Energy and Reserve Co-optimization of a Combined Cycle Plant Using Mixed Integer Linear Programming

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The growth in the importance of interruptible sources of energy is increasing the concerns of many electricity market regulators with respect to the reliability and stability of electricity supply. Decisions such as that to increase the number of reserve markets, their reserve requirements, or the role of reserve prices in the final electricity price have meant that generation plants are currently often operating with strategies to obtain not only large energy market quotes but also reserve ones. In this paper, a mixed integer linear programming (MILP) model is proposed to obtain the energy and reserve dispatch of a real combined cycle plant (CCP) to optimize its use on a weekly or annual basis. The dispatch is optimal in the sense that it maximizes the joint energy and reserve profits, including an estimation of the energy and reserve prices. The detailed technical and economic characteristics of the plant have been considered, such as start-ups, shut-downs, minimum hours for steam generation, supplementary firing, or natural gas contracts. The cases studies validate the main features of the mathematical model and analyze the computational efficiency in a realistic simulation. [DOI: 10.1115/1.4028002]

1 Introduction

Over recent years, the increasing investment in interruptible energy sources, such as renewable energy, has emphasized the need for new market mechanisms to guarantee that the supply of electricity meets the demand within a stable electricity supply network. Examples of such mechanisms are the establishment of higher reserve requirements for generation companies (GENCOs) in existing ancillary reserve markets or the creation of new auctions to ensure a system's reliability (such as the upward reserve market recently created in Spain for power plants that have not been cleared in the day-ahead market). Ancillary reserve services must respond almost simultaneously to system load fluctuations and restore the generation and load balance in the event of a contingency such as the loss of a generator or a line, the correction of demand, or solar or wind generation forecasting errors. Reserves are usually classified according to their technical characteristics: The response timescale in which they must be provided, the control mechanisms that ensure their coordination, and the type of event they must respond to. In Europe, for example, these services are classified into three different types: primary, secondary, and tertiary. Primary is a virtually instantaneous service for frequency variations controlled by an automatic generation control that coordinates multiple generators at different power plants. Secondary reserve responds basically to small load changes that must be compensated in a period ranging from a few seconds to a few minutes. Finally, tertiary reserve must be supplied within minutes and it is used to replace the secondary energy that is being used (see Ref. [1] for a review of the current types of reserves and for a classification of these services implemented in various countries).

Prior to the existence of large amounts of interruptible energy, fossil-fuel power plants with high variable costs were mainly relegated to running only in peak hours, particularly in oversized generation capacity systems. With the significant penetration of wind or other nondispatchable energy, these fossil plants will have to stop for very long time periods, compromising their capacity to

recover their investments. Nevertheless, the fact that renewable energy is subject to multiple uncertainties (mainly related to weather conditions) increases the importance of reserve markets and brings new opportunities for such fossil plants, especially for those with the ability to provide a large quantity of reserve in a very short period of time. This is precisely the case of CCPs, since they have fast ramp-rates due to the physicochemical properties of the natural gas burned and their gas/steam cycles [2].

In this new scenario, new powerful models which represent CCP operation and take into account both energy and reserve supply services have become increasingly necessary. Consequently, the majority of the research presented in the literature related to CCP modeling focuses on the representation of the process of selecting and facilitating the control strategies of CCPs in real time. This research models the technical aspects of the main design components of CCPs, such as air compressors, boilers, gas turbines (GTs), heat recovery steam generators (HRSGs), steam turbines (STs), or alternators, and even circuitry, shafts, valves, pipes, and so on. It also analyzes CCP operating modes and control characteristics. The main resolution methodology of these approaches is based on the application of dynamic simulation by means of the use of different standard commercial simulators (see, for example, Ref. [3] for a steady-state simulation of a CCP, applying software from Lehigh University) or by using soft computing methods (see, for example, Ref. [4] for the optimization of the start-up costs of a CCP using fuzzy-logic and evolutionary computation).

Nevertheless, few CCP models optimize the operation decisions of CCPs in larger time frames. Economic dispatch (ED) and unit commitment (UC) are the main approaches used in these cases. ED provides the optimal electricity production for all scheduled generating units (including CCPs) in order that the system load can be supplied in the most economical way, subject to transmission and operational constraints [5]. In Ref. [6], the theoretical aspects of applying constrained optimization to obtain the ED of CCPs are discussed, and a maximum level of greenhouse gas emissions is proposed (though CCPs can be considered to be environmentally friendly). The resolution of the UC problem, on the other hand, determines the generation schedule for a given combination of generating units, satisfying a set of dynamic operational

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and environmental constraints at minimum cost. In particular, UC optimizes the multiple operating configurations of CCPs, that is, the different modes in which certain elements or functions of CCPs are turned on or off (such as the state of the ST, see Ref. [7]). The objective of the UC approach is to supply a fixed generation program that has been assigned to each CCP as well as computing the optimal CCP operating configurations, and usually for larger planning periods than in ED (typically 1 week for UC, see Ref. [8]). Since the operating configurations involve discrete decisions, UC always entails integer variables that complicate the resolution. In Refs. [7] and [9], a UC is solved by means of using dynamic programming to analyze a state-transition diagram in which each node represents a particular configuration. Reserve commitments and network constraints are included in Ref. [7] and solved applying Bender decomposition [10] to minimize the network violation in each Bender subproblem. Nevertheless, suboptimal solutions can be obtained, especially if interhourly constraints, such as ramps, are included. In addition, large computational times are sometimes obtained if a large number of states are considered (as occurs when modeling start-up and shut-down decisions, or minimum on/off time constraints). These conditions are included in Ref. [11], where a very interesting UC model is presented. This model optimizes the operating modes by using a joint energy and reserve MILP model, while taking into account other complex conditions such as ramping constraints and limits on the number of turbines running. In addition, computational efficiency is not compromised, thank to the use of the powerful optimizer Cplex [12] in one of its latest versions.

This paper proposes a price-based UC model for the optimization of generation and secondary reserve operations, using a similar MILP to Ref. [11]. Instead of minimizing costs, GENCO profits in the day-ahead and reserve markets are maximized by

considering energy and reserve prices to be input data. This means that no demand meeting constraints are required, and only the CCP to be analyzed is represented. In contrast to Ref. [11], gas contracts, daily and monthly gas capacities (tolls), and CO₂ emissions costs have been taken into account. Several peculiarities of CCP operation are also considered, such as the minimum time and the minimum production levels required in order to start-up the ST or supplementary firing processes; however, network constraints are omitted. As in the CCC model of Ref. [11], the turbines have been represented as individual components rather than by means of a finite number of operation modes. Spinning reserve is also represented but for the CCP under study there is no necessity to include additional ramps constraints in an hourly schedule, since the ramp-rates of the CCP are sufficiently high (today's fast start CCPs can reach full load in 18 min or less, see Ref. [13]). See Ref. [14] to include ramp constraints when modeling CCPs.

This document is organized as follows. In Sec. 2, an overview of the analyzed CCP is presented. Section 3 describes the main assumptions and modeling issues with respect to CCPs and gas contracts. The detailed mathematical formulation of the proposed MILP is also presented. Finally, several case studies and conclusions will end this paper.

2 General Description of the CCP

The CCP modeled in this paper has four GT and four HRSG, but only one ST. Figure 1 shows the basic scheme of the CCP studied here, avoiding, for the sake of simplicity, the detailed representation of the high pressure (HP) and the low pressure (LP) water/steam circuitry, or other more specific components such as pumps, drums, or economizers.

Fig. 1 CCP design

The CCP in this study can operate with any combination of GTs turned on or off, and each supplementary burner in the HRSG for the ST running can also be turned on or off. Each HRSG introduces two streams of steam that are expanded by the ST: HP and LP flows. These gas volumes depend on whether the supplementary burner is used or not. If the burner is turned on, the total electricity generated in the ST is increased at a certain cost, since some quantity of additional natural gas is consumed. In fact, these supplementary burners are often utilized only in peak hours. Finally, each GT has a bypass stack that allows the turbine to run in simple cycle mode, isolating the HRSG used in combined cycle.

3 CCP Modeling With MILP

This section describes the main assumptions and the mathematical formulation of the proposed MILP problem. Super index $\sim$ refers to the ST, capital letters are used for the decision variables, and small letters for the parameters.

3.1 Decision Variables. The main decisions to be optimized in the CCP operation provide answers to the following questions:

- Considering a time horizon of 1 yr, is it necessary to turn on the ST in addition to GTs (that is, in this last case the CCP running in combined cycle)?
- Binary variables $U_{u,h}$ indicate if each GT u is turned on at hour h and $U_h^\sim$ indicate if the ST is running.
- When does the ST have to be turned on, started up or shut down, in addition to each GT?
- In this case, binary variables ($U_{u,h}$ and $U_h^\sim$) indicate if each GT and the ST are turned on, respectively, while ($Y_{u,h}$ and $Y_h^\sim$) and ($Z_{u,h}$ and $Z_h^\sim$) are the corresponding binary variables for start-up and shut-down decisions, respectively.
- Is the CCP running in a combined cycle and is it necessary to apply supplementary firing to the gases resulting from the combustion in each HRSG of each GT?
- The binary variable $U_{u,h}^{i,\sim}$ indicates whether each GT is in combined cycle and whether it is generating without ($i=0$) or with supplementary firing ($i=1$).
- With supplementary firing, what is the degree $U_{u,h}^{2,\sim} \in [0,1]$ of supplementary firing?

It is also necessary to quantify

- The gross ($Q_{u,h}$ and $Q_h^\sim$) and net ($P_{u,h}$ and $P_h^\sim$) energy generated by each GT and by the ST (and therefore to know what the total energy sold by the CCP in the day-ahead market is).
- The upward ($UR_{u,h}$ and $UR_h^\sim$) and downward reserve ($DR_{u,h}$ and $DR_h^\sim$) provided by each GT and the ST and sold in the reserve market.
- The total consumption $G_{u,h}$ of natural gas by each GT, taking into account the supplementary firing process (if this is applicable).
- The monthly J_m and daily J_d capacities of gas contracted by the GENCO to be able to deal with unpredictable closer-in-time gas demands by the CCP.

Some other less significant variables are described in the Nomenclature section of this paper.

3.2 Objective Function. The objective function consists in the maximization of the company's margin as the difference between revenues and costs. The total revenues of the company consist of the following items (see Nomenclature for a better understanding of their formulations).

- (1) Revenues from the day-ahead energy market

$$\sum_h \{P_h \cdot p_h\} \quad (1)$$

- (2) Revenues from the secondary reserve market

$$\sum_h \{(UR_h + DR_h) \cdot p_h^s\} \quad (2)$$

- (3) Revenues from the energy bought or sold within the reserve

$$\sum_h \{p_h \cdot (PUR_h \cdot n^{ur} - PDR_h \cdot n^{dr})\} \quad (3)$$

When considering Eq. (3), it is important to remember that the model optimizes the remuneration of the reserve in terms of providing opportunities to sell or buy energy, taking into account the energy finally sold or bought within such a reserve (here called regulating energy). This is especially relevant in the Spanish case, in which regulating energy is remunerated using the marginal price of the tertiary market (see Ref. [15] for more details).

The cost function to be considered consists of the following:

- (1) Through a forward natural gas supply contract, GENCOs are required to pay the gas supplier an agreed price, cng , for the natural gas burned in his CCP, instead of a fluctuating spot price. This allows GENCOs to manage the risk associated with the highly volatile pricing of natural gas and produces the following fuel cost (natural gas cost):

$$\sum_h \{cng \cdot (G_h + GUR_h - GDR_h)\} \quad (4)$$

- (2) The maintenance cost computed through the number of hours of each turbine being committed

$$\sum_{u,h} \{cm_u \cdot U_{u,h}\} + \sum_h \{cm^\sim \cdot U_h^\sim\} \quad (5)$$

- (3) Start-up and shut-down costs

$$\begin{aligned} & \sum_{u,h} \{cst_u \cdot Y_{u,h}\} + \sum_h \{cst^\sim \cdot Y_h^\sim\} \\ & + \sum_{u,h} \{csh_u \cdot Z_{u,h}\} + \sum_h \{csh^\sim \cdot Z_h^\sim\} \end{aligned} \quad (6)$$

- (4) CO₂ emissions costs

$$\sum_{u,h} \{p^{emis} \cdot c_u^{emis} \cdot (G_{u,h} + GUR_{u,h} - GDR_{u,h})\} \quad (7)$$

- (5) By means of another contract, GENCOs also pay the network operator (who is also normally the pipeline company, as in most European countries), for the use of the infrastructure needed in the delivery. The unit cost in this case, referred to as "variable tariff" (v), depends on the quantity of natural gas to be delivered and is as follows:

$$\sum_h \{(G_h + GUR_h - GDR_h) \cdot v\} \quad (8)$$

- (6) The main flaw in these contracts is that v may not be sufficient for the recovery of the investment in pipeline. A further problem is inflexibility in demand and fluctuation in supply. To mitigate this, the network operator and the GENCO usually agree on some specific clauses in contracts. One of these consists in a commitment on the part of the GENCO to specify not only monthly capacities J_m for the gas pipeline but also daily ones J_d , to deal with the demand for gas, the corresponding daily tariffs being higher than the monthly ones. Daily and monthly tariffs are represented in this paper by monthly coefficients (d_m and m_m ,

respectively) with respect to a base annual tariff (f), called “fixed tariff.” These tariffs result in the following fixed costs:

$$\sum_{d,m:d \in m} \{J_d \cdot d_m \cdot f\} + \sum_m \{J_m \cdot m_m \cdot f\} \quad (9)$$

3.3 Constraints. The objective function described above must be optimized subject to the following constraints.

GT Modeling. Relation between gross and net power

$$P_{u,h} = Q_{u,h} \cdot (1 - ca_u) \quad (10)$$

Each GT is required to reach different minimum power generation levels, depending on whether the GT is running without producing steam (minimum denoted as $q_u^{\min}$), producing steam without supplementary firing ($q_u^{\min,0,\sim}$), or producing steam and applying supplementary firing ($q_u^{\min,1,\sim}$). The first constraint of Eq. (11) models the relationship between the final minimum gross power $Q_{u,h}^{V,\min}$ and the different minimum power limits $q_u^{\min}$, $q_u^{\min,0,\sim}$, and $q_u^{\min,1,\sim}$ for each of the mentioned three states, by using their corresponding binary variables $U_{u,h}$, $U_{u,h}^{0,\sim}$, and $U_{u,h}^{1,\sim}$. The second constraint represents the power $Q_{u,h}^{\min}$ above $Q_{u,h}^{V,\min}$, and the last constraint defines the total gross power $Q_{u,h}$ generated

$$\begin{aligned} Q_{u,h}^{V,\min} &= \begin{cases} q_u^{\min} \cdot U_{u,h} \\ + (q_u^{\min,0,\sim} - q_u^{\min}) \cdot U_{u,h}^{0,\sim} \\ + (q_u^{\min,1,\sim} - q_u^{\min,0,\sim}) \cdot U_{u,h}^{1,\sim} \end{cases} \\ Q_{u,h}^{\min} &\leq (q_u^{\max} \cdot U_{u,h} - Q_{u,h}^{V,\min}) \cdot d_{u,h} \\ Q_{u,h} &= Q_{u,h}^{\min} + Q_{u,h}^{V,\min} \cdot d_{u,h} \end{aligned} \quad (11)$$

The gas consumption $GW_{u,h}$ (without taking into account the consumption in the HRSG) is modeled as a linear function of the gross power $Q_{u,h}$, and considering a minimum gas consumption $g_u^{\min}$ necessary to turn on the CCP (this linear function, see Fig. 2, have been estimated based on a linear regression using experimental data directly obtained from the CCP operation and with a coefficient of determination not lesser than 0.92)

$$GW_{u,h} = g_u^{\min} \cdot U_{u,h} + \alpha_u \cdot Q_{u,h} \quad (12)$$

Unlike the CCP under study in this paper, Ref. [16] introduces nonlinear equations for the modeling of these performance curves for other CCPs and proposes an approximation of these curves by means of piecewise linear functions, which leads to a MILP optimization model, as in this research.

Fig. 2 Gas consumption as a linear function of gross power

Limits on the upward and downward spinning reserves

$$\begin{aligned} UR_{u,h} &\leq q_{u,h}^{\max} \cdot d_{u,h} \cdot (1 - ca_u) \cdot U_{u,h} - P_{u,h} \\ DR_{u,h} &\leq P_{u,h} - Q_{u,h}^{V,\min} \cdot d_{u,h} \cdot (1 - ca_u) \\ UR_{u,h} + DR_{u,h} &\leq b_u^{\max} \end{aligned} \quad (13)$$

Apart from the use of supplementary firing mode, there are occasions, especially when the reserve price is high and the day-ahead price is low, when some GTs may be configured in order to provide only upward reserve without being turned on for a whole hour. This is called the “fast start” operation mode. The relation between the upward reserve and the binary variable $UF_{u,h}$ for this mode is represented as

$$\begin{aligned} UR_{u,h} &\leq bs_u^{\max} \cdot (U_{u,h} + UF_{u,h}) \\ U_{u,h} + UF_{u,h} &\leq 1 \end{aligned} \quad (14)$$

The first two constraints of Eq. (15) model the cleared energy within the upward and downward reserves by using the percentages q_{ur} and q_{dr} of energy cleared within each type of reserve. Equation (15) also includes the relationship between the gross and the net cleared energy within each reserve by means of the auxiliary services coefficient ca_u

$$\begin{aligned} PUR_{u,h} &= UR_{u,h} \cdot q^{ur} \\ PDR_{u,h} &= DR_{u,h} \cdot q^{dr} \\ QUR_{u,h} &= PUR_{u,h} / (1 - ca_u) \\ QDR_{u,h} &= PDR_{u,h} / (1 - ca_u) \end{aligned} \quad (15)$$

Definition of the gas consumed within the reserve

$$\begin{aligned} GUR_{u,h} &= \alpha_u \cdot QUR_{u,h} \\ GDR_{u,h} &= \alpha_u \cdot QDR_{u,h} \end{aligned} \quad (16)$$

Coherency between the start-up, shut-down, and turn-on decisions

$$Y_{u,h} - Z_{u,h} = U_{u,h} - U_{u,h-1} \quad (17)$$

HRSG Modeling. In this paper, there are assumed to be three main operation modes for each HRSG, depending on if the supplementary burner is turned off (no supplementary firing, $i=0$); it is turned on, burning a minimum natural gas quantity (minimum supplementary firing mode, $i=1$); or it is turned on and burning a quantity above the minimum quantity mentioned above (supplementary firing mode, $i=2$). Thus, if $U_{u,h}^{i,\sim}$ is a binary variable indicating mode i activation for $i \in \{0,1\}$, and $U_{u,h}^{2,\sim}$ a continuous variable in $[0,1]$ graduating the natural gas burned between these minimum and maximum quantities, then the total natural gas consumption in terms of the gas used in the HRSG (without supplementary firing, $i=0$ or additional consumption) is represented as

$$G_{u,h} = GW_{u,h} + \sum_{i=1,2} U_{u,h}^{i,\sim} \cdot gc_i^{\sim} + fs_u \cdot g_u^{\min} \cdot UF_{u,h} \quad (18)$$

Note that Eq. (18) includes the gas consumed during the fast start mode. This consumption is estimated for each hour using the fraction fs_u of the hour in which the GT will actually be turned on (because the system operator requires the use of the upward reserve). As mentioned, the unit gas consumption decreases as the output gross power increases and, thus, for this fraction of the hour, the corresponding minimum gas consumption $g_u^{\min}$ (see Fig. 2) has to be taken into account.

In addition, in Eq. (18), it is assumed that gas consumed $gc_i^{\sim}$ is an incremental value with respect to the previous mode $i-1$ and therefore,

$$U_{u,h}^{i-1,\sim} \geq U_{u,h}^{i,\sim} \quad (19)$$

If the GT is not turned on, then it cannot provide steam

$$U_{u,h}^{0,\sim} \leq U_{u,h} \quad (20)$$

However, once it has been decided that a GT must apply supplementary firing, i.e., $U_{u,h}^{1,\sim} = 1$, the next important issue is to optimize the level of supplementary firing. In this paper, it is assumed that the continuous variable $U_{u,h}^{2,\sim}$ in $[0,1]$ represents this level, from a minimum firing (with $U_{u,h}^{2,\sim} = 0$ and $Q_{u,h} = q_{u,h}^{\min,1,\sim}$) to a maximum firing (with $U_{u,h}^{2,\sim} = 1$ and $Q_{u,h} = q_{u,h}^{\max}$). Intermediate values of $U_{u,h}^{2,\sim}$ (in $(0,1)$) are assumed to be proportional to $Q_{u,h} - q_{u,h}^{\min,1,\sim}$ in a linear way, that is

$$U_{u,h}^{2,\sim} = \frac{Q_{u,h} - q_{u,h}^{\min,1,\sim}}{q_{u,h}^{\max} - q_{u,h}^{\min,1,\sim}} \quad (21)$$

Since this expression must be true only when $U_{u,h}^{1,\sim} = 1$, Eq. (22) presents the linear formulation as a function of $U_{u,h}^{1,\sim}$. Note that if $U_{u,h}^{1,\sim} = 0$, Eq. (22) does not constrain

$$\begin{aligned} U_{u,h}^{2,\sim} - \frac{Q_{u,h} - q_{u,h}^{\min,1,\sim}}{q_{u,h}^{\max} - q_{u,h}^{\min,1,\sim}} &\leq (1 - U_{u,h}^{1,\sim}) \cdot 1 \\ U_{u,h}^{2,\sim} - \frac{Q_{u,h} - q_{u,h}^{\min,1,\sim}}{q_{u,h}^{\max} - q_{u,h}^{\min,1,\sim}} &\geq (1 - U_{u,h}^{1,\sim}) \cdot (-1) \end{aligned} \quad (22)$$

Another technical requirement is that if a GT provides reserve, that is, $UR_{u,h} + DR_{u,h} > 0$, and steam, i.e., $U_{u,h}^{0,\sim} = 1$, at the same time supplementary firing is necessary, that is, $U_{u,h}^{1,\sim} = 1$, which is modeled as

$$U_{u,h}^{1,\sim} \geq (U_{u,h}^{0,\sim} - 1) + \frac{UR_{u,h} + DR_{u,h}}{b_{u,h}^{\max}} \quad (23)$$

One important technical constraint (on each GT) is the need to produce above an amount of power during a number of hours $t^{\sim}$ previous to the production of steam. It is also necessary that the GT is turned on (above its minimum technical power) 1 h before this time period. If the amount of power to be supplied during this period is imposed in terms of a load $u^{\sim}$, Eq. (24) uses the binary variable $V_{u,h}^{\sim}$ as an auxiliary variable such that $V_{u,h}^{\sim} = 1$ if the load of each GT is above $u^{\sim}$ in h and turned on in $h-1$

$$2 \cdot V_{u,h}^{\sim} \leq \frac{(Q_{u,h}/q_{u,h}^{\max})}{u^{\sim}} + U_{u,h-1} \quad (24)$$

Using $V_{u,h}^{\sim}$, Eq. (25) means that if a GT is producing steam ($U_{u,h}^{0,\sim} = 1$), then the previous $t^{\sim}$ variables $V_{u,h-1}^{\sim}, \dots, V_{u,h-t^{\sim}}^{\sim}$ must be equal to 1

$$t^{\sim} \cdot U_{u,h}^{0,\sim} \leq \sum_{h-t^{\sim} \leq h' < h} V_{u,h'}^{\sim} \quad (25)$$

ST Modeling. Relation between the net and gross powers

$$P_h^{\sim} = Q_h^{\sim} \cdot (1 - ca^{\sim}) \quad (26)$$

Maximum and minimum power

$$q^{\min,\sim} \cdot U_h^{\sim} \leq Q_h^{\sim} \leq q_h^{\max,\sim} \cdot U_h^{\sim} \quad (27)$$

Coherency between the binary variables which indicate if a GT is producing steam and the status of the ST

$$U_h^{\sim} \leq \sum_u U_{u,h}^{0,\sim} \leq 4 \cdot U_h^{\sim} \quad (28)$$

In this paper, it has been assumed that the total generated gross production at each hour by the ST can be characterized as a linear regression of the HP and LP flows, that is

$$Q_h^{\sim}(hpf_h, lpf_h) = k_0 + k_1 \cdot hpf_h + k_2 \cdot lpf_h \quad (29)$$

the estimation of coefficients k_i being based on historical information. The HP and LP flows are at the same time expressed using incremental flows $hpf_{i,u,h}$ and $lpf_{i,u,h}$ depending on the three operation modes described in the HRSG modeling (no supplementary firing, $i=0$; minimum supplementary firing mode, $i=1$; or supplementary firing mode, $i=2$)

$$\begin{aligned} hpf_h &= \sum_{i,u} U_{u,h}^{i,\sim} \cdot hpf_{i,u,h} \\ lpf_h &= \sum_{i,u} U_{u,h}^{i,\sim} \cdot lpf_{i,u,h} \end{aligned} \quad (30)$$

Flows $hpf_{i,u,h}$ and $lpf_{i,u,h}$ are also modeled via linear regressions,¹ in this case using the gross production generated by each GT as the explanatory variable

$$\begin{aligned} hpf_{i,u}(Q_{u,h}) &= a_{i,u}^{hp} + b_{i,u}^{hp} \cdot Q_{u,h} \\ lpf_{i,u}(Q_{u,h}) &= a_{i,u}^{lp} + b_{i,u}^{lp} \cdot Q_{u,h} \end{aligned} \quad (31)$$

Since the three operation modes described are incremental, Eq. (30) can be expressed using Eq. (31) as

$$\begin{aligned} hpf_h &= \sum_{i,u} U_{u,h}^{i,\sim} \cdot \left\{ (a_{i,u}^{hp} - a_{i-1,u}^{hp}) + (b_{i,u}^{hp} - b_{i-1,u}^{hp}) \cdot Q_{u,h} \right\} \\ lpf_h &= \sum_{i,u} U_{u,h}^{i,\sim} \cdot \left\{ (a_{i,u}^{lp} - a_{i-1,u}^{lp}) + (b_{i,u}^{lp} - b_{i-1,u}^{lp}) \cdot Q_{u,h} \right\} \end{aligned} \quad (32)$$

Equation (29) is therefore equivalent to

$$Q_h^{\sim} = k_0 + \sum_{i,u} U_{u,h}^{i,\sim} \cdot \{ ka_{i,u,h} + kb_{i,u,h} \cdot Q_{u,h} \} \quad (33)$$

being

$$\begin{aligned} ka_{i,u} &= k_1 \cdot (a_{i,u}^{hp} - a_{i-1,u}^{hp}) + k_2 \cdot (a_{i,u}^{lp} - a_{i-1,u}^{lp}) \\ kb_{i,u} &= k_1 \cdot (b_{i,u}^{hp} - b_{i-1,u}^{hp}) + k_2 \cdot (b_{i,u}^{lp} - b_{i-1,u}^{lp}) \end{aligned} \quad (34)$$

Finally, Eq. (35) is the linear formulation of the ST total power defined in Eq. (33) and included in the proposed MILP

$$\begin{aligned} Q_h^{\sim} &= k_0 \cdot U_h^{\sim} + \sum_i Q_{u,h}^{i,\sim} \\ Q_{u,h}^{i,\sim} &\leq ka_{i,u} \cdot U_{u,h}^{i,\sim} + kb_{i,u} \cdot Q_{u,h} \\ Q_{u,h}^{i,\sim} &\leq \{ ka_{i,u} + kb_{i,u} \cdot q_{u,h}^{\max} \} \cdot U_{u,h}^{i,\sim} \end{aligned} \quad (35)$$

Limits on the upward and downward spinning reserves are also imposed

¹Statistical analyses using real data taken directly from the CCP confirm that all the regressions are very accurate, with coefficients of determination always higher than 0.9 even in pessimistic scenarios.

$$\begin{aligned} UR_h^\sim &\leq q_h^{\max,\sim} \cdot (1 - ca^\sim) \cdot U_h^\sim - P_h^\sim \\ DR_h^\sim &\leq P_h^\sim - q_h^{\min,\sim} \cdot (1 - ca^\sim) \cdot U_h^\sim \\ UR_h^\sim + DR_h^\sim &\leq b^{\max,\sim} \end{aligned} \quad (36)$$

The definition of the cleared energy within the reserve is

$$\begin{aligned} PUR_h^\sim &= UR_h^\sim \cdot q^{\text{ur}} \\ PDR_h^\sim &= DR_h^\sim \cdot q^{\text{dr}} \end{aligned} \quad (37)$$

Coherency between the start-up, shut-down, and turn-on decisions is represented as

$$Y_h^\sim - Z_h^\sim = U_h^\sim - U_{h-1}^\sim \quad (38)$$

Modeling of Gas Contracts. Apart from the specific price provisions modeled in Eqs. (8) and (9), a nonpricing clause that reduces the supplier's risk and that has received much attention is the take-or-pay clause. This clause requires GENCOs to pay for a specific minimum quantity of natural gas, even if this is not burned. Therefore, GENCOs either use the product or simply pay the supplier for such a minimum quantity. Take-or-pay clauses are represented in this paper using the minimum daily and monthly natural gas quantities ($c_d^{\min}$ and $c_m^{\min}$) to be supplied. Maximum quantities may also be imposed

$$\begin{aligned} c_d^{\min} &\leq \sum_{h \in d} (G_h + GUR_h + GDR_h) \leq c_d^{\max} \\ c_m^{\min} &\leq \sum_{h \in m} (G_h + GUR_h + GDR_h) \leq c_m^{\max} \end{aligned} \quad (39)$$

Daily consumption is limited by the total daily contracted capacity

$$\sum_{h \in d} \{G_h + GUR_h - GDR_h\} \leq J_d + \sum_{m: d \in m} J_m \quad (40)$$

Other Constraints. Definition of aggregated variables

$$\begin{aligned} Q_h &= \sum_u Q_{u,h} + Q_h^\sim \\ G_h &= \sum_u G_{u,h} \\ UR_h &= \sum_u UR_{u,h} + UR_h^\sim \\ DR_h &= \sum_u DR_{u,h} + DR_h^\sim \\ GUR_h &= \sum_u GUR_{u,h} \\ GDR_h &= \sum_u GDR_{u,h} \\ PUR_h &= \sum_u PUR_{u,h} + PUR_h^\sim \\ PDR_h &= \sum_u PDR_{u,h} + PDR_h^\sim \end{aligned} \quad (41)$$

If this model is used to represent a Spanish CCP (as occurs in the case studies), a single reserve price must be considered since upward and downward reserves are related

$$UR_h = DR_h \cdot \text{urdr}_h \quad (42)$$

A certified maximum net power can be also considered

$$\left\{ \sum_u (P_{u,h} + UR_{u,h}) \right\} + (P_h^\sim + UR_h^\sim) \leq cq^{\max} \quad (43)$$

An upper bound $cb^{\max}$ over the total reserve generated with the CCP is required to be certified by the system operator. Equation (44) models $cb^{\max}$ taking into account the fact that it is given by each GT turned on

$$UR_h + DR_h \leq cb^{\max} \cdot \sum_u (U_{u,h} + UF_{u,h}) \quad (44)$$

4 Case Studies

This section presents several executions of the proposed MILP model to simulate the day-ahead and secondary reserve markets in Spain [17] for the whole of 2013.

Execution Modes. Three execution modes for the whole year were tested:

- Weekly optimization (WO): one representative week for each month is optimized and the results are extrapolated to the rest of weeks of the month, taking into account the day of the week.
- Monthly optimization (MO): each month is sequentially optimized, the CCP state at the end of previous executed months being considered as an output.
- Joint optimization (JO): with all the hours of the year optimized at the same time.

The executions were run on a 64-bit intercore CPU at 3.4 GHz, programmed in GAMS² and solved using Cplex solver. Interior point methods for the linear problems [18] and the branch and bound algorithm [19] available in Cplex with the default cut identification strategies have been applied.

Input Data. A real Spanish CCP with the structure presented in Fig. 1 is analyzed. The following is a set of technical inputs used in the analysis with slight differences with respect to the real values used in the real operation, for reasons of confidentiality:

- The nominal output of the CCP with the ST turned off is $\sum_u q_{u,h}^{\min} = 200$ MW, while the ST is able to increase the power by an extra $q_u^{\max,\sim} = 100$ MW.
- Total minimum gross power for the GT with the ST turned off and on are $\sum_u q_{u,h}^{\min} = 52$ and $\sum_u q_{u,h}^{\min,0,\sim} = 50$ MW, respectively. Minimum gross power when generating with the ST and with supplementary firing is $\sum_u q_{u,h}^{\min,1,\sim} = 75$ MW.
- Maximum secondary reserve is $b_u^{\max} = 48.2$ MW and $b^{\max,\sim} = 80$ MW for the ST.
- Auxiliary services coefficients are $ca_u = 1.8\%$ and $ca^\sim = 5\%$, respectively.
- CO₂ emission factors have been established to 0.2 tCO₂/MWh.
- Minimum monthly gas consumption $c_m^{\min,T}$ is 600 MWh.
- To produce steam, each GT must be above a load $u^\sim$ during $t^\sim = 3$ h.

The actual day-ahead and secondary reserve Spanish prices for Jan. 2013 are depicted in Fig. 3.

High Price Scenario Analyses. Profiles of Fig. 3 have been modified to test how the CCP operates in a high price scenario. The baseline case corresponds to the real market prices of 2013 (first month in Fig. 3), while the scenario labeled as "IncP" assumes an increase of 25% in energy prices, maintaining the reserve price fixed at its base value. Scenario "IncPs" is similarly defined but with an increase in the reserve price of 50%. Scenario IncP is reasonable⁷⁴¹ given that renewable technologies are to lose their current regulated incentives and given a possible growth in demand after the current economic crisis. IncPs is also reasonable given the premise of increasing reserve requirements and

²<http://www.gams.com/>.

Fig. 3 Day-ahead and reserve prices for Jan. 2013

prices that encourage CCPs to run as a backup for intermittent technologies.

For Jan. 2013, Fig. 4 shows the CCP final revenues and costs for each scenario in euros applying the MO execution mode. The labels used and their significance are as follows:

- Rev_md: revenues in the day-ahead market
- Rev_ms: revenues in the reserve market
- Rev_rs: revenues due to regulating energy
- Cost_vac: maintenance costs
- Cost_ng: cost of natural gas
- Cost_sud: start-up and shut-down costs
- Cost_emis: cost of emissions
- Cost_peajv: variable cost of contracted capacities of gas
- Cost_peajfm: cost of monthly contracted capacities of gas
- Cost_peajfd: cost of daily contracted capacities of gas

Table 1 presents the number of hours and the CCP efficiency for each operation mode, in which

- Cycle denotes if the CCP is running with the ST turned on (CC) or not (CS).
- Postc indicates whether the HRSG is running with (CP) or without (SP) supplementary firing.
- Numgt indicates the number of GTs turned on and not running in fast start mode (GT*i* indicating *i* GT turned on).
- Fast indicates the number of GTs turned on in fast start mode.
- Func indicates whether the CCP is providing reserve (REG) or not (running at PEAKS).
- STOP stands for the number of hours where the CCP is totally turned off.
- Efic (%) indicates the efficiency of each operation mode as the ratio between the total net energy generated and the thermal energy used in this generation. As average value approximations, the plant's thermal efficiency is approximately 30–35% in its single cycle, with an overall efficiency of 40–45% using the combined cycle. This efficiency is reduced to 35–40% with supplementary firing but compensated by a higher reserve generation which enables the plant to respond to fluctuations in electricity load.
- Nhours (h) indicates the number of hours running in each operation mode.

The total gas consumption (Cons_gas), the net and gross productions (Prod and Prodg), and the upward and downward reserves (Reser_uw and Reser_dw) are depicted in Fig. 5 for Jan. 2013.

As can be seen, since the CCP has been running for very few hours at the current energy and reserve prices (see the baseline scenario in Table 1), revenues are almost null (see Fig. 4, which is completely in line with the actual operation of many Spanish CCPs). Revenues in the IncP and IncPs scenarios rise and fuel costs are almost covered by the day-ahead market, reserve market revenues being a key issue for the economic survival of the CCP. An increase in reserve revenues would be justified by a scenario in which high penetration of renewable energy meant that CC technology played a decisive role as backup generation to provide reserve in Spain (unlike other conventional technologies, CCP ramp-rates are faster when regulating). Furthermore, delivered gas costs are higher in IncP than at IncPs (see Cost_ng in Fig. 4) since in the former a greater quantity of fuel is burned in the energy market due to the opportunity energy price (see Fig. 5). In Fig. 5, it can be seen that reserves provided at IncP and IncPs are similar, but the CCP operation also achieves energy production goals at IncP. Table 1 also shows that efficiency increases from values of 23% to 40–43% when the two thermodynamic cycles of the CCP are combined, increasing the efficiency of the process.

Figure 6 shows the evolution of the net production, gas consumption, upward and downward reserves, and turn-on and start-up binary decisions, during 6 h on Jan. 3, 2013 and for the IncP scenario. It depicts the operation of the CCP from a turned-off operation mode (hour h14) to a full operation mode with every GT turned on (hour h17 onwards), with the exception of two GTs (un2 and un3) that are providing upward reserve but they are not turned on (fast start mode).

For Jan. 3, 2013, the evolution of the net production and the total reserve provided by all the GTs (GT.Prod and GT.Reser) and by the ST (ST.Prod and ST.Reser) in relation to the energy and reserve prices is depicted in Fig. 7.

For the first hours of the day (from h01 until h09), the CCP runs in regulation mode since reserve prices are high. In particular, two GTs are in fast start mode and another GT at maximum power to provide downward reserve. When the reserve price decreases, the CCP shuts down (from h10 to h14) until the energy price begins to increase (from h15 onwards). More specifically,

Fig. 4 Economic results for Jan. 2013

Table 1 Number of hours and efficiency for each operation mode and for Jan. 2013

Scenario	Cycle	Postc	Numgt	Fasts	Func	Efic (%)	Nhours (h)
Baseline	CS	SP	GT1	2	REG	25.60	47
	STOP	STOP	GT0	0	STOP	0	697
IncP	CC	CP	GT0	1	REG	31.20	5
				2	REG	30.20	146
				3	REG	27.50	18
			GT2	1	REG	39.20	12
				2	REG	34.91	115
				1	REG	34.80	5
			GT1	0	PEAKS	40.40	25
			GT2	1	REG	40.00	48
			GT3	2	REG	32.68	51
				0	PEAKS	43.80	23
				1	REG	38.04	26
	CS	SP	GT1	1	REG	26.60	9
				2	REG	25.60	97
				3	REG	23.00	4
	STOP	STOP	GT2	2	REG	26.77	29
			GT0	0	STOP	0	131
			GT1	1	REG	30.60	4
IncPs	CC	CP	GT1	2	REG	30.20	201
				1	REG	35.10	4
	CS	SP	GT2	1	REG	27.40	37
			GT1	2	REG	25.60	163
	STOP	STOP	GT2	2	REG	26.42	14
			GT0	0	STOP	0	321

Fig. 5 Production, reserve and gas consumption for Jan. 2013

Fig. 6 Dispatch results for 6 h on Jan. 2013

from h15 to h17, the CCP progressively starts up three GTs, providing reserve during this starting. From h18 to h24, the CCP operation aims to take advantages of high energy prices, relegating reserves to lower levels. Note that the ST satisfies the minimum number $t^* = 3$ h (from h15 to h17) to be turned on.

Sensitivity to Gas Contracts. This subsection presents the sensitivity of the proposed model with respect to daily and monthly tariffs (v and f). Scenario IncP, with an increase of 50% in energy prices, has been executed to turn on the CCP during a longer period of time.

Fig. 7 Dispatch results for Jan. 3, 2013

Fig. 8 Daily and monthly contracted capacities of gas for Jan. 2013

Fig. 9 Gas consumption, and daily and monthly contracted capacities, for Jan. 2013

Applying WO execution mode, Fig. 8 shows the daily and monthly contracted capacities (J_d and J_m) of natural gas corresponding to Jan. 2013 for three different scenarios: The baseline scenario, with reference daily and monthly tariffs; scenario “Incd,” doubling the daily reference tariff; and “Decd,” with the reference tariff halved. Pd_“esc” and Pm_“esc” in Fig. 8 refer to the daily and monthly capacities mentioned above for scenario “esc.”

It can be seen that only monthly capacities are contracted when the daily reference tariff is increasing (Pd_Incd is null), whilst the opposite occurs when it is decreasing (scenario Decd). In this last case, monthly capacity is 600 MWht, since this is exactly the minimum amount of gas that must be consumed according to the take-or-pay signed contract. An intermediate situation between

these two extreme behaviors is the baseline scenario, where the monthly toll is above 600 MWht and the Pd_baseline is not null. Applying MO execution mode, Fig. 9 shows the monthly and daily contracted capacities for this last scenario. Gas consumption is also depicted.

Note that a base monthly capacity of gas is contracted as well as an additional daily capacity for those days with a gas consumption over that base capacity (see, for example, days D3, D4, and D5; in contrast, the consumption on some other days is below this monthly capacity; see, for example, day D1).

Computational Results. For each of the execution modes, the total number of continuous variables, binary variables, constraints,

Table 2 Computation results

	#Continuous variables	#Binary variables	#Constraints	Execution time (min)
WO	149,508	46,320	270,420	72
MO	661,284	205,296	1196,916	408
JO	680,722	169,412	1142,179	na

and the total execution times are presented in Table 2. In the case of the MO execution mode, these results solved sequentially.

As Table 2 shows, JO cannot be solved because the Cplex resource limit was exceeded. Indeed, only WO and MO modes are currently being executed by one price-taker electricity generator in Spain, with satisfactory results in terms of accuracy and execution time requirements for 1 yr planning.

5 Conclusions

This paper has proposed a price-based UC model for the optimization of the generation and reserve operation of a CCP, using a MILP model. Its objective function consists in the maximization of energy and reserve joint profits, while the main technical constraints represent several peculiarities of the CCP operation, such as the Brayton and Rankine cycles, the minimum time, and production levels to start-up the ST or supplementary firing processes in the HRSG.

Flexible and take-or-pay contracts can also be modeled, as well as CO₂ emissions costs. The proposed MILP model can be used to estimate the behavior of a CCP on the basis of energy and reserve prices forecasted using statistical and econometric models for short-term planning [20]. It would also be suitable for use with more fundamental representations (such as those set up on Nash games [21]) with longer horizons.

The cases studies are focused on the Spanish day-ahead and secondary reserve markets. The operation of a real Spanish CCP is simulated, showing that, with the current electricity and reserve prices, the CCP is turned off a large amount of time, which reflects the current operation of this kind of CCPs in Spain. A principal conclusion is that an increase of 50% in the reserve price would imply a higher utilization rate for this type of CCPs, which could be a stimulus for the creation of new regulatory frameworks that improve the remuneration of reserve, ensuring the recovery of the CCP investment costs. Furthermore, this could also encourage new investments in renewable technologies since the CCP may be running as a backup for these technologies.

The examples also show the improvement in the overall efficiency of the CCP as a result of the application of the combined operation mode, as well as the behavior of gas contracts in different daily and monthly tariff scenarios. Several operation modes, such as the fast start mode, have also been tested in addition to compliance with minimum start-up times for STs. Though the computational efficiency in real applications can be subject to discussion for large temporal scopes, because of the large number of binary decisions, execution times are acceptable for monthly or weekly sequential executions.

Future developments will focus on the inclusion of more operating modes for the HRSG and also stochasticity in the ST operation when modeled as a function of the output HP and LP flows.

Nomenclature

CCP = combined cycle plants
 ED = economic dispatch
 GT = gas turbine
 HP = high pressure
 HRSG = heat recovery steam generator
 LP = low pressure

MILP = mixed integer linear programming
 ST = steam turbine
 UC = unit commitment

Indexes

d = days
 h = hours
 i = HRSG state (without applying supplementary firing: $i = 0$, with minimum supplementary firing: $i = 1$, and above minimum supplementary firing, $i = 2$)
 m = months
 u = gas turbines

System Data Parameters

c_{ng} = cost of the natural gas (€/MWh)
 n^{ur}, n^{dr} = coefficients to obtain the energy reserve price with respect to p_h (pu)
 p_h = day-ahead energy price (€/MWh)
 p^{emis} = CO₂ emission price (€/TCO₂)
 p_h^s = reserve price (€/MW)
 q^{ur}, q^{dr} = percentage of reserve energy cleared (pu)
 $urdr_h$ = upward/downward reserve relation (pu)

GT and ST Technical and Economic Data Parameters

$b_u^{max}, b^{max,\sim}$ = maximum reserve (MW)
 $ca_u, ca^{\sim}$ = auxiliary services coefficient (pu)
 $cm_u, cm^{\sim}$ = maintenance cost (€/h)
 $csh_u, csh^{\sim}$ = shut-down cost (€)
 $cst_u, cst^{\sim}$ = start-up cost (€)
 cb^{max} = maximum certified net reserve per GT turned on (MW/GT)
 cq^{max} = maximum certified net power (MW)
 $q_u^{min}, q^{min,\sim}$ = minimum gross power (MW)
 $q_{u,h}^{max}, q_h^{max,\sim}$ = maximum gross power (MW)

Other GT Data Parameters

bs_u^{max} = maximum upward reserve (MW)
 c_u^{emis} = CO₂ emission factor (TCO₂/MWh)
 $d_{u,h}$ = availability coefficient (pu)
 fs_u = percentage of $q_u^{min,T}$ in fast start (%)
 g_u^{min} = gas consumed at minimum gross power (MWht)
 $q_u^{min,I,\sim}$ = minimum gross power providing steam without ($i = 0$) or with supplementary firing ($i = 1$) (MW)
 α_u = consumed gas per gross MW (MWht/MW)

Other ST Data Parameters

$t^{\sim}$ = minimum number of hours above $u^{\sim}$ to run the ST (h)
 $u^{\sim}$ = minimum load to test if the ST can run (pu)
 fs_u = fraction of hour in which the GT u is turned on in fast start mode (pu)

Combined Cycle Data Parameters

$a_{i,u}^{hp}, a_{i,u}^{lp}$ = HP/LP independent regression coefficients between the HP/LP flow and each GT gross power (kg/s)

$b_{i,u}^{hp}, b_{i,u}^{lp}$ = HP/LP regression coefficients between the HP/LP flow and each GT gross power ((kg/s·MW))
 $gc_i^{\sim}$ = gas consumed with minimum ($i = 1$) and maximum ($i = 2$) supplementary firing (MWht)
 hpf_h, lpf_h = total HP and LP flows (kg/s)
 $hpf_{i,u,h}, lpf_{i,u,h}$ = HP and LP flows per HRSG state (kg/s)
 k_i = regression coefficients between the ST gross power and the total HP and LP flows (MW for $i = 0$, and (MW/(kg/s)) for $i > 0$)

Gas Contracts Data and Tariffs

$c_d^{\min}, c_d^{\max}$ = minimum and maximum daily gas consumption (MWht)
 $c_m^{\min}, c_m^{\max}$ = minimum and maximum monthly gas consumption (MWht)
 d_m, m_m = monthly coefficients for the fixed tariff (pu)
 v, f = variable and fixed tariffs (€/MWht)

Gas Consumption Continuous Variables

G_h = total (MWht)
 $G_{u,h}$ = by each GT (MWht)
 GUR_h, GDR_h = upward/downward regulating energy (MWht)
 $GUR_{u,h}, GDR_{u,h}$ = by each GT (MWht)
 $GW_{u,h}$ = without supplementary firing (MWht)

Energy and Power Continuous Variables

$P_{u,h}, P_h^{\sim}, P_h$ = net power (MW)
 PUR_h, PDR_h = net energy within the reserve (MW)
 $PUR_{u,h}, PDR_{u,h}$ = net energy within the reserve by each GT (MW)
 $PUR_h^{\sim}, PDR_h^{\sim}$ = net energy of the ST within the reserve (MW)
 $Q_{u,h}, Q_h^{\sim}, Q_h$ = gross power (MW)
 $Q_{u,h}^{i\sim}$ = additional gross power with or without supplementary firing (MW)
 $Q_{u,h}^{\min}$ = gross power above the minimum (MW)
 $Q_{u,h}^{V,\min}$ = minimum gross power (MW)
 $QUR_{u,h}, QDR_{u,h}$ = gross energy within the reserve by each GT (MW)

Reserve Continuous Variables

UR_h, DR_h = upward and downward reserves (MW)
 $UR_{u,h}, DR_{u,h}$ = by each GT (MW)
 $UR_h^{\sim}, DR_h^{\sim}$ = by the ST (MW)

Contract Continuous Variable

J_d, J_m = maximum daily and monthly quantity of natural gas contracted (MWht)

HRSG Continuous Variables

$Q_{u,h}^{i\sim}$ = gross power with or without supplementary firing (MWht)

$U_{u,h}^{2,\sim}$ = percentage of gross power in relation to the maximum and the minimum gross power produced with supplementary firing (pu)

Binary Variables

$U_{u,h}, U_h^{\sim}$ = turn-on decision
 $U_{u,h}^{i\sim}$ = steam generation decision without ($i = 0$) or with minimum supplementary firing ($i = 1$)
 $UF_{u,h}$ = fast start decision
 $V_{u,h}$ = if the load is above $u^{\sim}$ in h and the GT is on in $h - 1$
 $Y_{u,h}, Y_h^{\sim}$ = start-up decision
 $Z_{u,h}, Z_h^{\sim}$ = shut-down decision

References

- [1] González, P., Villar, J., Díaz, C. A., and Campos, F. A., 2014, "Joint Energy and Reserve Markets: Current Implementations and Modeling Trends," *Electr. Power Syst. Res.*, **109**, pp. 101–111.
- [2] Daneshvar, K., Behbahani-nia, A., and Khazraei, Y., 2012, "Transient Modeling of Single-Pressure Combined," *Int. J. Model. Optim.*, **2**(1), pp. 64–70.
- [3] Ahluwalia, K. S., and Domenichini, R., 1990, "Dynamic Modeling of a Combined-Cycle Plant," *ASME J. Eng. Gas Turbines Power*, **112**(2), pp. 164–167.
- [4] Bertini, I., De Felice, M., Pannicelli, A., and Pizzuti, S., 2011, "Soft Computing Based Optimization of Combined Cycled Power Plant Start-Up Operation With Fitness Approximation Methods," *Appl. Soft Comput.*, **11**(6), pp. 4110–4116.
- [5] Chowdhury, B. H., and Rahman, S., 1990, "A Review of Recent Advances in Economic Dispatch," *IEEE Trans. Power Syst.*, **5**(4), pp. 1248–1259.
- [6] Rifaat, R. M., 1998, "Economic Dispatch of Combined Cycle Cogeneration Plants With Environmental Constraints," *International Conference on Energy Management and Power Delivery (EMPDP'98)*, Singapore, March 3–5, Vol. 1, pp. 149–153.
- [7] Lu, B., and Shahidehpour, M., 2004, "Short-Term Scheduling of Combined Cycle Units," *IEEE Trans. Power Syst.*, **19**(3), pp. 1616–1625.
- [8] Padhy, N. P., 2004, "Unit Commitment—A Bibliographical Survey," *IEEE Trans. Power Syst.*, **19**(2), pp. 1196–1205.
- [9] Bjelogrić, M. R., 2000, "Inclusion of Combined Cycle Plants Into Optimal Resource Scheduling," *IEEE Power Engineering Society Summer Meeting*, Seattle, WA, July 16–20, Vol. 1, pp. 189–194.
- [10] Benders, J. F., 2005, "Partitioning Procedures for Solving Mixed-Variables Programming Problems," *Comput. Manag. Sci.*, **2**(1), pp. 3–19.
- [11] Liu, C., Shahidehpour, M., Li, Z., and Fotuhi-Firuzabad, M., 2009, "Component and Mode Models for the Short-Term Scheduling of Combined-Cycle Units," *IEEE Trans. Power Syst.*, **24**(2), pp. 976–990.
- [12] IBM, 2013, "CPLEX Optimizer," IBM Corp., Armonk, New York, accessed Apr. 17, 2013, <http://www-01.ibm.com/software/commerce/optimization/cplex-optimizer/>
- [13] Gülen, S. C., 2013, "Gas Turbine Combined Cycle Fast Start: The Physics Behind the Concept," *Power Eng.*, **117**(6), pp. 40–45, available at: <http://www.power-eng.com/articles/print/volume-117/issue-6/features/gas-turbine-combined-cycle-fast-start-the-physics-behind-the-con.html>
- [14] Mitra, S., Sun, L., and Grossmann, I. E., 2013, "Optimal Scheduling of Industrial Combined Heat and Power Plants Under Time-Sensitive Electricity Prices," *Energy*, **54**, pp. 194–211.
- [15] Saiz-Marin, E., Garcia-Gonzalez, J., Barquin, J., and Lobato, E., 2012, "Economic Assessment of the Participation of Wind Generation in the Secondary Regulation Market," *IEEE Trans. Power Syst.*, **27**(2), pp. 866–874.
- [16] Bisch, A., Taccari, L., Martelli, E., Amaldi, E., Manzolini, G., Silva, P., and Macchi, E., 2014, "A Detailed MILP Optimization Model for Combined Cooling, Heat and Power System Operation Planning," *Energy* (in press).
- [17] Villar, J., Munoz, A., Sanchez-Ubeda, E. F., Mateo, A., Casado, M., Campos, A., Mate, J., Centeno, E., Rubio, S., Marcos, J. J., and Gonzalez, R., 2001, "SGO: Management Information System for Strategic Bidding in Electrical Markets," *IEEE Porto Power Tech Proceedings*, Porto, Portugal, September 10–13, Vol. 1, pp. 6.
- [18] Esteban Bravo, M., 2004, "Computing Equilibria in General Equilibrium Models Via Interior-Point Method," *Universidad Carlos III de Madrid*, Madrid, Spain.
- [19] Wolsey, L. A., 1998, *Integer Programming*, Wiley, New York.
- [20] Cuaresma, J. C., Hlouskova, J., Kossmeier, S., and Obersteiner, M., 2004, "Forecasting Electricity Spot-Prices Using Linear Univariate Time-Series Models," *Appl. Energy*, **77**(1), pp. 87–106.
- [21] González, P., Villar, J., Díaz, C. A., and Campos, F. A., 2013, "Hourly Energy and Reserve Joint Dispatch With a Hydro-Thermal Technological Based Representation," *IEEE 10th International Conference on the European Energy Market (EEM2013)*, Stockholm, Sweden, May 27–31.