04	0.02	0.01
O	*1.78*	*1.40*	*1.40*	2.15	1.56	1.56	2.80	2.10	2.10	2.82	2.09	2.09	0.11	0.04	0.04
N	*1.68*	*1.44*	*1.44*	2.18	1.62	1.62	2.54	1.96	1.96	2.35	1.98	1.98	0.16	0.05	0.05
D	*0.94*	*0.46*	*0.46*	0.82	0.53	0.53	2.91	2.79	2.79	3.53	2.81	2.81	0.77	0.08	0.08
X	0.60	0.60	0.60	1.03	0.57	0.56	0.85	0.60	0.60	1.42	0.57	0.56	0.00	0.00	0.00
Y	0.94	0.65	0.65	0.76	0.62	0.62	1.22	0.65	0.65	1.54	0.62	0.62	0.00	0.00	0.00
Z	1.25	0.58	0.57	0.72	0.68	0.68	1.15	0.57	0.57	1.36	0.68	0.68	0.00	0.00	0.00
							Portfolio 3								
J	*1.66*	*1.23*	*1.23*	2.03	1.41	1.41	1.49	1.05	1.05	1.39	1.05	1.05	0.00	0.00	0.00
F	*1.08*	*0.92*	*0.92*	1.90	1.18	1.18	1.46	0.86	0.86	1.27	0.86	0.86	0.00	0.00	0.00
M	*1.74*	*1.58*	*1.58*	1.87	1.70	1.70	2.55	2.00	2.00	2.64	2.01	2.01	0.00	0.00	0.00
A	*2.95*	*2.23*	*2.23*	3.17	2.48	2.48	3.70	3.04	3.04	3.15	3.04	3.04	0.01	0.00	0.00
M	*1.79*	*1.47*	*1.47*	2.01	1.75	1.75	2.04	1.74	1.74	2.46	1.76	1.76	0.01	0.01	0.00
J	*1.37*	*0.70*	*0.70*	0.77	0.65	0.65	1.25	0.74	0.74	1.28	0.74	0.74	0.00	0.00	0.00
J	1.17	0.57	0.57	0.88	0.57	0.57	1.22	0.56	0.56	0.91	0.57	0.57	0.00	0.00	0.00
A	1.07	0.48	0.48	0.83	0.48	0.48	1.29	0.48	0.48	1.29	0.48	0.48	0.00	0.00	0.00
S	*1.28*	*0.69*	*0.69*	1.22	0.67	0.67	1.25	0.79	0.79	1.23	0.80	0.80	0.01	0.01	0.00
O	*1.40*	*0.86*	*0.86*	1.50	1.04	1.04	1.26	0.98	0.98	1.68	0.98	0.98	0.00	0.00	0.00
N	*1.25*	*0.94*	*0.94*	1.41	1.04	1.04	1.38	1.05	1.05	1.64	1.05	1.05	0.01	0.01	0.00
D	*2.00*	*1.34*	*1.34*	1.97	1.54	1.54	2.22	1.71	1.71	2.24	1.68	1.68	0.01	0.00	0.00
X	0.91	0.35	0.35	1.19	0.35	0.35	0.84	0.35	0.35	0.39	0.35	0.35	0.00	0.00	0.00
Y	0.93	0.36	0.36	1.31	0.36	0.36	0.94	0.36	0.36	0.38	0.36	0.36	0.00	0.00	0.00
Z	0.55	0.27	0.26	0.38	0.28	0.28	0.69	0.26	0.26	0.35	0.28	0.28	0.25	0.05	0.05

When the tightness of the different methodologies is studied, STG cases reveal that they often converge to the same IG if strict OGs are imposed. At the same time, greater variations are noticed in the more difficult-to-solve HRI cases. The three portfolios show that EDUC is tighter than EDUC-N, EDUC-N is tighter than GDUC, and GDUC-N is the less tight formulation, manifesting that inequalities in the balance constraint mean a more significant tightness loss than introducing NSE terms when medium-term horizons are considered.

Finally, similar trends are observed in each portfolio. It is important to mention that P3 achieves better IGs than P1 and P2. That must happen because of the 'thermal unit effect': a little operational infeasibility in the relaxed solution involves a more significant impact on smaller generation portfolios.

3.3.3. Compactness

The compactness of an optimization problem is given by the number of constraints (CT) and variables (binary, continuous, and/or integer) that constitute the problem. The approaches studied in this paper only differ in their balance equation and objective function formulations. In accordance, it would be expected that they manifest a similar compactness. The methodologies with NSE terms will aggregate one more continuous variable (CV) per time step. Meanwhile, PBUC formulation will lose a constraint per time step due to the absence of demand constraint.

Consequently, these approaches show a priori, a practically identical number of constraints, binary variables (BV), and continuous variables. However, the diverse nature of balance equations leads to different feasible regions, which the characteristics of the input data can also modify. Hence, it is necessary to evaluate the compactness after the generation of the polytope that the solver will work with.

Table 5 gathers the general equations to determine the quantity of CT, BV, and CV of each methodology. Note that these formulas do not consider a little constraint and variable reduction following the imposed initial conditions. (Including this reduction would be confusing if providing a general idea is desired). Moreover, Table 5 also illustrates the average number of constraints and variables for HRI and

Table 5
Compactness comparison: constraints, binary, and continuous variables of each methodology.

	EDUC			EDUC-N			GDUC			GDUC-N			PBUC		
	P1	P2	P3	P1	P2	P3	P1	P2	P3	P1	P2	P3	P1	P2	P3
CT Compactness	$(G^1 + (6+s) \cdot g + 1) \cdot t$			$(G^1 + (6+s) \cdot g + 1) \cdot t$			$(G^1 + (6+s) \cdot g + 1) \cdot t$			$(G^1 + (6+s) \cdot g + 1) \cdot t$			$(G^1 + (6+s) \cdot g) \cdot t$		
BV Compactness	$(3+s) \cdot g \cdot t$			$(3+s) \cdot g \cdot t$			$(3+s) \cdot g \cdot t$			$(3+s) \cdot g \cdot t$			$(3+s) \cdot g \cdot t$		
CV Compactness	$g \cdot t$			$g \cdot t + t$			$g \cdot t$			$g \cdot t + t$			$g \cdot t$		
Constraints & Variables Before Presolve [#]															
CT Average	41,872	48,975	90,103	41,872	48,975	90,103	41,872	48,975	90,103	41,872	48,975	90,103	41,128	48,231	89,359
BV Average	26,040	29,760	55,800	26,040	29,760	55,800	26,040	29,760	55,800	26,040	29,760	55,800	26,040	29,760	55,800
CV Average	5208	5952	11,160	5952	6696	11,904	5208	5952	11,160	5952	6696	11,904	5208	5952	11,160
Constraints & Variables After Presolve [#]															
CT HRI Avg.	*24,830*	*26,140*	*62,056*	29,694	30,403	64,549	34,763	38,113	76,827	37,410	40,950	78,525	5587	5745	5598
BV HRI Avg.	*11,625*	*13,768*	*31,861*	14,709	16,423	33,584	17,745	21,025	41,184	19,528	22,902	42,424	2854	2945	2862
CV HRI Avg.	*3724*	*4094*	*9011*	4657	4927	9699	4402	5050	9599	5076	5716	10,231	694	686	693
CT STG Avg.	28,425	32,377	76,480	39,894	42,903	82,053	28,423	32,377	76,478	39,892	42,903	82,051	5488	5669	5488
BV STG Avg.	10,590	15,874	37,780	19,566	22,934	42,500	10,588	15,874	37,778	19,564	22,934	42,498	2854	2945	2854
CV STG Avg.	5197	5865	11,159	5952	6696	11,904	5197	5865	11,159	5952	6696	11,904	671	671	671
After/Before Presolve Ratios															
CT HRI	*59%*	*53%*	*69%*	71%	62%	72%	83%	78%	85%	89%	84%	87%	14%	12%	6%
BV HRI	*45%*	*46%*	*57%*	56%	55%	60%	68%	71%	74%	75%	77%	76%	11%	10%	5%
CV HRI	*72%*	*69%*	*81%*	78%	74%	81%	85%	85%	86%	85%	85%	86%	13%	12%	6%
CT STG	68%	66%	85%	95%	88%	91%	68%	66%	85%	95%	88%	91%	13%	12%	6%
BV STG	41%	53%	68%	75%	77%	76%	41%	53%	68%	75%	77%	76%	11%	10%	5%
CV STG	100%	99%	100%	100%	100%	100%	100%	99%	100%	100%	100%	100%	13%	11%	6%

STG cases for each methodology, before and after presolve, and the percentage of reduction its performance involves.

The information obtained after the presolve performance reveals that the different formulations' polytopes do not similarly reduce their size at all. Furthermore, differences can be appreciated depending on the portfolio configuration and the nature of demand profiles.

In this section, it would be preferable to avoid comparing EDUC - HRI because its profiles can be reduced after presolve differently. Nevertheless, when the number of CT, BV, and CV are individually observed in the initially feasible EDUC - HRI cases, they experienced a practically identical performance to GDUC in each case study and portfolio. This trend is analogous to EDUC - STG cases.

Besides that, Table 5 shows that GDUC-N is the less compact formulation, as could be expected. However, the behaviors of EDUC-N and GDUC provide unforeseen conclusions. It can be affirmed that when NSE terms are included, STG problems barely reduce the problem size after presolve. Meanwhile, in their absence, it is possible to achieve a substantial reduction even if greater than equal balance constraints are employed. It could have an origin in that NSE terms bring many start-up and shut-down options to the problem, with the corresponding BV and CT associated. Thus, these approaches cannot be as notably reduced as EDUC and GDUC, in which STG profiles allow a remarkable suppression of unnecessary start-ups and shut-downs, preventing them from removing production decisions (modeled with CV).

Conversely, HRI profiles present EDUC-N as a more compact methodology than GDUC when medium-term horizons are evaluated. Here, the solver seems to identify NSE terms uniquely as an option to make problems feasible and gives a more ample optimization space.

Regarding PBUC, compactness results demonstrate that the solver breaks every generator interrelation, returning tiny problems after presolve. This reduction is more significant as the portfolio's size grows.

Finally, when results are compared at a portfolio level, the formulations that include a balance constraint reveal that greater compactness is achieved with the presence of small- and medium-thermal units (P2).

3.3.4. Arduousness

The mathematical analysis of the feasible region is a tough task that changes with every constraint reformulation. Furthermore, it is also susceptible to the nature of the problem's input data. Likewise, apparently-similar problems experience substantial variations after the presolve performance due to the complexity introduced by the election of particular balance equations, as demonstrated in [31]. There, an aggregated demand for the whole time horizon was imposed, leading to remarkably different problem sizes whose resolution processes were drastically complicated.

This scope has been further developed in this paper, given that great discrepancies are observed in analogous-size problems whose resolution would be expected to be uniform. That comes from the different solver's ability to explore the feasible regions induced by the permissivity of the demand constraint (or its absence). For this reason, the concept of 'arduousness' has been introduced as a metric of numerical optimization performances. It is defined as:

$$Arduousness\,(\#/s) = \frac{Number\,of\,Optimization\,Elements\,After\,Presolve\,(\#)}{Run\,Time\,(s)} \tag{37}$$

Hence, the ability of the solver to work with the polytope of a specific problem and to find an optimal solution can be measured and compared. Although the arduousness will be naturally influenced by the imposition of an optimality gap, presolve options, etcetera, it represents an excellent indicator in formulations' comparison. It is important to note that arduousness considers the problem generation, presolve, and solve time to provide a clear and entire idea of the implications when dealing with a specific methodology.

In this section, the arduousness of managing constraints, binary, and continuous variables has been individually calculated for each methodology, optimality gap, and portfolio, and they are exposed in Table 6 together with their corresponding average run times, distinguishing between HRI and STG cases. When arduousness is analyzed, surprising behaviors are brought to light, especially when compared with the expected approach performances according to their tightness and compactness.

A clear example is EDUC-N formulation. In previous sections, a better tightness has been obtained for EDUC and EDUC-N methodologies with HRI profiles. Additionally, EDUC-N HRI is more compact than GDUC HRI and GDUC-N HRI. Consequently, it would be expected for this approach to be more efficient than the other two. Nevertheless, it offers worse run times. Therefore, when the EDUC-N HRI performance with strict OGs is compared, it manifests a higher arduousness for CT, BV, and CV than the rest of the methodologies.

Another unpredictable fact is that EDUC HRI, whose tightness and compactness are very good, reflects a worse arduousness than GDUC

Table 6
Arduousness comparison: performance of each methodology and optimality gap.

	EDUC			EDUC-N			GDUC			GDUC-N			PBUC		
	P1	P2	P3	P1	P2	P3	P1	P2	P3	P1	P2	P3	P1	P2	P3
Average Run Time [s]															
HRI - OG 10	*15.5*	*27.4*	*174.5*	27.8	42.1	143.5	23.3	34.2	82.1	36.6	42.1	121.4	10.1	7.5	25.3
HRI - OG 10	*160.8*	*1877.7*	*6545.6*	474.1	2921.4	6786.7	824.4	2304.4	6696.7	653.1	3156.2	6654.3	10.1	7.5	25.4
HRI - OG 10	*172.7*	*2196.9*	*6661.7*	496.6	3199.7	7103.7	862.8	2651.1	7200.0	703.8	3652.7	7039.7	10.1	7.5	25.4
STG - OG 10	10.7	10.8	33.1	21.1	23.7	80.9	10.5	9.5	32.5	20.7	14.6	85.3	9.9	7.5	25.5
STG - OG 10	14.5	49.2	227.8	24.2	140.3	334.9	14.1	44.9	230.2	23.3	144.5	367.6	10.0	7.5	25.8
STG - OG 10	15.6	65.8	1507.0	24.7	162.4	4732.2	15.7	104.4	1407.1	25.0	165.8	4736.8	10.0	7.5	25.8
Arduousness Constraints [#/s]															
HRI - OG 10	*1598.5*	*953.1*	*355.5*	1068.8	723.0	449.8	1489.3	1114.4	935.8	1022.1	973.8	646.7	553.7	771.1	221.2
HRI - OG 10	*154.4*	*13.9*	*9.5*	62.6	10.4	9.5	42.2	16.5	11.5	57.3	13.0	11.8	553.2	768.5	220.7
HRI - OG 10	*143.8*	*11.9*	*9.3*	59.8	9.5	9.1	40.3	14.4	10.7	53.2	11.2	11.2	551.8	767.7	220.6
STG - OG 10	2648.3	3007.1	2312.9	1887.7	1812.8	1014.3	2706.9	3408.1	2353.2	1930.3	2931.9	961.5	552.5	752.5	215.2
STG - OG 10	1955.8	657.6	335.7	1650.8	305.8	245.0	2015.8	720.5	332.3	1709.7	296.9	223.2	547.0	752.5	213.0
STG - OG 10	1822.1	492.3	50.7	1613.0	264.2	17.3	1810.4	310.0	54.4	1595.7	258.7	17.3	547.0	752.5	213.0
Arduousness Binary Variables [#/s]															
HRI - OG 10	*748.4*	*502.0*	*182.5*	529.4	390.6	234.0	760.2	614.8	501.6	533.6	544.6	349.4	282.8	395.2	113.1
HRI - OG 10	*72.3*	*7.3*	*4.9*	31.0	5.6	4.9	21.5	9.1	6.1	29.9	7.3	6.4	282.6	393.9	112.9
HRI - OG 10	*67.3*	*6.3*	*4.8*	29.6	5.1	4.7	20.6	7.9	5.7	27.7	6.3	6.0	281.9	393.5	112.8
STG - OG 10	986.6	1474.4	1142.6	925.8	969.0	525.3	1008.4	1671.0	1162.4	946.6	1567.2	498.0	287.3	391.0	111.9
STG - OG 10	728.7	322.4	165.8	809.6	163.5	126.9	750.9	353.3	164.1	838.5	158.7	115.6	284.5	391.0	110.8
STG - OG 10	678.8	241.4	25.1	791.1	141.2	9.0	674.4	152.0	26.8	782.6	138.3	9.0	284.5	391.0	110.8
Arduousness Continuous Variables [#/s]															
HRI - OG 10	*239.8*	*149.3*	*51.6*	167.6	117.2	67.6	188.6	147.6	116.9	138.7	135.9	84.3	68.8	92.0	27.4
HRI - OG 10	*23.2*	*2.2*	*1.4*	9.8	1.7	1.4	5.3	2.2	1.4	7.8	1.8	1.5	68.8	91.7	27.3
HRI - OG 10	*21.6*	*1.9*	*1.4*	9.4	1.5	1.4	5.1	1.9	1.3	7.2	1.6	1.5	68.6	91.6	27.3
STG - OG 10	484.2	544.7	337.5	281.6	282.9	147.1	495.0	617.3	343.3	288.0	457.6	139.5	67.6	89.1	26.3
STG - OG 10	357.6	119.1	49.0	246.3	47.7	35.5	368.6	130.5	48.5	255.1	46.3	32.4	66.9	89.1	26.1
STG - OG 10	333.1	89.2	7.4	240.6	41.2	2.5	331.0	56.2	7.9	238.1	40.4	2.5	66.9	89.1	26.1

and GDUC-N. Finally, comparing GDUC and GDUC-N with HRI profiles, an expected result is obtained: GDUC-N's greater arduousness highlights that resolution processes struggle with NSE terms.

On the other hand, STG profiles exhibit a more rational behavior. The arduousness of EDUC-N and GDUC-N formulations is considerably higher than in EDUC and GDUC, which aligns with their greater tightness and compactness. In this way, EDUC performs a similar arduousness to GDUC and EDUC-N to GDUC-N when strict OGs are imposed.

In general terms, STG profiles entail a lower arduousness than HRIs when the approach has a balance equation. Additionally, the portfolio configuration also plays a role. Arduousness increases with the number of units. However, it can be appreciated that large units reduce arduousness when strict OGs are defined. The amount of thermal units is practically equal in P1 and P2. Nevertheless, P2 is more difficult to optimize due to its small- and medium-size generators.

Finally, PBUC formulation displays similar performances for CT, BV, and CV arduousness independently from the OG, given its fast convergence towards a tiny gap to finish the resolution process. Insignificant differences are found when comparing real market profiles and academic electricity prices.

As expected, the largest portfolio (P3) is the most arduous. However, it is remarkable that P2, which has more generation units and entails less compact problems than P1, performs a less arduous resolution. Then, it can be affirmed that when maximizing benefits, small thermal units are more difficult to handle, compared to larger generators, than when operational costs are minimized. This fact could not be concluded according to the tightness and compactness results of PBUC formulation.

3.3.5. Evolution of the optimality gaps & OF bounding

An analysis of the resolution processes of the different methodologies to identify simplicities and complications is described in this section, helping to provide a more detailed explanation for the arduousness results.

Fig. 4 illustrates the evolution of optimality gaps and objective function bounding, upper bound (UB), and lower bound (LB), which are examined along the run times for each generation portfolio. Some of these data are presented in per-unit magnitudes regarding their final OF value to establish a clearer comparison benchmark for computational performances. It is important to mention that the EDUC approach includes feasible-made cases, as the previous section did. Despite input profiles being different, feasible regions are neither the same for each formulation and insights into how the solver works can be discerned.

The illustration reveals that although EDUC-N and GDUC-N methodologies find 'better' solutions than EDUC and GDUC, they need considerably greater effort to reduce OGs, especially at the beginning. That can be explained through the initial OF raise (the primal bound corresponds to the UB in minimization problems) when NSE situations are allowed. It might happen because the solver focuses on finding a feasible solution soon, which internalizes NSE as a high cost and improves it step by step. Furthermore, it can be appreciated in Table A.8 that if the process finishes early (not too low OGs), the optimal solution is 'worse' than in some non-NSE cases. This fact demonstrates that the solver struggles to work with these kinds of 'slackness' but finally finds better solutions when strict OGs are imposed.

However, EDUC can also be stuck at some phases of the resolution process, consuming too much CPU time in improving the OG to continue with the optimization. That is seen in P3 and could have its origin in the inflexibility of the EDUC's balance equation for changing the generation schedule at some point in the branch & cut process, making the solver inefficient for moving to earlier nodes that were less promising a priori but that finally allow an OG reduction after an exhaustive possibilities examination.

Besides, the push-ups[1] in the lower bounds (dual/best bound in minimization problems) are more significant during the last conver-

[1] The term 'push-up' is introduced in this paper to denote an event of numerical optimization. It consists in redefining the best-bound value when

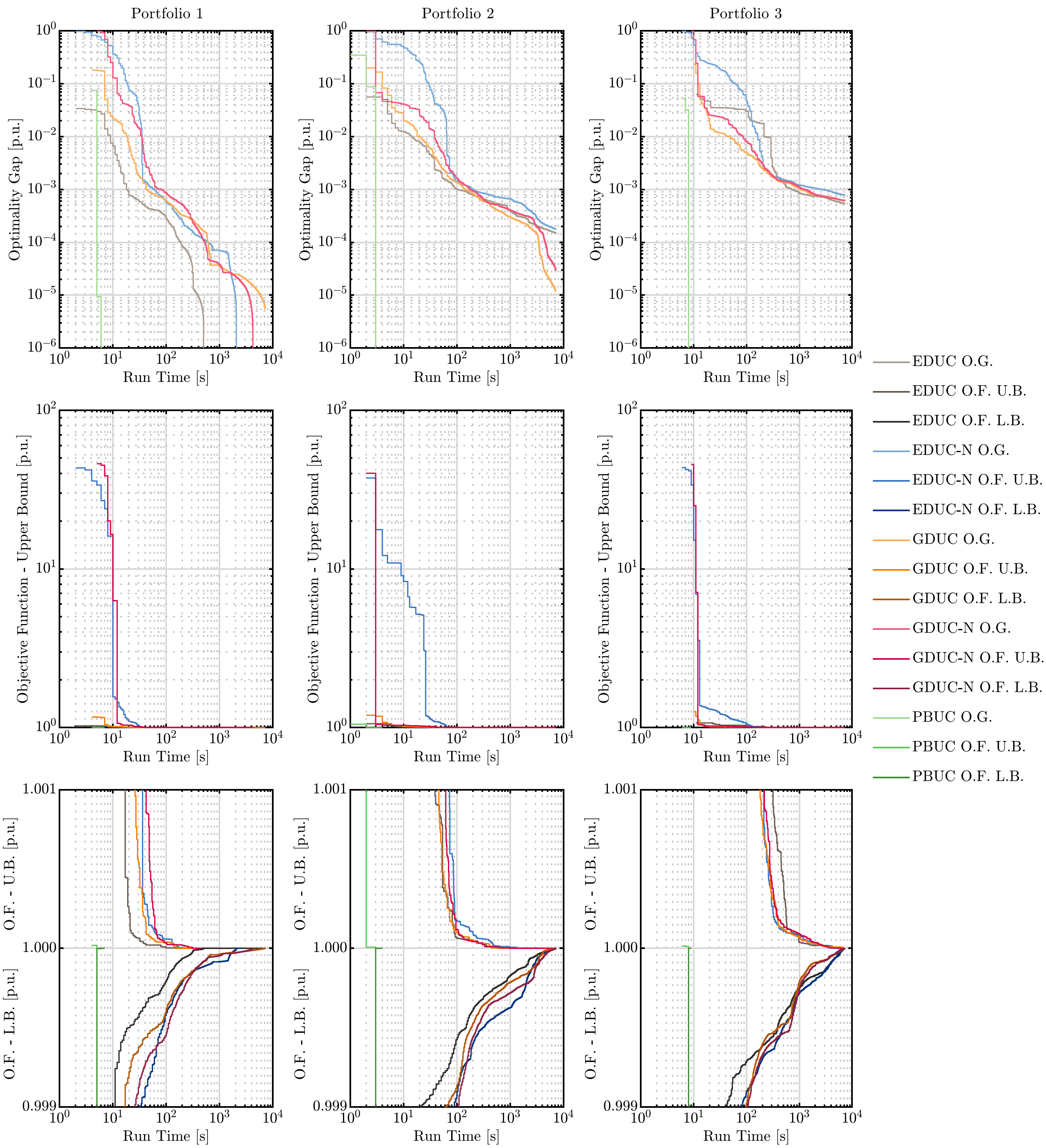

Fig. 4. Improvements in the optimality gaps versus run time and corresponding evolution of the objective-function bounding along the optimization processes.

gence stages for all the methodologies with demand constraints. It can be observed that when their OFs barely improve, the LBs start to perform higher jumps at the final phase, mainly with portfolios 1 and 2. This remarkable bouncing behavior is a consequence of the thermal-unit effect in these generation portfolios. Their smaller sizes imply that making a little operational infeasibility in the relaxed solution means a higher weight in the resolution performances. Hence, push-ups are proportionally greater and take more time to be accomplished, respecting the evolution of the upper bounds.

On the other hand, the LB slopes of EDUC and EDUC-N are generally greater than GDUC and GDUC-N when their optimization processes finish. It can be concluded that the solver identifies that it is worthless to keep trying to improve the OF and proceeds to perform these remarkable jumps in the best-bound assumptions. Meanwhile, GDUC and GDUC-N have more flexible optimization spaces, and their LB jumps are softer in time.

the solver understands that the found solution is not subject to many further developments and, consequently, the optimality gap is unreachable.

Hence, a relationship with the arduousness can be established: the greater absence of 'big LB jumps' when the OF (UB) is stabilized means a more accessible feasible region exploration, which is translated into a less arduous performance despite these approaches' worse tightness and compactness (especially in HRI cases with strict optimality gaps).

On the other side, OFs (primal bounds) are equivalent to the lower bounds in maximization problems, and the best bounds (dual solutions) correspond to the upper bounds. Nevertheless, analyzing the performance of the PBUC cases exposed in this section is not worthwhile due to their quick convergence toward the optimal solution.

3.4. Numerical results of the short-term case studies and comparison to the medium-term

This section evaluates the computational performance of traditional large-size case studies in the unit commitment problem with large generation portfolios and short-term horizons (24 h). It compares their results to the medium-term horizon (MH) cases described in Section 3.3.

Accordingly, 60 hourly cases with a daily horizon (DH) are run with each methodology (300 cases) to represent HRI short-term case studies. Besides, eight cases are run as STG-DH profiles in EDUC, EDUC-N, GDUC, and GDUC-N. In PBUC-STG-DH, three cases are considered. Then, a total of 35 STG short-term cases studies are evaluated.

These 335 DH cases utilize P4 as its generation portfolio to obtain large-size case studies with similar dimensions to MH-P3 before presolve in order to make a fairer comparison. DH-P4 cases are run on the same computer and under the same software as MH-P3. An optimality gap of 10^{-4} is chosen as the stopping criterion. Finally, Table 7 shows their results and compares them to those obtained with MH-P3-10^{-4}OG.

Regarding PBUC, DH and MH manifest similar run times when managing HRI and STG cases. The PBUC-DH-P4 run times are approximately half of PBUC-MH-P3's. The tightness of all these cases is extremely low, and their compactness before presolve is practically identical. Differences appear after presolve performance. DH cases achieve a ~50 times higher reduction of variables and constraints. It happens because of the internal individualization of generators in PBUC. Then, the shorter horizon in DH makes this size difference. Therefore, when arduousness is calculated, substantial differences are observed. Nevertheless, it is worthless to analyze them in-depth, given that PBUC's resolutions are practically instantaneous.

When run times are analyzed in the methodologies with a balance constraint (EDUC, EDUC-N, GDUC, and GDUC-N), a one-order magnitude difference is appreciated between DH-P4 and MH-P3, being the DH cases faster to solve. However, similar differences are observed in the run time ratio between HRI and STG cases for both situations. STG cases always show lower average run times. Moreover, they perform similarly in STG-DH-P4 independently from the methodology. On the other hand, higher run times are required in STG-MH-P3 for EDUC-N and GDUC-N when compared to EDUC and GDUC.

Regarding the tightness, STG cases manifest lower integrality gaps than HRI in both DH-P4 and MH-P3. However, it is important to note that tightness in HRI-DH-P4 is very close to STG-DH-P4 in every approach, while MH-P3 shows remarkable differences. Hence, the balance constraint and the load profile do not introduce a significant difference in tightness when evaluating short-term horizons.

Likewise, the constraint and variable reductions after presolve in DH-P4 manifest the same behavior. This lack of differences between HRI and STG cases can be explained by the absence of no-load days in the selection of the case studies. Their presence in MH-P3 cases involves a greater reduction after presolve performance. Moreover, it also plays a role in the difference between methodologies observed when medium-term horizons are employed.

Therefore, it can be concluded that problems' tightness and compactness are more predictable (by far) when short-term horizons are considered, involving a reduction in the complexity of the unit commitment problem. Meanwhile, it is important to highlight that these practically equal T&C trade-offs in DH-P4 do not lead to similar run times. Balance constraints also entail differences in these resolution processes. Consequently, determining arduousness is necessary to provide improved insights about the expected computational performance.

In this way, DH-P4 clearly distinguishes the methodologies' arduousness for constraints, binary, and continuous variables. EDUC is more arduous than EDUC-N. For its part, EDUC-N is more arduous than GDUC, and finally, GDUC-N is the less arduous model when working with short-term horizons. This behavior is appreciated in both HRI and STG cases, manifesting a difference between them of one order of magnitude.

4. Conclusions

MILP formulations of the unit commitment problem provide valuable schedules to optimally manage the thermal operation in current electricity markets. However, they also imply a high computational burden when detailed representations are needed. Consequently, multiple approaches are continuously being proposed in the literature.

This paper identifies several research gaps in the unit commitment literature, like the computational implications of choosing a specific balance-equation, dealing with high-intermittency demand profiles, or testing methodologies with large-size problems while avoiding introducing symmetry effects in the resolution processes.

For that reason, an in-depth analysis of the computational performance when utilizing different demand constraints (and price-based relationships) is presented, clarifying differences in feasible regions, resolution performance, and tightness & compactness of their corresponding methodologies. Moreover, some T&C limitations were detected when defining good practices in MIP numerical optimization. In accordance, the concept of arduousness was introduced to enhance computational comparisons.

Furthermore, the study exposed in this article also contemplates analyzing differences when working with stable load profiles in the unit commitment problem and with high intermittency demand curves. Meanwhile, diverse portfolio configurations were also tested to discover how they affect the resolution processes. In turn, medium-term horizons were employed to construct large-size problems without replicating thermal generators to avoid the presence of symmetry effects in this study. Later, they are compared to large-size short-term cases in which thermal units have been replicated to constitute problems whose sizes are similar to the medium-term cases.

Thereafter, the main findings of this paper when analyzing balance constraints in medium-term horizons are summarized below:

- *Objective function and run time trade-off:* although every formulation entails a different feasible region and their OFs cannot be directly compared, computational trade-offs between modeling detail (approaches, portfolios' size and configuration, optimality gaps, solution quality, etc.) and run times were established. HRI profiles involve more difficult-to-optimize polytopes than STG since they require higher computational resources to find a solution, frequently reaching the maximum run time when strict optimality gaps are imposed. Independently, PBUC quickly converges to tiny OGs, given that generation decisions are not interrelated between thermal units. Moreover, it was also detected that large-generators are easier to manage than small- and medium-units in minimization problems. However, when benefits are maximized, large units are more difficult to handle.
- *Tightness:* utilizing STG profiles results in tighter problems for each methodology than HRI curves. In turn, STG cases tend to reach similar IGs when strict OGs are imposed. All these cases are generally tighter than HRI. On the other hand, more significant differences between approaches were observed when working

Table 7
Computational performance comparison: metrics of each methodology with different time spans.

	EDUC		EDUC-N		GDUC		GDUC-N		PBUC	
	DH P4	MH P3	DH P4	MH P3	DH P4	MH P3	DH P4	MH P3	DH P4	MH P3
Run Times [s]										
HRI Average	520.0	6545.6	399.7	6786.7	229.1	6696.7	180.0	6654.3	13.9	25.4
STG Average	30.4	227.8	30.1	334.9	29.5	230.2	29.2	367.6	14.0	25.8
Integrality Gaps [%]										
HRI Average	0.12	1.09	0.12	1.21	0.11	1.25	0.11	1.25	0.00	0.00
STG Average	0.10	0.32	0.10	0.33	0.10	0.32	0.10	0.33	0.03	0.02
Constraints & Variables Before Presolve [#]										
CT Average	88,796	90,103	88,796	90,103	88,796	90,103	88,796	90,103	88,772	89,359
BV Average	63,000	55,800	63,000	55,800	63,000	55,800	63,000	55,800	63,000	55,800
CV Average	12,600	11,160	12,624	11,904	12,600	11,160	12,624	11,904	12,600	11,160
Constraints & Variables After Presolve [#]										
CT HRI Avg.	69,305	62,056	69,305	64,549	69,235	76,827	69,235	78,525	117	5598
BV HRI Avg.	36,234	31,861	36,234	33,584	36,164	41,184	36,164	42,424	54	2862
CV HRI Avg.	12,576	9011	12,600	9699	12,576	9599	12,596	10,231	24	693
CT STG Avg.	69,305	76,480	69,305	82,053	69,235	76,478	69,235	82,051	108	5488
BV STG Avg.	36,234	37,780	36,234	42,500	36,164	37,778	36,164	42,498	58	2854
CV STG Avg.	12,576	11,159	12,600	11,904	12,576	11,159	12,600	11,904	19	671
After/Before Presolve Ratios										
CT HRI	78%	69%	78%	72%	78%	85%	78%	87%	0.1%	6%
BV HRI	58%	57%	58%	60%	57%	74%	57%	76%	0.1%	5%
CV HRI	100%	81%	100%	81%	100%	86%	100%	86%	0.2%	6%
CT STG	78%	85%	78%	91%	78%	85%	78%	91%	0.1%	6%
BV STG	58%	68%	58%	76%	57%	68%	57%	76%	0.1%	5%
CV STG	100%	100%	100%	100%	100%	100%	100%	100%	0.2%	6%
Arduousness [#/s]										
CT HRI	133.3	9.5	173.4	9.5	302.2	11.5	384.6	11.8	8.4	220.7
BV HRI	69.7	4.9	90.7	4.9	157.8	6.1	200.9	6.4	3.8	112.9
CV HRI	24.2	1.4	31.5	1.4	54.9	1.4	70.0	1.5	1.7	27.3
CT STG	2282.6	335.7	2305.4	245.0	2350.9	332.3	2373.1	223.2	7.7	213.0
BV STG	1193.4	165.8	1205.3	126.9	1228.0	164.1	1239.6	115.6	4.2	110.8
CV STG	414.2	49.0	419.1	35.5	427.0	48.5	431.9	32.4	1.4	26.1

with HRI cases. There, EDUC is the tightest formulation, followed by EDUC-N. After that, GDUC is tighter than GDUC-N. It means that an inequality in the demand constraint entails a higher tightness loss than utilizing NSE terms in HRI cases. From a portfolio perspective, greater portfolios achieve better tightness because of their operational flexibility. Finally, PBUC cases offer the best tightness independently of the input data.

- *Compactness:* all the formulations have similar CT, BV, and CV numbers before presolve. Nevertheless, remarkable differences appear after the presolve performance. As expected, PBUC is always the most compact approach. Conversely, when NSE terms are considered in STG cases (EDUC-N & GDUC-N), the presolve barely reduces the optimization problem. Meanwhile, EDUC-N is more compact than GDUC in HRI cases. Hence, NSE terms apparently have a more significant impact in cases where start-up and shut-down processes were, a priori, less important. At the portfolio level, greater portfolios mean lower compactness in minimization methodologies. However, more substantial reductions are appreciated for P3 rather than P1 and P2 in PBUC cases.
- *Arduousness:* T&C metrics do not always meet the expected resolution process behaviors, especially in HRI cases. For that reason, the concept of arduousness is introduced to provide clearer notions about computational performances, making it also possible to measure and compare the ability of the solver to work with the problems' polytopes and to find optimal solutions. In this way, NSE terms increase the arduousness in STG cases. On the other hand, HRI performances are clarified: EDUC and EDUC-N formulations are more arduous to solve than GDUC and GDUC-N. This surprising result could not be concluded according to the T&C information. From the portfolio perspective, small- and medium-generators imply greater arduousness in minimization problems. Meanwhile, large units are more difficult to handle in maximization problems.
- *Resolution processes:* the utilization of NSE terms requires a greater effort to reduce the optimality gap, especially at the beginning of the optimization. On the other hand, when the solvers consider that the relaxed solution is unreachable and it is worthless to keep trying to improve the objective function (integer solution), they start to perform changes in the relaxed solution, establishing less strict targets. This phenomenon is generally more remarkable in the final steps of EDUC-N and EDUC cases due to the greater rigidness of their demand constraints.
- *Comparison to short-term problems when problem sizes are similar:* medium-term problems manifest greater differences because of their balance constraint than short-term cases do. They imply differences in tightness and compactness, which are negligible with short-term horizons independently from using HRI or STG profiles. This fact makes arduousness an even more important metric to provide realistic predictions of the methodologies and case studies' computational performance.

Finally, the conclusions manifest the usefulness and validity of the analysis presented in the article. These highlights define several good practices that should be taken into account when any formulation improvement of the unit commitment problem is proposed. Moreover, this article establishes a basis for analyzing future constraints that link all the generators involved in the unit commitment problem, such as reserve constraints. However, it is important to remember that solvers

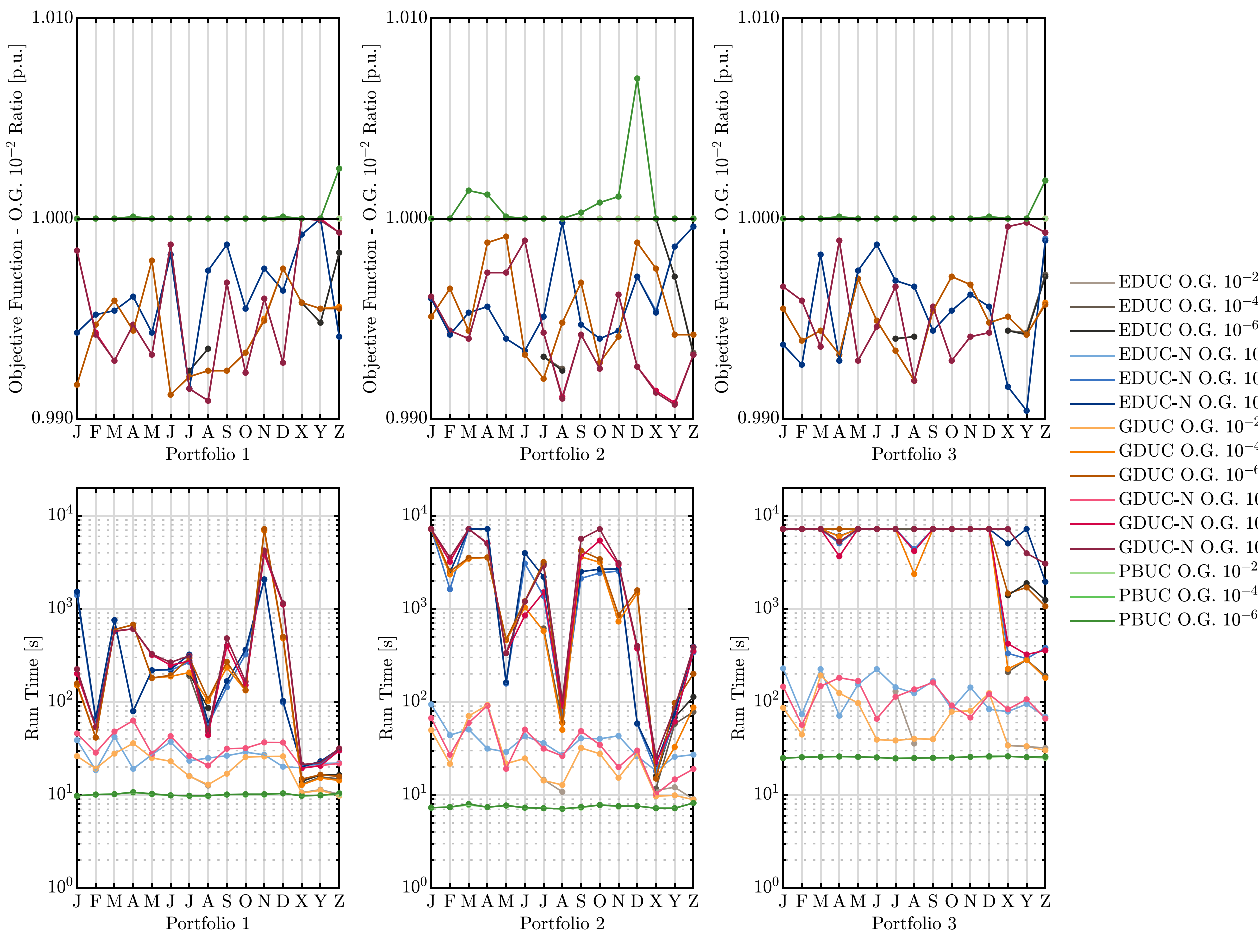

Fig. A.5. Improvements of the objective functions with optimality-gap reductions and their corresponding run-time increments.

have an internal-opaque functioning which entails some degree of unpredictability, making it imprudent to provide categorical recommendations. Despite that, computational implications of the different exposed methodologies have been clarified thanks to this study, and they can be leveraged in future research from a qualitative perspective in academics and also from a quantitative point of view for independent system operators (ISOs) and utilities.

CRediT authorship contribution statement

Luis Montero: Writing – original draft, Visualization, Validation, Supervision, Software, Resources, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. **Antonio Bello:** Writing – original draft, Supervision, Methodology, Investigation. **Javier Reneses:** Supervision, Methodology, Investigation.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

The authors are unable or have chosen not to specify which data has been used.

Appendix. Computational trade-offs

Fig. A.5 illustrates the improvements in the OF regarding the reduction of the optimality gap (the ratio between the solutions and those obtained with a 10^{-2} OG) and the run time of each case. Furthermore, the objective-function data are gathered in Table A.8 in $. It is important to highlight that it would be senseless to consider the feasible-made EDUC cases in these quantitative analyses. Nevertheless, the EDUC HRI cases of July and August are feasible as they are, and their solutions can be typically compared.

In this way, EDUC-N shows 'better' values than EDUC, as expected, since it is a less rigid problem. Consequently, GDUC manifests 'better' OFs than EDUC, and GDUC-N 'overcomes' both EDUC and GDUC when strict optimality gaps are defined. All these results are predictable. However, the point of this section is to determine an OF/run time trade-off per the imposed OGs for these methodologies.

Fig. A.5 reflects that OFs never improve more than 1% with the 10^{-4} and 10^{-6} OGs. However, this difference entails substantial revenues [32] and should not be ignored. Conversely, when OFs with 10^{-4} and 10^{-6} OGs are compared, results are practically equal, and their series overlap in the figure. That may happen because the found solution is not subject to many feasible further developments, and the solver starts to perform push-ups over the best bound, getting their values closer and satisfying the optimality gap.

Table A.8
Objective function comparison.

#	EDUC			EDUC-N			GDUC			GDUC-N			PBUC		
	Optimality gap			Optimality gap			Optimality gap			Optimality gap			Optimality gap		
	10^{-2}	10^{-4}	10^{-6}	10^{-2}	10^{-4}	10^{-6}	10^{-2}	10^{-4}	10^{-6}	10^{-2}	10^{-4}	10^{-6}	10^{-2}	10^{-4}	10^{-6}
								Portfolio 1							
J	–	–	–	26,234,045	26,084,172	26,084,172	25,705,274	25,493,070	25,493,070	25,532,302	25,491,217	25,490,480	52,125,153	52,125,153	52,125,153
F	–	–	–	23,982,465	23,867,627	23,867,627	22,577,953	22,458,768	22,458,768	22,588,914	22,459,089	22,458,768	45,250,583	45,250,583	45,251,006
M	–	–	–	13,135,058	13,075,135	13,075,135	12,419,026	12,367,810	12,367,810	12,453,940	12,365,597	12,365,597	32,137,133	32,137,133	32,137,470
A	–	–	–	16,363,859	16,299,429	16,299,413	7,050,090	7,010,362	7,010,362	7,045,959	7,008,632	7,008,632	11,116,399	11,117,588	11,117,588
M	–	–	–	7,881,066	7,835,772	7,835,772	6,249,570	6,236,324	6,236,324	6,279,280	6,236,324	6,236,324	32,177,327	32,177,327	32,177,358
J	–	–	–	19,926,539	19,891,519	19,891,519	18,501,756	18,339,137	18,339,137	18,362,576	18,338,428	18,338,428	52,665,666	52,665,666	52,665,666
J	23,963,272	23,780,779	23,780,779	23,983,850	23,780,739	23,780,739	23,970,807	23,780,779	23,780,779	23,983,535	23,780,739	23,780,739	68,077,447	68,077,447	68,077,447
A	37,952,583	37,707,084	37,707,055	37,788,007	37,688,987	37,688,815	37,997,583	37,707,228	37,707,055	38,036,039	37,689,008	37,688,815	57,290,698	57,290,698	57,290,698
S	–	–	–	40,023,159	39,970,685	39,970,685	39,176,075	38,878,340	38,878,340	39,002,078	38,876,849	38,876,849	59,849,333	59,849,333	59,852,311
O	–	–	–	31,341,735	31,200,757	31,200,757	28,922,470	28,727,772	28,727,772	28,932,594	28,710,224	28,710,224	34,266,833	34,266,833	34,267,393
N	–	–	–	35,215,029	35,127,755	35,127,755	33,322,741	33,154,538	33,151,875	33,260,898	33,129,417	33,129,417	40,549,121	40,549,121	40,551,026
D	–	–	–	30,487,726	30,377,857	30,377,556	28,042,265	27,972,430	27,972,430	28,137,905	27,936,687	27,936,687	24,802,028	24,804,985	24,804,985
X	36,321,028	36,167,522	36,167,522	36,198,272	36,169,806	36,167,522	36,320,254	36,167,522	36,167,522	36,167,522	36,167,522	36,167,522	54,785,675	54,785,675	54,785,675
Y	34,349,154	34,169,733	34,169,709	34,170,105	34,170,105	34,169,709	34,323,398	34,169,709	34,169,709	34,171,867	34,171,867	34,169,709	90,700,668	90,700,668	90,700,668
Z	36,317,268	36,257,122	36,256,547	36,472,552	36,255,870	36,255,870	36,418,841	36,257,214	36,256,547	36,281,333	36,256,101	36,255,870	11,636,906	11,665,932	11,665,932
								Portfolio 2							
J	–	–	–	12,718,723	12,668,436	12,668,436	12,348,056	12,287,818	12,287,818	12,334,599	12,287,009	12,287,009	15,156,326	15,156,326	15,156,326
F	–	–	–	11,226,578	11,161,341	11,161,341	10,828,788	10,790,854	10,790,854	10,851,848	10,790,639	10,790,639	13,848,659	13,848,659	13,848,659
M	–	–	–	6,235,892	6,206,589	6,206,589	5,976,565	5,942,899	5,942,899	5,979,015	5,942,851	5,942,851	9,132,476	9,144,846	9,145,337
A	–	–	–	6,220,505	6,193,192	6,193,192	3,357,734	3,353,731	3,353,731	3,362,010	3,352,834	3,352,834	2,563,022	2,566,206	2,566,206
M	–	–	–	3,526,456	3,505,331	3,505,331	3,001,345	2,998,770	2,998,770	3,006,799	2,998,770	2,998,770	10,654,678	10,655,782	10,655,896
J	–	–	–	9,689,371	9,625,512	9,625,512	8,931,246	8,870,811	8,870,811	8,880,220	8,870,648	8,870,648	18,083,544	18,083,544	18,083,544
J	11,605,534	11,525,377	11,525,377	11,582,373	11,525,231	11,525,231	11,618,641	11,525,377	11,525,377	11,591,233	11,525,231	11,525,231	24,216,076	24,216,076	24,216,076
A	18,477,256	18,338,099	18,337,390	18,341,836	18,337,390	18,337,390	18,432,761	18,336,931	18,336,910	18,503,124	18,337,556	18,336,910	17,991,688	17,991,688	17,991,688
S	–	–	–	19,559,726	19,456,067	19,456,067	18,968,637	18,908,430	18,908,430	19,017,502	18,906,494	18,906,494	18,406,008	18,410,852	18,411,649
O	–	–	–	14,218,127	14,132,410	14,132,410	13,865,077	13,765,669	13,765,669	13,865,285	13,761,656	13,761,656	8,502,650	8,509,342	8,509,342
N	–	–	–	16,754,318	16,660,525	16,660,525	15,948,276	15,853,870	15,853,870	15,912,407	15,852,515	15,852,515	10,087,096	10,098,607	10,098,607
D	–	–	–	14,077,949	14,037,212	14,037,212	13,159,152	13,142,995	13,142,995	13,238,350	13,140,278	13,140,278	4,558,342	4,590,361	4,590,361
X	17,461,012	17,461,012	17,460,488	17,528,597	17,447,488	17,446,705	17,503,760	17,460,488	17,460,488	17,599,095	17,448,091	17,446,705	18,345,911	18,345,911	18,345,911
Y	16,534,711	16,486,460	16,486,072	16,498,881	16,476,028	16,475,846	16,582,216	16,486,103	16,486,072	16,629,879	16,476,374	16,475,846	33,112,046	33,112,046	33,112,046
Z	17,684,060	17,564,883	17,563,778	17,567,082	17,559,776	17,559,776	17,667,054	17,564,142	17,563,778	17,680,273	17,559,776	17,559,776	3,389,575	3,389,575	3,389,575
								Portfolio 3							
J	–	–	–	37,314,972	37,080,271	37,080,271	36,450,705	36,288,498	36,288,498	36,410,643	36,287,900	36,287,900	67,281,479	67,281,479	67,281,479
F	–	–	–	33,507,031	33,261,615	33,261,615	32,433,205	32,234,937	32,234,937	32,365,925	32,234,187	32,234,187	59,099,242	59,099,242	59,099,665
M	–	–	–	18,132,780	18,100,592	18,100,592	17,531,643	17,432,817	17,432,817	17,546,595	17,433,788	17,433,788	41,282,470	41,282,470	41,282,807
A	–	–	–	19,396,587	19,259,445	19,259,445	9,731,651	9,665,358	9,665,358	9,674,955	9,664,520	9,664,520	13,682,701	13,683,794	13,683,794
M	–	–	–	10,339,028	10,312,362	10,312,362	8,819,026	8,792,765	8,792,765	8,855,678	8,792,618	8,792,618	42,832,036	42,832,036	42,833,254
J	–	–	–	28,692,156	28,655,218	28,655,218	26,500,649	26,364,274	26,364,274	26,507,973	26,363,953	26,363,953	70,749,210	70,749,210	70,749,210
J	34,597,663	34,388,674	34,388,674	34,493,760	34,386,299	34,386,299	34,615,623	34,386,299	34,386,299	34,505,331	34,386,590	34,386,590	92,293,522	92,293,522	92,293,522
A	54,925,281	54,598,894	54,598,894	54,785,977	54,598,629	54,598,629	55,044,626	54,598,894	54,598,894	55,044,269	54,598,629	54,598,629	75,282,386	75,282,386	75,282,386
S	–	–	–	58,156,481	57,832,038	57,832,038	56,420,098	56,162,188	56,162,188	56,410,049	56,162,203	56,162,203	78,260,982	78,260,982	78,263,960
O	–	–	–	43,120,489	42,921,059	42,921,059	41,086,711	40,968,519	40,968,519	41,258,674	40,967,132	40,967,132	42,776,175	42,776,175	42,776,734
N	–	–	–	49,652,478	49,465,237	49,465,237	47,590,671	47,431,749	47,431,749	47,712,257	47,429,769	47,429,769	50,647,728	50,647,728	50,649,633
D	–	–	–	42,152,530	41,966,586	41,966,586	39,638,448	39,431,730	39,431,730	39,645,911	39,419,323	39,419,323	29,392,389	29,395,346	29,395,346
X	52,448,348	52,155,356	52,155,044	52,597,222	52,155,044	52,155,044	52,411,627	52,155,044	52,155,044	52,175,032	52,155,428	52,155,044	73,131,586	73,131,586	73,131,586
Y	49,601,514	49,316,312	49,316,296	49,793,608	49,316,296	49,316,296	49,604,945	49,316,296	49,316,296	49,327,802	49,316,396	49,316,296	123,812,714	123,812,714	123,812,714
Z	52,529,411	52,380,055	52,378,012	52,433,342	52,378,636	52,378,012	52,601,880	52,379,417	52,378,012	52,415,555	52,378,425	52,378,012	15,026,481	15,055,507	15,055,507

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