

1 **Exergoeconomic analysis of potential magnet**
2 **configurations of a magnetic refrigerator operating**
3 **at 4.2K**

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14 **ABSTRACT:** This article compares the exergetic cost of cooling of an Adiabatic Demagnetization
15 Refrigerator (ADR) providing 1W of refrigeration at 4.2K, with two different magnetic field
16 sources: a Nb₃Sn superconducting (SC) magnet and a NdFeB permanent magnet (PM) Halbach
17 cylinder. The total cost of the system is assumed to be comprised of two components: the cost of
18 the magnetocaloric material (MCM), which is a function of the total volume of the MCM, and the
19 cost of the magnetic system, which depends on the MCM volume and the peak magnetic field.
20 The exergetic cost of cooling for different values of mass (volume) of MCM and hot source
21 temperatures are shown in the article, assuming a specific cost of the SC wire of 890\$/kg,
22 3500\$/kg for the MCM, and 100\$/kg for the PM. The SC appear to be the most cost-effective
23 solution for the system. However, if large temperatures spans are required between the hot source
24 and the cold source PMs emerge as a better option.

25 **KEYWORDS:** Magnetic refrigerator; Cryogenics; Superconducting magnet; Halbach cylinder.

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26	Contents	
27	1. Introduction	1
28	2. The magnetocaloric effect	2
29	2.1 Thermodynamics of the MCE	2
30	2.2 Carnot cycle (ADRs)	3
31	3. Components of magnetic refrigerator	4
32	3.1 Magnetic refrigerant	4
33	3.2 Magnetic field sources	4
34	3.2.1 Electromagnets: Superconducting magnets	5
35	3.2.2 Permanent magnets: Halbach cylinder	5
36	4. Exergoeconomic model	6
37	4.1 Superconducting solenoid cost	7
38	4.2 Permanent magnet cost	7
39	5. Results	7
40	5.1 Refrigerator cost at 4.2K	8
41	6. Conclusions	9
42		
43		

44 1. Introduction

45 The magnetocaloric effect (or MCE) [1], is a physical phenomenon that occurs in certain
46 materials, which under the exposure of a magnetic field suffer a significant change in entropy.
47 MCE was first observed by Warburg in 1881 [2]. In 1933, Giauque and MacDougall [3] surpass
48 the 1K barrier achieving a temperature of 250mK with a magnetic refrigerator (MR). Since then,
49 magnetic refrigeration has been employed to provide cooling over a wide temperature range.

50 For low temperatures applications, ranging from a few Millikelvin to a few Kelvin,
51 Adiabatic Demagnetization Refrigerators (ADRs) are used. The refrigeration process followed by
52 an ADR is analogous to a Carnot cycle. The thermodynamics of these devices are described in
53 [4]. For higher temperatures, from hydrogen liquefaction to room temperature refrigerators,
54 magnetic refrigerators have combined the refrigerant material and regenerator material into one
55 device which uses a regenerative cycle, such as Ericsson cycle, to provide cooling. These
56 refrigerators are denominated Active Magnetic Refrigerators (AMRs) [5].

57 As compared to conventional vapor compression refrigeration, magnetic refrigeration is
58 simple, safe, quiet, compact, and has a high cooling efficiency. Their efficiency superiority is
59 especially notable below 20K, as traditional gas refrigerators cease to operate efficiently, since
60 the specific heat of the regenerator materials drops off rapidly at these temperatures. This
61 phenomenon can be indirectly appreciated by the low Carnot efficiency reported by these devices
62 [6]. However, higher costs are still the main drawback for this technology, especially due to the
63 need of rare earth materials for the refrigerant material and the magnetic field source.

64 In this paper the basic principles of the MCE effect and ADR cycles will be reviewed.
 65 Afterwards, the main components of an MR will be examined, with special dedication to two
 66 possible magnetic field sources: superconducting and permanent magnets. Previous work has
 67 been conducted to compare the performance of MRs using both technologies, focusing only
 68 ambient temperature applications [7]. In this case, the cost needed for each magnetic configuration
 69 will be discussed with the objective of developing an exergoeconomic model for a 1W ADR type
 70 magnetic refrigerator operating at 4.2K. Such refrigerator could be used for an MRI machine, or
 71 scaled in power for other applications that make use of regenerative cryocoolers, with less than
 72 50W of cooling power, e.g.: for low temperature electronics, or certain particle accelerators
 73 applications [6].

74 2. The magnetocaloric effect

75 The entropy of a magnetocaloric material (MCM) can be divided in three components [8], the
 76 magnetic entropy S_m , the entropy of the lattice S_l , and the electronic entropy of the material's
 77 free electrons:

$$78 \quad S_T(B, T) = S_m(B, T) + S_l(T) + S_e(T) \quad (1)$$

79 where the lattice and electronic entropy depend on the material temperature, and the magnetic
 80 entropy is dependent on both the magnetic field and the temperature.

81 If an external magnetic field is applied adiabatically to a MCM, the magnetic moments will
 82 tend to align with the field, thereby decreasing the magnetic entropy of the material while
 83 maintaining the value of S_T . To compensate for the reduction in the magnetic entropy, lattice, and
 84 electronic entropy must increase, which causes an increase in the temperature of the sample. If
 85 the magnetic field is withdrawn, the process reverts, the magnetic moments will return to their
 86 original alignment capturing energy from the lattice and electronic system, thus reducing the
 87 temperature to its original value.

88 2.1 Thermodynamics of the MCE

89 A thermodynamic material can exchange energy with an external system through heat and work
 90 interactions, which can be expressed as a differential energy balance:

$$91 \quad dU = TdS + dW \quad (2)$$

92 Work interactions can be expressed more specifically if the energy is exchange in terms of
 93 mechanical, chemical or magnetic work:

$$94 \quad dW = -PdV + \sum \mu_j dN_j + \mu_0 V_m H dM \quad (3)$$

95 For a magnetic refrigeration system, where the volume is not modified, i.e $dV = 0$, and there
 96 is no exchange of chemical energy, Eq. (2) is expressed as:

$$97 \quad dU = TdS + \mu_0 V_m H dM \quad (4)$$

98 The total specific entropy change of the system can be represented as:

$$99 \quad ds = \left(\frac{\partial s}{\partial T} \right)_H dT + \left(\frac{\partial s}{\partial H} \right)_T dH \quad (5)$$

100 Using the definition of specific heat, $C_{p,H} = T \left(\frac{\partial s}{\partial T} \right)_H$, and the Maxwell relation $\left(\frac{\partial s}{\partial H} \right)_T =$
 101 $\mu_0 \left(\frac{\partial m}{\partial T} \right)_{p,H}$, Eq. (5) can be expressed as:

$$102 \quad ds = \frac{C_{p,H}}{T} dT + \mu_0 \left(\frac{\partial m}{\partial T} \right)_{p,H} dH \quad (6)$$

103 Under the condition that $ds = 0$, the following expression can be derived:

$$104 \quad \Delta T_{ad} = -\mu_0 \int_{H_I}^{H_F} \frac{T}{C_{p,H}} \left(\frac{\partial m}{\partial T} \right)_{p,H} dH \quad (7)$$

105 Where H_F and H_I are the final and initial magnetic fields. The expression is denominated as
 106 adiabatic temperature change and is the reversible change of temperature that a magnetocaloric
 107 material undergoes in an adiabatic process under certain magnetization conditions.

108 When the MCM undergoes an isothermal process ($dT = 0$), Eq. (5) yields:

$$109 \quad \Delta S_M = \mu_0 \int_{H_I}^{H_F} \left(\frac{\partial m}{\partial T} \right)_{p,H} dH \quad (8)$$

110 In this case, the change in entropy is equal to the magnetic entropy change. Both Eq. (7) and
 111 Eq. (8) are used to characterize the magnetocaloric effect of certain material. It can be derived
 112 from these expressions that the maximum value appears when a significant change of
 113 magnetization occurs. This is the reason why magnetocaloric materials are often used near a phase
 114 transition, in order to maximize the heat extraction.

115 To be able to compare among different materials a variable denominated refrigerant capacity
 116 (RC) or relative cooling power (RCP) is commonly used, which is defined as:

$$117 \quad RC(H) = \int_{T_{cold}}^{T_{hot}} \Delta S_m(T, H) dT \quad (9)$$

118 Where T_{cold} and T_{hot} are the temperature of the cold and hot reservoirs. The advantages of
 119 using RC over other parameters is discussed in [9].

120 **2.2 Carnot cycle (ADRs)**

121 Carnot cycles consist of four processes: two adiabatic and two isothermal processes as illustrated
 122 in the T-S diagram of Figure 1 (left). Other cycles are employed in MR, as Ericsson cycles Figure
 123 1 (right), although they won't be explored in this article.

124 The first step in the Carnot process (1-2) is an adiabatic magnetization where an external
 125 field is applied. The entropy remains constant during the magnetization process. The second step
 126 (2-3) is an isothermal magnetization where the heat produced is rejected to the hot source. The
 127 third step is an adiabatic demagnetization process lowering the temperature of the MC material.
 128 Finally, in the last step, the sample is demagnetized isothermally absorbing heat from the cold
 129 source. The area (1-2-3-4) represents the work done during the process, and is equal to:

$$130 \quad w = \oint_3^2 T dS - \oint_4^1 T dS = T_h(S_2 - S_3) - T_c(S_1 - S_4) \quad (10)$$

131 Where the first term of the right-hand side of the equation is the heat rejected to the hot
 132 source and the second term is the heat absorbed, i.e the cooling load of the refrigerator:

$$133 \quad q_c = \oint_4^1 T dS = T_c(S_1 - S_4) \quad (11)$$

134 The cooling power of the cycle is proportional to the frequency:

$$135 \quad P = f * q_c = f * T_c * \Delta S_c \quad (12)$$

136 The maximization of the cycle frequency is a key parameter in the design of a magnetic
 137 refrigerator; however, it is limited by the thermal losses produced in the heat exchange process
 138 between the refrigerant, and the hot and cold sources. Therefore, the minimization of thermal

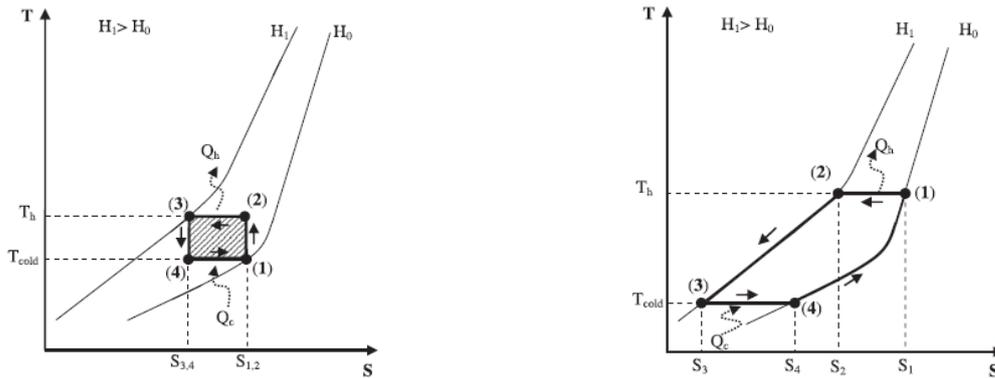
139 losses is essential for optimizing the refrigerator. Because of the reversibility of the Carnot cycle,
 140 the COP of the

$$141 \quad COP = \frac{T_c}{T_h - T_c} \quad (13)$$

142 In refrigerators, it is useful to define the exergetic cooling power, which is the work
 143 equivalent value of the heat flow to the cold source:

$$144 \quad Ex_c = q_c \left(\frac{T_h}{T_c} - 1 \right) \quad (14)$$

145 It shows that if the cooling power or the temperature difference between reservoirs is
 146 negligible the useful refrigeration is zero in either case.



147 Figure 1. T-S diagram of an MR Carnot cycle (left), and T-S diagram of an Ericsson cycle with
 148 regeneration (right). Reprinted from [10]

149 3. Components of magnetic refrigerator

150 3.1 Magnetic refrigerant

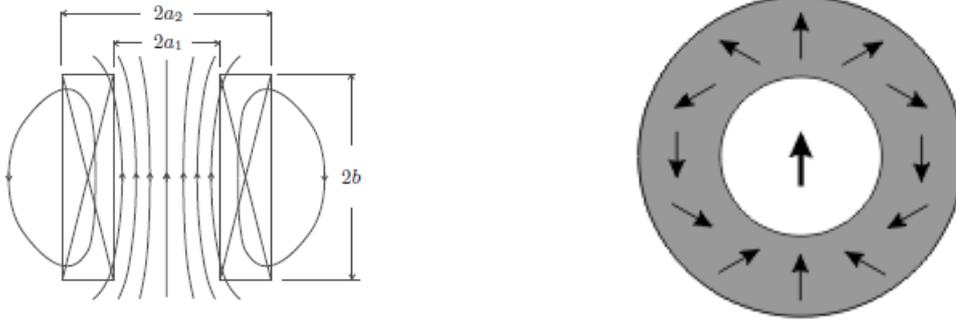
151 The MC material is an essential component on the design of a magnetic refrigerator, and two
 152 types can be distinguished regarding the order of the phase transition between the ferromagnetic
 153 and paramagnetic states near their Curie temperature: first order magnetocaloric materials
 154 (FOMT), and second order magnetocaloric materials (SOMT). The former undergoes a
 155 discontinuous change in magnetization with temperatures, while the latter undergoes a continuous
 156 change. Comprehensive reviews regarding the different MCM exist within the literature [11] from
 157 low to ambient temperatures applications.

158 Some authors have provided a practical set of selection rules for picking a magnetic
 159 refrigerant depending on the application [12]. Some of these rules are: the selection of a suitable
 160 Curie temperature, intensity of the magnetocaloric effect, high electrical resistivity (to prevent
 161 eddy currents), or good manufacturing and corrosion properties.

162 3.2 Magnetic field sources

163 The magnetic field is a crucial part of the magnetic refrigerator. There are two potential sources:
 164 electromagnets, or permanent magnet assemblies. Among the first, two types are recognized:
 165 superconducting or copper electromagnets. The design of the magnetic field source is key since
 166 it is usually the most expensive component, representing in some cases up to 85-90% of the cost
 167 [13], although it will vary depending on the application. In [12] a comprehensive review of the
 168 different magnetic field sources and their characteristics for MR is found. In the following section,
 169 superconducting magnets and a cylindrical Halbach array using permanent magnets will be

170 examined, as both are the most common options for MRs. Superconducting magnets are capable
 171 of providing high and stable magnetic fields with low space requirement, PMs provide smaller
 172 fields although it is not necessary to cool them down to cryogenic temperatures.



173 Figure 2. Two types of magnetic field sources: solenoid, a type of electromagnet (left) and a Halbach
 174 cylinder made of permanent magnets (right)

175 3.2.1 Electromagnets: Superconducting magnets

176 In MR applications, superconducting magnets have been utilized since the early beginning of the
 177 field. It have been used in refrigerators from very low temperatures to room temperature
 178 refrigeration, some of the developed prototype devices can be found in [11].

179 A typical superconducting system is composed of three main parts: a superconducting coil,
 180 a cryogenic system, and a power conditioning system. The topology of the superconducting coil
 181 in magnetic refrigerators is typically a solenoid, where the aperture of the magnet is a cylindrical
 182 volume where the MCM is placed. There is plenty of information in the literature concerning the
 183 design of superconducting solenoids [14].

184 The main variable when designing a magnet for MR is the magnetic field, which is not
 185 homogenous in all the volume, neither in magnitude nor direction. In a SC solenoid, where there
 186 is no ferromagnetic material, the magnetic field at the center is given by the following equation:

$$187 \quad B_z(0,0) = \mu_0 \lambda J a_1 F(\alpha, \beta) \quad (15)$$

188 Where μ_0 is the magnetic permeability in the vacuum, λ the field factor, J the current density,
 189 a_1 the internal radius, and $F(\alpha, \beta)$ the field factor which depends on the geometric parameters
 190 [14] ($\alpha = \frac{2a_2}{2a_1}$) and ($\beta = \frac{2b}{2a_1}$).

191 3.2.2 Permanent magnets: Halbach cylinder

192 A permanent material is a magnetic material which remain magnetized after the withdrawal of an
 193 external magnetic field. Permanent magnets materials are usually divided into: ceramics
 194 materials, rare-earth materials, Al-Ni-Co materials and polymer bonded materials. A review
 195 regarding their composition and critical properties can be found in [15].

196 A way to classify a permanent magnet array is to consider the figure of merit, M^* , which
 197 according to [16] is equal to:

$$198 \quad M^* = \frac{\int_{V_{field}} |\mu_0 H|^2 dV}{\int_{V_{mag}} |B_{rem}|^2 dV} \quad (16)$$

199 Where V_{field} is the volume where the magnetic field ($\mu_0 H$) is created, V_{mag} is the volume
 200 of the permanent magnets and B_{rem} the remanence, the magnetization left after the removal of

201 the external magnetic field. If it is assumed that the magnetic field is constant in all the volume,
 202 as well as the remanence, Eq. 16 gives:

$$203 \quad M^* = \frac{\left(\frac{\mu_0 H}{B_{rem}}\right)^2 V_{field}}{V_{mag}} \quad (17)$$

204 Which if V_{field} is substituted by the mass of the MCM divided by its mass density, and one
 205 minus the porosity of the regenerator, and also substituting the volume of the magnet by its mass
 206 divided by the density, the following expression can be derived:

$$207 \quad m_{magnet} = \left(\frac{\mu_0 H}{B_{rem}}\right)^2 \frac{m_{MCM} \rho_{mag}}{(1 - \varepsilon) \rho_{MCM} M^*} \quad (18)$$

208 In which the mass of the permanent magnets is related to its magnetic and mechanical
 209 properties, and to the mechanical properties of the MCM material.

210 **4. Exergoeconomic model**

211 The term exergoeconomics is used to describe the combination of an economic and exergy
 212 analysis [17]. In an exergoeconomic balance each exergy stream is associated with a cost. Using
 213 this methodology, the cost balance of a refrigerator would be:

$$214 \quad C_c = C_{capex} + C_{op} \quad (19)$$

215 Where C_c is the cost rate of cooling, C_{capex} is the cost rate of capital, the equipment, and
 216 C_{op} is the cost rate of operation of the device during its lifetime, which includes operation and
 217 maintenance costs. In a detailed analysis, operating costs should be included. However, as they
 218 represent a small fraction of the total cost, independently of the solution proposed, they will be
 219 neglected, e.g.: a 4.2K, 1 W MR with a 50% Carnot efficiency [12], will have an equivalent 150W
 220 electric consumption, which with an electricity cost of 0.1\$/kWh yields an operating cost of 131\$
 221 per year (less than 3% of the capital costs as will be shown later).

222 Capital costs should be amortized for the expected life of the device in order to transform
 223 them into a cost rate. For that purpose, it is used the capital recovery factor (CRF):

$$224 \quad C_{capex} = CRF * Z \quad (20)$$

225 Where Z are the absolute capital expenses, and CRF is given by:

$$226 \quad CRF = \frac{i(1 + i)^n}{(1 + i)^n - 1} \quad (21)$$

227 If c_c is defined as the cost per unit cooling, such as C_c is given by the product of c_c and the
 228 cooling exergy, E_c . Eq. 18, neglecting operating costs, would be expressed:

$$229 \quad c_c E_x c = CRF * Z \quad (22)$$

230 In the case of a magnetic refrigerator, the capital costs Z , are mainly due the cost of the
 231 magnet and the MCM. Other costs are ignored. Hence, the cost per unit of exergetic cooling is:

$$232 \quad c_c = CRF * \frac{Z_{magnet} + Z_{MCM}}{E_x c} \quad (23)$$

233 The cost of the refrigerant material is easily determined as it can be defined as the product
 234 of the cost per unit volume c_{MCM} and the total volume of material used, V_{MCM} . Establishing the
 235 cold source temperature, the cooling power needed, the lower magnetic field (typically 0 T), and
 236 the operating frequency, a relation between the mass of MCM needed, the hot source temperature
 237 and the value of the higher magnetic field, for an ADR, can be obtained with Eq. 12, and the
 238 isentropic equality in the process 3-4. If one of the three variables is fixed, the others can be

239 immediately obtained. The cost of the magnetic field source is related to the mass of the MCM,
 240 and the value of the higher magnetic field as will be shown in the following sections.

241 **4.1 Superconducting solenoid cost**

242 The capital cost of a superconducting magnet can be related to the superconducting material mass
 243 used in the magnet. To compute the mass, the following assumption will be made: that the length
 244 (2b) of the magnet is much greater than the internal radius. This is particularly true for a magnet
 245 of these characteristics, as with this configuration the magnetic field would be more homogenous
 246 in the internal volume, where the magnetocaloric refrigerant will be placed. With this assumption,
 247 the magnetic field at the center is:

$$248 \quad B_z(0,0) = \frac{\mu_0 NI}{2a_1\beta} \quad (24)$$

249 Full derivation of this terms can be found in [14]. It is also known that the current density
 250 should be equal to the total ampere-turns divided by the cross section of the magnet:

$$251 \quad \lambda J = \frac{NI}{a_1^2\beta(\alpha - 1)} \quad (25)$$

252 Therefore, being $V_{MCM} = 2\pi a_1^3\beta$ the volume of the magnetocaloric material, and $V_{Magnet} =$
 253 $2\pi a_1^3(\alpha^2 - 1)\beta$, the following expression can be derived:

$$254 \quad B_z(0,0) = \frac{\mu_0 J V_{mag} a_1}{V_{MCM}(\alpha + 1)} = \frac{\mu_0 J V_{mag} \rho_{MCM} a_1}{V_{MCM} \rho_{mag}(\alpha + 1)} \quad (26)$$

255 **4.2 Permanent magnet cost**

256 Eq. 18 gives a relation between the magnet mass and the mass of MCM, therefore if the unit cost
 257 per kg of the PM is known, it would be possible to establish the cost. However, M^* is not yet
 258 defined. For a Halbach cylinder of infinite length it can be shown through the relation of the field
 259 in the bore, $\mu_0 H = B_{rem} \ln\left(\frac{r_o}{r_i}\right)$, that the figure of merit M^* is [18]:

$$260 \quad M^* = \frac{\left(\frac{\mu_0 H}{B_{rem}}\right)^2}{e^{2\frac{\mu_0 H}{B_{rem}}} - 1} \quad (27)$$

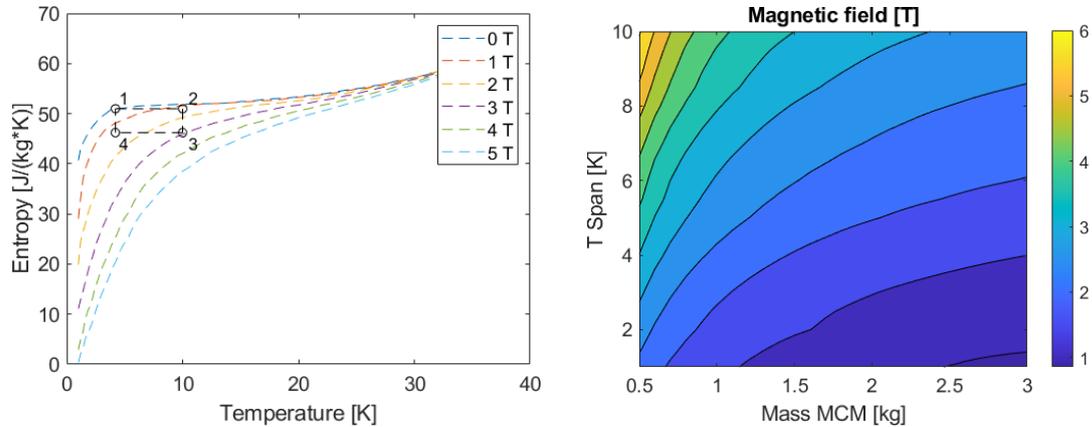
261 For NdFeB magnets, with a remanence of 1.2T, there is an optimum value at $\frac{\mu_0 H}{B_{rem}} \approx 0.8$,
 262 which yields a figure of merit $M^* \approx 0.162$. Although this equation could yield values over 3T for
 263 the Halbach cylinder, it would be limited to that value due to practical motives.

264 **5. Results**

265 In this section, the costs of two identical magnetic refrigerators in terms of cooling, one with a
 266 superconducting magnet and the other with a permanent magnet, will be compared in terms of
 267 cost per unit of exergetic cooling. Both systems will have a refrigeration power of 1W at 4.2K,
 268 operating at a frequency of 0.05Hz, with a regenerator porosity of 0.4, and will use GGG as
 269 magnetic refrigerant. GGG properties has been discussed in [19]. Figure 3 (left) shows the entropy
 270 dependence of GGG on temperature and magnetic field. The assumed unit cost of GGG is
 271 3500\$/kg, provided by American Elements [20].

272 The hot source temperature, and the mass of the magnetocaloric material will be the iterative
 273 variables, which will iterate between 1-10K and 0.5-3kg., respectively. Figure 3 (right) shows the
 274 peak magnetic field in the regenerator as a function of the mass of the MCM, and the hot source

275 temperature. Although, the direction of the magnetic field created by the superconducting magnet,
 276 and the Halbach cylinder, are not in the same plane, it has been assumed that the demagnetization
 277 factors are equal in both directions and the average field in the regenerator is equal to the peak
 278 field.



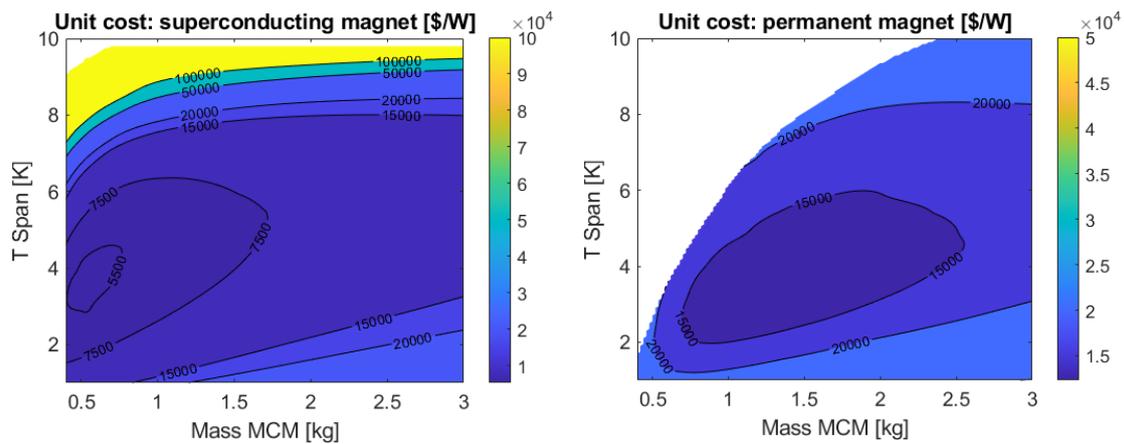
279 Figure 3. Entropy of GGG as a function of Temperature [K] and Magnetic Field [B] (left), and magnetic
 280 field dependence on mass and hot source temperature to provide 1W at 4.2K of cooling (right).

281 The superconducting magnet is considered to be operating at the temperature of the hot
 282 source, and the cost of refrigeration has not been included. In this case, Nb₃Sn technology has
 283 been considered [21], for which a fitting function [22] has been used to extrapolate the current
 284 density values to other temperatures and magnetic fields. In this case, NbTi is not suitable due to
 285 its reduced range of operating temperatures, however HTS could be of interest for the opposite
 286 reason. It is assumed that the filling factor λ , is equal to 0.8, and the working point of the magnet
 287 is 0.75 [14]. A unit price of 890\$/kg has been used for the SC magnet, provided by the
 288 manufacturer [21]. For the permanent magnet a price of 100\$/kg [7] is used. Since both devices
 289 are expected to have a similar lifetime, the capital recovery factor has been omitted.

290 5.1 Refrigerator cost at 4.2K

291 Having computed the maximum field in the regenerator the derivation of the costs of the magnetic
 292 systems is straightforward, using Eq. 25 for the superconducting magnet, and Eq. 16 and 26 for
 293 the permanent magnet configuration. Figure 4 shows that the minimum value cost per unit of
 294 heat transfer is obtained with the use of a superconducting magnet, with a MCM mass of around
 295 0.6 kg. and a hot source temperature of 8K. If this configuration is compared against Figure 3, it
 296 is observed that the optimum magnetic field is in the range of 3T, much lower than the maximum
 297 magnetic field achievable by a Nb₃Sn superconducting magnet, and used in previous refrigerators
 298 [23]. It is also seen that the cost rapidly increases as the hot source temperature increases. This is
 299 due to the deterioration of the current density with temperature.

300 On the other hand, the permanent magnet configuration shows a different performance. As
 301 expected, the minimum values of cost appear with higher mass of the MCM than in the previous
 302 configuration, this implies lower magnetic fields on higher volumes. It is also noteworthy to
 303 observe in the Halbach array configuration, the cost dependence with the hot source temperature.
 304 In this case, the increase with temperature is much slower than with a superconducting magnet,
 305 as the magnetic properties of the permanent magnet do not depend on temperature. For hot source
 306 temperatures over 12-13K, the permanent magnet configuration appears to be more cost-efficient
 307 if capable of providing the required magnetic field.



308 Figure 4. Cost per unit of heat transfer for a magnetic refrigerator providing 1W at 4.2K, with a
 309 superconducting magnet (left) and a permanent magnet (right). The white area, in both graphs, comes
 310 from the impossibility of achieving the required magnetic fields in the specific operating conditions.

311 6. Conclusions

312 The unit cost of a 1W ADR type magnetic refrigerator operating at 4.2K was determined over
 313 different hot sources temperatures, and values of MCM mass, for two possible magnetic sources:
 314 a Nb₃Sn superconducting magnet, and a NdFeB permanent magnet in a Halbach array. Assuming
 315 a cost of 890\$/kg for the SC, and 100\$/kg price for the NdFeB, it was shown that the most cost-
 316 effective solution was using a superconducting solenoid operating with an optimum magnetic
 317 field of 3T. However, if the temperature span, between the cold source and hot source increases
 318 over 8K, the permanent magnet configuration becomes more cost-effective. This creates the
 319 necessity (1) to further study the use of PMs in MRs with large temperature differences, spanning
 320 from LN₂ (77K), to LH₂ (20K) to LHe (4.2K). (2) to explore the possibility of using HTS
 321 superconducting materials over large temperatures spans for MRs. Likewise, further research is
 322 needed in the magnetic optimization of other types of thermodynamic cycles more appropriate to
 323 temperatures over 20K, such as AMRs.

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