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MEDIUM-TERM FORECASTING OF ELECTRICITY PRICES: A HYBRID METHODOLOGY BASED ON FUNDAMENTAL AND TECHNICAL ANALYSIS

Predicción del precio eléctrico en el medio plazo: Una metodología híbrida basada en análisis fundamental y técnico

Tesis para la obtención del grado de Doctor

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Agradecimientos

Contents

Dissertation

The thesis developments and original contributions have been presented in the following publications. These are: Bello et al. (2015); Bello et al. (2016); Bello et al. (2016); Bello et al. (2016a); Bello et al. (November 24-25, 2011); Bello et al. (June 10-14, 2012); Bello and Reneses (June 23-26, 2013); Sánchez et al. (May 20-22, 2015); Yeo et al. (2015); Bello and Reneses (2015).

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Bello, A., Bunn, D., Reneses, J., Muñoz, A., 2016a. Medium- term probabilistic forecasting of electricity prices using multifactor dynamic models in a hybrid framework. Invited for publication in Energies, –.

Bello, A., Bunn, D., Reneses, J., Muñoz, A., 2016b. Medium-term probabilistic forecasting of electricity prices: A hybrid approach. Accepted for publication in IEEE Transactions on Power Systems, –.

Bello, A., Reneses, J., Muñoz, A., 2016c. Medium-term probabilistic forecasting of extremely low prices in electricity markets: application to the spanish case. Energies 9 (3), 193 – 1 –193 – 27.

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Chapter 1

Introduction

Contents

In between the increased focus on short-term forecasting and real time balancing, and the emerging concerns about resource adequacy in the longer term, the task of medium-term planning remains an area of substantial business risk for power generators. This is even more relevant as a result of the high penetration of intermittent technologies and frequent regulatory and structural changes occurring in power markets. In this context, forecasting electricity prices in the medium term, from one month to several months ahead, is crucial for all market agents. This is not an easy task, since seasonal weather risks affecting both demand and supply, volatile fuel prices, as well as the uncertain behavior of competitors in the market must be carefully taken into account. All these factors combine to require accurate forecasts that go beyond point estimates to provide at least interval, if not full density, estimates of market price risks. This thesis is aimed at developing a new methodology to estimate in a precise and probabilistic manner the hourly price of electricity in the medium term, considering for this the hybridization between the two traditional approaches used in financial markets for the prediction of prices: fundamental and technical. This Chapter introduces the context and motivation of the thesis, defines the main objectives and describes the structure that has been followed in the document.

1.1 Motivation

In recent decades, a large number of countries have liberalized their electricity sectors. Before that, most energy sectors were not open to competition and prices were set by regulatory bodies on the basis of the costs of generation, transmission and distribution. The recent liberalization of power markets has led generation companies and wholesale buyers to assume a more intense risk exposure than in the traditional framework. In particular, all of them are subject to the uncertain spot price of electricity[1], which determines their revenues and costs, and therefore, their success. It is also necessary to emphasize that spot market prices are an important reference for the trading of forward, future and option

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[1] As stated in Weron (2006), the electricity 'spot market', unlike most other commodity or financial markets, typically consists of 24 hourly or 48 half-hourly auctions that take place simultaneously one day in advance and does not allow for continuous trading. It should be noted that although we use the terms spot and day-ahead interchangeably in this thesis, the former need not necessarily refer to the day-ahead market. As discussed in Weron (2014), the European convention is to refer to the day-ahead price as the spot price. However, in the US, the term spot price is typically reserved for the intra-day real-time market, while the day-ahead price is called the forward price.

contracts. In this context, electricity price forecasting has become essential for any market player in their decision-making process and strategy development. This in turn has fostered extensive research in electricity price forecasting.

It is usual to distinguish between short-, medium- and long-term electricity price forecasting, but there is no consensus in the literature regarding the scope of each horizon or lead time (see Weron (2014)). Electricity price forecasting in the short term, which generally covers from a few minutes up to a few days ahead, is of paramount importance in day-to-day market operations. More specifically, short-term forecasts are useful for market players to develop more efficient bidding and trading strategies, as well as production or consumption schedules. At the opposite extreme, long-term price forecasting, which comprises from a few months to several years ahead, is extensively used for investment decisions.

However, not only price forecasting for these lead times is interesting for market agents. In a medium-term horizon, which covers from one to two months, there is a great need for price estimates, as mentioned in Torbaghan et al. (2012). However, as discussed in Bello et al. (2015), very little research has been conducted in this field, in contrast to short-term price forecasting. In such time horizon, electricity producers must be able to anticipate the amount of energy they have to sell through bilateral contracts and through the wholesale market. Likewise, consumers must make similar decisions regarding the procurement of electricity. By means of a reliable price forecast, producers or retailers are able to negotiate favorable bilateral and financial contracts. Therefore, wherever there is a market for electricity with medium-term time of delivery (e.g., future or forward contracts) there will be a need for forecasting the prices. In addition, the applications of mid-term price forecasting also include: risk management support (at both generation and retail level), derivatives pricing [2], maintenance scheduling, balance sheet calculations, operational decisions that are oriented to the optimal exploitation of available resources, right signals to the short-term operation (i.e. preparing the signals needed for an optimal hydro management in the short term or providing signals for bidding thermal units subject to medium-term constraints such as take-or-pay clauses or emission quotas) and cost efficient fuel purchasing policies (in which Gencos have a natural short position).

Furthermore, not only is forecasting the expected value of the electricity price in-

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[2] Derivatives are financial instruments which derive their value from that of an underlying asset (for instance the spot electricity price). The four basic types are forward contracts, futures, swaps and options.

creasing in importance, but most recently, there is a growing interest on having a quantified picture of future price uncertainty. Point forecasts, unlike probabilistic forecasts, even when they are very accurate, provide no information on risk exposure. In addition, in order to fully and adequately examine the widest range of decisions, particularly in a context with many uncertainty sources (e.g. fuel prices, intermittent renewable energy generation and hydro conditions), decision makers need more a probabilistic prediction than just an expected value of the price. In spite of their practical importance, probabilistic forecasts, which can be in the form of quantiles, intervals, or density functions, have received much less attention than predicting expected prices.

This may be due to the fact that probabilistic electricity price forecasting is a difficult task. Note that electricity prices show a unique behavior, which is significantly different to other traded commodities. High-quality forecasts should be able to capture its distinctive features such as: mean reversion nature, multiple levels of seasonality, high volatility (which can be even two orders of magnitude higher than for other assets, as stated in Weron (2006)), non-constant mean and variance, strong correlation with other variables, as well as skewness and heavy tails.

Particularly relevant in electricity price dynamics is the increasingly common existence of outrageously accused movements (or "spikes")[3]. The occurrence of extreme price events is not an exclusive feature of electricity prices, but something that can also occur with other commodities such as natural gas (e.g. prices of Chicago in the winter of 1995 or California in the winter of 2000), or more generally what happened in financial historical crises (such as the Great Depression triggered by the stock market crash of 1929 or the Asian recession that began in 1989). Notwithstanding, in electricity markets this is not a punctual but a recurring phenomenon, which makes the dynamics of electricity price series more complicated than other commodities. Extremely high prices have been frequently associated with situations where the demand or fuel prices have grown considerably, where there has been network congestion, market power or adverse weather conditions. On the other hand, the recently increasing deployment of non-dispatchable generation is also leading to the collapse of thermal demand at certain moments, and as a consequence, the appearance of extremely low prices (zero or even negative prices depending on the considered regulatory

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[3] These spikes are sudden extreme upward or downward changes in the time series in a very short period of time followed by a recovery of the previous level.

framework)[4].

The occurrence of these events is becoming progressively more recurrent in those systems where there has been a rapid and remarkable development of renewable energy sources, like Germany and Spain (see (Würzburg et al. (2013), Edenhofer et al. (2013))). We should note that in the Spanish case, which can be chosen as a paradigmatic example due to the massive penetration of renewable energy sources, there is a regulatory framework in which negative bids are not accepted, so the occurrence of zero prices is more and more frequent. Proof of this is that during 2013 there were several days when prices were zero in virtually all hours of the day. These periods were followed by others in which the presence of very high prices (motivated by the attempt to recover losses) were a repeated event.

The presence of such extreme episodes can have significant consequences for market participants. For instance, even well-capitalized power firms, which may have large power price exposures, can suffer corporate default or bankruptcy when adverse price changes occur suddenly (see Bessembinder and Lemmon (2002)). A notable example is disclosed in Ullrich (2012), which refers to the episode that occurred in Texas during the summer of 2008, when electricity prices reached 4000 $/MWh and caused the bankruptcy of five retail companies. Another example occurred in June 1998, when power prices in the U.S. Midwest increased from 300 $/MWh to over 7000 $/MWh, leading to contract defaults and the near bankruptcy of some power firms.

Although the presence of this unusual volatility is often seen as negative inasmuch as it is associated with uncertainty and risk, it can also be seen as positive in the sense that market participants can expect to make a significant profit from it. One of the main issues arising is how to assess the risks prevailing in these markets, as well as how to make purchases and sales knowing that peaks can appear without notice. However, in spite of its great practical utility, predicting extreme prices in a given time horizon has received less attention than predicting expected average prices (under normal conditions). In this sense, higher-frequency (i.e. at hourly granularity) price predictions could provide a much richer source of information than with other time scales such as the daily average price.

In view of the previous discussion, it would be desirable for all market stakeholders to have a methodology to predict electricity prices in the medium term, with

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[4] It must be noted that from an economic perspective, negative or near zero prices (depending on the regulatory framework considered) can be rational. For example, if the costs of stopping and starting a power plant outweigh the loss caused by accepting extremely low prices.

an hourly basis, and in a probabilistic framework, which has not been touched upon in the literature yet (see Yan and Chowdhury (2014)). If trying to predict electricity prices entails great difficulties due to its particular characteristics, which have already been discussed, doing it in a medium-term horizon brings additional challenges. Thus, in a medium-term context, different factors should be considered. On the one hand, it is a well-known that in the medium term forecasting electricity prices is subject to several sources of uncertainty, such as structural changes, system demand, wind energy, hydrological conditions or fuel prices. On the other hand, both the technical characteristics of the system and the strategic behavior of market agents are of paramount importance.

In this context of price forecasting, two distinct trends of modeling approaches can be distinguished in the literature: technical analysis [5] and fundamental analysis. These approaches present conflicting characteristics. Overall, a pure fundamental analysis focuses on the study of the causes that govern price dynamics, taking into account all possible relevant factors, such as the interaction between supply and demand. This approach is based on an adequate representation of technical characteristics and operational constraints of the system, as well as the market rules.

By contrast, a pure technical analysis relies on the study of the consequences and tries to find functional relationships that explain the behavior of the prices. Among its basic premises are that history repeats itself and that all factors having some influence on the price (whether economic, regulatory, psychological or of any other nature) are explicitly reflected in the price series. However, there are numerous academic research studies that conclude that technical analysis has limitations to make predictions, especially in the medium term. One of the major criticisms made to this approach has to do with the assumption that financial markets are efficient[6], as well as the fact that some of the historical patterns of prices will not necessarily be repeated in the future because market conditions may change over time[7]. This is for instance the case of structural changes related to the market design (i.e. coupling of markets, generation expansion or moth-

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[5] It should be noted that some authors classify statistical models and econometric techniques as technical analysis tools. In this way, these terms are used interchangeably throughout this thesis.

[6] The efficient market hypothesis suggests that at any given time, prices fully reflect all available information on a particular stock and/or market. According to this assumption, no investor has an advantage in predicting a return on a stock price because no one has access to information that is not already available to everyone else.

[7] Unless such changes in market conditions have occurred previously and have been sufficiently reflected on other exogenous variables.

balling of generation units), participant merging or new participants, subsidies for renewable energy integration, drops in energy demand due to economic crisis or fuel price impacts. This is not the case of the fundamental approach, which is robust against these possible new conditions in the electricity markets.

Among the advantages of technical analysis are that it does not require such a profound knowledge of the market as fundamental analysis (so it is adaptable to any financial instrument), that is able to replicate more accurately the so-called stylized facts (for example mean reversion, seasonality or price spikes, as stated in Weron (2006)) of electricity price with high frequency data (i.e. hourly), that the theory behind it applies much in the same way for any time horizon and that is capable to capture behavioral, speculative and psychological aspects.

There are many studies that use a fundamental or a technical approach independently, but it is rare to find research that combines both focuses in a comprehensive methodology. This is because, in most cases, both approaches have been seen as mutually exclusive. In order to address the particularities of the medium term, it would be desirable to be able to use fundamental and technical approaches in a complementary manner for this time horizon, i.e. by leveraging the strengths of each individual technique to add value to the price prediction methodology. Fig. 1.1 summarizes the potential benefits of such a hybrid approach[8] as opposed to the traditional view of independence between the supporters of technical and fundamental analysis.

1.2 Objectives

In this section the main objectives to be met with this thesis are presented. A central objective and a series of specific objectives can be distinguished.

1.2.1 Main objective

The main objective of this thesis is to develop a new methodology to estimate the probability distribution function of electricity prices in the medium term (understanding that this corresponds to a time horizon spanning from one to two

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[8] This terminology is used within the context of this thesis to categorize those techniques that join a fundamental and a technical focus.

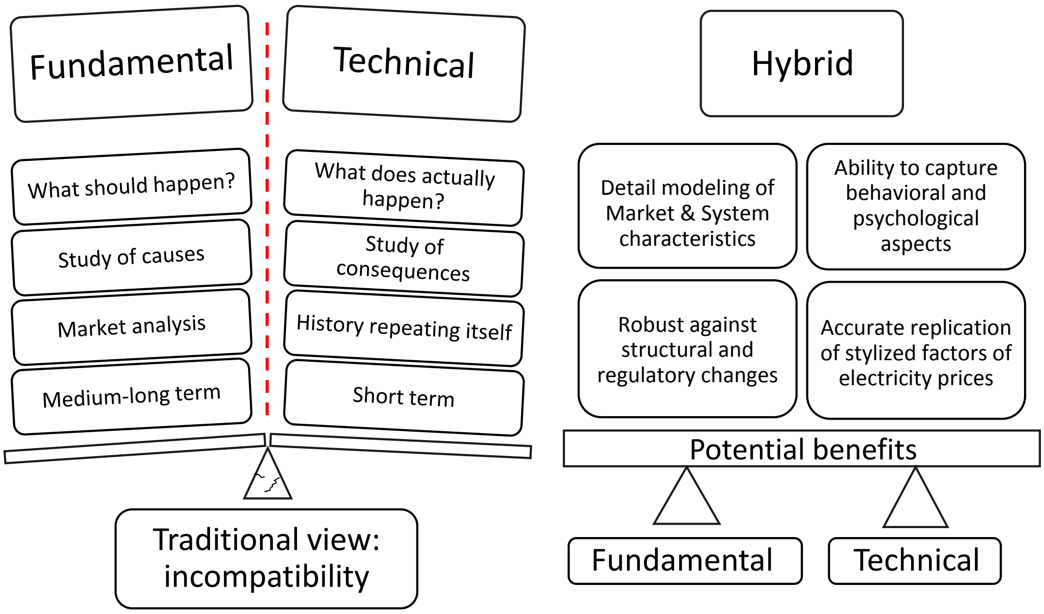

Figure 1.1: Advantages of a hybrid approach versus the traditional view of independence between fundamental and technical analysis

months) on a hourly basis. For this purpose, a fundamental approach and a technical / statistical approach will be combined in an effective way seeking to benefit from the advantages of both modeling strategies. With this hybrid methodology we aim to answer the question of what causes price movements and which variables are the drivers of these dynamics in a stable and robust way and finally, to model electricity price developments taking into account all its peculiarities.

Notable among these particularities are the existence of seasonality at multiple scales (intra-daily, weekly), calendar effects (such as holidays and weekends), extreme and erratic behavior and the presence of non-normality in its distribution. Spot prices also show an unusual volatility, with orders of magnitude much higher than the rest of commodities and financial assets.

1.2.2 Specific objectives

More specifically, the objectives that have been covered within the context of the thesis are described below.

- **Medium-term probabilistic forecasting of electricity prices on a hourly basis:** That is, being able to capture the distribution function of hourly prices over a period of time covering one to two months, taking into account the uncertainty associated with the input variables. At this point, it will be of great importance to propose probabilistic models that allow us to capture the tails and the higher order moments of the probability distributions. As will be seen in Chapter 4, it is true that most of the practitioners assume normal distributions and only focus on the first two moments of the electricity price distribution. The problem with these simplifications is that they miss the information contained in the higher-order moments[9] and that it presents inability to capture a wide range of shapes such as skewness, heavy tails, etc.

- **Proposal of effective hybrid schemes:** Bearing in mind the previous objective, another purpose of this thesis is to study the interaction between (fundamental) equilibrium market models and (technical) statistical models, i.e., the possible combination schemes of both perspectives. Taking into account the relationship between both approaches, the proposed methodology is aimed at allowing the assessment of the "communication possibilities" between them.

- **Assessment of probabilistic forecast combinations:** Due to the fact that the search of a proper hybrid model implies to compare between different options, an issue that immediately arises is how to best exploit information from different individual probabilistic forecasts. In particular, an additional objective is to evaluate the convenience of choosing between a single dominant forecast or a combination of the underlying forecasts of several competing models. It should be noted that although the idea of combining forecasts from several competing models in itself is not novel, it has been barely touched upon in the context of electricity spot prices, especially in a probabilistic framework. This issue will be further addressed in Chapter 4.

- **Prediction of extreme prices:** As it is expected that an important limita-

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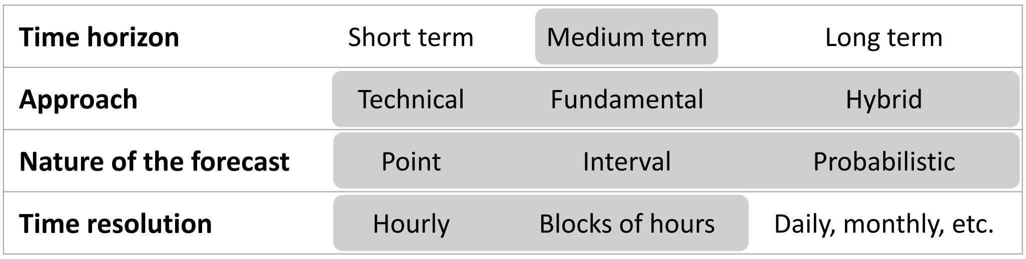
[9] Broadly speaking, any statistical distribution can be described in terms of its moments. As higher-order moments we refer to the third and fourth moments. On the one hand, the third moment gives and indication of the asymmetry or the skewness of the distribution. In general, a distribution with a longer tail to the right than the left has positive skewness, while one with a longer tail to the left has negative skewness. In this way, a symmetric distribution has zero skewness. On the other hand, the fourth moment, which is the kurtosis, gives a measure of the flatness of the distribution. A distribution with a heavy (i.e. fat) tails (relative to a normal distribution) will have high kurtosis, i.e. leptokurtosis, while a distribution with short (i.e. thin) tails will have low kurtosis, i.e. platykurtosis.

tion of the estimation of the price density function is the prediction of the tails, an additional objective is to propose and develop specific prediction models for extreme prices, trying to capture some of the features that current models do not include. In addition, a particular objective derived from this point would be the realization of a general methodology to distinguish between extreme and normal prices. It is important to emphasize that the prediction of extreme prices has not received the same relevance that has been dedicated to the estimation of expected average prices. Furthermore, the majority of the predictions focuses on a purely statistical analysis or at most incorporates some exogenous variable. On the other hand, there is not any technique that pays special attention to extremely low prices, which is becoming a more and more recurrent phenomenon in electricity markets.

- **Empirical evaluation of the proposed approaches:** That is, assessing in realistic case studies the predictive advantages of applying the hybridization of these techniques as opposed to the application of other benchmarking models traditionally used in the literature. In this regard, the methodology should be comprehensively tested in a market where, as a result of recurrent regulatory and structural changes (for example a remarkable growth of renewable energy production), the price dynamics reflects a greater complexity, with the recurrent presence of abrupt changes and extreme low and high prices. This is for instance the case of the Spanish electricity market.

In view of the above and in order to meet the objectives, the scope of the price forecasting methodology proposed in this thesis can be classified as shown in Fig. 1.2.



Time horizon	Short term	Medium term	Long term
Approach	Technical	Fundamental	Hybrid
Nature of the forecast	Point	Interval	Probabilistic
Time resolution	Hourly	Blocks of hours	Daily, monthly, etc.

Figure 1.2: Scope of the thesis

1.3 Outline and contents of the document

In order to address the previous objectives, this document is organized into five chapters and one appendix, as can be appreciated in Fig. 1.3. Besides this introductory chapter and Chapter 5, which presents the major conclusions, original contributions and lines for future research, each one of these sections includes specific conclusions as well as their own reference list.

More specifically, the remainder of the thesis is organized as follows:

Chapter 2.
This chapter proposes a novel methodology based on a fundamental model which incorporates Monte Carlo simulation combined with spatial interpolation techniques (as presented in Dueñas and Reneses (2011)) and a new definition of load levels (groups of hours which can be classified with the same market conditions in order to make the model computationally tractable, as stated in Wogrin et al. (2014)). This enables to predict, on an hourly basis, the distribution function of electricity prices in the medium term in a more precise and robust way.

More specifically, a fundamental market equilibrium model, which incorporates the strategic behavior of market agents, will be used to represent the electricity market with its main technical and economic characteristics. This approach is particularly useful for predicting expected price levels in those markets for which a long history of price information is not available and provides insights even when structural and regulatory changes are expected to happen in the market. However, although this approach outperforms the traditional statistical techniques used in the short term in a probabilistic framework, it generally does not fully capture important characteristics of electricity price series, such as volatility or the higher moments of the distributions of prices, which ultimately is what is needed to develop hedging strategies and effective risk-management tools.

Chapter 3.
As a consequence of the limitations of the market equilibrium model that is presented in Chapter 2, this chapter proposes an alternative hybrid methodology, based upon this fundamental model but extended with the econometric non-parametric technique of quantile regression to provide specific estimates of any percentile of the full density function of electricity prices, including the extremes.

Different hybrid schemes are presented in this chapter to effectively re-calibrate and re-shape the distribution function of the price predicted by the pure fundamental model. Overall, these hybrid specifications demonstrate their effectiveness when compared to other benchmarking techniques traditionally used in the field of probabilistic forecasting. In addition, they constitute nice inferential approaches that allow to incorporate the impact of both the projected fundamental changes in the market and behavioral aspects (such as strategic and speculative behavior). However, these techniques still fail when trying to capture the higher-order moments of the price's distribution function. This is due to both their observed sensitivity to data scarcity and the fact that some restrictions need to be imposed to make coherent probabilistic forecasts with these techniques.

Chapter 4.

In this chapter, more complex hybridization schemes that the proposed in Chapter 3 are investigated with the aim of overcoming the main deficiencies of non-parametric techniques. As a parametric representation of the price distribution function would be even more convenient for market agents, different flexible distributions of four parameters, which can dynamically accommodate the higher-order moments and the range of shapes that hourly prices can exhibit as functions of several exogenous drivers, are analyzed. This is of great importance since estimation and forecasting accuracy depends on the full specification of the distribution moments.

It should be noted that the hybridization process is extended to a three-stage methodology in which a non-parametric re-calibration of the price distribution function that has been previously estimated from the market equilibrium model is firstly carried out, and in a subsequent stage, the re-estimated percentiles are used as inputs for each distribution parameters. We also investigate if diverse combination schemes for probabilistic forecasts produce better predictions on average than methods based on the ex ante best individual forecasting method. In this way, the role of amalgamation under optimal weights and simple combinations that ignore correlations between forecast errors is fairly analyzed. Finally, the design for the architecture of the hybrid system that best characterizes the probability density function of the electricity prices when making real ex ante forecasts is discussed.

Chapter 5.

This final chapter of the thesis dissertation summarizes the main conclusions drawn from the thesis developments together with the major original contributions. Additionally, potential lines for future research are outlined.

Appendix .
As a concluding remark, it should be pointed out that although substantial progress has been made regarding the tails of the distribution, there remains considerable scope for improvement in the prediction of extremely low prices, where behavioral factors often matter more than the fundamentals. For this reason, in this appendix, a preliminary approach is presented to simultaneously accomplish punctual and probabilistic hourly predictions about the appearance of extremely low electricity prices in a medium-term scope that has been proposed by the author and has already been published in an international journal with JCR impact factor (Bello et al. (2016)). The goal in this paper is not only to predict when extreme values will occur, but also to anticipate the probability that a given threshold will be exceeded within a certain time frame.

The proposed approach is based on the statistical identification of the process key drivers. The procedure for making real ex ante forecasts consists of a nested compounding of different forecasting techniques, which incorporate Monte Carlo simulation, combined with spatial interpolation techniques. Logistic regression for rare events, decision trees, multilayer perceptrons and a hybrid approach, which combines a market equilibrium model with logistic regression, are used and compared to a Markov regime switching model and several naive methods.

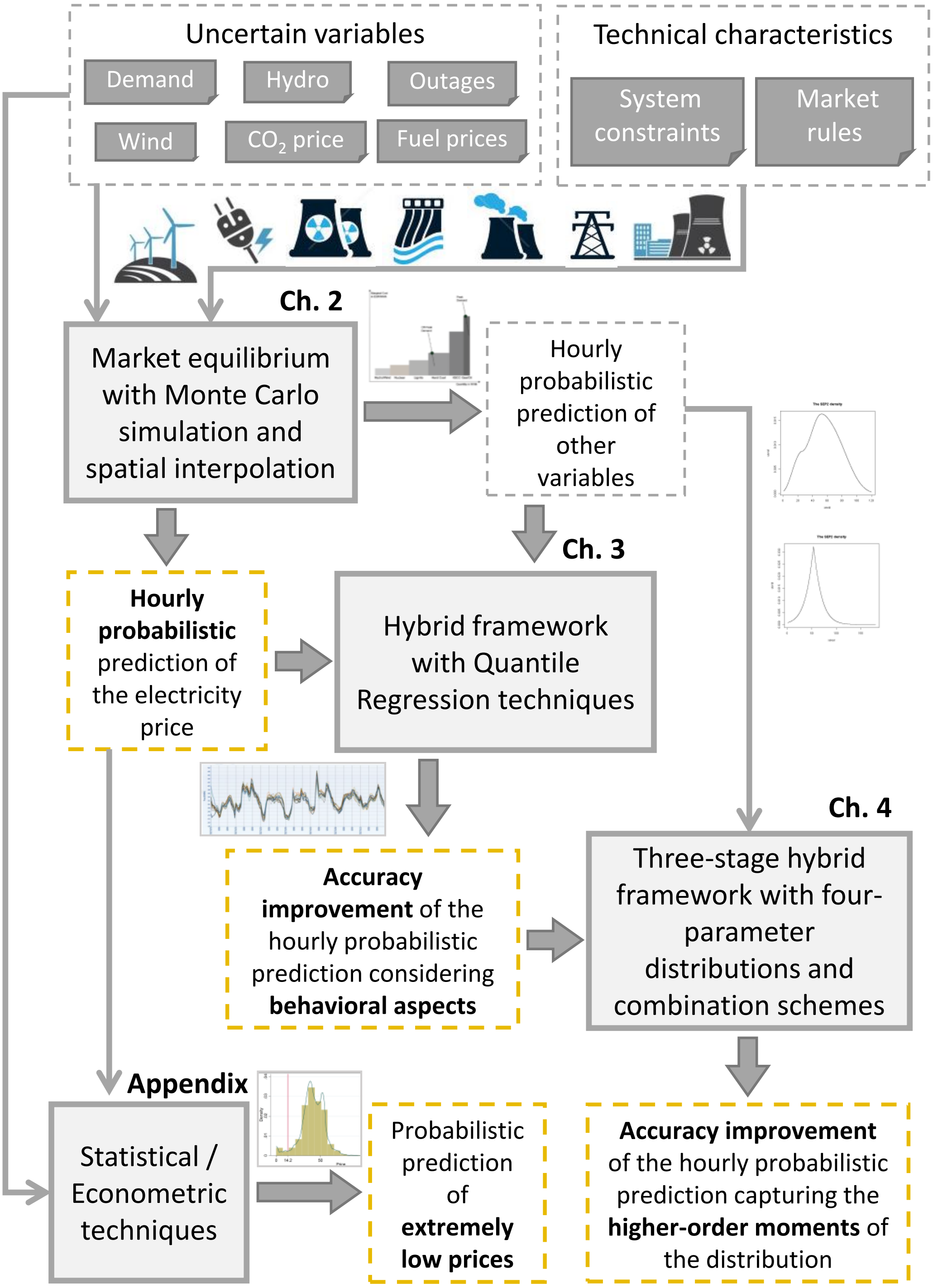


Figure 1.3: Overview of the thesis structure

1.4 Contributions

The development of this thesis has yielded several original contributions to the current state of the art of electricity price forecasting in the medium term. These are briefly summarized below:

1. Development of an original methodology that allows to simultaneously perform punctual and probabilistic hourly predictions by using a fundamental market equilibrium model. Essentially, this methodology is based on a market equilibrium model that incorporates Monte Carlo simulation, integrated with spatial interpolation techniques and a new definition of load levels.
2. A novel methodology for a medium-term wind production scenario generator is presented, which is based on a dynamic model that incorporates seasonal deterministic terms and an Autoregressive Moving Average component. It enables the simulation of a wide number of hourly wind production paths per each percentile of the monthly distribution function.
3. Several schemes are proposed to conduct the hybridization process by combining a market equilibrium model and probabilistic nonparametric techniques based on quantile regression. Moreover, in this hybridization process, different regions of the distributions functions of the explanatory variables are used in an innovative way as drivers for specific regions of the electricity price distribution.
4. A novel extension of the hybrid framework with flexible density functions of four parameters is presented. This enables to incorporate the information contained in the higher-order moments, as well as to accommodate the wide range of shapes that hourly electricity prices can take in such a way that the latent estimates of the moments change parametrically accoding to the behavior of exogenous variables.
5. Different ways of combining probabilistic forecasts from several competing four-parameter density functions within a three-stage hybrid framework are proposed. Furthermore, an extension of a finite mixture of normal distributions is applied in this hybrid framework.
6. The performance of all the suggested approaches to make probabilistic forecasts on a hourly basis are compared with a wide number of traditional

techniques when real ex ante forecasts are carried out in a real-size electricity system.

7. Specific techniques to make point and probabilistic forecasts of extremely low prices for a mid-term horizon on an hourly basis are developed. In particular, the forecasting capabilities of a novel hybrid approach that integrates behavioral and fundamental information, logistic regressions for rare events, decision trees and multilayer perceptrons is contrasted against the forecasts obtained from a traditional Markov regime switching model and different naive methods.

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Chapter 2

Probabilistic Forecasting of Hourly Electricity Prices in the Medium-Term

Contents

In the context of competitive electricity markets, medium-term price forecasting plays an essential role for market stakeholders. In contrast to short-term price forecasting, very little research has been conducted in this field. As market conditions can be very different from those observed historically when the prediction horizon is extended, the recent history can be almost useless and most data driven forecasting techniques, which are traditionally used in the short-term, cannot be directly applicable. Previous works regarding electricity price forecasting have tackled with the problem of mid-term prediction by using fundamental market equilibrium models over daily or, at most, averages of groups of hours. On the other hand, the limitations of point forecasts are widely recognized and the literature dealing with probabilistic forecasts is scarce. In this chapter, a novel methodology to deal with medium-term hourly forecasting of electricity prices is proposed. This methodology is unique in the sense that it attempts to simultaneously perform punctual and probabilistic hourly predictions. The approach consists of a nested combination of several modeling stages. The first stage consists of the generation of multiple scenarios of uncertain variables. In a second stage, a market equilibrium model which incorporates Monte Carlo simulation and a new definition of load levels is executed for a reduced combination of the generated scenarios. The application of spatial interpolation techniques allows us to estimate numerous feasible realizations of electricity prices from only several hundreds executions of the fundamental market equilibrium model without a significant lose in accuracy. The efficiency of the proposed methodology is verified in a real-size electricity system characterized by a complex price dynamics: the Spanish market.

2.1 Introduction

In a liberalized environment, as was already presented in the first chapter of this thesis, supporting risk-analysis and decision-making processes are essential for all market agents in electricity markets. For this reason, producers, retailers and large consumers need reliable probabilistic price predictions in the medium term (from one month to a few months) in order to optimize their operation, as well as to properly negotiate in the short-term markets[1] and sign profitable con-

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[1] Note that medium term projections are used in practise as right signals to the short-term operation. For instance, thermal units' bidding is subject to medium-term constraints, such as take-or-pay clauses.

tracts.

For this purpose, several econometric and fundamental models have been proposed in the literature. The former works have demonstrated their effectiveness in achieving promising results in short-term forecasting in systems with a long history of price information. However, in a medium-term scope, structural and regulatory changes (such as an increasing deployment of renewable energy sources and shale gas, nuclear moratorium, merges between companies, subsidies to domestic fuels, etc.) must also be taken into account. In these contexts, some of the historical price patterns are not necessarily repeated in the future as market conditions change over time and, hence, econometric techniques are not directly applicable. Instead, there is a widespread use of fundamental models to represent the electricity market with its main technical and economic characteristics. However, the existing approaches using this kind of models are not able to capture important features of the dynamics of electricity prices, such as their extreme volatility in comparison to other relevant commodities or the high-order moments of the probability distribution functions, required to develop an adequate hedging strategy.

This chapter proposes a novel methodology for medium-term electricity price forecasting, based on a fundamental market equilibrium model which incorporates Monte Carlo simulation combined with spatial interpolation techniques (as stated in (Dueñas and Reneses, 2011)) and a new definition of load levels (groups of hours which can be classified with the same market conditions in order to signficantly reduce the computational burden with reasonable loss of accuracy, as presented in (Wogrin et al., 2014)). This nested combination enables to make predictions on an hourly basis of the price in the medium-term.

It should be hightlighted that the content of this chapter is based on the journal paper ((Bello et al., 2015)) written during the development of this thesis. Furthermore, the proposed methodology is currently in operation in one of the major Spanish electricity companies. The methodology is comprehensively tested in a case study based on the Spanish market, which constitutes an interesting case where the price dynamics reflects a big complexity, with the recurrent presence of abrupt changes and extreme low and high prices, as a result of the financial crisis and the regulatory and structural changes that have occurred in the market.

It should be noted that the wholesale market in Spain is made up of an organized mechanism and a non-organized mechanism. The organised market is structured around a day-ahead market followed by six intraday auctions. The non-organized part consists of physical bilateral contracts. In Spain, most of the electricity is priced at the day-ahead hourly market[2]. The day-ahead market and the intra-day market both work as uniform-price auction markets. Prices for each hour are determined according to the marginal-price criteria (i.e. the price for the last accepted supply bid). Spanish regulation forbids submitting negative bids. Therefore, prices are constrained to be non-negative. Among the power plants with the lowest short-running costs are renewable energy sources (RES)[3], nuclear and run-of-the-river hydro[4]. On the contrary, the plants with the highest short-running costs are coal, head-dependent hydro and fuel oil.

Summarizing, the main contributions of this chapter are as follows:

1. Medium-term probabilistic forecasts with an hourly basis have been carried out by means of a fundamental model based on market equilibrium. The prediction accuracy of the proposed methodology has been compared with other results obtained by means of four benchmarking models traditionally used for short-term electricity price forecasting.
2. The performance of the suggested approach to make point forecasts is compared with the traditional methodology based on the combined use of an equilibrium model with load levels.
3. The efficiency of the proposed methodology is tested in a real-size electricity system.
4. A methodology for a medium-term wind production scenario generator has been proposed.

The chapter is structured as follows. In Section (2.2), a state of the art review in electricity price forecasting is presented. In Section (2.3), the proposed nested method is described, while applications of the model in a real case study and

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[2] For instance, according to National Markets and Competition Commission (CNMC), during 2013 bilateral contracts represented 26% of the sold energy in the daily base-load programme.

[3] It is important to clarify that RES are legislated under what is referred to as special regime. More specifically, RES are promoted through a feed-in tariff system.

[4] Run-of-the-river hydro production is that energy which has to be produced due to minimum flows of the river. The hydro production that can be managed by the company is referred as head-dependent hydro.

forecasting accuracy results are presented in Section (2.4). Finally, the conclusions and the main contributions of this chapter are highlighted in Section (2.5), as well as the field for future developments that will be addressed in the following chapters of this thesis.

2.2 Electricity price forecasting

Numerous forecasting approaches for electricity prices with different aims and time horizons have been proposed in the literature. Several classifications of these techniques are suggested in González and Muñoz (2005), Vehviläinen (2005) and Weron (2014, pp.1041). All these methods found in the literature can be classified into the following major categories in terms of the forecasting time horizon: those that are preferred for the short term and those that turn out to be more useful for a medium- or long-term[5] temporal scope. In general, short-term price forecasting relies on statistical techniques and recent past history of electricity prices while the medium- and long-term approach is usually based on the fundamental representation of the system dynamics.

Much research has been conducted in those statistical methods that are aimed at making short-term predictions of electricity prices. Within this group, the most widely used modeling approach is the application of time series models, but artificial intelligence techniques, such as Artificial Neural Networks (González and Muñoz (2005) or Mandal et al. (2007)) and data-mining (Chen and Deng (2008)) have also been reported in the past years. Pertinent examples of time series models for electricity price forecasting in the short term are ARIMA models (Conejo et al. (2005), Zareipour et al. (2006) and Jonsson et al. (2013)), Exponential Smoothing (Gardner (2006)), Linear Transfer Function (Nogales (2005), (Cruz et al., 2011)), GARCH (Koopman et al. (2007)), Vector models (Panagiotelis (2008)) and Functional Time Series, which are becoming more relevant ((Liebl, 2013)). A common limitation of these methods is the underlying assumption that all information about future prices is contained in past information, i.e. it is presumed that all possible future situations have occurred before.

The literature on medium- and long-term electricity price forecasting is relatively scarce in comparison to short-term forecasting research (Yan (2013)). This group

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[5] Short-term refers to a few minutes to one month and medium- to long-term (or simply medium-term), from one month to a few months.

encompasses a wide variety of simulation[6] (Bastian et al. (1999)) and market equilibrium models (Barquín and Centeno (2005); Centeno and Reneses (2007)), which are classified in more detail in Ventosa et al. (2005). These models are used to find out the market clearing prices by representing the strategic behavior of market agents subject to technical constraints. Many of these models apply game theory, as in the Cournot model[7] (Barquín (2008)), the Bertrand model[8] (Younes (1998)) or the Conjectural Variations approach (Díaz et al. (2010)). The latter approach aims to represent the fact that rival agents react to higher prices by increasing their production. An important characteristic of models based on conjectural variations is that they are able to model different degrees of market competition (from perfect competition to Cournot oligopoly). The supply function equilibrium models (Green (1992)), which are also based on game theory, deal with the calculation of the balance between supply and demand curves by means of a set of differential equations, instead of the traditional general systems of equations of the previous approaches. These models therefore have important limitations in terms of numerical treatment.

These fundamental models present the clear advantage of not depending on the availability of past information to make predictions about market situations that are not fully observable or can be hypothetical. One of the main drawbacks for their application is that they require a detailed description of the analyzed market in terms of structural and parametric information. Another shortcoming that is commonly recognized in the literature is their inability to provide accurate forecasts compared to data-driven methods (Bunn (2000), Aggarwal and Saini (2009)). The latter is mainly due to the need for grouping hours into load levels of similar characteristics in order to make the models computationally far less demanding.

An alternative approach to address long- and medium-term predictions is the application of short-term techniques to an extended forecasting horizon. The problem of this is that, as stated in García-Martos and Rodríguez (2011), the mod-

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[6] These models are designed to provide a detailed information of system prices, especially in those cases where nodal prices exist. Nonetheless, these models present two major shortcomings. First, a great volume of system operation data is required, and second, they are difficult to implement and their computational cost is very high.

[7] Within the frame of Cournot-Nash, the market equilibrium is determined by the decisions made by each company participating in the market in terms of the quantities they are willing to produce, and this is why this is often referred to as equilibrium in quantities. The main problem of these models is that they tend to provide prices that are higher than those observed in reality.

[8] Under Bertrand framework, market players compete on prices

els that perform well in the short run usually degrade when extending the forecasting horizon. Some good examples at medium-term and year-ahead predictions can be found in Vehviläinen (2005) and Torbaghan et al. (2012) for monthly electricity prices. Meanwhile, Conejo et al. (2010) describe prices throughout the year using 48 base values. Although there are some published works dealing with long-term predictions with high-frequency data in fields such as load forecasting (i.e. Hyndman (2010), Hong and Wilson (2014)), it is difficult to find examples of long-term price forecasting with hourly data with the exception of (Yan and Chowdhury, 2014) (they use support vector machine and ARMAX models for making point forecasts of just one month), Alonso et al. (2011) and García-Martos and Rodríguez (2011) (both of which extend Dynamic Factors Analysis (DFA) to the long run).

In relation to the nature of the forecasts, there is a wide variety of research focused on price point forecasts, which produce only single-valued expectations of the price of electricity, but there are quite less references dealing with probabilistic forecasting (Weron (2014, pp. 1065)). In comparison to the widely-used point predictions, the less common probabilistic forecasts provide additional quantitative information of the uncertainty that is always associated to forecasting, therefore overcoming the limitations observed in point forecasts (Hyndman (2006)).

Among the diversity of probabilistic forecasting approaches, interval forecasts and density forecasts can be distinguished, according to the method used to represent uncertainty. The former are more commonly found in the literature, e.g. Misiorek and Trueck (2006), Misiorek (2008), Zhao et al. (2008), González and Contreras (2012) and Wan et al. (2014) evaluate the performance of different forms of interval forecasts in the short-term. In contrast, methods aimed at providing forecasts of the entire probability density function have been barely touched upon. Some examples are: Panagiotelis (2008), who carried out short-term forecasts by means of a first order vector autoregressive model with exogenous effects and a skew t-distributed disturbance; Serinaldi (2011), who makes day ahead forecasts of the distribution of electricity prices by using the GAMLSS approach; and Jonsson et al. (2014), who generate prediction densities of the day-ahead electricity prices in Western Denmark using time-adaptive quantile regression for the 5-95% quantiles and a description of the distribution tails by exponential distribution.

All these examples deal with probabilistic forecasts in the short term. To the best knowledge of the author, no references can been found in the literature that propose a probabilistic forecasting methodology for the medium term. The only exception is Alonso et al. (2011), who in addition to applying Dynamic Factors Analysis (DFA) to the long run in an hourly basis, also makes an introduction to computing interval prediction of the electricity prices of the Spanish day-ahead energy market. The methodology provides good results for the single confidence interval analyzed even though a data-driven technique is used. This is because it is applied to a case study of one week in a year when no relevant structural or regulatory changes took place in the market. In addition to this, the methodology proposed in Alonso et al. (2011) does not account for exogenous variables and assumes that the relation between the vector of daily prices and its unobserved components remains constant in time.

The proposed work is unique in the sense that it attempts to produce medium-term forecasts with an hourly accuracy even in the event of structural and regulatory changes taking place in the market, providing a full estimation of the probability density function of electricity prices.

2.3 Methodology

This section describes the proposed methodology, which is based on 5 stages as shown in Fig. 2.1. This methodology comprises the solution of a Monte Carlo simulation of the electricity market equilibrium. Therefore, in a first step of the methodology, the operation and the behavior of the market is fairly represented using a fundamental market equilibrium model based on conjectural variations as stated in (Centeno and Reneses, 2007) and (Reneses, 2004). The model, which is further described in Section 2.3.3, is formulated as an equivalent cost-based optimization problem where each generation company tries to maximize its own profit. The decision variables are the dispatch of the generators, subject to the demand-balance equation and the technical constraints of all thermal and hydro groups (maximum and minimum power obtained, efficiency, variable costs, pumping, etc.). The system marginal price is computed as a result of the market equilibrium model.

One important characteristic of medium-term market equilibrium models is that an hourly representation is not often used in practice because the size and resolution times increase considerably for real power systems. Therefore, in this thesis and as will be introduced in Section 2.3.2, similar hours are grouped together and modeled using the so-called *system states* as proposed in Wogrin et al. (2014), significantly reducing the problem size with respect to an hourly representation. This novel alternative for time representation, unlike the traditional load levels, allows that the chronological information between individual hours is not lost. Each hour is assigned to a system state through a k-means clustering process. These system states are defined not only considering the electricity demand (as is usually done in this kind of models), but also the non-dispatchable generation, such as wind or solar.

The models supporting the decision-making process should also be able to consider uncertainty. It is well known that in the medium term there are many risk factors such as demand, CO_2 prices, fuel costs, hydro and wind generation, etc. with a great impact on prices. Therefore, a great amount of hourly scenarios for these random variables are generated based on historical data, as stated in Section 2.3.1. In order to use a Monte Carlo simulation to tackle with uncertainty, a large number of realizations of the model are needed, usually entailing huge computational time and effort (for instance, in a real representation of the Iberian electricity market, the model consists of about 350,000 equations and 800,000 variables). For this reason, an efficient method proposed in Dueñas and Reneses (2011) to cope with this drawback is used and is presented in Section 2.3.4, allowing us to obtain a large number of realizations by reducing the computational time and without substantial loss of accuracy.

2.3.1 Scenarios

It is a well-known fact that medium-term forecasting of electricity prices is subject to several sources of uncertainty. Although some of the risk factors (such as regulatory changes) are unpredictable, there are other factors such as system demand, wind energy, hydrological conditions or fuel prices that can be analyzed by studying historical data (Mosquera et al., 2008). The process of generating scenarios requires a thorough knowledge of the underlying probability distribution functions and the corresponding correlations between the different uncertain variables (Cabero et al., 2005). In this chapter, we have considered seven random variables as medium-term risk factors: system demand, wind generation, hydro conditions (modeled by the hydro inflows and the run-of-the-river

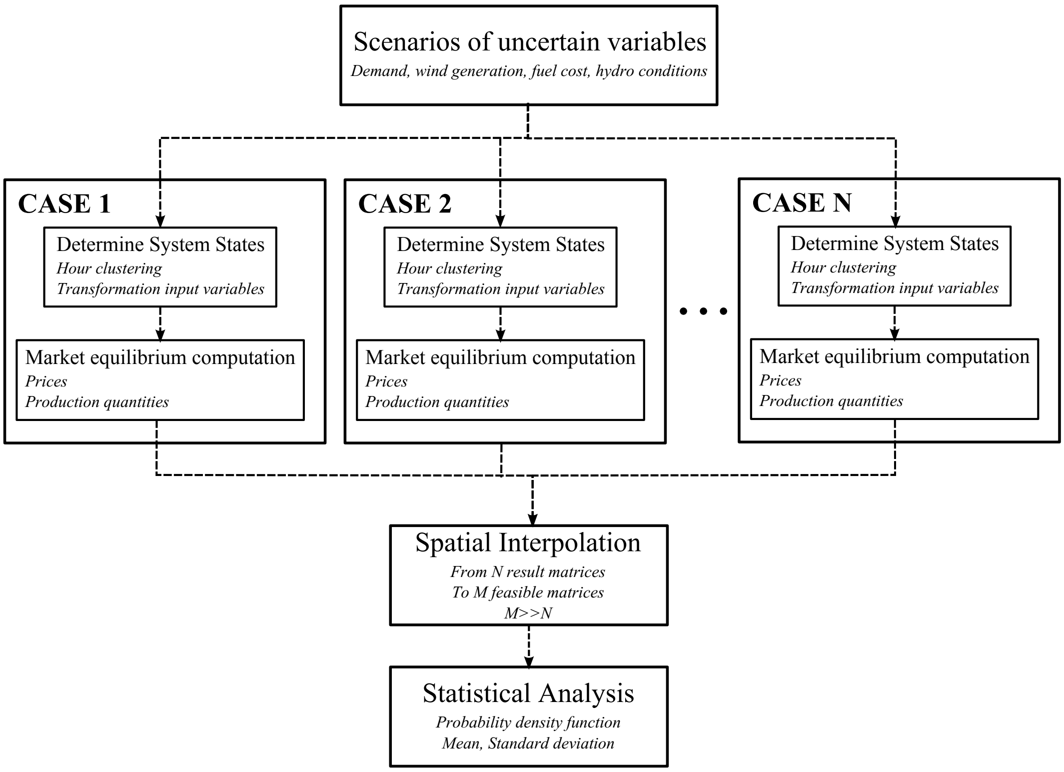

Figure 2.1: Global overview

generation), coal prices, natural gas prices, CO_2 emission allowance prices and the unplanned unavailability of thermal power plants. All these factors are characterized by their corresponding cumulative distribution function in such a way that each scenario can be represented by a percentile.

For each one of the risk factors, a number of scenarios has been considered, with the final result of 601,425 possible combinations of realizations. From all these scenarios, 250 representative cases have been selected using hypercube sampling as explained in detail in Section 2.3.4. The historical distribution function of each variable for each hour of the year has been used to build the scenarios. As stated in Hong and Wilson (2014), this methodology constitutes a more rigorous approach, although more intensive from a computational point of view. In the case of fuel costs and CO_2 prices, their distribution functions are centered in market future expectations when making real ex ante forecasts. The highly unpredictable and volatile nature (especially in the medium term) of wind generation, which is a key driver of prices in electricity markets such as the Spanish one, has required the adoption of a special methodology to build the scenarios. This is further explained in Section 2.3.1.1.

2.3.1.1 Wind scenario generation

Different wind power scenarios (represented by its historical distribution function) have been generated in order to characterize the uncertainty in the medium term and internalize in the market equilibrium model that is presented in Section 2.3.3 their potential capability to alter the behavior of the electricity price volatility.

Multiple approaches have been researched and tested in the past years regarding wind power forecasting (see Lei et al. (2009) and Foley et al. (2012)). There are two main branches of approaches: physical and statistical (Sideratos and Hatziargyriou (2007)). The physical methodologies are related to using meteorological variables, such as temperature, pressure, wind speed and direction, or even the physical characteristics of the wind turbine to estimate local wind speed, e.g. Cellura et al. (2008). The statistical approach typically employs recursive techniques based on previous power outputs (Sánchez (2006)).

Similar to electricity price forecasting, the majority of the techniques are related to short-term (24 to 48 hours ahead) forecasts with the purpose of efficiently integrating wind generation into a power system. In these cases, the combination of physical and statistical approaches has proved to be effective. However, with medium-term forecasting, it is not possible to have the advantage of accurate wind speed and meteorological predictions. Therefore, it is inadvisable to follow a physical approach when working with forecasts of production months in advance. Thus, the scenario generator follows a purely statistical approach by taking historical data and creating possible realistic forecasts from the past data.

For consistency's sake throughout this thesis, the proposed medium-term wind scenario generator methodology will be specifically presented for the Spanish electricity market, but of course, it could be equally extended to other markets (for instance in Yeo et al. (2015) the methodology proposed here was successfully applied to the German electricity market).

The historical data base includes the hourly production and installed capacity in Spain since the existence of records of a stable wind production (that is, since 1st January 2007). In order to remove the trend caused by changes in installed capacity $P^{ins}_{wind,h}$, an utilization factor w_h for each hour h is used. The utilization factor is then calculated by dividing the hourly production $P_{wind,h}$ by the installed capacity, as follows:

$$w_h = \frac{P_{wind,h}}{P^{ins}_{wind,h}} \tag{2.1}$$

It is important to highlight that the utilization factor w_h is not dynamic. Note that even though the wind generators that were initially installed were located in the most windy areas, there has been a compensation effect with the increase in the efficiency of the wind turbines.

With the aim of stabilizing the resulting w data (which presents a distribution with positive skew) and tranform it approximately into a normal distribution, a logit-transformation with an adjustment parameter η (which was optimally fixed at 0.33) was used. This is defined below:

$$x_h = log\left(\frac{w_h^{\eta}}{1-w_h^{\eta}}\right) \qquad : s.t.\ w_h \in (0,1), \eta \neq 0 \tag{2.2}$$

The seasonal trend and the cyclical component are modeled through an additive model as stated in Eq. 2.3. The general sinusoidal function chosen, where the term $\bar{x}_{daily}$ represents a daily component which is calculated as the arithmetic mean of each hour, is given below:

$$\tilde{x}_h = x_h - (a_0 + a_1 cos(\varphi t) + b_1 sin(\varphi t) + a_2 cos(2\varphi t) + b_2 sin(2\varphi t) + \bar{x}_{daily}) \tag{2.3}$$

where

$$\varphi = \frac{2\pi}{365.25 \times 24} \tag{2.4}$$

Then, the data is seasonally adjusted by removing the seasonal component that has been estimated by minimizing the weighted least squares equation. This leaves a stochastic series $\tilde{x}_h$ which is considered to be completely stationary, as can be seen in Fig. 2.2 for a period ranged between 1st January 2007 and 31st December 2013. This term is captured through an Autoregressive Moving Average (ARMA) model, which allows to take into account both the time correlations and the random nature of the phenomenon.

The generator is calibrated specifically for each month with the aim of having different response functions for generating scenarios that allow to better capture the dynamics of the serie. Because a model is fitted for each month, spline

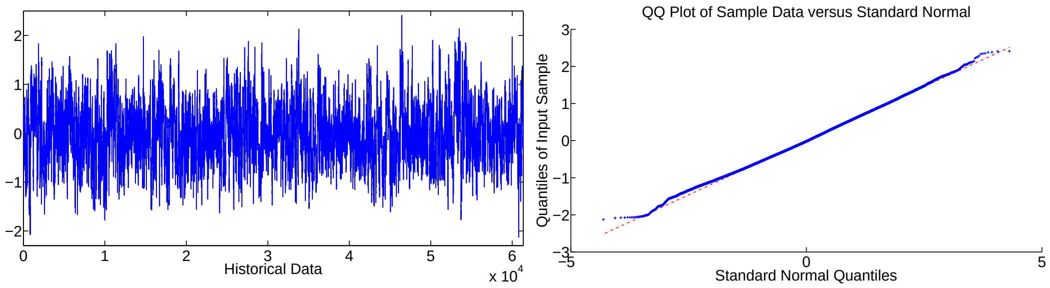

Figure 2.2: Graphical representation of $\tilde{x}_h$

interpolation is used to create a coherent transition between months. Spline interpolation is used instead of linear interpolation because it offers a more realistic interpretation. The percentiles are based upon the total production of the simulation per month. In order to capture the enormous variability of wind production, seven different profiles per each percentile have been built in the case studies presented in this thesis. On the one hand, Fig. 2.3 displays three different profiles of the median during a representative period of time. As can be seen, the transition between months is of a unique smooth form for every simulation done. On the other hand, Fig. 2.4 shows the evolution of three percentiles.

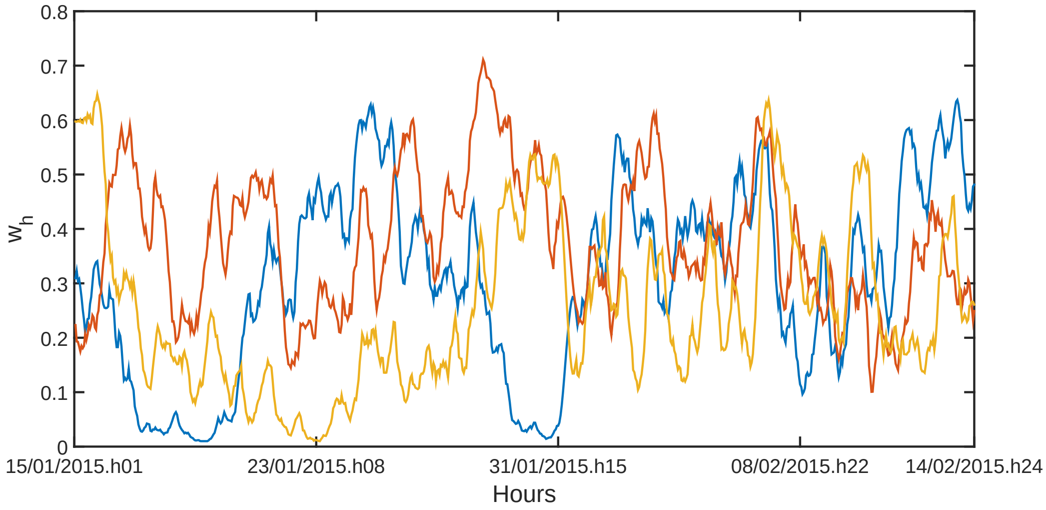

Figure 2.3: Wind dynamics of the median of the distribution for three runs

2.3.2 System states

In this chapter, the market equilibrium is computed in the medium term (the medium-term scope is assumed to cover from one month up to one year). Since an hourly representation of the market equilibrium in the medium term is com-

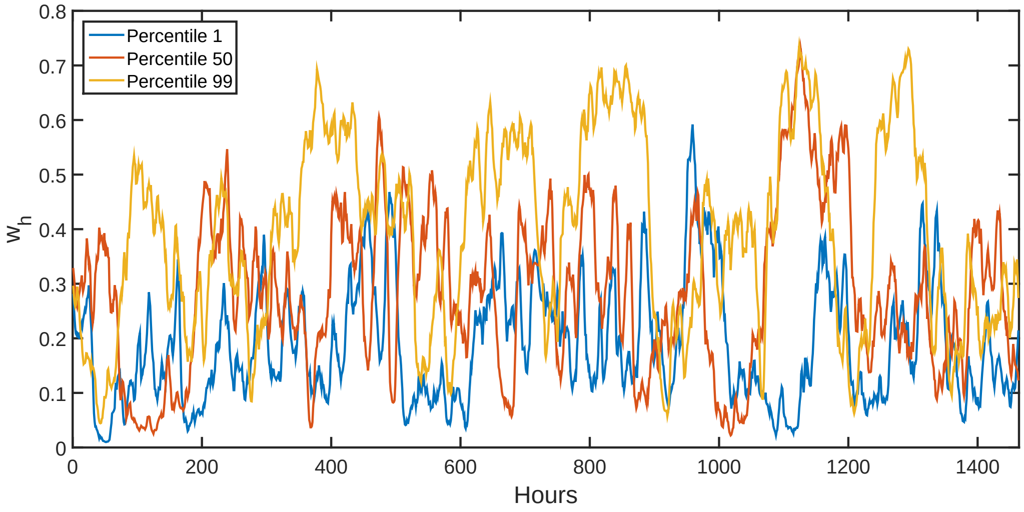

Figure 2.4: Wind dynamics of three different percentiles

putationally intensive and incompatible with a powerful Monte Carlo simulation, some models reduce the size of the problem by using load levels which gather hours with similar demand levels (Chen and Hobbs, 2005; Centeno and Reneses, 2007; Kazempour et al., 2011; Díaz et al., 2016). The advantage of this approach is that the number of load levels is much smaller than the number of hours considered, and therefore the computational burden is reduced. However, its main disadvantage is that the sequential information of the hours is lost in the hour ordering process, and thus chronology-based decisions such as starting-up and shutting-down variables are not adequately modeled.

The use of system states as proposed in Wogrin et al. (2014) is an alternative to the load-level representation. This approach allows a better representation of the chronological information. Moreover, system states unlike load levels can be defined considering more than one system characteristic, e.g. they can be defined using demand and wind generation at the same time.

By definition, a state of the system is an exogenous set predefined by k features that happen simultaneously and frequently in the system, and they characterize the whole system over a certain period p. Each one of the different characteristics may have several levels. The vector containing the levels of each feature is called a system state s. Therefore, it is possible to define a set S of different system states s, where each individual state is a vector of k elements, and each element corresponds to a particular level of the defined characteristics.

For the sake of clarity, imagine a power system that could be characterized using two features ($k = 2$): demand and renewable energy sources (RES) production. The levels of the demand can be defined as high demand and low demand. In the same way, the levels of the RES can be defined as high RES and low RES. If we define the states according to a variable that is calculated from the demand minus the RES production, we could observe that a system state s correspond to a level of this variable that implicitly takes into account a pair of the values of the demand level an RES level, e.g., (low demand, low RES). Therefore, the set S of the different systems states could be defined as S ={(low demand, low RES), (low demand, high RES), (high demand, low RES), (high demand, high RES)}. Fig. 2.5 shows a representative example of the assignment of any two features to different states by plotting the identified groups in different colors. Note that the figure also contains the representative points, which correspond to the centroids of the clusters.

All in all, the system states are calculated in this thesis using one feature that takes into account implicitly other variables: the net demand (total demand minus the aggregate production of renewable energy sources, nuclear and run-of-the-river hydro).

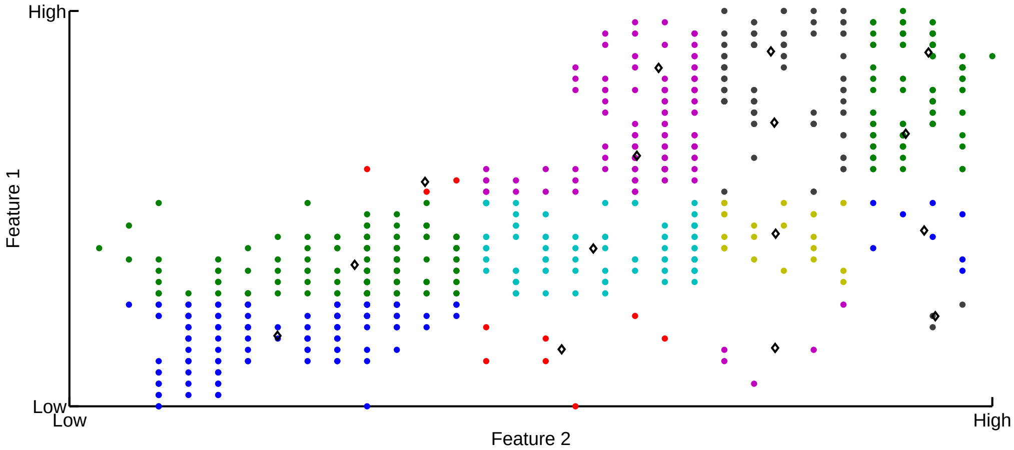

Figure 2.5: Representation of system states definition for two features

In the system states approach, the chronological information is preserved using the transition matrix X, in which each element $x_{s,s'}$ represents the expected number of times that there is a transition from state s to s'. It is important to note that this matrix is not necessarily symmetrical because it depends on the chronological information of the hourly data.

Wogrin et al. (2014) also propose a methodology based on a clustering process in order to determine the set S of system states and the transition matrix X using the hourly information of the k features. The methodology uses the k-means clustering algorithm which minimizes the distance between each datum of the cluster to its centroid. In that methodology each cluster represents a system state s, and the centroids are the levels of the k features. Since each hour is assigned to a specific system state, the determination of the transition matrix is straightforward. It is only necessary to count the number of jumps between system states for all hours.

As discussed before, the system states are calculated using the net demand. This representation enables a better treatment of the various important features that characterize the operation of current electricity markets like the Spanish one. Therefore, the result of this stage is that the input variables (demand, production costs, etc.) are transformed using the system states representation.

As electricity prices are among the most volatile commodities in the world and a correct estimation of their density function should take into account the occurrence of extreme prices, more weight has been given in this thesis to the states with extreme values. Although this could be obtained by using a high number of states, this would imply a significant increase in the number of variables, equations and runtimes. In order to overcome this disadvantage, in this thesis the process for determining the states comprises two successive steps. In the first stage, a high number of initial states is defined. Afterwards, those states with more similar centroids and with frequently occurring values are grouped, leaving the states that correspond to extreme values of the net demand as were estimated in the first step. This clustering process allows to reduce the number of states without a significant loss of information. In particular, in the case study that will be presented in Section 2.4, 16 states have been generated for each period (month)[9].

2.3.3 Market equilibrium model

A market equilibrium model like the one proposed in Barquín and Centeno (2005) and Centeno and Reneses (2007) is used in this thesis to represent the operation of the electric power system. This model is able to compute the market equilib-

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[9] It is important to clarify that by using this number of states, a compromise solution have been found between the computational burden and the accuracy.

rium in an oligopolistic framework by using an equivalent quadratic optimization problem. Under the framework of game theory, it is well known that the market equilibrium is reached at the point where each generation company i maximizes its own profit, taking into account that the rest of the companies also maximize their profits. This equilibrium point is known as the Nash Equilibrium (see Nash (1950)), in which the companies do not have an incentive to unilaterally change their strategic behavior because any deviation implies a decrease in benefits.

The main assumption of this model is that the strategic behavior of each generation company i is represented by a parameter known as the conjectured-price response θ_i. The value of this exogenous parameter can be estimated using historical data as proposed in García-Alcalde et al. (2002); Bunn (2004); de Haro et al. (2007a); Díaz et al. (2010). This parameter measures the market power of the companies and represents the change of electricity market price λ when the generation company changes its production quantity q_i, thus:

$$\theta_i = -\frac{\partial \lambda}{\partial q_i} \geq 0, \qquad \forall i \in I. \tag{2.5}$$

where I is the set of generation companies.

In the simplest case, the profit π_i of the generation company i at the market clearing price λ is equal to the revenues minus the costs of the company:

$$\pi_i = \lambda \cdot q_i - C_i\left(q_i\right) \qquad \forall i \in I \tag{2.6}$$

The equilibrium point is then calculated by expressing the first-order profit-maximization condition for each generation company, which yields:

$$\frac{\partial \pi_i}{\partial q_i} = \lambda + q_i \cdot \frac{\partial \lambda}{\partial q_i} - \frac{\partial C_i\left(q_i\right)}{\partial q_i} = 0 \qquad \forall i \in I \tag{2.7}$$

In order to demonstrate that this point corresponds to a maximum, the second order condition is:

$$\frac{\partial^2 \pi_i}{\partial q_i^2} = \frac{\partial \lambda}{\partial q_i} - \frac{\partial \lambda}{\partial q_i} - \frac{\partial^2 C_i\left(q_i\right)}{\partial q_i^2} = -\frac{\partial^2 C_i\left(q_i\right)}{\partial q_i^2} < 0 \qquad \forall i \in I \tag{2.8}$$

Under the hypothesis of increasing marginal cost MC_i functions for each agent, the existence of such maximum point in the profits of generation companies can be guaranteed.

If the conjectural variation of Eq.(2.5) is substituted into Eq.(2.7):

$$\lambda - \theta_i \cdot q_i = \frac{\partial C_i\left(q_i\right)}{\partial q_i} = MC_i\left(q_i\right) \qquad \forall i, \in I \tag{2.9}$$

In this way, the market equilibrium is reached when the marginal revenue MR_i (the left hand side on Eq.(2.9)) equals the marginal cost MC_i for each company i:

$$MR_i = \ MC_i \qquad \forall i \in I \tag{2.10}$$

Moreover, in electric power systems, the generation and the demand D must be balanced:

$$\sum_{i \in I} q_i = D \tag{2.11}$$

The inverse demand curve is the relationship between market price and demand presented in Eq.(2.12). To ensure the existence of the equilibrium point, the inverse demand curve must satisfy certain properties. This function has to be continuous, differentiable, monotone and strictly decreasing, which constitute reasonable assumptions.

$$\lambda = \lambda\left(D\right) \tag{2.12}$$

Taking into account all these premises, Barquín and Centeno (2005) demonstrated that the market equilibrium can be calculated as the solution of the minimization problem (2.13)-(2.15):

$$\min_{q_i} \sum_i \overline{C_i}\left(q_i\right) - U\left(D\right) \tag{2.13}$$

s.t.

$$\sum_i q_i = D \qquad : (\lambda) \tag{2.14}$$

$$\mathscr{H}\left(q_i\right) \geq 0 \tag{2.15}$$

where $\overline{C_i}\left(q_i\right)$ is the so-called effective cost function of agent i, and $U\left(D\right)$ is the utility demand function. More specifically, the effective cost function is defined as:

$$\overline{C_i}\left(q_i\right) = C_i\left(q_i\right) + \theta_i \cdot \frac{q_i^2}{2}, \qquad \forall i \in I \tag{2.16}$$

and therefore it includes the cost function $C_i\left(q_i\right)$ and a quadratic term that allows computing market equilibrium through an equivalent optimization problem taking into account the strategic behavior of the company or market agent i.

And the utility demand function is defined as:

$$U\left(D\right) = \int_0^D \lambda\left(D\right) dD \tag{2.17}$$

In some electricity markets (and specially for short- and mid-term time scopes), it is common to assume that the demand is inelastic, i.e., the demand is a known constant. In this situation, the optimization problem does not include the term $U\left(D\right)$. If $C_i\left(q_i\right)$ is a linear or a quadratic function then the minimization problem (2.13)-(2.15) is a quadratic optimization problem that can be solved efficiently using available commercial software. The outcome of this model is that the profit for each generation company i is maximized when they produce the quantities q_i at the market price λ. It is essential to note that the market clearing price λ can be calculated as the dual variable associated to the power balance constraint (2.14) which ensures that the total generation meets the demand D.

All the other constraints that model hydro-thermal coordination and the operation of the generation units (minimum generation, maximum generation, etc.) are represented in Eq. (2.15). Thermal units are modeled by means of their installed capacity and a minimum stable load level. Here standard unavailability rates representing the average limitations on the maximum power output of both maintenance programs and unforeseeable events are also considered. Thermal costs of each unit are approximated by a linear cost curve, including a fixed cost term for dispatched units, in addition to start-up and shut-down costs. Variable operating costs include fuel costs, operation and maintenance costs and carbon emission costs. Unit commitment decisions are linked to the start-up and shut-down decisions between states of the same period. The dispatch is also constrained by contract considerations. Meanwhile, all river basins are represented by an equivalent plant and aggregated reservoir with the total production and pumping capacity. Storage hydro production is also dispatched by the model.

As the case study presented in Section 2.4 is focused on the Spanish market, the model here introduced takes into account the single pricing mechanism joint to a market splitting algorithm that exists in the Iberian market [10] to determine not only the price in Spain, but also in Portugal. Then, the calculations for the Iberian two-area system are carried out in such a way that power flows between Spain and Portugal are endogenously determined by the cost minimization problem according to the economic dispatch of the available technologies in the system and limited by the interconnection capacities. Moreover, the transmission capacity limits between adjacent countries have been included in the model as another technical constraint.

2.3.4 Monte Carlo simulation with spatial interpolation techniques

Stochastic programming is not always tractable to incorporate uncertainty in market equilibrium models. This is because in the medium-term there are several risk factors which have to be taken into account with an adequate degree of complexity, and therefore, it is required a considerable amount of computational effort and time. In contrast, Monte Carlo simulation, which consists of computing many realizations of a deterministic model, is an alternative to cope with uncertainty. The problem is that multi-factor uncertainty requires a large amount of realizations in order to reach stable results. Since just one execution of the market equilibrium model, for $n = 1, 2, ..., N$ implies a significant computational time, in this thesis an efficient Monte Carlo simulation is used to deal with this drawback as proposed in Dueñas and Reneses (2011).

The basic idea behind this methodology consists of first computing a reduced number of N executions of the market equilibrium model, and then estimating a huge number of M outputs (e.g. spot prices) from the initial executions by means of a spatial interpolator based on local regression. As evidenced by Dueñas and Reneses (2011), there are no relevant differences between the intensive computation of all possible combinations of the R uncertain variables and the interpolation of realizations. The methodology, which can be divided into three steps, is depicted schematically in Fig. 2.6.

In the first step, the initial executions of the model are placed spatially in an R-

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[10] The Iberian Electricity Market (MIBEL), constitutes a joint initiative from the governments of Portugal and Spain, with a view to the construction of a regional electricity market.

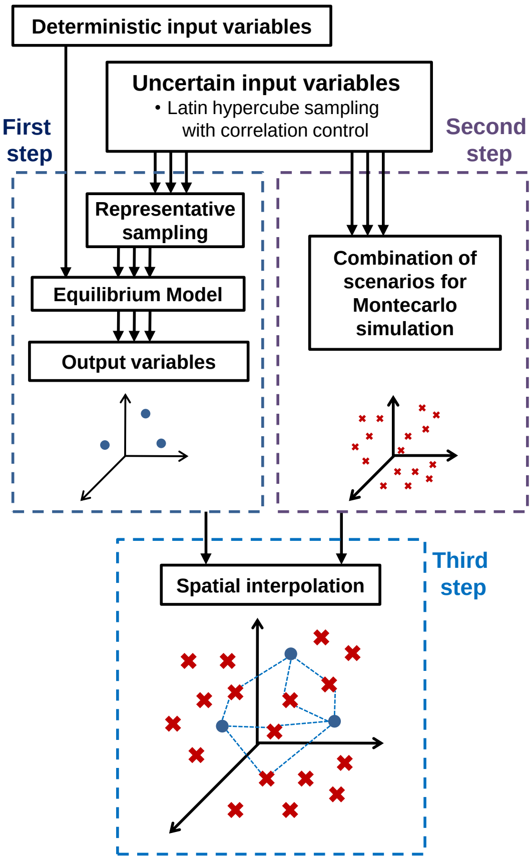

Figure 2.6: Visual overview of Monte Carlo simulation with spatial interpolation techniques

dimensional hypercube according to different combinations of scenarios. In the hypercube each dimension corresponds to a risk factor characterized by its cumulative distribution function, and a scenario is defined by the percentiles corresponding to the different risk factors. Therefore, the spatial co-ordinates of the nth point will be $c_n = (c_{n1}, c_{n2}, ..., c_{nR})$ and each nth point will represent a vector v_n with the values of different outputs (in this case, the electricity prices). As the performance of the spatial interpolator depends on the closeness of the neighbours to the point of interpolation, it is essential to have a well-sampled hypercube in which each scenario is used around $\frac{N}{R}$ times in the N executions of the equilibrium model. This well-sampled skeleton will be defined by means of Latin hypercube sampling. As Latin hypercube sampling does not ensure *per se* the construction of non correlated scenario combinations, (see for instance the left side of Fig.2.7, in which the scenario combination presents certain positive correlation), correlation control techniques are incorporated in order to obtain uncorrelated scenario combinations (see Helton and Davis (2003)). The right side of Fig. 2.7 shows an example of Latin hypercube sampling, in which non-correlation between the coal price and the demand has been guaranteed through control correlation techniques.

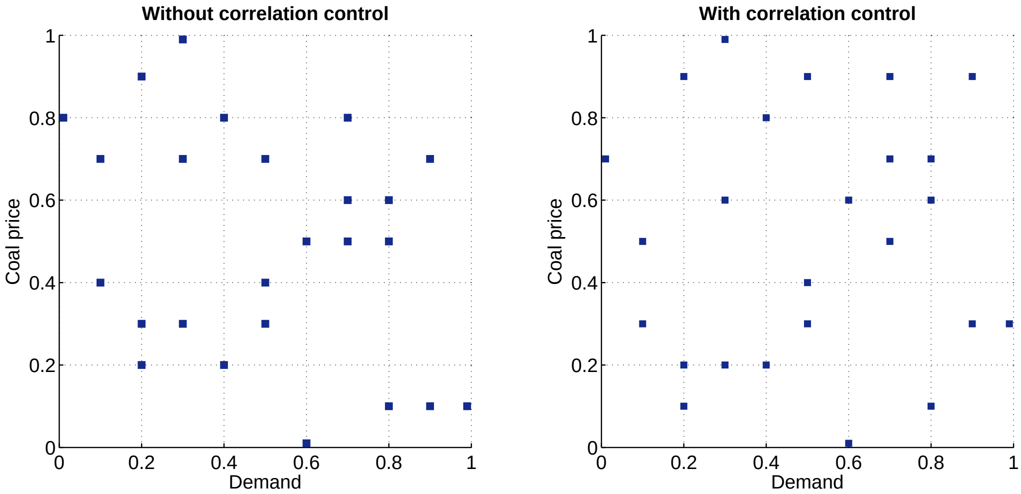

Figure 2.7: Example of Latin hypercube sampling

In this first stage, it is also of crucial importance to sample the extreme percentiles in an appropriate way. All this can be more clearly appreciated by checking the black thick points in Fig. 2.8.

In the second step, a huge amount of $m = 1, 2, ..., M$ (M>>N) random scenario combinations of the uncertain variables with a correlated structure and coordinates $c^*_m = (c_{m1}, c_{m2}, ..., c_{mR})$ are generated in order to determine the unobserved areas v^*_m of the hypercube to be estimated. The inclusion of correlation in this step is important because it allows to sample scenarios with a higher density is those areas which are most likely to occur (denoted by blue points in Fig. 2.8). For example, it is more common for higher coal costs to be linked with higher gas costs than with lower ones. The correlation between fuel prices has been calibrated by using market data. In contrast to the first stage, Latin hypercube sampling is used in this case as a variance reduction technique.

In the third step, the unobserved areas of the hypercube, which were established in the second step, are estimated. This is done by means of an interpolator based on local regression with Euclidean distance that considers the spatial structure of the initial realizations (see Roweis and Saul (2000)). More specifically, each unobserved point c^*_m is established as the center of coordinates and its value v^*_m is interpolated from a set of the nearest K neighbours in such a way that closer

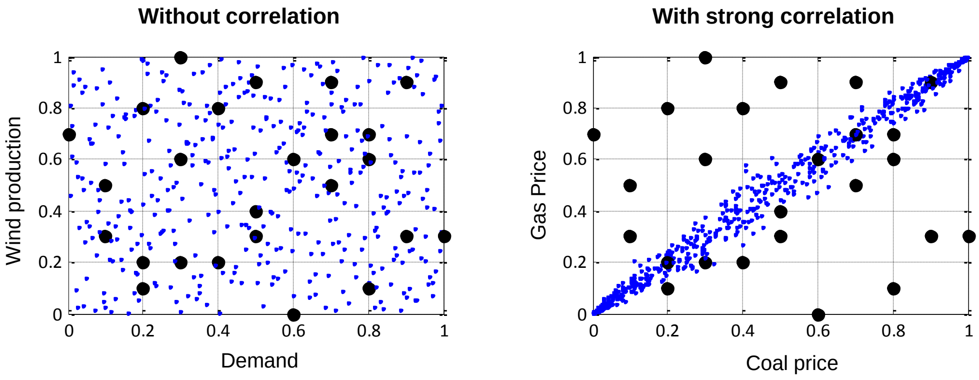

Figure 2.8: Latin hypercube sampling with correlation control

neighbours to c_m^* have larger weights $w_{n=1,\ldots,K}$. Eq. 2.18 presents the process that has been followed to compute those weights that allows us to recalculate the coordinates c_m^* from the coordinates c_n of the known neighbours. As can be seen, the weights are constrained to sum to one with the aim of avoiding bias distortion.

$$\min_{w_n}\left(c_m^* - \sum_{n=1}^{k} w_n \cdot c_n\right)^2 \tag{2.18}$$

$$\text{s.t.} \sum_{n=1}^{k} w_n = 1$$

Finally, the unknown vector of values v_m^*(i.e. prices) is estimated as follows:

$$v_m^* = \sum_{n=1}^{k} w_n \cdot v_n, \qquad \forall m \in M \tag{2.19}$$

2.3.5 Statistical Analysis

The proposed methodology could also be applied in order to obtain other outputs (such as the production of the different technologies) and not only the spot price. In addition, it can be useful to carry out several statistical analysis, such as sensitivity studies of the output variables with respect to the risk factors or the computation of their probability density function (pdf). The latter is one of the central cores of this chapter.

As the scenario definition is random and uniformly distributed and takes into account the correlation between risk factors, all the scenarios are considered to be equally probable and, thus, it is possible to compute the probability density function of all the variables of interest. A non-parametric estimation has been used because it does not make sense that the density function a priori belongs to a parametric family. More specifically, the Epanechnikov kernel[11] has been applied.

2.4 Illustrative case study

This section illustrates the performance of the proposed methodology in a real-size electricity system. The aforementioned equilibrium model has been adapted to the Spanish electricity market. This equilibrium model depicts the whole system in detail, including all the technical characteristics of the hydraulic and thermal generation units (for instance, variable costs, efficiency, maximum and minimum power, etc.). Although the problem size using system states is much lower than the hourly representation, the model consists of about 350,000 equations and 800,000 variables. The resolution time is around 2 minutes for just one realization of the uncertain variables using a PC with Intel Core Duo i7-2,600 CPU @3.4 GHz CPU and 8.0 GB RAM.

The covered time scope ranges from 1st August 2012 to 31st October 2013 (data corresponding to the Spanish market are available from the Market Operator (OMIE) website, www.omie.es). The actual prices are shown in Fig.2.9. This period of time provides an interesting example in which structural and regulatory changes have repeatedly occurred in the market. The most relevant are related to an increasing deployment of renewable energy sources, technological events such as the mothballing of Garoña nuclear power plant (16th December 2012), the implementation of regulatory policies designed to protect vulnerable sectors such as the subsidies for thermal power plants using national coal and the taxation of electricity production (law 15/2012 which entered into force on 1st January 2013). A more detailed statistical analysis of the complete data set used is presented in Table 2.1. This summary of statistics includes measures of central tendency such as the mean and median. Besides, measures of dispersion such

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[11] There are several types of kernel functions which are of general use: Epanechnikov, uniform, Gaussian, triangle, cosine, quartic (biweight), tricube and triweight. Thus, these estimators achieve smoothed density functions which are constructed in each point of the real axis in accordance with the sample values closest to it. In applications, the Epanechnikov kernel appears to be the most common. Here, the optimal bandwidth selection was chosen by minimizing the mean integrated squared error.

as the standard deviation are incorporated, as well as measures of distribution, such as kurtosis and skewness.

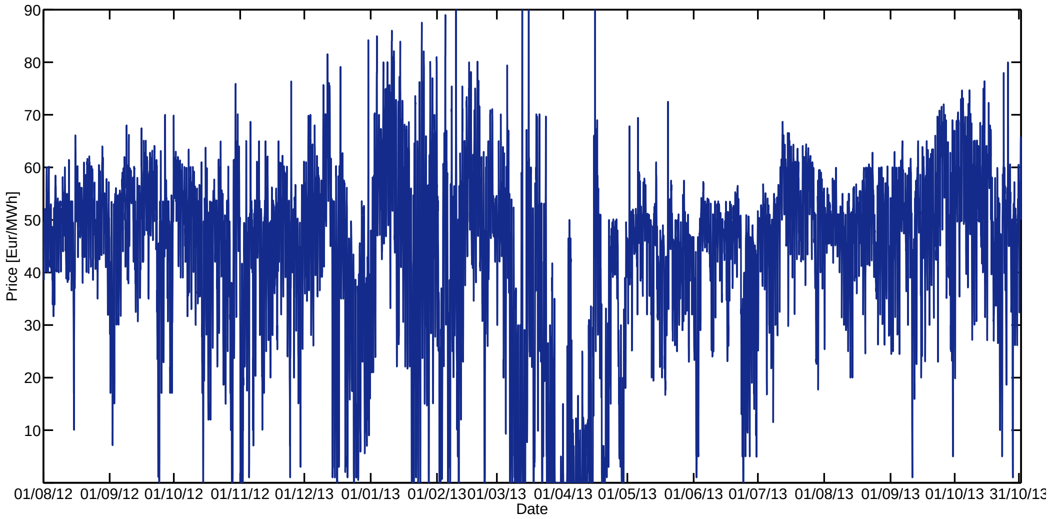

Figure 2.9: Spanish day-ahead electricity market prices

[€/MWh]	Mean	Std. dev.	Skew	Kurt.	Min.	Q1	Q2	Q3	Max.
Price	43.44	17.39	-0.98	0.61	0.00	36.57	47.97	54.2	90.00

Table 2.1: Statistical summary of the actual market spot price during the validation period

In the Monte Carlo simulations, 250 executions of the equilibrium model have been conducted for each month and then the electricity price is estimated by spatial interpolation in 100,000 points. Each period (month) is split into 3 subperiods (Working days, Saturdays and Holidays) with the objective of best capturing the weekly seasonality. The hours are represented by 8 system states in working days, 4 system states on Saturdays, and 4 system states on Holidays. These system states are calculated taking into account the net demand (total demand minus the aggregate production of renewable energy sources, nuclear and run-of-the-river hydro). This representation enables to better capture important features of the operation of electricity systems[12].

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[12] Note that there are generators that produce irrespective of the market price because of their low variable costs (nuclear) and / or their inflexible operation (run-of- the-river hydro, wind, solar photovoltaic, or solar thermal - having no capacity to store-).

2.4.1 Comparison with the traditional load levels representation

In this section, the Monte Carlo execution with system states and the Monte Carlo execution with traditional load levels representation are compared. The objective is to provide illustrative examples that may help to understand the detail in the price modeling that can be achieved with the proposed methodology as opposed to the traditional approach. This section also pursues to further clarify the methodology presented in Section 2.3.

A graphical comparison between both methodologies is presented in Fig. 2.10. This figure shows the expected hourly electricity prices for September 2013. It seems evident that the Monte Carlo execution using system states approach is able to capture the so-called stylized facts Weron (2006, Chapter 2, pp. 25-65) (for example mean reversion, seasonality or price spikes) of the hourly market price better than the Monte Carlo execution using the traditional load level approach. Note how the captured range of market prices is much broader in the system states representation than in the load level approach. The load level representation does not allow a proper hourly modulation because the demand curve is the same for each type of day (working days, Saturdays and Holidays), and there are only 15 price levels (5 levels on workdays, 5 levels on Saturdays, and 5 levels on Holidays). On the other hand, the Monte Carlo execution using system states performs a better hourly simulation because the price in each hour is different.

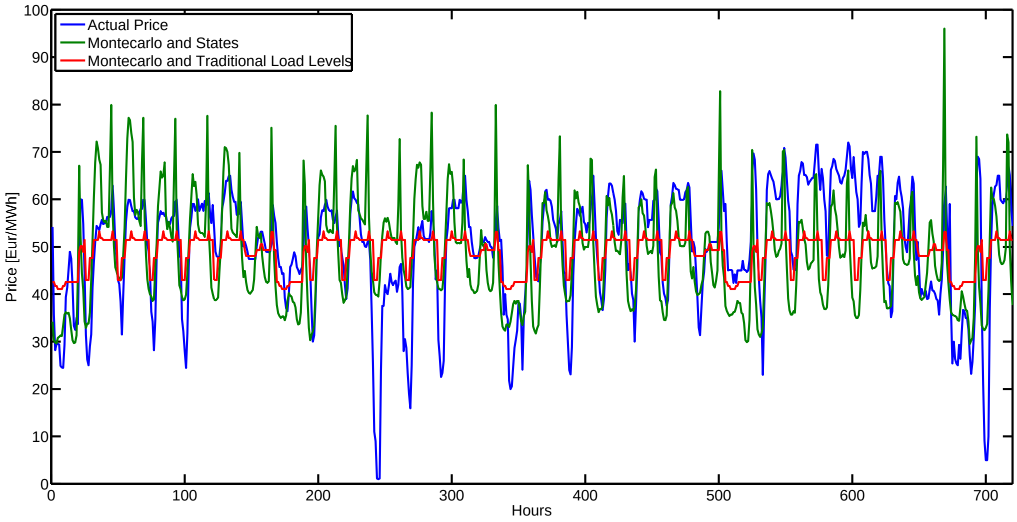

Figure 2.10: Comparison between Monte Carlo execution with system states implementation and the execution with traditional load levels

More specifically, since each Monte Carlo execution corresponds to a particular scenario of the variables subject to uncertainty, the hourly net demand is different for each execution. Consequently, the definition of the states vary in each run depending on the net demand and every hour of the day can be classified in different states. For instance, the price curve for three representative days in Fig 2.11 clearly illustrates that each hour of the day has a different average price.

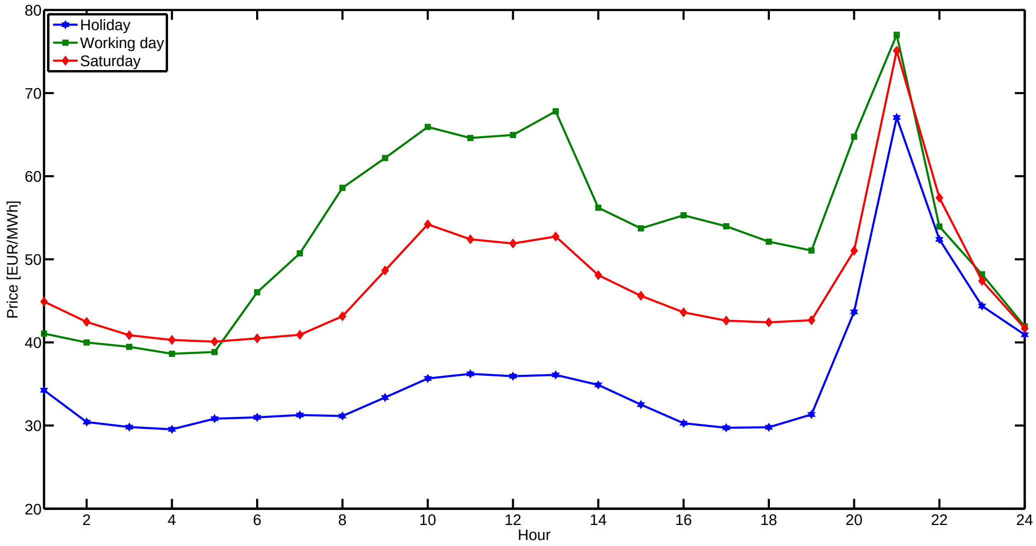

Figure 2.11: Representative price for each type of day with the proposed methodology

All of the above can be also fully appreciated in Table 2.2, in which the proportion of time that each hour belongs to each system state is provided. For the sake of clarity, only the results for one working day are shown. The states have been ordered from the lowest to the highest net demand as s1-s8. In this table it can be checked how each hour has different proportions, but also that adjacent hours have similar proportions (see hours 6 and 7, which only differ in 1 out of the 250 Montecarlo executions). The results presented in Table 2.2 confirm that the proposed methodology might be suited for making hourly predictions of the price of electricity.

[%]	Working day							
Hour	s1	s2	s3	s4	s5	s6	s7	s8
1	7.2	10.8	50	21.6	10.4	0	0	0
2	6	5.2	52	26.4	10.4	0	0	0
3	6	5.2	51.2	26	11.6	0	0	0
4	1.2	10	50	25.2	13.6	0	0	0
5	1.2	5.2	52.8	27.2	13.6	0	0	0
6	1.2	4.4	50.8	30	13.6	0	0	0
7	1.2	4.4	50.4	30.4	13.6	0	0	0
8	0	0	0	6.4	34.8	44.8	13.2	0.8
9	0	0	0	6.4	22.4	46	12	13.2
10	0	0	0	6.4	16	30.8	27.2	19.6
11	0	0	0	6.8	15.6	35.2	24.4	18
12	0	0	0.4	9.6	12.4	56	18.8	2.8
13	0	0	2.8	7.2	19.6	52.8	17.6	0
14	0	0	3.6	6.4	24.4	53.2	12.4	0
15	0	0	4	13.6	30.8	51.6	0	0
16	0	0	4	13.6	36.4	46	0	0
17	0	0	3.6	14	33.6	46	2.8	0
18	0	0	3.6	17.6	29.2	49.6	0	0
19	0	0	3.6	18.8	30.8	46.8	0	0
20	0	0	0	10	22.8	40	24.4	2.8
21	0	0	0	6.4	20.4	20.4	20.8	32
22	0	0	0	6.8	20	21.2	24.4	27.6
23	0	0	22.4	17.2	42	18.4	0	0
24	0	3.6	28	34	34.4	0	0	0

Table 2.2: Proportion of hours belonging to each state

2.4.2 Point Forecasts

After having shown the main differences between the Monte Carlo execution with system states and those one with traditional load levels representation, this section assesses their capabilities to provide point forecasts. To this end, fifteen Monte Carlo simulations (one per month of the analyzed time period) with each methodology have been conducted. The forecasting horizon varies from one to two months. More specifically, if the objective is the prediction of hourly prices for month m, the simulation is carried out in a single step in the first hour of month m-1. Thereafter, in order to make forecasts for the whole month of August 2012, the market equilibrium model have been executed on 1st July 2012 and so on until October 2013. It should be noted that it is out of the scope of this work to understand how the forecast errors in the construction of scenarios contribute to the error of ex ante forecasts[13].

In order to make point forecast, the average price of 100,000 points which have been interpolated from the 250 simulations has been computed in each execution and each hour. In this chapter, following Hyndman and Koehler (2006), the Mean Absolute Error (MAE) and the Median Absolute Error (MdAE) have been selected as accuracy measures of the deviation of the forecasts from the actual prices, allowing us to evaluate the performance of both methodologies. In Table 2.3 the monthly values of these measures are provided. The best value obtained with each approach is highlighted in bold.

[%]	MAE System states	MAE Load levels	MdAE System states	MdAE Load levels
Aug-12	**4.81**	6.18	**4.06**	5.65
Sep-12	**8.09**	8.66	6.91	**6.74**
Oct-12	**8.96**	9.51	7.49	**7.25**
Nov-12	**7.95**	10.39	**5.59**	6.82
Dec-12	**10.94**	14.22	**8.35**	10.49
Jan-13	**12.08**	12.99	**9.93**	11.39
Feb-13	14.80	**13.49**	9.80	**9.45**
Mar-13	24.59	**24.31**	25.68	**23.79**
Apr-13	27.22	**26.88**	32.28	**31.89**
May-13	**6.31**	7.36	**5.17**	5.30
Jun-13	10.19	**9.52**	6.69	**5.24**
Jul-13	7.69	**7.44**	6.89	**6.33**
Aug-13	**7.39**	7.55	**5.05**	5.94
Sep-13	**8.88**	9.22	**7.46**	7.95
Oct-13	11.66	**10.99**	10.68	**9.48**
Average	**11.40**	11.79	**7.57**	7.82

Table 2.3: Monthly prediction errors in point forecasts

For the whole period under study, the MAE and the MdAE suggest that the methodology which uses system states slightly outperforms the traditional approach. As seen, the MdAE tends to be much lower than the counterpart MAE.

Since the traditional methodology makes smoother predictions, it is evident that

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[13] As stated in (Hyndman, 2010), the term ex ante forecasts refers to those predictions that are made using only the information that is available in advance. On the other hand, ex post forecasts are those that are made using known information on the "driver variables", but should not assume knowledge of the data that are to be forecast. The difference between the ex ante forecasts and ex post forecasts provide a measure of the effectiveness of the model for forecasting (taking out the effect of the forecast errors in the input variables).

system states may incur in larger prediction errors during certain hours. The latter is even more important since the price dynamics during the considered span of time reflects a great complexity, with the recurrent presence of abrupt changes and extreme low and high prices. This is particularly relevant for the months of March and April 2013, where the obtained errors are particularly high and slightly worse in the system states methodology. However, as seen in Table 2.4, the new approach has been able to better represent extreme events and the high price volatility. The price of electricity during the mentioned months took exceptionally low average values. Thus, the average price was 25.92 €/MWh in March and 18.17 €/MWh in April, which accounts respectively a 45.5% and a 55.9% lower than those registered during 2012. This is a result of the extremely high production of hydro and wind generation, along with the fall in demand during the holidays. In addition, on two occasions, March 29 and April 1, the price was 0 €/MWh during the totality of hours of the day, which is an unprecedented event in the history of the Spanish market. As a result of the conjunction of exceptional events in these months (which will be explained in further detail in Section 2.4.3), some constructed risk factor scenarios were underestimated.

[€/MWh]	Actual		System states		Load levels	
	Min.	Max.	Min.	Max.	Min.	Max.
Mar-13	0	90	26.30	90.60	33.34	57.45
Apr-13	0	90	27	68	31.61	48.31

Table 2.4: Comparison between maximum and minimum predicted values with both methodologies and the actual prices

After setting the global results of the forecasting accuracy, let us now compare the actual electricity prices of a particular day (27th September 2013), with those yielded by the traditional load levels and the novel proposed methodology. When contrasting the performance of both strategies in Fig. 2.12, it seems that the latter approach might capture the dynamic of the hourly market price better than the traditional load level approach, especially in regard to predict more extreme prices. Notwithstanding, as stated in Table 2.4, it is not easy to forecast the appearance of extreme prices by using market equilibrium models. This result raises important questions about the further research which has to be undertaken in order to better capture price spikes. In any case, the potential of this new methodology is expected to be even more important in performing probabilistic predictions.

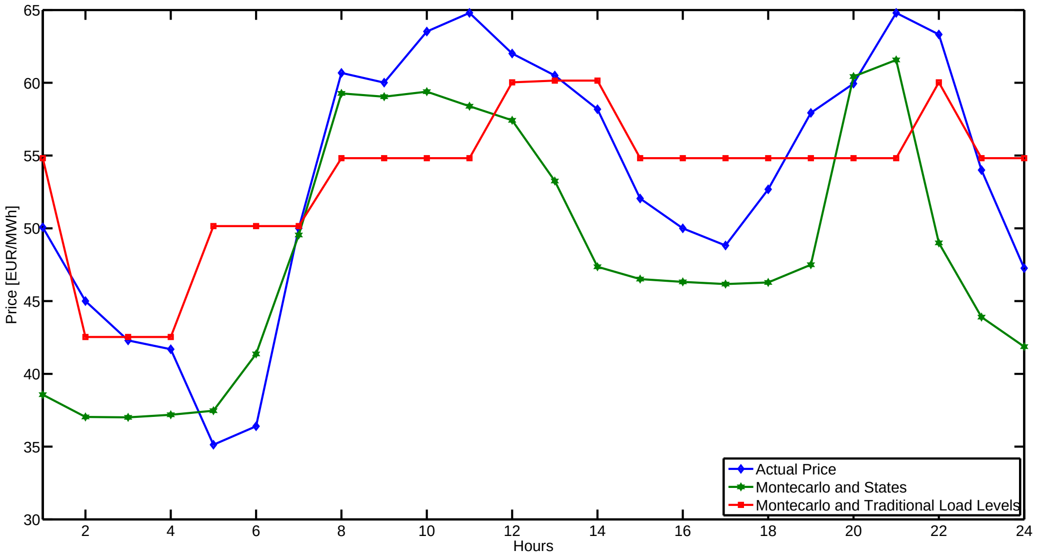

Figure 2.12: Comparison with actual prices

2.4.3 Probabilistic Forecasts

2.4.3.1 Detailed out-of-sample forecasting results

Probabilistic forecasts are particularly relevant for the decision-making process of all agents involved in power markets. Using the proposed methodology the probability density function (pdf) and the cumulative distribution function (cdf) have been forecasted for each hour of the aforementioned period.

Fig. 2.13 and 2.14 show the pdf that has been estimated in an hourly basis during two representative days. The red dashed line represents the actual price. There appears to be that the actual price always falls under the range of predicted prices. Moreover, for many of the hours the actual price is always near the mode value and the model assigns a high probability of occurrence to it.

The predicted pdf of electricity prices tend to have wider tails than those of normal distribution. This means that excessively high or low prices have a higher probability of occurrence than in the case of a normal distribution with identical mean and variance. During holidays, a more symmetrical shape is observed. The hourly pdf is significantly more skewed to the right during peak hours (e.g. hours 21, 22, 23 and 24 of Fig. 2.13 or hours 12 and 13 of Fig. 2.14) than those corresponding to off-peak hours, which are generally negative skew (e.g. hours 9 and 10 of Fig. 2.13 or hours 6 and 7 of Fig. 2.14). Looking at the figures, it is also clear that the price shows a higher variability during peak hours.

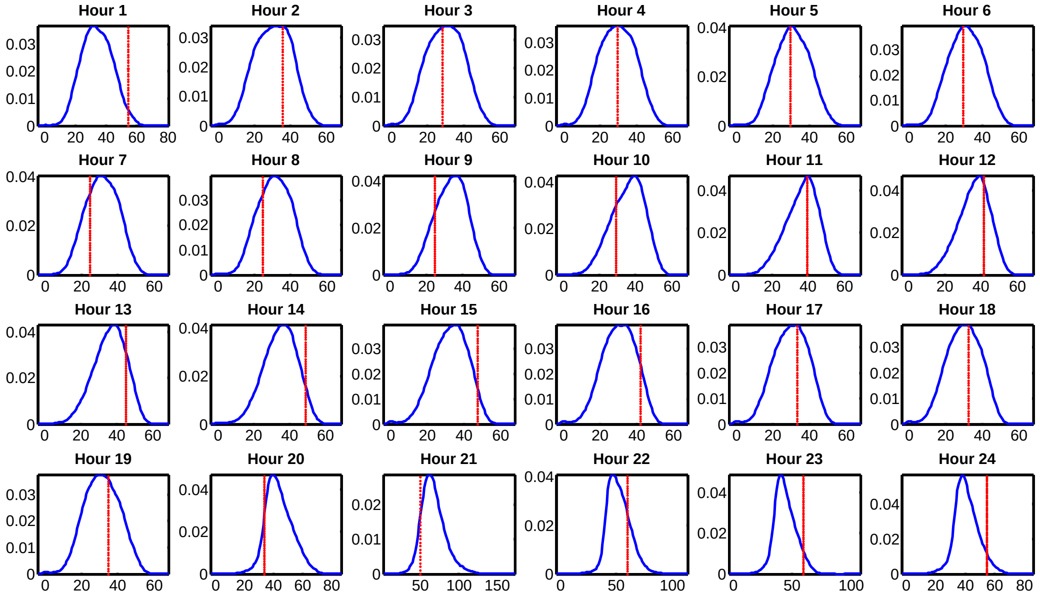

Figure 2.13: Probability density function for each hour of a typical day of holiday: 01/09/2013

A graphical comparison between the actual price of May 2013 and three percentiles that have been estimated using the proposed methodology is presented in Fig. 2.15. According to this figure, it seems that the model can provide quite good results when it is used to make predictions of the hourly distribution of the electricity prices. In order to understand the performance of the forecasting system, different metrics have been used. The first criterion used to evaluate the global quality of the probabilistic forecasts is based on counting how many observations of the sample left out of different percentiles of the cumulative distribution function. This is a measure very transparent and can be interpreted easily. Thus, in Table 2.5 the detail of the test results for percentiles 1%, 30%, 50%, 70% and 99% in each of the months of the validation sample is shown. For the sake of clarity, the ideal percentage of exceedances for the target percentiles is displayed in brackets.

Based on this results, there appears to be some bias in the estimated cdf. Furthermore, the predicted cdf is less volatile that the ultimately observed. Overall, the proposed methodology tends to underestimate the uncertainty associated to the occurrence of extreme prices. This behavior is more remarkable at lower percentiles. If we focus on the probability of exceedance of the median value, the percentile 70% and the percentile 99% it would seem that the forecasted values

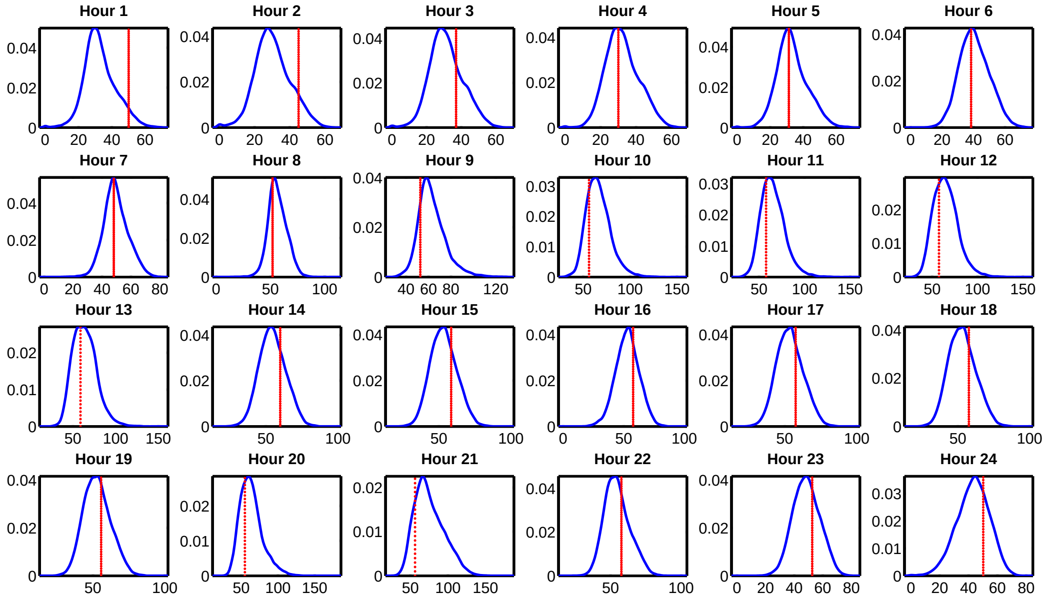

Figure 2.14: Probability density function for each hour of a typical working day: 09/09/2013

are quite acceptable. All the evidence suggests that this loss of accuracy in the lower tail is greatly affected by the presence of large differences with regard to real prices during the months of February, March, April and June 2013.

As mentioned above, during these months exceptional events concurred. On the one hand, renewable energy sources covered more than 46% of the electric demand. Thus, the maximum values of wind production achieved in previous years were significantly exceeded. Simultaneously, hydro production was significantly higher than those corresponding to its historic mean value. For example, the third month of 2013 became the wettest March since the existence of records according to the Meteorological Agency.

On the other hand, the demand continued falling down to the levels reached in 2006 and the system operator reduced the generation of nuclear power plants around 20% between March 29 and April 2, which is an exceptional and unprecedented event since 1997. As a result of all these facts the average prices significantly dropped and there were 413 hours at nil price. Moreover, they were highly volatile months, in which extremely low prices come followed with peak prices reaching 90 €/MWh.

The concurrence of these events serves to justify that some constructed scenarios for uncertain variables have been out of range of the training data and, as

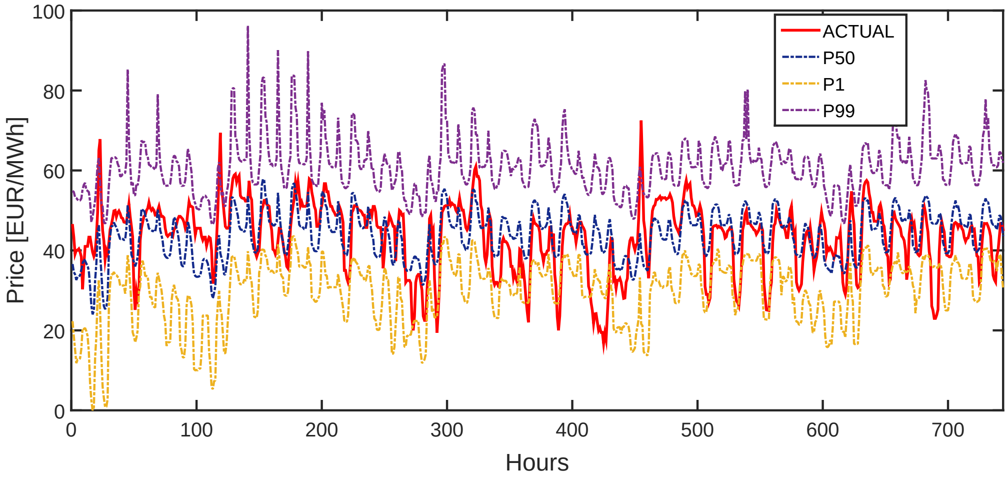

Figure 2.15: Comparison between the actual price and three representative percentiled predicted by the model for the particular case of May 2013

a result, their underlying probability distribution functions have been underestimated. More specifically, the risk factors that have been undervalued during these months have been hydro production, wind power generation and the unavailability of nuclear power plants. All this greatly worsens the quality of the predictions which have been carried out. Proof of this is that if these four months are taken out of the validation period, the overall percentage of the real prices which are inside the nominal values of the analyzed percentiles is generally better estimated (see Table 2.6). It also can be seen how the estimates are slightly less biased. The estimation of the lowest percentile is also significantly improved. However, it is still not easy to account for the occurrence of extreme events. For this reason, this research should be extended in order to explore better ways of capturing the tails of the cdf. This is one of the main targets of the next chapters.

2.4.3.2 Comparison with traditional econometric methods

Since there is no published work which makes medium-term probabilistic forecasts of electricity prices in an hourly basis, it is not possible to directly compare our results with other existing benchmark models. The only exception, as mentioned in Section 2.2, is the work of Alonso et al. (2011). However, this work only focuses on computing the forecasting intervals for one week in the Spanish market (from 24th to 30th of May 2004) by using a seasonal dynamic factor analysis. This period of time was not characterized by significant structural or regulatory

[%]	Percentile 1	Percentile 30	Median	Percentile 70	Percentile 99
Aug-12	94.76	62.63	42.74	28.90	1.61
Sep-12	93.61	72.22	61.94	49.86	3.33
Oct-12	93.28	75.54	62.90	50.40	6.05
Nov-12	90.97	64.03	51.81	33.06	2.92
Dec-12	85.48	59.95	49.46	38.44	8.47
Jan-13	83.06	53.63	43.28	33.06	3.90
Feb-13	77.97	39.29	21.58	8.48	0.30
Mar-13	47.72	25.67	18.68	11.69	0.40
Apr-13	38.33	15.97	6.67	2.92	0.28
May-13	94.09	65.19	49.73	35.75	0.94
Jun-13	75.56	35.28	20.14	8.47	0.00
Jul-13	94.49	77.28	66.40	50.67	9.27
Aug-13	91.67	55.91	38.71	21.77	0.40
Sep-13	97.22	74.44	59.86	45.83	2.78
Oct-13	97.31	80.11	68.82	54.70	13.44
Average	83.79 (99)	57.30 (70)	44.38 (50)	31.81 (30)	3.66 (1)

Table 2.5: Performance of the proposed methodology with probabilistic forecasts

[%]	Percentile 1	Percentile 30	Median	Percentile 70	Percentile 99
Average	92.34	67.33	54.13	40.22	4.85

Table 2.6: Performance of the proposed methodology with probabilistic forecasts when atypical months are removed from the data set

changes, which would represent a clear limitation for a model that is based on time series. As seen in Table 2.7, the price dynamics during this week was characterized by a relatively low volatility and great stability, which is not the case in the period under study presented in this work. Moreover, only the percentage of real prices which are inside the 95% prediction interval (89.88%) are provided in this chapter.

[€/MWh]	Mean	Std. dev.	Skew	Kurt.	Min.	Q1	Q2	Q3	Max.
Price	25.08	4.66	0.542	-0.63	16.95	21.35	23.67	29.00	36.96

Table 2.7: Statistical summary of market spot price from 24th to 30th of May 2004

For this reason, in order to evaluate the performance of the proposed methodology, a comparative study with other traditional electricity price forecasting techniques that goes beyond point forecasts has been conducted. In the modeling of the conditional mean, a transfer function model with a white noise ARIMA component (denoted as Benchmark 1) has been used. Transfer function models have been widely used and have demonstrated their effectiveness to be very accurate for the short term (see e.g. Nogales (2005), Zareipour et al. (2006), Cruz et al. (2011)). In this model, following the recommendation of Weron (2006, Chapter 4.3, pp. 106-136) and the procedure implemented in Misiorek and Trueck (2006), the load has been included as an exogenous variable.

As the assumption of constant conditional variance is not suitable to properly describe the behavior of electricity prices, a wide range of GARCH models and their extensions have been applied to model the time-varying conditional variance and the heavy tails of the underlying distribution of electricity prices. Specifically, it is common practice to simultaneously model the conditional mean and the conditional variance of spot prices by using several ARMAX and GARCH specifications (see e.g. Knittel and Roberts (2005), Misiorek and Trueck (2006), Koopman et al. (2007), Hickey and Loomis (2012)).

The ARCH-LM test for the residuals of the conditional mean specification has been computed (the ARCH test of Engle assesses the null hypothesis that a series of residuals exhibits no conditional heteroscedasticity). This test provides significant evidence in support of heteroscedasticity as the null hypothesis needs to be strongly rejected with a p-value very close to zero.

For this reason, several of the most widely recognized alternatives to model the disturbance variance have also been compared. Consequently, four alternative benchmark models have been constructed. These models are:

- Benchmark 1: Transfer Function with ARIMA noise and constant variance.
- Benchmark 2: Transfer Function with ARIMA noise, with GARCH conditional variance (Bollerslev (1986)). The representation of the GARCH*(p,q)* variance is presented in Eq.2.20, in which ε_t is white noise (here it is assumed that $\varepsilon_t \sim N(0,1)$) and the coefficients α_i, $\beta_j \geq 0$, $w > 0$ to ensure that the conditional variance σ_t^2 is strictly positive. The parameter estimation is based on maximizing the likelihood function for the considered data set. As seen, the conditional variance σ_t^2 is specified as a function of a constant term w, the last periods's predicted variance σ_{t-j}^2 and news about the volatility during the previous periods. The ARCH component (α_i parameter) quantifies the size of the effect of how much volatility increases

irrespective of the direction of the shock. The GARCH effect (β_i parameter) captures the degree of volatility persistence.

$$h_t = \varepsilon_t \sigma_t, \qquad with \qquad \sigma_t^2 = w + \sum_{i=1}^{q} \alpha_i h_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 \tag{2.20}$$

- Benchmark 3: Transfer Function with ARIMA noise, with EGARCH conditional variance (Nelson (1991)). The EGARCH specification, unlike the GARCH one, allows that not only the magnitude but also the direction of the lagged residuals can influence the conditional variance. In this case, the specification for the conditional variance σ_t^2 is represented in Eq.2.21, where $\alpha + \beta < 1$, $\alpha \geq 0$, $\beta \geq 0$ and $w > 0$. In this model, the asymmetric effect (which is captured by γ) is exponential, rather than quadratic such as in the classical GARCH model. More specifically, if $\gamma_k < 0$ the presence of leverage effects can be assured, while $\gamma_k > 0$ implies an inverse leverage effect. Note how the forecasts of σ_t^2 are guaranteed to be nonnegative. The presence of autoregressive conditional heteroskedasticity in the residuals raises the possibility of the inverse leverage effect[14] suggested in the literature Bowden (2008).

$$log\left(\sigma_t^2\right) = w + \sum_{j=1}^{q} \beta_j log\left(\sigma_{t-j}^2\right) + \sum_{i=1}^{p} \alpha_i \left| \frac{\varepsilon_{t-i}}{\sigma_{t-i}} \right| + \sum_{k=1}^{r} \gamma_k \frac{\varepsilon_{t-k}}{\sigma_{t-k}} \tag{2.21}$$

- Benchmark 4: Transfer Function with ARIMA noise, with the volatility following an Asymmetric Power ARCH (APARCH) (Ding and Granger (1993)). The APARCH model, which is represented in Eq. 2.22, is an alternative specification that allows that the power parameter δ of the standard deviation to be a generic value, rather than imposed. This is because it has been often argued that given the existence of leptokurtosis in electricity prices, other power terms besides the square could be suitable. In Eq. 2.22, $\alpha + \beta < 1$, $\alpha \geq 0$, $\beta \geq 0$ and $w > 0$. The parameters γ_i are added to capture asymmetry.

$$\sigma_t^\delta = w + \sum_{i=1}^{p} \alpha_i \left(|\varepsilon_{t-i}| - \gamma_i \varepsilon_{t-i}\right)^\delta + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^\delta \tag{2.22}$$

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[14] As stated in Knittel and Roberts (2005); Bowden (2008), the inverse leverage effect reflects the fact that positive shocks increase volatility more than negative shocks. A positive shock to prices represents in essence an unexpected positive demand shock. Due to the convexity of marginal costs, a positive demand shock has a greater impact on prices relative to a negative demand shock of equal magnitude.

The methodology proposed by Pankratz (1991, Chapter 5, pp. 167-201) has been used to identify the transfer function models with ARIMA noise that allow to capture the conditional mean. The AIC and SIC information criteria have been used to select the best final structure of the transfer function model with ARIMA noise (this term is more specifically a SARIMA$(1,0,1)_{168}(1,0,1)_{24}(1,1,0)_1$ model with logarithmic transformation) among all the possible candidates, and its associated volatility model. The resulting model is shown in Eq. 2.23, in which p_t is the spot price at hour t, d_t is the demand at hour t, L^{-1} is the backshift operator and ε_t is uncorrelated noise at hour t. In this model, variables p_t and d_t have been normalized between the same values.

$$ln\left(p_t\right) = \alpha_1 ln(d_t) + \frac{\left(1-\beta_1 L^{-24}\right)\left(1-\beta_2 L^{-168}\right)}{\left(1-L^{-1}\right)\left(1-\beta_3 L^{-1}\right)\left(1-\beta_4 L^{-24}\right)\left(1-\beta_5 L^{-168}\right)}\varepsilon_t \qquad (2.23)$$

Multi-step forecasts with re-estimation of model parameters in an expanding window of one month have been carried out. In this way, the benchmark models have been firstly fitted using data from the Spanish market for the period from 1st February 2011 to 30th June 2012. Then, forecasts for August 2012 are accomplished. Thereafter, the models have been estimated with data from 1st February 2011 to 31st July 2012. Subsequently forecasts for September 2012 are made and so on until October 2013. In order to make predictions in the same way as in real life, the expected load scenarios for the next month have been used. Due to the very large number of estimated parameters, they are not graphically displayed, but they can be requested to the author.

The autocorrelation of the squares of the residuals and the Information Criteria suggest that GARCH (2,2), EGARCH (2,2) with an asymmetric order of 2 and APARCH (2,2) with an asymmetric order of 1 are specifications that might be more suitable to assess the characterization of the variance process. Moreover, the estimated parameters are always significant at the 10% level, according to the t-test.

The asymmetric parameters of EGARCH and APARCH are always significant and positive, which denotes that positive shocks increase volatility more than negative shocks. This is consistent with the literature about short-term forecasting in which there is a general consensus that electricity prices, unlike most of the financial assests, present an inverse leverage effect (see e.g. Knittel and Roberts (2005),Karakatsani (2008),Bowden (2008)). The significance of the Ljung-Box Q-statistics have been computed on the squared standardized residuals in order to verify that there is no remaining heteroscedasticity.

In order to evaluate the performance of the models in terms of probabilistic forecasts, different percentiles of the error term density have been analytically determined. This approach is equivalent to the one developed in Misiorek and Trueck (2006) to estimate the prediction interval assessing the short-term forecasting power of ARX, ARX-GARCH and TARX models in terms of the quality of the interval forecasts.

Another method chosen for evaluating the forecasts for the target percentiles is the pinball loss function (PLF), which is denoted as a comprehensive measure in the forecasting community[15] (see Hong and Fan (2016)). For a percentile forecast for hour h denoted by $p_{a,h}$ with $a = 1, 2, ..., 99$, this score is expressed as:

$$PLF\left(p_{a,h}, y_h\right) = \begin{cases} \left(1 - \frac{a}{100}\right)\left(p_{a,h} - y_h\right) & if\ y_h < p_{a,h} \\ \left(\frac{a}{100}\right)\left(y_h - p_{a,h}\right) & if\ y_h \geq p_{a,h} \end{cases} \tag{2.24}$$

where y_h is the actual price. The lower the score, the better the forecasts are. This score has been averaged over each of the target quantiles considered in this chapter, for all models over all out-of-sample periods. As the data set is sufficiently large, the observed differences between the models will be statistically significant. The results of the estimates for all specifications are reported in Table 2.8.

As can be seen, the obtained results suggest that the proposed methodology produces superior probabilistic forecasts than the traditional alternative techniques, with the only exception of the percentile 1. In this sense, as already pointed out, further research should be carried out in order to capture the probability of occurrence of extreme low prices through this procedure. An overview of results from dynamic out-of-sample forecasts for the benchmark models clearly suggest that, as expected, the incorporation of a time-varying volatility into the Transfer Function model significantly improves the probabilistic forecasting performance. It is also clear that Benchmark 3, which is able to properly incorporate an asymmetric volatility clustering effect, can accommodate more complex dynamics than the other benchmark models. Another noticeable issue is that the simple GARCH model, which is able to satisfactorily account for time-varying volatility

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[15] For instance, the PLF has used as the evaluation measure in the Global Energy Forecasting Competition.

and volatility clustering, outperforms a more complex model such as APARCH, in which the choice of the power parameter of the standard deviation process is not obvious. In this analysis, the estimated power coefficients are significant for all months and range from 1.36 to 2.17, which are not significantly different from the value 2 of a GARCH specification. Overall, it seems that the proposed methodology constitutes a promising technique for characterizing the full pdf in comparison with other traditional methods.

PLF	Percentile 1	Percentile 30	Median	Percentile 70	Percentile 99	Average
Proposed model	1.57	**5.61**	**5.70**	**4.50**	**0.37**	**3.55**
Benchmark 1	2.43	5.75	6.65	7.11	4.88	5.36
Benchmark 2	0.58	6.01	6.65	5.53	0.77	3.91
Benchmark 3	**0.46**	5.91	6.65	5.58	0.51	3.82
Benchmark 4	0.64	6.17	6.65	5.70	0.61	3.95

Table 2.8: Comparison of the proposed model with traditional techniques in terms of PLF

2.5 Conclusions

In this chapter, a novel methodological approach to simultaneously address two challenging topics that have been barely touched upon in the field of electricity price forecasting has been presented. On the one hand, the proposed methodology is able to make medium-term forecasts with an hourly basis by means of a fundamental model based on market equilibrium. On the other hand, it allows to perform probabilistic hourly predictions of electricity prices. The methodology has been comprehensively tested in a real-size case study based on the Spanish market, but it could be equally extended to other real electricity markets.

The presented nested combination of a market equilibrium model with Monte Carlo simulation, integrated with spatial interpolation techniques and a new definition of load levels, has demonstrated its effectiveness in making hourly predictions with a meaningful reduction of the computational burden and a reasonable loss of accuracy. Additionally, it has been stated that this new proposal, in comparison to the traditional approaches that are based on market equilibrium models trying to make point forecasts over averages of groups of hours, significantly enhances the ability to capture the so-called stylized facts of electricity prices, such as seasonality, mean reversion, or price spikes. The results have also shown a slight improvement in the point forecasting capability when the proposed methodology is used.

Further research has been undertaken in order to evaluate whether the presented approach is suitable for making probabilistic forecasts. The probability density function and the cumulative distribution function have been entirely estimated on an hourly basis. As seen, the methodology is able to provide reliable and satisfactory density forecasts in comparison with other traditional electricity price forecasting techniques, even in the event of recurrent regulatory and structural changes in the market when recent history is almost useless.

As a concluding remark, it should be pointed out that there remains considerable scope for improvement in the prediction of extreme prices. Further research should be conducted to explore better ways of capturing the tails of the price density function, where behavioral factors often matter more than the fundamentals. This naturally links with the joint use of the market equilibrium that has been presented in this chapter with statistical techniques in a hybrid framework as an interesting field for future work. By combining fundamental and statistical models, it can be possible to incorporate the impact of both the projected fundamental changes in the market and the revealed behavioral aspects (such as strategic and speculative behavior). This is the central research topic of the next chapters.

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Chapter 3

Medium-Term Probabilistic Forecasting with a Non-Parametric Hybrid Approach based on Quantile Regression Techniques

Contents

This chapter, which constitutes a natural extension of Chapter 2, also provides a focus upon forecasting electricity prices in the medium term (from one month to a few months ahead) in which accurate estimates of tail risks, e.g. at the 1%, 5%, 95% and 99%, are also important. Medium term forecasting and risk analysis are important for operational scheduling, fuel purchasing, trading and profit management. Here, the research on hybrid forecasting methods, which link the detailed fundamental price formation models, using optimization techniques and market equilibrium considerations, with econometric re-calibration to the time series data is extended. This chapter is innovative in its use of non-parametric econometric techniques such as quantile regression to undertake the re-calibration of the market equilibrium model presented in Chapter 2 and provide accurate risk estimates. It is shown that probabilistic outputs from the fundamental model add value over expected value inputs to the quantile regressions and that if the fundamental model is itself well-specified to diurnal variation through the inclusion of relevant explanatory variables such as demand or climatic conditions, then it is not necessary to undertake the quantile regressions separately for each hour of the day. A real application of the proposed methodology is also successfully tested on the Spanish electric power system, in which the high penetration of intermittent generation creates extreme price risks. The hybrid method outperforms the more conventional fundamental model presented in Chapter 2 and other probabilistic techniques traditionally used in the short term, by making particular use of previous price estimation and several fundamental information data sources in the quantile re-calibrations.

3.1 Introduction

As was seen in Chapter 2, the medium-term horizon presents special forecasting challenges from a methodological perspective. Short term statistical methods, e.g. Conejo et al. (2005), Zareipour et al. (2006), do not tend to extrapolate reliably when the forecasting horizon is extended to such lead times (as was shown in Section 2.4.3 and can be seen García-Martos and Conejo (2013)), especially when market conditions evolve over time. On the other hand, longer-term simulation and market equilibrium models Bastian et al. (1999); Barquín and Centeno (2005); Ventosa et al. (2005), which are often based upon stylized assumptions of economic behavior Green (1992); Barquín (2008); Younes (1998); Bello et al. (2015), provide insights when structural and regulatory changes are expected to happen in the market, but, unlike statistical models, these are not well calibrated to actual data at hourly granularity. As was seen in Section 2.2, this is because fundamental models are not generally focused upon predictions with high-frequency time series data, but instead usually group hours into load levels of similar characteristics.

It is easy therefore to understand the convenience of hybrid price forecasting methods, in which two or more forecasting techniques are aggregated to increase predictive capability, for the medium term (see Yan (2013)). These processes are usually grounded in fundamental models of price formation from the supply stack of generating units, run through unit commitment and dispatching algorithms, and then subject to empirical calibrations to actual data through second stage econometric estimations (for example González and Contreras (2012); Vehviläinen (2005); García-González et al. (2008)). Thus, of the few references that show a certain complementarity between the two focuses are: González and Contreras (2012), who combine a fundamental model, formulated with the modeling of the supply stack, with an econometric model for forecasting day-ahead baseload prices; Vehviläinen (2005), who model spot prices in the medium term on a monthly basis by using a fundamental methodology that represents the existing relations between supply and demand and stochastic techniques to represent the evolution of the underlying factors such as demand, gas prices or weather variables. None of the analyzed works do in-depth modeling of the market characteristics. In addition, risk analysis from such models is, however, only achieved to some degree by running a variety of scenarios. Hence, as an open research question, creating an effective approach to adapt the hybrid formulation into a more fully functional probabilistic method could have substantial methodological and practical value. That is the theme of this chapter.

As was demonstrated in Section 2.4.3, due to the skewness and spikiness of electricity spot prices, it is very difficult to capture its full distribution using the traditional methods based on normal distributional assumptions. Notwithstanding, most of the literature about forecasting modeling offers mainly parametric methods that rely on these assumptions. In general, probabilistic price forecasting is under-researched compared to point forecasts Weron (2014), and even within the probabilistic forecasting approaches, methods aimed at making forecasts of the entire probability density function, e.g. Panagiotelis (2008) and Serinaldi (2011), are less common than interval forecasts, e.g. Misiorek (2008), Zhao et al. (2008), González and Contreras (2012) and Wan et al. (2014). Moreover, these references only address probabilistic forecasts of electricity prices in the short term.

For the medium term, as was already pointed out, the author is only aware of the methodologies proposed in Alonso et al. (2011) and Bello et al. (2015), the latter being the basis for Chapter 2. The analysis in Alonso et al. (2011) is restricted to a 95% prediction interval, computed using a seasonal dynamic factor analysis, without accounting for exogenous variables, and focusing upon a short period of one week during which there were no structural changes. In contrast, as showed in Chapter 2, the work of Bello et al. (2015) developed medium-term forecasts with an hourly granularity, accommodating structural and regulatory changes taking place in the market, and specifying a full probability density for electricity prices. It should be recalled that this model is based on a market equilibrium approach, which uses Monte Carlo simulation combined with interpolation between hourly clusters for tractability. Notwithstanding, as seen in Chapter 2, this model is apparently not able to adequately replicate the risks of extreme prices, where price formation may depart from fundamentals. Another relevant deficiencies of the model presented in Chapter 2 are the following: i) the presumption that all market agents, which are rational[1], have perfect information, and ii) the grouping of hours into system states of similar characteristics (although only a few hours are assigned to each state, it is evident that the resulting price will be less volatile than the ultimately observed). As a consequence of these limitations, in this chapter we seek to develop an alternative hybrid methodology, based upon the equilibrium model but extended to provide specific estimates of the full density function, including the extremes.

Hence, the motivation of this chapter is to enhance the fundamental methodology presented in Chapter 2 with non-parametric econometric techniques such as quantile regression in a hybrid framework, in order to calibrate more effectively the full distribution function and in particular the extreme risk levels. According to Xiao et al. (2015), there is a growing interest towards the development of more robust estimators of conditional quantiles. In this sense, quantile regression is the most prominent technique among the so called non-parametric estimation methods. This is because other alternatives such as kernel or nearest neighbour methods present limitations to deal with several covariates.

Quantile regression is a well-established technique in the context of climate studies, ecology, medicine, survival analysis and so on, for estimating specific per-

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[1] A rational player takes the actions that lead toward his highest expected payoff

centiles of the distribution as a function of exogenous drivers (Koenker and Bassett (1978); Koenker (2005)). As such it has recently become an attractive method in electricity price forecasting in the short term Bunn et al. (2015); Jonsson et al. (2014), where extreme risk levels at 5% of 95% may be required. It has not however, so far, been linked to fundamental inputs in order to provide a hybrid, medium term probabilistic forecast on an hourly basis. By combining fundamental and statistical models, we expect to incorporate the impact of both the projected fundamental changes in the market and the revealed behavioral aspects (such as strategic and speculative behavior). In addition, it is expected that this hybrid approach would also mitigate the effect of any bias in the construction of the risk factors scenarios or in the estimation of the conjectured-price response θ_i of each generator within the fundamental model, as well as other modelling deficiencies of the approach proposed in Chapter 2. Finally, the fact that quantile methods inherently associate a separate regime for each quantile provides a valuable alternative to regime-switching models[2] to capture different formation process for normal and extreme prices (see Bunn et al. (2015)). The results presented in this chapter show that it can indeed be very effective, as a transparent, credible and accurate approach for this purpose. It should be hightlighted that the content of this chapter is mainly based on the journal paper (Bello et al. (2016)) written during the development of this thesis. To summarise, the main contributions of this chapter are as follows:

1. Medium-term probabilistic forecasts with an hourly basis have been carried out by combining a market equilibrium model and probabilistic non-parametric techniques based on rolling quantile regression in a hybrid framework. As several schemes have been proposed to conduct the hybridization process, the work here developed also constitutes a useful tool to guide the combination of fundamental and behavioral information in a probabilistic field.

2. The prediction accuracy of the different hybrid methods has been compared with the pure fundamental market equilibrium model presented in Chapter 2 and other results obtained by means of four benchmark models recently used for probabilistic short-term electricity price forecasting.

3. The proposed methodology is tested when real ex ante forecasts are carry

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[2] Note that techniques such as Markov regime-switching models (see for instance the one presented in the Appendix A) assume the existence of an unobserved variable representing each state or regime.

out in a real-size electricity system particularly unpredictable due to the high penetration of intermittent renewable energy. In addition, the impact of various rolling windows of different sizes has been examined.

The chapter is structured as follows. In Section 3.2 the proposed non-parametric hybrid method is described, while applications of the model in a real case study and forecasting accuracy results are presented in Section 3.3. Finally, the conclusions and the main contributions of this chapter are highlighted in Section 3.5.

3.2 Methodology

Essentially, the proposed methodology consists of the steps shown in Fig. 3.1. Steps from 1 to 3 correspond to the procedure implemented in Chapter 2. The subsequent steps, which are further explained in Section 3.2.1, present the main contribution of this chapter to improve probabilistic forecasts in the medium-term.

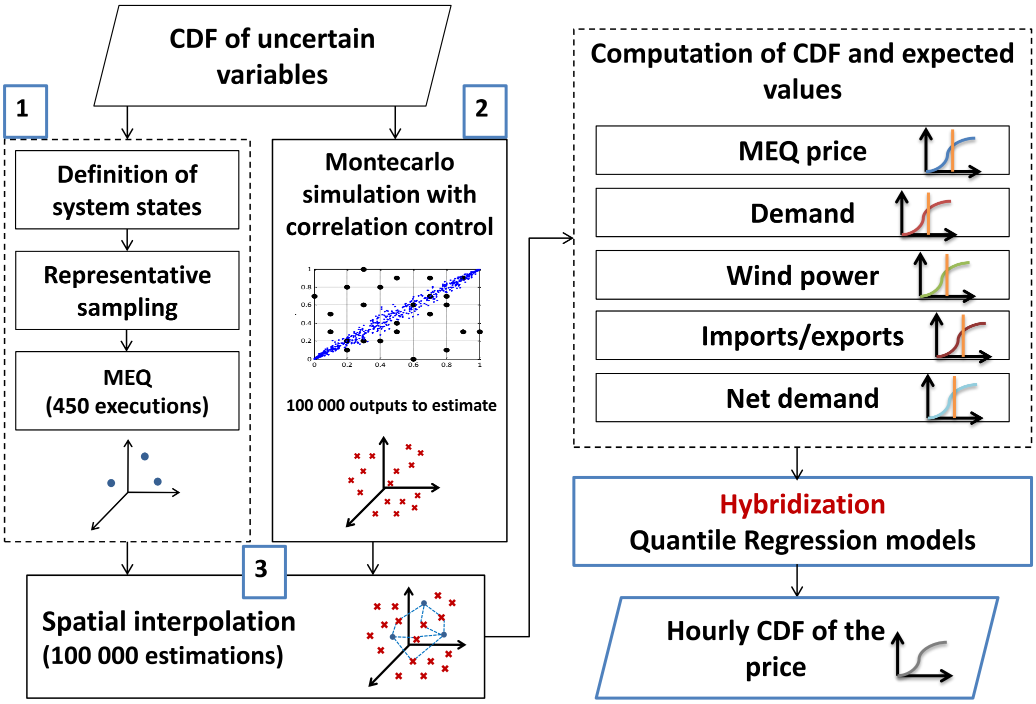

Figure 3.1: Overview of the methodology

As can be seen in Fig. 3.1, the detailed market price formation model based upon fundamentals presented in Chapter 2 is the core of the hybrid approach. As was already introduced, this market equilibrium model based on conjectural variations (denoted here as MEQ) is specified in detail for the Spanish system. As was seen in Chapter 2, this model, unlike econometric techniques, enables the simultaneous estimation of prices (denoted as λ), company profits, generation levels, emissions and transmission flows, even when structural changes have occurred in the underlying market conditions.

In the same way, in order to cope with the computational requirements of Monte Carlo simulation, the efficient method presented in Chapter 2, Section 2.3.4 is used. This method initially computes a manageable number of simulations (in the case study presented in this chapter, 450) from the fundamental model, and then estimates a much larger number (100,000) of outputs (e.g. spot prices, production of each technology) from the initial simulations by means of an interpolator based on local regression that considers the spatial structure of the initial simulations. It should also be remembered that with the combination of system states and Monte Carlo simulation, each hour can be classified into a different state depending on the executed scenario. Finally, in the last stage of the Monte Carlo simulation, a statistical analysis of the results is performed by computing the expected values and the probability distribution functions of all the output variables of interest such as market prices and production of the generation units. This is possible because the scenario definition accounts for the existence of correlation between risk factors and all the huge number of scenario combinations are considered to be equally probable.

3.2.1 Hybridization

This Section presents the main contribution of this chapter, a methodology to re-calibrate and re-shape the distribution function more effectively, especially to the tails, where behavioral factors often matter more than the fundamentals. This requires us to complement the approach of Chapter 2 which is not able to properly capture the high-order moments of the distribution functions, with econometric techniques processing the actual data into a hybrid framework. This hybrid approach provides a valuable methodological synthesis in which the relevant drawbacks of each forecasting method are overcome.

The hybrid approach uses the probabilistic forecasts from the fundamental model as inputs to quantile regression models[3]. The overall scheme is indicated in Fig. 3.1. As is detailed in Section 3.2.1.3, a wide number of possible specifications of quantile regression models have been tested.

3.2.1.1 Selection of the explanatory variables

Apart from regressing price quantiles upon the outputs from the fundamental model, which is the main purpose of this proposed approach, the quantiles can additionally be estimated as a function of the various variables known to influence price formation, as Jonsson et al. (2014); Bunn et al. (2015) do for short-term forecasting.

It is important to emphasize that the evolution of electricity prices is related to a large set of variables. These variables differ substantially across different markets, but in all of them the price prediction techniques must reflect at the same time the production costs and the strategic behavior of market players. This fact has prompted the use of explanatory variables in some of the models proposed in the literature with the aim of defining price distributions. These structural models are not a new category, but a generalization of existing pure technical analysis (or statistical) techniques.

In this sense, many articles show how the demand, fuel prices or weather conditions (such as precipitation, temperature and wind), are relevant in the construction of econometric pricing models (see for instance Karakatsani (2008), Knittel and Roberts (2005), Kosater and Mosler (2006), Koopman et al. (2007) and Schmutz and Elkuch (2004)). Moreover, there is a great deal of research that explores the relationship between electricity prices and other variables using cointegration techniques (see Engle and Granger (1987)) and by using techniques based on the principle of Granger causality (Granger (1969)). In this way, a great deal of research has explored ways to analyze the relationship between Spanish electricity prices and other variables (see Monteiro et al. (2015)). For example, Muñoz and Dickey (2009) probes the relationship between the Spanish electricity prices, U.S. dollar-euro exchange rate and oil prices and Moutinho et al. (2011) explores the relationship between electricity and several commodities prices. There are other studies such as Gelabert et al. (2011) that examine the

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[3] The term hybrid used in this chapter may be also denoted as a post-processing of the fundamental model in a two-stage approach.

impact caused by the implementation of renewable energy sources in Spanish electricity prices. Recently, Bello and Reneses (June 23-26, 2013) use a vector error correction model (VECM)[4] to analyze the long-term relationship between the Spanish electricity price and variables such as the demand, fuel costs, producible of different technologies and the exchange rate.

In the hybrid approach here presented, the influence of the different price drivers must be thoroughly analyzed. This is because some of these factors, such as demand, fuel prices and reserve margin, should have been previously internalized in the fundamental (MEQ) estimation. Therefore, to the extent that they also became significant in the quantile regression, this would indicate a potential misspecification of the MEQ when making price forecasts.

As can be seen in Figure 3.1, in addition to the MEQ price λ, the other explanatory variables considered here are the hourly expected values or the predicted percentiles of the demand, the net demand, the wind generation and the energy imports and exports. The main characteristics of these variables are summarized as follows:

- *MEQ price λ:* This variable, which is obtained by the simulation of the fundamental model, simultaneously reflects the production cost and the strategic behavior of market agents. Moreover, as could be seen in Chapter 2, it is able to capture some of the widely recognized characteristics of electricity prices such as seasonality, mean reversion and leptokurtic behavior of the distribution.
- *Demand:* It is evident that the electricity price is strongly correlated with electricity demand. The systematic shape of the load curve for each day together with the existing within-week seasonal cycle are two relevant characteristics that have been taken into account in the generation of scenarios.
- *Wind generation:* Its high penetration level in Spain makes it a meaningful explanatory variable for modeling and forecasting electricity prices. This variable is characterized by its high volatility and its low correlation with demand. The wind distribution is skewed to the right with occasionally very high wind infeed.

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[4] Using what is known as vector error correction model we can relate the short term behavior of several cointegrated variables with their long-term behavior. We should note that a cointegration relationship expresses a long-run equilibrium but obviously in the short term imbalances can arise.

- *International exchanges:* Spain's interconnection capacity with neighboring countries is very limited. Therefore, this relative isolation can play a major role. Energy is traded on the border with Portugal, France, Andorra and Morocco.

- *Net demand:* There are generators that produce irrespective of the market price because of their low variable costs (nuclear) and / or their inflexible operation (run-of- the-river hydro, wind, solar photovoltaic or solar thermal). The net demand is defined as the portion of total demand which has to be covered by gestionable generators: combined cycle gas turbines (CCGT), coal, fuel and head-dependent hydro. Therefore, net demand variable is equal to the total system demand minus the aggregate production of renewable energy sources (RES) and other baseload power sources such as nuclear and run-of-the-river hydro. Within the RES group, which has become an important part of the Spanish energy system, the wind generation is the most important variable. However, the rest of technologies, which includes electricity generation though solar, mini-hydro, combined heat and power, biomass and waste, suppose an additional source of uncertainty and contribute to increase price volatility.

As the task of variable selection is of combinatorial nature, especially when several percentiles of the aforementioned explanatory variables are used, a "general-to-specific" variable selection approach (Gilbert (1986)) has been followed. More specifically, backward elimination methods under 10% significance tests have been carried out for a series of different post-processing techniques, explained in detail in Section 3.2.1.3

In order to avoid multicollinearity, Belsley collinearity diagnostics (see Belsley et al. (2005)) have been carried out. This may be of great importance in variables that are constructed from other regressors. This is, for instance, the case of net demand. From this perspective, for example, it is possible to simultaneously use any percentile of the net demand and its comprising variables as regressors (i.e. system demand or wind), because it is evident that collinearity might not necessarily exist between them. Another important aspect is that the constant term has been included in all specifications since it can incorporate those effects omitted that may be relevant.

Another critical aspect of the hybridization is to guarantee the existence of non-spurious regressions. For this reason, at least one of the predicted percentiles from the MEQ price distribution, or the MEQ expected system marginal prices λ, has always been used as a deterministic regressor. The Augmented Dickey-Fuller test[5] Dickey and Fuller (1979) on the difference between the actual price and the predicted quantile is stationary in all cases, and cointegration between them was further verified by applying the Johansen's test Johansen (1988). This specification process ensured that the quantile regression would add non-spurious predictive value to the MEQ outputs.

3.2.1.2 Quantile regression inference

It is well known the exclusive focus of classical least-squares regression on the conditional mean. Regarding probabilistic forecasting, the classical theory of linear regression assumes that the conditional quantile functions of the response variable, given covariates, are all parallel to one another, implying that the slope coefficients of distint quantile regressions will be identical. Covariate effects shift the location of the response distribution but do not change its scale or shape.

Quantile regression is a valuable alternative to classical least squares regression, which allows the location, scale and shape of the conditional density to change with the covariates. The fact that the quantile regression model allows to study the impact of predictors on different quantiles τ of the response distribution and that it is a technique which does not assume a distribution for the dependent variable makes it versatile and reduces the bias caused by assuming a parametric distribution[6]. In addition, this technique is able to trace the entire distribution of the variable of interest even when the data set is characterized by a large heterogeneity with extreme outliers, which is a distinctive feature of electricity prices. Thus, it is easy to understand that the conditional mean and median fits are quite different in the case of electricity prices. This fact is explained by the asymmetry of the conditional density and by the strong influence of extreme prices on the least squares.

Following Koenker and Bassett (1978), the standard quantile regression model can be formulated as the search for the best predictor of the variable of interest

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[5] The null hypothesis of this test is that the variable contains a unit root, and the alternative is that the variable was generated by a stationary process.

[6] In the literature, quantile regression is often regarded as a non-parametric method in the sense that no distributional assumption is required. However, other authors, come up with term semi-parametric because in most applications parametric forms are assumed for the relationship between the response and the regressors.

Y given a set of parametric functions $\xi(X,\beta)$ under the asymmetric least absolute deviation loss $\rho_\tau(u) = u(\tau - I(u < 0))$, where $I(.)$ is the usual indicator function[7]. This is equivalent to solving the following optimization problem:

$$Q_Y(\tau|X) = arg\,\min_{\beta\in\mathbb{R}} E\left[\rho_\tau(Y - \xi(X,\beta))\right] \tag{3.1}$$

In Eq. 3.1 $Q_Y(\tau|X)$ are the conditional quantile functions of Y and E is the mathematical expectation.

For the sake of clarity, the function $\rho_\tau(u)$ is illustrated in Figure 3.2 for three representative quantiles τ. As can be seen, the median is the solution to the problem of minimizing a sum of absolute residuals (symmetric case of absolute value loss). Meanwhile, for the other quantiles, we have an asymmetric weighting of the number of observations with positive and negative residuals. More specifically, a penalty $(1-\tau)|u_i|$ is given for overprediction and $\tau|u_i|$ for underprediction in the observation i. Therefore, it is logical that an optimal point estimator for asymmetric linear loss should lead us to the quantiles (see Koenker (2005)). In order to prove that the optimization problem presented in Eq. 3.1 yields the sample quantiles as its solutions, it is only necessary to compute the directional derivative of the objective function with respect to $\xi(X,\beta)$, taken from the left and from the right (see Koenker and Hallock (2001)).

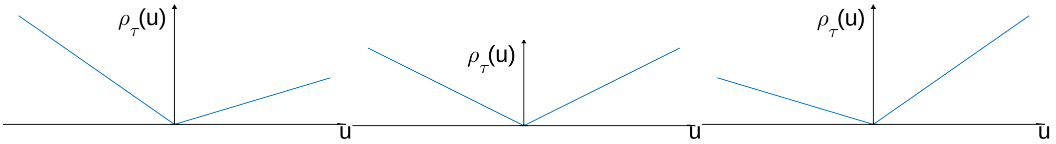

Figure 3.2: Asymmetric least absolute deviation loss for τ=0.3(left), τ=0.5(center) and τ=0.7(right)

With the objective of finding a trade-off between the model complexity and the goodness of data fit, the final selection of explanatory variables has been based on the minimization of Schwartz Information Criterion (SIC) that was proposed by Koenker et al. (1994) for the quantile regression problem. In Eq. 3.2, in which n is the sample size and df is the number of parameters to be estimated, the SIC is formally defined.

$$SIC = log\left(\frac{1}{n}\sum_{i=1}^{n}\rho_\tau(Y_i - \xi(X_i,\beta))\right) + \frac{log(n)}{2n}df \tag{3.2}$$

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[7] $I(u\geq 0) = \begin{cases} 1 & if\ u \geq 0 \\ 0 & if\ u < 0 \end{cases}$

Since the most common approach to estimate quantile regression curves is to fit a function for each target percentile individually, the quantile curves can cross when multiple percentiles are estimated. This can lead to a lack of monotonicity[8] and therefore, to an invalid distribution for the response. Typically, this undesirable phenomenon only occurs in outlying regions of the input space where the observations are scarce[9]. The problem of quantile crossing is well known in the literature, but not trivial, and that is why several approaches have been proposed to deal with this issue. For instance, Koenker (1984) considered parallel quantile planes and Schnabel and Eilers (2012) quantile sheets. Wu and Liu (2009) proposed an stepwise algorithm to ensure non-crossing that consists of fitting the quantiles in a sequential way and constraining the current curve to not cross the previous one. One drawback of this approach is its dependence on the order that the different quantiles are fitted. Reference Neocleous and Portnoy (2008) discuss interpolation of the typical regression quantiles to ensure that, asymptotically, the probability of crossing will tend to zero for the full quantile process. The method presented in Chernozhukov et al. (2010) is based on sorting or monotone rearranging the original estimated non-monotone curve into a monotone rearranged curve. In this thesis, a direct correction to the quantile regression optimization problem, which is the approach proposed in Bondell et al. (2010), has been adopted.

If in Eq. 3.1 a linear quantile model is assumed in which the τth condtional quantile of the response is given by $z_i^T \beta^\tau$, the classical estimator of the regression coefficients $\hat{\beta}^\tau$ will be given by Eq. 3.3. Let be $X = (X_1, ..., X_P)^T$ with P predictors, and $z = \left(1, X^T\right)^T$. Let $D \subset \mathbb{R}^P$, be a closed convex polytope, represented as the convex hull of N points in P-dimensions. Recall that n is the sample size.

$$\hat{\beta}^\tau = arg\,min_\beta \sum_{i=1}^{n} \left[\rho_\tau(Y_i - z_i^T \beta)\right] \tag{3.3}$$

As was already stated, the typical analysis solves the problem presented in Eq. 3.3 separately for each one of the q target percentile levels, which yields that the estimated curves often cross in finite samples. This is particularly problematic when a large number of explanatory variables is used. This violation of the monotonicity condition at any value of the covariate x is formally given by $z_i^T \beta^{\tau_t} < z_i^T \beta^{\tau_{t-1}}$

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[8] The monotonicity requirement refers to the fact that the quantile curve should be increasing as a function of the probability index.

[9] The number of quantile crossings will typically depend on the model complexity, the sample size, the number of estimated quantiles, and the distance between the chosen probabilities (see Taieb et al. (2015))

for at least one $t\epsilon(2,...,q)$. With the aim of providing a solution to the problem of crossing, the approach followed in this thesis is based on estimating the target quantiles simultaneously under a non-crossing restriction. The resulting optimization problem, which can be solved via standard linear programming[10], is presented in Eq. 3.4.

$$\hat{\beta^{\tau}} = arg\,min_{\beta} \sum_{t=1}^{q} w(\tau_t) \sum_{i=1}^{n} \left[\rho_{\tau}(Y_i - z_i^T \beta)\right] \tag{3.4}$$

s.t.

$$z_i^T \beta^{\tau_t} \geq z_i^T \beta^{\tau_{t-1}} \qquad \forall X \in D, \forall t = 2,...,q$$

for some weight function $w(\tau_t)$, such that $w(\tau_t) > 0$ for all $t = 1,...,q$. Here, for practical issues, it has been decided that this weight function has to be the same for all t.

It should be noted that in a previous step, the optimization problem is solved without a non-crossing restriction with the objective of first finding the optimum set of explanatory variables for each percentile as stated in Section 3.2.1.1 and taking into account the SIC criteria (Eq. 3.2). In this way, the coefficients of those variables which are not significant are fixed to 0 at this step. It should be clarified that the estimated coefficients of the covariates do not differ considerably between Eq. 3.3 and Eq. 3.4.

3.2.1.3 Quantile regression specifications

From the quantile regression class of models, four different functional forms have been used: HM1, HM2, HM3 and HM4. Specifications HM1, HM2 and HM3 constitute three different alternatives that have been tested as post-processing techniques of the MEQ for predicting the full cumulative distribution function. In contrast, HM4, which is presented in Section 3.2.1.4 is a specific approach focusing upon the tails. Specifications HM1, HM2 and HM3 are described as follows:

- Model HM1: This model considers the classical linear functional form for

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[10] Several algorithms for obtaining a solution to the standard quantile regression problem have been proposed in the literature: i) the simplex method (Koenker et al. (1994)), which is typically used for moderate data size, ii) the interior point method (Portnoy and Koenker (1997)), which is computationally efficient for large data size, and iii) the interior point method with preprocessing (Portnoy and Koenker (1997)), which is especially suitable for very large data sets (i.e. hundreds of thousands).

the conditional quantile function, as in Koenker (2005). This functional form is quite flexible in the sense that it has good approximation properties and is computationally convenient. In this case, only the predicted expected value of the P exogenous variables X is used (as well as the MEQ). It should be noticed that each of the quantile coefficients β_i^τ is not specific to a particular hour h, as shown in Eq. 3.5, but they are, of course, quantile-specific. Consequently, this model can display variations in scale, i.e. the slope coefficients, and location, i.e. the intercept.

$$Q_\tau(Y_h) = \beta_0^\tau + \sum_{k=1}^{P} \beta_k^\tau E[X_{k,h}] \tag{3.5}$$

In addition to the linear functional form, other nonparametric versions of quantile regression could be used. In this thesis, the possibility of using smoothing splines was also explored[11]. However, no benefits were found over the linear approach.

- Model HM2: This model considers the same functional form as model HM1. The difference is that, instead of the expected values, a wide number of percentiles τ' of the output variables of the market equilibrium model, and exogenous variables, are used as explanatory variables (Eq. 3.6). This is motivated by a consideration that different regions of the distributions of the explanatory variables could be identified as drivers for specific regions of the price distribution. For instance, price spikes have frequently been associated with situations in which the demand or fuel prices are high, or when network congestions, market power or adverse weather conditions have occurred. In contrast, the appearance of extremely low prices is linked with the collapse in thermal demand and high wind production. In the case studies that will be presented in the next sections, seven percentiles for each one of the explanatory variables are considered as possible regressors for the quantile models.

$$Q_\tau(Y_h) = \beta_0^\tau + \sum_{k=1}^{P} \sum_{j=1}^{\tau'} \beta_{k,j}^\tau X_{k,j,h} \tag{3.6}$$

- Model HM3: This model, which constitutes a natural extension of model HM2, is motivated by variable segmentation. More specifically, 24 separate

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[11] As a spline of order b is a piecewise polynomial of order b constrained so that the spline and all of its derivatives to order b-1 are continuous across all the input space, the estimation of the conditional quantile model is not remarkably difficult.

hourly quantile regression models have been used to frame the problem. The major advantage of this approach is that it allows us to have d different hourly response functions by using diverse explanatory variables and that the model parameters switch according to the different behaviors identified in the course of the day. However, this model entails estimating many more parameters and is considerably more computationally intensive and data sensitive.

$$Q_{\tau,d}(Y_h) = \beta_0^{\tau,d} + \sum_{k=1}^{P} \sum_{j=1}^{\tau'} \beta_{k,j}^{\tau,d} X_{k,j,h} \quad \forall d = 1, ..., 24 \tag{3.7}$$

3.2.1.4 Extreme quantile regression

This model, which will be denoted as HM4, is based on the one proposed in Chernozhukov (2005), which develops inferential methods based on extreme value (EV) theory for quantile regression when the quantile index τ is close to 0 or to 1. In reference Chernozhukov (2005) it is assumed that the tails of the conditional distribution of the outcome variable have Pareto-type behavior. Such type of tails are prevalent in economic data, as was discovered by Vilfredo Pareto in 1895. Pareto-type tails encompass or approximate a rich variety of tail behavior, including that of thick-tailed and thin-tailed distributions, having either bounded or unbounded support. Thus, the tail of a distribution function has Pareto-type behavior if it decays approximately as a power function, or more formally, a regularly varying function.

Developing specific models is often necessary (see also Jonsson et al. (2014)) in the tails of the distribution since, as observed in Chernozhukov and Umantsev (2001), data scarcity problems in these extremes are amplified by the presence of covariates, which cause biased quantile estimates in the tails. Thus, Chernozhukov and Fernández-Val (2011) estimate the impact of smoking and maternal behavior on extremely low birthweights in the U.S. using extremal quantile regression. In order to describe the methodology of extremal conditional quantiles from the linear functional form that has been used in this thesis, reference Chernozhukov and Du (2006) has been followed.

Without loss of generality, the main theoretical aspects are discussed in the following for the lowest percentiles. The main assumption is that the response variable Y_h, which has a continuous distribution function F_Y, transformed by some auxiliary regression line has regularly varying tails with extreme value index ξ. Therefore, the starting point is the classical linear function form for the conditional quantile function of Y_h (Eq. 3.8) given $X = x$. In addition to this, it is assumed that there is an auxiliary regression with parameter coefficients β_e such that the disturbance $U_h \equiv Y_h - X_h'\beta_e$ has conditional end-point 0 or $-\infty$ a.s. (almost surely). It is important to note that its conditional quantile function $F_U^{-1}(\tau \mid x)$ satisfies the tail-equivalence relationship given by Eq. 3.9 as $\tau \searrow 0$, uniformly for x in the support of X.

$$F_Y^{-1}(\tau \mid x) = x'\beta(\tau) \tag{3.8}$$

$$F_U^{-1}(\tau \mid x) = F_Y^{-1}(\tau \mid x) - x'\beta_e \tag{3.9}$$

In Eq. 3.9 F_U^{-1} is a quantile function such that

$$F_U^{-1}(\tau) \sim L(\tau)\tau^{-\xi} \tag{3.10}$$

where $\xi \neq 0$ and $L(\tau)$ is a non-parametric slowly-varying function[12] at 0. Note that the absolute value of ξ, which is often referred as extreme value index, measures the heavy-tailedness of the distribution function. A distribution F_Y with Pareto-type tails necessarily has a finite lower support point if $\xi < 0$ (this includes distributions such as the uniform and the Weibull) and an infinite lower support point if $\xi > 0$ (specific examples are the Student's t, the stable and the Pareto distributions). It should be remembered that in the case of being $\xi = 0$, the distribution function will have exponentially light tails and will belong to the so called rapidly varying ones, such as the normal and exponential distributions.

Eq. 3.10 imposes Pareto-type behavior on the conditional law, while Eq. 3.9 requires this behavior to hold uniformly across conditioning values. Since this assumption only affects the tails, it allows covariates to impact the extremal quantiles and the central quantiles very differently. Thus, the impact of regressors on extremal quantiles is approximated by β_e, which differ sharply from the impact on the central percentiles, which are given by $\beta(\tau)$. The extreme value distribution can be calculated following an analytical approach as proposed in Chernozhukov and Fernández-Val (2011).

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[12] A function is said to be slowly-varying at a if $lim_{l \searrow a}[L(l)/L(ml)] = 1$ for any $m > 0$.

	Average		Std. deviation	Skewness		Kurtosis	
	Statistic	Std. Error		Statistic	Std. Error	Statistic	Std. Error
Actual	47.231	0.140	13.133	-1.188	0.026	1.993	0.052
MEQ	47.816	0.095	8.899	-1.730	0.026	12.056	0.052

	Min.	Q25	Q50	Q75	Max.
Actual	0.000	40.770	50.000	55.000	90.130
MEQ	0.000	44.615	49.770	53.154	128.529

Table 3.1: Statistical summary of market spot price

3.3 Empirical Analysis

Evidently, the quality of the hybrid approach will depend substantially on the quality of the basic fundamental model. The calibration of the fundamental equilibrium model is ultimately determined by the estimation of the conjectured-price response θ_i of each generator. Historical prices in Spain during 2012 have been used to estimate this strategic parameter. Having been calibrated to give similar average values, Table 3.1 shows the overall fit of the MEQ model, with the observed values of the exogenous variables being utilized. Thus, instead of simulating fuel prices, demand values and other risk factors, the actual realizations of the input variables during the year 2012 have been used. It is clear that the distribution of electricity prices is not normally distributed and exhibits fat tails, with an excess of kurtosis and negative skewness. The MEQ output has lower variance than the actuals, but higher skewness and kurtosis. This is consistent with the findings of Chapter 2, where the fundamental model tends to underestimate the tails of the electricity price distribution. The data set for specifying and testing the quantile models was constructed from monthly executions of the market equilibrium model for the Spanish day-ahead market over the period ranging from 1st April 2013 to 30th June 2014. Thus, 15 executions of the fundamental model have been accomplished, one per month. As in the case study of Chapter 2, the forecasting horizon varies from one to two months. More precisely, for the hourly predictions for month m, each execution is carried out in a single step in the first hour of month m-1. In order to thoroughly investigate the forecasting capability of the proposed hybrid models, a series of multistep forecasts with re-estimation of model parameters in a moving window, month by month, have been carried out. In order to explore the robustness of the methodology and assess the sensitivity to the sample size, different sizes of moving windows have been used to update the model parameters: 12 months (12 M), 6 months (6M) and 3 months (3M). This leaves us with a validation set of 2 months, 8 months and 11 months, respectively, for which probabilistic price predictions are computed.

It is important to highlight that a window size greater than 12 M was not considered appropriate as it could diminish the potential benefits of a hybrid approach. This is because a re-calibration with such distant windows can be very sensitive to possible structural changes that are already incorporated in a natural way with the fundamental model. In addition, strategic and speculative behavior are expected to be better captured with nearest market information.

3.3.1 Probabilistic scoring methods

The performance of the proposed approaches is evaluated through the out-of-sample forecast quantile accuracies. In order to evaluate and benchmark the proposed method three different probabilistic scoring methods are used: the exceedance rate and relative performance, the Pinball loss function and the Winkler score. They are described below.

3.3.1.1 Exceedance rate and relative performance

This method consists of recording the number of observations of the sample that for each model exceed the estimated percentiles of the cumulative distribution function. The exceedance rate is investigated in each period[13] defined by the rolling window of forecasts at quantiles 1%, 5%, 30%, 50%, 70%, 95% and 99%. Then, the reliability of a model can be defined as the deviation of the empirical from the nominal probabilities. The relative performance $PI_{p,\tau}$ of the proposed methodology with respect to the benchmark model MEQ presented in Chapter 2 (which represents the only other published approach Bello et al. (2015) to medium-term probabilistic forecasts of electricity prices on an hourly basis), is

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[13] This term should be understood differently for the in-sample and the out-of-sample data sets. In the former case, a period corresponds to 12, 6 or 3 months depending on the window size used for calibrating the models. In the latter case, a period is always one month.

computed in each period p and each quantile τ as Eq. 3.11.

$$PI_{p,\tau} = \frac{|PEH_{p,\tau} - (1-\tau)\cdot 100|}{|PEF_{p,\tau} - (1-\tau)\cdot 100|} \tag{3.11}$$

where $PEH_{p,\tau}$ and $PEF_{p,\tau}$ are the observed percentage of exceedances of the hybrid approach and the fundamental model in each target quantile, respectively. Then, the geometric average of the $PI_{p,\tau}$, which is denoted as $\overline{PI}_{\tau}$, is calculated as a measure of the average improvement. Evidently, this measure will be less than 1 when the hybrid methodology outperforms the market equilibrium model. This measure has the advantage of being intuitive and self-explanatory.

3.3.1.2 Pinball loss fuction

As was already discussed in Chapter 2, the pinball function (PLF) is a widely used measure in the context of probabilistic forecasting which penalizes for observations lying far from a given quantile (Liu et al. (2015)). Since it is also the function to be minimized in quantile regression models (Eq. 3.1), we have used this measure as an additional test for assessing the post-sample predictive performance for the target percentiles. Remember that for a percentile forecast for hour h denoted by $p_{a,h}$ with $a = 1, 2, ..., 99$, this score is expressed as,

$$PLF\left(p_{a,h}, y_h\right) = \begin{cases} \left(1 - \frac{a}{100}\right)\left(p_{a,h} - y_h\right) & if\ y_h < p_{a,h} \\ \left(\frac{a}{100}\right)\left(y_h - p_{a,h}\right) & if\ y_h \geq p_{a,h} \end{cases} \tag{3.12}$$

where y_h is the actual price, and as such, the lower the score, the better the forecasts. As this evaluation measure is most sensitive around the 50th percentile, the PLF has been provided for specific percentiles.

3.3.1.3 Winkler score

This measure, proposed by L.Winkler (1972), is widely used (e.g. Liu et al. (2015)) and jointly assesses the interval width and unconditional coverage. For a central

$(1-\alpha)\cdot 100\%$ prediction interval, the Winkler score for an hour h is defined as:

$$W_h = \begin{cases} \delta_h & for\ y_h \epsilon\, [L_h, U_h] \\ \delta_h + \frac{2}{\alpha}\left(L_h - y_h\right) & for\ y_h < L_h \\ \delta_h + \frac{2}{\alpha}\left(y_h - U_h\right) & for\ y_h > U_h \end{cases} \tag{3.13}$$

where L_h and U_h are the lower and upper bounds of the prediction interval respectively and $\delta_h = U_h - L_h$ is the interval width. The Winkler score gives a penalty if the actual price y_h lies outside the constructed interval, and rewards a forecaster for a narrow prediction interval. Naturally, the lower the score the better the prediction interval. Then the average W_h score for all predicted hours, $\overline{W}$, is taken as an overall measure of the accuracy of the interval forecasts. This takes into account both "sharpness"[14] and "calibration"[15], as identified in Gneiting et al. (2007) as the desirable characteristics for the evaluation of density forecasts.

3.4 Evaluation

A comparison of the benchmark MEQ model and the proposed methodology based on the simplest version of the hybrid approach (HM1) is reported in Table 3.2 for the three calibration window sizes. It should be remembered that values below 1 are desired because they would indicate more accuracy when the hybrid methodology is used. The analysis reveals that the in-sample estimates are always significantly more precise when the hybrid methodology is used. Substantial improvements are made in the lower tails of the distribution, 1%, 5%, as well as (but not always) in the other quantiles.

A graphical comparison between the actual price of March 2014 and the P1, P50 and P99 that have been estimated using HM1 is presented in Figure 3.3. This figure shows that HM1 can provide very good results when it is used to make predictions of the hourly distribution of electricity prices. More specifically, for the P99, the P50 and the P1, the number of exceedances are respectively 98.39%, 49.73% and 1.48%. It seems also evident that this approach is able to properly capture the previously referred to as stylized facts (for example seasonality, mean

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[14] This term refers to the ability of the forecast model to concentrate probabilities

[15] This term refers to the reability or probabilistic correctness of the forecasts

Window		P1	P5	P30	P50	P70	P95	P99
12 M	In-sample	0.1608	0.1605	0.0225	0.0053	0.1106	0.1451	0.0032
	Out-of-sample	0.4201	0.8168	1.9736	1.3066	0.7558	0.8491	0.1017
6 M	In-sample	0.1177	0.1131	0.0138	0.0044	0.1450	0.2496	0.0076
	Out-of-sample	0.2645	0.3995	0.5342	0.6547	1.0840	1.1268	0.7504
3 M	In-sample	0.0871	0.0784	0.0331	0.0001	0.0559	0.8123	0.0268
	Out-of-sample	0.2974	0.6127	0.7936	0.8678	0.9183	1.8711	2.3665

Table 3.2: Forecasting results of HM1 in terms of $\overline{PI}_\tau$

reversion or price spikes).

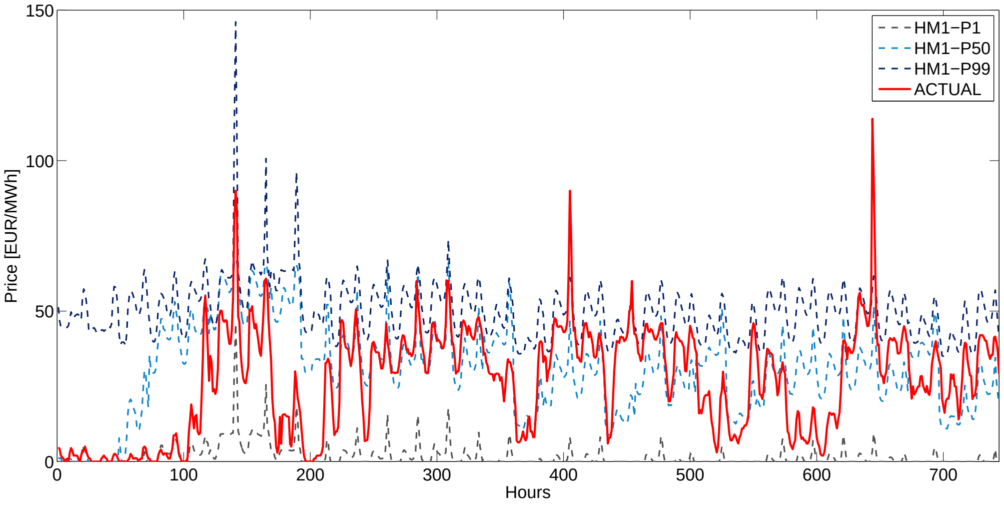

Figure 3.3: Comparison between the actual price and three representative percentiles predicted by model HM1 for the particular case of March 2014

In Fig. 3.4 is examined how the coefficients estimated for the MEQ expected price vary over the target quantiles, along with 95% confidence intervals[16]. As can be seen, the coefficient estimates are significantly different from 0 throughout all quantiles of the market electricity price. Moreover, there is a clear positive relationship between the percentile value and the adjusted coefficients. The positive elasticity of MEQ expected price present an increased effect as prices gets higher.

Table 3.3 shows the results when the conditional quantile is estimated using as explanatory variables not only the expected values, but also the quantiles of the

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[16] It should be noted that, as discussed in Koenker (2005), there are several alternatives to compute covariances, such as: the direct estimation, the rank score method or ressampling methods. In the empirical applications here showed, bootstrapping with 100 replications and the rank score method have been used. As was checked, negligible differences were observed between both approaches.

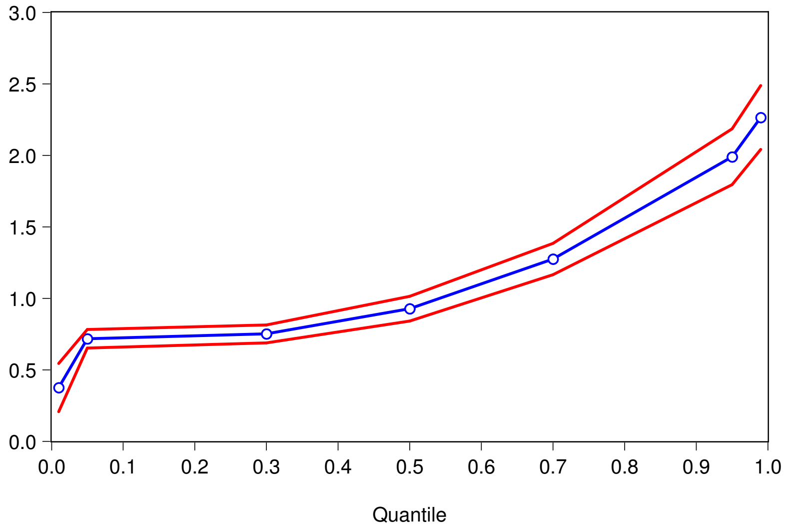

Figure 3.4: Process quantile graph for the coefficient of the MEQ price

Window		P1	P5	P30	P50	P70	P95	P99
12 M	In-sample	0.1603	0.1536	0.0168	0.0050	0.0856	0.0637	0.0032
	Out-of-sample	0.4201	0.7859	1.6481	1.2773	0.6421	0.7955	0.0337
6 M	In-sample	0.1120	0.1097	0.0096	0.0044	0.0960	0.1478	0.0076
	Out-of-sample	0.2499	0.1683	0.4221	0.3592	1.0830	1.0139	0.5775
3 M	In-sample	0.0766	0.0516	0.0269	0.0002	0.0046	0.7933	0.0243
	Out-of-sample	0.2544	0.4997	0.8202	1.0585	0.9824	1.4232	1.9318

Table 3.3: Forecasting results of HM2 in terms of $\overline{PI}_\tau$

different variables (HM2). It can be seen that the forecasts have lower $\overline{PI}_\tau$ values than the model HM1 (compare Table 3.2) in almost all cases, suggesting that the effectiveness of the hybrid technique is amplified if the previously estimated quantiles of the market equilibrium model are used, as well as the other exogenous variables, in the quantile regression.

This finding is also confirmed when comparing Tables 3.4 and 3.5, which show, respectively, the average exceedance rates of HM1 and HM2 for the target percentiles throughout the whole validation sample in each one of the calibration windows. If each table is independently analyzed, we observe that the different calibration periods produce results that are not directly comparable, because, for example for a window size of 12 months, the final 1464 hours are evaluated while, for a window size of 3 months, the final 8016 hours are covered. This comparison, together with the resolution and the sharpness in the evaluation will be further developed in Section 3.4.1.

Window	P1	P5	P30	P50	P70	P95	P99
12 M	100	99.38	96.26	82.71	75.34	20.83	15.35
6 M	92.23	82.64	55.71	33.77	24.95	20.40	18.89
3 M	90.67	87.85	72.12	60.80	39.10	30.91	26.30

Table 3.4: Forecasting results obtained with HM1 for the target percentiles in terms of the exceedance rates

Window	P1	P5	P30	P50	P70	P95	P99
12 M	100	96.99	92.14	79.40	59.61	14.69	0.74
6 M	93.30	84.37	55.82	39.65	33.54	16.49	13.96
3 M	95.37	91.07	69.38	50.63	45.10	25.92	21.00

Table 3.5: Forecasting results obtained with HM2 for the target percentiles in terms of the exceedance rates

Let us now focus on the performance of the periodic (hourly segmented) quantile regression model (HM3). Figure 3.5 shows an example of the evolution of the coefficients. They have been estimated on an hourly basis from the sample that covers 1st May 2013 to 30th April 2014, for making out-of-sample predictions of P1 for June 2014. Given the most relevant predictors, the average variation of the output variable with respect to the input variable is displayed as positive (brown-red scale), null (white scale) or negative (blue scale). Recall that the exogenous variables here are at quantile levels and that they are acting as corrective adjustments to the MEQ forecasts, which already take into account these variables. Hence, their signs and patterns of significance may be complicated to interpret, since they are serving as correction factors. Evidently, the MEQ price is generally significant, but the importance of the P1 wind output during the night can be noted, when the lowest prices occur. It can be also observed that, in some hours, percentiles other than P1 of the exogenous variables, net demand, imports, exports and wind are significant, suggesting that the tail risks need more than simple point forecast inputs into the quantile regressions.

Window		P1	P5	P30	P50	P70	P95	P99
12 M	In-sample	0.0893	0.0794	0.0300	0.0008	0.0334	0.0481	0.0475
	Out-of-sample	0.0225	0.1708	1.5593	0.6630	0.6285	0.7349	0.1828
6 M	In-sample	0.1417	0.1054	0.0517	0.0003	0.0380	0.0815	0.0835
	Out-of-sample	0.1761	0.1698	0.3954	0.3536	0.9920	0.9893	1.7884
3 M	In-sample	0.2274	0.0921	0.0161	0.0001	0.1226	0.8343	1.3483
	Out-of-sample	0.2069	0.1635	0.5102	0.4231	0.9754	1.3791	7.6340

Table 3.6: Forecasting results of HM3 in terms of $\overline{PI}_\tau$

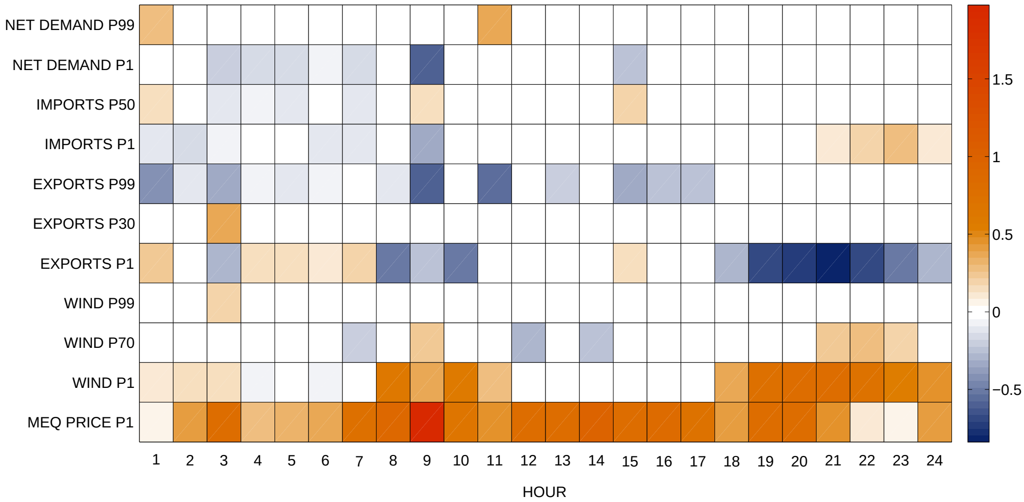

Figure 3.5: Evolution of the coefficients of the explanatory variables over all hours in the particular case of an annual adjustment of percentile 1 for making real ex-ante forecasts for June 2014

Window		P1	P5	P30	P50	P70	P95	P99
12 M	MEQ	0.43	0.84	5.43	6.55	4.55	**1.34**	**0.94**
	HM3	**0.41**	**0.81**	**4.93**	**6.02**	**4.37**	1.77	1.21
6 M	MEQ	2.22	3.78	7.88	8.98	**7.32**	3.55	**1.30**
	HM3	**1.79**	**3.18**	**7.85**	**8.76**	8.03	**3.20**	2.12
3 M	MEQ	2.26	3.94	6.55	**6.99**	**5.99**	**3.67**	**1.05**
	HM3	**1.05**	**2.73**	**5.94**	8.10	7.42	5.25	4.61

Table 3.7: Forecasting results of HM3 and MEQ in terms of PLF

Table 3.6 and 3.7 report the accuracy metrics for the hybrid approach HM3. Table 3.6 reveals that significant improvements over the MEQ are achieved when specific quantile regression models are used for each hour of the day, the only exception being the P99. This is mainly linked to data scarcity problems in the tails, as identified in Chernozhukov and Umantsev (2001).

In Table 3.7, as a summary, the PLF scores have been computed for the seven target quantiles (P1, P5, P30, P50, P70, P95, P99). The results suggest that in general the proposed HM3 produces superior probabilistic forecasts than MEQ, with the only exception being the 3-months rolling window and the right tail of the distribution, as in the previous Table. The accuracy gains compared to MEQ are more relevant in the percentiles which are below the median of the distribution. It seems that 3 months provide a too short window for recalibrating the MEQ with HM3, and this will reflect not only the number of observations but also perhaps seasonality. Our analysis therefore suggests that the hybridization process with HM3 should be performed for a rolling window of 12 months in order to estimate risks more effectively than with the probabilistic simulation of the MEQ.

Finally, to focus more on the tail risks, following Chernozhukov (2005), the application of HM4, which extends HM3 but with a Pareto distribution fitted to the extreme quantiles, is reported. Table 3.8 reports the comparative results in terms of PLF for the P99. For a 12-month rolling estimation window, this use of extreme value theory improves the accuracy over HM3. It can be observed, however, that HM2 is surprisingly the most accurate overall. In other words, using the quantiles of the input variables adds value, but seeking to discriminate between intraday hours as separate time series in the recalibration does not. It seems that the volatility is adequately encapsulated through the cumulative distribution function of the exogenous factors, and therefore, adding volatility as an explanatory variable is not necessary[17]. It is possible, therefore, that the fundamental model captures the intraday pattern sufficiently well and that the recalibration is invariant to a particular hour, for a month-ahead forecasting accuracy, but it is also possible that other exogenous variables might be useful in the recalibration to predict spikes. With the latter consideration in mind, the out-of-sample results having trimmed the high spikes from the time series have been recomputed. In this way, the values marked with an asterisk refer to the PLF that is obtained when those periods are removed from the validation sample. This is for instance the case of December 2013, in which the price took exceptionally high average values as a result of adverse meteorological conditions with low hydro and wind production. Even a spike of 112 €/MWh, the maximum hourly price since November 2008, was recorded. Moreover, the average price in that month was 63.64 €/MWh, 52.5% and 52.2% higher than in December 2012 and November 2013, respectively. Whilst this did reduce the metrics substantially, the qualitative conclusion remained that HM2 performed best and that a 12 month rolling window is preferred. Moreover, the overall results from the test suggest that it could be better to use the MEQ when the constructed scenarios for uncertain variables are out of range of the training data used for the calibration of the data driven methods.

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[17] It should be noted that the volatility has been used in the short-term as an additional explanatory variable in Bunn et al. (2015) for the peak price distribution

Window		MEQ	HM2	HM3	HM4
12 M	In-sample	0.9248	0.3580	0.4316	**0.3082**
	Out-of-sample	0.9383	**0.2601**	1.2120	0.4375
6 M	In-sample	1.1830	0.3450	0.2848	**0.2426**
	Out-of-sample	1.2985	2.1042/**0.4971***	2.3305/0.9053*	2.1226/0.8477*
3 M	In-sample	0.9690	0.2970	0.2123	**0.1744**
	Out-of-sample	1.0475	1.8044/**0.7569***	4.6128/3.6152*	4.2275/3.5494*

Table 3.8: Forecasting results of P99 in terms of PLF

3.4.1 Comparison with benchmark methods

In order to further evaluate the performance of the proposed methodology in an out-of-sample forecasting context, a comparison has also been conducted with other well-established electricity price forecasting techniques that have been widely used in the context of probabilistic prediction. Moreover, in order to be realistic, ex ante forecasts have also been undertaken by including only the information that is available to the market one month in advance. It is important to recall the difference of this approach with the one based on ex post forecasts (see Hyndman (2010)), in which predictions are made using information on the driver variables that becomes available during the forecast period (i.e. the actual net demand).

As observed, established research on electricity price forecasting focuses mainly on the short term, and does not provide, apart from MEQ-like methods, a range of obvious medium-term comparators. Thus, following García-Martos and Conejo (2013), some of the best short-term methods have been extended, as in Bunn et al. (2015), to the medium term as further benchmarks for evaluation.

3.4.1.1 Benchmark 1

A fully parametric location scale model (denoted as BM1) is used in which the conditional mean is a linear function of the lagged price (in this case, as the analysis is focused in the medium term this corresponds with the price P of the same hour h and the same type of day one month m before, i.e. between four and five weeks ago) and the expected net demand ND for the predicted hour, which allows to capture seasonal effects. The volatility follows a GARCH(1,1) process, $\sigma_h^2 = \alpha_0 + \alpha_1\sigma_{h-1}^2 + \alpha_2\varepsilon_{h-1}^2$. The conditional density of z, $g_z(.)$, is Gaussian.

$$P_{h,m} = \beta_0 + \beta_1 P_{h,m-1} + \beta_2 ND_{h,m} + \sigma_{h,m} z_{h,m} \tag{3.14}$$

3.4.1.2 Benchmark 2

Due to the spikiness and skewness of electricity spot prices, it is expected that the Gaussian GARCH model of BM1 may not be sufficiently accurate. This specification (denoted as BM2) constitutes an extension of BM1 in which the conditional density of z is set to skew Student-t. This distribution extends the Student-t distribution (Fernández and Steel (1998)) by adding a skewness parameter with the aim of accommodating the skewness and the excess of kurtosis. This distribution is represented in Eq. 3.16, in which μ is the mean, $\sigma^2$is the variance, ν is the shape parameter with $2 < \nu < \infty$ and λ is the skewness parameter with $-1 < \lambda < 1$. The constants a, b and c are given by the following expressions, in which Γ denotes de Gamma function:

$$a = 4\lambda c\left(\frac{\nu-2}{\nu-1}\right), \quad b = 1+3\lambda^2-a^2, \quad c = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\pi\left(\nu-2\right)\Gamma\left(\frac{\nu}{2}\right)}} \tag{3.15}$$

$$f\left(z;\mu,\sigma,\nu,\lambda\right) = \begin{cases} bc\left(1+\frac{1}{\nu-2}\left(\frac{b\left(\frac{z-\mu}{\sigma}\right)+a}{1-\lambda}\right)^2\right)^{-\frac{\nu+1}{2}} & if \quad z < -\frac{a}{b} \\ bc\left(1+\frac{1}{\nu-2}\left(\frac{b\left(\frac{z-\mu}{\sigma}\right)+a}{1+\lambda}\right)^2\right)^{-\frac{\nu+1}{2}} & if \quad z \geq -\frac{a}{b} \end{cases} \tag{3.16}$$

As can be checked, this parametric distribution is continuous and presents a mode at $-a/b$. Moreover, if $\lambda < 0$ the variable is skewed to the left, and vice-versa when $\lambda > 0$.

3.4.1.3 Benchmark 3

Here the residual terms ε_h are incorporated to the conditional expectation used in Eq. 3.14 following an indirect GARCH(1,1) CAViaR (Conditional Autoregressive Value at Risk) model (see Engle and Manganelli (2004)). Reference Engle and Manganelli (2004) proposed CAViaR models of the quantiles Q_τ in which the assumption of i.i.d. realizations is relaxed. CAViaR models, as nonlinear dynamic quantile models in which quantiles themselves follow an autoregression, aim to derive the evolution of the target quantiles rather than extracting the quantile from a volatility estimate or from a conditional distribution. As stated in Jeon and Taylor (2013), this approach has the advantage of allowing the shape of the conditional price distribution to be time varying, and therefore, to be different for different quantiles of the distribution.

The model is calibrated following Bunn et al. (2015) in such a way that the conditional mean is first estimated using ordinary least squares (OLS) regression, and then the CAViaR model is fitted to the residuals of the regression.

$$Q_\tau(P_{h,m}) = \beta_0 + \beta_1 P_{h,m-1} + \beta_2 ND_{h,m} + Q_\tau(\varepsilon_{h,m}) \tag{3.17}$$

where

$$\begin{aligned} Q_\tau(\varepsilon_{h,m}) \quad = \quad & (1 - 2I(\tau < 0.5))\left(\alpha_1 + \alpha_2 Q_q\left(\varepsilon_{h-1,m}\right)^2 + \right. \\ & \left. \alpha_3 \varepsilon^2_{h-1,m}\right)^{0.5} \end{aligned} \tag{3.18}$$

The value of the parameters of the Eq. 3.18, which allows to estimate the quantile conditioned to the information set up to time $h-1$ ($Q_\tau(\varepsilon_{h,m})$), are computed through the classical quantile regression minimization problem that was presented in Eq. 3.1. Therefore, although this model is similar to the traditional GARCH(1,1) in form, it is not estimated by the maximum likelihood approach method.

3.4.1.4 Benchmark 4

As quantile methods are widely used in risk analysis because of their distribution-free property, here a non paramatric quantile regression approach is also used as a benchmark (BM4 in the following). The functional form includes the lagged price and the net demand as explanatory variables.

$$Q_\tau(P_h) = \beta_0^\tau + \beta_1^\tau P_{h,m-1} + \beta_2^\tau ND_{h,m} \tag{3.19}$$

In Table 3.9 the Winkler scores of the proposed models are showed. The overall results from the tests suggest that the proposed hybrid models yield narrower intervals than the analyzed benchmarks in almost all confidence levels and window sizes. As seen, hybrid models give a better fundamental understanding of

Window	P.I.	HM1	HM2	HM3	BM1	BM2	BM3	BM4
12 M	98%	77.65	**68.41**	70.63	231.66	92.43	140.11	95.96
	90%	49.34	**46.78**	47.55	72.67	69.37	51.90	77.62
	40%	32.75	26.63	**21.23**	42.41	36.22	31.24	40.37
6 M	98%	92.22	**83.37**	213.75	1383.41	240.46	356.89	253.75
	90%	70.12	**66.59**	85.43	314.54	132.97	134.06	134.78
	40%	47.37	**47.33**	55.34	66.15	60.94	61.15	60.42

Table 3.9: Forecasting results of proposed models and different benchmarks in terms of Winkler score for each predicted interval (P.I.)

how different factors affect various quantiles. It can be concluded that adding fundamental information to a pure data driven method such as quantile regression serves to create a more robust model, especially in market situations that are not completely observable in the market. Note that the achieved improvements are more significant in the 98% and 90% prediction intervals, which are crucial in risk analysis and power systems planning. It can be seen that the proposed HM2 produces superior probabilistic forecasts in terms of Winkler scores to those from the alternative techniques in all windows. The only exception corresponds to a 40% predicted interval, in which HM3 is able to better capture the central part of the distribution.

Regarding the predictive capabilities of benchmark models, their performance is adequate for the 40% confidence level in all instances. However, they tend to deteriorate in higher confidence levels. In the particular case of BM3, the results are quite impressive with a window size of 12 months since it is able to provide high quality 40% and 90% prediction intervals. An overview of the results from the benchmarks BM1 and BM2 clearly suggests that, as one would expect, choosing the normal innovation tends to understimate the occurrence of extreme events and the tail risks. In fact, when compared to the other three benchmark models, BM2 performs remarkably better in a 6 month rolling window.

Regarding the rolling estimation periods, comparing window sizes of 12 months (see upper part of Table 3.9) and 6 months (Table 3.10), it can be concluded that a longer calibration improves all models, with the exception of HM1. This may be due to the fact that the expected values of uncertain variables are able to better incorporate seasonal patterns in comparison with the percentiles of the corresponding distribution functions.

P.I.	HM1	HM2	HM3	BM1	BM2	BM3	BM4
98%	71.52	**71.11**	232.89	147.94	119.30	83.98	102.62
90%	43.76	**41.19**	104.85	98.76	61.36	44.00	87.53
40%	31.92	**31.07**	66.77	52.63	41.93	33.55	54.11

Table 3.10: Forecasting results of proposed models and different benchmarks in terms of Winkler score with a calibration window size of 6 Months for each predicted interval (P.I.)

As seen, substantial improvements, in particular, are achieved in the performance of HM3 when a 12-month rolling window is used, suggesting that the intraday pattern captured by the fundamental model should be only supplemented when similar behaviours were observed in the past. This fact is even more important in the tails of the distribution.

3.5 Conclusions

This chapter has presented a novel approach for making medium-term probabilistic price forecasts on an hourly basis by recalibrating the fundamental model introduced in Chapter 2 with quantile regression techniques. For lead times of one month on the Spanish data, the use of a rolling quantile regression estimation process has demonstrated substantial improvement in forecasting the tail risks, when compared to the approach of probabilistic simulation of the fundamental price formation model presented in Chapter 2. In particular, with wind generation as an exogenous variable in the quantile regression (as well as demand, net demand and import/exports), the lower tail risks estimates were improved. A comparative study with conventional time-series and optimization-based models, has demonstrated the effectiveness of the proposed methodology across the entire distribution function.

Variations on the hybrid approach revealed several technical insights. Using inputs to the quantile regression that were expressed as quantiles themselves (e.g. the 1% level of the predicted wind output distribution rather than the mean) improved the accuracy, and furthermore several of the quantiles (e.g. the 1% 50% and 90%) were sometimes useful as predictors for specific price quantiles. Generally, it was found that a rolling window of one year is most appropriate for the re-calibration of the fundamental model to give accurate tail risk estimates, in order to have sufficient data and a full seasonal spread.

Furthermore, it was found that seeking to use separate time series models for each hour of the day was more accurate for the low tails and less accurate for the high tails, than modelling all the hours together in one time series. Apparently, for the high quantile values, the basic fundamental model captures the intraday pattern sufficiently well, for lead times of one month, and whilst the recalibration coefficients vary by quantile, modelling their variability by hour does not lead to more robust predictions. Notwithstanding, for the low tails, the importance of the time of day effect would reflect the way in which high wind production and low demand at night are the major ingredients for the occurrence of low prices. Thus, in practice, it would appear to be prudent to specify an hourly model first and then compare its performance with a hourly invariant estimation.

As a concluding remark, it seems that the hybrid approaches here presented constitute nice inferential methods that do not make rigid assumptions about the distribution of the data. In addition, they allow to successfully incorporate the impact of both the projected fundamental changes in the market and the revealed behavioral aspects (such as strategic and speculative behavior). However, the restrictions imposed to avoid non-crossing between the percentiles and data scarcity problems in the tails can lead to biased quantile predictions. In addition, a parametric representation of the price distribution could be even more desirable for risk management and derivative pricing. For this reason, in Chapter 4 it is investigated not only other hybridization schemes, but also if the use of flexible distributions, which can dynamically accommodate the higher moments and the range of shapes that hourly prices can exhibit, contribute to improve probabilistic forecasting accuracy.

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Chapter 4

Medium-Term Probabilistic Forecasting Electricity Prices: a Hybrid Approach Based on Parametric Techniques

Contents

In the current worldwide context of growing complexity of electricity markets, it is well-known that spot price distributions are getting more non-normal, skewed and heavy-tailed. However, in the forecasting literature there is a lack of research devoted to other characteristics of the unknown distribution beyond its conditional mean and variance. As seen in Chapter 2, the traditional fundamental models used for medium-term forecasting fail to capture the increasing volatility and spikiness of electricity prices. In addition, although fundamental market equilibrium models and statistical methods are frequently seen as polar opposites, this chapter extends the research presented in Chapter 3 on hybrid forecasting methods, with not only the aim of combining them more effectively to benefit from the strengths that each focus offers separately, but also for providing parametric formulas that enable to compute the probability density function more accurately. This chapter is innovative in its use of a wide diversity of general four-parameter distributions for hourly spot prices, in which the four moments are dynamically estimated as functions of several plausible exogenous drivers, to undertake the re-calibration of the probabilistic forecasts. This work also constitutes a useful tool to guide the hybridization procedure as it proposes other alternatives and more complex models that the ones presented in Chapter 3. As a result, it is convenient to perform the re-calibration process using a three-stage methodology that also incorporates combination schemes for probabilistic forecasts. The proposed approach demonstrates its effectiveness across the full range of percentiles of the cumulative distribution function of the price, particularly in the tails. A real application of the proposed methodology is also successfully tested and benchmarked against the most prominent hybrid non-parametric method presented in Chapter 3 and the market equilibrium model of Chapter 2 in an empirical case study for the Spanish electric power system, particularly unpredictable due to the high penetration of intermittent renewable energy.

4.1 Introduction

As seen in Chapter 3, the quantile regression approach, as a non-parametric method, does not make specific assumptions about the price distribution, which constitutes a nice inferential mode (Rigby et al. (2013)). However, quantile regression also has some potential disadvantages such as: i) different quantile curves can cross and the constraints imposed to deal with this issue reduce its flexibility and can lead to an increase of the bias, ii) quantile regression methods do not present easy ways of comparing the fitted models, which means that requires the practitioner to be aware about assumptions made regarding the functional form chosen, and iii) the quantile curves in the tails of the distribution tend to be more irregular than the ones generated by parametric methods (van Buuren (2007)). Finally, perhaps most important, the fact that the quantile regression models lack an explicit formula that enables calculating, for instance, the quantiles, can be a handicap for asset pricing, options valuation or portfolio selection, in which it is fundamental to properly evaluate the uncertainty via distribution functions (Nicolau (2011)). In terms of empirical specifications, following a Taylor expansion of the expected utility functions, Bessembinder and Lemmon (2002) express the forward premium[1] as a linear combination of the variance and the skewness of expected spot price Bessembinder and Lemmon (2002).

Apart from the need of parametric models that can overcome these limitations of the methodology presented in Chapter 3, we must be able to incorporate the influence of higher-order moments. The significance of higher-order moments has been revealed in a series of dramatic market effects[2] and of course, in wholesale electricity markets, in which spot price distribution is only partially understood due to the idiosyncrasies of wholesale electricity markets and the instantaneous nature of the commodity. The main challenge, as will be discussed in Section 4.2, is that most of the economic forecasting literature has traditionally only focused on the first two moments when producing probabilistic forecasts of some variable of interest such as the electricity price.

Taking into account all of the above, this chapter is focused on the proposal of an alternative general hybrid framework based on the estimation of full parametric distributions. Therefore, one of the core objectives is to compare the most accurate distribution free model of Chapter 3 with a multifactor dynamic estimation that incorporates flexible parametric distributions that are able to capture a wide

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[1] The forward premium is the difference between forward and expected spot prices. As stated in Bunn and Chen (2013), the non-storability of electricity prices means that market agents cannot link spot and forward prices through storage costs, but must choose instead to conceive forward prices in terms of expected spot plus risk premia. Intuitively, this market premia emerges as the net hedging costs from the different risk aversions of generators and retailers in the wholesale market. As discussed in Redl and Bunn (2012), the forward premium in electricity is a complex function of different factors such as fundamental (for example, the scarcity or the risk premia of fuels), behavioral (for instance, spikes or oil volatility), market power and shock components between forward and future spot trades.

[2] This is for instance the case of the Great Depression (October 29, 1929-1939), the Black Monday (October 19, 1987), the subprime mortgage crisis (December 2007 – June 2009), oil crashes (i.e. 1973 or 2015-2016) and a long etcetera.

range of shapes such as skewness, heavy tails, bimodality, etc. While it might be agreed that it is desirable to allow the parametric conditional density function to depend on explanatory variables, it is probably not fully clear how to achieve this goal in a closed form, particularly when the attention is devoted to other moments of the distribution besides the mean and the variance. Thus, one of the main targets of this chapter is to find flexible density functions that can accommodate the wide range of shapes that hourly electriciy prices can take in such a way that the latent estimates of the moments changes parametrically to exogenous variables. In parallel with these research focuses, this chapter investigates a fruitful general framework to combine the probabilistic predictions of several competing parametric models for obtaining an optimal performance.

The chapter is structured as follows. In Section 4.2, a state-of-the-art review that is focused on the higher-order moments of the distributions of economic series in general and electricity prices in particular is presented. In Section 4.3, an overview of the proposed hybrid methods is described, while theoretical details and empirical applications of the two- and three- stage approaches are presented in Section 4.4 and Section 4.5, respectively. In Section 4.6, a variety of forecast averaging techniques are thoroughly examined in the context of parametric probabilistic forecasts. Finally, the conclusions and the main contributions of this chapter are highlighted in Section 4.7, as well as the field for future developments that are addressed in part in the Appendix A of this thesis.

4.2 Literature Review

It is well known that most of the practitioners only focus on the first two moments of the electricity price distribution. The problem with this simplification is that it misses the information contained in the higher order moments. For example, a full specification may be particularly relevant in the context of forward, futures and options pricing, in which the price is determined not only by the conditional mean and variance, but also by other features of the conditional distribution such as the skewness and kurtosis. In particular, Bessembinder and Lemmon (2002) demonstrated that the skewness increases the equilibrium forward premium and therefore the optimal forward positions. As stated in Hansen (1994), models of pricing are incomplete unless the full conditional model is specified. In this sense, some authors have found that higher-order moments can serve as explanatory variables for modeling for instance stock returns (Ranaldo and Favre (2005), Boyer et al. (2010)). Furthermore, the characterization of higher-order moments is important for properly determining portfolio management (Vinod (2004), Harvey et al. (2010)). Thus, studies such as Malevergne and Sornette (2005) demonstrated that it is possible to simultaneously increase the expected return and decrease the risks on a portfolio by incorporating higher-order moments risk.

In order to take into account the distributional anomalies of stocks or commodity prices and dealing with higher order moments, some models based on GARCH-type specifications have been proposed in the econometric literature (Engle and Kroner (1995), Malmsten and Teräsvirta (2010), León et al. (2005)). For instance, Hansen (1994) was the first to propose a skew Student-t to accommodate the conditional bias and a significant work regarding the conditional kurtosis is White Jr et al. (2008). According to Chiang and Li (2015), some existing references are capable of explaining the conditional distribution from information contained in the higher-order moments, but they have not completely resolved the distinctive characteristics of financial time series.

As seen, the effects of high-order moments have been relatively analyzed in the case of some financial assets, but they have been barely touched upon in the case of spot electricity prices. One of the few exceptions is the work of Serinaldi (2011), who used a Johnson's U distribution with time varying means and variances, but constant skewness and kurtosis, to predict short-term electricity prices in the California Power Exchange and the Italian Power Exchange. Another exception is the recent work of Gianfreda et al. (2015), where a Skewed t distribution is used in which the four moments are dynamically estimated for making day ahead predictions in the German wholesale market.

Therefore, to the best of the author's knowledge, the work here developed is the first study that evaluates real out-of-sample density forecasts from a wide diversity of parametric models with time-varying higher moments. In addition, none of the existing works focus on the medium term and uses a hybrid framework in which not only probabilistic fundamental information from a market equilibrium model is incorporated, but also that information coming from non-parametric statistical techniques. Finally, this work is innovative also in the sense of combining the probabilistic forecasts from several competing parametric distributions.

4.3 Overview of the methodology

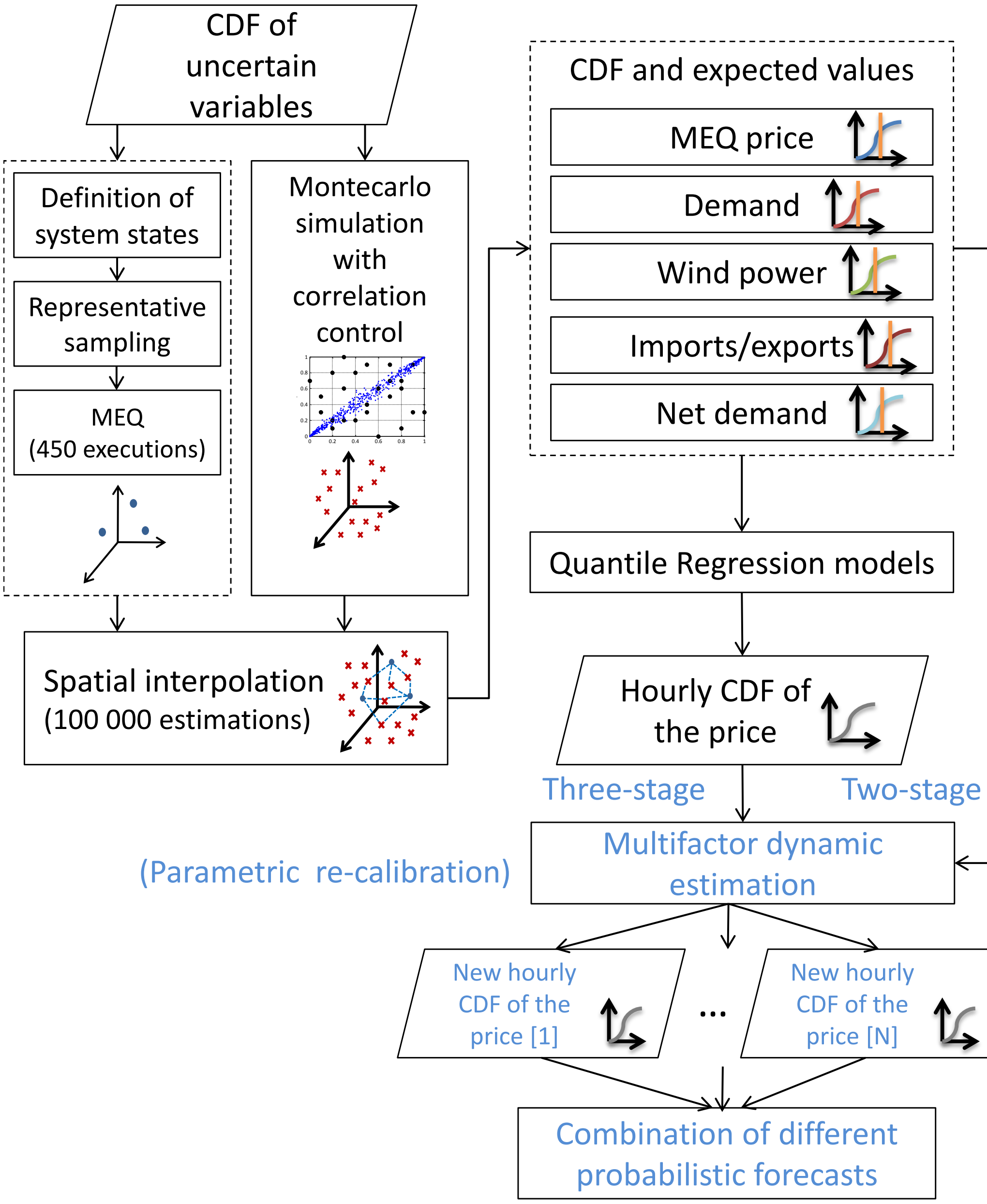

Figure 4.1: Overview of the methodology

This chapter introduces a new hybrid framework that extends far beyond the methodology presented in Chapter 3. This novel approach comprises different blocks. On the one hand, the hybrid approach uses the probabilistic forecasts obtained as the outputs of the fundamental model as inputs to several general four-parameter distribution for hourly prices, in which the four moments are dynamically estimated as latent state variables and furthermore modelled as functions of several exogenous drivers. The latter issue, which is denoted as two-stage approach, is addressed in Section 4.4.

On the other hand, a three-stage approach is proposed in which the percentiles of the price cumulative distribution function of the fundamental model are firstly recalibrated with quantile regression (as suggested in Chapter 3), and in a subsequent stage, linked up with the aforementioned two-stage forecasting model. This subject is covered in detail in Section 4.5.

Finally, we investigate if in the presence of multiple probabilistic forecasts of the same variable it is better to combine the forecasts or rather simply to attempt to identify the single best forecasting model (Section 4.6). A general overview of the hybrid framework is indicated in the flowchart represented in Fig. 4.1. As can be seen, the text in blue represents the contribution of this chapter upon the developments presented in the previous chapters.

4.3.1 Implementation of the proposed methodology

The case studies of this chapter, as in Chapter 3, are underpinned by a data set that has been constructed from executions of the market equilibrium model for the Spanish day-ahead market for the period ranging between 1st May 2013 and 30th June 2014. It should be recalled that this period of time constitutes an interesting case in which structural and regulatory changes have repeatedly happened in the Spanish market. As the aim is to make medium-term predictions in the same way as in real life, 14 executions of the fundamental model have been accomplished, one per month. Therefore, the forecasting horizon varies from one to two months. More concretely, in order to accomplish hourly predictions for month m, each execution of the fundamental model is carried out in a single step in the first hour of month m-1.

In order to thoroughly investigate the forecasting capability of all considered hybrid models here presented, a series of multi-step forecasts with re-estimation of model parameters in an expanding window of one month have been carried out. It is worth highlighting that in Chapter 3, a rolling window, instead of an expanding window, was used. It should be recalled that in Chapter 3 it was found that a rolling window size of at least 6 months, but not greater than to 12 months, is generally most appropriate for the re-calibration of the fundamental model. For this reason, the idea now is to evaluate the robustness of the hybrid methodology when a expanding window of around such sample sizes is used.

Therefore, with the aim of estimating the parameters of the initial models for making predictions for January 2014, the data set ranging from 1st May 2013 to 30th November 2013 has been used. Thereafter, in order to make forecasts for February 2014, the models have been estimated from 1st May 2013 to 31st December 2013 and so on until June 2014. This means that the forecast evaluation sample runs from 1st January 2014 to 30th June 2014. This leaves us with a validation set of 6 months for which real ex ante probabilistic price predictions are computed. As can be appreciated, the window sizes used in the calibration process range from 7 to 12 months.

4.3.2 Forecast Evaluation

Similarly to the previous chapters, the criterion used to evaluate the global quality of the probabilistic forecasts is based on counting the number of observations of the sample left out of the estimated representative percentiles of the cumulative distribution function, with both the market equilibrium model and the proposed hybrid approaches. The violation rate is investigated in each period defined by the expanding window at the same target quantiles of Chapter 3: 1%, 5%, 30%, 50%, 70%, 95% and 99%. As an additional method chosen for assessing the post-sample predictive performance for the target percentiles the pinball loss function (PLF) is used.

4.4 Two-stage approach

With the attractive flexibility that a parametric approach could provide for supporting the risk management and decision making processes, the consequent research question is if the combination of a fundamental market equilibrium model and multifactor specifications with time-varying higher moments can provide any probabilistic forecasting benefit over the counterpart approaches presented in Chapter 2 and 3. In order to answer this question, multifactor specifications based on Generalized Additive Models for Location, Scale and Shape are investigated in the context of hybrid models.

4.4.1 Generalized Additive Models for Location, Scale and Shape

Generalized Additive Models for Location, Scale and Shape (GAMLSS) is a general framework that was proposed by Rigby and Stasinopoulos (2005) to overcome some relevant limitations of the well known Generalized Linear Models[3] (GLM, as proposed by Nelder and Wedderburn (1972)) and Generalized Additive Models[4] (GAM, as introduced in Hastie and Tibshirani (1990)), in which most of the current econometric studies rely. One of the most remarkable shortcomings of the GLM and GAM approaches that is overcome with GAMLSS is that the variance, the skewness and the kurtosis are only implicitly represented through their dependence on the mean μ, but are not explicitly formulated as a function of exogenous variables. Moreover, in GAMLSS, the assumption that the dependent variable follows an exponential familily distribution is relaxed.

Thus, the highly flexible GAMLSS models assume that the response variable presents a general parametric distribution[5] $F\left(\mu,\sigma,\upsilon,\tau\right)$, in which μ and σ are location and scale parameters and υ and τ represent the shape parameters. These parameters can be characterized by means of a wide number of functional forms and can change over time as a function of several covariates. This is of particular importance since electricity price distributions exhibit a great heterogeneity and it can be expected that its shape and its scale change with explanatory variables.

From a mathematical point of view, let Y^S be the vector of y_h independent observations of the response variable for the hour $h = 1,...,S$, with distribution function $F_Y(y_h;\theta_{kh})$, where θ_{kh} are the distribution parameters to predictors η_{kh} for $k = 1,2,3,4$. Let g_{kh} be a known monotonous link function relating the distribution parameters to explanatory variables and random effects through

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[3] The most relevant characteristics of this approach are the following: i) the response variable can represented by the exponential distribution family and ii) the relationship between the mean and the covariates is modelled by a monotonic link function.

[4] This method constitutes an extension of GLM in which smoothing techniques can be used.

[5] A GAMLSS model is parametric in the sense that it requires a parametric distribution assumption for the response variable, but this does not mean that the functions of explanatory variables cannot involve using non-parametric smoothing functions such as cubic splines or penalized splines (this term refers to piecewise polynomials defined by B-spline basis functions in the exogenous variable, where the coefficients of the basis functions are penalized to guarantee enough smoothness).

$$g_{kh}\left(\theta_{kh}^{S}\right) = \eta_{kh}^{S} = X_{kh}\beta_{k} + \sum_{j=1}^{J_k} Z_{jk}\gamma_{jk} \tag{4.1}$$

where θ_{kh}^{S} and η_{kh}^{S} are vectors of length S; X_{kh} is a known matrix of regressors of order $S \times J_k$; β_k is a vector of coefficients of length J_k; Z_{jk} is a fixed known $S \times q_{jk}$ design matrix and γ_{jk} is a q_{jk} dimensional random variable. The first term represents a linear function of explanatory variables and the second one represents random effects. It should be noted that Eq. 4.1 can be equally extended to non linear functional terms.

4.4.2 Multifactor Estimation

The estimation method for the vector of coefficients β_k and the random effects γ_{jk} is based on the maximum likelihood principle through the generalization of the algorithm presented in Cole and Green (1992), which uses the first and (expected or approximated) second and cross derivatives of the likelihood function with respect to the distribution parameters, $\theta^{S}\left(\mu,\sigma,\upsilon,\tau\right)$. Due to the proven fact that computation of cross derivatives is sometimes problematic when the parameters $\theta^{S}\left(\mu,\sigma,\upsilon,\tau\right)$ are orthogonal[6], a generalization of the algorithm developed in Rigby and Stasinopoulos (1996a) and Rigby and Stasinopoulos (1996b) has been used. This algorithm, which does not compute the expected values of the cross derivatives, is more stable (especially in the first iterations) and faster than the one developed by Cole and Green (1992).

For instance, if in Eq. 4.1 a distribution function of four parameters for the price $Y_i \sim D\left(y_i|\mu_i,\sigma_i,\upsilon_i,\tau_i\right)$ without random effects is considered, the likelihood to be maximized with respect to the β_k coefficients is represented as:

$$L\left(\beta_1,\beta_2,\beta_3,\beta_4\right) = \prod_{i=1}^{S} f\left(y_i|\beta,\beta_2,\beta_3,\beta_4\right) \tag{4.2}$$

where S is the number of observations. The likelihood is then maximized with an iterative algorithm that has both an outer and an inner cycle (the former one calls repeatedly the last one).

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[6] This is to say, the expected values of the cross derivatives in the likelihood function are null.

On the one hand, the outer cycle is in charge of fitting the model of each distribution parameter $\theta^S(\mu,\sigma,\upsilon,\tau)$ while the rest of the distribution parameters are fixed at their latest estimates values. On the other hand, for each fitting of a distribution parameter $\theta^S(\mu,\sigma,\upsilon,\tau)$, the inner cycle ckecks the maximization of the whole likelihood with respect to the β_k coefficients, for $k = 1,2,3,4$. The outer cycle is continued until the change in the likelihood is sufficiently small. It should be noted that this algorithm requires initialization of the distribution parameter $\theta_0^S(\mu_0,\sigma_0,\upsilon_0,\tau_0)$, but does not need initial values for the β_k parameters.

Following this approach, very occasional difficulties have arised regarding algorithm convergence. Mainly, these problems have occurred when: i) the parametric distribution function for the electricity price is not adequate or not flexible enough, ii) the starting values of the variables are not correctly defined, iii) the structure of the functional form chosen is unnecessarily complex, particularly when trying to fit the higher moments υ and τ, and iv) the step length in the Fisher's scoring algorithm[7] is too wide. Some of these problems can be easily solved by fitting a series of models of increasing complexity. Thus, for instance, simpler models can provide starting values for the more complicated ones. Moreover, the possible existence of multiple maxima has been also investigated by using several widely varying starting values. This point is particularly critical when the data set is small. Overall, the algorithm has been found to be fast and stable, especially when explicit derivatives are used[8].

As seen, the inference procedure is not trivial and implies the selection of a suitable distribution family of the dependent variable, the explanatory variables, the link functions, and the structure of the systematic part (i.e., linear and/or nonlinear, parametric and/or nonparametric additive functions between parameters and covariates). The model fitting and the selection of these models are discussed in further detail in Stasinopoulos and Rigby (2007).

Keeping in mind the objectives of this thesis, different distribution families (this issue will be further covered in Section 4.4.3) and functional relationships have

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[7] This scoring algorithm is a form of quasi-Newton method particularly suitable for solving maximum likelihood equations numerically when the second derivatives are difficult to compute or not available. Therefore, a general purpose of quasi-Newton algorithms is to build up a working approximation to the second-derivative matrix from successive values of the first derivative. Note that it could be possible to extend the derivative calculations further with the aim of obtaining an exact Hessian, but the benefits of an exact Hessian are not as substantial as the benefits of exact first derivatives.

[8] Note that numerical derivatives can be used instead, but this results in higher computational time.

been compared in terms of their goodness of fit. It should be remembered that in this case, unlike in Chapter 3 with quantile regression techniques, it is possible to use a global measure of goodness of fit and directly compare model adequacy.

The final configuration has been defined searching for increased predictive power while minimizing the risk of overfitting. Thus, the model selection has been carried out by performing a K-fold cross validation analysis that allows to check the significance of the forecasting improvement in terms of the validated global deviance (equal to minus twice the current fitted log likelihood, that is to say, $VGD = -2L(\hat{\theta})$, where $\theta(\mu, \sigma, \upsilon, \tau)$). In order to carry out K-fold cross validation, the sample was randomly divided into 10 subsets, SS_1, SS_2, ..., SS_{10} of training data that corresponds to 70% of the in-sample data set. For each group, the proposed model is adjusted with the training data and afterwards the VGD is computed for the remaining 30% of the data that was omitted from the fit (validation data set). Finally, an average of the 10 measures obtained for the VGD is computed for comparison purposes between the different candidates that have been previously defined. Definitely, choosing those specifications that minimise this information criterion results in a consistent model selection.

4.4.3 Family Distributions

In this work, 32 continuous distribution families with two, three, and four parameters among those listed by Stasinopoulos and Rigby (2007) have been considered. Briefly speaking, some distributions which have been traditionally used in the econometric literature were beforehand discarded. This is for instance the case of symmetric distributions (i.e. the t-distribution or the power exponential) that clearly fail when addressing the issue of skewness. This is also the case of other distributions with three parameters (such as the skew normal), that did not show enough flexibility.

Overall, and as it was to be expected, four-parameter distributions, which are able to model both skewness and kurtosis in addition to the location and scale parameters[9] , demonstrated best performance in terms of the global deviance

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[9] It should be stressed that two-parameter distributions (such as the Normal or the Gumbel distributions) are only able to model independently the scale and the location of the distribution. Meanwhile, the skewness and kurtosis are implicitly defined from those two parameters. On the other hand, three-parameter distributions can model explicitly either skewness or kurtosis in addition to the location and scale parameters.

criterion. For a consistent selection and extensive comparison, those four density functions that minimize the global deviance criterion were tested in the hybrid approaches proposed along this chapter: the box-cox power exponential, the skew t type 3 and the skew exponential power type 2 and 3. Essentially, the attractive feature of the analyzed flexible family distributions is that they allow likelihood inference while coping with continuous changes in the location, scale, asymmetry and kurtosis.

4.4.3.1 Box-Cox Power Exponential

The Box-Cox power exponential family distribution (denoted by BCPE), which was introduced by Rigby and Stasinopoulos (2004), provides a model for the response variable Y exhibiting both skewness (positive or negative) and kurtosis (leptokurtosis or platykurtosis).

The BCPE() distribution is specified for the positive random variable Y through the transformed random variable Z given by:

$$Z = \begin{cases} \frac{1}{\sigma \nu}\left[\left(\frac{Y}{\mu}\right)^{\nu} - 1\right] & if\ \nu \neq 0, \\ \frac{1}{\sigma} log\left(\frac{Y}{\mu}\right) & if\ \nu = 0, \end{cases} \tag{4.3}$$

for $0 < Y < \infty$ where $\mu > 0$, $\sigma > 0$ and $-\infty < \nu < \infty$, and where the random variable Z is assumed to follow a standard power exponential distribution with power parameter, $\tau > 0$, treated as a continuous parameter.

The probability density function (pdf) of Z, a standard power exponential variable, is given by:

$$f_Z(z) = \frac{\tau}{c^{2(1+1/\tau)}\Gamma(1/\tau)} \exp\left\{-0.5\left|\frac{z}{c}\right|^{\tau}\right\} \tag{4.4}$$

for $-\infty < z < \infty$ and $\tau > 0$, where $c^2 = 2^{-2/\tau}\Gamma(1/\tau)\,[\Gamma(3/\tau)]^{-1}$. This parameterisation, used by Nelson (1991), ensures that Z has zero mean and standard deviation 1 for all $\tau > 0$. Note that $\tau = 1$ and $\tau = 2$ correspond to the Laplace (i.e. two-sided exponential) and normal distributions, respectively, while the uniform distribution is the limiting distribution as $\tau \to \infty$. Strictly, the exact distribution of Z in Equation 4.3 is a truncated standard power exponential distribution. From Equation 4.3 the pdf of Y is given by:

$$f_Y(y) = f_Z(z)\left|\frac{dz}{dy}\right| = \frac{y^{\nu-1}}{\mu^{\nu}\sigma} f_Z(z) \tag{4.5}$$

for $y > 0$. The parameters μ, σ, ν and τ may be interpreted as relating to location (median), scale (approximate coefficient of variation), skewness (transformation to symmetry) and kurtosis (power exponential parameter which determines the fatness of the tails), respectively. Therefore, these four distribution parameters define the shape of the curve. For the sake of clarity, Fig.4.2 plots the BCPE(μ,σ,ν,τ) distribution for different values of the parameters.

An identity link function has been assumed for $g_1(\cdot)$ and $g_3(\cdot)$, whereas logarithmic link functions have been assumed for $g_2(\cdot)$ and $g_4(\cdot)$ to ensure positivity for the parameters σ and τ of hourly prices. Remember that the terms $g_k(\cdot)$ for the k moments were previously defined in Eq. 4.1.

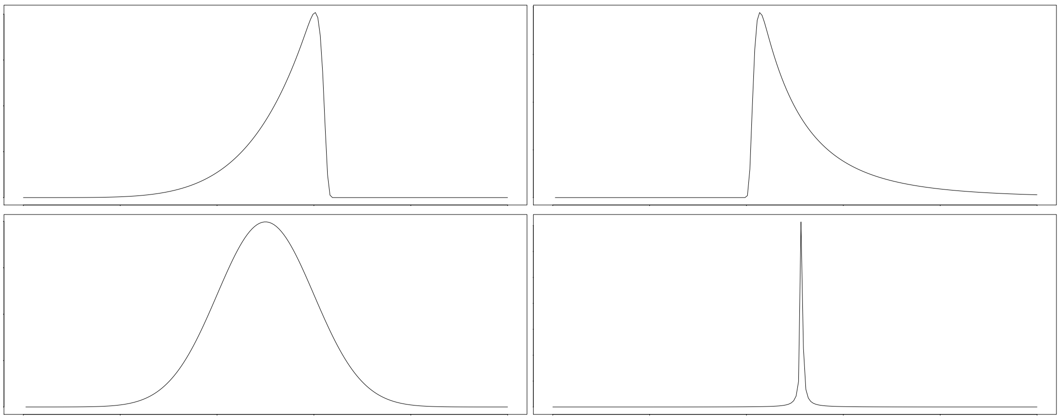

Figure 4.2: The pdf from a BCPE distribution for specific parameter values. Upper-left: BCPE (5,0.2,6,10), upper-right: BCPE (5,0.2,-4,10), lower-left: BCPE (5,0.2,1,2) and lower-right: BCPE (5,0.2,6,0.1).

4.4.3.2 Skew Exponential Power Type 2

The pdf of the skew exponential power type 2 distribution, denoted by SEP2(μ,σ,ν,τ) is defined as follows:

$$f_Y(y \mid \mu,\sigma,\nu,\tau) = \frac{2}{\sigma} f_{Z_1}(z)\phi(\omega) \tag{4.6}$$

for $-\infty < y < \infty$, where $-\infty < \mu < \infty$, $\sigma > 0$, $-\infty < \nu < \infty$, and $\tau > 0$, and where $z = (y-\mu)/\sigma$ and $\omega = sign(z) \mid z \mid^{\tau/2} \nu\sqrt{2/\tau}$ and $\phi(\omega)$ is the cdf of a standard normal variable evaluated at ω and $f_{z_1}$is the pdf of $Z_1 \sim PE2(0,\tau^{1/\tau},\tau)$ being $PE2$ the power exponential type 2.

This distribution, which is able to incorporate a wide range of shapes, was introduced by Azzalini (1986) as his type 2 distribution and was further developed by DiCiccio and Monti (2004). The parameter υ determines the skewness of the distribution with $\upsilon > 0$ indicating positive skewness and $\upsilon < 0$ negative. The parameter τ determines the kurtosis of the distribution, with $\tau > 2$ for platykurtic data and $\tau < 2$ for leptokurtic. Fig. 4.3 shows the flexibility of this distribution for representative values of each one of the four parameters.

Here $E(Y) = \mu + \sigma E(Z)$ and $Var(Y) = \sigma^2 V(Z)$ where

$$E(Z) = \frac{2\tau^{1/\tau}\upsilon}{\sqrt{\pi}\Gamma(\frac{1}{\tau})(1+\upsilon^2)^{(2/\tau)+(1/2)}} \sum_{n=0}^{\infty} \frac{\Gamma(\frac{2}{\tau}+n+\frac{1}{2})}{(2n+1)!!}\left(\frac{2\upsilon^2}{1+\upsilon^2}\right)^n \tag{4.7}$$

and $E(Z^2) = \tau^{1/\tau}\Gamma(\frac{3}{\tau})/\Gamma(\frac{1}{\tau})$, where $(2n+1)!! = 1 \cdot 3 \cdot 5 ... (2n-1) \cdot (2n+1)$, as stated in DiCiccio and Monti (2004). Note how for $\upsilon = 1$ and $\tau = 2$ the SEP2(μ,σ,υ,τ) distribution is the normal density function N(μ,σ).

An identity link function has been assumed for $g_1(\cdot)$ and $g_3(\cdot)$, whereas logarithmic link functions have been used for $g_2(\cdot)$ and $g_4(\cdot)$ to ensure positivity for the σ and τ of hourly prices.

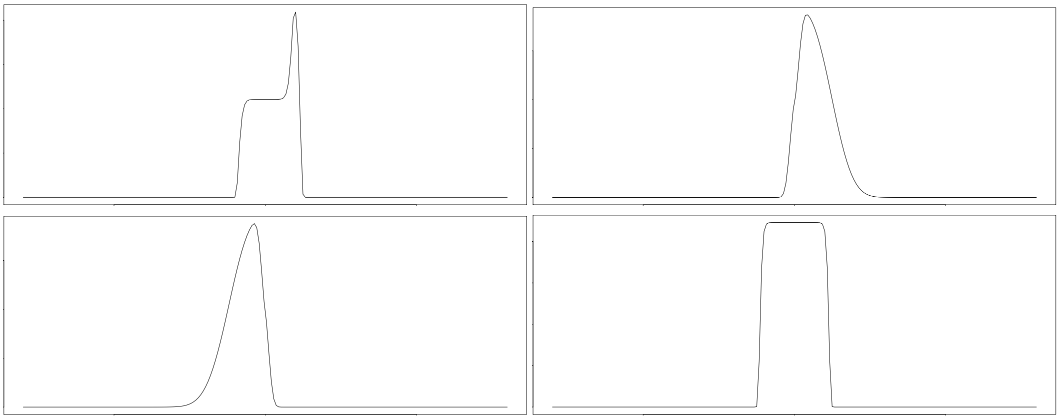

Figure 4.3: The pdf from a SEP2 distribution for specific parameter values. Upper-left: SEP2 (0,1,10,3), upper-right: SEP2 (0,1,10,1), lower-left: SEP2 (0,1,-10,1) and lower-right: SEP2 (0,1,0,3).

4.4.3.3 Skew Exponential Power Type 3

This is a "spliced-scale" distribution with pdf, denoted by SEP3(μ,σ,υ,τ). It is defined by:

$$f_Y(y \mid \mu,\sigma,\upsilon,\tau) = \frac{c}{\sigma}\left\{exp\left[-\frac{1}{2}\mid \upsilon z \mid^{\tau}\right] I(y<\mu) + exp\left[-\frac{1}{2}\mid \frac{z}{\upsilon}\mid^{\tau}\right] I(y \geq \mu)\right\} \tag{4.8}$$

for $-\infty < y < \infty$, where $-\infty < \mu < \infty$, $\sigma > 0$, $\upsilon > 0$, and $\tau > 0$, and where $z = (y-\mu)/\sigma$ and $c = \upsilon\tau / \left[\left(1+\upsilon^2\right) 2^{1/\tau}\Gamma\left(\frac{1}{\tau}\right)\right]$ (see Fernández et al. (1995). Note that $I()$ is an indicator function, where $I(u) = 1$ if u is true and $I(u) = 0$ if u is false.

Note that μ is the mode of Y. Here $E(Y) = \mu + \sigma E(Z)$ and $Var(Y) = \sigma^2 V(Z)$ where $E(Z) = 2^{1/\tau}\Gamma\left(\frac{2}{\tau}\right)\left(\upsilon - \frac{1}{\upsilon}\right)/\Gamma(\frac{1}{\tau})$ and $E(Z^2) = 2^{2/\tau}\Gamma\left(\frac{3}{\tau}\right)\left(\upsilon^3 + \frac{1}{\upsilon^3}\right) / \left[\Gamma\left(\frac{1}{\tau}\right)\left(\upsilon + \frac{1}{\upsilon}\right)\right]$.

An identity link function for $g_1(\cdot)$ has been used, whereas logarithmic link functions have been assumed for $g_2(\cdot)$, $g_3(\cdot)$ and $g_4(\cdot)$ to ensure positivity for the σ, υ and τ of hourly prices. Finally, Fig. 4.4 reflects some representative shapes of SEP3 distribution for different parameter values.

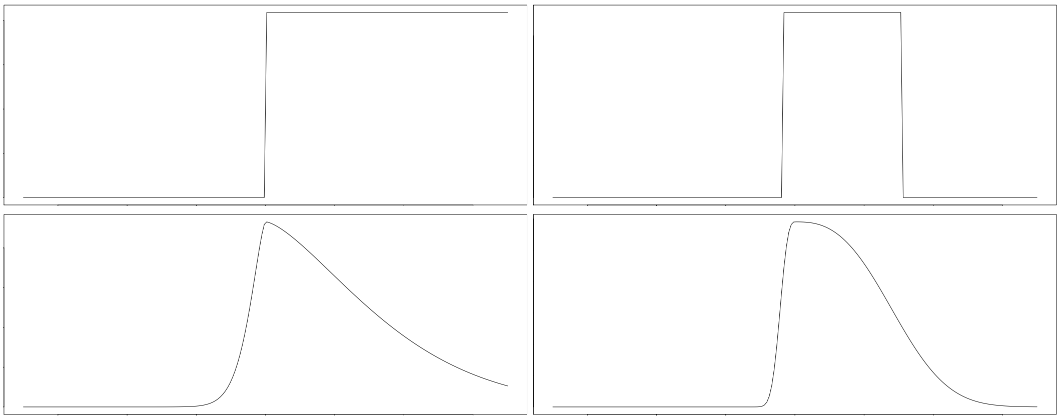

Figure 4.4: The pdf from a SEP3 distribution for specific parameter values. Upper-left: SEP3 (0,1,7,7), upper-right: SEP3 (0,1,1,7), lower-left: SEP3 (0,1,1,0.1) and lower-right: SEP3 (0,1,1,1).

4.4.3.4 Skew t Type 3

This is again a "spliced-scale" distribution with pdf, denoted by ST3(μ,σ,υ,τ) defined by:

$$f_Y(y \mid \mu,\sigma,\upsilon,\tau) = \frac{c}{\sigma}\left\{1 + \frac{z^2}{\tau}\left[\upsilon^2 I(y<\mu) + \frac{1}{\upsilon^2} I(y \geq \mu)\right]\right\} \tag{4.9}$$

for $-\infty < y < \infty$, where $-\infty < \mu < \infty$, $\sigma > 0$, $\upsilon > 0$, and $\tau > 0$, and where $z = (y-\mu)/\sigma$ and $c = 2\upsilon / \left[\sigma\left(1+\upsilon^2\right) B\left(\frac{1}{2}, \frac{\tau}{2}\right)\tau^{1/2}\right]$, (see Fernández and Steel (1998)).

Note that μ is the mode of Y. The mean and variance of Y are given by $E(Y) = \mu + \sigma E(Z)$ and $Var(Y) = \sigma^2 V(Z)$ where $E(Z) = 2\tau^{1/2}(\upsilon^2 - 1) / \left[(\tau - 1) B\left(\frac{1}{2}, \frac{\tau}{2}\right)\upsilon\right]$ and $E(Z^2) = \tau\left(\upsilon^3 + \frac{1}{\upsilon^3}\right) / \left[(\tau - 2)\left(\upsilon + \frac{1}{\upsilon}\right)\right]$.

This four-parameter distribution, unlike the previous ones, is able to model only leptokurtosis. This can be seen more easily in Fig. 4.5 for specific parameter values. It should be noted that an identity link function for $g_1(\cdot)$ has been assumed, whereas logarithmic link functions has been used for $g_2(\cdot)$, $g_3(\cdot)$ and $g_4(\cdot)$ to ensure positivity for the σ, υ and τ of hourly prices.

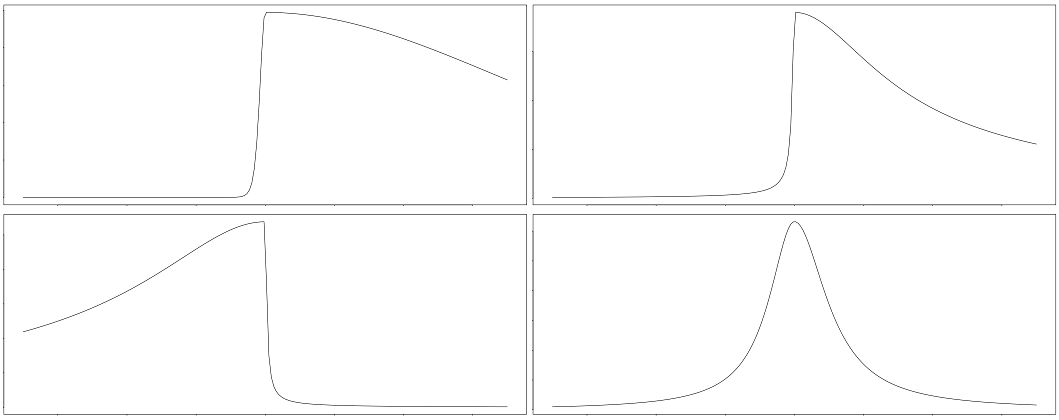

Figure 4.5: The pdf from a ST3 distribution for specific parameter values. Upper-left: ST3 (0,1,7,7), upper-right: ST3 (0,1,7,0.1), lower-left: ST3 (0,1,0.1,0.1) and lower-right: ST3 (0,1,1,1).

4.4.4 Selection of the fundamental drivers

As noted above, apart from analysing the temporal evolution of the mean and the variance, we are also very interested in introducing explanatory variables in the skewness (persistence) and the kurtosis process. Therefore, one of the advantages of the proposed approach is that the exogenous variables can be different for each distribution parameter. As the issue of variable selection for each distribution parameter is of combinatorial nature, stepwise procedures have been applied to select the meaningful explanatory variables. Despite the fact that stepwise methods do not guarantee to find the best model, it is very likely to reach it, or at least a model that contains the true one. These methods simply try to find the best models using a one-step at a time approach. These procedures can only be considered when trying to find the best predictors Miles and Shevlin (2001).

It is commonly accepted that the approach of moving from a more general model to a more restricted one is usually more convenient than the specific-to-general methodology (see for instance Gilbert (1986)). For this reason, only backward elimination methods have been used[10]. Note that with this approach it is expected to obtain a valid model containing only statistically significant regressors (see Brooks (2008)). The contrast for removal is based on the individual significance of each variable at 95% confidence.

In order to test whether a specific predictor parameter is different from zero, a standard Chi-squared test is used, comparing the deviance change when the parameter is set to zero with a $\chi^2_{0.05}$ critical value. Confidence intervals for individual parameters θ can be constructed in several ways, including, for example: i) the usual symmetric way using the standard errors obtained from the observed information matrix (this is, the inverse of the Hessian matrix), and ii) using bootstrap.

It should be emphasized that seasonal dummy variables were not included in the final specifications because this would significantly increase the number of regressors and it has been proved that they do not marginally improve the forecasting performance. Another remarkable fact is that it was not found useful to include smooth non-parametric functions for the distribution parameters with the aim of accounting for non-linearities in the relationship between the parameters of the distribution and the explanatory variables. Regarding the strategic and the speculative behavior of market agents, it is well known that it is very difficult to capture them by fundamental variables. It is believed that at least part of such information may be reflected in the constant terms.

The signs of the full model specification are summarized in Tables 4.1 and 4.2 for two representative cases. These results present in general a coherent interpretation, mostly consistent with previous expectations. As new insights, it is noteworthy to mention that a direct and logical relationship between the percentiles of the MEQ price and the moments of the parametric distributions has been observed across all the tested windows.

For instance, it is interesting that the kurtosis measure, which in risk measurement practice is usually taken to be an indicator of the fatness of the tails of the distribution, in the case of BCPE distribution is dependent on the 1st and 99th percentiles of the fundamental price. This is coherent and very similar to the interpretation of many quantile-based measures of peakedness given by some ref-

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[10] In this thesis, a method involving forward selection has not been considered to be suitable. The reason lies in that once a variable has entered in the model, it cannot be deleted even if its coefficient becomes statistically insignificant when adding other variables, which is possible.

erences Andrews et al. (1973), Kim and White (2004),Crow and Siddiqui (1967)[11]. Note also how the 99th percentile of the net demand, which is associated to demand shocks and the collapse of thermal demand, has a negative impact in the kurtosis of SEP2 distribution.

Regarding skewness, which conceptually describes which side of the distribution has a longer tail, it is also appealing to observe its direct relationship with the left tail of the MEQ distribution. This fact is related to a certain extent with some centile based measures of skewness proposed in the literature[12]. In addition, it seems that the expected wind production, which is a significant variable in the SEP2 distribution for increasing skewness, helps to justify changes in the skewness between different hours. Finally, it can be seen that the MEQ price and the net demand variables (both mean and median) are positive on price levels, indicating adaptive behavior.

	μ	σ	ν	τ
MEQ PRICE P50	+			
NET DEMAND P50	+			
MEQ PRICE P1		-	+	-
MEQ PRICE MEAN		+		
MEQ PRICE P30			-	
MEQ PRICE P99				+
INTERCEPT 1	-			
INTERCEPT 2		-		
INTERCEPT 3			+	
INTERCEPT 4				+

Table 4.1: Summary of significant signs from the proposed multifactor model-BCPE

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[11] For instance, in Crow and Siddiqui (1967) the coefficient of kurtosis is given by:

$$KR = \frac{q_4 - q_0}{q_3 - q_1} - 2.91 \tag{4.10}$$

where $q_0 = F^{-1}(0.025)$, $q_1 = F^{-1}(0.25)$, $q_3 = F^{-1}(0.75)$, $q_4 = F^{-1}(0.975)$ and $F(y) \equiv P_0\left[Y_t < y\right]$ is the unconditional CDF of Y_t.

[12] For example, the Hinkley's measure of skewness (Hinkley (1975)), which is independent of position and scale and is given by:

$$S = \frac{(q_\alpha + q_{1-\alpha})/2 - q_{0.5}}{(q_\alpha - q_{1-\alpha})/2} \tag{4.11}$$

where $q_\alpha = F^{-1}(\alpha)$ and $F(y) \equiv P_0\left[Y_t < y\right]$ is the unconditional CDF of Y_t. Note that a common value widely used for the α is 0.75.

	μ	σ	υ	τ
MEQ PRICE MEAN	+			
NET DEMAND MEAN	+			
EXPORTS MEAN	-			
MEQ PRICE P1		-	+	
MEQ PRICE MEAN		+		
WIND MEAN		+		
MEQ PRICE P30			+	
NET DEMAND P99				-
INTERCEPT 1	+			
INTERCEPT 2		+		
INTERCEPT 3			-	
INTERCEPT 4				+

Table 4.2: Summary of significant signs from the proposed multifactor model-SEP2

4.4.5 Performance Analysis

In this subsection, the accuracy of the best four hybrid models that have been built using each of the aforementioned distribution functions is empirically evaluated when making real ex ante forecasts. Therefore, using the proposed hybrid approaches, the probability density function has been forecasted for each hour of the validation period (1st January 2014 to 30th June 2014).

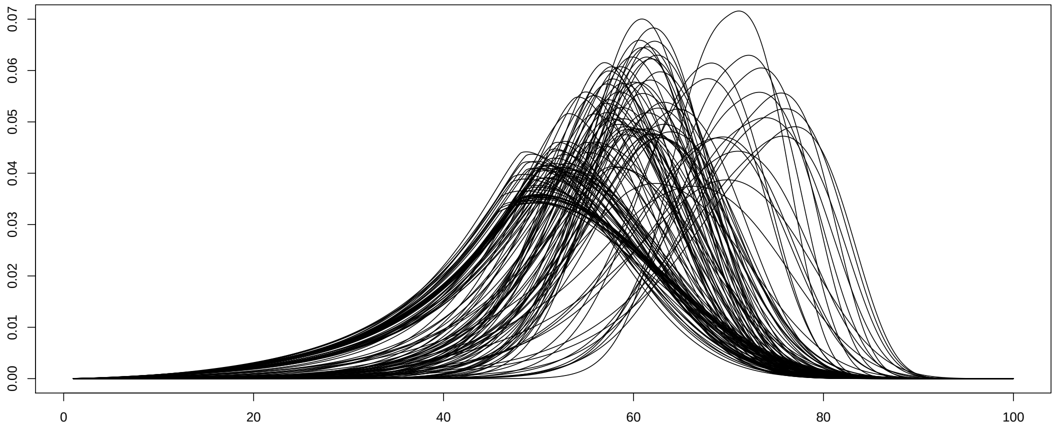

Figure 4.6: Predicted probability density function for a representative week-BCPE

Above all, the great flexibility of these models in capturing a wide diversity of shapes of the probability density function (pdf) is graphically displayed in Fig. 4.6 and Fig. 4.7. On the one hand, Fig. 4.6 presents the predicted pdf for the particular case of the BCPE model during one week of June 2014. On the other hand, Fig. 4.7 shows the pdf that has been forecasted for two days of February 2014 using a SEP3 distribution. As can be seen, these models are able to encompass, among other characteristics, the features of asymmetry and fat tails.

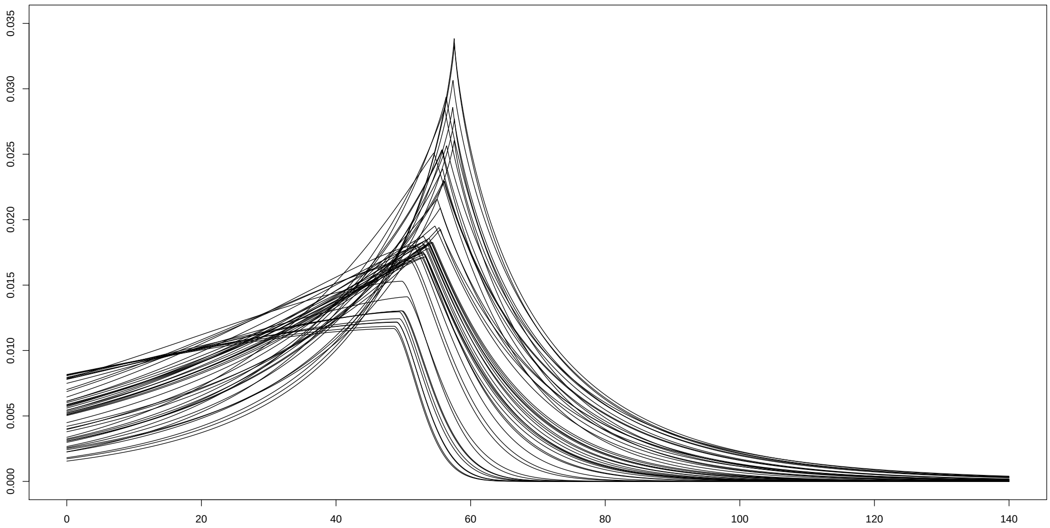

Figure 4.7: Predicted probability density function for two days of February 2014-SEP3

In addition to this, Fig. 4.8 shows the pdfs that have been estimated on an hourly basis with the SEP2 model. The color scale ranges from white (null probability) to blue (the highest predicted probability). It can be seen how the actual price always falls under the range of predicted prices. Moreover, the actual price for most days is near the mode value, and the model assigns a high probability of occurrence to it.

Table 4.3 shows the violation rate for seven target percentiles throughout the validation sample for each of the proposed family distributions. The proposed methodology is directly compared with the results of the other relevant benchmark models which deal with medium-term probabilistic forecasts of electricity prices on an hourly basis: the market equilibrium model MEQ presented in Chapter 2 and the hybrid approach which is based on quantile regression denoted as MEQ-QR (this corresponds to specification HM2 of Chapter 3). Remember that the functional form HM2 was the most robust and accurate among those presented in Chapter 3.

Overall, as can be appreciated on Table 4.3, the proposed methodology is not outperformed by the nonparametric MEQ-QR benchmark, which does not offer

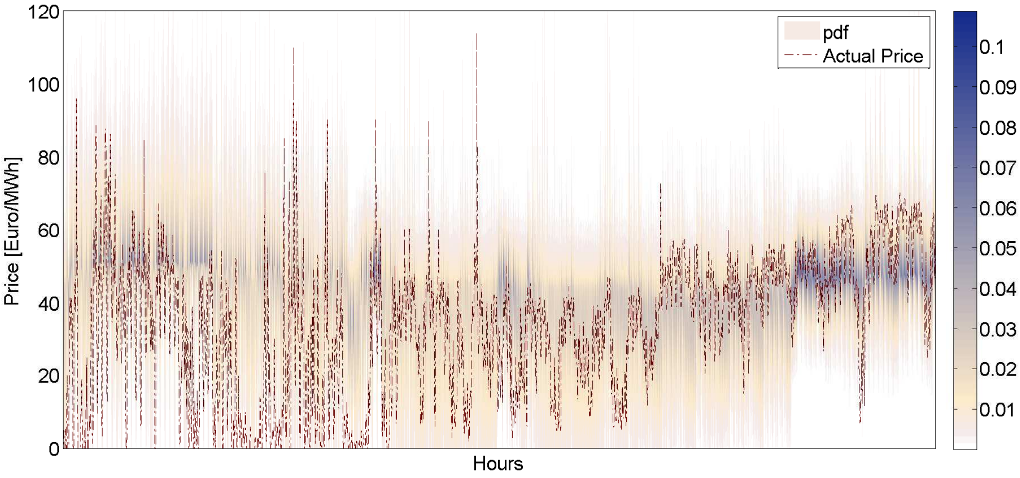

Figure 4.8: Comparison between the predicted pdf using SEP2 model and the actual price during the period from 1st January 2014 to 30 th June 2014

analytical solutions, but would have been expected to estimate the percentiles more precisely (Gianfreda et al. (2015)). Further, the extremes quantiles appear to be better calibrated with the proposed parametric approach, which is crucial in risk analysis. This is logical given the less reliable behaviour of quantile regression in the extreme tails because of the scarcity of data points. In general, it seems that the fitted parametric distributions show a quite acceptable degree of effectiveness in the centre and the tails of the distribution. However, there is still scope for further improvement as it seems that probabilistic forecasts are shifted to the right. That is the ultimate goal of next section.

[%]	P1	P5	P30	P50	P70	P95	P99
BCPE	93.43	81.95	55.50	34.63	19.86	0.18	0.02
SEP2	**96.20**	**90.27**	56.29	38.48	11.61	0.42	0.05
SEP3	91.82	85.84	56.64	39.00	22.38	3.34	0.11
ST3	91.93	84.71	58.17	**40.57**	22.77	**4.02**	**0.16**
MEQ	77.97	70.06	48.20	37.34	28.01	12.34	7.65
MEQ-QR	93.14	80.43	**61.81**	36.76	**30.92**	8.20	2.39

Table 4.3: Forecasting results for the target percentiles in terms of the violation rate

4.5 Three-stage approach

In this section, we investigate if the use of a three-stage approach may provide more accurate probabilistic predictions. Quantile regression is firstly used to undertake a first re-calibration of the price cdf given by the market equilibrium model as suggested in Chapter 3. As the fundamental model is itself well-specified to hourly variation, here it was not considered necessary to undertake the quantile regressions separately for each hour of the day. Therefore, the functional form HM2 that was presented in Chapter 3 has been used.

As a following step, we link up with the two-stage approach of Section 4.4 in such a way that the percentiles of the re-calibrated price are used as inputs in the distribution parameters, $\theta\left(\mu,\sigma,\upsilon,\tau\right)$ of the parametric multifactor models. Note that this hybridization scheme allows to relax the constraints imposed to avoid the crossing between the quantile curves as these restrictions are naturally forced in the parametric distributions. Consequently, this fact leads to a major flexibility and a decrease of the bias.

Table 4.4 reports a summary of the violation rate across the aforementioned representative quantiles. The analysis reveals that the proposed three-stage approach is able, in general, to provide more accurate probabilistic forecasts than the two-stage approaches presented in Section 4.4.5 and the specification MEQ-QR. As seen, meaningful improvements are made in the lowest tails and in the centre of the distribution. These findings demonstrate that the effectiveness of the hybrid technique is dramatically amplified if the previously re-calibrated quantiles of the market equilibrium model are used, as well as the other exogenous variables, in the parametric multifactor models.

It can be also observed that the MEQ-QR-BCPE aproach, which is clearly the poorest specification in terms of forecasting accuracy, presents some bias in the estimated cdf. When looking at Table 4.4, there appears to be no clear winner. It is appealing to observe that the three-stage approach with SEP2 distribution improves the forecasting accuracy in the centre of the distribution, but losses precission at the tails in comparison to the two-stage approach. It is also of interest to note that a distribution such as ST3, that is not able to model platykurtosis in certain hours, gains accuracy in the upper tail of the distribution. It may be concluded that there is a dominant hybrid technique for each one of the parts of the distribution. This fact naturally leads to the idea of combining probabilistic forecasts, which is addressed in next section.

[%]	P1	P5	P30	P50	P70	P95	P99
MEQ-QR-BCPE	94.54	82.19	56.22	32.18	21.93	4.19	0.53
MEQ-QR-SEP2	95.30	93.04	**64.85**	**47.06**	28.91	6.84	2.57
MEQ-QR-ST3	95.19	93.29	77.19	58.45	31.65	**4.72**	**0.91**
MEQ-QR-SEP3	**95.59**	**94.74**	79.23	58.61	32.45	3.06	0.27
BCPE	93.43	81.95	55.50	34.63	19.86	0.18	0.02
SEP2	96.20	90.27	56.29	38.48	11.61	0.42	0.05
SEP3	91.82	85.84	56.64	39.00	22.38	3.34	0.11
ST3	91.93	84.71	58.17	40.57	22.77	4.02	0.16
MEQ-QR	93.14	80.43	61.81	36.76	**30.92**	8.20	2.39

Table 4.4: Forecasting results for the target percentiles in terms of the violation rate

4.6 Three-stage approach with averaging schemes

It is obvious that an important question that must be answered in the presence of multiple forecasts of the same variable is whether it is suitable to try to identify the single best forecasting model or rather attempt to combine their forecasts. It is well known in the literature for both point (Timmermann (2006)) and density (Aastveit et al. (2014),Hall and Mitchell (2007),Wallis (1986)) forecasting that combinations of forecasts may offer diversification gains and can provide insurance against possible model misspecification, data sets that are not enough informative and smooth structural changes (see Hjort and Claeskens (2003)). In addition, in situations in which it is not possible to identify a single dominant forecasting method, it makes sense to combine forecasts because as stated in Makridakis and Winkler (1983) the risk diminishes rapidly when more methods are considered and their forecasts are averaged. Hence, and for all these reasons, it could be attractive to combine the information contained in the different forecasting models proposed along this chapter.

Although the idea of combining forecasts in itself is not new, it has been barely touched upon in the context of electricity spot prices (see the discussion of Weron (2014)). One of the few exceptions is the work of Nowotarski and Weron (2015), in which a novel method has been recently introduced for computing prediction intervals (PI) and it was dubbed as Quantile Regression Averaging (QRA). The method involves applying quantile regression to a pool of point forecasts of individual (i.e., not combined) forecasting models. Another extension of this approch is presented in Maciejowska et al. (2015), in which quantile regression is also applied, but to point forecasts of the principal components (i.e., the common factors) rather than to point forecasts of the individual models.

In the light of the results discussed before there is no apparent reason to restrict ourselves to one method or another and a density combination approach seems a natural avenue to follow. Therefore, we are now interested in how well the approach of combining forecasts performs in the context of probabilistic forecasting of electricity prices. For this purpose, different combination schemes for the predictions provided by the proposed individual models of the three-stage approach (MEQ-QR-SEP2, MEQ-QR-SEP3 and MEQ-QR-ST3) have been investigated. Note that including MEQ-QR-BCPE, which is the worst model, in the shrinkage methods leads to poor forecasting performance. This is consistent, as expected, with Zou and Yang (2004), who point out that it is not a good idea to blindly combine all possible models available.

Unlike in Nowotarski and Weron (2015) and Maciejowska et al. (2015), we propose to use directly the probabilistic forecasts instead of the point predictions. As stated in Timmermann (2006), combinations of probability density or distribution forecasts impose new requirements beyond those that have been highlighted for combinations of point forecasts. The fundamental requirement is that the combination must be convex with weights restricted to the zero-one interval so that the probability forecast never becomes negative and always sums to one. Consistently with this prerequisite, the following weights have been employed in order to generate a single probabilistic forecast: 1) equal weights; 2) fit-based weights, that is, weights which are inversely proportional to the least absolute deviation (LAD). Note that we have not considered an approach based on common factors since the number of models to combine is not very large.

It should be emphasized that the forecast evaluations of these schemes are also based on comparisons of ex ante forecasts with increasing windows of one month. Therefore, adaptive weights are reestimated at every time step. In this way, note how the initial window is chosen to cover the period from 1st May 2013 to 30th November 2013 for making probabilistic forecasts for January 2014.

4.6.1 Equally weighted combinations

The most natural approach to forecast averaging is the use of the arithmetic mean of all forecasts produced by the different models. This scheme, which is denoted as EWC, is highly robust and is widely used in forecast combination literature (see e.g. Raviv et al. (2013), Genre et al. (2013), Stock and Watson (2004)). Note that this approach does not take into account the historical performance of the individual forecasts. In Eq. 4.12 the EWC combination scheme is represented, where W is the number of methods used (three in the case study here presented: MEQ-QR-SEP2, MEQ-QR-SEP3 and MEQ-QR-ST3), $\hat{y}^{\alpha}_{h,w}$ is the forecast of the price in hour h for the percentile α from parametric method w (i.e. $\hat{y}^{\alpha}_{h,w} = F^{-1}_{h,w}(\alpha)$, in which $F_{h,w}$ is the unconditional CDF) and $\hat{y}^{\alpha}_{h,EWC}$ is the final weighted prediction for the percentile α and hour h.

$$\hat{y}^{\alpha}_{h,EWC} = \frac{1}{W}\sum_{w=1}^{W}\hat{y}^{\alpha}_{h,w} \tag{4.12}$$

4.6.2 Least absolute deviation averaging

Since the best combination of candidates is expected to depend on percentiles, the objective here is the formation of composite forecasts for an optimal performance at each given percentile. Therefore, another pragmatic method for combining distributions is to use weights based on the LAD (remember the graphical representation of Fig. 3.2). The methodology applied, which hereinafter will be referred to as quantile regression averaging (QRA), is explained below:

1) As in the previous combination approach, for each proposed parametric distribution function w (MEQ-QR-SEP2, MEQ-QR-SEP3 and MEQ-QR-ST3), the corresponding quantile functions $\hat{y}^{\alpha}_{h,w}$ are derived for the usual target percentiles, so that $\hat{y}^{\alpha}_{h,w} = F^{-1}_{h,w}(\alpha)$. Note how as the distribution functions vary with time, i.e. as the information set of explanatory variables evolves, so will the quantile functions.

2) In a second step, the predicted quantile functions $\hat{y}^{\alpha}_{h,w}$ are combined for each percentile α using the asymmetric absolute loss function to yield the LAD regression. The LAD regression may be viewed as a particular case of quantile regression which allows to develop explicit models for specific quantiles of the distribution of the dependent variable Koenker (2005). Intuitively, this methodology

assigns specific weights for each percentile depending on the inverse of the absolute deviation error, so that larger weights are given to models that show smaller deviation error during the in-sample data set. We should recall that the weights are sequentially updated after each additional moving window. As constructed quantile functions are sample unbiased, then we might expect that the weights sum to unity and there is strong intuitive appeal for omitting the constant (see Taylor and Bunn (1998)).

4.6.3 Mixture of distributions as a benchmark

As a natural benchmark to the combination schemes that have been presented in the context of probabilistic forecasting arises the concept of finite mixture models (Titterington et al. (1985)). For this reason, we apply the methodology of finite mixture models in the three-stage hybrid framework. The main idea of mixture models is that the probability density function of the final prediction is a linear combination of other probability density functions $f_w(y)$, which can be represented as follows:

$$f_Y(y) = \sum_{w=1}^{W} \phi_w f_w(y) \tag{4.13}$$

where the mixing probability of each component w (for $w = 1, ..., W$) is ϕ_w. In the same way as the weights of the rest of combination schemes previously discussed, the mixing probability of each of the linear terms, which satisfies that $0 \leq \phi_w \leq 1$, are constrained to sum to one.

One of the advantages of mixture models is that they allow for more exotic and complex shapes than other traditional density functions. More generally, by combining the properties of the individual probability density functions, mixture models are capable of approximating any arbitrary distribution.

It is important to highlight that we assume that the k distribution parameters θ_{kh} of each pdf $f_w(y)$ (component of the mixture) depend on hourly explanatory variables X_{kh}. Moreover, these distribution parameters are different for each component of the mixture. This makes sense since there are several plausible reasons why we might expect different regimes in the functional specification of power price formation (see Chen and Bunn (2010) and Chen and Bunn (2014))[13].

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[13] As discussed in these references, there are reasonable propositions why power prices can react nonlinearly, such as: i) the existence of market power, ii) substantial changes in fuel prices or

This is also coherent with the focuses of González and Muñoz (2005) and Cruz (2013), which propound that the dynamic of the prices can evolve through different hidden regimes that are mainly determined by the interaction among resources, demand, and participants' strategies.

The likelihood function given S observations from the mixture model presented in Eq. 4.13 can be expressed as follows:

$$L = \prod_{i=1}^{S} f_{Y_i}(y_i) = \prod_{i=1}^{S} \left[\sum_{w=1}^{W} \phi_w f_w(y_i) \right] \tag{4.14}$$

and the the log likelihood function is given by:

$$l = \sum_{i=1}^{S} log \left[\sum_{w=1}^{W} \phi_w f_w(y_i) \right] \tag{4.15}$$

As seen, we want to maximize Eq. 4.15 with respect to the distribution parameters and the mixing probability ϕ_w. In order to achieve this objective, finite mixture distributions are fitted using the Expectation-Maximization (EM) algorithm, which alternates between the so-called E-step and the M-step until convergence (for further details see Dempster et al. (1977) and McLachlan and Krishnan (1997)).

As normal mixture distributions are undoubtedly the most widely used mixture models, we propose to use a mixture of normals in which the distributional shape of the mixture change along with exogenous variables. Among the diversity of forecasting approaches based on this concept, the works of González and Muñoz (2005) and Cruz (2013), which are focused on the short term, can be highlighted. On the one hand, González and Muñoz (2005) proposes an Input–Output Hidden Markov Model (IOHMM) in which each market state is characterized by a particular normal distribution. Only the mean of this distribution is time-variant and dependent on the input variables. The switches among these states have been also explained according to Markovian transition probabilities that are conditioned to the evolution of several explanatory variables. On the other hand, Cruz (2013) proposes a similar approach to the one presented in González and Muñoz (2005). The main differences are that in this case it is not assumed a Markovian

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renewable energy sources production, and iii) interaction between demand and discontinuous, steeply increasing supply functions.

process and that the dynamic of the price conditioned to each regime is modelled in the state space that takes into account both the mean and the variance of the process.

As further discussed in Dowd (2005) and McLachlan and Peel (2000), normal mixture approaches have a number of merits. Apart from being more intuitive from a conceptual point of view, they are flexible enough to approximate many different kinds of conditional distributions and can capture, for example, conditional skewness and kurtosis. For instance, note how in Fig. 4.9 with two Gaussian densities as components, it is possible to model a skewed density, a bimodal density or a heavy tail density.

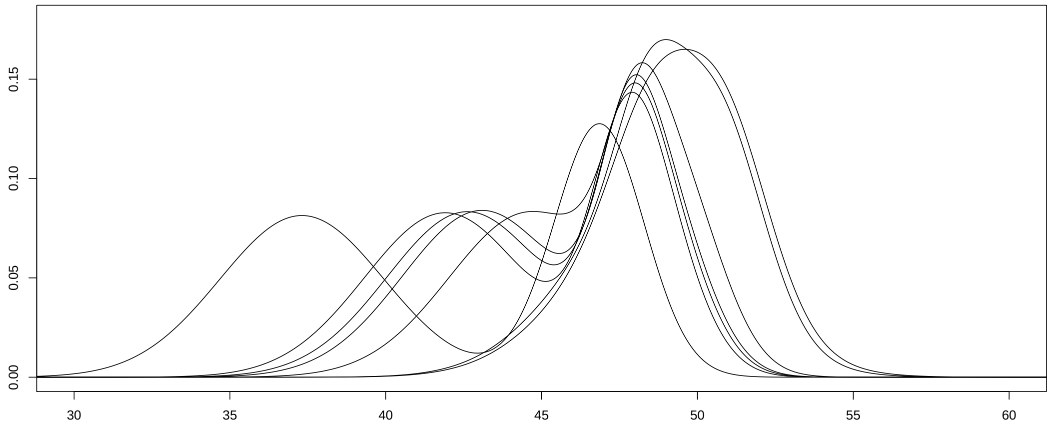

Figure 4.9: Illustrative example of a mixture of two Gaussian densities for the prediction of 6 hours

We have focused on the cases in which the number of components are assumed to be known. More specifically, and for the sake of parsimony, the number of components has been assumed to be optimal with a value of three. We have also tried with more components, but this resulted in a loss of generalization capabilities and in numerical instabilities in the estimation process, due to the very low probability assigned to the fourth component. For illustration purposes, Fig. 4.10 shows the predicted conditional density for 13 representative hours of June 2014 with a mixture of three Gaussian densities. It can be checked how this approach provides a very flexible model which can capture a richer class of density models. Moreover, this mixture model can incorporate for instance a regime with extremely low prices by having a distribution with very low probability and with a low mean.

We should note that specific variables for the mean and the variance processes have been used. On the one hand, the chosen variables for the mean are the ex-

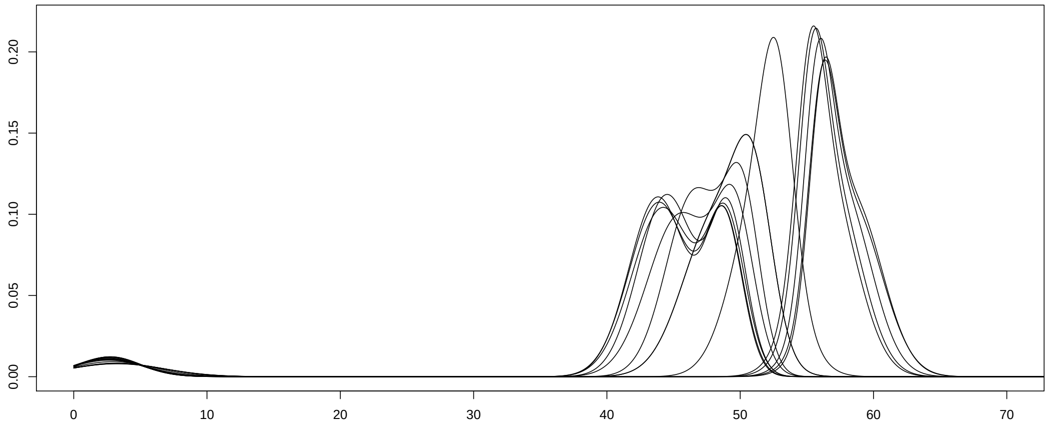

Figure 4.10: Illustrative example of a mixture of three Gaussian densities for 13 hours of June 2014

pected net demand and the re-calibrated median price that has been estimated from quantile regression in the two-stage approach. On the other hand, the variance depends on the expected wind production and the 30th percentile of the re-calibrated distribution function of the price. In addition to this, further analyses were carried out with some specifications in which the mixing probabilities were not set constants but depend on explanatory variables such as the net demand. The problem is that this approach resulted in worst generalization capabilities.

4.6.4 Performance Analysis

Table 4.5 report the accuracy metric for the amalgamation of the approaches considered, as well as the mixture of the three Gaussians used as a benchmark (MEQ-QR-GM3) and the individual three-stage hybrid methods. We should note that here the best model has been defined in terms of the violation rate for its easy interpretability, but it was also checked that the same conclusions could be drawn by other measures such as the PLF and Winkler score.

As can be appreciated on Table 4.5, the proposed hybrid approaches based on four-parameter distributions clearly outperform the benchmark model. Unfortunately, there is not a clear winner in terms of forecasting performance. However, it seems that model averaging, which can capture different aspects of market conditions (particulary when the different approaches contain distint information), can be viewed as a model diversification strategy that can improve forecasting robustness in the same manner that asset diversification improves portfolio performance.

It can be seen that the simple combination EWC could outperform more sophisticated rules that rely on estimating optimal weights such as QRA, which is coherent with the discussion of Bordignon et al. (2013) and the findings of Bunn (1985)[14] and Timmermann (2006). A possible explanation for this finding is the strong similarity of the probabilistic predictions provided by the individual models. However, it seems that an optimal combination scheme such as QRA can accommodate better the central part of the distribution.

Overall, the simulation over the real case study that has been carried out with the proposed models has given very satisfactory and promising results across the whole distribution function. The only exception correspond to the 1st percentile of the distribution. It strongly encourages to the proposal of models specifically focus on the lower tail of the distribution. This issue is addressed in a preliminar way in the JCR article that is presented in Appendix A.

[%]	P1	P5	P30	P50	P70	P95	P99
QRA	95.14	90.46	74.52	**52.38**	31.80	3.53	1.97
EWC	95.36	93.61	**72.94**	54.57	**30.73**	5.80	1.59
MEQ-QR-GM3	85.43	82.79	48.82	42.00	22.22	14.56	11.64
MEQ-QR-SEP2	95.30	93.04	64.85	47.06	28.91	6.84	2.57
MEQ-QR-ST3	95.19	93.29	77.19	58.45	31.65	**4.72**	**0.91**
MEQ-QR-SEP3	**95.59**	**94.74**	79.23	58.61	32.45	3.06	0.27

Table 4.5: Forecasting results for the target percentiles in terms of the violation rate

4.7 Conclusions

This chapter has presented a novel hybrid framework for medium-term probabilistic price forecasting on an hourly basis, which constitutes a parametric alternative to the non-parametric recalibration of the fundamental model that was presented in Chapter 3. The parametric models here developed extend the existing literature on electricity price forecasting by considering more general distributions that allow to incorporate not only a time-varying conditional mean

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[14] In this reference it is shown that simple combination schemes perform better than more sophisticated rules relying on estimating optimal weights that depend on the full variance-covariance matrix of forecast errors.

and variance, but also conditional skewness and kurtosis. The methodology that has been followed allows that the location, scale, skewness and kurtosis of the conditional distribution change over time as a function of the percentiles of multiple exogenous variables. As a result of the research, it can be seen that these ambidextrous distributions can provide, for instance, an alternative way to value derivative instruments.

In the empirical application here developed, it is found that linear multifactor representations of the first four moments of several density functions produce significantly better probabilistic forecasts than the competing models that have been proprosed in the previous chapters. The benefits of this hybrid parametric approach are significantly amplified when a non-parametric re-calibration of the price distribution function that has been previously estimated from the market equilibrium model (as in Chapter 2) is carried out.

Another contribution of this chapter is the proposal of practical ways of combining forecasts from several competing density functions, which constitutes an area barely touched upon in the context of electricity price forecasting literature. The analysis reveals that the proposed amalgamation schemes provide more stable and less risky probabilistic forecasts than trying to identify the best model. From the perspective of potential practitioners, optimal adaptive weights based on the least absolute deviation averaging can deliver empirical gains in the central part of the distribution. However, composite forecasts as simple equally weighted averages, which perform better in the tails, seem to be more robust in those areas where data scarcity problems may arise.

As a concluding remark, it should be emphasized that the methodology finally proposed is able to reliably provide accurate forecasts of the full density function, incluing the tails. Certain scope of improvement exists in the prediction of extremely low prices. Thus, further research could be conducted to explore better ways of capturing the lower tail of the price density function. This naturally links with the use of specific techniques based on the concept of threshold forecasting as an interesting field for future developments. In this regard, Appendix A presents a preliminary approach to simultaneously accomplish punctual and probabilistic hourly predictions about the appearance of extremely low electricity prices in a medium-term scope. This methodology has already been published in an international journal with JCR impact factor.

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Chapter 5

Conclusions, contributions and future research

Contents

This last Chapter is dedicated to present the main conclusions that arise from the research that has been conducted in this thesis. It includes a brief summary of the analysis, as well as the developments and findings that constitute the core of this work. It also specifies the original contributions that have resulted while pursuing the general objective of this thesis. Finally, several future lines of research stemming from the developments of this thesis are identified.

5.1 Summary and conclusions

The new competitive framework of electricity markets and the great deployment of intermittent generation technologies have led to the emergence of more complex dynamics in the electricity price series. In this context, in which market stakeholders face new challenges, trying to predict the electricity price in the medium term is crucial for decisions such as fuel purchasing, operational scheduling, hedging and trading. However, it is not an easy task since electricity price is far more volatile than other commodities.

As discussed throughout this thesis, very little research has been previously conducted in the field of medium-term forecasting in contrast to short-term price forecasting. In addition, the existing fundamental models in the literature of medium-term prediction are not able to adequately capture the dynamics or the commonly called stylized facts of electricity prices, especially with high-frequency data (i.e. hourly prices). Moreover, the vast majority of forecasting literature has traditionally focused on making point forecasts. In this context, a novel methodology to deal with medium-term probabilistic forecasting of electricity prices has been presented in this thesis.

Initially, in Chapter 2 a novel methodology based on a fundamental model which incorporates Monte Carlo simulation combined with spatial interpolation techniques and a new definition of load levels has been presented. In a first step, this pure fundamental model, which allows us to predict the electricity price on an hourly basis with reasonable execution times, has been compared with the traditional fundamental approach that makes point forecasts over averages of group of hours. On account of this analysis, it has been demonstrated that this new proposed approach has a significantly enhanced ability to capture the stylized facts of electricity prices with respect to the traditional fundamental approach.

In addition, the results have empirically demonstrated a slight improvement in point forecasting capabilities when the proposed methodology is used.

The fundamental methodology has been extended with the aim of obtaining probabilistic forecasts. In this way, the probability density function and the cumulative distribution function have been estimated entirely on an hourly basis. As has been seen, the proposed methodology is able to outperform some well-recognized techniques traditionally used in the short term for modeling conditional mean and variance. More specifically, Transfer Function models with ARIMA noise have been used for the conditional mean, while four different GARCH specifications have been used for the conditional variance. The analysis of the obtained results confirms that pure statistical techniques tend to degrade when the forecasting horizon is extended, especially in the event of recurrent structural and regulatory changes in the market. This is not the case of a fundamental model that represents the main technical and economic characteristics of the electricity market. However, even such a detailed model has not been able to capture the higher moments of the distribution. Basically, this is because price formation usually tend to depart from fundamentals, and instead, behavioral aspects such as speculative or strategic conduct become more relevant. In addition, the grouping of hours into system states results in a less volatile price.

As a consequence of these observed limitations, the research has been extended in Chapter 3 to hybrid forecasting methods, based upon different functional forms of quantile regression techniques. By using four hybrid schemes, the distribution functions predicted by the fundamental model have been non-parametrically re-calibrated and re-shaped. It has been observed that these innovative hybrid approaches outperform the pure fundamental model when making real ex ante probabilistic forecasts. Therefore, they have allowed to successfully mitigate the effect of any bias in the estimation of the conjectured-price response θ_i of each generator or in the construction of the risk factors scenarios, as well as other modelling deficiencies of the fundamental approach. In addition, these hybrid methods also produce superior probabilistic forecasts in comparison with other fully parametric location scale econometric methods such as ARMAX-CAViaR or the ARMAX-skew Student-t GARCH.

Focusing on the link between fundamental and statistical techniques, it has been proved that probabilistic outputs computed from the fundamental model, such as the price, the demand, or the wind production, add value over expected values when has been used as exogenous variables in the quantile regression models. Furthermore, it seems that dynamic models, in which separate econometric models are calibrated for each hour of the day, do not offer more robust predictions. In view of this finding, it is reasonable to presume that the fundamental model is able to properly capture the intraday dynamics of prices. In Chapter 3 the sensitivity of the methodology when different sizes of moving windows are used for calibrating the model parameters has also been explored. It was found that, in order to have full seasonal spread and sufficient informative data, rolling windows of at least six months, but less than one year, are preferable.

Although the hybrid methodology that has been presented in Chapter 3 shows that the probabilistic forecasting certainly benefits of combining fundamental and technical analysis, it seems that some improvements could still be done. This is primarily due to the fact that the constraints imposed to avoid crossing between the different percentiles and the observed data scarcity problems in the tails lead to biased quantile predictions. As these issues should be overcome, further research has been conducted in Chapter 4 to propose a hybrid framework that incorporates flexible four-parameter distributions. As it has been empirically demonstrated, all the density functions that have been used can capture a wide range of shapes in such a way that the four distribution parameters can change in each hour with the value of several exogenous variables. On the basis of the results, it is clear that this parametric hybrid framework provides better probabilistic forecasts than the rest of techniques that had been already proposed. Moreover, significant improvements have been observed when a three-stage hybrid framework is used, i.e. when the parametric price distribution function depends on the non-parametric re-calibration of Chapter 3.

It has been also examined the suitability of combining the probabilistic forecasts of the competing density functions instead of trying to identify the best hybrid method. On account of this analysis, there is not a clear winner. Moreover, there is not a clear consensus regarding the adequacy of using adaptive weights that rely on simple combination schemes or more sophisticated rules that are based on estimating optimal weights for each percentile of the distribution. However, from the perspective of potential practitioners, our suggestion is that, in order to have less risky and more robust probabilistic forecasts, it is better to choose one of the amalgamation schemes, preferably a simple average.

Overall, this three-stage hybrid methodology, which produces parametric predictions by also encompassing a non-parametric re-calibration, has empirically demonstrated its effectiveness to provide very satisfactory and reliable forecasts in the medium term across the entire distribution function. This is particularly apparent relative to the rest of electricity price forecasting techniques that have been presented throughout the thesis. Moreover, the fact that the probabilistic predictions are parametric makes the proposed methodology even more valuable for market agents. It seems that in order to enhance even more the predictive capabilities, this work could be complemented with other techniques that allow us to better capture the lower tail of the price density function.

For this reason, Appendix A presented a preliminary approach that is very useful to simultaneously obtain punctual and probabilistic hourly predictions about the appearance of extremely low prices. More especifically, the prediction accuracy of a novel hybrid approach that integrates fundamental and behavioral information has been carefully compared with logistic regressions for rare events, decision trees, multilayer perceptrons, traditional Markov regime switching models and different naive methods. Overall, all of the proposed models present reasonable errors taking into account the complex nature of the phenomenon and substantially outperform naive techniques in both the in- and out-of-sample datasets. Encouraging results have been obtained from real ex ante forecasts of the distribution function of the exogenous variables used to predict the phenomenon. The results reveal that the integration of a market equilibrium model and logistic regression for rare events in a hybrid approach provides a significant improvement in the prediction accuracy in comparison to the individual models.

In the light of the results that have been discussed, we may conclude that the probabilistic forecasting performance in the medium term can be improved by incorporating fundamental and statistical information within a hybrid complementary framework. This efficient use of the various sources of information allows us to internalize projected structural changes (such as major new investments in generation or transmission infrastructure, shifts in technology, changes in the market rules or industry structure) and psychological or behavioral aspects (such as the strategic and speculation-based behavior of traders that is difficult to catch by fundamental variables) in a natural way. It is important to highlight that a complete methodology that allows to support decision-making process in the medium term and provide the practitioners with accurate forecasts would require a constant monitoring and should be re-estimated month by month. In short, models and results arising from this thesis may be useful for a wide number of market agents (whether these are generators, retailers or large consumers) or even to regulatory authorities and policymakers. This way, it would be possible to analyze and interpret the evolution of the electricity sector and reactions (fair competition), as well as provide regulatory and policy recommendations (regarding e.g. market design).

5.2 Original contributions

The development of this thesis has yielded a number of original contributions that can be categorized as follows:

1. The reasoning process followed throughout the development of this thesis can be seen as a contribution itself. It starts by clearly defining the context and the motivation of this thesis. In particular, an analysis of the relevance of probabilistic medium-term forecasting for market agents has been carefully carried out. Also, a **discussion of the main research gaps** that this thesis fulfills has been presented in parallel with **different literature reviews**. Additionally, the logic behind the need for a hybrid methodology adopted in this thesis has also been theoretically justified from the perspective of overcoming the particular challenges of medium-term lead times.

2. The development of a novel methodology that allows to simultaneously perform **punctual and probabilistic hourly predictions by using a fundamental market equilibrium model.** More specifically, the methodology is based on a nested combination of a market equilibrium model with Monte Carlo simulation, integrated with spatial interpolation techniques (as proposed in Dueñas and Reneses (2011)) and a new definition of load levels (following Wogrin et al. (2014)). It has demonstrated its effectiveness in capturing the stylized facts of electricity prices and in making reliable probabilistic hourly predictions with a meaningful reduction of the computational burden.

3. An original methodology for a **medium-term wind production scenario generator** has been presented. In order to capture the enormous variability of wind production, the generator allows to simulate different hourly profiles per each percentile of the monthly wind production distribution.

The percentiles of the wind probability distribution are independently estimated by means of a dynamic model that incorporates seasonal deterministic terms and an Autoregressive Moving Average component. Coherent hourly transitions between consecutive months has been achieved with spline interpolation techniques.

4. Another relevant contribution of this thesis is that several schemes have been proposed in a nonparametric probabilistic framework to conduct the **hybridization process by combining a market equilibrium model and techniques based on quantile regression.** The lack of a clear guideline to carry out a proper prediction of electricity price distribution has led to the proposal of four different functional forms of quantile regressions. They include a dynamic approach and a method based on extreme value theory. It is important to highlight that none of these techniques had been previously used in a hybrid framework. In addition, the consideration that different regions of the distributions functions of the explanatory variables can be used as drivers for specific regions of the electricity price distribution is totally innovative.

5. As discussed, most of the practitioners assume normal distributions and only focus on the first two moments of the electricity price distribution. This gap has been addressed by extending the **hybrid framework with flexible density functions of four-parameters** that allows to incorporate the information contained in the higher-order moments. This approach brings the opportunity to accommodate the wide range of shapes that hourly electricity prices can take in such a way that the latent estimates of the moments change parametrically to exogenous variables. In addition, this thesis has provided a satisfactory guideline for the choice of those families of probability distributions that better explain the electricity prices, as well as those variables that are the most relevant for explaining the distribution parameters.

6. As already remarked, the idea of **combining forecasts from several competing models** in itself is not novel, but it had been barely touched upon **in the context of electricity spot prices**, especially in a probabilistic framework. This issue has been further addressed in this thesis. In addition, the performance of combination schemes within a three-stage hybrid framework has also been compared with a mixture of normal distributions. The novelty here is that this kind of mixture, in which the distributional shape

of the mixture changes along with explanatory variables, has been applied in a hybrid way.

7. Both the pure fundamental methodology and the several hybrid approaches that have been proposed in this thesis have been thoroughly compared with many of the most widely recognized econometric techniques that are traditionally used in short-term time horizons. The extension of these statistical models to medium-term lead times when making real ex ante probabilistic forecasts in a real-size electricity system can be also recognized as a methodological contribution.

8. Specific techniques to make **point and probabilistic forecasts of extremely low prices** for a mid-term horizon on an hourly basis have been developed. More specifically, the accuracy of a novel hybrid approach that integrates fundamental and behavioral information, logistic regressions for rare events, decision trees and multilayer perceptrons has been compared to the results obtained by means of a traditional Markov regime switching model and different naive methods. In addition, as the definition of an extremely low price is in many occasions a subjective choice, this thesis provides an alternative approach to those used in the literature in order to categorize spikes. This way, extremely low prices are classied by choosing a threshold from the characteristic modes of the electricy price distribution function.

5.3 Future research

The developments that have been carried out in this thesis lead to a number of future lines of research, the exploration of which is likely to yield interesting results. This section tries to summarize some of them and justify their relevance.

5.3.1 Modeling challenges

Some of the research lines stemming from this thesis that would constitute interesting modeling developments are summarized below:

1. A straightforward extension of the work presented in this thesis would be

based on implementing other methods for density combinations. This could be done by letting the combination weights on the component densities follow other schemes based on the maximization of scores such as the GAIC or the logarithmic scoring rule of the competing forecasting models. Moreover, a more appealing research line would be the assessment of the extension of the three-stage hybrid framework by using finite mixtures of four-parameter distributions in which distribution parameters and the mixing probability would change with different predictors.

2. The potential benefits of nonlinear models are under-researched in the field of medium-term probabilistic forecasting and the value of regime-switching models for forecasting the evolutionary nature of electricity prices is not clear Bunn et al. (2015). Therefore, it remains as an open question whether the quantile regression techniques used within the hybrid framework could be extended in such a way that the conditional percentiles were subject to regime switching.

3. As shown in the thesis, different regions of the probability distributions of the explanatory variables are efficient predictors for specific regions of the electricity price distribution. In this thesis, the regions of the distribution have been characterized by specific percentiles. Since functional data analysis is becoming more relevant in the field of forecasting, it would be of great interest from a methodological point of view to take advantage of this approach to estimate the hourly probability distribution functions of the electricity price. This could be done by maximizing the likelihood of the data and directly modeling price dependency with the whole probability distribution functions of the exogenous variables, which should be treated as functional times series. Furthermore, in the context of the hybrid framework that has been proposed in this thesis, this aproach could be used as an alternative to the quantile regression methods.

4. As the effects of renewables or fuel price volatility are likely to become stronger in the future, further work is needed to explore better ways for capturing the tails of the density function. A first research line may include the extension of the methodology presented in this thesis to deal with extremely low prices to predict extremely high priced hours. Additional quantitative methods could be explored to complement the prediction given by the four-parameter distributions. For intance, the existing literature on extreme value theory could be extended with the aim of pro-

viding asymptotic models which could properly capture the tails of a distribution. Moreover, encouraging results could be obtained by considering a finite mixture of a four-parameter distribution and one distribution focus on the tail behavior.

5. In this thesis, as the main objective is to provide reliable probabilistic price predictions, we have assumed an unidirectional hybrid framework. That is to say, the outputs of the fundamental model are inputs for different statistical techniques. However, it could be also interesting to feedback the probabilistic prediction of the price in such a way that the fundamental model allows to compute the generation schedule or to take other operation decisions.

5.3.2 Further applications

This section briefly outlines possible potential applications that, taking the work presented in this thesis as a reference, could be valuable for different market stakeholders:

1. The hybrid methodology here developed has been applied to the Spanish market. However, we could also apply the proposed methodology to other markets worldwide. To this end, it would be necessary to adapt the fundamental model to other market designs. In order to avoid that the resolution times increase considerably for large or integrated power systems (this could be, for instance, the case of the current project of creating a single competitive electricity market at a European level), it could be interesting to evaluate the performance of the hybrid method when the fundamental model is simplified and presents a lower level of detail. Several issues could be explored for this purpose. For instance, generation technologies can be used instead of individual generation units.

2. In spite of the special characteristics of electricity prices, the proposed hybrid framework resulting from this thesis, as well as the way of estimating the distribution functions flexibly, could be extended to other commodities and financial products.

3. As already discussed, modeling the conditional skewness of spot prices is not only important to properly forecast the distribution function of spot

prices, but also for derivative valuation and for quantifying the risk of a given position (see Escribano et al. (2002) and Bessembinder and Lemmon (2002)). Furthermore, future markets and forward trading are expected to increase their liquidity due to the hedging neccessity of market agents against power prices that are getting more and more volatile. Within this new context in which there is aversion to extreme outcomes and an increasing interest in fat tails, it could be also interesting to empirically demonstrate the importance of the higher order moments of the spot price distributions presented in this thesis for the risk assessment of market stakeholders. For instance, it might be appealing to investigate the influence of the kurtosis of spot prices on the forward premium.

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Appendices

Appendix A

Article - Medium-Term Probabilistic Forecasting of Extremely Low Prices in Electricity Markets: Application to the Spanish Case

This appendix brings new light on the controversial issue of extremely low prices appearance in the current global context of growing complexity of electricity markets. Other articles have been published in the literature dealing with the prediction of electricity prices in general, and spikes in particular, but no other references have been found in the literature making a quantitative model to simultaneously accomplish punctual and probabilistic hourly predictions about the appearance of extremely low electricity prices in a medium-term scope. This novel methodology for making real ex ante forecasts consists of a nested compounding of different forecasting techniques which incorporate Monte Carlo simulation, combined with spatial interpolation techniques. The procedure, which is based on the statistical identification of the process key drivers, compares the prediction accuracy of a novel hybrid approach that integrates fundamental and behavioral information, logistic regressions for rare events, decision trees and multilayer perceptrons with a traditional Markov regime switching model and different naive methods.

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Article

Medium-Term Probabilistic Forecasting of Extremely Low Prices in Electricity Markets: Application to the Spanish Case

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Abstract: One of the most relevant challenges that have arisen in electricity markets during the last few years is the emergence of extremely low prices. Trying to predict these events is crucial for market agents in a competitive environment. This paper proposes a novel methodology to simultaneously accomplish punctual and probabilistic hourly predictions about the appearance of extremely low electricity prices in a medium-term scope. The proposed approach for making real *ex ante* forecasts consists of a nested compounding of different forecasting techniques, which incorporate Monte Carlo simulation, combined with spatial interpolation techniques. The procedure is based on the statistical identification of the process key drivers. Logistic regression for rare events, decision trees, multilayer perceptrons and a hybrid approach, which combines a market equilibrium model with logistic regression, are used. Moreover, this paper assesses whether periodic models in which parameters switch according to the day of the week can be even more accurate. The proposed techniques are compared to a Markov regime switching model and several naive methods. The proposed methodology empirically demonstrates its effectiveness by achieving promising results on a real case study based on the Spanish electricity market. This approach can provide valuable information for market agents when they face decision making and risk-management processes. Our findings support the additional benefit of using a hybrid approach for deriving more accurate predictions.

Keywords: electricity markets; medium-term electricity price forecasting; probabilistic forecasting; extremely low prices; spikes; hybrid approach

1. Introduction

In the current global context of the growing complexity of electricity markets, trying to predict electricity prices is essential for all market agents. However, this is not an easy task, since the price of electricity is far more volatile than other commodities. The presence of extremely high prices has been a recurrent phenomenon in markets worldwide. Nevertheless, the recent increasing deployment of non-dispatchable generation is also leading to the appearance of extremely low prices (zero or even negative prices depending on the considered regulatory framework).

This paper focuses on improving the understanding of the factors that contribute to the occurrence of these extreme price events and also their accurate forecasting with a medium-term scope. More specifically, the aim of this paper is to propose a novel methodology that allows one to predict not only the expected number of hours with very low prices in the medium term, but also the associated probability density function. The proposed methodology relies on a thorough in-sample analysis to adjust the models and an out-sample simulation approach to test its performance when making real *ex ante* forecasts.

The covered time horizon is from one month up to one year. In general, retailers and large consumers need reliable medium-term predictions to optimize their operation, as well as to properly negotiate in the short-term market and accomplish beneficial bilateral contracts. In addition, producers need medium-term predictions to optimize their generation programs and negotiate favorable bilateral and financial contracts. On the other hand, it is essential to anticipate the occurrence of these abnormally low priced hours, because this situation significantly increases the exposure of industry participants to price risk. Even in extreme cases, these unanticipated large changes in the spot price can lead to bankruptcies of energy companies if they are not prepared to tackle such risks. For this reason, an effective risk management support for the operation of electrical systems must also be able to foresee extremely low values.

The proposed methodology, which is currently in operation in one of the major Spanish electricity companies, is tested in a real case: the Spanish electricity market. The Spanish electricity market constitutes one of the most interesting cases in which the remarkable growth of renewable energy production frequently pushes the most expensive thermal power stations outside the generation program of the wholesale market. The consequent reduction in thermal production, coupled with a decline in the demand curve (especially in off-peak hours) due to the financial crisis and a low interconnection capacity to evacuate the surplus of non-dispatchableenergy, causes at certain times a sharp reduction in the clearing price. Apart from the oversupply of generation technologies with zero opportunity cost (renewable energy sources (RES), run-of-the-river hydro and nuclear), an excess of gas (due to take or pay clauses) can make combined cycles have zero opportunity cost. The conjunction of these events causes the emergence of a scenario in which the matching of supply and demand is occurring at 0 €/MWh (note that in Spain, unlike other countries, such as Australia and Germany, negative prices are not allowed).

The main contributions of this paper can be summarized as follows:

1. A general methodology has been developed to make real *ex ante* forecasts (point and probabilistic) of extremely low prices for a mid-term horizon on an hourly basis. The methodology combines different forecasting methods and spatial interpolation techniques within a Monte Carlo simulation of multiple predicted scenarios for the considered risk factors.
2. The accuracy of a novel hybrid approach that integrates fundamental and behavioral information, logistic regressions, decision trees and multilayer perceptrons has been compared to the results obtained by means of a traditional Markov regime switching model and different naive methods. This comprehensive comparison has been carried out in both in-sample and out-of-sample datasets. It has also been examined if the use of periodic models helps to improve prediction capabilities.
3. The performance of the proposed methodology has been tested in a real-sized electricity system. Note that the empirical application presented in this paper is in a single price market that does not incorporate distribution network constraints in the market clearance. In the Spanish electricity market, the high complexity of the electricity price dynamics is mainly due to the huge penetration of renewable energy sources in the generation mix and the limited interconnection capacity with France. These aspects have been taken into account in all of the forecasting models presented in this paper. However, in order to extend the methodology to other markets, where locational marginal prices may exist, and for which this methodology could be applicable, the impact of variables related to local distributed generation should be taken into account (as [1] investigates in the electrical system in Italy). In this sense, another paper that presents the influence of distributed generation (DG) on congestion and locational marginal price (LMP) is [2].

The paper is structured as follows. After a state of the art review, Section 3 describes the methodology developed in the paper. Section 4 introduces the proposed forecasting techniques, as well as the in-sample results obtained. In Section 5, the case study and the real *ex ante* forecasting results are presented. Finally, the conclusions and the main contributions of the paper are summarized in Section 6.

2. Previous Work

Diverse models have been proposed in the literature to forecast electricity prices with different aims and time horizons. The wide number of forecasting techniques is likely to be grouped by various criteria that have been proposed in several studies [3,4]. According to [5], electricity price forecasting models include statistical and non-statistical models. The latter group, which is classified in more detail in [6], comprehends simulation models and equilibrium analysis models [7]. These approaches are preferred in a medium- to long-term horizon, as they can provide price predictions even when there are structural or regulatory changes in the market. However, as they are highly demanding computationally, they tend to group hours of similar characteristics. The latter makes that the forecasts not be as accurate as data-driven methods [8]. On the other hand, statistical methods, which rely on historical data, are useful for short-term price forecasting, but they degrade when are used for medium- or long-term horizons [9]. They include time series models and artificial intelligence techniques.

A great number of time series models has been successfully implemented. In this way, the ARIMA (autoregressive integrated moving average) models are the most representative, with different particularizations. Thus, there are references that accommodate the seasonality using the same set of parameters for all hours of the day [10,11]; and others that perform ARIMA model fitting (or its variants, AR or ARMA) for each time slot of the day [12,13]. Other generalizations of the ARIMA models are the so-called linear transfer function or transfer function models with ARIMA noise [14,15], which have the peculiarity of including past and present influence of other series. Other kinds of time series are the multiple-input multiple-output models, which predict the n-dimensional price vector in a single step [16]. Artificial intelligence techniques, which can be classified into artificial neural networks (ANN) [17], fuzzy logic and their combination, the neuro-fuzzy method [18], are more powerful for complex, nonlinear time series analysis than the rest of the statistical models. The methods presented before show a considerable ability to forecast the expected electricity prices under normal market conditions. So far, however, none of these techniques can effectively deal with spikes or extreme prices in electricity markets [19]. Among the first references that address these specific features of electricity prices is [20], where spikes are modeled by introducing large positive jumps together with a high speed of mean reversion. Other authors model spikes by allowing signed jumps [21]. According to [22], spike forecasting techniques can be classified into traditional and non-traditional approaches. Traditional approaches fall, broadly speaking, into three categories: (i) traditional autoregressive time series models; (ii) nonlinear time series models with particular emphasis on Markov-switching models; and (iii) continuous-time diffusion or jump diffusion models. Non-traditional approaches include artificial neural networks or other data-mining techniques.

Traditional autoregressive time series models treat spikes through Poisson and Bernoulli jump processes [23], the inclusion of thresholds [24] or the use of different multivariate error distributions [16]. Meanwhile, regime-switching models are the nonlinear extension of traditional time series. These models are capable of identifying the nonlinearities of the dynamics and distinguish the normal chaotic motion from the turbulent and spike regime. One of the most representative model of this class is the threshold autoregressive (TAR) one, which determines the regime by the value of an observable variable corresponding to a threshold value. In the case of including exogenous (fundamental) variables, TAR processes lead to the TARX model. An alternative is the self-exciting threshold autoregressive (SETAR) model, which arises when the threshold variable is taken as the lagged value of the price series itself [25]. Markov switching models are the most prominent among those in which the switching mechanism between the states cannot be determined by an observable variable. For the treatment of spikes, they suggest different states in which at least one is consistent with its appearance [26]. With regard to continuous-time diffusion processes, spikes are essentially captured by the combination of a Poisson jump component and an intensity parameter. This parameter can be constant [27] or can be driven by deterministic seasonal variables [28]. Recently, in [22], a nonlinear variant of the autoregressive conditional hazard model has been used to estimate the probability of a spike with a short-term horizon, and in [29], a spike component is predicted in the short term using a linear approximation based on consumption and wind.

Some other approaches are based on the namely nontraditional techniques, which include: decision trees and rule-based approaches; probability methods, such as Bayesian classifiers [30]; neural network (NN) methods, such as spiking NN [31]; example-based methods, such as k-nearest neighbors [19,32]; and SVM (support vector machine) [33].

To the best knowledge of the authors, no references have been published dealing with the problem of medium-term price spikes or extreme price forecasting. The proposed work is unique in the sense that it proposes to use several forecasting techniques for making both point and probabilistic medium-term prediction of extremely low prices with an hourly accuracy.

3. Methodology

Essentially, the steps of the methodology suggested in this paper are the following:

1. The choice of a threshold to define what is considered as an extreme low price event. This point is discussed in depth in Section 3.2. It is important to point out that the methodology is not materially affected by the choice of the threshold.
2. The selection of explanatory variables that contribute to explain the phenomenon of the emergence of very low prices from a perspective that takes into account the market behavior and their statistical significance. This is further discussed in Section 3.3.
3. The adjustment of a forecasting technique for predicting the occurrence of extremely low variables in terms of a probability value from actual market data (in-sample dataset). In Section 4, we detail all of the forecasting techniques that have been used and calibrated for this purpose. Due to the fact that in our study, the dependent variable (occurrence of extremely low prices) is dichotomous in nature, the potential models to apply for the analysis are restricted to binary choice models. The proposed models classify observations based on a cutoff value. If the probability predicted by the model is greater than this cutoff value, the observation will be classified as a normal price. Otherwise, it will be deemed as an extremely low price. The choice of this cutoff point is discretional, and it will influence the sensitivity and specificity, which vary inversely with the probability value chosen. These statistics, as well as the rest of the Cooper statistics [34], can be calculated from a contingency table (Table 1) as shown in Table 2. In this paper, the cutoff point was chosen so as to provide a balance between sensitivity and positive predictivity (*i.e.*, a failure to predict an actual extremely low price is penalized as heavily as a false alarm). As a result of this step, the parameters and the optimal cutoff value for each forecasting technique are obtained and will be used in the following stage.
4. The development of probabilistic real *ex ante* forecasts through cross scenario analysis, which is the basis of Section 5. In order to use Monte Carlo simulation to tackle uncertainty in the medium term, a large number of realizations of the model are needed, usually entailing a huge computational time and effort. In order to cope with this inconvenience, we have adapted an efficient method proposed in [35] for making market equilibrium models tractable (a practical implementation can be also found in [7]) to other forecasting techniques. This method, which is illustrated schematically in Figure 1, allows one to compute a huge number of simulations by decreasing the computational time and without a major loss of accuracy. As can be seen in the figure, the first step of the methodology consists of computing a reduced number of executions (m simulations) of each of the proposed forecasting models. As a result, we obtain m result matrices about the appearance or not of extremely low prices (the classification is

made with the cutoff value previously estimated) in each specific hour of the simulation time horizon. These initial m simulations of the model are spatially placed in a hypercube of N dimensions according to the combinations of scenarios. More specifically, each dimension of the hypercube corresponds to an uncertain variable. For the sake of clarity, N is equal to three in Figure 1. Note that each risk factor is distinguished by its cumulative distribution function (CDF), and a particular scenario is defined by the pertinent percentiles of the considered risk factors. Latin hypercube sampling with correlation control techniques has been used with the aim of having a well-sampled hypercube in which each scenario is used at least once in the m executions of the statistical models. In the second stage, a vast amount (M >> m) of correlated random scenario combinations of the risk factors is generated to establish those unobserved areas of the hypercube. Here, the correlation structure between the variables is determined by using historical data. In the third step, these unsimulated areas (M feasible matrices about the appearance of extremely low prices) of the hypercube are interpolated from the initial executions by means of an interpolator based on local regression that considers the spatial structure of these initial executions. Finally, as the scenario definition is random and considers the correlation structure between the uncertain variables, all of the scenarios can be considered to be equally probable, and thus, it is possible to make both point and probabilistic forecasts of the variables of interest.

Table 1. Contingency table.

Observed Price	**Predicted Price**		**Marginal Totals**
	Extremely Low	**Rest**	
Extremely low	a	b	a + b
Rest	c	d	c + d
Marginal Totals	a + c	b + d	a + b + c + d

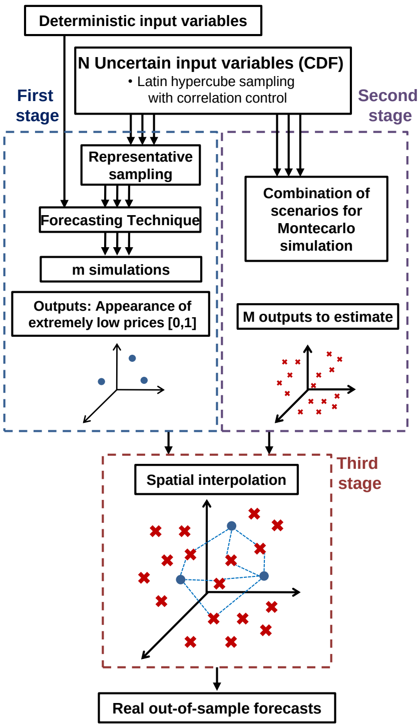

Figure 1. Global overview of Monte Carlo simulation with spatial interpolation techniques.

Table 2. Definitions of the Cooper statistics.

Statistic	**Definition**
Sensitivity: a/(a + b)	Proportion of the extremely low prices that the model predicts to be extreme
Specificity: d/(c + d)	Proportion of the normal prices that the model predicts correctly
Accuracy: (a + d)/(a + b + c + d)	Proportion of prices that the model classifies correctly
Positive predictivity (Pos. Pred.): a/(a + c)	Proportion of the prices predicted to be extremely low prices by the model that give positive results in observed prices
Negative predictivity (Neg. Pred.): d/(b + d)	Proportion of the prices predicted to be normal prices by the model that give negative results in observed prices
False positive rate: c/(c + d)	Proportion of the normal prices that are falsely predicted to be extreme by the model
False negative rate: b/(a + b)	Proportion of the extreme prices that are falsely predicted to be normal by the model

3.1. The Times Series Dataset

In this paper, a dataset from the Spanish day-ahead market, which comprises the period ranging between 1 January 2009 and 31 March 2012, is used. This period has been chosen because it is the moment that marked the inflexion point in relation to the appearance of extremely low prices in the Spanish market. However, the methodology would be equally extrapolated to other subsequent time periods. The complete data consisted of hourly spot prices and the actual production for each technology. The data corresponding to the Spanish market are available from the Iberian Energy Market Operator (OMIE [54]).

In order to thoroughly investigate the forecasting capability of each model, the data were divided into in-sample and out-of-sample datasets. The former set, which includes the training and testing sets, encompasses from 1 January 2009–30 November 2011. Thus, in Section 4, the generalization capabilities of the models with the actual data of the exogenous variables are carried out. In Section 5, an out-of-sample analysis for the period ranging between 1 December 2011 and 31 March 2012 with estimated scenarios of the explanatory variables is conducted. As no major structural or regulatory changes occurred during this period, it can be possible to capture the price dynamics by using a common statistical model.

A more detailed statistical analysis of the in-sample dataset is presented in Table 3. As noted, the distribution of electricity prices is not normal, presenting excess kurtosis and negative skewness. This means that excessively high or low prices have a higher probability of occurrence than in the case of a normal distribution. Moreover, prices below the average are more likely to occur than prices above the mean value.

Table 3. Statistical summary of market spot price.

Average	**Standard Deviation**	**Skewness**	**Kurtosis**	**Min.**	**Quartile 1**	**Quartile 2**	**Quartile 3**	**Max.**
41.041	13.280	−0.521	1.479	0.000	34.450	41.170	50.230	145.000

3.2. Extremely Low Prices Threshold

Due to the fact that market agents are not only interested in trying to model the incidence of zero prices, the objective of this section is the choice of a threshold for distinguishing extremely low prices from the rest. There are several approaches in the literature to classify whether an observation is extreme or not [36,37]. In fact, the choice of a reasonable threshold is still a subjective choice. In Spain, for instance, the threshold defining an extreme low price event is generally regarded between 10 and 15 €/MWh. In this article, a point of the characteristic modes of the distribution function has been used. In order to estimate the probability density function, the Epanechnikov kernel, which minimizes the asymptotic mean integrated squared error, has been used.

Figure 2 illustrates a graphical representation of the distribution function. This representation enables one to derive a threshold of 14.2 €/MWh. This point, which lies below the 5th percentile of the the unconditional distribution of the price series, fulfills two features. On the one hand, it is an inflection point of the density function on the left tail. On the other hand, it is a point that is away from the average a distance of more than two standard deviations. The last premise is consistent with that adopted in [19]. It should be noted that the methodology is not restricted by the choice of the threshold. The resulting indicator variable is coded 0 for extremely low prices and 1 for the remainder prices.

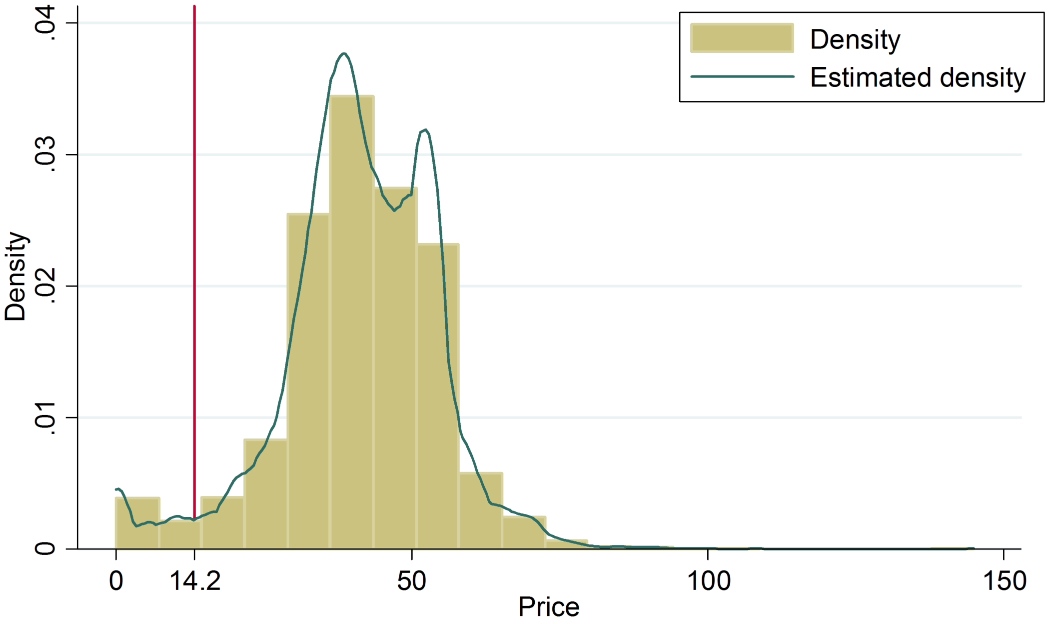

Figure 2. Density estimation

3.3. Explanatory Variables

Several studies have been conducted in order to detail the explanatory variables affecting spot market prices [28,38–40] and price spikes [41]. However, none of these references focus on the appearance of extremely low priced hours in the medium term. Therefore, in this paper, a critical point is not only the choice of the variables that help to better explain the aforementioned phenomenon, but also those factors that allow one to operationalize the forecasting model in a feasible way. This entails that it is possible to use only those variables that can be characterized with a reasonable accuracy at this medium-term time horizon. This is, for instance, the case of technologies that usually behave as price takers.

In the particular case of Spain, where most of the electricity is priced at the day-ahead hourly market, the price is set by the marginal generator bid. Among the power plants with the lowest short-opportunity costs are renewable energy sources (RES), nuclear and run-of-the-river hydro. On the contrary, the plants with the highest opportunity costs are gas, coal, head-dependent hydro and fuel oil. During the horizon of the study, RES were legislated under what was referred to as a special regime, promoted through a feed-in tariff system. Thus, apart from the special regime, the technologies that usually behave as price takers are run-of-the-river hydro and nuclear. Due to the zero variable cost of the former and the inflexibility of the latter, both of them usually bid at low prices to collect then the marginal price. It should be noted that it is well known that hydro conditions are one of the most important sources of uncertainty in the Spanish market. In this study, run-of-the-river hydro production has been considered as a reasonable approach for hydro conditions. This is because, run-of-the-river hydro production, unlike head-dependent hydro, behaves as a price taker, and therefore, it is expected to be relevant for the appearance of extremely low prices.

The selection of explanatory variables, which has been done taking into account the principle of parsimony, was based on backward elimination methods, which evaluate different statistics in order to control the exit of variables. Thus, the adjusted coefficient of determination and several model selection criteria (such as AIC and BIC) were measured.

Before constructing the proposed models, the presence of unit roots in the candidates for exogenous variables has been tested. To this end, augmented Dickey–Fuller has been used. For all considered explanatory variables, the absence of stationarity was strongly rejected beyond a 5% significance level. The absence of multicollinearity between the regressors has also been confirmed by applying the variance inflation factor.

4. Forecasting Techniques

The main target of this section is to compare several novel models with one of the most prominent models used in the prediction of spikes: Markov regime-switching models. In order to estimate their generalization capabilities, the in-sample data were divided into two different sets: 80% for training and 20% for testing. The validation set comprises the training and testing sets (from 1 January 2009–30 November 2011). Note that this section analyzes the models' generalization capabilities with actual data of the exogenous variables.

It is well known that electricity prices exhibit seasonal fluctuations, which can be collected by including relevant explanatory variables. Still, despite a well-specified inclusion of demand in the models that could explain the weekly effects, a separate modeling for (1) working days, (2) Saturdays and (3) Sundays and holidays has also been carried out with the aim of testing if the forecasting performance is improved. The main benefit of this approach, which uses periodic models, is that it allows the model parameters to switch according to the different behaviors identified in the course of the week. Moreover, the construction of separate models has the advantage of selecting an optimum and specific threshold value for classifying an observation as a very low price or not depending on the predicted likelihood given by the statistical technique used in each case. This could be particularly relevant for the accuracy of the forecasts.

4.1. Logistic Regression

Logistic regression is a well-known supervised learning algorithm that can allow us to estimate the probability of the occurrence of extremely low prices, which is a dichotomous outcome. This technique is very useful to analyze the potential impact of the independent variables on the dependent variable. For each constructed model, the estimated coefficients of the explanatory variables are also shown. The Wald test was utilized to check their validity. The models goodness-of-fit is checked by means of the Cox and Snell R square and the Nagelkerke R square.

4.1.1. Model 1

As has been shown previously, the addressed classification problem is unbalanced with a proportion of hours with very low prices only accounting for 4.32%. For this reason, a model based on the traditional logistic regression procedures could sharply underestimate the probability of this rare event, and therefore, it could lead to erroneous results. One of the most popular techniques to correct these effects when the occurrence of the events is less than 5% was the bias correction method proposed in [42]. This procedure estimates the same logit model as the traditional one, but with an estimator that provides lower mean square error in the presence of rare events data for coefficients, probabilities and other quantities of interest.

Model 1 uses the explanatory variables referred to in Table 4 without taking into account the effects of work activity in a specific way. Table 5 and Table 6 show that the overall accuracy of the model is acceptable. Note that a naive model, for simple chance, would have a specificity and sensitivity of 50%. For example, the statistic R square is 65.6%, which is quite acceptable in predictive terms. According to Table 4, the estimated coefficient of the dependent variables, as shown by the Wald test, are significant from a statistical point of view.

Table 4. Variables in the equation: Model 1. ND, net demand.

Variable	Coefficient	S.E.	Wald	Signification
ND	0.0005860	0.0000176	1113.7107	0.000
H	−0.0016148	0.0000422	1463.4780	0.000
N	−0.0003917	0.0000587	44.5235	0.000
W	−0.0007231	0.0000226	1023.5278	0.000
CONST	1.7120790	0.4079335	17.6144	0.000

Table 5. Contingency table with a cutoff value of 0.65: Model 1.

Observed Price	Predicted Price		Correct Percentage
	Price [0,14.2]	Price > 14.2	
Price [0,14.2]	752	350	68.24
Price > 14.2	361	24,070	98.52
Global percentage			97.22

Table 6. Summary of Model 1.

−2 Log of the Likelihood	Cox and Snell R Square	Nagelkerke R Square
3501.401	0.196	0.656

4.1.2. Model 2

This model is an extension of the previous one. The novelty is that the effect of the working patterns in prices has been incorporated by using a periodic logistic regression model. Thus, a regression model is estimated for weekdays, another one for Saturdays and a different one for the holidays. Table 7 shows that the model goodness-of-fit is better, and the variation explained by the model is slightly higher than the previous one.

In accordance with Table 8, the explanatory variables are significant from a statistical point of view (except the constant term in the case of the working days). Table 9 shows the performance of the three models separately and for the global model. Furthermore, the optimal cutoff points that have been calculated are presented in parenthesis. As seen, this model presents a power for prediction slightly higher than Model 1.

Table 7. Summary of Model 2.

−2 Log of the Likelihood			Cox and Snell R Square			Nagelkerke R Square		
WORK	SAT	HOL	WORK	SAT	HOL	WORK	SAT	HOL
1954.41	464.10	981.57	0.18	0.22	0.23	0.69	0.70	0.59

Table 8. Variables in the equation: Model 2.

Variable	**Coefficient** ($\times 10^{-4}$)			**Signification**		
	WORK	**SAT**	**HOL**	**WORK**	**SAT**	**HOL**
ND	6.45	5.15	6.14	0.00	0.00	0.00
H	−16.41	−18.55	−16.55	0.00	0.00	0.00
N	−2.26	−6.16	−5.62	0.01	0.00	0.00
W	−7.50	−4.69	−7.99	0.00	0.00	0.00
CONST	−5531	38,342	32,099	0.33	0.00	0.00

Table 9. Cooper statistics of Model 2.

(%)	**WORK (0.66)**	**SAT (0.66)**	**HOL (0.61)**	**Global**
Sensitivity	69.85	78.31	61.54	68.97
Specificity	98.87	98.97	97.06	98.60
Positive Predictivity	70.06	78.31	61.11	68.97
Negative Predictivity	98.86	98.97	97.12	98.60
Accuracy	97.81	98.03	94.59	97.32

4.1.3. Model 3

This particular case is a variant of Model 2, not considering the correction proposed in [42]. Instead, it was decided to reduce the data with the aim that the number of low prices had a greater significance in the sample. In this way, according to Table 10, those intervals of data in which it is guaranteed the absence of extremely low prices in the dataset were eliminated. Hence, the sample was reduced by 42.3%. The model, although still slightly better than Model 1, cannot overcome the suitability of Model 2. Table 11 presents Cooper statistics broken down individually and globally. Note that there are categories with two values. The values on the left side refer to those corresponding to the simplified model, while the values on the right side refer to the correction made taking into account that the removed values of the training set have been successfully predicted. Meanwhile, Table 12 confirms the goodness-of-fit of each one of the regression models. Furthermore, Table 13 presents the coefficients associated with each explanatory variable, as well as the statistical significance of each one of them. As in Model 2, the constant term for weekdays is not significant.

Table 10. Intervals without very low prices.

Variable	**WORK**	**SAT**	HOL
ND	>32,500	>28,000	>26,000
ND′	>28,000	>23,000	>21,000
ND″	>13,000	>16,000	>9000

Table 11. Cooper statistics of Model 3.

(%)	**WORK (0.66)**	**SAT (0.66)**	**HOL (0.62)**	**Global**
Sensitivity	70.00	74.70	60.84	68.33
Specificity	97.53–98.84	98.73–98.98	95.45–96.73	97.44–98.57
Positive Predictivity	69.57	75.61	61.27	68.33
Negative Predictivity	97.58–98.86	98.67–98.93	95.37–96.67	97.44–98.57
Accuracy	95.48–97.78	97.53–98.00	91.79–93.91	95.26–97.27

Table 12. Summary of Model 3.

−2 Log of the Likelihood			Cox and Snell R Square			Nagelkerke R Square		
WORK	**SAT**	**HOL**	**WORK**	**SAT**	**HOL**	**WORK**	**SAT**	**HOL**
1948.50	460.18	972.42	0.26	0.23	0.27	0.64	0.70	0.55

Table 13. Variables in the equation: Model 3.

Variable	Coefficient ($\times 10^{-4}$)			Signification		
	WORK	**SAT**	**HOL**	**WORK**	**SAT**	**HOL**
ND	6.34	4.90	5.77	0.00	0.00	0.00
H	−16.32	−18.55	−16.55	0.00	0.00	0.00
N	−2.18	−5.98	−5.48	0.01	0.00	0.00
W	−7.44	−4.53	−7.88	0.00	0.00	0.00
CONST	−4721	41,169	36,893	0.41	0.00	0.00

4.1.4. Comparison between the Models

As seen, the three models have a quite acceptable predictive accuracy. Note that Model 1 showed slightly worst global performance than the other models. However, it presents the advantage that it is a much simpler model. The results obtained in this comparison demonstrate that the inclusion of weekly seasonality slightly improves the predictive capabilities of the forecasting methods. A remarkable fact is that there are discrepancies regarding the significance of the explanatory variables depending on the day of the week under consideration. Another finding to note is that the correction proposed in [42] is an effective tool when dealing with unbalanced problems. Finally, a greater difficulty in achieving effective predictions on Sundays and holidays has been found. This may be explained by a different market behavior during these days.

4.2. Decision Trees

Decision trees classify observations based on a set of decision rules, applied in a sequential manner. The probability of occurrence of extremely low prices is allocated to each end of a branch in the tree. The way of estimating probabilities does not require the assumption of specific probability distributions for the variables, which is an advantage of this methodology. In order to not over-fit the data, stopping rules control the growing process, and the over-fitted parts were pruned. In this paper, an ID3 (Iterative Dichotomiser 3) algorithm has been used, with splitting criteria based on the entropy [43].

4.2.1. Model 1

A global model ignoring the effects of weekly patterns has been built. According to Figure 3, the most representative variable is the hydro production. Note that under a scenario below the 88th centile, there is a negligible probability of occurrence of very low prices. Another interesting aspect is that nuclear production is not representative in this case. Table 14 shows the classification success rate of the tree. As seen, the model is able to accurately predict an acceptable number of the events that occurred during this period of time.

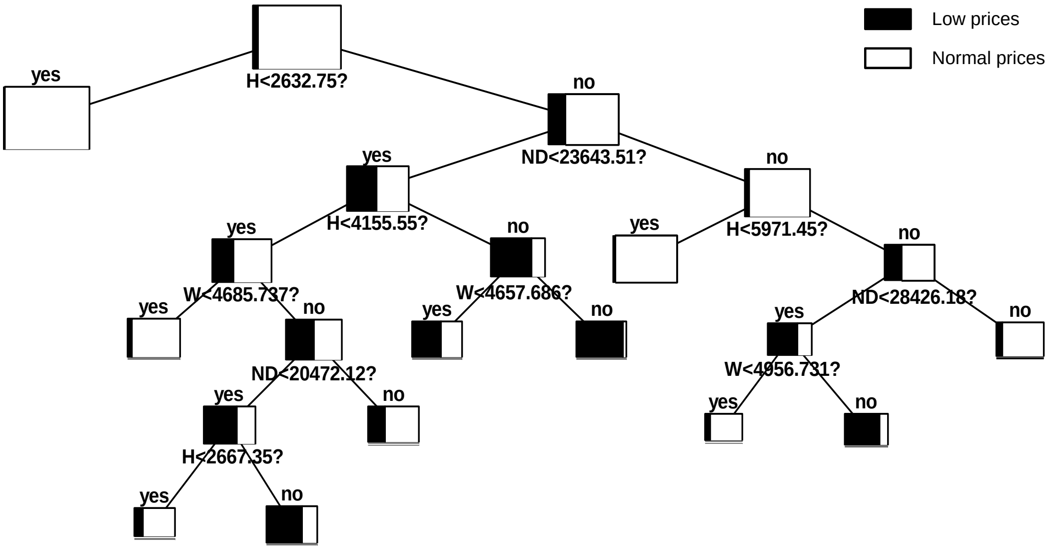

Figure 3. Decision tree: Model 1.

Table 14. Contingency table with a cutoff value of 0.85: Model 1.

Observed Price	Predicted Price		Correct Percentage
	Price [0,14.2]	Price > 14.2	
Price [0,14.2]	657	445	59.62
Price > 14.2	401	24,030	98.36
Global percentage			96.69

4.2.2. Model 2

In this case, the weekly seasonality has been taken into account by building three different trees. If we analyze the constructed trees in comparative terms, we can observe clear similarities and differences between them. First, it is clear that hydro production is the most relevant variable. Moreover, the scenarios that lead to hours with very low prices are similar. However, a more detailed individual analysis of each tree allows one to establish significant differences in the representativeness of the explanatory variables and characteristic values of the selected splits.

Figure 4 shows the tree for Saturdays. As can be seen in the figure, those hourly scenarios of hydro production that fall below 3767.2 MW have a negligible probability of occurrence of very low prices. One of the most remarkable facts is that, from a statistical point of view, hydro production seems to be even more important than in the other types of days, such as Sundays and holidays. Furthermore, nuclear production is irrelevant. The typical behavior of Sundays and holidays is reflected in Figure 5. An interesting aspect is that wind power is not significant. In contrast, nuclear production itself is useful in explaining the output's behavior. Finally, Figure 6 corresponds to working days. According to the figure, the main factors influencing the appearance of low priced hours are the hydro production and the system net demand.

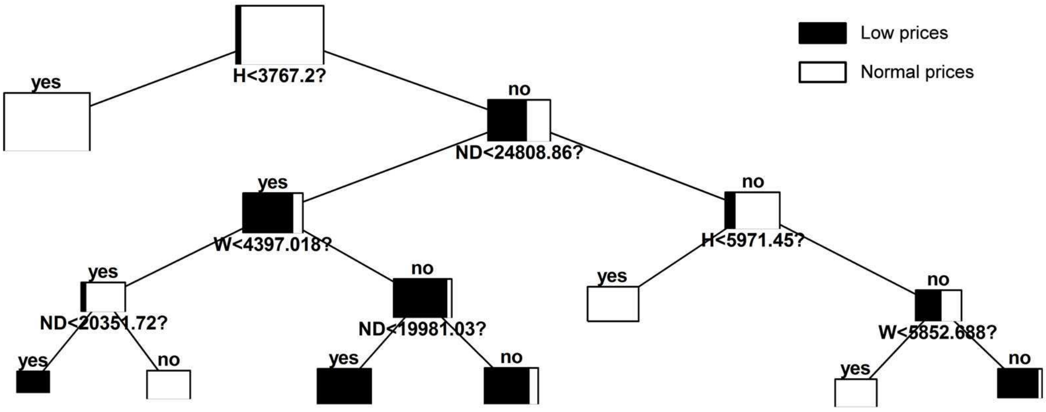

Figure 4. Decision tree: Saturdays.

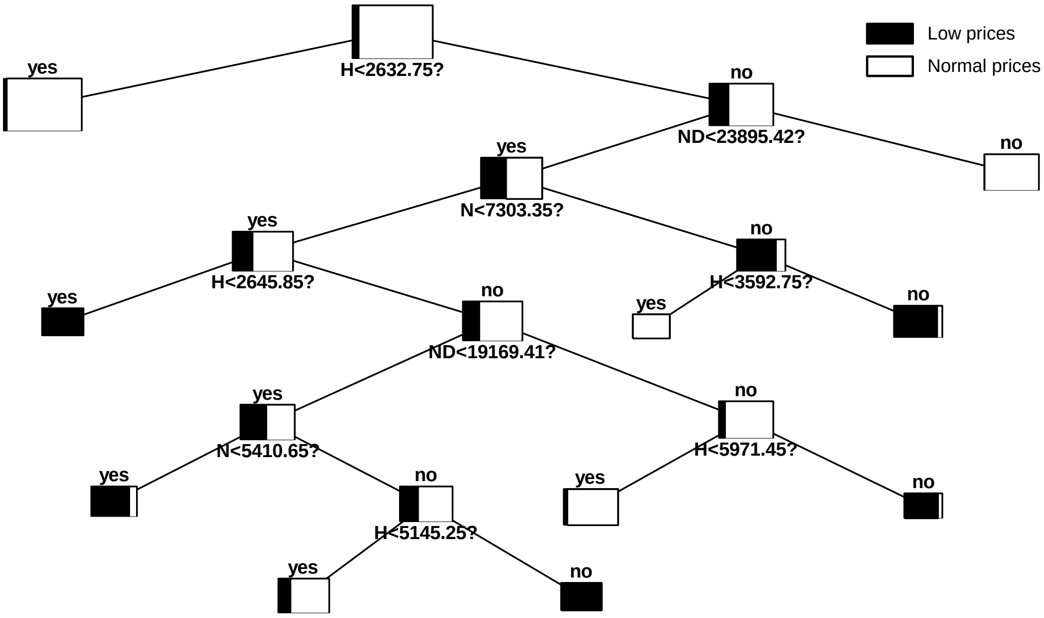

Figure 5. Decision tree: Sundays and holidays.

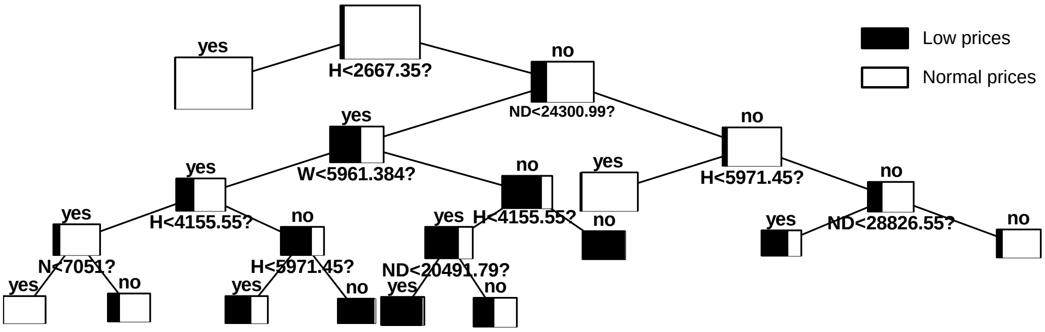

Figure 6. Decision tree: working days.

Table 15 shows the global performance of the model derived from each of the trees, which has been constructed separately. As the optimal cutoff value was set independently for each tree, it is not possible to achieve a balance between positive and negative predictivity.

Table 15. Contingency table: Model 2.

Observed Price	**Predicted Price**		**Correct Percentage**
	Price [0,14.2]	**Price > 14.2**	
Price [0,14.2]	669	433	60.71
Price>14.2	308	24,123	98.74
Global percentage			97.10

4.2.3. Comparison between the Models

Table 16 shows a comparison between the proposed models. The percentages of success in the training, test and validation sets are shown, as well as the optimal cutoff value for each tree. In general, both models are acceptable, since the overall accuracy is superior to that obtained by chance (95.68%). The second model performs slightly better. A remarkable aspect is that the tree constructed for Sundays and holidays is only able to identify 50% of the hours with very low prices.

Table 16. Comparison among the different decision trees.

Strategy	**Decision Tree**	**Percentage of Success (%)**			**Cutoff**
		Training	**Test**	**Validation**	
Without weekly effect	FULL	96.671	99.001	97.137	0.85
Weekly effect	WORK	97.449	99.353	97.830	0.75
	SAT	97.875	100	98.246	0.8
	HOL	95.427	96.590	95.660	0.76

Decision Tree	**Cooper Statistics (%)**				
	Sensitivity	**Specificity**	**Pos. Pred.**	**Neg. Pred.**	**Accuracy**
FULL	59.619	98.359	62.098	98.182	96.687
WORK	62.923	98.815	66.830	98.597	97.503
SAT	70.482	99.397	84.783	98.604	98.081
HOL	50.000	97.800	62.996	96.309	94.465

4.3. Multilayer Perceptron

Multilayer perceptrons constitute a useful tool for regression and classification [44]. In this case, its use is justified because we expect that there could exist a nonlinear relationship between the proposed inputs and output.

There are several problems associated with local minima and decisions over the size of the network to use. Thus, the use of this technique usually implies experimenting with different architectures, as the determination of the number of hidden layers and the number of hidden neurons in each hidden layer. It has been shown in practice that one hidden layer configuration is enough for most applications. For this reason, a topology with two layers of adaptive weights (a hidden layer and an output layer) has been selected.

The activation function of neurons is the hyperbolic tangent, both for the hidden and the output layer. With the aim of providing a probabilistic interpretation, the outputs have been scaled to the interval [0,1].

Regarding the proper number of hidden neurons to be used, a sweep computing the validation error was carried out to find its optimal value. As said, one of the main problems in the multilayer perceptron is getting stuck in local minima. This problem has been solved by initializing the weights with random values and by repeating the process several times. Before training, the inputs were normalized by a simple linear rescaling. The fitting criterion was the quadratic error minimization, as it penalizes large errors more than small ones.

Using this technique, two models were constructed: a single MLP for the whole set of data and three different MLP to take into account the weekly seasonality. In Table 17, the structure of the proposed models is summarized. In order to evaluate the forecast performance, the Cooper statistics are displayed for the optimal cutoff value that has been computed. It is evident that the prediction ability of this model is significantly improved if the weekly seasonality is included. In any case, each of the models based on MLP is clearly superior in terms of prediction performance to the other proposed techniques. Curiously, this model demonstrates a better ability to capture the dynamics of the very low prices on Sundays and holidays.

Table 17. Comparison of proposed models: MLP.

Strategy	**MLP**	**Network Size**	**Cutoff**	**Cooper Statistics** (%)			
				Sensitivity	**Specificity**	**Pos. Pred.**	**Neg. Pred.**
Without weekly effect	FULL	4-25-1	0.65	78.131	98.993	77.778	99.013
Weekly effect	WORK	4-24-1	0.64	82.308	99.311	81.930	99.329
	SAT	4-28-1	0.78	92.169	99.627	92.169	99.627
	HOL	4-28-1	0.59	91.259	99.318	90.941	99.345

4.4. Hybrid Approach

The hybrid approach presented in this section is a novel forecasting methodology that combines a fundamental market equilibrium model with the logistic regression approach implemented in Section 4.1.1, with the ultimate objective of benefiting from the advantages that each of them offers separately. Fundamental models, which are preferred in a medium- and long-term horizon, can provide useful insights for the analysis of the strategic behavior in electricity markets and constitute a valuable tool to represent the electricity market with its main technical and economic characteristics, especially when there are structural or regulatory changes in the market. However, as stated in [7], market equilibrium models fail when the aim is to capture the high-order moments of the probability distribution functions compared to the data-driven methods. This is because in the tails of the distribution, fundamentals are less important than behavioral factors. This naturally leads to complementing the fundamental approach with some of the statistical methods discussed before, which is one of the ultimate goals of this paper.

4.4.1. Fundamental Market Equilibrium Model

In the first step of the methodology, the operation and the behavior of the Spanish electric power system are fairly represented using a fundamental market equilibrium model based on conjectural variations as stated in [45,46]. The model, which is equivalent to the one used in [7], is formulated as a traditional cost-based optimization problem where each generation company i tries to maximize its own profit. In this model, the strategic behavior of each generation company i is represented by means of a parameter known as the conjectured-price response θ_i. This exogenous positive parameter has been valued by using historical data following [47,48]. This parameter, which measures the market power of the various companies taking part in the market, is the minus derivative of the electricity market price λ with respect to the production q_i of the generation company (Equation (1)).

$$\theta_i = -\frac{\partial \lambda}{\partial q_i} \geq 0, \qquad \forall i \tag{1}$$

As was shown in [45], the market equilibrium (note that under game theory, the market equilibrium is the point in which each market agent maximizes its own profit, but bearing in mind that the rest of the agents also maximize their profits) can be calculated by solving an equivalent quadratic optimization problem (Equations (2)–(4)):

$$\min_{q_i} \sum_i \overline{C_i}\left(q_i\right) \tag{2}$$

s.t.

$$\sum_i q_i = D \qquad : (\lambda) \tag{3}$$

$$\mathcal{H}\left(q_i\right) \geq 0 \tag{4}$$

The term $\overline{C_i}\left(q_i\right)$ is the so-called effective cost function of agent i, which is defined as:

$$\overline{C_i}\left(q_i\right) = C_i\left(q_i\right) + \theta_i \cdot \frac{q_i^2}{2}, \qquad \forall i \tag{5}$$

As seen, the effective cost function takes into account a linear or quadratic cost function $C_i\left(q_i\right)$ and a term that models the strategic behavior of the company i. Therefore, the minimization problem Equations (2)–(4) is a quadratic optimization problem that can be effectively solved using readily available commercial software. The decision variables of this problem are the dispatch of the generators, subject to the demand-balance equation (Equation (3)) and the technical constraints (Equation (4)) of the operation of all hydro and thermal groups (emission limits, variable costs, minimum and maximum power, efficiency, *etc.*).

Since in medium-term market equilibrium models, an hourly representation is not often used in practice because the size and resolution times increase considerably for a real-sized electricity systems, the hours within each month have been grouped into l = 1,2,. . . ,16 net demand levels (denoted as ND in this paper), or system states, by means of a k-means clustering process, as explained in [49]. System states consist of a number of hours in which market conditions are considered to be the same.

The use of system states as proposed in [49] is an alternative approach to the traditional representation based on load levels, which prevents the loss of chronological information between individual hours. This is very important for decision variables, such as the starting-up and the shutting-down of thermal groups. It should be noted that load levels, unlike system states, are only defined based on system demand. The consideration of the net demand for the computation of system states allows us to better represent the operation in power systems with a high penetration of renewable energy sources. Furthermore, as stated in [7], this novel approach based on system states enables one to reach a better forecasting accuracy and allows one to successfully capture the so-called stylized facts [50] of electricity prices. However, even in this complex model, there are many difficulties to properly account for the occurrence of extreme events, and that is why a complementary approach with a statistical model is needed.

4.4.2. Communication between the Models

For the hybridization of the models, the system marginal price for each state λ is firstly estimated by computing the dual variable of the power-balance constrain (Equation (3)) of the market equilibrium model. Hereinafter, the price is allocated to the hours that belong to the corresponding state, and it is used as an explanatory variable in the logistic regression model for rare events, which was detailed above in Section 4.1.1. As stated in Table 18, the market price λ shows the statistical significance thereof. Another factor to highlight is that the sign of the considered variables coincides with what would be expected *a priori*. The fact that the fundamental model seems not to add very much to the logistic model implies, as was expected, poor fundamental specification at low prices. However, as can be seen in Table 19, the obtained results suggest that this approach performs slightly better than the individual models presented in Section 4.1. This finding suggests that λ, which simultaneously captures the production cost and the strategic behavior of market agents, can provide useful insights to predict extremely low prices.

Table 18. Variables in the equation: hybrid model.

Variable	Coefficient	S.E.	Wald	Signification
ND	0.0007084	0.0000618	131.3093	0.000
H	−0.0007399	0.0004128	3.2121	0.073
N	−0.0003914	0.0002106	3.4540	0.063
W	−0.0008067	0.0000741	118.4120	0.000
λ	0.0510856	0.0111722	20.9082	0.000
CONST	2.5112069	1.5278314	2.7015	0.100

Table 19. Contingency table with a cutoff value of 0.66: hybrid model.

Observed Price	Predicted Price		Correct Percentage
	Price [0,14.2]	Price > 14.2	
Price [0,14.2]	796	306	72.23
Price >14.2	306	24,125	98.75
Global percentage			97.60

It should be stressed that the hybridization may also be performed with the rest of the techniques that have been previously presented. The major reasons why logistic regression has been used are based on the commitment between accuracy, transparency and simplicity of implementation that this technique has demonstrated. This is of great importance, since, as will be explained ahead in Section 5, the implementation of the methodology to make real predictions in the medium term requires simulating multiple scenarios of the variables subject to uncertainty, which is computationally highly intensive in real-sized electricity systems. Note that although the problem size using system states is much lower than the hourly representation, the model for the Spanish electricity market consists of more than 300,000 equations and 800,000 variables. The optimization problem is formulated in GAMS (General Algebraic Modeling System) and is solved using CPLEX 12.4. The resolution time is almost two minutes for just one realization of the risk factors using a PC with Intel Core Duo i7-4790 CPU @3.6 GHz CPU and 32.0 GB RAM.

4.5. Markov Regime-Switching Model

Markov regime-switching models (MRS) assume the existence of an unobserved variable representing the state or regime, which governs a given dataset at each point in time. The usefulness of MRS models for power market applications has already been recognized. However, their effectiveness for forecasting has been vaguely proven, and only lately has this issue been approached in the literature [39,41,51].

Markov switching models do not require a previous dating of which periods are considered extreme. Therefore, fixed imposed thresholds are not needed. The model is able to capture changes in the mean and the variance between state processes. The motions of the state variable between the regimes are governed by an underlying Markov process.

In this paper, one of the most popular specifications of MRS models in the energy economics literature is followed. Specifically, as proposed in [51,52], the specification includes two independent regimes (R_1 and R_2) and a mean-reverting heteroskedastic process for the base regime dynamics.

In Equation (6), where the base regime is described, ϵ_t is supposed to be N(0,1)-distributed. On the other hand, a Gaussian distributed spike regime is assumed (Equation (7)).

$$X_{t,R_1} = \alpha_{R_1} + \beta_{R_1} X_{t-1} + \sigma_{R_1} \mid X_{t-1} \mid^{\gamma_{R_1}} \epsilon_t \tag{6}$$

$$X_{t,R_2} \sim N(\alpha_{R_2}, \sigma_{R_2}) \tag{7}$$

Following the recommendations provided in [51], the prices themselves instead of the log-prices are modeled. Moreover, the deseasonalization of prices is conducted in a similar way as was stated in [51]. Thus, an additive model is considered. Equation (8) represents that the hourly spot price Y_t can be decomposed into a stochastic part X_t and a predictable component (trend and seasonal component). On the one hand, for estimating the trend T_t, a wavelet filtering-annual-smoothing technique is used. On the other hand, weekly periodicity is considered for the seasonal component S_t. This component is removed by applying a variation of the moving average technique, using the median instead of the mean value. The reason is that the median is more robust than the mean in the presence of outliers. Figure 7 graphically shows the decomposition process, which has been performed.

$$Y_t = T_t + S_t + X_t \tag{8}$$

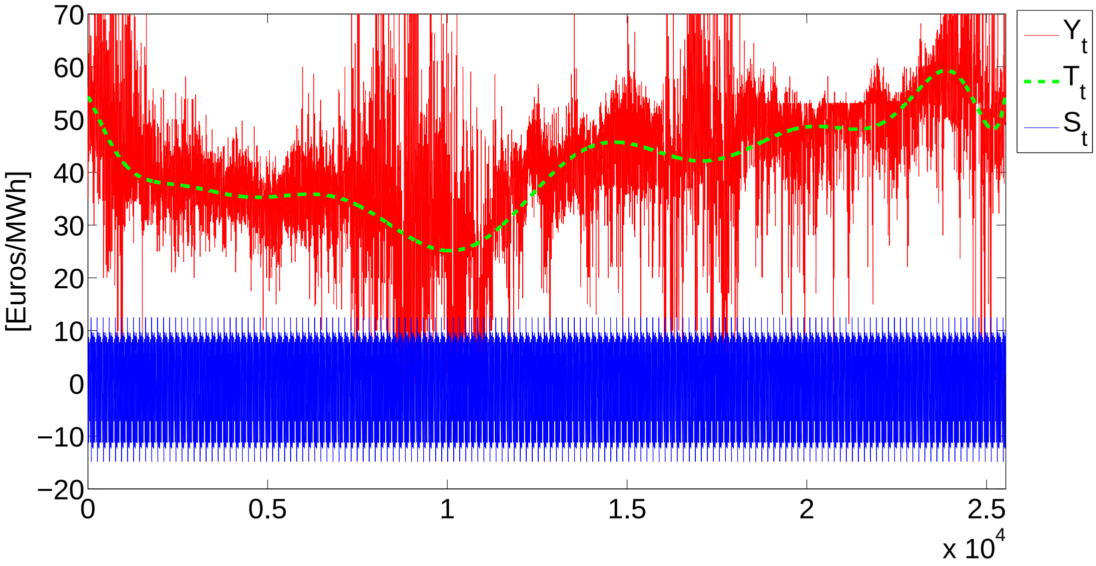

Figure 7. Decomposition of the price series using an additive model.

Next, MRS models are fitted to deseasonalized prices X_t. The calibration of parameters is accomplished by an iterative procedure based on the application of the expectation-maximization algorithm proposed in [53]. The estimated model parameters are given in Table 20. As can be seen, the parameters obtained for the base regime suggest a high speed of mean-reversion (which is represented by the parameter β_{R_i}) and that extremely low prices increase volatility more than extremely high prices (this is captured by the parameter γ_{R_i}). Regarding the probabilities q_{ii} of staying in the same regime, high values in both regimes are observed. This suggests that several consecutive observations in each regime will be appreciated, which represent an advantage in comparison to jump diffusion models.

Table 20. Parameter estimates and descriptive statistics of the Markov regime-switching (MRS) model.

R_i	Parameters				Statistics		
	α_{R_i}	β_{R_i}	${\sigma_{R_i}}^2$	γ_{R_i}	$E\left(X_{t,R_i}\right)$	q_{ii}	P (R = i)
Base	4.59	0.91	11,654.67	−0.96	50.62	0.98	0.84
Spike	47.38		259.01		47.372	0.89	0.16

Figure 8 shows the deseasonalized prices X_t and the spikes that have been identified. The lower picture displays the probabilities of being in the spike regime. The deseasonalized series has been shifted so that its minimum coincides with that of the original series Y_t.

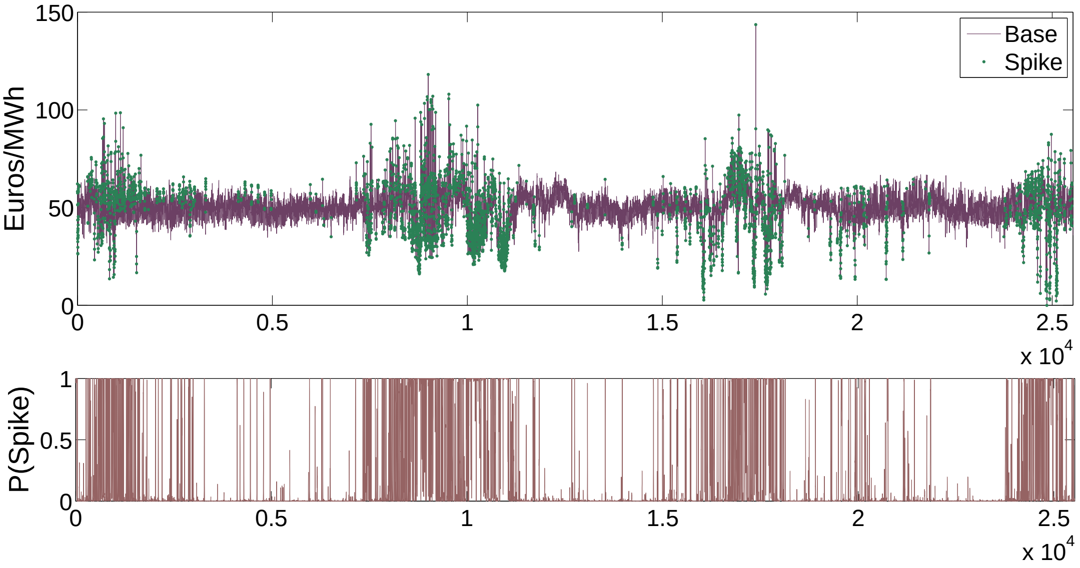

Figure 8. Calibration results for the MRS model.

In order to test the ability of the model to predict extremely low prices in the in-sample dataset, 5000 price trajectories have been simulated. The performance of the model is analyzed with two measures typically used in spike classification: sensitivity (85.75%) and positive predictivity (45.88%). Although the model is able to predict many of the extremely low prices correctly, this technique tends to classify many non-spikes as spikes. Therefore, it seems that this model has no advantages over the other proposed models.

5. Real *Ex Ante* Forecasts

This section is aimed at making real *ex ante* forecasts in a probabilistic way by using predicted scenarios of the risk factors and the parameters that have been estimated for the forecasting techniques presented in Section 4. On the one hand, Section 5.1 presents a description of how the last stage of the methodology explained in Section 3 is actually implemented with the proposed forecasting techniques. On the other hand, in Section 5.2, we briefly discuss the simulation with the MRS model, which is used as a benchmark. Finally, the presented case study is presented in Section 5.3.

It is important to highlight that the medium-term horizon is referred here to a forecasting scope that varies from one to two months. More specifically, if the primary objective is the prediction of extreme hourly prices for month m, the simulations are carried out in a single step in the first hour of month m-1.

5.1. Simulation with the Proposed Models

In order to make simulations with the proposed models, a multi-scenario analysis has been conducted. Therefore, the first step of this methodology is to generate scenarios for those random variables based on historical data. As was stated in Section 3, all risk factors are represented by their corresponding cumulative distribution function (CDF) in such a way that each scenario corresponds to a percentile. Different strategies have been used for the hybrid approach and the remaining techniques. This is because in the market equilibrium model, it is also important to incorporate the uncertainty related to fuel prices and the unavailability of thermal plants.

5.1.1. Logistic Regression, Decision Trees and Multilayer Perceptrons

In the presented case study, possible hourly realizations for water inflows (five scenarios), power demand (five scenarios) and wind production (55 scenarios) have been generated. In order to obtain a well-sampled spatial structure, representative percentiles ranging from the 1st to the 99th of its CDF have been chosen. Meanwhile, it is assumed that the international exchanges, as well as the production of the rest of the technologies belonging to the special regime are completely determined by their expected values. It should be noted that it is out of the scope of this work to understand how the prediction errors in the generation of scenarios contribute to the error of *ex ante* forecasts. Taking into account all of the possible combinations of the generated scenarios, a total of 1375 simulations have been performed with each forecasting technique. For each scenario and for each hour, the likelihood of the appearance of low priced hours has been computed. If this value is lower than the optimal cutoff value defined for each particular model in Section 4, the observation is classified as an extremely low price. Because of its practical interest, an additional variable that indicates the number of extremely low prices per month has also been constructed for every simulation on the basis of the sum of all hourly indicator variables. In the next step, a huge amount (more specifically, 100,000) of random scenario combinations of the percentiles of the well-known uncertain variables is generated in order to establish the unobserved areas of the hypercube, which have to be estimated by means of the spatial interpolator. Finally, probabilistic forecasts are calculated taking into account that all of the 100,000 scenarios are equiprobable.

5.1.2. Hybrid Approach

In the specific case of the hybrid approach, we have taken into account as medium-term risk factors, in addition to the three variables considered in Section 5.1.1, the natural gas prices (11 scenarios), the CO_2 emission allowance prices (11 scenarios), the coal prices (11 scenarios) and the unplanned unavailability of thermal power plants (three scenarios). As a result, there are 5,490,375 possible combinations of uncertain variables. In the initial stage, a representative sample of 1375 uncorrelated scenario combinations has been defined by means of Latin hypercube sampling as stated in Section 3. The next step is to perform 1375 simulations of the hybrid approach by including the well-sampled scenario combinations of the uncertain variables and the deterministic inputs. Finally, probabilistic forecasts are computed in a similar way to what is done in the previous section. The main difference is that in this case, the 100,000 random scenario combinations that have been generated for the Monte Carlo simulation present a correlated structure. This is particularly important, since it is well established that commodity prices are correlated, and it would be unrealistic to consider certain combinations when making the spatial interpolation. For the sake of clarity, a general outline of the methodology followed is provided in Figure 9.

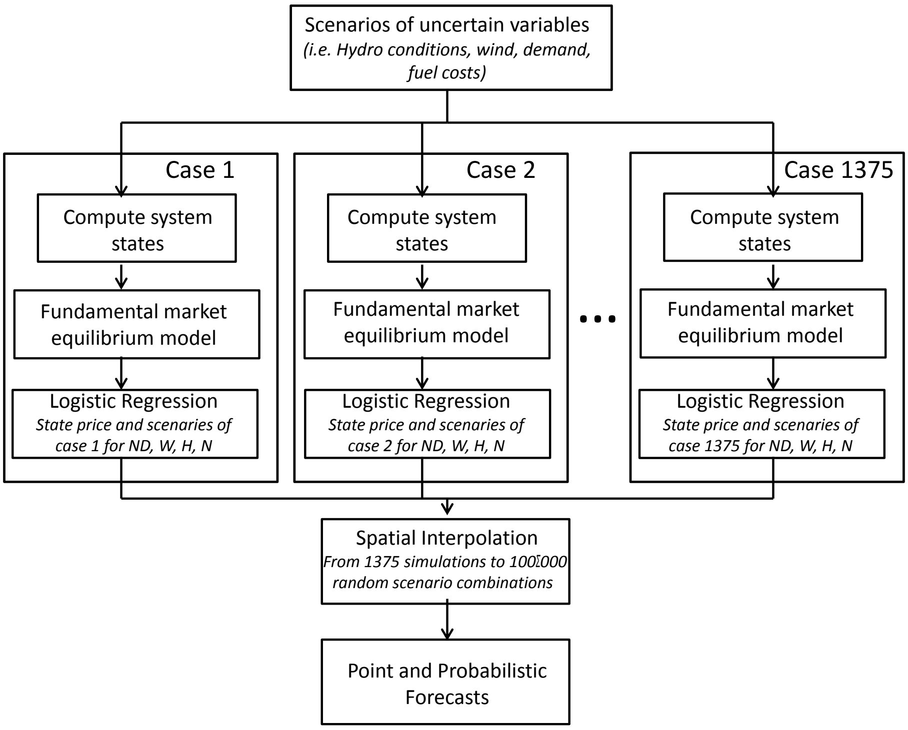

Figure 9. Global overview of the out-of-sample simulation with the hybrid approach.

5.2. Simulation with the Markov Regime-Switching Model

Several price trajectories are simulated in order to guarantee the stability of the results. Specifically, 5000 different paths have been used for each month. Then, using the simulated forecasts for the spot price, the corresponding probabilistic forecasts have been determined. Forecasting based on decomposition methods has been performed by extending each of the predictable components. The trend is the component that presents major problems, since wavelets are functions that are quite localized in time and space. In order to extend the signal, polynomial extrapolation or a spline fit might be utilized. In this case, as this component is closely related to expectations about fuel price levels, climate and consumption conditions, an adjusted linear model based on futures prices information being traded is used. This approach is suitable to properly internalize the expectations of all market agents. Regarding the seasonal component, it has been extended through the duplication of the last seasonal period. This can be considered as appropriate, since the seasonal component does not vary with time.

5.3. Case Study and Results Analysis

This section firstly assesses the capabilities of the proposed techniques to provide real *ex ante* point forecasts. The number of hours with very low prices per month has been selected, due to its interest in practical applications, as accuracy measure to evaluate the performance of the different approaches. For this assessment, a comparative study with two naive methods has been conducted. On the one hand, Naive 1 makes forecasts for month m by taking into account the proportion of extreme low prices that have taken place from 1 January 2009 to the last hour of month m-2. On the other hand, Naive 2 considers the proportion of events in similar months of the in-sample dataset.

In Table 21, the values of these measures are provided. Comparing the predictions of each proposed model with those that actually occurred, it can be concluded that the hybrid approach seems to be superior to the rest of models. This result suggests that the inclusion of the prediction of the market equilibrium price as an input of logistic regression in each scenario can provide useful information about the economic and technical characteristics of the market. This is even more important when possible structural changes can occur in the market. As can be seen, models based on logistic regression are able to achieve high levels of accuracy and slightly outperform multilayer perceptrons. Furthermore, note that MLP models perform significantly worse in the out-of-sample test. Regarding decision trees, it seems that they do not provide satisfactory results from the constructed scenarios. In turn, Markov regime-switching models show acceptable results. However, they have the well-known disadvantage of being very sensitive to the predictable component estimation. As seen, all models are successful when facing the naive test. It is also interesting that, unlike in the in-sample dataset, the prediction ability is not improved when periodic models are used.

Table 21. Number of hours with very low prices expected per month.

Month	**Logistic Regression**			**Decision Trees**		**MLP**		**Hybrid**	**MRS**	**Naive 1**	**Naive 2**	**Actual**
	M1	**M2**	**M3**	**M1**	**M2**	**M1**	**M2**					
December	3.92	3.87	3.81	0.00	0.00	0.77	9.84	**4.14**	3.31	31.81	69.00	6
January	**8.81**	7.44	7.51	0.00	0.05	9.67	16.99	7.89	3.50	32.11	77.33	9
February	10.04	9.94	10.14	0.00	0.10	9.13	14.27	**11.59**	2.48	29.24	43.85	11
March	20.96	18.65	18.56	0.00	0.13	21.06	23.07	**15.38**	4.53	30.65	98.14	11

Since probabilistic forecasts are crucial for an adequate risk management, the proposed methodology has also been used to compute the probability of an extremely low price for each hour in the forecast period. Figure 10 shows the probability of the appearance of extremely low prices, which has been estimated on an hourly basis during a representative month.

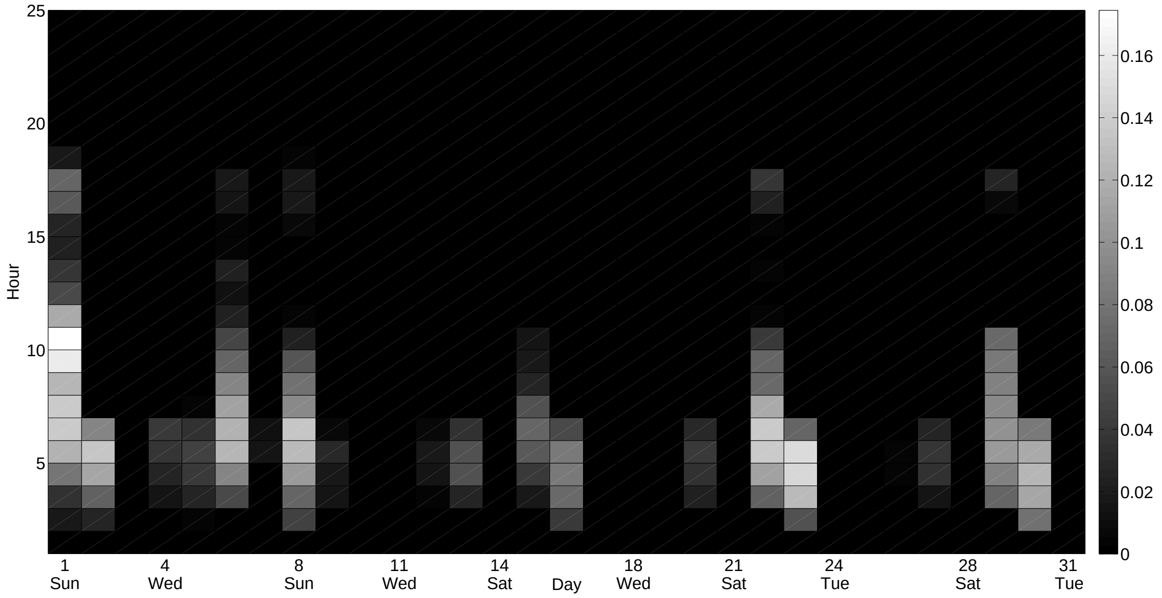

Figure 10. Probability of the appearance of extremely low prices predicted by the Logistic Regression M1 for January 2012.

Similarly, the proposed methodology has been applied to estimate the probability density function (PDF) associated with the number of hours with very low prices throughout the projection period. An illustrative example of the forecasted PDF for February 2012 by using the hybrid approach appears in Figure 11. In this figure, the dashed line represents the actual number of hours that occurred in the market. As shown, the distribution is unimodal and right-skewed. In this particular case, it is evident that the actual value always falls under the range of the most likely values. The real value (11 h) is near the mode value (8 h) and the expected value (11.59 h).

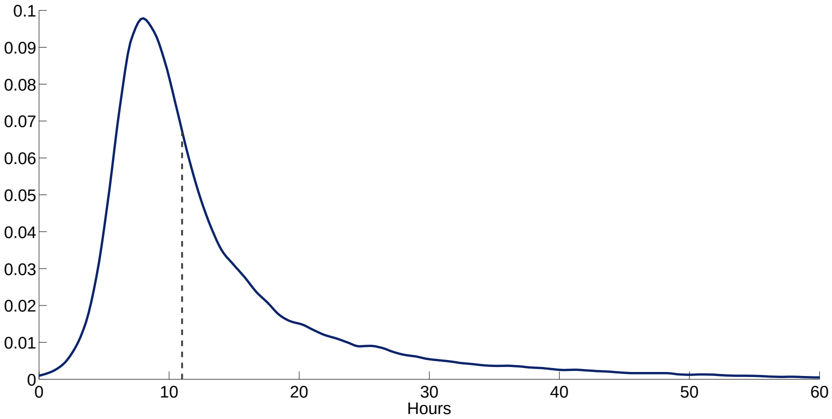

Figure 11. Probability density function for the number of hours with extremely low prices that has been predicted by the hybrid model for February 2012.

In order to compare the forecast quality of the proposed models, the Brier Score (BS) has been used. The BS is probably the most commonly-used verification measure for assessing the accuracy of binary probabilistic predictions. It is the mean squared error of the probability forecasts over the verification sample and it is expressed as:

$$BS = \frac{1}{S}\sum_{i=1}^{S}(p_i - o_i)^2 \tag{9}$$

where S is the sample size, p_i is the predicted probability of the event occurring according to the i-th hourly forecast and o_i is equal to one or zero, depending on whether the event subsequently occurred or not during that hour. The BS ranges from zero for a perfect forecast to one for the worst possible forecast.

With the objective of making it easier to interpret the results, two naive models have also been used as benchmarks. Naive 3 is based on the previous similar month, while Naive 4 relies on taking the historical values of month m-2 as forecasts of future prices for month m. A comparison of the results for the probabilistic estimates for all specifications is reported in Table 22. As can be seen, the obtained results suggest that the proposed hybrid methodology produces superior probabilistic forecasts than the rest of the alternative techniques. Naive techniques are clearly outperformed by all of the proposed procedures, which demonstrates the practical interest of the developed methodology. In this case, a slight increase in accuracy was obtained when considering different dynamics for each day of the week through the periodic models.

Table 22. Comparison of the proposed models in terms of the Brier Score (BS).

Logistic Regression			**Decision Trees**		**MLP**		**Hybrid**	**MRS**	**Naive 3**	**Naive 4**
M1	**M2**	**M3**	**M1**	**M2**	**M1**	**M2**				
0.01239	0.01213	0.01205	0.01264	0.01261	0.01246	0.01241	**0.01195**	0.01258	0.04611	0.03005

6. Conclusions

In this paper, a novel methodological approach to analyze and make real *ex ante* forecasts of the occurrence of extremely low prices in electricity markets with a medium-term horizon has been presented. The proposed methodology, which is a mixture of different forecasting techniques with a Monte Carlo simulation that integrates a spatial interpolation tool, is able to simultaneously perform punctual and probabilistic predictions with an hourly basis. The methodology has been specifically applied to the Spanish wholesale market, but may be extended equally to other electricity markets worldwide.

Logistic regression for rare events, decision trees, multilayer perceptrons and a novel hybrid approach, which is able to incorporate both fundamental and behavioral information, have been compared to a Markov regime switching model and several naive methods. Further research has been undertaken in order to evaluate whether periodic models, in which parameters switch according to the day of the week, can provide better prediction capabilities.

Overall, all of the proposed models present reasonable errors taking into account the complex nature of the phenomenon and substantially outperform naive techniques in both the in- and out-of-sample datasets. Encouraging results have been obtained from real *ex ante* forecasts of the distribution function of the exogenous variables used to predict the phenomenon. The results reveal that the integration of a market equilibrium model and logistic regression in a hybrid approach provides a significant improvement in the prediction accuracy in comparison to the individual models. We also found that the inclusion of a prior estimation of the market equilibrium price can provide valuable information when used as an input in a statistical technique, such as logistic regression, especially when there are structural or regulatory changes in the market. Logistic regression with the correction for rare events data has proven to be a simple, but effective tool enabling one to outperform multilayer perceptrons and decision trees in terms of forecasting accuracy. When the real explanatory variables are used, it is clear that MLP performs better than the other models, but it behaves significantly worse when making real *ex ante* forecasts. However, still, MLP is superior to decision trees. With respect to decision trees, they have shown that they can provide valuable information and offer great interpretability for probabilistic approaches.

Another interesting conclusion is that a meaningful improvement of the prediction capability is reached when considering different dynamics for working days, Saturdays, Sundays and holidays. Open research lines may include the extension of this methodology to extremely high priced hours and to other markets, where locational marginal prices may exist and for which the impact of variables related to local distributed generation should be taken into account.

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Nomenclature

D	Electric demand
H	Run-of the-river hydro production
HOL	Sundays and holidays
IE	Difference between exports and imports
N	Nuclear energy production
ND	Net demand (D-SR)
ND′	Net demand (ND + IE-N)
ND″	Net demand (ND-W-N-H)
SAT	Saturdays

SR	Special regime energy production
W	Wind energy production
WORK	Working days

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