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Ranking stability under uniform preference intensification in pairwise comparison matrices

Luis Ángel Calvo [a,b,*], Jiří Mazurek [c]

[a] Department of Quantitative Methods, Comillas Pontifical University, C/ Alberto Aguilera 23, 28015 Madrid, Spain
[b] Institute for Research in Technology (IIT), Comillas Pontifical University, C/ Santa Cruz de Marcenado 26, 28015 Madrid, Spain
[c] School of Business Administration in Karviná, Silesian University in Opava, Univerzitnínáměstí 1934/3, 733 40 Karviná, Czech Republic

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ABSTRACT

This paper examines ranking stability in multiplicative pairwise comparison matrices under uniform preference intensification, represented by the entrywise power transformation $A \mapsto A^{(k)} = [a_{ij}^k]$. The associated invariance requirement is known as scale invariance; here, we study its failure at the level of the induced ranking, referred to as intensity-of-preference rank reversal. We combine theoretical observations, illustrative examples, and Monte Carlo experiments to analyse how this phenomenon depends on matrix order, inconsistency, and the priority derivation method. The row geometric mean method is used as the known scale-invariant benchmark, since its ranking is preserved under uniform intensification. In contrast, the eigenvector method and several other commonly used procedures may change the induced ranking, including the top-ranked alternative. The simulations indicate that such instability becomes more frequent as the number of compared objects increases, persists even among matrices satisfying conventional consistency-ratio thresholds, and differs substantially across priority derivation methods. These results show that robustness to uniform preference intensification is distinct from consistency screening and should be considered separately when evaluating priority derivation methods.

1. Introduction

Multi-criteria decision-making (MCDM) methods are widely used to structure and support complex decisions in engineering, management, and public policy. Among them, the Analytic Hierarchy Process (AHP) [1] remains one of the most influential and extensively studied approaches. Despite its popularity, AHP has long been associated with various forms of ranking instability, most notably rank reversal triggered by the addition or removal of alternatives [2–4].

Here, we study a different source of instability. Consider a fixed decision set and assume that a decision maker's ordinal judgements do not change: alternative i is preferred to j, or tied with j, exactly as before. Only the numerical strength of every strict preference is uniformly increased or decreased. Should the resulting ranking remain the same?

In multiplicative pairwise comparisons, such a uniform recalibration corresponds to the entrywise power transformation

$$A = [a_{ij}] \mapsto A^{(k)} = [a_{ij}^k], \qquad k > 0,$$

which we call *uniform preference intensification*. If a priority derivation method changes the ranking after such a transformation, we call the phenomenon *intensity-of-preference (IOP) rank reversal*. The ordinal preference pattern is unchanged, but the ranking of alternatives may change because the numerical calibration of preference intensities changes.

The invariance requirement underlying this question is not new. Genest et al. (1993) had already shown, in the context of eigenvector scaling of ordinal pairwise preferences, that rankings may depend on the numerical ratios used to encode the same ordinal information. Related effects for incomplete pairwise comparisons were later discussed by Csató and Rónyai [5]. Petróczy and Csató (2021) subsequently formalised the corresponding requirement as scale invariance, established the contrast between the scale-sensitive eigenvector method and the scale-invariant row geometric mean method, and illustrated its practical relevance through a Formula One revenue allocation example. Thus, the possibility that eigenvector-based rankings may react to a uniform recalibration of preference intensities is already known. In the present paper, the term IOP rank reversal is used only to describe the induced rank-level change, not to introduce a new invariance axiom.

The present paper provides a systematic rank-level analysis of uniform preference intensification in pairwise comparison matrices. After

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[*] Corresponding author.
E-mail address: lacalvo@comillas.edu (L.Á. Calvo).

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recalling the priority derivation methods considered, we relate the proposed terminology to the existing scale invariance framework. We then use a NASA capability prioritisation case [6] as an applied illustration and provide low-dimensional examples showing that rank changes may affect not only intermediate positions but also the top-ranked alternative.

The empirical part of the paper quantifies the phenomenon by Monte Carlo simulations. In discrete and continuous settings, we examine how the frequency of IOP rank reversal varies with matrix order, baseline inconsistency, and the priority derivation method. In addition to changes in the induced ranking, we record whether the top-ranked alternative changes, since this is directly relevant in applications where the main objective is to select the best option.

The paper also provides a homogeneity-based interpretation of intensification-stable rules. Within a natural covariance class, uniform intensification can be represented, after a monotone change of scale, as degree-one homogeneity. This recovers the known scale invariance of multiplicatively separable procedures, such as the row geometric mean method, and helps distinguish them from procedures based on sums, normalisations, or globally coupled criteria. Overall, the paper extends the existing scale invariance perspective to a broader rank-level comparison across priority derivation methods, matrix orders, inconsistency levels, and top-ranked alternative changes.

The paper is organised as follows. Section 2 reviews the notation, priority derivation methods, and uniform preference-intensification framework used throughout the paper. Section 3 relates the IOP rank-reversal terminology to scale invariance, presents the NASA Gateway capability prioritisation case as an applied illustration, provides low-dimensional examples of top-rank reversal, and develops the homogeneity interpretation underlying rank preservation under uniform intensification. Section 4 reports the Monte Carlo experiments. Section 5 discusses the empirical patterns, the distinction between consistency and robustness, and the dependence on intensification strength. Finally, Section 6 summarises the main findings.

2. Preliminaries

This section introduces the notation, basic concepts, and priority derivation methods used throughout the paper.

2.1. Pairwise comparison matrices

Let $n \geq 2$ denote the number of objects (typically criteria or alternatives) to be compared pairwise. For each ordered pair (i, j), let $c_{ij} > 0$ represent the relative importance of object i over object j. In multiplicative pairwise comparisons, c_{ij} expresses how many times object i is judged to be more important than object j. In particular, $c_{ij} > 1$ indicates a strict preference for i over j, while $c_{ij} = 1$ denotes indifference.

All pairwise judgements can be collected in an $n \times n$ *pairwise comparison matrix* (PCM)

$$C = \begin{bmatrix} 1 & c_{12} & \cdots & c_{1n} \\ c_{21} & 1 & \cdots & c_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ c_{n1} & c_{n2} & \cdots & 1 \end{bmatrix}.$$

A PCM is said to be *reciprocal* if

$$c_{ij} = \frac{1}{c_{ji}}, \qquad \forall\, i, j \in \{1, \dots, n\}. \tag{1}$$

Reciprocity is a standard assumption in the Analytic Hierarchy Process and related pairwise comparison methods.

A reciprocal matrix is *consistent* if

$$c_{ij}\, c_{jk}\, c_{ki} = 1, \qquad \forall\, i, j, k \in \{1, \dots, n\}. \tag{2}$$

For reciprocal PCMs, consistency is equivalent to the existence of a positive weight vector $w = (w_1, \dots, w_n)$ such that $c_{ij} = w_i/w_j$ for all i, j. In practice, however, perfect consistency is rarely achieved due to judgemental noise and cognitive limitations.

In AHP, a standard summary measure of inconsistency is the consistency ratio CR. For a reciprocal PCM C, let $\lambda_{\max}$ denote its principal eigenvalue. The consistency index is defined as

$$\mathrm{CI} = \frac{\lambda_{\max} - n}{n - 1},$$

and the consistency ratio is

$$\mathrm{CR} = \frac{\mathrm{CI}}{\mathrm{RI}_n},$$

where RI_n is Saaty's random index for matrices of order n [1]. The conventional AHP rule of thumb regards matrices with $\mathrm{CR} < 0.10$ as acceptably consistent.

The output of a priority derivation method applied to a PCM C is a *priority vector* $w = (w_1, \dots, w_n)$, usually normalised so that $\sum_i w_i = 1$, assigning a numerical weight to each object.

2.2. Priority derivation methods

The most widely used weighting procedure in the AHP is the *eigenvector method* (EVM), introduced in the AHP context by Saaty [1,7]. Under the EVM, the priority vector w is defined as the principal right eigenvector of C:

$$Cw = \lambda_{\max} w, \tag{3}$$

where $\lambda_{\max}$ denotes the principal eigenvalue of C. By the Perron–Frobenius theorem, w can be chosen strictly positive and is unique up to a multiplicative constant. The resulting vector is typically normalised so that $\sum_i w_i = 1$.

An alternative and extensively studied approach is the *geometric mean method* (GMM), also known as the row geometric mean method (RGMM) or the logarithmic least-squares method (LLSM). The method was studied and popularised in the pairwise-comparison literature by Crawford and Williams [8,9]. The GMM/LLSM also admits several axiomatic characterisations; see, for example, [10–13]. Under the GMM, the unnormalised score of the ith alternative is

$$s_i(C) = \left(\prod_{j=1}^{n} c_{ij} \right)^{1/n}, \qquad i = 1, \dots, n, \tag{4}$$

followed by normalisation, e.g. $w_i = s_i(C) / \sum_{\ell=1}^{n} s_\ell(C)$.

In addition to the EVM and GMM, the paper considers the following methods. We distinguish between aggregation-based and optimisation-based procedures. The row sum, column sum and harmonic mean methods assign unnormalised scores $s_i(C)$, which are subsequently normalised to obtain a priority vector w. The cosine maximisation method is formulated as an optimisation problem over positive vectors; whenever the normalisation is not imposed in the optimisation problem itself, the resulting vector is normalised so that $\sum_i w_i = 1$.

Row sum method (RSM). This method is related to the classical row-sum procedure in pairwise comparison and ranking problems [14]. In the context of priority derivation from pairwise comparison matrices, row-based aggregation procedures are also discussed within the common framework of Choo and Wedley [15]. It assigns to each alternative the sum of the entries in the corresponding row:

$$s_i(C) = \sum_{j=1}^{n} c_{ij}, \qquad i = 1, \dots, n.$$

The priority vector is then obtained by normalising these scores, for example, by $w_i = s_i(C) / \sum_{\ell=1}^{n} s_\ell(C)$.

Column sum method (CSM). Also known as the column-normalisation method or additive normalisation, this method corresponds to the usual procedure of normalising each column of a matrix C so that it sums to one, and then averaging the rows of the column-normalised matrix. This procedure is often used as a simple approximation to AHP priorities; see, for example, Saaty [1], Saaty and Vargas [16], and the common framework of Choo and Wedley [15]. Each column of C is first normalised so that it sums to one:

$$\hat{c}_{ij} = \frac{c_{ij}}{\sum_{k=1}^{n} c_{kj}}.$$

The score of alternative i is then obtained as the row sum of the normalised matrix, equivalently n times the row average:

$$s_i(C) = \sum_{j=1}^{n} \hat{c}_{ij}, \qquad i = 1, \dots, n.$$

Since the columns of $\hat{C} = [\hat{c}_{ij}]$ sum to one, the final priority vector is obtained by $w_i = s_i(C)/n$, or equivalently by normalising the scores $s_i(C)$.

Harmonic mean method (HMM). The harmonic mean method is an aggregation-based procedure, based on applying the harmonic mean to each row of the pairwise comparison matrix; see, e.g., the common framework of Choo and Wedley [15]. It assigns to each alternative i the harmonic mean of its pairwise comparison entries:

$$s_i(C) = \left(\frac{1}{n} \sum_{j=1}^{n} \frac{1}{c_{ij}} \right)^{-1}, \qquad i = 1, \dots, n.$$

Since C is reciprocal, this can also be written as $s_i(C) = n / \sum_{j=1}^{n} c_{ji}$, so the score of alternative i is large when the average strength of the comparisons against i is small. The priority vector is then obtained by normalising these scores, for example, by $w_i = s_i(C) / \sum_{\ell=1}^{n} s_\ell(C)$.

Cosine maximisation method (CMM). The cosine maximisation method was proposed by Kou and Lin [17] as a similarity-based procedure for deriving priorities from pairwise comparison matrices. In the formulation used here, the priority vector is chosen so that the original matrix C is as close as possible, in cosine similarity, to the consistent matrix generated by w, namely $[w_i/w_j]$. Thus, $w > 0$ is obtained by solving

$$\max_{w>0,\ \sum_i w_i = 1} \frac{\sum_{i,j} c_{ij} \frac{w_i}{w_j}}{\sqrt{\sum_{i,j} c_{ij}^2} \sqrt{\sum_{i,j} \left(\frac{w_i}{w_j} \right)^2}}.$$

The normalisation $\sum_i w_i = 1$ fixes the scale of the priority vector.

Several other priority derivation methods have been proposed in the literature (see, e.g., Mazurek [18]). Comparative studies indicate that the GMM often exhibits favourable robustness and order-preservation properties when compared to the EVM, especially in the presence of inconsistency [15,19–23].

2.3. Uniform preference intensification

Let $A = [a_{ij}]$ be a reciprocal PCM and let $k > 0$. Following the power transformation underlying scale invariance [24], we consider the matrix

$$A^{(k)} := \left[a_{ij}^{k} \right]. \tag{5}$$

In the present paper, we refer to the mapping $A \mapsto A^{(k)}$ as *uniform preference intensification*. This transformation preserves reciprocity and the direction of all comparisons: $a_{ij} = 1/a_{ji}$ implies $a_{ij}^{k} = 1/a_{ji}^{k}$, and $a_{ij} > 1$ (resp. $a_{ij} < 1$) implies $a_{ij}^{k} > 1$ (resp. $a_{ij}^{k} < 1$) for any $k > 0$. Hence A and $A^{(k)}$ encode exactly the same ordinal preference pattern, while uniformly modifying the numerical intensity of every strict preference. In particular, $k > 1$ strengthens all strict preferences and $0 < k < 1$ weakens them, without changing any ties.

Transformations of the form (5) arise naturally when ordinal judgements are mapped to numerical ratios through different comparison scales, or when a given scale is globally recalibrated.

3. Intensity-of-preference rank reversal

This section recalls the scale invariance requirement of Petróczy and Csató [24] in the notation used in this paper and introduces the corresponding rank-reversal terminology. It then illustrates the associated rank-level effect with an additional applied decision-making example. We then present additional synthetic examples and develop the theoretical representation underlying rank preservation under uniform intensification.

3.1. Definition

Let $A = [a_{ij}]$ be a reciprocal pairwise comparison matrix. A priority derivation method assigns to A a positive (unnormalised) score vector $s(A) = (s_1(A), \dots, s_n(A)) \in (0, \infty)^n$, which induces a ranking $\sigma(A)$ by ordering the alternatives according to their scores.

Definition 1 (*IOP-invariance*)**.** Following the scale invariance property introduced by Petróczy and Csató [24], a priority derivation method is *invariant under uniform preference intensification* if for every reciprocal PCM A and every $k > 0$,

$$s_i(A) > s_j(A) \iff s_i(A^{(k)}) > s_j(A^{(k)}) \qquad \forall\, i, j.$$

Equivalently, the induced ranking satisfies $\sigma(A^{(k)}) = \sigma(A)$, for all $k > 0$.

We use the term *IOP rank reversal* for the failure of this scale invariance requirement, that is, for the case in which there exist a reciprocal PCM A, a parameter $k > 0$, and two alternatives i, j such that the strict order between i and j differs in A and $A^{(k)}$.

This terminology emphasises the rank-level consequence of scale sensitivity: the underlying ordinal preference pattern is preserved, but the ranking may change when the numerical intensity of all strict comparisons is uniformly rescaled. The connection with the previous literature on scale sensitivity and scale invariance has been discussed in the Introduction; here, IOP rank reversal is used only as a descriptive term for the resulting change in the induced ranking.

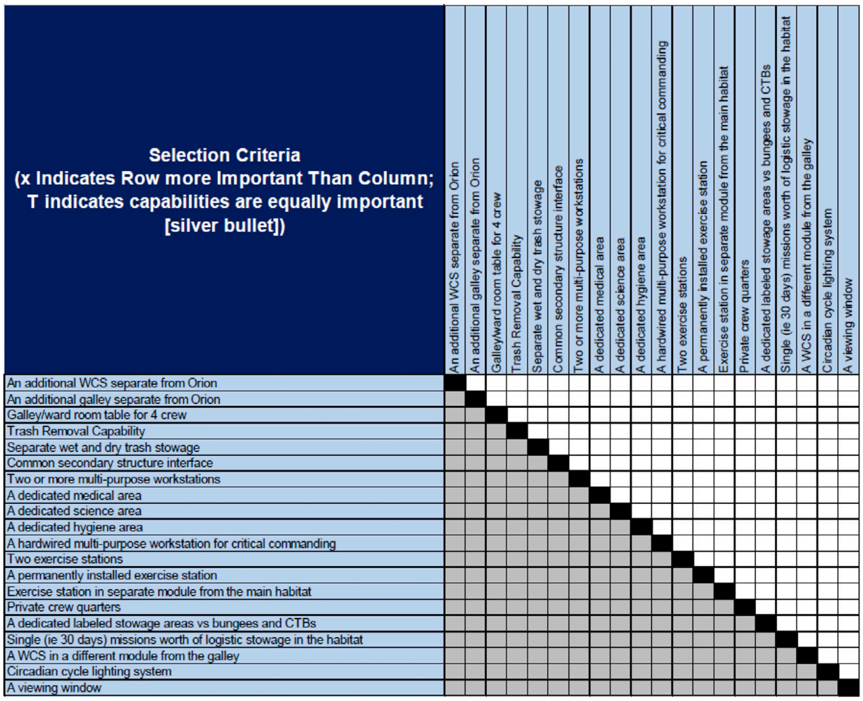
Unlike classical rank reversals triggered by adding or removing alternatives, IOP rank reversal arises with a fixed decision set and reflects dependence on preference-intensity calibration rather than on changes in the set of alternatives. The role of the present paper is to study this rank-reversal behaviour systematically across priority derivation methods, matrix order, inconsistency levels, and changes in the top-ranked alternative.

3.2. Illustrative examples of IOP rank reversal

As discussed above, [24] illustrated the practical consequences of scale sensitivity through a Formula One revenue allocation example. As an additional applied illustration from a different decision-making domain, we consider a decision-support problem reported in the NASA technical manual *Deep Space Habitability Design Guidelines Based on the NASA NextSTEP Phase 2 — Ground Test Program* [6]. In Section A.3.3, 20 candidate habitat capabilities for the Lunar Gateway are evaluated using pairwise comparisons (see Fig. 1).

Example 1 (*NASA Gateway Capability Prioritisation*)**.** To examine the effect of preference intensification, consider the first six capabilities and denote by $S > 1$ the numerical value assigned to strict preference, while $T = 1$ represents ties. The corresponding reciprocal matrix is

$$A(S) = \begin{bmatrix} 1 & S & S & 1 & S & 1 \\ 1/S & 1 & S & S & 1 & S \\ 1/S & 1/S & 1 & S & 1 & S \\ 1 & 1/S & 1/S & 1 & S & S \\ 1/S & 1 & 1 & 1/S & 1 & S \\ 1 & 1/S & 1/S & 1/S & 1/S & 1 \end{bmatrix}.$$



Selection Criteria (x Indicates Row more Important Than Column; T indicates capabilities are equally important [silver bullet])	An additional WCS separate from Orion	An additional galley separate from Orion	Galley/ward room table for 4 crew	Trash Removal Capability	Separate wet and dry trash stowage	Common secondary structure interface	Two or more multi-purpose workstations	A dedicated medical area	A dedicated science area	A dedicated hygiene area	A hardwired multi-purpose workstation for critical commanding	Two exercise stations	A permanently installed exercise station	Exercise station in separate module from the main habitat	Private crew quarters	A dedicated labeled stowage areas vs bungees and CTBs	Single (ie 30 days) missions worth of logistic stowage in the habitat	A WCS in a different module from the galley	Circadian cycle lighting system	A viewing window
An additional WCS separate from Orion	■																			
An additional galley separate from Orion	▨	■																		
Galley/ward room table for 4 crew	▨	▨	■																	
Trash Removal Capability	▨	▨	▨	■																
Separate wet and dry trash stowage	▨	▨	▨	▨	■															
Common secondary structure interface	▨	▨	▨	▨	▨	■														
Two or more multi-purpose workstations	▨	▨	▨	▨	▨	▨	■													
A dedicated medical area	▨	▨	▨	▨	▨	▨	▨	■												
A dedicated science area	▨	▨	▨	▨	▨	▨	▨	▨	■											
A dedicated hygiene area	▨	▨	▨	▨	▨	▨	▨	▨	▨	■										
A hardwired multi-purpose workstation for critical commanding	▨	▨	▨	▨	▨	▨	▨	▨	▨	▨	■									
Two exercise stations	▨	▨	▨	▨	▨	▨	▨	▨	▨	▨	▨	■								
A permanently installed exercise station	▨	▨	▨	▨	▨	▨	▨	▨	▨	▨	▨	▨	■							
Exercise station in separate module from the main habitat	▨	▨	▨	▨	▨	▨	▨	▨	▨	▨	▨	▨	▨	■						
Private crew quarters	▨	▨	▨	▨	▨	▨	▨	▨	▨	▨	▨	▨	▨	▨	■					
A dedicated labeled stowage areas vs bungees and CTBs	▨	▨	▨	▨	▨	▨	▨	▨	▨	▨	▨	▨	▨	▨	▨	■				
Single (ie 30 days) missions worth of logistic stowage in the habitat	▨	▨	▨	▨	▨	▨	▨	▨	▨	▨	▨	▨	▨	▨	▨	▨	■			
A WCS in a different module from the galley	▨	▨	▨	▨	▨	▨	▨	▨	▨	▨	▨	▨	▨	▨	▨	▨	▨	■		
Circadian cycle lighting system	▨	▨	▨	▨	▨	▨	▨	▨	▨	▨	▨	▨	▨	▨	▨	▨	▨	▨	■	
A viewing window	▨	▨	▨	▨	▨	▨	▨	▨	▨	▨	▨	▨	▨	▨	▨	▨	▨	▨	▨	■

Fig. 1. Pairwise comparison of candidate Gateway capabilities.
Source: Reproduced from Figure 7 in Gernhardt et al. [6, p. 29].

For the value used in the manual, $S = 2$, the matrix is moderately consistent (CR = 0.0796). The eigenvector method yields

$w_{A(2)} = (0.231,\ 0.202,\ 0.161,\ 0.163,\ 0.139,\ 0.103),$

inducing the ranking

WCS ≻ galley ≻ trash removal ≻ room table ≻ trash storage ≻ interface.

If all strict preferences are uniformly strengthened, for instance, to $S = 6$, the ordinal structure of the matrix remains unchanged. However, the eigenvector method produces

$w_{A(6)} = (0.340,\ 0.241,\ 0.146,\ 0.137,\ 0.086,\ 0.050),$

leading to

WCS ≻ galley ≻ room table ≻ trash removal ≻ trash storage ≻ interface.

Thus, despite identical ordinal information, the third and fourth alternatives exchange positions. This NASA example, therefore, provides a further concrete instance of IOP rank reversal under the eigenvector method: the decision set and the ordinal judgements are fixed, the baseline matrix satisfies the conventional CR < 0.10 threshold, but a uniform recalibration of strict preference intensities changes the induced ranking.

The examples below were obtained within the ordinal class of reciprocal matrices with entries in $\{1, \alpha, 1/\alpha\}$, using $\alpha = 2$ as the baseline and $\alpha \in \{3, \dots, 9\}$ as the intensified values. The search was exhaustive over the relevant matrix orders within this class. The order-four EVM example below was selected, among the order-four EVM examples exhibiting IOP rank reversal, as the one with the lowest baseline value of CR(A). The subsequent examples illustrate cases in which the top-ranked alternative changes. For each method, all smaller orders were checked exhaustively, so these examples are minimal with respect to matrix order within the searched ordinal class. These examples are not intended to establish minimality over all possible reciprocal pairwise comparison matrices.

The following example shows that IOP rank reversal can already occur for the eigenvector method in a matrix of order 4.

Example 2 (*Order-Four IOP Rank Reversal Under EVM*)**.** Consider the reciprocal matrix

$$A = \begin{bmatrix} 1 & 2 & 1/2 & 1/2 \\ 1/2 & 1 & 1 & 2 \\ 2 & 1 & 1 & 1/2 \\ 2 & 1/2 & 2 & 1 \end{bmatrix}.$$

For this matrix, the consistency ratio is CR(A) = 0.2123. The eigenvector method yields the priority vector

$w_A = (0.218,\ 0.258,\ 0.235,\ 0.289),$

which induces the ranking $4 \succ 2 \succ 3 \succ 1$. Let $k = \log 4/\log 2 = 2$, and define the uniformly intensified matrix $A^{(k)} = [a_{ij}^k]$. For $A^{(k)}$, the eigenvector method yields

$w_{A^{(k)}} = (0.212,\ 0.267,\ 0.211,\ 0.311),$

which induces the ranking $4 \succ 2 \succ 1 \succ 3$. Hence, uniform preference intensification changes the induced ranking, although the top-ranked alternative remains alternative 4. The rank reversal occurs because alternatives 1 and 3 exchange their relative order.

Examples 1 and 2 leave the top-ranked alternative unchanged, but they show that uniform intensification may alter the induced ranking. The following examples show that the effect can also change the top-ranked alternative for several priority derivation methods. In contrast, the scale-invariant GMM is not included among these examples, because its ranking is preserved under uniform preference intensification.

Example 3 (*IOP Top-Rank Reversal Under EVM*)**.** Consider the reciprocal matrix

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 & 2 \\ 1 & 1 & 1 & 1 & 1/2 \\ 1 & 1 & 1 & 1 & 1/2 \\ 1 & 1 & 1 & 1 & 1/2 \\ 1/2 & 2 & 2 & 2 & 1 \end{bmatrix}.$$

For this matrix, the eigenvector method yields the priority vector

$$w_A = (0.241,\ 0.166,\ 0.166,\ 0.166,\ 0.262),$$

and hence the top-ranked alternative is alternative 5. Now, let $k = \log 9/\log 2$ and define the uniformly intensified matrix $A^{(k)} = [a_{ij}^k]$. For $A^{(k)}$, the eigenvector method yields

$$w_{A^{(k)}} = (0.424,\ 0.086,\ 0.086,\ 0.086,\ 0.319),$$

and hence the top-ranked alternative is alternative 1. Therefore, uniform preference intensification changes the top-ranked alternative from 5 to 1.

Example 4 (*IOP Top-Rank Reversal Under RSM*)**.** Consider the reciprocal matrix

$$A = \begin{bmatrix} 1 & 1 & 2 & 1/2 & 1 & 1 \\ 1 & 1 & 1/2 & 2 & 1 & 1 \\ 1/2 & 2 & 1 & 1/2 & 1 & 1 \\ 2 & 1/2 & 2 & 1 & 1/2 & 1/2 \\ 1 & 1 & 1 & 2 & 1 & 1 \\ 1 & 1 & 1 & 2 & 1 & 1 \end{bmatrix}.$$

For this matrix, RSM gives the priority vector

$$w_A = (0.165,\ 0.165,\ 0.152,\ 0.165,\ 0.177,\ 0.177),$$

which induces the weak ranking $\{5,6\} \succ \{1,2,4\} \succ 3$. Let $k = \log 9/\log 2$ and define the uniformly intensified matrix $A^{(k)} = [a_{ij}^k]$. For $A^{(k)}$, RSM gives

$$w_{A^{(k)}} = (0.153,\ 0.153,\ 0.143,\ 0.225,\ 0.163,\ 0.163),$$

inducing the weak ranking $4 \succ \{5,6\} \succ \{1,2\} \succ 3$. Hence, uniform preference intensification changes the top-ranked set from $\{5,6\}$ to $\{4\}$.

Example 5 (*IOP Top-Rank Reversal Under CSM*)**.** Consider the reciprocal matrix

$$A = \begin{bmatrix} 1 & 1 & 1 & 2 \\ 1 & 1 & 2 & 1/2 \\ 1 & 1/2 & 1 & 2 \\ 1/2 & 2 & 1/2 & 1 \end{bmatrix}.$$

For this matrix, CSM gives the priority vector $w_A = (0.273,\ 0.261,\ 0.246,\ 0.220)$, which induces the ranking $1 \succ 2 \succ 3 \succ 4$. Let $k = \log 9/\log 2$ and define the uniformly intensified matrix $A^{(k)} = [a_{ij}^k]$. For $A^{(k)}$, CSM gives $w_{A^{(k)}} = (0.243,\ 0.307,\ 0.223,\ 0.227)$, inducing the ranking $2 \succ 1 \succ 4 \succ 3$. Hence, uniform preference intensification changes the top-ranked alternative from 1 to 2.

Example 6 (*IOP Top-Rank Reversal Under HMM*)**.** Consider the reciprocal matrix

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 & 2 & 1/2 \\ 1 & 1 & 1 & 2 & 1 & 1/2 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1/2 & 1 & 1 & 1 & 2 \\ 1/2 & 1 & 1 & 1 & 1 & 1/2 \\ 2 & 2 & 1 & 1/2 & 2 & 1 \end{bmatrix}.$$

For this matrix, HMM gives the priority vector $w_A = (0.165,\ 0.165,\ 0.178,\ 0.165,\ 0.134,\ 0.195)$, which induces the weak ranking $6 \succ 3 \succ \{1,2,4\} \succ 5$. Let $k = \log 9/\log 2$ and define the uniformly intensified matrix $A^{(k)} = [a_{ij}^k]$. For $A^{(k)}$, HMM gives $w_{A^{(k)}} = (0.144,\ 0.144,\ 0.315,\ 0.144,\ 0.086,\ 0.167)$, inducing the weak ranking $3 \succ 6 \succ \{1,2,4\} \succ 5$. Hence, uniform preference intensification changes the top-ranked alternative from 6 to 3.

Example 7 (*IOP Top-Rank Reversal Under CMM*)**.** Consider the reciprocal matrix

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 1/2 & 1 & 2 \\ 1 & 1/2 & 1 \end{bmatrix}.$$

For this matrix, CMM gives the priority vector $w_A = (0.403,\ 0.329,\ 0.268)$, which induces the ranking $1 \succ 2 \succ 3$. Let $k = \log 9/\log 2$ and define the uniformly intensified matrix $A^{(k)} = [a_{ij}^k]$. For $A^{(k)}$, CMM gives $w_{A^{(k)}} = (0.148,\ 0.838,\ 0.015)$, inducing the ranking $2 \succ 1 \succ 3$. Hence, uniform preference intensification changes the top-ranked alternative from 1 to 2.

3.3. A homogeneity result under uniform intensification

Eigenvector-based sensitivity to ordinal-to-cardinal transformations had already been shown by Genest et al. [25]. The corresponding invariance requirement was later formalised as scale invariance by Petróczy and Csató [24], who introduced it as Axiom 3.1 and proved that the row geometric mean method satisfies it. The purpose of the present subsection is to give a structural homogeneity interpretation of intensification-stable procedures. Under a natural covariance assumption, uniform intensification can be linearised by a change of scale, yielding a degree-one homogeneity representation. This recovers the known invariance of multiplicatively separable rules, such as the geometric mean method, and contrasts them with procedures whose scores are affected by sums, normalisations, or globally coupled optimisation criteria.

Assumption 1. There exists a family $\{\phi_k\}_{k>0}$ of strictly increasing homeomorphisms $\phi_k : (0,\infty) \to (0,\infty)$ such that for every reciprocal PCM A and every i,

$$s_i(A^{(k)}) = \phi_k\big(s_i(A)\big) \qquad \forall\, k > 0. \tag{6}$$

Moreover, the family is compatible in the sense that

$$\phi_1 = \mathrm{id}, \qquad \phi_{k\ell} = \phi_k \circ \phi_\ell \quad \forall\, k, \ell > 0. \tag{7}$$

Finally, we assume $\phi_k(1) = 1$ for all $k > 0$.

Assumption 2. Define $\psi_k : \mathbb{R} \to \mathbb{R}$ by

$$\psi_k(t) := \ln\big(\phi_k(e^t)\big).$$

We assume that the action is non-trivial in the sense that

$$\begin{aligned} &\exists\, t_+ > 0 : \ \{\psi_k(t_+) : k > 0\} = (0,\infty), \\ &\exists\, t_- < 0 : \ \{\psi_k(t_-) : k > 0\} = (-\infty,0). \end{aligned} \tag{8}$$

Remark 1. Assumption 1 is satisfied by methods whose scores transform componentwise under uniform intensification, such as multiplicatively separable scoring rules. It fails for methods in which intensification interacts with sums, normalisations, or globally coupled solutions in a matrix-dependent way.

Proposition 1. *Under Assumption 1, uniform preference intensification cannot produce IOP rank reversal. That is, for every reciprocal PCM A, every $k > 0$, and all i, j,*

$$s_i(A) > s_j(A) \quad \Longleftrightarrow \quad s_i(A^{(k)}) > s_j(A^{(k)}). \tag{9}$$

Proof. By (6), $s_i(A^{(k)}) = \phi_k(s_i(A))$ and $s_j(A^{(k)}) = \phi_k(s_j(A))$. Since ϕ_k is strictly increasing, it preserves strict inequalities, hence the ranking is unchanged. ☐

Theorem 1. *Under Assumptions 1 and 2, there exists a strictly increasing homeomorphism $h : (0,\infty) \to \mathbb{R}$ with $h(1) = 0$ such that for every reciprocal PCM A, every i, and every $k > 0$,*

$$h\big(s_i(A^{(k)})\big) = k\, h\big(s_i(A)\big). \tag{10}$$

Equivalently, the transformed scores $r_i(A) := h(s_i(A))$ satisfy $r_i(A^{(k)}) = k\, r_i(A)$, i.e., they are positively homogeneous of degree 1 under uniform intensification.

Proof. Passing to logarithmic coordinates linearises the multiplicative structure. For $x > 0$ write $t = \ln x$ and define

$$\psi_k(t) := \ln\big(\phi_k(e^t)\big).$$

Then the covariance relation $s_i(A^{(k)}) = \phi_k(s_i(A))$ is equivalent to

$$\ln s_i(A^{(k)}) = \psi_k\big(\ln s_i(A)\big).$$

Hence uniform intensification induces a family $\{\psi_k\}_{k>0}$ of strictly increasing homeomorphisms of $\mathbb{R}$. Moreover, by (7),

$$\psi_1 = \mathrm{id}, \qquad \psi_{k\ell} = \psi_k \circ \psi_\ell \quad \forall\, k, \ell > 0,$$

and $\psi_k(0) = 0$ since $\phi_k(1) = 1$. Thus $\{\psi_k\}_{k>0}$ forms a multiplicative one-parameter group acting on $\mathbb{R}$.

Under the non-degeneracy condition (8), a classical order-preserving linearisation theorem (see [26]) yields a strictly increasing homeomorphism $g : \mathbb{R} \to \mathbb{R}$ with $g(0) = 0$ such that

$$g(\psi_k(t)) = k\, g(t) \qquad \forall\, t \in \mathbb{R},\ \forall\, k > 0.$$

Finally, define $h : (0, \infty) \to \mathbb{R}$ by $h(x) := g(\ln x)$. Then h is strictly increasing, $h(1) = 0$, and

$$h(\phi_k(x)) = g(\psi_k(\ln x)) = k\, g(\ln x) = k\, h(x).$$

Applying this identity to $x = s_i(A)$ yields (10). ☐

Remark 2. Proposition 1 shows that rank preservation is immediate within the covariance class of Assumption 1. The substantive content of Theorem 1 is that, under the mild non-degeneracy condition in Assumption 2, the intensification action admits a change of scale turning it into a pure dilation. In particular, since h is strictly increasing, (10) also implies (9).

Remark 3. In the sense of Stevens' theory of measurement scales [27], Theorem 1 can be read as a logarithmic linearisation: after the log change of variables (and an order-preserving re-scaling), uniform intensification becomes a pure rescaling $t \mapsto kt$ on an interval scale.

Corollary 1 (*Geometric Mean Method (No IOP Rank Reversal)*). *For the geometric mean method,*

$$s_i(A) = \left(\prod_{j=1}^{n} a_{ij}\right)^{1/n},$$

we have $s_i(A^{(k)}) = s_i(A)^k$ for all $k > 0$. Thus (10) holds with $h(x) = \ln x$, and $\sigma(A^{(k)}) = \sigma(A)$ for all $k > 0$. This recovers, in the present notation and homogeneity framework, the known scale invariance of the row geometric mean method proved by Petróczy and Csató [24, Lemma 3.7].

4. Numerical experiments

This section reports numerical simulations designed to assess the frequency of intensity-of-preference (IOP) rank reversal under uniform preference intensification. We first analyse the phenomenon under the eigenvector method and then compare its frequency across several priority derivation procedures. The experiments also examine how IOP instability depends on matrix order and inconsistency.

Two experimental frameworks are considered. First, in the *discrete* setting, random ordinal reciprocal comparison patterns are generated and then cardinalised through common strict-preference intensities $\alpha \in \{2, \dots, 9\}$, with ties encoded by 1. For each matrix order $n = 3, \dots, 9$, we generated a fixed set of 5000 ordinal reciprocal matrices, yielding 35000 base ordinal matrices in total. The baseline is $A(2)$, and the matrices $A(\alpha)$, $\alpha = 3, \dots, 9$, are uniformly intensified versions of this baseline.

Second, in the *continuous* setting, strict preference intensities are heterogeneous and pair-specific. For each strict comparison, the intensity is drawn independently from $U_{ij} \sim \mathrm{Unif}[1.2, 9]$, while ties remain equal to 1. Uniform preference intensification is then applied by comparing each matrix A with $A^{(2)} = [a_{ij}^2]$. For each matrix order $n = 3, \dots, 9$, a fixed set of 5000 continuous reciprocal matrices was generated. The same fixed set was evaluated by all priority derivation methods, so that differences across methods are not affected by differences in the sampled matrices.

4.1. Discrete preference intensification

4.1.1. Experimental design

In the discrete experiment, we generated a fixed set of random ordinal reciprocal comparison patterns for $n = 3, \dots, 9$, with 5000 independent matrices for each order. For each unordered pair (i, j), $i < j$, one of the relations $i \succ j$, $i \sim j$, and $j \succ i$ was selected independently with equal probability $1/3$. Equivalently, the tie probability is $p_{\mathrm{tie}} = 1/3$, where p_{tie} denotes the probability that an unordered pairwise comparison is sampled as a tie. This symmetric choice is used as a neutral benchmark over the three possible ordinal relations, not as an empirical claim about the frequency of ties in applied AHP data. To check that the main conclusions are not driven by this specific benchmark, we also performed additional EVM sensitivity checks with lower tie probabilities, $p_{\mathrm{tie}} = 0$ and $p_{\mathrm{tie}} = 0.10$. These checks, reported in the replication repository, yielded the same qualitative pattern: rank reversal is absent for $n = 3$ and increases with matrix order. Each pattern was then encoded by an exponent matrix $O = [o_{ij}]$, with $o_{ii} = 0$, $o_{ji} = -o_{ij}$, and $o_{ij} \in \{-1, 0, 1\}$.

For each $\alpha \in \{2, \dots, 9\}$, the associated cardinal reciprocal matrix was

$$A(\alpha) = \alpha^{O}, \qquad a_{ij}(\alpha) = \alpha^{o_{ij}}.$$

Thus, all matrices generated from the same ordinal pattern share the same preference directions and differ only in the common intensity assigned to strict preferences. We used $\alpha = 2$ as the baseline and compared its ranking with those obtained for $\alpha = 3, \dots, 9$. Equivalently,

$$A(\alpha) = A(2)^{\log(\alpha)/\log(2)},$$

so the matrices $A(\alpha)$, $\alpha = 3, \dots, 9$, are uniformly intensified versions of the baseline $A(2)$. The same set of ordinal matrices was used for all priority derivation methods.

Rankings were compared using the tolerance $\tau = 10^{-10}$. For a priority vector w, we set

$$\varepsilon(w) = \tau \max\{1, \max_i |w_i|\},$$

and defined the pairwise order matrix $R(w) = [r_{ij}(w)]$ by

$$r_{ij}(w) = \begin{cases} 1, & \text{if } w_i - w_j > \varepsilon(w), \\ -1, & \text{if } w_i - w_j < -\varepsilon(w), \\ 0, & \text{otherwise}. \end{cases}$$

A rank reversal was recorded whenever $R(w(A(\alpha)))$ differed from $R(w(A(2)))$ for at least one $\alpha \in \{3, \dots, 9\}$. We also recorded whether the top-ranked alternative changed; in the presence of ties within tolerance, a top-rank change was counted only if the top set after intensification had empty intersection with the baseline top set.

4.1.2. Frequency of rank reversal

We first applied the eigenvector method to the fixed matrix set. Fig. 2 reports, in a single plot, the matrix-level percentages of IOP rank reversal and top-ranked alternative change. Each ordinal matrix was counted once if at least one $\alpha \in \{3, \dots, 9\}$ changed the baseline ranking at $A(2)$. No rank reversal was observed for $n = 3$. This is consistent with the known equivalence between EVM and the scale-invariant GMM for 3×3 reciprocal matrices [8, p. 393]. The rank reversal percentage then increases rapidly with matrix order, from 5.0% for $n = 4$ to 36.3% for $n = 5$, 60.9% for $n = 6$, and 94.5% for $n = 9$. Hence, under the EVM, IOP rank reversal becomes the majority outcome already from $n = 6$.

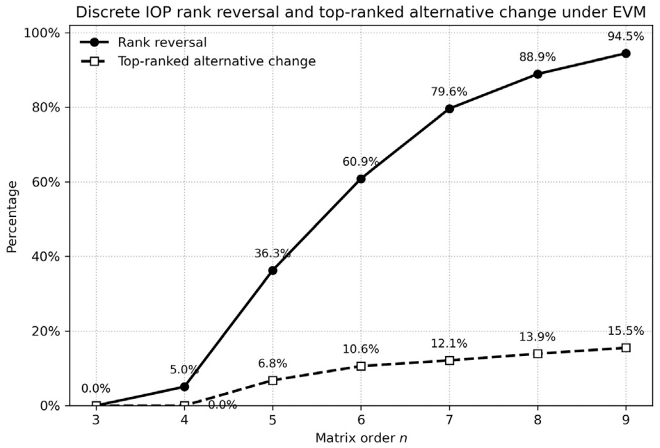


Fig. 2. Empirical matrix-level percentages of discrete intensity-of-preference (IOP) rank reversal and top-ranked alternative change under the eigenvector method. For each matrix order n, percentages are computed over the 5000 ordinal matrices by comparing the baseline $A(2)$ with the uniformly intensified matrices $A(\alpha)$, $\alpha \in \{3,\dots,9\}$. A rank reversal is recorded when the pairwise order matrix changes; a top-ranked alternative change is recorded when the top set after intensification has empty intersection with the baseline top set.

Table 1
Baseline consistency-ratio groups and conditional rank-reversal percentages under the eigenvector method in the discrete experiment. The consistency ratio is computed for the baseline matrix $A(2)$. For each matrix order n, the fixed set contains 5000 ordinal matrices. Percentages in parentheses report the frequencies of the groups, whereas Reversal denotes the percentage of matrices within the corresponding CR group exhibiting rank reversal for at least one strict-preference intensity $\alpha \in \{3,\dots,9\}$.

	$\mathrm{CR}(A(2)) < 0.10$		$\mathrm{CR}(A(2)) \geq 0.10$	
n	Matrices	Reversal	Matrices	Reversal
3	3550 (71.0%)	0.0%	1450 (29.0%)	0.0%
4	3010 (60.2%)	0.0%	1990 (39.8%)	12.6%
5	2975 (59.5%)	26.5%	2025 (40.5%)	50.7%
6	3056 (61.1%)	52.3%	1944 (38.9%)	74.3%
7	3187 (63.7%)	77.6%	1813 (36.3%)	83.1%
8	3470 (69.4%)	88.1%	1530 (30.6%)	90.7%
9	3623 (72.5%)	94.2%	1377 (27.5%)	95.2%

The same figure also reports the corresponding percentage of changes in the top-ranked alternative. Such changes are decision-relevant because they may alter the alternative selected as the best. No top-rank change was observed for $n = 3$ or $n = 4$, whereas the percentage rises from 6.8% for $n = 5$ to 15.5% for $n = 9$. Thus, uniform preference intensification may affect not only intermediate positions but also the alternative selected as the best.

4.1.3. Relation to inconsistency

The consistency ratio was computed for the baseline matrix $A(2)$. We therefore distinguish between matrices satisfying the conventional baseline threshold $\mathrm{CR}(A(2)) < 0.10$ and those with $\mathrm{CR}(A(2)) \geq 0.10$.

Table 1 reports the size of each baseline CR group and the corresponding conditional rank-reversal percentage. Percentages in parentheses refer to group sizes, whereas percentages in the reversal columns refer to rank-reversal percentages conditional on belonging to the corresponding CR group.

Table 1 shows that the conventional consistency threshold does not guarantee ranking stability under uniform preference intensification. For example, even among matrices satisfying $\mathrm{CR}(A(2)) < 0.10$, the rank-reversal percentage increases from 26.5% for $n = 5$ to 94.2% for $n = 9$. Hence, baseline consistency screening alone does not eliminate IOP instability.

Table 2
Observed and CR-standardised rank-reversal percentages under the eigenvector method in the discrete experiment. Observed reversal denotes the empirical percentage of matrices exhibiting rank reversal for at least one strict-preference intensity $\alpha \in \{3,\dots,9\}$. Reversal at common CR denotes the fitted percentage obtained from a ridge logistic model including baseline $\mathrm{CR}(A(2))$ and matrix order, evaluated at the common median baseline value $\mathrm{CR}(A(2)) = 0.0902$.

n	Mean CR	Median CR	Observed reversal	Reversal at common CR
4	0.0944	0.0687	5.0%	2.1%
5	0.0907	0.0879	36.3%	34.2%
6	0.0908	0.0906	60.9%	61.8%
7	0.0913	0.0914	79.6%	80.5%
8	0.0903	0.0902	88.9%	89.6%
9	0.0906	0.0908	94.5%	94.7%

To examine whether the increase in rank-reversal percentage with matrix order could be explained by changes in baseline inconsistency, we also carried out a descriptive CR-standardised check under the eigenvector method. A ridge logistic model was fitted with rank reversal as the response variable and baseline $\mathrm{CR}(A(2))$ and matrix order as explanatory variables. Matrix order was included as a categorical predictor. The fitted probabilities were then evaluated at the common median baseline value $\mathrm{CR}(A(2)) = 0.0902$ for all matrix orders. This standardisation is descriptive and should not be interpreted as a causal decomposition.

The CR-standardised results preserve the same qualitative pattern as the observed percentages. In particular, for $n = 5,\dots,9$, mean and median $\mathrm{CR}(A(2))$ remain close to 0.09, while the rank-reversal percentage increases sharply. At the common baseline CR, the fitted reversal percentage rises from 34.2% for $n = 5$ to 94.7% for $n = 9$. Thus, in this discrete experiment, the matrix-order pattern cannot be attributed solely to an increase in baseline CR.

4.1.4. Comparison across priority derivation methods

The discrete experiment was then applied to five priority derivation methods: EVM, RSM, CSM, HMM, and CMM. The EVM is retained in this comparative analysis as the standard AHP reference method, so that the behaviour of the other methods can be interpreted relative to it. For all methods, the same fixed set of 5000 ordinal matrices per matrix order was used.

Fig. 3 reports, in a single grouped bar chart, the average matrix-level percentages of discrete IOP rank reversal and top-ranked alternative change across the five methods. The percentages are averaged over matrix orders $n = 3,\dots,9$. Substantial differences are observed across methods. For rank reversal, the largest average percentage among the methods is obtained for CSM (55.5%), followed by EVM (52.2%) and CMM (44.7%). HMM and RSM are less sensitive, with average percentages of 22.6% and 22.3%, respectively. Hence, IOP rank reversal is not specific to the eigenvector method; it also appears in aggregation-based, normalisation-based and similarity-based procedures.

The same figure also reports changes in the top-ranked alternative. The largest average top-change percentage among the methods is observed for CSM (14.8%), followed by CMM (10.6%) and EVM (8.4%). Such changes are rare for HMM (1.1%) and RSM (0.2%). Thus, uniform preference intensification may affect not only intermediate ranking positions but also the alternative selected as the best.

The corresponding method-by-order percentages are reported in Table 3.

4.2. Continuous preference intensification

4.2.1. Experimental design

To complement the discrete-scale experiment, we considered a continuous setting with heterogeneous cardinal preference intensities. For each matrix order $n = 3,\dots,9$, we generated a fixed set of 5000 reciprocal PCMs. For each unordered pair (i,j), $i < j$, one of three

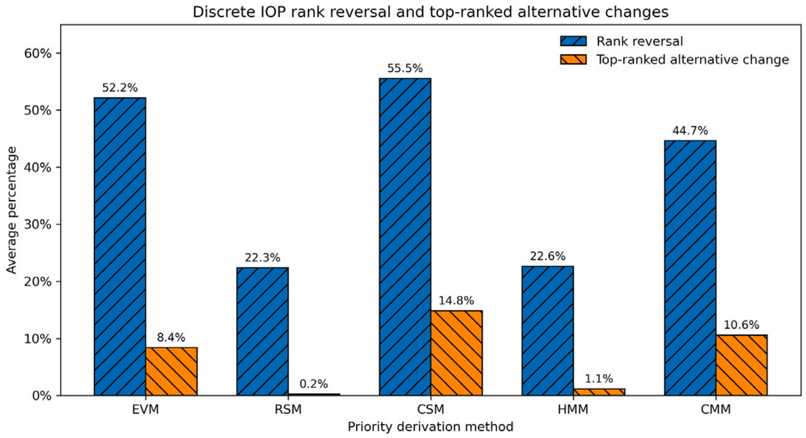


Fig. 3. Average matrix-level percentages of discrete intensity-of-preference (IOP) rank reversal and top-ranked alternative change across priority derivation methods. For each method, percentages are first computed separately for each matrix order $n = 3, \ldots, 9$ and then averaged across matrix orders, using the same fixed set of 5000 ordinal matrices for each n. A rank reversal is recorded when, for at least one strict-preference intensity $\alpha \in \{3, \ldots, 9\}$, the pairwise order matrix differs from the baseline $A(2)$; a top-ranked alternative change is recorded when the top set after intensification has empty intersection with the baseline top set.

Table 3
Discrete intensity-of-preference rank-reversal and top-ranked alternative change percentages by matrix order and priority derivation method. Percentages are computed over the same fixed set of 5000 ordinal matrices for each matrix order n. A matrix is counted as exhibiting IOP rank reversal, respectively top-ranked alternative change, respectively, if the event occurs for at least one uniformly intensified matrix $A(\alpha)$, $\alpha \in \{3, \ldots, 9\}$, relative to the baseline $A(2)$.

	IOP rank reversal					Top-ranked alternative change				
n	EVM	RSM	CSM	HMM	CMM	EVM	RSM	CSM	HMM	CMM
3	0.0%	0.0%	0.0%	0.0%	14.8%	0.0%	0.0%	0.0%	0.0%	7.2%
4	5.0%	0.0%	14.7%	0.0%	46.2%	0.0%	0.0%	6.6%	0.0%	14.0%
5	36.3%	6.2%	41.0%	5.8%	40.6%	6.8%	0.0%	10.4%	0.0%	11.6%
6	60.9%	15.8%	64.2%	16.7%	44.9%	10.6%	0.0%	15.4%	0.6%	10.8%
7	79.6%	30.9%	81.5%	30.5%	49.3%	12.1%	0.2%	20.3%	1.1%	10.8%
8	88.9%	45.4%	91.5%	45.2%	54.5%	13.9%	0.6%	25.4%	2.6%	10.1%
9	94.5%	58.0%	95.9%	60.0%	62.2%	15.5%	0.9%	25.6%	3.8%	9.7%

outcomes was selected independently with equal probability 1/3: a tie, a strict preference of i over j, or a strict preference of j over i. As in the discrete experiment, this corresponds to $p_{\text{tie}} = 1/3$, where p_{tie} denotes the probability that an unordered pairwise comparison is sampled as a tie. This symmetric choice is used as a neutral benchmark over the three possible ordinal relations, rather than as an empirical assumption about the prevalence of ties in applied AHP data. The additional EVM sensitivity checks with $p_{\text{tie}} = 0$ and $p_{\text{tie}} = 0.10$, reported in the replication repository, also cover the continuous design. In the case of a tie, we set $a_{ij} = a_{ji} = 1$. In the case of a strict preference, an intensity $U_{ij} \sim \text{Unif}[\alpha_{\min}, \alpha_{\max}]$ was drawn independently, with $\alpha_{\min} = 1.2$ and $\alpha_{\max} = 9$, and the corresponding entries were set to either $(a_{ij}, a_{ji}) = (U_{ij}, 1/U_{ij})$ or $(a_{ij}, a_{ji}) = (1/U_{ij}, U_{ij})$, according to the selected direction. Diagonal entries were set equal to one. Thus, the ordinal directions are sampled as in the discrete experiment, but strict preference intensities are continuous and pair-specific.

Uniform preference intensification was then applied by the entry-wise power transformation

$$A \mapsto A^{(k)} = [a_{ij}^{k}].$$

In the continuous experiment reported below, we set $k = 2$ and recorded an IOP rank reversal whenever the pairwise order matrix induced by A differed from that induced by $A^{(2)}$. Changes in the top-ranked alternative were recorded using the same top-set criterion as in the discrete experiment. Rankings were compared using the same pairwise-order tolerance $\tau = 10^{-10}$.

The same fixed set of continuous reciprocal matrices was used for all priority derivation methods. Hence, differences across methods are not affected by differences in the sampled matrices.

4.2.2. *Frequency of rank reversal*

Using the continuous design described above, we first evaluate the eigenvector method by comparing each matrix A with its uniformly intensified counterpart $A^{(2)} = [a_{ij}^{2}]$. As in the discrete experiment, a matrix-level IOP rank reversal is recorded whenever the pairwise order matrix changes, and top-ranked alternative changes are recorded using the same top-set criterion.

Fig. 4 shows that the percentage of IOP rank reversal increases sharply with matrix order. No reversal is observed for $n = 3$. As in the discrete setting, this is consistent with the known equivalence between EVM and the scale-invariant GMM for 3×3 reciprocal matrices [8, p. 393]. The percentage rises to 14.1% for $n = 4$, 37.9% for $n = 5$, 61.3% for $n = 6$, 78.7% for $n = 7$, reaching 94.8% for $n = 9$. Thus, under the EVM, uniform preference intensification produces frequent ranking instability even if strict preference intensities are continuous and pair-specific.

Top-ranked alternative changes also occur and become more common for larger matrices. Since such changes may alter the alternative selected as the best, they are especially relevant from a decision-making perspective, with the percentage increasing from 5.0% for $n = 4$ to 20.2% for $n = 7$, and 24.5% for $n = 9$. Hence, the effect is not limited to intermediate ranking positions; in a substantial number of cases, it also changes the alternative selected as the best.

4.2.3. *Relation to inconsistency*

We also examined the relation between inconsistency and IOP instability in the continuous uniform experiment. The consistency ratio was computed for each baseline matrix A, before intensification, and rankings were compared between A and $A^{(2)} = [a_{ij}^{2}]$.

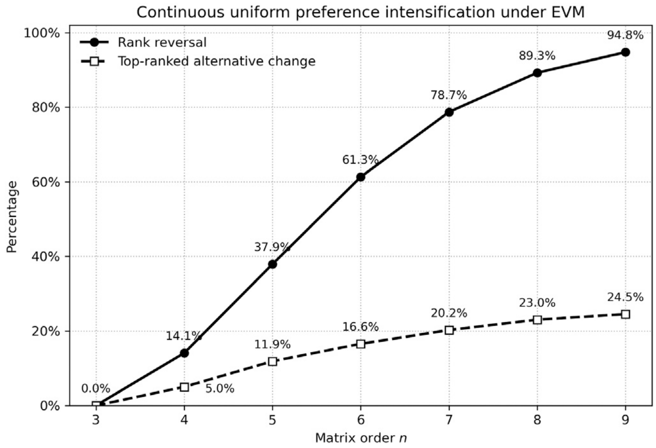


Fig. 4. Matrix-level percentages of rank reversal and top-ranked alternative change under continuous uniform preference intensification for the eigenvector method. For each matrix order n, percentages are computed over a fixed set of 5000 continuous reciprocal pairwise comparison matrices with heterogeneous strict-preference intensities $U_{ij} \sim \mathrm{Unif}[1.2, 9]$. A rank reversal is recorded when the pairwise order matrix induced by $A^{(2)} = [a_{ij}^2]$ differs from that induced by the baseline matrix A; a top-ranked alternative change is recorded when the top set after intensification has empty intersection with the baseline top set.

Table 4
CR-stratified analysis of continuous uniform preference intensification for $n = 7$. The strata are empirical quartiles of the baseline CR distribution; hence each stratum contains 1250 matrices. These quartiles represent relative inconsistency levels within the continuous simulation design, not AHP acceptability classes. Reversal denotes the percentage of matrices for which the pairwise order matrix induced by $A^{(2)} = [a_{ij}^2]$ differs from that induced by the baseline matrix A. Top change denotes the percentage of matrices for which the top set after intensification has empty intersection with the baseline top set.

CR stratum	CR range	Median CR	Reversal	Top change
Q_1 lowest	$[0.1292, 0.4614]$	0.3744	67.1%	14.3%
Q_2	$[0.4614, 0.6087]$	0.5390	76.6%	19.0%
Q_3	$[0.6087, 0.7614]$	0.6813	83.8%	24.6%
Q_4 highest	$[0.7614, 1.4713]$	0.8695	87.4%	23.0%

Unlike in the discrete experiment, conventional AHP consistency thresholds do not provide balanced comparison groups in the continuous design. For example, for $n = 7$, no matrix satisfies $\mathrm{CR}(A) < 0.10$, and only 6 out of 5000 matrices satisfy $0.10 \le \mathrm{CR}(A) < 0.15$. We therefore use empirical quartiles of the baseline CR distribution for the representative case $n = 7$. These quartiles should be interpreted as relative inconsistency strata within the continuous simulation design, not as AHP acceptability classes.

Table 4 reports the corresponding CR-stratified percentages. The rank-reversal percentage increases from 67.1% in the lowest-CR quartile to 87.4% in the highest-CR quartile, while top-ranked alternative changes remain non-negligible across all quartiles. Thus, higher relative inconsistency is associated with higher IOP instability, although even the lowest-CR quartile exhibits a substantial rank-reversal percentage.

To examine whether the increase in rank-reversal frequency with matrix order could be explained by changes in baseline inconsistency, we carried out an analogous descriptive CR-standardisation in the continuous experiment. A ridge logistic model was fitted with rank reversal as the response variable and baseline $\mathrm{CR}(A)$ and matrix order as explanatory variables. Matrix order was included as a numerical predictor. The fitted probabilities were then evaluated at the common median baseline value $\mathrm{CR}(A) = 0.5878$ for all matrix orders. This standardisation is descriptive and should not be interpreted as a causal decomposition.

Table 5
Observed and CR-standardised rank-reversal percentages under the eigenvector method in the continuous uniform experiment. Observed reversal denotes the empirical percentage of matrices for which the pairwise order matrix induced by $A^{(2)} = [a_{ij}^2]$ differs from that induced by the baseline matrix A. Reversal at common CR denotes the fitted percentage obtained from a ridge logistic model including baseline $\mathrm{CR}(A)$ and matrix order, evaluated at the common median baseline value $\mathrm{CR}(A) = 0.5878$. The standardisation is descriptive.

n	Mean CR	Median CR	Observed reversal	Reversal at common CR
4	0.5901	0.4668	14.1%	12.4%
5	0.6091	0.5504	37.9%	37.4%
6	0.6085	0.5813	61.3%	61.9%
7	0.6193	0.6087	78.7%	78.9%
8	0.6069	0.6015	89.3%	89.5%
9	0.6217	0.6210	94.8%	94.7%

The CR-standardised results again preserve the same qualitative pattern as the observed frequencies. For $n = 4, \ldots, 9$, mean CR remains close to 0.61, while the observed rank-reversal percentage increases from 14.1% to 94.8%. At the common baseline CR, the fitted reversal percentage similarly rises from 12.4% for $n = 4$ to 94.7% for $n = 9$. Thus, also in the continuous experiment, the matrix-order pattern cannot be attributed solely to an increase in baseline CR.

4.2.4. *Comparison across methods*

We finally compare several priority derivation methods under continuous uniform preference intensification. The EVM is again retained as the standard AHP reference method. Each reciprocal matrix A is drawn from the fixed continuous set with heterogeneous pair-specific intensities $U_{ij} \sim \mathrm{Unif}[1.2, 9]$, and is compared with $A^{(2)} = [a_{ij}^2]$. For each method, a matrix-level rank reversal is recorded whenever the induced pairwise order matrix changes; top-ranked alternative changes are recorded using the same top-set criterion as above. The GMM is not included in this empirical comparison because its invariance under uniform intensification follows from the known scale invariance result of Petróczy and Csató [24, Lemma 3.7], recalled here in Corollary 1.

Fig. 5 summarises the average percentages over $n = 3, \ldots, 9$. Rank reversal is frequent for all empirical methods considered, but its magnitude is method-dependent. Among the methods, CMM exhibits the largest average rank-reversal percentage, 64.2%. The remaining methods show lower and rather similar average percentages: CSM reaches 55.2%, RSM and HMM both reach 54.4%, and EVM reaches 53.7%.

Top-ranked alternative changes show a different pattern. CMM again has the largest average percentage among the methods, 23.0%, followed by RSM (14.9%) and EVM (14.5%). CSM and HMM show lower top-change percentages, 11.3% and 7.9%, respectively. Thus, methods with similar overall rank-reversal percentages may differ in the extent to which this instability reaches the top-ranked alternative.

The results by matrix order indicate that instability tends to increase with matrix order n in this simulation design. At $n = 9$, the rank-reversal percentage reaches 98.0% for CMM, compared with 94.8% for EVM. The corresponding top-change percentages also differ across methods; for example, at $n = 9$, the top-change percentage is 36.0% for CMM. Overall, all empirically compared methods are affected by uniform preference intensification, but they differ in how often this instability propagates to the top of the ranking.

The corresponding method-by-order percentages are reported in Table 6.

5. Discussion

This paper has investigated *intensity-of-preference (IOP) rank reversal*, that is, ranking instability induced by *uniform preference intensification* (modelled as $A \mapsto A^{(k)} = [a_{ij}^k]$) of reciprocal pairwise comparison matrices. Unlike classical rank reversal phenomena triggered by changes in the set of alternatives, IOP rank reversal arises even if the decision problem remains fixed and the ordinal preference pattern is preserved.

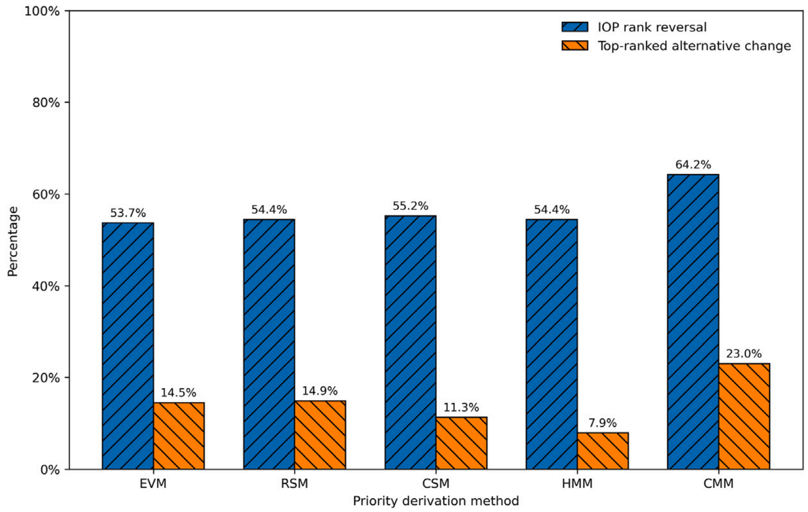


Fig. 5. Average matrix-level percentages of rank reversal and top-ranked alternative change under continuous uniform preference intensification, by priority derivation method. Percentages are averaged over matrix orders $n = 3, \ldots, 9$, using a fixed set of 5000 continuous reciprocal matrices for each n. Each matrix A is compared with its uniformly intensified version $A^{(2)} = [a_{ij}^2]$.

Table 6
Continuous intensity-of-preference rank-reversal and top-ranked alternative change percentages by matrix order and priority derivation method. Percentages are computed over the same fixed set of 5000 continuous reciprocal matrices for each matrix order n, with heterogeneous strict-preference intensities $U_{ij} \sim \mathrm{Unif}[1.2, 9]$. A matrix is counted as exhibiting IOP rank reversal, respectively top-ranked alternative change, respectively, if the event occurs when comparing the baseline matrix A with its uniformly intensified version $A^{(2)} = [a_{ij}^2]$.

	IOP rank reversal					Top-ranked alternative change				
n	EVM	RSM	CSM	HMM	CMM	EVM	RSM	CSM	HMM	CMM
3	0.0%	4.9%	2.9%	5.3%	6.7%	0.0%	4.1%	1.3%	0.7%	3.5%
4	14.1%	20.9%	21.2%	20.5%	30.5%	5.0%	12.3%	5.6%	2.7%	11.9%
5	37.9%	41.8%	42.7%	40.7%	56.0%	11.9%	16.2%	9.3%	5.6%	19.6%
6	61.3%	60.0%	61.5%	60.1%	75.0%	16.6%	17.7%	12.8%	7.6%	26.3%
7	78.7%	75.1%	77.3%	76.3%	87.9%	20.2%	17.2%	14.9%	10.8%	30.2%
8	89.3%	85.7%	87.3%	85.4%	95.1%	23.0%	18.4%	16.7%	12.6%	33.2%
9	94.8%	92.7%	93.3%	92.5%	98.0%	24.5%	18.5%	18.5%	15.1%	36.0%

5.1. Structural interpretation of IOP rank reversal

Our analysis suggests that IOP rank reversal is not merely an empirical peculiarity of the eigenvector method (EVM), but rather reflects a structural distinction between classes of priority derivation rules. Uniform intensification preserves reciprocity and all ordinal relations, and can be viewed as a coherent global recalibration of preference intensity. Consequently, if a method fails to preserve rankings under the transformation $A \mapsto A^{(k)}$, the induced ordering depends not only on the qualitative structure of comparisons, but also on their numerical calibration.

The homogeneity result of Section 3.3 makes this distinction explicit. Within the covariance framework, compatible intensification actions can be linearised after a monotone change of scale, yielding degree-one homogeneity in the intensification parameter. This delineates a structural boundary: multiplicatively separable rules, such as the geometric mean method, are inherently compatible with uniform intensification, whereas methods based on sums, normalisations, or globally coupled optimisation problems need not be. This interpretation recovers the known scale invariance of the GMM [24, Lemma 3.7] and clarifies why procedures such as the EVM, RSM, CSM, HMM, and CMM may display IOP rank reversal.

5.2. Empirical patterns across matrix order and methods

The numerical experiments reveal three empirical patterns. First, matrix order n is associated with increasing IOP instability in the present simulation design. This association should not be interpreted as isolating a pure dimension effect, since the average inconsistency of the randomly generated matrices may also vary with n. However, the analyses reported in Tables 2 and 5 suggest that the increasing pattern cannot be attributed solely to increasing baseline CR. These checks remain descriptive and should not be interpreted causally. Under the eigenvector method, rank reversal is absent for $n = 3$ in both experimental settings, consistently with the known equivalence between EVM and GMM for 3 × 3 reciprocal matrices [8, p. 393]. In the discrete experiment, the reversal percentage rises from 5.0% for $n = 4$ to 60.9% for $n = 6$ and 94.5% for $n = 9$. A similar pattern appears in the continuous heterogeneous experiment, where the corresponding percentage increases from 14.1% for $n = 4$ to 61.3% for $n = 6$ and 94.8% for $n = 9$. Thus, within the present simulation design, IOP rank reversal becomes increasingly frequent as the number of compared objects grows, even though the ordinal preference pattern is kept fixed.

Second, top-ranked alternative changes are decision-relevant because they may affect the final choice recommended by the method. Some applications require a full ranking of all alternatives, whereas others mainly use the priority vector to identify the best option. In the EVM experiments, top-ranked alternative changes remain rare for small matrices, but increase with matrix order, reaching 15.5% in the discrete setting and 24.5% in the continuous setting for $n = 9$. Hence, uniform preference intensification may affect not only intermediate positions but also the alternative that would be selected as the best.

Third, the phenomenon is strongly method-dependent. In the discrete comparison, the largest average rank-reversal percentage is observed for CSM, followed by EVM and CMM, whereas RSM and HMM are less sensitive on average. In the continuous heterogeneous comparison, CMM displays the largest average reversal percentage among the methods. The other methods have lower and relatively close average percentages, with EVM showing the lowest value. However, this lower average value should not be interpreted as scale invariance, since the EVM percentage is still high, reaching 94.8% for $n = 9$. The impact on the top-ranked alternative is also method-dependent: methods with similar overall rank-reversal percentages may differ in how often this instability affects the top of the ranking.

5.3. Consistency versus robustness

A striking empirical finding is that ranking instability persists even among matrices satisfying the conventional threshold $\mathrm{CR} < 0.10$ in the discrete experiment, where the consistency ratio is computed at the baseline intensity $A(2)$. This demonstrates that consistency screening alone is insufficient to ensure robustness under global recalibration of preference intensities.

Consistency measures the internal transitivity of judgements within a fixed numerical encoding. IOP-invariance, by contrast, concerns the behaviour of the priority derivation method under coherent scale transformations (in particular, uniform preference intensification). These are conceptually distinct properties: a pairwise comparison matrix may satisfy conventional inconsistency thresholds and yet the induced ranking may remain sensitive to global exponentiation. Thus, a low value of CR should not be interpreted as a guarantee that the ranking is robust to a uniform recalibration of preference intensities.

This distinction is also useful in relation to the condition of order preservation studied by Bana e Costa and Vansnick [28]. Their condition requires a priority vector to preserve both the ordinal preference between alternatives and the ordering of stated preference intensities: if the preference of alternative i over j is stronger than that of p over q, then w_i/w_j should exceed w_p/w_q. They show that the eigenvector method may violate these requirements even for matrices satisfying $\mathrm{CR} < 0.10$. The present paper addresses a different issue. Rather than asking whether priority ratios preserve the internal ordering of intensities within a fixed cardinal matrix, we ask whether the ranking of alternatives is preserved when a fixed ordinal preference pattern is globally recalibrated through $A \mapsto A^{(k)}$. Thus, both analyses concern preference intensity, but Bana e Costa and Vansnick (2008) study internal order preservation within a given numerical matrix, whereas IOP-invariance concerns rank preservation under a uniform transformation of that matrix.

IOP-invariance is also distinct from independence-of-irrelevant-alternatives type requirements, since it concerns stability under a coherent rescaling of intensities with a fixed set of alternatives, rather than stability under changes in the decision set.

This distinction suggests that robustness under preference intensification should be treated as a separate methodological criterion in AHP practice.

More broadly, rank reversal and related ranking irregularities in AHP and other multi-criteria decision-making methods have been extensively documented in the literature. Classical discussions include the legitimacy debate of rank reversal [29–31] and critical analyses based on independence-type axioms [32,33]. Numerous empirical and numerical investigations have examined structural sources of instability, including scale sensitivity and normalisation effects [34–36], simulation-based evidence [37–39], and comprehensive literature surveys [40,41]. Related robustness analyses under uncertainty or perturbations of the comparison matrix can be found, for example, in Arbel and Vargas [42],Faramondi et al. [43],Górecki et al. [44],Tu and Wu [45],Wang and Triantaphyllou [46].

In this broader context, the present study complements this stream of research by analysing the rank-level consequences of uniform preference intensification across several priority derivation methods, and by recovering the known scale invariance of the row geometric mean method within a structural homogeneity framework. This perspective also helps interpret the empirical patterns reported above, including the increase of rank instability with matrix order, the distinction between general rank reversals and top-ranked alternative changes, the persistence of instability below conventional consistency thresholds, and the differences observed across priority derivation methods.

5.4. Dependence on intensity and limiting regimes

The experiments reported above focus on finite and practically relevant intensification levels. In the discrete setting, the baseline matrix $A(2)$ is compared with $A(\alpha)$, $\alpha = 3, \ldots, 9$, which corresponds to uniform intensification exponents $\log(\alpha)/\log(2)$. In the continuous heterogeneous setting, each matrix A is compared with its uniformly intensified version $A^{(2)} = [a_{ij}^2]$. These results show that substantial rank instability can already arise for moderate intensification levels.

It is also useful to distinguish these finite-intensity results from the limiting behaviour as the intensification parameter becomes large. Heuristically, as strict preferences are strongly intensified, the PCM approaches a dominance-type regime in which the strongest directed comparisons increasingly determine the behaviour of the priority vector. Indeed, writing $a_{ij} = e^{b_{ij}}$ yields $A^{(k)} = [e^{kb_{ij}}]$, so comparisons with $b_{ij} > 0$ are exponentially magnified while those with $b_{ij} < 0$ are exponentially damped. In such regimes, the principal eigenvector may become increasingly influenced by a dominance-driven structure, depending mainly on the induced strict-preference pattern and on dominant comparison cycles rather than on fine-grained cardinal intensity differences. This asymptotic behaviour, however, is not addressed by the present simulations. Nevertheless, substantial instability persists at the moderate intensification levels examined in the simulations, which constitute a practically relevant range for AHP applications.

The matrix-order pattern observed in both experiments also suggests that limiting behaviour should be studied jointly in n and k. Increasing the number of alternatives creates more opportunities for pairwise order changes, whereas increasing k changes the relative dominance of the strongest comparisons. A rigorous asymptotic characterisation of eigenvector rankings under extreme intensification, for example as $k \to \infty$, remains an interesting direction for further research. A systematic study of how reversal percentages vary with the exponent k, including possible non-monotonic behaviour and dominance-regime transitions, is also left for future work.

6. Conclusions

This paper studied *intensity-of-preference (IOP) rank reversal* in multiplicative pairwise comparison methods. The phenomenon occurs when the ranking induced by a priority derivation method changes under the uniform power transformation $A \mapsto A^{(k)} = [a_{ij}^k]$, although the decision set and the ordinal preference pattern remain unchanged. The corresponding invariance requirement has previously been studied as scale invariance, with earlier eigenvector-based examples under ordinal-to-cardinal transformations given by Genest et al. [25] and the scale invariance axiom later formalised by Petróczy and Csató [24]. The present paper focused on the associated rank-level behaviour across matrix orders, priority derivation methods, inconsistency levels, and changes in the top-ranked alternative.

The results show that uniform preference intensification can produce substantial ranking instability under the eigenvector method. This instability is not restricted to the bottom of the ranking: in some cases, the top-ranked alternative also changes. The illustrative examples further show that similar top-rank changes can arise for several other priority derivation methods considered in the paper.

The simulations also show that conventional consistency screening does not guarantee robustness to preference-intensity recalibration. A matrix may satisfy the usual consistency-ratio criterion and still yield a ranking that changes after uniform intensification. Thus, consistency and robustness to uniform scale recalibration are distinct methodological properties.

Finally, the findings provide a strong argument in favour of the row geometric mean method whenever robustness to uniform preference intensification is regarded as important. Unlike the other priority derivation methods considered, the row geometric mean method is protected from IOP rank reversal by its scale invariance property. Robustness to uniform preference intensification should therefore be treated as a separate criterion when assessing priority derivation methods.

CRediT authorship contribution statement

Luis Ángel Calvo: Writing – review & editing, Writing – original draft, Visualization, Validation, Supervision, Software, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. **Jiří Mazurek:** Writing – review & editing, Writing – original draft, Visualization, Validation, Supervision, Methodology, Investigation, Formal analysis, Data curation, Conceptualization.

Code availability

The code used to generate the examples of Section 3, the simulations and figures of Section 4, and the additional tie-probability sensitivity check assessing the robustness of the results to alternative values of p_{tie}, where p_{tie} denotes the probability of sampling a tie in each unordered pairwise comparison, is available at: https://github.com/LuisAngelCalvoPascual/IOP-rank-reversal. The tie-probability sensitivity check is provided in the Jupyter notebook `tie_probability_sensitivity_evm.ipynb`.

Declaration of Generative AI

During the preparation of this work the authors used ChatGPT, developed by OpenAI, in order to support language editing, improve readability, and refine the wording of selected parts of the manuscript. After using this tool, the authors reviewed and edited the content as needed and take full responsibility for the content of the publication.

Declaration of competing interest

The authors have declared no conflict of interest

Data availability

Data will be made available on request.

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