Universidad Pontificia Comillas ICADE

Breaking the loop: Handling circular references in LBOs

Code: 202005078
Author: Victoria Inda González
Tutor: Carlos Bellón Núñez-Mera

MADRID | April 2025

ABSTRACT:

This study consists in an analysis of what Leverage Buyouts (LBO) are, focusing on its financial model. The LBOs can turn out to be a very complex product in the banking and the Private Equity (PE) world. In order to work, when doing this kind of deals, a firm must analyze in detail every relevant information and forecast all of the affected financials of the target company we want to acquire, this is made mainly by developing an LBO financial model. However, as the deal gets more complicated, so the relations between its financials, many times resulting in the appearance of "circular references". As for today, the existing solutions to solve these circularities between the formulas present many deficiencies.

The goal of the present study is to propose a new solution to this crucial issue, that must be solved in order to be able to set the deal in motion. The solution is based on a program, created with coding, that will (after answering to a series of questions model related) generate a "formula sheet" with the needed equations to solve algebraically the circular references in your model.

Key words: debt, equity, model, iteration.

RESUMEN:

Este estudio consiste en un análisis de qué son los Leverage Buyouts (LBO), enfocándose en su modelo financiero. Los LBO pueden resultar ser un producto muy complejo en el mundo bancario y del Private Equity (PE). Para que funcionen, al realizar este tipo de operaciones, una empresa debe analizar en detalle toda la información relevante y proyectar todos los estados financieros afectados de la empresa objetivo que se desea adquirir; esto se hace principalmente desarrollando un modelo financiero LBO. Sin embargo, a medida que la operación se vuelve más compleja, también lo hacen las relaciones entre sus estados financieros, lo que muchas veces resulta en la aparición de "referencias circulares". Hasta el día de hoy, las soluciones existentes para resolver estas circularidades entre fórmulas presentan muchas deficiencias.

El objetivo del presente estudio es proponer una nueva solución a este problema crucial, que debe resolverse para poder poner en marcha la operación. La solución se basa en un programa, creado mediante programación, que (tras responder una serie de preguntas relacionadas con el modelo) generará una "hoja de fórmulas" con las ecuaciones necesarias para resolver algebraicamente las referencias circulares en tu modelo.

Palabras clave: deuda, capital, modelo, iteración.

GLOSSARY:

AI: Artificial Intelligence

BS: Balance Sheet

CE: Capital Employed

CF: Cash Flow

COGS: Cost of Goods Sold

D&A: Depreciation & Amortization

EBT: Earning Before Taxes

EBITDA: Earnings Before Interest, Depreciation and Amortization

EV: Enterprise Value

FCF: Free Cash Floe

IRR: Internal Rate of Return

IS: Income Statement

LBO: Leverage Buyout

LLM: Large Language Models

MOIC: Multiple on Invested Capital

Mn: Million

OCF: Operating Cash Flow

PE: Private Equity

SG&A: Selling, General and Administrative Expenses

UFCF: Unlevered Free Cash flow

Table of Contents

CHAPTER 1: LEVERAGE BUYOUT

1.1. What's an LBO?

Leverage buyouts, which are mostly used in corporate finance[1] and private equity[2] (PE), consist in the acquisition of a company using a significant percentage of leverage in order to pay for its price. This leverage is borrowed money, usually a loan from a bank. The rest of the price is paid with equity (the acquirers' own funds) and that, after buying, take control of the target company.

The debt taken in this operation is secured by the assets and the cash flows of the target company, they are the collateral for the loans that are going to be used in the transaction to cover the payment. This operational tactic allows the acquirer to make large purchases with a relatively small percentage of equity, a small percentage of its own investment. This is what gives this kind of operations its name of a "leverage buyout". Its basic principle and objective being that the target company generates enough future cash flows in order for the acquirer to pay down the debt over time, the company must pay for itself.

LBO's or Leverage Buyouts are typically used to take: a public company and turned it into private, a private company and also, when separating a segment of the company to operate independently. The latter is called a carve-out, and according to Corporate Finance Institute (n.d.), is when "a company tactically separates a subsidiary from its parent as a standalone company."

This kind of deals are usually led by a PE firm (called also financial sponsors), that invests equity and arranges the debt by borrowing the rest of the price from the lenders, such as banks or bond investors, for example. The debt to equity ratio is usually very high, acquirers try to use as much debt as possible, so they can amplify to the max the returns on their equity. The main reason that it is better to finance with debt than with equity is that debt is cheaper, as its interest is most of the times tax

[1] A branch of finance that focuses on how companies manage their financial resources, including funding, investment, and capital structure, with the goal of maximizing shareholder value.
[2] It involves investing in companies that are not publicly traded, often with the goal of improving their performance and eventually selling them for a profit.

deductible and the returns on lenders are usually lower than the returns on shareholders (equity). Also, in terms of risk it has an advantage, as debt must be secured by a collateral and lenders have a legal right to be repaid. A typical example of a debt to equity ratio for an LBO can be of 80% debt and 60% equity. However, it also has its disadvantages, as now the new acquired company is linked to a large amount of debt that must be paid down, regardless of the company's performance ending up being good or bad.

During the 80's, LBOs gained a bad reputation because they mostly acquired companies that where maybe in a bad situation by using its own assets as leverage to obtain the company itself. Also, a successful LBO can generate for the new owners a great benefit while the company bears the debt burden. These deals were claimed to be "predatory". An example[3]: if a PE firm buys a company for $100 Mn, where $30 Mn are equity while $70 Mn consist in debt. If during the next years, the company grows in value to $150 Mn and they've repaid $40 Mn of the debt, only $30 Mn remains. Now, the net equity will be $150 Mn - $30 Mn = $120 Mn, being this a 4x return.

In order to understand these deals better we will next be talking about its origin and evolution, what we will call its historical background.

1.2. Historical background

The first LBO in history may have been the purchase by McLean Industries, Inc. of Pan-Atlantic Steamship Company in January 1955 and Waterman Steamship Corporation in May 1955 (Cheffins & Armour, 2008). Early leverage buyouts were known as "bootstrap"[4] acquisitions, and they remained rare for several decades.

The boom happened in the 1980's, where the modern LBO appeared. The pioneering PE firms at that moment were Kohlberg Kravis Roberts & Co. (KKR) and Thomas H. Lee Co., both funded at the time, as they anticipated a profit opportunity by

[3] For this and all-forward examples or explanations involving currency, we will be using US dollars. However, all examples are translatable to any currency the lector may prefer.

[4] "Bootstrap" acquisitions in the 1960s and 1970s because they were considered to involve "bootstrap" investments. "Bootstrap" investments are investments made with little money.

acquiring companies with borrowed money instead of its own. During these years, many public companies were traded at very low values (e.g., low stock prices relative to their asset values or to their cash flows), making them attractive targets for buyouts. Early LBO users realized they could acquire underperformed or undervalued companies, break them up or improve their operations and resell them at a higher price, gaining a great profit. A common product used in the 80's was the so called "junk bonds", these were high yield bonds, high risk and rated poorly by the rating agencies. It's use in LBOs was led by Michael Milken, an American economist, father of high yield bonds. They were a new source of financing large buyouts. This era saw a dramatic rise in the LBO activity, with four LBO deals worth $1.7 Bn in 1980 growing into 410 deals worth $188 Bn in 1988, this represents a 10,150% increase in only 8 years, and was a time captured in many books and movies such as "Barbarians at the Gate", a 1989 book that portrays the fall of RJR Nabisco. Nevertheless, by the late 80's, concerns about excessive leverage started to arise, slowing the economy. Resulting in the collapse of the junk bond market in 1989, after the political campaign and bankruptcy of Drexel Burnham Lambert, an investment bank. This affected the LBO market, slowing down all operations.

Between the 1990's and the 2000's, after the boom and crash of the 80's, LBO didn't just disappear, but the market needed to evolve. During these years, there were less of the previous megadeals and lenders were more cautious when giving out money, obliging the PE firms to contribute with a larger percentage of equity than before. According to NYU Stern (n.d.), the average equity contribution increased from 9.7% in 1988 to 38.4% in 2000. LBO firms could not rely purely on high leveraged returns and had to focus on improving their companies to increase profitability and create value.

At the beginning of the 2000's, the second wave of LBOs started, pushed by the favorable credit markets that led to a growth of low price debt obligations. This added to the investors' appetite for leverage deals prompted the LBO boom between 2004 and 2007, also known as the "Private Equity boom". Numerous companies were taken private in record breaking buyouts such as the $45 Bn worth TXU Corporation deal (Vistra Energy), the largest leverage buyout in history (Andrews & Sorkin, 2007). This wave of LBO's exceeded in ways we could not have imagined in terms of dollar value deals. Nonetheless, it also ended with the 2007-2008 global financial crisis, which caused a freeze in the credit markets.

After the 2008 crisis, LBO activity slowed, this was because using the debt to finance become scarcer as its price and regulations grew, increasing scrutiny and requirement strictions on high leveraged deals. However, in 2010 PE activity started growing again and in the late 2010's the LBO deal structures had evolved with innovations that made financing easier to obtain in a low interest rate market. An example of these innovation is the "cov-lite" loans (Ivashina & Vallee, 2019), that had less of the protective covenants[5] for lenders.

In the recent years, firms have been operating with large LBOs again, as credit markets have improved their requirements. For example: in 2021 a group of PE firms led by Blackstone (the world's largest alternative asset manager, with more than $1 trillion in AUM[6], that serve institutional and individual investors by building strong businesses that delivers lasting value) made a $34 Bn LBO of Medline Industries. This was one the largest deals since the crisis of 2008. Also, in 2022 there were a lot of huge deals made such as the $13 Bn LBO of Citrix Systems. Bain & Company (2025) highlights that the number of buyout funds meeting or exceeding their fundraising targets in 2024 edged up to 85% from 80% in 2023. In addition, McKinsey & Company (2025) notes that global private equity dealmaking rebounded significantly in 2024, rising by 14% to $2 trillion after two years of decline.

Overall, while the LBO's basic concept has remained the same, the practice has evolved over time. Going from giving more importance to the operational improvements and moderate leverage to the explosion of deal sizes, a consequent crash and last, a modern resurgence under new market conditions.

1.3. LBO's structure

The leverage buyout is a transaction deal in which several parties are involved, essentially these are:

---

[5] A covenant from a bank is a condition in a loan agreement that outlines the borrower's financial obligations. Covenants are also known as debt covenants or banking covenants.

[6] AUM (Assets Under Management) is the total value of assets that an investment firm manages for its clients.

- Financial Services (PE firm). This is the acquirer that organizes and initiates the LBO. The PE firm creates a new shell or holding company that will be the one absorbing the target company. This shell is what's known as the SPV (Special Purpose Vehicle), the PE firm does not acquire the company directly, but creates this legal entity so that it becomes the buyer. The reason of this practice is that with this legal financial separation, the SPV isolates the liabilities and simplifies the accounting of the transactions. If the deal fails or the target company goes bankrupt, the losses will be contained by the SPV. Also, the SPV raises all the funds needed for the transaction (debt and equity), reducing the complexity of the operation.

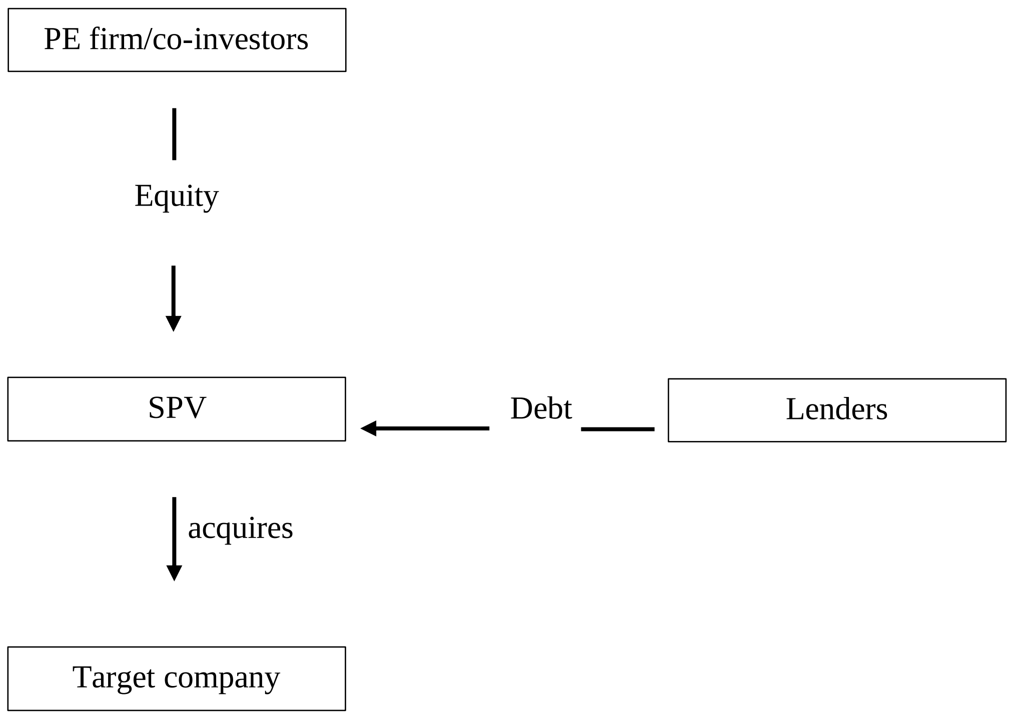
  Another reason for creating this holdco[7] is the tax efficiency it has, as in many jurisdictions, routing the transaction through an SPV enables favorable tax effects, such as interest deductibility or deferral of capital gain taxes. In interest deductibility, if the debt is taken by the PE firm its interest will not be deductible as the PE does not generate taxable operating income (it's just an investment vehicle), but when using the SPV the debt is pushed onto the target company acquired, that will be the one paying the interest and being then able to deduct its taxes. The other example of favorable tax effects is the deferral of capital gains, this occurs when the managers or the shareholders of the company exchange some of their shares for equity in the SPV, it is called a "rollover" transaction, but it cannot be made with the PE firm as usually it is not structured for it, the firm is not a corporate entity with operational business purposes.

  The SPV eases the transaction by also simplifying its legal acquisition process, allowing a clean integration after the merger between the SPV and the target company.

  Usually the procedural is: first, the PE firm creates a new SPV, then this SPV borrows money from lenders and receives the equity from the PE firm. Later the SPV uses these funds to acquirer the 100% of the target company, being wholly owned by the SPV as a subsidiary. The debt is owned by this target's Balance

[7] A company that owns shares in other companies rather than producing goods or services.

Sheet through merger or downstream assumption, and the PE firm holds the equity ownership though the SPV. We should clarify that although most of the time the equity comes from the PE firm, it can also come from the management of the target company (if it's a management buyout) or from co-investors. Next, an image of the company's structure:



The goal of the PE firm is essentially to, after a few years, exit the investment though a sale or an IPO at a higher value than it was bought, with a strong return on the equity they had invested.

- Lenders. They provide the debt percentage of the purchase. These lenders can be banks with products such as leveraged loans, term loans, revolving credit facilities, etc; institutional investors or other debt providers (mezzanine[8] funds, etc.). The LBO's debt is issued for the acquisition against the assets and the cash

[8] A collection of investments in mezzanine finance, which is a combination of debt and equity financing. Mezzanine funds are a way for businesses to raise money for projects or acquisitions.

flows of the target company. If this company has assets like buildings, working equipment, receivables, etc, those can serve as collateral for the loans and the lenders will then have claims against the company's cash flows, through the interested and the principal payments of the loans. The lenders expect to be repaid overtime from the company's operations and they charge interest and fees because of the risk this generates for them, as the target company could go south. Because of the high levels of debt in an LBO, it usually carries higher interest rates and may be below investment-grade (below BBB-), often taking the form of the so called "junk bonds" mentioned before.

- Target company. This is the company that is being acquired in the transaction, the one that the financial services want to buy. If the LBO is successful, its shareholders will sell their equity, usually at a premium to the market price if it's a public company, and the target company will be taken private (if it was public) or will become a 100% owned subsidiary of the SPV. After the merger the target's Balance Sheet is restructured, including a large amount of new debt and having changed its ownership. It is important to highlight again that the target's own assets and its future cash flows will be the ones that paydown the debt that has been acquired to purchase the company itself, as this is the key difference that an LBO has from other kind of acquisitions.

- Existing owners/management. In many LBOs the existing owner of the target is kept managing the company after the buyout and may even participate in the equity of the purchase or invest alongside the PE firm, aligning their incentives in order to improve the target's portfolio. When the target consists in a public company, the existing shareholders will receive cash for their shares and will cease to be the owners once the deal is closed. Sometimes, LBO can be called hostile, this means that the acquisition is done without the full consent of the target's management, although they can also be friendly or via auction. The result is that the target company passes from being owned by the public shareholders or parent company to being owned by the PE firm through the SPV, and now having much higher debt on its Balance Sheet.

In practice, making an LBO deal involves not only several different parties, but also several different steps and documents. The SPV will be the one collecting the final agreements and commitments, often in the commitment letters before closing the deal. Also, a percentage of the loan might be senior secured loans, that have the first priority claims on assets, while the other part of the loans could include junior high yield bonds, etc. Simply put, loan seniority refers to the order in which a company's debts are repaid if the company goes bankrupt or is liquidated. Senior debt has priority over other types of debt, and is paid back before subordinated or junior debt. Now, the capital structure of the company after the buyout usually has multiple tranches of debt, this can be for example, revolving, term loan A, term loan B, mezzanine, etc.

On the other side of the funds, the equity comes (as we have already stated) mostly from the financial services, with a few exceptions such as the target's management. At closing day, these structures are used to pay the acquisition price, typically paying off any existing debt that the target company held and buying its shares/equity, plus covering the transaction fees.

Once done, the target company operates with the new capital structure, its main priority becomes the payment of the new existing debt with its cash flows, since this is the key goal of the transaction, to delever[9] the company and increase its Equity value. If in the next years, the company performs well (grows earning and reduces its debt), the equity owner (PE firm) will gain when it's the time to sell the company.

Another way to understand this could be thinking of this transaction as a simil to buying a house with a mortgage with the intention of reselling in the next 5-10 years. When buying a house, you put a small down payment (this is what the PE firm does, the equity) and the rest of the price is paid by asking for a mortgage at the bank (the loans, the debt). The house value and homeowners' income (assets and cash flows from the target company) are what secures the mortgage. This allows the buyer to control a big asset with limited initial cash, magnifying the returns on equity if the value of the house increases in the next years, but with the risk being that if the value

[9] Reduce the level of one's debt by rapidly selling one's assets.

drops or the income stops for any matter, you will not be able to repay the debt you asked for at the beginning (your mortgage). But if the value grows and your income increases or stays stable, you will maybe be able to sell the house in 5 years and benefit from the difference between the price you paid when you bought it and the price you sell it for.

The end of an LBO deal comes when the PE firm exits the target company. The SPV is usually ditched or becomes inactive, unless reused because of tax or legal purposes (although this is rare). The SPV sells the equity of the target company or merges with the new buyer. With the buyer's payment, the SPV pays all its remaining debt and distributes between the equity holders the remaining amount. The Equity value at exit is the result of subtracting SPV's net debt to the enterprise value (EV). Now the SPV has no more assets left (as it has sold all of them to the buyer), nor liabilities (paid in full) or purpose, so the PE firm will either:

- legally dissolve the SPV, officially shutting it down and deregistering it;
- or, leave it as a dormant entity if there is a tax/legal advantage.

With this we will conclude our LBO deal. When explained in simplifying terms, it may seem easier than it is, but the reality is that in order to make this kind of deals correctly we must consider many different factors: market factors, economical, financial, etc. One of the central documents of the whole deal is the financial model, were we (acting as a PE firm) must previously of initiating a deal, forecast and prepare all of the historic and future financial performance of the target company, so that we can be almost sure or close to sure that this deal will eventually be profitable for us.

In the next section we will be explaining in detail how a financial model is made and which are the key steps in order to make one correctly, before starting the LBO deal and in order to profit from it at exit.

1.4. Financial Models

An LBO model is a financial model, made usually in an Excel document, in which we (as a PE firm) must forecast the performance of the target company, its cash flows, and capital structure over the LBO timeline, that is usually between 3-7 years.

This is made in order to estimate the returns that the equity investors will be obtaining from the deal. We will be dividing the model in 5 key steps.

1.4.1. Step 1: set the key assumptions

First of all we must lay out all the major assumptions we will be making for the transaction and for the target's operations. This usually includes:

- The acquisition price and the financial mix that this involves. An example would be the entry valuation of the target company, based on the EBITDA[10] multiple (e.g., 8x EBITDA). This gives as a purchase price, that is what we will be paying to buy the target company.

  With this we must then decide which percentage of that price we will be paying with debt and which percentage we will be paying in equity. But one question you might have, is how we will be making this decision. Well, here there are different factors taken into account, such as:

  > Which is the debt capacity of the target company, as it must generate enough cash flows to pay down the debt. Here we could use multiples like the leverage ratio (debt/EBITDA) or the interest coverage ratio (EBITDA/interest expense). The more stable the company is or the more cash it generates, more percentage of debt we can take.

  > The market conditions, as with low interest conditions the lenders are more likely to provide a high level of leverage.

  > The limits from the lenders, as for example different banks in different times in the economy will have different restrictions or covenants, less or more favorable to us, that will move us to take less or more debt in our LBO model.

  > Your PE firm strategy, as there are firms that have riskier strategies, so they will take on more debt.

  > And in some of the cases, the flexibility that the sellers will have about maybe accepting part of the payment, as for example, a

[10] Earnings before interest, taxes, depreciation, and amortization.

rollover, into the new company. This would reduce for the need of upfront equity.

When deciding this we can create what is usually called in the model the "Sources of funds" (debt and equity tranches) and the "Uses of funds" (the total acquisition price, plus any fees and perhaps the refinancing[11] of existing debt).

Every dollar that comes from any source must be allocated to a specific use, and with the "Sources & Uses" statement, we will be able to balance the money that will be going in and out of the transaction. An example: if we estimate that the EV of the target company is $500 Mn, and the PE firm objective of debt/equity ratio is 70/30. Then we will raise $350 Mn of debts and invest $150 Mn of equity, these will be the sources. And the uses would be the whole $500 Mn to pay the sellers (plus additional fees).

- Operating assumptions. With these we refer to the revenue growth rates, the margin profits, and all the other parameters with which we will project the target's financial statements. It's important to know that many of the key line items are often projected as a percentage of the revenue or based on other drivers (e.g., to assume that the sales of the target company will increase +5% YoY[12], the EBITDA margins (EBITDA/Revenue) will be around 20% and the CAPEX[13] will be 15% of the sales). We will feed all of these assumptions into the income statement and the cash flows projections. As a quick remark, there are 3 main financial statements: Income Statement (IS), Balance Sheet (BS) and Cash Flow Statement (CF statement). We will be talking about all of them in this and in the following chapters.

- Debt assumptions. As the debt is key to this deal, we must carefully decide which will be the conditions of each one of the tranches or

[11] When you replace an existing loan with a new loan that has better terms. The new loan is used to pay off the old loan.
[12] Year on Year.
[13] Capital expenditure, the money a business spends on long-term assets, like equipment, property, or research and development.

different kinds of debt that we will be taking. We must specify the interest rate on each one of them, any amortization (by scheduling the principal payments of each year) and any concrete relevant features such as whether a revolving credit line[14] is available for short term liquidity, or if any debt has specific payment requirements. Also, is important to note if any debt has maintenance covenants that might limit how much debt can be repaid early, or other specific covenants.

As we have already said, we must assume the interest rates, but these can change from one tranche of debt to another, while the annual senior debt might have a 5% interest rate, the subordinated debt could have an 8%, the junior debt a 7%, an going on. Last, we will also note the maturity of each of the debts, the objective being that its maturity coincides with the exit date.

- Exit assumptions. Here we will be deciding which will be the exit scenario. Usually, analysts assume that the company will be sold after the projection period at a certain exit EBITDA multiple, that could be the same or a different one from the entry EBTDA multiple, depending on which are the market conditions in each of the moments. And this, same as before as with the entry EV, will allow us to calculate the exit EV. Another exit assumption is the minimum cash that the company will keep at the exit (as any excess cash is usually assumed to go to the debt payment or to the equity). Last, the exit Equity Value is equal to the exit EV minus the remaining net debt, this is what the financial sponsors will recover after the sale.

  From that, the model can calculate the IRR (Internal Rate of Return) and the MOIC (Multiple on Invested Capital) for the equity investors. The IRR is a measure of the annual percentage rate of profitability on a project or solution when compared to the original amount spent or invested.

[14] A type of credit that allows a borrower to access funds up to a pre-approved limit, and then pay them back over time. It's also known as open-ended credit.

While the MOIC is the ratio of exit equity to initial equity (e.g., 2,0x means doubling the money).

All of these assumptions are usually organized in a clear section at the top of the financial model so that it has easy access for when you would like to modify them or for anyone in order to understand the calculations of the model. On many occasions it is used Excel data validation tools to organize this, as for example when creating a drop-down menu for some assumptions to be able to select different scenarios or leverage levels depending on what is useful for your own deal.

To conclude, next to these assumptions, it is important to state clearly which are all the units and measure systems (percentages, dollars, years, etc.) that you will be using in all of your calculations, so that anyone reading the model can understand these assumptions without fail, and with that, the model itself.

1.4.2. Step 2: finance history of the company and projecting the income statement

The second step when making your financial model is to look at the financial statements of the past years from your target company. You will use the first step (Section 1.4.1) to build year by year (or quarter by quarter) its financial projections. Starting with the IS and following up with the other two financial statements. The key calculations and formulas that must be included in all LBO models are:

- EBITDA, one of the most common measures in finance. It is the operating cash flow before the capital expenses such as COGS, SG&A, etc. (specifying each of the expense ratios to make the calculations), but it can also be calculated by taking last year's EBITDA and growing it if you have assumed a margin growth.

- Depreciation & Amortization (D&A). These are non-cash expenses for the write-down of assets and the amortization of the intangibles. These can be projected based on the historical CAPEX and a previously assumed depreciation schedule. Sometimes there is built a separate depreciation schedule (when making a detailed model) that may use past CAPEX and an assumed depreciation life to roll forward. In order to simplify, you can also assume the D&A as a percentage of the revenue or as a percentage of the previous CAPEX. D&A's correct calculation is very important as it will impact the IS and the BS (e.g., accumulated depreciation).

- Interest expense. This is crucial in an LBO model, but we will explain it later in detail when describing the schedule of the debt. For now, know that the interest expense on the debt of the LBO will be added to the IS by reducing its net income.

- Taxes. We cannot forget this essential effect. In our model we must apply an assumed tax rate to the income before taxes (EBT) by either using a statutory corporate tax rate or using the target's historical tax rate. Keeping always in mind that the interest expense will reduce the taxable income. Here, LBOs usually create tax shields due to the interest expense being tax deductible, and this can increase our CFs. But the model might also incorporate this effect by using the tax rate on the EBT.

- Net income, the amount of money a business (or individual) has left after deducting all expenses, including taxes and interest, from its total revenue. It's the bottom line of the IS and, it will go at the top of the CF statement and into the retained earnings of the BS.

All of the three financial statements (IS, BS and CF statement) are linked together. For example, after projecting the IS to the net income we can also project some of the BS calculations as some of them depend on the IS (retained earnings, that increase by the net income and decreases with dividends, if any). Other items on the BS as working capital (account receivables, account payables and inventory) might be forecast with the assumptions of the inventory turnover or using the percentage of sales method. And the CF statement will track how the CF from operations affects the CAPEX and our remining debt for each one of the periods. The CF statement is crucial as then we can determine which will be our debt payments in each period.

Once we have forecasted in detail the IS we will move onto step 3, building the BS at the closing day of the deal (this is at year 0, before year 1).

1.4.3. Step 3: building the Balance Sheet at closing (year 0)

Often called "Pro Forma BS" or transaction adjustments, it's the second financial statement of the three mentioned before. We must start with the target company's last BS report and then add the impact that the acquisition will have in the company. It must always include:

- The removal of the old equity. The equity that was in the BS before the sale, as the new one will be the one from the SPV. The target's shareholder equity is removed, as it is being bought out. In accounting terms, the common stock and the retained earnings are eliminated or replaced because the company is now under new ownership (SPV => PE firm).

- The addition of the new debt, added to the liabilities of the target but by differentiating between the several kinds of debts that we have. For example, if the new debt is $350 Mn you must add this into the liabilities splitting it into the different debt tranches such as senior, junior, subsidiary, etc. Also, if you have decided to refinance any old/existing debt, this must be removed (paid off as part of the "Uses of funds") and replaced with the new one.

- The addition of Goodwill/intangibles[15]. In most of the LBOs, the price exceeds the book value of the company's asset. The reason is because of the difference generated by goodwill or other intangible assets on the BS (the purchase price allocation or PPA[16]). Goodwill is essentially an accounting item that ensures that the new BS is balanced correctly. For

[15] The value of a company that's above its physical assets. It's an intangible asset that includes a company's reputation, customer base, and other non-physical assets.
[16] The process of assigning the purchase price of a company to its assets and liabilities.

example: if the target's net identifiable assets are valued at $100 Mn, but the price of the company is of $500 Mn, then $400 will be allocated to goodwill. Assets must be equal to liabilities plus equity after the transaction. And the new equity is calculated by subtracting the debt raised to the acquisition price, while the assets gained are calculated by adding the new equity to the liabilities.

Goodwill makes the assets' side match, remaining on the books and maybe being subjected to periodic impairment test, but usually not amortized when using modern accounting.

- Fees (banker, legal, etc.). In the Pro Forma it may be deducted from the cash or added to the debt when financed and expensed or capitalized appropriately. Mostly, financing fees are capitalized as an asset (deferred financial costs) and amortized over the life of the loan. The LBO model must incorporate these then by: reducing the cash or creating a deferred cost asset that amortizes into the interest expenses. Although, if you would like to simply this, you could always treat all fees as cash uses that reduce the cash at the closing.

Now, after these adjustments you have calculated a post-LBO Balance Sheet that reflects the new capital structure the target company will have after the buyout. The new BS will have new high debt plus the new equity, and it will be the starting point for all of our future projections as it will be the starting point of your Year 1; the closing of this period, the starting point of your Year 2; and moreover. Now, we only have one last financial statement without explaining, the CF statement, this will be step 4.

1.4.4. Step 4: building the CF statement and the debt schedule

This is the analysis and forecast of how the model will handle the down payments of the leverage over time. We can track the debt year by year, or whatever time measure you find more comfortable. Here will be key to calculate the interest expenses and determine the mandatory and optional repayments of the debt. In order to do this:

- We must first begin at the debt balance for each of the tranches with the balance at closing that we have previously calculated in step 3 in the Pro Forma BS.
- Then we calculate the interests for each tranche in each one of the periods. Usually, the interest is equal to the interest rate multiplied by the average balance of debt over the period (this is the average of the total amount of debt owed to a lender over a period of time). Some of the simpler models will use the initial debt of the period to calculate the interest, which avoids circular references, but this will be discussed later on in Chapter 2. An example using the average balance of debt is: if a loan has an annual interest of 5% and the average balance of interest in year 1 is of \$300 Mn, then the interest expense will be of \$15 Mn. When determined for all the tranches, we will add all of the different interest expenses and this number will go into the interest line of the IS.

- Amortization and maturities. If a tranche has scheduled the amortization of its principal payments, then we must subtract this amount each period (e.g., a term loan that amortizes 5% of the initial balance each year until maturity). In addition, if any debt reaches maturity within the forecast (e.g., year 3), then the model should reflect that this debt is fully paid off by that time or refinanced. However, basic LBO models usually assume the debt is paid off when reaching the exit of the transaction.

- Cash sweep, an optional debt payment (very common in LBOs). This is when the excess CFs of the target company, after paying all of its mandatory obligations, is used to pay the debt faster than expected, in order to delever the new company faster. To implement this, the model needs to compute the company's available Free Cash Flow (FCF) for the payment of debt in every period:
    - OCF – CAPEX – mandatory payments – dividends = cash that can be used to pay the debt of that period. If this calculation is positive and assuming the target company does not retain it as excess cash

above the required minimum[17], then it will go towards the down payment of our most expensive or least senior debt first (depending on the priorities of payments we have assumed for the model, although usually senior debt is paid first because junior debt has restrictions).

> In the debt schedule we would then calculate also optional repayments for each of the tranches. When using the formulas, you can be sure you are not paying more than the outstanding balance for each of the debts acquired.

- Now you have to end the debt balance by calculating the ending figure for each tranche that is equal to its initial balance minus the principal paid during the period (scheduled and optional principal). Then, the ending balance becomes the next period's initial balance. By structuring the debt this way, we will generate looping dependencies, as optional payments depend on the CFs, CFs on the interests and interests on the balance. Nevertheless, we will address this also in Chapter 2. For now, the debt schedule gives us a forecast for the debt we will paying and that will be remaining each year/period.

After finishing step 4, we will have the forecast of the full three financial statements for the target company, showing year by year how the debt is paid, how then the interest expense declines as the debt decreases, and how the target's equity builds up as (hopefully) the profit also rises and are accumulated.

Before moving onto the exit explanation, as a recommendation, it is considered good practice to check in every period if Assets = liabilities + equity, which it should, if the financial statements are correct. And to add, in some models they include some key metrics as the leverage ratio in order to evaluate how well/bad the deal will be performing each year.

[17] The amount of cash that a company commits to retain over and above what a business requires to fulfil its daily operational cash and requirements.

When finished, the final step is to sell the company and calculate the acquirer's returns, we will be talking about this in the next step.

1.4.5. Step 5: exit and calculating the investors' returns

Firstly, for the exit of the company we must use our assumptions and set an exit EV. We use the EBITDA calculated in our final period and multiply it by a previously assumed (step 1) EBITDA multiple (e.g., if the EBITDA in year 5 is equal to $120 Mn and the exit multiple assumed is 8x, then the exit EV will be $960 Mn). Another way would be using the perpetuity growth method or even other valuations, but the multiple strategy is the most common one.

Having calculated the exit EV we can now determine the exit Equity value that is equal to the exit EV minus the net debt at exit (total debt – cash). If debt is higher than the cash at exit, then it will decrease the Equity value; and if its lower, it will increase it. This Equity value represents the return amount that the PE firm will recover when selling the target company and paying off any remining debt. At this point it is also key to calculate the IRR and the MOIC, so that we can calculate the returns correctly.

Finally, having completed all of the steps, we have essentially completed our LBO model. That includes our projected financials and the future payments of the debt acquired at the beginning. We can use it now to test the model in different assumptions or scenarios, to be prepared for any given circumstance, this is called making a "sensitivity analysis". Consciously evaluating, if the model meets our required thresholds and our expected profitability, weather the LBO deal is worth pursuing or not.

1.5. Innovation on modelling

Thanks to AI, every sector is experiencing a great transformation. In finance we can already see how it is evolving, being able to do tasks automatically, improve the decision-making process and even personalize customer experiences. But it does not only stop there, as we are also implementing AI in security, crucial for the financial sector. Nowadays is already being used in cases of fraud detection, credit decisions, risk management, portfolio protection and making more efficient it’s the security systems.

LBO models are starting to see these changes, and this will continue to evolve in the years to come. The integration of AI and LLM (Large Language Models) has resulted in several innovations and improvement of the LBO modelling process. As Asimiyu (2024) discusses in *The Convergence of AI Optimization and Financial Engineering*, LLMs are used to process and analyze large amounts of financial data (market reports, news, articles, research, periodic results, etc.), allowing to have a better comprehensive analysis of this information. Also, with its use, the financial analysts are able to increase the accuracy in the decision-making process of the LBO deals, processing complex scenarios and forecasting the results with precision. Not only that, AI can also able be able to identify the risks of the model and make an accurate statement of the issues it presents, while then giving you ways in order to fix these founded issues. Assisting the analyst in charge of making the LBO model, so that the firm does not miss any detail.

Nevertheless, AI is not the only innovation that is affecting LBOs, also we can see the effects of the changes in management strategies. Bertoni, Le Nadant, and Perdreau (2019) analyzes how management innovation evolves in companies after LBOs. This article talks about how these improvements are implemented in the target companies that go under new management after an LBO. The study is focused on the changes centered in the organization structures, alignment of incentives, restructuring of the management overtake, etc.

As we have seen when talking about the history of the LBOs, this product has evolved throughout the years and these recent innovations teaches us that we should not think that we have reached the ceiling in improvement. We can always try to innovate or perfectionate the LBO process, and in the case of our study, the LBO model. The goal of our study to analyze a concrete issue we identify in the LBO models, circular references, and propose a more effective solution than the tools most of the firms are using nowadays, reaching always for improvement.

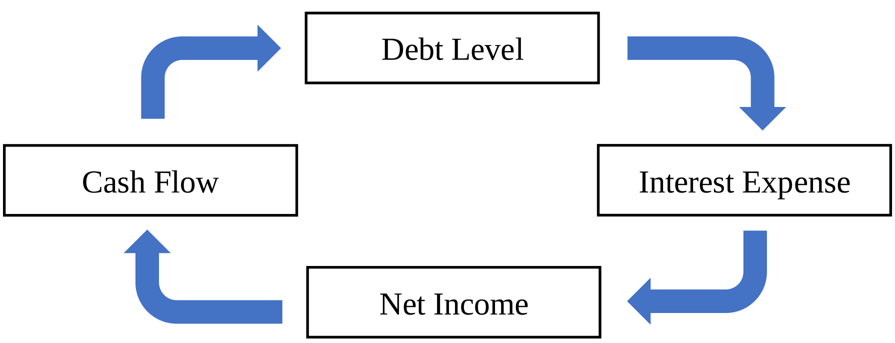
CHAPTER 2: THE PROBLEM ON FINANCIAL MODELS

2.1. Circular references

When you are constructing an LBO deal a main part of the process consists in making the LBO financial model, this is the one we have explained in detail throughout Chapter 1. However, because of the three financial statements (IS, BS and CF statement) being linked and the nature of the accounting equations needed to create this model, on many occasions we encounter with the same problem, a circular reference.

But what is a circular reference? Well, in simple terms, is when in order to resolve a formula we must resolve a second equation that loops back to the first one, being then unable to resolve both of them. In the loop there can be many parties, for example a loop of three parties would be if A is equal to B, B is equal to C, but C is equal to A. In an Excel spreadsheet is when (directly or indirectly) a formula refers to its own output, creating then a feedback loop. In LBOs one of the most common circular references is the one that appears because of the calculation of the interest expense when using the average debt or cash balances.

Getting into the accounting reason, the interest expense depends on the amount of debt we have in the period we are calculating it. The amount of outstanding debt at the end of the period depends on how much of the debt was paid during that period with the target's CFs. However, the CFs generated are also affected by the interest expense, as interest reduces the net income and net income reduces then the OCF. The circular logic would be:



An example would be when calculating the interest of a revolving credit line by using the average debt balance. The formula of the interest expense is then (initial debt + end debt) divided by 2; but the end debt is calculated with initial debt + net borrow – repayments. And repayments are made by the available CF after interest, going back with the interest to the end debt. Creating as such, a circular reference. For the model to work, we must solve this dependance.

However, this is not the only accounting formulas that experience the issue, the same can happen if the company has a sizeable cash balance. The interest earned on cash creates the cash balance but would also yields more interest income. Although this effect is not the most common one in LBOs as usually the companies are do not have so much excess cash, the foundation is similar to the previous example, and therefore, also its solution.

In LBO models the circular references occur in three main circumstances:

1. The interest expense is calculated based on the average debt balances.
2. The PE firm is targeting a specific capital structure (as could be the leverage ratio λ = debt/capital employed).
3. The debt repayments depend on the FCF which at the same time depends on the interest, which depends on debt.

Number 1 is the situation when a current period's value depends on an end of period's value, that at the same time depends on the current value.

For number 3, net income, that goes into the retained earnings and to the cash, is used to calculate the debt payments, while the interest (part of the net income) depends on debt. A more explicit cash circularity being when we have a debt "cash sweep" mechanism. Here the interest affects the CFs, the latter affects the debt payments, and the debt payments affect, again, the interest, closing the loop.

Nevertheless, we will be solving in detail all of the three scenarios by using algebra, as this is the foundation of the tool we will be proposing in the present study.

Under the basic principle of Assets = Liabilities + Equity, it is key to solve all of these circular references. When we do not solve them, Excel will not calculate the model correctly. And unresolved circular references make the model unstable and unreliable. This is the reason why modelers try always to avoid circular references if possible. They do this by using simplifications like calculating the interest on the initial period debt only, breaking then the link at the cost of a small accuracy trade-off. Or also by using an alternate that fixes either the interest or the cash flow.

Another example would be when including a minimum cash balance, also explained previously, that the company must maintain. Then, if the CF is high, you must pay less debt because you want to keep a cushion of cash. But excess cash depends on the interest and on the debt, creating also a loop. Breaking Into Wall Street (n.d.) explains how circular references generated in the interest income and the interest expense appear when calculating these two by using the average cash/debt balances rather than the beginning of the period ones. As more interest is earned, more cash balance there is, and more interest can be earned, closing the loop. Same happens with interest expense, lower interest expense fees generate more cash to pay the debt, that also lowers the end of the period debt, resulting then in even lower interest expenses.

2.2. Solution tools used nowadays

Today, probably the most used solution for solving circular references is Excel Iterative Solver. Excel provides a built-in mechanism so that we can deal with circular references when they appear, this is the iterative calculation feature.

When a circular reference is generated in the spreadsheet, Excel (by default) will show you a warning error, and if you want it to solve the loop, you have the option of activating its iteration tool. In order to do this, we must go to: File > Options > Formulas and check the "Enable iterative calculations" cell, setting a maximum number of iterations (100 by default) and what it is called a tolerance (this is the change between iterations under which it will stop). When this feature is turned on, Excel will then repeatedly calculate the circular formulas by looping through them until it finds the solution or until it reaches the maximum number of iterations, stopping on the last try when reached to the iteration number 100 (by default). It starts in an initial guess for the circular reference cells, often using a value from the last calculation made in the spreadsheet or if none, using the value of zero, and then continuously updates the values, hopefully getting more consistent values with each of the iterations.

If the model is what is called as “well-behaved”, then the circular reference will be linear or convergent. Excel’s iterative tool will find a number that satisfies the relationships to within the specific tolerance. For example, it might iterate 100 times and find that the interest expense settles at $10,2 Mn and the debt at $200 Mn and using those to calculate each other no longer changes the outcome more than a small fraction, so Excel stops and returns this as the result.

2.3. Why Excel’s solution fails?

Although is a possible solution, easy and quick, as we only have to select a cell in order to activate it, is a solution that presents many deficiencies. Sometimes it does not work. Excel’s iterative tool cannot guarantee an accurate solution in every case, having more problems as the model becomes more complex, hitting its maximum number of iterations without converging or giving you a close solution. Excel will just stop and give you whatever figures it tried for the last iteration, which could be, and mostly is, wrong. This would mess with the equilibrium of the BS by a few million or make the results related to the figures slightly off.

Also, although the figures in which it has settled are incorrect, they might be stable, maybe causing a less experienced analyst to not locate the issue at first sight. Another issue is that if a portion too large of the debt is being paid in a way that one iteration overshoots and then the next one it tries, undershoots the correct figure, the solver will oscillate. By default, as we have already mentioned, Excel makes 100 iterations and a tolerance of approximation only, however, in several cases it can even slow to a crawl or freeze because of the size of the calculations and of the model.

Lastly, it also presents practical limitations, as the model is sensitive to these circular references. So, if one person in the team has used Excel’s tool, but another one has the iterative featured turned off in their Excel, when the latter tries to open the LBO model, it will break. In addition, even if they have it turned on, if they change even the slightest thing from the model and this affects these circular formulas in any way, Excel will start again the iterative process in order to solve what it thinks is a new circular reference, giving us a slightly different solution every time, with every change. But it does not stop there, as another complication is that Excel can only solve one circular problem at a time, and if you have various circularities, you must go one by one. And if you have two that are connected for any reason, and then change something you must now need to start from the beginning, by first letting it solve one and then, the other. All of these practical complications make Excel's tool not worth it if at the end, it is not going to give us an exact correct solution every time.

Because we think that solving this is crucial in order maintain the balance of the model and therefore, make a good LBO model, so that your deal functions correctly, in this study we will be proposing an alternative tool to solve the circular references. We will be explaining this tool in the next chapter.

CHAPTER 3: SOLUTION PROPOSITION

3.1. Description of the logic used

The circular references are, essentially, a formula. And, by using algebra, we can solve them. Bellón (2020), in *Solving Circular References in Financial Models*, explains in detail how these formulas can be solved in the different scenarios. As we have already mentioned in Chapter 2, there are three main situations where the circular references occur in an LBO model. By using Bellón's (2020) analysis, we will be explaining each of them.

First of all, number one, when the interest expense is calculated based on the average debt balances, also called the mid-period interest. Here the formulas are:

$$Interest\_t = r\_D * [(Debt\_t\text{-}1 + Debt\_t)/2]$$

r_D = interest rate
UFCF = unlevered free cash flow
Tc = tax rate
t = periods

And FCF after interest becomes:

$$FCF^{post}\_t = UFCF\_t + \Delta Debt\_t - (1\text{-}Tc) * Interest\_t$$

-- Now, by applying algebra we solve and obtain:

$\Delta FCF\_t = (1 - [(1-Tc) * r\_D]/2) * \Delta Debt\_t$

Being (1- [(1-Tc) * r_D]/2) equal to α (alpha).

The debt issuance increases cash, but the interest on the new debt partially affects this. The coefficient α, that is equal to (1 – [(1-Tc) * r_D]/2), reflects this net cash gain from the new debt.

The second scenario is the one when the firm is targeting a specific capital structure (as could be the leverage ratio λ = debt/capital employed). In this situation the first step is to define the CE:

a. $CE\_t = IC\_t + Cash\_t\text{-}1 + FCFf^{ixed}\_t + Debt^{now}\_t$

The FCF^{fixed} in formed by the CF independent at the end of the year capital structure as fixed dividends. And the $Debt^{new}$ is the new debt issued to reach the target leverage. The second step is to determine the target leverage policy:

b. $Debt^{new}\_t = \lambda * CE\_t - Debt^{pre}\_t$

The $Debt^{pre}$ is the debt at the beginning of the period, after the meeting the mandatory payments. The third and last step is to solve the CE_t algebraically (by substituting formula “b” into formula “a”.

c. $CE\_t = IC\_t + Cash\_t\text{-}1 + FCF^{fixed}\_t + \lambda * CE\_t - Debt^{pre}\_t \Rightarrow$

$\Rightarrow CE\_t = (IC\_t + Cash\_t\text{-}1 + FCF^{fixed}\_t - Debt^{pre}\_t) / (1- \lambda)$

And substituting back to obtain the debt:

d. $Debt^{new}\_t = [\lambda - (1 - \lambda)] * (IC\_t + Cash\_t\text{-}1 + FCF^{fixed}\_t) - [1 - (1 - \lambda)] * Debt^{pre}\_t$

This formula eliminates the need for the iterative circular solver from Excel by expressing the new debt issued in a closed form.

And to conclude, the third scenario, that is when the debt repayments depend on the FCF which at the same time depends on the interest, which depends on debt. This is if cash compensating balance is required, when the lenders require the firm to maintain a cash level (% of the debt = ϒ). The starting equation is the past FCF.

$$FCF^{past}\_t = UFCF\_t + (1 - \Upsilon) * \Delta Debt\_t - (1 - Tc) * [(Debt\_t\text{-}1 + Debt\_t)/2] * r\_D$$

So, the net effect on FCF from ΔDebt is:

$$\Delta FCF\_t = \{1 - \Upsilon - [(1 - Tc) * r\_D]/2\} * \Delta Debt\_t$$

Where the coefficient α is equal to {1 – ϒ – [(1 – Tc) * r_D]/2}. This formula shows the cash balance requirement reduces more the positive CF impact of the new debt.

3.2. How we will create this solution

The alternative solution we are proposing in the study, in order to solve circular references more efficiency, is to create a program which will start with five yes or no questions being asked to the user about its LBO financial model. And when the user answers the questions, the program will generate, based on its answers, a formula sheet with the equations the user will be needing to include in its financial model to solve its circular references.

We will be making this program by programming the tool's code and to do that we will be using Python. This is because this programming language presents many advantages when achieving our objective. The main ones being:

- Its efficiency, Python is easy to read and write, being able to build a code that is useful by using fewer lines than other languages might need.
- This language has a great number of libraries, from finance to machine learning, and these libraries can be used in the favor of our code.
- Some of its tools, such as Jupyter Notebook, are perfect for prototyping a program, being able to test ideas quickly.
- And last, it's very important to highlight, as it is one of the main reasons, we have chosen this language, its close connection with Excel. As the financial models we will be working on are going to be made on excel.

Therefore, in the next Chapter 4 we will be analyzing which coding we have used and how the program we have created operates.

3.3. Proposition of the 5 Questions

As the program starts by asking five general questions about its LBO financial model to the user, I find it relevant to, previously, explain which are these five questions and why I have chosen them instead of others.

The five questions are:

1. Do you keep a stable leverage ratio of Debt/Capital Employed?
2. Is your interest expense calculated using average debt during the year?
3. Do you have a cash compensating balance that is tied to your debt?
4. Is your Free Cash Flow (FCF) adjusted to tax on interest?
5. Is your excess of cash or debt directly associated to your end of the year Balance Sheet values?

As we can deduct from the questions itself, the reason of choosing these is that they cover most of the relevant information the program needs to classify the model in one or more of the three previously explained possible scenarios where a circular reference may appear. Covering, in my opinion, all the relevant points.

However, and of course, it is possible that some very complex models are not covered in full by using this program. In the future, we could specify more how the user's model works so that we can create an even more accurate formula sheet. But for now, these cover all the essential issues and is viable and useful for almost all of the LBO financial models.

CHAPTER 4: PROGRAMMING AND DEVELOPMENT OF THE SOLVING TOOL

4.1. Code of the program

In Appendix 1 you will find the needed code to input in your Jupyter Notebook in order to create the program we have designed in this study and so you can generate you LBO formula sheet, personalized to your financial model.

In the code use, we first state which will be the five questions, and just after that a list of the formulas we will be using in the formula sheet for each one of the questions.

When running the code, this is the first thing that appears, so that the user can answer the questions:

```
=== Circular Reference Solver Tool ===
Make your own LBO formula sheet!

First, you must answer the following questions about your LBO model

1. Do you keep a stable leverage ratio of Debt/Capital Employed? (Yes/No): ↑↓ for history. Search history with c-↑/c-↓
```

Appearing each question one by one, as you keep answering them with: Yes or No.

Next, in the line that starts: “formulafile =…” and in the following lines the code is focused on generating a file with the results of the program, so that you can see which are the formulas you need, this file will save automatically in the Jupyter folder where you have runed your program.

After explaining the code, we have used, we must analyze now if it works and the results it will gives us when running the program.

4.2. Compilation of the following tests

For example, the result when answering the 5 questions: Yes / No / No / No / No

```
=== Circular Reference Solver Tool ===
Make your own LBO formula sheet!

First, you must answer the following questions about your LBO model

1. Do you keep a stable leverage ratio of Debt/Capital Employed? (Yes/No):  yes
2. Is your interest expense calculated using average debt during the year? (Yes/No):  no
3. Do you have a cash compensating balance that is tied to your debt? (Yes/No):  no
4. Is your Free Cash Flow (FCF) adjusted to tax on interest? (Yes/No):  no
5. Is your excess of cash or debt directly associated to your end of the year Balance Sheet values? (Yes/No):  no

Done!! Your generated LBO formula sheet has been saved as 'Your_LBO_formula_sheet.txt'.
```

The file is saved in your Notebook folder as: Your_LBO_formula_sheet

Your_LBO_formula_sheet.txt

And if you open it, you will find the formulas personalized to your case and answer to the questions:

LBO Formula Sheet, a practical solution to Circular References

This sheet includes the equations you need for your model in order to solve your Circula References. It is based on the inputs on the previous questions.

Formula 1: Recalculation of Capital Employed

Explanation: This equation reclaculates the Capital Employed (CE) in order to separate it from its circular effect. It uses cash and FCF as known inputs and solves CE algebraically based on a constant target leverage.

Equation: $CE_t = (IC_t + Cash_{t-1} + FCFfixed_t - DEBTpre_t) / (1 - \lambda)$

Formula 2: Debt required to meet our target leverage

Explanation: According to the target leverage ratio (λ) you've chosen, this equation calculates how much new debt should be added. With this we can remove dependencies on unknown end of the year variables in order to break the circular reference.

Equation: $DEBTnew_t = \lambda / (1 - \lambda) * (IC_t + Cash_{t-1} + FCFfixed_t) - DEBTpre_t / (1 - \lambda)$

Also, when we answer: No / No / No / No / No, so we don't have any of the situations in our LBO model, the program will give you this answer:

There has been not found any formulas related to solving circular references, based on your responses.

And lastly, if you answer with a Yes to all of the questions, it will give you the formulas to all of the possible scenarios. Even if the answers to some questions are not compatible, the program will still give you the solution to both of your scenarios, just in case you are not sure in which of them you are, so that when understanding it better, you can solve either of them.

Your will be able to do this, and to run the program as many times as you want, always being saved in the same folder, under the same name.

In addition to this, we have created the code needed, after generating the formula sheet, to transform the file to a PDF (Appendix 2). By running it after answering the questions as we have done before, only Question 1 = Yes, then it gives us the next result:

LBO Formula Sheet

LBO Formula Sheet, a practical solution to Circular References

This sheet includes the equations you need for your model in order to solve your Circula References. It is based on the inputs on the previous questions.

<u>Formula 1:</u> Recalculation of Capital Employed

Explanation: This equation reclaculates the Capital Employed (CE) in order to separate it from its circular effect. It uses cash and FCF as known inputs and solves CE algebraically based on a constant target leverage.

Equation: CE_t = (IC_t + Cash_t-1 + FCFfixed_t - DEBTpre_t) / (1 - λ)

<u>Formula 2:</u> Debt required to meet our target leverage

Explanation: According to the target leverage ratio (λ) you've chosen, this equation calculates how much new debt should be added. With this we can remove dependencies on unknown end of the year variables in order to break the circular reference.

Equation: DEBTnew_t = λ / (1 - λ) * (IC_t + Cash_t-1 + FCFfixed_t) - DEBTpre_t / (1 - λ)

And to Excel format (Appendix 3):

Your_LBO_formula_sheet.xlsx

If we download this, it will give us an Excel that looks like this:

LBO Formula Sheet

This sheet includes the equations you need for your model in order to solve your Circular References.

It is based on the inputs on the previous questions.

Formula #	Title	Explanation	Equation
Formula 1	Recalculation of Capital Employed	This equation reclaculates the Capital Employed (CE) in order to separate it from its circular effect. It uses cash and FCF as known inputs and solves CE algebraically based on a constant target leverage.	$CE_t = (IC_t + Cash_{t-1} + FCFfixed_t - DEBTpre_t) / (1 - \lambda)$
Formula 2	Debt required to meet our target leverage	According to the target leverage ratio (λ) you've chosen, this equation calculates how much new debt should be added. With this we can remove dependencies on unknown end of the year variables in order to break the circular reference.	$DEBTnew_t = \lambda / (1 - \lambda) * (IC_t + Cash_{t-1} + FCFfixed_t) - DEBTpre_t / (1 - \lambda)$

4.3. Feedback and additional proposition for future academics

From my point of view, this is a very complete solution and program, that works for the mains three scenarios we are presented when solving circular references. And with this formula sheet we will be able to solve, not only in a more efficient way the equations of the model, but also correctly (not by guessing).

The only critic I could make is that, as said at the beginning of the study, we can always look for improvement, being possible to extend even more the program by doing maybe more specific with the starting questions so that you can cover every possible circular formula.

Although I also think that, as we can modify the program whenever we want, maybe is more coherent to add/eliminate/modify the questions and the formulas at the same time that the new unexpected circularities appear in our LBO model, so that then it maintains in line with what we will be using in our day to day modelling process.

CHAPTER 5: RESULTS

5.1. Explanation of the proposed tool's results

The result of the tool shows us how, with the code, anyone that understands how to make an LBO financial model and the formulas needed to balance the numbers, can create a personalized formula sheet to solve its circular equations.

The user only needs two things: being the first one (as said) to understand how the LBO model works, but with this study, that knowledge can be reached, as we explain in detail an LBO and how the financial model is created in Chapter 1 and Chapter 2.

The second thing they will need is the Python program. They must download the application called "Anaconda", that includes the "Jupyter Notebook" tool we have used from Python, so that they are able to run the program. And in addition to the main tool, they will also need to download some of Python's packages in order to be able to transform the formula sheet into a PDF or into Excel format.

Nevertheless, when both of these conditions are met by the user, the program we have created in this study is very intuitive and of easy use, offering an attractive solution for circular references.

Declaración de Uso de Herramientas de Inteligencia Artificial Generativa en Trabajos Fin de Grado (Statement on the Use of Generative Artificial Intelligence Tools in Undergraduate Final Projects)

ADVERTENCIA: Desde la Universidad consideramos que ChatGPT u otras herramientas similares son herramientas muy útiles en la vida académica, aunque su uso queda siempre bajo la responsabilidad del alumno, puesto que las respuestas que proporciona pueden no ser veraces. En este sentido, NO está permitido su uso en la elaboración del Trabajo fin de Grado para generar código porque estas herramientas no son fiables en esa tarea. Aunque el código funcione, no hay garantías de que metodológicamente sea correcto, y es altamente probable que no lo sea.

Por la presente, yo, Victoria Inda González, estudiante de E3 Analytics de la Universidad Pontificia Comillas al presentar mi Trabajo Fin de Grado titulado "Breaking the loop: Handling circular references in LBOs", declaro que he utilizado la herramienta de Inteligencia Artificial Generativa ChatGPT u otras similares de IAG de código sólo en el contexto de las actividades descritas a continuación:

1. **Brainstorming de ideas de investigación:** Utilizado para idear y esbozar posibles áreas de investigación.
2. **Crítico:** Para encontrar contra-argumentos a una tesis específica que pretendo defender.
3. **Referencias:** Usado conjuntamente con otras herramientas, como Science, para identificar referencias preliminares que luego he contrastado y validado.
4. **Metodólogo**: Para descubrir métodos aplicables a problemas específicos de investigación.
5. **Revisor:** Para recibir sugerencias sobre cómo mejorar y perfeccionar el trabajo con diferentes niveles de exigencia.
6. **Traductor:** Para traducir textos de un lenguaje a otro

Afirmo que toda la información y contenido presentados en este trabajo son producto de mi investigación y esfuerzo individual, excepto donde se ha indicado lo contrario y se han dado los créditos correspondientes (he incluido las referencias adecuadas en el TFG y he explicitado para que se ha usado ChatGPT u otras herramientas similares). Soy consciente de las implicaciones académicas y éticas de presentar un trabajo no original y acepto las consecuencias de cualquier violación a esta declaración.

Fecha: 10 de abril de 2025

Firma: VICTORIA INDA GONZÁLEZ

BIBLIOGRAPHY

Andrews, E. L., & Sorkin, A. R. (2007, February 26). *TXU deal marks record buyout*. The New York Times. https://archive.nytimes.com/dealbook.nytimes.com/2007/02/23/kkr-plans-record-buyout-with-txu-acquisition/

Asimiyu, Z. (2024). *The convergence of AI optimization and financial engineering: Exploring LLM applications in LBO markets*. ResearchGate. https://www.researchgate.net/publication/387179868_The_Convergence_of_AI_Optimization_and_Financial_Engineering_Exploring_LLM_Applications_in_LBO_Markets

Ayres, I., & Talley, E. (2020, July 29). *The rise of cov-lite loans and its implications*. Harvard Law School Forum on Corporate Governance.

Bain & Company. (2025). *Global private equity report 2025*. https://www.bain.com/insights/outlook-is-a-recovery-starting-to-take-shape-global-private-equity-report-2025/

Beattie, A. (2023, July 17). *Corporate kleptocracy at RJR Nabisco*. Investopedia. https://www.investopedia.com/articles/stocks/09/corporate-kleptocracy-rjr-nabisco.asp

Bellón, C. (2020). *Solving circular references in financial models*. Carlos Bellón.

Bloomberg News. (2007, February 26). *KKR, TPG to buy TXU in record $45 billion LBO*. Bloomberg. https://www.bloomberg.com/news/articles/2007-02-26/kkr-led-group-to-buy-txu-for-45-billionbusinessweek-business-news-stock-market-and-financial-advice

Breaking Into Wall Street. (n.d.). *How to fix circular references in financial models*. https://breakingintowallstreet.com/kb/excel/circular-reference-excel/

Cheffins, B. R., & Armour, J. (2008). *The origins of the market for corporate control: The scientific evidence. University of Cambridge.*

Corporate Finance Institute. (n.d.). *Equity carve-out*. Corporate Finance Institute. https://corporatefinanceinstitute.com/resources/valuation/equity-carve-out/

Corporate Finance Institute. (n.d.). *LBO modeling – Definition, guide, steps, use, Excel*. Corporate Finance Institute. https://corporatefinanceinstitute.com/resources/financial-modeling/steps-to-lbo-modeling/

DataCamp. (2019). *Python Excel: A guide with examples*. DataCamp. https://www.datacamp.com/tutorial/python-excel-tutorial

Fetter, T., Snowberg, E., & Henderson, R. (2019). *TXU (A): Powering the largest leveraged buyout in history* (HBS Case No. 320-064, Rev. 2022). Harvard Business School.

Gazoni, E., & Clark, C. (2024). *openpyxl - A Python library to read/write Excel 2010 xlsx/xlsm files*. https://openpyxl.readthedocs.io/en/stable/

Ivashina, V., & Vallee, B. (2019). *Weak standards or pro-cyclical leverage? The role of cov-lite loans in the credit cycle* (Staff Report No. 837). Federal Reserve Bank of New York. https://www.newyorkfed.org/research/staff_reports/sr837.htm

Kaplan, S. N., & Strömberg, P. (2009). *Leveraged buyouts and private equity*. Journal of Economic Perspectives, 23(1), 121–146. https://www.aeaweb.org/articles?id=10.1257/jep.23.1.121

Kenton, W. (2024, June 8). *Leveraged buyout (LBO): Definition, how it works, and examples*. Investopedia. https://www.investopedia.com/terms/l/leveragedbuyout.asp

Liang, H. S. (2021). *Create a custom PDF using Python*. CodeX. https://medium.com/codex/create-a-custom-pdf-using-python-219a7e11c78b

Macabacus. (2023). *Conquering circular references in Excel*. https://macabacus.com/blog/conquering-circular-references-in-excel

McKinsey & Company. (2025). *Global private markets review 2025*. https://www.mckinsey.com/industries/private-capital/our-insights/global-private-markets-report

Monigatti, L. (2022). *How to create a PDF report for your data analysis in Python*. TDS Archive. https://medium.com/data-science/how-to-create-a-pdf-report-for-your-data-analysis-in-python-2bea81133b

NPR. (2007, February 26). *At a glance: The TXU deal*. NPR. https://www.npr.org/2007/02/26/7604407/at-a-glance-the-txu-deal

Oran, O. (2013, December 12). *Hilton Worldwide raises over $2.3 billion in biggest-ever hotel IPO*. Reuters. https://www.reuters.com/article/world/americas/hilton-worldwide-raises-over-23-billion-in-biggest-ever-hotel-ipo-idUSBRE9BA17H/

Powell, S. G., & Schultz, K. H. (2023). Spreadsheet modeling and wrangling with Python. *INFORMS Transactions on Education.*

Pregueiro, P. (2019). A guide to Excel spreadsheets in Python with openpyxl. *Real Python*. https://realpython.com/openpyxl-excel-spreadsheets-python/

Renneboog, L., & Vansteenkiste, C. (2017, February 8). *Leveraged buyouts: An overview of the literature*. Harvard Law School Forum on Corporate Governance. https://corpgov.law.harvard.edu/2017/02/08/leveraged-buyouts-an-overview-of-the-literature/

Rosenbaum, J., & Pearl, J. (2020). *Investment banking: Valuation, leveraged buyouts, and mergers & acquisitions* (2nd ed.). Wiley.

Talla, G. (2022). *Working with PDFs in Python*. Honeybadger Developer Blog. https://www.honeybadger.io/blog/python-pdf

WSJ Staff. (2007). *Private-equity firms set record with TXU buyout*. The Wall Street Journal. https://www.wsj.com/articles/BL-DLB-1746

APPENDIX 1. Code of the program

```
#Now we will be writing the code necessary in order to create the solution program we have proposed.

print("=== Circular Reference Solver Tool ===")
print("Make your own LBO formula sheet!\n")
print("First, you must answer the following questions about your LBO model\n")

#Step 1: Questions (we will be using only 5 questions as a first try, as for now we think this will be enough to generate a concrete formula sheet.

answers = {}

answers["Question 2"] = input("2. Is your interest expense calculated using average debt during the year? (Yes/No): ").strip().title()
answers["Question 3"] = input("3. Do you have a cash compensating balance that is tied to your debt? (Yes/No): ").strip().title()
answers["Question 4"] = input("4. Is your Free Cash Flow (FCF) adjusted to tax on interest? (Yes/No): ").strip().title()
answers["Question 5"] = input("5. Is your excess of cash or debt directly associated to your end of the year Balance Sheet values? (Yes/No): ").strip().title()


#Step 2: We will define the algebraic formulas that we are using to solve these circular references
Question 1"] = input("1. Do you keep a stable leverage ratio of Debt/Capital Employed? (Yes/No): ").strip().title()
answers["Q
formulas = []

if answers["Question 1"] == "Yes" or answers["Question 5"] == "Yes":
    formulas.append({
        "Title": "Recalculation of Capital Employed",
        "Description": (
            "This equation reclaculates the Capital Employed (CE) in order to separate it from its circular effect. "
            "It uses cash and FCF as known inputs and solves CE algebraically based on a constant target leverage."),
        "Formula": "CE_t = (IC_t + Cash_t−1 + FCFfixed_t - DEBTpre_t) / (1 - λ)"})

    formulas.append({
        "Title": "Debt required to meet our target leverage",
        "Description": (
            "According to the target leverage ratio (λ) you've chosen, this equation calculates how much new debt should be added. "
            "With this we can remove dependencies on unknown end of the year variables in order to break the circular reference. "),
        "Formula": "DEBTnew_t = λ / (1 - λ) * (IC_t + Cash_t−1 + FCFfixed_t) - DEBTpre_t / (1 - λ)"})

if answers["Question 2"] == "Yes" and answers["Question 4"] == "Yes":
    formulas.append({
        "Title": "Effect of the new Debt on Cash Flow after Tax",
        "Description": (
            "When your interest is calculated with average debt and the taxes are applied to the interest expense, "
            "this formula will give you the net impact that borrowing on available cash has."),
        "Formula": "ΔFCF_t = [1 - (1 - τc) * rD / 2] * ΔDEBT_t"})
```

```
if answers["Question 3"] == "Yes":
    formulas.append({
        "Title": "Cash Flow effect when the compensating balance in required",
        "Description": (
            "If your LBO  model includes a rule about a portion of debt that must be held as cash, "
            "this formula will show how much net cash you actually get when you borrow money, "
            "after subtracting things as interest and required cash reserves."),
        "Formula": "ΔFCFₜ = [1 - γ - (1 - τc) * rD / 2] * ΔDEBTₜ"})

#Step 3: Now is when we must program the tool to write, whith the previous questions answered, a formula sheet.

formulafile = "Your_LBO_formula_sheet.txt"

with open(formulafile, "w", encoding="utf-8") as file:
    file.write("LBO Formula Sheet, a practical solution to Circular References\n\n")
    file.write("This sheet includes the equations you need for your model in order to solve your Circula References. ")
    file.write("It is based on the inputs on the previous questions.\n")
    file.write("--------\n\n")

    if not formulas:
        file.write("There has been not found any formulas related to solving circular references, based on your responses.\n")
    else:
        for i, f in enumerate(formulas, start=1):
            formula_title = f"Formula {i}: {f['Title']}"
            file.write(f"{formula_title}\n")

            file.write(f"Explanation: {f['Description']}\n")
            file.write(f"Equation: {f['Formula']}\n\n")

print(f"\nDone!! Your generated LBO formula sheet has been saved as '{formulafile}'.")
```

APPENDIX 2. Code in order to pass to PDF

```
from fpdf import FPDF
import unicodedata
#Here what we are doing is exporting the formula sheet that has been generated into a PDF.

def clean_text(text):
    text = text.replace("ₜ", "_t")
    text = text.replace("₋", "-")
    text = text.replace("₁", "1")
    text = text.replace("₂", "2")
    text = text.replace("λ", "λ")

    text = unicodedata.normalize("NFKD", text)
    return ''.join(char for char in text if ord(char) < 65536)

class PDF(FPDF):
    def header(self):
        self.set_font("FreeSerif", "B", 12)
        self.cell(0, 10, "LBO Formula Sheet", ln=True, align="C")
        self.ln(10)
```

```
    def body_text(self, text):
        self.set_font("FreeSerif", "", 12)
        self.multi_cell(0, 8, text)
        self.ln()

pdf = PDF()

pdf.add_font("FreeSerif", "", "FreeSerif.ttf", uni=True)
pdf.add_font("FreeSerif", "B", "FreeSerifBold.ttf", uni=True)
pdf.set_font("FreeSerif", "", 12)
pdf.add_page()

with open("Your_LBO_formula_sheet.txt", "r", encoding="utf-8") as file:
    lines = file.readlines()

for line in lines:
    line = clean_text(line.strip())

    if line.startswith("Formula"):
        if ":" in line:
            section, title = line.split(":", 1)
            section = section.strip()
            title = title.strip()

            pdf.set_font("FreeSerif", "U", 12)
            pdf.cell(pdf.get_string_width(section + ":"), 10, section + ":", ln=0)

            pdf.set_font("FreeSerif", "", 12)
            pdf.cell(0, 10, " " + title, ln=True)
            pdf.ln(2)

        else:
            pdf.set_font("FreeSerif", "U", 12)
            pdf.cell(0, 10, line.strip(), ln=True)
            pdf.set_font("FreeSerif", "", 12)
            pdf.ln(2)

    elif line.startswith("Title") or line.startswith("Explanation"):
        pdf.body_text(line)
    elif line == "":
        continue
    else:
        pdf.body_text(line)

pdf.output("Your_LBO_formula_sheet.pdf")

print("Your LBO formula sheet has been saved as a PDF")
```

APPENDIX 3. Code in order to pass the sheet to Excel format

```
#Now, as our finanacial model is most probably done in an Excel document, we will also save the formula sheet is this format.

from openpyxl import Workbook

wb = Workbook()
ws = wb.active
ws.title = "LBO Formula Sheet"

with open("Your_LBO_formula_sheet.txt", "r", encoding="utf-8") as file:
    lines = [line.strip() for line in file if line.strip()]

ws.append(["LBO Formula Sheet"])
ws.append(["This sheet includes the equations you need for your model in order to solve your Circular References."])
ws.append(["It is based on the inputs on the previous questions."])
ws.append([])

ws.append(["Formula #", "Title", "Explanation", "Equation"])
formula_number = ""
title = ""
explanation = ""
equation = ""

for line in lines:
    if line.startswith("Formula") and ":" in line:
        if formula_number and title:
            ws.append([formula_number, title, explanation, equation])
        formula_number, title = line.split(":", 1)
        formula_number = formula_number.strip()
        title = title.strip()
        explanation = ""
        equation = ""

    elif line.startswith("Explanation:"):
        explanation = line.replace("Explanation:", "").strip()
    elif line.startswith("Equation:"):
        equation = line.replace("Equation:", "").strip()

if formula_number and title:
    ws.append([formula_number, title, explanation, equation])

excel_filename = "Your_LBO_formula_sheet.xlsx"
wb.save(excel_filename)

print(f"Your Excel has been saved as Your_LBO_formula_sheet. ")
```