BACHELOR’S DEGREE IN ENGINEERING FOR INDUSTRIAL TECHNOLOGIES

UNDERGRADUATE THESIS

APPLYING THE LIU MODEL TO ANALYSE PATENT CROSS-LICENSING

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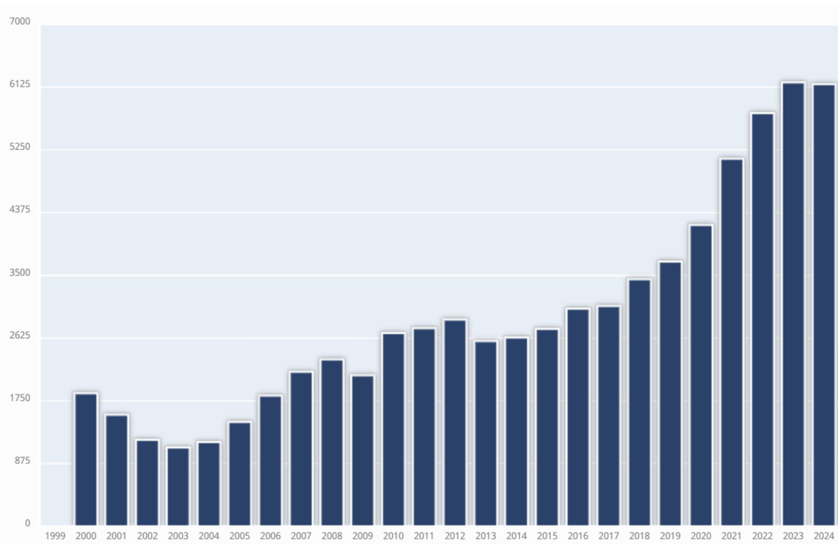
Abstract

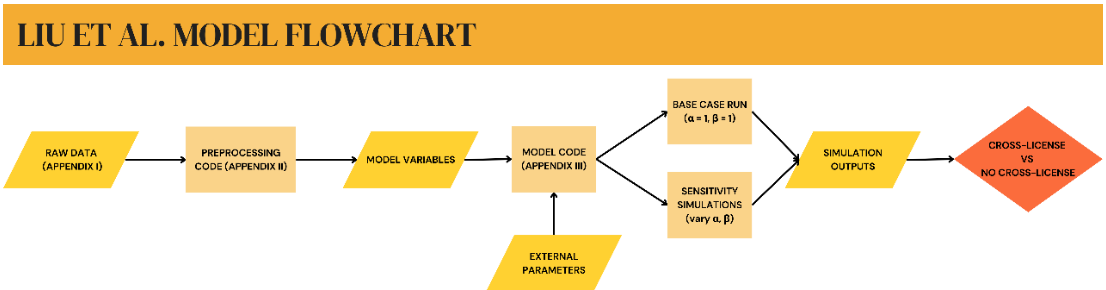
In today's innovation-driven economy, patents play a central role in protecting technological development and ensuring competitive advantage. However, the increasing number of patents in technology-intensive industries has created what are known as patent thickets: complex webs of overlapping rights that obstruct innovation, increase litigation risks, and raise entry barriers for new competitors. As a practical response to these challenges, cross-licensing agreements have emerged as a strategic tool that allows firms to share patented technologies, reduce legal uncertainty, and promote collaborative innovation.



Domain name dispute cases per year
[Source: WIPO, 2025]

This thesis studies cross-licensing agreements through the use of game theory, which provides a rigorous framework for analysing strategic interactions between firms. After reviewing the existing literature, the project develops and applies the model of Liu et al., which captures the decision to engage in cross-licensing, subsequent price setting, and consumer choice. The model was implemented computationally, including a data-processing pipeline, simulation code, and sensitivity analyses, in order to evaluate the outcomes under different market conditions.

The framework was applied to the case of the Airbus A320neo and Boeing 737 MAX, representing a clear example of duopolistic competition in an innovation-intensive industry. The results show that cross-licensing increases joint profitability, shifts demand in favour of the initially weaker competitor, and reduces quality differences between firms. At the same time, it was observed that while such agreements can justify moderate price increases due to quality improvements, they also entail a risk of reducing competitive intensity if not properly regulated.



Liu et al. Model Flowchart
[Source: Own elaboration, 2025]

The conclusions drawn from this work demonstrate that cross-licensing can be both a powerful mechanism to promote innovation and a potential source of anti-competitive behaviour. The thesis therefore contributes not only to understanding the economic and strategic implications of cross-licensing agreements, but also to informing firms and policymakers about the conditions under which these agreements are beneficial and the safeguards that are necessary to ensure fair competition.

Aplicando el Modelo de Liu para analizar Licencias Cruzadas de Patentes

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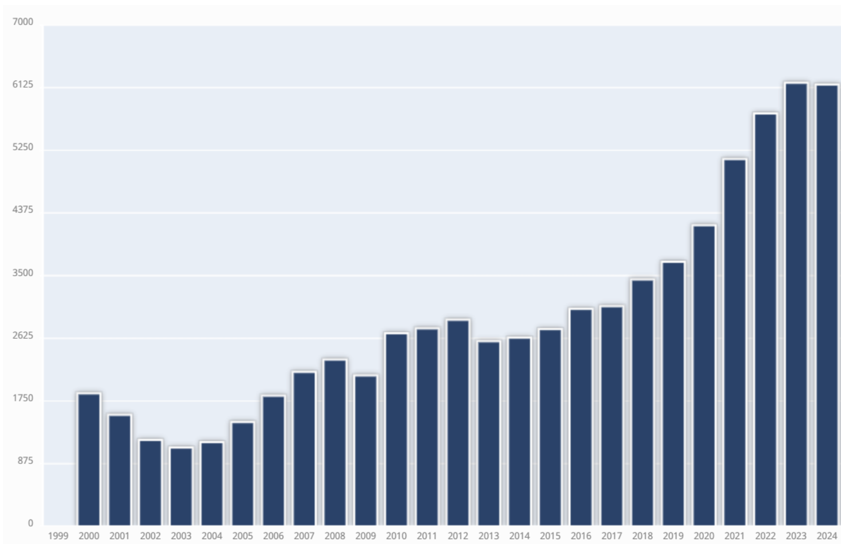
Resumen Ejecutivo del Proyecto

En la economía actual, impulsada por la innovación, las patentes desempeñan un papel central en la protección del desarrollo tecnológico y en la garantía de ventajas competitivas. Sin embargo, el creciente número de patentes en las industrias de alta tecnología ha dado lugar a lo que se conoce como *patent thickets (marañas de patentes)*: marañas complejas de derechos superpuestos que obstaculizan la innovación, aumentan los riesgos de litigio y elevan las barreras de entrada para nuevos competidores. Como respuesta práctica a estos desafíos, los acuerdos de *cross-licensing (licencias cruzadas)* han surgido como una herramienta estratégica que permite a las empresas compartir tecnologías patentadas, reducir la incertidumbre legal y promover la innovación colaborativa.



Casos de disputas anuales por nombres de dominio
[Fuente: WIPO, 2025]

Este proyecto estudia los acuerdos de licencias cruzadas mediante el uso de la teoría de juegos, la cual ofrece un marco riguroso para analizar las interacciones estratégicas entre empresas. Tras una revisión de la literatura existente, el proyecto desarrolla y aplica el modelo de Liu et al., que recoge la decisión de participar en un acuerdo de licencias cruzadas, la posterior fijación de precios y la elección del consumidor. El modelo fue implementado computacionalmente, incluyendo una capa de procesamiento de datos, el código de simulación y análisis de sensibilidad, con el fin de evaluar los resultados bajo diferentes condiciones de mercado.

El marco se aplicó al caso del Airbus A320neo y el Boeing 737 MAX, que representan un claro ejemplo de competencia duopolística en una industria intensiva en innovación. Los resultados muestran que las licencias cruzadas incrementan la rentabilidad conjunta, desplaza la demanda a favor del competidor inicialmente más débil y reduce las diferencias de calidad entre las empresas. Al mismo tiempo, se observó que, aunque dichos acuerdos pueden justificar aumentos moderados de precios debido a las mejoras en calidad, también conllevan el riesgo de reducir la intensidad competitiva si no están adecuadamente regulados.

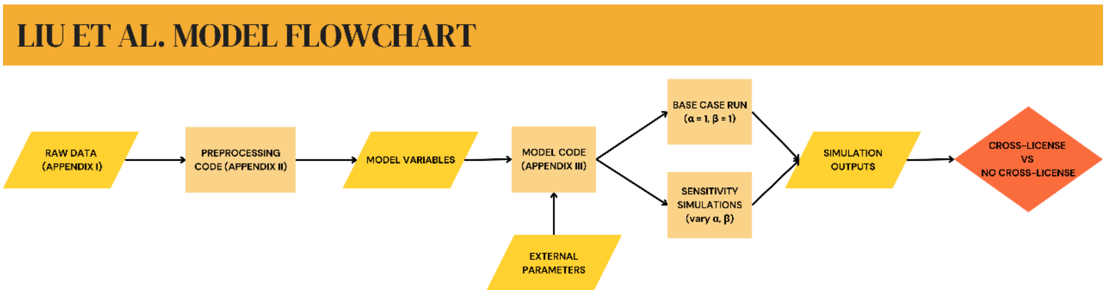


Diagrama de flujo del Modelo de Liu et al.
[Fuente: Elaboración propia, 2025]

Las conclusiones obtenidas a lo largo de este trabajo demuestran que las licencias cruzadas pueden ser tanto un mecanismo poderoso para fomentar la innovación como una posible fuente de conductas anticompetitivas. El trabajo contribuye, por tanto, no solo a comprender las implicaciones económicas y estratégicas de los acuerdos de licencias cruzadas, sino también a orientar a las empresas y a los responsables de políticas públicas sobre las condiciones bajo las cuales dichos acuerdos resultan beneficiosos y las salvaguardas necesarias para garantizar una competencia justa.

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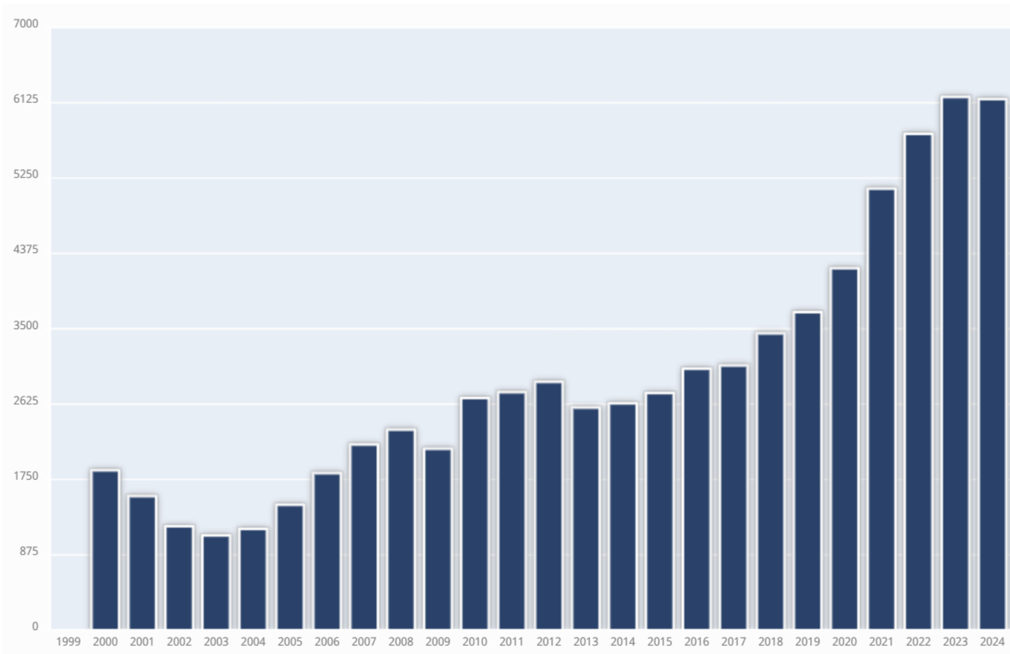
1. Introductory Framework

1.1. Introduction

In today's innovation-driven economy, patents play an incredibly important role in allowing technological advancement and securing competitive advantages. Patents grant their holders exclusive rights over a specific technology or invention, allowing firms to protect their products and money invested in R&D (Research and Development), while also being able to profit from licensing or commercialization[1]. While the aim of patents is to incentivize innovation while protecting the innovators, their vast proliferation over the years, especially when it comes to high-tech industries, has led to increasingly complex legal and strategic landscapes and resulting in counterproductive effects and making the legal steps required for new creations far more complicated.

The main problem and challenge that arises from this system overflown by patents is the issue known as “patent thickets”. Patent thickets are the dense webs of overlapping intellectual property rights that lead to a more difficult innovation, raising litigation risks and complicate market entry for new players[2]. Patent thickets emerge in those sectors in which rapid technological advancement leads to constant innovation, and therefore a constant filing for new patents. In these industries the vast number of patents constantly required to develop just a single product often leads to legal challenges and disputes on top of the operational difficulties. In such environments, a single product might require access to multiple patents held by different firms and as a result, innovators are often caught in a dilemma: to develop and commercialize new products, they must navigate a costly and uncertain web of legal constraints[3].

[1] Shahid, Suddle & Qureshi, 2025
[2] Rani, 2011
[3] Shapiro, 2000



Graph 1: Domain name dispute cases per year
[Source: WIPO, 2025]

In response to this clear problem, cross-licensing agreements have emerged as a strategic solution, allowing two or more firms to grant each other rights to use their respective patented technologies, reducing the risk of infringement lawsuits, facilitating innovation, and often leading to increased market efficiency. However, cross-licensing arrangements also introduce complex strategic interactions. Firms must decide whom to license to, on what terms, and how to balance cooperation with competition, considering several factors. Game-theoretic models allow us to understand the incentives, possible outcomes, and stability of cross-licensing agreements under different market conditions[4].

The aim of this project is to offer a strategic solution, by studying several game-theoretic models, and focusing on one to model how different licensing strategies (such as fixed fees or per-unit royalties) affect competition, firm profitability, and market stability. It aims to provide insights into how firms can navigate the increasingly intricate world of intellectual property rights, and how policy makers might design regulations that promote innovation while preserving fair competition.

[4] Nagaoka & Nishimura, 2014

1.2. Motivation

As previously seen in state of the art, there is a growing complexity of managing intellectual property in technology-intensive industries, where the increase of patent intensity has led to overlapping patents, known as patent thickets, which obstruct and impede innovation and raise legal and development costs[5]. Patents are a key factor of the industry in incentivizing development and innovation, with the main goal of protecting intellectual property so as to allow for technological advances and progress.

This project is motivated by the aim of studying the mentioned cross-licensing agreements as a practical solution in reducing litigation risk and enabling cumulative innovation thanks to cooperation between firms[6], by allowing to share patented technologies. However, these agreements involve complex strategic decisions. Factors such as firm size, technology asymmetries, and market conditions influence whether cooperation is beneficial or sustainable. Game theoretic models provide a powerful framework to analyse these dynamics, helping to identify stable and efficient licensing outcomes[7] , and applying a specific model to a real-life case would help prove the efficiency of cross-licensing as a potential solution.

1.3. Project objectives

The main goal of this project is to explore the strategic dynamics of patent cross-licensing agreements through Game Theory, in order to understand their effects on firm behaviour, market competition, and innovation efficiency. The project aims to provide theoretical and practical insights into how these agreements are structured and under what conditions they lead to mutually beneficial outcomes.

Specifically, the project aims to carry out the following objectives:

1. Conduct a literature review which communicates the current context of intellectual property and the use of different models and their applications

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5 Galasso, 2007
6 Shapiro, 2000
7 Galasso, 2012

2. Compare licensing strategies and introduce the model with which the study of cross-licensing will be carried out
3. Establish the specific market in which the model will be applied as a case example to apply the model
4. Study the economic effects and implications drawn from the results obtained from the modelled case
5. Use the obtained results to establish the benefits and downsides posed by the use of cross-licensing

1.4. Alignment with the Sustainable Development Goals (SDGs)

This project also aligns with global development objectives such as the UN Sustainable Development Goals (SDGs), specially focusing on the following:

- **SDG 9 – Industry, Innovation and Infrastructure:** This project promotes innovation by exploring more efficient ways to manage and share intellectual property, supporting inclusive and sustainable industrial development through collaboration and knowledge sharing.
- **SDG 17 – Partnerships for the Goals:** This project reinforces the importance of cooperation between firms through strategic partnerships such as cross-licensing.

1.5. Project methodology

In order to carry out this project the first step will be a thorough literature review, looking into the concept of patent thickets and the industries mostly affected[8], as well as the different models that have already been used and developed to carry out cross-licensing. The different concepts relevant to the project will be extensively looked into in order to properly develop a theoretical framework on which to base the study. Drawing from reviewed literature and

[8] Lin, 2011

real-world licensing practices, one or more representative game-theoretic models will be selected or developed.

Out of these few selected models one will be chosen to base the study on, firstly doing the development of the theoretical model. Having done the development of the theoretical model it will then be used applied to a practical case: using it to predict firm behaviours in different conditions and adapting the model as deemed necessary, in real-life situation in a specific market with its main competitors and using public data obtained from these firms.

Through the results obtained the project will then study the economic viability and draw the final conclusions, finding the potential benefits of cross-licensing or lack thereof, drawing conclusions on whether it would be a realistic and advantageous solution to the problems found in Intellectual Property laws.

1.6. Resources employed

The development of this project relies on a combination of academic, computational, and institutional resources necessary for the construction, analysis, and presentation of the chosen model. Scientific literature and published journals, in particular peer-reviewed academic articles and working papers in the fields of industrial organization, intellectual property, and game theory are key to the study in question. The use of institutional and online resources is also extremely helpful, taking advantage of the wide range of databases the university library grants access to, such as JSTOR, ScienceDirect, SpringerLink, SSRN, and Wiley Online Library, enabling the retrieval of high-quality, up-to-date research papers. There will also be a need for usage of analytical tools and software in order to treat the information obtained and carry out the model.

2. Literature review

In this section research will be done into the current available information about the relevant and necessary topics on which this project will be developed: such as patents (patent thickets and trolls, cross-licensing of patents, patent pools, etc) and Game-Theory. To show the application of Game-Theory within the optimization of patent licensing, different models previously used with such a purpose will be explained.

2.1. Patents

Patents are a critical mechanism for protecting intellectual property rights, with the purpose of protecting inventors and firms and their exclusive right to exploit their inventions for a certain period. Patents incentivize and allow for innovation, making them crucial to any industry and its development, they play a vital role in promoting technological advancement and economic growth [9].

These rights provide the inventors and firms who hold them with a monopoly over their creations, prohibiting a third party from making, using, selling, or distributing the patented innovation without authorization, which is where licensing agreements come into play. Patents incentivize innovation by allowing inventors the chance to recover the costs of R&D while profiting from their innovation. In exchange, patents require complete access and disclosure of the invention, making it public knowledge and helping further advance future inventions, even if profit cannot be made from this patented invention. There are different types of patents, based on their use and functionality, and they are only enforceable within the jurisdiction where they have been granted, which points out the need for multinational firms to carry out global patent strategies. In order to secure a patent, there is an intricate process which entails: meeting the novelty, non-obviousness, and industrial applicability criteria, as well as passing an examination by a relevant patent office. The systems in place responsible for regulating patent filings and approvals play a very important role in protecting the interests of inventors, competitors, and society, making sure that the

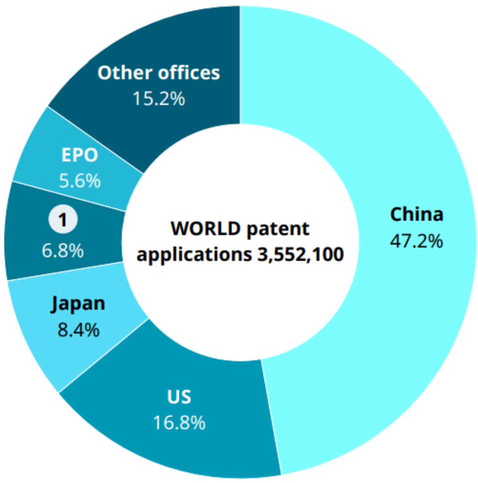
[9] Zhao et al., 2024

advancement of knowledge is not slowed down by overprotection or misuse of intellectual property rights.

2.1.1. Patent filing

Filing a patent is a formal process through which an inventor seeks legal protection for an invention considered to be new by certain standards. This process grants the inventor exclusive rights to exploit their invention for a limited period, typically 20 years, in exchange for publicly disclosing the technical details of the innovation.

To file a patent, the inventor submits an application to the intellectual property office in question depending on the kind of patent, such as the European Patent Office (EPO), the United States Patent and Trademark Office (USPTO), or the Spanish Patent and Trademark Office (OEPM). The application includes a detailed description of the invention and all the required information, and once filed, it undergoes a rigorous examination process to assess whether it meets the legal criteria for patentability.



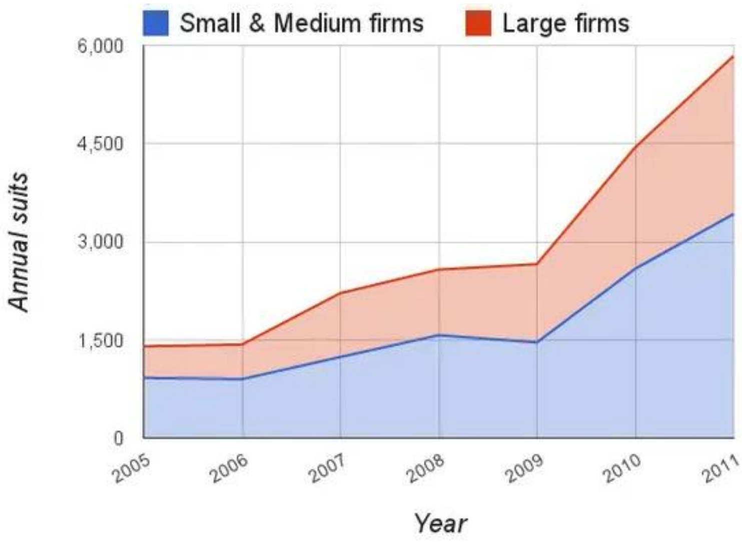
Graph 2: Share of patent applications by top five offices (2023)
[Source: WIPO, 2024]

In most jurisdictions, including Spain and the broader European Union, a patentable invention must meet three key requirements: novelty, inventive step, and industrial applicability. The novelty criteria states the invention must be new, having never been disclosed publicly prior to the filing date, being compared against the “state of the art” pertinent to the area of the invention. If even a single part of the invention has been publicly disclosed, it may no longer be considered novel. Inventive step, usually the more complicated criteria to assess, states that the invention must not be obvious to a professional in the corresponding field. Its aim is to ensure that the patent rewards a genuine advancement, rather than a trivial modification to existing knowledge. The examiners must assess whether the invention would have been an obvious solution which they themselves would have reached, the key distinction lying in whether they *would* have reached rather than *could* have reached. The last requirement, industrial applicability, states that the invention must have some use in any kind of industry, assuring that the invention has a practical utility and is not simply a theoretical or abstract idea.

2.1.2. Patent thickets

However, a problem arises because of the increasing complexity of modern technologies: patent thickets. Patent thickets refer to a dense web of overlapping patents held by several firms which make it incredibly hard for new innovations not to breach these licensing agreements and avoid infringement [10]. In this already complex environment created by patents, firms are often faced by additional challenges caused by "*patent trolls*": which are non-practicing entities (NPEs) that acquire patents strategically with the sole purpose of litigation and extracting royalties from other active firms, rather than for the protection of their own invention[11]. This is a very active, though unpleasant, practice that leads to exacerbating and already complex environment of patents and license negotiations [12].

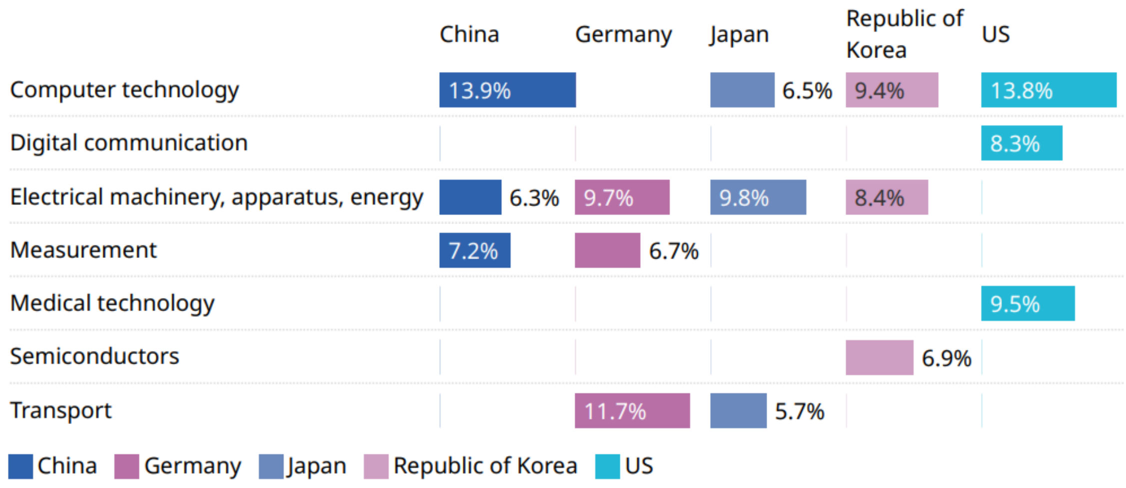
[10] Shapiro, 2001
[11] Spulber, 2016
[12] Shapiro, 2000



Graph 3: Annual defences against patent troll lawsuits
[Source: Business insider, 2013]

So as to address the several challenges posed by patents and license agreements, many firms have taken to cross-licensing agreements as a useful and strategic tool. Cross-licensing entails a good reciprocal licensing arrangement between at least two firms that grant each other the rights to use specific technologies to which they hold the patents. These agreements are beneficial to the firms involved, reducing litigation risks, promoting collaborative innovation and allowing firms to bypass restrictive patent barriers [13]. There are certain industries where cross-licensing has become a key element to ensuring the natural functioning of the industry and allowing technological progress [14], mainly the Information and Communications Technology (ICT) industry and the Biotech and Pharmaceutical industries, as they are industries holding an immense amount of patents which result in an inevitable patent thicket.

[13] Jeon & Lefouili, 2018
[14] Liu et al., 2024



Graph 4: Top technology fields where patents are filed for each of the top five origins [Source: WIPO, 2024]

2.1.3. Cross-licensing agreements

Patent thickets emerge in those sectors in which rapid technological advancement leads to constant innovation, and therefore a constant filing for new patents, such as those previously mentioned: ICT, Pharmaceuticals, Biotechnology, Semiconductors, etc [15]. In these industries the vast number of patents constantly required to develop just a single product often leads to legal challenges and disputes on top of the operational difficulties[16]. Cross-licensing agreements allow firms to navigate these thickets and avoid the drawn-out costs derived from patent litigation[17]. Although businesses do not always turn to cross-licensing and prefer to endure the downsides of endless litigation, as is a clear example Samsung vs. Apple who have been drawn into court constantly over the years on different patent infringement lawsuits without reaching neither unilateral licensing nor cross-licensing agreements. But a good example of a mutually beneficiary cross-licensing agreement reached in the same industry that led to a healthy exchange would be that between Apple and HTC in 2010 [18], where Apple allowed HTC the use of UI elements and functional software covered by their patents, and Apple was granted access to HTC's Wi-Fi patented technologies.

[15] Galasso, 2007
[16] Castaño Martinez, 2022
[17] Jeon & Lefouili, 2018
[18] Liu et al., 2024

Before entering cross-licensing agreements, the firms involved must assess the potential benefits and downsides. Since these agreements are found between firms in the same industry usually in competition with each other, each firm must find that the benefits granted by the agreement outweigh the potential boost given to the competition which could also result in losses to the firm. Therefore, cross-licensing agreements are reached when the quality improvements granted to each party involved are symmetrical to the different variables considered [19].

Firms are most likely to enter cross-licensing agreements when technological improvements are considerable and in markets with great growth potential, which allow firms to expand into new areas and benefit from their royalty agreements [20]. But there is also a certain concern that an unrestricted practice of cross-license agreements has the potential of leading to collusion and anticompetitive behaviour, replicating a monopolistic outcome in which the lack of market competition causes higher prices for consumers.

Another strategic approach to battling patent thickets are licensing pools, similar in many ways to cross-licensing. In a licensing pool multiple firms holding several patents agree to bundle their patents and license them as a collective amongst themselves or to third parties who might hold interest. The main advantage of licensing pools is the fact they simplify the licensing process immensely by reducing the transaction costs and avoid the "royalty stacking" problem: meaning when several royalties from multiple patent holders make the adoption of the technologies under these patents excessively expensive. While cross-licensing is based on a bilateral relationship between firms sharing their patented technologies, licensing pools allow for a broader multilateral participation, with a more standardized and inclusive environment since, unlike cross-license agreements, they aren't tailored to the specific needs of a limited number of firms [21]. But still, they share common objectives, the main being reducing litigation risks and avoiding patent infringement, while facilitating innovation through such collaboration. It must also be pointed out that patent pools are more regulated than cross-licensing agreements, they often involve

[19] Liu et al., 2024
[20] Hosseini et al., 2018
[21] Zhao & Wang, 2024

oversight from the regulatory entities responsible for the given industry to ensure that they meet with the antitrust laws and adequate licensing terms.

2.2. Game-Theory

Game-Theory is a branch of mathematics used to analyse strategic interactions between game participants (these can be individuals or entities), where each participant's outcome depends on their own decisions as well as the other participants' [22]. It provides the tools to model and predict hypothetical scenarios and behaviours in both competitive and cooperative environments. Its applications are endless, used very often in economics as well as in biology, politics, computer science, and many other fields. The concepts on which Game-Theory's foundation is built include players, strategies, payoffs, and equilibrium. The players are the entities capable of decision-making, the strategies state the possible courses of action that can be taken, payoffs would be the rewards or consequences associated with the outcome obtained, and the equilibrium states the stable position at which no party involved has interest in deviating from its course of action. Its applications extend to auction design, market competition analysis, negotiation tactics, resource allocation, etc, and is greatly useful in the context of patents and intellectual property.

Game-Theory helps analyse licensing agreements, litigation strategies, and the dynamics of cross-licensing agreements as well as patent pools in order to navigate patent thickets. By considering the relevant variables such as legal costs, innovation incentives, market dynamics, game-theoretic models provide optimal solutions during decision-making processes, making sure that the outcomes in complex environments with several stakeholders are fair and efficient.

2.2.1. Lihui Lin

The study carried out by Lihui Lin[23] focuses on developing optimal licensing strategies in the presence of patent thickets, exploring the challenges and decisions firms face when operating in such environments. Lin's study develops different models with the aim of evaluating the performance and implications of

[22] Galasso & Schankerman, 2010
[23] Lin, 2011

different licensing schemes. Lin structures licensing schemes divided by the payment methods, such as fixed-fee, per-unit royalty, and hybrid (fixed-fee and per-unit royalty simultaneously), as well as studying the models depending on whether there is a single patent holder or multiple patent holders who are licensing the patents to the producer. During the article, the author considers the patent holders and the producers to whom they are licensing the patents to be two different parties. The model developed in this article provides a framework for understanding how licensing contracts can be structured to minimize inefficiencies, especially in the absence of cooperative agreements like patent pools or cross-licensing.

2.2.1.1. Model structure and functionality

The model is set in the context of a vertical market structure involving the downstream firms producing the products requiring the patents, and the upstream patent holders. The downstream firm must obtain licenses from all patent holders before it can sell its product, and each upstream firm independently sets its own licensing strategy without coordinating with others.

The model's main stage is the licensing stage, in which the upstream firms decide on the form of licensing contract to offer. As previously mentioned, these may take the following forms: fixed-fee contracts, where the downstream firm pays a one-time lump sum in order to obtain the patent, per-unit royalty contracts, where the payments depend on the quantity of units sold, and hybrid contracts (two-part tariff), which combine both a fixed fee and per-unit royalty. Before carrying out the model in the different licensing contract possibilities, certain key parameters and assumptions are set in place in order to complete the model's framework. All patents are assumed to be complementary and therefor the downstream producer requires all of them in order to operate. Unlike in other models, there is no cooperation or coordination between the licensors, meaning each patent holder decides on the which licensing agreement to employ independently of the other licensors decisions.

The second stage would be the production stage, in which after licensing terms have been established, the downstream firm chooses its output level based on the combined cost structure from all licensing agreements. The downstream firm is monopolistic and profit-maximizing, in order to simplify the model by removing price competition from the analysis and focusing attention on how licensing fees affect production and pricing decisions.

2.2.1.2. Findings and conclusions

The study conducted by Lin draws several conclusions on the effectiveness and efficiency of different licensing strategies. One of the key conclusions is that fixed-fee licensing contracts, where the producer pays a single payment regardless of amount of product produced, are generally more efficient from both the beneficial perspective of the producer, and the social welfare perspective in terms of the access of affordable products provided to consumers as well as the effects on innovation. Fixed-fee licensing contracts avoid the doubts that come up in production decisions when firms pay per-unit royalties, allowing the producing firm to operate at the profit-maximizing output level without additional marginal costs from licensing. However, their usefulness is limited in cases in which there is high demand uncertainty, where both the licensors and the licensees may be reluctant to accept a fixed price due to the unpredictable nature of market performance.

The per-unit royalty contracts, in the other hand, bring about several inefficiencies. The effect they have on the marginal cost of production leads to a reduced output, increased prices for the consumers, and a general loss in welfare. The negative effects are exacerbated in the case of having multiple licensors, since each firm imposes their own royalty and leads to a phenomenon known as *royalty stacking*, whose resulting cumulative fees may lead to unprofitable production and as a result reduce innovation incentives.

To find a balance between the two extremes, Lin evaluates the performance of hybrid contracts, combining fixed fee with a per-unit royalty. These hybrid contracts lead to an equilibrium between efficiency and risk sharing, allowing the patent holding firms to benefit from potential high-demand of the product while also reducing the negative effects of output-based pricing. As a result, the study finds two-part tariffs to be preferable in most real-life scenarios, especially those in which market demand is uncertain or where licensors and licensees have unequal bargaining power.

The paper ultimately concludes that in industries suffering from high fragmentation of intellectual property rights, there is the need for some form of coordination, like cross-licensing agreements, patent pools, or collective licensing mechanisms, so as to avoid market failure. In the absence of such collective licensing mechanisms, relying on individually negotiated per-unit royalty contracts could stagnate innovation due to excessive production costs. Lin's model highlights the importance of strategically designed licensing schemes and points out the need for further exploration of cooperative mechanisms to enhance efficiency in heavily patented environments.

The value of Lin's model lies in his way of understanding licensing inefficiencies in industries with fragmented patent ownership, where access to multiple patents is essential for production. The model provides clear insights into the effects of different licensing schemes, emphasizing how fixed-fee or hybrid licensing can reduce distortions in production incentives. In contrast to models that assume cooperative agreements (like Liu et al., 2024 or Galasso, 2007), Lin's approach is particularly useful when firms operate independently and collective mechanisms like patent pools are not available.

2.2.2. Liu, Li, Feng & Feng

The study carried out by Liu, Li, Feng & Feng focuses on investigating cross-licensing agreements in the ICT sector (Information and Communications Technology) using a model to find the conditions benefiting the competing firms. The purpose of the model developed in the research article[24] is to evaluate the effects that cross-licensing agreements may have on certain factors of the firms involved in these agreements: profitability, value of the firm perceived by customers, and market efficiency. The study is based in the context of a duopoly between two competing firms that are characterized by substitutable products that have different quality. Taking into account market characteristics such as customer sensitivity to both price and quality, the model aims to find the effects on cross-licensing agreements and under what circumstances do these benefit the companies and the customers.

[24] Liu et al., 2024

2.2.2.1. Model structure and functionality

The market in which the model is based is setup as follows: the model assumes two competing firms which each produce ICT products with the same purpose (directly competitive), each differing in their quality due to different technological capacities of each firm. The firms are engaged in price competition and the two factors determining demand are product quality and price.

There are several key parameters in the model, used in functions that determine demand in different conditions and its potential, profits achieved by the firms, etc. The first parameter is the measure of customer sensitivity to the quality of the products (noted as $\boldsymbol{\alpha}$). As previously mentioned, the measure of customer sensitivity to the price of the products is also a parameter (noted as $\boldsymbol{\beta}$). The overall market demand potential shared between the firms is also one of the main factors involved (noted as $\boldsymbol{D}$), and if the firms were to sign cross-licensing agreements there would be improvements achieved by each respective firm A and B that would be **ΔA** and **ΔB**. The different notations used in the model and their meaning are the following:

Notation	Description
D	Common market potential shared by the two firms
α	Customer sensitivity to the quality difference between two products
β	Customer sensitivity to the price difference between two products
q_i	Product quality of Firm i ($i \in \{A, B\}$) without cross-licensing
p_i, $p_{cl,i}$	Product price set by Firm i without and with cross-licensing, respectively
n_i, $n_{cl,i}$	Market demand of Firm i without and with cross-licensing, respectively
π_i, $\pi_{cl,i}$	Profit of Firm i without and with cross-licensing, respectively
D_i, $D_{cl,i}$	Intrinsic demand potential of Firm i without and with cross-licensing, respectively
Δ_i	Quality improvement of Firm i through cross-licensing
$\frac{q_i}{p_i}$, $\frac{q_i+\Delta_i}{p_{cl,i}}$	Quality-to-price ratio of Firm i without and with cross-licensing, respectively

Table 1: Key notations of the model
[Source: Liu, Li, Feng & Feng, 2024]

There are two cases considered in the model: the first case implies no cross-licensing agreements, meaning that each firm relies solely on their own proprietary technologies, and the second case implies cross-licensing, where the firms share each of their patented technologies allowing for the quality of their products to increase. The decision to either engage in cross-licensing or not would be the first stage of the model. In the next stage the firms set the prices of their product which is determined by the quality levels observed. And in the last stage, the customers make their purchasing decisions depending on both price and quality. The model also includes the possibility of side payments, where one firm compensates the other for unequal benefits from cross-licensing, helping balance the uneven advantages which encourages agreements in scenarios which otherwise wouldn't engage in cross-licensing.

The outcome of this three-stage game is determined by efficiency metrics depending on the comparative results between engaging or not in cross-licensing: a bilateral efficiency in which both firms benefit from cross-licensing by obtaining an increase in profits, unilateral efficiency in which there is only one firm that benefits while the other is negatively affected, and the customer perceived value which is obtained by the quality-to-price ratio (quality improvement divided by price increase).

2.2.2.2. Findings and conclusions

The study determines the efficiency metrics obtained depending on whether the market is price-sensitive or quality-sensitive. In price-sensitive markets (those with high β and low α) cross-licensing is most likely to be beneficial to both the firms since the customers are more sensitive to price differences than to quality differences, and therefore the firms are able to improve the quality of their product without significantly compromising their competitiveness. While in quality-sensitive markets (those with high α and low β) in which the customers prioritize the quality of the product, cross-licensing is beneficial only when both firms have a similar improvement in the quality of their product, since otherwise it creates a competitive imbalance which leads to unilateral efficiency. The study finds that when cross-licensing leads to an asymmetric quality improvement between the firms, meaning that one firm's quality improvement is significantly larger than that of the other firm, cross-licensing favours the firm which has improved most and disadvantages the competitor: even though both have improved the quality of their products, the unevenness leads to one firm being negatively affected despite an increase in the quality of its product.

Another of the factors considered is the costs that firms may incur when engaging in cross-licensing and therefore implementing the shared technologies. There are both R&D and integration costs that come from implementing the shared technologies, which the study determines could negate the benefits brought by cross-licensing if these costs are disproportionately high.

One of the conclusions reached by the authors during the study is that even though cross-licensing tends to lead to higher quality products, it often does so at the expense of also increasing product prices. They enhance the quality-to-price ratio by finding that while the companies may profit, the customers only benefit when the quality improvements outweigh the price increases. The ideal outcome considered in the article is the “win-win-win” scenario, in which not only both firms but also the customers are benefited. The model identifies that the conditions for this ideal outcome are that the competition intensity must be moderate and the quality improvements between the firms must be symmetrical.

The authors apply the model to the ICT industry, to which it is particularly relevant since products like phones or other electronic devices require patents from multiple firms because of the complexity of their development including endless components. The model offers a manner of analysing cross-licensing decisions in the ICT industry but is limited by the fact that it is based in a duopoly and cannot include several firms simultaneously. The article highlights the importance of market characteristics, the relative improvement of quality and the variation of costs and prices, in determining the outcomes of cross-licensing. The model identifies the conditions required for both the firms involved to benefit and provides useful insights which could be used by firms belonging to the industry and by policymakers who aim to foster innovation and fair competition.

2.2.3. Galasso

Galasso's study[25], investigates cross-licensing's correlation with patent litigation in the semiconductor industry. The study provides both a theoretical model and also the empirical evidence to explain the conditions under which the firms decide whether to engage in cross-licensing or litigation. As previously stated, cross-licensing allow firms to avoid litigation due to patent disputes with other firms or patent holders, helping them obtain the capacity to operate freely in an industry filled with intellectual property barriers, which we have referred to as patent thickets, and differently to the model developed by *Liu et al.,* the decision made by firms is not only to engage in cross-licensing or not but also takes into account the litigation implied by not entering cross-licensing agreements. The goal of the model developed by Galasso in this article is to find the trade-offs between litigation and cross-licensing, taking into account the influence of firms' sunk costs (their technological investments) in their decisions to litigate or enter a cross-licensing agreement, as well as other determining factors such. The article looks into a concept known as "persuasive litigation", where firms us litigation as a tool with the purpose of obtaining better cross-licensing terms and not to enforce their intellectual property rights, and studies the economic and strategic inefficiencies that it creates. Using the findings obtained during the study, Galasso goes on to investigate the broader implications of cross-licensing and litigation on innovation and policy.

2.2.3.1. Model structure and functionality

Galasso's model focusses on the semiconductor industry, which is ideal for studying cross-licensing agreements due to its high patent density as a rapidly progressing technological industry with constant innovation. The semiconductor industry is fundamentally linked with the ICT industry which *Liu et al.*'s article is based on since semiconductors are core components in the ICT industry, and like products in the ICT industry semiconductor products can be covered by hundreds or even thousands of patents, leading to frequent litigation disputes and patent infringement claims which can result in production shutdowns.

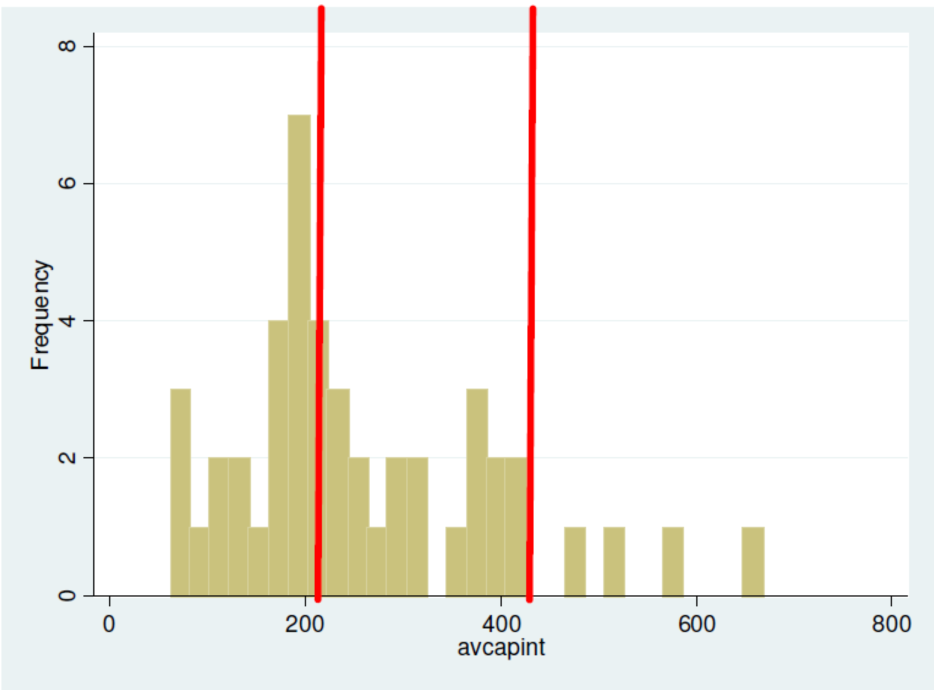
[25] Galasso, 2007

The model starts out from the premise of certain key assumptions, which are crucial in shaping the theoretical framework from which the model sets out from. Galasso assumes that firms with higher sunk costs (those with more expensive equipment) are more affected by production being halted because of litigation, caused by patent disputes that result in a significant financial and operational cost on the firm. The firms therefore have to consider the benefits and downsides of either maintaining monopoly profits and risking litigation or sharing duopoly profits in a cross-licensing agreement. Galasso references a model used by another author (Yildiz, 2003), pointing out its shortcomings and then developing an improved model. The model uses a bargaining game in which several firms make the decision of cross-licensing or litigating, those firms with higher sunk costs prefer avoiding litigation and are more prone to cross-licensing while firms with lower sunk costs may be more likely to go into litigation, and there are also firms with intermediate sunk costs which may engage in the mentioned "persuasive litigation" to delay cross-licensing and try obtaining better terms.

Firms in Galasso's game can reach certain positions which each have a different efficiency metric. Firms may reach perpetual litigation, in which a cross-licensing agreement is never reached, which is only efficient when litigation yields higher payoffs than cross-licensing by taking into account the different profits and costs incurred by each possibility. Persuasive litigation in the other hand is simply inefficient, as it reduces the joint benefit obtained by the firms involved by simply delaying the agreements. Immediate cross-licensing is the optimal decision when both firms are able to recognize early on the benefits of collaboration. Galasso makes predictions based on these efficiency metrics which he then goes on to prove; those firms with low sunk costs will engage in perpetual litigation, firms with high sunk costs in the other hand will prefer immediate cross-licensing to avoid litigation, and those with intermediate sunk costs will delay agreements and engage in persuasive litigation.

2.2.3.2. Findings and conclusions

In order to carry out the game, Galasso uses a dataset from the U.S. semiconductor industry to test his model's predictions. The model finds that, as Galasso had predicted, firms with higher sunk costs are in fact more likely to cross-license immediately in order to avoid litigation, and those with intermediate sunk costs engage in litigation before reaching cross-licensing agreements, which is consistent with Galasso's theory of persuasive litigation.



Graph 5: Capital Intensity distribution and Litigation
[Source: Galasso, 2007]

As shown in the graph extracted from Galasso's study, where capital intensity can be interpreted as a proxy for a firm's sunk costs, firms with higher capital intensity are unlikely to enter in litigation while those with low capital intensity are far more likely, and those finding themselves in the intermediate region also engage in litigation which reflects the idea of persuasive litigation.

Galasso's study highlights the inefficiencies of the current patent system and how it facilitates and even incentivizes persuasive litigation, and he suggests that if there were patent litigation cost reforms it could help reduce the amount of disputes causing this inefficiency and promote innovation, which is the main objective of patents even though it may often be forgotten.

By identifying the relevance of sunk costs, bargaining dynamics and technological overlap, Galasso's model offers a framework that firms finding themselves in patent thickets can use to assess their optimal decisions on cross-licensing and litigation based on their capital structure a market position. It is not only firms who can benefit from the insights provided by Galasso's study but also policymakers, as Galasso and his findings point out the importance of incentivizing innovation while keeping a balance with the costs of Intellectual Property enforcement, and the need for a more effective patent system.

Galasso's model is especially useful in industries involving high R&D investment and fragmented patent ownership, where there is a significant strategic consideration to bare in mind when deciding to litigate or cross-license. That makes it best applied in contexts like the semiconductor or biotechnology sectors, where there is a high risk of infringement disputes due to patent thickets and the firms involved have varying capital intensities. Unlike more abstract models, Galasso provides empirical validation for firms to assess whether their position in terms of sunk costs justifies aggressive litigation or cross-licensing, making the model especially insightful for firms that fall into the intermediate range of sunk costs, as it identifies when and why they might resort to "persuasive litigation" which can distort market efficiency[26] as further demonstrated in the applied analysis by *(Moreno Jurado, 2024)*.

2.2.4. Jeon & Lefouili

Jeon & Lefouili carry out a study in which they investigate the competitive effects of cross-licensing, as the previously analysed models do, in an oligopolistic market. The game-theoretical model developed[27] is used to find the agreements' influence in the market outcomes, by taking into account not only prices but also product quantities, focusing on Cournot competition. In the context of a Cournot competition, meaning firms determine the output of production quantities, which in the study is used to establish competition between different firms, each holding one patent covering a technology which can serve to reduce costs, and they can access the competitors' patents through cross-licensing. Taking into account royalties and the terms of the agreements, Jeon & Lefouili explore whether cross-licensing could allow firms to reach outcomes similar to a monopolistic situation in competitive industries even with there being several equivalent firms.

[26] Moreno Jurado, 2024
[27] Jeon & Lefouili, 2018

The study's model is developed with the aim of analysing the balance between the competitive and cooperative dynamics through cross-licensing, emphasizing on the agreements' impact on marginal costs and output quantity decisions. Like most articles investigating cross-licensing and its potential benefits, Jeon & Lefouili also examine policy implications, and in their study the particularly focus on the role of antitrust regulations in overseeing the agreements.

2.2.4.1. Model structure and functionality

The model sets from the basis of an oligopoly market with N symmetric Cournot firms, each holding an individual patent which has the capacity of reducing production costs when used. The firms involved can access additional patents through cross-licensing, which specify the royalties in the agreements and in some cases can also include a larger fixed payment. The game-theory model is made out of two main stages, firstly having to negotiate the cross-licensing agreements to access the different patented technologies of other firms and agreeing on the licensing terms and royalties, and then entering the Cournot competition, during which the firms compete in product quantity considering the cost reductions achieved through cross-licensing. The firms are assumed to have marginal costs, which decrease when gaining access to other patented technologies, and the agreements reached are private to the parties involved, meaning that the terms reached in each agreement are not known by the other firms in the market. The model develops a demand function that ensures the possibility of a unique Cournot equilibrium in which each firm has a determined outcome based on their marginal costs and their competitors' quantities.

In the study's game bilateral efficiency is reached when the agreements maximize the joint profit obtained by the firms entering the agreement, while holding the other agreements in the market of the game setting constant. There is also the possibility of collusion occurring between two firms, by them restricting their joint output in order to increase their profits, referred to as the coordination effect. The firms also have the capacity to reduce internal royalties, helping them lower costs and therefore increasing competitiveness by royalty-saving.

2.2.4.2. Findings and conclusions

As they had set out to do, Jeon & Lefouili find through their generalizable theoretical analysis that bilateral cross-licensing can replicate outcomes similar to those in monopolistic markets, where the firms charge royalty prices similar to monopoly pricing. As long as these royalties are mutually efficient, a monopoly-like outcome is possible even in markets with several Cournot oligopolists. The model describes two opposing forces which cancel out when reaching specific royalty levels: the coordination effect and the royalty-saving effect, and the theoretical analysis extends to scenarios of multilateral cross-licensing agreements, showing that even when all the firms involved participate in symmetric agreements a monopolistic outcome can still be sustained.

Unlike other articles, it focuses on a generalized theoretical analysis through the developed model without applying it to a specific industry in an empirical manner. The authors show that while agreements may improve efficiency by reducing costs, they can also lead to collusion and non-competitive behaviour. The inclusion of Cournot competition emphasizes how the output decisions made by each firm are clearly influenced by cross-licensing. It would be interesting to directly test the model in a particular industry which is relevant to the findings provided by the article, such as markets where patent thickets are common and marginal cost reductions are considerable, meaning that access to a patented technology could potentially decrease a firm's production costs significantly.

Jeon & Lefouili's model is useful for understanding how cross-licensing can lead to coordinated outcomes in markets with several firms, even under competition, specifically in those that compete through output quantities rather than prices. It shows that firms can achieve monopoly-like results through royalty setting, revealing how licensing affects both cooperation and output decisions. This insight is especially relevant in markets where marginal cost reductions from patent access strongly shape firm behaviour, offering a clear warning about the potential for hidden collusion through bilateral licensing, a risk mentioned in most articles a further demonstrated in this case.

2.3. Summary

The literature on the use of game-theory models for cross-licensing agreements, although not too extensive, is made out of several studies developing different models and offers a comprehensive perspective on cross-licensing and the potential benefits it offers in different industries[28]. The three different studies each use different models which set out from different assumptions and taking into account different parameters depending on the context of the industry the model is to be applied to and on the aim of the study.

Cross-licensing agreements serve as a strategic tool to enhance the competitive positions of the firms involved, especially in industries with dense patent thickets such as ICT and semiconductors, which as previously reviewed in in different studies tend to be some of the most commonly studied industries when it comes to cross-licensing. Liu et al. (2024) highlights that these agreements help foster technological advancement, and avoid knowledge blockades caused by patent thickets, by facilitating access to essential intellectual property. Firms enter cross-licensing agreements with the aim of optimizing profitability when the market conditions favour these agreements and the quality improvements obtained by the firms engaging in the agreements are symmetrical. The studies developed focus on different kinds of hypothetical markets, with models that can be applied differently, while Liu et al. (2024) focus on a duopolistic market setting, analysing the effects of cross-licensing on profitability and consumer value, Jeon & Lefouili (2018) expand the scope of the market setting to an oligopolistic context. Their findings point out the potential of cross-licensing replicating monopolistic outcomes even in competitive markets, demonstrating how these agreements result in lower litigation costs and better resource allocation, but also compromising market competitiveness.

Litigation is one of the main components taken into account in most models and considered when analysing the impacts of cross-licensing, as seen by Galasso (2007) which goes on to further study litigation as a delay tactic and its relation to firms' sunk costs. The outcomes of the different studies tend to by evaluated by

[28] Siegrist Echeverría, 2023

different efficiency metrics which depend on the possible scenarios, each study uses its own efficiency metrics depending on the parameters used and the objectives of the study. And while each study focuses on different efficiency metrics, Liu et al. (2024) introduce the concept of a "win-win-win" scenario, where cross-licensing benefits both firms involved as well as the consumers. Immediate cross-licensing, when mutually beneficial, leads to optimal economic outcomes. However, Jeon & Lefouili (2018) caution against potential anticompetitive effects, stating that cross-licensing can facilitate collusion between firms, therefor undermining market efficiency.

The collective insights from these studies stress the need for newer policy frameworks that balance the promotion of innovation while also preventing anticompetitive practices, incentivizing cooperative licensing mechanisms such as patent pools or structured negotiation platforms as stated by Lin (2011). Galasso (2007) suggests patent litigation reforms to reduce strategic litigation inefficiencies. Additionally, Jeon & Lefouili (2018) emphasizes that the authorities in question should scrutinize cross-licensing agreements to mitigate the risk of market monopolization. The literature makes it very clear that while cross-licensing agreements can significantly enhance innovation and reduce litigation costs, they also pose risks related to market competition and efficiency. There is a need for future research to explore and refine cross-licensing's cross-licensing multifaceted impacts as well as its optimal implementation.

3. Defining the “Liu et al.” model

In recent years, game-theoretic models have become a useful area of interest in analysing the dynamics of patent cross-licensing, particularly within innovation-intensive sectors burdened by patent thickets. The literature is not as extensive as it should be but is increasing exponentially over the years. Various models, as seen in the literature (Chapter 2), offer insight into how firms respond strategically to overlapping intellectual property rights. Although these provide valuable and useful information across different contexts and licensing architectures, they often overlook the quality differentiation and price sensitivity that are central to consumer behaviour. This is where the model developed by Liu, Li, Feng, and Feng (2024) stands out. Their work offers a perspective incorporating both quality and price competition, capturing the trade-offs firms face in realistic competitive settings, comparing the results in a market absent of cross-licensing and one with, to also analyse the potential quality improvements accomplished by the firms involved due to cross-licensing.

3.1. Foundations for the model design

This section lays out the theoretical foundations on which the model developed by Liu et al. (2024)[29] is built. It explains the economic context, market characteristics, and strategic considerations that justify the model’s structure, including the firms’ process of decision-making when engaged in cross-licensing within a duopolistic setting. By detailing the key variables, game stages, and efficiency metrics used, this section provides a clear basis for understanding how the model functions and why it is suited for analysing patent-related decisions in technology-intensive industries.

The model is built on a set of key economic and strategic assumptions with the aim of reflecting the dynamics of cross-licensing in the Information and Communications Technology (ICT) sector, particularly within a duopolistic market structure. The foundations of the model are based on the need to understand the cost-benefit considerations that competing firms face when deciding whether to enter into cross-licensing agreements. Unlike other models

[29] Liu et al., 2024

previously reviewed, such as Galasso's litigation vs. licensing approach[30] or Jeon & Lefouili's oligopolistic Cournot framework[31], Liu et al.[32] focus specifically on bilateral interactions between two asymmetric firms competing on both price and quality. The model's objective is not just to identify optimal licensing behaviour but also to capture how customer preferences and technological differences influence all the decisions involved at a firm level.

The model assumes a market with two firms producing substitutable products of different quality levels. These differences in quality levels are due to the previously mentioned asymmetries in technological capability, and hold great importance in understanding how licensing affects competitive positioning. The firms operate in a price competition environment, also known as Bertrand competition, and consumer demand depends on both quality and price sensitivity. Accordingly, two crucial parameters are introduced: **α**, representing the degree to which consumers are sensitive to quality differences, and **β**, indicating sensitivity to price. Additionally, the model incorporates **D**, the total market demand potential, and **ΔA** and **ΔB**, representing the quality improvements each firm could potentially achieve if engaged in a cross-licensing agreement. In the following table the different key notations used by Liu et al. are shown:

[30] Galasso, 2007
[31] Jeon & Lefouili, 2018
[32] Liu et al., 2024

Notation	Description
D	Common market potential shared by the two firms
α	Customer sensitivity to the quality difference between two products
β	Customer sensitivity to the price difference between two products
q_i	Product quality of Firm i ($i \in \{A, B\}$) without cross-licensing
p_i, $p_{cl,i}$	Product price set by Firm i without and with cross-licensing, respectively
n_i, $n_{cl,i}$	Market demand of Firm i without and with cross-licensing, respectively
π_i, $\pi_{cl,i}$	Profit of Firm i without and with cross-licensing, respectively
D_i, $D_{cl,i}$	Intrinsic demand potential of Firm i without and with cross-licensing, respectively
Δ_i	Quality improvement of Firm i through cross-licensing
$\frac{q_i}{p_i}$, $\frac{q_i+\Delta_i}{p_{cl,i}}$	Quality-to-price ratio of Firm i without and with cross-licensing, respectively

Table 2: Key notations
[Source: Liu et al., 2024]

The model consists of a three-stage game. In the first stage, the firms decide whether to cross-license. In the second, based on the product quality levels altered and determined on the first stage, each firm sets its product price. Finally, consumers choose which product to purchase based on their preferences. The model accounts for side payments (transfer payments between firms) to address the likely imbalance in gains from licensing, in order to account for scenarios where one firm may benefit significantly more than the other from the agreement. The results are evaluated through efficiency metrics: bilateral efficiency (both firms benefit), unilateral efficiency (only one firm benefits), and customer-perceived value, which is based on quality to price ratios.

This model is particularly relevant in environments where patent thickets create barriers to innovation and where firms must navigate complex decisions around cooperation and competition, offering a tool to assess when cross-licensing enhances market efficiency and firm profitability.

3.2. Proposed model and its development

This section introduces Liu et al.'s theoretical three-stage model, one that is based on the strategic interaction between two ICT firms deciding if they are going to entre a cross-licensing agreement or not. Detailing the workings and all the necessary information of the model, this section aims to set a theoretical basis for the posterior application of the model in a practical matter. While the study conducted by Liu et al.[33] skips some derivations or concepts as considered to be assumed knowledge and information, or describing aspects of the model verbally without using the corresponding equations at times, this section also has the objective of laying out the model and its process in an understandable manner.

The section is structured as follows. Firstly, all the hypotheses drawn from Liu et al., although they are not all explicitly mentioned, will be structured and explained, also detailing their relation and relevance to the ICT sector in which the model is originally employed. Next, the game itself is to be studied in reverse, starting with the last stage (Consumer demand behaviour), moving on to the second stage (Firm-level price setting), and finally going into the first stage (Licensing decision).

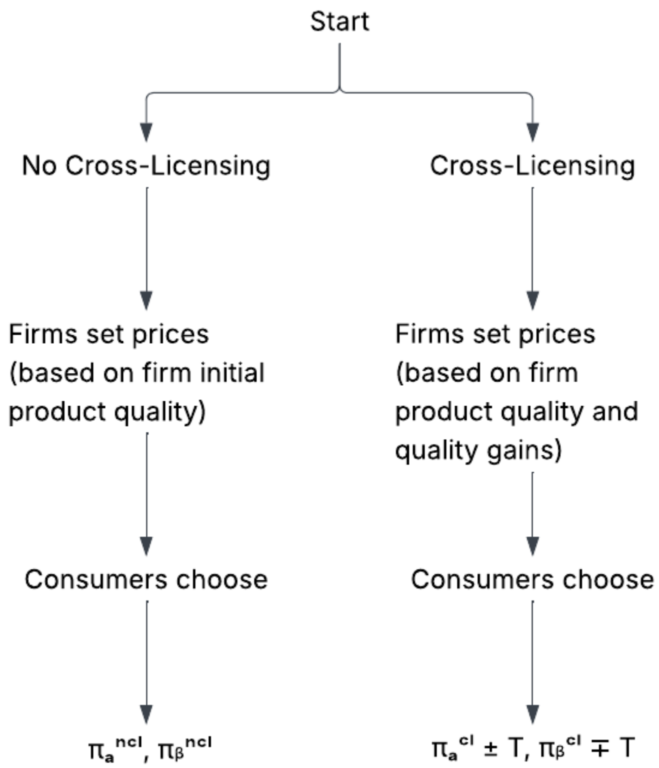


Figure 1: Game Tree of the cross-licensing decision process
[Source: Own elaboration, 2025]

[33] Liu et al., 2024

The reason for following the sequence of the game in inverse order is that in a game where decisions are made over time, it is usual for firms to anticipate the consequences of their actions by considering how future stages will play out in order to make the best choice at the beginning. At each stage, the relevant and necessary mathematical expressions will be obtained in order to compute the equilibrium outcomes on which the model is based.

3.2.1. Model assumptions

Before going into the game's structure, it is key to understand the main assumptions on which the model hinges, defining the scope and logic. The model assumes a duopolistic environment, where the products produced by each firm differ in quality and engage in price competition. Below, the different assumptions extracted from the Liu et al.[34] model are listed as the key hypotheses:

HYPOTHESES

- **H-1. Market structure:** The industry is a *duopoly* (Firms A and B) where both firms produce horizontally differentiated products that are imperfect substitutes for one another.
- **H-2. Initial quality asymmetry:** Before engaging in any quality agreement, basic quality levels satisfy:

  $$q_A^0 > q_B^0 > 0 \qquad [\boldsymbol{Hip.2}]$$

  Reflecting that there is a starting quality gap between the two firms.

  H-3. Single production cost: Marginal production cost is constant and identical for both firms, while also excluding the potential sunk R&D costs:

  $$c_A = c_B = c > 0 \qquad [\boldsymbol{Hip.3}]$$

- **H-4. Consumer utility:** Consumers make purchasing decisions based on price and quality, by comparing the net *utility* of each firm's product, which is represented as follows:

  $$U_i = \alpha q_i - p_i \quad where \quad \alpha > 0 \qquad [\boldsymbol{Hip.4}]$$

[34] Liu et al., 2024

Where utility increases with quality, which is weighted by the consumer's sensitivity to quality (α), and decreases with the price.

- **H-5. Market demand rule:** Total market demand D is distributed between the two competing firms based on their products net *utility,* where the demand share for firm i is represented as follows:

$$D_i = D \cdot \frac{\alpha q_i - p_i}{(\alpha q_A - p_A) + (\alpha q_B - p_B)}$$

$$D_i = D \cdot \frac{U_i}{U_A + U_B} \qquad [\boldsymbol{Hip.5}]$$

- **H-6. Timing of moves:** The sequential structure of the game follows:
  1. Licencing stage → Firms simultaneously decide to cross-license (CL) or stay independent (NCL).
  2. Bertrand stage → Observing qualities, the firms set prices.
  3. Consumer stage → Consumers choose.
- **H-7. Quality gains from cross-licensing:** Once a cross-licensing agreement is reached between the firms, the effective qualities become:

$$q_A = q_A^0 + \Delta_A \;\; and \;\; q_B = q_B^0 + \Delta_B \quad with \;\; \Delta_A, \Delta_B \geq 0 \qquad [\boldsymbol{Hip.7}]$$

Showing the benefits obtained by both firms in terms of product quality, quality improvements that each firm gains from accessing the other's patents, these improvements are assumed to depend on the relative size of the rival firm's patent portfolio. Specifically the gain for firm*s* is:

$$\Delta_A = \delta \cdot \frac{Pt_B}{Pt_A + Pt_B} \; and \; \Delta_B = \delta \cdot \frac{Pt_A}{Pt_A + Pt_B} \qquad [\boldsymbol{Hip.7.2}]$$

Where δ is the maximum achievable quality gain through full licensing, which will be calibrated based on tech performance data and industry benchmarks.

- **H-8. Transfer payment:** In the case of one firm obtaining more quality gains than the other from cross-licensing ($\Delta_A \neq \Delta_B$), there is the possibility of compensation with a transfer payment T from firm benefiting most to the other to keep the agreement mutually beneficial.

$$T > 0 \ if \ \Delta_A < \Delta_B \ \rightarrow \ Firm\ A\ receives\ payment$$

$$T < 0 \ if \ \Delta_A > \Delta_B \rightarrow \ Firm\ B\ receives\ payment \qquad [\boldsymbol{Hip.8}]$$

- **H-9. Complete information and rationality:** all parameters of the model, including cost c, quality gains Δ_i, and demand sensitivity α, are common knowledge to both firms. Firms are fully rational with the sole intention to maximize profit.

3.2.2. Model structure

As mentioned previously, the model will be solved using backward induction, starting from the final decisions (consumer choice), moving backwards from there.

STAGE 3: Consumer choice

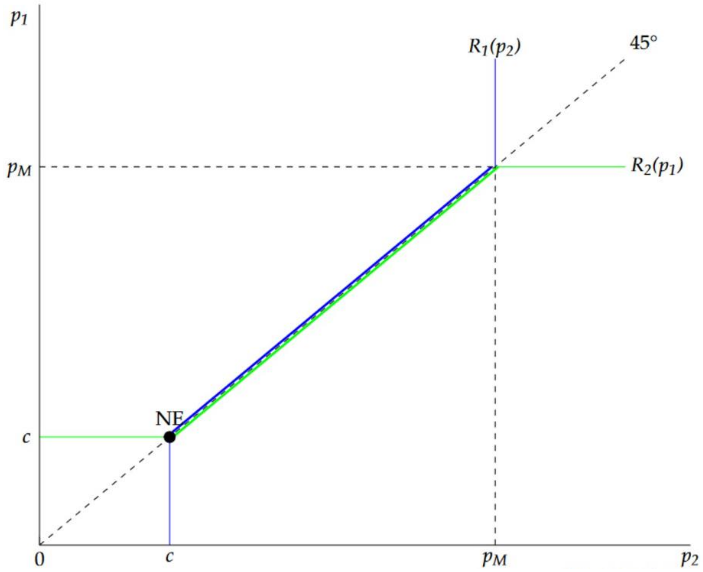
The last stage of the game is where consumers choose between the two different products provided by the two different firms based on the net utility of each. As shown in the hypotheses, Liu et al. use the utility function of the form $U_i = \alpha q_i - p_i$, ensuring the consumers weigh product quality improvements against the corresponding price increases. Taking consumer preferences into account, the market demand is divided between the two firms accordingly. The market demand is split according to:

$$D_i = D \cdot \frac{\alpha q_i - p_i}{(\alpha q_A - p_A) + (\alpha q_B - p_B)}$$

As stated in ***[Hip. 5]***, ensuring that each firm holds its share of the market based on the value they offer.

STAGE 2: Price competition

In the second stage, each firm takes the quality levels into account, including the improvements obtained if engaged in cross-licensing, and chooses the price that would maximize profit. Although Liu et al. do not explicitly define what kind of pricing competition the firms engage in, it can be assumed that the model follows a Bertrand Game pricing competition. Since the firms choose prices rather than quantities, demand depends on relative prices and quality, and each firm maximizes profit taking the rival's price into consideration, the features are consistent with a Bertrand competition framework with product differentiation.



Graph 6: The Bertrand Model and Equilibrium
[Source: INOMICS, 2021]

The presence of quality differences ensures that this Bertrand competition does not drive prices down to marginal cost. Instead, equilibrium prices depend on both the firm's own quality and its competitor's quality. Given the qualities q_A, q_B, each firm chooses price p_i to maximize profit:

$$\pi_i = (p_i - c) \cdot D_i \qquad [\boldsymbol{Eq.1}]$$

Finding the equilibrium price by maximizing the profit function:

$$\hat{p}_A = \frac{(\alpha q_A)(\alpha q_B - c) + c(\alpha q_A - c)}{2\alpha q_A - c + 2\alpha q_B - c}$$

$$\hat{p}_B = \frac{(\alpha q_B)(\alpha q_A - c) + c(\alpha q_B - c)}{2\alpha q_B - c + 2\alpha q_A - c} \qquad [\boldsymbol{Eq.2}]$$

By obtaining these values they can be entered into the demand from which we can draw the equilibrium demands $(\hat{D}_A, \hat{D}_B)$ and profits $(\hat{\pi}_A, \hat{\pi}_B)$.

STAGE 1: Licensing decision

At the first stage of the game, each firm must make the main decision, whether to cross-license or not. If both firms agree, they each benefit from quality improvements (Δ_A and Δ_B) reflecting the additional patented technologies they have gained access to.

The firms compare the expected profits when cross-licensing versus no cross-licensing.

- Without licensing: $q_i = q_i^0 \Longrightarrow \pi_i^{NCL}$
- With licensing: $q_i = q_i^0 + \Delta_i \Longrightarrow \pi_i^{CL}(T)$

If both firms improve their profits under cross-licensing, then the agreement is considered to be bilaterally efficient. In the case of only one firm benefitting, it is considered unilaterally efficient, where the firm that doesn't benefit will turn down the agreement. To address these scenarios of asymmetry, the model allows side payments between firms:

If $\Delta_A \neq \Delta_B$, a transfer T is introduced. Using *Nash bargaining*, the fair transfer is:

$$T^* = \frac{1}{2}[(\pi_B^{CL}(0) - \pi_B^{NCL}) - (\pi_A^{CL}(0) - \pi_A^{NCL})] \qquad [\boldsymbol{Eq.3}]$$

Ensuring both participation constraints are met:

$$\pi_A^{CL}(T^*) \geq \pi_A^{NCL}, \qquad \pi_B^{CL}(-T^*) \geq \pi_B^{NCL} \qquad [\boldsymbol{Eq.4}]$$

Firms will agree to cross-license if profits under licensing (adjusted by T^*) exceed those obtained under non-cooperation.

3.3. Conclusions

The Liu et al. model captures the trade-offs involved in cross-licensing decisions between competing firms. By working backward from consumer behaviour, through firm pricing, and to licensing decisions, we are able to reconstruct the model and its structure obtaining the key findings.

In this chapter, the model proposed by Liu et al. is presented and developed, breaking it down into its three main stages: consumer choice, price competition, and licensing decision. The model captures a duopolistic market with horizontally differentiated products, where the two firms compete in price and quality. These two factors drive consumer utility, and demand is then distributed accordingly. Each firm choses its pricing strategy, knowing how consumers respond, and anticipates the effect cross-licensing would have on its product quality and profitability. If the firms do decide to cross-license, they both benefit from quality improvements, though not necessarily to the same extent. In these cases, a transfer payment is introduced as a way to compensate the firm that is less benefited, and a Nash bargaining solution is used to determine its value.

The model allows to evaluate under which conditions firms prefer to cooperate by entering into cross-licensing agreements or to compete independently. Depending on the parameters, the outcome may be mutual cooperation, unilateral refusal, or no licensing at all.

This chapter has laid out the theoretical basis of the thesis, reconstructing the Liu et al. model and adapting it for posterior application. The next chapter focuses on a real-world case on which to apply the model, using parameter values, data from public databases on the targeted market and firms. The next chapter will help evaluate how the model behaves under realistic conditions, to then explore the practical and economic implications of cross-licensing strategies in a specific competitive context.

4. Boeing vs Airbus, The Aircraft Case

This chapter will apply the theoretical framework for the model developed throughout the previous chapter to a practical case. As seen previously, the Liu et al.[35] model is based on a duopolistic market, and therefor the focus of the study will be on a specific product where competition takes place in duopolistic setting, with the aim of testing the Liu et al. model against an industry where strategic licensing decisions and quality differentials are very relevant.

The rivalry between *The Boeing Company* and *Airbus SE*, the two main firms dominating the market for large commercial aircraft, meets the main assumptions on which the Liu et al. model is based on: two firms operating in a duopoly, offering horizontally differentiated products, competing both in price and quality, and working within a dense network of patents and proprietary technologies. The chapter will first present the relevant characteristics of the market and products pertaining to the case study, then applying the Liu et al. framework to simulate scenarios and discuss the implications of cross-licensing or not.

4.1. Case study presentation

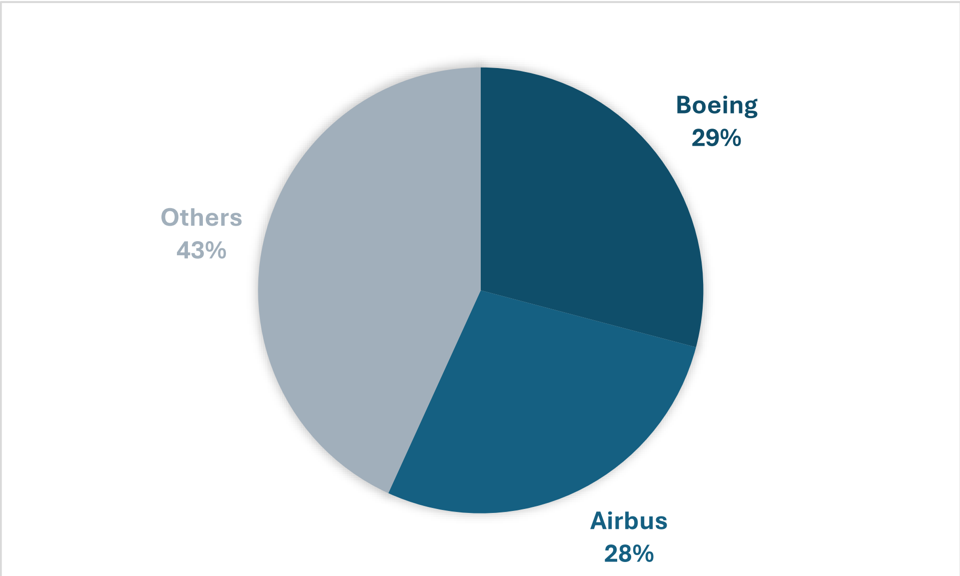
4.1.1. Aircraft manufacturing market

The global aircraft manufacturing market, which includes commercial, military, and general aviation segments, was valued at 661.5 billion dollars in 2024 (661,500,000,000 $) and is expected to reach 707.1 billion in 2025[36]. It is a massive and industry market, where the large commercial aircraft market was valued at around 170 billion dollars in 2024[37]. It is constantly growing, due to rapid growth in air travel as there is rising demand in growing economies and a recovering post-pandemic drive fleet expansion. Passenger traffic is expected to grow at 5 to 12% annually[38], pushing airlines to order more fuel-efficient narrow-body jets.

[35] Liu et al., 2024
[36] Business Research Insights, 2024
[37] Allied Market Research, 2024
[38] Deloitte, 2025

4.1.2. Boeing vs Airbus

The global market for large commercial aircraft has, for decades, been dominated by two firms: *The Boeing Company* (United States) and *Airbus SE* (Europe). Together, they form one of the most recognized and studied industrial duopolies, not only shaping the competitive dynamics of the aviation sector but also influencing technology development, airline economics, and even international trade policy. Their duopoly is particularly evident in their narrow-body, single-aisle families, which account for the majority of global commercial aircraft orders and deliveries, which is the specific product the study will be based on.



Graph 7: Largest Aircraft Manufacturers by Market Cap
[Source: Own elaboration, 2025]

The Boeing-Airbus rivalry is often described as a "duopoly" because the two manufacturers account for nearly all deliveries in the large commercial aircraft market, particularly in the narrow-body segment, which represents the majority of airline orders worldwide. Their competition is not limited to sales; it extends to patented technologies, R&D investment, and regulatory approval. Both companies maintain vast patent portfolios, which has resulted in an environment where access to rival technologies can significantly affect each company's product quality and market performance.

In recent years, Airbus has overtaken Boeing as the world's leading commercial aircraft manufacturer, both in terms of deliveries, revenue, and innovation focus. In 2023 Airbus generated approximately $71 billion in revenue, an 11% increase over the previous year and with an operating profit of nearly $6 Billion, while Boeing 2024 revenue dropped to $66.5 billion and recorded a loss of nearly $12 billion[39]. These significant losses stem from a series of crises over the past years, mainly the grounding of the 737 MAX in 2019 following two fatal crashes which resulted in severe consequences to their reputation and production. This was followed by the pandemic, which depressed global air travel and took a big toll on aircraft orders. More recently Boeing has struggled with production quality and other issues which have caused delays in their 737 Max output. These several challenges have not only reduced their revenues but increased their costs, forcing the company into restructuring and recovery efforts. Despite the mentioned setbacks, Boeing has been showing signs of a gradual recovery, showing a revenue growth of 35% year-over-year on their second quarter of 2025, and reducing their net loss to $612 million, showing their recovery strategy may be taking effect[40]. Meanwhile, Airbus continues to consolidate its lead, benefiting from a reputation for reliability in the situation of Boeing's setbacks.

The specific case selected within the Boeing vs Airbus rivalry will be the market for narrow-body single-aisle commercial aircraft, represented by the *Airbus A320neo* family and the *Boeing 737 MAX* family. These two product lines dominate global single-aisle aircraft sales and are the most direct competitors in this segment.

4.2. Model Application

The game developed in this thesis follows the theoretical structure proposed by Liu et al.[41] and adapts it to the case under study. It models the strategic interactions between two firms in a duopolistic environment with horizontally differentiated products, in this case the two firms being Boeing and Airbus, and the two products being the *Airbus A320neo* family and the *Boeing 737 MAX*

[39] Deutsche Welle, 2024
[40] FT, 2025
[41] Liu et al., 2024

family. Each firm's strategy depends not only on its own decisions but also on the anticipated responses of its competitor, making this a sequential game of strategic interdependence.

The game unfolds in three stages, solved through backward induction so that each firm anticipates the potential outcomes when making earlier choices. At the final stage, consumers decide between the two products based on utility, which increases with product quality and decreases with price ***[Hip. 4]***. At the second stage, the firms set their prices under Bertrand competition with product differentiation, aiming to maximize profits given consumer sensitivity to quality (α) and price (β). At the first stage, the firms decide whether to enter a cross-licensing agreement, weighing the potential quality gains (Δ_i) from their rival's patents and the need for a transfer payment (T) if benefits are asymmetric ***[Hip. 7, Hip. 8]***.

4.2.1. Model Inputs

Market parameters

- D : Total potential market demand
- α : Sensitivity of customers (airlines) to product quality
- β : Sensitivity of customers (airlines) to product prices

Firm characteristics

- q_A, q_B : Baseline quality levels
- p_A, p_B : Product prices
- c : Common marginal production cost per unit
- Pt_A, Pt_B : Number of relevant patents held by each firm, used to calculate potential quality gains

Licensing-related variables

- Δ_A, Δ_B : Potential quality improvements from cross-licensing, modelled as functions of rival patent portfolios
- δ : Maximum achievable quality gain through full licensing
- T : Transfer payment to balance asymmetrical gains from licensing

The model employs different methods in order to obtain the different information and values for the previously laid out variables. The total potential market demand will be based on the number of aircraft units, pertaining to the narrow-body single-aisle category, ordered yearly, using data from 2024. While the sensitivity of customers to product prices will be used as a baseline ($\beta = 1$), the sensitivity of consumers to product quality will be calculated in the model by taking into account list prices, fuel efficiency of each firm and market share ratio. The baseline quality levels will also be calculated in the model, taking into account fuel efficiency, passenger capacity, maximum range and reliability, as well as the maximum achievable quality, which is based as a normal composite of multiple KPIs and set as $\delta \approx 0.5$ as an upper bound scenario of the current quality index as introduced in the model code. The common marginal production cost per unit can be easily estimated by using the firms' financial statements and transaction prices for the products in question. The number of patents held by each firm will be specific to the narrow-body single-aisle aircrafts, belonging to the B64C category. The B64C category refers to the section of the International Patent Classification (IPC) system that covers 'aeroplanes; helicopters.' It includes patents related to the structure, design, and functional aspects of aircraft, such as wings, fuselages, control surfaces, and other components. By focusing on patents within this category, the model isolates innovations directly relevant to the narrow-body single-aisle aircraft segment, ensuring that the technological content of the patents obtained is tied specifically to aeronautical engineering advancements.

The following table shows the different data inputs that the model needs in order to obtain the different variables the model works with:

Category	Model variable	Data needed
Market Demand	D	Global annual demand for narrow-body aircraft
Consumer Sensitivities	α, β	Airbus vs Boeing market shares
		Aircraft list and transaction prices
		Fuel efficiency
		Seat capacity and range

Baseline Qualities	q_A, q_B	Fuel efficiency
		Seat capacity and range
		Dispatch reliability
Marginal Production Cost	c	Average transaction prices
		Industry operating margins
		Cost Of Goods Sold
		Deliveries per year
Cross-Licensing Gains	Δ_A, Δ_B	Patent counts in narrow-body single-aisle aircrafts, CPC → B64C
		Maximum achievable quality gain (δ)
Prices	p_A, p_B	Net transaction prices

Table 3: Data inputs for Liu et al. model
[Source: Own elaboration, 2025]

4.2.2. Game resolution

The game follows the next flowchart:

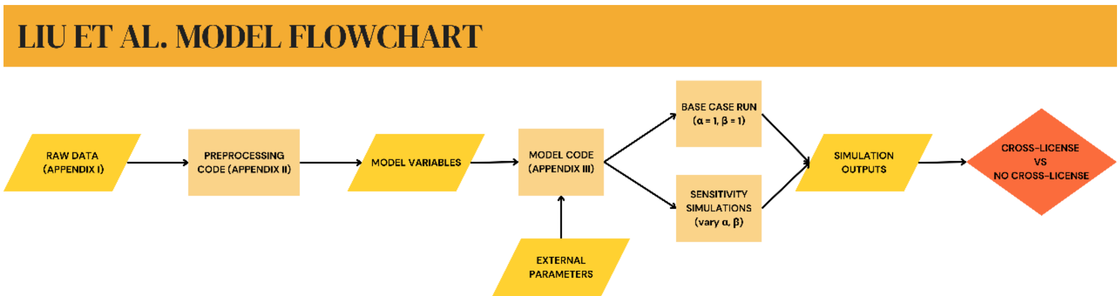
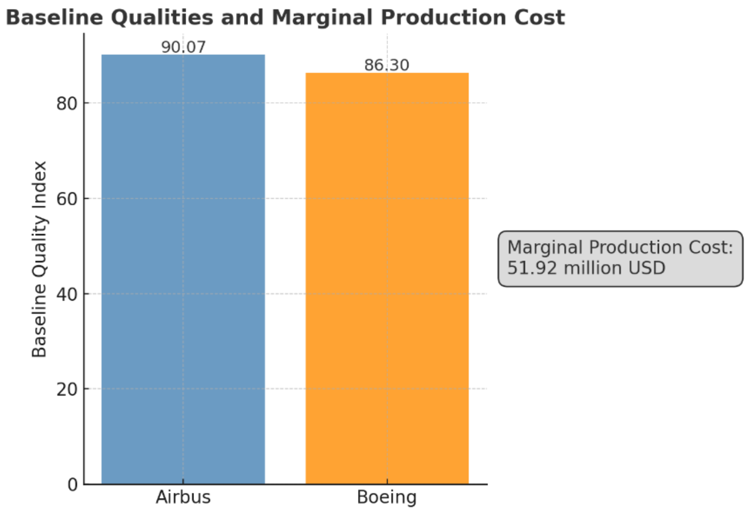


Chart 1: Liu et al. Model Flowchart
[Source: Own elaboration, 2025]

4.2.2.1. Data conversion

In order to carry out the game within the described duopoly, the first step is converting the raw data obtained from both aircraft manufacturers [Appendix I] into the variables used in the model. The data conversion process ensures the model operates on consistent and comparable inputs, allowing for the game to be executed correctly.

The data conversion code computes the baseline qualities of each firm's product, as well as the marginal production cost that will be used in the game's price-setting and profit calculations. The outputs of this step (baseline qualities for Airbus and Boeing and their common marginal production cost) serve as the direct inputs for the game-theoretic resolution in the following subsection.



Graph 8: Baseline Qualities and Marginal Production Cost
[Source: Own elaboration, 2025]

The graph shown indicates the baseline quality indices for both firms, alongside the estimated value for the marginal production cost expressed in millions of dollars. The quality values are derived from fuel efficiency, seating capacity, range and reliability. As shown by the results, Airbus currently achieves a higher baseline quality than Boeing, suggesting an advantage in the attributes most considered by market consumers. Meanwhile, the marginal production, which is assumed to be common to both firms for the sake and functioning of the game, is calculated at nearly $52 million per aircraft.

4.2.2.2. Game running

Then inputting the variable values and parting from a base case we find the resulting profits obtained by each firm from potential cross-licensing when setting price and quality sensitivities to 1 ($\alpha = \beta = 1$). This initial controlled setting allows for the model to be initially interpreted and the structural effects to be expected if the firms engage in cross-licensing. Comparing the results obtained from the model provides a first insight into the potential benefits and drawbacks of engaging in cross-licensing.

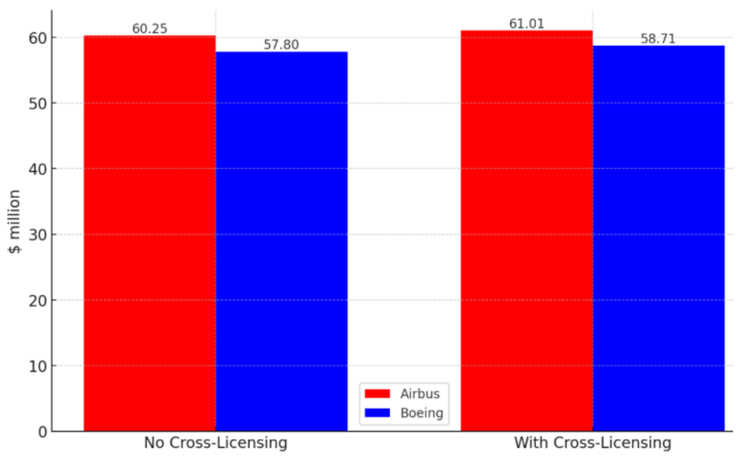
For each set of inputs, the model calculates:

- Product pricing
- Demand allocation
- Profits obtained
- Transfer payment

These values are given under both cross-licensing and non-licensing scenarios, and only providing a transfer payment when the profits obtained by one of the firms after cross-licensing decrease from the initial profits. It is very important to bear in mind that while the model does provide concrete numerical calculated results for each value, the significance of the outputs does not lie on the absolute values themselves, but on the relevance of the comparative perspective they offer. Although the equations and calculations the program uses to reach the results are coherent and justified, the object of this study isn't to predict the exact allocation of market shares after cross-licensing or calculate the exact potential profit, but to use the results as analytical tools to illustrate the relative effects of cross-licensing. Interpreting the effects of cooperation in product quality, demand, and profitability between the two firms.

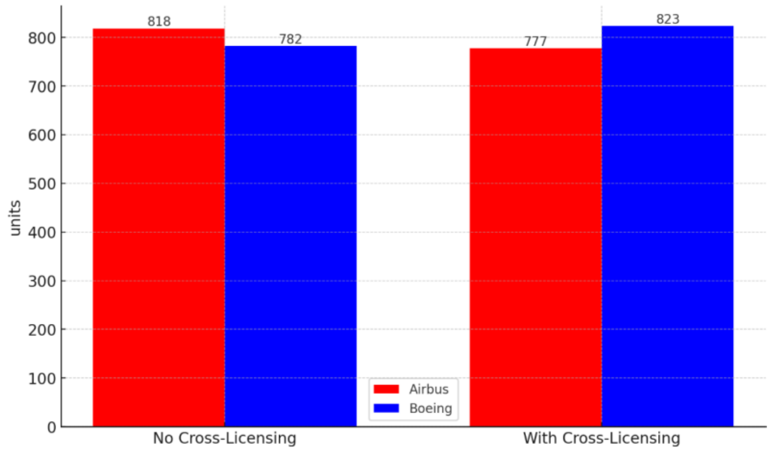
BASE CASE RESULTS

Running the model with both sensitivities set to one provides the baseline scenario against which later variations will be compared. The results are shown below:



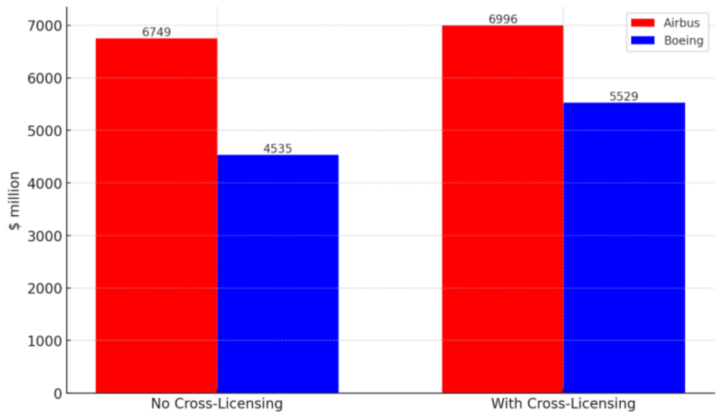
Graph 9: Product prices ($ million)
[Source: Own elaboration, 2025]

As the graph shows, the prices remain essentially constant, with a slight price increase in both firms, which is due to the quality increase obtained by each firm due to cross-licensing. Boing's prices due have a slightly higher increase, which is because of them benefitting more from the patent access as Airbus is the bigger patent holder.



Graph 10: Demand Allocation (Units)
[Source: Own elaboration, 2025]

This graph shows how demand is distributed between the two firms. Airbus initially holds an advantage over Boeing, but after cross-licensing the demand shifts noticeably in Boeing's favour, reflecting the fact that Boeing benefits more from the quality improvements gained through the agreement. Although it does not reflect that in real life the changes would be so noticeable, it does indicate how in the case of cross-licensing, demand allocation would benefit Boeing.

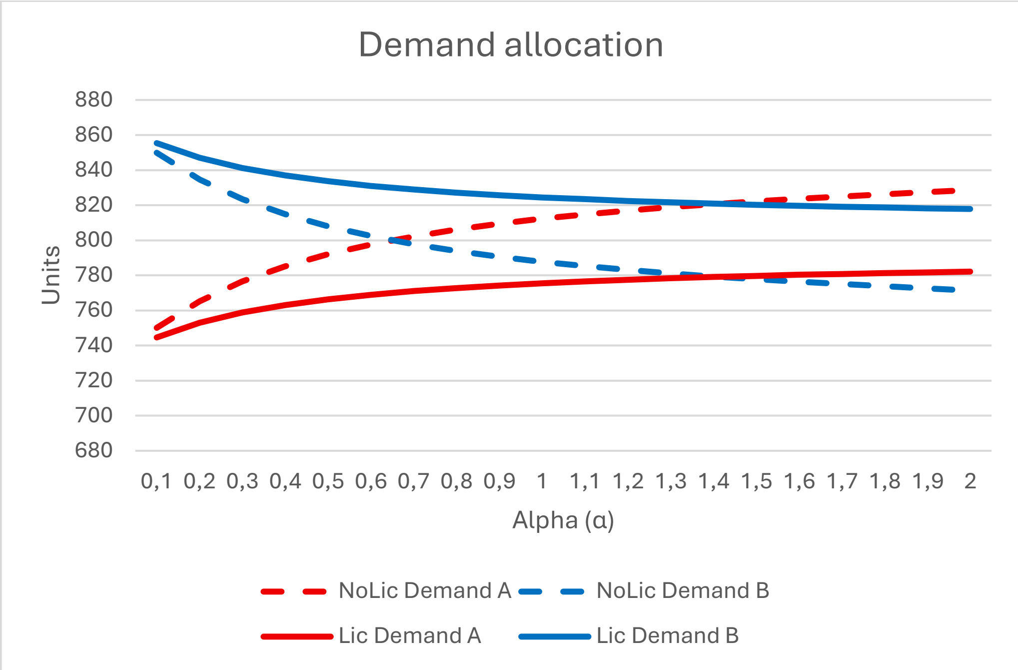


Graph 11: Firm profits ($ million)
[Source: Own elaboration, 2025]

The last graph shows the profits obtained by each firm, it must be pointed out that the profits do not represent the real-life values, but are values calculated from within the model following ***[Eq.1]***. Current profits for each firm are very different to the point where Boeing has found themselves obtaining negative results at the end of 2025, but the model provides values based on the potential effects of the variables and give justified projections on whether cross-licensing is beneficial or not. Both Airbus and Boeing see an increase in profits under cross-licensing, therefor not needing a transfer payment following ***[Hip.8]***, but the gain is more significant for Boeing. Airbus's profits rise slightly, despite decreasing product demand, while Boeing experiences a major improvement, narrowing the profitability gap between the two firms.

QUALITY SENSITIVITY SIMULATIONS

The following graph shows the evolution of demand allocation between Firm A and Firm B (Airbus vs Boeing) as the quality sensitivity parameter (α) increases, both under a no-licensing scenario and with cross-licensing.

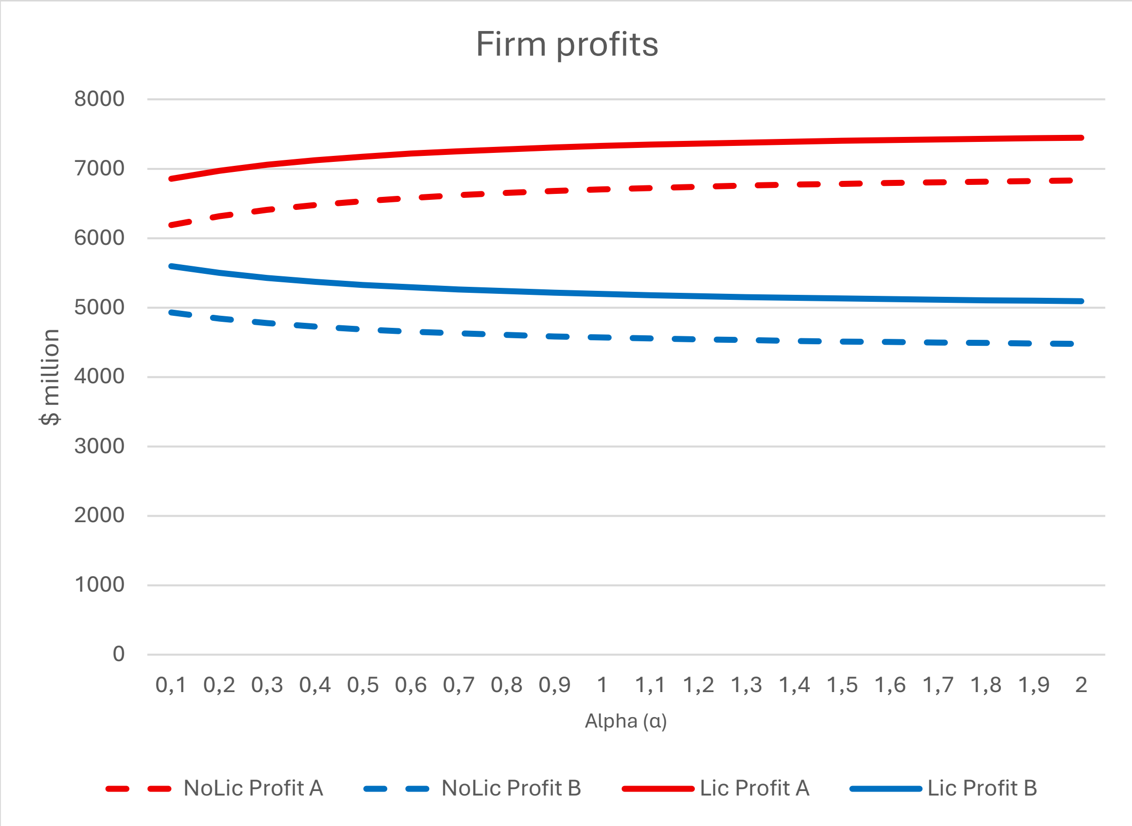


Graph 12: Alpha vs Demand Allocation
[Source: Own elaboration, 2025]

The graph shows how at low quality sensitivity values, when consumers are relatively insensitive to quality differences, Boeing benefits largely as it is modelled to have lower intrinsic quality. The pricing game results in Boeing setting lower equilibrium prices, making them more attractive to price driven consumers. As α increases consumers give more importance to product quality in their purchasing decisions. This shift progressively favours Airbus, whose product is perceived to have higher quality. The price advantage of Boeing becomes less decisive, and its demand declines, while Firm A's demand rises, following the demand distribution established in ***[Hip. 5]***. These changes are especially evident in the no-licensing case, where the absence of quality sharing keeps Boeing's perceived quality low but its price advantage strong.

Entering into cross-licensing reduces the quality gap between the firms, calculated by quality gains provided in ***[Hip. 7.2]***, enabling Boeing to compete more effectively in quality-sensitive markets and softening the decline in its demand as α grows. This also limits Airbus' gains from increasing α, as its quality advantage is reduced. With cross-licensing, the demand curves for both firms are closer together across the range of α values, reflecting a more balanced market.

The following graph shows the evolution of firm profits between Firm A and Firm B (Airbus vs Boeing) as the quality sensitivity parameter (α) increases, both under a no-licensing scenario and with cross-licensing:



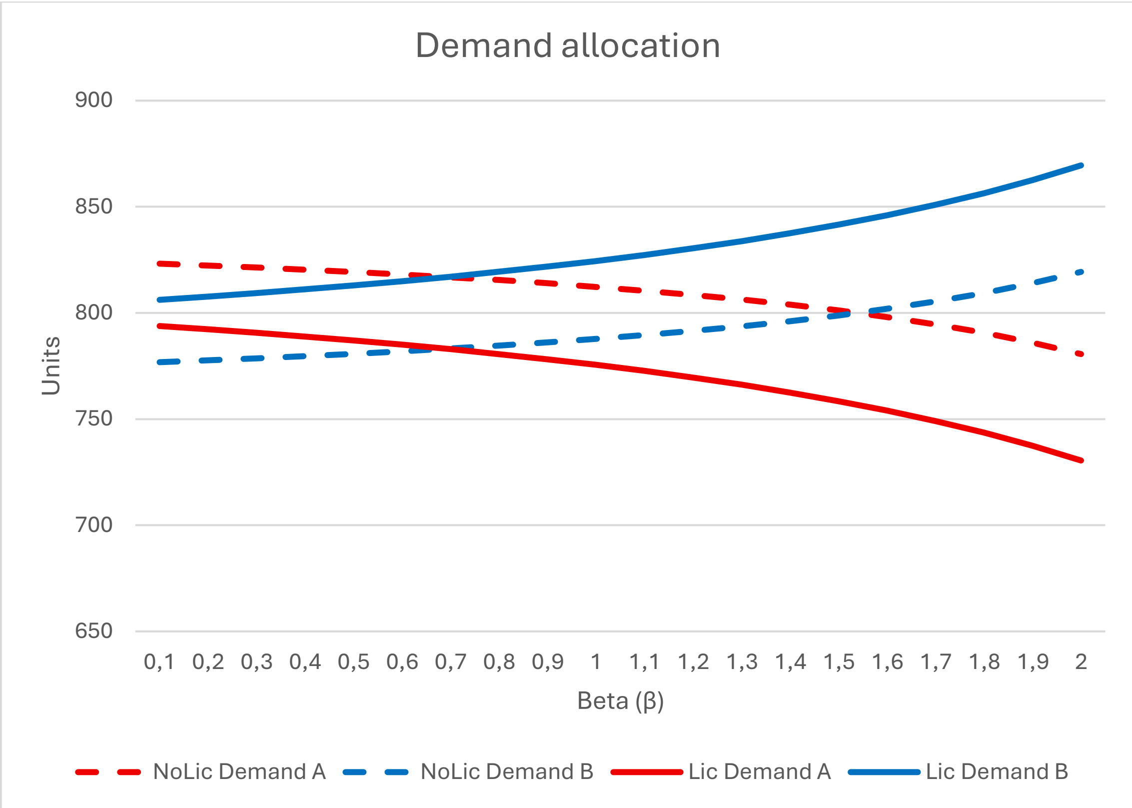
Graph 13: Alpha vs Firm Profits
[Source: Own elaboration, 2025]

As seen in the previous graph, when the market is relatively insensitive to quality differences (low α) Boeing is able to secure a larger demand share, but still their profits are lower than Airbus' due to their lower pricing. As α increases we have seen that it leads to a larger market share and pricing power for Airbus, which allows for a steady increase in profits for Airbus while Boeing's profits gradually decline due to its shrinking demand. The licensing scenario allows for a substantial and consistent profit boost for Boeing compared to the no-licensing simulation.

Cross-licensing leads to higher total industry profits, as calculated by ***[Eq.2],*** across all quality sensitivity values, still the distribution of these profits shifts towards the lower quality firm (Boeing), making licensing more appealing to them, and less so to the higher quality firm (Airbus). Though both firms are benefitted by entering cross-licensing agreements and therefor there is no need for a transfer payment, it should be considered since there is still an unequal benefit obtained from the agreement.

PRICE SENSITIVITY SIMULATIONS

The following graph shows the evolution of demand allocation between Firm A and Firm B (Airbus vs Boeing) as the price sensitivity parameter (β) increases, both under a no-licensing scenario and with cross-licensing:



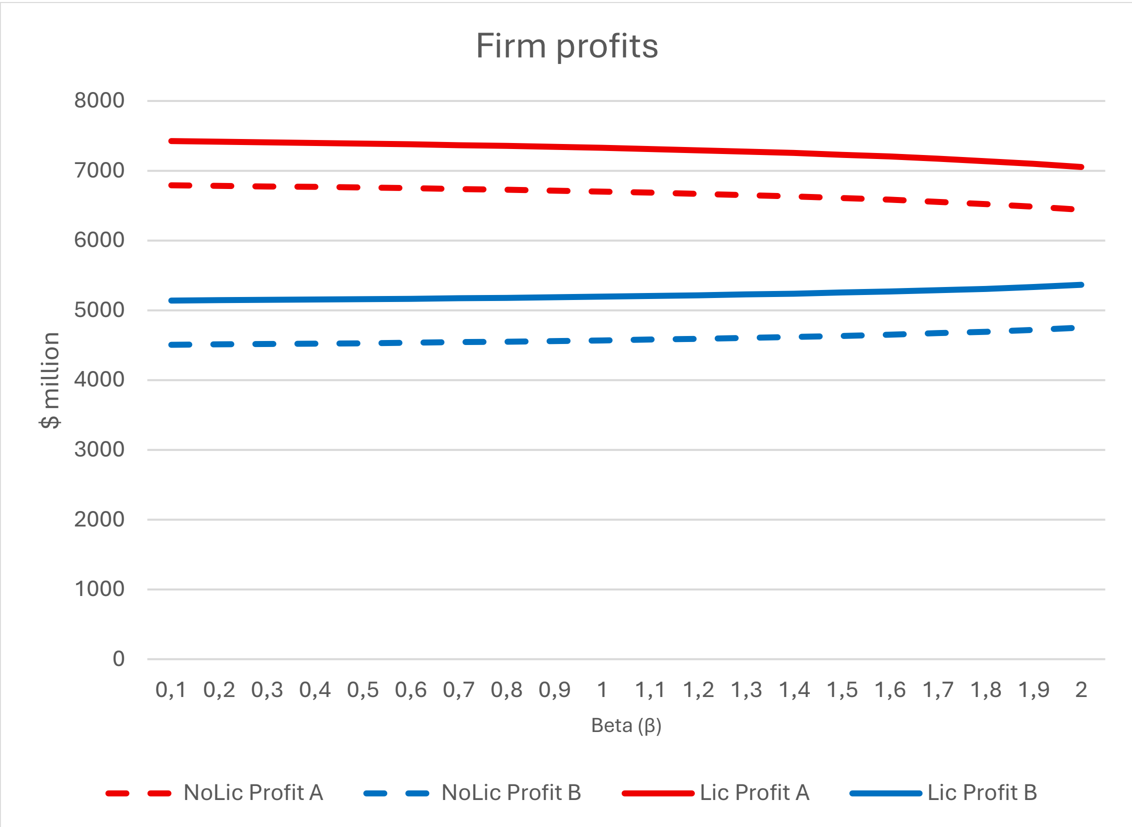
Graph 14: Beta vs Demand Allocation
[Source: Own elaboration, 2025]

The graph shows how at low price sensitivity (beta) values, when consumers are relatively insensitive to price differences, Airbus benefits from its higher perceived quality, while Boeing, despite offering lower prices, captures a smaller share of demand, as distributed by ***[Hip. 5]***. As β increases, consumers give more importance to price in their purchasing decisions, which progressively favours Boeing, whose lower pricing becomes increasingly attractive to price-driven consumers. The quality advantage, reflected by the utility function established at

[Hip. 4] , of Airbus becomes less decisive, and its demand declines, while Boeing's demand rises. These changes are especially evident in the licensing case, where the technology sharing improves Boeing's quality, allowing it to gain even more ground in price-sensitive markets.

In the no-licensing scenario, Airbus maintains a stronger position at lower β values due to its higher quality, but as price sensitivity grows, Boeing's demand overtakes Airbus'. Cross-licensing amplifies Boeing's gains by combining its price advantage with a reduced quality gap, leading to a more pronounced increase in its demand curve. As a result, the demand curves for both firms diverge more sharply at higher β values under cross-licensing, reflecting a stronger competitive shift towards Boeing.

The following graph shows the evolution of firm profits between Firm A and Firm B (Airbus vs Boeing) as the price sensitivity parameter (β) increases, both under a no-licensing scenario and with cross-licensing:



Graph 15: Beta vs Firm Profits
[Source: Own elaboration, 2025]

As seen in the previous graph, when the market is relatively insensitive to price differences (low β), Airbus secures higher profits, following ***[Eq.2]*** , due to its ability to charge premium prices supported by its higher perceived quality. Boeing, despite capturing a smaller share of demand, maintains stable but lower profits because of its lower pricing strategy. As β increases, the growing importance of price competitiveness shifts demand towards Boeing, which in turn results in a gradual rise in its profits under the no-licensing scenario. In contrast, Airbus' profits steadily decline as its market share erodes in a more price-sensitive environment.

The licensing scenario provides a notable profit boost for Boeing across all β values by allowing it to combine its price advantage with improved quality perception, enabling stronger competition against Airbus. For Airbus, while licensing initially helps soften the negative impact of rising β on profits, the overall effect remains a gradual decline as price sensitivity dominates consumer choice.

Cross-licensing leads to higher total industry profits throughout the range of β values, but the distribution of these gains becomes increasingly skewed towards Boeing at higher β. This makes licensing particularly advantageous for the lower-priced firm, while the higher-quality firm benefits less and sees its competitive edge reduced. As in the quality sensitivity simulations, there is no need for a transfer payment in this case since both firms benefit, though the asymmetry in profit gains could warrant its consideration.

4.3. Summary and Conclusions

This chapter applied the Liu et al. model[42] to the strategic rivalry between Airbus and Boeing in the narrow-body, single-aisle commercial aircraft market, with the objective of evaluating how cross-licensing agreements could influence competitive dynamics, demand allocation, and profitability. By framing the Airbus A320neo family and the Boeing 737 MAX family within a duopolistic game structure, the model reproduced the sequential decision-making process of licensing, price setting, and consumer choice.

[42] Liu et al., 2024

The base-case scenario demonstrated that cross-licensing benefits both firms, though the magnitude of gains is asymmetrical: Boeing, with a lower baseline quality, experiences greater relative improvement, while Airbus sees smaller but still positive changes. Price adjustments following licensing were modest, driven by the quality gains that justified slightly higher equilibrium prices for both manufacturers. Demand shifted more noticeably, favouring Boeing in the licensing scenario, which in turn narrowed the profitability gap between the two firms.

Simulations varying the quality sensitivity parameter (α) revealed that when customers pay less attention to quality, Boeing's lower price positioning allows it to have a greater demand, despite its lower baseline quality. As α increases, Airbus's higher quality becomes more decisive, reversing the demand advantage. Cross-licensing reduces this quality gap, enabling Boeing to compete more effectively in quality-driven markets and moderating Airbus's gains from higher α values.

Price sensitivity simulations (β) showed the opposite dynamic: when customers are less sensitive to price, Airbus retains an advantage through its perceived quality; as β rises, Boeing's pricing competitiveness attracts a growing share of demand. Cross-licensing amplifies Boeing's gains in price-sensitive environments by adding quality improvements to its existing cost advantage, while Airbus experiences a gradual erosion of market share and profits in these conditions.

Across all scenarios, the model predicted an increase in total industry profits under cross-licensing, though the distribution of these gains consistently favoured the lower-quality firm. This asymmetry, while not sufficient in the presented simulations to require transfer payments, highlights the potential need for side-payment mechanisms in real-world negotiations to ensure mutual willingness to cooperate.

It is important to note that the significance of these results does not lie in the precise numerical outputs of prices, demand, or profits, but in the patterns and relationships they reveal. The model's purpose is not to forecast exact market shares or revenues, but to illustrate how changes in consumer sensitivities and product quality differentials may influence the strategic value of licensing. The trends identified, such as the greater appeal of licensing to the lower-quality firm, the narrowing of quality gaps in high-α markets, and the amplification of price advantages in high price-sensitivity markets, provide actionable strategic insights for both industry actors and policymakers.

In conclusion, the application of the Liu et al.[43] model to the Airbus vs Boeing duopoly suggests that cross-licensing can enhance overall industry profitability, reduce quality differences, and rebalance competitive positions. However, it also shows that the benefits are rarely symmetric and may require contractual mechanisms to maintain long-term cooperation. For policymakers, these findings reinforce the need to consider both the innovation-promoting and competition-limiting aspects of licensing agreements when assessing their broader market impact. It must also be pointed out how in a duopolistic setting, the quality gains achieved by both firms through cross-licensing may justify moderate increases in product prices, as improved performance can command a higher market value. However, when these gains occur symmetrically and price adjustments follow a similar pattern, there is a risk that such parallel movements in quality and pricing could reduce the intensity of competition. In these cases, the cooperative benefits of cross-licensing may unintentionally approach a cartel-like outcome, where market rivalry is softened, consumer choice is limited, and prices remain elevated beyond what would occur under competitive pressure. The possibility of such unethical behaviours points out the importance of ensuring that cross-licensing agreements are structured to foster innovation and efficiency without facilitating tacit collusion or undermining market fairness.

[43] Liu et al., 2024

5. Economic Report

5.1. Introduction

The Economic Report frames the research and work carried out in this thesis as if it were a consulting engagement delivered to the client firms involved (Airbus and Boeing). The objective of the project has been to analyse the role of patent thickets and cross-licensing agreements in shaping competition within the commercial aircraft industry, with a focus on the narrow-body, single-aisle aircraft type.

In order to reach the desired results and conclusions, the project adapts and interprets the game-theoretic model introduced by Liu et al. (2024) [44] to the context of large commercial aircraft. The model was designed to predict the strategic effects of cross-licensing on product quality, pricing, demand allocation, and profitability between two dominant firms, which are simplified and considered to be engaged in a duopolistic market.

The work carried out in the development of this project required a combination of thorough theoretical research, data gathering, coding, simulation, and data interpretation and economic analysis. Each phase of the project involved a significant amount of hours, comparable to what would be expected in a professional consulting project assignment.

The Economic Report provides a breakdown of the scope of work, the time allocation in the different sections carried out in the project, cost estimation for the value of the time dedicated, and the outsourced services that in the case of a consulting project would have been required. The Economic Report highlights the economic value of the project, both in terms of the academic contribution to the literature in question, and as a strategic resource for the client firms.

5.2. Scope of the work and hour distribution

The project was structured into a different set of phases, each corresponding to a set of deliverables required to successfully carry out the Liu et al. model

[44] Liu et al., 2024

adaptation and implementation. The work carried out in each section is composed of both academic and technical contributions.

Phase	Description	Deliverable	Hours
1	Literature Review	Structured theoretical framework on patent thickets and licensing, as well as game-theoretic cross-licensing models	40
2	Model Development	Liu et al. model analysis and development	35
3	Market Analysis	Selection of Airbus vs Boeing as duopoly case and product and company research	20
4	Data Gathering	Raw dataset on patents, costs, demand, and industry metrics	10
5	Model Coding	Python implementation of Liu et al. game-theoretic model	20
6	Simulations	Outputs for base case and varying sensitivity variables (quality and price)	10
7	Result Interpretation	Strategic insights for Airbus & Boeing	10
8	Economic Report	Final engagement output estimating costs of services provided	5

Table 4: Scope of Work
[Source: Own elaboration, 2025]

5.3. Cost estimation

This section presents a detailed estimation of the total costs associated with the development of this project. The analysis accounts for all relevant components, including consultant time, outsourced services, and indirect expenses. The objective is to provide a transparent and structured overview of the financial resources required to carry out the project.

5.3.1. Assumptions

Based in Spain, a set of assumptions are established in line with industry standards in order to value the work carried out. The work has been valued as if it were done by a first-year consultant in Spain, earning an average annual gross salary ranging between €30,000 and €35,000. Considering a standard annual workload of around 1,800 effective hours, by assuming 40-hour working weeks and also discounting 3 weeks' vacation, and also including the additional costs associated with social security contributions, employee benefits, and firm overheads, the resulting fully loaded cost can be estimated to approximately €30 per hour.

As previously stated, the project development has been based on a standard 40-hour working week, and the total hours have been distributed across the main stages of the study as drawn out in the Scope of work section. In addition to the consultant's hours, the project has required the use of specialized technical support, particularly in programming and visualization tasks, which following the current market rates for IT services in Spain can be estimated to be worth between €40 and €50 per hour.

Finally, overhead costs such as software licenses, database subscriptions, and general office resources are assumed to represent approximately 10% of total direct consulting costs, following common practice in Spanish consulting firms. In this case all these licenses, database subscriptions and resources are provided by the university, but will be considered as overhead costs as they would be in an assigned project. All figures are presented without accounting for taxes (IVA, 21%), which would only be applied at the invoicing stage and does not form part of the internal project costing.

5.3.2. Cost distribution breakdown

In order to provide a clear and transparent breakdown of the resources involved in the project, this section presents a detailed allocation of the costs incurred. Distinguishing between the consultant's time distributed between the different tasks, the outsourced technical services for specific tasks, and the indirect costs associated with support resources and general overheads.

CONSULTANT TIME BREAKDOWN

Phase	Hours (h)	Rate (€/h)	Cost (€)
Literature Review	40	30	1200
Model Development	35	30	1050
Market Analysis	20	30	600
Data Gathering	10	30	300
Model Coding	20	30	600
Simulations	10	30	300
Result Interpretation	10	30	300
Economic Report	5	30	150
Total	**150**		**4500**

Table 5: Consultant Time Breakdown
[Source: Own elaboration, 2025]

OUTSOURCED SERVICES

Service	Description	Cost (€)
Programming Support	Assistance in debugging and structuring Python codes	1000
Specialist Advisory Support	Senior guidance and validation of project progress and delivery	2500

Total Outsourced Services		**3500**

Table 6: Outsourced Services Breakdown
[Source: Own elaboration, 2025]

INDIRECT COSTS

The category of indirect costs includes expenses related to the use of equipment and access to resources that, although not directly billed as hours of consulting or external services, are essential to the execution of the project.

The project was carried out using a Lenovo YOGA laptop with a purchase value of around €1800 and an estimated useful life of 5 years. Assuming an average annual use of 1000 hours, the amortized cost amounts to approximately €0,36 per hour of use. Since the project required around 150 hours of active work, the total cost attributed to the use of the computer is estimated at €54.

The project also relied on access to scientific journals and databases (such as JSTOR, ScienceDirect, SpringerLink, and SSRN), provided by the university. The amortized value of these institutional subscriptions must be recognized as part of the indirect costs. Estimating the market value of equivalent private subscriptions to be around €800 per year, and considering the share of resources used for the project, a proportional cost of €800 has been allocated to account for both the databases and computer amortization. To reflect the use of digital platforms and the general administrative support usually considered in consultancy practice, an additional €400 has been estimated.

Category	Description	Cost (€)
Software & Tools	Computer hours, access to research databases, Colab/Cloud services	800
Administrative Costs	Internal overhead	400

Total Indirect Costs		**1200**

Table 7: Indirect Costs
[Source: Own elaboration, 2025]

TOTAL COSTS

Category	Cost (€)
Consultant Time	4500
Outsourced Services	3500
Indirect Costs	1200
Total Indirect Costs	**9200**

Table 8: Total Cost Estimate
[Source: Own elaboration, 2025]

5.4. Executive Summary of Value Delivered

The project provides significant value from both an academic perspective and as a practical consultancy assignment for firms in the aerospace sector. By adapting the Liu et al.[45] framework to the case of Airbus and Boeing, the thesis demonstrates how game-theoretic modelling can be applied in understanding the effects and benefits of cross-licensing agreements in highly concentrated, innovation-driven markets, particularly based in a duopoly as dictated by the Liu et al. model.

The main contribution lies in determining how intellectual property sharing, known as cross-licensing, affects the market dynamics between the firms involved. The simulations reveal that while cross-licensing benefits both firms by increasing overall industry profits, also disproportionately benefiting the lower-quality competitor (Boeing), reducing the quality gap and softening the Airbus dominance. It provides strategic insights into how firms with different

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technological positions may view licensing agreements, and why such agreements are more often attractive to the weaker competitors.

The model developed and materialized delivers particular value to the aerospace sector and the case study, but is also applicable to any other industry where the patent-intensive competition can be considered detrimental. The firm enables firms to assess not only the direct profits implications but also its impact on the competitive positions under the different consumer preference scenarios.

The project delivers:

- A clear framework to interpret complex patent and competition dynamics
- Quantitative simulations highlighting the risks and opportunities of cross-licensing
- Strategic insights applicable to both Airbus vs Boeing as well as other industries

In summary, the thesis translates theoretical modelling into a practical decision-making tool.

6. Conclusions and future developments

This chapter points out the main findings of the research and work carried out throughout this thesis, highlighting the specific and concrete contributions and results that have been achieved throughout the work. It provides a summary of the thesis' accomplishments, while assessing their significance in economic and strategic terms. This chapter also outlines the potential avenues for improvement and potential avenues for future developments, indicating how the model and insights presented can be further improved and expanded.

6.1. Conclusions

This thesis has achieved its primary goal: it has demonstrated, with a clean and reproducible game-theoretic framework, that patent cross-licensing in innovation-intensive duopolies systematically raises total industry profitability while rebalancing competitive positions toward the initially lower-quality firm. The Airbus vs Boeing case application confirmed that cross-licensing increases joint profits and narrows quality gaps. These results were shown in the base case and in extensive sensitivity simulations, where licensing consistently increased profits even as it shifted relative advantages.

This thesis has delivered a complete state of the art synthesis on patent thickets, cross-licensing architectures, and game-theory, and has translated that synthesis into a tractable three-stage model tailored to the Liu et al.[46] game-theoretic model. It has made the modelling assumptions explicit and formalized the decision rule for bilateral versus unilateral efficiency, turning a literature review into an operational decision tool.

The thesis has developed the complete technical framework required to operationalize the proposed model. A data-conversion layer was created to transform market and technical inputs into model variables, the codebase was implemented including both the core structure and the simulation engine, and a set of visual representations was generated to analyse prices, demand

[46] Liu et al., 2024

distribution, profits, and transfers under cross-licensing and non-licensing scenarios. In addition, the baseline qualities were quantified, common marginal costs for the aircraft case were calibrated, and automated sensitivity analyses across the parameters α and β were carried out in order to test the strategic conclusions.

The framework was successfully applied to the Airbus A320neo and Boeing 737 MAX duopoly, leading to clear strategic implications. The analysis has shown that: firstly, cross-licensing only justified moderate price variations while providing utility gains mainly through quality improvements; secondly, demand reallocation favoured Boeing (the firm with lower quality advantage) under licensing, since access to Airbus's broader patent base reduced the quality gap; and lastly, the firm with lower quality benefited more consistently from cross-licensing, particularly in markets with higher price sensitivity (higher β), while Airbus's initial quality advantage prevailed under conditions of higher quality sensitivity (higher α) without licensing. These findings explain why followers (the firms considered to be the lower-quality competitor) are more inclined to initiate or accept cross-licensing agreements, whereas leaders might require compensatory transfers to do so.

The thesis has also produced conclusions relevant for governance and policy. It has been established that although symmetric improvements in quality can justify moderate price increases, simultaneous rises in both quality and price may reduce competitive pressure and bring outcomes closer to collusion if safeguards are not ensured. Therefore demonstrating that cross-licensing agreements should be carefully designed and, where necessary, supervised in order to preserve competition while enabling innovation and reducing litigation risks.

From an economic perspective, the thesis has quantified its own cost of execution and demonstrated its professional value. The work has been assessed as equivalent to a consultancy engagement, with €4,500 corresponding to analyst time, €3,500 to external technical and advisory support, and €1,200 to indirect costs, giving a total of €9,200. In return, the thesis has delivered a reusable model and codebase, a calibrated duopoly case with scenario analysis, and strategic insights that firms can directly apply in licensing negotiations.

In summary, the thesis has achieved four main contributions: firstly the operationalization of a cross-licensing model consistent with the academic literature but applicable in practice; secondly the development of the necessary data infrastructure and computational tools; thirdly the validation of the framework through its application to a major duopoly with results of clear significance; and finally the formulation of implications for both firm strategy and public policy. Taken together, these results transform theoretical concepts into a practical decision-making tool that allows organizations to assess under which conditions cross-licensing agreements generate maximum joint value while maintaining competitive and regulatory balance.

6.2. Future developments

The work carried out in this thesis leaves room for further development that could broaden both the applicability of the model and the depth of the analysis. There are two possible directions that stand out as particularly relevant ways in which this thesis could be expanded.

On one hand, since the analysis has shown that cross-licensing may under certain conditions reduce rivalry and approach collusive outcomes, a natural continuation would be to examine this risk in more detail. Future research could therefore focus on the regulatory perspective, analysing under which circumstances cross-licensing shifts from being efficiency-enhancing to a threat to competition. This would also involve a deeper study of existing legislation, antitrust considerations, and possible ways of monitoring and controlling such agreements in order to avoid collusive behaviour, ensuring that the benefits of innovation are preserved without undermining market competition.

On the other hand, the model could be adapted to go beyond duopolistic competition and consider markets in which several firms coexist and compete simultaneously. While the duopoly framework has provided clarity and tractability, most real industries are better represented by oligopolistic structures with three or more main competitors. Extending the model to such contexts would make it more useful and applicable to many other cases. Although this would involve additional complexity in the formulation of the stages and very complex computational implementation and adaptation of the already developed codes, it would also increase the scope of the results and their practical usefulness.

In this way, both extensions would complement the current work: the first by addressing the legal and regulatory dimension that accompanies the economic results obtained and risks involved, and the second by adapting the framework to a wider variety of market structures.

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APPENDIX I: Raw data inputs

	AIRBUS	BOEING
Global Demand (units/year)	1600	
Adjusted Market share	0,582	0,418
Transaction price (million $/unit)	60,25	57,8
Fuel efficiency (L/100seat-km)	2,2	2,25
Seats (number/unit)	180	178
Range (km)	6300	6570
Dispatch Reliability	0,997	0,995
Operating margin (%)	12	
Patents (Narrow-body single-aisle B64C)	83	69
Patents (B64C)	53260	29765

APPENDIX II: Preprocessing code (Raw Data into Model Variables)

```
import pandoc

raw_inputs = {
    'trans_price_A': 61e6,         # Airbus transaction price (USD)
    'trans_price_B': 57e6,         # Boeing transaction price (USD)
    'fuel_eff_A': 2.10,            # liters per 100 seat-km (lower
is better)
    'fuel_eff_B': 2.30,
    'seats_A': 180,
    'seats_B': 178,
    'range_A': 6300,
    'range_B': 6570,
    'reliability_A': 0.997,
    'reliability_B': 0.995,
    'max_seats': 180,              # normalization max
    'max_range': 6570,
    'operating_margin': 0.12       # industry estimate
}

# Function to compute marginal production cost
def compute_marginal_cost(raw_inputs):
    avg_price = (raw_inputs['trans_price_A'] + raw_in-
puts['trans_price_B']) / 2
    c = avg_price * (1 - raw_inputs['operating_margin'])
    return c/1e6

# computing quality index
def compute_quality(fuel_eff, seats, range_km, reliability,
max_seats, max_range, w1=0.7, w2=0.05, w3=0.2, w4=0.05):
    return 200*(w1 * ((1 / fuel_eff)**2) + w2 * (seats / max_seats)
+ w3 * (range_km / max_range) + w4 * reliability)

# Run preprocessing
def preprocess_inputs(raw_inputs):
    qA = compute_quality(raw_inputs['fuel_eff_A'],
                         raw_inputs['seats_A'],
                         raw_inputs['range_A'],
                         raw_inputs['reliability_A'],
                         raw_inputs['max_seats'],
                         raw_inputs['max_range'])
    qB = compute_quality(raw_inputs['fuel_eff_B'],
                         raw_inputs['seats_B'],
                         raw_inputs['range_B'],
```

```
                             raw_inputs['reliability_B'],
                             raw_inputs['max_seats'],
                             raw_inputs['max_range'])
    c = compute_marginal_cost(raw_inputs)

    return {
        "Baseline Quality A": round(qA,4),
        "Baseline Quality B": round(qB,4),
        "Marginal Production Cost": round(c,2)
    }

processed_vars = preprocess_inputs(raw_inputs)
for k, v in processed_vars.items():
    print(f"{k}: {v}")
```

APPENDIX III: Model Code

```
def liu_model(D, qA, qB, c, pA_init, pB_init, alpha, beta, Pa-
tentsA, PatentsB, G, transfer=True):

    # Utility and demand calculation
    def compute_equilibrium(qA, qB, pA, pB):
        UA = (qA ** alpha) - (pA ** beta)
        UB = (qB ** alpha) - (pB ** beta)
        sA = UA / (UA + UB)
        sB = UB / (UA + UB)
        nA = D * sA
        nB = D * sB
        piA = (pA - c) * nA
        piB = (pB - c) * nB
        return sA, sB, nA, nB, piA, piB

    # Without Licensing
    sA_no, sB_no, nA_no, nB_no, piA_no, piB_no = compute_equili-
brium(qA, qB, pA_init, pB_init)

    # With Cross-Licensing
    dA = G * (PatentsB / (PatentsA + PatentsB))
    dB = G * (PatentsA / (PatentsA + PatentsB))
    qA_L = qA + dA
    qB_L = qB + dB

    price_increase_factor = 0.05
    pA_new = pA_init * (1 + price_increase_factor * (dA / qA))
    pB_new = pB_init * (1 + price_increase_factor * (dB / qB))

    sA_L, sB_L, nA_L, nB_L, piA_L, piB_L = compute_equilib-
rium(qA_L, qB_L, pA_new, pB_new)

    # Transfer Payment (Only if one firm profits and other doesn't)
    T = 0
    if transfer:
        deltaA, deltaB = piA_L - piA_no, piB_L - piB_no
        if deltaA * deltaB < 0:  # one gains, one loses
            joint_gain = deltaA + deltaB
            T = (joint_gain / 2) - deltaA
            piA_L += T
            piB_L -= T

    return {
        "No_Licensing": {"Prices": (pA_init, pB_init), "Demands":
(nA_no, nB_no), "Profits": (piA_no, piB_no)},
```

```
        "Licensing": {"Prices": (pA_new, pB_new), "Demands": (nA_L,
nB_L), "Profits": (piA_L, piB_L), "Transfer": T}
    }


# Inputs
if __name__ == "__main__":
    results = liu_model(
        D=1600,
        qA=90.07,
        qB=86.3,
        c=52,
        pA_init=60.25,
        pB_init=57.8,
        alpha=1.0,
        beta=1.0,
        PatentsA=83,
        PatentsB=69, G=50,
        transfer=True
    )

    print("=== No Cross-Licensing ===")
    print("Prices:", (f"{results['No_Licensing']['Pric-
es'][0]:.3f}",
                      f"{results['No_Licensing']['Pric-
es'][1]:.3f}"))
    print("Demands:",(f"{results['No_Licensing']['De-
mands'][0]:.3f}",
                      f"{results['No_Licensing']['De-
mands'][1]:.3f}"))
    print("Profits:",(f"{results['No_Licensing']['Prof-
its'][0]:.3f}",
                      f"{results['No_Licensing']['Prof-
its'][1]:.3f}"))

    print("\n=== With Cross-Licensing ===")
    print("Prices:", (f"{results['Licensing']['Prices'][0]:.3f}",
                      f"{results['Licensing']['Prices'][1]:.3f}"))
    print("Demands:",(f"{results['Licensing']['Demands'][0]:.3f}",
                      f"{results['Licensing']['Demands'][1]:.3f}"))
    print("Profits:",(f"{results['Licensing']['Profits'][0]:.3f}",
                      f"{results['Licensing']['Profits'][1]:.3f}"))

    print("Transfer Payment:", results["Licensing"]["Transfer"])
```

APPENDIX IV: Simulation Code

```
if __name__ == "__main__":

    alpha_values = [0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9,
1.0, 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9, 2] #sensitivities
    table_data = []

    for a in alpha_values:
        res = liu_model(
          D=1600,
          qA=90.07,
          qB=86.3,
          c=52,
          pA_init=60.25,
          pB_init=57.8,
          alpha=a,
          beta=1.0,
          PatentsA=83,
          PatentsB=69, G=50,
          transfer=True
        )

        table_data.append({
            "Alpha": a,
            "NoLic Price A": round(res['No_Licensing']['Pric-
es'][0],2),
            "NoLic Price B": round(res['No_Licensing']['Pric-
es'][1],2),
            "NoLic Demand A": round(res['No_Licensing']['De-
mands'][0],2),
            "NoLic Demand B": round(res['No_Licensing']['De-
mands'][1],2),
            "NoLic Profit A": round(res['No_Licensing']['Prof-
its'][0],2),
            "NoLic Profit B": round(res['No_Licensing']['Prof-
its'][1],2),
            "Lic Price A": round(res['Licensing']['Prices'][0],2),
            "Lic Price B": round(res['Licensing']['Prices'][1],2),
            "Lic Demand A": round(res['Licensing']['De-
mands'][0],2),
            "Lic Demand B": round(res['Licensing']['De-
mands'][1],2),
            "Lic Profit A": round(res['Licensing']['Prof-
its'][0],2),
            "Lic Profit B": round(res['Licensing']['Prof-
its'][1],2),
            "Transfer": round(res['Licensing']['Transfer'],2)
```

```
        })

# Print data table
df_results = pd.DataFrame(table_data)

print(tabulate(df_results, headers="keys", tablefmt="pretty",
showindex=False))
```

APPENDIX V: Sensitivity Simulation Results

Alpha	NoLic Price A	NoLic Price B	NoLic Demand A	NoLic Demand B	NoLic Profit A	NoLic Profit B	Lic Price A	Lic Price B	Lic Demand A	Lic Demand B	Lic Profit A	Lic Profit B	Transfer
0.1	60.25	57.8	750.1	849.9	6188.32	4929.42	61.01	58.71	744.61	855.39	6855.28	5596.38	146.94
0.2	60.25	57.8	765.3	834.7	6313.69	4841.28	61.01	58.71	752.99	847.01	6971.64	5499.23	187.87
0.3	60.25	57.8	776.53	823.47	6406.4	4776.11	61.01	58.71	758.81	841.19	7057.26	5426.97	221.05
0.4	60.25	57.8	785.18	814.82	6477.71	4725.98	61.01	58.71	763.09	836.91	7122.89	5371.16	248.12
0.5	60.25	57.8	792.03	807.97	6534.25	4686.23	61.01	58.71	766.37	833.63	7174.8	5326.78	270.48
0.6	60.25	57.8	797.6	802.4	6580.18	4653.94	61.01	58.71	768.96	831.04	7216.88	5290.64	289.2
0.7	60.25	57.8	802.21	797.79	6618.22	4627.19	61.01	58.71	771.06	828.94	7251.69	5260.66	305.07
0.8	60.25	57.8	806.09	793.91	6650.24	4604.68	61.01	58.71	772.8	827.2	7280.95	5235.39	318.67
0.9	60.25	57.8	809.4	790.6	6677.57	4585.46	61.01	58.71	774.26	825.74	7305.9	5213.79	330.46
1.0	60.25	57.8	812.26	787.74	6701.17	4568.87	61.01	58.71	775.51	824.49	7327.42	5195.13	340.76
1.1	60.25	57.8	814.76	785.24	6721.75	4554.41	61.01	58.71	776.58	823.42	7346.18	5178.84	349.83
1.2	60.25	57.8	816.95	783.05	6739.85	4541.68	61.01	58.71	777.52	822.48	7362.67	5164.5	357.89
1.3	60.25	57.8	818.9	781.1	6755.9	4530.4	61.01	58.71	778.34	821.66	7377.28	5151.78	365.08
1.4	60.25	57.8	820.63	779.37	6770.23	4520.32	61.01	58.71	779.07	820.93	7390.32	5140.41	371.55
1.5	60.25	57.8	822.19	777.81	6783.1	4511.28	61.01	58.71	779.72	820.28	7402.02	5130.2	377.39
1.6	60.25	57.8	823.6	776.4	6794.72	4503.11	61.01	58.71	780.31	819.69	7412.59	5120.98	382.69
1.7	60.25	57.8	824.88	775.12	6805.26	4495.69	61.01	58.71	780.83	819.17	7422.17	5112.6	387.53
1.8	60.25	57.8	826.05	773.95	6814.88	4488.93	61.01	58.71	781.31	818.69	7430.91	5104.96	391.95
1.9	60.25	57.8	827.11	772.89	6823.68	4482.75	61.01	58.71	781.75	818.25	7438.9	5097.97	396.02
2.0	60.25	57.8	828.09	771.91	6831.76	4477.06	61.01	58.71	782.15	817.85	7446.24	5091.55	399.76

Beta	NoLic Price A	NoLic Price B	NoLic Demand A	NoLic Demand B	NoLic Profit A	NoLic Profit B	Lic Price A	Lic Price B	Lic Demand A	Lic Demand B	Lic Profit A	Lic Profit B	Transfer
0.1	60.25	57.8	823.2	776.8	6791.4	4505.44	61.01	58.71	793.83	806.17	7425.28	5139.32	273.52
0.2	60.25	57.8	822.31	777.69	6784.06	4510.6	61.01	58.71	792.28	807.72	7417.25	5143.79	279.48
0.3	60.25	57.8	821.36	778.64	6776.21	4516.12	61.01	58.71	790.63	809.37	7408.68	5148.59	285.74
0.4	60.25	57.8	820.34	779.66	6767.8	4522.03	61.01	58.71	788.88	811.12	7399.51	5153.74	292.35
0.5	60.25	57.8	819.24	780.76	6758.76	4528.39	61.01	58.71	787.02	812.98	7389.67	5159.29	299.32
0.6	60.25	57.8	818.06	781.94	6749.03	4535.23	61.01	58.71	785.03	814.97	7379.1	5165.3	306.68
0.7	60.25	57.8	816.79	783.21	6738.5	4542.63	61.01	58.71	782.9	817.1	7367.69	5171.81	314.47
0.8	60.25	57.8	815.41	784.59	6727.1	4550.64	61.01	58.71	780.61	819.39	7355.36	5178.9	322.72
0.9	60.25	57.8	813.9	786.1	6714.7	4559.36	61.01	58.71	778.16	821.84	7341.99	5186.64	331.47
1.0	60.25	57.8	812.26	787.74	6701.17	4568.87	61.01	58.71	775.51	824.49	7327.42	5195.13	340.76
1.1	60.25	57.8	810.47	789.53	6686.34	4579.3	61.01	58.71	772.65	827.35	7311.51	5204.47	350.63
1.2	60.25	57.8	808.49	791.51	6670.02	4590.78	61.01	58.71	769.54	830.46	7294.05	5214.81	361.12
1.3	60.25	57.8	806.3	793.7	6651.96	4603.47	61.01	58.71	766.17	833.83	7274.81	5226.31	372.3
1.4	60.25	57.8	803.87	796.13	6631.89	4617.58	61.01	58.71	762.48	837.52	7253.48	5239.17	384.2
1.5	60.25	57.8	801.14	798.86	6609.44	4633.36	61.01	58.71	758.43	841.57	7229.72	5253.65	396.88
1.6	60.25	57.8	798.08	801.92	6584.16	4651.14	61.01	58.71	753.98	846.02	7203.09	5270.06	410.38
1.7	60.25	57.8	794.6	805.4	6555.48	4671.3	61.01	58.71	749.05	850.95	7173.01	5288.82	424.72
1.8	60.25	57.8	790.63	809.37	6522.68	4694.36	61.01	58.71	743.56	856.44	7138.78	5310.46	439.92
1.9	60.25	57.8	786.03	813.97	6484.78	4721.0	61.01	58.71	737.42	862.58	7099.47	5335.68	455.93
2.0	60.25	57.8	780.67	819.33	6440.52	4752.12	61.01	58.71	730.5	869.5	7053.84	5365.44	472.64