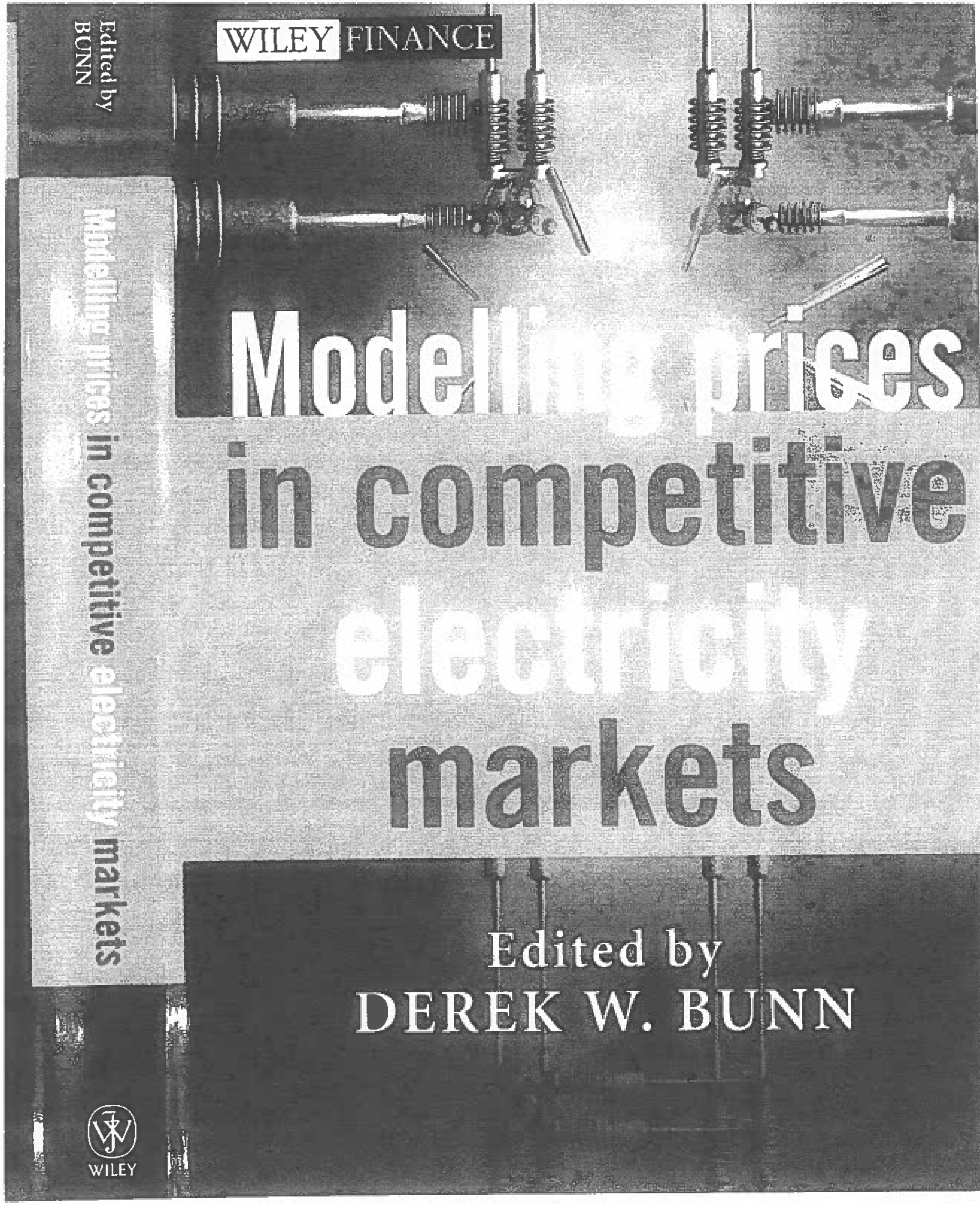

2 Competitors' Response Representation for Market Simulation in the Spanish Daily Market

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ABSTRACT

Since the 1998 liberalization, the Spanish daily electricity market has been based on an hourly auction process managed by a market operator, through which buyers and sellers submit their bids and offers. A common approach within companies for elaborating this is to model competitors' behaviour by means of past market public results. This chapter provides a detailed description of different techniques to address this problem.

2.1 INTRODUCTION

The objective of this chapter is to introduce some different techniques that can be used to characterize electricity bid-based markets when price is considered as an endogenous variable. This premise makes it necessary to represent, explicitly, the expected competitive behaviour. Two different groups of methods are described. The first group includes those that are based on a short-term competitors' response, represented by means of a detailed representation of residual demand function, based on historical bidding information. Once residual demand is estimated, the market can be simulated for different values of own firm's bids. The second group uses historical price information to establish the competitive bids' conjectural variations. In this case, the market is simulated by computing a market equilibrium. Detailed information about bidding is not necessary. Several real examples are presented, extracted from real and successful Spanish electricity market studies.[1]

[1] The authors would like to express their gratitude to the Spanish electrical utilities Endesa, Iberdrola and Union Eléctrica Fenosa for their cooperation in some parts of the models described herein.

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Table 2.1 Spanish firms' market share at the end of the last intra-daily market (September 2002)

Agent	Endesa	Iberdrola	Unión Fenosa	Hidrocantábrico	Viesgo	Others
Market share	33.5%	24.5%	9.9%	5.6%	3.7%	22.8%

Table 2.2 Production by technologies (2001)

Nuclear	Coal	Hydro	Fuel-gas	Special regime	Importations
30%	31%	18%	3%	15%	3%

2.2 HOURLY BIDDING-BASED SPANISH ELECTRICITY MARKETS

This section introduces some basic concepts in order to understand and analyse hourly bid-based markets, with special emphasis on short and medium-term analysis. It uses the Spanish electricity market as an example, including a description of its distinctive features.

2.2.1 Structure of the Spanish electricity generation market

The Spanish electricity generation market was established in 1998 and since then, four firms have competed: Endesa, Iberdrola, Unión Fenosa and Hidrocantábrico. A fifth firm was added in 2001 as a result of the sale of part of Endesa's assets (Table 2.1). The system meets a maximum peak load close to 30 000 MW and a yearly energy demand of about 175 000 GWh, and the average hydro energy available is over 30 000 GWh (Table 2.2).

In Spain, the wholesale electricity business is organized in a sequence of different types of markets (daily, intra-day and ancillary services such as secondary and tertiary reserves, etc.). These markets are managed by two different operators, the market operator (MO) and the system operator (SO), which are responsible for receiving participants' offers to sell and to buy, and for determining the market clearing price (OMEL, 2001).

The daily market's purpose is the execution of the electric energy transactions for a scheduling horizon of the following day, divided into 24 consecutive hourly periods. The intra-day markets' purpose is to ensure adjustments to the result of the daily market, and they are currently convoked six times per day. These markets allow both the generation and the demand agents to react to unforeseen changes (unit total or partial outage, errors in forecasted demand, etc.) that arise after the daily market has closed. After daily and intra-day markets, some adjustments can be made due to network constraints, safety or reliability of the electrical system. Then, successive markets are convoked to ensure system reserve and other ancillary services. A deviation market can be convoked in case of extra energy requirements. As the major part of the total energy negotiation takes place in the daily market (96.7% in 2001), this chapter is devoted only to this main market. The average daily market energy price was 31.5€/MWh in 2001. Final wholesale energy price is formed by the daily market price (81.6%), technical operation (6.5%) and capacity payments (11.9%).

The Spanish electricity market presents some features that make it especially difficult to model. A strong horizontal and vertical concentration remains in place. A 10-year transition period was adopted in order to fully liberalize the market. During this period, a competition

transition charge (CTC) will be collected in order to pay the stranded costs of the utilities (all private, except one that was more recently privatized). The CTC remuneration mechanism interferes with the market so that market prices and productions cannot be fully explained without them. A detailed review of the factors that condition the Spanish electricity market can be found in Kahn (1998).

2.2.2 Bidding structure

The daily market is based on a sealed-bid double auction. It is summoned once a day. Bids cannot be modified after they are submitted. Selling and buying offers (and bids) are sent simultaneously by every participant, so each participant's submissions are unknown to the others.

Daily market bids consist of a limited number of hourly blocks, with a different price for each one. Each selling block must be associated with a physical generation unit, but several blocks with different increasing prices can be submitted for a single unit. Thermal units are usually divided into two or more blocks, the cheaper one covers minimum unit power and the others the rest of the power to the maximum. Bidding blocks can be simple or complex. Simple blocks for one hour are independent of the other hourly blocks, while complex blocks can be affected by temporal conditions. For example, the matching of the first block of the sale offer for a single generating unit can be set to ensure that starting units are not stopped whilst their starting-up is in process. It is also possible to include an "income constraint", which sets a minimum daily income for a bid to be accepted, accompanying each generation unit bid. Other complex constraints may be added: inseparable bidding blocks and ramp-rates.

2.2.3 Clearing and price determination

Each single bid is defined by a pair (q_i, p_i) where p stands for price and q stands for quantity. The market operator receives a set of single bids for each hour (inclusion of complex bidding will be treated later). They are divided into selling offers **S** and buying biddings **B**. An aggregated supply function for each hour h is built using this set. It represents the total amount of energy the generators are offering for a given price. Selling offers must be ordered increasingly to build this function:

$$S^{-1}(p) = \sum_{(p,q)\in \mathbf{S}} \{q_i \mid p_i \le p\} \tag{2.1}$$

An aggregated demand function for each hour h is built in a similar way using buying bids. It represents the total amount of energy the buyers would buy for a given price. In this case, bids must be ordered decreasingly:

$$D^{-1}(p) = \sum_{(p,q)\in \mathbf{B}} \{q_i \mid p_i \ge p\} \tag{2.2}$$

Market clearing is obtained as the intersection of these two functions. The marginal price is determined by the point of intersection. Selling offers with a price lower than market clearing price are accepted and buying bids are accepted above this value. Additional rules must be stated to deal with offers that exactly match the clearing price.

Supply and demand functions can also be defined for each firm. They will be denoted as $S_f(q)$ and $D_f(q)$. Aggregated supply and demand functions can be obtained as:

$$S^{-1}(p) = \sum_f S_f^{-1}(p)$$
$$D^{-1}(p) = \sum_f D_f^{-1}(p) \quad (2.3)$$

Supply and demand functions, as previously defined, are step-shaped functions. This kind of function is discontinuous and non-invertible. In order to make the analysis easier, they will be considered as continuous and strictly monotonic approximations of the defined functions. Under this assumption, the market clearing point (q^*, p^*) can be expressed as:

$$p^* = D(q^*) = S(q^*)$$
$$q_f = S_f^{-1}(p^*) \quad (2.4)$$
$$q_f = D_f^{-1}(p^*)$$

The previous expression must be replicated for each bidding hour. Complex bidding complicates the analysis, but it is usually possible to build up an equivalent single bidding that provides a good approximation for market clearing.

2.2.4 Residual demand

A generation firm f can represent the other firms' bidding as a single function. It is the so-called residual demand. It is built from the aggregated demand function and the other offers (denoted as $-f$):

$$R_f^{-1}(p) = D^{-1}(p) - S_{-f}^{-1}(p) \quad (2.5)$$

Residual demand represents the potential capacity of a single firm to modify the market price. Market clearing can be obtained from a generating firm's point of view, from residual demand and firm's offers, represented by means of the supply function:

$$p^* = R_f(q^*) = S_f(q^*) \quad (2.6)$$

If a firm knows beforehand its residual demand function, it could determine the optimal quantity to be sold in the market to maximize profit. Remuneration of accepted offers is made at marginal price, so the profit function (assuming no contracts have been signed by the company) can be expressed as firm income minus firm cost $C_f(q_f)$. The problem to solve is to find the value of q_f that maximizes firm profit Π_f:

$$\Pi_f = R_f(q_f)q_f - C_f(q_f) \quad (2.7)$$

Short and medium-term market analyses differ from each other in the way they represent the residual demand functions as a consequence of their different scope and objectives, as well as the meaning given to marginal cost value.

2.2.5 Short-term market analysis

Short-term market analysis is commonly performed by generation companies with a scope of one or two weeks. These studies supersede traditional unit commitment studies in which a centralized operator decides the production schedule for every generating unit according

to minimum cost and reliability criteria. In a competitive framework, each company has to forecast the part of demand that it wishes to supply as a result of the bidding and clearing process, and plan which units to run for this purpose. This kind of analysis uses information provided by the medium-term analysis, such as the quantity of hydro resources to be used and the marginal cost of these resources. The objective is to fulfil as much as possible these referential values, with a feasible operation of generation units.

The main result of these studies is the complete set of offers and bids that should be sent to the market to obtain, as a result of market clearing, the previously computed unit commitment. As a secondary result, useful information for subsequent markets, the availability and marginal cost of reserve, can be obtained. Furthermore, this provides the basis for short-term price forecasting.

A common method to address the short-term analysis is based on obtaining an accurate estimation of the firm's residual demand. It can be managed as a set of offers and bids, but it is also possible to use some kind of approximation as linear, linear piecewise or sigmoid functions. The use of continuous and strictly monotonic approximations of the real functions has some advantages. Firstly, they are easier to manage from a mathematical point of view: clearing algorithms can be expressed more easily and their derivatives can be obtained to provide sensitivity information. Secondly, they need less memory to be stored and manipulated in a computer.

Once the residual demand function has been built, alternative bidding sets for the particular firm are evaluated by obtaining the clearing result with a market clearing algorithm. Uncertainty in residual demand can be taken into account, by considering the result of the different bidding sets under different residual demand values.

The method to estimate the residual demand depends on information availability about previous market bids. Although every single hour is represented separately, some classification and characterization methodology is needed to deal with historic bidding information. Time series clustering techniques and neural networks are some of the most useful alternatives, as will be shown.

2.2.6 Bidding information availability

Residual demand estimation can be performed from historic information, if available. The Spanish market operator provided all of the bidding information during the first months of market, with the objective of allowing a fast learning of market rules operation. Every market agent could obtain the bidding of the rest of the agents with a delay of one day. This period was later extended to one month. From July 2001 to date, aggregated supply and aggregated demand functions are published, with a one-day delay, and complete agents' biddings are made public after three months.

Although transparency is a basic principle in any competitive market, there is no commonly accepted criterion about the desirability of publishing the agents' full submissions. On the one hand, historical data on bidding may help the market participants to understand the behaviour of their competitors, but on the other hand, transparency should be restricted to non-strategic features in order to avoid anti-competitive actions.

This controversy has demonstrated various levels of information availability in different countries. Thus, information is not available in Nord pool, for example. In contrast, the market operator published on its web page aggregated supply and demand functions for each hourly period in California, as in Alberta pool. The Australian market operator also makes public bidding information, distinguishing those made by each agent. In New Zealand, offers were originally anonymous, but they started to be identified from April 1999.

2.2.7 Medium-term market equilibrium analysis

Medium-term market analysis usually deals with market operation from the generating company's point of view with a yearly horizon. Maximum production capacity is fixed by previous long-term decisions including: new plant building, seasonal hydro operation and energy sales or fuel purchases under long-term contracts. The objective is to decide the firm's market share objective, fuel and hydro resources management (also known as water allocation). These studies can also be oriented to forecast prices and associated values (costs, incomes, taxes, etc.), to evaluate medium-term contracts and to assess risk coverage. In Spain, special attention must be paid to take-or-pay gas contract management and to the use of subsidized national coals.

Short-term methods are not suitable over this scope. The study of every hour in a year separately is not only complex and time-consuming but also difficult to understand at the end. An adequate hourly aggregation is required. Besides, the meaning of marginal cost from a medium-term point of view is different to the short-term focus. In the short term, marginal cost is closely related to the variable cost of the most expensive unit that has been committed for production by the firm. However, medium-term analysis includes such considerations as fulfilment of yearly production requirements (produced by coal subsidy quotas, take-or-pay contracts and/or market-share objectives, for example; see Reneses *et al.*, 1999 or Barquín *et al.*, 2004). Finally, another weak point of detailed hourly analysis is that it does not reflect properly firms' long-term strategies, as daily bidding is frequently distorted by transitory situations.

The concept of market equilibrium is very useful in the mid-term analysis. It represents an ideal point, defined by the value of firms' output such that there exists no possibility for any firm to improve its profit by means of a unilateral decision. Market dynamics are heavily affected by external and strategic factors and this equilibrium point is seldom reached when we analyse the daily market in the short term. However, in the mid-term approach, it can be considered as a good estimation of the average behaviour of market variables as well as facilitating sensitivity analysis. Market equilibrium is computed as the solution of the set of equations:

$$\frac{\partial \Pi_f}{\partial q_f} = 0 \tag{2.8}$$

This derivative may be obtained from equation (2.7):

$$\frac{\partial \Pi_f}{\partial q_f} = R_f(q_f) + \frac{\partial R_f(q_f)}{\partial q_f} q_f - \frac{\partial C_f(q_f)}{\partial q_f} = 0 \tag{2.9}$$

Assuming that the derivative of cost (marginal cost) is known, solving this set of equations requires some assumption about the residual demand function's shape. Considering that $R_f(q_f)$ represents price, only an assumption about $\partial R_f(q_f)/\partial q_f$ is needed. This function represents the variation of equilibrium price when one firm's output changes. A conjecture about price behaviour around market equilibrium is called a conjectural variation. A set of conjectural variations for every firm determines a mid-term market behaviour. Conjectural variations can be obtained without using competitors' bidding information.

2.2.8 Chapter overview

The next sections present four different methods devoted to managing the problem of obtaining a representation of competitors' behaviour by constructing a residual demand function (RDF) with the final objective of forecasting market behaviour. The first of them is based on a cluster procedure to group similar bid functions. The second model uses several time-series-based techniques. The third one utilizes input–output hidden Markov methods. Finally, the fourth example makes use of a market equilibrium representation. These methods use different focuses both in market behaviour analysis and RDF representation, and are designed for different time horizon market analysis. The chapter is completed with some conclusions and a comparison of the different approaches.

2.3 A TWO-PHASE CLUSTERING PROCEDURE FOR THE ANALYSIS OF BID FUNCTIONS

This section introduces a method for the analysis of bid functions (including RDF) based on a two-phase clustering technique. The first stage is to convert the original hourly step functions into piecewise linear approximations. Subsequently, the second stage performs a cluster analysis of the piecewise linear functions in order to discover some patterns along the temporal scope considered.

For the sake of simplicity, the term *bid function* (BF) will be used hereafter to refer to any of the following concepts: supply function $S(q)$, demand function $D(q)$ or residual demand function $R(q)$.

This section is organized as follows. Firstly, the proposed methodology to analyse historic offer data is presented. Then, the two-phase clustering algorithm is explained and finally, its application to a real case is presented.

2.3.1 Proposed methodology

The initial difficulty that appears when analysing historical offers data is that the quantity of available information can be extraordinarily high. There is no common agreement about the convenience of publishing the offers presented to the market by the different participants, as mentioned before. Nowadays, the aggregated supply and demand functions are published the day after the market clearing, allowing each participant to build its own residual demand functions. When this kind of information becomes public, it is necessary to have automatic procedures to extract outstanding information from all the stored data.

Given a particular hour, the offer stack $\{(q_1, p_1), (q_2, p_2), \ldots, (q_n, p_n)\}$ can be modelled as a stepwise process (increasing for supply functions and decreasing for demand or residual demand functions). As the number of submitted blocks of quantity–price can be very high, many steps with different sizes can be observed in the original stepwise functions.[2] The proposed analysis methodology consists of two stages.

The first stage is to convert the original hourly stepwise functions into piecewise linear approximations. Besides the advantage of summarizing the input data without losing relevant information, another important advantage is that piecewise linear approximations can be derived along the quantity axis. Therefore, instead of having just two possible first derivate values (0 for the horizontal steps and $\pm\infty$ for the vertical ones), the linear approximations provide intermediate values, reflecting the result after fitting consecutive steps by straight lines. The slopes of these segments will play a very important role in the equilibrium-based models, as will be shown.

[2] Notice that in the Spanish case each generating unit can present up to 25 blocks of quantity–price for every hour of the day. Therefore, the analysis of the offers of 100 generating units during a whole year could require managing up to $100 \times 365 \times 24 \times 25 = 21.9 \times 10^6$ offers.

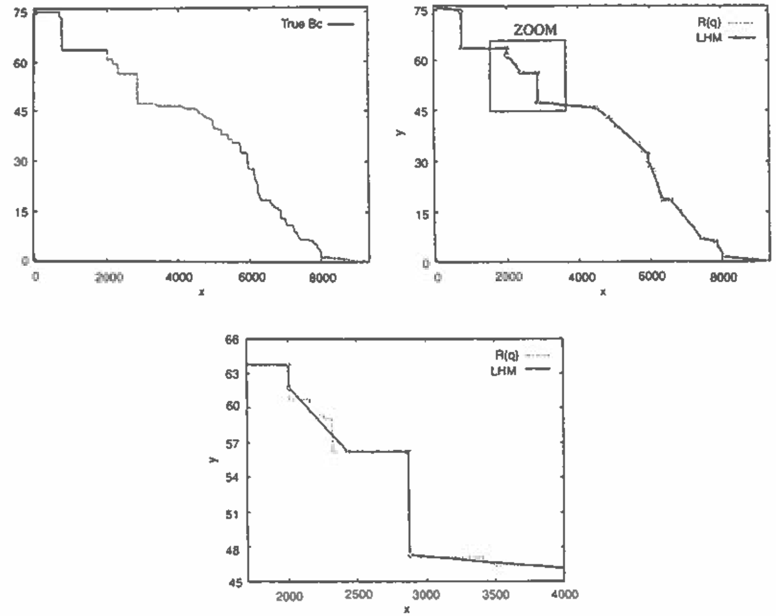

Figure 2.1 Fitting the original stepwise function by a piecewise linear approximation

In order to fulfil this stage, the *linear hinges model* (LHM) can be useful. Basically, the LHM is a very specialized algorithm to find piecewise linear approximations from scatter-plot data (Sánchez-Úbeda, 1999). Figure 2.1 shows an example where the LHM is applied to approximate a real BF. In the augmented zoom, it can be seen how one linear segment filters a set of small consecutive steps, where large ones are preserved.

The second stage of the proposed methodology is to make a cluster analysis of the piecewise linear functions in order to discover some patterns along the temporal scope considered. Hereafter, the clustering procedure is described in detail.

2.3.2 Cluster analysis of bid functions

The objective of the cluster analysis is to find natural groupings among a set of observations or samples, so that observations with similar characteristics are grouped in the same cluster whereas different or heterogeneous ones are grouped in different clusters. Each observation is normally represented by a vector with a dimension equal to the number of attributes characterizing it. Therefore, when this dimension is higher than 3, an inherent difficulty of cluster techniques is to check the goodness of the obtained results. In the case of BF, this difficulty does not appear since they are two-dimensional observations, on quantity–price, and thus a simple visual inspection could validate the correct operation of the algorithm. However, it is very important to highlight that the decision to group a pair of observations depends very much on the kind of study being carried out. This means that a subjective component always exists in this type of procedure, which it will be necessary to automate.

2.3.3 Two-phase procedure: hourly and daily clustering

A natural correlation exists among BF belonging to the same day. This empirical evidence can be justified because, when companies define their strategies, temporal horizons greater than one day are considered. As a consequence, given a particular day, the shapes of the supply functions for the same type of hours (off-peak, peak or medium load hours) are very similar, reflecting the fact that the underlying parameters used to build the functions (the load, the set of started-up generating units, the fuel costs, etc.) are analogous. It is possible to take advantage of this fact to facilitate the search of the clustering algorithm, diminishing notably its computational effort.

Let's assume that it is desired to analyse the BF of a whole month. Instead of defining the samples just as all the hourly BF simultaneously, it is proposed here to split the problem into two phases:

- **Hourly clustering.** For each day included in the studied period, perform a clustering of the 24 functions to obtain a small number (three to six) of clusters and their representatives, also called centroids.
- **Daily clustering.** Taking as samples the set of representatives obtained in the previous step, the daily clustering could reveal the pattern of the BF along the temporal scope considered. Moreover, the centroids of the new clusters formed during this phase can be stored in a library of historic "typical functions" which might be accessed later from some other forecasting models, scenario generators, etc.

In order to perform the clustering in whichever of the two phases, it is necessary first to define how to code each function, how to measure the dissimilarity between a pair of functions, and how to obtain a centroid representative of a set of functions.

2.3.4 BF codification

Usually, the observations used in cluster analysis are quantified as vectors $\mathbf{x} \in \mathbb{R}^p$, where p denotes the number of attributes used to characterize each sample.[3] If the total number of samples is N, the objective of the clustering is to find a partition of the set $\{\mathbf{x}_i \in \mathbb{R}^p : i = 1, \ldots, N\}$, trying to maximize the dissimilarity among samples belonging to different clusters (between-cluster dissimilarity), and to minimize the dissimilarity among samples belonging to the same cluster (within-cluster dissimilarity).

In order to perform a clustering of bid functions, the first step would be the codification of each function as a vector. The selected components should express common attributes, as for instance the quantities related to a fixed set of prices. In the case of Figure 2.1, if the reference prices are {0, 15, 30, 45, 60, 75}, the vector representing that function should contain the quantities related to them, i.e. $\mathbf{x} = (9.1 \quad 7.0 \quad 6.0 \quad 4.4 \quad 2.1 \quad 0.6)$ [GWh]. In this case, each vector representing a BF could be viewed as a point in the multi-dimensional space $\mathbb{R}^6$. As all the functions are coded homogeneously, this approach allows the application of conventional cluster algorithms.[4] However, the necessity of selecting *a priori* the set of reference prices is a great disadvantage because wide steps could exist between them, and therefore, outstanding information on the function could be lost or filtered. Instead of increasing

[3] When these attributes measure different magnitudes, it is convenient to normalize and homogenize the data in order to avoid the influence of the units adopted.

[4] In Kauffman and Rousseeuw (1999), clustering algorithms are classified as "partitioning methods" and "hierarchical methods". For instance, distance-based methods such as k-means analysis, nearest-neighbour clustering, etc.

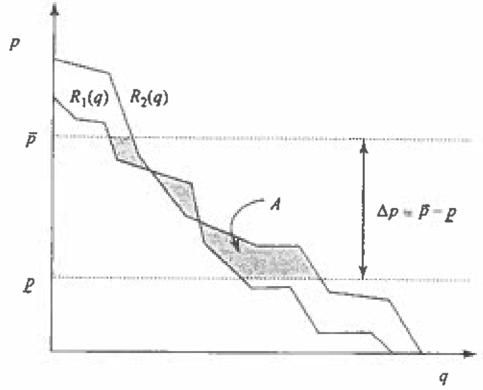

Figure 2.2 Defining the dissimilarity between a pair of BF as the area enclosed by them

the number of reference prices, i.e. the sampling density, here it is proposed to code each BF as the coordinates of every point in the quantity–price plane defining the continuous piecewise linear function. For instance, if the original function is a stepwise residual demand function $R(q)$, its piecewise linear approximation obtained with the LHM can be expressed as:

$$\hat{R}(q) \equiv \{(q_k, p_k) : k = 1, \ldots, K\} \tag{2.10}$$

where K is the number of "knots", i.e. the joining points between consecutive linear segments. This number is determined automatically by the LHM, and each piecewise function would have a different one depending on the complexity of the original function. This heterogeneous codification makes traditional measures of dissimilarity non-applicable.[5]

2.3.5 Defining the dissimilarity between BF

The dissimilarity between a pair of observations i and j is defined as a positive number $d(i, j)$ which is very small for similar observations and very large for dissimilar ones. In García-González (2001) it is proposed to compute the dissimilarity between a pair of bid functions using the area enclosed by them: nearby functions in the quantity–price plane would enclose a small area, and accordingly, faraway functions a bigger one (Figure 2.2). Moreover, when the functions are very close, but their shapes are different, the area would be greater than for the case of similar shapes.

Although the functions could be defined for a wide range of prices, the cluster analysis might be interesting just for a particular price interval, $[\underline{p}, \overline{p}]$. This happens because the uncertainty faced by market participants can be bounded, and therefore, their strategic behaviour could be revealed only in the part of the function covering the most expected marginal prices. For that reason, the area should be computed along the price axis. For instance, let $R_1(q)$ and $R_2(q)$ be two residual demand functions. The area enclosed by them in the interval $[\underline{p}, \overline{p}]$ is obtained as:

$$A = \int_{\underline{p}}^{\overline{p}} \left| R_1^{-1}(p) - R_2^{-1}(p) \right| dp \tag{2.11}$$

[5] Given a pair of observations i and j coded as vectors $\mathbf{x}_i$ and $\mathbf{x}_j \in \mathbb{R}^p$, the dissimilarity can be computed using any measure of distance, such as for instance the Euclidean one $d(i, j) = \|\mathbf{x}_i - \mathbf{x}_j\|_2$.

where $R^{-1}(p)$ denotes the inverse function. Notice that as $R_1(q)$ and $R_2(q)$ are piecewise linear functions, the numeric integration can be achieved easily. In order to normalize the results, it is proposed to divide the mentioned area by the width of the price interval of interest. Therefore, the dissimilarity between functions $R_1(q)$ and $R_2(q)$ can be evaluated as:

$$d(R_1, R_2) = \frac{\int_{\underline{p}}^{\overline{p}} \left| R_1^{-1}(p) - R_2^{-1}(p) \right| dp}{\overline{p} - \underline{p}} = \frac{A}{\Delta p} \tag{2.12}$$

2.3.6 Centroid calculation of a cluster of bid functions

In order to implement the two-phase clustering procedure, it is necessary to calculate the representative element, i.e. the centroid, of each formed cluster. The intuition behind the centroid is the averaging over all elements in the cluster. As the functions are coded heterogeneously, it is necessary to define how to average bid functions, as a traditional mean vector is not applicable. In García-González (2001) it is proposed to obtain the centroid by applying again the LHM. In this case, the scatter-plot data used to build the model is obtained by sampling the set of original functions belonging to the cluster (Figure 2.3).

2.3.7 Example case

The example case presented in this section shows how the proposed methodology can be successfully applied to analyse real offer data. In particular, observations used as input data were the Californian hourly supply functions of the whole system during the week from 6th to 12th September 1999. Similar analysis could be applied to bidding on the Spanish market.

Arbitrarily, it has been decided to study the similarity of the functions just within the price interval 5–60$/MWh.

2.3.7.1 Hourly clustering

Among the different techniques available to perform a cluster analysis, an agglomerative linkage algorithm has been implemented. Each iteration of the algorithm, the two previously formed

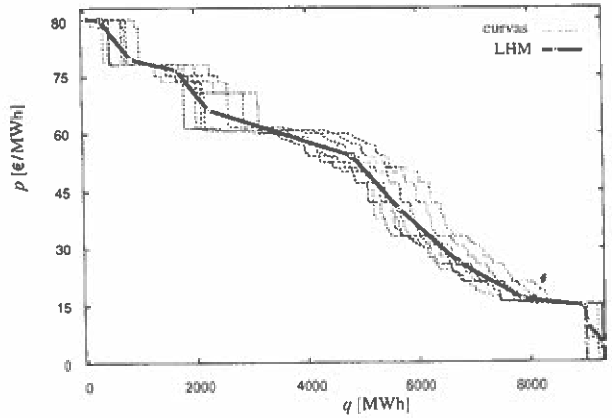

Figure 2.3 Centroid computation from the set of original functions belonging to the cluster

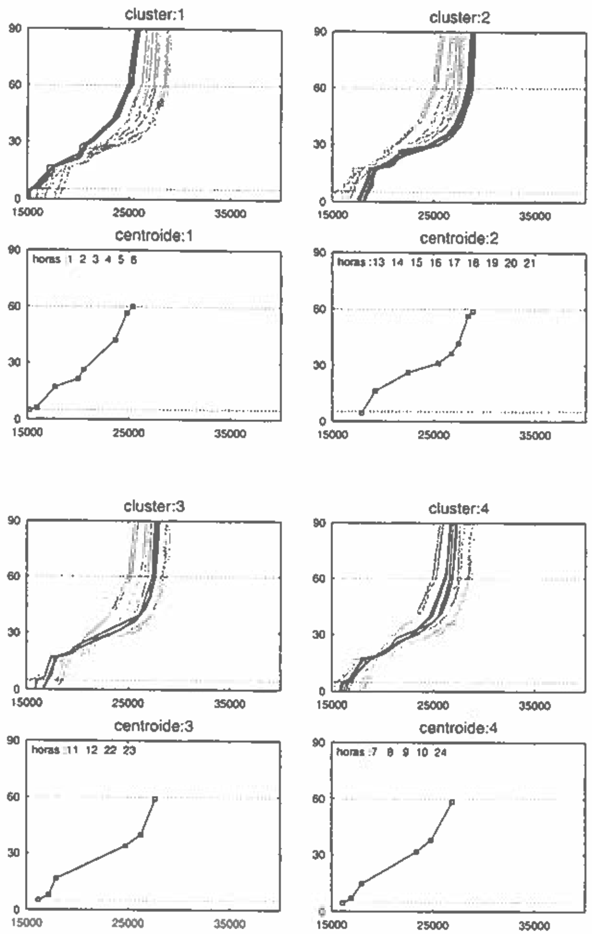

Figure 2.4 Obtained results for the 6th September hourly clustering

clusters with the lowest dissimilarity values (i.e. the most similar) are merged, continuing the process until the goodness of the partition satisfies a stop criterion.

Figure 2.4 shows the results of the hourly clustering of the 6th September. For each one of the four obtained clusters, two graphs are presented. In the superior ones, all the 24 offer functions used as observations are drawn, but only the functions belonging to the corresponding cluster are coloured in black. On the other hand, the inferior graphs present the representative centroid of each cluster, drawn and computed just for the interesting range of prices. The set of hours belonging to each cluster has also been indicated. For instance, the first cluster gathers the supply functions of hours 1, 2, 3, 4, 5, 6, i.e. off-peak hours.

Table 2.3 Number of centroids obtained for each day

Day	No. centroids
Monday	4
Tuesday	5
Wednesday	6
Thursday	6
Friday	5
Saturday	6
Sunday	5
Total	37

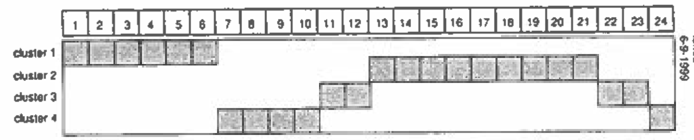

Figure 2.5 Obtained results for the 6th September hourly clustering

Another way to represent these results is shown in Figure 2.5, where hours (columns) belonging to each cluster (row) are shadowed. This chart can easily be interpreted, as the clustering results stem from a physical meaning: clusters 1 and 2 correspond to off-peak and peak hours respectively; clusters 3 and 4 correspond to medium hours, depending on whether they are close-to-peak or off-peak hours, respectively.

The same procedure described here should be repeated for every day included in the temporal scope in order to generate the representatives to be used as samples during the next phase.

2.3.7.2 Daily clustering

The number of samples in this phase is the sum of the number of centroids obtained after performing the hourly clustering for each day of the week (see Table 2.3).

The number of daily clusters has been preset to eight. Therefore, the set of 37 centroids obtained during the first phase is grouped on eight clusters. Note that in this case, the linkage algorithm finishes when this number of clusters is reached, without it being necessary to quantify the goodness of the partition at each iteration.

Similar plots to the ones shown in Figure 2.4 could also be generated here, where the centroids would now be the "representatives" of the "daily representatives". However, in order to capture some pattern along the temporal scope, it is proposed to show the clustering results in a *chromogram*, as presented in Figure 2.6.

For each day considered, a similar chart to the one shown in Figure 2.5 (6th September) can be obtained. In order to build the chromogram, it would be necessary to join all those daily charts in a consecutive order, but using the same colour for the rows (i.e. the centroids) that have been grouped within the same cluster during the second phase. In this way, a simple visual inspection of the chromogram allows us to:

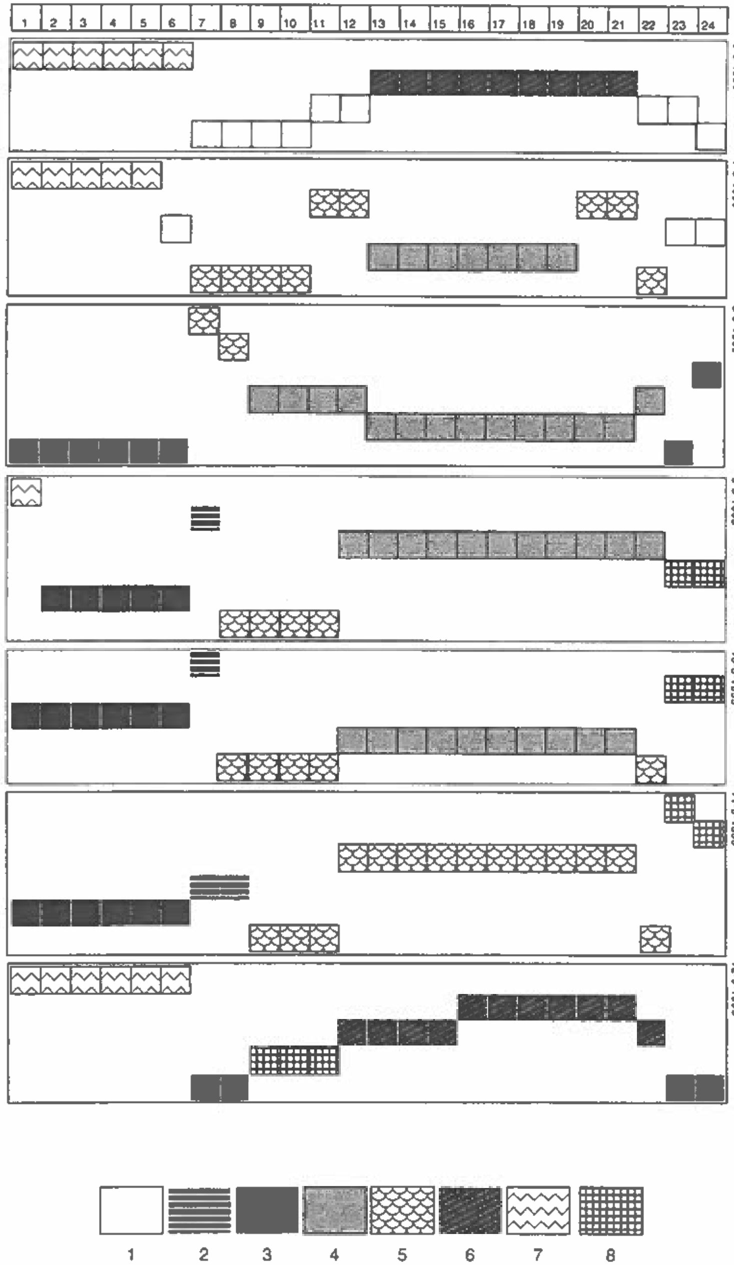

Figure 2.6 Chromogram summarizing two phases' results

- Represent the hourly clustering results for each day. This may help the user to easily identify the groups of similar hours belonging to the same day.
- Represent the daily clustering results. Hours belonging to different days, but sharing the same colour, would have a very similar BF. Therefore, if a pattern of behaviour along the time exists, it would be captured with the chromogram.

Figure 2.6 shows the obtained chromogram corresponding to the studied week. It can be seen how hours assigned to different clusters during the first phase are grouped later on during the second one. This is due to the fact that the hourly clustering is more refined than the daily clustering, as only the 24 functions of the day are considered each time. However, when considering the whole week, the differences among those functions are not big enough to separate them into different clusters, but rather they group together. For example, when making the Monday clustering, hours 11, 12, 22 and 23 form the cluster number 3 while hours 7, 8, 9, 10 and 24 form the cluster number 4. However, when carrying out the daily clustering, all those hours are grouped together with hours 6, 23 and 24 of Tuesday to constitute the "definitive" cluster labelled as 1 and marked with white frames.

The most significant conclusions from the inspection of the chromogram are the following:

- Cluster 1 contains off-peak hours of Monday and some medium-load hours of Tuesday.
- Cluster 3 contains off-peak hours of Wednesday, Thursday, Friday and Saturday.
- Cluster 4 contains peak hours of Tuesday, Wednesday, Thursday and Friday.
- Cluster 5 contains medium-load hours of Tuesday, Wednesday, Thursday and Friday as well as peak hours of Saturday.
- Cluster 6 contains peak hours of Monday and Sunday.
- Cluster 7 contains off-peak hours of Monday, Tuesday and Sunday.

From the previous analysis, operational optimization and bidding elaboration can be carried out avoiding the use of the whole set of bidding data (García-González, 2001). The description of these methods goes beyond the scope of this chapter.

The presented method performs a static analysis of bidding functions based on a linear piecewise approximation of residual demand functions, grouped by means of a clustering method. The next two sections are devoted to methods that are also oriented to short-term analysis and with a simpler RDF representation, but which deal with dynamic phenomena appearing in the daily market evolution.

2.4 FORECASTING METHODS FOR RESIDUAL DEMAND FUNCTIONS USING TIME SERIES (ARIMA) MODELS[6]

RDFs are important inputs in order to carry out the dispatch optimization, which every utility has to solve in order to maximize its expected profits. A wrong prediction of these functions may provide misleading results in the dispatch optimization process. An accurate forecasting of the day-ahead residual demand function can be carried out using *time series analysis*.

The present section is focused on the forecasting and estimation of the *hourly* RDFs. For generation agents of a moderate size, the shape of the RDFs can usually be approximated using a linear interpolation. Thus, the problem is reduced to the analysis of the time evolution of

[6] The authors gratefully acknowledge the contributions of the staff at *Centro de Gestión de la Energía* of *Unión Fenosa Generación, S.A.* for their work and collaboration in this research. Also we appreciate the helping hand of SCA company members William Lattyak and Professor Lon Mu Liu, who have provided great support and brilliant ideas.

two variables, the *slope and the intercept of the linearized RDFs*. Both of them can fluctuate sharply depending on the hour of the day and also the day of the week. Thus, hours with a strong demand for electricity tend to have greater values of slope and intercept than the rest, due to the higher level of demand served and the more expensive generation technologies dispatched. As the prediction's horizon is very short (day-ahead market or spot market), tools like ARIMA time series are useful to achieve a good RDF estimation. In particular this approach will allow the following analysis:

- Determining whether a unique time series model for each one of the variables is enough or whether more detailed models (i.e. distinction between labour days and weekends or between peak/valley demand hours) lead to more accurate results. In the latter case a weighted estimation will be developed in this section that allows us to obtain optimal parameters for each of the time zones considered. The study of data seasonality is considered as well.
- Evaluating the influence of some explanatory variables in the shape of the residual demand functions: expected system demand, expected level of run-of-the-river power or the expected amount of nuclear power in the system. This fact leads to the use of transfer function (TF) models.
- Classification/filtering of outliers that can come up in the data.
- Analysis of calendar variations (i.e. holidays like Easter or Christmas) and known disturbances (like a sudden increase in the run-of-the-river power level due to heavy rainfalls), which can distort the performance of the models used.

This section is structured as follows: in subsection 2.4.1 a first approach to the problem is carried out by performing a classical ARIMA analysis of both intercept and slope corresponding to the linearized RDFs. In subsection 2.4.2 a useful combination of explanatory variables is looked at in order to improve the performance of the original ARIMA model. Besides this, a weighted estimation method is presented that will allow the achievement of optimal models for each of the different time intervals that can be considered depending on the type of day (labour, weekend) and the hour (peak, valley, plateau). After this, subsection 2.4.3 shows a case study where each of the above model's (ARIMA, TF and weighted estimation) performance is compared. Finally, subsection 2.4.4 contains a summary of conclusions obtained by means of a detailed analysis of the models' performance.

2.4.1 Analysis using an ARIMA model

Firstly, it must be determined whether a transformation of the original series is required in order to deal with the conditions required to perform an ARIMA analysis. Standard ARIMA analysis relies on the assumption that the time series is stationary. This means that the mean and the variance of the data series are constant through time. The first of these (stationary mean) is usually solved by differencing the series and it is clearly visible in the pattern of the ACF (autocorrelation function). Possible modifications to induce a stationary variance must be applied (when needed) before any further analysis of the data. It is fairly common to find data series where the variance is proportional to its level. In this case a constant variance is induced by transforming the data, where the Box–Cox transformations are the most widely used (Pankratz, 1991; Liu, 2001).

An analysis of a typical two-month sample (Figure 2.7) of the linearized RDC intercept (a similar graph is obtained using the variable slope) shows that the variance tends to be

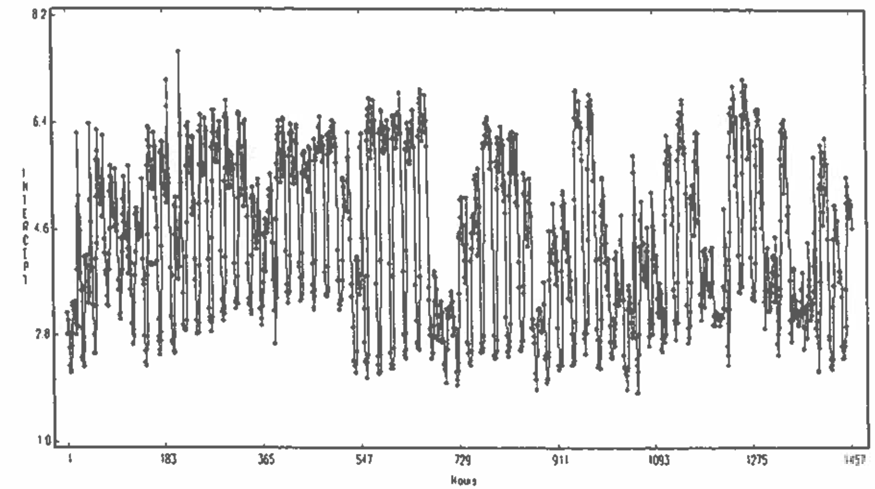

Figure 2.7 Variable intercept (c€/KWh), hourly evolution during two months
Source: Association for Studies on the Quality of Electric Energy. Reproduced with permission.

more or less constant, regardless of the level reached. So a transformation in this case is not necessary. It should be noticed that if the data span is extended (i.e. a year or more instead of two months) there will be periods with higher volatility than the others, thus the variance would not be stationary. Nevertheless, up to three or four months the constant variance assumption is usually right, and there are enough data contained in this period to perform adequate analysis.

Once these previous steps have been accomplished, it is time to analyse the ACF and PACF (partial autocorrelation function) of the model, in order to identify the autoregressive (AR) and moving average (MA) parameters of the model. During the identification procedure the principle of parsimony is applied. According to this principle, models with as few coefficients as necessary to adequately explain the behaviour of the data will be selected.

When a tentative model is proposed, then it must be estimated and checked to verify if it complies with the conditions of stationarity (mean, variance and ACF are constant through time) and invertibility (which ensures that more recent data have more weight than the distant past). Our ACF/PACF graphs correspond to the variable intercept. The graphs obtained with the variable slope present the same kind of pattern and behaviour.

The remarkable features of the ACF shown in Figure 2.8 are a slow decay at the first seasonal lags (24, 48, 72) mixed with a slow growth during the following seasonal lags (96, 120, 144) until it reaches a local maximum at lag 168. Then the procedure repeats itself from lag 168 to lag 336 but with slightly lower values. So it seems that there is a double periodicity, one with a daily scope (this is the reason for the peaks of the ACF function at lags $t = 24k$, with $k = 1, 2, 3, \ldots$) and the other with a weekly horizon (ACF peaks at lags $t = 168k$, with $k = 1, 2, 3, \ldots$). Due to the ACF pattern at these seasonal lags, a double seasonal differencing is needed. Regular differencing (i.e. between adjacent values) is not necessary because the ACF decays fairly quickly at the first lags.

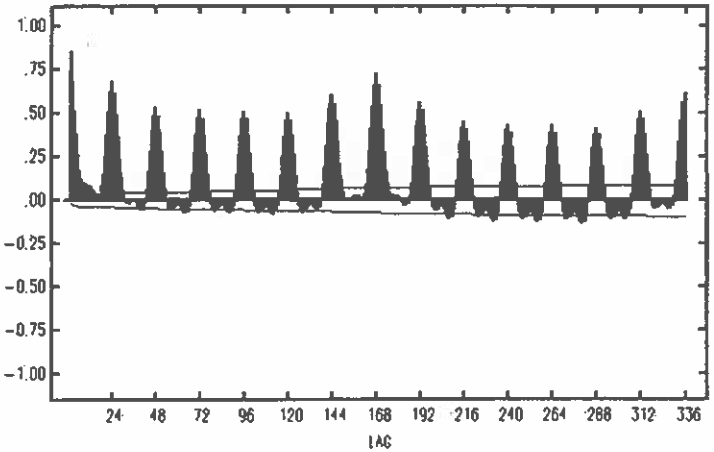

Figure 2.8 ACF for the variable intercept. 1 LAG = 1 hour
Source: Association for Studies on the Quality of Electric Energy. Reproduced with permission.

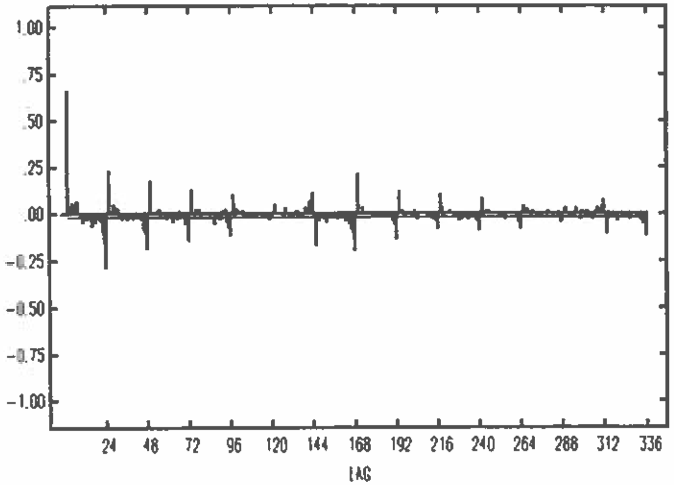

Figure 2.9 PACF for the variable intercept with double seasonal differencing. 1 LAG = 1 hour
Source: Association for Studies on the Quality of Electric Energy. Reproduced with permission.

Figure 2.9 shows a slow decay in the PACF pattern at the seasonal lags $t_1 = 24k$ and $t_2 = 168k$ ($k = 1, 2, 3, \ldots$). Figure 2.10 shows high spikes at lags $t = 24$ and $t = 168$ and a lower one at lag $t = 48$. This means that a moving average behaviour is present at these lags. In contrast, the first lags show a different pattern with a spike at lag $t = 1$ in the PACF (Figure 2.9) and a decay in the ACF (Figure 2.10), so a first-order autoregressive parameter looks adequate. Many of the other large autocorrelations (e.g. lags 144 and 192) observed

Table 2.4 SCA results for the variable intercept. ARIMA model

Coefficients	Type	Order	Value	t-Value
θ_1	MA	24	0.6379	67.81
θ_2	MA	48	0.1551	16.27
θ_3	MA	168	0.7708	118.26
ϕ_1	AR	1	0.7368	114.98
k		0	−0.0006	−0.41
RSE			0.82	

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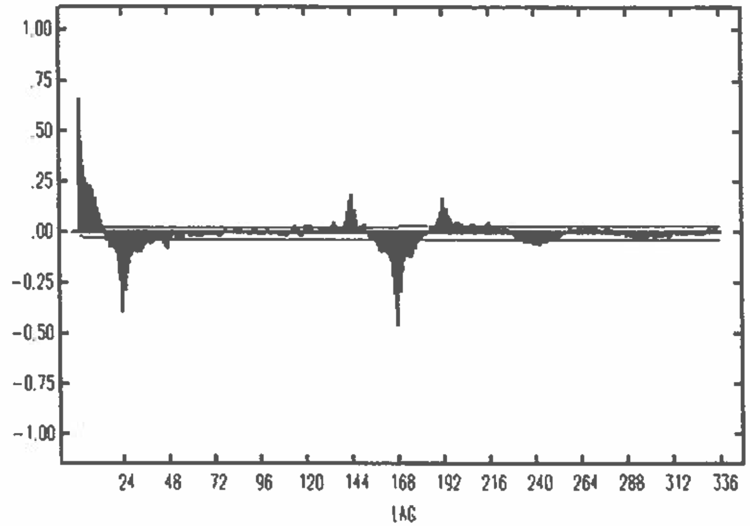

Figure 2.10 ACF for the variable intercept with double seasonal differencing. 1 LAG = 1 hour
Source: Association for Studies on the Quality of Electric Energy. Reproduced with permission.

in Figure 2.4 are probably a reflection of the weekly seasonality, thus it is not necessary to consider more differencing.

It is also very important to avoid over-differenced models, which always drive the identification process to pure moving average models (Box, 1976; Liu, 1997). Thus, a tentative model which complies with all the above characteristics is proposed:

$$\nabla_{24}\nabla_{168}Y = k + \frac{(1 - \theta_1 B^{24} - \theta_2 B^{48})(1 - \theta_3 B^{168})}{(1 - \phi_1 B)} a_t \quad (2.13)$$

where Y corresponds to the model output variable (either intercept [c€/kWh] or slope [c€/MW*kWh] of the linearized RDFs) and k is a constant. ∇_x is a differencing operator of order x, a_t is a white noise (random shock) and B^x represents the backshift operator, that delays x lags when applied to a given variable. SCA™ (the statistical software program used) provides particular values for the model given in equation (2.13): see Tables 2.4 and 2.5.

Table 2.5 SCA results for the variable slope. ARIMA model

Coefficients	Type	Order	Value	t-Value
θ_1	MA	24	0.6406	68.04
θ_2	MA	48	0.1455	15.24
θ_3	MA	168	0.8006	126.63
ϕ_1	AR	1	0.7208	109.75
k		0	−0.0963	−0.4
RSE			151.65	

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The t-value indicates whether a parameter of the model is meaningful (t-value greater than two) or not by means of a Student-t significance test. As was pointed out, both intercept and slope present the same characteristics (same periodicities and differencing). It is straightforward to check that both determined models deal successfully with the stationarity and invertibility conditions required for an ARIMA model – all the roots of the AR and MA polynomials lie outside the unit circle (Pankratz, 1991; Box, 1976; Box *et al.*, 1994). Once a tentative model is specified, it is mandatory to check if the residuals obtained are normally distributed. There are a great variety of tests to accomplish this task – e.g. the histogram of the residuals, the normal probability plot or the Box–Ljung test (Pankratz, 1991; Box, 1976; Box *et al.*, 1994).

The histograms of the intercept and slope variable residuals (not shown here) are quite symmetrical, suggesting that the residuals are normally distributed. The normal probability plot, not shown here (Pankratz, 1991; Liu, 1997), does not have large deviations from a straight line except from very large residuals, again suggesting that the residuals are normally distributed and also that an outlier test should be done to analyse the largest residuals (this task will be briefly commented on in subsection 2.4.2.3).

Finally, it was also checked that the ACF of the residuals series was non-significant (i.e. each residual autocorrelation falls well short of its two standard error limit), concluding that the model adequately captures the autocorrelation patterns in the data. The ARIMA models built in this section will be used as a baseline comparison for an improved model, which will be proposed in the following section.

2.4.2 Analysis using TF models and weighted estimation

In order to improve the performance of the above ARIMA model, further explanatory variables should be identified. In order for an explanatory variable to be useful, it should be easily available and, if possible, predictable with relatively little uncertainty. Three meaningful variables have been determined in order to explain the evolution (level and slope) of the linearized RDFs (results will only be shown for the variable intercept, but the interpretation with the variable slope is straightforward). For the sake of clarity *results are shown in weekly average values although real data are hourly values*. Thus, these explanatory variables are (in order of importance):

- **The national hourly electricity demand.**
  It is obvious that the higher the demand, the higher and more abrupt should be the RDFs. At present, existing demand forecasting models are very accurate, so the demand will be treated as a non-stochastic variable.

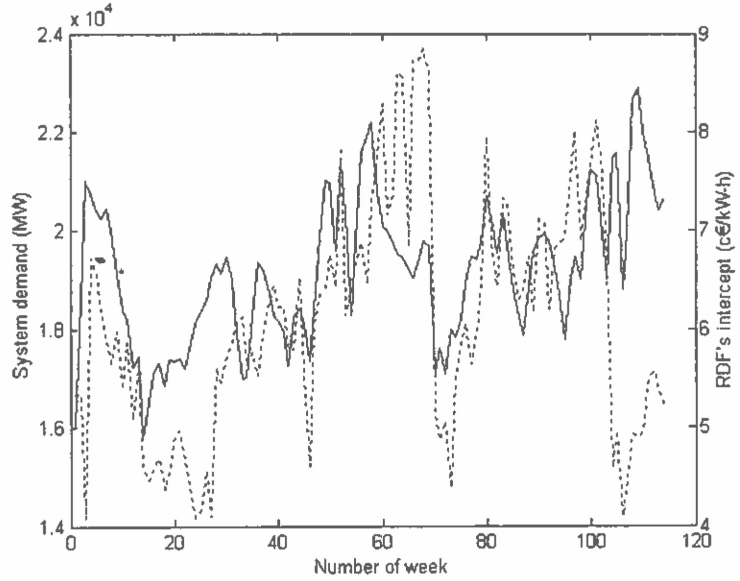

Figure 2.11 Evolution of the variable system demand (continuous line) versus variable intercept (dashed line). Weekly average values
Source: Association for Studies on the Quality of Electric Energy. Reproduced with permission.

Although it can be seen in Figure 2.11 that there is a meaningful correlation between both variables, there are periods where the demand was high and the intercept low, and *vice versa*. So there are more factors involved in the RDF time evolution.

- **Run-of-the-river power.**
  There is always a fraction of the whole generation power available which is offered at the lowest prices. This portion of power (known as base generation) corresponds to the amount offered by both nuclear power plants and run-of-the-river units (hydro plants without regulating capacity, because they do not have dams). This base hydro generation remains rather constant through a day and it is easy to forecast, knowing the dam's hydro conditions and the weather forecast. Figure 2.12 shows the correlation between the variable intercept and the variable run-of-the-river power with a weekly average value basis.
  It can be observed that when there has been a sudden increase in the level of the run-of-the-river power (due to heavy rainfalls), the values of the variable intercept have fallen dramatically. Also in this situation the RDFs tend to be flatter so the values of the variable slope are lower than under normal conditions.
- **The level of nuclear power.**
  Nuclear power is also a base generation component. The scheduled maintenance of the nuclear power plants is often known in advance, so this variable becomes a useful explanatory variable. The effect of the level of this variable is quite similar to the run-of-the-river power.

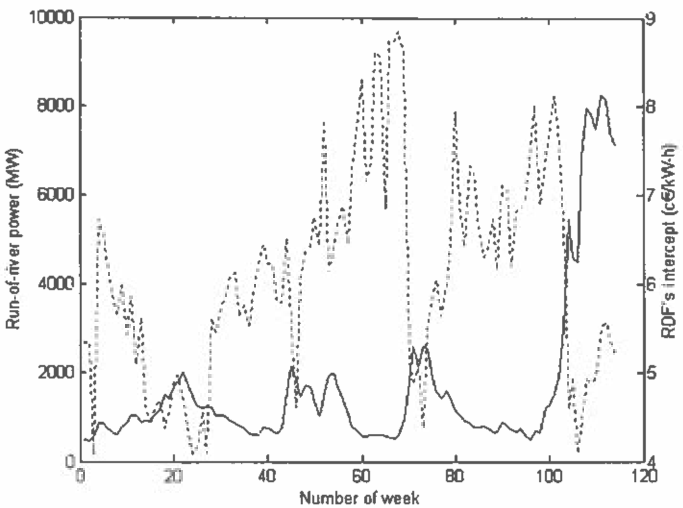

Figure 2.12 Evolution of the variable run-of-the-river power (continuous line) versus variable intercept (dashed line). Weekly average values
Source: Association for Studies on the Quality of Electric Energy. Reproduced with permission.

2.4.2.1 Aggregation with a single explanatory variable

If a TF model is tried using separately the above three explanatory variables, all the variables except from the electricity demand become considered as non-significant. The reason is clear, on the one hand the two output variables (intercept and slope) change greatly from hour to hour, but the variables run-of-the-river and nuclear power are rather constant during the whole day, so the estimated correlation between the output variables and these two input variables will be very small. On the other hand, if these variables are compared on a weekly basis (as in Figure 2.12), it is obvious that there exists a relationship between the output variables and the level of run-of-the-river power (or nuclear power).

Therefore an explanatory variable with an hourly scope and that takes into account all the above characteristics needs to be defined. If the sum of run-of-the-river power and nuclear power (nearly constants for the whole day) is subtracted from the hourly system demand, a useful explanatory variable is obtained. This can be expressed as the non-base demand. Formally, the proposed explanatory variable is defined for each hour h as:

$$NBD_h = D_h - F_h - N_h \tag{2.14}$$

where NBD stands for non-base demand, D is the demand, F is run-of-the-river power and N is nuclear power. All the values in this expression must be interpreted as expected values.

This new explanatory variable covers all the possible cases. It will take high values if the system demand is high and/or there is a lack of base generation (due to low values of run-of-the-river power and/or nuclear power) and lower values if the system demand is low and/or there is a high level of base generation, so it presents an adequate hourly variation. For the sake of clarity, Figure 2.8 shows in weekly values the relationship between the new explanatory variable and the variable intercept.

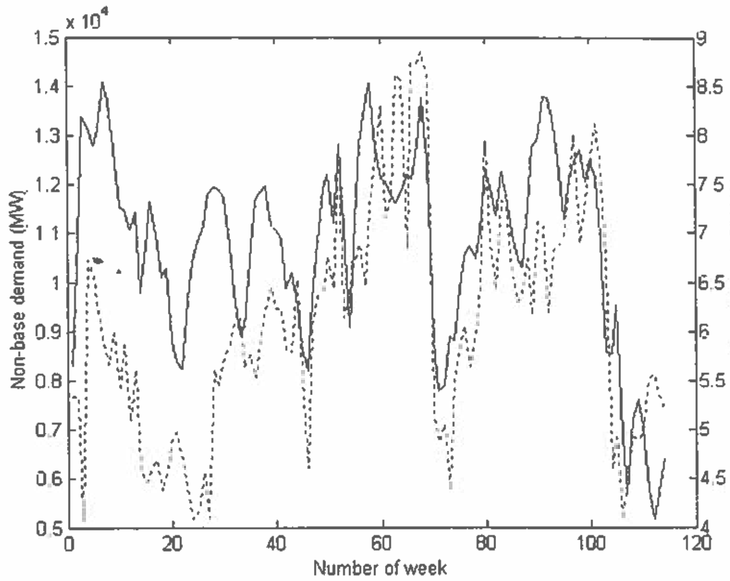

Figure 2.13 Evolution of the variable non-base demand (continuous line) versus variable intercept (dashed line). Weekly average values
Source: Association for Studies on the Quality of Electric Energy. Reproduced with permission.

It is observed in Figure 2.13 that the peaks and valleys of the new explanatory variable evolution bear a strong resemblance with those of the variable intercept, and their correlation is much better than when only the variable demand (Figure 2.11) is considered. For example, focusing on the last weeks of Figure 2.11 and Figure 2.13, which corresponds with a period of both high levels of system demand and run-of-the-river power, the new explanatory variable is able to capture the sudden drop of the variable intercept, while if only the system demand is used the results will be poor, because higher prices are expected. The same result occurs with the variable slope of the linearized RDFs, which has low values (i.e. the RDFs are flatter) if the explanatory variable non-base demand also has low values, and *vice versa*.

2.4.2.2 TF models and a weighted estimation approach

Firstly, a non-weighted TF model with non-base demand as explanatory variable will be proposed. The LTF approach (Liu, 1996, 2001) has been applied to adjust the model stated in equation (2.15), because it presents some advantages with respect to the more classical prewhitening method (Box, 1976; Box *et al.*, 1994). Using this method and skipping the non-meaningful lags of the explanatory variable, the following structure results:

$$\nabla_{24}\nabla_{168}Y = \nu_0\nabla_{24}\nabla_{168}[NBD] + \frac{(1-\theta_1 B^{24} - \theta_2 B^{48})(1-\theta_3 B^{168})}{(1-\phi_1 B)}a_t \tag{2.15}$$

The TF model shown in equation (2.15) does not dramatically reduce the RSE (residual standard error) in comparison with the ARIMA model proposed in subsection 2.4.1, but when an outlier analysis is carried out, a great number of the outliers caused by changes in the input variable are explained, so its total amount is strongly reduced using the TF model. Regarding the model considered in equation (2.15), it has some serious drawbacks:

(a) If the terms containing differencing are fully developed, the model shows that the values of the output and input variables at current time and the values which took place 24 hours ago (the day before) and 168 hours ago (the week before) are related. In fact there are some more relations than these, but they are much less important.
(b) The fact stated above can give problems when one is focused on days like Mondays or Saturdays, when there are great differences between their RDFs pattern and those of the previous day (Sunday and Friday respectively). So it seems reasonable that Saturdays, Sundays and Mondays will follow a different model than that stated in equation (2.15), therefore a daily discrimination must be considered.
(c) The TF model shows a linear and unique relation between the output and the input variable – coefficient υ_0 in equation (2.15) – regardless of the type of hour considered. This is not fundamentally true, because it is obvious that the ratio between the values of the output and input variables will change depending on the hour and the day of the week. Thus both daily and hourly discriminations should be taken into account.

The *weighted estimation method (WE)* provides a good and easy way to deal with problems like non-linear relations or saturations between the variables (Liu, 2001). In order to use the weighted estimation approach, a prior classification of the available data series divided in time ranges must be done:

Daily discrimination. One independent model for each of the following groups: (1) Saturday, (2) Sunday, (3) Monday and (4) Tuesday through Friday.
Hourly discrimination. One independent model for each of the following groups, according to the pattern of the system demand function: (1) from 2 a.m. to 7 a.m., (2) from 8 a.m. to 9 a.m., (3) from 10 a.m. to 2 p.m., (4) from 3 p.m. to 6 p.m., (5) from 7 p.m. to 10 p.m., and (6) from 11 p.m. to 1 a.m.

An example of the Spanish system electricity demand is provided in Figure 2.14. It can be seen that the hourly discrimination is based on grouping together those periods of time with approximately the same characteristics. Nevertheless, these properties should not be considered as fixed, due to the fact that they depend on seasonality (peak–valley hours are not exactly the same in summer as in winter, there is always a certain displacement). A clustering process could be used in order to obtain an adequate hourly discrimination regarding the current period of the year.

The weighted estimation method searches for the optimal values of the parameters which minimize the RSE at the time range considered (Liu, 2001). It is important to note that, despite the weights applied, the whole series is still considered, so the dynamic pattern of the data is not lost. The objective is to adequately represent the non-linear relationship between output and input variables as a group of linear models focused in different time intervals, taking advantage of the simplicity and clarity of linearity. The following model equations (2.16)–(2.18) and Tables 2.6–2.8 are focused on the time period 8 a.m.–9 a.m. and results are displayed for different days and for the variable intercept. Firstly, the adjusted model and its estimated parameters focusing on Saturdays are shown. The model has been adjusted using

Table 2.6 SCA results for time interval Saturday, 8 a.m.–9 a.m. Weighted estimation. Model equation (2.16)

Coefficients	Type	Order	Value	*t*-Value
υ_0		0	0.0002	14.62
θ_1	MA	1	0.5746	10.21
ϕ_1	AR	1	0.8962	32.16
Differencing	None			
RSE (time zone)			0.51	

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Table 2.7 SCA results for time interval Tuesday through Friday, 8 a.m.–9 a.m. Weighted estimation. Model equation (2.17)

Coefficients	Type	Order	Value	*t*-Value
υ_0		0	0.0008	13.07
θ_1	MA	1	0.2802	3.04
θ_2	MA	24	0.7904	13.86
ϕ_1	AR	1	0.7426	11.01
ϕ_2	AR	24	0.4392	6.01
Differencing	Yes	24		
ϕ_3	AR	168	0.1562	3.6
RSE (time zone)			0.9	

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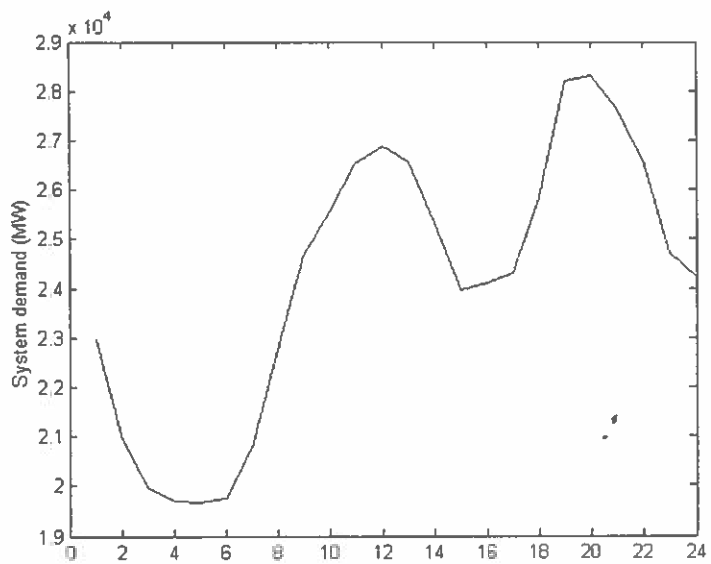

Figure 2.14 Hourly evolution of the Spanish system electricity demand (12/12/2001)

Table 2.8 SCA results for time interval Sunday, 8 a.m.–9 a.m. Model equation (2.18)

Coefficients	Type	Order	Value	t-Value
v_0		0	0.0001	4.41
θ_1	MA	1	0.2285	2
θ_1	AR	1	0.848	14.43
ϕ_3	AR	168	0.1727	2
Differencing	None			
RSE (time zone)			0.59	

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the SCA™ statistical software. As has been stated, Y stands for the output variable (intercept or slope of the RDFs), the explanatory variable *non-base demand* is measured in MW, and v_0 is expressed either in [c€/(MW*kWh)] for the RDF intercept equation or [c€/(MW^2*kWh)] for the slope equation:

$$Y = v_0\,[NBD] + \frac{(1-\theta_1 B)}{(1-\phi_1 B)} a_t \tag{2.16}$$

Next, the optimal model for the same hourly range (8 a.m.–9 a.m.) focusing on Tuesday through Friday is developed:

$$\nabla_{24} Y = v_0 \nabla_{24}\,[NBD] + \frac{(1-\theta_1 B)\,(1-\theta_2 B^{24})}{(1-\phi_1 B)\,(1-\phi_2 B^{24})\,(1-\phi_3 B^{168})} a_t \tag{2.17}$$

Finally, the optimal model focused on Sundays with the previous hourly range (8 a.m.–9 a.m.) is expressed:

$$Y = v_0\,[NBD] + \frac{(1-\theta_1 B)}{(1-\phi_1 B)\,(1-\phi_{31} B^{168})} a_t \tag{2.18}$$

Tables 2.6–2.8 show a great variety in the structure of the models depending on the time interval considered. They should be interpreted in the following way: for instance, focusing on the row corresponding to parameter ϕ_2 in Table 2.7, it means that a seasonal daily (order 24) autoregressive (AR) parameter labelled ϕ_2, which takes a value of 0.4392 is considered in the model, because it is a meaningful parameter (its t-value is greater than two).

Although the base model used has always been the one stated in equation (2.15), the weighted estimation method focused on the time zone selected skips in each case the non-significant parameters and differencings or adds new parameters like ϕ_2 and ϕ_3 in Table 2.7, until an optimal parsimonious model which deals with the stationarity and invertibility conditions has been reached (Pankratz, 1991; Box, 1976; Box *et al.*, 1994).

It can be noted that Table 2.6 and Table 2.8, which correspond to weekend days, do not include any differencing, because due to the strong variation in the RDF pattern from Friday to Saturday and from Saturday to Sunday, there does not exist any relation between these days and previous days.

In order to avoid confusion, it is necessary to remark that the ARIMA model developed in subsection 2.4.1, which includes two seasonal differencings, did not discriminate the data series in time intervals, so a model, which minimizes the overall RSE, was obtained.

However, using the WE method, it is possible to see that depending on the time interval analysed, some of them do not need any differencing at all (like the ones detailed in Table 2.6 and Table 2.8), while others may need one (Table 2.7) or more differencings.

Each time zone model also includes its own RSE, whose magnitude will usually have a strong resemblance to the range of variation of the values of each time interval (i.e. the lower the output variable dispersion, the lower the value of the RSE). In contrast to the ARIMA model, where only one overall RSE was used, the WE method provides one different weighted RSE (calculated focusing only on the residuals of those observations belonging to the analysed time period, i.e. their weight equals one) for each one of the previously selected time periods.

The weighted RSE can be used to generate multiple RDF scenarios (instead of using only a base scenario for the dispatch optimization problem, upper and lower scenarios could also be generated. The separation between them would be proportional to the standard deviation of the weighted RSE. Thus, valley hour scenarios would tend to be less dispersed than the ones obtained for peak hours) in order to build up the 24 optimal bid functions as required by the MO.

2.4.2.3 A brief comment on outliers

The SCA™ software package provides the capability to detect the most common type of outliers (for example a level shift or a temporary change in the data series) and its deviation from the expected value (i.e. obtained under normal conditions) in order to evaluate their impact. The effects of outliers should always be considered, because they can bias the estimation of the parameters of the model. The problem is that the outlier detection procedure takes a lot of time if the data series has a big size, as is usually common with hourly data.

A possible approach to this topic would be to build auxiliary daily and weekly TF models, in which both input and output variables are the daily/weekly average values, so the size of the series is shortened. The daily model would detect those days where there has been an abnormal behaviour (usually non-labour days different from the weekends) and the weekly model would be suitable for more longer special pattern events like Easter or Christmas.

Finally, as the RDFs must be generated hourly, the average daily or weekly deviations should be converted into hourly deviations taking into account the different characteristics of each period of time.

2.4.3 Case study

In this case study the advantages of an accurate prediction of the variables slope and intercept corresponding to linearized RDFs are going to be shown. Firstly, a week of year 2001, with a step increase of run-of-the-river power with respect to the previous week, has been chosen. It is expected that the RDF forecasts of the classical ARIMA model without explanatory variables described in Section 1.2 will deviate greatly from the real RDF functions, while the more complex weighted estimation model which uses the explanatory variables described in Section 1.3 (denoted as WE in the next figures) should capture the change in the RDF pattern caused by the new hydro conditions. Figures 2.15–2.17 display the different performance of the forecasting models in three different types of hours of a Wednesday (one valley hour, one plateau hour and one peak hour).

As was expected, ARIMA model forecasts present big departures from the real RDF functions. In this case, forecasted RDFs tend to be higher and more abrupt, whereas weighted estimation model forecasts match better to the real pattern.

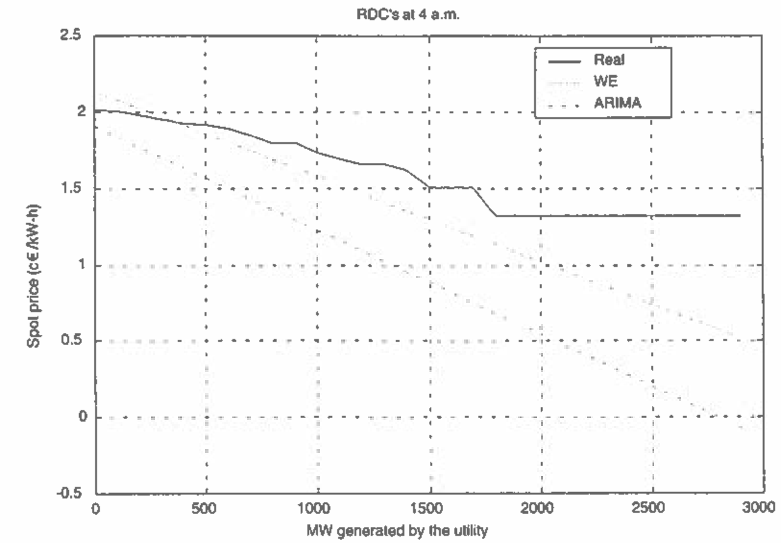

Figure 2.15 Real residual demand function (continuous line) versus WE forecast (dashed line) and ARIMA forecast (dotted line). Valley hour
Source: Association for Studies on the Quality of Electric Energy. Reproduced with permission.

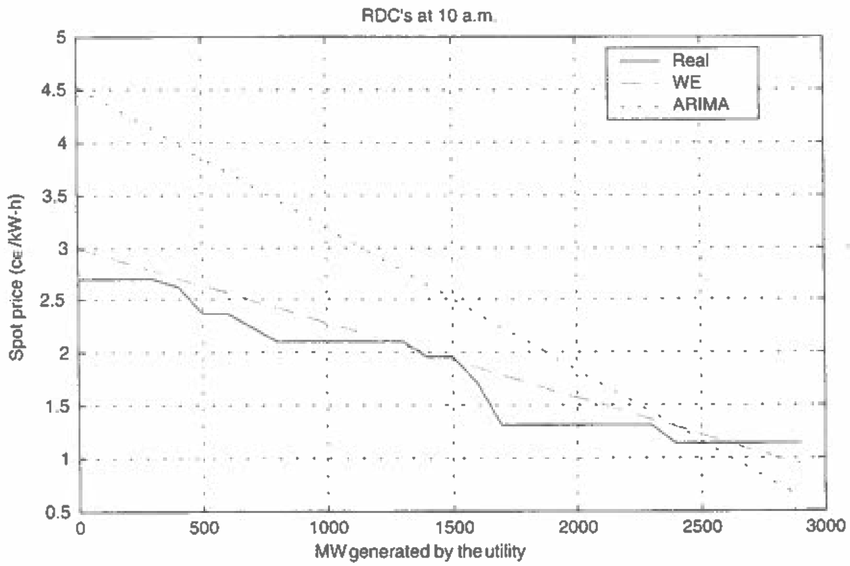

Figure 2.16 Real residual demand function (continuous line) versus WE forecast (dashed line) and ARIMA forecast (dotted line). Plateau hour
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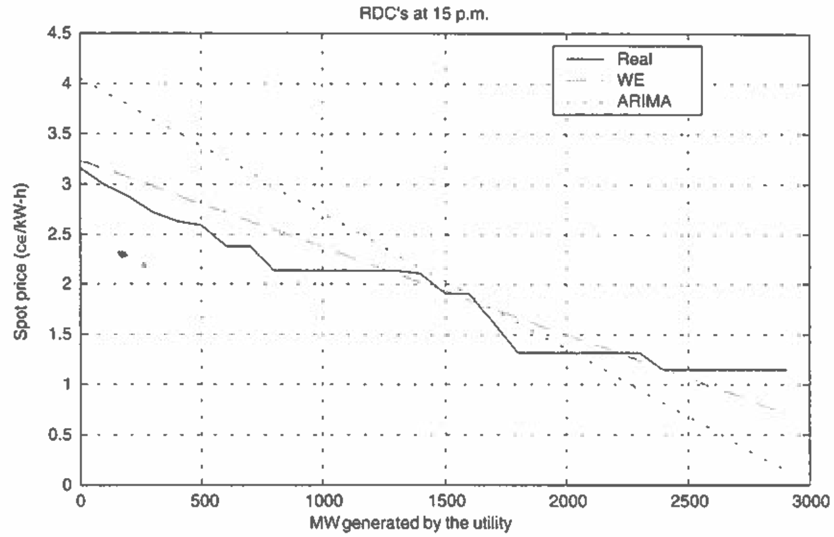

Figure 2.17 Real residual demand function (continuous line) versus WE forecast (dashed line) and ARIMA forecast (dotted line). Peak hour
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2.4.4 Section conclusions

In this section a method for an accurate prediction of the residual demand functions of a given utility has been shown. An adequate hourly explanatory variable has been defined which takes into account different situations of the national system. A weighted estimation method has been proposed in order to deal with the non-linear relationship between the explanatory and the input variables, depending on the time period (day and hour) considered. Finally, a case study has been carried out, showing the advantages of the weighted estimation using the proposed explanatory variable. The next section presents an alternative method to deal with the multi-regime aspect of the electricity price series.

2.5 DISCOVERING ELECTRICITY MARKET STATES FOR FORECASTING THE RESIDUAL DEMAND FUNCTION USING INPUT–OUTPUT HIDDEN MARKOV MODELS

In the previous section, the problem of forecasting the residual demand function has been approached from the point of view of time series analysis, with the primary aim of maximizing the accuracy of the forecasts. The model proposed in this section takes a step forward. The objective is not only to propose a successful model in terms of accuracy, but also to extract dynamical information about the market by identifying and characterizing different regimes in competitor's behaviour. Since market participants often apply the same strategy for several days, the residual demand time series can reflect a switching nature related to discrete changes in a participant's behaviour. This phenomenon gives rise to piecewise stationary time series where the series switches between different regimes (Weigend and Mangeas, 1995).

Two fundamental groups of switching models appear in the literature. The first one comes from the economic and financial fields, and their application to electricity markets has been focused mainly on the analysis of system marginal price time series. The most important model of this group for the purpose of this section was introduced by Hamilton (1990). In that paper, the time series is modelled by a Markovian algorithm for switching among autoregressive regimes, adapting to occasional discrete shifts in the level, variance and autoregressive dynamics of the series. The probability law governing these shifts is fixed, time-invariant and non-conditioned to exogenous variables. Therefore this model presents a serious shortcoming to capture the idiosyncrasy of electricity markets. In the model proposed in this section, a set of exogenous input variables are considered in such a way that both the distribution of the output variable in each regime and the transition probabilities among regimes are conditioned to them.

The second group is related to models based on artificial neural networks. The most relevant model in this group is the one presented in Weigend and Mangeas (1995). In that paper, the authors present a gated experts architecture in order to discover different regimes in the electricity load time series. This gated experts model consists of a non-linear gating network and several competing experts. Each expert learns to predict the conditional mean of the output variable and adapts its width to match the noise level in its corresponding regime. The gating network learns to predict the probability of each expert, given the input. Although in this approach exogenous variables are considered at each time step and therefore the probability of each regime is time-variant, the previous regime is not considered to forecast the next expert.

The model proposed in this section combines both, focusing on trying to solve their main drawbacks. The proposed model is a probabilistic model with a fixed number of states corresponding to different conditional distributions of the output variables given the input variables, and with transition probabilities between states that can also depend on the input variables.

2.5.1 Hidden Markov models and the analogy with electricity markets

2.5.1.1 Introduction to hidden Markov models

Hidden Markov models (HMM) were first introduced in the late 1960s (Baum and Petrie, 1966) as statistical methods extremely useful for modelling sequentially changing behaviours. A hidden Markov model (Levinson *et al.*, 1982; Rabiner, 1989) is a double embedded stochastic process:

1. An *underlying process* defined by a Markov chain with a finite number of states $S = \{S_1, S_2, \ldots, S_N\}$, which is hidden from observation.

   A Markov chain (Berger, 1993) describes a system which at any time step is in one of a set of mutually exclusive states. Let $\{Y_n\}_{n\geq 0}$ be a sequence of random variables taking values in a finite set $S = \{S_1, S_2, \ldots, S_N\}$. These random variables are said to be a Markov chain if there is no dependence between states that are not immediately consecutive (order 1). That is:

$$P(s_{t+1} = S_i | s_0 = S_j, \ldots, s_t = S_k) = P(s_{t+1} = S_i | s_t = S_k) \tag{2.19}$$

   Equation (2.19) is known as the Markovian property, where P stands for probability. The Markovian property implies that given the present state, the future probabilistic behaviour is independent of its past history.

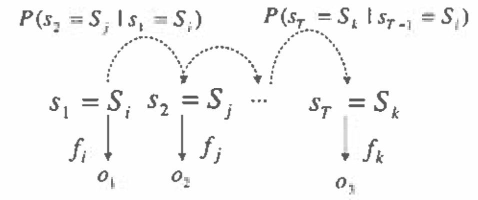

Figure 2.18 Hidden Markov model process

   The conditional probabilities $P(s_{t+1} = S_i | s_t = S_k)$ are referred to as the "transition probabilities", and when they are time-invariant, the Markov chain is said to be homogeneous.

2. An *observable process* which is determined by the underlying Markov chain. At each state a unique probability density function (*pdf*) governs the emission of observations.

Mathematically, a hidden Markov model is defined by the following four elements:

- The number of states in the model N.
- An initial state probability distribution:

$$\Gamma = (\gamma_i = P(s_0 = S_i)) \in [0, 1]^N \tag{2.20}$$

- The transition state probability matrix:

$$A = (a_{ij} = P(s_{t+1} = S_i | s_t = S_j)) \in [0, 1]^{N\times N} \tag{2.21}$$

- A set of N emission probability density functions $(f_i)_{i=1,2,\ldots,N}$.

The HMM process of generating observations is as follows (Figure 2.18). The first state is selected according to the initial state distribution, which contains the probabilities of starting from each state S_i, $\forall i \in \{1, \ldots, N\}$. For each time step t the system shifts from the state $s_{t-1} = S_k$ to the state $s_t = S_j$ according to the transition probability matrix. At this state S_j, a new observation o_t^j is emitted using its corresponding *pdf* f_j. After that, the system evolves to time $t + 1$, and a new state $s_{t+1} = S_i$ is reached according again to the transition probability matrix. On that state, another observation O_{t+1}^i is emitted according to the *pdf* f_i. This process evolves until the last stage T is reached.

When a sequence of length T is generated, the result of the HMM process will be both the observations sequence $o_1, o_2, \ldots, o_T$ and the states sequence $s_1, s_2, \ldots, s_T$. However, whereas the observations sequence is visible, the states sequence remains hidden and no information about it is available for an observer.

Given an observation sequence $o_1, o_2, \ldots, o_T$ (or several defined over the same temporal scope), the problem of adjusting the HMM is concerned with finding the set of parameters $\Theta = \{\Gamma, A, (f_i)_{i=1}^N\}$ which best fits all these training data. For this issue the expectation maximization algorithm (Dempster *et al.*, 1977; Geoffrey, 1997) and its simplified version, the Baum–Welch algorithm (Rabiner, 1989), are widely proposed in the related literature.

At this point it is important to remark that in standard HMM the probability law governing the transitions between states is fixed, time-invariant and non-conditioned to exogenous variables. The *pdf* of the output variable is also non-conditioned to input variables. Therefore this model presents a serious shortcoming to capture the idiosyncrasy of electricity markets. As a generalization of HMM, IOHMM considers at each time step a set of input variables. The

Table 2.9 Analogy between IOHMM and electricity markets

IOHMM	Electricity market
States $S = \{S_1, S_2, \ldots, S_N\}$	"*Market states*" $M = \{M_1, M_2, \ldots, M_N\}$
Observations O_i	Residual demand functions *RDF* (slope *m* and intercept *I*)
pdf at state s_i (conditioned to a set of input variables)	Probability of generating a particular residual demand function from the *market state* m_i given the current explanatory variables
Initial probability	Probability of selecting the market state m_i at time $t = 1$
Transition matrix (conditioned to a set of input variables)	Probability of market state m_i at time t given the market state m_j at time $t-1$ and the set of current explanatory variables

main difference between HMM and IOHMM is that in IOHMM both the distribution of the output variable in each regime and the transition probabilities among regimes are conditioned to a set of explanatory variables. In this section IOHMMs are proposed to model electricity markets.

2.5.1.2 Applying IOHMMs to electricity markets

In the case of electricity markets, the observations and states of IOHMMs can be interpreted as follows.

Observations: Residual demand functions will be considered as the observations of the electricity market bidding process treated in this section.[7] In the Spanish daily electricity market, residual demand functions are published by the market operator after market clearance.

Market states: Each market state is characterized by a different residual demand function *pdf*, conditioned to a set of input variables such as load, hydro, thermal and nuclear resources, etc. In that sense, different states correspond to different functional relationships between input variables and residual demand functions. These functional relationships are associated with the interaction of competitors' strategies.

Table 2.9 shows the analogy between IOHMM and electricity markets.

2.5.2 Model description

2.5.2.1 Architecture

IOHMMs were first introduced by Bengio and Frasconi (1996). The architecture is based on a traditional HMM including a set of input variables at each time step. Whereas HMMs are focused on estimating the *pdf* of the output sequence $f(o_1^T)$, IOHMMs are trained to fit the output sequence *pdf* $f(o_1^T|u_1^T)$ conditioned to the input variable sequence. Using the *Markovian*

[7] Instead of the residual demand function other variables to study the market, such as for example the market clearing price, participant market shares, etc., could also be used.

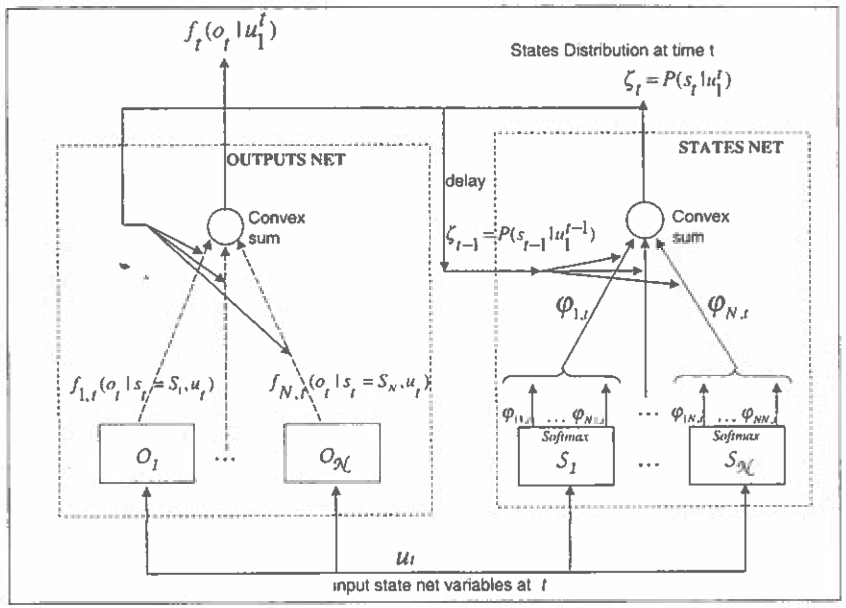

Figure 2.19 IOHMM architecture

property (2.19), the conditional distribution at each time step t can be expressed as:

$$f(o_t|u_1^t) = \sum_{i=1}^{N} P(s_t = S_i|u_1^t) f(o_t|s_t = S_i, u_t) \tag{2.22}$$

$$\text{where} \quad u_1^t = (u_1, \ldots, u_t)$$

that is, sums of products of the conditional output distributions and the conditional states distribution.

The basic idea of the IOHMM architecture (Figure 2.19) relies on taking advantage of the above expression in order to distribute the learning tasks. In that sense, a modular architecture is composed of a states network and an outputs network. Both networks share the learning problem but each of them is specialized on estimating just one of the product factors in equation (2.22).

The *states network* is composed of a set of states subnetworks $\{S_j\}_{j=1}^N$, one for each state defined in the model. The aim of each state subnetwork S_j is to propose a prediction of the next state distribution assuming that the previous state was its own state, and given the current input variables, i.e. $\varphi_{jt} = P(s_t|s_{t-1} = S_j, u_t)$. Hence, at each time step, every state subnetwork has r inputs $u_t \in R^r$ and N outputs (one for each state), $\varphi_{jt} = [\varphi_{1j,t}, \ldots, \varphi_{Nj,t}]$ where $\varphi_{ij,t} = P(s_t = S_i|s_{t-1} = S_j, u_t)$.

In this application, each states subnetwork is implemented as a multilayer perceptron (Bishop, 1995) with a single hidden layer and sigmoidal activation functions (Bishop, 1995). As proposed in the original architecture, the softmax (Bishop, 1995) function has also been implemented at the output units as a normalization function, which guarantees that the outputs of states subnetworks are non-negative and sum to one. It is important to highlight on the one hand that the transition probabilities $\varphi_{ij,t}$ are estimated by the set of states subnetworks. On the other hand, combining the candidates of each subnetwork and the previous state distribution, the current state distribution is computed as $\zeta_t = [\zeta_{1,t}, \ldots, \zeta_{N,t}]$ where $\zeta_{j,t} = \sum_{j=1}^{N} \zeta_{j,t-1}\varphi_{j,t}$.

The *output network* is composed of a set of output subnetworks $\{O_j\}_{j=1}^{N}$ each associated with a unique state of S. The task of the output subnetworks is concerned with predicting the expected output value given the current state and the current input variables. For this issue a conditional *pdf* f_j is implemented in each output subnetwork O_j. The output of O_j is computed as the distribution expectation $f_{j,t} = E[f_j(o_t|, u_t)] \in \mathbb{R}^r$. Several distributions seem to be well suited (Lauzon, 1999) for the output subnetworks. In this application a conditional Gaussian distribution $G^j(\mu^j, \sigma^j)$ with diagonal covariance matrix has been adopted. The distribution expectation is modelled as a linear function of the input variables, $\mu = W\,u + b$, where $W \in \mathbb{R}^N \times \mathbb{R}^r$ and $b \in \mathbb{R}^N$.

The combination of the output subnetworks generates an output distribution $f(o_t|, u_t)$ which is a mixture of probabilities (Titterington *et al.*, 1985), where each component is conditional on a particular state and the mixing weights are the current state probabilities (states network outputs) conditioned to the input. That output can be expressed as follows: $f_t = \sum_{j=1}^{N} \zeta_{j,t}\eta_{j,t}$.

Concerning the learning algorithm, the expectation maximization algorithm has been implemented. A detailed description of this algorithm is presented in Bengio and Frasconi, 1996).

2.5.3 Example case

2.5.3.1 Description

The problem of analysing the RDF is reduced in this case to the analysis of the time evolution of two variables, the slope and the intercept of the linearized RDF. In this section the studied period was from January 2001 to July 2001 divided into hourly intervals. The two time series of this study are represented in Figure 2.20.

A different IOHMM model has been adjusted for the slope m and for the intercept I, but only the intercept variable analysis will be presented (similar results have been obtained for the slope variable). Concerning the explanatory variables, the model has to be restricted to public information. In that sense, past values of hourly production by technology, hourly system demand and marginal price have been used to generate the input variables. The set of explanatory variables for the IOHMM model is summarized in Table 2.10 and shown in Figure 2.21.

Table 2.10 Explanatory variables definition

Variable	Description
$D_{d,h}$	Load for the day d and hour h
$N_{d,h}$	Nuclear generation for the day d and hour h
$H_{d,h}$	Hydro generation for the day d and hour h
$P_{d,h}$	Marginal price value for the day d and hour h
$NBD_{d,h}$	Non-base demand for the day d and hour h

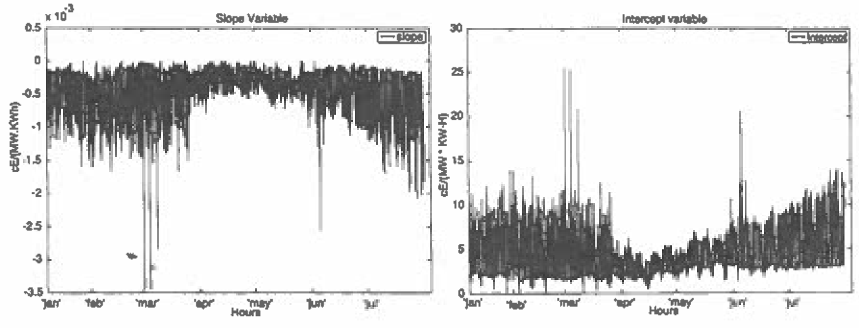

Figure 2.20 Slope and intercept for the linearized residual demand functions

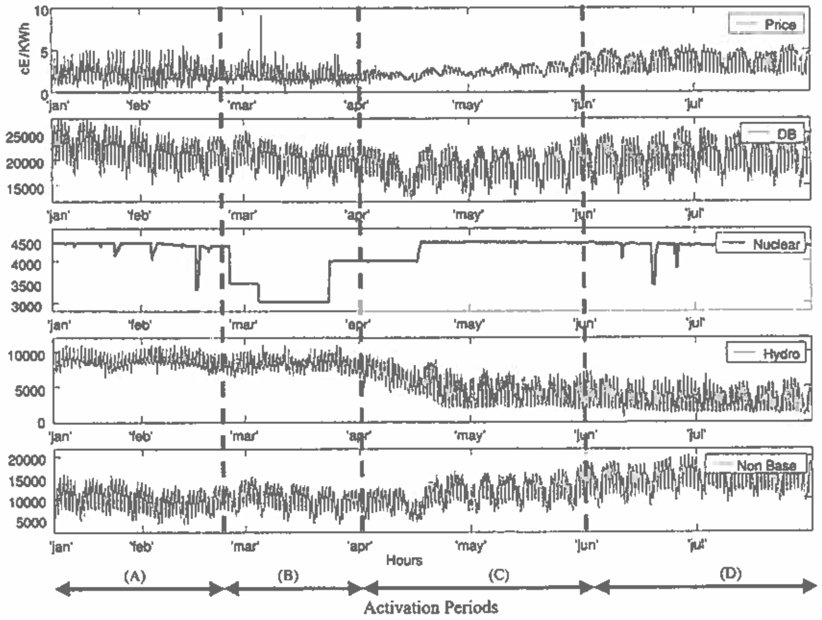

Figure 2.21 Explanatory variables for the studied period

At this point it is important to note that the original IOHMM proposed in Bengio and Frasconi (1996) considers the same input variables for the state and output networks. By contrast, in this application different variables for each type of network are allowed. This small improvement makes it easier to interpret and understand model results. For instance, states input variables capture the market states and switches among them, whereas input variables for output networks accurately fix the RDF intercept and slope values given the probability

Table 2.11 Input variables for each type of network

Network	Input variables
States	$p_{d-1,h}$, $D_{d,h}$, $D_{d-1,h}$, $N_{d-1,h}$, $H_{d-1,h}$, $NBD_{d-1,h}$, $I_{d-1,h}$
Output	$p_{d-1,h}$, $D_{d,h}$, $D_{d,h-1}$, $D_{d-1,h}$, $I_{d-7,h}$, $I_{d-1,h}$, $I_{d,h-1}$

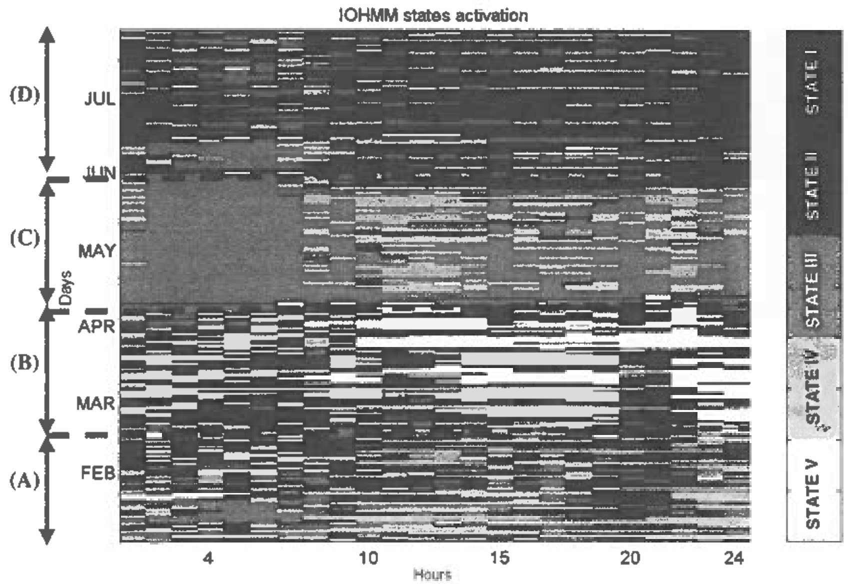

Figure 2.22 Activation of IOHMM states

states distribution. Under this approach, the user can intentionally split input variables using their knowledge and experience. Table 2.11 presents the input data used for state and output networks.

2.5.3.2 Discovering and analysing market states

The first step to apply the IOHMM is to select the number of states. In this example a five states model was selected according to accuracy and interpretability.

In Figure 2.22 the activation of the most probable state for each hour is represented. Each row corresponds to one day and each column to one hour. States are represented in different colours. This figure provides useful information since it can be interpreted from two standpoints:

1. The first analysis consists of fixing the y-axis values (days) and moving along the x-axis (hours). A first sight of Figure 2.22 reveals a state activation pattern related to off-peak, peak and medium-load hours. For instance, during the period April–May, hours from 2:00 to 7:00 are activated in most cases with state III. In the same period medium-load hours from 11:00 to 13:00 are activated with state IV, and the 15:00 and 20:00 peak hours are also clearly identified with state III.
2. The opposite point of view consists of fixing the x-axis values and moving along the y-axis. In that analysis the evolution of the different market states along the temporal scope shows discrete changes related to the different patterns of a competitor's behaviour. This visual inspection provides mainly four activation periods. The first one (A) goes from January to mid-February. In this period states III and IV are mainly activated. The second period (B) is identified because state V is the most probable and goes from mid-February to the beginning of April. The third activation period (C) is characterized by state III, which is the most representative of this period. And finally, during June–July (D), state II seems to exhibit the higher activation probability. From this point of view, it may be observed that specific patterns are assigned to weekends. For example in the last period from 8:00 to 24:00, Saturdays and Sundays are activated with most probability with state III whereas workdays are activated with state II.

2.5.3.3 States and explanatory variables

The identification of four activation periods (A, B, C and D in Figure 2.22) is related to the occurrence of causal episodes in the states network input variables. In the first activation period (A), explanatory variables do not suffer abrupt changes. In the second fortnight of February (A→B), nuclear production is drastically reduced due to the maintenance of several units. The joint effect of this reduction of base production and the decreasing tendency of the load is reflected as a shift in the variance of the RDF intercept and slope time series. At the end of March (B→C), some of the nuclear units in maintenance are recovered but water resources present an important drop. This new situation results in an increase in thermal production (see Figure 2.21). Finally, at the end of May (C→D), a new market condition takes place due to an increase in the load while available production resources are stable.

On the other hand, Figure 2.23 provides information about the normalized significance of each explanatory variable in the outputs subnetworks. For this sake, coefficients of the regression model for the conditioned mean of the Gaussian model, as well as standard deviation values, are presented. The analysis of the signs of the coefficients (positive for $D_{d,h}$, negative for $D_{d,h-1}$ and $D_{d-1,h}$ and positive for $I_{d-7,h}$, $I_{d-1,h}$ and $I_{d,h-1}$) reveals an autoregressive integrated component of the form:

$$I_{d,h}-\alpha_1 I_{d,h-1}-\alpha_2 I_{d-1,h}-\alpha_3 I_{d-7,h}=\beta_1 D_{d,h}-\beta_2 D_{d,h-1}-\beta_3 D_{d-1,h} \quad \text{with} \quad \alpha_i,\beta_j>0 \tag{2.23}$$

The comparative analysis of the coefficients of each state reveals the similarities between states I and II, IV and V, and the specificity of state III. States I and II stand out for their first-order autoregressive component (coeff($I_{d,h-1}$) > 0.5). The difference between these two states comes from the daily and weekly autoregressive components (coeff($I_{d-1,h}$) and coeff($I_{d-7,h}$)). States IV and V are the only ones to consider the marginal price as a relevant input variable. The difference between them also comes from the daily and weekly autoregressive components. State III has been characterized by the high significance assigned to the load input variable.

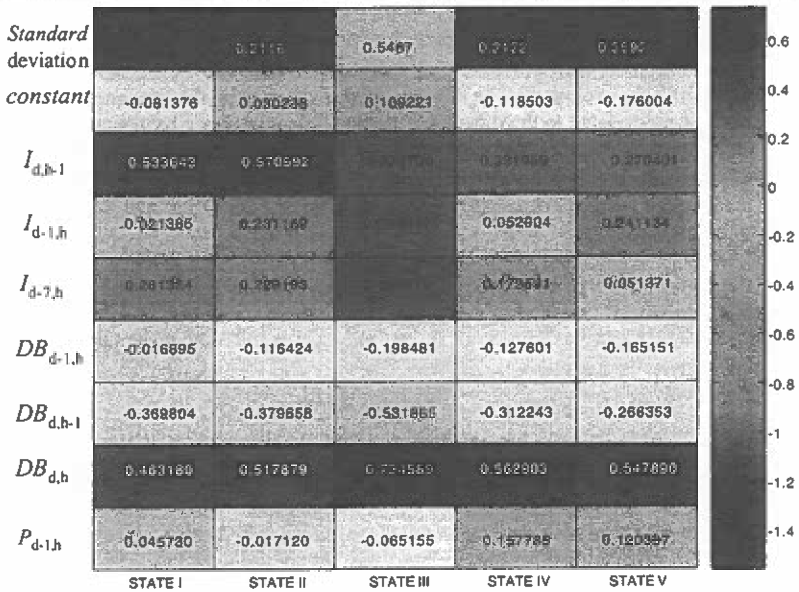

Figure 2.23 Explanatory variables coefficient for each state

2.5.3.4 Forecasting residual demand functions

Examples of slope and intercept forecasts for each activation period are presented in Figure 2.24. The mean absolute percentage error (MAE) defined as

$$MAPE = \frac{1}{N}\sum_{i=1}^{N}\frac{|y_i - \widehat{y_i}|}{y_i} \tag{2.24}$$

was 0.85 for the intercept value and 0.57 for the slope.

Figure 2.25 shows examples of the residual demand function forecasts for each activation area for different types of hours in the four periods.

2.6 CONJECTURAL VARIATIONS APPROACH FOR MODELLING ELECTRICITY MARKETS

This section presents a fitting procedure designed for medium-term electricity markets unlike the previous sections that were oriented towards short-term analysis. It uses as input data only public information. This feature makes the model especially useful in situations when detailed bidding submissions are not available.

The method is based on the conjectural variations approach. This approach allows a more flexible representation of firms' behaviour and a more accurate price generation process than the more commonly applied Cournot equilibrium formulations (Vives, 1999). These improvements are achieved by means of modelling firms' residual demand functions, which provide information about the energy that firms are able to sell at each price. As we have seen in

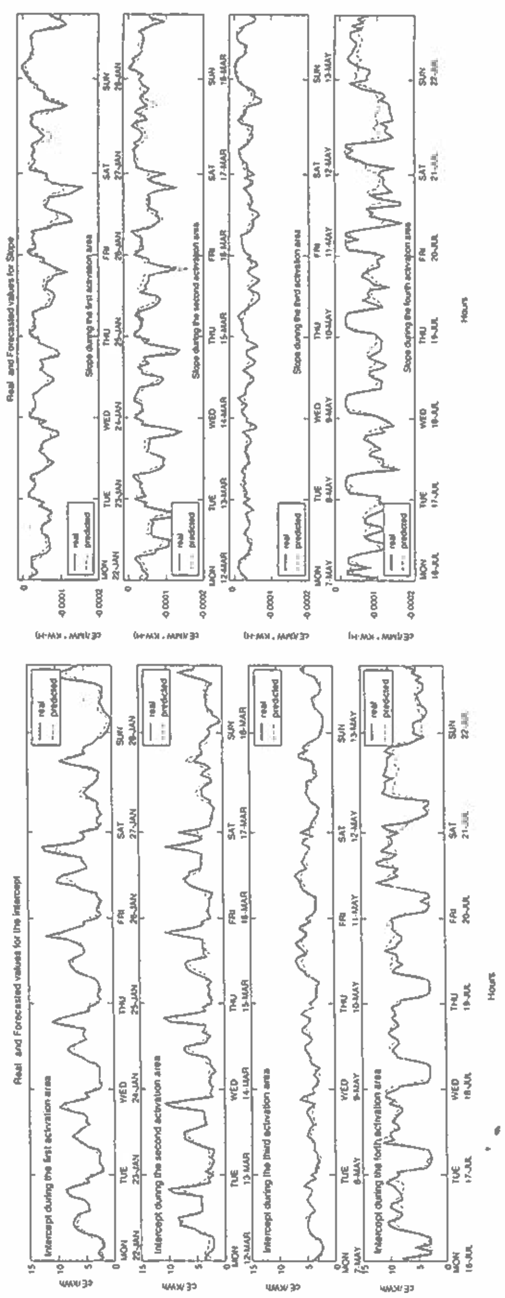

Figure 2.24 IOHMM slope (a) and intercept (b) forecasts

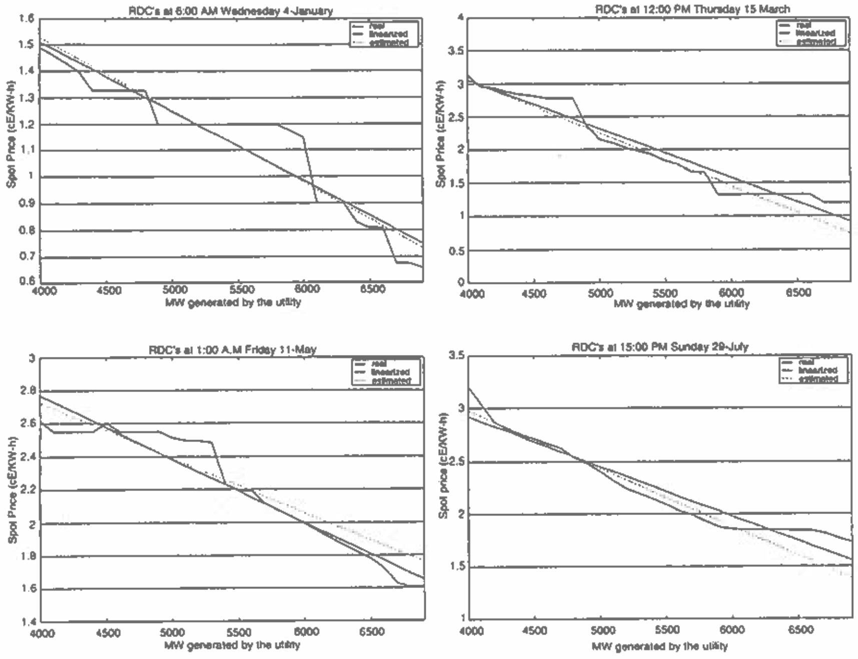

Figure 2.25 IOHMM RDF forecasts

previous sections, these residual demand functions are difficult to estimate because they are related to the other firms' supply functions.

The proposed methodology characterizes each residual demand function by its elasticity and estimates the implicit values of this parameter. Thereby, this procedure computes the long-term residual demand elasticity that each firm has been inferring when it has been bidding. The inference about firms' bidding perceptions is based on clearing market information provided by the market operator, instead of the real supply functions submitted by the firms. An estimation process based on short-term supply functions has been dismissed because they do not reflect properly firms' long-term strategies but just firms' short-term tactics.

2.6.1 Conjectural variations

There exist a number of long-term electricity market models based on Cournot competition (Scott and Read, 1996; Bushnell, 1998; Wei and Smeers, 1999; Hobbs, 2001). However, it is well known that these models provide very high, barely credible prices. These high prices are due to neglecting competitors' supply functions (Day *et al.*, 2002).

The conjectural variations approach considers the reaction of competitors when a firm is deciding its optimal production. This reaction comes out from firms' supply functions and demand function and can be modelled by means of the residual demand function.

2.6.1.1 Firms' optimization problem

In this section, the optimization problem of each firm is stated as its profit maximization program facing its own residual demand function. Thereby, conjectural variations are considered in the firms' strategic behaviour.

Since the system marginal price is set by the decisions made on the supply and demand sides, it is possible to relate this price to the aggregated production by means of the demand function d:

$$p = D\left(\sum_f q_f\right) \tag{2.25}$$

In order to calculate the fth firm optimal production, the derivative of the profit function (2.7) with respect to the decision variable (q_f) is equalized to zero:

$$\frac{\partial \Pi_f}{\partial q_f} = p + \frac{\partial p}{\partial q_f} q_f - MC_f(q_f) = 0 \tag{2.26}$$

The first two terms of the equation make up the firms' marginal revenue MR_f. The marginal revenue measures how a firm's revenue increases (or decreases) when the firm increases (or decreases) its production in one unit. Likewise, the firm's marginal cost can be defined as how a firm's cost increases (or decreases) when the firm increases (or decreases) its production in one unit. Therefore, a firm's optimal production is achieved when its marginal revenue is equal to its marginal cost.

A firm's marginal revenue can be expressed in terms of its residual demand elasticity ε_f instead of the residual demand slope

$$MR_f(q_f) = p + \frac{\partial p}{\partial q_f} q_f = p\left(1 + \frac{1}{\varepsilon_f}\right) \tag{2.27}$$

where the residual demand elasticity is defined as the quotient between the unitary change in the firm's production caused by a unitary change in market price:

$$\varepsilon_f = \frac{\partial q_f / q_f}{\partial p / p} \tag{2.28}$$

Note that the residual demand elasticity is different for each firm. This parameter takes into account how the market price changes when each firm changes its production unilaterally.

To conclude, this parameter expresses the market conjecture of each firm. The variation of this parameter encompasses different types of competition, providing great flexibility of modelling.

2.6.1.2 Flexibility of modelling

Depending on the value of a firm's residual demand elasticity, a different assumption is made about how the firm's marginal revenue is conceived, and consequently, about how firms behave. In this way, some widely used market models can be stated in terms of the residual demand elasticity as follows.

Perfect competition: Perfect competition can be defined as a market situation where firms are not able to change the market price by modifying their production unilaterally. In terms of market modelling, each firm's residual demand function becomes horizontal. The elasticity

Table 2.12 Market conjecture

Model	Market conjecture
Perfect competition	$\varepsilon_f = \infty$
Cournot	$\varepsilon_f = \varepsilon_d/\alpha_f$
Conjectural variations	$\varepsilon_f \neq \varepsilon_d$

value of a fixed-price residual demand function is infinity – see equation (2.28). From equation (2.26), the firm's marginal revenue is equal to the market price. As expected, the firm's optimal production takes place when its marginal cost reaches the market price.

Cournot model: The Cournot model is one of the most common approaches to represent competition amongst just a few firms. In contrast to perfect competition, these firms are able to modify the market price by means of changing their own production. The market model conjecture that states a Cournot equilibrium can be written in terms of a demand elasticity equal to every firm competing in the market scaled by its market share ($\varepsilon_f = \varepsilon_d/\alpha_f$), where ε_d is the demand function elasticity and α_f is the fth firm market share. Cournot prices are usually far higher than real market prices.

The different values of the residual demand elasticity depending on the theoretical market model are summarized in Table 2.12.

2.6.2 Implicit elasticity estimation

In this subsection, a methodology suitable to estimate residual demand elasticity is presented. The resulting estimations will allow the fitting of medium-term, equilibrium, electricity market models.

2.6.2.1 Estimation overview

The proposed methodology is based on fitting the residual demand elasticity by means of an evaluation of the conjectural variations model on past data. It is firstly supposed that firms behaved via a conjectural variations pattern during the fitting period. Therefore, the decisions – productions – they made can be assumed to be optimal. This optimal schedule can be expressed equalizing the firm's marginal revenue with its marginal cost – see equation (2.26).

Consequently, we can infer the past residual demand elasticity by using these decision variables as acknowledged values. This procedure is known as implicit valuation. Thus, the residual demand elasticity obtained is called *implicit elasticity*. It is important to remark that the implicit values of the elasticity measure firms' perception about their market positions in a conjectural variations context. In fact, this perception does not need to coincide with the real residual demand computed by the supply and demand functions submitted to the market by all the firms.

2.6.2.2 Implicit elasticity

The procedure to obtain the implicit value of firms' residual demand in a conjectural variations context is stated along these lines. Let p be the actual system market price and MR_f be the marginal revenue of the firm f. The implicit value of the residual demand elasticity ($\widehat{\varepsilon_f}$) is

derived from expression (2.27) and can be written as follows:

$$\widehat{\varepsilon_f} = \frac{p}{MR_f - p} \tag{2.29}$$

The value of firm's marginal revenue is impossible to estimate from market data. However, it can be approximated to the firm's marginal cost through the assumption that each firm is generating optimally (2.26). The firm's marginal cost can then be approximated as the generation cost of the most expensive committed unit.[8] Therefore, the implicit expression of the residual demand elasticity can be stated in terms of the market price and firm's marginal cost as follows:

$$\widehat{\varepsilon_f} \simeq \frac{p}{MC_f - p} \tag{2.30}$$

2.6.2.3 Estimation

By means of valuing the implicit residual demand elasticity, it is possible to infer firms' long-term behaviour. The estimation procedure uses public market information provided hourly – prices and production – and an estimation of the firms' marginal costs. Applying equation (2.30) to this information, the hourly time series of the implicit residual elasticity of each firm is directly assessed. These implicit values not only consider long-term behaviour, but also are very influenced by short-term uncertainty sources and short-term operational constraints.

Some relevant short-term uncertainty sources that have some bearing on the implicit demand elasticity are plant outages, hydraulic inflows, deviations from the forecasted demand and supply functions submitted by the rest of the firms. Short-term operational constraints such as plant ramps, minimum stable output, limited reservoir capacity and network constraints also have a significant effect on implicit values.

Therefore, the previous short-term issues must be sieved in order to properly estimate the implicit values that will model firms' long-term behaviour. It is necessary to process all the information about the implicit residual demand evolution looking for a long-term trend value. Statistics make available several data-analysis methodologies that are able to deal with long-term behaviour estimation from short-term data. Depending on the nature of the variables used to do so, statistical methods can be classified as follows.

Relational models: By means of these models, the implicit elasticity of the residual demand is related to one or several relevant variables – explanatory variables – whose long-term values are easy to infer. This relationship is expressed by a function among the variables. Having a long-term estimation of the set of explanatory variables – e.g. demand, hydraulic inflows – a trend value of the implicit elasticity can easily be obtained by evaluating the former function. Some of the statistical methods that can be defined as *relational models* are regression, neural networks and decision trees.

Classification models: In contrast to relational models, the set of values of implicit elasticities is categorized by the different levels of several discrete factors. The long-term value of the implicit elasticity for each factor combination – i.e. class – can be expressed as a statistical measure – average, median or mode – of the past data distribution. Variance analysis and clustering are some statistical methodologies based on classification processes.

[8] A more precise marginal cost estimation requires considering units' generation constraints (i.e. ramps and minimum stable output) and other relevant market issues (i.e. capacity payments).

Table 2.13 Spanish firms' production structure (1999)

Type	# Units	Power (MW)	Iberdrola	Endesa	Unión Fenosa	Hidrocantábrico
Thermal	82	32 107	29%	54%	13%	5%
Hydro	38	16 628	51%	36%	3%	10%

Time series models: These models consider implicitly the evolution of the elasticities and infer a trend value of the time series. Time series methodology generally precludes taking into account other variables or factors when estimating the long-term values of the implicit residual demand elasticity. Some of the time series methodologies that deal with long-term trends are time series decomposition and exponential smoothing methods.

2.6.3 Case study

A complementarity-based equilibrium model (Rivier *et al.*, 2001) in which firms compete in a Cournot manner has been adapted to consider conjectural variations (García-Alcalde *et al.*, 2002). This model has been applied to the Spanish electricity market to validate the estimation and fitting procedures detailed in previous subsections. A comparison with firms' Cournot behaviour is also shown.

2.6.3.1 System description

Data used to fit this estimation model is based on the market results of the year 1999. The annual scope of the model has been divided into 12 periods – months – with five load levels for each one.

There are 82 thermal generators grouped into 42 thermal plants. The hydro units have been grouped into 28 equivalent units plus 10 pumped-storage units (Table 2.13).

All the information that has been used in this case study is public. Units of production and market prices have been obtained through the Spanish market operator. Fuel costs and unit thermal rates are respectively based on international fuel prices and standard technology rates.

2.6.3.2 Numerical results

Two different Cournot scenarios have been considered by changing the aggregated demand's elasticity value. For the first one ε_d is equal to 0.3, while for the second one ε_d is equal to 0.5. A conjectural variations approach has also been implemented and fitted. The value of firms' residual demand elasticities has been estimated using a classification model where each value was labelled depending on the firm, demand level and the expected hydraulic inflows.

The model-obtained prices (€/MWh) and the real market results can be seen in Table 2.14. Note that conjectural variations provide a realistic estimation of the real market prices for every demand level. In contrast, the Cournot equilibrium gives deficient results when demand elasticity decreases.

For the first Cournot scenario, estimated prices are eight times higher than the real ones. It is important to remark that a demand elasticity value of 0.3 is very high in electricity markets.

Table 2.14 Model's average prices (€/MWh) for each demand level

Model	On-on-peak	On-peak	Plateau	Off-peak	Off-off-peak
Cournot $\varepsilon_d = 0.3$	281.21	258.80	231.34	168.47	116.76
Cournot $\varepsilon_d = 0.5$	79.09	74.24	68.45	54.61	39.99
Conjectural variations	43.92	38.01	31.59	19.56	13.77
Market prices (2000)	46.08	40.21	32.61	22.16	16.53

Table 2.15 Comparison of presented methods

Method	RDF representation	Time scope for market behaviour representation	Analysis type
Clustering techniques	Linear piecewise functions	Short-term	Static
Time series (ARIMA models)	Linear function (slope and intercept)	Short-term	Dynamic
Input–output hidden Markov models	Linear function (slope and intercept)	Short-term	Dynamic
Market equilibrium	Conjectural variations (slope)	Medium-term	Static

For instance, the average demand elasticity in the Spanish electricity market in 1999 was 0.03. Therefore, the use of a Cournot model in which its demand elasticity estimation is based on the submitted demand functions is completely inappropriate. Note that although only four firms compete in the Spanish electricity market, actual clearing prices are much lower than those provided by a theoretical Cournot model. In fact, price shocks as occurred in California have not been observed in the Spanish electricity market.

Regarding the results obtained from the case study, the conjectural variations approach and the proposed implicit estimation methodology of the residual demand elasticities provide a flexible and accurate tool to infer realistic firms' productions and market prices. The advantages with respect to Cournot-based models are notable.

2.7 CONCLUSIONS

Four different methods devoted to managing the problem of obtaining a representation of competitive behaviour by constructing an RDF have been presented (Table 2.15). The first one is based on a cluster procedure to group similar bid functions. The second method uses time series techniques. The third one utilizes input–output hidden Markov methods. Finally, the fourth approach makes use of a market equilibrium representation.

There are several differences between the four models that can be pointed out. RDF is approximated by means of a linear piecewise function in the first case, with a linear function in the second and third and using a conjectural variation in the fourth. As for the time scope, the first three approaches represent RDF from a short-term analysis point of view (however they can manage data for long time periods), while the fourth focuses on a medium-term market representation. This analysis is made from a static point of view in the first and fourth examples, while the other two address the dynamic behaviour of bidding processes.

APPENDIX: NOMENCLATURE

Subscript	Meaning
d	Day
$f, -f$	Firm (generation company), all firms but f
h	Hour
t	Time step

Symbol	Meaning
a	Random shock (white noise)
ACF	Autocorrelation function
AR	Autoregressive term
ARIMA	Autoregressive integrated moving average
B	Backshift operator. It delays x lags when applied to a given variable
BF	Bid function (supply or demand)
C	Cost
d	Dissimilarity
D	Demand
F	Run-of-the-river power
HMM	Hidden Markov model
IOHMM	Input–output hidden Markov model
LHM	Linear hinges model
LTF	Linear transfer funcion. Method used to adjust TF (transfer function) models
m	Slope of a linear function
MA	Moving average term
MC	Marginal cost
MO	Market operator
MR	Marginal revenue
N	Nuclear power
NBD	Non-base demand
H	Hydro generation
I	Intercept of a linear function
p	Price
pdf	Probability density function
P	Probability
PACF	Partial autocorrelation function
q	Quantity (energy production)
R, RDF	Residual demand function
RSE	Residual standard error
s	Markovian process state
S	Supply function
SCA™	Comercial software for time series analysis
SO	System operator
TF	Transfer function model
WE	Weighted estimation method
Y	Random variable
ε	Elasticity
∇	Differencing operator
Π	Profit

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3

Complementarity-Based Equilibrium Modeling for Electric Power Markets

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ABSTRACT

Complementarity-based power market models represent the constrained optimization problems of electricity generators, consumers, arbitragers and transmitters. These models directly solve a system of conditions that include each player's first-order optimality conditions plus market clearing. We introduce basic complementarity modeling concepts and a general energy model, and then compare alternative specifications of oligopolistic power markets subject to transmission constraints. Computational advances allow equilibria to be obtained for very large problems, such as the North American Eastern Interconnection case study described herein.

3.1 INTRODUCTION

Price simulation using computable network equilibrium market models is one promising approach to understanding the complexity of market power in electricity markets.[1] There is a rich literature on small-scale equilibrium analyses of this type. But while the regulated energy sector was a focus of large-scale modeling for many years, only recently has the availability of commercial software that includes efficient, robust algorithms for computation of equilibria allowed for growth in sophisticated large-scale simulation of deregulated national and multinational energy markets. In Europe and North America, modeling of regional electricity markets – using simulation, empirical analysis and types of concentration analysis – has been increasingly important in *ex ante* and *ex post* evaluation of market competitiveness and in resolving disputes over market outcomes.[2]

[1] In the USA and some European countries, electricity industry regulators generally have concluded that market power policy is necessary given the high concentration of supply in some markets, the lack of price-responsive (elastic) demand, and the market-narrowing effects of transmission constraints. There are two basic policy approaches: to require structural changes in the market or to impose behavioral restrictions, often called market power "mitigation", on a permanent or transitory basis. If the behavioral approach is chosen, then a great degree of sensitivity is required in implementation to ensure measures that limit prices, such as bid caps in the spot auction markets, are set at levels that allow for both short-term and long-term efficiency. These measures should also be consistent with other elements of the market design, for example, markets of ancillary services such as installed capacity or operating reserves.

[2] Until recently, most *ex ante* market power simulations of regional US markets included little network detail (e.g., Borenstein *et al.*, 2000). Helman (2003) undertakes a simulation of the eastern part of the US electricity grid with a more detailed DC load flow network model, as discussed in Section 3.5. Examples of detailed *ex post* empirical analysis to measure market power in US markets with different market designs include Borenstein *et al.* (2003) and Joskow and Kahn (2002) on the California market, Bushnell and Saravia (2002) on the New England market and PJM (2001) for the Pennsylvania–Maryland–New Jersey interconnection (the PJM reports are available for every year of market operation). Such analyses often compare observed market prices with a competitive (marginal-cost pricing) "benchmark", constructed with the help of simulation models.

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Modelling prices in competitive electricity markets

Edited by
Derek W. Bunn

Electricity markets are structurally different to other commodities, and the real-time dynamic balancing of the electricity network involves many external factors. Because of this, it is not a simple matter to transfer conventional models of financial time series analysis to wholesale electricity prices.

The rationale for this compilation of chapters from international authors is, therefore, to provide econometric analysis of wholesale power markets around the world, to give greater understanding of their particular characteristics, and to assess the applicability of various methods of price modelling.

Researchers and professionals in this sector will find the book an invaluable guide to the most important state-of-the-art modelling techniques which are converging to define the special approaches necessary for unravelling and forecasting the behaviour of electricity prices. It is a high-quality synthesis of the work of financial engineering, industrial economics and power systems analysis, as they relate to the behaviour of competitive electricity markets.

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