Stochasticity in Electric Energy Systems Planning

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Abstract. Electric energy systems have always been a continuous source of applications of planning under uncertainty. Stochastic parameters that may strongly affect the electric system are demand, natural hydro inflows and fuel prices, among others. A review of some estimation methods used to approximate those parameters is presented. Reliability and stochastic optimisation are widespread techniques used to incorporate random parameters in the decision-making process in electric companies. A unit commitment, a market-based unit commitment, a hydrothermal coordination and a risk management model are typical models that can incorporate uncertainty in the decision framework.

1 Introduction

Uncertainty may be originated, in a broad sense, by the lack of reliable data, measurement errors or parameters representing future information. In electric energy systems planning, uncertainty appears mainly in demand, natural hydro inflows, fuel prices, system availability, electricity prices, competitors' strategies, and regulatory framework. Electric demand has a cyclic pattern, with seasonal, weekly and daily variations along the year. Besides, demand also presents a locational variation depending on the local or regional economic activity. Natural hydro inflows are subject to climate conditions every year and, therefore, also the water flowing into the reservoirs that can be used for electricity production. Fossil fuel prices are subject to geopolitical circumstances. System elements such as power plants and transmission lines are subject to random failures that can affect the capability of the system to supply electricity to final customers. Because of the previous stochastic parameters, electricity prices resulting from market clearing are also subject to stochasticity. Finally, the regulatory frameworks under which many electric energy systems are currently operating are subject to changes to adapt them to new requirements (i.e., emissions market) or to improve their performance by changing some market rules.

Planning[1] and operation decisions of electric systems are certainly complex, with very different time scopes. They can include decades in the case of generation and transmission expansion or just several minutes for the economic dispatch. These decisions are coordinated to achieve the objective of optimal operation of the electric system. This general objective is separated into several others for different time horizons that are implemented in hierarchical decision support tools. In power systems planning the time scope is usually divided into the following levels:

- *Very long term*: for any time ranging from five to fifteen years
- *Long term*: for any time ranging from two to five years
- *Medium term*: any time ranging from one month to two years
- *Short term*: from one week to one month
- *Very short term*: any time below one week

This division is required by the practical infeasibility of finding a model detailed enough to characterize the system. At the same time, the nature of the whole problem is well suited to be functionally decomposed. Longer the time period lower the detail in modelling the system. The purpose of this hierarchical process is to represent adequately the main variables, parameters and characteristics of the electric system affecting each decision level. Besides, it allows managing the complexity of the whole problem. The previously mentioned stochastic parameters can affect the system planning in different time horizons. As previously established, only the relevant stochastic parameters are considered in each time horizon and decision level. For example, stochasticity in demand may affect all the decisions. However, it seems that this influence can be more relevant in the very long term (where expansion decisions are taken) and in the very short term (where unit commitment decisions must be adopted). Uncertainty of natural hydro inflows seems to be relevant in the medium term due to its yearly cycle.

Firstly, in section 2 we present some tables that show the importance of some stochastic parameters. We have used Spain as the case study for presenting real data. In section 3 we present some of the methods that have been used so far to predict future values of stochastic parameters. In section 4 we show some of the mathematical techniques that can be used to deal with uncertainty in electric energy systems incorporated in decision support tools. In section 5 we summarize some of the classical applications and we present how they take into account the uncertainty. Finally, we extract some conclusions and recapitulate the work presented in this chapter.

2 Uncertainty Impact

For proximity and data accessibility, we have chosen to show the impact of the uncertainty of the Spanish electric energy system (see [20]). As a matter

[1] Planning is used here for any time horizon for taking decisions apart from the online system operation.

of fact, the energy demand increase for the last five years is shown in the following table. The energy load has increased at an approximate 5 % rate for the last five years (a cumulative 21 %) mainly due to the economic activity. An increment correction is made to include the effect of working vs. non working days and temperature.

Year	Energy TWh	Yearly increment %	Corrected yearly increment %
2000	195.0	5.8	6.5
2001	205.6	5.4	4.9
2002	211.5	2.8	3.9
2003	225.8	6.8	5.4
2004	235.4	3.5	3.6

Peak load has also increased as shown in the next table. From year 2000 to year 2004 the winter peak load has increased in 4.5 GW (an increment of 13.5 % with respect to winter peak load in year 2000) and the summer peak load in 7.2 MW (an increment of 24.6 % with respect to summer peak load in year 2000) and it is almost the same that the winter peak load. The main reason for this huge increment in summer peak load is the high penetration of air conditioning in new home and hotel developments in Spain. This peak load increment in five years would be equivalent to approximately ten combined cycle gas turbines, which implies two units per year.

Year	Winter peak load GW	Summer peak load GW
2000	33.2	29.4
2001	34.9	31.2
2002	34.3	31.9
2003	37.2	34.5
2004	37.7	36.6

The annual energy coming from natural hydro inflows shows also a great variation along the last years. For example, the hydro energy in year 2003 was 160 % the energy available in year 2002. The percentage of being exceeded corresponds to the value of the cumulative distribution function for that hydro energy.

Year	Hydro energy TWh	Index	Percentage of being exceeded %
2000	26.2	0.90	62
2001	33.0	1.14	27
2002	21.0	0.73	88
2003	33.2	1.15	26
2004	24.6	0.85	68

3 Estimation Methods

In this section we present different techniques used for estimating the evolution of some stochastic parameters along the time, namely demand and natural hydro inflows. With these two parameters, we have tried to show a variety of complementary prediction techniques used in the context of electric energy systems.

3.1 Demand

Load forecasting has always been an important concern for long term expansion decisions, mainly related to yearly peak demand. At this time horizon, the main influence factors are related to the use of electricity by different customers and to the general socioeconomic and demographic parameters. Besides these, weather conditions strongly influence the electric load. In the short term, not only peak is important but also the demand profile and its variation for each day of the week need to be estimated.

Forecasting methods differ depending on the time range they are dealing with, see book [9] for a detailed review. For long term forecasting, *end-use models* and *econometric models* are primarily used. For short term forecasting a large variety of methods from statistical and artificial intelligent techniques are used. Among them, we can mention *regression methods*, *time series analysis*, *artificial neural networks*, *fuzzy logic*, and combinations of them.

End-use models explain the electric demand as a function of the direct use of electricity by different customers (for example, in appliances for domestic users, electric motors or aluminium tons for industrial customers, and air conditioning for commercial customers). So load forecasting is reinterpreted as the estimation of end-user devices and their evolution along the time. Theoretically, this approach is very precise. However, it requires a huge amount of data and can be very sensitive to their quality.

Econometric models use general economic data as factors for explaining electricity consumption. So load forecasting is estimated as a function of economic parameters (such as gross domestic product, customer price index, etc.) obtained by using statistical techniques.

Regression is used to determine the relationship between load consumption and factors such as weather, temperature, day of the week, etc., see [12,23].

Time series methods are based on detecting the intrinsic structure of load data regarding correlation, trend, seasonal variation, etc. ARMA[2] and ARIMA[3] techniques use time and past load as input parameters, see [1].

Artificial neural networks (ANN) are devices able to do nonlinear curve fitting. The outputs of an ANN are nonlinear functions of the inputs. These usually are load, temperature, humidity and weather. Its use in load forecasting has received a lot of attention, see [13] for a recent and exhaustive review of papers.

Fuzzy logic generalizes the classic Boolean logic by associating qualitative ranges to a number value. Therefore, this technique allows the introduction of qualitative data in load forecasting, for example in ANN, see [21].

After the deregulation process that has been carried out by the electricity industry in many countries an important additional factor that may affect load forecasting is price. So sensitivity analysis needs to consider as well demand elasticity in load forecasting.

3.2 Natural Hydro Inflows

Another important source of stochasticity in electric energy systems are natural hydro inflows. Two different techniques are used to include their stochasticity. One is *scenario generation* and the other is *scenario tree generation.*

The first tries to create plausible scenarios for future outcomes of hydro inflows. It usually resorts to time series analysis or other forecasting techniques, see [11]. The second tries to detect the internal structural dependence of the different scenarios previously generated. The scenario tree is then incorporated in multistage decision tools, which are going to be described in the following section. In these models, whose resolution relies on the use of LP, NLP and MIP solvers, uncertainty given by parameters with continuous distributions complicates its resolution because of the necessity of combining simulation techniques with optimisation techniques. For that reason, the choice of an appropriate discrete distribution is crucial for obtaining good results of the associated stochastic optimisation problem.

Among the existing techniques for generating scenario trees, they appear those based on *moment adjustment* [14]. These techniques consist of minimizing the distance between statistical properties of the discrete outcomes given by the scenario tree and those of the underlying distribution. This minimization is carried out through the resolution of a NLP problem. Although this method has been extended to multistage and multivariate distributions [15], the nonlinearity of resulting mathematical problem experiences difficulties

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[2] AutoRegressive Moving Average.
[3] AutoRegressive Integrated Moving Average.

when a large number of time periods and dimensions in the multivariate distribution needs to be approximated. Another type of methods uses *clustering techniques* to generate the scenario tree [19,16]. This technique adapts iteratively the tree branches to the original data series as a function of its vicinity to a series randomly chosen.

4 Decision Making Methods

In this section we present some of the mathematical methods used to incorporate the uncertainty in the decision support process in electric energy systems. These techniques are *reliability* and *stochastic optimisation.*

4.1 Reliability

It is evident that cost and reliability criteria can be conflicting. A strict reliability criterion may derive in over investment. On the other hand, under investment usually leads to highly unreliable systems. Reliability evaluation in electric energy systems has been for many years an area of research, see the classical reference book from Billinton and Allan [4]. Recently, under deregulated electricity markets it has been a renewed interest in the topic due not only to the recent important blackouts occurred in several systems (for example, in New York, UK and Italy in 2003) but also to new concepts like transmission open access that are being explored. Even more, networks are currently led to operate close to physical limits. The main objective of reliability is to determine some measures or criteria to be used in generation and network capacity or operation planning.

Important aspects to be considered in reliability evaluation are:

1. Load forecast and capability of the system to supply it
2. Possible generator locations for new generators, generation commitment and maintenance scheduling and other unit requirements including fuel availability
3. Possible contingencies in generation or transmission systems and ways to alleviate them

Generation reliability is usually evaluated by analytical methods such as *probabilistic production simulation*, see the seminal papers of Baleriaux [3] and Booth [5] and a comparison of algorithms in [17]. This technique is based on obtaining the cumulative distribution function of the sum of random variables corresponding to load and generation unit failures. Dispatch of generating units is made by iteratively convoluting the random variables. The most common reliability measures obtained by this method are loss of load probability (LOLP) and expected energy not served (EENS). These reliability indexes are frequently used as adequacy criteria for generation expansion and operational planning. For example, a classical planning criterion used for generation expansion has been to have a LOLP lower than 1 day in 10 years.

However, the method only considers the forced outage rate of the units and ignores the frequency and duration of these outages and the operating constraints that play a significant role in short term operations, for example, startup and shutdown time and minimum uptime and downtime. *Monte Carlo simulation* can incorporate some of these characteristics in probabilistic simulation models [22] or in chronological or sequential planning models [10].

Monte Carlo simulation with *variance reduction techniques* (VRT) is also used to evaluate generation and transmission composite reliability, see references [18,6]. Control and antithetic variables are some of the VRT frequently used.

4.2 Stochastic Programming

Within a decision-making framework, many problems can be posed as optimisation problems. This way of modelling considers a set of decision variables, relations among these variables (termed constraints) and an expression of the variables whose value needs to be optimised (the objective function). A problem set in this form is known as a mathematical programming problem. The algebraic expressions that form the constraints and objective function may lead to a LP or NLP problem. Additionally, the nature of the decision variables leads to a continuous problem or to a mixed-integer one. These problems are solved by using a collection of algorithms that are the wide-range subject of research of mathematical programming community. These algorithms include simplex methods, branch & bound methods, methods of feasible directions, etc. From a practical point of view, there exists a wide collection of algorithms already implemented in computer codes available for being used by decision makers. In addition, current algebraic languages give the possibility of modelling a mathematical programming problem and test these algorithms quickly.

The difficulty of the resolution of mathematical programming problems increases when stochasticity is introduced in the problem parameters. The introduction of uncertainty in the context of energy planning is aimed at providing a collection of optimal decisions that have to be taken prior to uncertainty disclosure. This type of stochastic problem is usually denoted as two-stage program and its purpose is to give a solution, which hedges against the uncertain future. This is the most extended way of dealing with uncertainty. There also exist other methods, like those of probabilistic constraints, which produce a solution of a mathematical program such that their constraints are satisfied with some given probability.

Random parameters in stochastic programming (SP) appear as scenarios. The use of scenarios is extended and is a common way of representing stochasticity in multistage problems. These scenarios share part of their stochastic information and create a graph structure, which is denoted in the literature as scenario tree. Contrary to deterministic problems, for which a collection of well-studied algorithms exists, for the moment there is no algorithm that outstands as the leading algorithm to solve stochastic problems. Users and researches are focused on the resolution of the deterministic equivalent problem and in the combination of decomposition techniques to create ad hoc algorithms for specific problems. SP has been widely used as a mathematical programming technique for planning under uncertainty in electric energy systems. Next section describes some examples that deal with uncertainty in different ways depending on the time scope of the model.

5 Characteristic Models

The type of stochastic parameters that enter within energy planning mathematical programming models heavily depends on the considered model. This section reviews classical models, focusing on the presence of stochasticity:

- a unit commitment (UC) model
- a market-based unit-commitment model
- a mid-term hydrothermal coordination model
- a mid-term risk management model

With these models, we try to introduce the treatment given in SP to random parameters like demand, hydro inflows and fuel prices.

Short-term models consider uncertainty in electricity demand. A classical cost-minimization UC model considers uncertainty in the chronological weekly load demand. A market unit-commitment model represents competitors' behaviour by means of their residual demand curve. Uncertainty in competitors' behaviour can be represented as a discrete random variable whose values are the different residual demand curves. In mid-term models, besides uncertainty in demand information, models incorporate uncertainty of hydro inflows and fuel costs. Typically, hydrothermal models use SP to obtain robust decisions for the set of future hydro scenarios. The use of SP is also necessary for risk management models. Finally, stochasticity in fuel costs is employed in one of the presented problems to model future contracts with the purpose of exercising control over minimum benefit scenarios.

The authors have developed the models presented in this section and their references are given in the corresponding sections. These models have been implemented in computer applications and applied to the Spanish electric system.

5.1 Unit Commitment

This problem has to decide the set of generating units that need to be committed as well as their generation levels. In these problems, total variable cost is minimized. Demand appears in classical models as a known parameter and the problem decides the subset of committed units that will provide the required demand. This modelling reflects the traditional regulation framework where an Independent System Operator (ISO) orders to the different companies the amount of energy they had to produce.

The operating cost of thermal units is modelled as a straight line with a fixed operating cost (the intercept) and a variable cost (the slope). This operating cost represents the fuel and operation and maintenance costs.

A weekly model is interpreted as a multiperiod problem where each period comprises a set of hours. A possibility is to consider one period for each hour, summing up 168 periods. The nature of the decision variables turns this optimisation problem into a mixed-integer one. Variables that represent the commitment status of the units are binary and those that represent operating levels are continuous.

The remaining section describes the algebraic model of a weekly UC problem. Consider the following collection of sets, indices, parameters and variables.

Sets

T	Set of periods
I	Set of thermal units

Indexes

t	Index for periods
h	Auxiliar index for periods
i	Index for thermal units

Parameters

D_t	Demand of period t	[MW]
R_t	Spinning reserve coefficient for thermal production in period t	[%]
Dur_t	Duration of period t	[h]
P_i^{max}	Maximum rated capacity of thermal unit i	[MW]
P_i^{min}	Minimum rated capacity of thermal unit i	[MW]
L_i^{up}	Upwards ramp limit of thermal unit i	[MW/h]
L_i^{down}	Downwards ramp limit of thermal unit i	[MW/h]
F_i	Fixed operating cost of thermal unit i	[€/h]
V_i	Variable cost of thermal unit i	[€/MWh]
C_i^{up}	Startup cost of thermal unit i	[€]
C_i^{down}	Shutdown cost of thermal unit i	[€]
τ_i	Minimum uptime of thermal unit i	[h]
κ_i	Minimum downtime of thermal unit i	[h]

Variables		
p_{ti}	Operating level of thermal unit i in period t	[MW]
u_{ti}	Commitment status of thermal unit i in period t	{0,1}
s_{ti}^{up}	Startup decision of thermal unit i in period t	{0,1}
s_{ti}^{down}	Shutdown decision of thermal unit i in period t	{0,1}

The UC problem must satisfy the load profile in each load level considered

$$\sum_{i=1}^{I} p_{ti} = D_t \qquad \forall t \tag{1}$$

requiring a spinning reserve operating margin that can be modelled as

$$\sum_{i=1}^{I} (P_i^{max} u_{ti} - p_{ti}) \geq R_t D_t / 100 \qquad \forall t \tag{2}$$

Each thermal unit operating level is bounded between its minimum and maximum rated capacity

$$P_{ti}^{min} u_{ti} \leq p_{ti} \leq P_{ti}^{max} u_{ti} \qquad \forall t, i \tag{3}$$

Variation in a thermal unit power generation is controlled through the ramp constraints

$$L_i^{down} Dur_t \leq p_{ti} - p_{t-1i} \leq L_i^{up} Dur_t \qquad \forall t, i \tag{4}$$

Startup and shutdown decisions are managed with the following constraints

$$u_{ti} - u_{t-1\ i} = s_{ti}^{up} - s_{ti}^{down} \qquad \forall t, i \tag{5}$$

Some advanced UC models include minimum uptime and downtime requirements for switched-on and switched-off thermal units. Committed units are usually required to produce a minimum number of hours before they can stop. Similarly, once they stop, they must also remain offline a minimum number of hours, before they can produce again. These minimum uptime and downtime requirements can be formulated as follows:

$$u_{t+h_t\ i} \geq u_{ti} - u_{t-1\ i} \qquad \forall t, h_t, i \tag{6}$$

$$u_{t+h_t\ i} \leq 1 + u_{ti} - u_{t-1\ i} \qquad \forall t, h_t, i \tag{7}$$

where the set of shifted indexes, controlled by h_t, maybe reduced for those values $h_t \geq 1$ such that

$$\tau_i \leq \sum_{l=0}^{h_t-1} Dur_{t+l} \tag{8}$$

$$\kappa_i \leq \sum_{l=0}^{h_t-1} Dur_{t+l} \tag{9}$$

Given the above variables and constraints, the UC model minimizes the total variable operating cost, given as:

$$\sum_{t=1}^{T}\sum_{i=1}^{I}(Dur_t F_i u_{ti} + Dur_t V_i p_{ti} + C_i^{up} s_{ti}^{up} + C_i^{down} s_{ti}^{down}) \tag{10}$$

Uncertainty in a weekly UC model appears in the randomness of demand load profiles that a generation company faces. For this reason, instead of formulating a single-scenario problem, the company may analyze its decision-making problem by means of a SP problem. Stochasticity in demand profiles can be modelled as a discrete random variable in the form of a scenario tree. A load profile scenario tree is presented in figure 1. It is represented the possible evolution of the demand for a week that begins on Tuesday. It is not considered being uncertain along the very first day. For the second day, Wednesday, two branches appear. These branches branch at the end of the second day producing four scenarios that represent the evolution of the demand profiles for the remaining days of the week.

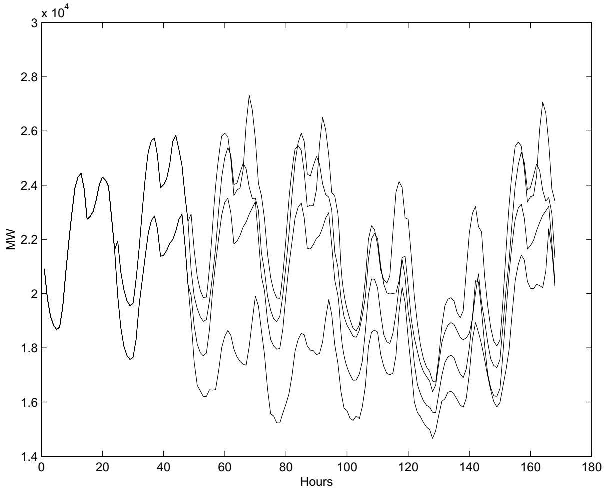

Fig. 1. Load demand profile scenario tree.

5.2 Market-Based Unit Commitment

Classical UC problem changes dramatically if the company operates in an electricity spot market. In this new framework, companies are responsible of their total production, which is no longer decided by an ISO. A day-ahead market is the market that takes place one day before the physical delivery of power production. This market is based on offers submitted by power producers and bids submitted by power purchasers. Offers and bids indicate the price at which producers are willing to sell and purchasers to buy.

In this new context, the objective function of an energy planning problem changes from the traditional cost minimization to a maximization of the company's benefit.

The company's benefit $B(p)$ is defined as the difference between revenues and operating cost $c(p)$. In addition, company's incomes depend on the market price π at which the energy p is sold.

$$B(p) = \pi p - c(p) \tag{11}$$

The energy price is a function of the total amount of energy sold. Similarly, the energy amount that each company is able to sell depends on the final price. Observe that the energy demand (understood as a function of price) needs to be equal to the energy supplied (also understood as a function of price).

$$D(\pi) = \sum_{agents} S^{agents}(\pi) \tag{12}$$

Under the assumption that competitor's behaviour is given by their supply energy functions, the amount of power a single company is able to sell depends on the demand at that price, $D(\pi)$, and the offers of the rest of agents, $S^{rest}(\pi)$

$$R(\pi) = D(\pi) - \sum_{rest} S^{rest}(\pi) \tag{13}$$

expression that gives the residual demand faced by the company, $R(\pi)$. The company's benefit is now given as

$$B(p) = R^{-1}(p)p - c(p) \tag{14}$$

The inverse residual demand function is a staircase function that can be approximated by means of a piecewise linear function. The revenue function is also a non-concave function that can be modelled as a piecewise linear function fig.2). This function is modelled by considering a collection of binary variables to represent the total amount of energy produced as a sum of the quantities of each segment. Price and revenue values can also be modelled in the same way.

Uncertainty is again a relevant ingredient of these new market-based UC models. However, the main source of uncertainty is now the wholesale electricity market, because the decisions made by the rest of agents are not known

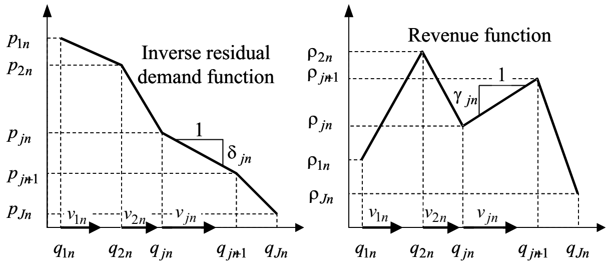

Fig. 2. Piecewise linear residual demand function and revenues' function.

in advance. This uncertainty is implicit into the residual demand function that may be considered a random variable within a SP problem.

When having a completely known residual demand function, the benefit maximization problem is deterministic. This problem determines company's optimal production and price for selling that production. However, if a random residual demand function is given, the benefit maximization problem turns into a SP problem. It should provide an optimal quantity for each one of the residual demand functions involved. This obeys the rules of a supply energy function, although additional conditions about non decreasing values need to be imposed.

The multistage stochastic problem we are about to present considers a realization of uncertainty to be a set of residual demand functions, one function for each period of the problem scope (fig. 3). The reader should note the difference in uncertainty management in this model with respect to that of the weekly UC model and forthcoming models.

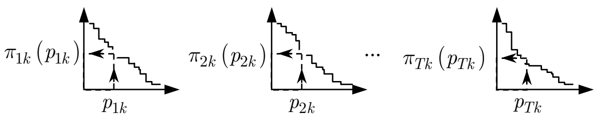

Fig. 3. Single scenario of residual demand functions.

Consider the next collection of sets, indexes, parameters, variables and constraints used in the formulation of a market-based UC problem.

Sets	
T	Set of periods
I	Set of thermal units
J	Set of segments to represent the residual demand function
K	Set of scenarios

Indexes	
t	Index of periods
i	Index of thermal units
j	Index of segments
k	Index of scenarios

Deterministic parameters		
Dur_t	Duration of period t	[h]
P_i^{max}	Maximum rated capacity of thermal unit i	[MW]
P_i^{min}	Minimum rated capacity of thermal unit i	[MW]
L_i^{up}	Upwards ramp limit of thermal unit i	[MW/h]
L_i^{down}	Downwards ramp limit of thermal unit i	[MW/h]
F_i	Fixed operating cost of thermal unit i	[€/h]
V_i	Variable cost of thermal unit i	[€/MWh]
C_i^{up}	Startup cost of thermal unit i	[€]
C_i^{down}	Shutdown cost of thermal unit i	[€]

Stochastic parameters		
δ_{tj}^k	Slope of segment j of the residual demand function in period t and scenario k	[€/MW]
γ_{tj}^k	Slope of segment j of the revenue function in period t and scenario k	[€/MW]
π_{tj}^k	Price at segment j of the residual demand function in period t and scenario k	[€]
$\bar{p}_{tj}^k$	Quantity at segment j of the residual demand function in period t and scenario k	[MW]
$\bar{b}_{tj}^k$	Benefit at segment j of the revenue function in period t and scenario k	[€]
$Prob^k$	Probability of scenario k	

The load demand constraints adopt the next expression in this case

$$\sum_{i=1}^{I} p_{ti}^k = p_t^k \qquad \forall t, k \tag{15}$$

where the total amount of energy produced p_t^k in period t and scenario k is modelled by

$$p_t^k = \bar{p}_{t0}^k + \sum_{j=1}^{J-1} p_{tj}^k \tag{16}$$

The total revenue is modelled as a piecewise linear function similarly to the total amount of energy produced.

Variables

v_{tj}^{k}	Binary variable corresponding to segment j in period t and scenario k	{0,1}
p_{t}^{k}	Total production in period t and scenario k	[MW]
p_{tj}^{k}	Total production of segment j in period t and scenario k	[MW]
p_{ti}^{k}	Operating level of thermal unit i in period t and scenario k	[MW]
π_{t}^{k}	Price in period t and scenario k	[€]
b_{t}^{k}	Benefit in period t and scenario k	[€]
u_{ti}^{k}	Commitment status of thermal unit i in period t and scenario k	{0,1}
$s_{ti}^{up\ k}$	Startup decision of thermal unit i in period t and scenario k	}0,1}
$s_{ti}^{down\ k}$	Shutdown decision of thermal unit i in period t and scenario k	}0,1}
$x_{t}^{kk'}$	Binary variable related with monotonicity of the supply function in period t and scenarios k and k'	{0,1}

$$b_t^k = \bar{b}_{t0}^k + \sum_{j=1}^{J-1} \gamma_{tj}^k p_{tj}^k \tag{17}$$

as well as the price obtained when considering the optimal production p_t^k in period t and scenario k.

$$\pi_t^k = \bar{\pi}_{t0}^k + \sum_{j=1}^{J-1} \delta_{tj}^k p_{tj}^k \tag{18}$$

This piecewise linear modelling requires the next constraints, which force a monotonic use of variables representing segment values.

$$(\bar{p}_{tj}^k - \bar{p}_{tj-1}^k) v_{t+1\ j}^k \leq p_{tj}^k \leq (\bar{p}_{tj}^k - \bar{p}_{tj-1}^k) v_{tj}^k \tag{19}$$

$$v_{tj}^k \geq v_{tj+1}^k \qquad j = 1, \ldots, J-1 \tag{20}$$

Due to uncertainty, limits for thermal units power output are introduced for any of the scenarios considered. Similarly, ramp constraints and startup and shutdown constraints are independently introduced for each scenario k.

$$P_{ti}^{min} u_{ti}^k \leq p_{ti}^k \leq P_{ti}^{max} u_{ti}^k \qquad \forall t, k \tag{21}$$

$$L_i^{down} Dur_t \leq p_{ti}^k - p_{t-1\ i}^k \leq L_i^{up} Dur_t \qquad \forall t, k \tag{22}$$

$$u_{ti}^{k} - u_{t-1\,i}^{k} = s_{ti}^{up\,k} - s_{ti}^{down\,k} \qquad \forall t, k \tag{23}$$

The former set of constraints is the core of the market-based UC problem with stochasticity in the parameters modelling the residual demand functions. As it has already been commented, the optimal solution provided by this SP problem is a set of quantities and prices that form an offer curve. This curve has to be non decreasing. The following set of constraints is introduced into the model for that reason.

$$p_t^k - p_t^{k'} \geq -x_t^{kk'} M^p \qquad \forall t, k, k' \quad k' > k \tag{24}$$

$$\pi_t^k - \pi_t^{k'} \geq -x_t^{kk'} M^\pi \qquad \forall t, k, k' \quad k' > k \tag{25}$$

$$p_t^k - p_t^{k'} \geq -(1 - x_t^{kk'}) M^p \qquad \forall t, k, k' \quad k' > k \tag{26}$$

$$\pi_t^k - \pi_t^{k'} \geq -(1 - x_t^{kk'}) M^\pi \qquad \forall t, k, k' \quad k' > k \tag{27}$$

The SP model is completed with the objective function that maximizes the expected benefit.

$$\max \sum_{t=1}^{T} \sum_{k=1}^{K} Prob^k [b_t^k - c(p_t^k)] \tag{28}$$

where $c(p_t^k)$ indicates the production cost in each period t and scenario k. This cost can be modelled as it has been presented in previous section. The optimal solution for this problem is an offer curve for each period (fig.4). For simplicity in the exposition, a pure thermal generating system has been considered. However, the model has been extended to more complex systems comprising hydro units as well as futures and options [2]. It is necessary to outline that building the offer curve necessarily implies the consideration of stochasticity. This model represents uncertainty in a different way that the weekly UC problem and the next models, where stochasticity is introduced by means of a scenario tree.

5.3 Hydrothermal Coordination

A hydrothermal coordination model considers a generating system with thermal units as well as hydro units, see [8] for further details. Hydro units provide the capability for energy reserve management. In hydrothermal models, a constant coefficient of efficiency for each hydro unit is usually considered and hydro reserves are expressed in terms of energy stored, in MWh. A difference between short-term models and mid-term models appears in the way of

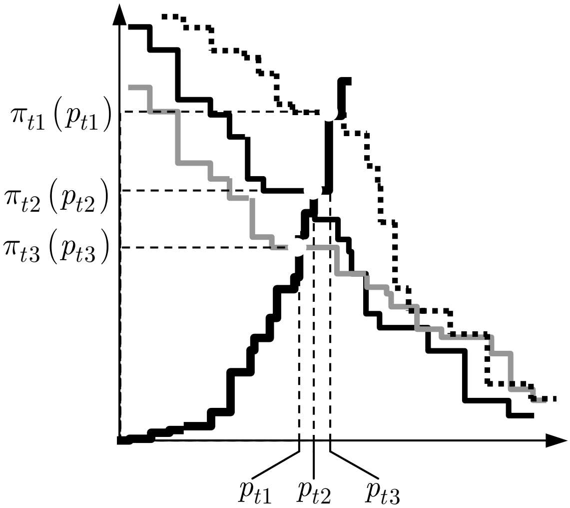

Fig. 4. Stochastic residual demand function and offer curve for a single period.

considering electricity demand. Short-term models usually consider a chronological load profile, while mid-term models use to represent the demand aggregated. Thus, mid-term models usually gather the demand in blocks of peak, shoulder and off-peak hours. Another difference with short-term models appears in the stochastic parameters considered. In short-term models, demand profile (together with units' outage) is the main source of uncertainty. In mid-term models, hydro inflows and fuel costs represent additional sources that must be taken into account when looking for optimal solutions to hedge against uncertainty. In a mid-term model, like the presented in this section, stochasticity enters as a scenario tree. Figure 5 shows a hydro inflows scenario tree. The tree represents an initial inflow value that branches into two possibilities in the second month of the model. The scenario tree branches again in the second and third months producing a final eight-scenario tree.

One of the objectives of a mid-term model is to schedule hydro reserves. A model that minimizes the expected operation cost over the complete time scope can achieve this. Hydro units have a very low cost that is usually neglected. Operating cost is limited to variable costs of thermal units. Reservoir levels are bounded in order to prevent spillage and dramatic scenarios of low reserves.

Thus, hydrothermal models include equations that represent the evolution of the reserves. Let us consider the next collection of sets, indexes, parameters and variables in order to model them.

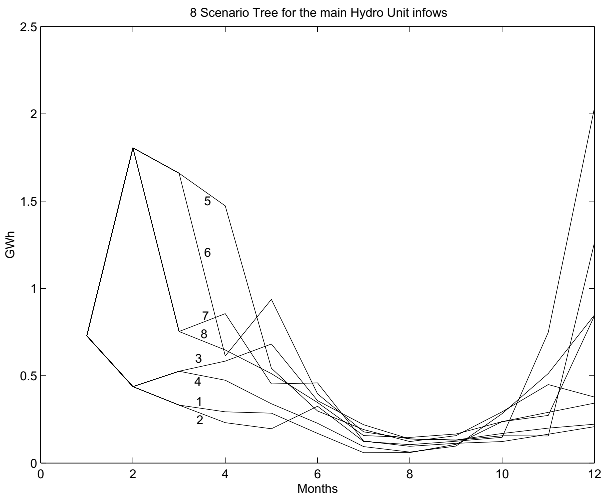

Fig. 5. Scenario tree for hydro inflows to a reservoir.

Sets	
T	Set of periods
H	Set of hydro units
K	Set of scenarios

Indexes	
t	Index of periods
h	Index of hydro units
k	Index of scenarios

Deterministic parameters		
R_j^{max}	Maximum storage capacity of hydro reserve j	[MWh]
R_j^{min}	Minimum storage capacity of hydro reserve j	[MWh]
L_j^{max}	Maximum rated capacity of hydro unit j	[MW]
L_j^{min}	Minimum rated capacity of hydro unit j	[MW]
ρ_j	Pumping efficiency of hydro unit j	[%]

Stochastic parameters		
I_j^k	Natural inflows of hydro unit j in period t and scenario k	[MWh]
$Prob^k$	Probability of scenario k	

Variables		
r_{tj}^k	Level of hydro reserve j in period t and scenario k	[MWh]
s_{tj}^k	Production level of hydro unit j in period t and scenario k	[MW]
ω_{tj}^k	Pumping level of hydro unit j in period t and scenario k	[MW]

As mentioned, scheduling of hydro reserves can be obtained by introducing some constraints that represent the dynamics of water reserve evolution.

$$r_{tj}^k = r_{t-1,j}^k + I_j^k - Dur_t(s_{tj}^k - \rho_j \omega_{tj}^k)/100 \tag{29}$$

with

$$R_j^{min} \leq r_{tj}^k \leq R_j^{max} \tag{30}$$

$$L_j^{min} \leq s_{tj}^k \leq L_j^{max} \tag{31}$$

$$L_j^{min} \leq \rho_j \omega_{tj}^k/100 \leq L_j^{max} \tag{32}$$

A stochastic mid-term hydrothermal coordination problem gives the possibility of verifying the reserve evolution for the set of hydro scenarios analyzed. The SP problem provides a solution for the first stage that does not anticipate the uncertainty given by natural hydro inflows. An example of this solution is given in the next figure 6. It is depicted the evolution of the hydro reservoir storage level for the hydro unit whose natural inflows are given in figure 5.

5.4 Risk Management Model

Risk is implicit to all activities that take place in energy operation business and planning activities must consider this risk. SP is a suitable tool to carry on with this risk, which appears under different forms depending on the activity considered. As outlined at the beginning of the chapter, short-term operation suffers from the unit failure risk and demand fluctuation. Mid-term operation has to deal with uncertain hydro inflows and fuel prices, see for example [7], and long-term models pay careful attention to different factors, for example demand evolution and regulatory changes.

A risk management model controls the variability of the random variable that represents the operating cost function or the profit function. A variety of methods to measure risk can be introduced into a SP problem. A possibility consists of penalizing those scenarios in which the company cost is greater than a certain reference cost. Similarly, those scenarios whose profits are less than a certain reference profit can be penalized.

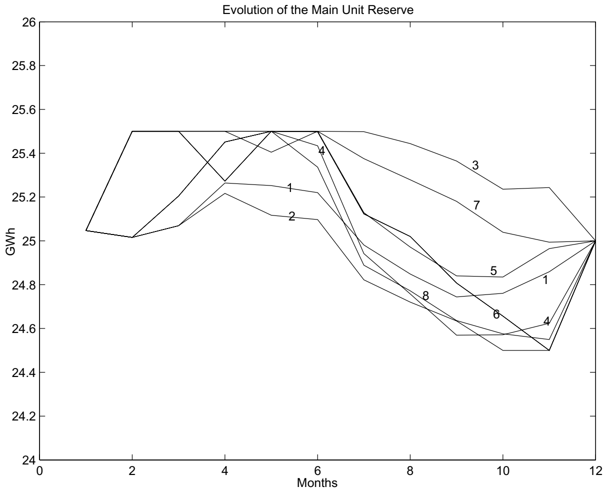

Fig. 6. Reserve evolution of a stochastic mid-term hydrothermal model.

Another alternative consists on introducing a hard limit for the quantile of the distribution function at a given confidence level. This quantile is usually referred to as Value at Risk (VaR) in risk management models. VaR has the additional difficulty, for SP problems, that it requires the use of binary variables for its modelling. Conditional Value at Risk (CVaR) computes the average of scenario profit values that lie under the quantile given by the VaR. CVaR computation does not require the use of binary variables and it can be modelled by the simple use of linear constraints. Figure 7 illustrates the concepts of VaR and CVaR.

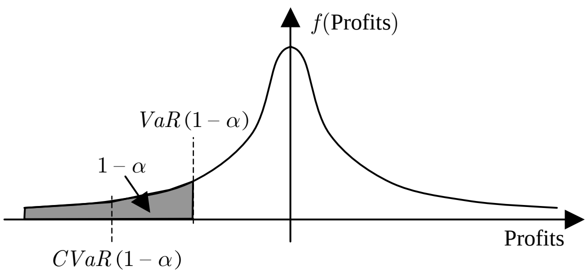

Fig. 7. VaR and CVaR illustration.

A SP model that incorporates risk measures obtains a final solution (cost or profit random variable) with less volatility than the final solution of a model that does not incorporate any measure of risk control. This is observed in the next figure 8. Different distribution functions are depicted for the profit random variable in a mid-term operation planning. It can be observed that the higher the upper limit imposed to the CVaR, the more concentrate the scenarios' profit values. In the following figure 9 the efficient frontier curve is obtained for expected profit and CVaR for the same case.

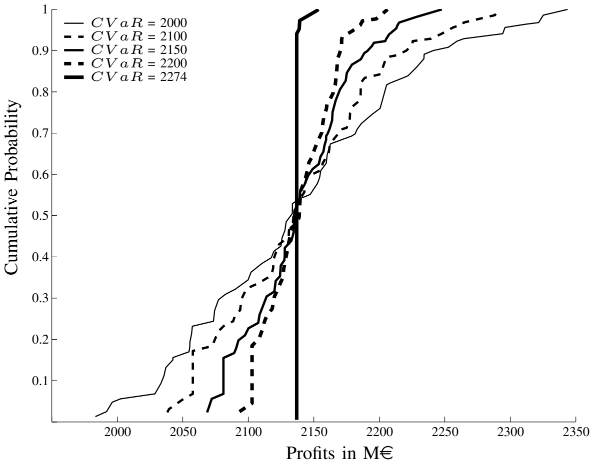

Fig. 8. Comparison of profit's distributions.

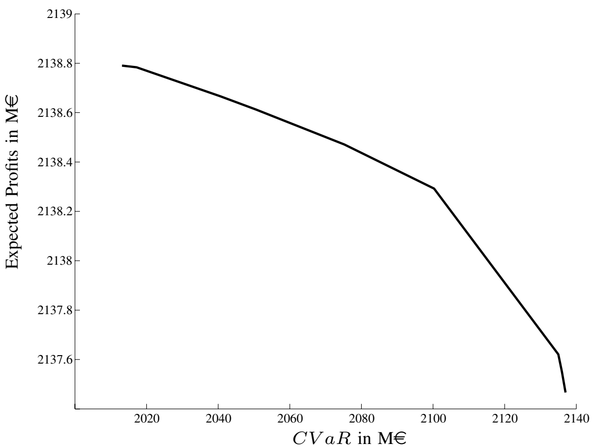

Fig. 9. Efficient frontier.

6 Conclusions

Electric energy systems have been for long time a continuous source of advances and applications of planning under uncertainty and a test bed for many developments to include stochastic parameters. In this chapter, we have presented a review and summary of the impact that the uncertainty may have in electric energy systems. We have presented the methods used to estimate the main stochastic parameters to be considered in power systems, namely demand and hydro inflows. Then, we have examined the two main methodologies that deal with uncertainty. One is reliability computation and the other is stochastic optimisation. Finally, we have presented some characteristic models that include an explicit treatment of parameter uncertainty.

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