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Unit Commitment with Analytical Under-frequency Load-shedding Constraints for Island Power Systems

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ABSTRACT This paper presents a novel corrective frequency-constrained unit commitment (C-FCUC) for island power systems implementing analytical constraints on underfrequency load shedding (UFLS). Since UFLS is inevitable for sufficiently large disturbances, it can be argued that less spinning reserve could be held back since UFLS takes place nonetheless. Congruently, the reserve criterion should consider UFLS likely to occur under disturbances. The C-FCUC can be converted into a preventive frequency-constrained unitcommitment (P-FCUC) or a standard security-constrained unit commitment. Thus, the C-FCUC is a generalization. The proposed formulation is successfully applied to two representative Spanish island power systems of La Palma and La Gomera. Results confirm that the proposed model can reduce generation costs while reducing the expected amount of UFLS.

INDEX TERMS Island power system, frequency stability, unit commitment, under frequency load shedding

Acronyms

BESS	battery energy storage systems
C-FCUC	corrective frequency-constrained unit commitment
ED	economic dispatch
MILP	mixed integer linear program
P-FCUC	preventive frequency-constrained unitcommitment
ReLU	rectified linear unit
RES	renewable energy source
SUC	standard unit commitment
UC	unit commitment
UFLS	underfrequency load shedding

Nomenclature

C-FCUC:

γ_j	binary operator of affine segments [$\in\{0,1\}$]
$\tilde{K}^s_{t,\ell}$	Sum of turbine-governor gain after losing unit ℓ
λ_j	weight associated with breaking point j
$a'_{t,i,s}$	Auxiliary variable [$\in \mathbb{R}^+$]
A_j	breaking point
$a_{t,l}$	Auxiliary variable [$\in \mathbb{R}^+$]
$H^s_{t,\ell}$	Available inertia after losing unit ℓ [s]
H_i	Inertia of unit i [s]
J	number of the breaking points
j	breaking point index
K_i	Base power [MW]
M	A relatively big number
$p^{\text{UFLS}}_{t,\ell}$	Amount of UFLS after outage of unit ℓ [MW]
$p^c_{t,\ell}$	Maximum power with no UFLS [MW]
S^{base}	Base power [MW]
T	Delivery time of units [s]
u'	Auxiliary variable [$\in\{0,1\}$]
u	Auxiliary variable [$\in\{0,1\}$]

standard unit commitment (SUC):

ℓ	Index of the lost unit
$\mathcal{D}$	Demand [MW]
$\mathcal{I}$	Set of all generators
$\mathcal{T}$	Set of all time intervals
$\overline{\mathcal{P}}_i$	Maximum power output of generator i [MW]
$\overline{\mathcal{R}}_i$	Maximum ramp-up of generator i [MW]
DT	Minimum down-time of generators [hours]
gc(.)	Generation costs [€]
sc(.)	Start-up costs [€]
UT	Minimum up-time of generators [hours]
$\underline{\mathcal{P}}_i$	Minimum power output of generator i [MW]
$\underline{\mathcal{R}}_i$	Maximum ramp-down of generator i [MW]

C_ℓ	Multiplier for the required reserve
i	Index of generators
p	Power variable [MW]
p^{solar}	Solar generation variable [MW]
p^{wind}	Wind generation variable [MW]
r	Online reserve power variable [MW]
s	Alias index for time intervals
t	Index of time intervals
x	Commitment variable [$\in\{0,1\}$]
y	Start-up variable [$\in\{0,1\}$]
z	Shut-down variable [$\in\{0,1\}$]

I. INTRODUCTION

OPERATIONAL planning for island power systems primarily occurs through a centralized, sequential approach managed by the system operator across various timeframes. Optimal operational planning faces constraints, notably concerning the security of the power supply. Typically, ensuring the security of supply entails mandating a minimum level of spinning reserve. This reserve is tapped into when unforeseen disturbances occur [1].

Currently, the spinning reserve criterion remains static, ensuring that the reserve capacity is adequate to cover disturbances up to $N-1$ in terms of power [2]. However, primary frequency control may not always activate the spinning reserve promptly enough to mitigate frequency excursions effectively. Consequently, under substantial or even moderate imbalances in active power, UFLS occurs to stabilize frequency decay in island power systems. Subsequently, non-spinning generation is rapidly initiated to counterbalance the imbalance. Given that UFLS becomes unavoidable during significant disturbances in small systems [3], one could argue for a reduction in the amount of spinning reserve held, as UFLS is bound to occur regardless. Therefore, the reserve criterion could incorporate the likelihood of UFLS during disturbances. The aim here is not to unnecessarily increase UFLS occurrences but rather to decrease the necessary spinning reserve capacity.

The literature has extensively addressed the timely activation of spinning reserves through primary frequency control under the framework of P-FCUC, which incorporates constraints on post-disturbance frequency indices. P-FCUC methodologies can broadly be categorized into analytical (e.g., [4]–[6]) or data-driven approaches (e.g., [7], [8]).

In [4], the frequency nadir is approximated with high precision using a piece-wise linear function, and the resulting constraint is reformulated as a mixed integer linear program (MILP) problem using separable programming. [5] utilizes a stochastic unit commitments (UCs) to optimize various frequency-related services concurrently. The approach is designed for low inertia systems characterized by substantial renewable energy source (RES) integration. The stochastic model leverages scenario trees generated through a quantile-based scenario generation method. To linearize the frequency nadir constraint, an inner approximation method is applied to one side of the constraint, while a binary expansion technique is used to approximate the other side as a MILP problem employing the big-M method. [6] introduces a two-stage chance-constrained stochastic optimization method to determine the optimal thermal UC and placement of virtual inertia. Among the data-driven approaches, optimal classifier tree is utilized in [8], deep neural network is employed in [9], and logistic regression is used in [3], among other approaches. [7] compares an analytical frequency-constrained UC with data-driven models leveraging machine learning, highlighting their respective advantages and disadvantages.

However, relying solely on primary frequency control may not always suffice to restrain frequency deviations promptly within island power systems. Particularly, in scenarios of significant active power imbalances, such as those resulting from the outage of a single generating unit, UFLS may become necessary to stabilize the frequency. Non-spinning generation is swiftly engaged to rectify the imbalance and replenish the spinning reserve. Given the inevitability of UFLS during substantial disturbances [10], [11], one might argue that holding a reduced amount of spinning reserve is reasonable since UFLS will occur regardless. Thus, the reserve criteria could incorporate the probability of UFLS during disturbances, thereby reducing the ongoing requirement for spinning reserve and subsequently lowering the generation costs of the system.

A methodology presented in [12] combines economic dispatch (ED) with dynamic simulations to assess the cost of spinning reserve against the risk of load shedding for a known UC schedule. Integrating UFLS into security-constrained UC results in C-FCUC, which, despite its potential advantages, has received limited attention. A preliminary step toward C-FCUC is discussed in [13], where an analytical expression is introduced to estimate UFLS, assuming a linear increase in total generation over time and a known disturbance. The resulting non-linear expression is approximated by predefined discrete UFLS blocks, which are further linearized using K-block piecewise linear functions. In a recent advancement [14], the previous approach is expanded by distinguishing between fast and slow generation, resulting in a nonconvex expression for estimating UFLS. Again, discrete UFLS blocks are assumed, enabling the definition of a set of convex second-order cones based on the considered blocks. This approximation of the nonconvex expression leads to a mixed-integer second-order cone program.

Another solution to alleviate the inertia scarcity and improve the frequency response is by adding fast frequency response devices to the system. [15] introduces a probabilistic optimization scheme for configuring battery energy storage systems (BESS) to effectively manage uncertain power fluctuations and ensure frequency deviation remains within predetermined thresholds. Then in [16] a multi-objective optimization approach for determining the optimal siting and sizing of BESS is introduced to effectively manage frequency excursions and alleviate line overload during significant disturbances.

While most of the mentioned P-FCUC formulations strive to circumvent UFLS, which is achievable in larger systems but smaller and particularly island power systems it's either impractical or prohibitively costly. Previous C-FCUC proposals have assumed convexification of the formulation using UFLS blocks and presumed an overall linear increase in generation responses over fixed time intervals. However, it's crucial to note that generation responses vary based on the generation dispatch in small island power systems. Moreover, these proposals often either consider known disturbance sizes or focus solely on the outage of the largest generation units, disregarding the fact that even the failure of medium-sized generation units can trigger UFLS [17]. The incorporation of constraints depicting potential UFLS, thereby leading to a C-FCUC approach, has received limited attention. Based on the argument in [18] the power flow constraints are ignored in the presented paper, firstly because the frequency is a system-wide metric. Secondly, Operators typically address the unit commitment and energy dispatch challenges sequentially, first establishing nominal plant operation and subsequently adjusting for line flow deviations.

This paper contributes to bridging this gap by formulating analytical constraints for UFLS. The proposed C-FCUC approach transforms into a P-FCUC approach when faced with significant UFLS costs, while the SUC is obtained by omitting the UFLS-related constraints. The contributions of this paper are as follows:

- a C-FCUC framework that integrates and optimizes potential UFLS alongside inertia and spinning reserves to ensure system security following the outage of each generation unit.
- An analytical formulation designed to estimate the potential amount of UFLS by considering individual generation responses and constraints, rather than relying on overall generation responses.
- Exploration of two real-world Spanish island power systems, selected to represent stereotypical sizes that mirror numerous existing islands worldwide.

The rest of the paper is organized as follows: first, the SUC formulation is briefly reviewed in section II. The formulation of the proposed C-FCUC is presented in section III. Section IV applies the proposed C-FCUC to two Spanish island power systems. Section V concludes the paper.

II. SUC FORMULATION

The operational planning problem of island power systems is typically formulated as a Unit Commitment (UC) problem, aimed at minimizing variable operation costs over a specified horizon. The UC is tasked with determining the hourly schedule of generating units for the given time horizon, which could range from weekly UC to daily UC schedules, among others. The SUC is formulated as a MILP problem, as described in [3].

$$\min_{x,p} \ \mathrm{sc}(x_{t,i}) + \mathrm{gc}(p_{t,i}) \tag{1a}$$

$$x_{t,i} - x_{t-1,i} = y_{t,i} - z_{t,i} \qquad t \in \mathcal{T},\ i \in \mathcal{I} \tag{1b}$$

$$y_{t,i} + z_{t,i} \leq 1 \qquad t \in \mathcal{T},\ i \in \mathcal{I} \tag{1c}$$

$$\sum_{s=t-\mathrm{UT}_i+1}^{t} y_{s,i} \leq x_{t,i} \qquad t \in \{\mathrm{UT}_i, \ldots, \mathcal{T}\} \tag{1d}$$

$$\sum_{s=t-\mathrm{DT}_i+1}^{t} z_{s,i} \leq 1 - x_{t,i} \qquad t \in \{\mathrm{DT}_i, \ldots, \mathcal{T}\} \tag{1e}$$

$$p_{t,i} \geq \underline{\mathcal{P}}_i x_{t,i} \qquad t \in \mathcal{T},\ i \in \mathcal{I} \tag{1f}$$

$$p_{t,i} + r_{t,i} \leq \overline{\mathcal{P}}_i x_{t,i} \qquad t \in \mathcal{T},\ i \in \mathcal{I} \tag{1g}$$

$$p_{t-1,i} - p_{t,i} \leq \underline{\mathcal{R}}_i \qquad t \in \mathcal{T},\ i \in \mathcal{I} \tag{1h}$$

$$p_{t,i} - p_{t-1,i} \leq \overline{\mathcal{R}}_i \qquad t \in \mathcal{T},\ i \in \mathcal{I} \tag{1i}$$

$$\sum_{i \in \mathcal{I}} (p_{t,i}) + p_t^{\mathrm{wind}} + p_t^{\mathrm{solar}} = \mathcal{D}_t \qquad t \in \mathcal{T} \tag{1j}$$

$$\sum_{i \in \mathcal{I}, i \neq \ell} r_{t,i} \geq C_\ell \times p_\ell \qquad t \in \mathcal{T},\ \forall \ell \tag{1k}$$

The objective is to optimize eq. (1a) subject to the constraints outlined ineqs. (1b) to (1k). Equations (1b) and (1c) encode the binary logic constraints inherent in the UC problem. Equations (1d) and (1e) represent the minimum up-time and minimum downtime constraints for the units, respectively. Equation 1f ensures that each generating unit operates at or above its minimum power generation level. Equation 1g imposes an upper limit on the power generation, ensuring that the sum of power generation and power reserve for each online unit does not exceed its maximum output capacity. Equations (1h) and (1i) enforce the ramp-down and ramp-up constraints on the units, respectively. Equation (1j) represents the power balance equation, ensuring that power generation matches power demand. Finally, Equation 1k is the spinning reserve constraint, guaranteeing sufficient reserve to compensate for active power disturbances in the event of the loss of generating unit ℓ. The coefficient C_ℓ is set to 1.0 for thermal unit loss, while for modeling a single equivalentRESs generation, C_ℓ represents the expected fraction of RESs generation to be lost.

III. C-FCUC FORMULATION

The C-FCUC method estimates the required amount of UFLS following a disturbance, although it does not determine its distribution among the loads. This section elucidates the procedure for estimating UFLS and integrating it into the SUC formulation.

A. ESTIMATION OF UFLS

Short-term frequency dynamics predominantly hinge on inertia (whether physical or emulated) and turbine-governor systems. The pivotal generation loss resulting from the outage of the generating unit ℓ, denoted as $p^c_{t,\ell}$, which induces the nadir frequency deviation to breach a predetermined acceptable threshold $\Delta F^{\mathrm{nadir}}$, can be approximated using eq. (2) (refer to [19]).

$$2H^s_{t,\ell} \times \hat{K}^s_{t,\ell} \times (\Delta F^{\mathrm{nadir}})^2 \times (S^{\mathrm{base}})^2 = (p^c_{t,\ell})^2 \tag{2}$$

where

$$H^s_{t,\ell} = \sum_{i\in\mathcal{I},i\neq\ell} H_i \cdot x_{t,i} \tag{3}$$

And

$$\hat{K}^s_{t,\ell} = \sum_{i\in\mathcal{I},i\neq\ell} \frac{K_i}{T_i} \times x_{t,i} \tag{4}$$

If the outage of a generation unit, $C_\ell \times p_{t,\ell}$, exceeds $p^c t, i$, UFLS is triggered. Conversely, if the outage is less than $p^c t, \ell$, no UFLS is necessary. In cases where the nadir frequency deviation ΔF^{nadir} is sufficiently large, the critical imbalance always surpasses the outage of each generation unit, resulting in no UFLS. Ideally, the amount of shed load, $p^{\text{UFLS}}_{t,\ell}$, can be determined as follows:

$$p^{\text{UFLS}}_{t,\ell} = \begin{cases} C_\ell \times p_{t,\ell} - p^c_{t,\ell}, & \text{if } \ C_\ell \times p_{t,\ell} > p^c_{t,\ell} \\ 0, & \text{otherwise.} \end{cases} \tag{5}$$

This ideal value of $p^{\text{UFLS}}_{t,\ell}$ serves as an estimate for both advanced and conventional UFLS schemes, as proposed in the existing literature [19].

Equation (2) holds under the condition that generation output limits are not exceeded. This assurance is ensured by imposing the following constraint on the absolute generation output, allowing each generation unit i to supply the necessary power during the transient.

$$p_{t,i} + \frac{\hat{K}_i x_{t,i}}{\hat{K}^s_{t,\ell}} p^c_{t,\ell} \leq \overline{\mathcal{P}}_{t,i} x_{t,i} \quad i \in \mathcal{I},\ i \neq \ell \tag{6}$$

To formulate the C-FCUC, eqs. (2), (5) and (6) need to be linearized and integrated into the SUC formulation described in section II for the outage of each generation unit ℓ.

B. LINEARIZING UFLS-RELATED EXPRESSIONS

The non-linear equations that are proposed to estimate the amount of UFLS involve decision variables from SUC. In the following subsections, we will address the non-linearities in Equations eq. (2), eq. (5), and eq. (6) to derive linear, equivalent expressions.

1) Bi-linear terms:

Considering the definitions of $H^s t, \ell$ and $\hat{K}^s t, \ell$ as outlined in eq. (3) and eq. (4), the product of these variables will introduce binary-on-binary non-linearities. Addressing these non-linearities is crucial for achieving a linearized formulation.

$$H^s_{t,\ell} \times \hat{K}^s_{t,\ell} = \begin{bmatrix} \frac{K_1}{T_1} \\ \vdots \\ \frac{K_\ell}{T_\ell} \\ \vdots \\ \frac{K_n}{T_n} \end{bmatrix}^{-1} \begin{bmatrix} x_1x_1 & x_1x_2 & \dots & x_1x_\ell & \dots & x_1x_n \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x_\ell x_1 & x_\ell x_2 & \dots & x_\ell x_\ell & \dots & x_\ell x_n \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x_n x_1 & x_n x_2 & \dots & x_n x_\ell & \dots & x_n x_n \end{bmatrix} \begin{bmatrix} H_1 \\ \vdots \\ H_\ell \\ \vdots \\ H_n \end{bmatrix} \tag{7a}$$

To linearize each of the binary-on-binary products, let $u_{ij} = x_i x_j$. Then the following set of constraints can be used to compute u_{ij},

$$u_{ij} \leq x_i \tag{8a}$$
$$u_{ij} \leq x_j \tag{8b}$$
$$u_{ij} \geq x_i + x_j - 1 \tag{8c}$$

It's noted that the matrix of x's is symmetric, and the diagonal elements are in the form of $x_i x_i$, which simplifies to x_i. This characteristic aids in reducing the computational workload.

2) Quadratic term:

A piece-wise linearization method is employed to linearize the quadratic term $(p^c_{t,\ell})^2$.

$$p^c_{t,\ell} = \sum_{j=0}^{J} A_j \lambda_j \tag{9a}$$
$$(p^c_{t,\ell})^2 \approx \sum_{j=0}^{J} (A_j)^2 \lambda_j \tag{9b}$$
$$\sum_{j=0}^{J} \lambda_j = 1 \tag{9c}$$
$$\sum_{j=1}^{J} \gamma_j = 1 \tag{9d}$$
$$\lambda_0 \leq \gamma_1 \tag{9e}$$
$$\lambda_j \leq \gamma_j + \gamma_{j+1} \qquad j \in \{1, ..., J-1\} \tag{9f}$$
$$\lambda_J \leq \gamma_J \tag{9g}$$

Here, the A_j are fixed constants that control the approximation.

3) Conditional term:

Equation (5) essentially computes $\max(0, C_\ell \times p_{t,\ell} - p^c t, \ell)$, which is equivalent to the rectified linear unit (ReLU) function. Although the ReLU function is nonlinear, it has been utilized in MILP problems previously. The approach proposed here is inspired by [20]. Accordingly, $p^{\text{UFLS}} t, \ell$ can be estimated as follows:

$$C_\ell \times p_{t,\ell} - p^c_{t,\ell} = p^{\text{UFLS}}_{t,\ell} - a_{t,\ell} \tag{10a}$$
$$p^{\text{UFLS}}_{t,\ell} \leq M u'_{t,\ell} \tag{10b}$$
$$a_{t,\ell} \leq M(1 - u'_{t,\ell}) \tag{10c}$$

where $a_{t,\ell}$ is a positive auxiliary variable and $u'_{t,\ell}$ is a binary activation variable. The solution of eq. (10a) is not unique, unless either $p^{\text{UFLS}}_{t,\ell}$ or $a_{t,\ell}$ is zero. Equations (10b) and (10c) make sure that at least one of $p^{\text{UFLS}}_{t,\ell}$ and $a_{t,\ell}$ is zero. A more detailed discussion can be found in [20].

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4) Transient power output

Equation (6) can be written as,

$$p_{t,i}\hat{K}^s_\ell + \hat{K}_i x_{t,i} p^c_{t,\ell} \leq \overline{\mathcal{P}}_{t,i} x_{t,i} \hat{K}^s_\ell \quad i \in \mathcal{I},\ i \neq \ell \tag{11}$$

Given the definition of $K^s\ell$ in eq. (4), eq. (11) incorporates numerous binary-on-binary and binary-on-continuous non-linearities. An important consideration here is that we are solely concerned with enforcing eq. (4) for online units. As previously mentioned, eq. (4) ensures that the remaining online units can utilize their headroom effectively. Hence, we can assume that xt,i is equal to one in eq. (11). However, we will introduce an additional term to the constraint to ensure it is always satisfied for offline units (where their $x_{t,i}$ is zero). After the modifications, eq. (11) will become,

$$p_{t,i}\hat{K}^s_\ell + \hat{K}_i p^c_{t,\ell} \leq \overline{\mathcal{P}}_{t,i} \hat{K}^s_\ell + M(1 - x_{t,i}) \quad i \in \mathcal{I},\ i \neq \ell \tag{12}$$

where M is a sufficiently large value. Now, the only remaining non-linear term to address is $p_{t,i}\hat{K}^s\ell$, which involves continuous-into-binary non-linearities. Each $pt,ix_{t,s}$ can be linearized using the following set of constraints,

$$a'_{t,i,s} \leq M x_{t,s} \tag{13a}$$
$$a'_{t,i,s} \leq p_{t,i} \tag{13b}$$
$$a'_{t,i,s} \geq p_{t,i} - M(1 - x_{t,s}) \tag{13c}$$

where $a'_{t,i,s}$ is an auxiliary variable that stores the product $p_{t,i}x_{t,s}$. s is an alias index of i.

C. PROPOSED C-FCUC METHOD

After calculating $p^{\text{UFLS}}_{t,\ell}$, the associated cost can be incorporated into the objective function. The objective function of the proposed C-FCUC is as follows,

$$\min_{x,p} \text{suc}(x_{t,i}) + \text{gc}(p_{t,i}) + C^{\text{UFLS}} \times p^{\text{UFLS}}_{t,\ell} \tag{14}$$

Where C^{UFLS} represents the cost of UFLS. This cost function can be based on factors such as the probability of outages or a constant cost associated with UFLS. Equation (14) is subject to the binary logic constraints of the problem (eqs. (1b) to (1e)), minimum and maximum capacity constraints (eqs. (1f) and (1g)), ramping constraints (eqs. (1h) and (1i)), power balance (eq. (1j)), as well as the linear expressions to estimate the amount of UFLS (eqs. (8) to (10), (12) and (13)) which are introduced in section III-B. Since the amount of UFLS can be subtracted from the reserve requirement, the reserve constraint is modified as follows:

$$\sum_{i \in \mathcal{I}, i \neq \ell} r_{t,i} \geq C_\ell \times p_\ell - p^{\text{UFLS}}_{t,\ell} \quad t \in \mathcal{T},\ \forall \ell \tag{15}$$

IV. RESULTS

This section investigates the effect of the proposed C-FCUC formulation on the operational planning of two Spanish island power systems: La Palma and La Gomera.

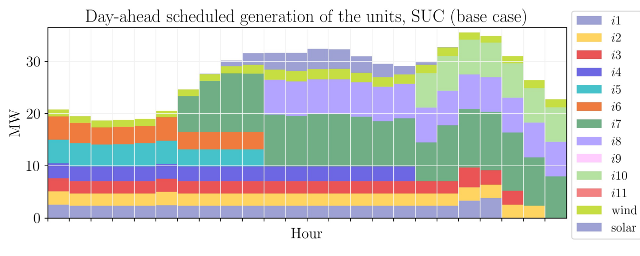
A. LA PALMA

The La Palma power system comprises 11 conventional generation units, with a peak demand of approximately 40 MW. Currently, about 6% of the installed generation capacity is attributed to RESs. The technical parameters of the 11 generating units are detailed in Table 1.

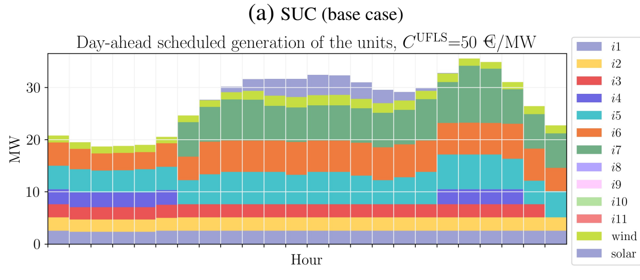
TABLE 1: Technical Parameters of the Generating Units in La Palma

Unit	$\overline{\mathcal{P}}$ (MW)	$\underline{\mathcal{P}}$ (MW)	M_{base} (MVA)	H (s)	K (pu)	T (s)
1	3.82	2.35	5.4	1.75	20	8.26
2	3.82	2.35	5.4	1.75	20	8.26
3	3.82	2.35	5.4	1.75	20	8.26
4	4.3	2.82	6.3	1.73	20	8.26
5	6.7	3.3	9.4	2.16	20	8.26
6	6.7	3.3	9.6	1.88	20	8.26
7	11.2	6.63	15.75	2.1	20	8.26
8	11.5	6.63	14.5	2.1	20	8.26
9	11.5	6.63	14.5	2.1	20	8.26
10	11.5	6.63	14.5	2.1	20	8.26
11	21	4.85	26.82	6.5	21.25	3.28

The active power imbalances considered encompass the loss of any connected generation unit. ΔF^{nadir} is established at -2.5 Hz. We will analyze the influence of C^{UFLS}. fig. 1a illustrates the generation schedule derived from the SUC, while fig. 1b depicts the generation schedule when the UFLS cost is set to 50 €/MW.



(a) SUC (base case)

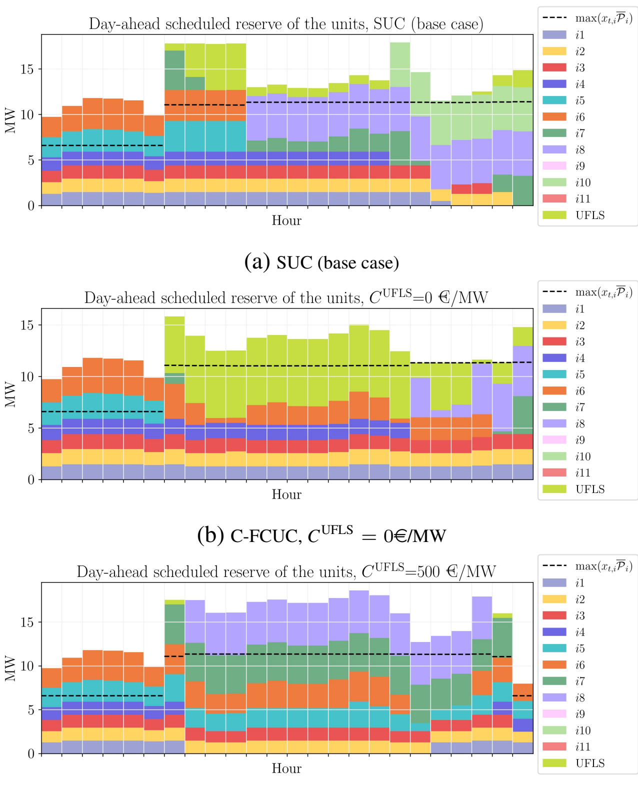


(b) C-FCUC, $C^{\text{UFLS}} = 50$€/MW

FIGURE 1: Day-ahead generation schedule of the units

Comparing fig. 1a and fig. 1b, it's evident that the proposed C-FCUC begins to distribute generation more evenly among the units. Unit $i7$ generates less power, while units $i6$ and $i5$ remain online for longer durations to prevent UFLS resulting from the outage of $i7$ in the base case.

The scheduled reserve for the SUC, the proposed model with $C^{\text{UFLS}} = 0$ €/MW, and the proposed model with $C^{\text{UFLS}} = 500$ €/MW are depicted respectively in figs. 2a to 2c. Although UFLS occasionally occurs in the SUC, it is



(a) SUC (base case)

(b) C-FCUC, $C^{UFLS} = 0$€/MW

(c) C-FCUC, $C^{UFLS} = 200$€/MW

FIGURE 2: Day-ahead reserve schedule of the units

not considered as a potential reserve. In the proposed model, when the cost of UFLS is set to 0 €/MW, UFLS is factored in to partially offset potential active power imbalances (i.e., generation outages) in certain hours. As less reserve is held back, lower generation costs can be anticipated. Conversely, if the cost of UFLS is higher (500 €/MW), smaller units are enlisted to avoid any potential UFLS. Since smaller units typically incur higher costs, it is expected that the generation cost will increase to avoid UFLS, thereby resulting in a more reliable generation schedule. The outcomes for the SUC and the proposed model with varying UFLS costs are summarized in table 2, where SUC cost is the summation of the generation cost and the UFLS cost considering the specified C^{UFLS}. Higher UFLS costs lead to a reduction in potential UFLS occurrences but an increase in generation costs. To better compare the proposed C-FCUC with the SUC, the increments and decrements are also shown in percentage in table 2.

The calculation of UFLS in the proposed model is performed using eq. (5), which is nonlinear. As previously elaborated, linearization was imperative to utilize this equation in the UC problem. Most of the linearizations employed in this paper are exact, except for the piece-wise linearization of the quadratic term, as explained in section III-B2. Figure 3 illustrates the accuracy of calculating UFLS using eq. (5) compared to the exact value of $p^{c}_{t,\ell}$ in eq. (2), for every outage throughout the day in the scenario where $C^{UFLS} = 0$. Figure 3

	generation cost	$\sum p^{UFLS}$	SUC total cost
SUC	120.68 k€	82.43 MW	-
$C^{UFLS} = 0$ €/MW	117.86 k€ (-2.34%)	123.74 MW (+50.11%)	120.68 k€
$C^{UFLS} = 50$ €/MW	118.99 k€ (-1.40%)	36.62 MW (-55.57%)	124.80 k€
$C^{UFLS} = 500$ €/MW	123.20 k€ (+2.09%)	1.03 MW (-98.75%)	161.89 k€
$C^{UFLS} = 1000$ €/MW	123.80 k€ (+2.58%)	0 MW (-100%)	203.11 k€

TABLE 2: Obtained results

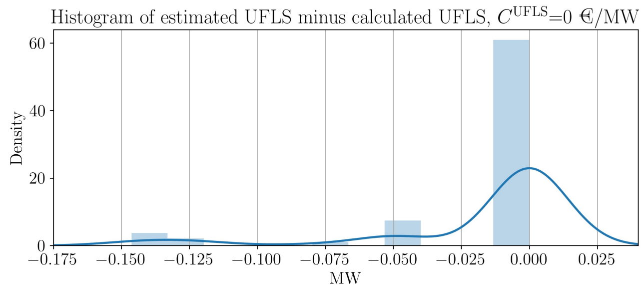
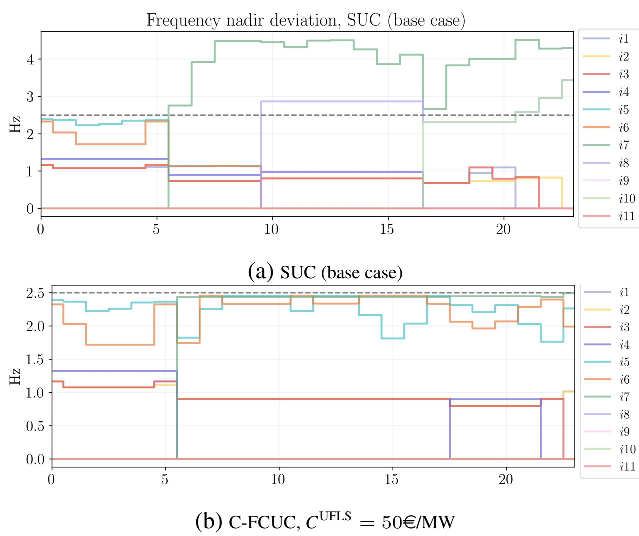


FIGURE 3: Histogram of the difference between the exact calculation of UFLS and the proposed approximation.

demonstrates that the approximation error is predominantly close to zero in most instances, and for this specific case study, it remains consistently smaller than 0.15 MW.

The frequency nadir deviation resulting from each outage, calculated using eq. (2) for both SUC and $C^{UFLS} = 50$ €/MW scenarios, are depicted in figs. 4a and 4b. It is evident that in



(a) SUC (base case)

(b) C-FCUC, $C^{UFLS} = 50$€/MW

FIGURE 4: The frequency nadir deviation

the case of SUC, numerous active power imbalances lead to violations of the maximum frequency deviation of 2.5 Hz. In contrast, in the case of C-FCUC, frequency deviations remain within bounds.

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B. LA GOMERA

The power system of La Gomera is smaller than La Palma. The peak demand is around 10.5 MW. Again the units in La Gomera are diesel units. The technical parameters of the units are detailed in Table 3.

TABLE 3: Technical Parameters of the Generating Units in La Gomera

Unit	$\overline{\mathcal{P}}$ (MW)	$\underline{\mathcal{P}}$ (MW)	M_{base} (MVA)	H (s)	K (pu)	T (s)
1	1.4	0.85	1.4	2	16.39	2.84
2	1.4	0.85	1.4	2	16.39	2.84
3	1.84	0.96	1.84	2	20.41	3.72
4	1.84	0.96	1.84	2	20.41	4.78
5	2.5	1.44	2.5	2	16.39	2.84
6	2.5	1.44	2.5	2	16.39	2.84
7	3.1	1.73	3.1	2	10.0	2.75
8	3.1	1.73	3.1	2	16.13	2.80
9	1.0	0.63	1.0	2	20.0	3.65
10	0.97	0.39	0.97	2	16.67	2.88

To analyze the effectiveness of the proposed model, simulations are carried out for the system of La Gomera. Most arguments are similar to the previous section, but testing the proposed model on different case studies further proves the model's practicality. The reserve provision for the case that UFLS has no cost, is shown in fig. 5. As shown in

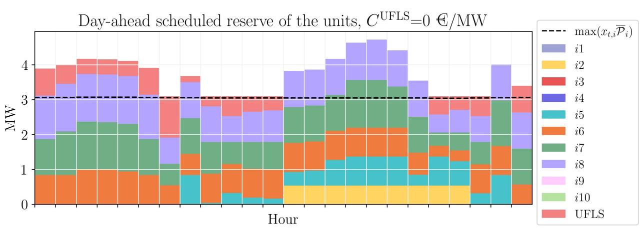


FIGURE 5: Day-ahead reserve schedule of the units, when $c^{\text{UFLS}} = 0$

fig. 5 in many hours the amount if UFLS has been able to contribute in providing the required reserve. A summary of the obtained results is presented in table 4. Table 4 confirms

	generation cost	$\sum p^{\text{UFLS}}$	SUC total cost
SUC	40.76 k€	10.28 MW	-
$C^{\text{UFLS}} = 0$ €/MW	39.88 k€ (-2.16%)	23.36 MW (+127.24%)	40.76 k€
$C^{\text{UFLS}} = 50$ €/MW	40.11 k€ (-1.56%)	13.78 MW (+34.05%)	41.50 k€
$C^{\text{UFLS}} = 500$ €/MW	41.35 k€ (+1.45%)	1.05 MW (-89.79%)	47.65 k€
$C^{\text{UFLS}} = 1000$ €/MW	41.37 k€ (+1.50%)	1.02 MW (-90.08%)	54.54 k€

TABLE 4: Obtained results

again that the proposed method can reduce the generation costs considerably when the cost of UFLS is low. For higher costs of UFLS the generation cost will increase, while the summation of UFLS will decrease. Note that in the case of the La Gomera power system, the summation of UFLS never recedes to zero.

In fig. 6 the accuracy of calculating UFLS using eq. (5) compared to the exact value of $p^c_{t,\ell}$ in eq. (2), for every outage throughout the day in the scenario where $C^{\text{UFLS}} = 0$ is shown for La Gomera. As it can be seen, the approximation error is

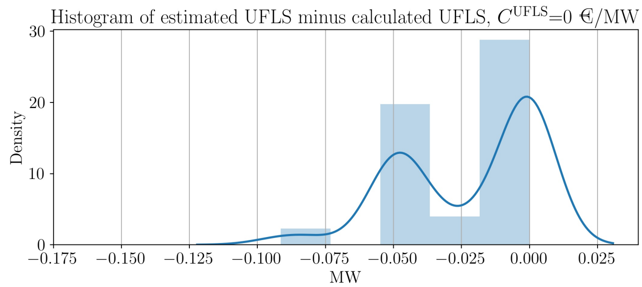
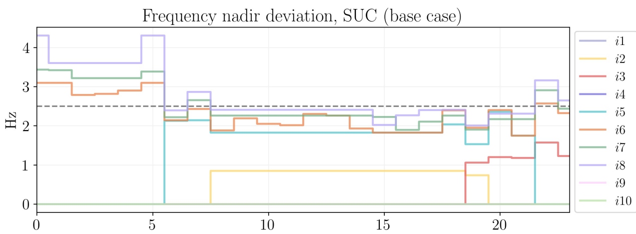
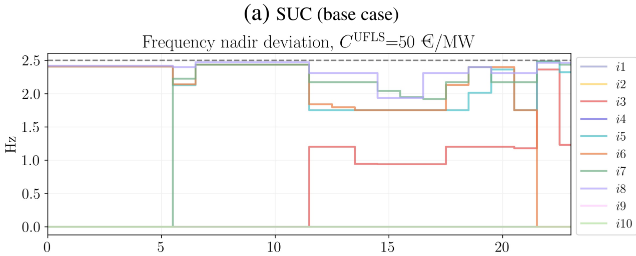


FIGURE 6: Histogram of the difference between the exact calculation of UFLS and the proposed approximation.

very low. The frequency nadir deviation resulting from each outage, for both SUC and $C^{\text{UFLS}} = 50$ €/MW scenarios in La Gomera power system, are shown in figs. 7a and 7b. It



(a) SUC (base case)



(b) C-FCUC, $C^{\text{UFLS}} = 50$€/MW

FIGURE 7: The frequency nadir deviation

is evident that in the case of SUC, numerous active power imbalances lead to violations of the maximum frequency deviation of 2.5 Hz. In contrast, in the case of C-FCUC, frequency deviations remain within bounds.

V. CONCLUSIONS

This paper introduces a novel C-FCUC approach tailored for island power systems, incorporating analytical constraints on UFLS. Depending on the specified cost of UFLS and the incorporation of UFLS constraints, the C-FCUC can seamlessly transition into either a P-FCUC or a security-constrained UC, rendering it a versatile solution. The proposed formulation has been successfully implemented in two Spanish island power systems. Results demonstrate the accuracy of the linearized calculation of UFLS and showcase that the proposed model not only reduces generation costs but also mitigates the expected amount of UFLS. In smaller islands, where UFLS is bound to happen after the outages, co-optimizing generation cost and UFLS cost is necessary, as they can cover a considerable percentage of the required reserve.

REFERENCES

[1] G. Liu and K. Tomsovic, "Quantifying spinning reserve in systems with significant wind power penetration," *IEEE Transactions on Power Systems*, vol. 27, no. 4, pp. 2385–2393, 2012.
[2] A. M. L. da Silva, J. F. C. Castro, and R. A. González-Fernández, "Spinning reserve assessment via quasi-sequential monte carlo simulation with renewable sources," in *2016 International Conference on Probabilistic Methods Applied to Power Systems (PMAPS)*. IEEE, 2016, pp. 1–7.
[3] M. Rajabdorri, E. Lobato, and L. Sigrist, "Robust frequency constrained uc using data driven logistic regression for island power systems," *IET Generation, Transmission & Distribution*, vol. 0, no. 0, pp. 1–15, 2022.
[4] C. Ferrandon-Cervantes, B. Kazemtabrizi, and M. C. Troffaes, "Inclusion of frequency stability constraints in unit commitment using separable programming," *Electric Power Systems Research*, vol. 203, p. 107669, 2022.
[5] L. Badesa, F. Teng, and G. Strbac, "Simultaneous scheduling of multiple frequency services in stochastic unit commitment," *IEEE Transactions on Power Systems*, vol. 34, no. 5, pp. 3858–3868, 2019.
[6] M. Shahidehpour, T. Ding, Q. Ming, J. P. Catalao, and Z. Zeng, "Two-stage chance-constrained stochastic unit commitment for optimal provision of virtual inertia in wind-storage systems," *IEEE Transactions on Power Systems*, 2021.
[7] M. Rajabdorri, B. Kazemtabrizi, M. Troffaes, L. Sigrist, and E. Lobato, "Inclusion of frequency nadir constraint in the unit commitment problem of small power systems using machine learning," *Sustainable Energy, Grids and Networks*, vol. 36, p. 101161, 2023.
[8] D. Lagos and N. D. Hatziargyriou, "Data-driven frequency dynamic unit commitment for island systems with high res penetration," *IEEE Transactions on Power Systems*, 2021.
[9] Y. Zhang, H. Cui, J. Liu, F. Qiu, T. Hong, R. Yao, and F. Li, "Encoding frequency constraints in preventive unit commitment using deep learning with region-of-interest active sampling," *IEEE Transactions on Power Systems*, vol. 37, no. 3, pp. 1942–1955, 2021.
[10] C. Concordia, L. Fink, and G. Poullikkas, "Load shedding on an isolated system," *IEEE Transactions on Power Systems*, vol. 10, no. 3, pp. 1467–1472, 1995.
[11] O. Moya, "A spinning reserve, load shedding, and economic dispatch solution by bender's decomposition," *IEEE Transactions on Power Systems*, vol. 20, no. 1, pp. 384–388, 2005.
[12] J. O'Sullivan and M. O'Malley, "A new methodology for the provision of reserve in an isolated power system," *IEEE Transactions on Power Systems*, vol. 14, no. 2, pp. 519–524, 1999.
[13] F. Teng and G. Strbac, "Full stochastic scheduling for low-carbon electricity systems," *IEEE Transactions on Automation Science and Engineering*, vol. 14, no. 2, pp. 461–470, 2017.
[14] C. O'Malley, L. Badesa, F. Teng, and G. Strbac, "Probabilistic scheduling of ufls to secure credible contingencies in low inertia systems," *IEEE Transactions on Power Systems*, vol. 37, no. 4, pp. 2693–2703, 2022.
[15] Y. Cao, Q. Wu, H. Zhang, C. Li, and X. Zhang, "Chance-constrained optimal configuration of bess considering uncertain power fluctuation and frequency deviation under contingency," *IEEE Transactions on Sustainable Energy*, vol. 13, no. 4, pp. 2291–2303, 2022.
[16] Y. Cao, Q. Wu, H. Zhang, and C. Li, "Multi-objective optimal siting and sizing of bess considering transient frequency deviation and post-disturbance line overload," *International Journal of Electrical Power & Energy Systems*, vol. 144, p. 108575, 2023.
[17] S. Padrón, M. Hernández, and A. Falcón, "Reducing under-frequency load shedding in isolated power systems using neural networks. gran canaria: A case study," *IEEE Transactions on Power Systems*, vol. 31, no. 1, pp. 63–71, 2016.
[18] C. O'Malley, L. Badesa, F. Teng, and G. Strbac, "Probabilistic scheduling of ufls to secure credible contingencies in low inertia systems," *IEEE Transactions on Power Systems*, vol. 37, no. 4, pp. 2693–2703, 2021.
[19] L. Sigrist, I. Egido, and L. Rouco, "Principles of a centralized ufls scheme for small isolated power systems," *IEEE Transactions on Power Systems*, vol. 28, no. 2, pp. 1779–1786, 2013.
[20] M. Fischetti and J. Jo, "Deep neural networks and mixed integer linear optimization," *Constraints*, vol. 23, no. 3, pp. 296–309, 2018.

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