UNIVERSIDAD PONTIFICIA DE COMILLAS

ESCUELA TÉCNICA SUPERIOR DE INGENIERÍA (ICAI)

INGENIERÍA INDUSTRIAL

MODELING, PRICING AND HEDGING DERIVATIVES ON NATURAL GAS: AN ANALYSIS OF THE INFLUENCE OF THE UNDERLYING PHYSICAL MARKET

Autor: D. Enrique Colín Betancourt

Director: Dr. D. Álvaro Baíllo Moreno

Madrid

Mayo 2014

Proyecto realizado por el alumno:

D. Enrique Colín Betancourt

Fdo.: …………………… Fecha: ……/……/……

Autorizada la entrega del proyecto cuya información no es de carácter confidencial

EL DIRECTOR DEL PROYECTO

Dr. D. Álvaro Baíllo Moreno

Fdo.: …………………… Fecha: ……/……/……

Vº Bº del Coordinador de Proyectos

Fernando de Cuadra

Fdo.: …………………… Fecha: ……/……/……

A mi Director de Proyecto, por los consejos y la orientación que me ha brindado, en el ámbito de este Proyecto y en muchos otros.

A mis abuelos, por los padres que me han dado.

A mi hermana y a Andrea.

MODELO DE VALORACIÓN Y COBERTURA PARA DERIVADOS DE GAS NATURAL: ANÁLISIS DE LA INFLUENCIA DEL MERCADO FÍSICO SUBYACENTE.

Autor: Colín Betancourt, Enrique.

Director: Baíllo Moreno, Álvaro.

Entidad Colaboradora: Banco Santander.

RESUMEN DEL PROYECTO

Objetivos del Proyecto

En un entorno energético cambiante marcado por estrictas políticas regulatorias y por una preocupación generalizada por la garantía de suministro energético, el gas natural ha logrado posicionarse como la fuente de energía primaria con mayor potencial de crecimiento. Este posicionamiento es debido a las bajas emisiones de carbono por unidad de energía producida y al aumento de la producción como consecuencia del desarrollo de nuevas tecnologías de exploración y producción de hidrocarburos.

El importante papel que desempeña el gas natural en el panorama energético global ha traído como consecuencia un proceso de transformación desde la vinculación a barriles de referencia mediante contratos a largo plazo, hacia la generación de precios en mercados organizados, donde los precios responden a principios a corto plazo. Con este proceso de transformación surge la necesidad de redefinir un marco de análisis que demuestre ser coherente con los nuevos principios del mercado. Este Proyecto ofrece un profundo análisis de estos mercados en Norteamérica y Europa, con el objetivo de comprender los fundamentos que originan los precios y la volatilidad de los mismos. Tras un profundo análisis del impacto que tiene el mercado físico subyacente, es preciso representar esta influencia en un modelo coherente que pueda ser posteriormente implantado a través de una metodología fiable. La metodología que propongo es la de un Análisis Mensual por Componentes Principales, la cual logra representar fielmente la estacionalidad del producto a través de un modelo multifactorial de volatilidad.

El enfoque propuesto y la metodología desarrollada en este Proyecto han demostrado cumplir los objetivos iniciales y puede ser considerada de útil aplicación como herramienta de gestión de riesgos para los distintos agentes del mercado. Además, dada su robustez y versatilidad, puede ser a su vez extrapolable a otras materias primas cuyos precios estén determinados por procesos estacionales.

Análisis Fundamental de los mercados *Henry Hub* y *NBP*

Los mercados organizados buscan corregir los desequilibrios entre la oferta y la demanda. La magnitud de dichos desequilibrios depende de la velocidad de respuesta por parte de los agentes del mercado. Los factores que determinan las causas de los desequilibrios y la velocidad de respuesta por parte de los agentes son específicos para cada mercado, de ahí la necesidad de estudiar los fundamentos y el comportamiento de los mercados subyacentes.

En el mercado Norteamericano, el *Henry Hub*, los precios se ven influenciados por los niveles de los inventarios, las importaciones y los huracanes (en lo que refiere a la garantía de suministro); y por las condiciones meteorológicas (las cuales determinan la demanda). Estos factores se combinan para generar una estructura temporal de volatilidad, marcada por una tendencia estacional con máximos en invierno y mínimos en verano. Dado que el sistema gasista americano es un sistema prácticamente aislado, los inventarios son esenciales para cubrir las variaciones de carga en el mismo. A lo largo de los últimos años ha tenido lugar una profunda transformación del sector en este país debido al desarrollo de nuevas tecnologías de producción de gas natural no convencional, lo cual ha permitido que EEUU pase a ser un país autosuficiente en lo que refiere al suministro de gas natural.

El mercado del gas natural en el Reino Unido (el *NBP*) está caracterizado por la alta dependencia de esta fuente de energía primaria para la generación de electricidad y por la baja capacidad de almacenamiento a nivel nacional. El estudio de la evolución de los precios y su volatilidad desde finales de los años noventa hasta la fecha permite distinguir dos periodos: un primer periodo comprendido entre finales de los años noventa hasta 2007, marcado por la dependencia en la producción nacional y del Este de Europa; y un segundo periodo desde el 2007 en adelante, caracterizado por una intensa diversificación de las vías de suministro. En cuanto a la estructura temporal de volatilidad, ésta es similar a la que presenta el análisis del *Henry Hub*. No obstante, cabe destacar que el *NBP* presenta un máximo local en verano producto del aumento en el consumo de gas para generación eléctrica durante esos meses (debido a la demanda para climatización, principalmente).

Modelos de Valoración para Gas Natural

En este Proyecto se ha llevado a cabo un análisis de los principales modelos empleados para la valoración de productos energéticos y materias primas. Tras estudiar los modelos *spot* de regresión a la media y modelos *forward* de un solo factor y multifactoriales, se ha elaborado un caso particular de estos últimos.

El desarrollo de un modelo multifactorial ha sido la mejor opción para modelar la curva *forward* de un producto con alta estacionalidad como el gas natural. El modelo multifactorial concibe la existencia de múltiples fuentes de incertidumbre que definen la evolución de la curva en el tiempo. Cada una de estas fuentes tendrá asociada una función de volatilidad que servirá para determinar la dirección y magnitud del movimiento de cada punto de la curva ante la llegada de acontecimientos asociados a cada una de estas fuentes. El modelo desarrollado propone una función de volatilidad que presenta los siguientes términos:

- Un término exponencial negativo que representa la disminución de la volatilidad en el tiempo y que es necesario para obtener un proceso de Markov.

- Un término periódico estacional multiplicado por un segundo término exponencial decreciente, ya que el efecto de estacionalidad decae con el tiempo.
- Un nivel constante hacia el cual converge la volatilidad.

Implementación de una Metodología de Valoración

Actualmente existen varias metodologías enfocadas a la valoración de derivados de energía. Con el objetivo de ofrecer un estudio comparativo entre las más extendidas, se ha optado por implantar tres de estas técnicas: un entorno de simulación mediante Monte Carlo, un proceso de generación de árboles trinomiales y una calculadora de opciones mediante Black-Scholes. Estas técnicas han sido implantadas a través de Matlab, VBA y Excel, respectivamente. No obstante, debido a que una correcta valoración de derivados energéticos pasa por considerar más de un precio futuro, se ha optado por una metodología que considere el movimiento conjunto de varios puntos de la curva forward y su evolución en el tiempo. El hecho de que el gas natural presente un menor número de fuentes de incertidumbre que de precios *forward*, nos sugiere la viabilidad de implementar un modelo de Análisis por Componentes Principales (PCA). Este enfoque reduce el número de dimensiones del problema a una serie de factores que capturan las fuentes de incertidumbre más representativas. Además, este análisis ofrece la capacidad de cuantificar el peso relativo de cada una de estas fuentes con respecto al comportamiento absoluto de la curva *forward*.

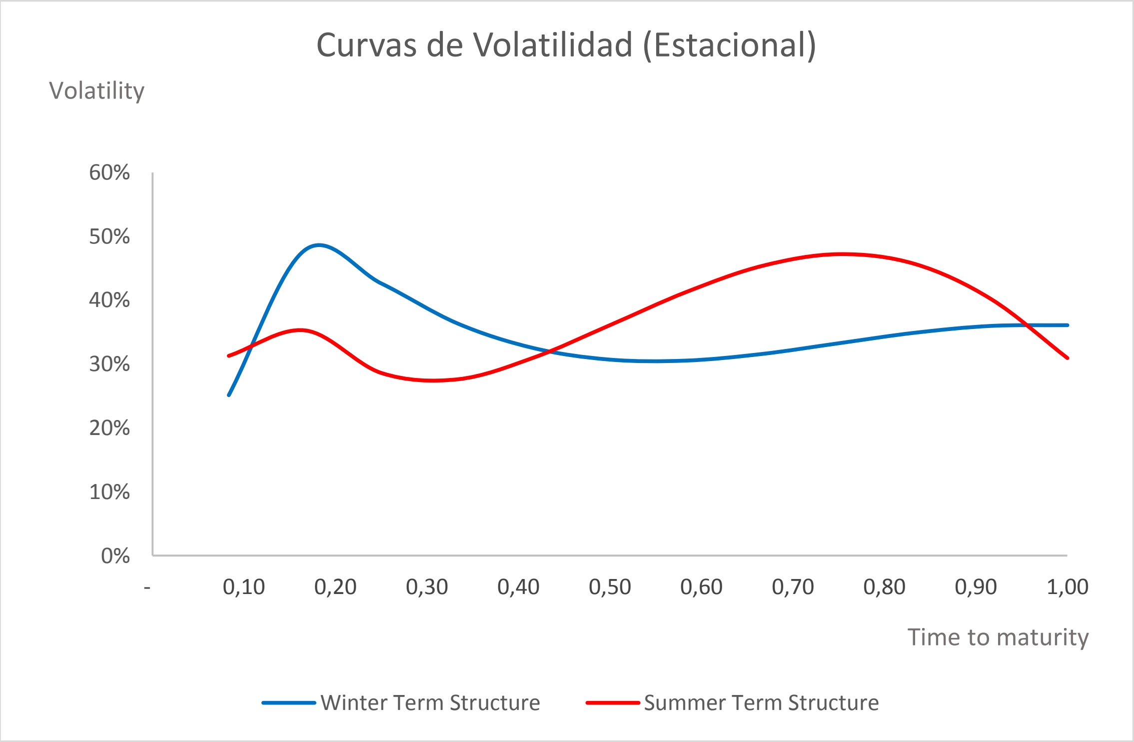
Atendiendo a la dificultad inherente a la interpretación de los resultados derivados de un PCA sobre un producto fuertemente estacional como el gas natural, este Proyecto propone el empleo de un marco de análisis dinámico que segregue la información de entrada y analice únicamente franjas temporales similares entre sí. De esta forma, se llevará a cabo un PCA sobre cada unidad temporal predefinida, de forma que sean concebidas como productos independientes entre sí. Esta metodología parte de un PCA Estacional en el cual se segrega la información de entrada en dos referencias, correspondientes a los meses de alta volatilidad (invierno) y a los de baja (verano). Una vez realizado este análisis, las series temporales de entrada son discretizadas a la mínima unidad temporal coherente (un mes), constituyendo así el denominado PCA Mensual. Esta metodología ha sido estandarizada en seis pasos:

1. Segregación de las series temporales e introducción en el modelo.
2. Cálculo de la matriz de covarianza de las series introducidas.
3. Factorización de la matriz en autovalores.
4. Selección de los principales autovalores y autovectores.
5. Calibración de los parámetros de cada función de volatilidad frente a su Componente Principal.
6. Simulación de la curva *forward* a través de la suma de todas las funciones de volatilidad ponderadas por sus factores de carga, el intervalo de tiempo considerado y una muestra aleatoria independiente.

Resultados y Conclusiones

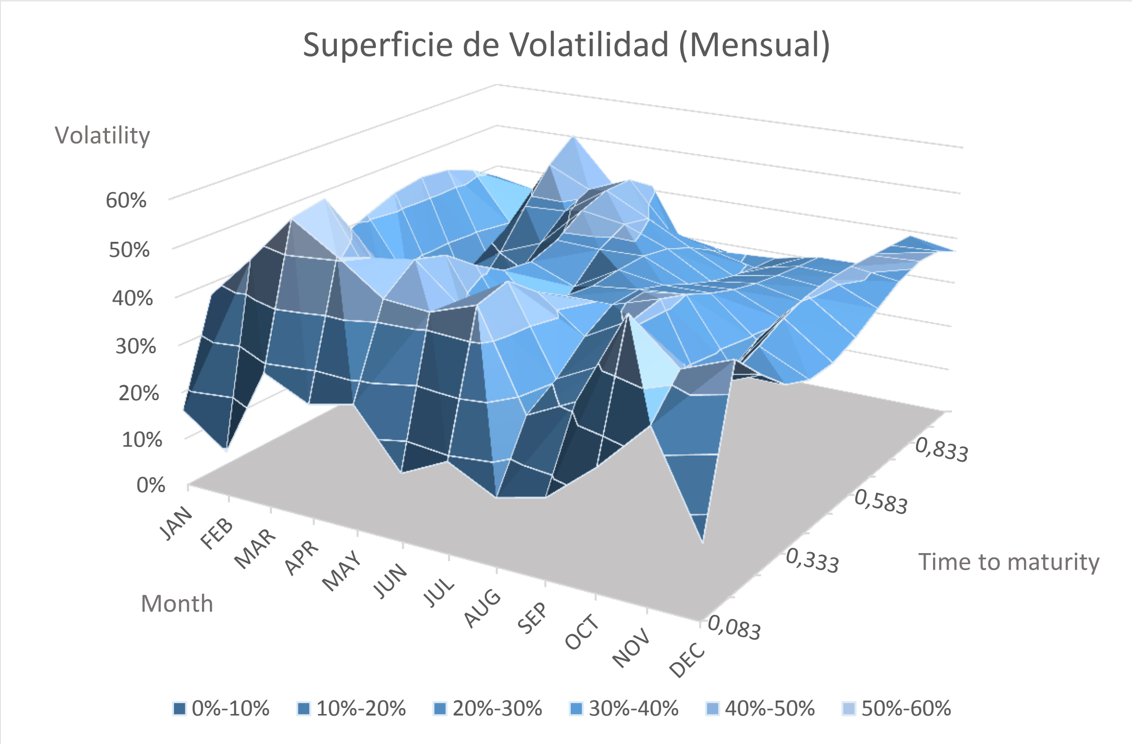
El PCA Estacional sirve como punto de partida para sustentar la metodología propuesta y su aplicación a la valoración de derivados de gas natural. Este análisis ofrece las siguientes conclusiones:

- La primera función de volatilidad (común a los análisis de invierno y verano) presenta un máximo absoluto en torno al segundo mes *forward*, y decrece de ahí en adelante para ambos casos. Esta estructura se debe al efecto amortiguador de volatilidad que ofrecen los inventarios a lo largo del primer mes. No obstante, este efecto desaparece y la preocupación en la garantía de suministro repunta a partir del segundo mes.
- La segunda función de volatilidad representa la componente estacional y su máximo varía desde los meses próximos a vencimiento en la curva de invierno, hasta la mitad de la curva *forward* en el análisis de verano.
- Mientras que para el análisis de invierno son necesarias únicamente dos funciones de volatilidad para alcanzar el nivel de confianza mínimo del 95%, el análisis de verano requiere cuatro funciones de volatilidad para cubrir dicho nivel. De este hecho se puede deducir que son necesarias más fuentes explicativas para analizar la curva desde los meses de verano, debido a que el impacto que tiene la demanda asociada a factores meteorológicos durante esos meses es menor.



La segregación de las series temporales de entrada a unidades mensuales nos permite capturar la evolución de los doce primeros meses *forward* desde la perspectiva de cada uno de los meses del año como si se tratara de productos independientes. Además de las conclusiones extraídas del PCA Estacional, el PCA Mensual nos ofrece las siguientes:

- Cualquier acontecimiento sistémico tendrá un mayor impacto en aquellos meses limítrofes al invierno, independientemente del mes desde el que se observe la evolución de la curva.
- El efecto amortiguador de la primera función de volatilidad es común para todos los meses, si bien los meses de verano presentan una mayor capacidad para absorber los picos de volatilidad.
- La evolución de la segunda función de volatilidad es una gran fuente de información para analizar el comportamiento de las diferencias de precios entre los distintos meses.
- El PCA Mensual ofrece una mayor sensibilidad que el PCA Estacional, el cual producirá por tanto curvas más suaves que el primero.
- Por último, la segregación de información de entrada reduce el número de componentes principales y el número de fuentes explicativas de incertidumbre, facilitando la tarea de interpretación de los resultados.



MODELING, PRICING, AND HEDGING DERIVATIVES ON NATURAL GAS: AN ANALYSIS OF THE INFLUENCE OF THE UNDERLYING PHYSICAL MARKET.

SUMMARY

Project's Main Objectives

In a changing global energy environment marked by tougher than ever regulatory policies and a generalized concern about security of energy supply, natural gas has made its way through to become the fastest growing primary source of energy due to its low carbon emissions per unit of energy produced and the development of new production technologies that have dramatically increased the volume of economically viable natural gas reserves.

This relevant role that natural gas plays today in many developed countries has caused a departure from the traditional long-term oil price indexation approach towards the use of increasingly liquid natural gas price references through the creation of local spot and forward wholesale markets where prices are based on short-term fundamentals. This change in the price reference has created the need to redefine a robust modelling framework which proves to be consistent with the new market drivers and fundamentals. Hence, the first objective of this project is to provide a profound fundamental analysis of the American and European natural gas markets in order to understand their price drivers and sources of volatility. Having addressed the influence of the underlying physical market, it will then be necessary to represent it through a consistent pricing model and develop a reliable valuation methodology. Regarding the latter, the pricing methodology that I propose is based on both a Seasonal and a Monthly Principal Component Analysis which has proven to cope with seasonality in running time, and a Multi-Factor volatility function to fit volatility in time to maturity.

The valuation methodology that I suggest fulfils these basic goals and should be useful as a risk management tool for all participants along the natural gas value chain, from the wellhead to the final consumer. Given its trustworthiness and versatility, the proposed methodology could be further used for other seasonal commodities and energy products, by simply calibrating the volatility functions with the corresponding product's historical time series. This broadens the scope of the project to all players with exposure or interests in energy markets, from power generators and oil companies, to financial institutions seeking to deliver tailored solutions to their specific client's needs.

Fundamental Analysis of Henry Hub and NBP

Effective, liberalized markets evolve in order to match the imbalances of supply and demand. The magnitude of these imbalances and their impact on prices is determined by the speed of reaction of market participants. Each market will be driven by specific

fundamentals, which fall into two main categories: structural factors (affecting supply) and consumption patterns (affecting demand).

According to this, the North American Henry Hub natural gas market is mainly driven by hurricanes, storage dynamics and imports (from the supply side) and by weather (from the demand side). These factors define a volatility term structure which tends to follow a yearly seasonal pattern marked by winter highs and summer lows. Moreover, volatility is likely to peak in late autumn months due to the fact that storage inventories have reached their highs and gas surplus are then viewed as finite. In addition, a second but lower spike will occur in late winter months, as storage facilities have been nearly depleted and supplies have to meet uncertain late cold weather. Given the isolated nature of the North American natural gas network, storage facilities are essential to meet load variations and to balance the system. The enhancement of exploration and production technologies in the US has prompted an increase in unconventional gas production and the US has shifted from being a net gas importer towards a self-sufficient country. This has had as direct impact on prices and volatility, which have steadily decreased from the year 2009 onwards.

Regarding the NBP natural gas market, UK's high dependence on gas for power generation and its low storage capacity must be taken into consideration. Two main structural regimes can be distinguished when analysing the evolution of volatility in this market: from late 90's till 2007, the UK natural gas market was characterized by a high dependence on domestic and Eastern Europe production and scarce capacity to face import shortages; and from 2007 onwards, it has evolved to a generalized diversification of supply through the commissioning of importing infrastructure to secure LNG supplies from Qatar and pipeline imports from continental Europe and Norway. The yearly seasonal volatility term structure is similar to that of Henry Hub, but differs from it in that year highs are reached towards the end of the winter due to colder than usual weather and also in that summer months feature a second but lower high due to an increase of gas-fired power generation for air conditioning.

Natural Gas Valuation Models

Several valuation models have been discussed in this project. This range from Spot mean-reverting convenience-yield models to Single and Multi-Factor Forward models. The latter approach has been selected for the scope of this project, as the evolution of the forward curve analysis framework is based on the availability of prices quoted in liquid exchanges such as NYMEX (Henry Hub) and ICE (NBP), where the closest-to-maturity contract is used as the spot reference.

The Single-Factor model falls short when attempting to replicate a complex seasonal curve such as that of natural gas. The Multi-Factor model, in contrast, assumes the existence of multiple independent sources of uncertainty which drive the evolution of the forward curve with time. Each of these sources will have a volatility function which will determine the direction and magnitude of the movement of each point of the forward curve at the arrival of information associated with that particular source of uncertainty. Our proposed model has been developed as a special case for this general framework. Each of the proposed volatility functions consists of the following terms:

- A negative exponential term, to measure the exponential decrease of volatility over time and to obtain a Markovian spot price process.
- A seasonal periodic component multiplied by a second negative exponential term to represent the fact that the seasonality effect dies out with time.
- A constant level to which volatility converges with time.

Implementation of a Pricing Methodology

A variety of numerical techniques can be used in order to value derivatives in the absence of closed-form solutions. These range from binomial and trinomial methods to Monte Carlo simulations, which provide traceable and straightforward approaches. In order to compare the consistency of these methodologies and their deployment feasibility, this project has implemented a Monte Carlo framework, a Trinomial Tree generation procedure and a Black-Scholes pricing calculator to price European Call options in Matlab, VBA and Excel, respectively. Nevertheless, given the fact that energy derivatives depend on more than one forward price, it is necessary to develop a model of the forward curve which captures the joint movement and evolution with time of all forward prices. In the case of natural gas the fact that there are fewer sources of risk than price references leads to the advantage of implementing a Principal Component Analysis (PCA), as it reduces the number of dimensions of the problem to only two or three factors which retain the most representative uncorrelated sources of uncertainty. Moreover, as the PCA yields information on the average weight of each source, it is possible to quantify the impact of each of them in the absolute behaviour of the forward curve.

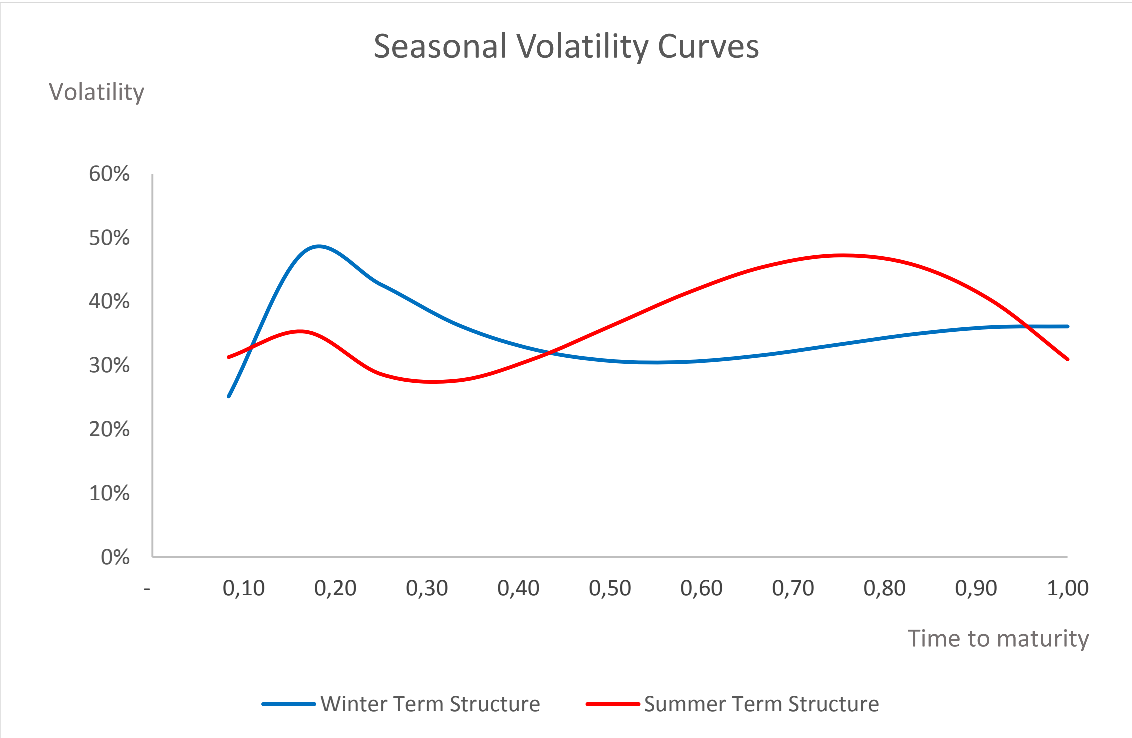
Given the fact that natural gas exhibits strong seasonality, attempting to capture the evolution of the forward curve through the correlations between different points along the curve may result in an overload of information difficult to compute and understand. Hence, to cope with seasonality in running time, this project has developed a dynamic framework which analyses the full set of sample data in seasonal slices in order to compare only similar periods with each other. This statement brings forth the Seasonal PCA, which considers two seasonal benchmarks corresponding to winter and summer, and the Monthly PCA, which discretizes input data into monthly units in order to offer a closer fit. The proposed methodology has been developed to be as standardised and straightforward as possible, and has been structured into six steps:

1. Loading the segregated data into the model.
2. Computing the covariance matrix for the input data.
3. Carrying out the eigenvalue decomposition of the covariance matrix.
4. Ranking eigenvalues and selecting Principal Components.
5. Calibrating each volatility function against their respective PC.
6. Simulating forward curve movements through the sum of all factor loadings scored by their respective factor scores, time step and uncorrelated random sample.

Results and Conclusions

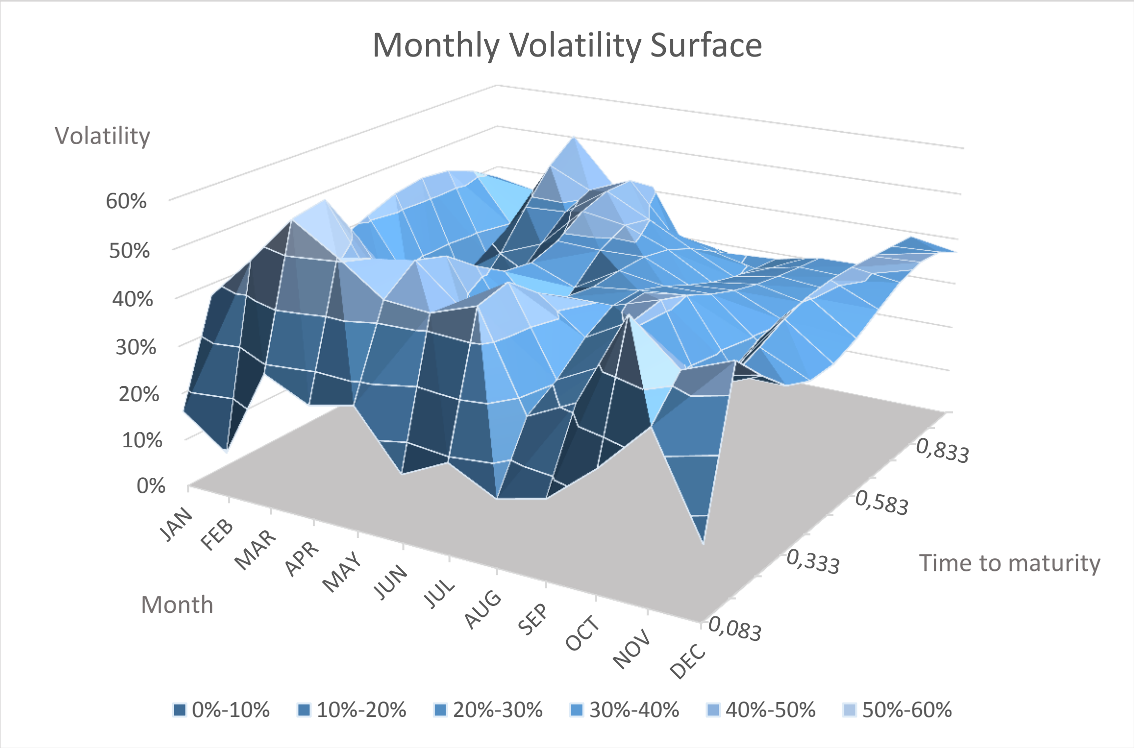
The Seasonal (Winter-Summer) PCA serves as a comprehensive and reliable baseline to support the analysis of natural gas through the developed methodology. This analysis draws forward the following conclusions:

- The first volatility function (which is structurally similar in both the winter and the summer analysis) spikes around the second month forward and then decreases exponentially over time to maturity. This term structure is due to the buffering effect of storage, which mitigates volatility of the front month forward but raises concern on supply on the second month forward.
- The second volatility functions define seasonality and their highs range from the closest to maturity months in the winter benchmark to the further-down-the-curve months in the summer analysis.
- Whereas only two sources of uncertainty have been necessary to reach a 95% confidence level in the Winter PCA, the Summer PCA has required four sources of volatility to reach the same level of confidence. This statement draws forth the conclusion that more factors are necessary to explain market behaviour when viewing the curve from summer months, as closer to maturity months are less affected by weather demand in comparison to winter months.



Stressing out the segregation of input data into monthly units enables us to capture the evolution of the 12 front months forward viewed from the perspective of each month of the year. In addition to the conclusions drawn from the Seasonal PCA, the Monthly PCA brings forth the following:

- Any price shock will have a major impact on winter boundaries, regardless of these contracts being close or not to maturity.
- The buffering effect of storage is common for all months, despite summer months cope with spikes more easily than winter months, as expected.
- The evolution of the second volatility function from one month to the following serves as a great source of information when analysing the behaviour of calendar spreads.
- The monthly PCA provides more sensible curves than the Seasonal PCA, which therefore generates smoother curves.
- Segregating input data allows us to capture more information from fewer components, reducing the number of explanatory sources of uncertainty.



Index

Index

Table of figures

1

Motivation

1 Motivation

This project has been conceived to offer a profound analysis of the fundamentals which drive the underlying physical natural gas market and to represent this influence through a consistent valuation model. In a fast changing energy environment marked by stronger than ever regulatory policies and a generalized concern on security of supply, natural gas has climbed its way through to become the fastest growing energy source. This boost in natural gas production has been fostered by the enhancement of new technologies which have enabled the exploitation of reservoirs which had not been economically feasible till date.

With a changing energy outlook comes the need to redefine and develop a robust modelling framework which proves to be consistent with the new market fundamentals. This project is meant to offer a trustworthy analysis of the changing regimes in the US and the UK. These analysis have considered the evolution of prices and volatility from the early 2000's onwards, focusing on the events occurred over the past decade. The conclusions drawn from the previous process are widely representative of the key drivers for each the American and European gas markets.

This project aims to develop a reliable methodology which can be further implemented into other seasonal commodities and energy products. The approach taken in this project has proven to fulfil these goals and would be useful as a risk management tool for all participants along the natural gas value chain, from the wellhead to the final consumer. Moreover, the proposed methodology would also result useful for all shareholders with exposure or interests in natural gas markets, varying from power generators and distribution companies to banks and financial institutions seeking to deliver tailored solutions to their specific client's needs.

2

Natural Gas Trading Hubs

2 Natural Gas Trading Hubs

2.1 Introduction

The structure of natural gas has changed dramatically since the mid 80's due to technological improvements along the value chain. At the beginning, Exploration and Production (E&P) companies explored and drilled for natural gas and later sold their production at the wellhead to large transportation pipelines. These pipelines transported the gas at a high pressure across throughout the countries and would then sell it to local distribution companies, who then transported the product at a lower pressure and sold it to final consumers. The prices at which each element in the supply chain purchased the gas was federally regulated, as was the final price charged by retail companies to their customers. According to this, the structure of the natural gas industry was very straightforward, with assured monopolies for transportation and distribution companies, little competition in the marketplace and low incentives to improve and innovate.

Nevertheless, after deregulation and pipeline unbundling the industry has changed drastically as it is now more opened to competition and choice. Wellhead prices are no longer regulated, leading to inelastic dependence on supply and demand interaction. Transportation pipelines are no longer owners of the physical commodity at any stage from the wellhead to the distribution company, instead they only charge a transportation fee which is still under federal regulation. A turning point in the industry has come with the appearance of natural gas marketers. This figure aims to facilitate movement of natural gas from the producer to the final consumer. They serve as a middleman between two parties, offering either the sale or purchase of natural gas, but can also contract for transportation and storage.

Natural gas trading started gaining momentum in the late 90's with an exponential increase in the number of participants and their trading volumes, plus the emergence of new derivatives such as futures, swaps and options in addition to traditional OTC contracts. With the opening of the IUK interconnector pipeline between UK and Belgium, the existence of a pan-European gas market seemed to be feasible. Nevertheless, after the collapse of Enron and the near collapse of Eastern Gas Marketing, gas trading suffered a downturn on trading volumes. From 2006 gas trading began to recover and to attract new players into the marker as more companies were seeking to limit their risk exposures while trying to optimize their portfolios through new trading strategies. Trading volumes and deals peaked by 2010 after overcoming the 2008-2009 recession, which had little overall effect on the market in what refers to volumes traded but lead to an increase in Exchange traded contracts against OTC. Since the year 2008 there has been a growing transition from long term oil-indexed contracts towards spot pricing from benchmark hubs. For a hub to become a robust and reliable price reference it requires to have amongst other attributes, liquidity, transparency and the capacity to attract a significant number of market players.

The following sections are conceived to analyse the general structure of major natural gas hubs across Europe and the US, an overview of their trading activities and an outlook for their price evolution and its relationship with the underlying physical market.

2.2 Gas Hubs in Europe

Gas markets in Europe have seen significant evolutionary changes over the past ten years. In general terms, two main areas can be distinguished according to their pricing fundamentals:

North West Europe and Central Europe. UK's National Balancing Point (NBP) has had the liquidity and transparency to become a pan-European benchmark, but continental marketers have been reluctant to adopt this referral due to the currency exchange risk arising from NBP's pence/therm in comparison to Continental hubs' €/MWh.

Britain's NBP started operating in the year 1996 and was followed by Belgium's Zeebrugge after the Interconnector pipeline between these two countries (Bacton - Zeebrugge) was completed. Along came the Dutch TTF and the Italian PSV, the Austrian CEGH and the German EGT. The overall landscape was completed in the year 2009 with the opening of the German Gaspool and NCG. Minor developments have been carried in countries where progress has been slower due to geographical issues such as in Spain, which will be analysed further in this section.

From the European Hubs mentioned above, two of them stand out in what refers to maturity and reliability as benchmarks to manage market risk in gas portfolios: NBP and TTF. As trustworthy wholesale price indicators, they are liquid enough, they have proven to be transparent in their reports and they are opened to a wide number and variety of participants. The emergence of NBP and TTF has been closely related to their linkage to two mayor exchanges: ICE and APX-Endex, respectively.

2.2.1 The National Balancing Point (NBP)

The National Balancing Point is Europe's longest established (over fifteen years) spot-traded natural gas market and the virtual trading location for the exchange of this commodity in the UK. The price of NBP is widely used as an indicator for Europe's wholesale gas market, alongside the fast growing Dutch TTF.

The NBP model establishes that gas in any part of the national transmission system counts as NBP gas, bringing sellers and buyers together to simplify trade. National Grid plc operates this network and is responsible for the physical transportation across the system, as shippers are required to appoint only for volumes entering and exiting the system. Unlike other Continental trading hubs, shippers are not required to be balanced nor are penalized for being out of it. At the end of each trading day, shippers out of balance are automatically settled by a cash-out procedure that involves selling or buying the required quantity of gas to clear their position at the marginal system. When the shippers as a whole are out of balance and the system is short on gas, prices tend to be driven up. On the contrary, if the system is long on gas NBP prices will be driven down. Given its market liquidity, NBP it is frequent for continental shippers to balance their positions throw the Bacton - Zeebrugge interconnector.

UK's gas market liquidity is ensured through a wide range of supplies such as internal production, piped imports from Norway and Continental Europe, local storage and liquefied natural gas from the Middle East. From the buyer's point of view, there is also a broad variety of participants that go from international oil companies (IOC), utility and distribution companies, power generators, industrial consumers and financial traders. Traded volumes have grown marginally on a year to year basis for the last decade, with a stronger increase in paper ICE futures trades over physical transactions. These increments in traded volumes have shown a clear seasonality pattern with two high spikes in October and March (due to the beginning and end of the winter) and year lows in the summer months (June to August). A variety of delivery periods are traded in this market, from within-day or day-ahead deliveries, to months and quarters, summers and winters or annual contracts.

Since its commencement in the year 1996, the NBP has seen a steady growth in volume and liquidity, mainly due to a generalized increase in its linkage to other markets worldwide. As mentioned before, the start-up of the UK – Belgium interconnector pipeline in 1998 allowed oil-indexed gas flow from Continental Europe and the possibility to balance both systems. Latter enhancements came in the mid 2000's by the opening of the BBL pipeline from the Netherlands and the Norwegian Langeled pipeline to the UK. LNG imports from major global suppliers as Qatar became feasible after the opening of the Grain LNG terminal in Kent and the South Hook and Dragon LNG terminals in southwest Wales in 2009. The following exhibits the current picture for the UK:

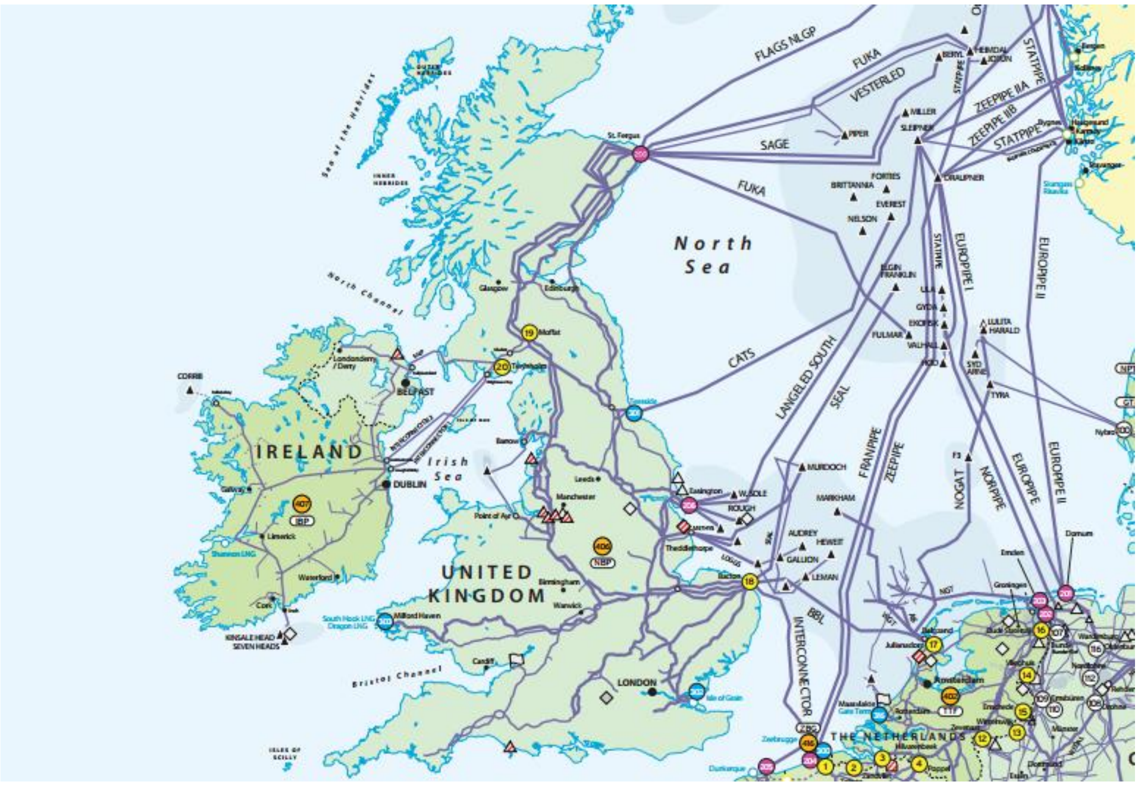


Figure 2.1 NBP Virtual Trading Hub

2.2.2 The Title Transfer Facility (TTF)

The Title Transfer Facility is a virtual trading point which covers the whole Dutch gas grid. Shippers can choose whether to enter the virtual point or not, by transiting from a border Entry Point to another Exit Point and avoiding to pay any extra fee.

There have been several factors which have enhanced the development of gas trading in the Netherlands. One of these key factors was the allowance of quality conversion by the Transmission System Operator, which meant that shippers could now trade either high or low calorific value gas. By this, traders started trading pure energy instead of different types or grades of gas. This procedure was accepted as a robust and reliable methodology by individual market players whose trading volumes soon started to increase exponentially. It is worth mentioning that this trades were executed along the forward curve from Day Ahead up to three years out, which was a clear indicator that traders were carrying out hedging and balancing operations on TTF. Another turning point in gas trading in Europe came with the implementation of an innovative real time balancing regime: the Market Based Balancing. This system takes from the electricity market model the division of each trading session into one-hour blocks at which market participants offer operational flexibility on a neutral basis by monitoring their own portfolio balance with regard to the total system balance. This approach has increased the transparency of whole market and has consolidated the trust of new participants, especially financial players. Another differentiating factor is the fact of TTF being the first cross-border market between two EU Members (due to Gasunie's ownership of both Transmission System Operators): the Dutch GTS and the German GDU. This cross-border bundled capacity enables counterparties to enter into only one transport agreement and capacity payment to transport gas across the whole grid. The conception of the TTF market as a solid trading center for Europe is backed up by its geographical location, its high imports due to their LNG facilities, its developed transport network and storage capacity, its liquid market model and innovative balancing system.

The following shows the wide linkage between the UK and Continental Europe, where the area marked in red represents the virtual boundaries for TTF:

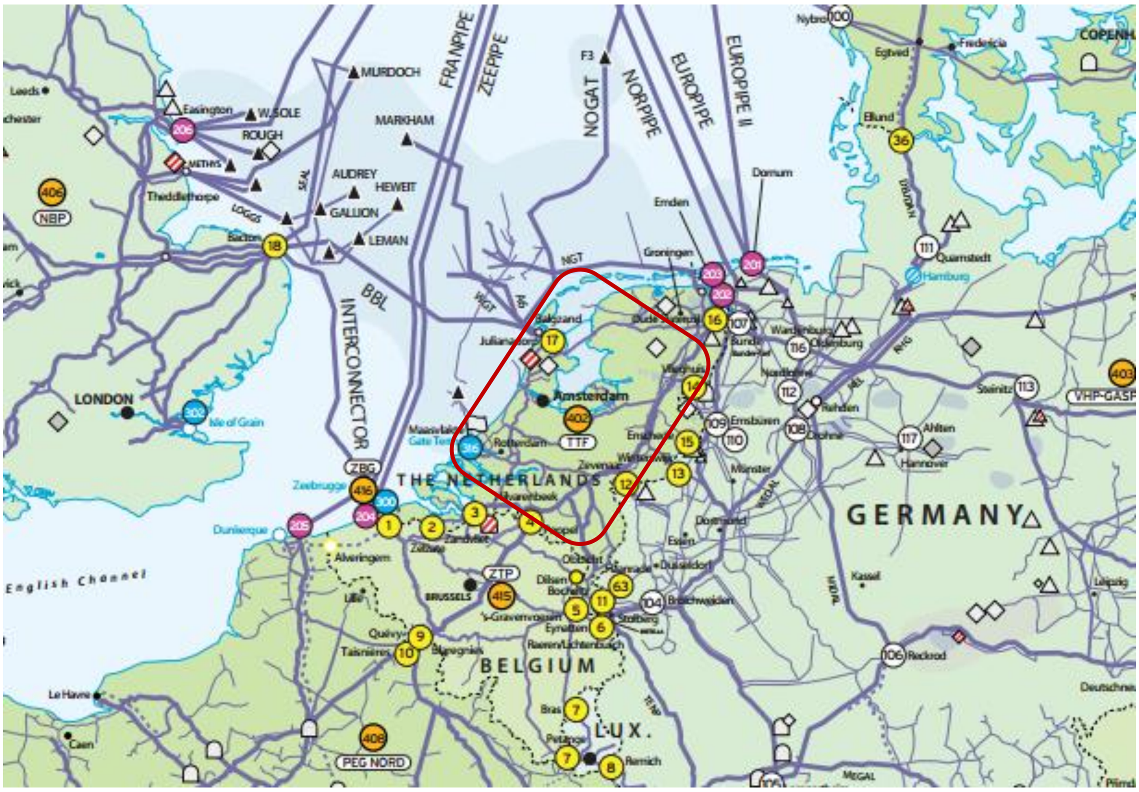


Figure 2.2 TTF Virtual Trading Hub

2.3 Gas Hubs in the US

The late 90's saw the spread and development of market hubs across the North American gas network as a response to the unbundling of the interstate pipeline company and looking forward to provide new gas shippers with the physical capabilities and administrative support previously monopolized by the company. These capabilities included transportation form between interconnected pipelines and physical coverage of short term balancing needs.

As for 2005, there are 37 operational market centres in North America from which 28 are located in the US and the rest are situated in Canada. According to the Federal Energy Regulatory Commission, these markets hubs are in charge of the following services:

- Transportation: between two interconnected pipelines.
- Parking and storage: which stand for a temporary holding of the shipper's gas for redelivery at a future date, and differ only in the length of the holding period.
- Loaning: a short term advance of gas to be repaid later by the shipper.
- Peaking: a short term injection to meet unexpected demand peaks.
- Balancing: which is provided in conjunction with the previous to fix temporary imbalanced regimes.
- Title transfer: when ownership of a specific gas volume changes and therefore requires to be recorded and cleared.
- Electronic trading: to match buyers and sellers through legally binding transactions.
- Risk management and general administration: nominations, confirmations and exchanges of futures for physicals.

The concept of market centres was fostered by the capacity to provide shippers with the supply, transportation and storage services to manage their portfolios, while promoting the individual development of each independent hub. Moreover, the availability of different wholesale spot prices has enabled shippers to optimize their gas flows while minimizing price arbitrages across the network. The most active market hub is North America is the Henry Hub, which provides services across 14 interconnecting pipelines:

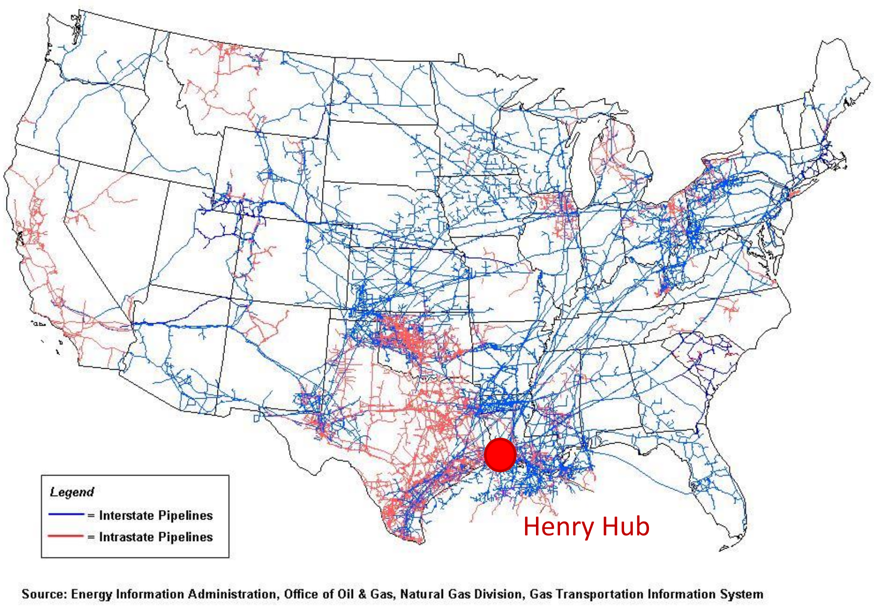


Figure 2.3 US Gas Network

2.3.1 The Henry Hub (HH)

The Henry Hub is the largest centralized point for natural gas spot and futures trading in the United States and a major reference benchmark worldwide. The New York Mercantile Exchange establishes the Henry Hub as the delivery point for its natural gas future contracts, and therefore serves as price benchmark for spot trades across the country. The NYMEX natural gas futures contract began trading on April 1990 and is currently traded 72 months forward with average churn rates around fifty, whereas European gas hubs account for less than twenty. This measure is a key indicator of the liquidity of the market, and stands for the ratio between the total volume of trades and the physical volume of gas consumed by the hub.

Whereas European NBP and TTF are virtual trading hubs in what refers to a lack of physical infrastructure with delimited entry-exit boundaries, the Henry Hub is physically situated at Sabine's Henry Gas Processing Plant located in Erath, Louisiana. It is owned and operated by Sabine Pipe Line, LLC, which is a wholly owned subsidiary of Chevron. This pipeline starts in eastern Texas near Port Arthur and ends in Vermillion Parish, Louisiana, near Erath:

Figure 2.4 Sabine's Henry Gas Processing Plant

The Henry Hub interconnects nine interstate and four intrastate pipelines, which collectively provide access to markets in the Midwest, Northeast, Southeast and Gulf Coast regions of the United States. According to the EIA, approximately half of US wellhead production either occurs near the Henry Hub or passes across it in its way to downstream consumption markets.

The Henry Hub spot price is represented by the natural gas contracted for next day delivery and title transfer at the Henry Hub, and is measured downstream of the wellhead, once the natural gas liquids have been removed and transportation costs have been incurred. On the other hand, wellhead price, as reported in EIA's *Natural Gas Monthly*, includes the value of natural gas liquids but does not reflect transportation costs. These prices are quoted in in dollars per thousand cubic feet and prove to have a close correlation to Henry Hub prices for obvious reasons.

2.4 Natural Gas Outlook in Spain

From the year 2008 onwards, the Spanish natural gas market has functioned as a liberalized market except for the regularized *TUR* tariff, which comprises small consumers with annual consumptions below 50MWh and 4 bar. From April 2012, Spanish OMEL and Portuguese OMIP (both from the OMI group of companies) decided to launch the MIBGAS initiative to develop and implement an operating model for the Iberian natural gas market, under the principles enshrined in the *European Gas Target Model*. This model has been laid down by the joint collaboration of the European Commission, the Agency for the Cooperation of Energy Regulators and regulators from each Member State.

The *European Gas Target Model* conceives and European gas market framework formed by interconnected entry-exit virtual hubs. These entry-exit zones would allow shippers to trade gas freely across all the European territory, minimizing all possible internal physical congestions within each hub. This single market requires a high interconnection between all hubs and a consistent framework to provide sufficient revenues to cover System Operators' costs. It is necessary to maximize capacity in order to guarantee an easy access to shippers on a non-discriminatory basis and at a transparent and fair price. This capacity must be considered both in the short and long term, looking forward to meet all shippers' individual requirements. This accessibility will foster liquidity across all trading hubs.

In order to achieve the model stated above, the MIBGAS initiative seeks to develop an organised model in which the main product to be traded will be the delivery of gas at one virtual hub with different time horizons (daily and intraday) and in which all market participants (transporters, distributors, retailers and end consumers) may freely trade any surplus or shortfall at a virtual exchange hub according to their volume and time requirements.

The Iberian gas supply is covered both through sea (65%) and pipeline imports (35%), from a diversified portfolio of suppliers leaded by Algeria, Nigeria, Qatar and Norway. Given that the Spanish law dictates that total imports from a same country must be limited to a 60% of total supply, there is a continuous trend of diversifying supplying sources.

March 2014 has seen the approval of a joint venture agreement between OMI Group and Iberian Gas Hub (IBGH, an OTC gas broker) for the development of an Iberian Gas Hub in compliance with the European framework and under the guidelines laid out by the Spanish and Portuguese governments. This initiative considers the development of a virtual gas hub which comprises exchange traded products and OTC contracts, all settled through a common counterparty and clearing house.

2.4.1 The Iberian Market Operator

The structure of the Iberian Market Operator, OMI, comprises the Spanish OMEL and the Portuguese OMIP. Under these companies, OMIClear acts as clearing house and central counterparty for the settlement system in the Iberian market.

The objective of the market operator is to manage a spot gas market in a transparent manner and public prices that allows all companies and markets agents to conduct their gas exchanges. This process passes through the correct execution of the following activities:

- Reception and validation of bids.
- Processes involving contracting sessions and bid-offer matching.
- Transparent management of different products.

- Developing reports on participating agents.
- Settling matched bids.

The Iberian gas market offers a variety of traded products which range from natural gas delivered at the virtual hub, to other products such as LNG, secondary capacity and storage capacity. Natural gas volumes delivered in the virtual hub are differentiated by delivery horizon and time of trading. According to this, there are two main frames in which natural gas volumes may be traded:

- Daily market: it is a trading platform which provides access to buy or sell natural gas for each day, for a number of days in advance (typically between two days and a week)
- Intraday market: it is a short-trading platform which gives access to trade gas with delivery on the same supply day. This short time frame seeks to marry supply and demand at any moment.

Despite these are the typical trading time frames, several other products with different time horizons are also considered in this market. These range from weekend contracts (which consider supply for Saturday and Sunday as a whole), to weekly, monthly, quarterly and yearly supply contracts. Among these contracts, the Iberian market allows trading several products with other underlying assets. These products are mainly: LNG, secondary capacity (which stands for the availability of supply in the intraday market) and storage capacity.

The pricing follows a continuous contracting process which involves the immediate matching of bids and offers followed by a record of the trade, a notification to the gas system's operator, and a settlement and payment. The daily trading session is preceded and followed by an auction sessions.

2.4.2 The Iberian System Operator

Enagás is certified as the Transmission System Operator (TSO) in the Spanish gas network by the European Union. As such, it is primarily responsible for operating the basic and secondary network, and ensuring the continuous gas supply through the correct management of all entry/ exit points, transmission and distribution networks, and storage facilities. This activity must be carried out in a transparent framework and independently from producers and consumers.

Enagás, in its capacity as technical manager of the gas system is also in charge of the following activities:

- Controlling the availability of supply on the short and mid-term, determining the rate of usage of all facilities and pipelines according to the expected demand.
- Calculating daily settlements for all users of the gas network as well as managing all gas entries and exits in the national grid.
- Schedule and launch maintenance and conservation plans across all infrastructure in order to guarantee a correct and safe operation of the system, as well as developing expansion and adaptation plans for the network.

3

Natural Gas Fundamentals

3 Natural Gas Fundamentals

3.1 Introduction

Structural pricing shift from long term oil indexation towards wholesale spot pricing through trading hubs has had as result a higher volatility of natural gas prices compared to oil. This premise has been the main reason for why many have remained reluctant to end traditional gas linkage to oil as pricing methodology. Nevertheless, oversupply in the US and increasing traded volumes in the UK and Continental Europe has pushed back in establishing hub pricing as referral. On average, Henry Hub and NBP volatility levels excel on over 50% to WTI and Brent oil levels. This significant increase is given due to several factors which may be organized into two categories: structural factors (affecting supply) and consumption patterns (on the demand side).

Factors affecting supply are:

- Variations in the amount of natural gas being produced or imported (exported): increased production levels may be the result of more efficient, cost-effective techniques and the discovery of new economically exploitable reservoirs. Increased gas supply will dampen natural gas prices. Geopolitical events regarding LNG or pipeline exporting countries (Qatar and Russia, for example) will also have a direct impact on natural gas prices.
- Transportability: natural gas lower density in comparison to oil and other energy sources is a constraint for its transportation. This limitation involves the development of capital intensive transportation assets to ease with issue and to deliver energy from the wellhead to the end consumer. The decommissioning of these assets, which vary from pipelines, liquefaction facilities, regasification terminals to vessels and LNG cargoes, is therefore a key factor driving natural gas supply.
- Storage dynamics: similarly to transport limitations, natural gas storage is more capital intensive and operationally dreadful than oil storage for energy unit. Natural gas can be stored in salt caverns, depleted gas reservoirs and artificial tanks, which require a minimum gas level (cushion) for gas to be injected or removed. These facilities serve as a buffer for price volatility, as they can offer and immediate and flexible source for this utility. Nevertheless, this extra capacity is expensive and can result in higher volatility spikes in periods with high demand, when gas storage are emptied to their cushion levels.
- Market structure: unlike the oil market, which is characterized by being global and having two highly correlated price referrals worldwide, natural gas markets are much more regional. This is ought to the transportation and storage limitations mentioned above. In addition to this, price formation structures differ from market to market: whereas the US and the UK are fully liberalized markets, Continental Europe remains closely linked to long term oil contracts. Asian market products are also long term indexed to a basket of imported crudes which are known as the Japanese Crude Cocktail. These pricing structure differences are being steadily mitigated through the LNG market, whose enhancement and development is minimizing arbitrage opportunities between regional markets. Increased linkage between UK and Continental Europe through pipelines has also reduced price volatility in both regions.

On the other side, natural gas demand is generally driven by several consumption factors:

- Economic growth: strength of economy is a major factor influencing natural gas markets. Periods of economic wealth will have as result an increase in demand for goods and services from the commercial and industrial sectors.
- Seasonality and cycles: natural gas exhibits high seasonality patterns due to is usage for residential and commercial heating and power generation. Cold winter weather will place a strong upward pressure on natural gas prices as it is directly burned to produce heat. On the other hand, extremely hot summers will increase power generation plants demand for natural gas, as they will need to produce more electricity to meet the cooling requirements.
- Competition with other fuels: flexible power generating plants can have the possibility of switching from lower to heavier hydrocarbons according to market prices. Natural gas prices will drop if power generation shift to coal, fuel oil or gasoil over a period of high gas prices.

3.2 Natural Gas Price Volatility

In an effective liberalized market prices vary to meet the imbalances of supply and demand. The magnitude of these imbalances and their impact on prices is defined by the consequent response of market participants to rebalance the equilibrium. This ability of response is limited in natural gas markets, and can create inelastic supply and demand. The impact of volatility varies across all market players and depends on the time period being examined. Hourly price spikes for example will have little impact on wellhead producers but will be crucial for midstream traders. It is important to define a consistent framework to analyse price volatility upon the specifics being examined. Following (Henning, 2003) we will define the characteristics of the prices being examined according to:

- Geographic market: location of the energy market being examined.
- Time interval of the prices: time period over the one price statistics are averaged.
- Investment horizon: volatility will affect market participants according to their investment's time view.

(Henning, 2003) also evaluates price volatility according to the point in the energy supply chain where the product is traded, and in terms of spot and retail prices. This characteristics involve a specific regulation framework so, for the course of this project, we will only analyse price volatility in the terms mentioned above.

3.2.1 Measuring Natural Gas Price Volatility

Measuring price volatility is not an easy task as it by itself is not a precisely defined term. There are two different approaches towards measuring this variable, those measuring price volatility in absolute terms and those based on the change in price relative to the initial price. The first point is aimed at offering a general outlook of the market and to evaluate its volatility over an investment horizon. The second approach is referred to as the "returns" method, and measures volatility as a percentage change in daily prices. This framework is usually associated with financial markets, and can be viewed as a measure of expected return on investment by those market players concerned with shorter term price fluctuations.

The key statistical indicators for measuring volatility are described below:

- *Daily Price Range:* it represents the spread in prices over a given period. In standardized exchange traded products such as Henry Hub in NYMEX or NBP in ICE, range is usually defined as the difference between the bid and the offer price, over a specific period. For OTC traded products and spot wholesale contracts, range is conceived as the difference between the daily high and low. In general terms, an increase in the range will typically, *ceteris paribus*, represent an increase in volatility and a decrease in liquidity.
- *Standard Deviation:* it represents the expected deviation from the average market price during a given period. The returns method is calculated as a standard deviation of daily relative changes in price, which is in turn computed using a natural log transformation. Volatility is calculated by multiplying the standard deviation of the daily logarithmic price changes for all trading days within the time period.
- *Coefficient of Variation:* it is a relative measure of price movement calculated as the ratio between the standard deviation and the mean value. It is useful to compare different underlying commodities, with different baseline prices and units.

- *Parkinson's Measure of Volatility:* it used range rather than midpoint to estimate price volatility. It therefore provides a measure of volatility based on the difference between high and low prices within a given time frame. It is often used to measure exchanged traded products where all trades are made at a single price and with same credit risk premiums. Changes in Parkinson's Volatility over time can be used as an indicator of volatility between different time periods. It is calculated through the following:

$$Var(Pice) = \frac{(Ln(Day\ High) - Ln(Day\ Low))^2}{4Ln2}$$

- *Returns:* as mentioned before, returns are defined as a percentage change in prices and reflect the expected return in an investment. They are normally calculated in a log-normal basis:

$$Return(x) = Ln(Price_a / Price_{a-1})$$

- *Annualized returns:* which are used to compare volatility of price series with different time periods (daily spot, weekly, monthly, etc.). Typically, 21 trading days in a month and 252 trading days per year conventions are used.
- *Skewness:* provides a measure of the asymmetrical market impact of directionally different effects. Variables such as price, which have a theoretical minimum value of zero but no theoretical maximum will be expected to have a skewed distribution. In the case of natural gas, an increase in demand due to colder weather will typically have a stronger upward impact on gas prices than a similar decrease in demand due to the same temperature difference on warmer weather.
- *Kurtosis:* it is a measure of the peakedness, tail weight and lack of shoulders in a random variable probability distribution. It is measured through the fourth moment of the distribution, and can result to be platykurtic or leptokurtic according to its negative or positive excess, respectively.

3.2.2 Impact of Natural Gas Volatility on Market Players

Natural gas price volatility has a broad range of impacts on market players according to their position in the value chain. From upstream to downstream, market participants can see their business models threatened due to price uncertainty:

- Natural Gas Producers: price volatility has a direct impact on the total revenue that can be obtained from an exploration and production project. This uncertainty is viewed from a long-term scope, considering long run seasonality patterns, periods of intensive investment and economic activity and transportation constraints. The risks and uncertainties an E&P company faces when developing a new project can be divided into four:
  - Geological risks of dry holes (Success Rate on gas fields is currently around 80%).
  - Engineering risks in recovery per successful gas well.
  - Financing risks in exploiting the well.
  - Economic risks of the value of the gas produced.

A proper evaluation of these risks will serve as input to calculate cumulative probability distributions of the major financial decision criteria such as NPV and IRR in order to evaluate the project through an expected return on investment. Financing projects in the Oil & Gas industry can be a challenging task due to high volatility of market prices, which can increase average cost of capital for these projects.

- Transportation companies: whereas natural gas producers are mainly concerned about the long-term price volatility of prices, transportation (pipeline) companies are also concerned about the short-term, as if affects its throughput and the midstreamers willing to pay for transportation services. Holders of pipeline capacity will benefit from pipeline constraints, as they will be able to sell their extra capacity at a higher fee. This short-term profitability has a side effect: it will increase uncertainty in the development of new projects and will push away risk averse investors in the upstream and in the downstream. This lack of new projects and developments will narrow transportation companies' revenues on the long run.

- Power generators: natural gas price volatility impact in power generators depends on their location (as electricity markets remain decentralised) and their regulatory framework. In regulated markets, where price of gas (or any other fuel) represents a cost of service for the power utility, price fluctuations will be passed to the final electricity consumer. On the other hand, deregulated wholesale power markets, natural gas prices will have a direct impact on the marginal production costs a power plant offers in the daily market. Combined cycle power plants and other gas burning utilities will seek to cover the marginal power generation capacity in the intraday market, meeting shoulder and peak period loads. As mentioned before for transportation companies, high price volatility will tend to keep investors reluctant to undertake new power generation projects unless they can offer a fuel flexibility to shift to cheaper hydrocarbons when required. The development of these project will also bear an increase in cost of capital to raise funds.

- Local Distribution Companies (LDC): downstream companies are subject to natural gas price fluctuations due to seasonality patterns. Moreover, when prices rise from year to year, LDC face additional hurdles and risk:

    - Financial risk derived from decreased system throughput: as utility ratemaking is designed to cover incurred costs plus a premium, high volatility periods have a direct impact on an LDC's financial performance. Continued periods of low prices after winter spikes can incur in a generalized cut on revenues, inhibiting distributors to cover fixed costs and shrinking their profit over the period.
    - Credit risk derived from increases in uncollectable accounts receivable: winter spikes can often drag distribution companies into an increase in uncollectable accounts receivable due to law protections which seek ensure basic household utility services.
    - Increases in operating costs associated with continuous turn-on and shut-off connection activity.

3.2.3 Natural Gas Price Volatility compared to Crude Oil's and other Products

Following (Alterman, 2012), crude oil exhibits an overall lower volatility than natural gas. Regarding the two principal crude oil reference barrels, WTI show an approximately 0.8% higher average volatility when compared to Brent (10.2% and 9.4%, respectively). This increase in average volatility can be explained by the physical constraints (in storage and pipeline capacity) around Cushing, Oklahoma, which is the main entry hub for Gulf Coast suppliers to northern consumers. Despite being extreme liquid, the North American crude oil market remains geographically isolated for exports, as since 1970 domestically produced crude oil exports have been regulated and kept to their minimums. The aim of this legislation was to keep domestic oil reserves while discouraging foreign imports. Nevertheless, domestic oversupply in North America has opened the debate on whether federal lawmakers should amend this policy. This structural isolation confers WTI a higher standard deviation than Brent, as it exhibits higher peaks and lower bottoms than the latter. December 2008 brought forth the highest volatility episode for crude oil prices over the past decade, as result of Lehman Brothers' collapse and the beginning of the global economic recession.

Shown below are price and volatility correlations for main crude oil reference barrels and their geographically associated natural gas hub. Correlations with Heating Oil and Gasoil have also been computed for Henry Hub and NBP, respectively:

Price Correlations (%)	Brent	WTI	NBP	HH	Gasoil	HO
Brent		92.4	85.2		99.5	
WTI	92.4			32		98.8
NBP	85.2			27.1	86.6	
Henry Hub		32	27.1			28.1
Gasoil	99.5		86.6			96.9
Heating Oil		98.8		28.1	96.9	

Figure 3.1 Correlations Table

The correlation charts for prices and volatilities for Henry Hub and NBP against main crude oil barrels and against Heating Oil and Gasoil are exhibited below:

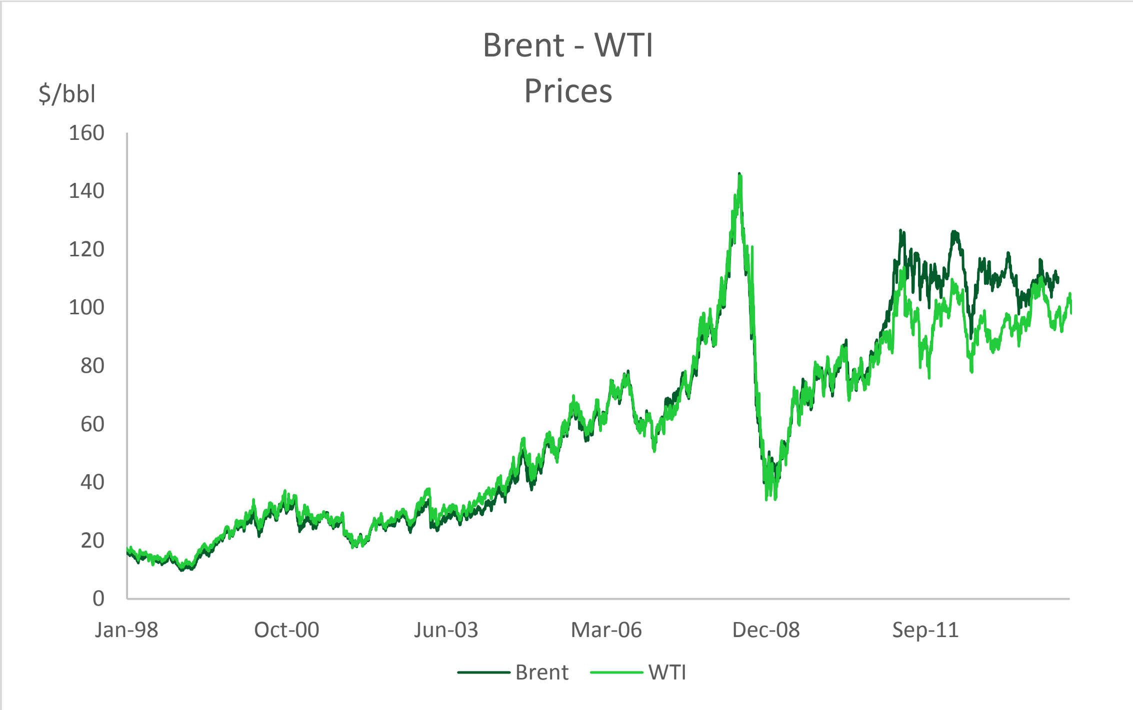


Figure 3.2 Brent-WTI (Prices)

$$\rho_{\textit{Brent-WTI (Prices)}} = 92.4\%$$

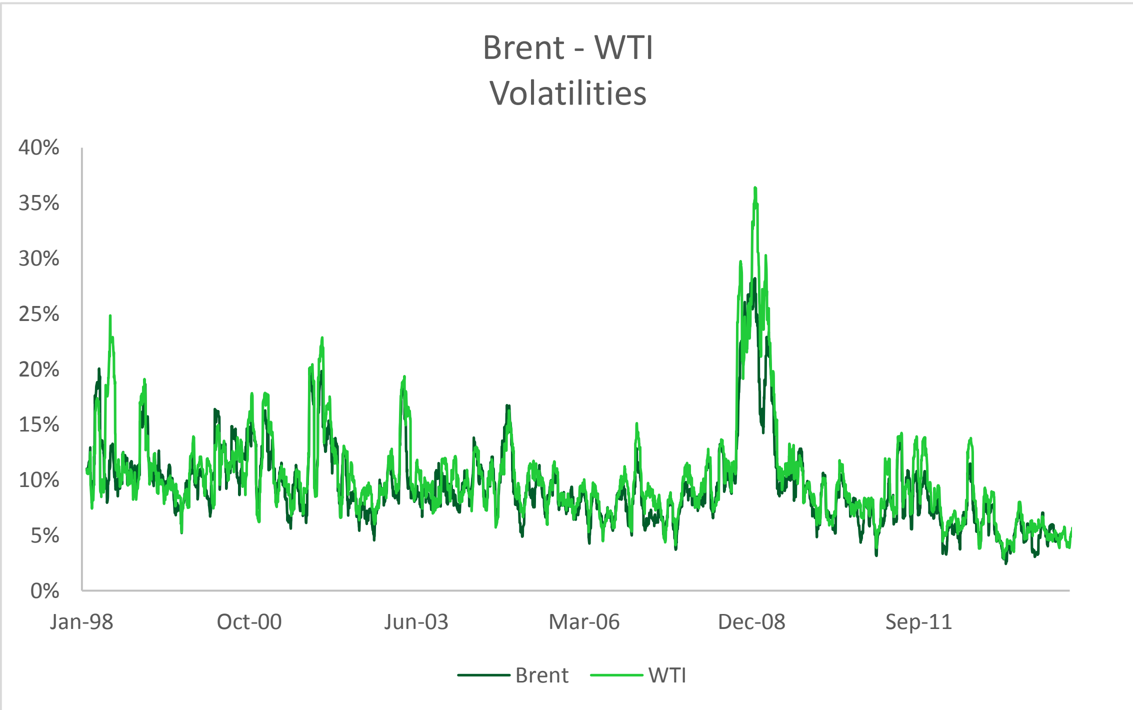


Figure 3.3 Brent-WTI (Volatility)

$$\rho_{\textit{Brent-WTI (Vol)}} = 73.1\%$$

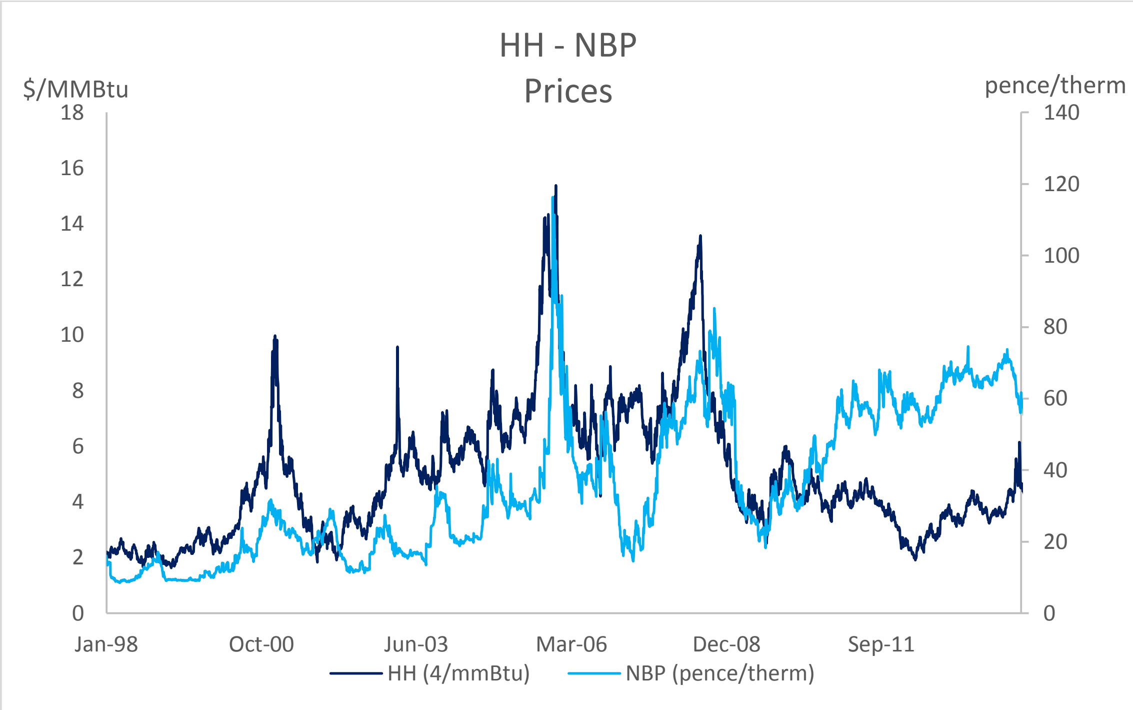


Figure 3.4 HH-NBP (Prices)

$\rho_{HH\text{-}NBP(Prices)}$= 27.1%

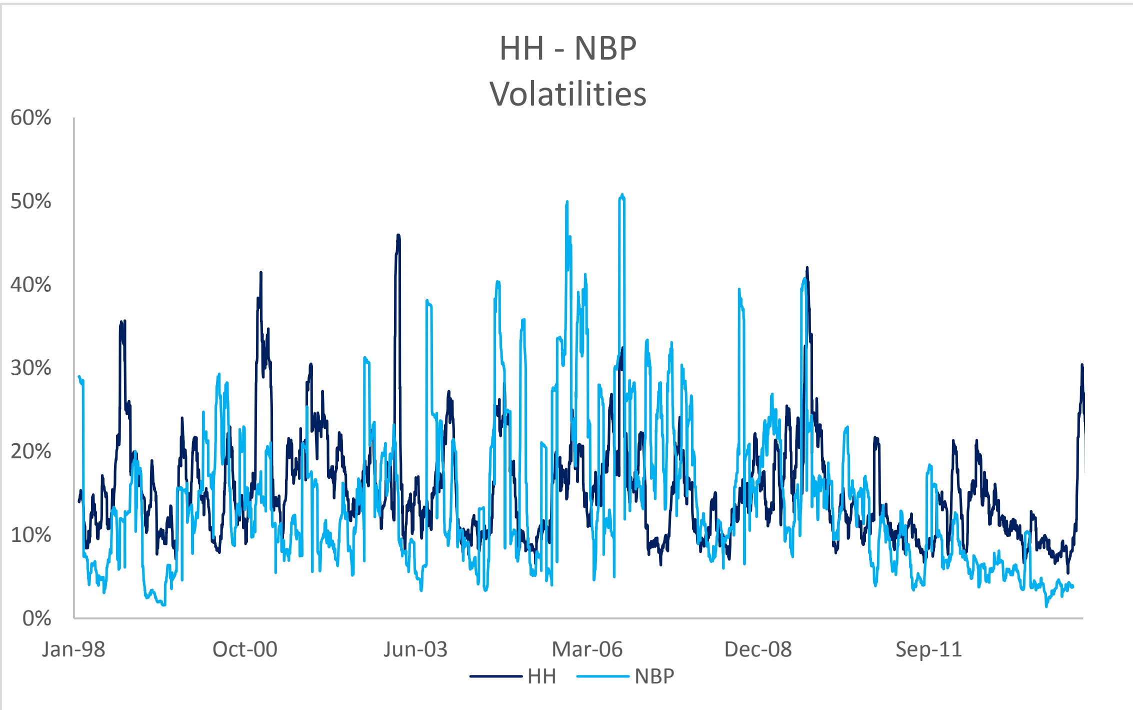


Figure 3.5 HH-NBP (Volatilities)

$\rho_{HH\text{-}NBP(Vol)}$= 26%

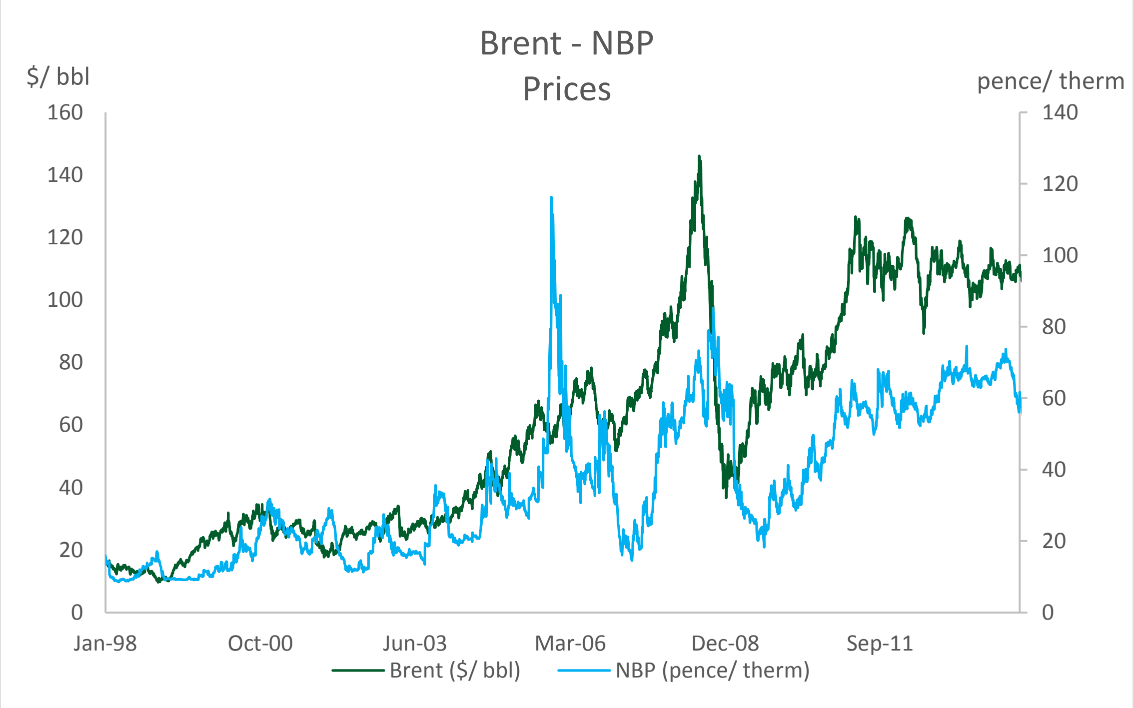


Figure 3.6 Brent-NBP (Prices)

$\rho_{Brent\text{-}NBP(Prices)}$ = 85.2%

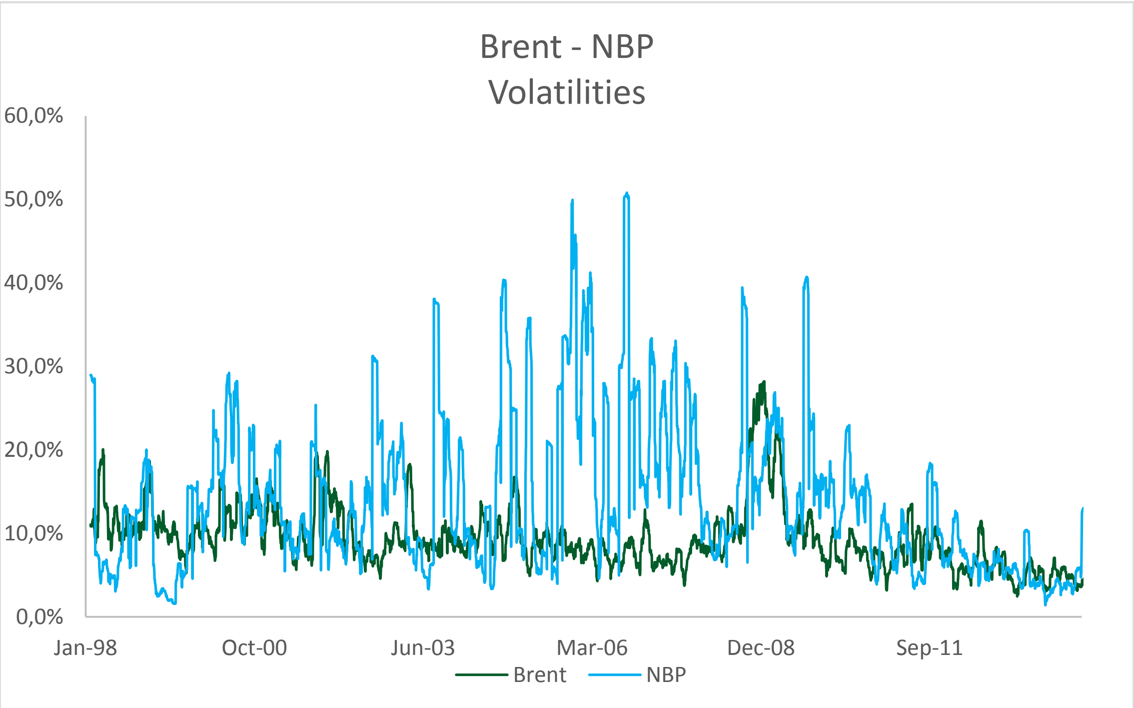


Figure 3.7 Brent-NBP (Volatilities)

$\rho_{Brent\text{-}NBP(Vol)}$= 3.8%

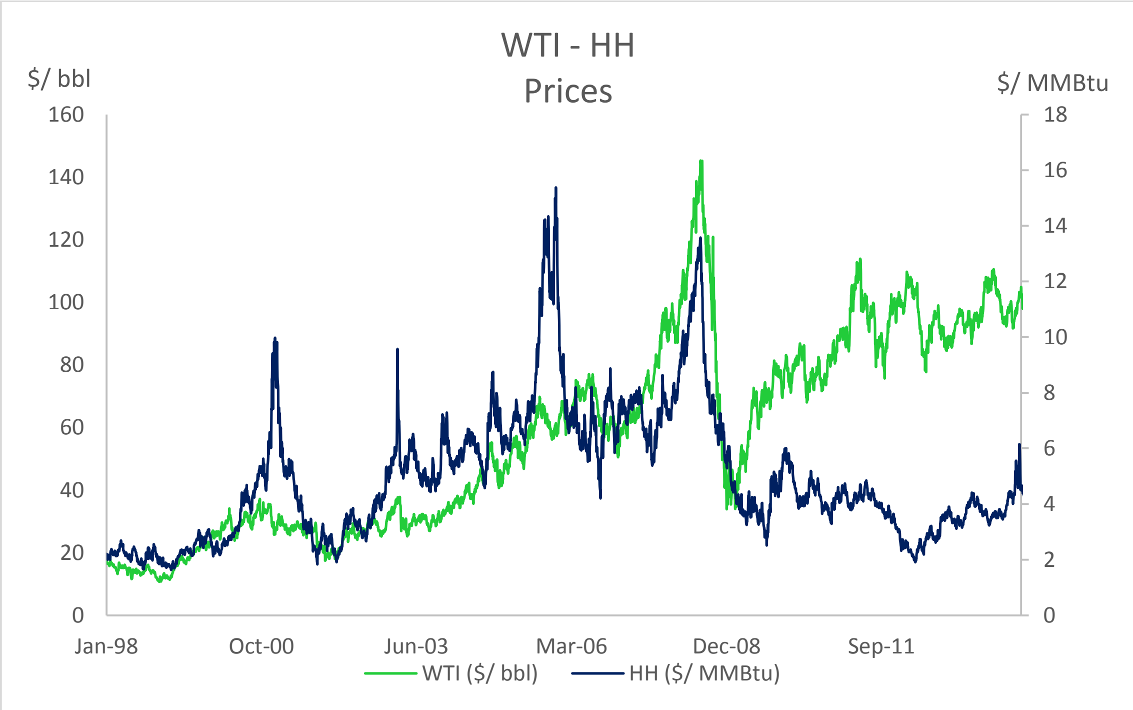


Figure 3.8 WTI-HH (Prices)

$\rho_{WTI\text{-}HH(Prices)}$= 32%

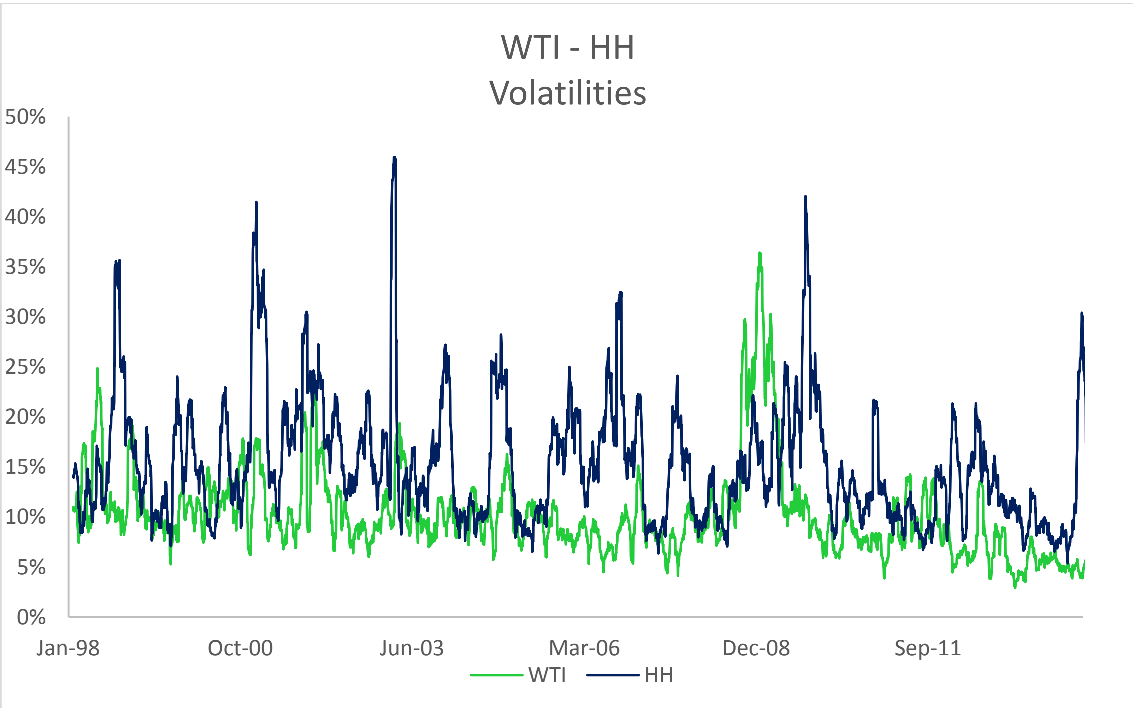


Figure 3.9 WTI-HH (Volatilities)

$\rho_{WTI\text{-}HH(Vol)}$= 16.9%

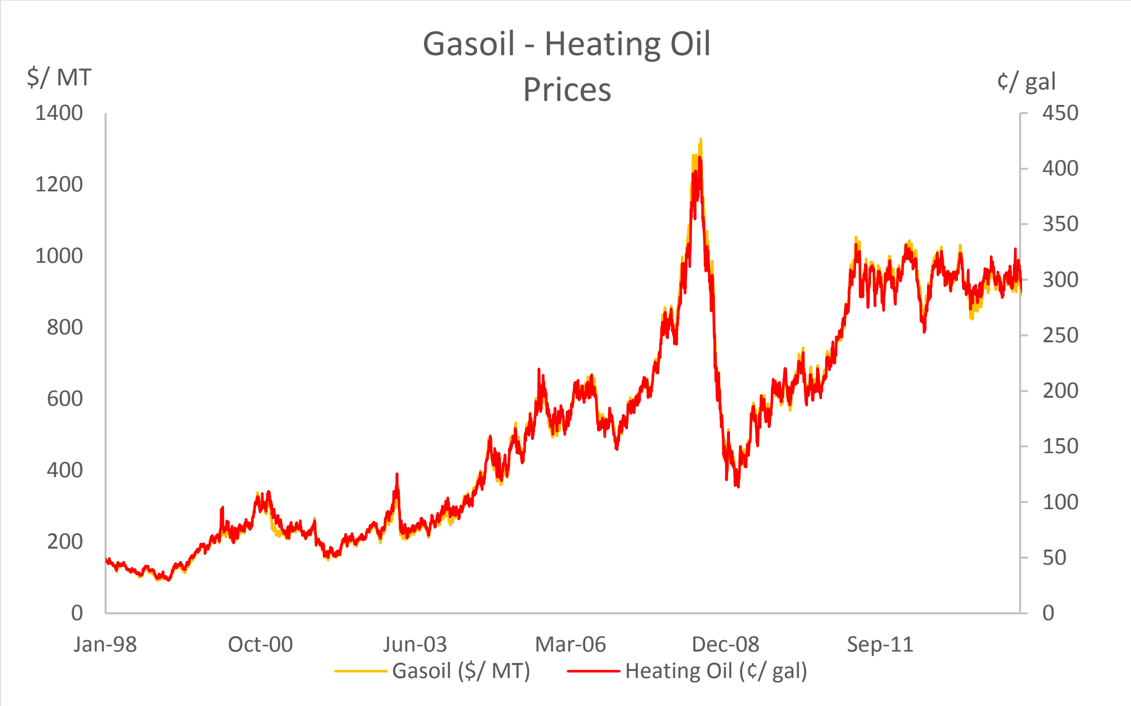


Figure 3.10 Gasoil-HO (Prices)

$\rho_{Gasoil\text{-}HO(Prices)}$= 96.9%

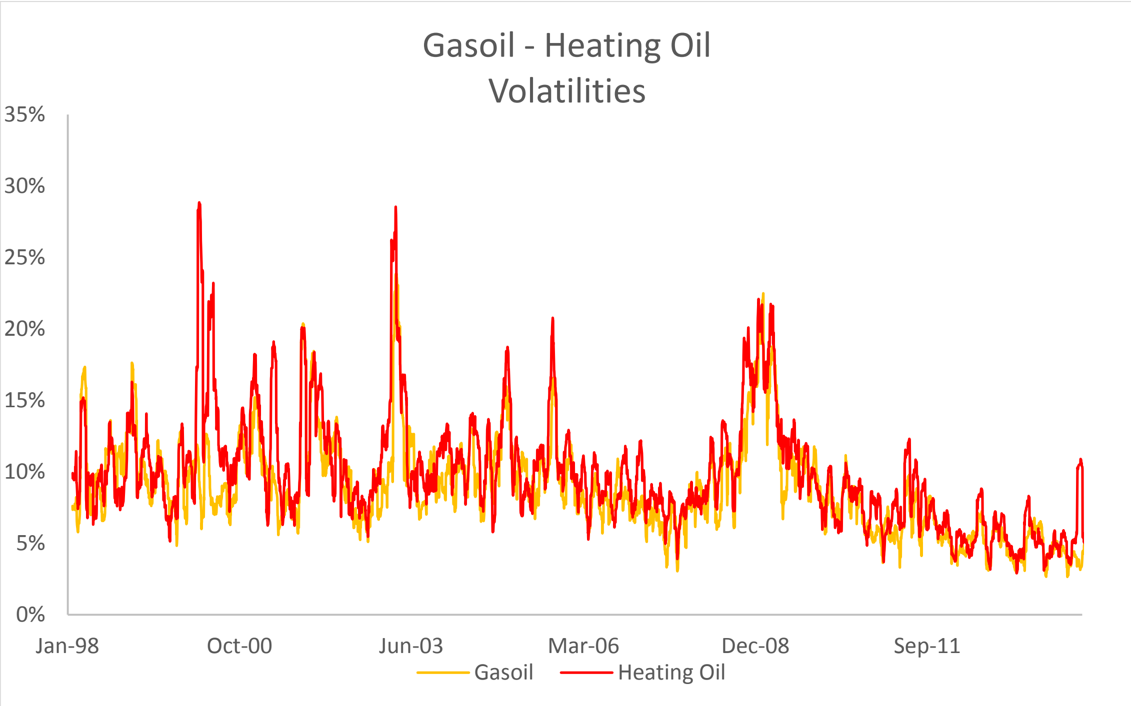


Figure 3.11 Gasoil-HO (Volatilities)

$\rho_{Gasoil\text{-}HO(Vol)}$= 59.9%

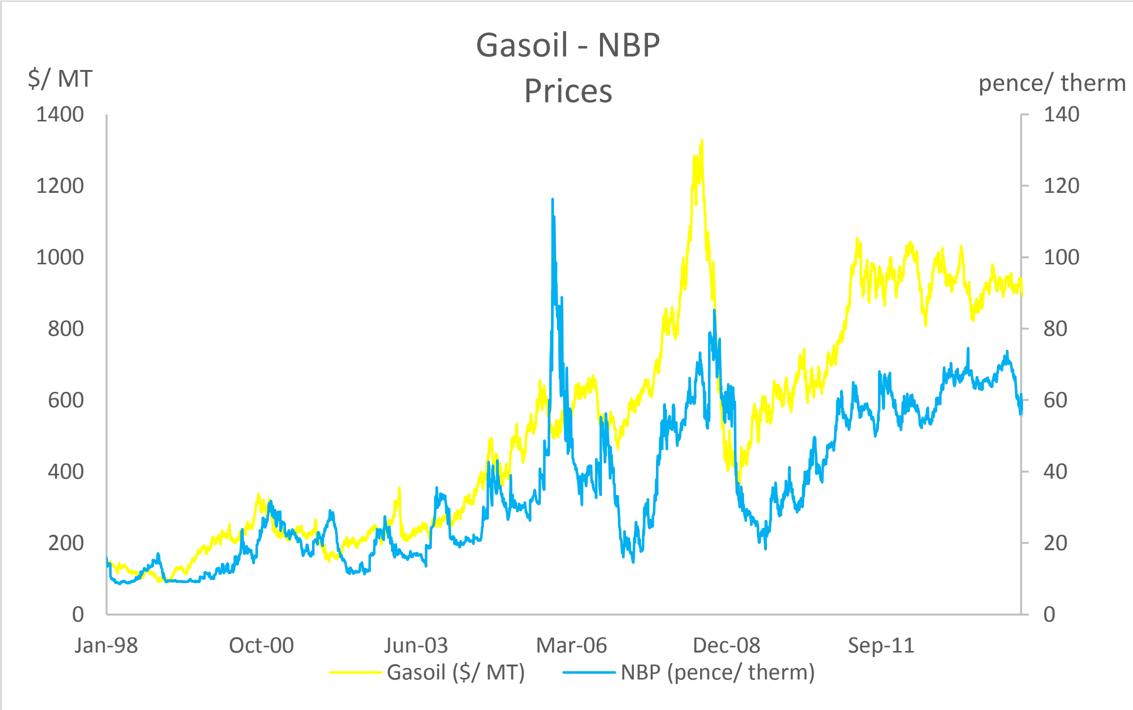


Figure 3.12 Gasoil-NBP (Prices)

$\rho_{Gasoil\text{-}NBP(Price)}$= 86.6%

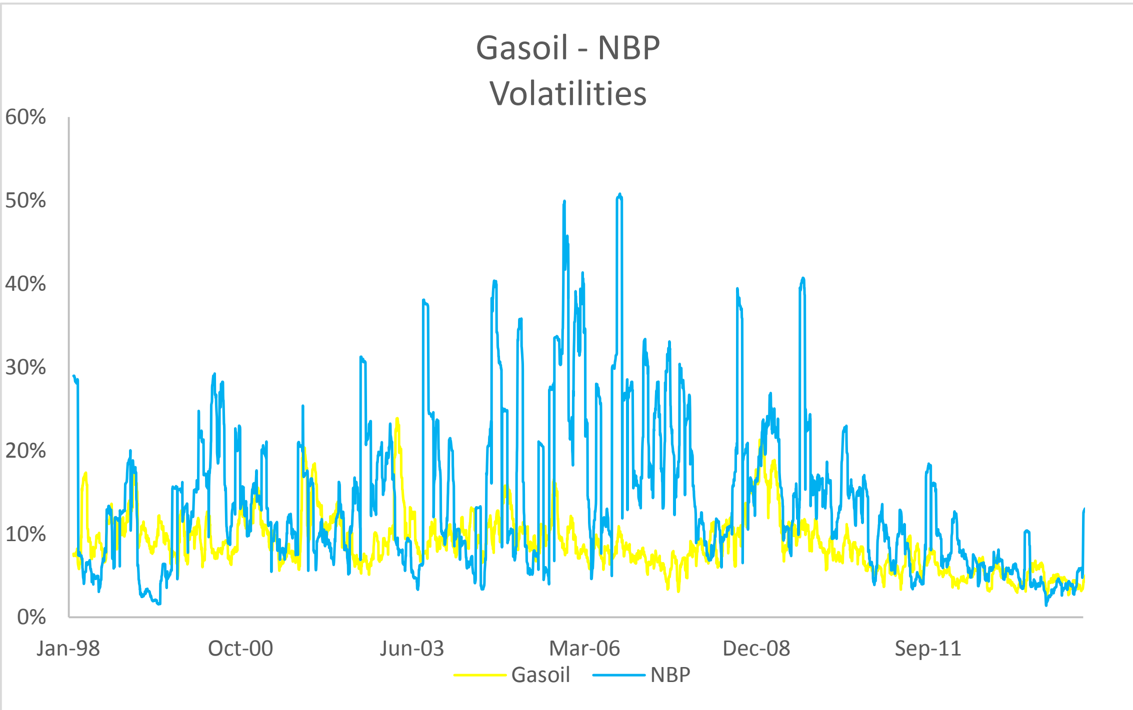


Figure 3.13 Gasoil-NBP (Volatilities)

$\rho_{Gasoil\text{-}NBP(Vol)}$= 20%

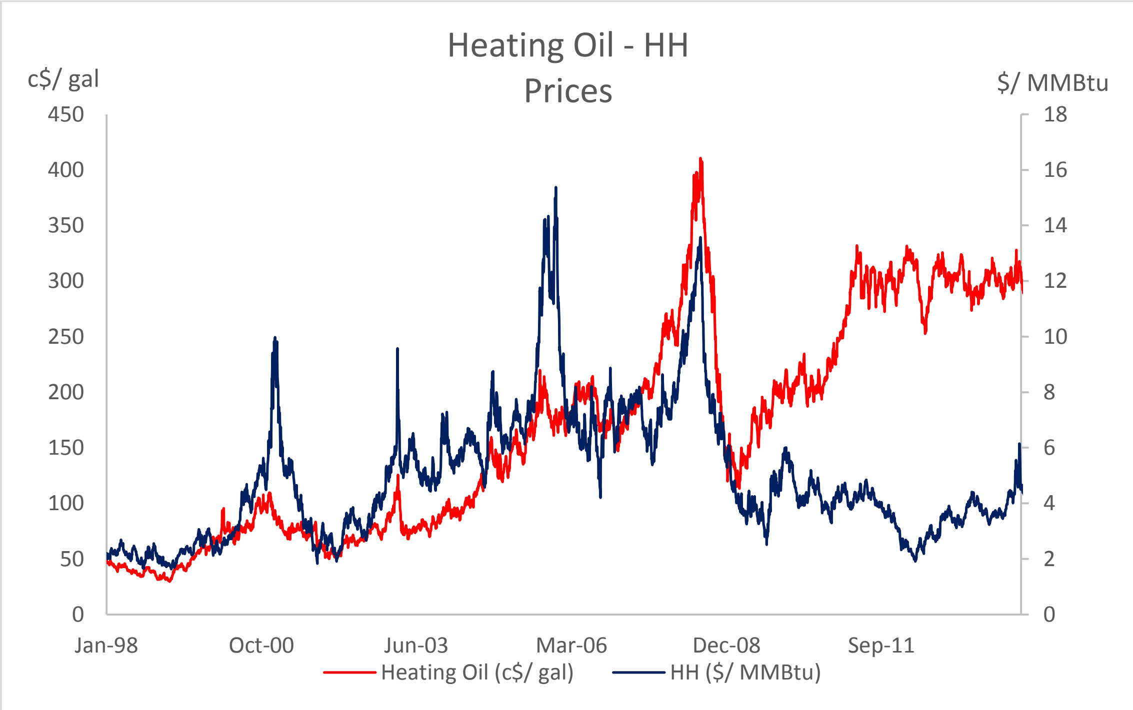


Figure 3.14 HO-HH (Prices)

$\rho_{HO\text{-}HH(Price)}$= 28.1%

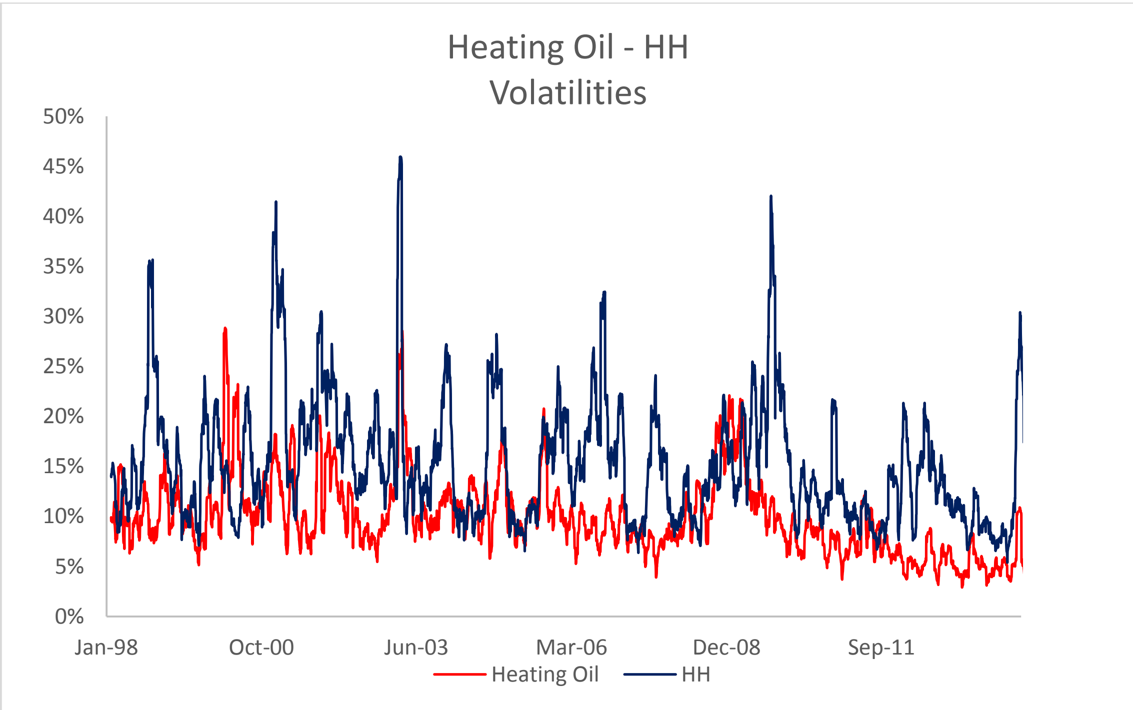


Figure 3.15 HO-HH (Volatilities)

$\rho_{HO\text{-}HH(Vol)}$= 30.2%

3.3 Natural Gas Fundamentals and Price Volatility in the US

Natural gas is traded across the US territory in over 30 different locations or markets hubs, which are situated at the intersection between principal pipelines. The Henry Hub, which has been explained in the previous chapters, is the main referral for pricing natural gas in the US and price differences between local hubs and Henry Hub are usually known as location differential.

The following figure exhibits two main structural periods which are separated by hurricanes Katrina and Rita and are mainly distinguished by an increase in unconventional gas production over the second period. This structural fundamentals are further explained in this chapter.

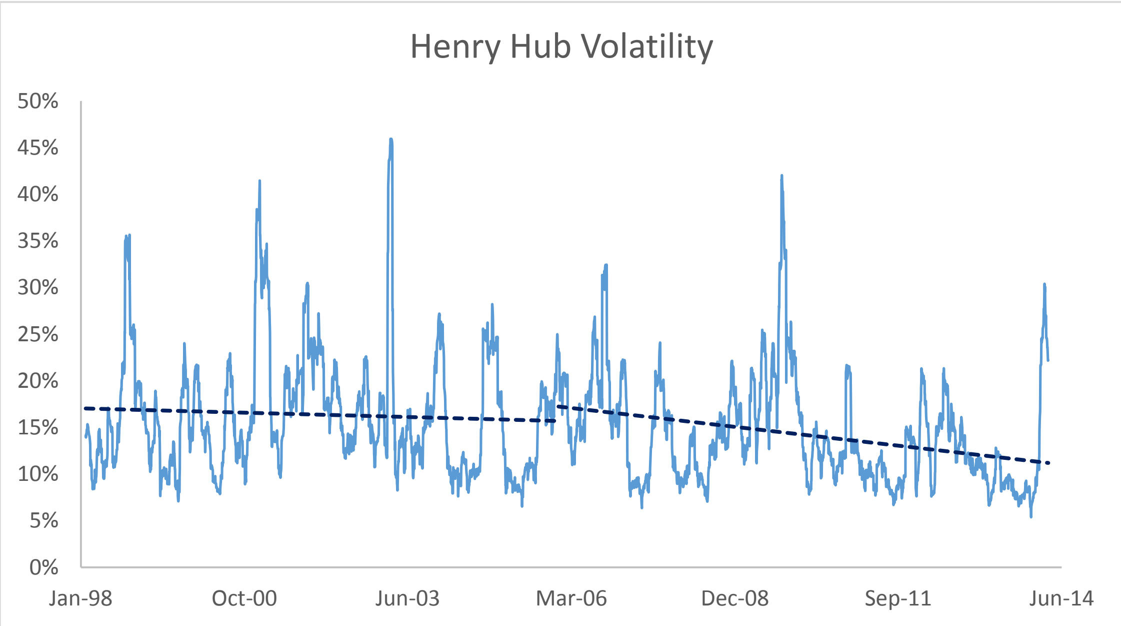


Figure 3.16 Henry Hub Volatility

Natural gas volatility term structure tends to follow a yearly seasonal pattern with highs in the winter months and lows in the summer. According to (Alterman, 2012) and in addition to this pattern, volatility tends to peak in September and October. This is likely to occur due to the fact that by those months natural gas storage inventories have already reached their highs and gas surplus are then viewed as finite to meet the winter demand. A second but lower volatility spike takes part in the late winter months, when gas storage inventories have been nearly depleted and a gas supplies are to meet uncertain late cold weather. The following figure has been developed by averaging daily volatilities from 1999 to 2013, resulting into the seasonal volatility term structures mentioned above.

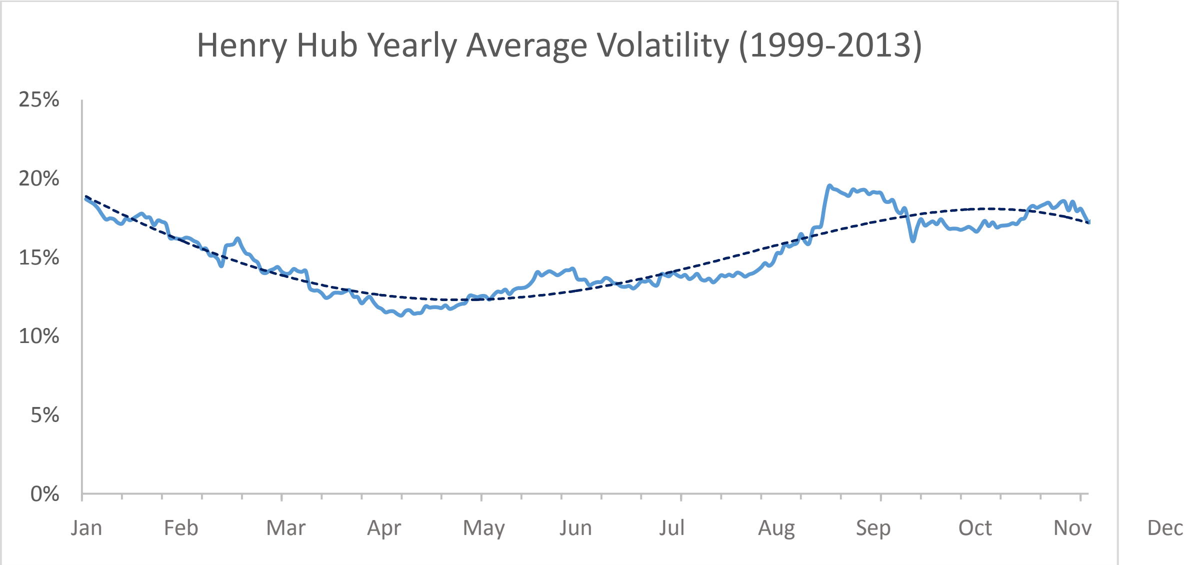


Figure 3.17 Henry Hub Yearly Average

North American natural gas market is characterized by having high churn rates which enable the market to have a high liquidity. These variety of active market participants with financial rather than physical interests can lead to speculative movements on prices. Nevertheless, given the volume of the market and assuming no individual manipulation, the natural gas market responds to several physical fundamentals from the supply and demand side. These physical market drivers are explained below.

3.3.1 Factors Affecting Supply

There are several key factors which determine the availability of natural gas supply in the US. These can be divided into three main groups: hurricanes, storage dynamics and imports.

North America's reliance on Gulf of Mexico's offshore natural gas production has had a direct impact on Henry Hub prices and volatility over the past decades. Nevertheless, a distinction must be made on how hurricane strikes on the Gulf of Mexico have determined these fluctuations. According to this statement, we must distinguish two main periods which have Hurricane Katrina (2005) as breaking point:

- During the late 1990s and early 2000s, offshore gas production accounted for around 25% of North America's total domestic production. When a hurricane struck the oil-producing areas of the Gulf of Mexico, oil rig managers were forced to evacuate their platforms to assure the safety of the operators. These shutdowns disrupted the oil and gas supply and lead to price and volatility spikes (e.g. Hurricanes Mitch, Isidore and Lily). The largest drop in offshore production came with the strike of Hurricane Katrina (with the destruction of 115 platforms and damage of other 52), followed by Hurricane Rita and a harsh winter season which prolonged volatility for several months.
- After this breaking episode, hurricane strikes on the Gulf of Mexico have had a minor effect on Henry Hub prices and volatility due to three main reasons:

- A general improvement of drilling technology and rig infrastructure, as well as a closer collaboration between de American Petroleum Institute (API) and government agencies to raise weather awareness.
- The boom of unconventional gas sources such as shale gas and technologies such as hydraulic fracking, which have lower offshore production to less than 10% of US total production.
- The fewer hurricane strikes which have occurred in the oil-producing area of the Gulf of Mexico.

Given the isolated nature of the North American natural gas markets, gas storage dynamics are essential to meet load variations and to balance the system. As mentioned before, gas storing capacity serves as a buffer for price volatility and its levels are a key indicator of demand. Inventory levels are published every Thursday by the US Energy Information Administration as *Working gas in underground storage* in reports such as the following, for the week ending March 14, 2014:

	Stocks (billion cubic feet)				Year Ago		5-year Average	
Region	**03/14/14**	**03/07/14**	**Net change**	**Implied flow**	**(Bcf)**	**% change**	**(Bcf)**	**% change**
East	395	430	-35	-35	790	-50.0	794	-50.3
West	167	169	-2	-2	339	-50.7	293	-43
Producing	391	402	-11	-11	756	-48.3	742	-47.3
Salt	62	54	8	8	179	-65.4	135	-54.1
Non salt	329	348	-19	-19	557	-43	607	-45.8
Total	953	1001	-48	-48	1885	-49.4	1829	-47.9

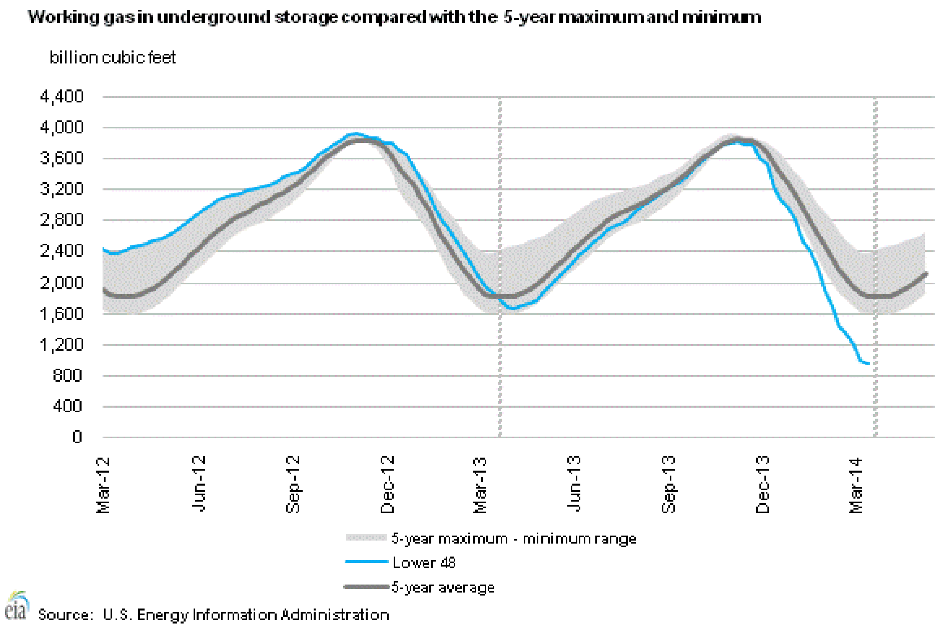


Figure 3.18 Working Gas in Storage

Where lower 48 stands for the inventories across all continental states in the US. Storage levels follow a seasonality term structure with a peak in October, which is the last injecting month before the winter. Storage levels will then decrease throughout the winter while gas is withdrawn to cover the heating demand. It is during the course of these months when inventory levels ought to be strictly monitored. Early cold weather will drive inventories down and a climate of uncertainty that will be materialized in an increased volatility over the following winter months. The lower turning point is crucial on market prices due to the fact that extraordinary cold weather and low inventories may lead to unforeseen price spikes. From 2005 onwards, minimum storage levels have kept rising on a yearly basis due to an increased domestic production from shale gas and an intensive exploration of new storage deposits. This upward trend has been disrupted since March 2013 due to a general slowdown in domestic production and outstanding late cold temperatures in North America during the beginning of 2014.

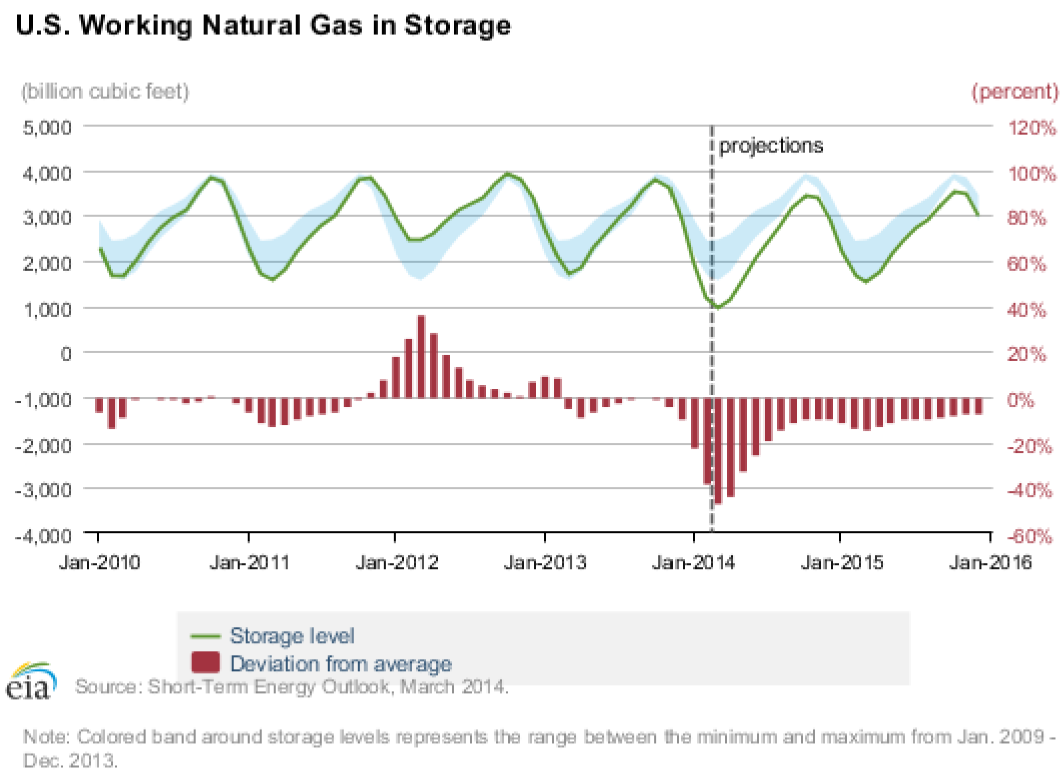


Figure 3.19 Working Gas in Storage II

According to (Alterman, 2012), US natural gas imports rose up to 16% of total demand over the early 2000s and have declined from 2007 onwards with the rise of unconventional gas sources. These imports are represented by pipeline imports from Canada (which account for over 95% of total imports) and LNG imports from Algeria, Australia, Canada, Egypt, Malaysia, Nigeria, Norway, Peru, Qatar, Trinidad and Tobago, Yemen and United Arab Emirates. These LNG imports enter the system through regasification terminals located across the Gulf Coast and throughout the East Coast (it is worth emphasizing that these LNG facilities are only conceived for imports). US natural gas import and export activity is regulated by the US Department of Energy (DOE) and the Federal Energy Regulatory Commission (FERC). While the latter is responsible for review and approval of the construction and operation of pipeline and

LNG import and export facilities, DOE is responsible for authorization of the natural gas importing and exporting contracts.

Particularly remarkable is the fact that US exports to Mexico are expected to double from 2013 to 2016, from an average of 2 Bcf/d to 4.5 Bcf/d (*Platts, October 2013)*. This expected increase in natural gas demand comes from an intensive expansion of the Mexican pipeline network driven by new power generation and industrial needs. Up to eight new major pipelines are expected to be built by 2017 and three of them will connect directly to the US grid.

The following figures exhibit the evolution of US natural gas imports over the period 1994-2013 and their price per volume:

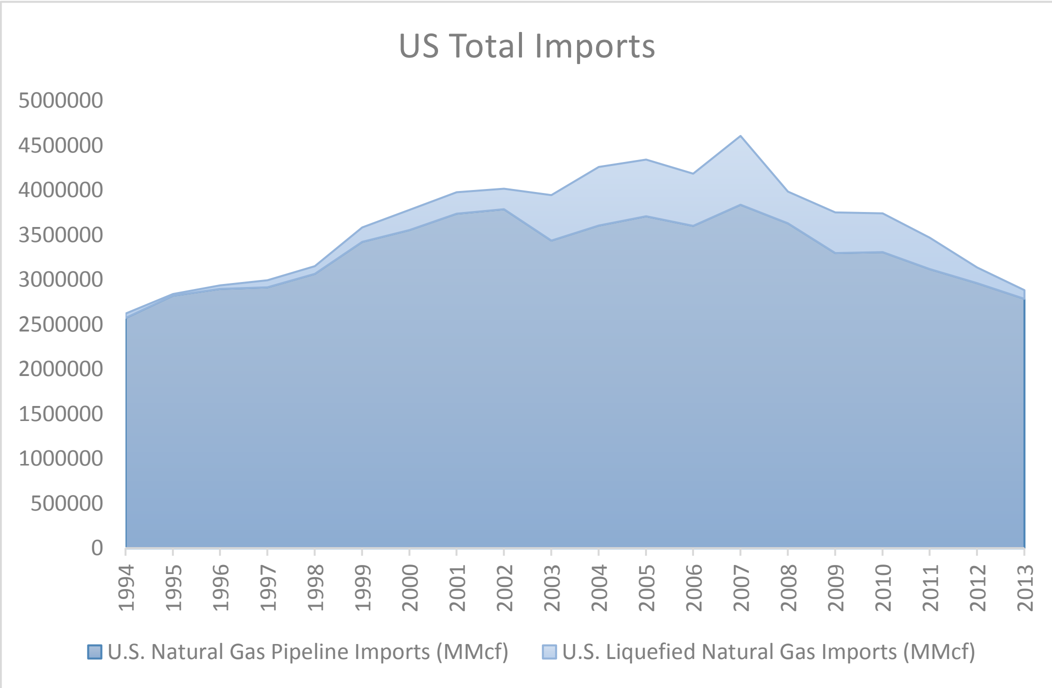


Figure 3.20 US Total Imports

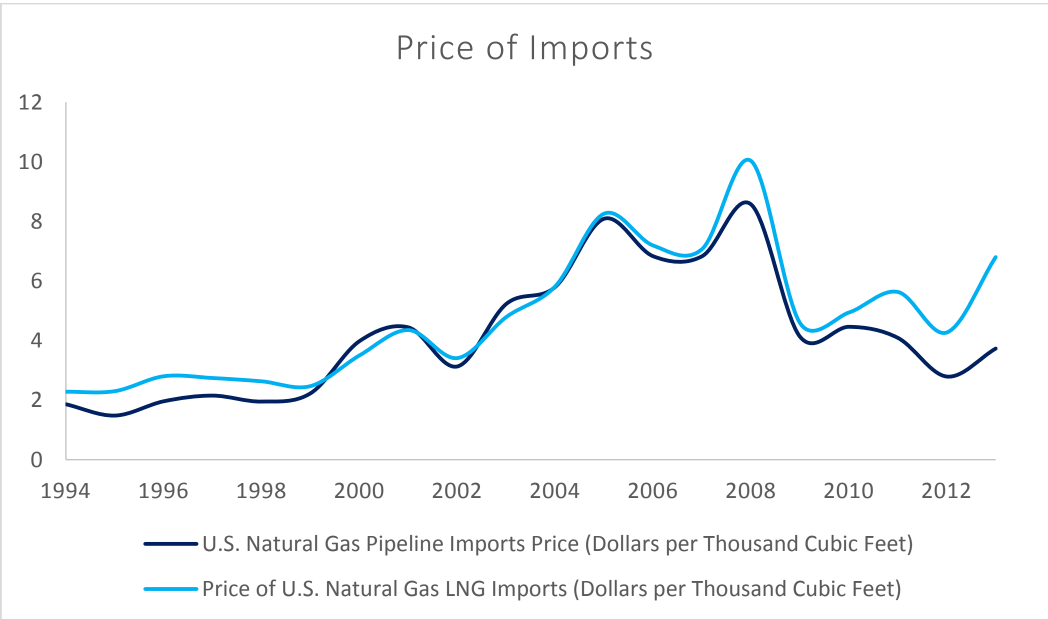


Figure 3.21 Price of US Imports

3.3.2 Factors Affecting Demand

Natural gas demand in the US is vastly affected by GDP growth and efficiency gains in power generation and industrial production. These effects will respectively increase and decrease demand. Nevertheless, despite its impact on natural gas term structure on a year to year horizon, these factors have little impact on seasonal volatility.

For the scope of this project, it more interesting to analyse the impact on demand due to weather. From the already defined seasonal term structure, warmer or colder than average years will lead to increased volatility during the summer or winter months, respectively. This is due to an increase in electricity demand for space cooling during warmer than average summers, whereas colder than usual winters will raise gas demand for heating. Deviation from the average are calculated through the number of Heating Degree Days in a year, which represent the number of days with average temperature below 65ºF (18ºC) and therefore require buildings to be heated.

3.3.3 Conclusions

Having explained US natural gas fundamentals, we can now follow (Alterman, 2012) and undertake an explanatory analysis of major volatility peaks on Henry Hub prices from 1998 to 2014:

1. October, 1998: Hurricane Mitch, concern about supply continuity.
2. December, 2000: Coldest November and December months on record, volatility due to increased demand for storage and consumption.
3. September, 2001: Terrorist attacks on World Trade Centre followed by the collapse of Enron a month after. Extraordinary systemic events which dropped prices due to global fear on recession.
4. February, 2004: Late cold weather plus low inventories peaked prices and volatility levels reached maximums. Volatility affected by storage constraints and increased demand.
5. September, 2006: Warmer than usual winter plus increased production due to shale gas lead to highest storage injections till date and prices to drop dramatically.
6. September, 2009: Mild summer heat and continued low demand due to global depression took inventory levels to their highs while plummeting prices.
7. February, 2014: Late cold weather in North America and lower production made prices to peak as inventory levels reached ten year minimums.

From the previous events we can draw the following conclusion:

Volatility peaks in the Henry Hub market show their highest correlation to extraordinary weather events which occur during early or late winter months, when concern on storage inventory levels is higher. Other systemic events will have a minor impact on prices due as long as storage buffering is guaranteed.

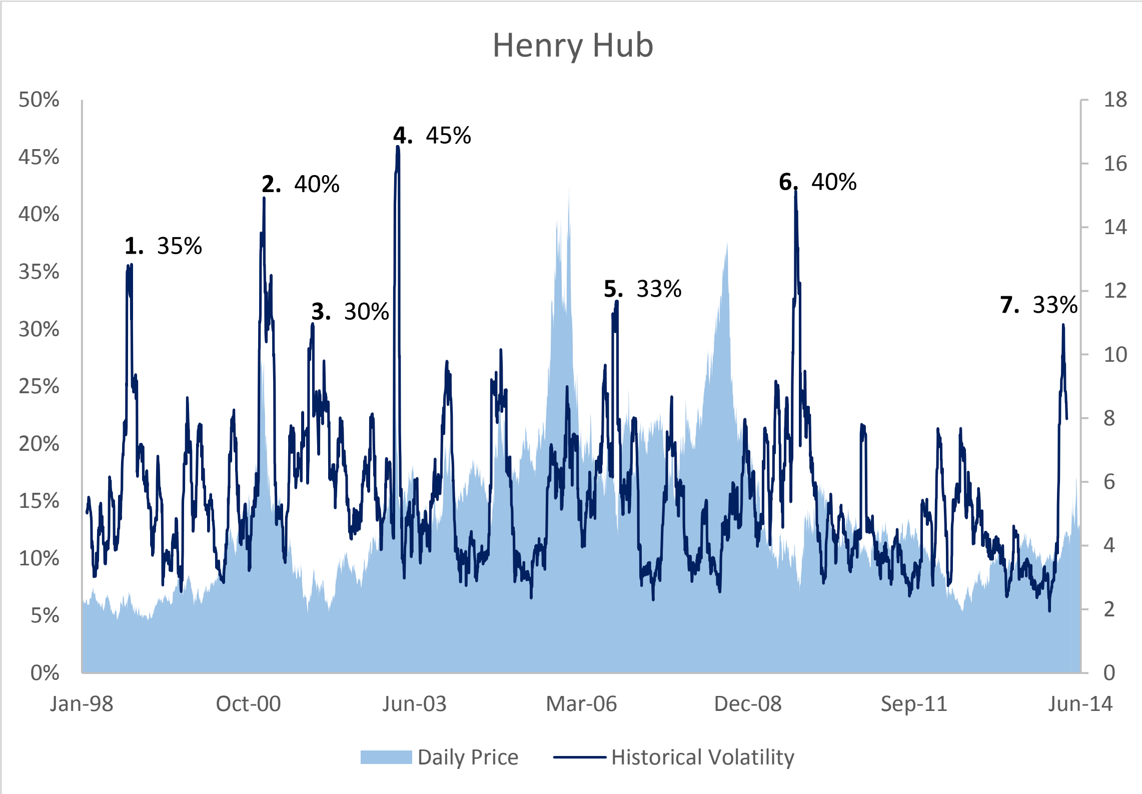


Figure 3.22 Henry Hub Prices and Volatility

3.4 Natural Gas Fundamentals and Price Volatility in the UK

The National Balancing Point (NBP) is a virtual trading hub for UK natural gas and serves as the pricing reference and delivery point for the Intercontinental Exchange (ICE Futures Europe) natural gas futures contracts under the name *UK Natural Gas Futures.*

An analysis of NBP's volatility from 1998 till March 2014 exhibits two main structural periods:

- From late 90's till 2007, which presented a steady volatility growth due to a continuous tightening of the market. Whereas the end of the twentieth century was characterized by an abundance of supply due to high domestic production, the beginning of 2000 came along with an increase in Continental Europe's oil indexed prices and lower production. Political instability in Eastern Europe during the mid-2000's lead to import shrinkage which immediately spiked prices. In addition, facility unavailability during this period lead to unprecedented volatility hikes.
- From 2008 onwards, linkage developments with Continental Europe have made prices converge to Continental Europe's oil-indexed contracts. Whereas prices have increased, arbitrage between these territories has been reduced, dragging volatility in both markets down. Punctual volatility spikes have come from geopolitical crisis, but have had less impact due to a greater diversification of supply.

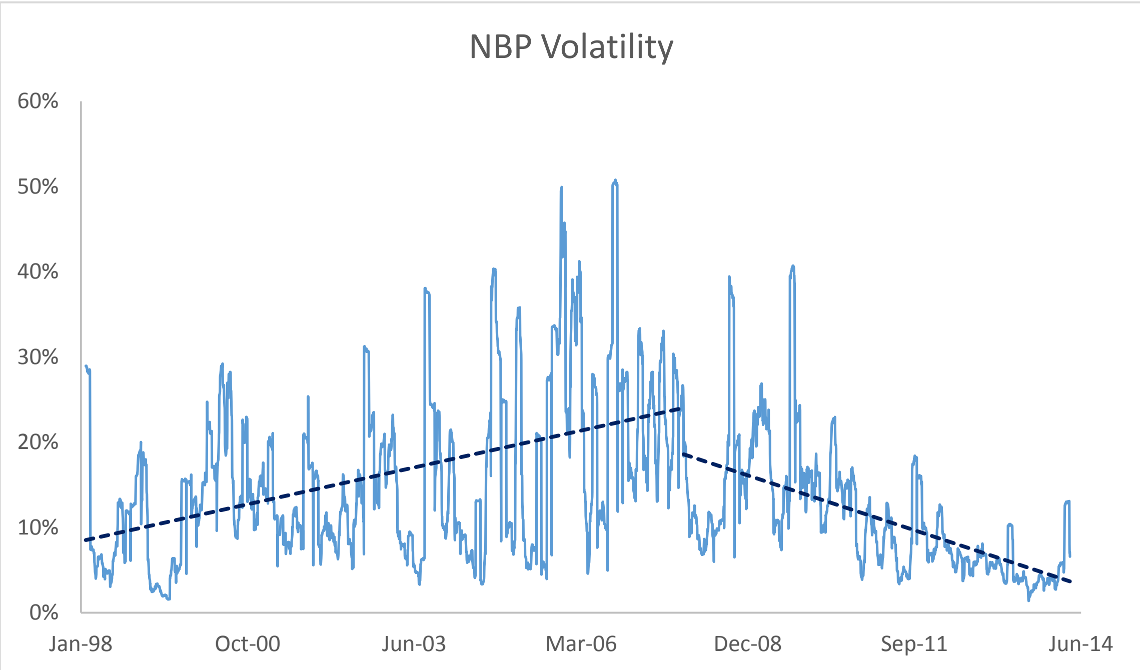


Figure 3.23 NBP Volatility

The UK natural gas market is a liberalized market and therefore price is fully driven by the interaction of supply and demand. This market fundamentals affect prices in a similar way as they affect Henry Hub settlements, but some facts must be taken into consideration:

- First of all, power generation fuelled by natural gas accounts for over 40% of UK's total electricity generation (followed by coal, 30%, and nuclear, 18%), which doubles in percentage the use of natural gas for power generation in the US.

- Secondly, the UK market differs from the North American market in what refers to availability of storage (4% of its annual consumption compared with US' 18%).

These facts affect the seasonal term structure for natural gas in the UK: despite the fact that winter months present higher volatility than summer months, the yearly average term structure exhibits a local peak during the summer months. This local maximum can be explained by the high share of natural gas in total electricity production. A second conclusion can be drawn from the average volatility term structure and is derived from the second factor above: year maximums are now reached in the mid to late winter months (January and February), as opposed to Henry Hub, where year maximums were reached during September and October. An explanation for this point is the low buffering effect in the market given its low storage capacity. This buffering constraint has as direct consequence the inability to ease price spikes derived from unexpected late cold weather.

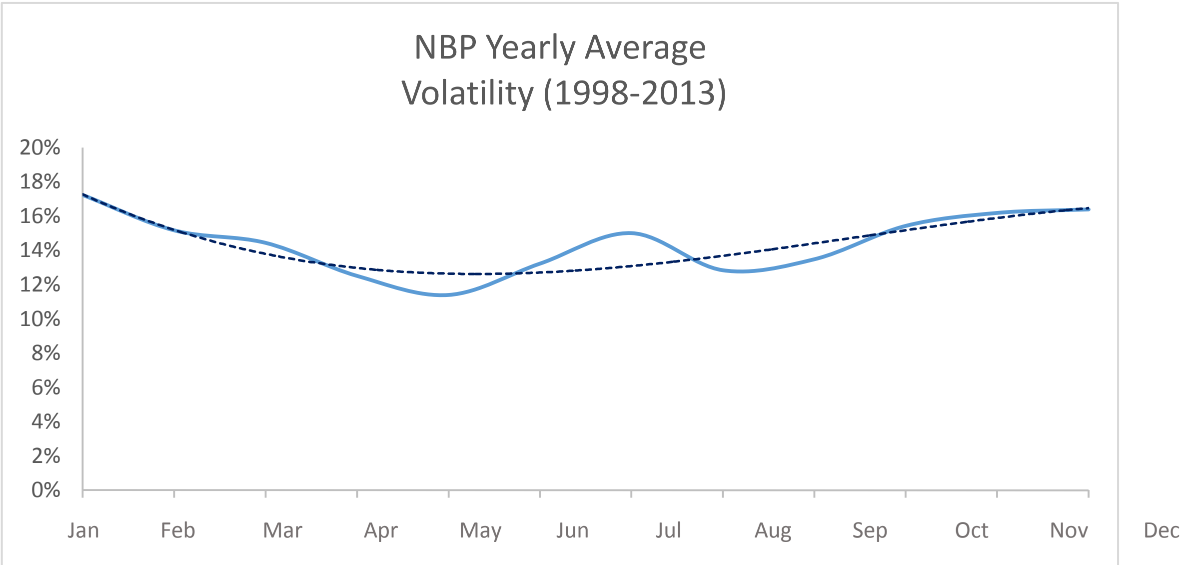


Figure 3.24 NBP Yearly Average Volatility

Similarly to Henry Hub and regardless of speculative activity, the NBP physical market responds to several drivers which are explained below.

3.4.1 Factors Affecting Supply

As it has been previously explained, natural gas storage (natural salt caverns and depleted fields or artificially built storing facilities) is a source of flexible gas which can be relied to during unexpected demand spikes and can play an important role in ensuring security of supply.

According to (Fevre, 2013), as of January 2013 Great Britain's working capacity amounted to 4.73 Bcm, from which 3.65 Bcm is provided by only one storage unit, the depleted field of Rough located in offshore Easington and operated by Centrica. The relatively low storage capacity in the UK has been claimed to be sustained on two factors:

- The historical ability of domestic fields to vary production levels according to seasonal demand, which has decreased over the past decade due to a greater dependence on foreign gas supply.
- UK's consolidated position as a net importer due to an intensive investment on transportation assets over the past decade, which has granted the system with a diversified portfolio to ensure supply continuity.

This reluctance to storage was threatened in February 2006 with a fire at the Rough facility which forced its service to be interrupted. No injection or withdrawals were possible during the following four months and therefore prices rose steeply. Prices peaked to 259p/therm form 70 p/therm during March 2006 due to a cold snap and the impossibility to withdraw any volume from the Rough facility.

Nevertheless, from 2006 onwards, security of energy supplies has come through the deployment of new projects and the commissioning of transportation infrastructure. The Pöyry Report (Morris, 2010) modelled the impact of a range of supply shocks such as losing the Rough storage facility. It concluded that Great Britain has now enough diversity and capacity to receive gas from LNG terminals, Norwegian pipelines and Continental Europe interconnectors to meet nearly all of UK's gas demand. Despite the report recognized that importing capacity is not the same as available gas inventories, it claimed that diversification of suppliers and a move towards a more liberalised market in Continental Europe were both favourable aspects to ensure UK's position as a net importer.

Due to the low percentage of storing capacity in the UK, supply flexibility to meet the country's seasonal variations has been found through a broadly diversified portfolio of importing infrastructure. Commissioning of these assets throughout the past decade has had a direct impact on NBP gas prices due to an increased availability of supply. Several episodes can be highlighted throughout UK's international grid expansion:

- The commissioning of UK Interconnector Pipeline across the North Sea between Bacton Gas Terminal and Zeebrugge, Belgium, in October 1998.
- The opening of Isle of Grain's LNG importing terminal in July 2005.
- The BBL Pipeline (Balgzand Bacton Line) between the Netherlands and the UK, in July 2006.
- The opening of the Langeled Pipeline from Ormen Lange gas field in Norway to Easington, from October 2006 to October 2007.
- The operational start of South Hook LNG terminal, the largest LNG facility in Europe, in March 2009.
- The commissioning of Dragon LNG terminal in July 2009, which together with South Hook terminal can handle up to 25% of UK's gas demand.

Late 90's were characterized by a high productivity in the UK Continental Shelf and the export of gas surplus to Continental Europe through the Interconnector Pipeline. Form the year 2001 onwards, domestic production began to fall as Norwegian imports rose heavily. Continental Europe imports also began to flow from Belgium in order to meet UK's winter demand. Nevertheless, in the year 2005 Gas supply from Continental Europe was interrupted as a cold snap threatened Europe's heating resources. The following years saw similar volatility spikes due to unusual cold weather and continuous geopolitical conflicts between Russia and Ukraine. The commissioning of the Langeled Pipeline was a key episode due to an oversupply of gas that could not be stored with the current infrastructure and dragged prices down to minimums. This bearish trend was reversed with the outage of the CATS UK import pipeline and the general concern on supply. With the global economic slowdown and the rise of shale gas, LNG demand in Asia and the US began to decline steadily. Unprecedented LNG flows then started to address the UK, resulting in low UK prices. This availability of LNG supply plus the ongoing balancing flows with Continental Europe have minimised NBP's volatility over the past years.

The following charts exhibit UK's net flow balance over the past years and the origin and destination of these flows:

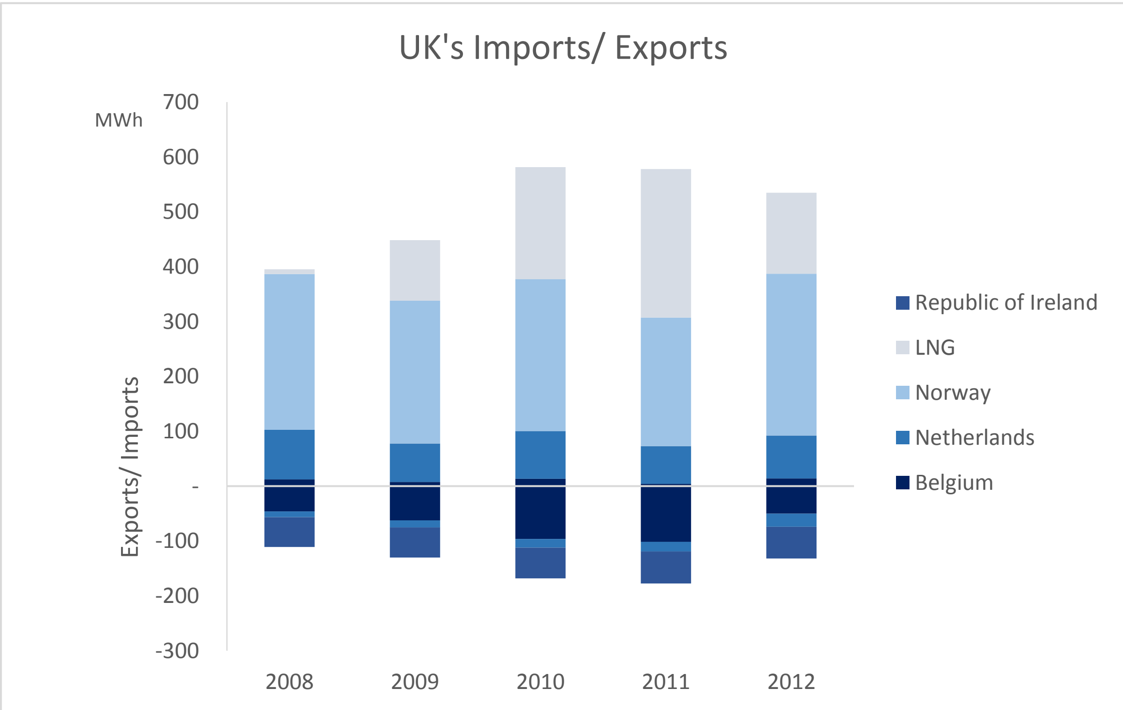


Figure 3.25 UK Exports-Imports

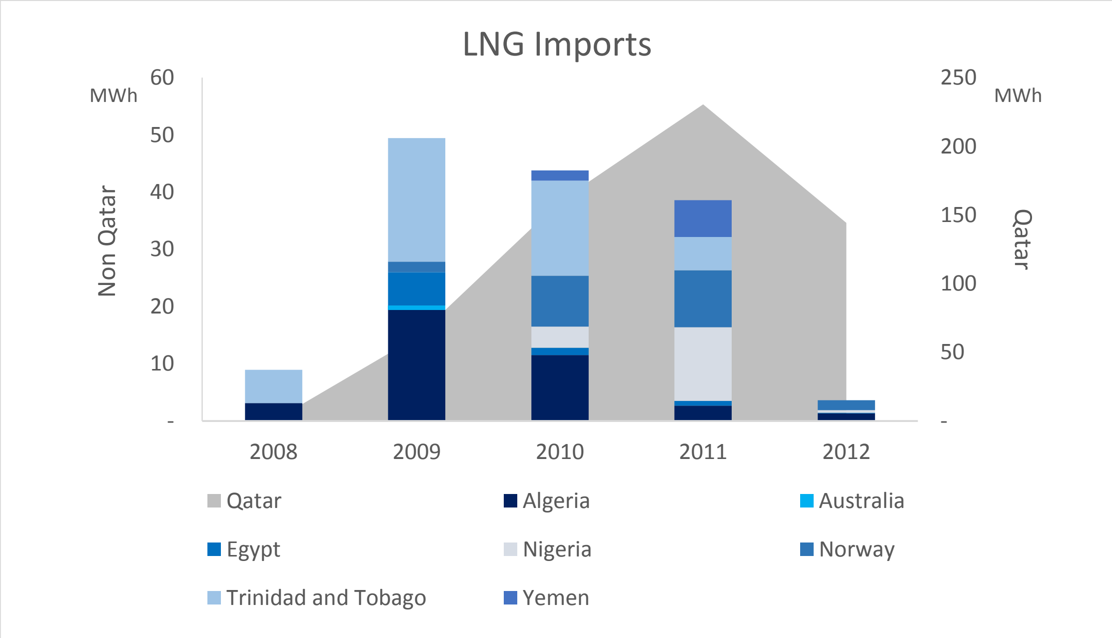


Figure 3.26 LNG Imports By Origin

According to (Fevre, 2013), major disruption to UK's gas supply from 2000 onwards were:

- February 2006: Fire in the Rough platform resulting in a four month outage. Prices peaked from 70p/therm to 259 p/therm due to a cold snap in March.
- July 2007: CATS pipeline was damaged by a ship's anchor and was shut for 64 days. Volatility spiked as priced were low at the time.
- February 2008: Fire at the Shell Bacton terminal. Immediate loss of supply and 25% increase in prices within a day.
- January 2009: Geopolitical tension between Russia and Ukraine which lead to an increase in exports to Continental Europe.
- January 2010: Norwegian supply interruptions during high demand lead prices to double NBP's winter average.
- February 2012: Russian supply restrictions during cold weather derived in price peaks across all European hubs.

3.4.2 Factors Affecting Demand

Given UK's natural gas major usage as space heating and power generation fuel, consumption patterns will be closely associated to weather. Moreover, unexpected colder than average weather has shown to have a high impact on prices due to low buffering capacity. On the other hand, unusual warm weather during the summer has also demonstrated to have a direct impact on prices due to the country's high dependence on natural gas for power generation.

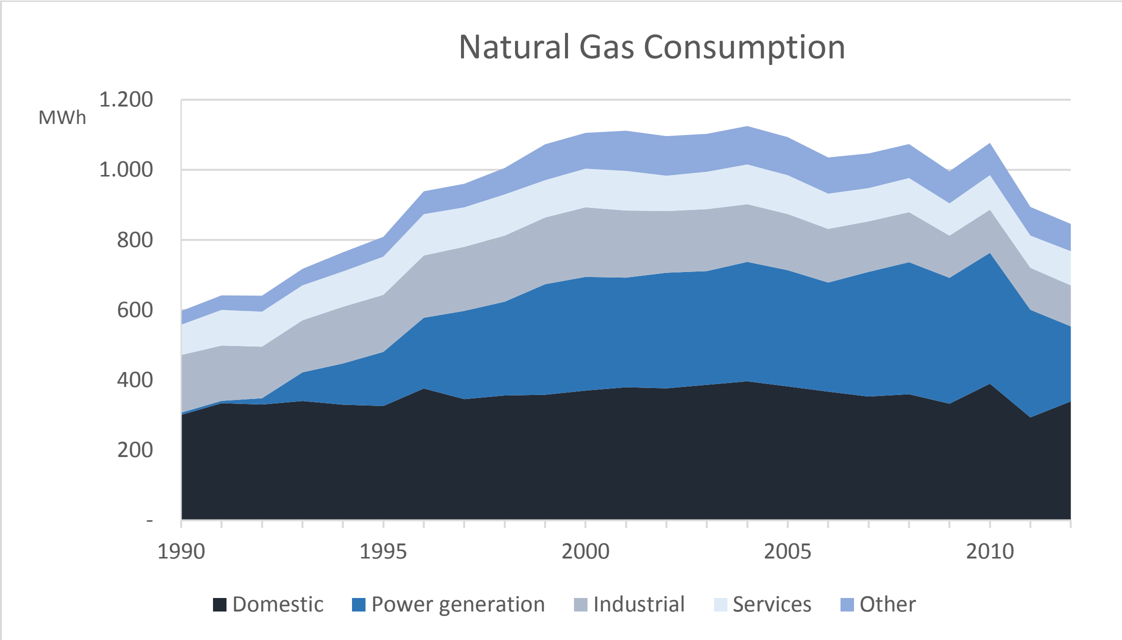


Figure 3.27 Natural Gas Consumption

3.4.3 Conclusions

Given NBP's fundamentals and market drivers, it is now possible to explain volatility spikes and lows throughout the period 1998-2013:

1. October, 1998: commissioning of the UK's Bacton - Zeebrugge interconnector pipeline, beginning of exports/ imports flow.
2. April, 2000: High oil prices in 1999 lead to an increase on oil-indexed prices in Continental Europe in 2000 (as these contracts are derived from the average oil prices in a 6 to 9 month period), which lead to an increase in UK prices due to a lower self-supply.
3. July, 2005: Commissioning of the Isle of Grain LNG import terminal.
4. November, 2005: Early cold weather in Europe reduced imports to the UK.
5. January, 2006: Political instability between Russia and the Ukraine lead Gazprom to reduce its gas flows through Ukraine to Europe.
6. February, 2006: Fire at the Rough storage facility resulted in a four month outage.
7. October, 2006: Commissioning of the Langeled pipeline from Norway.
8. July, 2007: Damage in the CATS pipeline left the infrastructure inoperative for two months.
9. October, 2007: Increased production in the Ormen Lange gas field in Norway increased gas imports from that country.
10. February, 2008: Fire at the Shell Bacton terminal.
11. October, 2008: Decrease in global LNG demand due to economic recession.

12. January, 2009: Gas imports from Russia through the Ukraine were cut off due to political conflict between these countries.
13. March, 2009: South Hook LNG import terminal begins operation.
14. July, 2009: Dragon LNG terminal begins operation.
15. January 2010: Norwegian supply interruptions during high demand in the UK.
16. February 2012: Russian supply restrictions during cold weather.
17. March, 2014: Russia takes over Ukrainian Crimea.

It is important to notice that, despite Russian supply cut offs due to geopolitical conflicts with the Ukraine have been a constant issue throughout the past decade, its impact on NBP gas prices has been lowered due to an increased diversification of suppliers, mainly from Norway and LNG imports form Qatar.

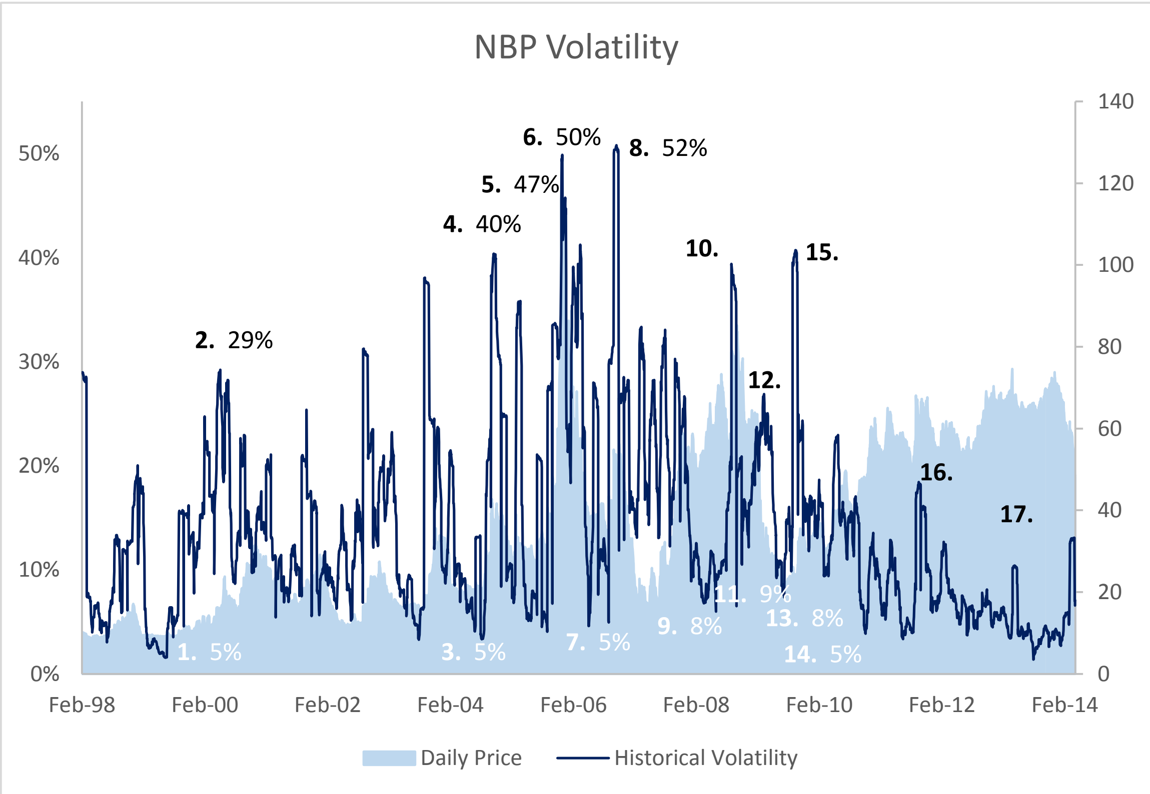


Figure 3.28 NBP Volatility

4

Managing Risk through Hedging Derivatives

4 Managing Risk through Hedging Derivatives

4.1 Risk quantification in energy portfolios

Managing uncertainty in a changing environment is vital in order to achieve growth in value of a company or a portfolio. Handling a wide variety of uncertain market variables can only be achieved through prescribing several scenarios which can closely explain market behaviour and estimate possible future values for financial contracts and physical assets. It is therefore a daunting task to be able to develop a consistent framework for analysing and measuring risk. Underestimating future variability of prices and asset values can lead to excessive exposure and unforeseen disastrous financial outcomes. On the other hand, overestimating the variability in future value of a portfolio can result in an inefficient expenditure of risk-managing budget.

Energy companies will often have exposures driven by portfolios which enclose both physical assets (pipelines, storage facilities, power plants, vessels, etc.) and financial contracts. Therefore, energy companies face a greater challenge than purely financial companies when it comes to modelling and valuating risk. This increase in complexity is caused by the mere dynamics of the underlying energy markets, whose prices fluctuate more than equity or fixed income securities due to seasonality patterns and price spikes. Moreover, energy markets are more complex for three reasons: they usually involve spreads (crack spreads, calendar spreads, spark spreads), they are subjected to volumetric uncertainty and they typically involve physical facilities (pipelines, cargoes, storage facilities, etc.). This variety of assets involves a wide range of variables and constraints and the need to develop individual approaches for each portfolio instead of generic standardized solutions.

4.1.1 Major risks faced by energy companies

Risks can be organized in two main categories, according to their feasibility to be measured and modelled with a certain degree of accuracy. We will therefore distinguish between quantifiable and non-quantifiable risk. The latter is usually related to operational and organisational risk, and are managed through the deployment and enhancement of operational processes. Regarding the first ones, quantifiable risks can be again divided into the following:

- *Market or price risk:* as it can be drawn from its name, this risk is related to changes in market prices. The behaviour of these prices involve several factors such as volatility, correlation to other prices and illiquidity.
- *Credit (default) risk:* it represents the risk of a counterparty failing to pay or deliver the underlying asset comprised in a contract.
- *Modelling risk:* it is derived from the lack of consistency in the results produced by the analytical framework used to model, valuate and price a portfolio.
- *Volumetric risk:* it represents the failure to deliver the amount of the underlying asset specified on a contract due to an ineffective operational performance or to external variables such weather, social or regulatory uncertainties.
- *Financial risk:* it represents the fact of initial financing costs forecasts mismatching real costs, resulting from changes in interest rates, currencies and a lack of cash flow.

4.1.2 Measuring quantifiable risks

Given the major quantifiable risks above, the principal methodologies used to measure these risks are the following:

- Measuring fair asset values, which passes through modelling revenues and costs of a specific asset over the life of that asset. This entails having access to historical market variables (mainly prices and volatilities) over periods where the asset was actively traded. These inputs can be employed in stochastic methods to develop net present value distributions and internal rates of returns through forecasting future asset cash flows.
- Measuring impact of volatility of asset prices on a whole portfolio, instead of valuating individual assets. This procedure is conceived to measure all supply and demand variables which have direct impact on a portfolio over a period of time, with the intent to measure the results of diversification.
- Measuring economic capital adequacy, which is defined as the capital required for a company in order retain and grow equity on time. Even if the present outlook for a company seems favourable on the short run, economic capital is required to overcome future uncertainties that may derive into unfavourable scenarios. These situations will represent a lack of equity growth in favour of an increase in debt. Measuring this variable enables companies to manage their asset’s increase in value from year to year without having to worry for long term value of their portfolios and allowing them to quickly react to short and midterm market events.
- Measuring cash flow adequacy, which is related to the financial liquidity and the requirements of cash over a period of time in order to meet payment needs. The disparity between short term payments and retail incomes over a larger period opens new exposures to companies. Being able to forecast cash requiring events creates value for the company while reducing its inefficient expenditures.
- Measuring budget uncertainty, which consists on spotting differences between budget results and real outlooks. This enables companies to explain results better to shareholders and to balance the cost of hedging strategies. This too combined with the measurement of earnings at risk allows the valuation of variability in a portfolio and in the budget outcome.
- Measuring performance of trading strategies, in order to valuate if these strategies are beating the market and the possible variance of the position. This performance is deployed through mark to market objectives for specific units, whereas the downside is usually controlled through a VaR measure. Improving individual performance is therefore the most effective way to improve profitability.
- Measuring credit exposure, which is understood as the probability of loss due to the default of a counterparty. This allows companies to place limits to their exposures to any counterparty on their OTC positions (exchange traded products will generally count with clearing houses to mitigate these risks). Moreover, establishing different time horizons in which to measure potential future exposure can provide an overview of the replacement value in the contracts of any counterparty.

4.1.3 VaR and its acceptance in energy risk management

Value at Risk (VaR) is a widely used risk measure in financial risk management. For a given portfolio, probability and time period, VaR is defined as a limit value such that the probability that the mark to market loss on the portfolio over the time period exceeds this value is the given probability level (assuming no trading over the period). For example:

Given a hypothetical portfolio whose Profit and Loss probability function follows a standard normal distribution, and is said to have a one day 5% VaR of 1M$, there will be a 0.05 probability that the portfolio falls in value by more than 1M$ over a one day period assuming no trading activity will take part over that day. From this we can expect a loss of 1M$ or more on one day of every twenty days, which would be defined as a VaR break.

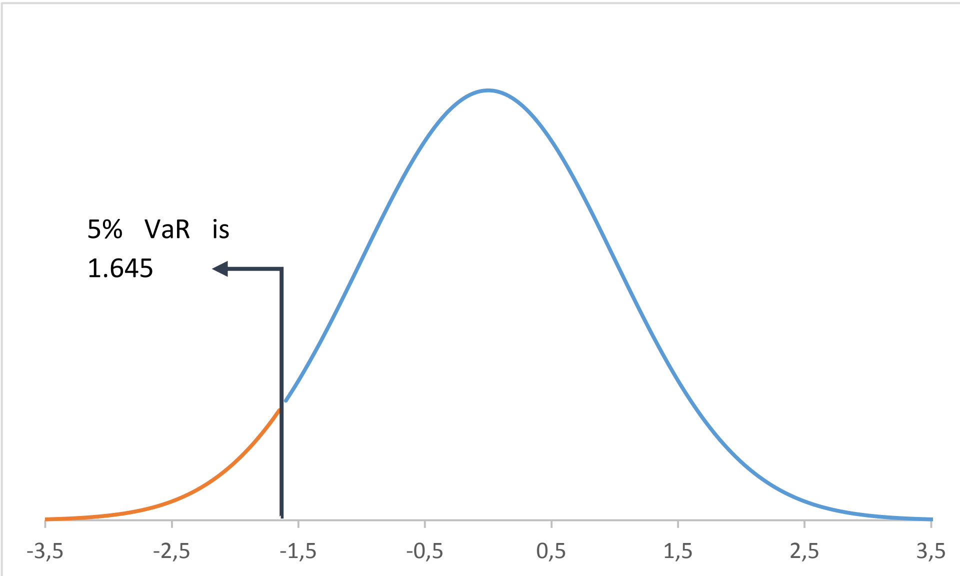


Figure 4.1 A 5% VaR

The probability level is usually represented as one minus the probability of entering into a VaR break. In the previous example, the portfolio would be said to have a one day 95% VaR. This measure has four main uses in finance: risk management, financial control, financial reporting and computing regulatory capital. For the course of this section we will focus on it use as a risk management tool.

VaR is the most commonly used metric for energy organisations, which can also be found as Delta or Analytic VaR. It is used as the simplest metric to make decisions, apply hedging strategies, face market volatility and enhance profitability. Not only it is used a risk metric for the day-to-day running of the business, but has expanded to become a company risk profile indicator for senior management and shareholders. The success of this metric among other studies relies in its simplicity and its straightforward approach towards diagnose a company's risk profile. Nevertheless, many have remained sceptic towards the use of this metric in an energy environment due to several factors:

- The assumption of normal Geometric Brownian Motions (GBM) price distributions for the underlying assets, where market variables follow strong mean reversion and seasonality patterns, along with price discontinuities which can´t be conceived through GBM.

- Short term horizon, which doesn´t couple in with typical long-term take or pay oil and gas contracts.
- Liquidity is taken for granted in the VaR method, assumption that is not always certain in OTC energy contracts.
- The combination of physical assets and financial contracts on a same energy portfolio can become a dreadful task to accomplish even with advance technology solutions. This limitation suggests leaving aside physical assets when calculating VaR of a portfolio, taking into account only those products whose mark-to-market can be easily calculated and therefore underestimating the total risk of the portfolio.
- The VaR method is fully driven by price variables, whereas energy companies risk profile is also driven by other variables such as generation levels, temperature, weather, storage dynamics, etc.

4.2 Managing Natural Gas Price Risk

Energy markets are generally more volatile assets than commodity markets which, at the same time, are more volatile than equity, fixed income and FX markets. The main reason why energy products and commodities in general are more volatile than the latter is given by the uncertainty on potential supply. The fact of commodities being expensive to store along with their intrinsic long recovering nature establish these products as the most uncertain and therefore volatile in the financial landscape. This price uncertainty leads individuals and organizations to develop hedging strategies through hedging instruments which seek to minimize the short or long term impact of price movements on their performance.

Smaller organizations will use exchange traded products such as futures and options which are traded on organized exchanges such as ICE (Intercontinental Exchange) an NYMEX (New York Mercantile Exchange) in order to hedge their transactions and portfolios against price movements. Larger organizations will rely on OTC contracts with banks and other large companies in the industry, developing tailored solutions that meet their specific needs. (Downey, 2009) clearly explains the main two reasons why hedging is essential: to reduce short-term cash flow volatility and to maximize return on invested capital for a target level of risk.

Regarding the first reason, reducing cash flow volatility reduces the risk of bankruptcy due to unexpected price movements, which in turn reduces cost of capital for companies. Due to the fact that their performance is now more foreseeable, investor’s confidence will increase and companies will be able to raise more funds. The second principle for hedging is related to portfolio theory, which is used to optimize the overall result of a portfolio containing a variety of assets for a specific level of risk. This theory can be applied to the expected result from a company for a level of risk which shareholders are confident with.

Implementing hedging strategies involves narrowing operating results. Nevertheless, hedging involves incurring in additional expenses, which lowers the overall revenue for the period. This effect can be clearly exhibited in the following figure:

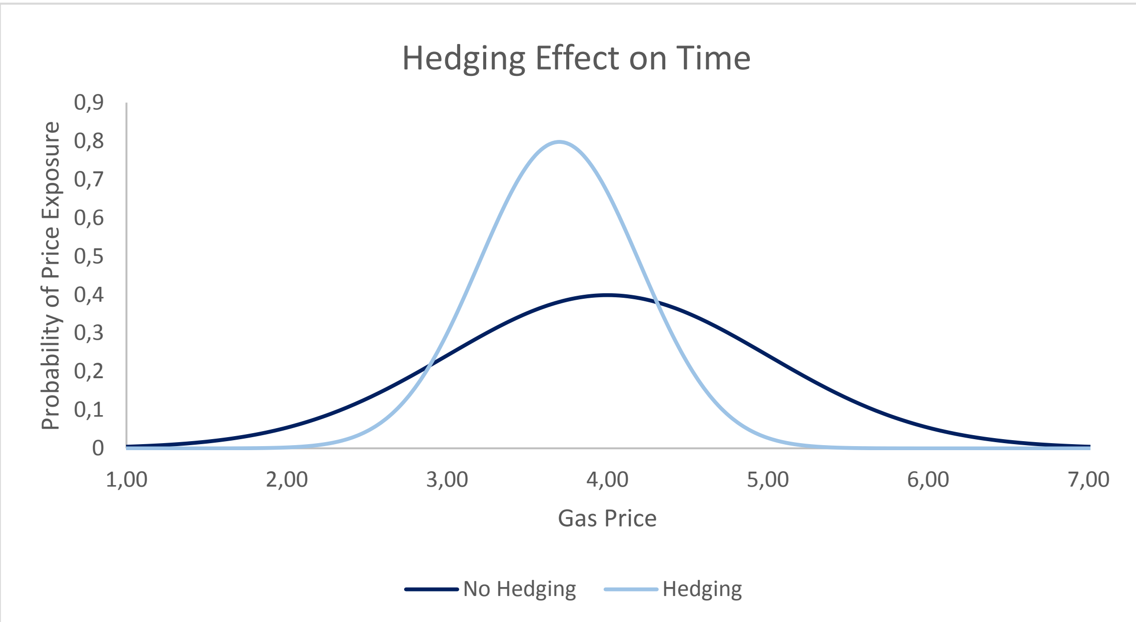


Figure 4.2 Hedging Effect on Time

Investment risk is be primarily reduced through diversification (by investing in cross correlated assets in a same portfolio) and by using hedging instruments which are called derivatives. These derivatives stand for financial products whose value depends on or is derived from the value of an underlying asset. Depending on where the derivatives contracts are exchanged we can distinguish two broad categories: Exchange Traded Markets and OTC (Over the Counter) contracts.

An Exchange Traded Market (or simply, *Exchange*) is a market where individuals trade standardized contracts that have previously been defined by the Exchange under specific conditions. These markets have evolved from physical *open outcry system* towards electronic platforms which connect sellers and buyers worldwide. When trading on an organized Exchange, market players do not have any contact with their counterparties. Therefore, Exchange Traded Markets require a separate institution responsible for settling trading accounts and clearing trades, which is called a *Clearing House*. This corporation is in charge of reducing credit risk from all counterparties by regulating funds to cover debit balances. Moreover, Clearing Houses must collect and maintain margin monies, regulate the delivery of underlying where necessary and reporting trading data at the end of every trading session.

An alternative to Exchange Traded Markets is the OTC Market, which is a telephone and online network of dealers. OTC trades usually involve two financial institutions or a financial institution and a corporate client. In what refers to the type of contracts that are traded OTC, they generally consider larger trading volumes than Exchange Traded contracts and can be fitted to suit corporate specific requirements.

4.3 Hedging Derivatives: Futures and Forwards

The price of an asset for immediate delivery is said to be the spot price. On the other hand, the price of an asset on a certain time in the future is called the forward price. The main difference between future and forward contracts is that futures are traded on Organized Exchanges whereas forward contracts are traded OTC. Given this distinction and according to the previous point, future contracts require a Clearing House in order to guarantee that the contract is honoured at expiration. Forward contracts will typically involve two financial institutions or a financial institution and its client as counterparties.

One of the counterparties in a forward contract will take a long (buying) position in the transaction, whereas the seller of the contract will assume a short (selling) position. The contract will contemplate the price at which the underlying asset will be traded at a specific time in the future. As useful feature of these products is that the physical underlying asset does not have to be delivered at expiration if the contract is conceived as a *paper* contract. Instead of undertaking the physical transaction of the underlying asset, the paper contract can be novated with a reverse contract which contemplates the trade of the same underlying at the spot price at maturity.

4.3.1 Forwards

A forward contract on the underlying asset $S(t)$, contracted at time t_0 with maturity T and with a forward price $f(t_0,T)$ decided at time t_0 will have zero value at time t_0. The payoff from the holder of the contract (long position) will receive at expiration, T, the stochastic payoff:

$$p_T = S(T) - f(t_0,T)$$

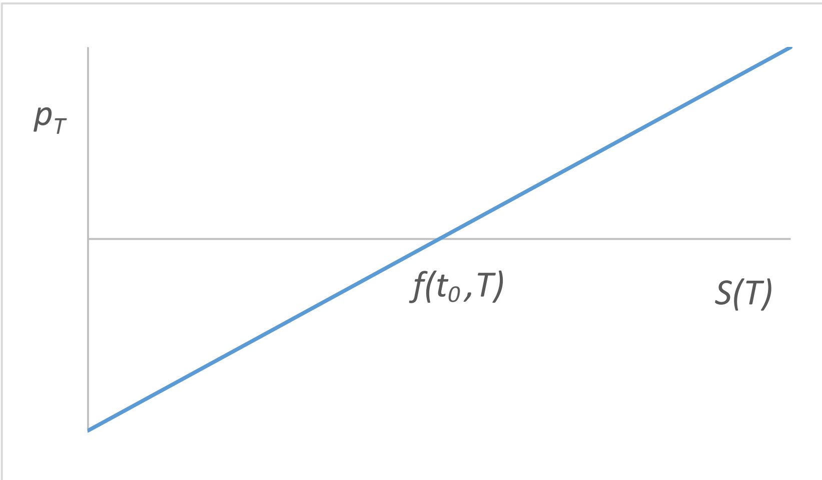


Figure 4.3 Long Future's Payoff

On the other hand, the seller of the contract will receive the stochastic payoff at expiration, *T*:

$$p_T = f(t_0, T) - S(T)$$

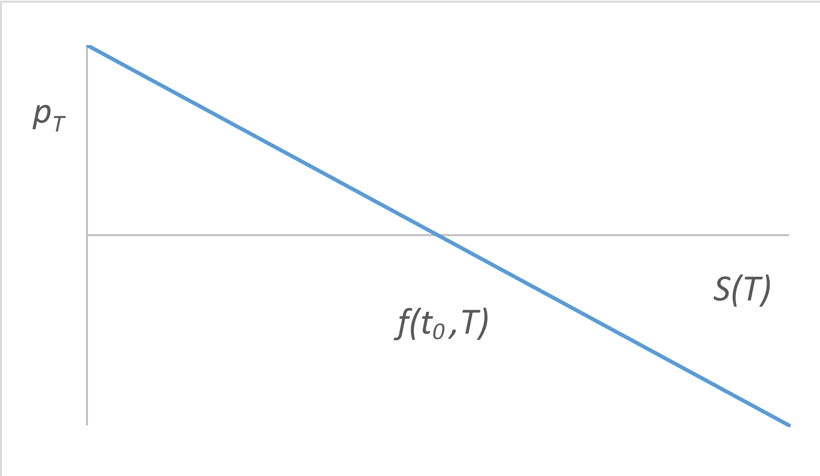


Figure 4.4 Short Future's Payoff

The forward price at time t_0 is given by the expected value at maturity, which is given by the risk neutral measure, E^T, the measure under which all assets scaled by the bond price are martingales:

$$f(t_0, T) = E_{t_0}^T[S(T)]$$

4.3.2 Futures

Due to the fact that future financial losses on a forward contact are unlimited and there is an intrinsic credit risk that a counterparty defaults its obligation, it is convenient to develop a daily basis settling contract: the futures contract. In a futures contract, differences with respect to settlement prices are cleared and counterparties automatically receive a position in a new futures contract with a new futures price. Accordingly, a futures contract on the underlying asset *S* contracted at time t_0, with maturity *T* at a futures price $f(t_0, T)$ decided at time t_0 will have zero value at each point in time. During a random time interval *(t1, t2)*, the holder (seller, respectively) of the contract will receive (pay) the amount:

$$p_{t_2} = F(t_2, T) - F(t_1, T)$$

At time of expiration T, the holder (seller) of the contact will receive (pay):

$$p_T = S(T) - F(T, T)$$

Under deterministic rates, the futures price will also be given by the expected forward neutral measure, E^T. Given the fact that natural gas is a consumption asset which produces no income, determining future prices from spot prices under no-arbitrage conditions is substantially different from calculating investment assets prices. According to (Hull, 2011), in order to calculate the futures price from the spot price, it necessary to include the storage costs incurred from owning the physical commodity. Assuming continuous compounding and a financing rate, *r*, storage cost, *u*, can be treated as a negative yield and expressed as a proportion to the price of the commodity:

$$F(t_0, T) = S(\ t_0)e^{(r+u)(T-t_0)}$$

Nevertheless, this assumption of equality is not representative enough to explain the inherent added value that ownership of a physical may provide in comparison to the ownership of a futures contract. Temporary local gas shortages will tend to increase the value of the physical asset as it enables producers to keep ongoing throughput at normal rates. The benefits from holding the physical asset is usually referred as the convenience yield provided by the product. The convenience yield, *y*, can be expressed as a proportion to the price of the commodity which is subtracted from the financing and storage costs from the previous equation:

$$F(t_0, T) = S(\ t_0)e^{(r+u-y)(T-t_0)}$$

This explanatory variable seeks to measure the extent to which the spot price is higher than the futures price. It reflects the market's concern on the future availability of the commodity in the future. High inventories (high availability) will be reflected by low convenience yields, as there will be little chance of shortages on the short run. On the other hand, low inventories and general concern on near availability will usually be reflected by a high convenience yield.

4.3.3 Term Structures: Contango vs Backwardation

Natural gas futures contracts are listed on a monthly basis from around 80 months forward for ICE futures to 118 months forward for NYMEX natural gas contracts. Charting future prices for various dates creates a forward curve of prices going out into the future, which is called the term structure. Closer to maturity contracts constitute what is called the front of the curve, whereas further contacts are referred as back of the curve. Differences between month spreads are calculated by subtracting the front month minus the back month, and are also standardized contracts by themselves. Forward curves may take the form of *Contango* or *Backwardation*, according to their upward or downward sloping, respectively.

In a Contango term structure, future prices (or further along the curve prices) will be above spot prices (closer to maturity contracts). This term structure usually occurs when gas supply is high in comparison to gas demand, and there may be a benefit opportunity from buying physical gas at low spot prices in order to store it and sell it in the future at a higher price. This term structure is characterised by lower convenience yields, as the added value of selling the physical commodity at the present is low due to oversupply and low prices. Convenience yield values in a contango term structure will be limited by:

$$0 \leq y \leq r + u$$

Contango intermonth spreads will usually be limited by the cost of building and operating storage facilities plus the opportunity cost of buying spot gas, all of which is called the cost of carry. When convenience yield is equal to zero, the marginal cost of carry will equal the intermonth spread and the market will be referred as to being at full carry. Contango term structure will therefore be limited to a maximum value, which last for brief periods of time due to the construction of new facilities and the increase of short positions in the back of the curve. Typical contango situation will have the following structure:

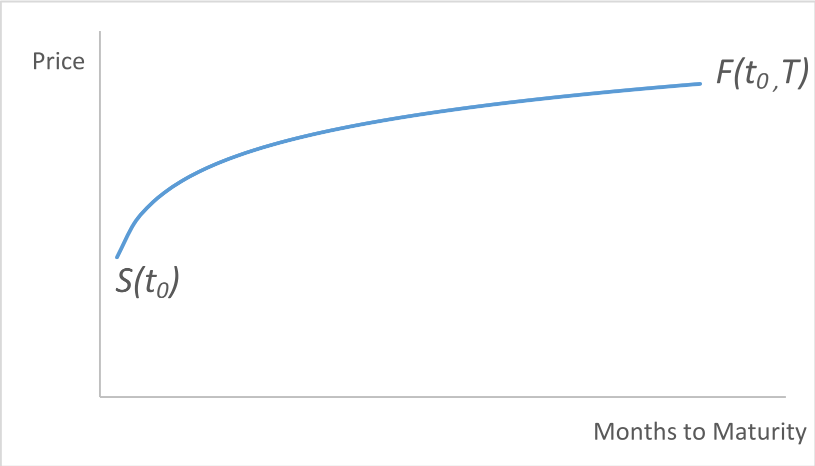


Figure 4.5 Contango Term Structure

Backwardation is a far more common term structure than contango as it occurs when the relative shortage of gas today increases the value of selling physical spot gas, discouraging the strategy of buying the physical asset to storage it and to sell it higher in the future. Whereas contango structures are limited to full carry costs, backwardation has no theoretical limit. Convenience yields will therefore be subject to only one restriction:

$$y \geq r + u$$

Typical backwardation situation will have the following term structure:

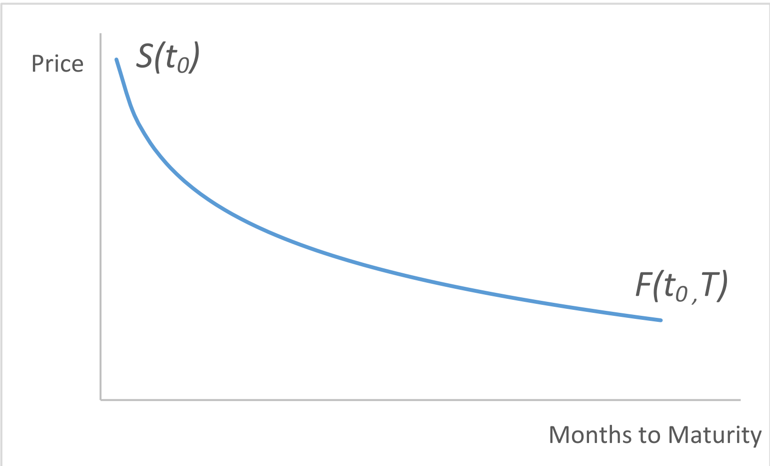


Figure 4.6 Backwardation Term Structure

J.M. Keynes (Keynes, 1930) stated that backwardation was a more normal situation due to the fact that if hedgers (generally producers) tend to hold more short future positions than consumers and speculators tend to hold long positions. Whereas speculators expect to make money on average from the risk they are bearing when taking long positions, hedgers will tend to loose on average as they aim is to reduce risk by entering into short future positions to sell their production.

Analysis of Henry Hub's historical prices for month ahead contract and 2 to 5 months ahead contracts during price peaks and lows exhibits a clear backwardation and contango term structure, respectively:

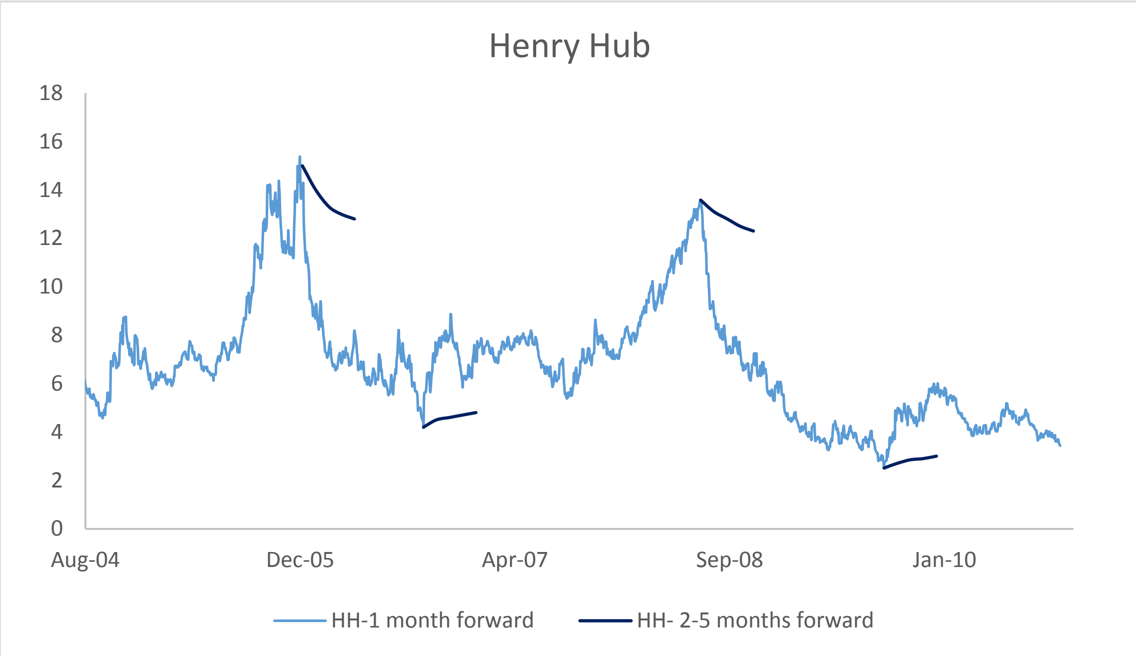


Figure 4.7 Contango and Backwardation on HH

4.4 Hedging Derivatives: Options

Options are a fundamentally different from future contracts in what refers to giving the holder of the option the right but not the obligation to exercise that option. In exchange, the buyer of an option is required an upfront payment, whereas in a futures contracts no initial outlay is required. We can divide options into two broad categories according to the holder's right to buy or sell an underlying asset:

- *Call option*: it gives the holder the right but not the obligation to buy the underlying asset at a previously specified price at a certain date in the future.
- *Put option*: it gives the holder the right but not the obligation to sell the underlying asset at a previously specified price at a certain time in the future.

The specified price in the contract is called the strike price, *K*; the exercise date is called the expiration date or maturity, *T*; the upfront premium payment is called the premium, *p*. from the previous definitions we can affirm that the maximum potential loss that the buyer of an option can face is the initial premium, whereas the potential losses for the seller (writer of the option) are unlimited.

From the buying point of view, the buyer of a call option will benefit form market prices rising at expiration, and will only receive a payoff once the option has expired above the strike price. On the other hand, the buyer of a put option will receive a payoff only if market prices are below the strike price at expiration. Therefore, the buyer of a call option will have a bullish view on the market, whereas the buyer of a put will feel bearish on the underlying asset. The same reasoning can be applied to the writers (sellers) of options, who will receive the premium at time t_0, when the contract is initiated. From that moment on, the writer of a call option will only benefit from market prices going below the strike price at maturity, as the buyer of the call will not make use of his right to execute the option. Likewise, the writer of a put will benefit from market prices rising above strike price, as the holder of the option will refuse to execute his right to sell at *K*. Hedging producers will usually buy put options in order to secure a future price to sell their production. On the other hand, wholesale gas consumers will buy call options to hedge their exposure and to fix a future price for their supply. The expectations on market prices from a speculators point of view can be summarised as follows:

	BUY	**SELL**
CALL	Bullish	Bearish
PUT	Bearish	Bullish

The value of a call option at expiration from the buyer's point of view is given below, where S_T is the value of the underlying asset at expiration and *K* is the strike price defined at t_0:

$$C_T = \max(S(T) - K, 0)$$

Net payoff resulting from the purchase of a call option will therefore have the following structure (assuming there is no initial disbursement):

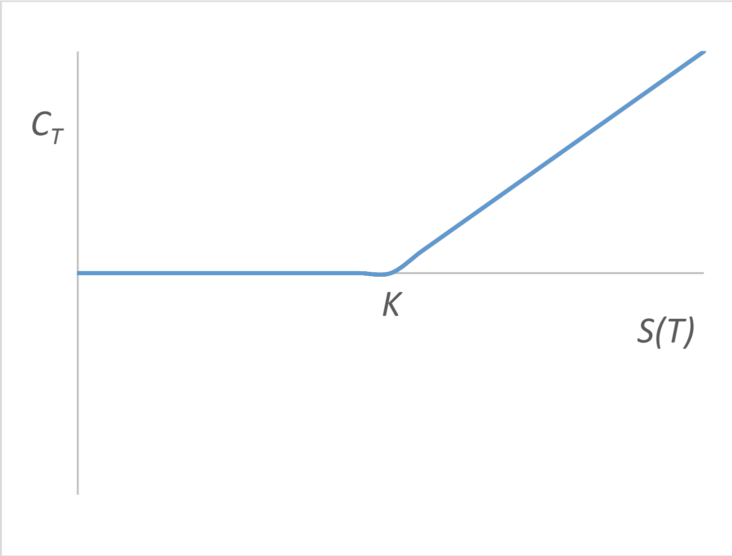


Figure 4.8 Long Call Payoff

In light of the above, the value of a put option at expiration from the buyer's view will be:

$$P_T = \max(K - S(T), 0)$$

Which can be represented by the following:

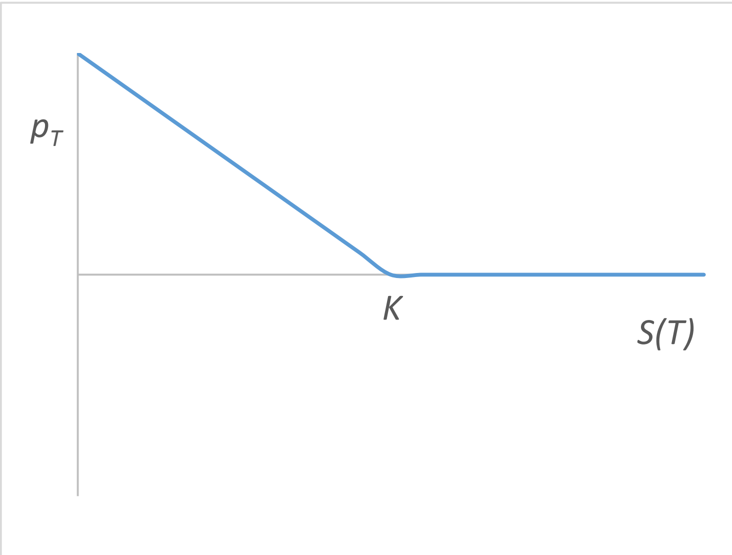


Figure 4.9 Long Put Payoff

Depending on the underlying's market price with respect to the strike price, options are said to be in the money, at the money or out of the money, and their value will change accordingly:

- *In the money:* the option is said to be in the money if the holder can make a profit from executing his right at that moment. A call option will be in the money if the underlying's market price is above the strike price. On the other hand, a put option will be in the money if the asset's market price is below the strike price.

- *At the money:* an option (call or put) is said to be at the money if the market price is equal to the strike price.
- *Out of the money:* an option is out of the money if the holder cannot make a profit from executing his right at that moment. For call option, it will be said to be out of the money when its strike price is above market price. Reversely, a put option will be out of the money when its strike is below market price.

Typical plain vanilla options can be divided into two categories according to their expiry:

- *European option*: the holder may only execute his right at maturity date. The fact of the option settling against a single price makes them the most volatile type of options.
- *American option*: the holder of the option may execute his right at any time from the purchase date up to the expiry date. These are the most commonly exchange traded options.

Due to the wider time window in which the holder of an American option can execute his right, these options will be more expensive than European ones. Nevertheless, American and European options may not fit specific OTC needs. Therefore, most common OTC transactions are *Asian options*, also known as average price options. These products are only exercised if the arithmetic average price over a period of time (month, quarter, year) is in the money, and can only be exercised at expiration. The main reasons why Asian options are so popular for trading natural gas are that producers and consumers follow daily producing and consuming patterns, instead of focusing on a single day period. Also, averaging prices over a period of days minimizes day spikes and volatility as a whole. This lower volatility is reflected in the price of the option, which is lower when compared to European and American options (in this last case also due to the longer exercise window).

4.4.1 Modelling Fundamentals: The Black-Scholes Formula

The option modelling approach from (Black & Scholes, 1973) was a breakthrough in the modern financial economics as it proved that the payoff of an option could be replicated with a continuously adjusted holding in the underlying asset and the risk free bond. For this approach all assets are assumed to earn the riskless rate of interest and options are assumed to be perfectly replicated by continuously trading the underlying asset, which is unrealistic for energy markets. Nevertheless, energy options depend on the futures price rather than the spot price, and futures can therefore be used to replicate positions allowing the risk neutral approach or at least, they can provide a good reference. This model parts from the definition of an infinite divisible non-dividend paying asset with constant volatility and interest rates, assuming costless trading in continuous time. Prices are modelled to evolve through time following a Geometric Brownian Motion with proportional changes in price defined by a constant instantaneous drift, μ, and volatility, σ:

$$dS = \mu S dt + \sigma dz$$

Where *dS* represents the increment of the asset price in an infinitesimal *dt*, with *dz* being the underlying uncertainty driving the model and represents an increment in a Wiener process during *dt*. A derivative whose value depends on the asset price will evolve according to the following equation, which is derived from Îto's lemma:

$$dC = \left(\frac{\partial C}{\partial S}\mu S + \frac{\partial C}{\partial t} + \frac{1}{2}\frac{\partial^2 C}{\partial S^2}\sigma^2 S^2\right)dt + \left(\frac{\partial C}{\partial S}\sigma S\right)dz$$

Which has two components that can be interpreted as deterministic drift term and a volatility term. It is important to note that the source of randomness for this process is the same as for the underlying asset. Due to this fact it is possible to combine the two securities in a same portfolio is such a way as to eliminate the uncertainty. Considering a portfolio *P* consisting of a short position in the call option and a long position in $\partial C/\partial S$ units of the underlying's asset price (the option's sensitivity to the underlying's price). The value of the portfolio will be:

$$P = -C + \frac{\partial C}{\partial S}S$$

Whose change over a period of time *dt* will be defined by:

$$dP = -dC + \frac{\partial C}{\partial S}dS$$

Substituting *dC* and *dS* by their respective values from the previous equations and collecting terms involving *dt* and *dz* together, we obtain:

$$dP = -\left(\frac{\partial C}{\partial t} + \frac{1}{2}\frac{\partial^2 C}{\partial S^2}\sigma^2 S^2\right)dt$$

Due to the fact that changes in value of the portfolio in *dt* are independent of the source of randomness *dz*, we can assume the portfolio to be riskless and it must therefore earn the riskless rate of interest, *r*, which is assumed to be constant:

$$\frac{dP}{P} = \frac{-\left(\frac{\partial C}{\partial t} + \frac{1}{2}\frac{\partial^2 C}{\partial S^2}\sigma^2 S^2\right)dt}{-C + \frac{\partial C}{\partial S}S} = rdt$$

Rearranging this formula leads to the Black-Scholes differential equation:

$$\frac{\partial C}{\partial t} + rS\frac{\partial C}{\partial S} + \frac{1}{2}\sigma^2 S^2\frac{\partial^2 C}{\partial S^2} - rC = 0$$

Surprisingly, the expected return on asset μ does not appear on the equation, from which we can conclude that the value of the derivative is independent from the investor's risk preferences. In line with this, we could assume the risk neutrality and therefore the asset would earn the riskless interest rate in time:

$$dS = rSdt + \sigma dz$$

The present value of any future random cash flow (payoff to an option) will be given by the expected value of the random future value discounted at the riskless rate, *e.g.* for an European call:

$$C = e^{-r(T-t)} E_T[\max(S_T - K, 0)]$$

We can replace the expectation term by the integral across all the possible asset prices at the maturity of the option, of the product of the payoff of the option by the probability of each price occurring:

$$C = e^{-r(T-t)} \int_0^{\infty} \max(S_T - K, 0)\, g(S_T) dS_T$$

Applying Îto's lemma to obtain the process for *x=Ln(S)* we will obtain:

$$dx = \left(r - \frac{1}{2}\sigma^2\right) dt + \sigma dz$$

Which (due to *dz* being normally distributed with mean 0 and variance *dt*) is also normally distributed with mean $(r - 1/2\ \sigma\text{^}2\ )$ and variance $\sigma\text{^}2\ dt$. The natural logarithm of *S* at time *T* will be normally distributed as it follows:

$$LnS_T \sim N\left(LnS + \left(r - \frac{1}{2}\sigma^2\right)(T-t), \sigma\sqrt{T-t}\right) dt + \sigma dz$$

The Black-Scholes equation applied for a European call will then be:

$$C = SN(d_1) - Ke^{-r(T-t)} N(d_2)$$

where

$$d_1 = \frac{Ln\frac{S}{K} + \left(r - \frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{T-t}}$$

$$d_2 = d_1 - \sigma\sqrt{T-t}$$

The corresponding equation for a European put will be given by:

$$P = Ke^{-r(T-t)} N(-d_2) - SN(-d_1)$$

Despite the Black-Scholes model flaws due to somewhat unrealistic assumptions, it is yet commonly used given its quick and straightforward approach towards closed-form solutions.

4.4.2 Implied Volatility

Assuming a liquid enough market where no individual agent may impact directly on prices, the only parameter in the Black-Scholes pricing formula that cannot be directly observed is the implied volatility, which must be distinguished from the historical volatility, which is estimated from the historic prices of an asset. Whereas historical volatilities are backward looking, implied volatilities seek to monitor the market's opinion about the volatility of a particular stock, which tends to be less variable than the price of the option. Option traders will monitor this variable in order to trade not only the direction of price movements but also the speed at which the price moves.

Calculating implied volatilities for options with different expiry dates into the future will generate a term structure or future curve of implied volatility. Volatility term structures are commonly used to price in the money, at the money or out of the money options. Closer to maturity commodity options will usually have higher volatility than further along the curve options. This phenomenon is given to the concern on short term supply, which is has a higher impact than long term uncertainty. Typical implied volatility curve will have the following term structure:

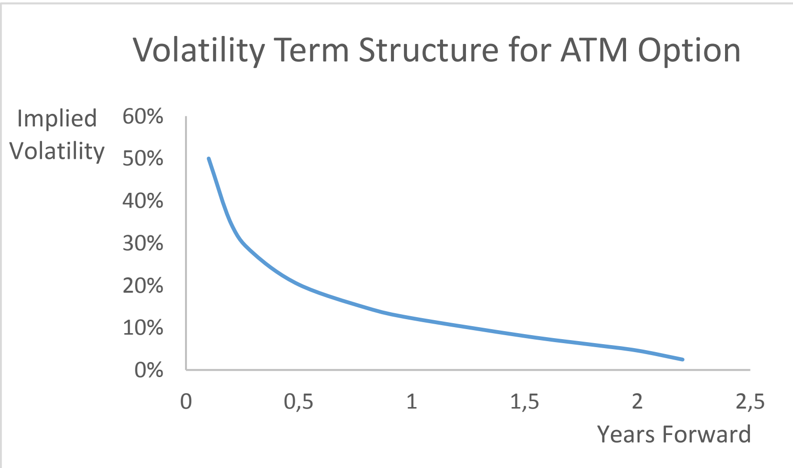


Figure 4.10 ATM Volatility

For a same expiry, at the money options will usually exhibit a lower volatility than in the money and out of the money options. This fact will generate what is known as a volatility skew or volatility smile, due to its shape:

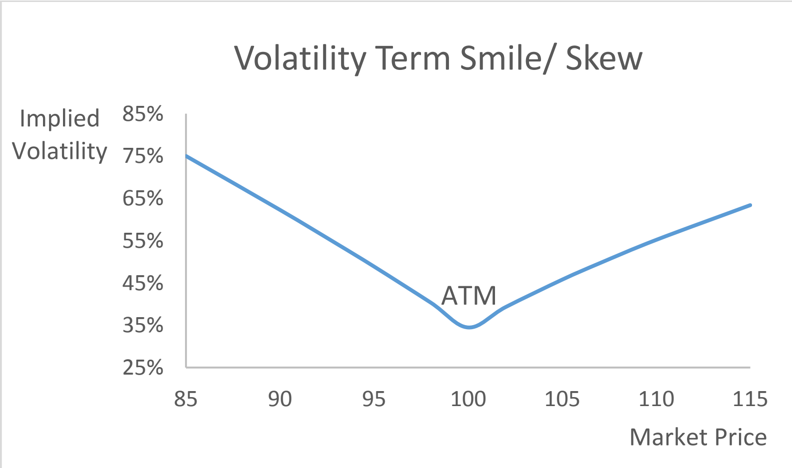


Figure 4.11 Volatility Smile

Combining both volatility structures and plotting them in a three dimensional representation will create what is called the volatility surface:

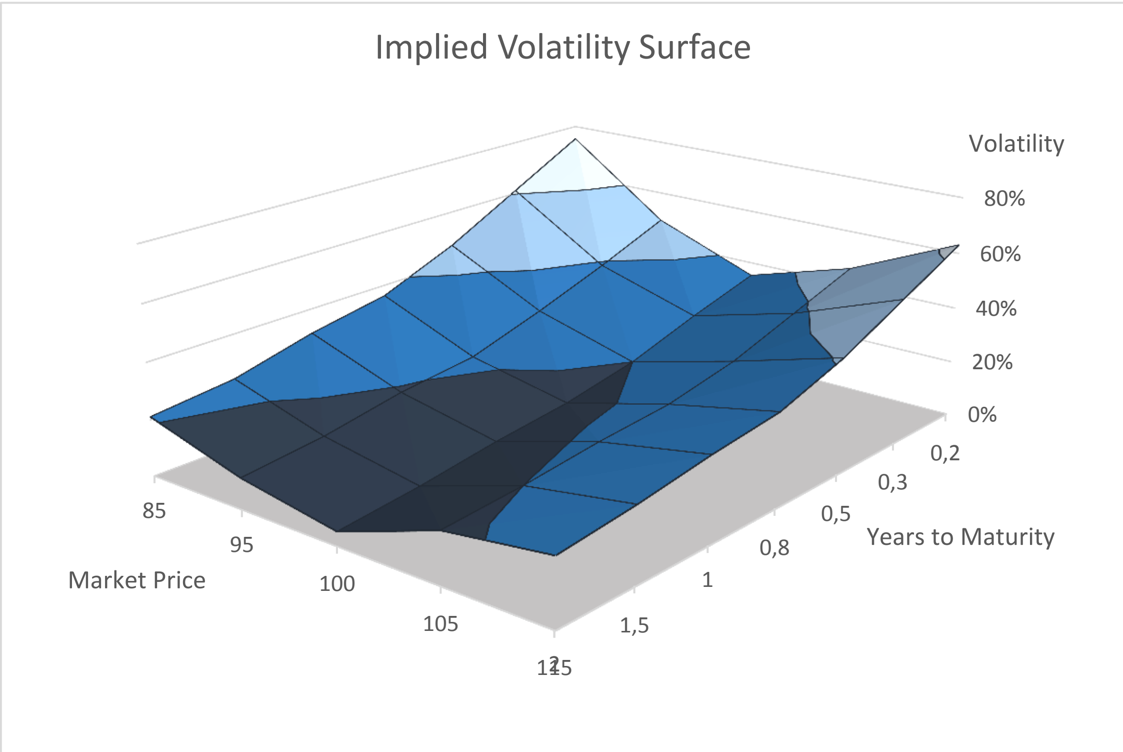


Figure 4.12 Volatility Surface

As the implied volatility of a European call option is the same as the implied volatility of a European put with the same strike price and maturity, the volatility smile will also be the same for both. This equality is derived from the put-call parity relationship that is based on a non-arbitrage assumption. This principle establishes that, for two options (call and put) with the same strike price and maturity, their relationship follows:

$$P + S_0 = C + Ke^{-r(T-t)}$$

We can stress this relationship to fit options on commodities by taking into account storage costs and the convenience yield to discount the strike price:

$$P + S_0 = C + Ke^{-(r+u-y)(T-t)}$$

4.4.3 Hedging an Option: Option Greeks

Following (Joshi, 2008) in the Black-Scholes model it is precise to make a distinction between variables and parameters. According to this, the spot price is the only variable in the model as it is the only term which is supposed to change. Nevertheless, this assumption does not hold true in reality, and it is therefore necessary to compute the derivative of the price of a portfolio with respect to all the other underlying parameters in order to hedge them through buying options which match those derivatives. These derivatives are denoted by Greek letters with initials corresponding to the parameters being differentiated:

Hedge Sensitivities: *Greeks*

Delta	$\frac{\partial C}{\partial S}$
Gamma	$\frac{\partial^2 C}{\partial S^2}$
Vega	$\frac{\partial C}{\partial \sigma}$
Theta	$\frac{\partial C}{\partial t}$
Rho	$\frac{\partial C}{\partial r}$

Delta is the most fundamental Greek. It is an increasing function of spot for call options (decreasing for put options) from zero for $S < K$ to one for $S > K$. The closer to maturity the more similar the Delta function will be to a step function at the strike price. The formula for the Delta of an option can be easily obtained by deriving Black-Scholes with respect to the price:

$$\Delta = \frac{\partial C}{\partial S} = N(d_1)$$

Gamma is the second derivative of the Delta with respect to the price, and it expresses how much hedging will cost being short or long an option over a small time interval. Writing an option will involve being short Gamma, and the procedure of hedging will cost us money over the life of the option. The closer to maturity the more spiked the Gamma function will be as Delta will change from zero to one in a shorter time interval. The formula for the Gamma is defined by:

$$\varGamma = \frac{\partial^2 C}{\partial S^2} = \frac{N'(d_1)}{S\sigma\sqrt{T-t}}$$

Where,

$$N'(x) = \frac{e^{-\frac{x^2}{2}}}{\sqrt{2\pi}}$$

The Vega (or Kappa) is the derivative with respect to the volatility, which is an uncertain parameter of the model. It expresses the trader's position on volatility: positive Vega will imply increasing volatility and negative Vega decreasing volatility. Due to the put-call parity principle, a call's Vega will be equal to the put's Vega with the same strike. The formula for a call's Vega is given by:

$$\varGamma = \frac{\partial C}{\partial \sigma} = S\sqrt{T-t}\, N'(d_1)$$

Theta and Rho are less representative than the previous: Theta stands for the change in value of an option due to passage of time whereas Rho is the change in value due to changes in interest rates. The loss of an options value is referred to as time decay, and is measured in monetary units per day. Rho for its part is measured in monetary units per basis points.

4.5 Hedging Derivatives: Swaps and Swaptions

As mentioned previously in this chapter, OTC contracts are traded mainly between large organization which are referred to as counterparties and offer them the possibility to fully customize the traded products to their own specific requirements. Despite these contracts being able to suit each counterparty, there are several recognized standards defined by the International Swaps and Derivatives Association (ISDA).

OTC contracts may use several benchmark references which are not typically exchange-traded (Platts trade journal, for example) and can also be customizable for any volume or expiry. This expiry will usually be held against the average prices over a certain period which can vary from a day to several years, but is commonly considered over the length of a given month. Instead of disbursing an upfront initial margin deposit and further clearing cash flows, OTC counterparties are effectively lending each other the margin and extending credit lines to each other.

Most widely spread OTC derivatives are *plain vanilla* swaps, which are named so due to their lack of complexity when compared to other more sophisticated *exotic* derivatives.

4.5.1 Swaps

The designation for this type of derivative stands for the fact that is allows market risk to be *swapped* from one counterparty to another. This swap transaction will leave a counterparty subject to all market price movement. A typical example for this type of contract is given by the airline whose profit is subject to fuel prices and therefore decides to enter into a swap transaction with a bank or other institution in order to hedge its operation from market prices rising. On the other hand, as future fuel prices have been fixed, a decrease in market prices will benefit the bank.

Similarly to the long and short terminology used in forward contracts, swap contracts lead place to two possible positions:

- Buyer of the swap: which is referred to as *paying fix, receiving variable*. The buyer of the contract will be buying the underlying product at a fixed price over the length of the contract. The value of this contract will increase as market price rise. A buying position will be typical of consumers willing to hedge from prices rising.
- Seller of the swap: which is referred to as *receiving fix, paying variable*. The seller of the swap contract will be selling the underlying at a fixed price over the length of the contract, benefiting from market prices falling. A selling position will be assumed by producers who are keen to hedge their throughput at a future fixed price.

OTC swap contracts will usually be cash settled at maturity, against the average price over a given period. Swap contracts can be viewed as a process of entering several forward contracts with different maturities at the same price for them all. The holder of a swap will receive, at any reset date T_i, $i=n,n+1,\dots,N$, for a contract initiated at time t_0, the stochastic positive of negative amount:

$$S(T_i) - K(t_0)$$

Where $S(T_i)$ is the price of the underlying asset at reset date T_i, and $K(t_0)$ the swap price defined at time t_0. By definition, the value of a swap at any time t must be equal to zero:

$$SW(t, K(t)) = 0; \ \forall t$$

Noting that, under deterministic rates,

$$SW(t, K(t)) = E_t^T\left[\sum_{i=n}^{N}(S(T_i) - K(t))e^{-(T_i-t)}\right] = \sum_{i=n}^{N}(F(t, T_i) - K(t))e^{-(T_i-t)}$$

It is possible to calculate the par swap price:

$$K(t) = \frac{\sum_{i=1}^{N} F(t, T_i)e^{-(T_i-t)}}{\sum_{i=1}^{N} e^{-(T_i-t)}}$$

4.5.2 Swaptions

A swaption is an option that grants its owner the right but not the obligation to enter into an underlying swap contract. As in options, there are two types of swaption contracts:

- A Call or Payer swaption: gives the owner of the contract the right but not the obligation to enter into a swap where he will be paying fixed and receiving variable.
- A Put or Receiver swaption: gives the owner of the contract the right but not the obligation to enter into a swap where he will be paying variable and receiving fixed.

There are three main types of swaption styles, according to the date at which the owner of the contract may enter into the underlying swap:

- European swaption: the owner of the contract is only allowed to enter the swap on the expiration date.
- American swaption: the owner is allowed to enter the swap at any time within a predefined period.
- Bermudian swaption: the owner of the contract is allowed to enter the swap on multiple previously specified dates.

4.6 Numerical Techniques

Several numerical techniques are used in order to value derivatives in absence of closed-form solutions. These techniques vary from binomial and trinomial tree building to Monte Carlo simulations, numerical integration and finite element methods. Monte Carlo simulations and tree methodologies, which will be deepen in this section, provide the possibility of pricing complicated payoffs not only as a function of the final price but also dependent on the path the underlying price follows.

As it has been previously mentioned, the Black-Scholes equation is widely used by market participants due to its straightforward approach and its high level of traceability. Nevertheless, this model is only applicable for European call and put options. American options (where there are early exercise opportunities and therefore the value of the premium must rise) and other more complicated derivatives will require the use of more computationally intensive numerical techniques. These exotic options are defined as path-dependent, in what refers to their pay-off depending on the path the asset draws before maturity. Closed-form models also assume that the price is monitored on a continuous basis, whereas real market observations tend to be discrete at a daily frequency. These models do not conceive several random factor such as stochastic volatility and interest rates, which can be closely modelled through numerical techniques.

The following sections aim to define two of these numerical techniques: Monte Carlo simulations and the Trinomial Tree method. These methodologies have been implemented through Matlab and VBA in order to offer a more detailed representation. Finally, several simulations have been carried with the implemented model in order to compare the different pricing methodologies.

4.6.1 Monte Carlo Simulation

Monte Carlo simulations where first used by (Boyle, 1977) and provide a relatively simple, representative and flexible approach towards valuating complex derivatives. Monte Carlo simulations have been boosted with the rise of computer systems, due to the fact that this methodology is technologically inefficient in its basic form. This methodology has established itself as the keystone in modern models due to its ability to offer more realistic asset price processes (jumps, for example), more realistic market conditions and a deeper analysis of the effectiveness of a hedge.

For the course of this project we will limit the study to the analysis and valuation of a European call, which will be considered to pay $C_{T,j}$ at time T for every j simulation. Assuming constant interest rates over the discounting period will simplify the expected payoff to:

$$C_{0,j} = e^{-r(T-t)} C_{T,j}$$

If M simulations are carried out and all outcomes are averaged, we can obtain an expected value for the European call of:

$$\widehat{C_0} = \frac{1}{M} \sum_{j=1}^{M} C_{0,j}$$

There is an inherent error due to the fact that the estimate has been calculated through an average of randomly generated samples. Calculating the sample standard deviation of $C_{0,j}$ and dividing it by the square root of the number of samples offers a measure of the error in the estimation, which is defined as the standard error:

$$SD(C_{0,j}) = \sqrt{\frac{1}{M-1}\sum_{j=1}^{M}\left(C_{0,j} - \widehat{C_0}\right)^2}$$

$$SE(\widehat{C_0}) = \frac{SD(C_{0,j})}{\sqrt{M}}$$

In order to implement Monte Carlo simulations it is precise to simulate the Geometric Brownian Motion (GBM) process for the underlying asset:

$$dS_t = rS_t dt + \sigma S_t dz$$

The best way to simulate this process is assuming the price is log normally distributed. Assuming $x=Ln(S_t)$, we will then have:

$$dx_t = \nu dt + \sigma dz$$

$$\nu = r - \frac{1}{2}\sigma^2$$

Which can be discretised by changing the infinitesimals to small changes Δx, Δt and Δz, resulting in:

$$\Delta x = \nu\Delta t + \sigma\Delta z$$

$$x_{t+\Delta t} = x_t + \nu\Delta t + \sigma(z_{t+\Delta t} - z_t)$$

And in terms of the asset price,

$$S_{t+\Delta t} = S_t e^{(\nu\Delta t+\sigma(z_{t+\Delta t}-z_t))}$$

The random increment $z_{t+\Delta t}$-z_t has mean zero and variance Δt, and can therefore be simulated by random samples of $\sqrt{\Delta t}\varepsilon$, where ε is a sample from a standard normal distribution. It is now possible to simulate S_T in (0, T) by dividing T into N intervals such as $\Delta t=T/N$, resulting in:

$$S_{ti} = S_{ti-1} e^{(\nu\Delta t+\sigma\sqrt{\Delta t}\varepsilon)}$$

We can now compute *M* simulated paths for the same parameters and calculate for each the pay-off *max*(S_T-*K,0)*, and later discount the average of these simulated pay-offs as it follows:

$$\widehat{C_0} = e^{-r(T-t)} \frac{1}{M} \sum_{j=1}^{M} \max(S_T - K, 0)$$

4.6.2 Implementing the Monte Carlo Simulation

Implementing Monte Carlo simulations is only feasible through a powerful software, as it demands a large number of randomly generated paths from which to average an expected pay-off for the option. We have therefore decided to use Matlab in order to program a basic Monte Carlo simulation code to price a European Call Option. This software has denoted to be faster in computing large number of random paths when compared to Excel VBA.

The default parameters for the European Call are the following:

- *Spot Price* ($S_{0,0}$): 100 monetary units.
- *Strike Price (K)*: 100 monetary units.
- *Volatility (σ)*: 20%.
- *Time to Maturity (T)*: 1 year.
- *Number of Periods (N)*: 12 periods.

Considering the discount risk free rate deterministic and equal to 3% for a total of 100000 simulated paths.

The code has been implemented following the underlying theory explained in section *4.6.1*, and is exhibited below:

```
%MONTE CARLO SIMULATION

%European Call Parameters

K=100;

T=1;

S=100;

sig= 0.2;

N=10;

r=0.03;

M=100000;


%Define Constants

dt=T/N;

nudt=(r-0.5*sig^2)*dt;

sigsdt=sig*sqrt(dt);
```

```
sum_CT=0;
sum_CT2=0;

%Calculate M random paths
for j=1:M
  lnSt=log(S)
  for i=1:N
    E=normrnd(0,1)
    lnSt=lnSt+nudt+sigsdt*E
  end
  ST=exp(lnSt)
  CT=max(0,ST-K)
  sum_CT=sum_CT+CT
  sum_CT2=sum_CT2+CT*CT
end

%Average generated paths and calculate error
call_value=(sum_CT/M)*exp(-r*T)
SD=sqrt((sum_CT2-sum_CT*sum_CT/M)*exp(-2*r*T)/(M-1))
SE=SD/sqrt(M)
```

The estimated value for the given European Call for a total of $M=10^5$ simulated paths is:

- Option Value *(call_value)*: 9.45
- Standard Error *(SE)*: 0.0447

The standard error indicates that in order to obtain an acceptably accurate estimation it is necessary to perform a larger number of simulations or to implement a variance reduction method. Nevertheless, the latter falls beyond the scope of this project and we will therefore focus on carrying a second simulation with an increased number of simulated paths.

For $M=10^6$, *ceteris paribus*, the estimation for the European Call is:

- Option Value *(call_value)*: 9.39
- Standard Error *(SE)*: 0.0445

4.6.3 The Trinomial Tree Method

The binomial model is a recognized alternative discrete time representation of the behaviour of prices to Geometric Brownian Motion mainly due to the fact that it serves as the basis for the dynamic programming solution to the valuation of American options, as explained below. Despite the binomial model is a robust approach towards both American and European option pricing, several studies have gone further by considering a trinomial model, which includes a third possible future movement of the asset price at each time period. Calibrating to market prices is also more feasible with the trinomial model as it presents a more regular and flexible grid than the binomial approach.

Following the stochastic differential equation for the risk neutral GBM process given for the Monte Carlo simulation, and assuming the price is log normally distributed, we will have the following process for $x=Ln(S_t)$:

$$dx_t = \nu dt + \sigma dz$$

Where,

$$\nu = r - \frac{1}{2}\sigma^2$$

The trinomial model is based on small time intervals, Δt, in which the price of an asset can increase by the space step, Δx, stay the same or decrease by this same step with probabilities p_u, p_m or p_d, respectively:

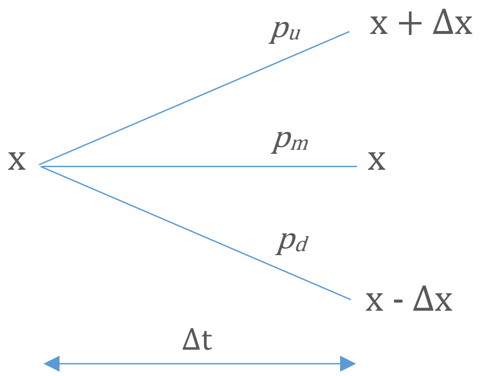


Figure 4.13 Trinomial Tree Generation

According to (Clewlow & Strickland, 2000) the relationship between the space step and the time interval is defined by:

$$\Delta x = \sigma\sqrt{3\Delta t}$$

Whereas the relationship between the parameters of the continuous time process and the trinomial process is obtained by is derived obtained by equating the mean and variance over Δt and requiring that the probabilities sum to one:

$$E[\Delta x] = p_u(\Delta x) + p_m(0) + p_d(-\Delta x) = \nu\Delta t$$
$$E[\Delta x^2] = p_u(\Delta x^2) + p_m(0) + p_d(\Delta x^2) = \sigma^2\Delta t + \nu^2\Delta t$$

$$p_u + p_m + p_d = \sigma^2 \Delta t + \nu^2 \Delta$$

Solving these equations we can obtain the value for the probabilities of the price going up, down or staying constant during *Δt:*

$$p_u = \frac{1}{2}\left(\frac{\sigma^2 \Delta t + \nu^2 \Delta t^2}{\Delta x^2} + \frac{\nu \Delta t}{\Delta x}\right)$$

$$p_m = 1 - \frac{\sigma^2 \Delta t + \nu^2 \Delta t^2}{\Delta x^2}$$

$$p_d = \frac{1}{2}\left(\frac{\sigma^2 \Delta t + \nu^2 \Delta t^2}{\Delta x^2} - \frac{\nu \Delta t}{\Delta x}\right)$$

Let $C_{i,j}$ be the value for the option at node *(i,j)*, where *i* stands for the number of time steps and *j* represents the asset price relative to the initial asset price. If *N* is defined as the number of time steps over the period (such that *T=NΔt*), the values for all branches at maturity will be given by:

$$C_{N,j} = \max\left(0, S_{N,j} - K\right); j \in [-N, N]$$

Option values can now be computed as discounted expectations of the values at the three following nodes:

$$C_{i,j} = e^{-r(T-t)}\left(p_u C_{i+1,j+1} + p_m C_{i+1,j+} p_d C_{i+1,j-1}\right)$$

The key advantage of the Trinomial Tree method relies on the fact that we can now easily evaluate the option at each node of the given period for both European and American options, by simply comparing the discounted expectation value and zero (for European options), and with the difference between the spot price at that node and the strike price (for American options):

- European call option value at node *(i,j)*:

$$C_{i,j} = max\left\{e^{-r(T-t)}\left(p_u C_{i+1,j+1} + p_m C_{i+1,j+} p_d C_{i+1,j-1}\right), 0\right\}$$

- American call option value at node *(i,j)*:

$$C_{i,j} = max\left\{e^{-r(T-t)}\left(p_u C_{i+1,j+1} + p_m C_{i+1,j+} p_d C_{i+1,j-1}\right), S_{i,j} - K\right\}$$

The final value for the option will be given at node *(0,0)* by $C_{0,0}$.

4.6.4 Implementing the Trinomial Tree Method

The implementation of the Trinomial Tree Method has been developed through Excel 2013 VBA, looking forward to offer a straightforward representation of the generated tree and its valuation. The feasibility of this approach has involved the reliance on a less powerful tool (when compared to the Monte Carlo simulations run in Matlab). Nevertheless, the aim of this tool is to be self-explanatory while being precise, and the chosen software has proven to fulfil these objectives.

The program's initial interface has been designed to be plain and simple, displaying three main command buttons to access the generation and valuation menus, and to clear the previous worksheet for new trees. The following picture exhibits the initial display when starting the program:

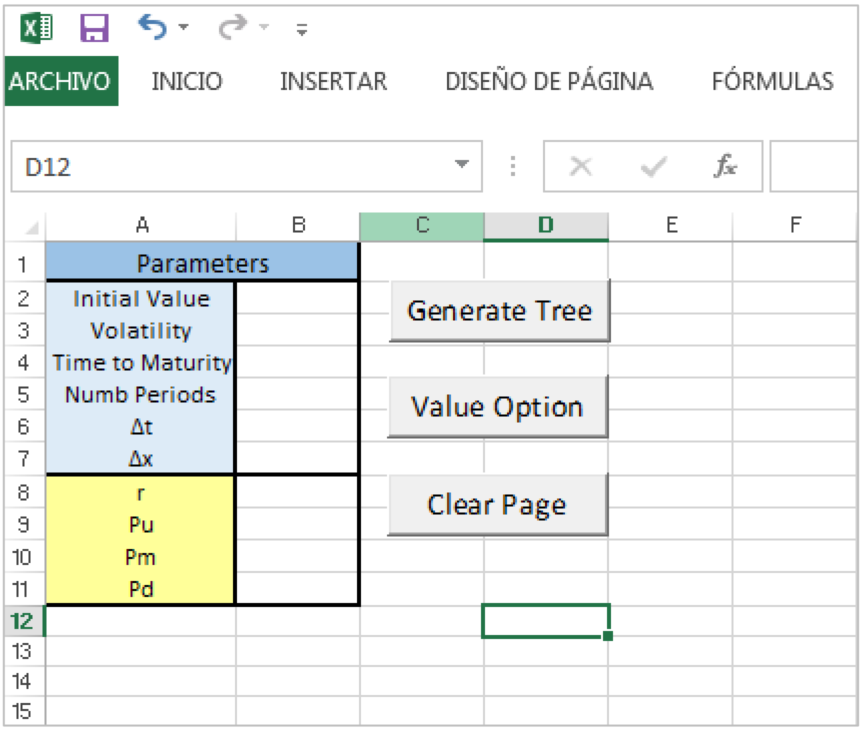


Figure 4.14 Initial Interphase

The first step in the process is to generate a trinomial tree that represents the behaviour of the asset over a given period. Initial (spot) price for the asset, historical volatility, time to maturity and number of periods are to be introduced through the following User Form:

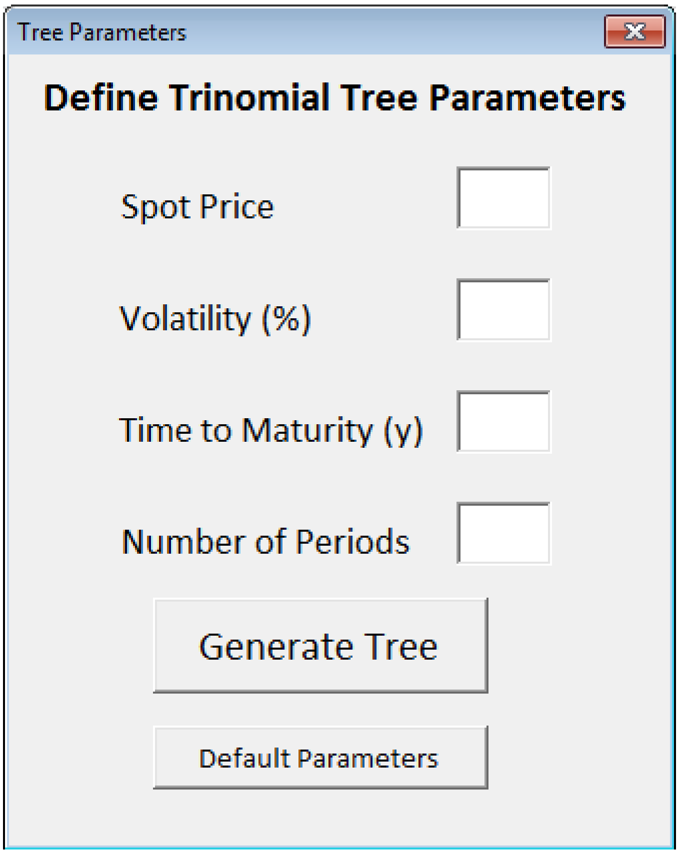


Figure 4.15 Parameters Display

Where historical volatility must be introduced as a percentage and time to maturity in years. Regarding the number of periods, precision will increase as a direct function of them but will depend on the capacity of the processor. There is option to set *Default Parameters* in order to carry out a test run. This option will set the following values:

- *Spot Price ($S_{0,0}$)*: 100 monetary units.
- *Volatility (σ)*: 20%.
- *Time to Maturity (T)*: 1 year.
- *Number of Periods (N)*: 12 periods.

Notice that the number of periods has been set to 12, which can be interpreted as new branch generation every month (as time to maturity has been set to 1 year). *N* has also been defined as 12 in order to meet the Monte Carlo simulation default parameters found earlier in this section.

The User Form's display after clicking on *Default Parameters* can be exhibited below:

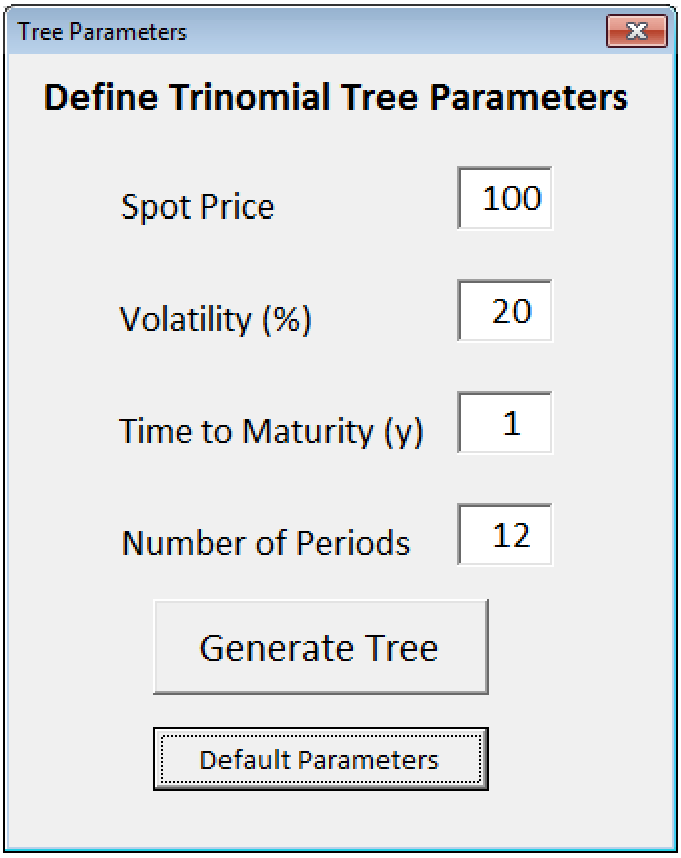


Figure 4.16 Defined Default Parameters

Generate Tree will run the *GenerateTrinomial* procedure receiving the user-defined input parameters and following the previously explained Trinomial Tree model. This procedure can be exhibited below:

```
Public Sub GenerateTrinomial(initial, vol, maturity, periods)
'Initialize Variables
   Dim deltat As Single
   Dim deltax As Single
   Dim i As Integer
   Dim j As Integer
   Dim c As Integer
'Define Tree Variables from input parameters
   deltat = maturity / periods
   Cells(6, 2) = deltat
   deltax = vol * (3 * deltat) ^ (0.5)
   Cells(7, 2) = deltax
'Start Tree Generation
```

```
    n = 12 + (periods)
    m = 1
    Cells(n, m) = initial
    Cells(n, m).Interior.ColorIndex = 24
    For j = 1 To periods
      c = 0
      For i = (-j) To j
        Cells(n + i, m + j) = Cells(n, m) * Exp(deltax * (j - c))
        Cells(n + i, m + j).NumberFormat = "0.##"
        Cells(n + i, m + j).Interior.ColorIndex = 24
        c = c + 1
      Next i
    Next j
End Sub
```

The generated tree parameters will appear on screen as well as the values for the generated tree at each value over the given period:

Parameters	
Initial Value	100
Volatility	20%
Time to Maturity	1
Numb Periods	12
Δt	0,08
Δx	0,10
r	
Pu	
Pm	
Pd	

												332,01
											300,42	300,42
										271,83	271,83	271,83
									245,96	245,96	245,96	245,96
								222,55	222,55	222,55	222,55	222,55
							201,38	201,38	201,38	201,38	201,38	201,38
						182,21	182,21	182,21	182,21	182,21	182,21	182,21
					164,87	164,87	164,87	164,87	164,87	164,87	164,87	164,87
				149,18	149,18	149,18	149,18	149,18	149,18	149,18	149,18	149,18
			134,99	134,99	134,99	134,99	134,99	134,99	134,99	134,99	134,99	134,99
		122,14	122,14	122,14	122,14	122,14	122,14	122,14	122,14	122,14	122,14	122,14
	110,52	110,52	110,52	110,52	110,52	110,52	110,52	110,52	110,52	110,52	110,52	110,52
100	100,	100,	100,	100,	100,	100,	100,	100,	100,	100,	100,	100,
	90,48	90,48	90,48	90,48	90,48	90,48	90,48	90,48	90,48	90,48	90,48	90,48
		81,87	81,87	81,87	81,87	81,87	81,87	81,87	81,87	81,87	81,87	81,87
			74,08	74,08	74,08	74,08	74,08	74,08	74,08	74,08	74,08	74,08
				67,03	67,03	67,03	67,03	67,03	67,03	67,03	67,03	67,03
					60,65	60,65	60,65	60,65	60,65	60,65	60,65	60,65
						54,88	54,88	54,88	54,88	54,88	54,88	54,88
							49,66	49,66	49,66	49,66	49,66	49,66
								44,93	44,93	44,93	44,93	44,93
									40,66	40,66	40,66	40,66
										36,79	36,79	36,79
											33,29	33,29
												30,12

Figure 4.17 Generater Tree and Parameters

The option valuation on the generated tree can be computed after defining the valuating parameters in the *Option Valuation* User Form, which can be accessed through the *Value Option* command button:

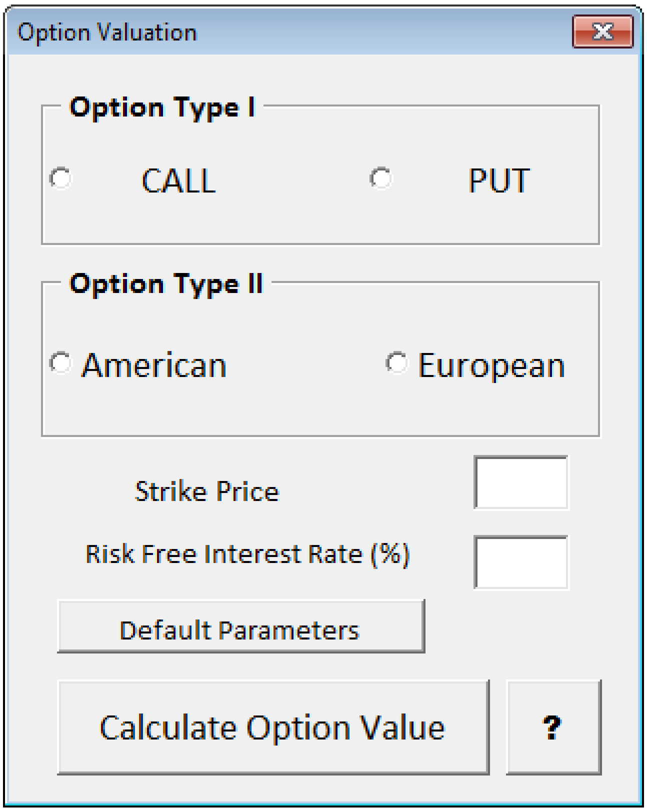
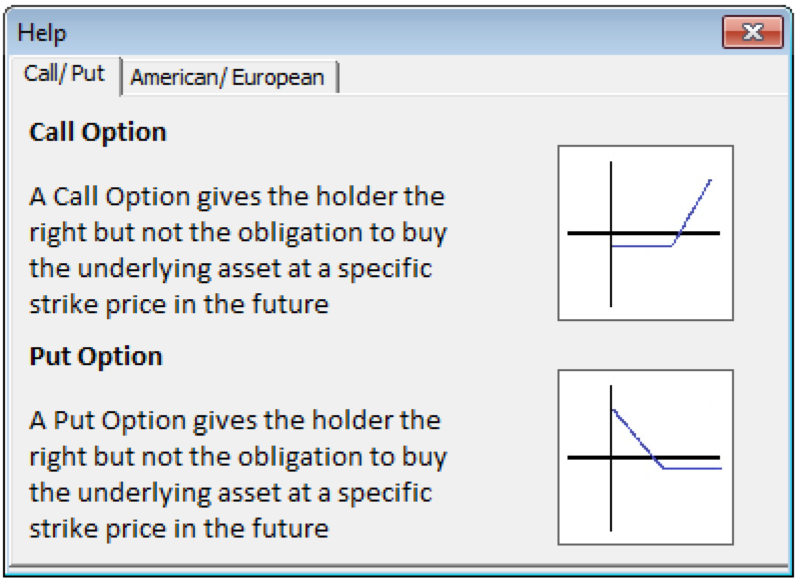


Figure 4.18 Option Valuation Display

Default Parameters for the option valuation process have been set to the following:

- *European Call Option*.
- *Strike Price*: 100 monetary units.
- *Risk Free Rate*: 3%.

As it can be viewed above, there is a link to an explanatory window which briefly defines the differences in between Call/ Put and European/ American options:



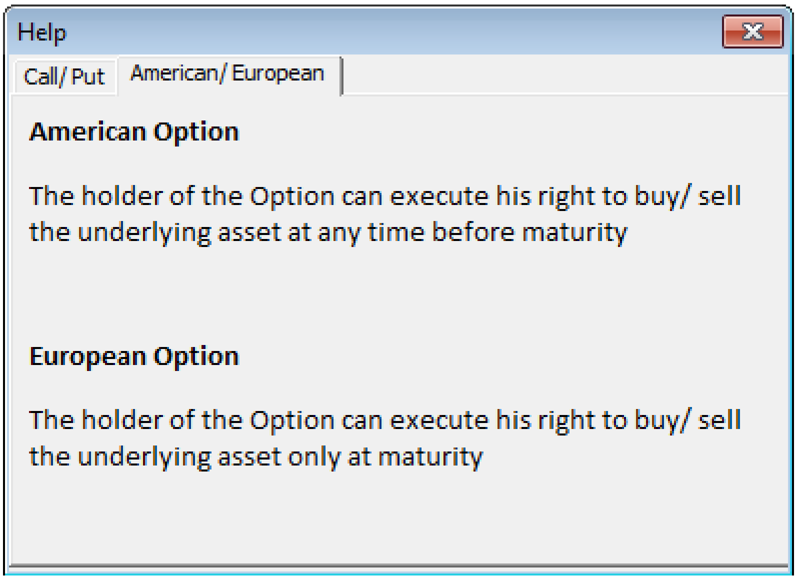


Figure 4.19 Help Box

Calculate Option Value will now compute de value for each branch at maturity and will continue by discounting each node from the latter by their probabilities and evaluating the option at each node. If the option is set to be European, the procedure run will be the following:

```
'Initialize Variables
    Dim v As Single
    Dim pu As Single
    Dim pm As Single
    Dim pd As Single
    Dim periods As Single
    Dim vol As Single
    Dim deltax As Single
    Dim deltat As Single
    Dim disc As Single
    Dim n As Integer
    Dim s As Integer
    Dim j As Integer
    Dim f As Integer
'Recover Tree parameters
    vol = Cells(3, 2)
    periods = Cells(5, 2)
    deltat = Cells(6, 2)
    deltax = Cells(7, 2)
'Define Valuating Variables
```

```
    v = Rate - 0.5 * (vol) ^ (2)
    pu = 0.5 * ((((vol) ^ (2) * deltat + (v) ^ (2) * (deltat) ^ (2)) / (deltax) ^ (2)) + v * deltat / deltax)
    pm = 1 - (((vol) ^ (2) * deltat + (v) ^ (2) * (deltat) ^ (2)) / (deltax) ^ (2))
    pd = 0.5 * ((((vol) ^ (2) * deltat + (v) ^ (2) * (deltat) ^ (2)) / (deltax) ^ (2)) - v * deltat / deltax)
    disc = Exp(-Rate * deltat)
    Cells(8, 2) = Rate
    Cells(9, 2) = pu
    Cells(10, 2) = pm
    Cells(11, 2) = pd
'Discount and Value Tree for each node
    If OptionType = True Then
        s = 1
    Else
        s = -1
    End If
    n = 14 + 2 * periods
    m = periods + 1
    d = 2 * periods + 2
    For i = n To (n + 2 * periods)
        Cells(i, m) = WorksheetFunction.Max(s * (Cells(i - d, m) - Strike), 0)
        Cells(i, m).NumberFormat = "0.00"
        Cells(i, m).Interior.ColorIndex = 36
    Next i
    For j = 1 To periods
        For i = 1 To (2 * periods + 1 - 2 * j)
            Cells(n + i, m - j) = disc * (pu * Cells(n + i - 1, m - j + 1) + pm * Cells(n + i, m - j + 1) + pd * Cells(n + i
+ 1, m - j + 1))
            Cells(n + i, m - j).NumberFormat = "0.00"
            Cells(n + i, m - j).Interior.ColorIndex = 36
        Next i
        n = n + 1
    Next j
    Cells(13 + 3 * periods, 1) = "OPTION PRICE"
    Cells(13 + 3 * periods, 1).Font.Bold = True
    Cells(14 + 3 * periods, 1).Font.Bold = True
    Cells(14 + 3 * periods, 1).BorderAround xlContinuous
End Sub
```

For an American option, the valuation process will involve comparing at each node the discounted option value and the difference between the spot price at that node and the strike price:

```
Public Sub Value_American(Strike, Rate, OptionType)
'Initialize Variables
    Dim v As Single
    Dim pu As Single
    Dim pm As Single
    Dim pd As Single
    Dim periods As Single
    Dim vol As Single
    Dim deltax As Single
    Dim deltat As Single
    Dim disc As Single
'Recover Tree parameters
    vol = Cells(3, 2)
    periods = Cells(5, 2)
    deltat = Cells(6, 2)
    deltax = Cells(7, 2)
'Define Valuating Variables
    v = Rate - 0.5 * (vol) ^ (2)
    pu = 0.5 * ((((vol) ^ (2) * deltat + (v) ^ (2) * (deltat) ^ (2)) / (deltax) ^ (2)) + v * deltat / deltax)
    pm = 1 - (((vol) ^ (2) * deltat + (v) ^ (2) * (deltat) ^ (2)) / (deltax) ^ (2))
    pd = 0.5 * ((((vol) ^ (2) * deltat + (v) ^ (2) * (deltat) ^ (2)) / (deltax) ^ (2)) - v * deltat / deltax)
    disc = Exp(-Rate * deltat)
    Cells(8, 2) = Rate
    Cells(9, 2) = pu
    Cells(10, 2) = pm
    Cells(11, 2) = pd
'Discount and Value Tree for each node
    If OptionType = True Then
        s = 1
    Else
        s = -1
    End If
```

```
    n = 14 + 2 * periods
    m = periods + 1
    d = 2 * periods + 2
    For i = n To (n + 2 * periods)
        Cells(i, m) = WorksheetFunction.Max(s * (Cells(i - d, m) - Strike), 0)
        Cells(i, m).NumberFormat = "0.00"
        Cells(i, m).Interior.ColorIndex = 36
    Next i
    For j = 1 To periods
        For i = 1 To (2 * periods + 1 - 2 * j)
            Cells(n + i, m - j) = WorksheetFunction.Max (s * (Cells(n + i - d, m - j) - Strike), disc * (pu * Cells(n +
i - 1, m - j + 1) + pm * Cells(n + i, m - j + 1) + pd * Cells(n + i + 1, m - j + 1)))
            Cells(n + i, m - j).NumberFormat = "0.00"
            Cells(n + i, m - j).Interior.ColorIndex = 36
        Next i
        n = n + 1
    Next j
    Cells(13 + 3 * periods, 1) = "OPTION PRICE"
    Cells(13 + 3 * periods, 1).Font.Bold = True
    Cells(14 + 3 * periods, 1).Font.Bold = True
    Cells(14 + 3 * periods, 1).BorderAround xlContinuous
End Sub
```

Risk Free Interest Rate, *r*, and valuating probabilities p_u , p_m and p_d, are independent from the option being Call/ Put, European/ American:

Parameters	
Initial Value	100
Volatility	20%
Time to Maturity	1
Numb Periods	12
Δt	0,08
Δx	0,10
r	3,00%
Pu	17,09%
Pm	66,66%
Pd	16,25%

Figure 4.20 Valuation Parameters

The resulting valuation tree for default European Call can be exhibited below:

												232,01
											200,67	200,42
										172,33	172,08	171,83
									146,71	146,46	146,21	145,96
								123,55	123,30	123,05	122,80	122,55
							102,62	102,37	102,12	101,87	101,62	101,38
						83,70	83,45	83,21	82,96	82,71	82,46	82,21
					66,61	66,36	66,11	65,87	65,62	65,37	65,12	64,87
				51,19	50,93	50,68	50,43	50,18	49,93	49,68	49,43	49,18
			37,42	37,11	36,81	36,53	36,25	35,99	35,73	35,48	35,24	34,99
		25,56	25,16	24,77	24,37	23,99	23,61	23,26	22,93	22,64	22,39	22,14
OPTION PRICE	16,08	15,61	15,12	14,62	14,10	13,56	13,00	12,43	11,84	11,27	10,77	10,52
9,25	8,78	8,30	7,80	7,27	6,72	6,13	5,49	4,78	3,98	3,03	1,79	0,00
	4,11	3,72	3,32	2,91	2,49	2,06	1,62	1,17	0,72	0,31	0,00	0,00
		1,37	1,13	0,90	0,68	0,48	0,30	0,16	0,05	0,00	0,00	0,00
			0,30	0,21	0,13	0,07	0,03	0,01	0,00	0,00	0,00	0,00
				0,03	0,02	0,01	0,00	0,00	0,00	0,00	0,00	0,00
					0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
						0,00	0,00	0,00	0,00	0,00	0,00	0,00
							0,00	0,00	0,00	0,00	0,00	0,00
								0,00	0,00	0,00	0,00	0,00
									0,00	0,00	0,00	0,00
										0,00	0,00	0,00
											0,00	0,00
												0,00

Figure 4.21 Discounted Trinomial Tree

Which offers an estimated value of 9.25 for the defined *default option.* Increasing the number of periods for the generated trinomial tree will increase the accuracy of the valuation technique:

- For *N=12*, *option value*= 9.25
- For *N=24*, *option value*= 9.33
- For *N=36*, *option value*= 9.36
- For *N=48*, *option value*= 9.37

4.6.5 Comparison between Valuation Methodologies.

In order to compare the numerical techniques that have been further explained in this chapter, we have decided to establish a third reference to which compare against. This reference is the Black-Scholes formula, as it has been explained in section *4.4.1*.

The Black-Scholes formula has been programmed in Excel with the following parameters:

- *Spot Price*: 100 monetary units.
- *Strike Price*: 100 monetary units.
- *Volatility*: 20%.
- *Time to Maturity*: 1 year.
- *Risk free interest rate*: 3%.

Notice that due to the fact that we are assuming the underlying to be a non-dividend paying asset, the annual dividend yield has been set to zero. Exhibited below is the interface with the programmed Black-Scholes model:

Black & Scholes

Parameters	
Spot Price	100
Strike Price	100
Risk Free Rate	3%
Volatility	20%
Time to Maturity	1

	Call	Put
Value	9,4134	6,458

Intermediate Calculations

d1	d2	nd1	nd2
0,25	0,05	0,5987	0,51994

Figure 4.22 Implemented Black-Scholes

Which draws a value of 9.41 for the default European Call option valued in sections *4.6.2* and *4.6.4*. This is the expected value to which Monte Carlo Simulations and Trinomial Tree Method valuations will tend to converge:

- For Monte Carlo Simulations, as the number of simulated paths increases, the expected value for the option will converge to the expected value given by Black-Scholes. Ideally, they would be equal when the number of randomly generated paths tend to infinity. Nevertheless, for values of *M* above 10^6, results seen to be consistent.
- For the Trinomial Tree Method, the expected option value will converge to Black-Scholes as the number of generated branches (number of periods within the time to maturity) increases. For the implemented model, tree generation is subject to our processing constraints. Despite this facts, the converging trend seems to be clearly followed as the number of generated branches increases

5

Natural Gas Valuation Models

5 Natural Gas Valuation Models

5.1 Spot Models

Convenience yield models introduce an unobservable variable to quantify the advantage on owning the physical asset rather than the futures contract. These models can be divided into two main categories: Spot models and Term Structure Factor models. Main representatives of the first category are Gibson and Schwartz, through their studies presented in (Gibson & Schwartz, 1990) and (Cortazar, Gibson, & Schwartz, 1994). On the other hand, the Term Structure approach has been discussed by Miltersen and Schwartz, and later by Bjork and Landen, and are similar to other studies developed for the analysis of fixed income markets.

Convenience Yield models stand on several assumptions on the behaviour of δ_t, which can be defined as the difference between the benefit of having direct access to the commodity (the convenience yield, *y*, as presented in section 4.3.2) and the cost of carry related to the physical ownership of the underlying:

$$\delta_t = y - u$$

These models start from the assumption that the spot price, S_t, for the underlying asset exists. Despite this fact can be true for certain commodities such as gold, for the majority of products there is no spot price due to physical constraints involved with the transportation and distribution of the asset. From a basic no-arbitrage reasoning, the price of a forward contract *F(t,T)* with payoff S_T at time *T* must be subject of the following equation:

$$F(t,T) = S_t E\left[e^{\int_t^T (r-\delta)dt}\right]$$

Notice that the convenience yield is only a correction factor to the drift of the spot process, and will remain as an unobservable stochastic variable throughout the period. Initial studies suggested the use of deterministic rates and convenience yields, through which the relation between forward contracts and spot prices will be subject to the following relation:

$$F(t,T) = S_t e^{(r-\delta)(T-t)}$$

Nevertheless, this approach is not consistent with reality and will therefore be viewed as a purely theoretical model to explain the use of convenience yields. The Gibson – Schwartz model will also assume interest rates to be deterministic, but will consider stochastic processes for the spot price and the convenience yield. This model is further explained below.

5.1.1 The Gibson – Schwartz Model

The Gibson – Schwartz spot model is a two-factor approach for which the risk neutral dynamics of the commodity spot price, S_t, is given by a geometric Brownian motion whose rate of growth is adjusted by a stochastic mean-reverting convenience yield, δ_t. The spot price is modelled as a lognormal-stationary distribution, and the dynamics of the state are given by a system of Îto stochastic differential equations of the form:

$$dS_t = (r_t - \delta_t)S_t dt + \sigma S_t dW_t^1$$

$$d\delta_t = \kappa(\theta - \delta_t)dt + \gamma dW_t^2$$

Where dW^1 and dW^2 are one-dimensional Wiener processes such that:

$$dW_t^1 dW_t^2 = \rho dt$$

Given the fact that convenience yields are typically an order of magnitude higher that interest rates, the assumption of deterministic interest rates seem to be consistent. On the other hand, if there is a positive correlation between W^1 and W^2, the stochastic convenience yield will introduce a mean reversion in the risk neutral dynamics of the spot price: and increase in S_t due to an increment in dW^1 will produce an increase in dW^2 and consequently, δ_t will rise reducing the drift of the spot price. The correlation between both Wiener processes is obtained empirically, being typical values within the 0.3 – 0.7 range.

To achieve an exact fit to futures prices, θ can be defined as a function of time, $\theta(t)$. This can be interesting in order to model seasonality term structure for natural gas.

5.1.2 The Eydeland – Geman Model

(Eydeland & Geman, 1998) suggests a more complex spot valuating model for electricity prices that can be implemented for natural gas derivatives. This model conceives stochastic volatility V, and is defined by the following system:

$$dS_t = \alpha(\beta - Ln(S_t))S_t dt + \sqrt{V} dW_t^1$$

$$d\delta_t = \gamma(\delta - V)dt + \varepsilon\sqrt{V} dW_t^2$$

Where dW^1 and dW^2 correlated Wiener processes as in the Gibson – Schwartz model. Further studies carried out by Geman have considered β to be a stochastic parameter.

5.2 Forward Models

A second stream of commodity valuation models focuses on the evolution of the forward curve. Given the fact that interest rates are assumed to be deterministic, forward prices will be equal to future prices and forward models can also be applied to futures curves. According to (Clewlow & Strickland, Valuing Energy Options in a One Factor Model Fitted to Forward Prices, 1999a), this framework stands on the availability to access prices which are quoted on liquid exchanges, establishing the closest to maturity contract as the spot reference to obtain implied convenience yields on longer time to maturity contracts.

The divergence of these authors from the spot model has its fundamentals on the fact that often the state variables which support the model are unobservable and therefore convenience yields have to be estimated. Moreover, as stated in (Clewlow & Strickland, A Multi-Factor Model for Energy Derivatives, 1999b), the forward price curve is an endogenous function of the model parameters and therefore will not necessarily be consistent with the market observable forward prices.

The modelling of the entire forward curve conditional to the initially observed forward curve starts with the single factor model proposed in (Clewlow & Strickland, Valuing Energy Options in a One Factor Model Fitted to Forward Prices, 1999a) and continues with a multi-factor model in (Clewlow & Strickland, A Multi-Factor Model for Energy Derivatives, 1999b). These enable a unified approach to the pricing and hedging of a portfolio of energy derivatives.

5.2.1 The Single-Factor Model

Following (Baíllo & Adroher, 2007), the SDE assumed for the forward price will have a trend equal to zero and will be given by:

$$\frac{dF(t,T)}{F(t,T)} = \sigma(t,T)dW_t$$

Where *F(t,T)* stands for the price at time t of the contract expiring at time T, and $\sigma(t,T)$ the instantaneous volatility for the same contract.

According to (Clewlow & Strickland, Valuing Energy Options in a One Factor Model Fitted to Forward Prices, 1999a), in a risk neutral world, as the forward contracts do not require any initial investment the expected change in the forward price must be zero. It also states that in order to obtain a Markovian spot price process the volatilities of forward prices must have a negative exponential form. In the resulting model volatility will tend to zero as time to maturity increases:

$$\frac{dF(t,T)}{F(t,T)} = \sigma e^{-\alpha(T-t)}dW_t$$

In this model, volatility is defined by to parameters: σ will determine the level of spot and forward price volatility, and α, the speed of mean reversion of the spot price. The estimation of these parameters can be carried out directly from the prices of option on the spot price or by fitting to historical volatilities of forward prices.

5.2.2 The Multi-Factor Model

The Multi-Factor Model is presented in (Clewlow & Strickland, A Multi-Factor Model for Energy Derivatives, 1999b) as framework designed to be consistent not only with the market observable price but also with the volatilities and correlations of forward prices.

The proposed multi-factor model is given by the following equation:

$$\frac{dF(t,T)}{F(t,T)} = \sum_{i=1}^{n} \sigma_i(t,T) dW_t^i$$

Where *F(t,T)* stands for the price at time t of the contract expiring at time *T*, and $\sigma_i(t,T)$ the volatility functions associated with the independent Brownian motions, W_t^i. This model will assume up to *n* independent sources of uncertainty which drive the evolution of the forward curve on time. Each of these sources will have associated a volatility function which will determine the direction and magnitude of the movement of each point of the forward curve at the arrival of information associated with that particular source of uncertainty. Several models are developed as special cases for this general framework.

Applying îto's lemma to the Multi-Factor Model it is possible to obtain a general logarithmic representation of the forward curve:

$$dLn\big(F(t,T)\big) = -\frac{1}{2}\sum_{i=1}^{n} \sigma_i(t,T)^2 dt + \sum_{i=1}^{n} \sigma_i(t,T) dW_t^i$$

Considering small time intervals, the previous equation can be discretized as:

$$\Delta Ln\left(F\big(t, t+\tau_j\big)\right) = -\frac{1}{2}\sum_{i=1}^{n} \sigma_i\big(t, t+\tau_j\big)^2 \Delta t + \sum_{i=1}^{n} \sigma_i\big(t, t+\tau_j\big) \Delta W_t^i$$

Where $\tau_j = 1, \dots, J$ are the relative maturities of the forward contracts. Following (Baíllo & Adroher, 2007), this equation can expressed in vector form:

$$\begin{bmatrix} \Delta Ln\big(F(t,t+\tau_1)\big) \\ \vdots \\ \Delta Ln\big(F(t,t+\tau_1)\big) \end{bmatrix} = \begin{bmatrix} -\frac{1}{2}\sum_{i=1}^{n} \sigma_i\big(t,t+\tau_j\big)^2 \\ \vdots \\ -\frac{1}{2}\sum_{i=1}^{n} \sigma_i\big(t,t+\tau_j\big)^2 \end{bmatrix} \Delta t + \begin{bmatrix} \sigma_1(t,t+\tau_1) & \dots & \sigma_n(t,t+\tau_n) \\ \vdots & \ddots & \vdots \\ \sigma_1(t,t+\tau_1) & \dots & \sigma_n(t,t+\tau_n) \end{bmatrix} \begin{bmatrix} \Delta W_t^1 \\ \vdots \\ \Delta W_t^n \end{bmatrix}$$

Notice that the increments of the logarithms of the futures prices for different maturities depend on a common vector of independent random components, $\Delta \boldsymbol{W}_t^T$. The parameters that express this dependency vary on time, *t*, and with the relative time to maturity, τ_j. This implies that the changes in the natural logarithms of the future prices are jointly normal distributed with a covariance matrix **Σ**, that changes in time. An estimation of this covariance matrix is computed through a sample of the following:

$$\hat{\sigma}_{ij} = \frac{1}{N}\sum_{k=1}^{N}(x_{ik} - \bar{x}_i)\big(x_{jk} - \bar{x}_j\big)$$

Where we have *k=1,…,N* quotes of futures prices $F\big(t_k, t_k + \tau_j\big)$ corresponding to relative maturities $\tau_j = 1, \dots, J$, whose logarithm's increments are defined by:

$$x_{jk} = Ln\left(F\big(t_k, t_k + \tau_j\big)\right) - Ln\left(F\big(t_{k-1}, t_{k-1} + \tau_j\big)\right)$$

The eigenvectors decomposition of the covariance matrix is given by the following:

$$\boldsymbol{\Sigma} = \boldsymbol{\Gamma\Lambda\Gamma}^{\boldsymbol{T}}$$

Where the columns of $\boldsymbol{\Gamma}$ are the resulting eigenvectors from the decomposition and $\boldsymbol{\Lambda}$ the vector of eigenvalues:

$$\boldsymbol{\Lambda} = \begin{bmatrix} \lambda_1 \\ \vdots \\ \lambda_n \end{bmatrix}$$

$$\boldsymbol{\Gamma} = \begin{bmatrix} v_{11} & \cdots & v_{1n} \\ \vdots & \ddots & \vdots \\ v_{n1} & \cdots & v_{nn} \end{bmatrix}$$

The eigenvector decomposition recovers the discretised volatility functions and yields the set of independent factors which drive the evolution of the variables underlying the covariance matrix. The eigenvalues represent the variances of the independent factors which drive the forward points in proportions determined by the eigenvectors. Forward volatility functions are therefore obtained through the following equation:

$$\sigma_i(t, t+\tau_j) = \nu_{ij}\sqrt{\lambda_i};\;\; j = 1, \dots, J$$

Where ν_{ij} are the eigenvector components for each eigenvalue, λ_i.

5.3 Analysing Forward Curves through Principal Component Analysis

According to (Blanco, Multi-Factor Models for Forward Curve Analysis, 2002), in order to value energy derivatives whose values depend on more than one forward price, it is necessary to develop a model of the forward price curve which measures the simultaneous evolution in time of all forward prices. These models seek to capture the joint movement of forward prices rather than their standalone behaviour. Given the fact that some energy forward prices exhibit strong seasonality, capturing the evolution of forward curves through the variances and correlations between different points along the curve may result in a correlated variables overload which makes it difficult to compute and understand. In the case of natural gas, prices for delivery at different future times are highly correlated but not perfectly, showing that there are fewer sources of risk than there are prices. This fact brings to light the advantage of implementing a Principal Component Analysis (from here on, PCA) against Black-Scholes or other one factor mean reverting models.

The efficiency of this methodology relies on the ability to reduce the number of dimensions of the problem to only two or three factors which retain the most representative uncorrelated sources of risk. Moreover, a PCA yields information on the importance or averaged weight of each uncorrelated source of randomness. It is therefore possible to quantify the impact of each of these sources in the absolute behaviour of the forward curve.

It is important to notice that PCA is not conceived to point out fundamental price drivers. Instead, this type of analysis relies on identifying uncorrelated sources which reveal the underlying structure of the original correlated data.

5.3.1 Factor Loadings in PCA

As introduced above, observation of historical forward curves drags out a set of principal components which mathematically drive price movements and rank their impact according to their contribution to the total variability of the data.

Each principal component is interpreted as an independent, uncorrelated source of risk, whose importance is given by its eigenvalue. Each eigenvalue is defined as the *score* of the respective source of risk. Associated with each independent source of randomness is a set of *factor loadings*, which stand for the volatilities of each relative maturity in the original dataset. For each principal component, its factor loadings will define how the price of each of the original contracts moves in response to a change in that component.

Typical uncorrelated sources of risk for energy price curves take the form of the following (in decreasing score value):

- First Principal Component: the Parallel Shift factor, which drives changes in the overall level of the curve. It will usually have the same sign (positive, for the case of natural gas) but will vary in magnitude, as its impact on closer to maturity contracts will usually differ from further in the curve contracts.

- Second Principal Component: the Slope factor, which defines the steepness of the curve. Is will often exhibit opposite movements in the front and back of the curve, and can be interpreted as a change in the overall level of the term structure of convenience yields.
- Third Principal Component: the Curvature factor, which introduces an inflexion point and results in a bending of the futures curve.

5.3.2 The Seasonal PCA

Despite some energy commodities such as crude oil are consistent with a PCA that uses as input the full set of sample data, this process is not valid for other underlying products which are highly driven by seasonal dependent factors. Capturing the average variability throughout the year in a typical month will not be realistic and will lack the capacity to accurately reproduce forward curve scenarios. Given this constraint, it is precise to develop a dynamic framework which captures the most representative patterns that drive the futures curve. (Blanco, Multi-Factor Models for Forward Curve Analysis, 2002) suggests analysing the full set of sample data in seasonal slices in order to compare only similar periods within each other. This approach allows us to analyse separately winter and summer months, seeking for a closer fit of forward curves.

5.3.3 Simulating through PCA

Having performed a seasonal PCA for a full set of sample data with the respective monthly factor scores (eigenvalues) and factor loadings (eigenvectors), it is now possible to simulate hypothetical forward curve movements that can be used to price derivatives and as a risk measurement instrument. As mentioned in chapter *5.2.2*, forward curve's volatility functions will be given by the eigenvectors' components and the eigenvalue for each of the uncorrelated sources of risk (this is, for each of the principal components derived from the PCA):

$$\sigma_i(t, t+\tau_i) = \nu_{ij}\sqrt{\lambda_i}$$

For $i = 1, \dots, n$ principal components and $j = 1, \dots, J$ maturity dates. Substituting in the derived SDE for the logarithm of the futures prices, and considering small enough time increments, we can obtain a Multi-Factor model to simulate the forward price curve using through principal components:

$$\Delta Ln\big(F(t, t+\Delta t)\big) = -\frac{1}{2}\sum_{i=1}^{n}\left(\nu_{ij}\lambda_i\right)^2\Delta t + \sum_{i=1}^{n}\left(\nu_{ij}\lambda_i\right)\Delta W_t^i$$

Substituting ΔW_t^i for $\sqrt{\Delta t}\varepsilon_i$, where ε_i is an uncorrelated random sample from a standard normal distribution for each principal component, the forward model will result:

$$F(t+\Delta t) = F(t)exp\left[-\frac{1}{2}\sum_{i=1}^{n}\left(v_{ij}\lambda_i\right)^2\Delta t + \sum_{i=1}^{n}\left(v_{ij}\lambda_i\right)\sqrt{\Delta t}\varepsilon_i\right]$$

Given the previous equation and following (Blanco, Multi-Factor Models of the Forward Price Curve, 2002), the PCA based multi-factor simulation process results quite straightforward:

1. Identify the number of relevant components.
2. Draw an uncorrelated random sample from a normal distribution for each of the principal components.
3. Scale each random sample by its associated factor score and rooted time step.
4. Compute the sum of all factor loadings scaled by their respective factor scores and time step.

6

Case Study: PCA on Henry Hub

6 Case Study: PCA on Henry Hub (2008-2013)

Having completed a general overview of the theory behind energy modelling and its fitting to natural gas derivatives, this chapter aims to offer a straightforward numerical example of a Principal Component Analysis carried out on this product. The developed framework has been kept simple in order to offer a wide scope which can be further applied to other seasonal commodities.

The Case Study has been structured into three phases:

- An initial PCA on the full set of sample data, discarding the influence of seasonality on the results.
- A second analysis on the disaggregated data, which has been previously divided into two seasonal categories (winter and summer months) and each of them has been analysed through an individual PCA.
- A third PCA which has further divided the seasonal benchmarks from the previous point into month units which have been in turn analysed through PCA.

6.1 The Data

The data consists of daily closing prices for the nearest 12 monthly contracts for NYMEX Henry Hub Natural Gas from January 2008 to December 2013 and have been obtained from Bloomberg. Several studies have already been developed on Henry Hub Natural Gas futures. Nevertheless, these studies have considered time frames between the late 90's and early 2000's. The change in fundamentals from the financial crisis onwards and the rise of unconventional gas sources have been the main drivers for conducting the proposed PCA in this period.

Henry Hub Natural Gas forward curves for the first 12 contracts can be joint together to from the surface below:

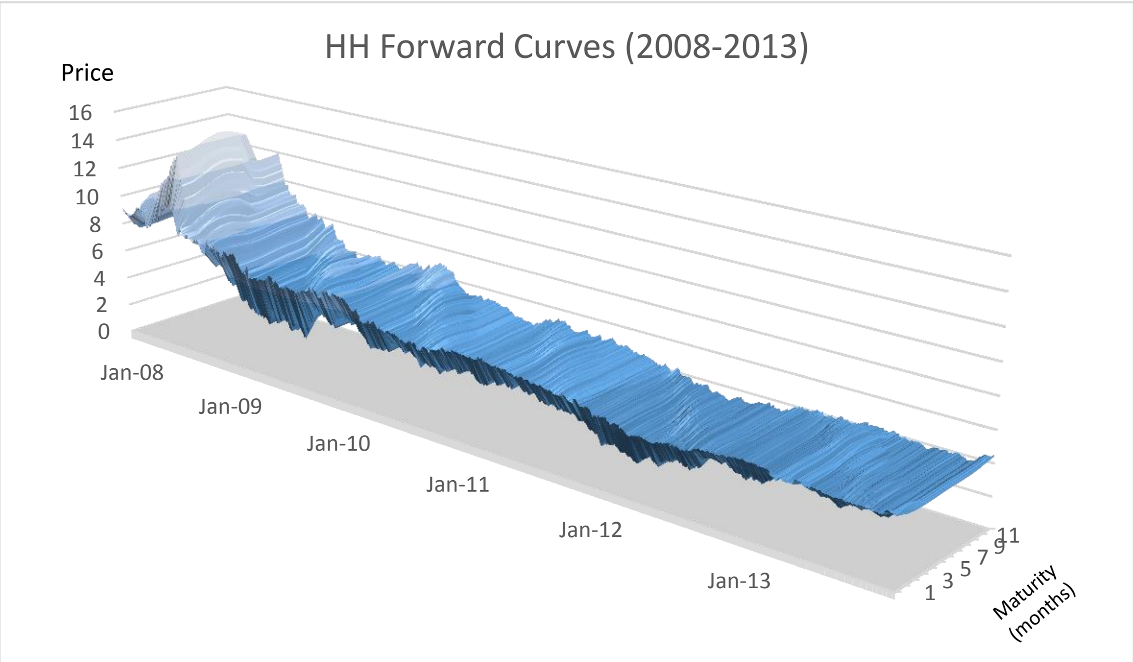


Figure 6.1 HH 12 Front Forward Curves

An in depth analysis of this surface denotes a continuous fluctuation of the forward curves and suggests the need to shorten the scope of the model in order to fit reality. Abrupt changes from contango to backwardation indicate that the underlying must be calibrated within a narrow time frame for results to be accurate:

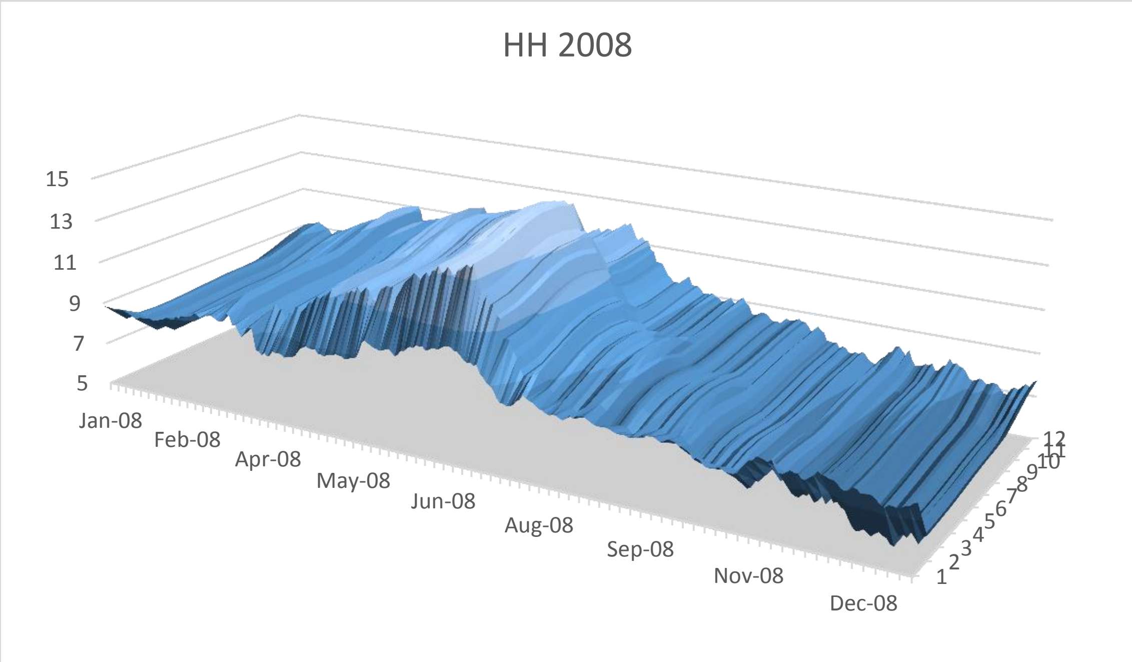


Figure 6.2 HH 2008 12 Months Forward

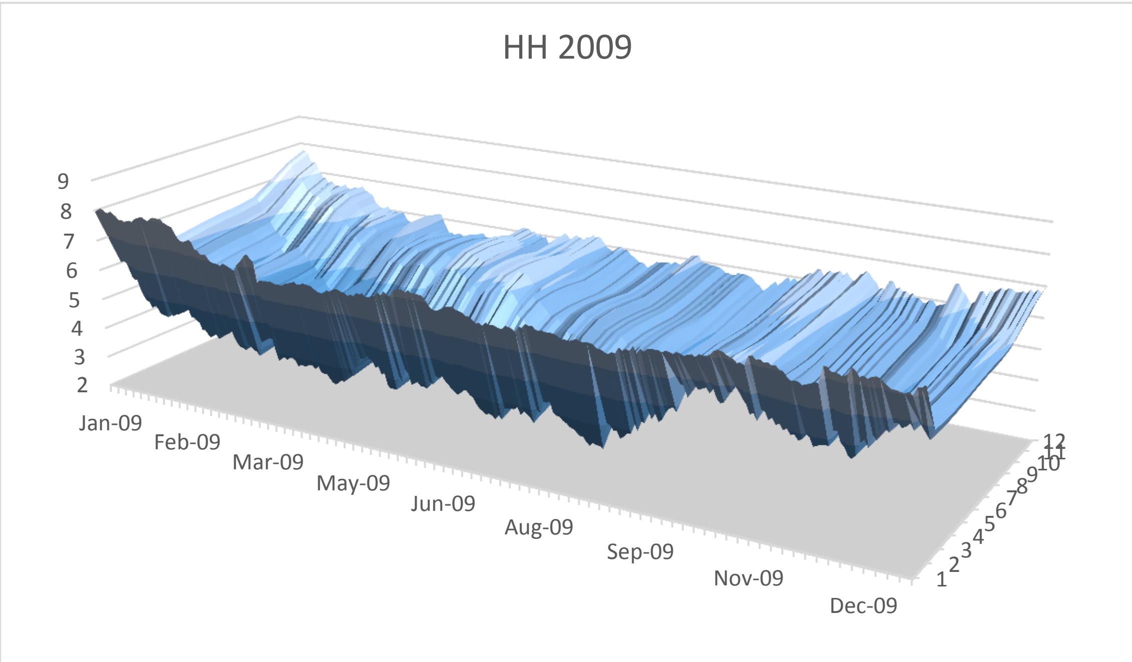


Figure 6.3 HH 2009 12 Months Forward

These sudden fluctuations are the reason why the PCA on Henry Hub has only considered as input the front 12 contracts instead of a larger time scope.

6.2 The Process

The Principal Component Analysis has been carried out in a straightforward and standardised process following the theory explained in chapter 5. It has been structured into six steps which have been implemented and systematised through Excel and VBA. These steps are given below:

1. Load the segregated input data into the model.
2. Compute de covariance matrix for the input data.
3. Carry out the eigenvalue decomposition for the computed covariance matrix.
4. Rank eigenvalues and select principal components.
5. Select a volatility functions to model the principal components.
6. Calibrate each volatility function against their respective principal components.

6.2.1 Loading the Model with Segregated Input Data

Aiming at developing a dynamic framework through a seasonal PCA, it is necessary to prepare the input data by segregating the full set of sample data into seasonal categories. In order to achieve this task, a function has been developed to read the data, organize it according to seasonal or monthly categories and to send it afterwards to the PCA model. For sections *6.4* and *6.5*, the data has been organized as follows:

- Seasonal categories: Henry Hub closing price quotes from 2008 to 2013 has been divided into *Winter Season*, which considers all prices from September to February (both included) throughout the time period, and *Summer Season*, which includes all closing prices from March to August. The definition of these two benchmarks has been established after analysing the yearly average term structure for this product that was explained in section 3.3.
- Monthly categories: to compute the seasonal PCA as stated in section 5.3.2, input data must be disaggregated into monthly units that cover all closing prices for each month within the analysed time period.

6.2.2 Computing the Covariance Matrix

Having entered the closing price quotes for the period, we now proceed to compute the increments of the logarithms of these prices. It is important to notice that these increments must be adjusted by a time factor to avoid steep changes from month to month. Considering *k=1,…,K* quotes of futures prices $F\left(t_k, t_k + \tau_j\right)$ corresponding to relative maturities $\tau_j = 1, \dots, J$, the logarithmic returns for these future prices will be calculated through the following:

$$x_{jk} = \frac{Ln\left(F\left(t_k, t_k + \tau_j\right)\right) - Ln\left(F\left(t_{k-1}, t_{k-1} + \tau_j\right)\right)}{\sqrt{\frac{t_k - t_{k-1}}{365}}}$$

The covariance matrix of the sample x_{jk} for *k=1,…,K* and $\tau_j = 1, \dots, J$ will provide an estimation of the covariance matrix, **Σ**.

6.2.3 Eigenvector Decomposition of the Covariance Matrix

Having estimated the covariance matrix, $\mathbf{\Sigma}$, through the logarithmic returns of the segregated historical data, we can now compute its eigenvector decomposition, $\mathbf{\Sigma} = \mathbf{\Gamma}\mathbf{\Lambda}\mathbf{\Gamma}^{\boldsymbol{T}}$, where $\mathbf{\Gamma}$ are the eigenvectors of the decomposition and $\mathbf{\Lambda}$ is the vector of eigenvalues.

Each of the columns of the eigenvector matrix, $\mathbf{\Gamma}$, will be associated to a respective eigenvalue from the vector, $\mathbf{\Lambda}$, and each of the rows will correspond to a specific time to maturity as it is shown below:

$$\mathbf{\Lambda} = \begin{bmatrix} \lambda_1 \\ \vdots \\ \lambda_N \end{bmatrix};\ \mathbf{\Gamma} = \begin{bmatrix} \nu_{11} & \cdots & \nu_{1N} \\ \vdots & \ddots & \vdots \\ \nu_{J1} & \cdots & \nu_{JN} \end{bmatrix}$$

6.2.4 Eigenvalue Analysis

Having computed the eigenvector decomposition of the covariance matrix and having obtained the corresponding eigenvalues, we can now analyse the importance of each eigenvector by rating the relative weigh of their respective eigenvalues. This procedure allows us to rank all eigenvectors according to their impact on the behaviour of the forward curve. The relative weight of each eigenvalue has been calculated through the following:

$$\omega_i = \frac{\lambda_i}{\sum_{k=1}^{N} \lambda_k};\quad i = 1, \dots, N$$

For the scope of this project, we have established a minimum target 95% of captured information from the covariance matrix.

6.2.5 Defining a Volatility Function

Following the theory for the Multi-Factor model from section *5.2.2*, after applying Îto's Lemma and considering small discrete time intervals, *Δt*, the Multi-Factor model take the form of:

$$\Delta Ln\left(F(t, t+\tau_j)\right) = -\frac{1}{2}\sum_{i=1}^{n} \sigma_i(t, t+\tau_j)^2 \Delta t + \sum_{i=1}^{n} \sigma_i(t, t+\tau_j)\Delta W_t^i$$

Having computed the eigenvector decomposition, we can now find a volatility function which marries the following:

$$\sigma_i(t, t+\tau_j) = \nu_{ij}\sqrt{\lambda_i};\quad j = 1, \dots, J;\quad i = 1, \dots, N$$

Where the eigenvectors are multiplied by the squared root of their corresponding eigenvalue in order to obtain discrete versions of the volatility functions in the sense of the Multi-Factor model. It is therefore necessary to find a normalized function which closely fits the layout of the principal eigenvectors.

At first, volatility functions were developed following those proposed in (Baíllo & Adroher, 2007) to model crude oil's volatility. Each of these functions consist of a constant term and a negative exponential term. Despite these function have proven to meet crude oil volatility term structure, they have fallen short for replicating that of natural gas. It was therefore necessary to find a new function which included the constant and negative exponential terms form the previous but also included a third term to fit the annual seasonality factor decreasing on time.

The proposed volatility function for each of the eigenvalues, λ_i, is the following:

$$\sigma_i(t, t+\tau_i) = \alpha_i e^{-\beta_i \tau_j} + \delta_i e^{-\zeta_i \tau_j} \sin\left(2\pi\tau_j + \varphi_i\right) + \gamma_i$$

Where:

- β_i and ζ_i must be strictly positive for the exponentials to be negative and to obtain a Markovian spot price process.
- The first term measures the exponential decrease of volatility over time.
- The second term provides a seasonal component multiplied by another negative exponential due to the fact that the seasonality effect dies out in time.
- The third term is the level to which volatility converges in time.

6.2.6 Calibrating Volatility Functions

Having selected the principal components and having defined a common volatility function to model these components, we can now calibrate the proposed function's parameters against the factor loadings (eigenvector components) adjusted by the squared root of the respective score factor (respective eigenvalue):

$$\sigma_i(t, t+\tau_i) = \alpha_i e^{-\beta_i \tau_j} + \delta_i e^{-\zeta_i \tau_j} \sin\left(2\pi\tau_j + \varphi_i\right) + \gamma_i = \nu_{ij}\sqrt{\lambda_i}; \;\; \forall i, \forall j$$

This calibration has been carried out for all eigenvector components and for each eigenvalue. Nevertheless, for practical purposes, the calibration process has considered the normalized parameters against the eigenvector components as it is exhibited below:

$$\frac{\alpha_i}{\sqrt{\lambda_i}} e^{-\beta_i \tau_j} + \frac{\delta_i}{\sqrt{\lambda_i}} e^{-\zeta_i \tau_j} \sin\left(2\pi\tau_j + \varphi_i\right) + \frac{\gamma_i}{\sqrt{\lambda_i}} = \nu_{ij}; \;\; \forall i, \forall j$$

This calibration process has been carried out with *Solver* by minimizing the sum of all squared errors between ν_{ij} and the estimated value from the model with normalized parameters. The calculated values for $\frac{\alpha_i}{\sqrt{\lambda_i}}, \frac{\delta_i}{\sqrt{\lambda_i}}$ and $\frac{\gamma_i}{\sqrt{\lambda_i}}$ have been later multiplied by their score factor, λ_i to obtain the function's parameters.

6.2.7 Numerical Example

This section aims to provide a numerical example of the full PCA and calibration process that has been explained in the previous points. The selected input data is the Henry Hub 2008-2013 August bucket, as it has proved to meet a close fit:

1. *Loading the segregated input data into the model:* Input data has been segregated into monthly categories and the August bucket has been entered into the model.

2. *Compute de covariance matrix for the input data:* Daily logarithmic returns have been calculated and the covariance matrix is then computed resulting in the following:

	1	2	3	4	5	6	7	8	9	10	11	12	13
1	0,0227	0,0347	0,0338	0,0298	0,0269	0,0263	0,0265	0,0256	0,0251	0,0246	0,0240	0,0237	0,0214
2	0,0347	0,3693	0,2934	0,1634	0,1097	0,0941	0,0905	0,0982	0,1024	0,1017	0,0936	0,0869	0,0577
3	0,0338	0,2934	0,2524	0,1489	0,1008	0,0869	0,0840	0,0926	0,0957	0,0946	0,0871	0,0807	0,0553
4	0,0298	0,1634	0,1489	0,1114	0,0888	0,0833	0,0821	0,0800	0,0791	0,0772	0,0732	0,0700	0,0488
5	0,0269	0,1097	0,1008	0,0888	0,0825	0,0813	0,0817	0,0716	0,0686	0,0665	0,0646	0,0634	0,0441
6	0,0263	0,0941	0,0869	0,0833	0,0813	0,0837	0,0847	0,0703	0,0663	0,0640	0,0628	0,0625	0,0433
7	0,0265	0,0905	0,0840	0,0821	0,0817	0,0847	0,0883	0,0705	0,0659	0,0635	0,0627	0,0626	0,0433
8	0,0256	0,0982	0,0926	0,0800	0,0716	0,0703	0,0705	0,0651	0,0623	0,0605	0,0585	0,0571	0,0405
9	0,0251	0,1024	0,0957	0,0791	0,0686	0,0663	0,0659	0,0623	0,0613	0,0592	0,0570	0,0553	0,0394
10	0,0246	0,1017	0,0946	0,0772	0,0665	0,0640	0,0635	0,0605	0,0592	0,0582	0,0555	0,0538	0,0385
11	0,0240	0,0936	0,0871	0,0732	0,0646	0,0628	0,0627	0,0585	0,0570	0,0555	0,0540	0,0522	0,0374
12	0,0237	0,0869	0,0807	0,0700	0,0634	0,0625	0,0626	0,0571	0,0553	0,0538	0,0522	0,0516	0,0367
13	0,0214	0,0577	0,0553	0,0488	0,0441	0,0433	0,0433	0,0405	0,0394	0,0385	0,0374	0,0367	0,0291

Figure 6.4 August HH Covariance Matrix

3. *Carry out the eigenvalue decomposition for the computed covariance matrix:* The eigenvector decomposition is computed and the obtained vectors are organized in descending order according to their eigenvalue's relative weight:

Time	Eigenvectors												
0,083	0,086	0,110	0,532	0,636	-0,364	0,228	-0,114	-0,296	0,003	0,085	0,018	-0,005	0,000
0,167	0,521	-0,594	-0,306	0,419	0,287	0,101	-0,110	0,038	-0,004	-0,036	-0,014	0,012	0,006
0,250	0,451	-0,371	0,268	-0,393	-0,490	-0,326	0,267	-0,115	0,023	0,008	0,021	-0,020	-0,006
0,333	0,314	0,081	0,142	-0,375	-0,105	0,623	-0,404	0,376	0,056	0,118	0,090	-0,059	-0,035
0,417	0,250	0,257	-0,180	-0,043	0,016	0,407	0,542	-0,205	-0,566	-0,124	0,014	0,015	0,008
0,500	0,234	0,323	-0,349	0,086	-0,155	0,123	0,314	-0,064	0,746	0,074	-0,061	0,062	0,026
0,583	0,232	0,346	-0,442	0,152	-0,441	-0,356	-0,384	0,155	-0,321	0,072	-0,081	0,012	0,022
0,667	0,224	0,220	0,070	-0,087	0,149	-0,116	-0,269	-0,272	0,117	-0,740	0,377	0,057	-0,007
0,750	0,223	0,175	0,160	-0,104	0,254	-0,076	-0,126	-0,192	0,026	-0,087	-0,756	-0,424	-0,035
0,833	0,218	0,160	0,182	-0,090	0,276	-0,121	-0,080	-0,100	-0,041	0,246	-0,181	0,811	-0,158
0,917	0,207	0,179	0,135	-0,023	0,283	-0,168	0,004	-0,040	-0,023	0,340	0,262	-0,145	0,769
1,000	0,198	0,200	0,079	0,051	0,260	-0,213	0,076	0,002	-0,012	0,357	0,397	-0,357	-0,616
2,000	0,138	0,157	0,305	0,250	0,072	-0,168	0,317	0,751	-0,005	-0,304	-0,087	0,044	0,027
	Eigenvalues												
	1,077	0,206	0,021	0,014	0,005	0,002	0,001	0,001	0,001	0,001	0,001	0,000	0,000
	Relative Weight												
	81,03%	15,49%	1,54%	1,02%	0,39%	0,17%	0,08%	0,07%	0,05%	0,05%	0,04%	0,04%	0,03%
	81,03%	96,52%	98,07%	99,08%	99,47%	99,64%	99,72%	99,79%	99,85%	99,89%	99,93%	99,97%	100,00%

Figure 6.5 August HH 12 Principal Components

4. *Rank eigenvalues and select principal components:* Having ranked all eigenvalues according to their relative weight, we can now select those which are more representative of the covariance matrix and which have a major impact in its behaviour. According to this, the first and second components show a relative weight of 81.03% and 15.49%, respectively. As they jointly represent more than 95% of the matrix's information, we will proceed to calibrate the volatility functions against them:

Time	Eigenvectors	
0,083	0,086	0,110
0,167	0,521	-0,594
0,250	0,451	-0,371
0,333	0,314	0,081
0,417	0,250	0,257
0,500	0,234	0,323
0,583	0,232	0,346
0,667	0,224	0,220
0,750	0,223	0,175
0,833	0,218	0,160
0,917	0,207	0,179
1,000	0,198	0,200
2,000	0,138	0,157
	Eigenvalues	
	1,077	0,206
	Relative Weight	
	81,03%	15,49%
	81,03%	96,52%

5. *Select a volatility functions to model the principal components:* The proposed volatility function from section *6.2.5* will be calibrated against the principal eigenvalues al later multiplied by the squared root of their respective factor scores, *λ_1=1.077* and *λ_2=0.206.*

6. *Calibrate each volatility function against their respective principal components:* The selected eigenvectors are shown below:

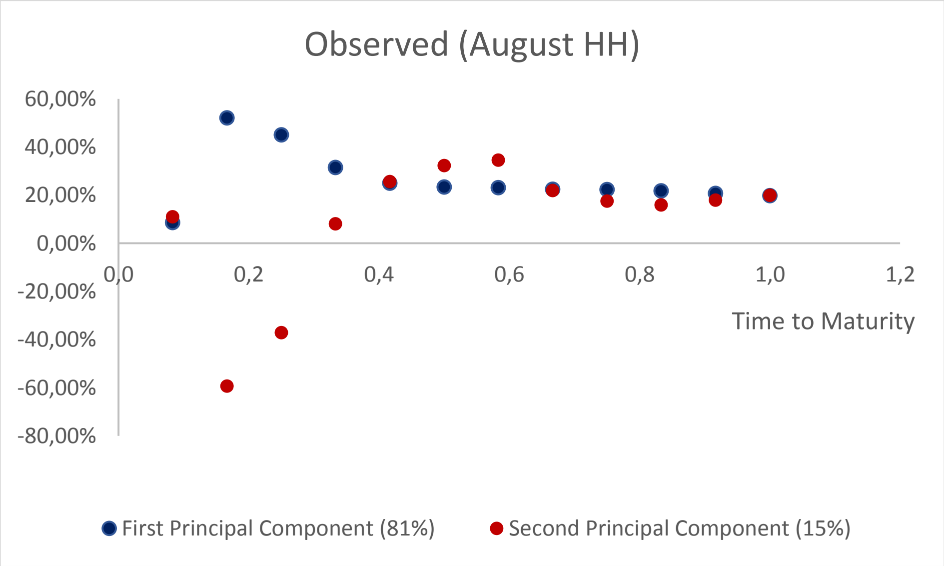


Figure 6.6 Observed Principal Components Example

Calibrating the normalized parameters to the observed eigenvector values as explained in section *6.2.6* will draw forth the following adjusted curve:

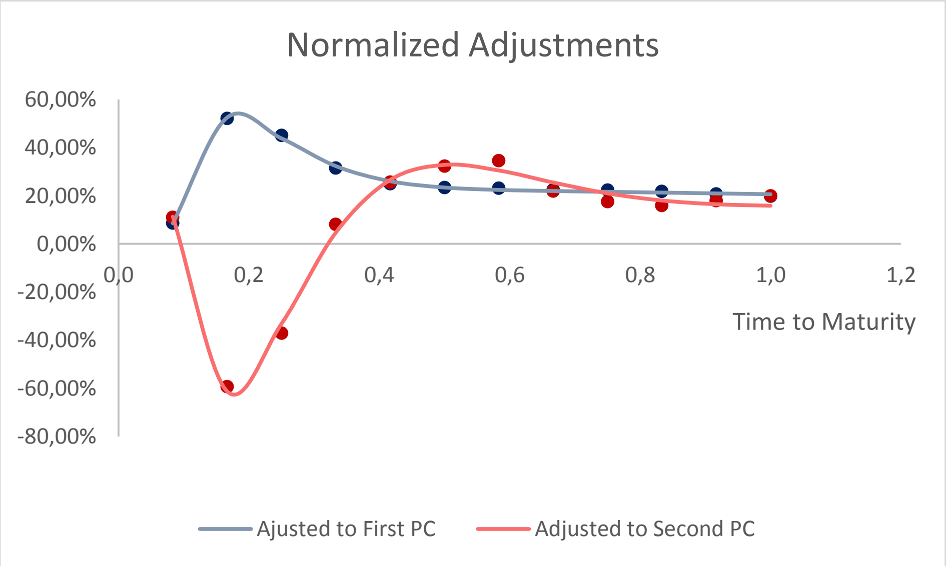


Figure 6.7 Normalized Adjustment for Observed Values

The resulting parameters for the model after multiplying the normalized parameters by the respective eigenvalues are:

Parameter	λ_1	λ_2
α	0.576	2.91
β	0.081	8.617
δ	5.243	-3.32
ζ	12.126	6.86
ϕ	-0.611	0.353
γ	-0.316	0.073

The resulting volatility functions that account for the 96% of August Henry Hub forward curve volatility structure are given below:

$$\sigma_1\left(t, t+\tau_j\right) = 0.576e^{-0.081\tau_j} + 5.243e^{-12.126\tau_j}\sin\left(2\pi\tau_j - 0.611\right) - 0.316; \;\; j = 1, \dots, 12$$

$$\sigma_1\left(t, t+\tau_j\right) = 2.91e^{-8.617\tau_j} - 3.32e^{-6.86\tau_j}\sin\left(2\pi\tau_j + 0.353\right) + 0.073; \;\; j = 1, \dots, 12$$

The sum of these two functions will represent the term structure for the 12 front monthly contracts of NYMEX Henry Hub natural gas during the month of August:

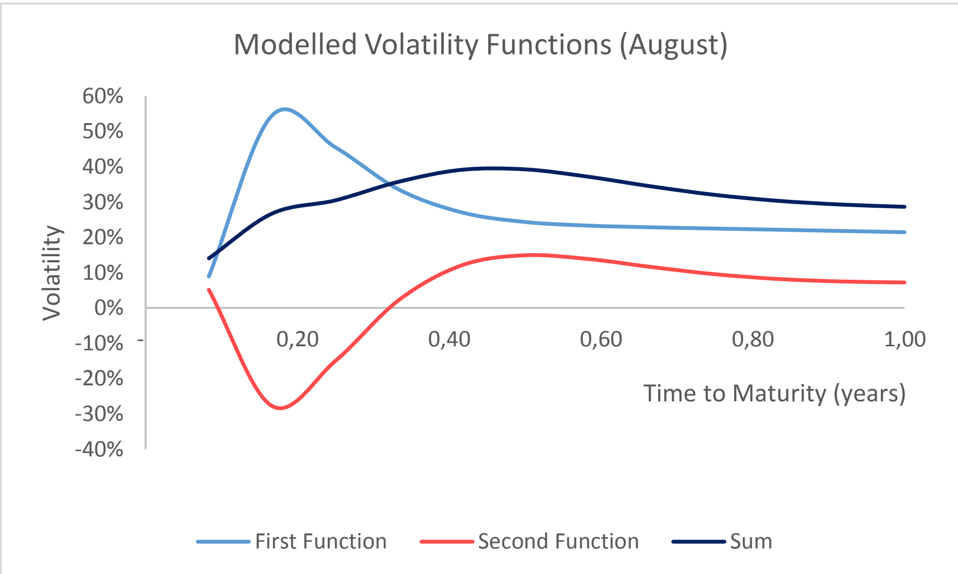


Figure 6.8 Modelled Volatility Functions

6.3 The Full Sample Data PCA

An initial PCA was conducted over the full sample data of Henry Hub closing prices from January 2008 to December 2013. For this analysis, we decided to bypass the effect of seasonality on the forward curve and assumed an ideal *typical month* which embraces all information on the front 12 forward contracts.

Time	Eigenvectors			
0,083	0,091	0,514	-0,160	0,069
0,167	0,392	-0,258	-0,452	-0,247
0,250	0,367	-0,241	-0,365	-0,188
0,333	0,338	-0,121	-0,182	0,053
0,417	0,319	-0,040	-0,046	0,380
0,500	0,301	-0,048	0,143	0,540
0,583	0,283	-0,106	0,324	0,331
0,667	0,267	-0,128	0,429	-0,063
0,750	0,257	-0,027	0,437	-0,391
0,833	0,244	0,172	0,264	-0,393
0,917	0,231	0,344	0,066	-0,183
1,000	0,215	0,427	-0,099	0,055
2,000	0,143	0,484	-0,131	0,056
	Eigenvalues			
	1,5133	0,0788	0,0506	0,043351
	Relative Weight			
	86,76%	4,52%	2,90%	2,49%
	86,76%	91,28%	94,18%	96,67%

Notice the role seasonality plays in the forward curve: in order to fulfil the 95% minimum target of captured data, it is necessary to consider the first four principal components. We proceed with the calibration of our volatility functions against the first four principal components.

The results show a close fit of the volatility function to the first and third eigenvectors:

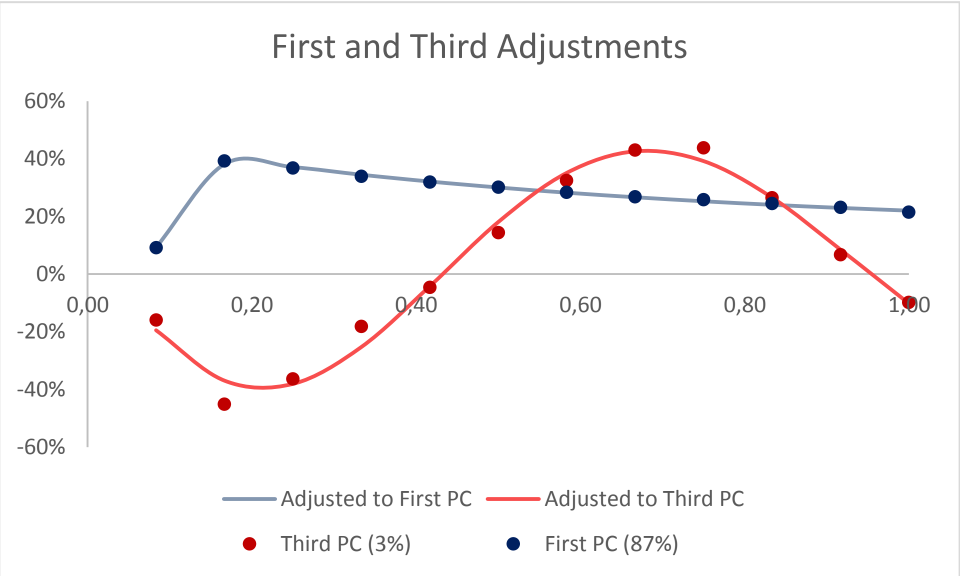


Figure 6.9 Adjustment to First and Third Full Data PCs

Nevertheless, the volatility function does not permit an adequate representation of the second and fourth eigenvectors under the constraint of the exponentials being negative. This fact sheds light upon the need to segregate the input data and to analyse principal components for each of the seasonal categories. The following sections will take this into consideration and will develop the Winter-Sumer Seasonal PCA and the Monthly PCA

6.4 The Winter-Summer PCA

Section *6.3* brought to light the need to develop a Seasonal PCA in order to reach a closer fit of the volatility functions to the forward curves. Our first approach consists on dividing input data into two reference benchmarks, corresponding to winter and summer months, as it has been further explained in section *6.2.1*. The result for the winter and summer benchmarks are deepen below.

6.4.1 Winter PCA

The PCA for all moths between September and February (both included) for the 2008-2013 trading period has yield the following eigenvectors as main independent sources of risk:

Time	Eigenvectors	
0,083	0,098	0,536
0,167	0,379	-0,114
0,250	0,340	-0,196
0,333	0,318	-0,162
0,417	0,304	-0,071
0,500	0,293	0,015
0,583	0,285	-0,005
0,667	0,285	-0,178
0,750	0,284	-0,247
0,833	0,267	-0,021
0,917	0,247	0,311
1,000	0,229	0,453
2,000	0,153	0,485
	Eigenvalues	
	1,513	0,0612
	Relative Weight	
	91,85%	3,72%
	91,85%	95,57%

Notice that for the winter benchmark, the two first components contribute with more than 95% of the covariance matrix information. This already indicates that analysing the full set of sample data instead of slicing it into seasonal categories will underestimate the contribution of the first principal components.

The calibration of the normalized volatility functions against the first two components and the later adjustment by their respective score factors draws forth the following term structure for the winter benchmark:

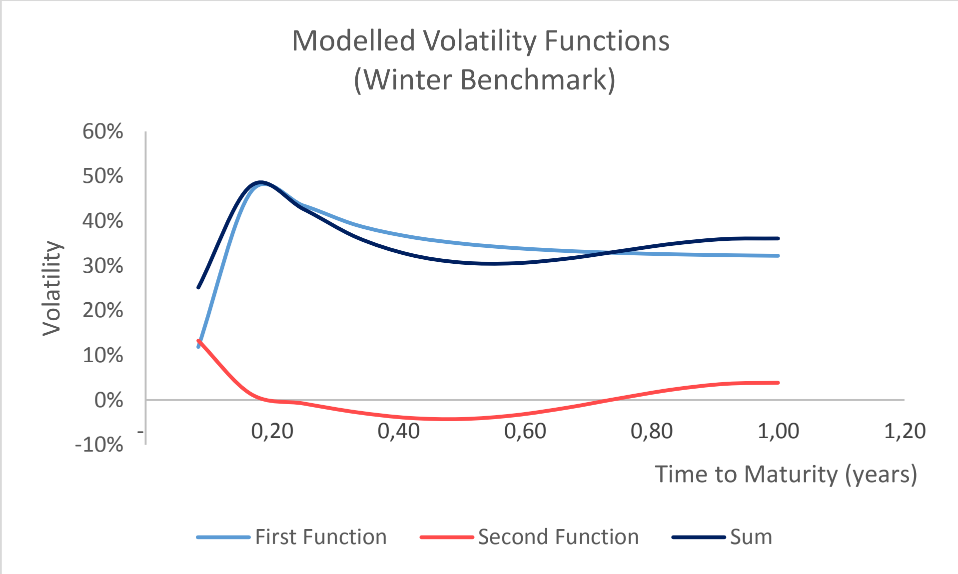


Figure 6.10 Modelled Functions for Winter Benchmark

These can be understood as the volatility term structure for the front 12 Henry Hub contracts viewed from September to February. Particularly remarkable is the inherent seasonality in the second principal component, which tends to decrease from the second to the sixth months forward to later increase as the winter season is back.

6.4.2 Summer PCA

The summer PCA has been conducted analogously to the winter PCA, considering all trading days between March and August from 2008 to 2013. The result obtained from this analysis are the following:

Time	Eigenvectors			
0,083	0,084	0,492	0,072	-0,149
0,167	0,403	-0,196	-0,441	0,020
0,250	0,392	-0,128	-0,461	-0,016
0,333	0,357	-0,023	-0,215	-0,190
0,417	0,332	-0,030	0,119	-0,424
0,500	0,307	-0,167	0,431	-0,350
0,583	0,280	-0,261	0,478	0,052
0,667	0,248	-0,189	0,297	0,382
0,750	0,230	0,026	0,106	0,527
0,833	0,221	0,240	-0,012	0,404
0,917	0,214	0,346	-0,007	0,174
1,000	0,200	0,405	0,093	-0,079
2,000	0,132	0,478	0,076	-0,112
	Eigenvalues			
	1,5155	0,0992	0,0782	0,0614
	Relative Weight			
	82,77%	5,42%	4,27%	3,35%
	82,77%	88,19%	92,46%	95,82%

Despite the 95% level is only reached through the joint contribution of the first four principal component, it is important to emphasize on the fact that the impact of the second, third and fourth components has increased in fifty per cent with respect to their relative weight in the Full Sample Data PCA. This can be viewed as an increase in the contribution of the seasonal factors, which now tend to increase volatility from the fifth month to maturity onwards, as can be viewed below:

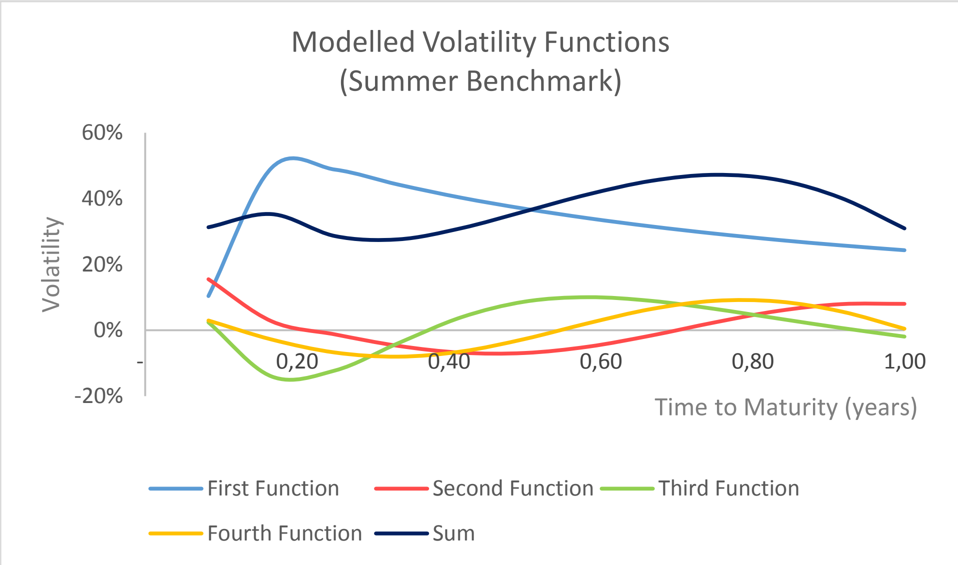


Figure 6.11 Modelled Functions for Summer Benchmark

6.4.3 Winter-Summer PCA Conclusions

The Winter-Summer PCA serves as a good example to support the analysis of natural gas from a Seasonal PCA. It also serves as a comprehensive and reliable baseline, which draws forth the following conclusions:

- The first volatility function represents an initial spike around the second month to maturity and the decrease of volatility as time to maturity increases. This is due to the buffering effect of storage, which mitigates volatility of the front month forward but raises concern on supply from the second month onwards. This function is structurally similar for both winter and summer benchmarks.
- The next volatility functions define seasonality and their highs range from closer to maturity months in the winter benchmark to further down the curve in the summer benchmark.

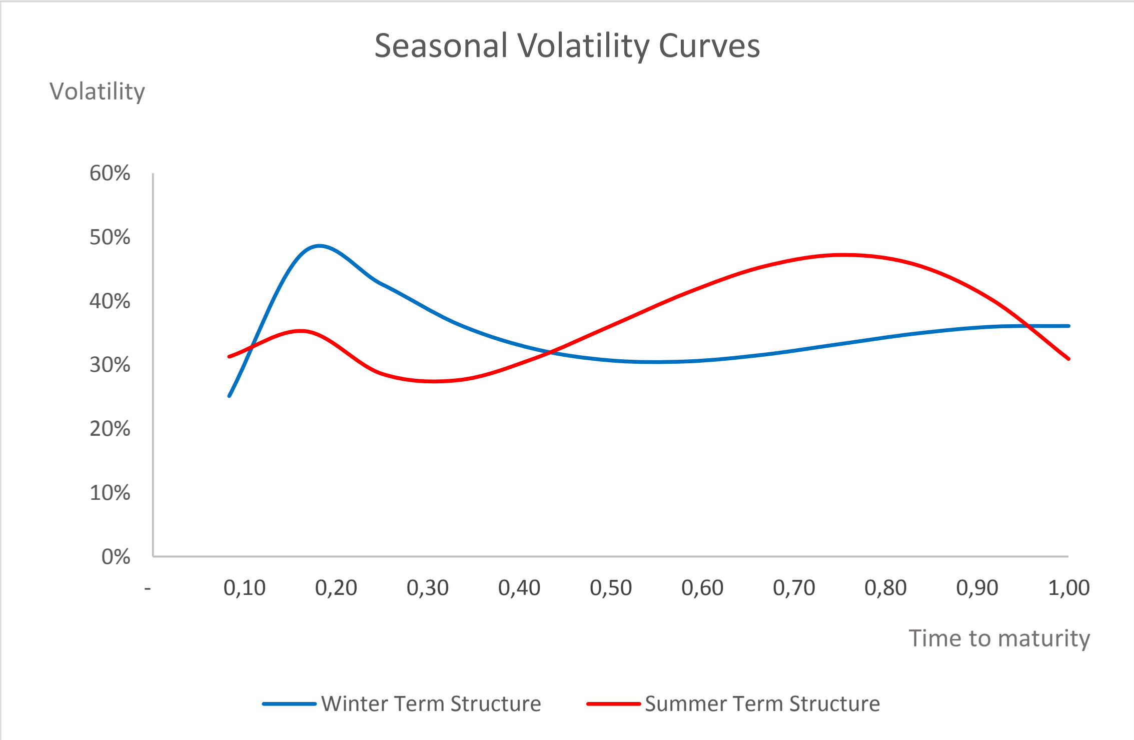


Figure 6.12 Modelled Functions Winter-Summer

6.5 The Monthly PCA

Having performed the Winter-Summer Seasonal PCA, we now aim at stressing out the segregation of input data by breaking it down into the smallest operationally feasible time unit. These units are constituted by all Henry Hub closing prices from each month of the year from January 2008 to December 2013. We will therefore break up the entire sample data into monthly units which will gather all observation made in each month during the observed period. This approach seeks to capture evolution in time of each of the 12 front months forward from the perspective of each month of the year.

6.5.1 January

Time	Eigenvectors	
0,083	0,058	-0,139
0,167	0,357	-0,278
0,250	0,333	-0,160
0,333	0,321	-0,105
0,417	0,311	-0,069
0,500	0,297	-0,128
0,583	0,285	-0,212
0,667	0,291	-0,067
0,750	0,327	0,524
0,833	0,300	0,624
0,917	0,234	0,191
1,000	0,192	-0,240
2,000	0,123	-0,193
	Eigenvalues	
	1,796	0,072
	Relative Weight	
	93,80%	3,76%
	93,80%	97,56%

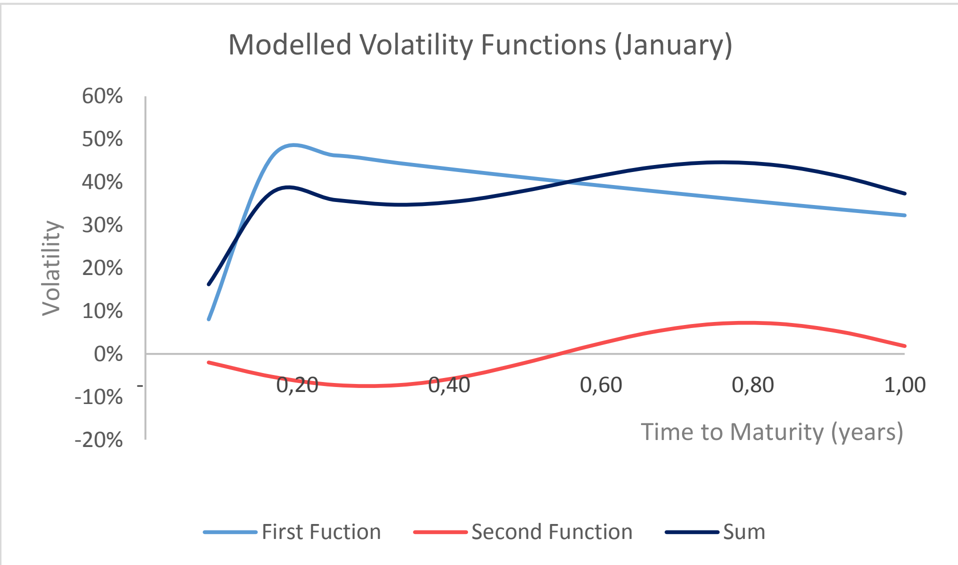


Figure 6.13 Modelled Functions for January

6.5.2 February

Time	Eigenvectors	
0,083	0,057	0,366
0,167	0,346	0,062
0,250	0,336	0,042
0,333	0,327	0,054
0,417	0,308	0,145
0,500	0,295	0,209
0,583	0,303	0,091
0,667	0,348	-0,396
0,750	0,326	-0,539
0,833	0,258	-0,167
0,917	0,208	0,198
1,000	0,184	0,351
2,000	0,113	0,382
	Eigenvalues	
	1,291	0,137
	Relative Weight	
	87,65%	9,32%
	87,65%	96,97%

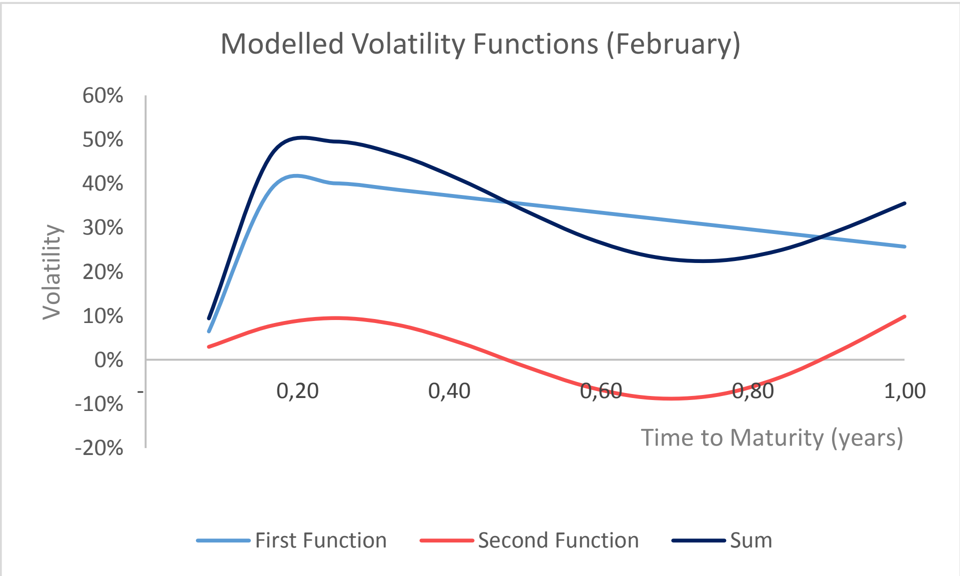


Figure 6.14 Modelled Functions for February

6.5.3 March

Time	Eigenvectors	
0,083	0,074	0,523
0,167	0,366	-0,060
0,250	0,356	-0,063
0,333	0,337	-0,012
0,417	0,327	0,035
0,500	0,327	-0,024
0,583	0,338	-0,244
0,667	0,297	-0,305
0,750	0,246	-0,110
0,833	0,219	0,072
0,917	0,209	0,151
1,000	0,193	0,493
2,000	0,120	0,530
	Eigenvalues	
	1,728	0,361
	Relative Weight	
	79,18%	16,54%
	79,18%	95,73%

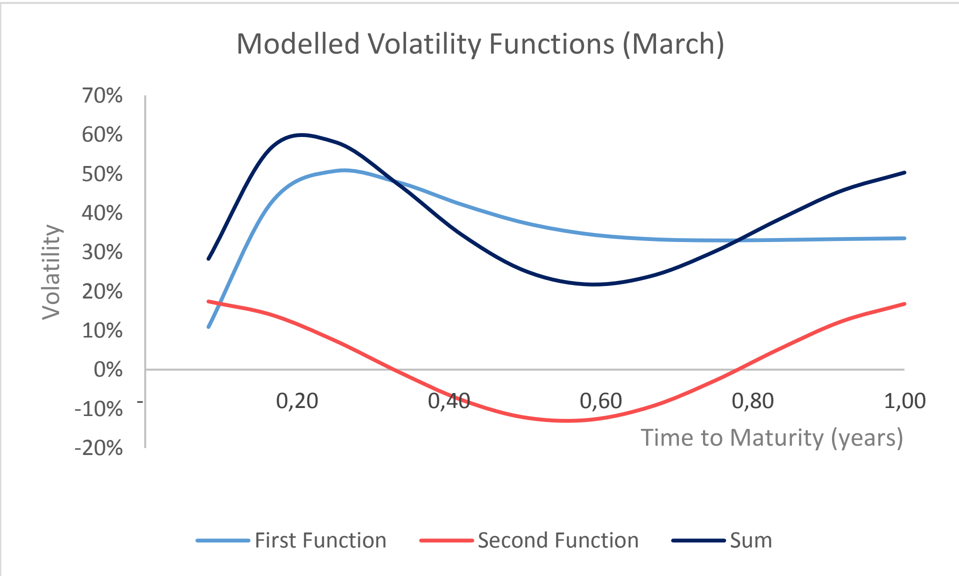


Figure 6.15 Modelled Functions for March

6.5.4 April

Time	Eigenvectors		
0,083	0,089	0,111	-0,069
0,167	0,367	-0,028	0,310
0,250	0,347	0,028	0,314
0,333	0,327	0,096	0,349
0,417	0,328	0,016	0,239
0,500	0,362	-0,434	-0,330
0,583	0,331	-0,463	-0,394
0,667	0,257	-0,129	-0,083
0,750	0,226	0,143	0,091
0,833	0,216	0,256	0,072
0,917	0,248	0,656	-0,559
1,000	0,196	0,145	-0,103
2,000	0,132	0,138	-0,116
	Eigenvalues		
	1,418	0,238	0,062
	Relative Weight		
	79,93%	13,43%	3,50%
	79,93%	93,36%	96,86%

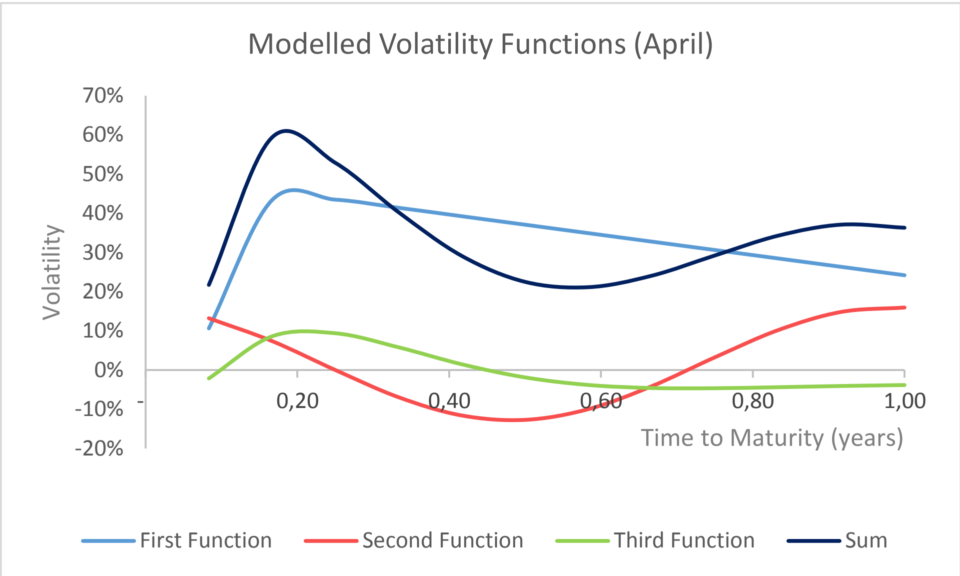


Figure 6.16 Modelled Functions for April

6.5.5 May

Time	Eigenvectors	
0,083	0,079	0,038
0,167	0,373	0,005
0,250	0,356	0,054
0,333	0,355	-0,013
0,417	0,406	-0,394
0,500	0,357	-0,417
0,583	0,270	-0,096
0,667	0,225	0,178
0,750	0,212	0,291
0,833	0,224	0,709
0,917	0,195	0,161
1,000	0,193	0,074
2,000	0,123	0,078
	Eigenvalues	
	2,089	0,205
	Relative Weight	
	86,64%	8,51%
	86,64%	95,16%

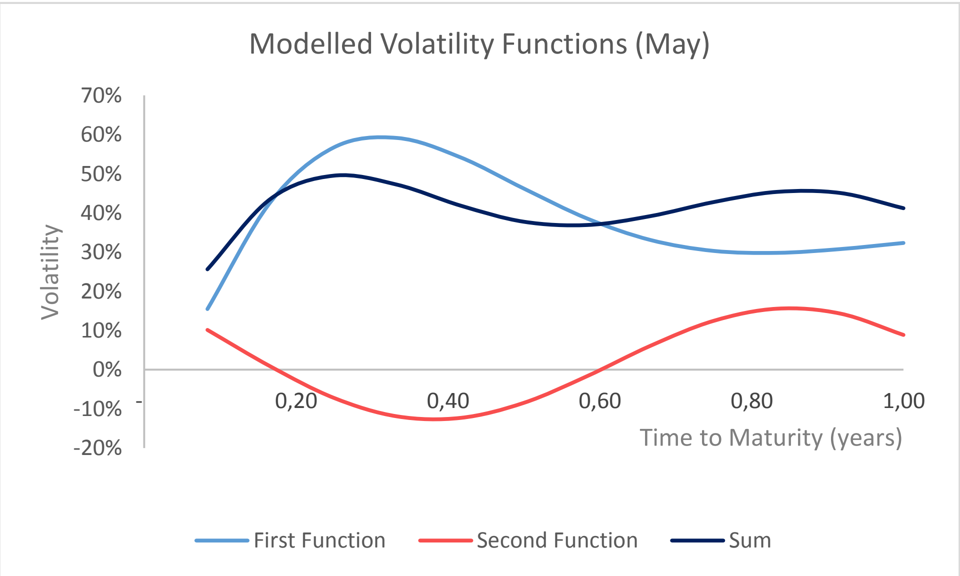


Figure 6.17 Modelled Functions for May

6.5.6 June

Time	Eigenvectors		
0,083	0,074	-0,014	0,472
0,167	0,417	-0,002	-0,432
0,250	0,401	-0,071	-0,345
0,333	0,380	-0,356	-0,145
0,417	0,317	-0,381	0,122
0,500	0,267	-0,134	0,186
0,583	0,254	0,130	0,113
0,667	0,250	0,235	0,087
0,750	0,256	0,787	-0,082
0,833	0,218	0,116	0,234
0,917	0,209	0,021	0,274
1,000	0,203	0,004	0,290
2,000	0,135	-0,008	0,400
	Eigenvalues		
	1,414	0,142	0,052
	Relative Weight		
	84,83%	8,53%	3,15%
	84,83%	93,37%	96,51%

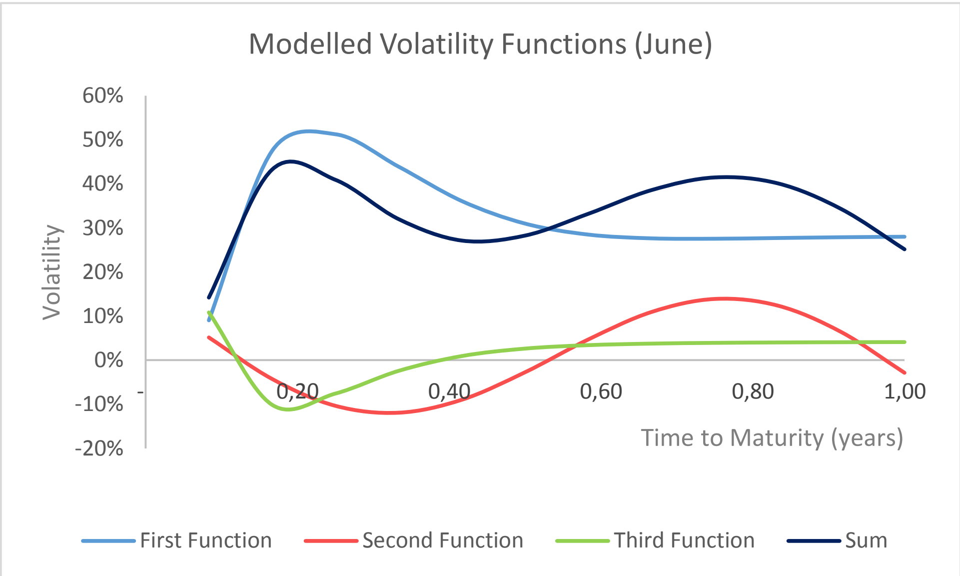


Figure 6.18 Modelled Functions for June

6.5.7 July

Time	Eigenvectors	
0,083	0,099	0,109
0,167	0,413	-0,001
0,250	0,466	-0,478
0,333	0,403	-0,417
0,417	0,302	-0,032
0,500	0,248	0,248
0,583	0,228	0,370
0,667	0,210	0,539
0,750	0,209	0,190
0,833	0,211	0,126
0,917	0,209	0,102
1,000	0,200	0,132
2,000	0,137	0,125
	Eigenvalues	
	1,685	0,101
	Relative Weight	
	90,40%	5,41%
	90,40%	95,81%

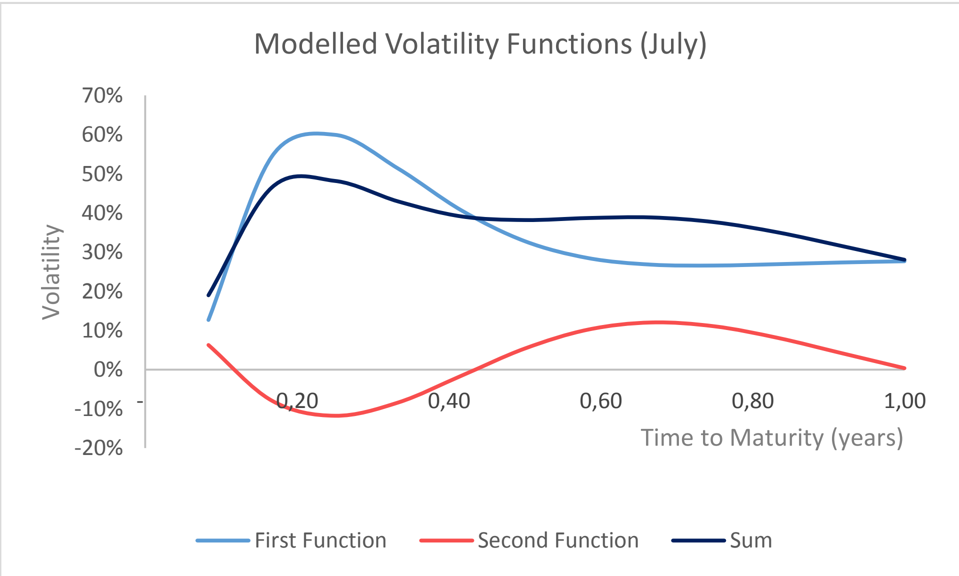


Figure 6.19 Modelled Functions for July

6.5.8 August

Time	Eigenvectors	
0,083	0,086	0,110
0,167	0,521	-0,594
0,250	0,451	-0,371
0,333	0,314	0,081
0,417	0,250	0,257
0,500	0,234	0,323
0,583	0,232	0,346
0,667	0,224	0,220
0,750	0,223	0,175
0,833	0,218	0,160
0,917	0,207	0,179
1,000	0,198	0,200
2,000	0,138	0,157
	Eigenvalues	
	1,077	0,206
	Relative Weight	
	81,03%	15,49%
	81,03%	96,52%

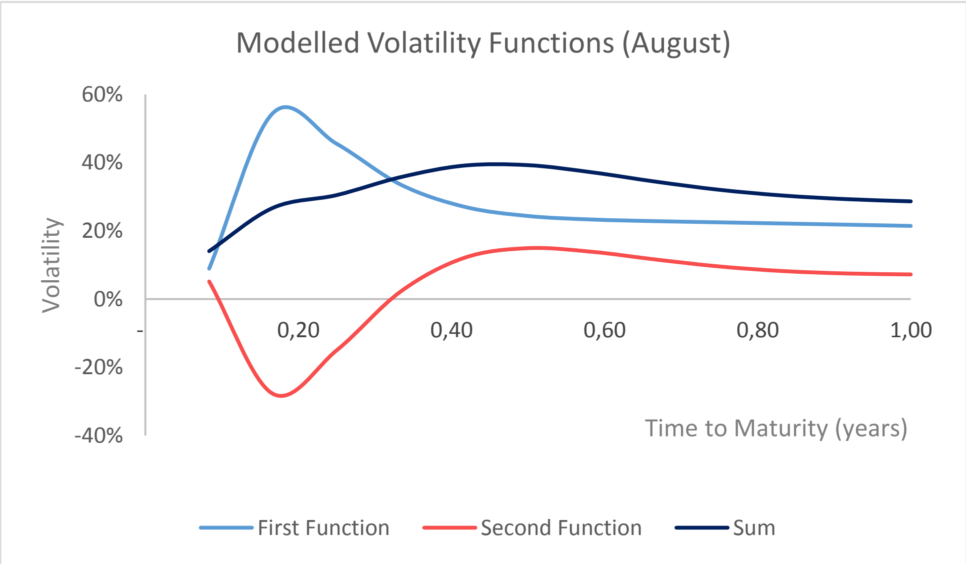


Figure 6.20 Modelled Functions for August

6.5.9 September

Time	Eigenvectors	
0,083	0,116	0,093
0,167	0,468	-0,783
0,250	0,355	-0,211
0,333	0,300	0,170
0,417	0,283	0,315
0,500	0,279	0,295
0,583	0,265	0,164
0,667	0,258	0,123
0,750	0,250	0,104
0,833	0,240	0,139
0,917	0,232	0,179
1,000	0,237	0,069
2,000	0,163	0,096
	Eigenvalues	
	1,388	0,090
	Relative Weight	
	91,52%	5,96%
	91,52%	97,48%

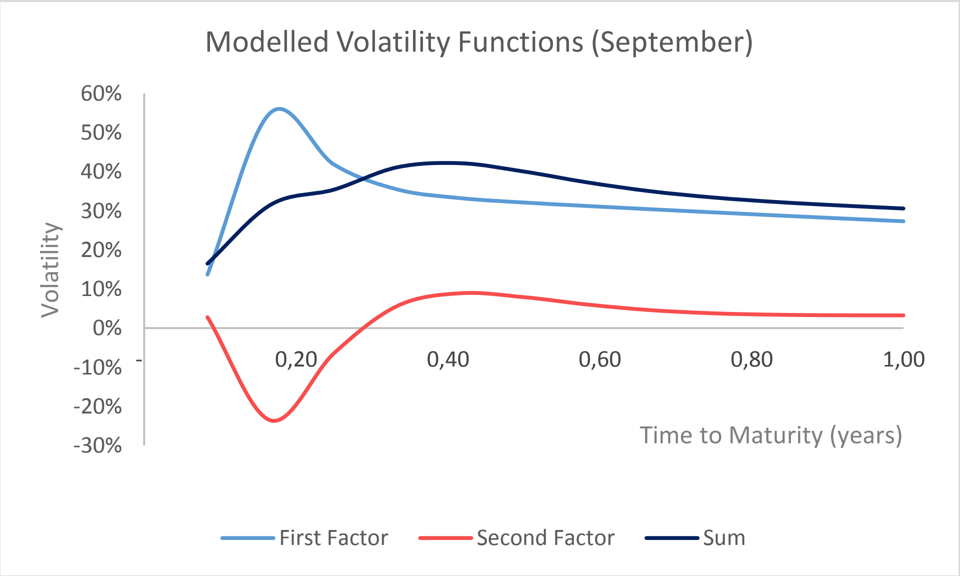


Figure 6.21 Modelled Functions for September

6.5.10 October

Time	Eigenvectors	
0,083	0,130	0,396
0,167	0,394	0,303
0,250	0,324	-0,242
0,333	0,303	-0,417
0,417	0,297	-0,346
0,500	0,288	-0,113
0,583	0,279	-0,029
0,667	0,271	0,015
0,750	0,259	-0,040
0,833	0,251	-0,111
0,917	0,259	0,034
1,000	0,277	0,456
2,000	0,189	0,404
	Eigenvalues	
	1,193	0,067
	Relative Weight	
	92,27%	5,21%
	92,27%	97,48%

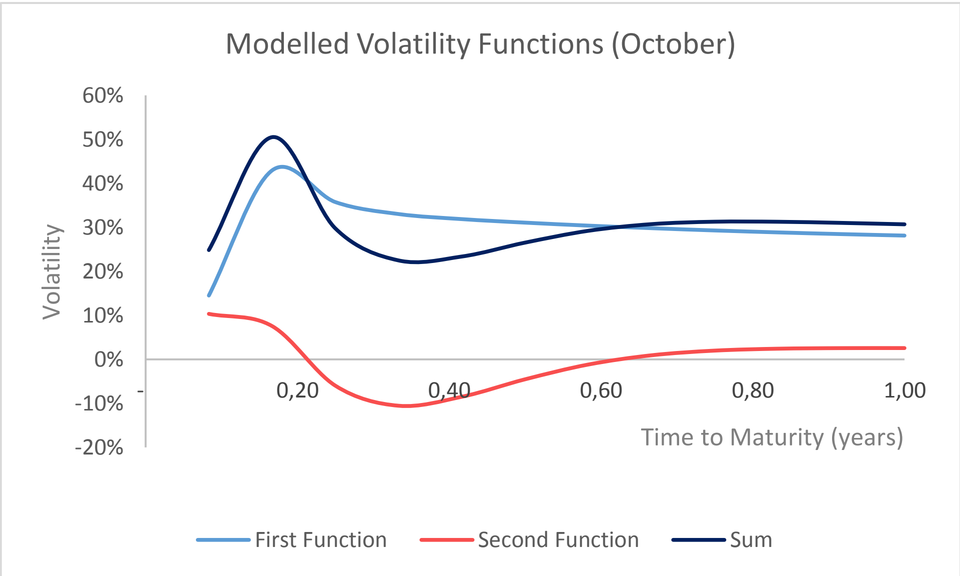


Figure 6.22 Modelled Functions for October

6.5.11 November

Time	Eigenvectors	
0,083	0,142	0,483
0,167	0,346	-0,202
0,250	0,327	-0,287
0,333	0,313	-0,252
0,417	0,300	-0,121
0,500	0,290	-0,065
0,583	0,281	-0,036
0,667	0,269	-0,076
0,750	0,263	-0,142
0,833	0,269	-0,029
0,917	0,285	0,322
1,000	0,265	0,498
2,000	0,190	0,426
	Eigenvalues	
	1,193	0,067
	Relative Weight	
	93,48%	5,11%
	93,48%	98,59%

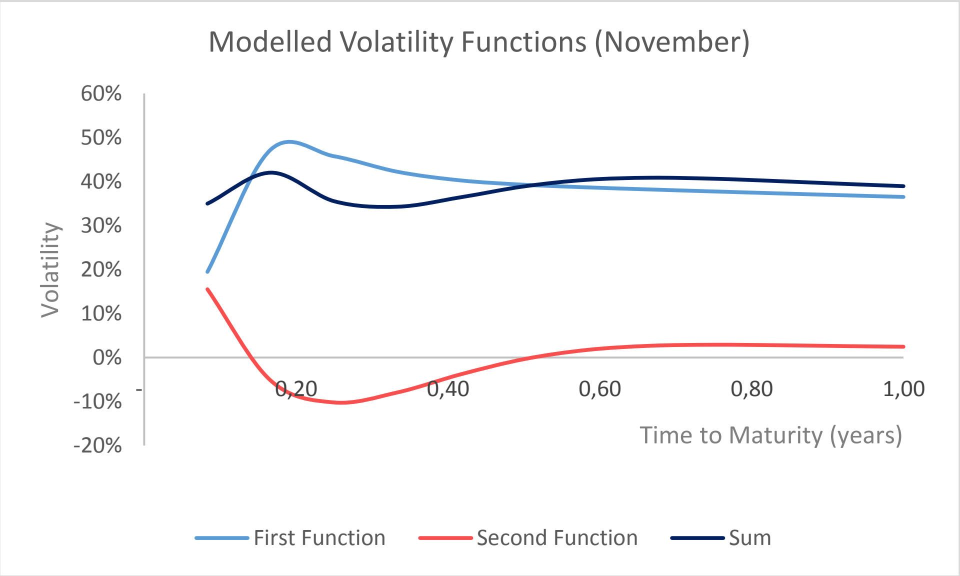


Figure 6.23 Modelled Functions for November

6.5.12 December

Time	Eigenvectors	
0,083	0,084	0,398
0,167	0,367	-0,343
0,250	0,353	-0,292
0,333	0,325	-0,146
0,417	0,311	-0,075
0,500	0,300	-0,025
0,583	0,289	-0,066
0,667	0,280	-0,128
0,750	0,279	-0,009
0,833	0,276	0,328
0,917	0,247	0,521
1,000	0,213	0,292
2,000	0,138	0,357
	Eigenvalues	
	1,527	0,093
	Relative Weight	
	92,06%	5,65%
	92,06%	97,72%

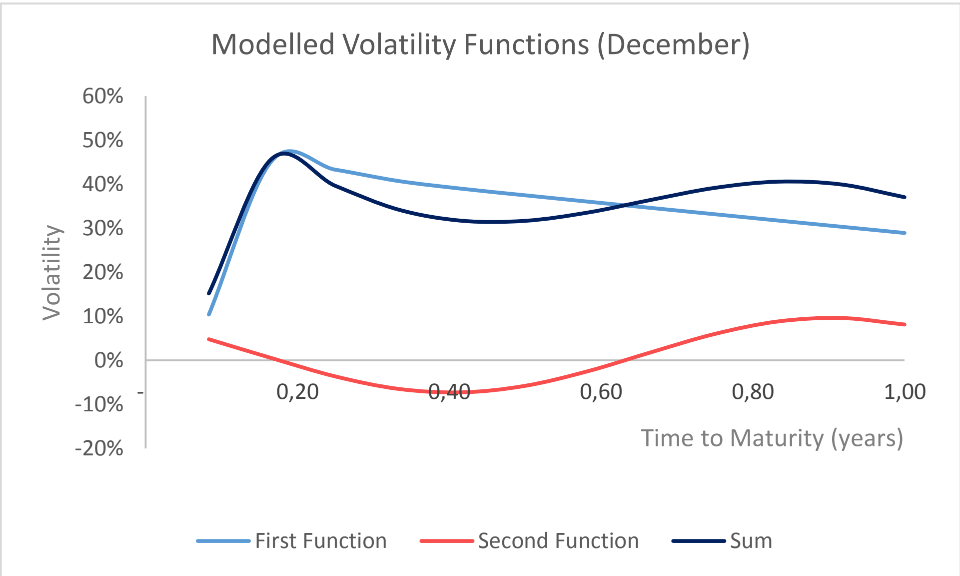


Figure 6.24 Modelled Functions for December

6.5.13 Monthly PCA Conclusions

Analysing the seasonal PCA's results draws the following conclusions:

- The monthly PCA confirms that any price shock will have a major impact on those months previous to winter or immediately after it, regardless of these contracts being close or not to maturity. On the other hand, volatility spikes will be buffered more easily during the summer months and its effect will be mitigated.
- It is important to notice the term structure for the first volatility function for all seasonal to monthly analysis, which tends to peak on the second month to maturity instead of the first one. This structure is attributable to the buffering effect of storage, which serves as a cushion for the nearest month forward but raises concern on security of supply in the second month forward.
- Particularly remarkable is the analysis of the term structure for the second volatility function from month to month. This function defines the evolution in time of calendar spreads, which represents a great source of information to understand the general behaviour of the forward curve.
- It is important to notice that the seasonal PCA on the summer-winter benchmarks produces smoother term structures than the monthly PCA. This is due to the fact that monthly analysis are more sensible to price evolution than the seasonal benchmarks.
- Segregating input data into monthly buckets enables the analysis to capture more information from fewer components, therefore reducing the number of sources of uncertainty which describe the evolution of volatility in time.

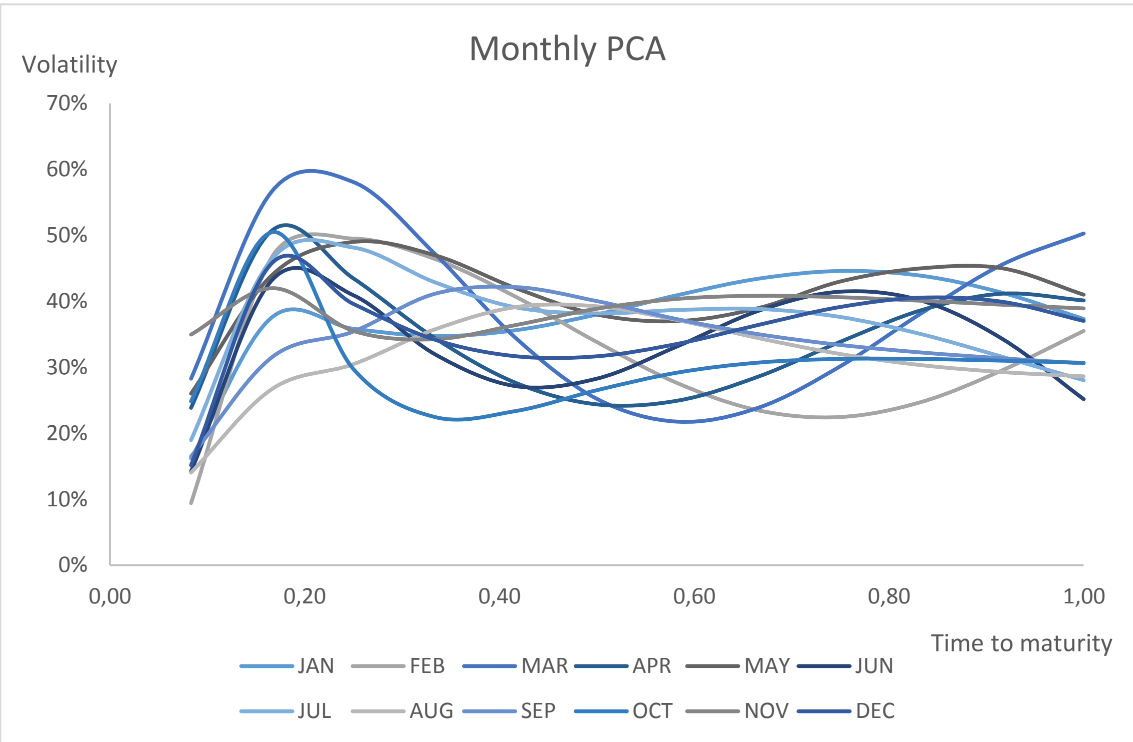


Figure 6.25 Monthly PCA

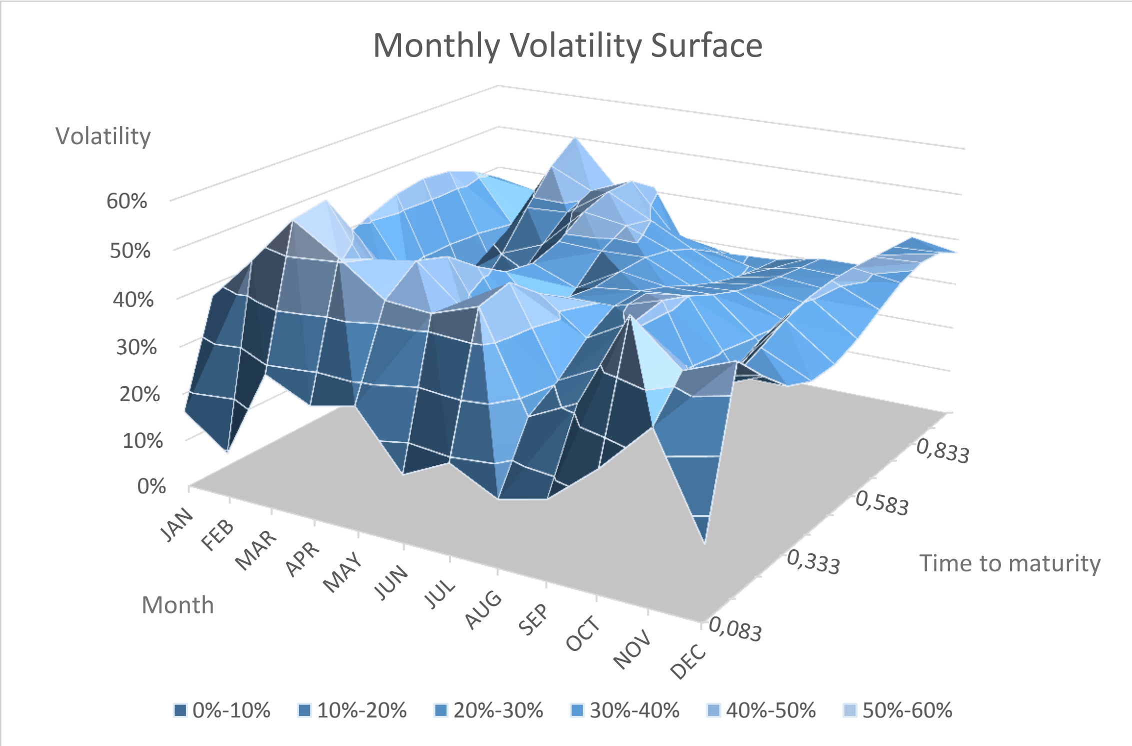


Figure 6.26 Monthly Volatility Surface I

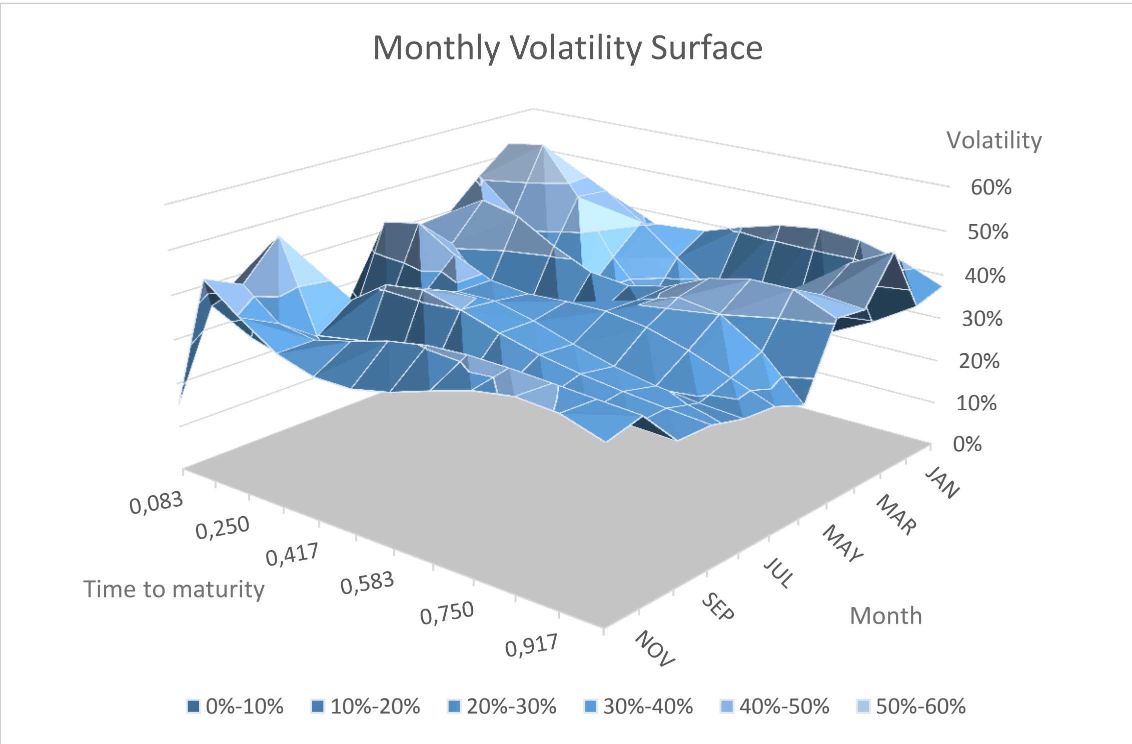


Figure 6.27 Monthly Volatility Surface II

7

References

7 References

Alterman, S. (2012). *Natural Gas Price Volatility in the UK and North America.* Oxford: Oxford Institute for Energy Studies.

Baíllo, Á., & Adroher, M. (2007). *A Model for the Evolution of the Forward Price of Crude Oil.* Madrid: Santander Central Hispano.

Black, F., & Scholes, M. (1973). *The Pricing of Options and Corporate Liabilities.* Chicago: Chicago Journals.

Blanco. (2002). *Multi-Factor Models for Forward Curve Analysis.* Commodities Now.

Blanco. (2002). *Multi-Factor Models of the Forward Price Curve.* Commodities Now.

Boyle, P. (1977). *Options: a Monte Carlo approach.* Vancouver: Journal of Financial Economics.

Clewlow, & Strickland. (1999a). *Valuing Energy Options in a One Factor Model Fitted to Forward Prices.* Lacima Group.

Clewlow, & Strickland. (1999b). *A Multi-Factor Model for Energy Derivatives.* Lacima Publications.

Clewlow, L., & Strickland, C. (2000). *Energy Derivatives: Pricing and Risk Management.* Lacima Group.

Cortazar, Gibson, & Schwartz. (1994). *The valuation of commodity contingent claims.* Journal of Finance.

Downey, M. (2009). *Oil 101.* Wooden Table Press.

Eydeland, & Geman. (1998). *Pricing Power Derivatives.* Risk.

Fevre, C. L. (2013). *Gas storage in Great Britain.* Oxford: Oxford Institute for Energy Studies.

Gibson, & Schwartz. (1990). *Stochastic convenience yield and the pricing of oil contingent claims.* Journal of Finance.

Henning, S. d. (2003). *Natural Gas and Energy Price Volatility.* Washington, DC: American Gas Foundation.

Hull, J. C. (2011). *Options, Futures and Other Derivatives (8th Edition).* Prentice Hall.

Joshi, M. S. (2008). *The Concepts and Practice of Mathematical Finance.* Cambridge: Cambridge University Press.

Keynes, J. M. (1930). *A Treatise on Money.* London: Macmillan.

Morris, M. (2010). *A report to Department of Energy and Climate Change.* Oxford: Pöyry Energy.