Energy 328 (2025) 136421

Contents lists available at ScienceDirect

Energy

journal homepage: www.elsevier.com/locate/energy

Single-level flexible robust optimal bidding of renewable-only virtual power plant in energy and secondary reserve markets

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ARTICLE INFO

Keywords:
Renewable-only virtual power plant
Electricity markets
Single-level model
Robust optimization
Uncertainty
Flexible worst-case profit

ABSTRACT

This paper proposes a novel single-level robust mathematical approach to model the Renewable-only Virtual Power Plant (RVPP) bidding problem in the simultaneous day-ahead and secondary reserve markets. In existing single-level models in the literature, different uncertainties are modeled separately, without explicitly considering their interactions, leading to worst-case energy scenarios. In this study, the *flexible* worst-case profit of RVPPs due to uncertainties related to electricity prices, Non-dispatchable Renewable Energy Sources (ND-RES) production, and flexible demand is captured. In order to find the flexible worst-case profit in a single-level model, the relationship between price and energy uncertainties leads to non-linear constraints, which are finely linearized. Simulation results show the superiority of the proposed robust model compared to alternatives found in the literature in terms of computational efficiency, without compromising the quality of results. Moreover, ND-RES and demand uncertainties exert a greater influence on the RVPP trading strategy and profitability compared to uncertainties in electricity prices. Specifically, for the studied RVPP, the sold and purchased energy decrease by 52% and increase by 74.2%, respectively, when only ND-RES and demand uncertainties are considered, as opposed to the case where solely uncertainties related to day-ahead and secondary reserve market prices are taken into account.

1. Introduction

1.1. Motivation

The penetration of ND-RES has experienced a remarkable growth in the last decades. However, the stochastic nature of these sources implies that ND-RES are less reliable when it comes to predictable and controllable power injection over a given period of time [1]. This makes ND-RES participation in the energy and SRM difficult, as failure to meet with the contracted energy and reserve in the market will lead to penalties if not suspension from future market activities. However, by integrating multiple portfolios of ND-RES and other flexible assets as an RVPP, the performance and competitiveness of ND-RES in these markets can be significantly improved [2].

The viability of RVPP depends on its economic performance, related to benefits and costs. Different markets bring different benefits according to the bidding/offering ability of RVPP and its ability to provide what is promised [3]. However, in addition to the internal uncertainties of RVPP units in their production and demand, there are various external uncertainties in the markets, such as the energy and reserve electricity price uncertainties, and also the combination of these uncertainties may affect the final profit of RVPP [4]. Therefore, the development of bidding approaches for RVPP participation in different markets taking into account the characteristics of RVPP units, market rules, and internal and external uncertainties has at most important for RVPP operators and researchers [5].

1.2. Background

Aligned with the European initiatives to expand ND-RES penetration and reduce greenhouse gas emissions [6], this study examines the operational strategies and market participation of an RVPP consisting solely of ND-RES and flexible demand. By aggregating and internally balancing the stochastic fluctuations of ND-RES, the RVPP enables more reliable market participation while optimizing its collective generation output. The proposed RVPP framework presents a viable and competitive approach to enhancing the profitability and integration of ND-RES into electricity markets. With its ability to coordinate multiple units and effectively manage uncertainties, the RVPP can participate in multiple markets, offering a diverse range of products. Other frameworks, such as energy hubs [7–9], also offer a way to coordinate multiple energy sources and enhance system flexibility, though this work specifically focuses on the RVPP approach.

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https://doi.org/10.1016/j.energy.2025.136421
Received 5 March 2025; Received in revised form 10 April 2025; Accepted 3 May 2025
Available online 19 May 2025

Nomenclature

This subsection presents the notation and nomenclature used in the remainder of the paper.

General Notation Concepts

- An uncertain parameter with a tilde symbol denotes the median value in the forecast distribution, representing a point where half of the observations are lower ($\tilde{A}$);
- the hat/inverse hat symbol on uncertain parameters signifies the greatest positive/negative permitted deviation from the forecast's median ($\hat{A}$, $\check{A}$);
- parameters with an upper/lower bar represent their upper/lower bounds of parameter A ($\bar{A}$, $\underline{A}$);
- upward/downward arrows indicate up/down direction of regulation in variables and parameters ($a^{\uparrow}$, $A^{\uparrow}/a^{\downarrow}$, $A^{\downarrow}$).

Indexes and Sets

$d \in \mathcal{D}$	Set of demands
$p \in \mathcal{P}$	Set of daily load profiles
$r \in \mathcal{R}$	Set of Non-dispatchable Renewable Energy Sources (ND-RES)
$t \in \mathcal{T}$	Set of time periods
$\Xi^{\mathrm{DA+SR}}$	Set of decision variables of Day-ahead Market (DAM) and Secondary Reserve Market (SRM)

Parameters

C_r^R	Operation and maintenance costs of ND-RES r (€/MWh)
$C_{d,p}$	Cost of load profile p of demand d (€)
E_d	Energy consumption of demand d throughout the planning horizon (MWh)
M	Very big positive value (€)
P_d	Power consumption of demand d (MW)
P_r	Power production of ND-RES r (MW)
$P_{d,p,t}$	Profile p of demand d prediction during period t (MW)
$P_{r,t}$	ND-RES r production prediction during period t (MW)
R_d	Ramp rate of demand d (MW/hour)
$R_{r(d)}^{\mathrm{SR}}$	Secondary Reserve (SR) ramp rate of ND-RES r (demand d) (MW/min)
T^{SR}	Required time for SR action (min)
Δt	Duration of periods (h)
$\Gamma^{\mathrm{DA/SR}}$	DAM/SRM price uncertainty budget (–)
$\Gamma_{r(d)}$	ND-RES r production (demand d) uncertainty budget (–)
κ	User-defined parameter to set the limit of up reserve traded in the SRM as a percentage of total power capacity of Renewable-only Virtual Power Plant (RVPP) (%)
ε	Very small positive value (€)
ϱ_t	Coefficient to calculate the ratio of down-to-up reserve requested by the Transmission System Operator (TSO) during period t (%)
$\beta_{d,t}$	Percentage of flexibility of demand d during period t (%)
$\lambda_t^{\mathrm{DA/SR}}$	DAM/SRM price prediction during period t (€/MWh/€/MW)

Continuous Variables

p_t^{DA}	Total traded power by RVPP in the DAM during period t (MW)
$p_{r(d),t}^{\mathrm{DA}}$	Production of ND-RES r (consumption of demand d) in the DAM during period t (MW)
r_t^{SR}	Total SR traded by RVPP for different TSO calls on conditions during period t (MW)
$r_{r(d),t}^{\mathrm{SR}}$	SR provided by ND-RES r (demand d) for different TSO calls on conditions during period t (MW)
$y_t^{(\prime)\mathrm{DA}}$	RVPP profit affected by DAM negative (positive) price uncertainty during period t (€)
y_t^{SR}	RVPP profit affected by SRM price uncertainty during period t (€)
$y_{r(d),t}$	RVPP profit (cost) affected by ND-RES r production (demand d) uncertainty during period t (€)
$\eta_t^{(\prime)\mathrm{DA}}$	Dual variable to model the negative (positive) price uncertainty of DAM during period t (€)
η_t^{SR}	Dual variable to model the price uncertainty of SRM during period t (€)
$\eta_{r(d),t}$	Dual variable to model the ND-RES r production (demand d) uncertainty during period t (€)
$\nu^{\mathrm{DA/SR}}$	Dual variable to model the price uncertainty of DAM/SRM (€)
$\nu_{r(d)}$	Dual variable to model the ND-RES r production (demand d) uncertainty during period t (€)

Binary Variables

$u_{d,p}$	Indicator of selection of profile p of demand d (–)
$\chi_t^{(\prime)DA}$	1/0 variable if DAM negative (positive) price robust constraint is active/inactive during period t (–)
χ_t^{SR}	1/0 variable if SRM price robust constraint is active/inactive during period t (–)
$\chi_{r(d),t}$	1/0 variable if ND-RES r (demand d) robust constraint is active/inactive during period t (–)

Electricity markets are generally divided into three main categories: long-term, short-term, and Real-time Market (RTM) (continuous market). Long-term electricity markets operate on time horizons ranging from weeks to years and primarily focus on large-scale planning and investment. Key participants include major electricity producers, TSOs, regulators, and investors. Transactions in these markets are typically conducted through electricity purchase agreements, capacity contracts, and bilateral deals [10]. In contrast, short-term electricity markets cover trading periods from a few hours to several days, ensuring system operation and supply–demand balance. These markets involve a diverse set of participants, including large and small-scale power producers, retailers, major consumers, demand response providers, and balance-responsible entities. Short-term trading mechanisms mainly rely on competitive auctions or pre-delivery market pools for electricity, capacity, and ancillary services [11]. Lastly, RTM optimization allows power units to dynamically adjust their operations in response to sub-hourly fluctuations in demand and renewable energy generation [12].

Given the relatively small scale of the RVPP analyzed in this study compared to the overall power system, the primary focus is thus on short-term electricity market bidding within a 24-hour horizon, rather than long-term market participation. As the main objective of this study is to evaluate the viability of RVPP, the analysis concentrates on two key short-term markets: the DAM and the SRM, while RTM participation is not considered. Moreover, although environmental aspects are not explicitly included in the objective function, the proposed model enhances the economic viability of RVPP, which predominantly relies on ND-RES. Consequently, by increasing the feasibility and competitiveness of RVPP, this approach indirectly contributes to economic efficiency and supports carbon neutrality goals in the power grid.

1.3. Literature review

Many papers in the literature use mathematical optimization models to capture different uncertainties associated with Virtual Power Plant (VPP) due to ease of implementation, convergence to the global optimum, and computational efficiency of these models [13]. In this context, Robust Optimization (RO) programming is an efficient way to deal with different sets of uncertainties that vary in their possible values. The goal of RO is to find the worst case of the optimization problem to minimize the negative impact of uncertainties on the solution [14]. However, the definition of the worst case can vary depending on how the optimization is implemented, whether it is single-level or multi-level, and can lead to different solutions in each approach. The authors in [9,12,15–25] develop a single-level optimization problem for the VPP market bidding problem to find the worst case of energy of ND-RES. The paper in [9] investigates the optimal scheduling of a renewable-oriented energy hub that integrates electrical, thermal, and cooling systems. A novel entropic Conditional Value at Risk (CVaR) approach is employed to account for various uncertainties, including electrical, thermal, and cooling demand fluctuations, as well as market price and ND-RES production uncertainties. The study in [12] introduces a continuous-time method for energy hub scheduling in the DAM and RTM energy and reserve dispatch. An Information Gap Decision Theory (IGDT) algorithm is utilized to model wind power uncertainties for both discrete-time and continuous-time approaches, with a comparative analysis provided. The study [15] employs a Mixed Integer Linear Programming (MILP) RO method to model VPP participation in energy and reserve markets. Uncertainties in demand and ND-RES are represented via bounded intervals, with robustness budgets adjusting the conservatism of the optimization model. The paper [16] formulates a multi-objective optimization model for VPPs, aiming to maximize operational revenue while minimizing carbon emissions and risk. The risk-averse strategy based on CVaR leverages RO to address uncertainties in demand, wind generation, and solar Photovoltaic (PV) output. In [17], a single-level model is used to formulate the DAM participation of a VPP, which includes a wind farm, demand, and Electrical Energy Storage (ESS). The model accounts for symmetric uncertainties in electricity prices and wind production using confidence bounds. The authors of [18] investigate a multi-energy VPP for energy and reserve scheduling, incorporating the capacity market, DAM, Ancillary Service Market (ASM), and Natural Gas Market (NGM). Uncertainties in PV unit production are addressed through an RO model, while market price uncertainties are represented using Probability Density Function (PDF). In [19], a two-stage stochastic RO problem is introduced to handle multiple uncertainties, including DAM electricity prices, ND-RES production, and demand within a virtual energy hub comprising industrial energy hubs and customers. The study in [20] proposes a single-level model for a VPP that includes ND-RES, ESS, and demand. A light RO approach is used to reduce the conservatism of RO and account for the uncertainties of ND-RES and load. The paper [21] presents a single-level RO model for RVPP participation in sequential energy and reserve markets, addressing the asymmetry of energy and reserve prices and defining the robustness budget over global scheduling horizons rather than individual time periods. The study in [22] introduces an RO model for a VPP incorporating power-to-hydrogen facilities to participate in both the DAM and RTM. Uncertainties in market electricity prices and wind power are addressed using an MILP framework. In [23], the optimal bidding strategy for electricity market participation of an RVPP, which includes ND-RES and dispatchable loads, is examined using a Mixed Integer Non-linear Programming (MINLP) approach based on IGDT. The work in [24] proposes a multi-objective optimization approach for a virtual energy hub that includes a data center and electric vehicles. A Stochastic Optimization (SO)-RO approach is utilized to model uncertainties in ND-RES, electricity demand, and electricity prices. In [25], a regret-based RO approach is proposed to model various uncertain parameters in the RVPP problem using economic factors rather than the inherent nature of the uncertainties.

The main advantages of the mentioned single-level RO programming in [9,12,15–25] are the possibility to consider multiple uncertainties, simplicity of implementation, global optimality, and calculation efficiency. However, a simplified definition of the worst case of energy for the severe scenarios is implemented. In fact, the worst case of energy defined for ND-RES in the above papers does not lead to the worst condition of profit, considering the possibility of different values of electricity prices. For instance, in a case where the electricity price is low in a certain period, even though the energy of a ND-RES can deviate significantly in this period, the resulting loss for RVPP might not be significant compared to a period with much higher electricity price and average or low energy deviation.

Multi-level RO models provide more flexibility to find the actual worst-case of the VPP bidding problem compared to single-level models. This is due to the definition of a new level for the optimization problem that models the behavior of uncertain parameters (both electricity price and energy uncertainties). Therefore, the objective function of this level can be defined to find the worst case of energy or profit of VPP. In addition, another level for the problem can be included to define the corrective or remedial actions after the occurrence of uncertainties. The literature on multi-level models proposes mathematical techniques, including RO [26–28], Adaptive RO (ARO) [29], SO-RO [30], Stochastic ARO (SARO) [4,31], Distributionally RO (DRO) [32–34], data-driven methods [35,36], and Stackelberg game [37] to account for various uncertainties. In [26], an RO approach is introduced to aggregate distribution systems as a VPP in the DAM, considering uncertainties in ND-RES and the dispatch order of the TSO. An enhanced Column & Constraint Generation (C&CG) algorithm, incorporating period decomposition and a connection method, is proposed to reduce the computational time of the RO problem. The work in [27] presents a bidding strategy for a VPP that integrates responsive-based electric vehicles to balance economic and environmental goals. A two-stage RO approach is employed to account for uncertainties in wind power, solar PV power, and load. In [28], the optimal scheduling of a VPP is investigated by considering the controllability of electric vehicles. A two-stage max–min–max RO approach is introduced to account for uncertainties in ND-RES and electric vehicles, with the C&CG algorithm used to solve the optimization problem. The study in [29] focuses on the participation of an aggregator, incorporating flexible resources and demands, across various energy and flexibility markets. In [30], a three-level model is used to capture source-side and load-side uncertainties of a multi-energy VPP through adjustable RO and clustering algorithms, respectively. The participation of a VPP in the DAM for both energy and reserve trading is modeled using a SARO problem in [4]. A modified Benders decomposition approach is developed to solve the proposed multi-level SARO model. Similarly, a SARO problem is presented in [31]. The first level of the optimization problem maximizes the worst-case expected profit of the VPP, while the second level minimizes the expected profit to model the worst-case uncertainties in wind generation and reserve deployment requests. The third level implements operational decisions following the first-level market participation and second-level uncertainty realizations to maximize the VPP's expected profit. In [32], a two-stage DRO framework is proposed for DAM and RTM scheduling of a VPP that includes ESS and electric vehicles. A tri-level optimization approach is employed to model uncertainties in ND-RES, and an enhanced nested C&CG algorithm is developed to solve the optimization problem. The paper [33] presents a two-stage RO approach based on data-driven methods for a rural VPP, incorporating carbon-green certificates. Uncertainties in wind and solar PV production are managed using the strong duality theorem and the C&CG algorithm. The study in [34] introduces a bi-level model for a price-maker VPP, where the first level focuses on minimizing the VPP cost, and the second level aims to minimize the social cost of market. Uncertainties in wind unit production are addressed using a DRO approach. The paper in [35] proposes a data-driven interval approach for a VPP participating in the DAM and RTM. A two-stage interval RO model, leveraging an improved C&CG algorithm, is adopted to handle uncertainties in ND-RES production, demand consumption, and electricity prices. The paper in [36] formulates the bidding problem of a VPP in the energy market using a two-stage stochastic MILP approach, with uncertainties in ND-RES and demand response addressed through a data-driven RO (sample RO) method. In [37], a multi-period Stackelberg game approach is introduced for RTM participation of VPP. The proposed method leverages the heat storage capabilities of power to heat demands to mitigate wind power fluctuations by dynamically adjusting price ceilings based on supply–demand balance. The main limitations of the multi-level approaches in general, and in the above works in particular, are the complexity of programming and the fact that the size of the problem grows with the number of iterations in the solving procedure. In addition, they usually imply long computational times, which can compromise applications such as sensitivity analysis. In contrast, our proposed model utilizes a single-level MILP approach, which simplifies the optimization process by eliminating the need for iterative master–slave problem solving. This reduction in complexity allows for faster computation and more straightforward implementation, making it easier to scale and apply to various real-world scenarios. The single-level structure also ensures that the problem size does not increase significantly, leading to more efficient solving procedures and shorter solution times compared to multi-level models.

1.4. Approach and contributions

Table 1 compares different aspects of the reviewed literature with this paper. To overcome the challenges and difficulties of implementing a multi-level optimization model, and to minimize the computational burden of the problem for real-world, practical applications, this paper proposes a flexible worst-case profit of RVPP against uncertainties by means of a novel single-level MILP problem. The uncertainties that characterize the problem include energy and ancillary service market prices, ND-RES generation, and flexible demand contribution. To account for the impact of such uncertainties in the problem (and in particular, their *couplings*), the methodology proposed builds on the works in [17,21,38], and by developing on the idea from the Big-M method [39]. The proposed implementation of robust constraints allows capturing the relationship between different uncertain parameters in the objective function and constraints of the optimization problem, finding the *exact* worst case profit of RVPP. In this process, defining the relationships and couplings between uncertain parameters leads necessarily to non-linear constraints, which are thoroughly linearized by using well-established methods.

The contributions of this paper are thus twofold:

- *Flexible Worst-Case Profit Robustness Modeling in a Single-Level MINLP Framework:* While single-level classical RO models exist in the literature [17–19,21–24], the proposed model in this paper introduces a new approach to handle multiple uncertainties and their interactions. In most existing single-level models, uncertainties related to electricity prices and energy (including ND-RES production and demand) are modeled separately, without explicitly considering their interactions. This separate treatment often results in the selection of worst-case energy scenarios that do not actually correspond to the worst-case profit scenarios for the RVPP. As a result, using these classical single-level methods prevents the RVPP operator from providing optimal and reliable bidding strategies across different markets. To address this shortcoming, the proposed model adopts a flexible RO approach based on the worst-case selection of the RVPP's profit, rather than focusing only on energy robustness. This profit-oriented robustness framework allows the RVPP to participate simultaneously in the DAM and SRM, while effectively capturing the impact of uncertainties in prices, energy production, and demand consumption, as well as their interactions.
- *Handling the Non-linear Couplings Between Different Uncertainties:* The proposed approach in this paper addresses how different types of uncertainties in the objective function and constraints interact with each other within the optimization framework. Modeling these interactions leads to non-linear constraints, particularly when capturing the joint impact of price and energy uncertainties on the RVPP's profit in a single-level model. In the existing literature, these types of interactions are typically handled by multi-level models [4,31,35], which are often complex, computationally demanding, and difficult to implement. In contrast, the proposed model captures these interactions between uncertain parameters through an exact linearization of the initial MINLP problem, ensuring a mathematically equivalent MILP formulation. By addressing the interactions between uncertainties within a single-level MILP model, the proposed approach represents a novel contribution that provide an effective and practical solution for RVPP bidding strategies across different markets in the presence of several uncertainties.

1.5. Paper organization

The reminder of the paper is organized as follows. A conceptual comparison of energy and profit robustness approaches is presented in Section 2. The proposed single-level robust bidding problem of RVPP for DAM and SRM participation is formulated in Section 3. An illustrative example is given in Section 4 to show the performance of the proposed robust model in finding the flexible worst-case profit. The simulation results are presented in Section 5. Finally, the conclusions are drawn in Section 6.

2. Comparing energy and profit robustness

Fig. 1 shows the structure of a deterministic RVPP bidding problem and a comparison between the energy and the profit robustness approaches. The top pane of this figure shows the deterministic RVPP problem. The left lower pane of this figure represents the energy robustness problem presented in [21]. The right lower pane of this figure shows the profit robustness problem presented in this paper. In the deterministic approach, a single value (usually the median or average) of the forecast data is considered to solve the optimization problem. The constraints are related mainly to the operation of the RVPP units, and supply–demand balance [3]. When considering the uncertainties, depending on whether the uncertainties affect the objective function or the constraints of the optimization problem, different sets of constraints need to be defined in each of the RO approaches. The uncertainties related to the energy/reserve electricity price affect the objective function

Table 1
Comparison of proposed approach in this paper and literature.

Ref.	Market		Uncertainty			Uncertainty couplings	Profit robustness	Computational efficiency[a]	Method & solution
	DAM	SRM	Price	ND-RES	Load				
[15]	✓	✓	×	✓	✓	×	×	High	Single-level, RO, MILP
[16]	✓	✓	×	✓	✓	×	×	High	Single-level, CVaR-RO, MILP
[17]	✓	×	✓	✓	×	×	×	High	Single-level, RO, MILP
[18]	✓	✓	✓	✓	×	×	×	High	Single-level, RO, PDF, MILP
[19]	✓	✓	✓	✓	✓	×	×	High	Single-level, SO-RO, MILP
[20]	✓	×	×	✓	✓	×	×	Medium	Single-level, Light RO, Non-linear
[21]	✓	✓	✓	✓	✓	×	×	High	Single-level, RO, MILP
[22]	✓	×	✓	✓	×	×	×	High	Single-level, RO, MILP
[23]	✓	×	✓	✓	×	×	×	High	Single-level, IGDT, PDF, MINLP
[24]	✓	×	✓	✓	✓	×	×	Medium–High	Single-level, SO-RO, MILP
[26]	✓	×	×	✓	×	×	✓	High	Multi-level, RO, C&CG, MILP
[27]	✓	×	×	✓	✓	×	✓	Low–Medium	Multi-level, RO, C&CG, MILP
[28]	✓	×	×	✓	×	×	✓	Low–Medium	Multi-level, RO, C&CG, MILP
[29]	✓	✓	×	✓	✓	×	✓	Low–Medium	Multi-level, ARO, MILP
[30]	✓	×	×	✓	✓	×	✓	Low–Medium	Multi-level, SO-RO, C&CG, MILP
[4]	✓	✓	✓	✓	×	✓	✓	Low–Medium	Multi-level, SARO, MILP
[31]	✓	✓	✓	✓	×	✓	✓	Low–Medium	Multi-level, SARO, C&CG, MILP
[32]	✓	×	×	✓	×	×	✓	High	Multi-level, DRO, C&CG, MILP
[33]	✓	×	×	✓	×	×	✓	Medium–High	Multi-level, DRO, C&CG, MILP
[34]	✓	×	×	✓	×	×	×	High	Bi-level, DRO, MILP
[35]	✓	×	✓	✓	✓	×	✓	Medium	Multi-level, Data-driven, RO, C&CG, MINLP
[36]	✓	×	×	✓	✓	×	×	Medium	Bi-level, Data-driven, SO-RO, MILP
This paper	✓	✓	✓	✓	✓	✓	✓	High	Single-level, RO, MILP

[a] High: under 60 s; Medium–High: between 1 and 10 min; Medium: between 10 and 30 min; Low–Medium: between 30 and 90 min; Low: over 90 min.

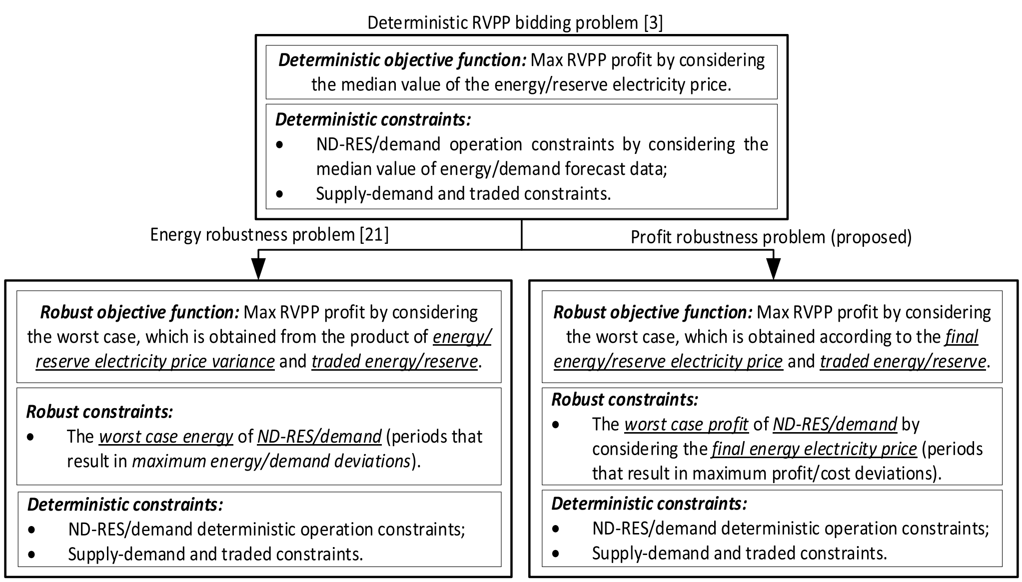

Fig. 1. A comparison between the energy and profit robustness approaches.

of the optimization problem, whereas the uncertainties associated with the ND-RES generation and demand consumption affect the constraints.

In the energy robustness approach, those periods that result in more deviation of the energy/reserve electricity price variance multiplied by the total traded energy/reserve of RVPP are selected as the worst-case scenarios of the electricity price [21]. In the energy robustness constraints, the periods that have higher deviation of energy are selected as the worst case of ND-RES production or demand regardless of the electricity price.

In the profit robustness approach proposed in this paper and for the uncertain parameters in the objective function of the optimization problem (energy/reserve electricity price), the worst case is defined according to the final value of the energy/reserve electricity price by means of binary variables. The final value of the energy electricity price is also used to calculate the worst case of profit/cost of each unit (uncertainty of ND-RES and demand in the constraints of the optimization problem). For this purpose, the final energy electricity price is multiplied by the energy variable of ND-RES/demand and is limited by the profit reduction effect due to ND-RES/demand uncertainty.

In the following section, the proposed profit robustness approach is formulated as a single-level optimization problem. In Section 4, these two approaches are compared using an illustrative example.

3. Profit robustness formulation

3.1. Price robustness (objective function)

The objective function of simultaneous RVPP participation in the DAM and SRM, as well as the associated robust constraints, are presented in this section.

The objective function (1) maximizes the benefits of RVPP in the DAM and SRM. The first and second lines of (1) calculate the expected RVPP incomes from bidding in the DAM and from up and down SR provision, respectively, considering the corresponding robustness cost of asymmetric electricity price uncertainties. The third line in (1) defines the operation costs of ND-RES, and the costs of selecting a particular load profile. Note that including variables y_t^{DA}, $y_t^{\prime DA}$, $y_t^{\mathrm{SR},\uparrow}$, and $y_t^{\mathrm{SR},\downarrow}$ in the objective function is one of the main differences between the proposed model and the common approach to model the price robustness in the literature [17,21]. By means of these variables, the final value of the DAM/SRM electricity price and the traded energy/reserve of RVPP are used to calculate the worst-case scenarios.

$$\max_{\Xi^{\mathrm{DA+SR}}} \sum_{t\in\mathcal{T}} \left[\tilde{\lambda}_t^{\mathrm{DA}} p_t^{\mathrm{DA}} \Delta t - y_t^{\mathrm{DA}} - y_t^{\prime \mathrm{DA}}\right] + \sum_{t\in\mathcal{T}} \left[\tilde{\lambda}_t^{\mathrm{SR},\uparrow} r_t^{\mathrm{SR},\uparrow} + \tilde{\lambda}_t^{\mathrm{SR},\downarrow} r_t^{\mathrm{SR},\downarrow} - y_t^{\mathrm{SR},\uparrow} - y_t^{\mathrm{SR},\downarrow}\right] - \sum_{t\in\mathcal{T}}\sum_{r\in\mathcal{R}} C_r^R p_{r,t}^{\mathrm{DA}} \Delta t - \sum_{d\in\mathcal{D}}\sum_{p\in\mathcal{P}} C_{d,p} u_{d,p} \tag{1}$$

3.1.1. DAM electricity uncertain constraints

The set of constraints (2) is related to the uncertainties of the DAM electricity price and are written by developing the approach in [17,21,38] and elaborating on the Big-M method [39].

Constraint (2a) determines the DAM electricity price in each time period according to the condition of binary variables χ_t^{DA} and $\chi_t^{\prime\mathrm{DA}}$, which are related to the negative and positive price volatility, respectively. Constraints (2b) and (2c) model the impact of the absolute value of negative and positive price volatility on profit reduction when the electricity price deviates from its median to its worst condition. Constraints (2d) and (2e) set a lower bound for the profit reduction variables y_t^{DA} and $y_t^{\prime\mathrm{DA}}$ due to the negative and positive price uncertainty, respectively. When the binary variable χ_t^{DA} $\left(\chi_t^{\prime\mathrm{DA}}\right)$ is 1, constraint (2d) (constraint (2e)) is active. Depending on whether RVPP sells or buys electricity on the market, the worst DAM price conditions occur at the price values $\tilde{\lambda}_t^{\mathrm{DA}} - \check{\lambda}_t^{\mathrm{DA}}$ and $\tilde{\lambda}_t^{DA} + \hat{\lambda}_t^{\mathrm{DA}}$, respectively.

The dual variables η_t^{DA} and $\eta_t^{\prime\mathrm{DA}}$, related to the negative and positive deviations of the electricity price, are logically constrained by (2f) and (2g), respectively, based on the active or non-active status of the periods to comply with the robustness budget defined in (2j). Constraints (2h) and (2i) define the lower and upper bounds for the differences between the possible profit reductions (due to the negative price deviation $\check{\lambda}_t^{DA} p_t^{DA} \Delta t$ and the positive price deviation $-\hat{\lambda}_t^{DA} p_t^{DA} \Delta t$) and the dual variable ν^{DA}. According to these constraints, the possible profit reductions must be greater than or equal to the dual variable ν^{DA} for those periods that the electricity price fluctuates to its worst case. Constraints (2h) and (2i) are thus essential to avoid selecting incorrect periods for the worst case of profit deviations, especially when other uncertain parameters such as ND-RESs production and demand (see Sections 3.3 and 3.4) affect the total power traded by RVPP (p_t^{DA}). The robustness budget Γ^{DA} in (2j) is a user-defined parameter that determines the number of periods in which the electricity price can deviate to its worst condition. Thus, Γ^{DA} is a particularly relevant parameter in this model for achieving flexible robustness: the higher Γ^{DA}, the worse the bidding scenario becomes. Constraint (2k) prevents positive and negative electricity price deviations in the same period. Constraints (2l) and (2m) define the nature of positive dual variables and binary variables, respectively. The use of two binary variables to define the negative and positive price deviations and the implementation of constraints (2h) and (2i) to avoid illogical condition for the price deviations is another significant improvement compared to previous robust formulations in the literature [17,21].

$$\lambda_t^{DA} = \tilde{\lambda}_t^{DA} - \check{\lambda}_t^{DA} \chi_t^{DA} + \hat{\lambda}_t^{DA} \chi_t^{\prime DA} \ , \quad \forall t \tag{2a}$$

$$\nu^{DA} + \eta_t^{DA} \geq \check{\lambda}_t^{DA} p_t^{DA} \Delta t \ , \quad \forall t \tag{2b}$$

$$\nu^{DA} + \eta_t^{\prime DA} \geq -\hat{\lambda}_t^{DA} p_t^{DA} \Delta t \ , \quad \forall t \tag{2c}$$

$$y_t^{DA} \geq \nu^{DA} + \eta_t^{DA} - M(1-\chi_t^{DA}) \ , \quad \forall t \tag{2d}$$

$$y_t^{\prime DA} \geq \nu^{DA} + \eta_t^{\prime DA} - M(1-\chi_t^{\prime DA}) \ , \quad \forall t \tag{2e}$$

$$\varepsilon(\chi_t^{DA}) \leq \eta_t^{DA} \leq M(\chi_t^{DA}) \ , \quad \forall t \tag{2f}$$

$$\varepsilon(\chi_t^{\prime DA}) \leq \eta_t^{\prime DA} \leq M(\chi_t^{\prime DA}) \ , \quad \forall t \tag{2g}$$

$$-M(1-\chi_t^{DA}) \leq \check{\lambda}_t^{DA} p_t^{DA} \Delta t - \nu^{DA} \leq M(\chi_t^{DA}) \ , \quad \forall t \tag{2h}$$

$$-M(1-\chi_t^{\prime DA}) \leq -\hat{\lambda}_t^{DA} p_t^{DA} \Delta t - \nu^{DA} \leq M(\chi_t^{\prime DA}) \ , \quad \forall t \tag{2i}$$

$$\sum_{t\in\mathcal{T}} (\chi_t^{DA} + \chi_t^{\prime DA}) = \Gamma^{DA} \ , \tag{2j}$$

$$\chi_t^{DA} + \chi_t^{\prime DA} \leq 1 \ , \quad \forall t \tag{2k}$$

$$\nu^{DA}, \eta_t^{DA}, \eta_t^{\prime DA}, y_t^{DA}, y_t^{\prime DA} \geq 0 \ , \quad \forall t \tag{2l}$$

$$\chi_t^{DA}, \chi_t^{\prime DA} \in \{0,1\} \ , \quad \forall t \tag{2m}$$

3.1.2. SRM price uncertain constraints

The set of constraints (3) related to the uncertainty in the up and down SRM price is defined similarly to (2). The main difference is that for both up and down SRM price, only the negative SRM price deviations due to uncertainty are evaluated in (3a) and (3b), respectively. This is due to the fact that the positive SRM price deviations usually result in more benefit for RVPP. Therefore, the maximum possible profit deviations $\check{\lambda}_t^{SR,\uparrow} r_t^{SR,\uparrow}$ and $\check{\lambda}_t^{SR,\downarrow} r_t^{SR,\downarrow}$ are calculated based on the negative upward and downward SRM price deviations $\check{\lambda}_t^{SR,\uparrow}$ and $\check{\lambda}_t^{SR,\downarrow}$ in constraints (3c) and (3d), respectively. Note that, in some cases, a positive deviation in the SRM price may have a negative impact on the objective function. For instance, in a deceptive solution where the SRM price increases, the RVPP may choose to provide less energy to allocate more capacity for reserve provision, which might not be as profitable as energy trading. Alternatively, if the RVPP does not have sufficient energy in real-time to meet its reserve obligations, bidding for additional reserve (driven by a higher SRM price) could lead to penalties for the RVPP. Constraints (3e) and (3f) establish a lower bound for the profit reduction due to the uncertainty in the up and down SRM prices, respectively. Constraints (3g) and (3h) define the bounds for the dual variables $\eta_t^{SR,\uparrow}$ and $\eta_t^{SR,\downarrow}$, which correspond to the up and down SRM electricity prices, respectively. Constraints (3i) and (3j) set the bounds for the differences between the possible profit reductions due to up and down SRM price deviations and the dual variables $\nu^{SR,\uparrow}$ and $\nu^{SR,\downarrow}$, respectively. The robustness budgets $\Gamma^{SR,\uparrow}$ and $\Gamma^{SR,\downarrow}$ for up and down SRM price uncertainty are assigned in (3k) and (3l), respectively. Finally, the nature of the positive dual variables and binary variables is determined by (3m) and (3n), respectively.

$$\lambda_t^{SR,\uparrow} = \tilde{\lambda}_t^{SR,\uparrow} - \check{\lambda}_t^{SR,\uparrow} \chi_t^{SR,\uparrow} \ , \quad \forall t \tag{3a}$$

$$\lambda_t^{SR,\downarrow} = \tilde{\lambda}_t^{SR,\downarrow} - \check{\lambda}_t^{SR,\downarrow} \chi_t^{SR,\downarrow} \ , \quad \forall t \tag{3b}$$

$$\nu^{SR,\uparrow} + \eta_t^{SR,\uparrow} \geq \check{\lambda}_t^{SR,\uparrow} r_t^{SR,\uparrow} \ , \quad \forall t \tag{3c}$$

$$\nu^{SR,\downarrow} + \eta_t^{SR,\downarrow} \geq \check{\lambda}_t^{SR,\downarrow} r_t^{SR,\downarrow} \ , \quad \forall t \tag{3d}$$

$$y_t^{SR,\uparrow} \geq \nu^{SR,\uparrow} + \eta_t^{SR,\uparrow} - M(1-\chi_t^{SR,\uparrow}) \ , \quad \forall t \tag{3e}$$

$$y_t^{SR,\downarrow} \geq \nu^{SR,\downarrow} + \eta_t^{SR,\downarrow} - M(1-\chi_t^{SR,\downarrow}) \ , \quad \forall t \tag{3f}$$

$$\varepsilon(\chi_t^{SR,\uparrow}) \leq \eta_t^{SR,\uparrow} \leq M(\chi_t^{SR,\uparrow}) \ , \quad \forall t \tag{3g}$$

$$\varepsilon(\chi_t^{SR,\downarrow}) \leq \eta_t^{SR,\downarrow} \leq M(\chi_t^{SR,\downarrow}) \ , \quad \forall t \tag{3h}$$

$$-M(1-\chi_t^{SR,\uparrow}) \leq \check{\lambda}_t^{SR,\uparrow} r_t^{SR,\uparrow} - \nu^{SR,\uparrow} \leq M(\chi_t^{SR,\uparrow}) \ , \quad \forall t \tag{3i}$$

$$-M(1-\chi_t^{SR,\downarrow}) \leq \check{\lambda}_t^{SR,\downarrow} r_t^{SR,\downarrow} - \nu^{SR,\downarrow} \leq M(\chi_t^{SR,\downarrow}) \ , \quad \forall t \tag{3j}$$

$$\sum_{t\in\mathcal{T}} \chi_t^{SR,\uparrow} = \Gamma^{SR,\uparrow} \ , \tag{3k}$$

$$\sum_{t\in\mathcal{T}} \chi_t^{SR,\downarrow} = \Gamma^{SR,\downarrow} \ , \tag{3l}$$

$$\nu^{SR,\uparrow}, \nu^{SR,\downarrow}, \eta_t^{SR,\uparrow}, \eta_t^{SR,\downarrow}, y_t^{SR,\uparrow}, y_t^{SR,\downarrow} \geq 0 \ , \quad \forall t \tag{3m}$$

$$\chi_t^{SR,\uparrow}, \chi_t^{SR,\downarrow} \in \{0,1\} \ , \quad \forall t \tag{3n}$$

3.2. Supply–demand and traded constraints

The supply–demand balancing constraint of RVPP units is defined in (4a). All RVPP units are assumed to be connected to a single node.

The variable r_t^{SR} related to the total traded reserve of RVPP and the variables $r_{r,t}^{SR}$ and $r_{d,t}^{SR}$ related to the reserve of RVPP units are defined according to different reserve activation scenarios similar to [21]. Constraints (4b) and (4c) assign the upper and lower bounds of traded energy and reserve by RVPP, respectively. Constraint (4d) defines the proportion of down and up reserve requested by TSO. The up reserve provided is limited by (4e) to a fraction of the total capacity of the generating units of RVPP.

$$\sum_{r\in\mathcal{R}}(p_{r,t}^{DA}+r_{r,t}^{SR})-\sum_{d\in\mathcal{D}}(p_{d,t}^{DA}-r_{d,t}^{SR})=p_t^{DA}+r_t^{SR}\ , \qquad \forall t \qquad (4a)$$

$$p_t^{DA}+r_t^{SR,\uparrow}\le\sum_{r\in\mathcal{R}}\bar{P}_r\ , \qquad \forall t \qquad (4b)$$

$$-\sum_{d\in\mathcal{D}}\bar{P}_d\le p_t^{DA}-r_t^{SR,\downarrow}\ , \qquad \forall t \qquad (4c)$$

$$r_t^{SR,\uparrow}=\varrho_t r_t^{SR,\downarrow}\ , \qquad \forall t \qquad (4d)$$

$$r_t^{SR,\uparrow}\le\kappa\sum_{r\in\mathcal{R}}\bar{P}_r\ , \qquad \forall t \qquad (4e)$$

3.3. ND-RES profit robustness

The profit robustness formulation of ND-RES is given in (5). Constraint (5a) is the lower bound on the ND-RES output power. Constraint (5b) sets the ND-RES output through the median forecast generation of ND-RES $\tilde{P}_{r,t}$ and the possible negative power deviation $\chi_{r,t}\check{P}_{r,t}$ (active when $\chi_{r,t}=1$). The binary variable $\chi_{r,t}$ is determined according to the profit robustness constraints (5c)–(5j) proposed in this work. Constraint (5c) limits the profit of ND-RES for each time period by considering the robustness of the problem against uncertain parameters of ND-RES production. The upper bound of this constraint is computed as the median profit minus the profit reduction due to the negative deviation of power forecast of ND-RES, $y_{r,t}$. The median profit is calculated by multiplying the electricity price λ_t^{DA} and the median production of ND-RES minus the provided up reserve, both multiplied by the time period duration $(\tilde{P}_{r,t}-r_{r,t}^{SR,\uparrow})\Delta t$. Constraint (5c) is a non-linear expression that is linearized in Section 3.5. Constraint (5d) assigns the upper bound of the dual variable $y_{r,t}$ to the negative profit deviation $\lambda_t^{DA}\check{P}_{r,t}\Delta t$ of each ND-RES due to uncertainty. To model the flexible worst-case scenarios of profit reduction for each ND-RES, only negative power deviations are considered in this constraint, since positive deviations will usually benefit the RVPP. Constraint (5e) determines the lower bound of the dual variable $y_{r,t}$ according to the dual variables v_r and $\eta_{r,t}$, and the condition of the binary variable $\chi_{r,t}^{DA}$. Constraint (5f) assigns the lower bound of the sum of the dual variables v_r and $\eta_{r,t}$ to the maximum profit reduction for each ND-RES in each time period. According to constraint (5g), the dual variable $\eta_{r,t}$ is defined based on the active or non-active status of the profit reduction due to the robustness of the production of ND-RES. Constraint (5h) defines the profit robustness budget for each ND-RES. Constraints (5i) and (5j) define the nature of the positive dual variables and the binary variables, respectively.

$$\underline{P}_r\le p_{r,t}^{DA}-r_{r,t}^{SR,\downarrow}\ , \qquad \forall r,t \qquad (5a)$$

$$p_{r,t}^{DA}+r_{r,t}^{SR,\uparrow}=\tilde{P}_{r,t}-\chi_{r,t}\check{P}_{r,t}\ , \qquad \forall r,t \qquad (5b)$$

$$\lambda_t^{DA}p_{r,t}^{DA}\Delta t\le\lambda_t^{DA}(\tilde{P}_{r,t}-r_{r,t}^{SR,\uparrow})\Delta t-y_{r,t}\ , \qquad \forall r,t \qquad (5c)$$

$$y_{r,t}\le\lambda_t^{DA}\check{P}_{r,t}\Delta t\ , \qquad \forall r,t \qquad (5d)$$

$$y_{r,t}\ge v_r+\eta_{r,t}-M(1-\chi_{r,t})\ , \qquad \forall r,t \qquad (5e)$$

$$v_r+\eta_{r,t}\ge\lambda_t^{DA}\check{P}_{r,t}\Delta t\ , \qquad \forall r,t \qquad (5f)$$

$$\epsilon\chi_{r,t}\le\eta_{r,t}\le M\chi_{r,t}\ , \qquad \forall r,t \qquad (5g)$$

$$\sum_{t\in\mathcal{T}}\chi_{r,t}=\Gamma_r\ , \qquad \forall r \qquad (5h)$$

$$v_r,\eta_{r,t},y_{r,t}\ge 0\ , \qquad \forall r,t \qquad (5i)$$

$$\chi_{r,t}\in\{0,1\}\ , \qquad \forall r,t \qquad (5j)$$

3.4. Demand cost robustness

The demand cost robust formulation is illustrated in (6), which is based on the deterministic model presented in [40]. Constraint (6a) assigns the demand for each period to predefined demand profiles, taking into account the median and positive demand forecasts. Only positive demand deviations are considered for the worst-case cost robustness scenarios, since the negative demand deviations (i.e., lower consumption) usually result in lower costs for RVPP. Constraint (6b) ensures that the algorithm selects only one demand profile among several profiles. When the binary variable $\chi_{d,t}$ in (6a) for a certain period is 1, the possible positive deviation of the demand becomes active. The binary variable $\chi_{d,t}$ is determined according to the cost robustness constraints (6c)–(6h) proposed in this work. Constraint (6c) sets the lower bound on the cost of buying electricity from DAM to meet demand, which equals the cost of buying electricity for the median demand forecast $\lambda_t^{DA}\sum_{p\in\mathcal{P}}(\tilde{P}_{d,p,t}u_{d,p})\Delta t$ plus the additional cost of positive demand fluctuation due to uncertainty represented by the dual variable $y_{d,t}$. The additional cost of buying electricity for positive demand fluctuation due to uncertainty $\lambda_t^{DA}\sum_{p\in\mathcal{P}}(\hat{P}_{d,p,t}u_{d,p})\Delta t$ is assigned as the upper bound of the dual variable $y_{d,t}$ by constraint (6d). On the other hand, the lower bound of the dual variable $y_{d,t}$ is given by constraint (6e) to find the worst cases of demand cost robustness. The dual variables v_d and $\eta_{d,t}$ are logically constrained in (6f) and (6g) to determine those periods that positive demand deviations lead to the worst cost robustness scenarios. Constraint (6h) assigns the user-defined parameter of the robustness budget Γ_d to set the number of periods allowed for positive deviations in demand due to cost robustness. Constraints (6i) and (6j) confine the demand up reserve according to the percentage of downward demand flexibility and the minimum possible demand, respectively. Constraints (6k) and (6l) are similarly defined to limit the down reserve considering the opposite direction of demand flexibility and the maximum possible demand. The worst conditions of ramp-up and ramp-down in two consecutive periods considering the reserve activation are defined in constraints (6m) and (6n), respectively. The capability of demand to provide up and down reserve is defined by constraints (6o) and (6p), respectively. Constraint (6q) limits the minimum energy that each demand should use for the entire period. Constraints (6r) and (6s) describe the nature of positive dual variables and binary variables, respectively.

$$p_{d,t}^{DA}=\sum_{p\in\mathcal{P}}(\tilde{P}_{d,p,t}u_{d,p}+\chi_{d,t}\hat{P}_{d,p,t}u_{d,p})\ , \qquad \forall d,t \qquad (6a)$$

$$\sum_{P\in\mathcal{P}}u_{d,p}=1\ , \qquad \forall d \qquad (6b)$$

$$\lambda_t^{DA}p_{d,t}^{DA}\Delta t\ge\lambda_t^{DA}\sum_{p\in\mathcal{P}}(\tilde{P}_{d,p,t}u_{d,p})\Delta t+y_{d,t}\ , \qquad \forall d,t \qquad (6c)$$

$$y_{d,t}\le\lambda_t^{DA}\sum_{p\in\mathcal{P}}(\hat{P}_{d,p,t}u_{d,p})\Delta t\ , \qquad \forall d,t \qquad (6d)$$

$$y_{d,t}\ge v_d+\eta_{d,t}-M(1-\chi_{d,t})\ , \qquad \forall d,t \qquad (6e)$$

$$v_d+\eta_{d,t}\ge\lambda_t^{DA}\sum_{p\in P}(\hat{P}_{d,p,t}u_{d,p})\Delta t\ , \qquad \forall d,t \qquad (6f)$$

$$\epsilon\chi_{d,t}\le\eta_{d,t}\le M\chi_{d,t}\ , \qquad \forall d,t \qquad (6g)$$

$$\sum_{t\in\mathcal{T}}\chi_{d,t}=\Gamma_d\ , \qquad \forall d \qquad (6h)$$

$$r_{d,t}^{SR,\uparrow}\le\underline{\beta}_{d,t}\sum_{p\in\mathcal{P}}(\tilde{P}_{d,p,t}u_{d,p})\ , \qquad \forall d,t \qquad (6i)$$

$$r_{d,t}^{SR,\uparrow}\le p_{d,t}^{DA}-\underline{P}_d\ , \qquad \forall d,t \qquad (6j)$$

$$r_{d,t}^{SR,\downarrow}\le\bar{\beta}_{d,t}\sum_{p\in\mathcal{P}}(\tilde{P}_{d,p,t}u_{d,p})\ , \qquad \forall d,t \qquad (6k)$$

$$r_{d,t}^{SR,\downarrow}\le\bar{P}_d-p_{d,t}^{DA}\ , \qquad \forall d,t \qquad (6l)$$

$$(p_{d,t}^{DA}+r_{d,t}^{SR,\downarrow})-(p_{d,(t-1)}^{DA}-r_{d,(t-1)}^{SR,\uparrow})\le\bar{R}_d\Delta t\ , \qquad \forall d,t \qquad (6m)$$

$$(p_{d,(t-1)}^{DA}+r_{d,(t-1)}^{SR,\downarrow})-(p_{d,t}^{DA}-r_{d,t}^{SR,\uparrow})\le\underline{R}_d\Delta t\ , \qquad \forall d,t \qquad (6n)$$

$$r_{d,t}^{SR,\uparrow} \le T^{SR} \underline{R}_d^{SR} \,, \qquad \forall d,t \qquad (6o)$$

$$r_{d,t}^{SR,\downarrow} \le T^{SR} \bar{R}_d^{SR} \,, \qquad \forall d,t \qquad (6p)$$

$$\underline{E}_d \le \sum_{t\in\mathcal{T}} (p_{d,t}^{DA} \Delta t - r_{d,t}^{SR,\uparrow}) \,, \qquad \forall d \qquad (6q)$$

$$v_d, \eta_{d,t}, y_{d,t} \ge 0 \,, \qquad \forall d,t \qquad (6r)$$

$$\chi_{d,t} \in \{0,1\} \,, \qquad \forall d,t \qquad (6s)$$

3.5. Coping with non-linear constraints

This section discusses the transformation from the non-linear terms in sets of Eqs. (5) and (6) to obtain a single-level MILP formulation with an exact solution (i.e. it is *not* a linear approximation).

The electricity price variable λ_t^{DA} is modeled by Eq. (2a). The Eq. (2a) represents different possible values for the electricity price depending on the status of the binary variables χ_t^{DA} and $\chi_t'^{DA}$. These binary variables are determined based on the worst-case scenario of the uncertain parameter for electricity price in the objective function of the optimization problem. Therefore, there are three possible combinations for the value of the electricity price. If $\chi_t^{DA} = \chi_t'^{DA} = 0$ for a specific period, that period is not the worst case, and $\lambda_t^{DA} = \tilde{\lambda}_t^{DA}$. If the RVPP operator acts as an energy seller during a specific period and that period represents the worst case, we have $\chi_t^{DA} = 1$ and $\chi_t'^{DA} = 0$. In this scenario, the lower bound of the electricity price is given by $\lambda_t^{DA} = \tilde{\lambda}_t^{DA} - \check{\lambda}_t^{DA}$. Lastly, if the RVPP operator acts as an energy buyer in a specific period and that period is the worst case, we have $\chi_t^{DA} = 0$ and $\chi_t'^{DA} = 1$. In this case, the price is obtained as the upper bound value $\lambda_t^{DA} = \tilde{\lambda}_t^{DA} + \hat{\lambda}_t^{DA}$. The electricity price variable λ_t^{DA} is used in Eqs. (5) and (6), which are related to the uncertain constraints of ND-RES and demand, to model the profit robustness formulation. By substituting the electricity price from Eq. (2a) into the constraints (5) and (6), nonlinear terms appear in the form of the multiplication of binary and continuous variables, as well as the product of two binary variables. These nonlinear terms need to be replaced with their exact equivalent linear constraints.

In this regard, Fig. 2 represents the general formulation presented in [39] for converting nonlinear equations arising from the multiplication of two variables (at least one of them being of binary nature) into a linear format. For the multiplication of a binary variable and a continuous variable, $x \cdot y$, two new positive variables, y^Q and y^A, need to be introduced to represent the final result of $x \cdot y$. These two new positive variables must be constrained by the values of ε and M, depending on the status of the binary variable x. For the multiplication of two binary variables, $x \cdot u$, a new binary variable, z, must be introduced. The value of z should be less than or equal to each of the binary variables x and u, and additionally, $z + 1$ must be greater than or equal to the sum of $x + u$. In the following sections, the above methods are used to obtain the exact equivalent linear formulation of the nonlinear terms in (5) and (6).

Product of binary and continuous variables $(x \cdot y)$:

$$y^Q = x \cdot y$$
$$y^Q = y - y^A$$
$$\varepsilon \cdot x \le y^Q \le M \cdot x$$
$$\varepsilon \cdot (1-x) \le y^A \le M \cdot (1-x)$$

Product of binary and binary variables $(x \cdot u)$:

$$z = x \cdot u$$
$$z \le x$$
$$z \le u$$
$$z + 1 \ge x + u$$

Fig. 2. Big-M method [39] to linearize multiplication of binary and continuous variables and binary and binary variables.

3.5.1. ND-RES non-linear constraints

The set of Eqs. (5) contains two non-linear terms on the left and right hand sides of (5c) due to the multiplication of the electricity price variable λ_t^{DA} and the continuous variables $p_{r,t}^{DA}$ and $r_{r,t}^{SR,\uparrow}$. By substituting the electricity price from constraint (2a) into the non-linear terms, the profit robustness constraint (5c) can be rewritten as (7a). In Eq. (7a), the non-linear terms $\check{\lambda}_t^{DA} \chi_t^{DA} (p_{r,t}^{DA} + r_{r,t}^{SR,\uparrow})\Delta t$ and $\hat{\lambda}_t^{DA} \chi_t'^{DA} (p_{r,t}^{DA} + r_{r,t}^{SR,\uparrow})\Delta t$ are multiplications of binary and continuous variables. Note that discrete rather than continuous values for the electricity price in (2a) is relevant to the robustness concept, since the flexible worst-case scenarios always occur in the boundary values of the electricity price. Finally, the non-linear equation (7a) can be replaced by the set of linear constraints (7b)–(7h) using the method in [39].

The auxiliary variables $p_{r,t}^{DA,Q}$ and $p_{r,t}^{DA,A}$ with the same possible lower and upper bounds as the term $p_{r,t}^{DA} + r_{r,t}^{SR,\uparrow}$ are defined to determine the final result of the non-linear term $\check{\lambda}_t^{DA} \chi_t^{DA} (p_{r,t}^{DA} + r_{r,t}^{SR,\uparrow})\Delta t$. When the binary variable χ_t^{DA} related to the negative electricity price deviation is 1, Eqs. (7c)–(7e) set $p_{r,t}^{DA,Q} = p_{r,t}^{DA} + r_{r,t}^{SR,\uparrow}$ and $p_{r,t}^{DA,A} = 0$. On the other hand, for $\chi_t^{DA} = 0$, Eqs. (7c)–(7e) lead to $p_{r,t}^{DA,Q} = 0$ and $p_{r,t}^{DA,A} = p_{r,t}^{DA} + r_{r,t}^{SR,\uparrow}$. Similarly, the auxiliary variables $p_{r,t}'^{DA,Q}$ and $p_{r,t}'^{DA,A}$ in Eqs. (7f)–(7h) can define the final result of the non-linear term $\hat{\lambda}_t^{DA} \chi_t'^{DA} (p_{r,t}^{DA} + r_{r,t}^{SR,\uparrow})\Delta t$ in (7a). Therefore, the linear Eqs. (7b)–(7h) can replace the non-linear constraint (7a). Note that since auxiliary variables $p_{r,t}^{DA,Q}$ and $p_{r,t}^{DA,A}$ represent the final results of the variable $p_{r,t}^{DA} + r_{r,t}^{SR,\uparrow}$ and the binary variable χ_t^{DA}, they have the same bounds as $p_{r,t}^{DA} + r_{r,t}^{SR,\uparrow}$. Therefore, the value of M in the Big-M method in Eq. (7) is exact and equal to the upper bound of $p_{r,t}^{DA} + r_{r,t}^{SR,\uparrow}$ $(\tilde{P}_{r,t})$.

$$\tilde{\lambda}_t^{DA} (p_{r,t}^{DA} + r_{r,t}^{SR,\uparrow})\Delta t - \check{\lambda}_t^{DA} \chi_t^{DA} (p_{r,t}^{DA} + r_{r,t}^{SR,\uparrow})\Delta t$$
$$+ \hat{\lambda}_t^{DA} \chi_t'^{DA} (p_{r,t}^{DA} + r_{r,t}^{SR,\uparrow})\Delta t \le \lambda_t^{DA} \tilde{P}_{r,t} \Delta t - y_{r,t} \,, \qquad \forall r,t \qquad (7a)$$

$$\tilde{\lambda}_t^{DA} (p_{r,t}^{DA} + r_{r,t}^{SR,\uparrow})\Delta t - \check{\lambda}_t^{DA} p_{r,t}^{DA,Q} \Delta t + \hat{\lambda}_t^{DA} p_{r,t}'^{DA,Q} \Delta t \le \lambda_t^{DA} \tilde{P}_{r,t} \Delta t - y_{r,t} \,, \qquad \forall r,t \qquad (7b)$$

$$p_{r,t}^{DA,Q} = p_{r,t}^{DA} + r_{r,t}^{SR,\uparrow} - p_{r,t}^{DA,A} \,, \qquad \forall r,t \qquad (7c)$$

$$\underline{P}_r \chi_t^{DA} \le p_{r,t}^{DA,Q} \le \tilde{P}_{r,t} \chi_t^{DA} \,, \qquad \forall r,t \qquad (7d)$$

$$\underline{P}_r (1 - \chi_t^{DA}) \le p_{r,t}^{DA,A} \le \tilde{P}_{r,t} (1 - \chi_t^{DA}) \,, \qquad \forall r,t \qquad (7e)$$

$$p_{r,t}'^{DA,Q} = p_{r,t}^{DA} + r_{r,t}^{SR,\uparrow} - p_{r,t}'^{DA,A} \,, \qquad \forall r,t \qquad (7f)$$

$$\underline{P}_r \chi_t'^{DA} \le p_{r,t}'^{DA,Q} \le \tilde{P}_{r,t} \chi_t'^{DA} \,, \qquad \forall r,t \qquad (7g)$$

$$\underline{P}_r (1 - \chi_t'^{DA}) \le p_{r,t}'^{DA,A} \le \tilde{P}_{r,t} (1 - \chi_t'^{DA}) \,, \qquad \forall r,t \qquad (7h)$$

3.5.2. Demand non-linear constraints

The demand robust cost formulation proposed in (6) includes non-linear terms in (6a), (6c), (6d), and (6f). The non-linear term $\lambda_t^{DA} p_{d,t}^{DA} \Delta t$ in (6c) can be linearized in the same way as in (7) by introducing new auxiliary variables. In addition, each of the non-linear terms $\sum_{p\in\mathcal{P}} (\chi_{d,t} \hat{P}_{d,p,t} u_{d,p})$ in (6a), and, by including the expanded term of the electricity price λ_t^{DA} from constraint (2a), the non-linear terms $\lambda_t^{DA} \sum_{p\in\mathcal{P}} (\tilde{P}_{d,p,t} u_{d,p})\Delta t$ in (6c), and $\lambda_t^{DA} \sum_{p\in\mathcal{P}} (\hat{P}_{d,p,t} u_{d,p})\Delta t$ in (6d) and (6f) includes only the multiplication of two binary variables. To linearize these binary multiplication terms, three new binary variables $z_{d,p,t}$, $w_{d,p,t}$, $w'_{d,p,t}$ are introduced as the final result of binary multiplications of $\chi_{d,t} u_{d,p}$, $\chi_t^{DA} u_{d,p}$, and $\chi_t'^{DA} u_{d,p}$, respectively. Furthermore, the set of linear constraints (8) is added to (6), which simulate the possible results of multiplying two binary variables by the newly defined binary

Table 2
RVPP units and DAM electricity price forecast data.

Time	ND-RES1		ND-RES2		Demand		DAM price		
	$\tilde{P}_{r,t}$ [MW]	$\check{P}_{r,t}$ [MW]	$\tilde{P}_{r,t}$ [MW]	$\check{P}_{r,t}$ [MW]	$\tilde{P}_{d,t}$ [MW]	$\hat{P}_{d,t}$ [MW]	$\tilde{\lambda}_t^{DA}$ [€/MWh]	$\hat{\lambda}_t^{DA}$ [€/MWh]	$\check{\lambda}_t^{DA}$ [€/MWh]
1	5	2	0	0	5	2	4	1	1
2	5	3	4	2	15	5	8	2	3
3	10	5	15	5	12	4	6	3	4
4	10	4	12	4	20	3	10	5	2
5	15	6	6	3	20	6	6	3	1

variables $z_{d,p,t}$, $w_{d,p,t}$, $w'_{d,p,t}$.

$$z_{d,p,t} \leq \chi_{d,t} \,, \qquad \forall d,p,t \tag{8a}$$

$$z_{d,p,t} \leq u_{d,p} \,, \qquad \forall d,p,t \tag{8b}$$

$$z_{d,p,t} + 1 \geq \chi_{d,t} + u_{d,p} \,, \qquad \forall d,p,t \tag{8c}$$

$$w_{d,p,t} \leq \chi_t^{DA} \,, \qquad \forall d,p,t \tag{8d}$$

$$w_{d,p,t} \leq u_{d,p} \,, \qquad \forall d,p,t \tag{8e}$$

$$w_{d,p,t} + 1 \geq \chi_t^{DA} + u_{d,p} \,, \qquad \forall d,p,t \tag{8f}$$

$$w'_{d,p,t} \leq \chi_t'^{DA} \,, \qquad \forall d,p,t \tag{8g}$$

$$w'_{d,p,t} \leq u_{d,p} \,, \qquad \forall d,p,t \tag{8h}$$

$$w'_{d,p,t} + 1 \geq \chi_t'^{DA} + u_{d,p} \,, \qquad \forall d,p,t \tag{8i}$$

Finally, by substituting the linear equivalent of non-linear constraints (5) and (6) with (7) and (8), problem (1)–(6) can be written as an MILP problem solvable with available MILP solvers such as CPLEX.

4. Profit robustness example

This section presents a simple illustrative example to show the performance of the proposed profit robust formulation in finding the worst-case profit robustness scenarios by considering the asymmetry of the DAM electricity price. The example provides a detailed description of how the worst cases of the electricity price deviations affect the worst cases of energy deviations. This illustrative example is particularly relevant as it highlights how the proposed profit robustness approach fundamentally differs from traditional energy robustness approaches in the literature, which typically focus only on energy worst-case selection. By explicitly demonstrating how price and energy uncertainties interact to shape profit robustness, this section helps the reader clearly understand the novelty of the proposed model before moving to the full-scale case study. This step-by-step demonstration also enhances the transparency and interpretability of the proposed approach, which is essential for first-time introduction of this modeling framework.

4.1. Defining illustrative example

In this context, an RVPP with two ND-RES and one demand in a sample period of 5 h is considered. The forecast bounds of production and demand of the RVPP units and the DAM electricity price are given in Table 2. Five cases are defined below to compare different conditions for the values of energy and price uncertainty budgets and to compare the results of the proposed model in [21]:

Case 1: Deterministic case with no deviations on electricity price, ND-RES production, and demand (i.e., $\Gamma^{DA} = \Gamma_r = \Gamma_d = 0$);
Case 2: Only the DAM electricity price uncertainty is considered. It is assumed that the values of the DAM electricity price can deviate from the median to the worst case values in three periods (i.e., $\Gamma^{DA} = 3$ and $\Gamma_r = \Gamma_d = 0$);
Case 3: Only the uncertainty of ND-RES units energy and demand is considered. It is assumed that the production values of ND-RES 1 and ND-RES 2 and the demand can deviate from the median to the worst case values in three, one, and two periods, respectively (i.e., $\Gamma^{DA} = 0$ and $\Gamma_{r1} = 3$, $\Gamma_{r2} = 1$, and $\Gamma_d = 2$);
Case 4: Both price and energy uncertainties are considered. The electricity price, the production of ND-RES 1 and ND-RES 2, and the demand values can deviate from the median to the worst case values ($\Gamma^{DA} = 3$ and $\Gamma_{r1} = 3$, $\Gamma_{r2} = 1$, and $\Gamma_d = 2$).
Case 5: The energy robustness problem presented in [21] is solved for the same uncertainty budgets as in Case 4.

4.2. Results of illustrative example

Fig. 3 shows the final results of DAM electricity price, ND-RES energy, and demand for different cases proposed in this example. The final values of the above variables, corresponding to the whole period in each hour, are shown by different bars in this figure. If the value of a variable is equal to the median of the forecast (solid black line in the figure), it means that the corresponding period is not selected as the worst case. Fig. 4 shows the values of the dual variables $y_t^{(\prime)DA}$ and $y_{r(d),t}$ related to the profit/cost affected by different uncertainties for all defined cases except for Case 5. Note that, in the model in [21], these variables are either not defined or defined for energy robustness; therefore, the comparison is only provided for the first four cases.

Case 1:

The RVPP obtains a profit of €56 by bidding its median values of ND-RES production according to Fig. 3. The final values for the DAM electricity price are also obtained as the median values as the length of all bars is equal to the median. As shown in Fig. 4, due to not considering the robustness, all dual variables $y_t^{(\prime)DA}$ and $y_{r(d),t}$ are equal to zero, since the problem is a deterministic optimization one.

Case 2:

In Case 2, the RVPP profit in the DAM is -€12, where the negative value means that the cost of buying electricity to meet demand is higher than the profit obtained by selling electricity on the market. The algorithm chooses periods 3 and 4 for the negative price fluctuation and period 2 for the positive price fluctuation. Therefore, the final electricity prices in periods 3 and 4 (2) are decreased (increased) to their minimum (maximum) values compared to Case 1.

Note that the maximum possible profit reduction in each period can be calculated by finding the maximum value of $\check{\lambda}_t^{DA} p_t^{DA} \Delta t$ for the negative price deviation and $-\hat{\lambda}_t^{DA} p_t^{DA} \Delta t$ for the positive price deviation. Therefore, the algorithm correctly identifies the periods that lead to the worst cases of profit reduction due to price uncertainty.

Case 3:

In Case 3, the RVPP profit in the DAM is -€166. The maximum possible profit reduction for each period can be calculated by $\lambda_t^{DA} \check{P}_{r,t} \Delta t$ for ND-RES and the maximum possible cost increase for demand can be calculated by $\lambda_t^{DA} \hat{P}_{d,t} \Delta t$. The worst cases of profit reductions for ND-RES 1 occur in periods 3, 4, and 5, whereas for ND-RES 2 this occurs in period 4. The worst cases of demand cost occur in periods 2 and 5, resulting in maximum demand in these periods. It can be easily verified that the algorithm correctly selects the worst periods in terms of profit reduction for ND-RES or cost increase for demand.

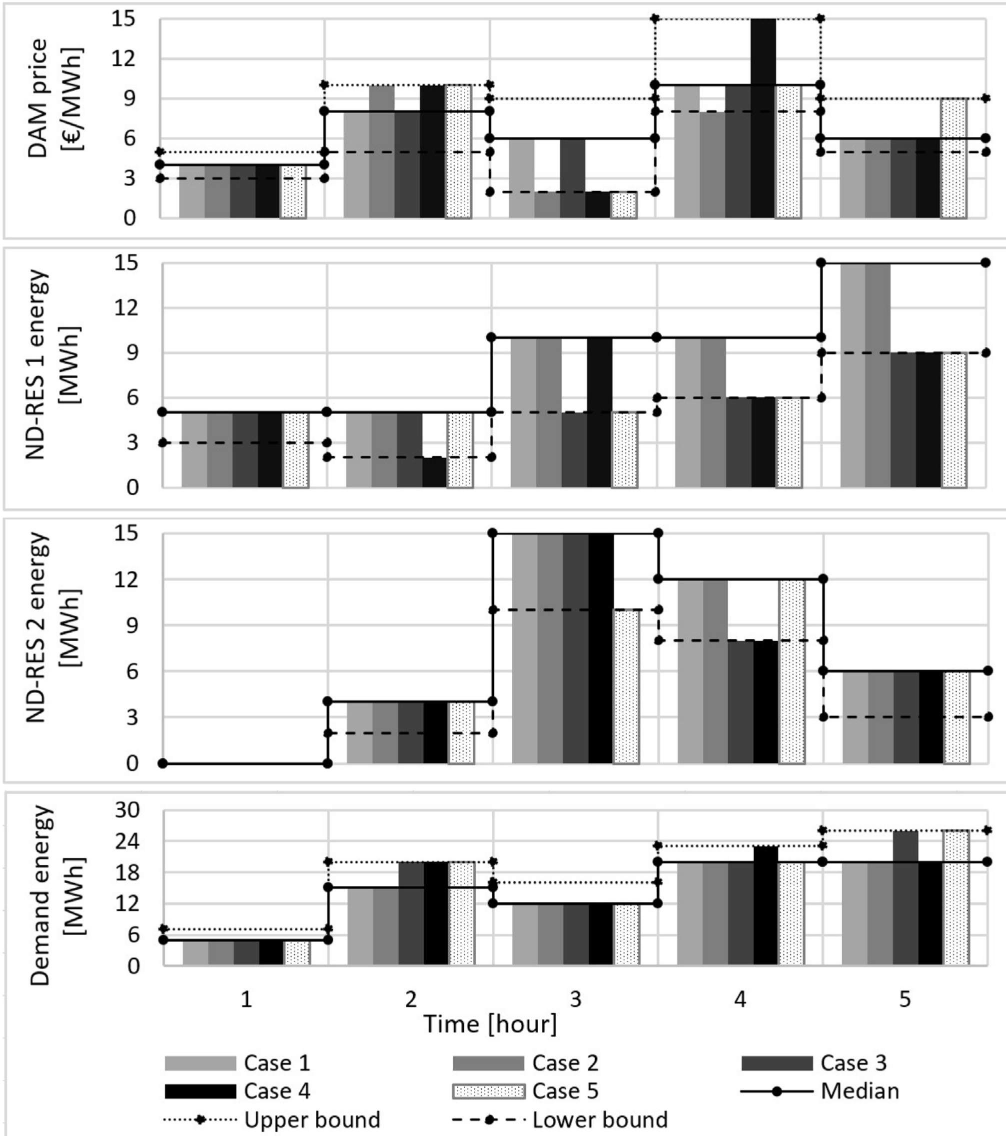

Fig. 3. Final values of DAM electricity price and RVPP units output energy in different case studies.

Case 4:

The RVPP profit in the DAM is -€279. This case shows one of the significant differences between the proposed model and the models in the literature [21] (by comparing the black (Case 4) and white (Case 5) bars), where instead of selecting the periods with higher energy reductions, the proposed algorithm selects the periods that result in higher profit reductions. For instance, the worst case of ND-RES 2 production occurs in period 4 with profit reduction of €60 and energy reduction of 4 MW. However, the period 3 with the highest amount of energy deviation (5 MW) in Case 5 is selected as the worst case.

Case 5:

In Case 5, those periods that result in more deviations of ND-RES production and demand are selected as the worst cases. Moreover, the worst cases of electricity price deviations are determined according to the final values of ND-RES production and demand. For example, for ND-RES 1 in Fig. 3, periods 3, 4, and 5 exhibit the highest energy deviation and are selected as the worst-case scenarios. For demand, hours 2 and 5 are chosen as the worst-case scenarios with the highest positive deviation in energy. For the DAM electricity price, deviations to higher values occur in periods 2 and 5, while a deviation to a lower value occurs in period 3, which are the worst-case scenarios. Considering the different selection of worst-case periods for Cases 4 and 5, the RVPP obtains a profit of -€279 in the former, which is lower than the profit of -€223 obtained in Case 5. Note that the profit obtained is for bidding in the market and is different from the actual profit after clearing the market. Suppose the RVPP uses the bidding strategy proposed in this paper, even though its profit is lower. In this condition, it reduces the risk of significant losses and penalties (e.g., due to buying energy in real time or penalties for the energy it promised to provide but cannot) for not considering the actual worst cases. Moreover, the results indicate that the energy robustness approach cannot fully cope with the actual worst cases for both energy (ND-RES production and demand) and price uncertainty. On the contrary, the profit robustness approach proposed in this paper considers the worst cases of profit/cost deviations for ND-RES/demand instead of the maximum energy deviation. As a final remark, illustrative results indicate that the proposed algorithm accurately selects the worst-case profit for different uncertainty budgets, and shows better performance in finding the worst-case scenarios compared to the model in [21]. These finding will be thoroughly analyzed in successive sections.

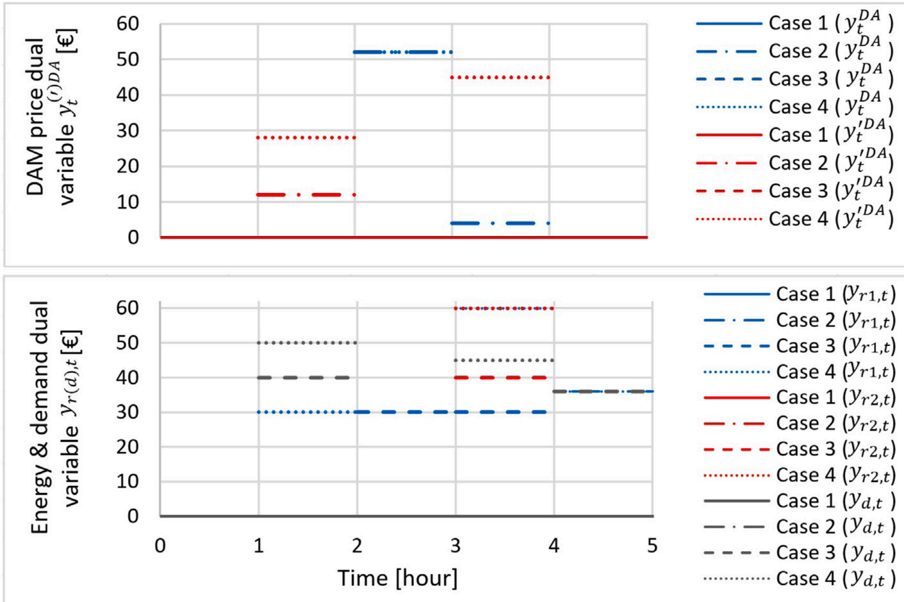

Fig. 4. The profit/cost dual variables affected by different uncertainties in different case studies.

5. Simulation results

This section presents the simulation results of the proposed single-level robust bidding model for different case studies. Simulations are performed on a Dell XPS with an i7-1165G7 2.8 GHz processor and 16 GB of RAM using the CPLEX solver in GAMS 39.1.1.

5.1. Data

The RVPP is located in southern Spain and includes a wind farm, two solar PV plants, and a flexible demand. The production forecast data of the wind farm and the solar PV plants, representing a sample day of the spring season in Spain, are taken from [41,42]. The solar PV plants and the wind farm each have a rated capacity of 50 MW and operating costs of 5 €/MWh and 10 €/MWh, respectively. These values are selected since they represent typical commercial-scale renewable energy projects in Spain, making them relevant for real-world applications. Additionally, using standard capacities ensures that the results are comparable with existing studies and industry practices. Although the simulations are performed for these specific units, the model is general and can accommodate units with different capacities. A residential aggregator profile for the flexible demand is considered according to [40]. The demand owner allows a 10% tolerance for additional demand flexibility, which is allocated for the possible SR provision. All energy forecast data related to RVPP units is shown in Fig. 5. The price forecast data for DAM and SRM are taken from the Red Eléctrica de España (REE) website, and are shown in Fig. 6 for illustration purposes [43]. Table 3 summarizes the above data and other RVPP data used in the simulations.

This section also evaluates the computational efficiency of the proposed model by considering a larger RVPP with a higher number (26) of units. This RVPP includes 12 PV plants, 11 wind farms, and 3 demands. The data for these units are generated by introducing specific modifications compared to the units in the *original* RVPP described above.

5.2. Case studies

Three case studies are performed to analyze the performance of the proposed model. In the first case study, different values for all uncertain parameters related to the DAM and SRM electricity prices, ND-RES energy, and demand are considered to show the behavior of the proposed model in different uncertain environments. In the second case, by means of an out-of-sample assessment, the bidding approach of this paper is compared with two models in the literature. In the third case, the computational performance of the proposed model is evaluated for the larger RVPP comprising 26 units.

The detailed description of the input parameters for the above cases is highlighted below:

Case 1: The behavior of the proposed model is evaluated by considering different combinations of uncertain parameters related to the DAM and SRM electricity prices, ND-RES energy, and demand.

- Case 1.1: Deterministic case ($\Gamma^{DA/SR} = \Gamma_r = \Gamma_d = 0$);
- Case 1.2: Only the uncertainties of the energy of the ND-RES units and the demand are considered ($\Gamma^{DA/SR} = 0$ and $\Gamma_r = \Gamma_d = 5$);
- Case 1.3: Only the DAM and SRM electricity price uncertainties are considered ($\Gamma^{DA/SR} = 5$ and $\Gamma_r = \Gamma_d = 0$);
- Case 1.4: Both DAM and SRM electricity price and energy uncertainties are considered ($\Gamma^{DA/SR} = \Gamma_r = \Gamma_d = 5$);
- Case 1.5: A sensitivity analysis is performed by considering DAM and SRM electricity price and energy uncertainties for $\Gamma^{DA/SR} = \Gamma_r = 0, 1, 2, \ldots, 9, \Gamma_d = 0$.
- Case 1.6: A sensitivity analysis is conducted to evaluate the performance of the proposed model under high-uncertainty conditions, considering uncertainty deviations as defined in Section 5.1, as well as 50% and 100% higher deviations.
- Case 1.7: A comprehensive sensitivity analysis is performed to evaluate operational profit and simulation time for different combinations of uncertainty budgets related to DAM and SRM electricity prices and energy uncertainties ($\Gamma^{DA/SR} = 0 - 24, \Gamma_r = 0 - 24, \Gamma_d = 0 - 24$).

Case 2: The results of the proposed model for $\Gamma^{DA/SR} = \Gamma_r = 0, 1, 2, \ldots, 9, \Gamma_d = 0$ are compared with models in [21,31] using an out-of-sample assessment.

Case 3: The computational performance of the proposed model is compared for RVPP with 4 and 26 units for $\Gamma^{DA/SR} = \Gamma_r = \Gamma_d = 0, 1, 2, \ldots, 9$.

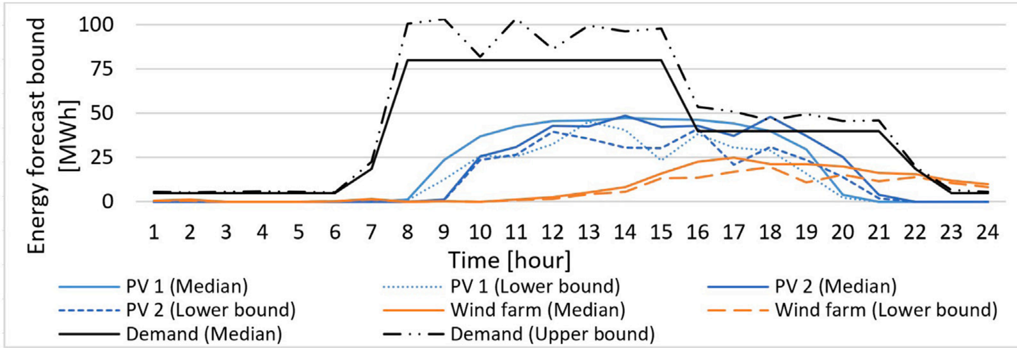

Fig. 5. The energy forecast data.

Table 3
RVPP units data.

PVs			Wind farm			Demand					RVPP	
$\bar{P}_r$ [MW]	$\underline{P}_r$ [MW]	C_r^R [€/MWh]	$\bar{P}_r$ [MW]	$\underline{P}_r$ [MW]	C_r^R [€/MWh]	$\bar{P}_d$ [MW]	$\underline{P}_d$ [MW]	E_d [MWh]	$C_{d,p}$ [€/MWh]	$\beta_{d,t}$ [%]	κ [%]	T^{SR} [min]
50	0	5	50	0	10	150	0	900	0	10	20	15

5.2.1. Case 1

Fig. 6 shows the RVPP traded energy and reserve versus the electricity price for Cases 1.1 through 1.4. The positive and negative bars in the upper figure show that RVPP is a seller or buyer of energy on the market. The positive and negative bars in the lower figure show the traded up and down reserve of RVPP. The general results for all cases show that between hours 8–11, when the demand is high and the production of ND-RES units is not enough to supply all demand, the RVPP is an energy buyer in the electricity market. Between hours 12–15, although the demand is high, the production and demand of RVPP are approximately equal, and RVPP does not trade too much energy in most cases. However, in these hours the consideration of different uncertain parameters in Cases 1.1 through 1.4 has a significant effect on the energy trading direction of RVPP (whether RVPP is a seller or a buyer of energy). Between hours 16–19, as the demand decreases, the RVPP becomes a seller of energy in most of the cases. The results for traded SR shows that between hours 9–20, that RVPP has high production, it provides more up and down SR to the market.

The total sold energy of RVPP in Cases 1.2 through 1.4 is decreased by 52.0%, 0%, and 51.7%, respectively, compared to Case 1.1, whereas the total bought energy of RVPP is increased by 74.2%, 0%, and 66.3%, respectively. The total up (down) SR provided by RVPP in Cases 1.2 through 1.4 is decreased by 0 (0)%, 2.2 (1.5)%, and 7.5 (7.1)%, respectively, compared to Case 1.1.

The results for each case study show that in the deterministic case (Case 1.1) and in the hours when RVPP is an energy seller in the market, RVPP usually sells more energy and reserve than in Cases 1.2 and 1.4. However, if RVPP is an energy buyer, the energy bought in Case 1.1 is usually less than in Cases 1.2 and 1.4. The reason is that in the deterministic case, the RVPP always takes an optimistic approach because it does not consider any uncertain parameter. In Case 1.2, considering the energy deviation of ND-RES and demand results in a lower amount of energy sold and a higher amount of electricity purchased from the market compared to Case 1.3, where only DAM and SRM electricity price uncertainties are considered. According to the comparison of Cases 1.1 and 1.3, considering only the electricity price uncertainty results in a lower amount of purchased energy only in some hours, e.g. hour 10, compared to Case 1.1. The reason is that this hour is one of the hours in which the electricity price goes to its worst case. Therefore, the RVPP prefers to supply its demand with its production and also to provide less SR. In other hours (except hours 1–6 and 12 with small differences) there are not too many differences between RVPP traded energy in Cases 1.3 and 1.1. The reason is that although the electricity price goes to the worst cases in some hours in Case 1.3, the RVPP must supply its demand or it can sell energy to the market with lower benefit. Considering all energy and price uncertainties in Case 1.4 results in a different bidding approach in some hours (e.g., hours 9, 10, 12, 14, and 15) compared to Case 1.2, which considers only energy uncertainty. Hours 9, 10, and 14 are exactly the hours where the worst cases of electricity price occur, forcing the RVPP to increase or decrease its bid amount. Note that the worst cases of DAM electricity price occur in hours 8–11 and 14 (which are different from the worst cases of electricity price in Case 1.3).

Fig. 7 shows the sensitivity analysis of RVPP decisions related to traded energy and reserve in Case 1.5. The decision variability is calculated by taking the standard deviation of RVPP decisions for $\Gamma^{DA/SR} = \Gamma_r = 0, 1, 2, \ldots, 9, \Gamma_d = 0$. The figure allows the operator to gain a more comprehensive understanding of the decisions made by the RVPP in each hour, taking into account the different uncertainty budgets. The hours in which RVPP decisions related to bought energy, sold energy, up reserve, and down reserve are more affected are represented by triangle, times, square, and circle signs, respectively. The results show that modifying the uncertainty budgets has negligible impact on the decisions of RVPP in the first eight hours, represented by very small dots. Although the variability of the RVPP decisions related to the bought energy is observed in a greater number of hours than its decisions variability related to the sold energy, it is typically observed that the standard deviation of the bought energy is lower than that of the sold energy. The standard deviation of the provided reserve is lower than that of energy. This is because it is usually beneficial for RVPP to provide the highest amount of requested reserve in most of the hours.

Fig. 8 presents the operational profit and simulation time for the original uncertainty bounds defined in Section 5.1 across different combinations of uncertainty budgets (black lines), for cases where uncertain parameters deviate 50% above the defined bounds (orange lines), and for cases where uncertain parameters deviate 100% above the defined bounds (blue lines).

The results show that an increase in ND-RES uncertainty bounds has a greater impact on the operational profit of the RVPP compared to other uncertainty bounds individually. Furthermore, when all uncertainties are considered, increasing the uncertainty bounds by 100% can significantly reduce the operational profit of the RVPP. This analysis highlights the importance of properly assigning the bounds of uncertain parameters in the proposed approach. Additionally, the simulation time in Fig. 8 demonstrates that the proposed approach remains computationally efficient under high-uncertainty conditions.

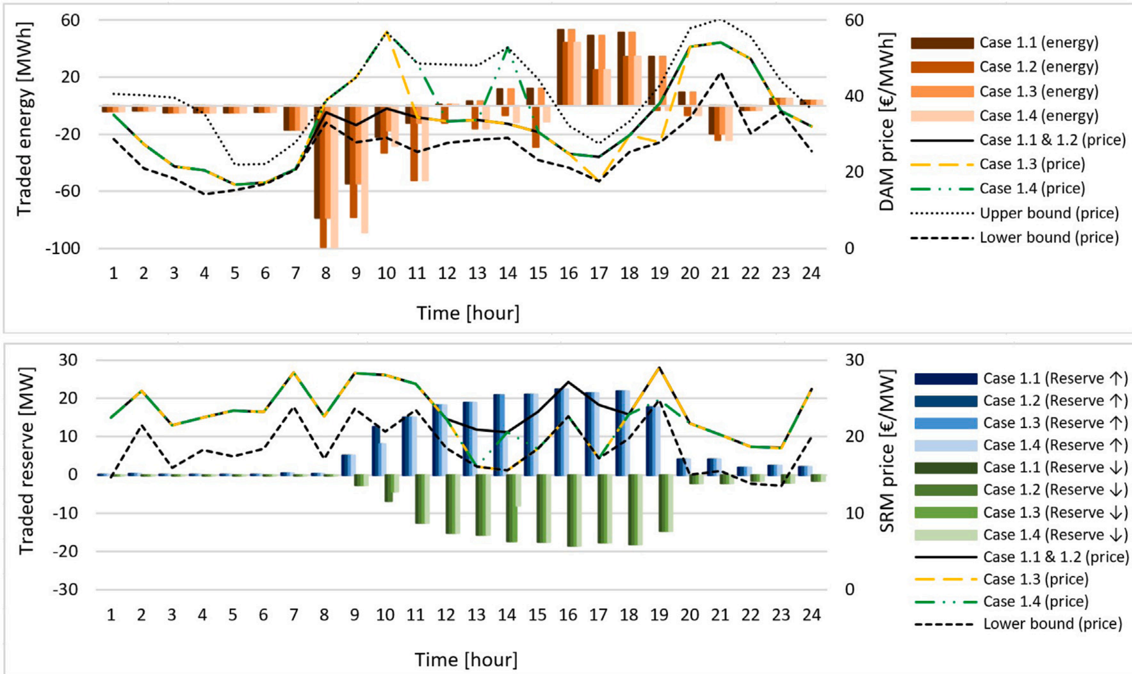

Fig. 6. RVPP traded energy and reserve versus electricity price in different case studies.

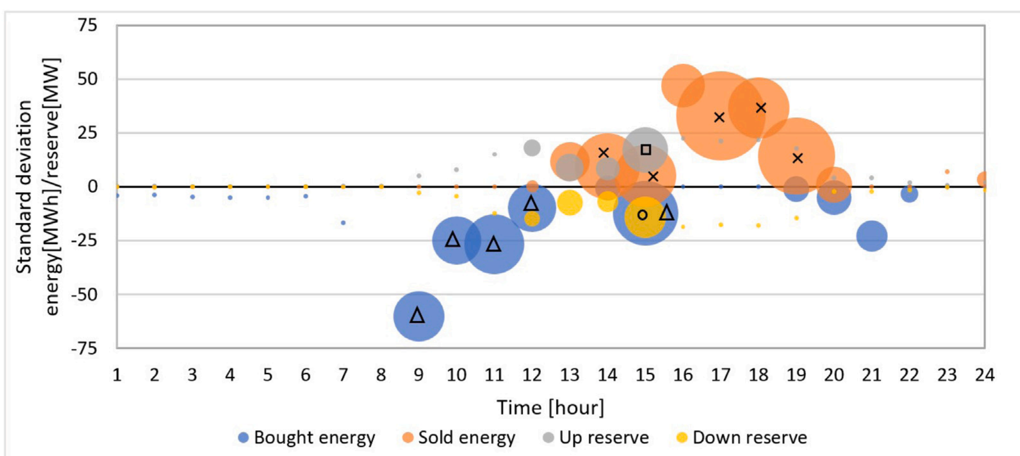

Fig. 7. Standard deviation of traded energy and reserve of RVPP for all $\Gamma^{DA/SR} = \Gamma_r = 0, 1, 2, \ldots, 9, \Gamma_d = 0$ in each period.

The simulation time remains below 140 s for different combinations of uncertainty budgets and uncertainty deviation bounds.

Fig. 9 illustrates the operational profit and simulation time for different combinations of uncertainty budgets related to DAM and SRM electricity prices, ND-RES production, and demand consumption. The figure presents results when some uncertainty budgets — those related to ND-RES production, demand consumption, and DAM and SRM prices — are set to zero, while others vary from 0 to 24 (black lines). Furthermore, Fig. 9 provides results when electricity price uncertainty parameters are fixed at higher values (4, 8, 16, and 24), and the uncertainty budgets related to ND-RES production and demand consumption increase from 0 to 24 (orange lines). Additionally, it shows the case where ND-RES production and demand consumption are fixed at higher values (4, 8, 16, and 24), while the uncertainty budgets for DAM and SRM prices increase from 0 to 24 (blue lines).

The results for operational profit show that, in general, increasing the uncertainty budget leads to a sharper decrease in operational profit at lower uncertainty levels until it reaches a saturation point, typically between uncertainty budgets of 9 and 15 for different combinations of selected uncertainty budget parameters. The results also indicate that ND-RES production uncertainty has a greater impact on the operational profit of the RVPP compared to electricity prices and demand uncertainties, as all RVPP production comes from ND-RES units. Furthermore, when all uncertainties are considered simultaneously, the decrease in operational profit is more significant than when each uncertainty is accounted independently.

The results for simulation time show that, in general, increasing the uncertainty budgets — by allowing more combinations for selecting worst-case scenarios — initially leads to an increase in simulation time, followed by a decrease. However, the figure indicates that the computational time for all combinations of uncertainty budgets remains at a highly acceptable level, staying below 80 s. This efficiency makes the approach suitable for RVPP market participation and enables several sensitivity analyses to be performed before submitting the final bid.

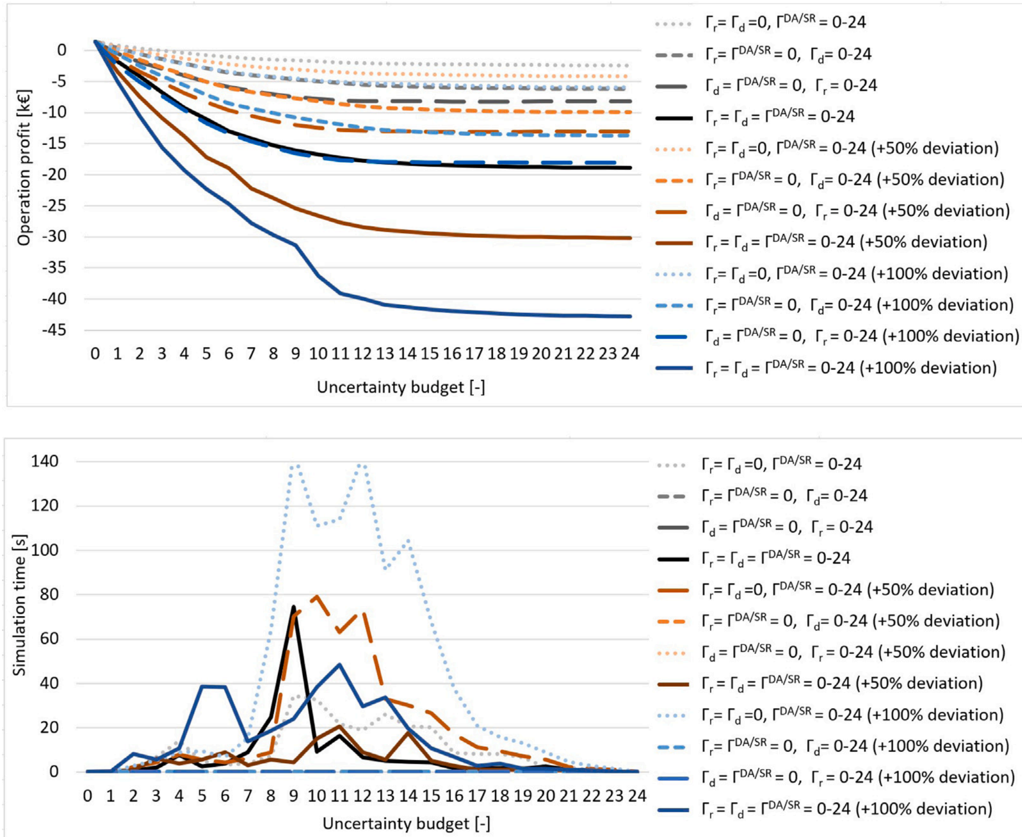

Fig. 8. Operation profit and simulation time for different uncertainty conditions and different uncertainty budgets in Case 1.6.

5.2.2. Case 2

Fig. 10 compares the results of an out-of-sample assessment for the proposed model and models in [21,31] for different values of uncertainty budgets between 0 and 9. In the figure, Π^{av} represents the operating profit (no penalization applied), K^{av} is the penalization cost for not complying with the energy bid, and the net profit of RVPP is represented by $\Pi^{av} - K^{av}$. In [21], the energy robustness approach is adopted. The authors in [31] use a multi-level optimization problem which implements an RO approach to model the ND-RES units uncertainties, and an SO to capture the electricity price uncertainties. Therefore, the uncertainty budget in Fig. 10 for model [31] refers only to the production of ND-RES units. To model the price uncertainty in their SO model, 200 scenarios are considered according to the REE website [43]. For the out-of-sample assessment, 1000 scenarios are generated based on the hourly distributions of uncertain parameters related to the DAM and SRM electricity prices and ND-RES production. The Weibull distribution, with its ability to model different degrees of skewness and tails, is used to generate scenarios to better capture the asymmetric behavior of uncertain parameters. Note that an equal value for all time periods, such as in [21,31], can be considered for the penalty cost. However, the penalty cost related to the energy that is not provided is set to three times the DAM median price forecast in this paper. In this way, the deviation in the hours when the electricity price is higher leads to more penalty for RVPP.

The net profit of RVPP for uncertainty budget 0 in the proposed model and model [21] is the same as the deterministic components in both models are the same. However, the model in [31] results in a lower value of net profit for uncertainty budget 0 compared to the proposed model and [21] due to the use of a different reserve provision strategy. In this paper and in [21], the production plus the reserve provided by each RVPP unit is limited by the maximum production of each unit, while in [31] this constraint is not defined and only the reserve provision limit by the entire RVPP is considered. Therefore, for an uncertainty budget of 0, using the proposed model or the model in [21] results in a lower energy bid in the DAM in several hours compared to [31]. By increasing the uncertainty budget, the proposed model leads to a higher net profit obtained compared to model [21]. The better results in terms of net profit by using the model [31] compared to the proposed model is, to some extent, expected. This is due to the use of a more sophisticated approach to find the worst case of uncertainties of ND-RES production, and the consideration of the possibility of rescheduling the RVPP units in the third level of the model [31]. However, the proposed model shows a closely aligned results compared to [31] even in some cases the obtained results in the proposed model are better than model in [31] (see e.g. results for uncertainty budgets 0, 3, 4, and 5).

To provide a more detailed comparison of the performance of the proposed model and the model in [31] in the DAM and SRM, as well as their ability to handle uncertainty, Fig. 11 is provided. This figure shows the traded energy and reserve of the RVPP under different values of uncertainty budgets. It can be observed that as the uncertainty budget increases, the sold energy of the RVPP decreases while the bought energy increases in the proposed model (solid lines). This occurs because the RVPP operator adopts a more conservative strategy to participate in the market, ensuring it can supply its demand. A similar trend is observed for the traded energy of the RVPP in the model from [31], where less energy is sold when the RVPP acts as a seller and more energy is purchased when it acts as a buyer. By comparing the results, it can be seen that although there are some differences in traded energy at certain hours, both models provide similar and effective bidding strategies in the DAM. The results for the traded reserve of the RVPP indicate that in certain hours (10, 12–15, 20, 22, and 23), the up and down reserves provided in the proposed model are lower than those in the model from [31], particularly at hours

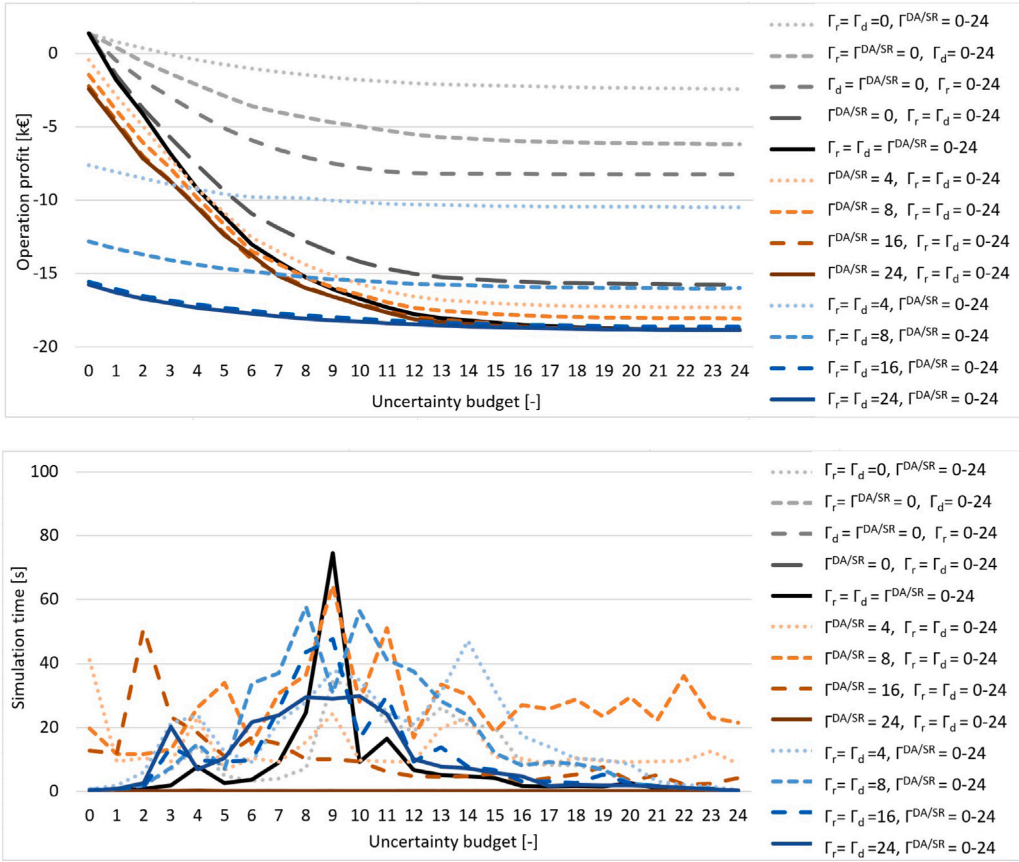

Fig. 9. Operation profit and simulation time for different uncertainty budgets in Case 1.7.

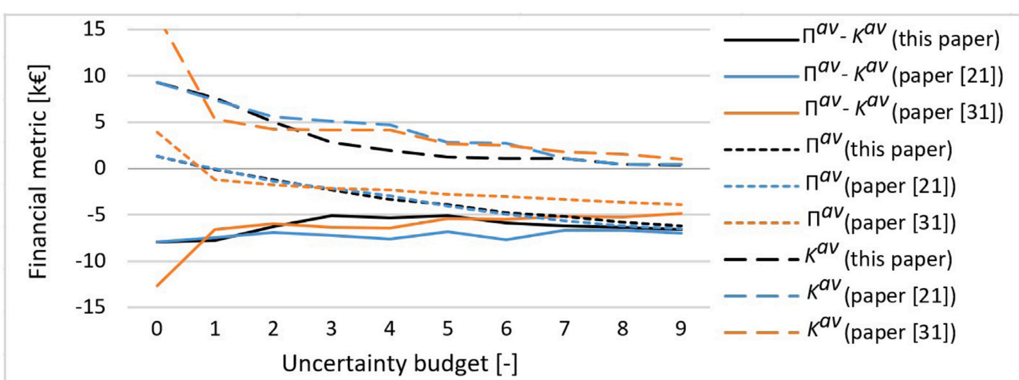

Fig. 10. The out-of-sample assessment for proposed model and the models in [21,31] ($\Gamma^{DA/SR} = \Gamma_r = 0, 1, 2, \ldots, 9, \Gamma_d = 0$).

10, 13, 14, and 20. This difference arises because the proposed model explicitly considers the technical capabilities of RVPP units in providing reserve, whereas the model in [31] neglects most of the technical constraints related to reserve provision and instead applies a general reserve bound for the RVPP. Additionally, the reserve provision in the proposed model is influenced by changes in the uncertainty budget. This is because variations in the uncertainty budget not only impact the energy output of the units but also affect their reserve provision accordingly. However, this relationship is not captured in [31], where a constant up and down reserve is maintained across all uncertainty budget levels.

From the computational standpoint in Case 2, the simulation time of different cases of the model [21] is less than 2 s due to the simplified approach to identify the worst case of the optimization problem. The simulation time of the model proposed in this paper is less than 30 s in all cases, which meets the acceptable criteria for the RVPP bidding problem when, e.g., different strategies need to be analyzed and compared before submitting the bid to the market operator. The simulation time of the model [31] reaches between 10 to 90 min. In summary, the proposed approach demonstrates outstanding computational performance against more intricate approaches such as [31], while providing results that compete in terms of profits with the model in [31] and reducing the risk of penalization compared with [21].

5.2.3. Case 3

Table 4 compares the operation profit and simulation time of the proposed bidding model for RVPP with 4 and 26 units. The number of equations, continuous variables, and binary variables in the simulations for RVPP with 4 units are, respectively, 3668, 2251, and 337. On the other hand, for the case of the 26-unit RVPP, these numbers are, 16558,

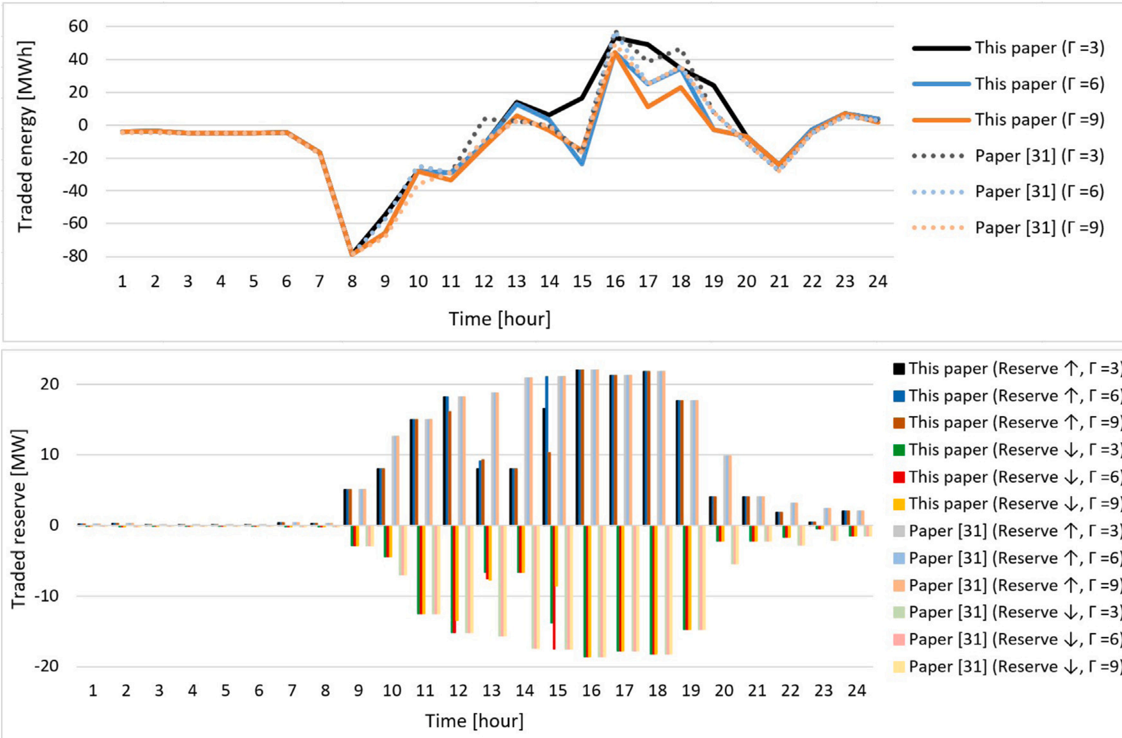

Fig. 11. RVPP traded energy and reserve for the proposed model and the model in [31] ($\Gamma^{DA/SR} = \Gamma_r = 3, 6, 9, \Gamma_d = 0$).

Table 4
Operation profit and simulation time for RVPP with 4 and 26 units.

Uncertainty budget [–]	Operation profit [k€]		Simulation time [s]	
	4-unit	26-unit	4-unit	26-unit
0	1.4	76.1	0.1	0.1
1	−1.8	51.2	0.1	1.1
2	−4.1	34.1	0.7	30.8
3	−6.8	18.9	2.2	36.9
4	−9.2	2.8	3.6	48.0
5	−11.1	−9.5	2.6	59.7
6	−13.0	−18.7	2.6	101.8
7	−14.2	−27.0	4.7	172.2
8	−15.2	−34.9	5.7	171.3
9	−16.0	−41.1	4.8	110.3

10339, and 1491. The results demonstrate that the proposed model exhibits a moderate simulation time for an RVPP with a high number of units. As the uncertainty budget is increased for the RVPP with 26 units, the computational time initially increases and then decreases. This is due to the fact that for a certain range of uncertainty budget, the optimization problem must identify the worst case by taking into account more combinations for variability of uncertain parameters. The maximum simulation time is around 3 min, and occurs for an uncertainty budgets of 7 and 8. Furthermore, a trend comparable to that observed in Case 2 for the operation profit of the RVPP is evident, whereby an increase in the uncertainty budget is associated with a decline in the operation profit. The proposed single-level model thus shows outstanding scalability for larger problems. We remark, as a reference, the 90 min of the multi-level problem considered in Case 2 above for a 4-unit RVPP. This feature, combined with the detailed information that can only be obtained from comprehensive sensitivity analysis (see Fig. 7), makes the proposed methodology a notably powerful tool for RVPP operators in their decision making.

Two additional factors — objective function value and optimality gap during the simulation — have been included in Fig. 12 to evaluate the effectiveness of the proposed model. The marked points in this figure indicate the objective function value and the optimality gap at different iterations of the solver. The results show that for both the 4-unit and 26-unit RVPP, the optimality gap is efficiently reduced, and the objective function value rapidly approaches the final optimal value. For example, in most cases, the optimality gap for the 4-unit and 26-unit RVPP falls below 1% in less than one second and 100 s, respectively. This demonstrates the stability of the proposed model in finding optimal solutions across different uncertainty budgets.

5.3. Remarks

The findings in Sections 4 and 5 are summarized in this section.

(1) *The tradeoff between simplicity and accuracy*. While a single-level MILP model is used to model the RVPP bidding in different markets, the results in the illustrative example of Section 4 show that the flexible worst case of different uncertain parameters associated with the electricity price as well as ND-RES and demand is obtained for the case where the combination of all uncertainties are considered or each set of uncertainties is considered.

(2) *Providing an exact linearization model*. The linearization method in this paper is developed for the case that different uncertainties in the objective function and constraints affect each other. The results obtained in Section 4 show that when all uncertainties are considered, which results in changing the worst condition, the proposed approach effectively finds the worst case.

(3) *Ability to perform comprehensive sensitivity analysis in an efficient time*. The RVPP operator must perform the sensitivity analysis with respect to the uncertainty budget to find its appropriate level of conservatism. The results presented for an RVPP with 26 units in Section 5 show the applicability of the proposed approach in real-world applications.

(4) *Hours with higher decision variability*. This paper presents a new graph that represents the most sensitive hours in terms of traded energy and reserve of RVPP. The results of Section 5 show that the traded energy of RVPP has higher deviations for different uncertainty budgets compared to the traded reserve of RVPP.

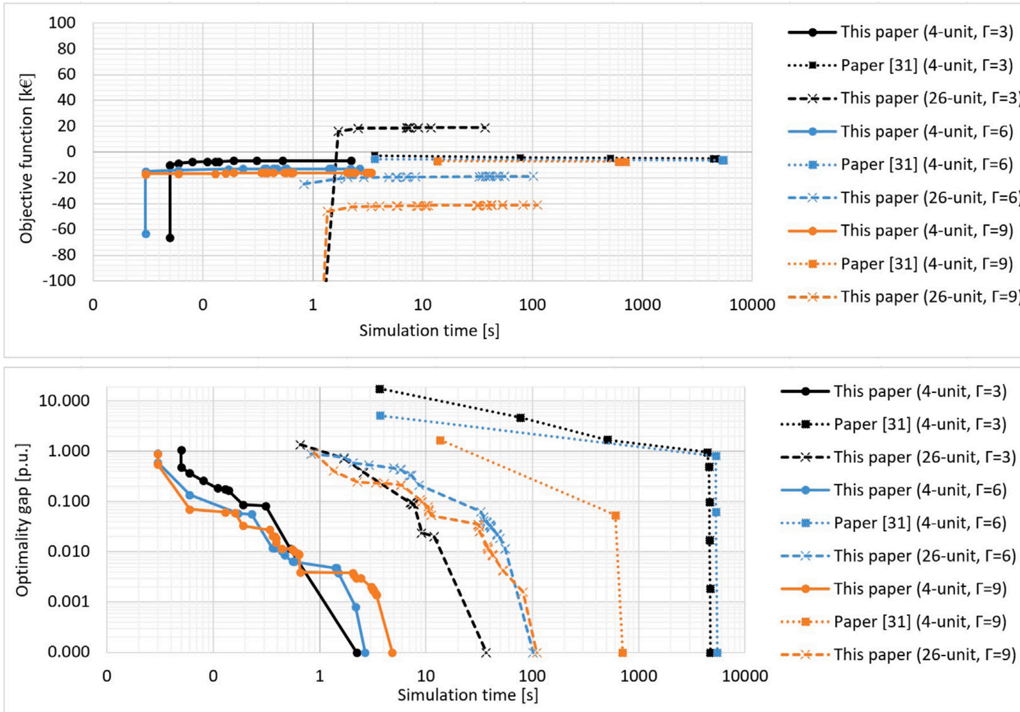

Fig. 12. Objective function and optimality gap versus simulation time for RVPP with 4 and 26 units.

(5) *The effectiveness of the bidding approach considering different market clearing scenarios*. The out-of-sample assessment is performed in Section 5 to compare the results of the model presented in this paper with models in the literature. The results show that the proposed approach provides competitive solutions in compared to more sophisticated approaches.

6. Conclusion

In this paper, a novel, computationally efficient, single-level robust bidding method for RVPPs is proposed to capture multiple uncertainties and their couplings, including DAM and SRM electricity prices as well as ND-RES production and demand consumption. The non-linear couplings between different uncertainties in the objective function and constraints of the optimization problem are addressed by developing an accurate linear model based on the Big-M method. The obtained results show that the uncertainty of ND-RES and demand has the highest impact on the bidding approach of RVPP compared to the electricity price uncertainty. Furthermore, the sensitivity analysis shows that the RVPP operator can significantly increase its net profit by considering even a low or median value for the risk measure parameter (uncertainty budget).

As a final remark, the proposed single-level MILP model offers significantly improved computational efficiency compared to multi-level optimization models available in the literature. By avoiding the need to explicitly solve iterative nested or hierarchical optimization problems, the proposed framework achieves faster solution times, making it more suitable for practical large-scale applications. In addition to faster computation, the proposed formulation offers a simpler and more straightforward implementation, enhancing its practical value for RVPP operators participating in simultaneous markets under uncertainty. The comparisons conducted in the simulations show that the proposed model produces results closely aligned with those obtained from multi-level models, while requiring significantly less computational time. This confirms that the proposed approach successfully balances accuracy and computational efficiency, making it well-suited for real-world operational environments.

The model proposed is not exempt from some limitations. The model assumes that the RVPP is a price-taker in the market, which may limit its applicability in scenarios where the RVPP has significant market power. Additionally, the model focuses primarily on short-term market participation, particularly in the DAM and SRM, and does not account for long-term planning or RTM dynamics.

In this regard, future research could explore the incorporation of long-term planning horizons and RTM participation to enhance the adaptability of the RVPP to dynamic market conditions. Additionally, it would be beneficial to integrate environmental and sustainability considerations, potentially through multi-objective optimization, to create a more holistic model. Another potential area for further investigation is the integration of hybrid models that combine robust optimization with other uncertainty modeling techniques, such as stochastic programming, to better capture complex risk profiles and system behaviors under extreme conditions. Finally, extending the current approach to integrate more flexible resources, such as hydrogen, hydropower, biomass, concentrated solar power plant, and ESS, as well as multiple energy sources and their associated uncertainties, appears as a promising direction for future research.

CRediT authorship contribution statement

Hadi Nemati: Writing – review & editing, Writing – original draft, Visualization, Validation, Software, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. **Pedro Sánchez-Martín:** Writing – review & editing, Visualization, Validation, Supervision, Methodology, Investigation, Conceptualization. **Ana Baringo:** Writing – review & editing, Validation, Software. **Álvaro Ortega:** Writing – review & editing, Visualization, Validation, Supervision, Project administration, Methodology, Investigation, Funding acquisition, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgments

This project has received funding from the European Union's Horizon 2020 research and innovation programme under grant agreement No 883985.

Data availability

Data will be made available on request.

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