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Characterizing affect dynamics with a damped linear oscillator model: Theoretical considerations and recommendations for individual-level applications

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Abstract

People show stable differences in the way their affect fluctuates over time. Within the general framework of dynamical systems, the *damped linear oscillator* (DLO) model has been proposed as a useful approach to study affect dynamics. The DLO model can be applied to repeated measures provided by a single individual, and the resulting parameters can capture relevant features of the person's affect dynamics. Focusing on negative affect, we provide an accessible interpretation of the DLO model parameters in terms of emotional lability, resilience, and vulnerability. We conducted a Monte Carlo study to test the DLO model performance under different empirically relevant conditions in terms of individual characteristics and sampling scheme. We used *State-Space Models* (SSM) in continuous-time. The results show that, under certain conditions, the DLO model is able to accurately and efficiently recover the parameters underlying the affective dynamics of a single individual. We discuss the results and the theoretical and practical implications of using this model, illustrate how to use it for studying psychological phenomena at the individual level, and provide specific recommendations on how to collect data for this purpose. We also provide a tutorial website and computer code in R to implement this approach.

Keywords: damped linear oscillator, negative affect, within-individual change, state-space modeling, continuous time modeling

Translational Abstract

Until recent years, psychological researchers have mainly focused on studying groups of people. However, applied practitioners in many fields usually work with individuals. This is often the case, for example, in clinical psychology. Statistical methods focused on single participants can capture the individuality of each person and can prove very useful for making treatment decisions. In this study, we focus on the damped linear oscillator model. We discuss how to interpret the model parameters in terms of emotional lability, resilience, and impact of daily events on emotional state, based on repeated measures provided by each single person. We applied it to the specific case of negative affect, which has been widely related to multiple psychological disorders. Based on the results of an extensive simulation study, we provide recommendations on how to collect data and use this model to extract relevant clinical information. We provide computer code in R that implements this approach.

Characterizing affect dynamics with a damped linear oscillator model: Theoretical considerations and recommendations for individual-level applications

Characterizing the dynamic nature of affective processes is an important goal for practitioners in various fields (Gross & Barrett, 2013). In recent years, affect dynamics have been frequently studied through daily diary, experience sampling, or ecological momentary assessment methods (Kuppens et al., 2022). These methods have led to the popularization of intensive longitudinal databases, which can be examined using various analytical approaches and techniques.

In this paper, we focus on negative affect. This outcome is a relevant variable for psychological science, especially in the field of psychopathology. Previous literature has shown that negative affect is a core feature of many types of psychopathology (Stanton & Watson, 2014). For example, it has been identified as a risk factor for adolescent substance use (Hussong et al., 2017). In fact, it plays an important role in the initiation and maintenance stages of smoking, and substance abusers generally experience more negative affect than abstainers (Kassel et al., 2007). Also, anxiety-related aspects of negative affect have been linked to rumination (Kirkegaard Thomsen, 2006). Another relevant outcome is that people with medical diseases such as systemic sclerosis (Leon et al., 2014) or psychological disorders such as depression (Peeters et al., 2006) usually report higher levels of negative affect. Importantly, participants with depression also show higher variability within a day in negative affect than healthy participants. These findings underscore the importance of studying negative affect dynamics and variability at the intra-individual level.

The Damped Linear Oscillator Model

The Damped Linear Oscillator (DLO) model, also termed Damped Harmonic Oscillator, is a powerful approach for studying intensive longitudinal data of negative affect. The DLO model is a system of equations that describes the dynamics of a process with oscillatory movements. Oscillatory fluctuations are a distinctive feature of affect processes, and a key aspect of the *Adaptative Equilibrium Regulation* framework (Boker, 2015). In this framework, the negative affect of an individual is considered an homeostatic system (Boker et al., 2010b) or a thermostat (Chow et al., 2005). These ideas highlight that the system is capable of regulating itself in response to perturbations in a short timescale, such as receiving bad news at work, and adapting to its environment in response to long-term persistent forces, such as suffering from a serious illness (Boker, 2015).

The first goal of this paper is to connect the key features of the DLO model with the clinical aspects that they capture. We aim to present the model in an intuitive way to clinical practitioners concerned about affect and affect dynamics and highlight its utility in the context of clinical psychology. Our second goal is to conduct a comprehensive Monte Carlo study to examine under what conditions the model is able to recover the dynamics underlying a time series of negative affect.

One noteworthy feature of the DLO model is that it can be fitted to data from a single person (Boker & Nesselroade, 2002). Therefore, it can be used as an individual-based approach to characterize the dynamics of change at the within-individual level through a set of statistical parameters. Until recently, most psychological research (e.g., in personality, individual differences, or psychopathology) has been conducted by gathering large samples and taking cross-sectional measures from them (i.e., from a nomothetic point of view; Hamaker, 2012). Very often, researchers draw longitudinal within-person conclusions based on the resulting cross-sectional between-person findings. However, this procedure is inadequate as it assumes ergodicity of the process under study. Ergodicity implies that the distribution of the variables in the population (and the statistical moments used to characterize it) accurately reflects their distribution within any single individual. This is very rarely the case in psychological data (Fisher et al., 2018; Molenaar, 2004). As a solution, many authors have advocated for the use of individual-level methods in psychological research. Such methods, typically termed idiographic, imply characterizing each individual instead of assuming population-based distributions (Hamaker, 2012; Molenaar, 2004). One example of this is the characterization of negative affect dynamics of single-subject time series by means of the DLO model proposed in this paper.

When specified as a State-Space Model in Continuous Time (SSM-CT), the DLO model has two main components: the dynamics equation (Equation 1) and the measurement equation (Equation 2). The dynamics equation is a second-order differential equation that describes change in two latent variables for an infinitesimally small time interval (*dt*):

$$\begin{bmatrix} \frac{dx}{dt} \\ \frac{d^2x}{dt^2} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \eta & \zeta \end{bmatrix} \begin{bmatrix} x \\ \frac{dx}{dt} \end{bmatrix} + \begin{bmatrix} 0 \\ q(t) \end{bmatrix} \tag{1}$$

where *x* is a latent variable that captures the *level* or *position* of the system, and dx/dt is the *rate of change* in *position*—that is, the *velocity*. Both *position* and *velocity* are derived with respect to time, with the derivative of *position* (*x*) being the *velocity* (dx/dt), and the derivative of *velocity* (i.e., the second derivative of position) being the *acceleration* (d^2x/dt^2). Interpreted in terms of negative affect at any given time, the *position* is the individual's current level of negative affect, the *velocity* is the direction and speed at which negative affect is changing, and the *acceleration* is how such speed is changing. For example, an individual recovering from bad news may still have a level of negative affect 5 points above their mean (position) but may be reducing their level at a given speed (velocity), and this reduction may become slower (acceleration or, more accurately in this case, deceleration) as the level gets closer to the person's average. Equation 1 includes three components: η, ζ, and $q(t)$. Each of them captures a particular feature of the individual's affect dynamics, which are described in the following section.

The second component of the DLO model is the measurement, output, or observation equation (Equation 2), which connects the latent variables defined in continuous time in Equation 1 with the observed measures (y) taken at a specific time point t, and includes one additional component, $r(t)$ (see further explanation in Data Analysis section):

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ \frac{dx}{dt} \end{bmatrix} + r(t) \qquad (2)$$

Equation 2 implies that the observed variable y is an indicator of the latent variable x, which captures the *level*, *position*, or momentary true score in negative affect. To define Equation 2, the second-order differential equation is reformulated as a two-dimensional system of first-order differential equations by defining the first-order derivative (dx/dt) as an extra latent variable without an underlying observed variable (see, for example, Voelkle & Oud, 2013).

Note that equations 1 and 2 define a univariate linear oscillator, which describes change in a single unobserved process x, linked to an observed variable y, and measured at time t. However, the model can be expanded to characterize systems with two or more processes (see, for example Reed et al., 2015; Steele & Ferrer, 2011). In the next sections, based on previous conceptualizations in the literature, we interpret the model parameters[1] and some of its key features.

Substantive Interpretation of the Parameters in the Univariate DLO Model

Equilibrium Point

In a DLO model, the change of a variable or process (in this case, negative affect) over time is conceptualized as a dynamical system that fluctuates around an equilibrium point. A metaphor that helps understanding this conceptualization is a ball in a bowl (Boker, 2015). Consider the system depicted in Figure 1. The black line represents the equilibrium of the system. This is the point to which the system tends. If the ball inside the bowl is pushed to the left (panel B of Figure 1), it will run along the surface of the bowl and, eventually, will return to the bottom of the bowl (panel A of Figure 1). In the case of affect scores for one individual, this equilibrium point is typically defined as the mean of all their scores over *T* measurement occasions. This implies that, even if different individuals can (and usually do) have different equilibrium points, the mean of each individual is usually subtracted from all their scores so that the equilibrium point is typically zero[2].

INSERT FIGURE 1 ABOUT HERE

Emotional Lability (η)

The *emotional lability* parameter has been also termed *frequency*. For clarity, we now simplify Equation 1 so that the acceleration of the system becomes $d^2x / dt^2 = \eta x$

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[1] Although other interpretations of the parameters are possible, the interpretation provided here is consistent with previous literature on the DLO model.

[2] Alternatively, the equilibrium point can be estimated, for example by adding an extra parameter in the measurement model

(i.e., we assume for the moment that the parameter ζ equals zero). If the parameter η is negative, the system has a stable equilibrium, meaning that the farther the position is from the equilibrium, the greater the acceleration it experiences to reduce the distance (see, for example, Boker et al., 2010b). In contrast, positive values of η lead to an unstable equilibrium: in the ball-bowl metaphor, the bowl would be upside down (Boker, 2015), meaning that the farther the ball is from the equilibrium point, the greater the acceleration it experiences to move away from it. According to previous descriptions of negative affect processes in the literature, in this paper we describe a system with a stable equilibrium, thus η is always negative.

In the context of emotion, this parameter is often assumed to capture *emotional lability* (Chow et al., 2005), which has been defined as mood stability, or lack of it (Harvey et al., 1989). Figure 2 depicts the trajectories of negative affect of three hypothetical individuals over time. All three individuals start off at the same point of non-equilibrium, but each of them has a different value of η. The individual with a value farther away from zero ($\eta = -.6$) oscillates faster and thus experiences faster changes in affect, leading to a higher oscillation *frequency*. If we think of the individuals as bowls, the individual with the closest η value to zero ($\eta = -.2$) would be a flatter bowl.

INSERT FIGURE 2 ABOUT HERE

***Resilience* (ζ)**

In Figure 2, the parameter ζ equals zero for all three individuals. In physics, this situation is called *Perfect Zero-Length Spring* and implies no dissipation of energy in the system. Once the system is displaced from the equilibrium, it keeps oscillating indefinitely, following an invariant trajectory over time (see, for example, Boker et al., 2010b). Of course, the *Perfect Zero-Length Spring* is not realistic in most empirical situations. In a real-life ball-bowl system, once the ball is displaced from the equilibrium, its oscillations will gradually decrease until it finally stops at the bottom of the bowl.

Changes in the amplitude of the oscillations are captured by non-zero values of parameter ζ. Such a system is called a *Zero-Length Spring with Linear Damping* (Boker et al., 2010b) in which acceleration is $d^2x/dt^2 = \eta x + \zeta\, dx/dt$ (see Equation 1). In theory, both positive and negative values are possible for ζ, but only negative values imply *damping* or *friction*. This is because, with a positive value of ζ, the faster the ball is moving, the more acceleration it experiences *in the same direction in which it is moving*, whereas a negative value of ζ implies that the faster the ball is moving, the more acceleration it experiences *in the opposite direction*. Thus, a positive ζ leads to an amplification of the oscillations: after a displacement occurs, the system moves away from the equilibrium with a gradually larger amplitude—that is, it reaches higher acceleration and velocity in each oscillation. In contrast, negative values of ζ result in a progressive reduction of the amplitude in each oscillation, because the faster the ball moves, the more *damping* or *friction* it experiences. Consider two bowls, one whose surface is made of velvet, and the other made of metal and covered by oil. The *damping* or *friction* parameter would be negative for both, but much farther from zero for the velvet bowl. Due to this friction, the system returns to the equilibrium point much sooner.

Fitting the DLO model to negative affect data typically leads to negative estimates of ζ (see, for example Bisconti et al., 2004; Boker et al., 2010b; Pettersson et al., 2013) representing the tendency of oscillations of affect to decrease in amplitude, until returning to the equilibrium, unless further perturbed. In the context of emotion, several interpretations are possible for this parameter. Here we consider it to capture *emotional resilience*. Resilience has been defined as the ability to resist, cope with, recover from, and succeed in the face of adverse life experiences (Montpetit et al., 2010). We consider that this parameter captures resilience because a more negative value of ζ implies a shorter time away from equilibrium after a displacement, given the same emotional lability. That is, if two individuals have the same emotional lability, the one with the more negative ζ will either resist better or recover faster from a change in negative affect and, consequently, will return to equilibrium earlier than the other individual. Figure 3 depicts the trajectories of negative affect of three hypothetical individuals over time, who have the same emotional lability (η = -.4). The individual with a ζ value farther away from zero (ζ = -.45) experiences more *damping* or *friction* and therefore returns to their equilibrium earlier than individuals with values closer to zero.

INSERT FIGURE 3 ABOUT HERE

Impact of Daily Events or Vulnerability to Daily Events **(σ^2_q)**

The trajectories of negative affect depicted in Figures 2 and 3 are not realistic because they only include one displacement from equilibrium at t = 0. However, negative affect is impacted by numerous daily events, such as having an argument with a romantic partner or a family member, failing an exam, receiving good or bad news, experiencing a problem at work, among many others. Each of these events cause a new displacement of negative affect from the equilibrium point. Back to the ball-bowl metaphor, these events are external forces that are constantly tapping the ball and moving it both to the right and to the left, leading to a system that never stabilizes at its "equilibrium point", regardless of how much time passes.

In the DLO model, the influence of these daily events is represented with the parameter $\boldsymbol{\sigma}^2_q$, which is usually termed *dynamic error variance*. Daily events cause fluctuations in the rate of acceleration that, in turn, influence the latent scores of negative affect. If such events are measured, they can be included in the model as observed time-specific variables. However, it is hardly possible to keep track of all the internal and external events to which an individual is exposed throughout the day. When daily events are not registered, which is often the case in applied research, their effects on the individual are not predictable. In the DLO model, these unpredictable pulls and nudges to the level of affect are modeled as random dynamic noise, which is represented as a Gaussian stochastic process with mean zero and variance $\boldsymbol{\sigma}^2_q$. The addition of random dynamic noises, also termed *innovations*, leads to a *stochastic differential equation*, and implies that the dynamics are not perfectly predicted by the previous state of the system, as random innovations can enter it at any moment. As shown in Equation 1, q is time-varying. This implies that, at each time t, the amount of dynamic error added to the system can be different, although it is always drawn from a normal distribution with time-invariant parameters: $q(t) \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & \sigma^2_q \end{bmatrix}\right)$.

In the context of emotion, dynamic error variance could be interpreted in two ways: individuals with larger variance are a) affected by more impactful events in their everyday life, or b) more vulnerable than other individuals to the same daily events. Figure 4 depicts the trajectories in negative affect of four hypothetical individuals over time who have the same emotional lability (η = -.4) and resilience (ζ = -.15). Unlike Figures 2 and 3, all individuals in Figure 4 start off at their equilibrium point at $t = 0$. A higher value of *dynamic error variance* means that larger displacements from equilibrium are more likely. Therefore, at any given time, the individual with $\boldsymbol{\sigma}^2_q = 6$ is likely to be displaced farther away from their equilibrium than the individuals with lower values of *dynamic error variance*.

INSERT FIGURE 4 ABOUT HERE

***Measurement Error* ($\boldsymbol{\sigma}^2_r$)**

As with any other psychological construct, the measures provided by an instrument designed to assess affect are not assumed to be the true values of negative affect. Any observed value is affected by a certain amount of measurement error, caused by multiple factors related to the measurement process (e.g., fatigue, reactivity, social desirability, etc.). This is represented in Equation 2 *by the measurement error variance* ($\boldsymbol{\sigma}^2_r$). In consequence, nonrelevant measurement error variance is added at each measurement occasion. Such errors are assumed to be drawn from a normal distribution $N\left(\left[0\right],\left[\sigma^2_r\right]\right)$ with mean zero and variance $\boldsymbol{\sigma}^2_r$.

Panels A and B of Figure 5 depict the latent trajectory of negative affect for one individual over time (with η = -.4, ζ = -.15 and $\boldsymbol{\sigma}^2_q = 2$) with a solid black line. The other two lines represent the observed trajectory in negative affect for this individual, and the values of *measurement error* variance are different in each panel. It can be noted how, for the same latent trajectory, different values of *measurement error variance* ($\boldsymbol{\sigma}^2_r$) lead to time-specific deviations of the latent trajectory of different size. Higher values of *measurement error variance* mean that larger errors are more likely in each measurement, leading to observed trajectories with larger deviations.

INSERT FIGURE 5 ABOUT HERE

Modeling Affect Dynamics in Continuous Time

In a dynamic system, the current state of negative affect is considered to depend on its preceding states—that is, the current negative affect is determined, at least in part, by the negative affect at the previous occasion. Two frameworks are available for describing this feature: (1) the discrete-time (DT) framework, in which the passage of time is represented in discrete steps and the change in the system is defined for a specific time interval (i.e., from $t-1$ to t), and (2) the continuous-time (CT) framework, in which time is considered a continuous variable and the change in the system is defined for an infinitesimally brief time interval (dt) (see, for example, Ryan et al., 2018; Voelkle et al., 2012). In the context of affect dynamics, parameters estimated in a DT framework capture the change in affect for a certain time lag (e.g., one day) whereas parameters estimated in CT capture the change for an infinitesimally brief time lag (Estrada & Ferrer, 2019). Theoretically, if a phenomenon is supposed to unfold continuously, as most of the psychological processes are, it seems more adequate to study it from a CT framework. Otherwise, the researcher is assuming that the process ceases to exist between observations (Oud & Delsing, 2010).

In this work, the dynamic model of a DLO is specified in CT. This presents some practical advantages over a DT specification (see, for example, Ryan et al., 2018). First, in a CT specification, the underlying process is assumed to develop continuously over time but observed at some discrete time points (Oud & Voelkle, 2014). Therefore equal sampling intervals are not necessary, furthermore, it can be advantageous to use unequal sampling intervals, in particular when the sampling rate is low (Voelkle & Oud, 2013). Second, in many scenarios, a model in discrete time is a particular case of the corresponding continuous-time model (Deboeck & Preacher, 2016; Ryan et al., 2018; Voelkle et al., 2012). Therefore, it is possible to rescale the parameters estimated in continuous time to obtain their corresponding values in discrete time for any sampling interval. Third, the application of CT models allows comparing parameter estimates from studies that used different time intervals (Deboeck & Preacher, 2016; Voelkle et al., 2012).

Purpose of the study

Previous research has shown that intraindividual variability is a very relevant factor in the study of negative affect (Bisconti et al., 2004; Chow et al., 2005). Therefore, we consider that a fully individual-based approach (i.e., fitting one separate model for each participant) is a promising avenue for the study of affect dynamics, especially in applied contexts typically focused on the individual, such as clinical psychology.

However, although some studies on the performance of the DLO model exist, there are no clear recommendations on how to use it to conduct single-participant studies. In particular, there is very little information about how to sample the process of interest in order to adequately capture its temporal dynamics, or how estimates are affected by the degree of stochasticity of the process. These aspects are especially important for researchers because insufficient sampling or a high degree of stochasticity may cause the dynamics of the system to be characterized incorrectly. Consequently, the researcher could draw inaccurate conclusions from the results with detrimental implications for their applied goals. On the other hand, a more frequent sampling rate than required would avoid mischaracterization but would also be less efficient and increase research costs. Accordingly, we conducted a Monte Carlo study with the following purposes:

1. Examine the performance of the DLO model under different empirically relevant conditions regarding the individual features and the sampling scheme.
2. Based on our results, make specific recommendations on how to design studies on negative affect, and how to study affect dynamics using a DLO model.

A tutorial website with computer code in R for applying this model, and notes on how to use it, are provided in the following link: https://marjfollero.github.io/DLO/Tutorial.html

Methods

Monte Carlo Study

We generated repeated measures of negative affect for one individual (*N*=1), modeled as a latent process that unfolds in continuous time. The generating process was defined by the DLO model described in the previous sections, with parameters η = -.4, ζ = -.15, and *r* = 0.75. Although previous empirical studies have found between-individual variability in these parameters, reported values for the frequency parameters (η) are typically close to the value selected here (Boker et al., 2010b; Montpetit et al., 2010; Steele & Ferrer, 2011). In a damped linear oscillator system, the period oscillation (time taken to complete a whole cycle, λ) is a function of the frequency and damping parameters:

$$\lambda = \frac{2\pi}{\sqrt{-\left(\eta + \zeta^2 / 4\right)}} \quad (3)$$

The values selected here for frequency and damping imply a period of oscillation of 9.86 days, meaning that, in the absence of large disturbances, the individual enters a new oscillation cycle approximately every 10 days. These values of period λ and damping ζ are consistent with previous estimates obtained from empirical data (Boker et al., 2010b).

Regarding the simulation conditions of the study, we manipulated the number of measurement occasions in two ways: (1) number of days the participant is under study and (2) number of measurement occasions per day. We also manipulated (3) the occasions at which these measurements were collected: either all measurements taken at the exact same time every day (i.e., fixed time intervals), or measurements taken at random times during the day (i.e., varying time intervals from one measurement to the next). Additionally, we manipulated (4) the value of *dynamic error variance* ($\boldsymbol{\sigma}^2_q$). As explained above, individual differences in this parameter would capture either the magnitude of daily events affecting the individual or their vulnerability to events experienced in their daily life.

1. Number of Days Under Study

The number of days an individual is measured affects the accuracy with which the parameters defining their affect dynamics can be recovered. We aim to determine the minimum number of days that a single person needs to be measured to accurately capture their affect dynamics with a DLO model. For this purpose, we generated data for 14, 30, 60, 90, 120, 150 and 180 days. We decided to include a condition with 14 days of data collection as the minimal setting because it is a common time frame in applied studies on affect. We extended our conditions up to 180 days because there was no previous information on what is the appropriate number of data points needed to recover the parameters correctly, and consequently we needed to achieve a high number of repeated measurements to examine what is the optimal duration.

2. *Number of Measurement Occasions per Day*

In most empirical studies on affect dynamics individuals are measured once per day. We hypothesize that more than one measurement per day might be helpful to characterize the dynamics of a single person, particularly if the process under study is affected by dynamic error. To investigate this aspect, we generated data in three different conditions: (1) one measurement occasion per day, (2) two measurement occasions per day, and (3) four measurement occasions per day.

3. *Measurement Occasions at Fixed Intervals or at Varying Intervals*

Most empirical studies omit whether measurements were taken at the same time each day. One of our goals is to compare the recovery of the parameters characterizing an individual's affect dynamics in conditions with equal versus unequal time intervals between measurements. In the conditions with equal time intervals, measurements were taken every day at the exact same times (i.e., the same hour every day) within a fixed 12-hour window: (1) in conditions with one measurement per day we selected the first hour of each day of data collection, (2) in conditions with two measurements per day we selected the first and twelfth hour of each day of data collection, and (3) in conditions with four measurements per day we selected the first, fourth, eighth and twelfth hour of each day of data collection. In contrast, in the conditions with unequal time intervals, we randomly selected the time of measurement for each day, with equal probability for all the hours within a fixed 12-hours period. This was done to reflect the fact that individuals are not usually available to be measured at any time (e.g., when they sleep). Instead, they are more likely to be measured during their active hours.

4. *Dynamic Error Variance*

As previously stated, *dynamic error variance* ($\boldsymbol{\sigma}^2_q$) can be interpretated as the impact of daily events that displace latent scores either to increase or decrease them. Individual differences in $\boldsymbol{\sigma}^2_q$ imply that people's negative affect is affected by such events to different extents. Larger values of *dynamic error variance* ($\boldsymbol{\sigma}^2_q$) mean that larger displacements not explained by any measured variable occur, thus it may be more difficult to estimate the parameters that characterize a trajectory. On the other hand, if $\boldsymbol{\sigma}^2_q$ is equal to zero, the system would eventually reach equilibrium and cease to oscillate afterwards. After this point, the level of negative affect would be constant and there would be no affect dynamics. However, this is a very unrealistic situation in which the individual's negative affect is not influenced by daily life events.

In order to understand how different values of *dynamic error variance* affect the recovery of the generating process, we imposed the following values for $\boldsymbol{\sigma}^2_q$: 0.5, 2, 4, and 6 (see Figure 4). In empirical applications of the DLO model, the measurement structure (Equation 2), is usually not included, and thus no distinction is made between *dynamic error variance* and *measurement error variance*. Because there is not a typical value of dynamic error variance in the literature, the values selected for the simulation were meant to cover a wide range of empirical scenarios.

The combination of the four simulation factors described above led to 168 simulation conditions. For each of the 168 combinations we generated five hundred replications (i.e., 500 individuals). This number was adequate to achieve consistent results.

Summary of Simulation Conditions

1. Number of days under study: 14, 30, 60, 90, 120, 150, and 180.

2. Number of measurement occasions per day: one measurement occasion per day; two measurement occasions per day; and four measurement occasions per day.
3. Measurement Occasions at: fixed time intervals or varying time intervals
4. Dynamic error variance ($\boldsymbol{\sigma}^2_q$): 0.5, 2, 4 and 6.

(Values fixed across all conditions: $\eta = -.4$, $\zeta = -.15$, and $\boldsymbol{\sigma}^2_r = 0.75$.)

Data Analysis

For each replication (i.e., individual) in each condition, we separately estimated the DLO model described previously. We specified the model as a continuous-time state space model, using the library *OpenMx* in *R* (Boker et al., 2018; Hunter, 2018; Neale et al., 2016). State space models (SSM) are composed of two equations: the *state equation*, which describes the system latent dynamics and corresponds to Equation 1 described previously, and the *output equation* which links the latent variables with their observed indicators, and corresponds to the measurement model described in Equation 2:

$$dx(t)/dt = \boldsymbol{A}x(t) + \boldsymbol{B}u(t) + q(t) \tag{4}$$

$$y_t = \boldsymbol{C}x_t + \boldsymbol{D}u_t + r_t \tag{5}$$

The state equation[3] (Equation 4) specifies the derivative of x_t with respect to time as $dx(t)/dt$. $x(t)$ is an $l \times \mathbf{1}$ vector of latent states at time t, where l is the

[3] The state equation (Eq. 4) is in continuous time, and the notation *x(t)* indicates that *x* is a continuous function of *t*, whereas the output equation (Eq. 5) is in discrete time and the notation x_t indicates that *x* is indexed by *t*. When estimating the model, the continuous differential equation in Eq. 4 is solved for all discretely observed values of *t*.

number of latent variables. $u(t)$ is an $m \times 1$ vector of covariates, where *m* is the number of covariates. $q(t)$ is an $l \times 1$ vector of dynamic noise with mean zero and covariance matrix $\boldsymbol{Q}$ (which includes the dynamic variance parameter $\boldsymbol{\sigma}^2{}_q$ in position 2,2). $\boldsymbol{A}$ is an $l \times l$ matrix of autoregressive dynamics (including parameters η and ζ) and $\boldsymbol{B}$ is an $l \times m$ matrix capturing the effects of the time-varying covariates $u(t)$(Hunter, 2018). In SSMs, the state equation is a first-order differential equation that describes how latent states change over time. In contrast, the dynamic equation of the DLO model is a second-order differential equation that describes how the negative affect (*position*) and the first derivative of negative affect (*velocity*) change over time. This apparent problem is solved by defining the first-order derivative (dx/dt) as an additional latent variable that is not linked to any observed indicator (see, for example, Voelkle & Oud, 2013). The specification used in this study includes no covariates, therefore $B = 0$ and $u(t) = 0$.

In the output equation (Equation 5), y_t is an $n \times 1$ vector of manifest variables, where *n* is the number of manifest variables. r_t is an $n \times 1$ vector of observation noise with mean zero and covariance matrix $\boldsymbol{R}$. $\boldsymbol{C}$ is an $n \times l$ matrix of factor loadings, and $\boldsymbol{D}$ is an $n \times m$ matrix of covariate effects on the observation (Hunter, 2018). Because no covariates are included in this study, $D = 0$ and $u_t = 0$.

We estimated this model through the functions *mxExpectationStateSpaceContinuousTime* and *mxFitFunctionML* in *OpenMx*. These functions estimate the model parameters through a set of recursive algorithms called the hybrid Kalman Filter. This process recursively estimates the state vector and state covariance matrix through a series of prediction and update steps. In the prediction step, it generates forecasts at any time *t* based on the latent state vector and covariance matrix at the previous time. In the update step, it uses the observed data to update the predictor from the previous step. At the end of this process, the Kalman Filter adjusts the parameters of the model reducing prediction error decomposition through *maximum likelihood prediction error decomposition* (Hunter, 2018; Kalman, 1960; Neale et al., 2016). A tutorial website with computer code in R for applying the model, and notes on how to use it, is provided in the following link: https://marjfollero.github.io/DLO/Tutorial.html

The next section reports the results obtained after applying this estimation method to the simulated data. For each of the 168 simulation conditions, we computed: the rate of improper solutions, the degree of estimation accuracy and efficiency for each parameter, the rate of 95% confidence interval (*CI*) coverage of the parameter estimates, and the relative bias of the *Standard Errors* (*SE*s) for each parameter.

Results

Improper solutions

In some of the replications, the estimation procedure led to convergence problems and improper solutions. A solution was considered improper when the optimization algorithm a) was stuck in a region of confusing geometry (*OpenMx* status code 5), or b) the optimizer could not find any way to improve the estimates (*OpenMx* status code 6).

The number of replications leading to improper solutions varied across the simulation conditions, with the duration of the study being the most important factor. Table 1 reports the percentage of improper solutions in the conditions with relevant rates of them. Improper solutions were very infrequent (<1.8%) in conditions with 60 days or more under study. In contrast, in conditions with 14 days under study, improper solutions were very frequent (range 1%-63%, median = 18%). This was also the case with 30 days under study when only one measurement occasion per day was available (rates ranging from 8% to 19%, median = 11%). With 30 or more days, and two or more time points per day, improper solutions never exceeded 5%.

In general, an increase in the number of measurements occasions per day led to a decrease in the percentage of improper solutions. Additionally, improper solutions were more frequent when $\boldsymbol{\sigma}^2_q$=0.5. Interestingly, no remarkable differences were observed between conditions with fixed vs. varying time intervals. Only proper solutions were used for the subsequent analyses.

INSERT TABLE 1 ABOUT HERE

Estimation Accuracy

As an indicator of accuracy, we computed Relative Bias (RB) for each parameter in the model as $RB = (\overline{\theta}_{est} - \theta)/\theta$, where θ is the true parameter value and $\overline{\theta}_{est}$ is the average estimated value of the parameter across all the replications in a given condition. Values of *RB* closer to zero imply unbiased estimates, positive values imply overestimation, and negative values imply underestimation. According to previous literature (Flora & Curran, 2004; Rhemtulla et al., 2012), we consider parameters with $|RB| < .10$ (bias under 10% of the parameter true value) as adequate estimates. We consider parameters with $.10 \leq |RB| \leq .15$ (bias between 10 and 15% of the parameter true value) as tolerable bias. Additionally, in each replication we computed the total relative bias for all parameters as $RMSRB = \sqrt{\sum_{k=1}^{K} RB_k^2 / K}$,where K=4 is the number of parameters in the model. The values of Root Mean Squared Relative Bias (RMSRB) are reported in Table 2 and depicted in Figure 6. The values of *RB* for each parameter are reported in Table 3.

In general, the total relative bias decreased as the number of days under study increased. The first important finding in this regard is that the results were always unacceptable with 14 days under study. In other words, even in the best conditions, at least 30 days were required to obtain unbiased estimates.

Furthermore, the number of measurement occasions per day had a large influence on the total relative bias across all conditions. Particularly, the total relative bias was much larger in conditions with one measurement occasion per day (1 O/D), compared to two or four O/D. We found *RMSRB* > .18 in all the conditions with 1 O/D (median *RMSRB* = .27), regardless of the dynamic error variance and the type of intervals (fixed vs. varying). The results from Table 3 indicate that this poor performance is due to heavily biased estimation of the dynamic and measurement error variances with 1 O/D. Therefore, the second important finding from our study is that 1 O/D is not enough to adequately characterize individual affect dynamics with a DLO model. The differences between the conditions with two and four measurement occasions per day were not large, although the total relative bias was generally lower in the conditions with four measurement occasions per day.

Interestingly, the dynamic error variance had an important effect on the estimation accuracy. We found substantial bias in most conditions with low dynamic error variance ($\boldsymbol{\sigma}^2_q$= 0.5). In such conditions, at least two O/D and 120 days were required to obtain overall bias below 10%. With higher values of $\boldsymbol{\sigma}^2_q$, the results were generally acceptable with 60 or more days (with fixed intervals), or 30 or more days (with varying intervals).

Another interesting finding was that, in conditions with $\boldsymbol{\sigma}^2_q$= 4 and $\boldsymbol{\sigma}^2_q$= 6, and only with varying time intervals, *RMSRB* increased with the number of days under study. This was not observed with fixed time intervals. This interaction pattern is another important finding from our study: with two or more O/D, using varying time intervals (i.e., sampling affect at random hours during the day) results in fewer study days required (i.e., 30 days in most conditions) to achieve unbiased estimates. However, and surprisingly, varying time intervals led to higher bias, compared to fixed intervals, in most of the conditions with 90 or more days.

INSERT TABLE 2 AND FIGURE 6 ABOUT HERE

Regarding the bias for the specific parameter estimates (reported in Table 3), the frequency parameter (η) was generally unbiased with 30 or more days, even in conditions where the overall relative bias (*RMSRB*) was not acceptable. The damping parameter (ζ) was acceptably recovered in most conditions with *RMSRB* < 0.10, with *RB* values generally below .10, or just above such value. The damping parameter was unbiased also in some conditions with 1 O/D, as long as a large number of days were available. In contrast, the dynamic error variance ($\boldsymbol{\sigma}^2_q$) was generally biased in the conditions where *RMSRB* > 0.10 and was frequently biased when $\boldsymbol{\sigma}^2_q$=0.5. This parameter was biased also in conditions with varying intervals and $\boldsymbol{\sigma}^2_q$=6. Finally, the measurement error variance ($\boldsymbol{\sigma}^2_r$) was better recovered with at least 2 O/D. In fact, it was unbiased in all the conditions with 4 O/D, regardless the number of days under study and the value of $\boldsymbol{\sigma}^2_q$. In summary, our results indicate that the two variance parameters in the model appear to be harder to estimate than the frequency and damping parameters.

INSERT TABLE 3 ABOUT HERE

Variability of the Parameter Estimates

To examine the efficiency of the parameter estimates, we computed the Standard Deviation of the Relative Bias (SDRB) as $SDRB = SD\left[(\theta_{est} - \theta)/\theta\right]$, where θ_{est} is the estimation in a given replication and θ is the generating value for that parameter. *SDRB* allows expressing the estimation inefficiency in the same scale for all the parameters: the index is always positive and values closer to zero indicate lower variability of the parameter estimates in any given condition. Table 4 reports the value of *SDRB* for all the conditions in our study.

As expected, more days under study led to lower *SDRBs*. This is because the model had more information (i.e., more measurement occasions, which in this scenario implies larger sample size) and therefore, the parameter estimates were more efficient. Regarding the number of measurement occasions per day, it was largely irrelevant in the conditions with $\boldsymbol{\sigma}^2_q \geq 2$ and 60 or more days. In contrast (and surprisingly), with $\boldsymbol{\sigma}^2_q$=0.5 and 14 or 30 days under study the estimate variability sightly increased with more O/D.

Additionally, the *SDRBs* were larger in conditions where $\boldsymbol{\sigma}^2_q$=0.5, compared to conditions with higher values of the dynamic error variance. Finally, comparing for parameters, the *SDRB* was generally higher for ζ and $\boldsymbol{\sigma}^2_q$ than for η and $\boldsymbol{\sigma}^2_r$ across all conditions.

INSERT TABLE 4 ABOUT HERE

Coverage and Relative Bias of the Standard Errors

Coverage was computed as the proportion of 95% confidence intervals around the estimated parameter value that include the true parameter value. It is considered optimal when at least the 95 % of the intervals include the true parameter within its limits, and adequate if at least the 90% of the intervals include the true parameter within its limits (Collins et al., 2001; Enders & Peugh, 2004). The coverage for each parameter across all conditions is reported in Table 5.

As expected, conditions with biased estimates led to poor coverage of the true parameter values. However, we found poor coverage also in conditions with adequate *RB* values. In fact, the coverage was below 95% for at least one parameter in all the conditions of our study.

INSERT TABLE 5 ABOUT HERE

Obtaining poor coverage indexes led us to investigate the possibility that the standard errors were underestimated. To examine this hypothesis, we computed the relative bias of the *SE*s (SERB) as $SERB = \left(\theta_{\overline{SE}} - SDRB\right) / SDRB$, where $SDRB$ is the empirical standard deviation of the relative bias of each parameter, computed across all the replications of each condition, and $\theta_{\overline{SE}}$ is the mean of the standard error estimates of each parameter in each condition. Note that *SDRB* is taken as a proxy of the true variability of the sampling distribution, whereas $\theta_{\overline{SE}}$ is the estimated variability. The results regarding Standard Error bias are reported in Table 6. We found that the standard errors of all parameters were underestimated across all conditions, except for the standard errors of $\boldsymbol{\sigma}^2_r$, which were anecdotally unbiased in some conditions with 1 O/D. Therefore, the bad coverage rates were due to the underestimation of the parameter standard errors. For this reason, even in replications with very accurate point estimates, the confidence intervals were too narrow and often excluded the true parameter value.

INSERT TABLE 6 ABOUT HERE

Discussion

Summary of Findings

In this paper, we had three objectives: (1) to connect the key features of the DLO model with the clinical aspects that they capture, and therefore to present the model in an intuitive way to applied practitioners, (2) to test the performance of the DLO model under different empirically-relevant conditions through a simulation study, and (3) based on our findings, to make specific recommendations on how to design individual-based studies on negative affect when the researcher plans to examine its dynamics through a DLO model. We addressed our first goal through a detailed step-by-step description of the model in the introduction. In this section, we summarize the most relevant results and discuss the model performance. In the last section of this paper, we make several recommendations for applied researchers.

The first relevant finding is that, from the standpoint of adequate model convergence, if the individual is measured once a day, at least 60 days are required to minimize the incidence of improper solutions. Higher values of dynamic error variance led to low rates of improper solutions with 30 days, but the value of $\boldsymbol{\sigma}^2_q$ for a given individual is unknown for the researcher before conducting the study, and therefore cannot be taken into consideration for planning it. Importantly, taking two or more daily measures, either at fixed or varying intervals, for at least 30 days, led to very low rates of improper solutions in the conditions examined here.

Regarding the accuracy of the parameter estimates, we found that at least two daily measurements are needed to obtain unbiased estimates. Indeed, overall model bias was unacceptable in all the conditions with one measurement occasion per day. We found acceptable bias (based on *RB* values) for all four parameters of the DLO model in only a restricted set of combinations of conditions (see the bold type results in Table 3, and the summary recommendations in the section “Conclusions and Recommendations”).

As expected, our results indicate that the estimation is more efficient (i.e., estimates show lower variability across replications) when the individual is measured for more days. However, and surprisingly, more measurement occasion per day lead to lower variability only in some (but not all) conditions. Dynamic error variance ≥ 2 led to similar efficiency across the conditions. Finally, coverage indexes were very poor in all conditions due to generalized underestimation of standard errors. Based on these findings, we make specific recommendations on how to apply the DLO model to single-case applied research in the section “Conclusions and Recommendations”.

Theoretical and Methodological Implications

This paper aims to increase the accessibility of the damped linear oscillator model for studying the dynamics of psychological processes at the individual level. Although we have previously discussed that, in some circumstances, an individual-based approach is necessary, we want to emphasize that it also implies different requirements, as typically more days under study and a higher sampling frequency are required in single-individual studies. In our simulation study, we generated data for hypothetical individuals with an oscillatory period of negative affect of approximately 10 days. In this context, taking two or four daily measurements either at fixed or varying time intervals within the same window of 12 hours each day led to the best results in terms of accuracy and efficiency of the parameter estimates.

Our results are in line with previous studies showing that a higher number of measurement occasions results in more accurate parameter estimates (McKee et al., 2018). A previous simulation study on the DLO model (Voelkle & Oud, 2013) reached the conclusion that unequal (instead of fixed) time intervals between measurements result in lower bias in the parameter estimates. In that study, one DLO model (without a parameter for measuring error variance) was fitted in continuous time to simulated samples of 200 individuals measured for 11 days. Our results are not directly comparable, because we fitted DLO models including measurement error variance to single individuals evaluated for at least 14 days. However, we did find somewhat better performance when sampling at varying time intervals under 30 days. Importantly, the performance under 14-day conditions was promising. Under these conditions, when a minimum of two measurements were available and the dynamic variance was greater than 0.5, the occurrence of improper solutions did not exceed 14%. Furthermore, although the overall relative bias rates were not satisfactory, η and $\boldsymbol{\sigma}^2{}_r$ were recovered well in several conditions and in some conditions, although to a lesser extent, $\boldsymbol{\sigma}^2{}_q$ and ζ. Nevertheless, we found that the performance of fixed time intervals sampling improved with more days under study, particularly with higher values of dynamic error variance, to the point of obtaining lower bias than varying sampling for some combinations of dynamic error and number of days (see Figure 6). It must be noted, though, that, under many conditions, whether the measurement occasions are collected at fixed or varying time intervals leads to fairly similar recovery of the parameter estimates.

In general, empirical and methodological studies do not report whether measurements are taken at the same time every day or at unequal intervals. In contrast to this, our study takes into account unequal intervals for data generation, allowing for more flexibility in data collection. Furthermore, methodological studies typically do not account for the fact that individuals are not always available to be measured. In our study, we considered that the individual was only available 12 hours each day, which is more realistic in empirical applications.

In brief, our study includes several factors that we believe facilitate the transmission of our results to the applied field. To begin with, we differentiate between dynamic error variance and measurement error. We think this distinction is necessary since, although they are both noise to the model, the source of origin is not the same and they impact the system under study differently. Given that, we provide the results for a wide range of dynamic error variance values giving information to the applied researcher on a variety of scenarios. In addition, we included conditions with measurement occasions at the same time and at different times of the days, but taking into consideration that participants are not available at any time of the day, thus generating data that are closer to substantive settings.

In this study, we considered scenarios with one, two, and four measurement occasions per day. However, the number of daily measurements required to characterize a phenomenon depends on the timescale of the phenomenon under study. If we want to uncover the dynamics of a phenomenon that has a shorter timescale, such as heart rate, we will need a higher sampling frequency each day and probably fewer days under study. In contrast, if the timescale is larger, as might be the case for weight fluctuations, then we might need fewer measurement occasions within a day, but we will probably need to measure the individual for more days. Otherwise, insufficient sampling frequency will lead to inaccurate parameter recovery and therefore we would not be able to characterize the dynamics of the phenomena (Haslbeck & Ryan, 2021). Thus, our results apply to negative affect and other phenomena having a similar oscillation timescale.

In the present study, we specified the DLO model as a state-space model in continuous time. As previously mentioned, the continuous-time specification has several advantages: (1) the underlying process is considered to evolve continuously over time and, therefore it is not necessary to use equal sampling intervals (although our results indicate that equal intervals may perform better in some conditions), (2) the parameters can be transformed to a discrete-time metric, and (3) it allows comparison of parameters estimated at different time intervals (Deboeck & Preacher, 2016; Ryan et al., 2018; Voelkle et al., 2012; Voelkle & Oud, 2013). In addition, a SSM in continuous time approach allows to estimate the model through the Kalman Filter. This is one of the innovations of this paper, as previous studies have used others methods such as the *local linear approximation* (see Steele & Ferrer, 2011) and the *general local linear approximation* (see Boker et al., 2010b). The estimation method applied in this paper has the important advantage over these other methods of simultaneously modeling both stochastic innovation (i.e., dynamic error variance, parameter $\boldsymbol{\sigma}^2_q$) and measurement error variance (parameter $\boldsymbol{\sigma}^2_r$) (Boker et al., 2010a).

The application of this model is very useful for several purposes. First, this model could be used to conduct applied research on affect dynamics or other phenomena that theoretically follow an oscillatory trajectory over time, both from an individual-based approach and from a group-based approach. Second, based on the *Adaptative Equilibrium Regulation* framework (Boker, 2015), after characterizing a person's affect dynamics, it would be possible to examine whether certain events lead to changes in the parameters characterizing affect dynamics (i.e., a regime change, Chow et al., 2018). For example, the effectiveness of a psychological intervention aimed at increasing the individual's emotional resilience could be assessed through a change in the resilience parameter estimate (ζ). Similarly, changes in the individual's environment could lead to larger and more frequent perturbations in negative affect, and this could be reflected by a higher value in the dynamic error variance. Relatedly, an adequate characterization of the individual's emotional lability and resilience would allow predicting the time needed for recovering their equilibrium (i.e., regulation) after a specific emotional shock (e.g., a one-time traumatic event such as the death of a close person). In fact, it would be possible to *empirically test* whether such a one-time shock leads to a stable regime change (i.e., an adaptation) or merely perturbs the trajectory at a given time point. Furthermore, in contexts with limited resources, characterizing the affective dynamics of several individuals affected by a recent event could inform how the resources should be allocated so the individuals most in need of a psychological intervention are prioritized based on their parameter estimates, as otherwise they would take longer to recover homeostasis after the shock.

We note that, although this model can be very useful in applied settings, accurate parameter estimation is difficult in some conditions. Therefore, we advise caution in the parameter interpretation (particularly of the dynamic error variance), especially if they were estimated under conditions found to be problematic.

Limitations and Future Directions

According to the *Adaptative Regulation Framework* (Boker, 2015), affect dynamics can experience both regulation and adaptation over time. Regulation is the ability of the system to recover equilibrium in response to perturbations on a short time scale, whereas adaptation is the ability to manage persistent forces over the long term. Importantly, the processes of adaptation and regulation occur simultaneously, but on different timescales. In this study we have focused on regulation, and therefore our results are constrained to such process. In addition, they are also limited to negative affect and other phenomena with similar timescale.

Of course, our findings apply to the specific conditions that we simulated. Although such conditions are empirically relevant, other scenarios are also possible in studies of emotion dynamics. For example, future research should examine the model's performance in situations with more frequent sampling (e.g., 8 or 10 measurement occasions per day), during 14 or fewer days. Here, it is important to take into account the timescale of what is being studied. If the phenomenon has a very large period oscillation (i.e., a cycle implies many days), then by increasing the sampling frequency further information will not be obtained. We recommend measuring during at least one cycle and we consider that future research could study how many measurements are needed within a day to characterize phenomena with different period oscillations.

Other authors have conducted simulation studies to provide recommendations on how to measure affect with versions of the DLO model different from the specification proposed here. For example, McKee et al. (2018) fitted an extension of the damped linear oscillator that includes both regulation and adaptation processes. On their part, Hu & Huang (2018) conducted a simulation study with the Driven DLO, which is an extension of the DLO model that focuses on regulation and includes a term that allows for a non-zero steady state. Additionally, the model could be extended to account for both time-dependent and time-invariant covariates, such as the day of the week or the presence of a psychological disorder, that could also explain the dynamics of negative affect (Adolf et al., 2017). Such covariates were not considered in our study. A special case worth mentioning is sleep/wake cycles. These cycles could influence the dynamics of negative affect by cyclically changing parameter values. This feature is difficult to assess because individuals are not available at any time of the day. We addressed this problem by considering an availability of 12 hours per day, allowing us to simulate a more realistic situation. Finally, the DLO can be extended to the bivariate case, which allows modeling two variables at the same time and examining the influence of each variable on the other. Some variables that would be susceptible to be modeled together with negative affect are, for example, stress or positive affect.

Although not considering these scenarios is a limitation of our work, we believe that it is necessary to examine the model performance in a simpler scenario – a univariate model with one timescale and no covariates– before adding more complexity. Therefore, we believe that future research should focus on developing formal theories to guide researchers on how to time the assessments to study negative affect in the applied context, while methodological research in particular should focus on accounting for such more complex situations.

A relevant limitation regarding the estimation method proposed in this study is the underestimation of the standard errors. Importantly, this limitation does not seem to affect the point estimates of the model parameters, but is certainly of concern, as hinders the construction of confidence intervals of the estimated parameters. Future research should explore when and why this underestimation of standard errors occurs, as well as compare different estimation methods. In addition, it would be particularly interesting to explore and develop new methods for characterizing the affect dynamics of individuals who have been measured during periods shorter than 60 days, either using different sampling schemes (e.g., measuring certain days per week), adopting an extension of the DLO model or applying alternative estimation methods.

Conclusions and Recommendations

Studying a dynamical system from and individual-based approach is necessary when: a) working with phenomena that present high inter- and intra-individual variability or b) in the context in which the research findings are to be applied, the focus is on the individual rather than the group. Our results support the DLO model as a valuable and appropriate tool for studying affect dynamics from a single-individual approach as long as the following requirements are met:

1. A 14-day data collection period can prove valuable for exploratory research, especially if the researcher is interested in estimating those parameters that can be adequately recovered under such conditions, according to our results. However, for other purposes, the individual should be measured at least for 30 days, two times per day.
2. In general, studies with a duration of 30 to 90 days led to better estimates when combined with varying time intervals (i.e., two or more measurements per day, taken at random times each day within the same 12-hour window).
3. In contrast, with more than 90 days under study, we found somewhat better results with fixed time intervals (i.e., two or more measurements per day, evenly spaced and taken at the same time every day within a fixed 12-hour window).
4. Characterization of affect dynamics appears to be more difficult with very high and, particularly, with very low degree of stochasticity, as captured by the dynamic error variance parameter. We advise the researcher to base on their theoretical expectations, together with preliminary visual inspection of the longitudinal trajectory, to get a sense of the degree of stochasticity in their data (see Figure 4 for several examples). If a low value for the dynamic error variance is expected, at least 4 measurements per day and 90 days of study are recommended. In any case, we recommend interpreting very high and very low estimates for dynamic error variance with special caution.

In recent years, single-individual research is making a resurgence and we encourage researchers to take part in it. In view of our results, it is possible to do applied research from this approach, and the gains at the theoretical level are large. We hope that researchers will find this study useful for designing and analyzing oscillatory dynamic processes in an individualized way.

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Figure 1

Illustration of the bowl-ball metaphor based on Boker (2015)

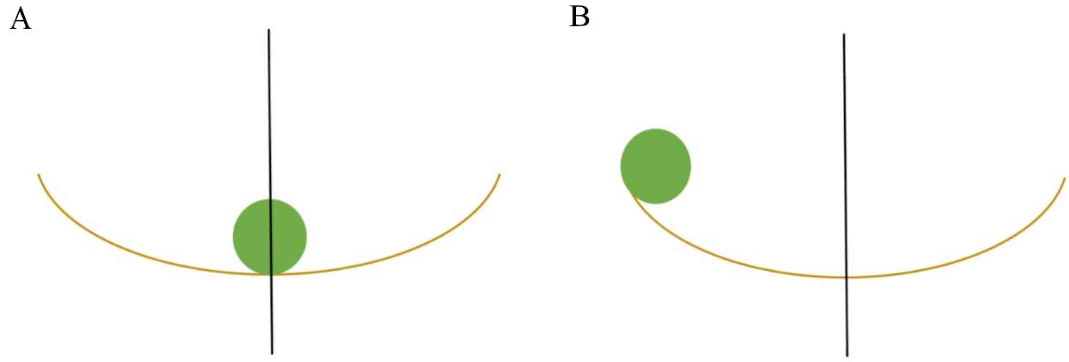


Figure 2

Trajectories for three different values of emotional lability (η)

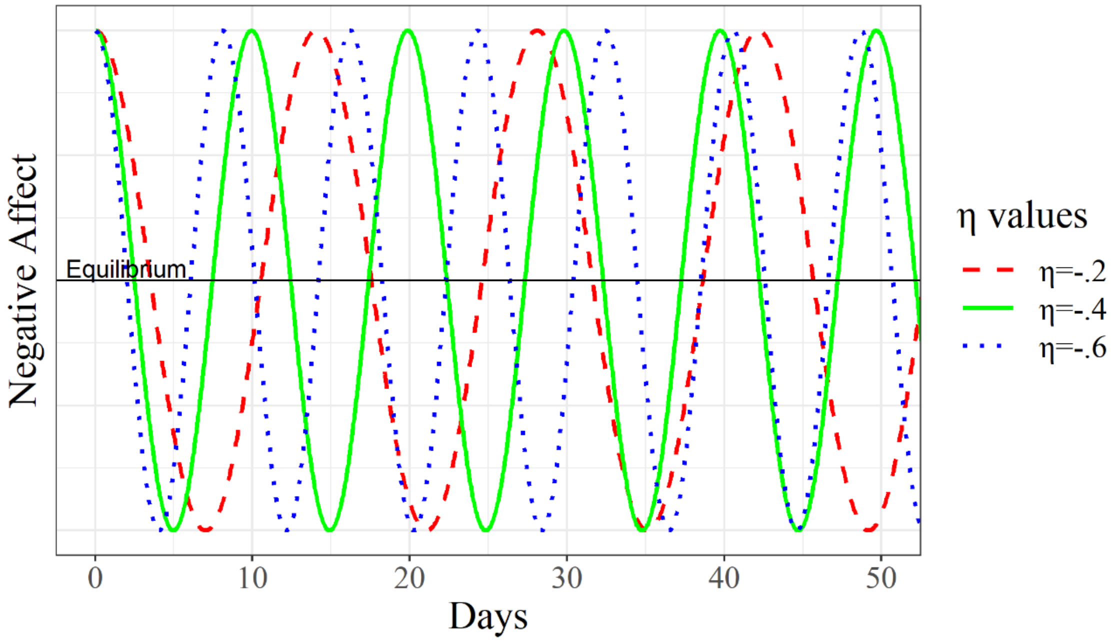


Figure 3

Trajectories for three different values of resilience (ζ)

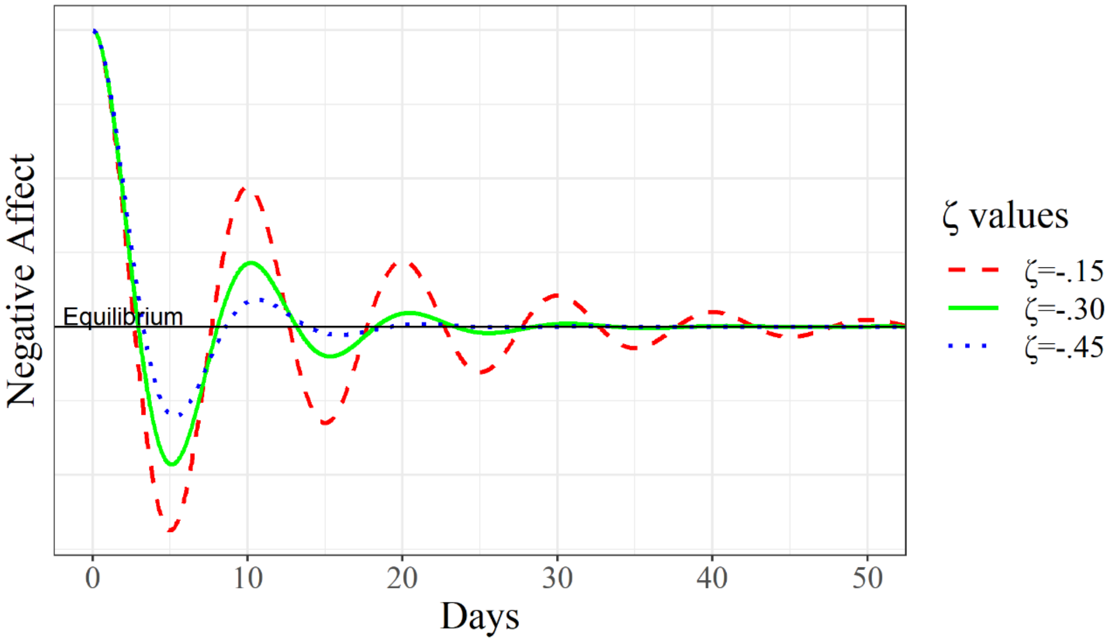


Figure 4

Trajectories for four different values of dynamic error variance ($\boldsymbol{\sigma}^2{}_q$*)*

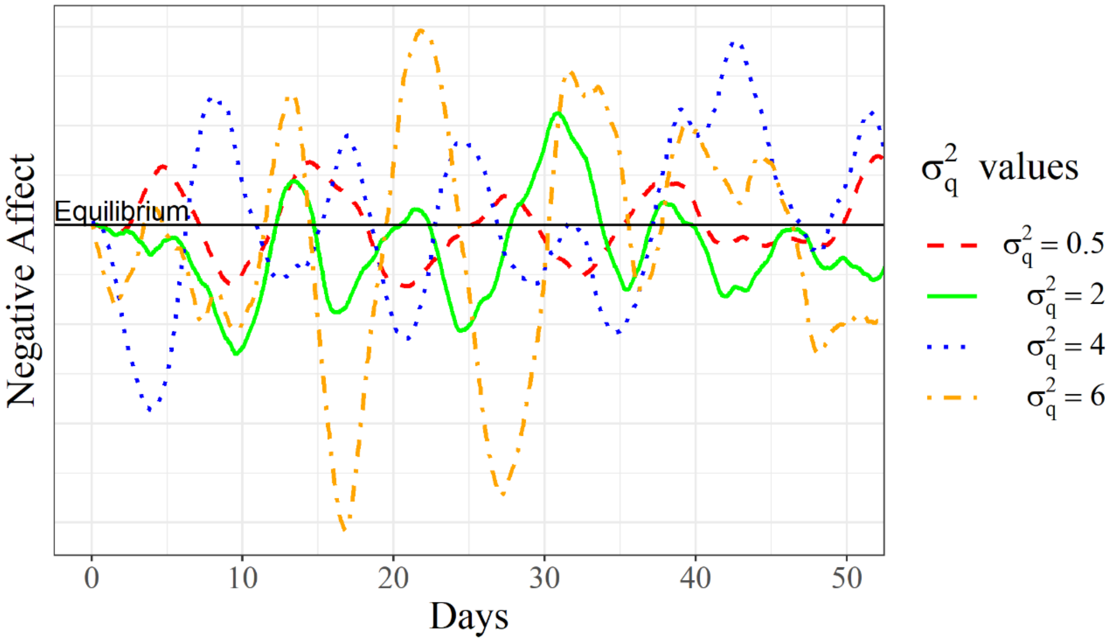


Figure 5

Trajectories for the same individual, without measurement error and with different values of measurement error variance ($\boldsymbol{\sigma}^2{}_r$)

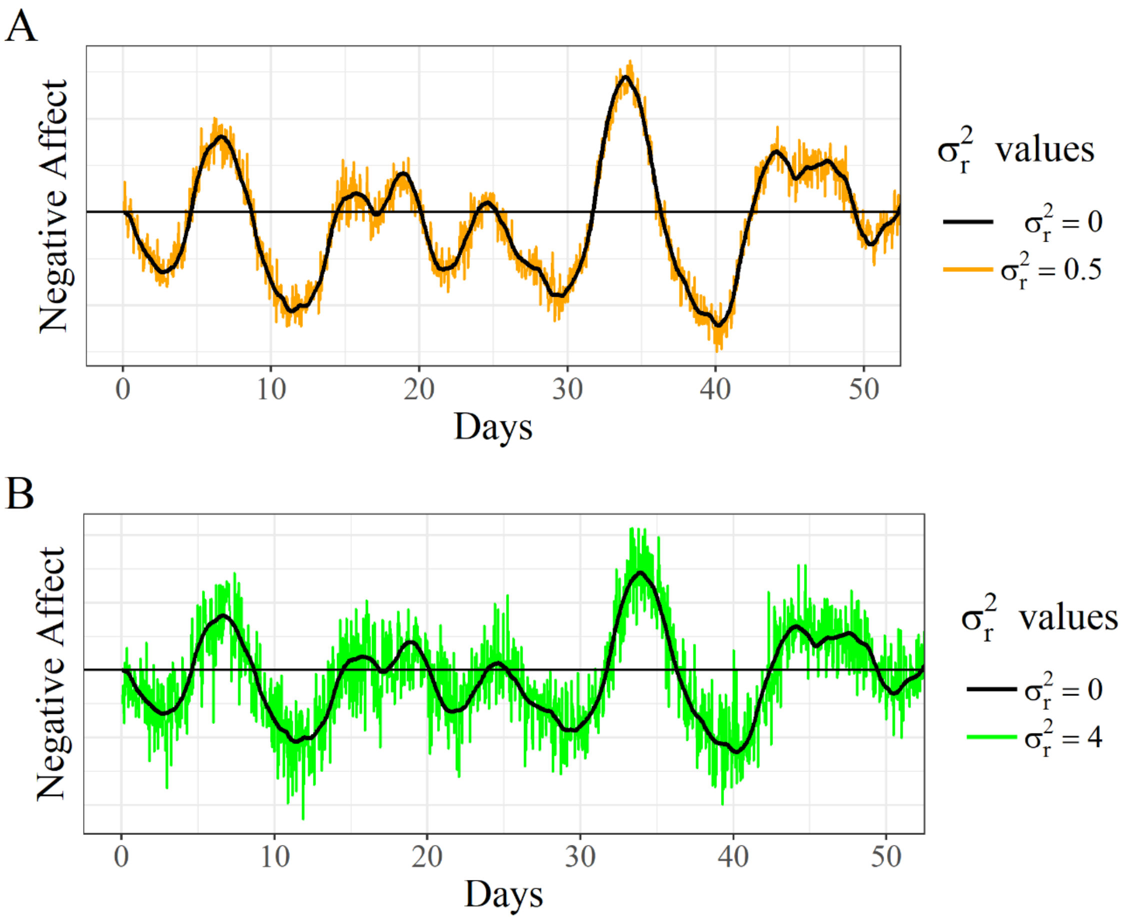
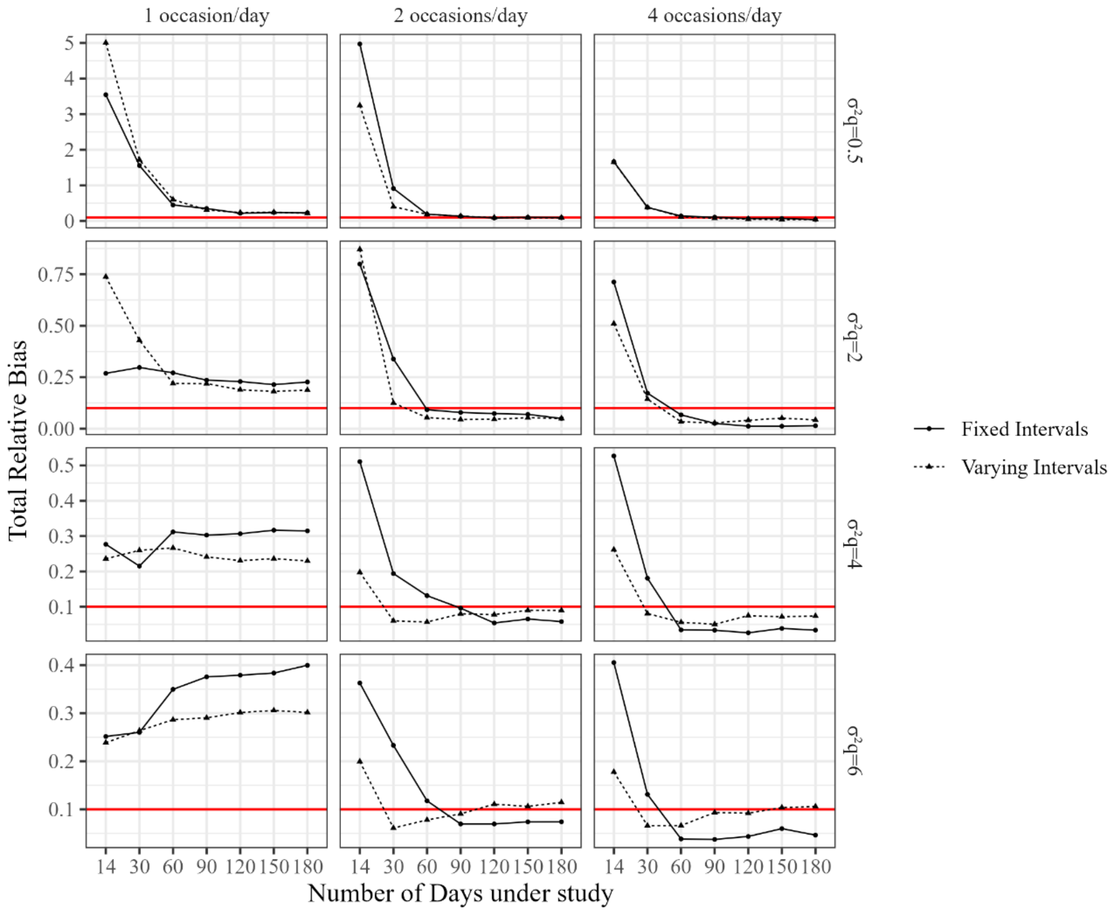


Figure 6

Total Relative Bias across all conditions



Note: Total Relative Bias = $RMSRB = \sqrt{\sum_{k=1}^{K} RB_k^2 / K}$ (where K = 4 is the total number of parameters in the model) is depicted for each condition. The horizontal red line indicates *RBMSRB* =.10. Note that the scale for the vertical axis is different for each value of $\boldsymbol{\sigma}^2_q$.

Table 1

Percentage of improper solutions across all conditions

O/D	σ^2_q	MO at Fixed Intervals			MO at Varying Intervals		
		14	30	60	14	30	60
1	**0.5**	63	19	2	53	16	2
	2	53	9	.4	43	4	-
	4	47	8	1	43	5	1
	6	46	13	1	42	9	1
2	**0.5**	35	2	.2	36	5	.2
	2	14	.2	-	17	-	-
	4	11	-	-	9	-	-
	6	9	-	-	5	-	-
4	**0.5**	19	1	.2	20	2	-
	2	6	-	-	7	-	-
	4	2	-	-	2	-	-
	6	1	-	-	1	-	-

Note: From 60 days under study the percentage of improper solutions did not exceed 1.8%. The dashed line indicates that there were no improper solutions in that condition. MO = Measurement occasions; O/D= occasions per day.

Table 2

Total Relative Bias of the DLO model across all conditions

O/D	σ^2_q	MO at Fixed Intervals							MO at Varying Intervals						
		14	**30**	**60**	**90**	**120**	**150**	**180**	**14**	**30**	**60**	**90**	**120**	**150**	**180**
1	**0.5**	3.54	1.56	.45	.35	.22	.24	.23	5.01	1.72	.60	.31	.24	.25	.22
	2	.27	.30	.27	.24	.23	.21	.23	.74	.43	.22	.22	.19	.18	.19
	4	.28	.22	.31	.30	.31	.32	.31	.24	.26	.27	.24	.23	.24	.23
	6	.25	.26	.35	.38	.38	.38	.40	.24	.26	.29	.29	.30	.31	.30
2	**0.5**	4.97	.91	.19	.13	.08	.10	.09	3.24	.40	.19	.13	.10	.09	.09
	2	.80	.34	.09	.08	.07	.07	.05	.87	.12	.05	.04	.05	.05	.05
	4	.51	.19	.13	.10	.05	.07	.06	.20	.06	.06	.08	.08	.09	.09
	6	.36	.23	.12	.07	.07	.07	.07	.20	.06	.08	.09	.11	.11	.11
4	**0.5**	1.66	.38	.14	.11	.07	.07	.04	1.65	.38	.12	.08	.05	.04	.04
	2	.71	.17	.07	.03	.01	.01	.01	.51	.14	.03	.03	.04	.05	.04
	4	.53	.18	.03	.03	.03	.04	.03	.26	.08	.06	.05	.07	.07	.07
	6	.41	.13	.04	.04	.04	.06	.05	.18	.07	.07	.09	.09	.10	.11

Note: Total Relative Bias, $RMSRB = \sqrt{\sum_{k=1}^{K} RB_k^2 / K}$ (where $K = 4$ is the total number of parameters in the model) is reported for each condition. Green shading (dark grey) indicates $RMSRB$ <.10 and yellow shading (light gray) indicates $.10 \leq |RBMSRB| \leq .15$. MO = measurement occasions; O/D= occasions per day. See the online article for the color version of this table

Table 3

Relative Bias of each parameter across all conditions

O/D	σ²q	Par	MO at Fixed Intervals 14	30	60	90	120	150	180	MO at Varying Intervals 14	30	60	90	120	150	180
1	0.5	ζ	3.74	1.22	.34	.27	.12	.12	.14	4.88	1.44	.48	.20	.12	.15	.08
		η	.55	.11	-.02	-.03	-.04	-.04	-.03	.71	.08	.01	-.03	-.04	-.04	-.04
		σ²q	5.97	2.85	.82	.63	.40	.44	.43	8.69	3.11	1.09	.57	.44	.45	.41
		σ²r	-.55	-.26	-.15	-.13	-.10	-.12	-.11	-.56	-.24	-.13	-.11	-.11	-.13	-.11
	2	ζ	.02	.23	.16	.08	.03	.04	.04	.92	.47	.11	.07	.02	-.03	-.02
		η	.00	-.05	-.04	-.05	-.05	-.05	-.05	.02	-.03	-.06	-.05	-.05	-.05	-.05
		σ²q	.27	.41	.38	.30	.28	.26	.28	1.06	.66	.31	.31	.26	.23	.24
		σ²r	-.47	-.36	-.35	-.35	-.36	-.33	-.35	-.46	-.27	-.29	-.30	-.27	-.27	-.28
	4	ζ	-.35	-.04	.10	.03	.00	-.01	-.01	.02	.20	.09	.02	-.03	-.09	-.07
		η	-.05	-.06	-.06	-.06	-.05	-.06	-.06	.00	-.08	-.06	-.07	-.06	-.07	-.06
		σ²q	-.04	.04	.22	.19	.19	.20	.20	.10	.23	.22	.18	.16	.13	.13
		σ²r	-.42	-.42	-.57	-.57	-.58	-.60	-.60	-.46	-.41	-.47	-.44	-.43	-.44	-.43
	6	ζ	-.35	-.12	-.02	-.02	-.07	-.07	-.06	.02	.05	.02	-.05	-.09	-.10	-.09
		η	-.09	-.09	-.07	-.07	-.06	-.06	-.06	-.07	-.07	-.07	-.08	-.08	-.07	-.07
		σ²q	-.19	-.05	.10	.11	.10	.12	.13	-.01	.11	.12	.11	.10	.10	.10
		σ²r	-.30	-.50	-.69	-.74	-.75	-.75	-.78	-.47	-.51	-.55	-.56	-.58	-.59	-.58
2	0.5	ζ	4.31	.84	.25	.17	.10	.11	.09	3.59	.43	.24	.14	.06	.05	.03
		η	.50	.07	.02	.00	-.01	-.01	.00	.39	.01	-.03	-.03	-.03	-.03	-.02
		σ²q	8.94	1.62	.28	.21	.13	.17	.16	5.39	.67	.28	.23	.18	.17	.17
		σ²r	-.22	-.04	-.04	-.02	-.02	-.03	-.02	-.16	-.05	-.03	-.03	-.03	-.03	-.03
	2	ζ	1.04	.53	**.13**	**.11**	**.08**	**.08**	**.02**	1.29	.21	**.06**	**-.01**	**-.06**	**-.09**	**-.07**
		η	.11	.02	**-.01**	**-.01**	**-.01**	**-.01**	**-.02**	.06	-.03	**-.04**	**-.05**	**-.04**	**-.05**	**-.05**
		σ²q	1.20	.41	**.12**	**.10**	**.11**	**.10**	**.08**	1.17	.12	**.05**	**.05**	**.02**	**.01**	**.01**
		σ²r	-.16	-.08	**-.05**	**-.05**	**-.06**	**-.06**	**-.06**	-.10	-.03	**-.05**	**-.05**	**-.05**	**-.04**	**-.04**
	4	ζ	.79	.34	.21	**.12**	**.00**	**.02**	**.00**	.32	**.07**	**-.04**	**-.12**	**-.12**	**-.15**	**-.15**
		η	.06	.01	.00	**-.02**	**-.02**	**-.01**	**-.01**	.00	**-.05**	**-.07**	**-.05**	**-.05**	**-.06**	**-.05**
		σ²q	.63	.15	.12	**.11**	**.04**	**.06**	**.05**	.21	**-.06**	**-.06**	**-.07**	**-.07**	**-.07**	**-.07**
		σ²r	-.16	-.11	-.10	**-.11**	**-.10**	**-.11**	**-.10**	-.10	**-.05**	**-.06**	**-.05**	**-.05**	**-.05**	**-.05**
	6	ζ	.63	.40	.16	**.05**	**.02**	**.01**	**-.03**	.38	**.07**	**-.10**	**-.13**	-.17	-.16	-.18
		η	.03	.01	-.02	**-.02**	**-.02**	**-.02**	**-.01**	-.02	**-.07**	**-.05**	**-.06**	-.06	-.06	-.07
		σ²q	.32	.19	.09	**.03**	**.03**	**.03**	**.03**	.09	**-.05**	**-.10**	**-.09**	-.12	-.11	-.11
		σ²r	-.16	-.15	-.15	**-.12**	**-.13**	**-.14**	**-.14**	-.07	**-.05**	**-.03**	**-.05**	-.05	-.05	-.06
4	0.5	ζ	2.35	.61	.25	.18	**.10**	**.10**	**.05**	2.29	.62	.20	**.12**	**.06**	**.01**	**-.01**
		η	.19	.04	.01	.00	**.00**	**.00**	**-.01**	.19	.02	-.01	**-.02**	**-.02**	**-.02**	**-.02**
		σ²q	2.35	.46	.12	.12	**.09**	**.10**	**.06**	2.38	.44	.13	**.10**	**.08**	**.07**	**.08**
		σ²r	-.05	.00	-.01	-.01	**-.01**	**-.01**	**-.01**	-.04	-.02	-.01	**-.01**	**-.01**	**-.01**	**-.01**
	2	ζ	1.31	.33	**.13**	**.05**	**.00**	**.00**	**-.02**	.95	.28	**.05**	**-.02**	**-.06**	**-.08**	**-.07**
		η	.10	.00	**.00**	**.00**	**-.01**	**-.02**	**-.01**	.04	-.02	**-.03**	**-.03**	**-.04**	**-.04**	**-.03**
		σ²q	.56	.10	**.01**	**.01**	**.02**	**.00**	**-.01**	.36	.03	**-.03**	**-.04**	**-.04**	**-.04**	**-.04**
		σ²r	-.02	-.01	**-.02**	**-.02**	**-.01**	**-.01**	**-.01**	.00	-.01	**-.01**	**-.01**	**-.02**	**-.01**	**-.01**
	4	ζ	.97	.36	**.06**	**-.03**	**-.02**	**-.05**	**-.05**	.49	**.13**	**-.06**	**-.04**	**-.10**	**-.11**	**-.12**
		η	.04	.00	**-.02**	**-.01**	**-.02**	**-.02**	**-.02**	.05	**-.03**	**-.04**	**-.04**	**-.05**	**-.03**	**-.04**
		σ²q	.40	.06	**-.03**	**-.05**	**-.04**	**-.05**	**-.04**	.18	**-.08**	**-.09**	**-.08**	**-.11**	**-.09**	**-.08**
		σ²r	-.03	-.01	**-.02**	**-.02**	**-.01**	**-.02**	**-.01**	-.03	**-.02**	**-.01**	**-.01**	**-.01**	**-.02**	**-.01**
	6	ζ	.80	.26	**.03**	**-.01**	**-.05**	**-.09**	**-.07**	.35	**.03**	**-.04**	**-.13**	**-.13**	-.16	-.16
		η	.04	.01	**-.01**	**-.02**	**-.02**	**-.03**	**-.02**	.02	**-.04**	**-.04**	**-.05**	**-.05**	-.05	-.05
		σ²q	.13	-.04	**-.07**	**-.07**	**-.07**	**-.08**	**-.06**	.04	**-.12**	**-.12**	**-.12**	**-.12**	-.13	-.13
		σ²r	-.01	-.02	**-.01**	**-.01**	**-.01**	**-.01**	**-.01**	-.01	**-.01**	**-.01**	**.00**	**-.01**	-.01	-.01

Note: $RB = (\overline{\theta}_{est} - \theta)/\theta$ is reported for each parameter. Green shading (dark gray) indicates $|RB| < .10$ and yellow shading (light gray) indicates $.10 \le |RB| \le .15$. Bold type indicates the conditions with acceptable, or very close to acceptable, bias in all four parameters ($|RB| \le .13$). MO = measurement occasions; O/D= occasions per day; Par=parameters

Table 4

Standard Deviation of the Relative Bias for each parameter across all conditions

O/D	$\sigma^2 q$	Par	MO at Fixed Intervals							MO at Varying Intervals						
			14	30	60	90	120	150	180	14	30	60	90	120	150	180
1	0.5	ζ	5.06	3.29	1.17	.83	.55	.48	.49	5.18	3.24	1.82	.85	.74	.52	.43
		η	1.15	.58	.20	.12	.10	.09	.08	1.27	.57	.29	.13	.11	.09	.08
		$\sigma^2 q$	8.14	7.44	2.19	1.31	.67	.63	.63	13.47	7.27	3.10	1.37	1.17	.69	.51
		$\sigma^2 r$	.48	.42	.28	.23	.20	.17	.16	.44	.41	.30	.25	.19	.17	.16
	2	ζ	2.24	1.24	.78	.58	.43	.37	.37	3.55	1.79	.69	.56	.43	.37	.32
		η	.50	.21	.13	.10	.08	.08	.07	.64	.29	.13	.10	.08	.07	.06
		$\sigma^2 q$	1.20	1.08	.77	.54	.42	.35	.33	3.47	1.92	.66	.52	.42	.35	.31
		$\sigma^2 r$	.53	.48	.38	.27	.25	.22	.20	.53	.50	.34	.27	.24	.22	.19
	4	ζ	2.06	.91	.67	.46	.42	.36	.30	2.30	1.09	.77	.56	.43	.32	.33
		η	.36	.21	.13	.09	.08	.07	.07	.53	.20	.12	.09	.08	.07	.06
		$\sigma^2 q$	1.30	.61	.50	.38	.33	.29	.27	1.11	.83	.58	.42	.32	.27	.26
		$\sigma^2 r$	.63	.59	.38	.33	.29	.25	.25	.62	.54	.35	.33	.28	.25	.23
	6	ζ	2.40	.86	.59	.46	.36	.33	.28	2.05	1.02	.58	.47	.38	.33	.29
		η	.38	.18	.12	.09	.08	.07	.06	.40	.19	.11	.09	.08	.07	.06
		$\sigma^2 q$	.97	.50	.38	.29	.24	.22	.21	.90	.68	.39	.31	.25	.24	.22
		$\sigma^2 r$	.82	.60	.37	.30	.27	.26	.24	.63	.54	.41	.33	.28	.26	.24
2	0.5	ζ	5.15	2.57	.95	.64	.48	.46	.41	5.00	1.84	.85	.63	.47	.42	.39
		η	1.04	.38	.16	.12	.10	.08	.08	.94	.24	.16	.11	.10	.08	.07
		$\sigma^2 q$	16.01	6.23	.83	.60	.39	.41	.34	10.47	2.86	.82	.55	.40	.35	.29
		$\sigma^2 r$	.34	.25	.15	.13	.11	.10	.09	.29	.23	.15	.12	.11	.09	.09
	2	ζ	3.38	1.74	.79	.52	.46	.39	.35	3.51	1.44	.67	.50	.38	.32	.30
		η	.53	.23	.13	.10	.09	.08	.07	.58	.22	.13	.10	.08	.07	.07
		$\sigma^2 q$	3.56	1.92	.53	.34	.30	.27	.23	4.31	1.18	.40	.30	.24	.21	.21
		$\sigma^2 r$	.37	.25	.17	.14	.12	.10	.09	.34	.24	.16	.13	.11	.10	.10
	4	ζ	3.19	1.21	.80	.55	.40	.34	.33	2.79	1.13	.54	.41	.35	.29	.26
		η	.43	.22	.13	.10	.09	.07	.07	.46	.21	.11	.10	.08	.07	.06
		$\sigma^2 q$	2.22	.75	.43	.31	.25	.21	.21	1.73	.55	.30	.24	.20	.18	.15
		$\sigma^2 r$	.41	.26	.18	.16	.13	.11	.10	.34	.24	.16	.15	.12	.11	.10
	6	ζ	2.73	1.35	.71	.49	.43	.35	.32	2.65	1.01	.52	.39	.33	.29	.26
		η	.41	.22	.13	.10	.09	.08	.07	.42	.19	.12	.10	.08	.07	.06
		$\sigma^2 q$	1.41	.74	.38	.27	.23	.21	.18	1.37	.47	.27	.20	.17	.16	.13
		$\sigma^2 r$	.41	.27	.18	.15	.13	.12	.11	.36	.24	.18	.14	.13	.11	.10
4	0.5	ζ	4.36	1.96	.92	.60	.49	.40	.36	4.53	1.88	.83	.57	.48	.37	.32
		η	.69	.29	.14	.12	.10	.08	.07	.71	.28	.14	.11	.09	.08	.07
		$\sigma^2 q$	5.96	1.91	.55	.41	.33	.28	.24	5.53	1.78	.63	.38	.31	.26	.24
		$\sigma^2 r$	.21	.14	.10	.08	.07	.06	.06	.21	.15	.09	.08	.07	.06	.06
	2	ζ	3.51	1.42	.70	.51	.39	.36	.33	3.32	1.21	.72	.44	.37	.30	.30
		η	.53	.22	.13	.10	.09	.07	.07	.47	.20	.13	.10	.08	.07	.07
		$\sigma^2 q$	2.28	.84	.35	.26	.22	.20	.18	1.92	.62	.34	.24	.20	.17	.17
		$\sigma^2 r$	.22	.14	.10	.08	.07	.07	.06	.21	.14	.11	.08	.07	.06	.06
	4	ζ	3.18	1.45	.61	.45	.37	.32	.28	2.78	1.09	.55	.45	.39	.32	.28
		η	.41	.21	.13	.10	.09	.07	.07	.41	.21	.12	.10	.09	.07	.07
		$\sigma^2 q$	2.01	.74	.30	.21	.19	.16	.15	2.14	.50	.26	.21	.18	.15	.15
		$\sigma^2 r$	.22	.15	.11	.08	.08	.06	.06	.21	.14	.10	.09	.08	.07	.06
	6	ζ	2.85	1.10	.62	.49	.38	.31	.28	2.55	.87	.57	.40	.34	.30	.27
		η	.43	.21	.13	.10	.09	.08	.07	.35	.19	.13	.10	.08	.07	.07
		$\sigma^2 q$	1.09	.46	.26	.21	.17	.14	.14	1.35	.38	.24	.18	.16	.15	.13
		$\sigma^2 r$	.23	.16	.10	.09	.07	.07	.06	.22	.14	.11	.08	.08	.07	.06

Note: $SDRB = SD\left[(\theta_{est} - \theta)/\theta\right]$ is reported for each parameter. MO = measurement occasions; O/D= occasions per day; Par=parameters.

Table 5

Coverage for each parameter across all conditions

O/D	$\sigma^2 q$	Par	MO at Fixed Intervals							MO at Varying Intervals						
			14	30	60	90	120	150	180	14	30	60	90	120	150	180
1	0.5	ζ	94	92	95	96	94	97	95	97	94	93	95	97	95	94
		η	96	90	89	91	91	90	91	95	88	90	92	89	89	90
		$\sigma^2 q$	96	88	95	96	98	99	98	98	91	93	96	98	98	99
		$\sigma^2 r$	79	87	89	87	89	87	87	85	85	91	86	86	83	86
	2	ζ	90	94	95	95	97	96	95	88	94	95	93	94	94	95
		η	90	89	91	90	88	86	84	93	90	89	91	85	87	85
		$\sigma^2 q$	84	88	95	98	98	98	98	83	88	95	96	98	97	97
		$\sigma^2 r$	82	86	81	72	65	64	58	84	90	81	75	75	68	65
	4	ζ	82	90	94	97	95	95	96	85	95	92	92	93	92	89
		η	85	88	89	88	86	82	82	85	87	89	83	85	76	80
		$\sigma^2 q$	76	85	97	96	98	98	96	79	88	94	96	98	96	97
		$\sigma^2 r$	88	88	78	63	52	41	36	89	90	76	68	65	54	50
	6	ζ	80	91	94	94	94	94	96	85	91	93	93	91	93	91
		η	86	85	86	86	85	82	82	85	87	89	84	80	76	71
		$\sigma^2 q$	72	88	95	97	98	99	99	78	89	94	96	97	97	97
		$\sigma^2 r$	92	94	83	62	44	31	21	93	93	76	62	53	41	37
2	0.5	ζ	94	91	96	95	94	97	95	96	91	94	94	95	96	94
		η	95	90	92	92	92	94	94	95	89	90	93	91	90	93
		$\sigma^2 q$	93	84	91	93	95	96	96	93	87	92	94	97	97	98
		$\sigma^2 r$	86	91	91	91	93	91	94	79	88	94	91	92	92	94
	2	ζ	90	94	96	96	95	96	95	92	93	95	93	93	92	92
		η	90	92	94	93	93	94	92	90	90	90	86	90	87	83
		$\sigma^2 q$	81	86	92	96	96	96	96	83	86	92	94	94	95	93
		$\sigma^2 r$	86	89	91	90	93	92	88	85	89	89	89	89	92	89
	4	ζ	87	95	94	96	95	95	94	85	88	94	90	89	91	89
		η	88	91	94	92	89	96	92	88	86	89	86	85	80	81
		$\sigma^2 q$	78	88	94	96	94	97	96	74	82	88	87	89	88	90
		$\sigma^2 r$	86	89	86	84	83	79	79	84	90	89	87	89	90	88
	6	ζ	90	94	95	96	94	94	92	87	91	91	91	87	87	86
		η	91	91	92	92	93	91	91	87	88	88	84	83	81	75
		$\sigma^2 q$	81	89	95	93	95	94	95	74	84	85	86	83	82	83
		$\sigma^2 r$	88	85	84	85	79	71	71	86	90	89	89	89	88	85
4	0.5	ζ	92	92	97	97	95	95	95	88	94	95	95	95	96	95
		η	91	90	94	92	93	94	94	91	91	90	92	92	93	93
		$\sigma^2 q$	85	84	93	94	94	97	95	84	88	91	95	94	95	97
		$\sigma^2 r$	91	95	93	94	94	95	94	89	90	95	95	92	95	94
	2	ζ	91	94	94	95	94	94	93	88	94	93	96	93	93	92
		η	88	92	94	93	90	93	94	88	92	90	90	91	91	88
		$\sigma^2 q$	81	89	89	93	95	92	92	75	85	89	91	91	93	91
		$\sigma^2 r$	92	93	92	94	94	93	95	94	95	92	95	93	95	95
	4	ζ	89	94	95	93	94	92	96	89	94	94	94	89	91	89
		η	91	95	92	92	91	93	92	91	88	91	89	84	89	85
		$\sigma^2 q$	79	89	89	90	90	90	92	73	82	85	86	84	86	84
		$\sigma^2 r$	92	94	93	93	94	93	94	93	94	95	93	92	93	96
	6	ζ	91	95	93	93	93	93	94	87	94	94	90	87	88	86
		η	90	90	93	91	92	89	92	90	88	87	87	88	87	81
		$\sigma^2 q$	83	86	88	88	89	88	89	76	81	84	82	80	75	75
		$\sigma^2 r$	93	92	94	93	95	95	93	93	95	92	95	94	95	93

Note: Coverage is the proportion of 95% confidence intervals around the estimated parameter value that include the true parameter value. Green shading (dark gray) indicates 95% or higher coverage rates. MO = measurement occasions; O/D= occasions per day; Par=parameters.

Table 6

Relative Bias of Standard Errors for each parameter across all conditions

O/D	σ^2q	Par	MO at Fixed Intervals							MO at Varying Intervals						
			14	30	60	90	120	150	180	14	30	60	90	120	150	180
1	0.5	ζ	-.76	-.90	-.87	-.88	-.86	-.86	-.87	-.77	-.89	-.88	-.89	-.90	-.87	-.86
		η	-.30	-.65	-.66	-.60	-.62	-.61	-.59	-.44	-.62	-.45	-.65	-.67	-.61	-.61
		σ^2q	.59	-.60	-.60	-.60	-.51	-.53	-.58	-.11	-.56	-.62	-.68	-.72	-.58	-.52
		σ^2r	.87	-.10	-.18	-.24	-.26	-.24	-.22	.53	-.11	-.24	-.32	-.26	-.25	-.26
	2	ζ	-.84	-.85	-.86	-.86	-.85	-.84	-.86	-.85	-.85	-.85	-.86	-.85	-.85	-.85
		η	-.67	-.64	-.60	-.63	-.61	-.63	-.61	-.71	-.65	-.62	-.62	-.62	-.63	-.60
		σ^2q	2.43	1.54	1.03	.94	.99	1.10	1.02	2.22	2.01	1.02	.94	.89	.92	1.02
		σ^2r	.15	-.08	-.21	-.17	-.25	-.21	-.25	.16	.09	-.24	-.25	-.26	-.31	-.25
	4	ζ	-.84	-.85	-.86	-.86	-.85	-.84	-.86	-.85	-.85	-.85	-.86	-.85	-.85	-.85
		η	-.67	-.64	-.60	-.63	-.61	-.63	-.61	-.71	-.65	-.62	-.62	-.62	-.63	-.60
		σ^2q	2.43	1.54	1.03	.94	.99	1.10	1.02	2.22	2.01	1.02	.94	.89	.92	1.02
		σ^2r	.15	-.08	-.21	-.17	-.25	-.21	-.25	.16	.09	-.24	-.25	-.26	-.31	-.25
	6	ζ	-.87	-.83	-.84	-.84	-.84	-.85	-.83	-.84	-.84	-.84	-.85	-.85	-.85	-.84
		η	-.68	-.63	-.63	-.62	-.63	-.64	-.61	-.67	-.63	-.61	-.63	-.65	-.64	-.64
		σ^2q	6.78	7.73	6.39	6.49	6.53	6.17	6.04	10.33	7.26	6.24	5.65	5.76	5.37	5.23
		σ^2r	.14	.30	.21	.15	.05	-.01	-.04	.60	.23	-.07	-.13	-.15	-.20	-.21
2	0.5	ζ	-.78	-.90	-.87	-.86	-.85	-.86	-.86	-.83	-.89	-.86	-.87	-.86	-.86	-.86
		η	-.61	-.72	-.62	-.63	-.62	-.60	-.62	-.63	-.64	-.65	-.60	-.64	-.60	-.61
		σ^2q	-.12	-.73	-.52	-.58	-.49	-.56	-.52	-.40	-.71	-.56	-.54	-.50	-.50	-.46
		σ^2r	-.05	-.30	-.24	-.28	-.28	-.25	-.23	-.22	-.30	-.23	-.25	-.25	-.20	-.23
	2	ζ	-.86	-.89	-.87	-.85	-.86	-.86	-.86	-.85	-.89	-.86	-.86	-.85	-.84	-.85
		η	-.69	-.62	-.62	-.61	-.62	-.61	-.61	-.69	-.66	-.63	-.64	-.62	-.62	-.63
		σ^2q	1.22	.20	.76	1.05	.98	.95	.99	.96	.23	.99	1.01	1.08	1.05	.87
		σ^2r	-.23	-.27	-.26	-.27	-.23	-.22	-.23	-.27	-.28	-.26	-.26	-.25	-.24	-.29
	4	ζ	-.86	-.86	-.88	-.86	-.85	-.84	-.86	-.88	-.87	-.84	-.85	-.85	-.84	-.84
		η	-.64	-.64	-.62	-.61	-.65	-.60	-.62	-.71	-.67	-.60	-.66	-.64	-.65	-.63
		σ^2q	3.84	2.61	2.82	2.99	2.88	3.28	2.86	2.21	2.64	3.02	2.90	3.07	2.87	3.13
		σ^2r	-.20	-.23	-.25	-.30	-.26	-.23	-.26	-.26	-.26	-.25	-.31	-.25	-.28	-.25
	6	ζ	-.86	-.88	-.87	-.85	-.86	-.85	-.86	-.86	-.86	-.85	-.84	-.84	-.84	-.84
		η	-.64	-.63	-.62	-.63	-.62	-.64	-.65	-.66	-.65	-.64	-.66	-.65	-.65	-.65
		σ^2q	6.13	4.37	4.81	5.06	4.84	4.87	5.18	4.66	4.77	4.87	5.39	5.05	4.80	5.37
		σ^2r	-.20	-.23	-.22	-.23	-.23	-.25	-.27	-.28	-.24	-.28	-.26	-.29	-.29	-.29
4	0.5	ζ	-.85	-.89	-.88	-.86	-.86	-.85	-.85	-.84	-.89	-.87	-.86	-.86	-.85	-.84
		η	-.67	-.67	-.61	-.64	-.63	-.61	-.62	-.65	-.67	-.63	-.62	-.62	-.61	-.61
		σ^2q	-.50	-.66	-.52	-.52	-.52	-.49	-.49	-.35	-.64	-.59	-.51	-.50	-.49	-.49
		σ^2r	-.26	-.20	-.27	-.26	-.24	-.26	-.23	-.27	-.30	-.20	-.24	-.29	-.25	-.25
	2	ζ	-.87	-.87	-.86	-.86	-.85	-.86	-.86	-.87	-.87	-.88	-.85	-.85	-.84	-.85
		η	-.68	-.64	-.63	-.60	-.62	-.62	-.61	-.67	-.62	-.63	-.63	-.63	-.61	-.63
		σ^2q	.86	.76	.94	1.03	1.07	.94	.97	.91	.80	.89	1.06	1.08	1.11	.96
		σ^2r	-.25	-.22	-.27	-.22	-.25	-.27	-.24	-.21	-.22	-.29	-.21	-.24	-.21	-.22
	4	ζ	-.87	-.88	-.86	-.85	-.85	-.85	-.84	-.87	-.87	-.85	-.86	-.86	-.85	-.85
		η	-.62	-.63	-.64	-.64	-.64	-.63	-.62	-.67	-.66	-.63	-.63	-.67	-.64	-.66
		σ^2q	2.35	2.42	2.76	3.18	3.02	3.09	3.00	2.85	2.38	2.91	3.00	2.92	3.03	2.79
		σ^2r	-.25	-.26	-.27	-.24	-.27	-.22	-.24	-.21	-.23	-.23	-.25	-.29	-.25	-.23
	6	ζ	-.87	-.86	-.86	-.86	-.85	-.84	-.84	-.88	-.85	-.86	-.85	-.85	-.85	-.85
		η	-.65	-.64	-.65	-.63	-.65	-.65	-.63	-.63	-.65	-.66	-.65	-.63	-.64	-.66
		σ^2q	5.15	4.51	4.92	4.88	5.06	5.52	5.05	4.06	4.86	5.04	5.05	5.04	4.66	4.96
		σ^2r	-.25	-.27	-.22	-.25	-.20	-.26	-.27	-.24	-.20	-.27	-.22	-.27	-.24	-.25

Note: $SERB = \left(\theta_{\overline{SE}} - SDRB\right) / SDRB$ is reported for each parameter (SDRB = Standard Deviation of the Relative Bias).. Green shading (dark gray) indicates |*SERB*|<.10. MO = measurement occasions; O/D= occasions per day; Par=parameters.