Pilot-Bus Selection for Secondary Voltage Control

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Abstract

The secondary voltage control determines control device actions based on reference voltage values set at certain load buses denominated pilot buses. A key factor for the appropriate functioning of secondary voltage control schemes is the selection of the pilot buses. This paper addresses this problem taking into account system-wide information, and different operating conditions regarding different load levels and different network topologies. The proposed approach improves previous approaches in two respects: the level of modelling detail and the computational efficiency. Two case studies based respectively on the IEEE 118-bus system and the New England 39-bus system are provided.

1 Introduction

The problem of controlling voltage magnitudes and reactive power flows in an electric network can be described as a large scale voltage control problem having a multi-level, multi-objective control structure. For ease of realization, the control levels are usually distributed in a hierarchy having a pyramidal structure. In such a pyramidal structure some of the controllers have only indirect access to the electric network; i. e. some controllers receive information from controllers at higher levels and use this information to control lower level controllers. A typical example of controllers having only indirect access to the electric network are the controllers of the secondary control level.

Currently, as realized by Electricité de France [1–4], large-scale voltage control is organized into three hierarchical levels:

- *Primary Control Level*

At this level, the control devices attempt to compensate rapid and random voltage variations by maintaining their output variables close to the desired reference values. Only local information is used for the computation of the control signals. A fast response is provided; the time constant is in the range of seconds.

- *Secondary Control Level*

Slow and large voltage variations, such as those produced by the hourly evolution of the load, are accounted for by the controllers of the secondary control level; the time constant is in the range of minutes. This level makes use of sub-system information to update the reference values of the controllers in the primary control level with the purpose of keeping voltages of the pilot buses at their optimal values. Through the secondary control level it is possible to keep an appropriate voltage profile throughout the transmission network in the face of the hourly evolution of the load and topological changes.

- *Tertiary Control Level*

System-wide information is used to compute optimal pilot-bus voltage magnitudes with the purpose of achieving an economic and secure operation of the electric network. This is mainly done by solving, either automatically or manually by operators, a large-scale optimization problem such as the optimal power flow with the objective of minimizing the active power losses of the electric network while taking security constraints into account. The tertiary control level is related to the idea of coordination, i. e. the whole network is taken into account, and it is much slower than the secondary control level; the time constant could be anything between 15 min and up to several hours.

It follows from the above presentation that a key factor for the appropriate functioning of a secondary voltage control scheme is the selection of the pilot buses.

This paper addresses the problem of selecting pilot buses taking into account system-wide information, and the effect of different operating conditions regarding different load levels and different network topologies.

Among previous approaches to the solution of this problem, *Thorp* et al. [5] used an exhaustive enumeration of the solution space to select one or two pilot buses so that a performance index is maximized. Results for a small power system are provided in that reference.

Note that an exhaustive enumeration of the solution space is not an appropriate solution technique for systems of realistic size. *Ilić-Spong* et al. [6] and *Stanković* et al. [7] proposed a time-consuming and cumbersome solution technique, named "simulated annealing", to select two or three pilot buses so that voltage deviations at load buses are minimized. It should be noted that the simulated annealing, which is a random search [8] is an inefficient solution technique which generates a solution that cannot be proved to be better than other solutions generated more efficiently via other heuristic procedures. *Lagonotte* et al. [4], *Vialas* and *Paul* [3], *Paul* et al. [2] and *Blanchon* [1] used the concept of electrical distance to divide the power system into control areas and then, again using electrical distances, one pilot bus is selected for every control area.

None of the above approaches considers different operating conditions regarding different load levels and different network topologies.

The approach proposed in this paper improves the previous approaches in two respects:

- It uses an efficient heuristic procedure which uses system-wide information. That is, the system is not beforehand arbitrarily divided into areas to reduce the dimensionality of the problem.
- It considers different operating conditions regarding different load levels (seasonal variation of load) and different network topologies.

2 Secondary Voltage Control

The secondary voltage control works in an incremental (digital) fashion and therefore every control step modifies the voltage magnitudes of the load buses by a relatively small amount. This makes possible the use of a linearized model. The validity of this argument is shown by performing appropriate time simulations in which the system is described through the non-linear load-flow equations.

The linearized model to be used considers the active power flows in the electric networks but neglects the effect of active power changes on voltage magnitudes. To analyze stressed systems this last effect can also be considered (see **Appendix**).

The considered linearized model is based on the following sensitivity equations

$$\begin{bmatrix} S_{GG} & S_{GL} \\ S_{LG} & S_{LL} \end{bmatrix} \begin{bmatrix} \Delta V_G \\ \Delta V_L \end{bmatrix} = \begin{bmatrix} \Delta Q_G \\ \Delta Q_L \end{bmatrix} \tag{1}$$

where

ΔV_G and ΔV_L are vectors of voltage magnitude changes, at generator buses and load buses, respectively,

ΔQ_G and ΔQ_L are vectors of injected reactive power changes, at generator buses and load buses, respectively, and

S_{GG}, S_{GL}, S_{LG}, and S_{LL} are sensitivity matrices defined in the Appendix.

Notice that vector ΔQ_L is considered to be the cause of change in the system, and that ΔV_G is the control vector.

The second of the above sensitivity equations renders

$$\Delta V_L = M \Delta Q_L + B \Delta V_G \tag{2}$$

where

$$M := S_{LL}^{-1}, \tag{3}$$

$$B := -S_{LL}^{-1} S_{LG}. \tag{4}$$

The information available to the controller are voltage magnitudes at pilot buses; the observation equation therefore becomes

$$\Delta V_P = C \Delta V_L \tag{5}$$

where

$C = [c_{ij}]$ is an $n_P \times n_L$ 0-1 matrix defined as

$$c_{ij} = \begin{cases} 1 & \text{should bus } j \text{ be the } i\text{–th pilot bus} \\ 0 & \text{otherwise,} \end{cases} \tag{6}$$

n_L is the number of load buses, and

n_P is the number of load buses selected as pilot buses.

Different approaches have been proposed to implement secondary voltage control schemes. Integral controls are proposed in [2] and [10]. A "one-step" optimal feedback control, which considers random load disturbances, is used in [6] and [7]. An improved approach to the optimal control of a discrete time-dynamic system was developed in [11], where both regulator and servomechanism problems are considered. This paper, however, does not address directly the implementation issue.

3 Pilot-Bus Selection

The aim of the secondary voltage control is to minimize voltage deviations at load buses using only the available output information, i. e. voltage magnitudes at pilot buses.

Load disturbances are modelled by a vector of Gaussian random variables with means equal to 0 and a covariance matrix denominated C_{LL}.

Control generators counteract load disturbances by maintaining in steady-state voltage magnitudes at pilot buses, i. e.

$$\Delta V_P = C B \Delta V_G + C M \Delta Q_L = 0. \tag{7}$$

This can be achieved in different ways because the number of control generators (control variables) is larger than the number of pilot buses (variables to be controlled). One convenient way is to achieve it while minimizing generator control actions, i. e. while minimizing

$$\sum_{i=1}^{n_G} \Delta V_{Gi}^2 \tag{8}$$

where n_G is the number of control generators.

Eqs. (7) and (8) result in the control law given by the expression

$$\Delta V_G = -F C M \Delta Q_L \tag{9}$$

where

$$F := (C B)^T \left(C B B^T C\right)^{-1}. \tag{10}$$

Pilot buses are selected so that in steady-state voltage deviations throughout all load buses are minimized, i. e.

$$\Delta V_L^T Q_x \Delta V_L$$

is minimized.

Q_x is a diagonal weighting matrix which is used to weight the relative importance of maintaining voltage magnitude in a given load bus with respect to the other load buses.

Using eqs. (2) and (9) the above expression renders the index below

$$\mathfrak{I}(C) = \text{trace}\left\{P_L (I - BFC)^T Q_x (I - BFC)^T\right\} \tag{11}$$

where

$$P_{\mathrm{L}} := M C_{\mathrm{LL}} M^T. \tag{12}$$

P_{L} is the covariance matrix of voltage deviations at load buses.

Index $\mathfrak{I}(C)$ can be written as a function of matrices of dimension $n_{\mathrm{P}} \times n_{\mathrm{P}}$ (i. e. matrices of reduced dimension) so that calculations are performed more efficiently, i. e.

$$\mathfrak{I}(C) = \mathrm{trace}\left\{P_{\mathrm{L}} Q_x\right\} - \mathrm{trace}\left\{\left(2H_1 - H_2 H_3^{-1} H_4\right) H_3^{-1}\right\} \tag{13}$$

where

$$H_1 = C P_{\mathrm{L}} Q_x B B^T C^T,$$

$$H_2 = C B B^T Q_x B B^T C^T,$$

$$H_3 = C B B^T C^T,$$

$$H_4 = C P_{\mathrm{L}} C^T.$$

The optimal selection of pilot buses therefore becomes

$$\underset{C}{\mathrm{Min}}\ \mathfrak{I}(C) \tag{14}$$

which in view of eq. (13) renders

$$\underset{C}{\text{maximize}}\ \mathrm{trace}\left\{\left(2H_1 - H_2 H_3^{-1} H_4\right) H_3^{-1}\right\}. \tag{15}$$

Notice that matrices H_1, H_2, H_3 and H_4 depend in general on sensitivity and weighting matrices. When realistic operating conditions are considered sensitivity matrices are random (i. e. multiple scenarios should be considered) and, as a result, matrices H_1, H_2, H_3 and H_4 are random as well. In what follows H_1, H_2, H_3 and H_4 are denoted as $H_1(\xi)$, $H_2(\xi)$, $H_3(\xi)$ and $H_4(\xi)$, respectively. ξ is a random variable capturing the stochastic nature of different operating conditions regarding different load levels and different network topologies.

The pilot-bus selection problem can be re-formulated to take into account different operating conditions, i. e.

$$\begin{aligned} &\underset{A_{\mathrm{P}}}{\text{maximize}}\ \underset{\xi}{E}\left\{f\left(C(A_{\mathrm{P}}), \xi\right)\right\}, \\ &\text{subject to}\ \ A_{\mathrm{P}} \subset A, \\ &\qquad\qquad\ \ \mathrm{card}\,(A_{\mathrm{P}}) \le n_{\mathrm{P}} \end{aligned} \tag{16}$$

where

A is the set of all load buses,

A_{P} is the set of load buses selected as pilot buses,

$C(A_{\mathrm{P}})$ is the $n_{\mathrm{P}} \times n_{\mathrm{L}}$ pilot bus selection matrix, previously defined and denoted by C; here, the fact that C depends on the set of load buses selected as pilot buses is stressed,

E is the expectation operator, and

"card" denotes cardinal number.

The function $f(C(A_{\mathrm{P}}), \xi)$ is defined as

$$\begin{aligned} &f\left(C(A_{\mathrm{P}}), \xi\right) \\ &:= \mathrm{trace}\left\{\left(2H_1(\xi) - H_2(\xi) H_3(\xi)^{-1} H_4(\xi)\right) H_3^{-1}(\xi)\right\}. \end{aligned} \tag{17}$$

An appropriate fashion to describe random variable ξ is by means of a finite set of scenarios [12]. Let Ω be a set which contains all considered scenarios and let p_s, $s \in \Omega$, be the probability (weight) associated with scenario s; then, eq. (16) becomes

$$\begin{aligned} &\underset{A_{\mathrm{P}}}{\text{maximize}}\ \ g(A_{\mathrm{P}}), \\ &\text{subject to}\ \ A_{\mathrm{P}} \subset A, \\ &\qquad\qquad\ \ \mathrm{card}\,(A_{\mathrm{P}}) \le n_{\mathrm{P}} \end{aligned} \tag{18}$$

where

$$g(A_{\mathrm{P}}) := \sum_{s \in \Omega} p_s f(A_{\mathrm{P}}, s). \tag{19}$$

It should be noted that every scenario is characterized by matrices $H_1(s)$, $H_2(s)$, $H_3(s)$ and $H_4(s)$, and probability p_s. It should also be noted that

$$\sum_{s \in \Omega} p_s = 1.$$

Notice that scenarios are defined regarding

- seasonal load levels, i. e. peak and off-peak load levels for the different seasons of the year, and
- different network topologies for every seasonal load level.

Different networks topologies should be considered as a result of forced and scheduled outages of generators, lines and transformers.

Notice also that for every considered scenario, appropriate sensitivity matrices are computed.

The selection of pilot buses is performed only once, and then they are used to implement a secondary voltage control scheme. For the selection of the pilot buses, it is therefore very important to take into account all relevant operating scenarios of the system under study.

4 Problem Properties

Some relevant properties of the above problem are:

- The decision variables are 0-1 integer: either a load bus is selected as pilot bus or it is not selected as that.
- The objective function is particularly non-linear, see eq. (17), and tough to evaluate.
- Matrices $H_1(\xi)$, $H_2(\xi)$, $H_3(\xi)$ and $H_4(\xi)$ are random and, therefore, the objective function is random as well.
- The dimension of the problem is very high. A typical system may consist of 500 load buses. Among them, for instance, 15 load buses may be selected as pilot buses. The total number of possible selections is $(500!)/[15!(500-15)!]$ which is not a small number.

The problem under consideration is therefore 0-1 integer, non-linear, and stochastic. The following conclusions can be stated:

- Given the size of the solution space, a technique based on a direct enumeration of this solution space is not appropriate.

- It is only possible to use heuristic procedures which directly, and hopefully cleverly, examine a small subset of the solution space.

5 Solution Algorithm

Eq. (18) can be approximately solved by using a "greedy" heuristic algorithm [8]. The idea of the proposed greedy algorithm is simple. Given a set of selected pilot buses, $A_P^{(v)}$, the next load bus to be chosen as pilot bus is the one which gives the greatest immediate increase in the value of the objective function, provided that such a load bus exists. Moreover, once a load bus is selected as pilot bus, it is kept as pilot bus throughout the algorithm.

The greedy algorithm proceeds as follows:

- Step 0:

  Set $A_P^{(0)} = \mathbf{0}$, and set $v = 1$.

- Step 1:

  Find $j_v = \arg\ \max_j\ g(A_P^{(v-1)} \cup \{j\})$,

  $j \in A \setminus A_P^{(v-1)}$.

- Step 2:

  If $g(A_P^{(v-1)} \cup \{j_v\}) \leq g(A_P^{(v-1)})$, stop;

  $A_P^{(v-1)}$ is a greedy solution.

- Step 3:

  If $g(A_P^{(v-1)} \cup \{j_v\}) > g(A_P^{(v-1)})$,

  set $A_P^{(v)} = A_P^{(v-1)} \cup \{j_v\}$.

- Step 4:

  If card $(A_P^{(v)}) = n_P$, stop, the maximum number of pilot buses has been attained;

  if $[g(A_P^{(v)}) - g(A_P^{(v-1)})]/[g(A_P^{(v)})] < \varepsilon$, stop, the rate of improvement of the objective function is small enough;

  otherwise, let $v \leftarrow v + 1$, and go to step 1.

Note that ε is a threshold on the rate of improvement of the objective function, and recall that "\" denotes set difference.

The proposed algorithm requires for the selection of the i-th pilot bus

- $n_L - i + 1$ evaluations of the objective function, and
- for every evaluation of the objective function, the inversion of an $i \times i$ matrix.

To select a number n_P of pilot buses, the total number of required evaluations of the objective function is

$$n_P\left[n_L + 1/2 - (1/2)\, n_P\right];$$

if $n_P \ll n_L$, the above number becomes $n_P n_L$. Note that $n_P n_L \ll \binom{n_L}{n_P}$ being this last number the total number of possible manners of selecting n_P pilot buses among n_L load buses.

On the other hand, for a given number of pilot buses, n_P, the required number of floating point operations (CPU time units) by the proposed algorithm changes linearly with the number of scenarios. For a moderate number of scenarios the algorithm is surprisingly fast.

6 Algorithm Extension and Validation

The greedy algorithm is a multi-stage optimization procedure which consists of a concatenated succession of breadth searches. In every optimization stage, it performs one breadth search and determines the best solution for this stage. This best solution is used as starting solution (root) for the next optimization stage.

A so-called less-greedy algorithm of order n keeps the same structure as the original greedy algorithm but broadens the search set. It memorizes the best n solutions obtained in every optimization stage, and uses these best solutions as starting solutions (roots) in the next optimization stage. Therefore, it performs n breadth searches in every optimization stage other than the first one (in the first one, it performs only one breadth search); and, as a result, its computational burden is approximately n times heavier than the computational burden of the original greedy algorithm.

A less-greedy algorithm of order n

- is a straightforward extension of the greedy algorithm, and
- it can be used to "locally" validate the original greedy algorithm.

For the optimal selection of pilot buses, a less-greedy algorithm of order two was used and compared with the original greedy algorithm. Different cases and systems were studied and no significant differences were found in the solutions provided by the two algorithms.

However, when computationally acceptable, a less-greedy algorithm of order n provides a higher guaranty of optimality than the original greedy algorithm.

The greedy algorithm is a heuristic algorithm which generates solutions similar to the solutions generated by other heuristic algorithms, for instance the one proposed in [6]. Therefore, the greedy algorithm cannot be proven to be more efficacious in generating a solution close or equal to the optimal one than other heuristic algorithms. The greedy algorithm is, however, very efficient from a computational point of view as it is apparent from its structural simplicity. This computational efficiency is important if the scope of analysis is to be broaden; that is, if many operating conditions regarding different load levels and different network topologies are considered, the computational burden of heuristic algorithms similar to the one proposed in [6] may become prohibitive even for off-line analyses.

On the other hand, it should be noted that

- the computational complexity of the greedy algorithm is well characterized,
- the greedy algorithm always finds a solution,
- a less-greedy algorithm of order n ($n > 1$) can be used to validate the original greedy algorithm, and

- the greedy algorithm is well suited to solve the problem of selecting pilot buses due to the behavior (evolution and saturation) of the proposed objective function.

It also should be noted that the solution provided by the greedy or a less-greedy algorithm can be improved through local searches. That is, a given pilot bus is interchanged with its neighbours in the hope of improving the objective function. This can be particularly efficacious for the first selected pilot buses because these pilot buses are the ones "less aware" of the history of the selection.

7 Case Study

A case study based on the IEEE 118-bus system [13] is presented to show the usefulness of the developed algorithms. The considered bus numbering is the one of that reference. Generating buses with reactive power generation capacity below 40 Mvar were considered load buses because these small generators lack relevant control capability. Due to the lack of data, the reactances of the step-up transformers at generating buses were supposed to be equal to 0.1 per unit of their respective rated values.

This case study comprises five scenarios. These scenarios consider only one load level and five network topologies. Appropriate sensitivity matrices are computed for every scenario. Matrix C_{LL} was assumed to be an $n_L \times n_L$ diagonal matrix with diagonal elements proportional to the reactive loads of the load buses. The considered scenarios were:

I. No line/transformer out.

II. Only line 12 – 14 out.

III. Only line 69 – 77 out.

IV. Only line 89 – 92 out.

V. Only line 38 – 37 out.

Six cases are considered. In every case the above scenarios are combined with different weighting factors (p). Pilot buses are selected for the six cases. The weighting factors for the six cases are described below.

- Case 1: (I) $p = 0.2$; (II) $p = 0.2$; (III) $p = 0.2$; (IV) $p = 0.2$; (V) $p = 0.2$.
- Case 2: (I) $p = 1$.
- Case 3: (II) $p = 1$.
- Case 4: (III) $p = 1$.
- Case 5: (IV) $p = 1$.
- Case 6: (V) $p = 1$.

Tab. 1 shows the pilot buses obtained after running the greedy algorithm for every case. The analyzed cases show that the set of selected pilot buses is robust with respect to topological changes and different weighting factors for the scenarios. Only local changes in the pilot-bus selection are observed when topological changes occur close to a selected pilot bus.

Case	Obtained pilot buses									
1	12	39	77	88	56	103	47	27	71	17
2	14	38	77	92	56	103	47	23	71	60
3	12	39	77	92	56	103	47	27	71	17
4	14	38	77	88	56	103	47	23	70	60
5	14	38	77	88	56	103	47	23	71	60
6	14	39	77	92	56	103	47	27	5	17

Tab.1. Obtained pilot buses for every analyzed case

The value of the objective function of cases 2, 3 ,4 , 5 and 6 evaluated for the set of pilot buses obtained for case 1 is similar to the objective function value obtained using the set of pilot buses obtained for cases 2, 3, 4, 5 and 6. This value ranges around 47 %; 100 % corresponds to the ideal case in which all voltage deviations in load buses are equal to zero by means of secondary voltage control actions. This means that the trade-off solution obtained in case 1 (which takes into account a set of possible network topologies) behaves properly for every particular scenario.

Fig. 1 shows the evolution of the objective function as a function of the number of selected pilot buses. Observe how the objective function "saturates" once a sufficient number of pilot buses has been selected. It practically attains its maximum value when more than nine or ten pilot buses are selected. This saturation criterion determines when a sufficient number of pilot buses has been selected and therefore it is used as a stopping rule in the search for pilot buses.

The criterion and the algorithm proposed to select pilot buses have been compared with the procedure proposed in [7]. In that paper the New England 39 buses system [14] is analyzed and the proposed set of pilot buses is

$A_P = \{5, 6, 12, 17\}$.

This set of pilot buses reaches a low value of the objective function used in this paper. This means that a secondary voltage control scheme based on the above set of pilot buses will have a bad performance. This is because this pilot-bus selection yields a bad observability of voltage disturbances occurring throughout the system because pilot buses are close to one another. Moreover, the controllability of that set of pilot buses by means of changes in voltage set points of control generators is difficult:

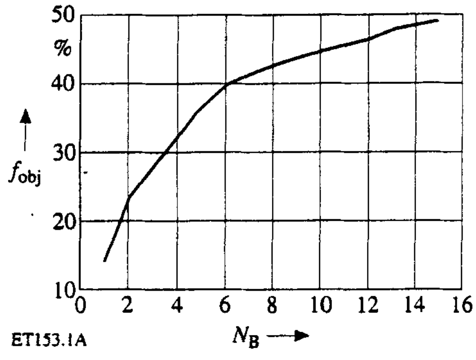

Fig. 1. Objective function evolution for the case study (f_{obj} with respect to feasible optimum; N_B number of selected pilot buses)

only two generators (buses 2, 3) are electrically close to control the voltage at three pilot buses (buses 5, 6, 12).

Using the criterion and the algorithm proposed in this paper the following set of pilot buses was obtained

$A_P = \{5, 16, 12, 20\}$.

The above set of pilot buses reaches a 78.7 % of the objective function maximum value. Both, observability and controllability with the proposed set are good: the pilot buses are well distributed among the different zones of the electric energy system and they are close to control generators.

8 Conclusions

The secondary voltage control determines control device actions based upon reference voltage values set at certain load buses denominated pilot buses.

The selection of pilot buses is crucial to ensure the adequate functioning of centralized or decentralized secondary voltage control schemes.

The selection of pilot buses is however a complicated and high-dimensional problem. It is actually a 0-1 integer, non-linear, stochastic problem.

Only heuristic algorithms can be used to attack such a problem; this paper proposes an efficient heuristic algorithm to solve it.

The procedure proposed in this paper improved the previous approaches in two respects: it uses system-wide information while keeping computational efficiency, and it considers the effect of different operating conditions regarding different load levels and different network topologies.

The proposed procedure is illustrated using the IEEE 118-bus system.

9 List of Symbols

- S_{GG} — Jacobian submatrix which relates voltage magnitude changes at generator buses with reactive power changes at generator buses
- S_{GL} — Jacobian submatrix which relates voltage magnitude changes at load buses with reactive power changes at generator buses
- S_{LG} — Jacobian submatrix which relates voltage magnitude changes at generator buses with reactive load changes at load buses
- S_{LL} — Jacobian submatrix which relates voltage magnitude changes at load buses with reactive load changes at load buses
- $\boldsymbol{M}$ — sensitivity matrix which relates reactive load changes at load buses with voltage magnitude changes at load buses
- $\boldsymbol{B}$ — sensitivity matrix which relates voltage magnitude changes at generator buses with voltage magnitude changes at load buses
- $\boldsymbol{C}$ — 0-1, $n_P \times n_L$ matrix to select pilot buses, its elements ij is equal to 1 if bus j is the i-th pilot bus and equal to 0 otherwise
- $C(A_P)$ — matrix $\boldsymbol{C}$ for a given set of selected pilot buses A_P
- $\boldsymbol{F}$ — pseudo-inverse matrix of matrix $\boldsymbol{C} \times \boldsymbol{B}$
- $\boldsymbol{Q}_x$ — diagonal weighting matrix used to weight the relative importance of maintaining voltage magnitude in a given load bus with respect to the other load buses
- $\boldsymbol{C}_{LL}$ — co-variance matrix of reactive power disturbances at load buses
- $\boldsymbol{P}_L$ — co-variance matrix of voltage magnitude deviations at load buses
- $\boldsymbol{H}_1, \boldsymbol{H}_2, \boldsymbol{H}_3, \boldsymbol{H}_4$ — intermediate matrices to perform calculations efficiently
- ΔV_G — vector of voltage magnitude changes at generator buses
- ΔV_L — vector of voltage magnitude changes at load buses
- ΔQ_G — vector of injected reactive power changes at generator buses
- ΔQ_L — vector of injected reactive power changes at generator buses
- n_L — number of load buses
- n_P — number of load buses selected as pilot buses
- n_G — number of control generators
- A — set of all load buses
- A_P — set of load buses selected as pilot buses
- Ω — set of all scenarios
- $\Im(\cdot)$ — objective function index
- $f(\cdot)$ — objective function for a given scenario
- $g(\cdot)$ — objective function for all scenarios
- p_s — probability (weight) associated with scenario s
- s — scenario index
- v — greedy algorithm counter
- j_v — index of the load bus selected as a pilot bus by the greedy algorithm at step v
- $E\{\cdot\}$ — expectation operator
- card — cardinal operator
- ξ — random variable capturing the stochastic nature of different operating conditions regarding different load levels and different networks topologies

Appendix: Sensitivity Equations

A linearized model of the load flow equations is:

$$\begin{bmatrix} H_{GG} & H_{GL} & N_{GG} & N_{GL} \\ H_{LG} & H_{LL} & N_{LG} & N_{LL} \\ J_{GG} & J_{GL} & L_{GG} & L_{GL} \\ J_{LG} & J_{LL} & L_{LG} & L_{LL} \end{bmatrix} \begin{bmatrix} \Delta\theta_G \\ \Delta\theta_L \\ \Delta V_G \\ \Delta V_L \end{bmatrix} = \begin{bmatrix} \Delta P_G \\ \Delta P_L \\ \Delta Q_G \\ \Delta Q_L \end{bmatrix} \quad (20)$$

where

$\Delta\theta_G$ and $\Delta\theta_L$ are vectors of voltage phase changes in generator (excluding the slack bus) and load buses, respectively,

ΔV_G and ΔV_L are vectors of voltage magnitude changes in generator and load buses, respectively,

ΔP_G and ΔP_L are vectors of active power changes in generator (excluding the slack bus) and load buses, respectively,

ΔQ_G and ΔQ_L are vectors of reactive power changes in generator and load buses, respectively,

$H_{GG}, H_{GL}, H_{LG}, H_{LL}, N_{GG}, N_{GL}, N_{LG}, N_{LL}, J_{GG}, J_{GL}, J_{LG}, J_{LL}, L_{GG}, L_{GL}, L_{LG}$ and L_{LL} are Jacobian sub-matrices.

From the above equations, it can be obtained that

$$\begin{bmatrix} S_{GG} & S_{GL} \\ S_{LG} & S_{LL} \end{bmatrix} \begin{bmatrix} \Delta V_G \\ \Delta V_L \end{bmatrix} = \begin{bmatrix} \Delta Q_G \\ \Delta Q_L \end{bmatrix} - \begin{bmatrix} T_{GG} & T_{GL} \\ T_{LG} & T_{LL} \end{bmatrix} \begin{bmatrix} \Delta P_G \\ \Delta P_L \end{bmatrix} \quad (21)$$

where

$$\begin{bmatrix} S_{GG} & S_{GL} \\ S_{LG} & S_{LL} \end{bmatrix} = \begin{bmatrix} L_{GG} & L_{GL} \\ L_{LG} & L_{LL} \end{bmatrix} - \begin{bmatrix} T_{GG} & T_{GL} \\ T_{LG} & T_{LL} \end{bmatrix} \begin{bmatrix} N_{GG} & N_{GL} \\ N_{LG} & N_{LL} \end{bmatrix} \quad (22)$$

and

$$\begin{bmatrix} T_{GG} & T_{GL} \\ T_{LG} & T_{LL} \end{bmatrix} = \begin{bmatrix} J_{GG} & J_{GL} \\ J_{LG} & J_{LL} \end{bmatrix} - \begin{bmatrix} H_{GG} & H_{GL} \\ H_{LG} & H_{LL} \end{bmatrix}^{-1}. \quad (23)$$

Different alternatives can be used to formulate the relationship between incremental variations of voltages and (active and reactive) power changes:

- The simplest one is the one considered in the solution of the fast decoupled load flow problem, in which the sensitivity matrix which relates voltage and reactive power variations is made equal to the negative nodal susceptance matrix of the global system.
- A more comprehensive approach considers the active power flows in the electric network, but neglects the effect of active power changes on voltage magnitudes. That is, the submatrices T_{GG}, T_{GL}, T_{LG}, T_{LL}, which are sensitivity matrices relating active power and voltage magnitude changes, are assumed to be equal to zero in eq. (22), but they are taken into account in eq. (22). This approach is used in this paper.
- The most comprehensive approach considers the effect of active and reactive power changes on voltage magnitudes and corresponds to the above complete formulation.

These last two alternatives of the sensitivity matrix are adequate to detect voltage stability problems when static analysis is performed in stressed systems.

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