

A Generation Simulation Tool for Concentrated Solar Power with Thermal Energy Storage



Source: Gemasolar CSP plant in Sevilla, Spain

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Table of contents

I

Resumen	1
Abstract	2
Introduction	3
Part 1. DNI Modeling	12
Part 2. CSP Modeling	17
Simulation Study Results	22
Conclusion	26
References	27
Appendix A. Nomenclature	29
Appendix B. Matlab code	32
Appendix C. Explanatory Drawings	49

Herramienta de simulación para la generación de Energía Solar de Concentración con almacenamiento térmico

Resumen

A pesar de que la mayor parte de la producción energética actual es a través de combustibles fósiles, el sector de las energías renovables ha experimentado un importante auge en los últimos años. La conciencia de un cambio climático global debido al CO_2 y otros contaminantes, así como el reconocimiento del problema ligado al agotamiento de los recursos fósiles podrían dar lugar a un cambio en la distribución de la red eléctrica tal como la conocemos. La energía solar se postula como una solución viable.

Este documento centra su atención en la Energía Solar de Concentración (CSP), concretamente en la producida por centrales solares de torre central. La principal limitación para cualquier tipo de generación solar es el desfase que existe entre los patrones de demanda eléctrica y los de energía solar. Sin embargo, el crecimiento de los sistemas de almacenamiento térmico de energía (TES), plantea una solución práctica y hace que este recurso sea parcialmente despachable. La limitada flexibilidad de la energía solar se resuelve usando TES, en tanto que permite producir energía más allá del periodo del amanecer al atardecer.

Nuestro objetivo es desarrollar una herramienta que permita simular una programación eficiente de la generación eléctrica de un sistema CSP con TES. No solamente debemos representar la variabilidad y la naturaleza cambiante de la energía solar, sino además la interacción entre el CSP y el resto de la red.

Primero debemos recoger los datos de radiación directa normal (DNI) para una zona específica, y usar técnicas de clasificación (*Clustering*) que nos permitan establecer patrones. A partir de estos grupos construiremos nuestro modelo. Después, haciendo uso de conocimientos eléctricos y termodinámicos básicos continuaremos diseñando el modelo de CSP. A partir de ahí podremos resolver el problema de optimización, que en nuestro caso tratará de maximizar la producción eléctrica de la planta. Las conclusiones del estudio aseguran el correcto funcionamiento de nuestra herramienta, valoran las ventajas de contar con un sistema de almacenamiento térmico (TES) y establecen las ideas clave para continuar con futuras investigaciones sobre el tema.

A Generation Simulation Tool for Concentrated Solar Power with Thermal Energy Storage

Abstract

Although most of power production is nowadays still produced using fossil fuels, the world has recently experienced an enthusiastic increase in renewable sources. Awareness of climate change due to CO_2 and other pollutants, as well as wisely recognition that fossil fuels will eventually run out, might lead to a change in power grid distributions as we know it. Solar energy is one of the rapidest growing enforcements in the market.

On this paper, we will focus our attention on Concentrated Solar Power (CSP), although there are considerably large guidelines for solar technology development. The limited coincidence between the solar resource and the load demand pattern could be a limiting factor for most solar. However, the growth of Thermal Energy Storage (TES) systems for CSP turns it into a partially dispatchable asset. Limited flexibility of solar is solved using TES, so as it enables CSP to produce power output after the sunrise-sunset interval.

Our goal is to develop a simulation tool that permits an efficient schedule of the CSP with TES power output. Not only has it to represent the uncertainty and somehow capricious nature of the solar energy source, but also the interaction between the CSP and the rest of the grid.

First of all, we have to collect the direct normal irradiation (DNI) data for a specific location of the CSP, and use clustering techniques to establish various patterns, so we can build the CSP model based on those groups. Then, aided by basic thermodynamic and electrical concepts, we will design a CSP model, so we can solve the optimization problem to maximize the power output produced. The research conclusions assure the correct operation of our simulation tool and provide key ideas on the relation between CSP along with their integrated systems.

Introduction

Solar is the biggest energy resource on earth. All other sources such as wind, hydro, fossil or biomass depend intrinsically on sunlight. To measure the enormity of the sun we might consider that the whole world's energy demand for 20 years can be met collecting the sunlight that impact the earth in one day [1]. If we consider that worldwide demand for energy is supposed to grow at a rate of 5% each year, solar energy is the only choice that can satisfy this demand.



Figure 1: World Energy Demand and Forecast

Source: Lynn Orr. Changing the World's Energy Systems. Stanford University Global Climate & Energy Project (after John Edwards. American Association of Petroleum Geologist); SPI Consulting



Figure 2: Expectation growth of Solar Energy Capacity in the future

Source: German Advisory Council on Global Change

Although photovoltaic solar panels (PV) and CSP are the two most mature technologies, we should not forget the other kinds currently available and summarized in *Figure 3*.





Concentrated Solar Power is unique among other renewable sources because it can be coupled with Thermal Energy Storage, making it higher dispatchable. However, it is not the aim of this paper to despise other solar technologies such as PV. It must be said that each has its own advantages and drawbacks. PV major advantage over CSP is the ability to collect both direct and diffuse irradiation or its installation versatility.

CSP, on the other hand, is often preferred by power grids. More importantly, hen connecting the sectors to the grid, CSP is more predictable energy and able to provide network ancillary services while PVs need more control features or additional equipment. The concentration process is affected by the weather conditions. Unlike Concentrated photovoltaic (CPV) technologies; CSP has an inherent capacity to store heat energy for short periods of time for later conversion to electricity which enhances energy security. The battery used to store electricity energy generated by PV is very expensive. When combined with thermal storage capacity, CSP plants can continue to produce electricity even when clouds block the sun or after sundown.

Recent estimate of the cost of adding TES to a CSP plant varies between 72 and 240\$/KWh of electricity storage capacity [4], which can be compared to the cost of electromechanical batteries, that usually cost over 300\$/KWh [5]. High efficiencies that typically exceed 98% are also an advantage. This is reasonable, because the energy do not have to go through a conversion process to be stored. The heat transfer between the CSP and the TES is performed in the heat exchanger by the heat transfer fluid (HTF). This enables the TES to store thermal energy from the CSP, but not electric energy from the rest of the power grid.

CSP concentrate a large area of sunlight, or solar thermal energy, onto a small are using mirrors or lenses. Then, electrical power is produced when the concentrated light is converted to heat, which drives a heat engine, usually a steam turbine referred also as Power Block and connected to an electrical power generator. There are typically four CSP designs: parabolic trough collectors (PTC), linear Fresnel collectors (LFC), Solar Towers (heliostat field collectors) and parabolic dish reflectors (PDR).

In this paper we will focus our attention on Solar Towers CSP, so let's give a brief description of this technology. Large amount of thermal energy is directed into the cavity of a steam generator to produce steam at high temperature and pressure. The concentrated heat energy absorbed by the receiver is transferred to a circulating fluid, which can be stored and later used to produce power. Some benefits of this systems are: the fact that there is only one receiver minimizes transportation issues; they are quite large (10MW) and thus benefit from economies of scale; they conveniently store thermal energy and they have high efficiency rates both collecting energy and converting it into electricity; they allow working at high temperatures of over 1500°C which make thermal cycles more efficient; they can easily integrate fossil fueled plants

for hybrid operation; using TES they can operate half the hours of each year at rated capacity.



Figure 4: Solar Tower CSP

In order to satisfy demand, CSP needs a storage if we are to provide electricity during night time.





Source: Geyer, 2007, Solar PACESS Annual Report.

When coupling TES with CSP there are a few decisions to be made. As the system consist on three main interrelated blocks, the sizing decisions concern this parts.

The first one would be the size of the Solar Field, which can be measured by its area or using the concept of solar multiple (SM), which normalizes the solar field based on the Power Block size [7].

The second decision element is the size of the TES determined by its rated power and energy capacities. Power capacity of the TES is set to allow the Power Block to operate at its rated capacity using only energy from TES. Energy capacity of the TES is usually measured either in megawatts hour of thermal energy [MWh-t], or either by the number of hours of storage.

Finally the steam turbine or Power Block is determined by its rated power [MW-e]. The sizing of those three elements will determine the capacity factor and possible power output of the plant, and so it is not a trivial issue.



Figure 6: Four kinds of Storage System to enhance CSP capacity factor

Source: Julien Octobre and Frank Guihard, Systèmes Solaires, 2009

CSP is being widely commercialized and the CSP market has seen about 740 MW of generating capacity added between 2007 and 2011. More than half of this (about 478 MW) was installed during 2010, bringing the global total to 1095 MW [8]. Spain added 400 MW in 2010, taking the global lead with a total of 632 MW, while the US ended the year with 509 MW after adding 78 MW, including two fossil–CSP hybrid plants.



Figure 7: Production and Consumption of CSP by 2050

Source: International Energy Agency (IEZ), Technology Roadmap-Concentrated Solar Power

More recent news include the first international CSP bid won by the Spanish company *Abengoa* in Chile. The 110MW Cerro Dominator Project will definitely help interconnecting the central-electricity system (SIC) and the north -electricity system (SING) of the Chilean power grid [10]. The US is also making huge effort to expand its renewals energy sector with the *Ivanpah Solar Power Facility*. This enormous CSP plant (392MW) located in the Mohave Desert is planned to be fully operating by the end of 2014 and will reduce carbon dioxide emissions by more than 400,000 tons annually [11].



Source: http://www.abengoasolar.com/web/en/nuestras_plantas/plantas_en_construccion/estado s_unidos

This fast growth around the world makes us start thinking about developing a probabilistic simulation tool which reflects the behavior of those systems that integrate CSP. The mixture of CSP arises certain issues due to the variability of DNI, which has a huge impact in the quantity and schedule of the CSP. Coupling TES to CSP gives an undoubtedly beneficial feature, but still CSP will be able to contribute whenever either solar or TES thermal energy is available. Unlike DNI uncertainty, load patterns are much easier to predict. Statistical forecast based on historical loads measurements is being done to detect consumption patterns. On a typical weekday load demand will be low at night and dawning period, until 6 to 7 a.m., when activities begin. Later we might observe a steadily growth until 12 a.m., when we will find a peak. Then it usually declines until 6 p.m., time at which consumption increases until 8 p.m. during a second peak. From 9 p.m. demand decreases until the end of the day. This behavior is due to work schedules established in our society and might vary very little within countries. In Spain, for instance the evening peak will take place at 9:30 p.m. rather than 8 p.m.



Figure 9: Spanish electricity demand on March 30 2014

Source: https://demanda.ree.es/movil/peninsula/demanda/total/2014-03-30

Load consumption during the week is higher than in weekends, and days like Mondays are expected to have higher demands than Fridays. In *Figure 10* we compare the load predictable patterns [13] with the uncertain DNI data [14] for a random winter week at Midland (TX).

Figure 10: Load and DNI data for the 5-11 January 2009 week at Midland, Texas





The relation between load and DNI patterns is a key issue to bear in mind when developing our simulation tool. Other important aspects are DNI uncertainty and operational schedule of CSP-TES. One way to quantify the variable effects of the systems with integrated CSP is by analyzing representative results such as CO2 emissions, production cost, and reliability indicators like Loss of Load Probability (LOLP) or Expected Unserved Energy (EUE).

The regime-based CSP power output model is built in our simulation tool to represent the uncertainty of CSP-TES outputs. After we will make use of statistical clustering theory to classify DNI data in a seasonal basis. Only introducing a common time scale can we compare DNI data in a meaningful way to find those clusters. Future researchers on this field can continue our work by incorporating the model into a probabilistic framework, using conditional probability concepts. By that time, it would be clear that our model will efficiently simulate the uncertainty CSP output. The potential of our model can be measured when applying the extended simulation tool to capture some illustrative results. This application will help us to measure the effects of the systems with integrated CSP resources over long-time periods.

On the next part of this paper we explain how DNI data is modeled. Later, we focus on CSP power output design. After, we introduce the optimization problem and expose the simulation study results. Finally we conclude the research by giving an outline of our work and encouraging future researcher to extend the simulation framework into the probabilistic field.

Part 1. DNI Modeling

The aim is to construct a generation model quantifying the impact of CSP in integrated power systems. As discussed previously, the CSP power output model will have to reflect the level of uncertainty due to variability DNI. This intermittency on sunlight isolation, which changes significantly in a seasonal basis, can be perfectly observed in *Figure 11*. Here we have plotted the DNI *spaghetti-data* for the hole 2008 at Midland, Texas [15], just to give the reader an idea of the need to classify the DNI in different patterns.





The reason for choosing this location and not another is purely arbitrary. When exploring the US Solar Radiation Data Base (NSRDB) website we realize that they have classified the data according to classes. The 1991–2010 NSRDB contains data for 1,454 sites, which are subdivided into three classes of stations.

Class I Stations have a complete period of record (all hours 1991–2010) for solar and key meteorological fields and have the highest-quality solar modeled data (242 sites).

Class II Stations have a complete period of record but significant periods of interpolated, or otherwise lower-quality input data for the solar models (618 sites).

Class III Stations have some gaps in the period of record but have at least 3 years o data that might be useful for some applications (594 sites).

We choose a random site that at least is disposed as Class I.

Figure 12: Distribution of NSRDB sites by class and measured solar data



Source: National Solar Radiation Database

We analyze DNI data for D days in a season at our location. Let's denote by U(d) as the random vector of DNI[Wh/m²] data for a certain day. Each day's sunrise-sunset period has M equal duration sub-periods. Let's denote as u(d,m) as the per unit DNI[pu] data for a certain day-d at a certain sub-period-m of the *M* duration sunrise-sunset period; then the vector u(d) would collect the DNI[pu] for a specific day.

As an example let's analyze the DNI and ETRN data for a random day-*d*:

 $U(d) = [0,0,0,0,0,0,0,4,,79,818,907,933,981,976,923,816,759,340,0,0,0,0,0,0]^T [W/m^2]$

and so U(d, 10) = 818 [Wh/m²] is the DNI of the day-d at 10am.

To obtain u(d) be divide U(d) by the Base or Benchmark-B, which we set to be the maximum of the ETRN data for each specific day-*d*. ETRN or Extraterrestrial Direct Normal Radiation is defined as the amount of solar radiation in [Wh/m²]

received on a surface normal to the sun at the top of the atmosphere. For our one-hour period analysis the units of ETRN would be $[Wh/m^2]$.

 $max{ETRN}=1415 [W/m^{2}]$

u(d) = [0,4,79,818,907,933,981,967,923,896,749,349,0]/1415 [pu]

The fact that the duration of sunrise-sunset period varies throughout the seasons and years requires the introduction of scaling scheme, to allow the comparison of the DNI data in a meaningful way. The scaling scheme maps each u(d) into a common time-scaled, with *J* equal sub-periods for each day in a season. Then we will construct the vector $a(d) = [a(d,1),a(d,2),...a(d,j),...,a(d,J)]^T$, collecting the DNI[pu] for the *J* scaled sub-periods of a day-*d* in the scaled time frame. Each a(d,j) corresponds to the DNI[pu] of a certain day-*d* at a certain equally scaled sub-period-*j*.

We can represent the scaling as follows:



Figure 13: Time scaling process scheme

Now we are ready to identify different patterns comparing the scaled data in a meaningful way [16]. In order to do so we have used the K-means algorithm [17], although some other clustering methods can be deployed. The number of groups k is an input parameter, an inappropriate choice can lead to bad results. It is therefore crucial when we run the k-means algorithm to take into account the importance of determining the numbers of groups to a dataset. A reasonable number for the researcher is 4.

Figure 14 shows the 4 different cluster DNI patterns for the spring season.



Conceptually, we view the cluster R(k) to consist of realizations of the r.v.s of the DNI for the scaled periods j. In addition the probability of each subset R(k) is estimated by:

$$prob\{R(k)\} = \frac{|R(k)|}{D}, \qquad k = 1, 2, \dots, K.$$

Both subset R(k) and its probability can be referred as the regime denoted by R(k).

Figure15: Clustering explanation scheme



Once we deploy the regime time-scaled a=DNI[pu] we have to take into account that such samples must be inverse-scaled to get the actual $Y=DNI[Wh/m^2]$ and expressed in the time frame of the production simulation [hours]. The regime based DNI model allows us to use the samples in the subset R(k) to compute the realization of the random variable vector:

$$Y(d) = [Y(d, 1|k), Y(d, 2|k), \dots, Y(d, j|k), \dots, Y(d, J|k)]^T$$

with Y(d, h|k) representing the DNI [Wh/m²] for the inverse-scaled sub-period-*h* of day-*d* conditioned on regime R(k).



Figure 16: De-scaling process explanation scheme

We have developed a regime-model for the DNI, scaling, clustering and de-scaling the data. The idea can be easily understood studying the first part of Appendix C.

From Matlab, we get the next plots reflected in *Figure 17*, representing the DNI input, then the scaled data and finally the de-scaled info.

Figure 17: DNI data collecting, scaling and de-scaling process



Part 2. CSP Modeling

Now, our next step would we to build the generation model of the CSP power output, intrinsically related with the DNI previous model. The nature of the solar input will determine the output power of the CSP, which might be also affected by the specific details of the CSP configuration and the operational schedule. Assuming that our plant has TES, it is clear that the solar input is zero outside the sunrise-sunset period. The power output is also zero before sunrise, but might be non-zero after sunset.

Given the solar input previously studied through the DNI regime-model, we define the power output of the CSP as the vector:

$$p'(d) = [p'(d, 1'), p'(d, 2'), \dots p'(d, h'), \dots, p'(d, H')]^T$$

with p'(d, h') representing the power output [MW] for the sub-period-h' of day-d. Note that the number of sub-periods comprehend the sunrise-midnight period of a day-d. Also important is to take into consideration that the CSP power output-p'(d) is a random variable, as it is the DNI solar input-Y(d).

The power output of a system with integrated depends on the DNI at its site, the CSP configuration, the utilization of the TES and its associated operational schedule [18]. Solar energy is converted by the CSP into thermal energy, used instantaneously to either generate electricity in the turbines or be stored into the TES for later conversion. A smart use of the TES enables the CSP to produce electricity when the solar energy is not available (either outside the sunrise-to-sunset periods or in a cloudy day). The TES deployment also allows the CSP operator to set up the schedule to meet the specific operational objectives. This can be for example maximizing profits, selling more electricity during load peaks; or maximizing the total energy produced by the CSP. In this paper we have decided to maximize the power output generated by the CSP. This optimization problem is subject to certain constraints though. We cannot violate the thermal energy storage limits. Both charging and discharging capacities must be respected. And we should not forget that there might be some losses too.

We state an optimization problem, in which our objective function will provide the value of the CSP generated energy. Our goal is to maximize this energy (1) subject to all constraints (2 up to 7i). Notation is summarized in Appendix A and reasonable values for the input parameter have been assume according to [19].

The conversion of solar to thermal energy is represented by the non-linear mapping (β -beta). This function's input is the DNI represented by the variable Y [kW/m²], and its output is the thermal energy T [MWh/h]. Knowing that from 1kW/m² we get 140MWh we build this function. The thermal-to-electrical conversion is represented by the function α (alpha). Its input is the thermal energy Z [MWh/h] used to produce electricity, and its output p' [MW] is determined knowing that from 70MWh/h we get approximately 30MW.

We assume that the change of the stored thermal energy only occurs at the end of each sub-period and so E(d,h) represents the stored thermal energy at the end of the sub-period h. We refer to the charging (discharging) efficiency as η (μ) and the sub-period thermal energy loss factor as ψ . The TES operations cannot violate the physical and operational capability limits E_{min} . The sub-period charging (discharging) rate K(d,h) (Q(d,h)) can take values only within its range K_{min} - K_{max} (Q_{min} - Q_{max}). With the input DNI data from before we are ready to look for the optimal solution. In our case, as we are not pursuing maximal profits but power, we can forget about the function $\gamma(d,h)$ (gamma) which represent the selling prices of electricity.

Now we are ready to state our optimization problem and constraints:

$$\max \sum_{h=1}^{H} \gamma(d,h) p'(d,h) \Delta$$

{K(h,d), Q(h,d), E(h,d), Z(h,d), p'(h,d), h=1,2, ..., H}

$$st: \begin{cases} h = 1, 2, ..., H\\ E(h, d) = (1 - \psi)E(h - 1, d) + \eta K(h, d)\Delta - Q(h, d)\Delta\\ Z(h, d) = \beta (U(h, d)) - K(h, d)\Delta + \mu Q(h, d)\Delta\\ p'(h, d) = \alpha (Z(h, d))\\ Emin \le E(h, d) \le Emax\\ Kmin \le K(h, d) \le Kmax\\ Qmin \le Q(h, d) \le Qmax \end{cases}$$

To make the optimization problem and its constraint more clear it might be useful to consider *Figure18*, in which the CSP, TES and Power Block's subsystems play important roles.



We use the notations described above to formulate the scheduling problem (SP) to determine the optimal operational trajectory of each CSP with TES for the day *d*, using the DNI input Y(d, h|k). However it is important to note that, when running the simulation, we should perform the optimization algorithm for each day, and not for each hour indepently. The reason is that the thermal energy stored depends on the stored energy in the previous sub-period, as stated in the first constraint (2). The consequence of this is that instead of having just five variables to optimize K(h,d), Q(h,d), E(h,d), Z(h,d), p'(h,d), we will have five super-variables K(d), Q(d), E(d), Z(d), p'(d), which can be also seen as 5 times 24 variables (120 variables).

To solve the problem we use the matlab command *fmincon*. A detailed explanation is provided in Appendix C.

To clarify how we have defined the matrixes due to the constraints it can be useful to analyze Appendix C.

Once we have found the optimal solution we can perform different simulations and see how the generation might vary if we change the thermal energy storage hours of our system. We will analyze two cases. The first one without storage, and the second one with 4 hours of storage. All results are presented in next section.

Before disclosing the results let us give some insights for developing a future Probabilistic Simulation Approach.

In order to solve the problem we are considering not only the DNI model, which provided the Y(d), but also the inverse scaled patters clusters R(k). We now compute the realizations of the CSP power output random variable vector conditioned to the regime R(k).

$$P(d|k) = [P(d, 1|k), P(d, 2|k), \dots, P(d, h|k), \dots, P(d, H|k)]^{T}$$

With P(d, h|k) representing the power output [KW] for the sub-period-h of day-*d* conditioned on regime R(k). Note that now, the number of sub-periods do not comprehend the sunrise-midnight period of a day-*d* as it was with the vector p'(d,h). Instead, and to keep consistency with the representation of the loads, the sub-periods will be midnight-midnight.

We can bring together the P(d,h/k) realization and build the CSP power output sample space, using the values for each sub-period-h, p(d,h), to estimate the cumulative distribution function (c.d.f.). Here it is important to note that the CSP is not going to supply energy before sunrise. Therefore the probability that the values of the first H-H^{\prime} elements in P(d,h|k) is 1.

$$P(d) = [0,0,0,...,0,P'(d,1'),P'(d,2'),...,P'(d,h'),...,P'(d,H')]^{T}$$

At this point, the future researcher on CSP will be ready to extend our work approaching the problem from a probabilistic point of view.

Simulation Study Results

After running our program we can state the optimal power output generation for a CSP with and without TES power plant at Midland, Texas. However if we plot the results for the all year we will obtain a *spaghetti* un-understandable figure similar to *Figure 11*. This is why we need to plot the power output of the DNI centroids and not the centroids of the power outputs.

To see the difference within days let us focus our attention in *Figure 19*. The intrinsic relation between the power output and the DNI can be seen comparing the following figures.



Figure 19: Power output comparison between three random days without TES

Figure 20: DNI comparison between three random days



From *Figure 21* it is possible to understand the importance of adding storage to the CSP. The aim of this plot is to Compare the DNI data and power output for a random day and prove that the production goes outside the sunrise-sunset period. Note that those variables have different units, the DNI of Wh/m^2 and the power output of MW.





We first evaluate the simulation tool with a four hours TES, and after without thermal energy storage, so as to contrast the results. We can see how clustering the power output with and without storage in *Figure 22* and *Figure 23*. Although there is no much difference at the first glance when comparing both images, a more accurate glaze will ensure that adding storage enlarges power output in all cases, giving the CSP better reliability overall. This can be seen in *Figure 25*, where the bold lines represent the clusters with 4 hours TES, and the weak ones stand for the clustering without storage.



Figure 22: Power Output Clustering with 4 hour storage



Figure 23: Power Output Clustering without storage







From this figure it is clear that the storage not only expands the generation beyond sunset, but in all cases provides more power output to the CSP system.

Conclusion

We have presented a Generation Simulation tool to evaluate the different effects of power systems with integrated CSP over long-time period. The results prove that adding thermal storage to the CSP system will increase the generation of the plant and enable the scheduler to produce electricity beyond the sun sets.

The outcome presented respond to the pursuit of comparing the impacts of adding thermal energy storage to a CSP system. This paper presents some understanding on how CSP can deploy a key role when trying to benefit from solar energy, and until what point an optimal use of TES enhances its dispatchability

Moreover, other studies can be done with our tool, choosing different locations or input parameters. Again, as explained at the end of Part 2, forthcoming studies might find this research as an appropriate starting point to deploy a simulation approach and reflect the variability of DNI data into a probabilistic framework. We hope our labor can enhance future achievements on the thrilling renewal's scope.

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Appendix A. Nomenclature

D	number of days for DNI data collection
М	number of non-overlapping sub-periods over the sunrise-sunset period of a day d
U(d,m)	DNI[W/m ²] observed for the sub-period m of a day d
U(d)	DNI vector $[U(d,1), U(d,2),, U(d,M)]$ [W/m ²] for <i>M</i> sub-periods over the sunrise-sunset period of day <i>d</i>
<i>u</i> (<i>d</i> , <i>m</i>)	DNI[pu] observed for the sub-period m of a day d
u(d)	DNI vector $[u(d,1),u(d,2),, u(d,M)]$ [pu] for <i>M</i> sub-periods over the sunrise-sunset period of day <i>d</i>
J	number of time-scaled sub-periods over the sunrise-sunset period
a(d,j)	DNI observed for the time-scaled sub-period j of a day d
<i>a</i> (<i>d</i>)	DNI vector $[a(d,1),a(d,2),, a(d,J)]$ for J time-scaled sub-periods over the sunrise-sunset period of day d
R(k)	DNI pattern cluster k, where $k=1,2,K$
R(k)	DNI pattern regime, , where $k=1,2,K$
Y(d,h)	DNI observed for the inverse-time-scaled sub-period h of a day d
<i>Y</i> (<i>d</i>)	DNI vector $[Y(d,1), Y(d,2),, Y(d,H)]$ for H inverse-time-scaled sub- periods over the sunrise-sunset period of day d
Y(d,h/k)	DNI random variable conditioned on $R(k)$ for the inverse-time-scaled sub-period h of a day d
Y(d k)	DNI random variable vector $[Y(d, 1/k), Y(d, 2/k),, Y(d, H/k)]$ for <i>H</i> inverse-time-scaled sub-periods over the sunrise-sunset period of day <i>d</i> conditioned on $R(k)$.
Η´	number of non-overlapping sub-periods over the sunrise-midnight period
p´(d,h´)	CSP power output [MW] in the sub-period-h of day-d
p´(d)	CSP power output vector $[p'(d, 1), p'(d, 2),, p'(d, H)]$ [MW] for the sub-periods over the sunrise-midnight period of day- <i>d</i>

Δ	duration for one sub-period in the TES schedule optimization problem
γ(d,h)	objective function coefficient from past average hourly system marginal price data for the sub-period h of day d
K(d,h)	charging rate in the sub-period-h of day-d
Q(d,h)	discharging rate in the sub-period-h of day-d
E(d,h)	stored thermal energy at the end of the sub-period h of day-d
Z(d,h)	thermal energy used to produce electricity in the sub-period-h of day- d
K _{min} -K _{max}	limit values for $K(d,h)$
Qmin-Qmax	limit values for $Q(d,h)$
E_{min} - E_{max}	limit values for $E(d,h)$
K(d)	charging rate vector $[K(d,1),K(d,2),,K(d,H)]$ for the sub-periods over the sunrise-midnight period of day- <i>d</i>
Q(d)	discharging rate vector $[Q(d,1),Q(d,2),,Q(d,H)]$ for the sub-periods over the sunrise-midnight period of day- <i>d</i>
<i>E</i> (<i>d</i>)	stored thermal energy vector $[E(d,1),E(d,2),,E(d,H)]$ for the sub- periods over the sunrise-midnight period of day- <i>d</i>
Z(d)	thermal energy used to produce electricity vector $[Z(d,1),Z(d,2),,Z(d,H)]$ for the sub-periods over the sunrise-midnight period of day- <i>d</i>
η	charging efficiency of TES
μ	discharging efficiency of TES
ψ	thermal energy loss factor of TES
β(·)	non-linear mapping of solar DNI into absorbed thermal energy flow rate
α(·)	non-linear mapping of thermal energy flow rate into net electrical power output
P´(d,h´)	CSP random variable power output in the sub-period-h ^{\prime} of day-d
<i>P</i> (<i>d</i>)	CSPpoweroutputrandomvariablevector $[P'(d, 1), P'(d, 2), \dots, P'(d, H)]$ for the sunrise-midnightsub-periodsH' of day-d over the sunrise-midnightperiod of day-d
P(d,h k)	CSP random variable power output in the sub-period-h of day-d conditioned on regime $R(k)$.

P(d|k) CSP power output random variable vector [P(d, 1), P(d, 2), ..., P(d, H)] for themidnight-midnight sub-periods H of day-d over the sunrisemidnight period of day-d conditioned on regime R(k).

Appendix B. Matlab code

Part 1 includes:

Program 1. Main program in which we call the functions to collect, scale and de-scale the DNI data. We store the key variables, perform the k-means clustering and plot some interesting results too.

Function Data. We collect the DNI and ETRN data from the Excel provided by NSR.

Function Scaling. We scale the DNI data.

Function De-scaling. We de-scale the DNI data

Part 2 includes:

Program 2. Taking the de-scaled data from Part 1, this program aims to obtain the power output. We perform some clustering, collect valuable data and make some plots too.

Function Optimization. Optimization problem definition and solving. Interesting to take a look to check some input parameters such as the lower and upper limits for the variables.

Function FUN. Objective function solving.

Function Alfa. Thermal to electrical energy non-linear mapping. Knowing that from 70MWh/h we get 30MW.

Function beta. Solar to thermal energy non-linear mapping. Knowing that from 1kW/m2 we get 140MWh/h.

```
Program 1
```

```
clear all
clc
close all
$$$$$$$$$$$$$$$$$$
global J H D
J=50;
H = 24;
D = 365;
୧୧୧୧୧
୧୫୫୫୫୫୫୫୫୫୫୫୫୫୫୫
%PROGRAMME 1
%DATA FUNCTION CALLING
*24-1 1-2 2-3 3-4 4-5 5-6 6-7 7-8 8-9 9-10 10-11 11-12 12-13 13-14 14-
15 15-16 16-17 17-18 18-19 19-20 20-21 21-22 22-23 23-24
[U,ETRN]=function_data; %U[Wh/m^2] ETRN[Wh/m^2]
%we assume h=1 DNI=U[W/m^2]
save('U.mat');
save('ETRN.mat');
%REPRESENTATION 1
figure(1)
colorstring =
'ymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrg
bwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymc
rgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwky
mcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwk
kymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrg
bwkymcrgbwkymcrg';
for(d=1:1:D)
plot([1:1:H],U(:,d), 'Color', colorstring(d))
title('DNI DATA')
xlabel('time [hours]')
ylabel('U=DNI [W/m^2]')
grid('on')
hold on
          stairs([1:1:H],U(:,d),'Color', colorstring(d))
2
hold on
end
%K-MEANS
k=4;
[IDX, CU] = kmeans(transpose(U), k);
save('CU.mat');
%REPRESENTATION 3
figure(2)
colorstring2 = 'ybcr';
for(i=1:1:k)
plot([1:1:H],CU(i,:), 'Color', colorstring2(i))
title('k-means Clustering')
xlabel('time [hours]')
ylabel('U=DNI [W/m^2]')
grid('on')
hold on
stairs([1:1:H],CU(i,:), 'Color', colorstring2(i))
hold on
end
```

```
%SCALING FUNCTION CALLING
a=zeros(J,D);
for d=1:1:D
    a(:,d)=function_scaling(U(:,d),ETRN(:,d)); %function [output]
=scaling(input)
end
save('a.mat');
%REPRESENTATION 3
figure(3)
for(d=1:1:D)
stairs([1:1:J],a(:,d),'Color', colorstring(d));
hold on;
title('SCALED DATA')
xlabel('scaled time') %units may vary from season to season, could be
1h or 2h...
ylabel('a=DNI [pu]')
grid('on')
end
%K-MEANS SCALED DATA
k=4;
[IDX, Ca] = kmeans(transpose(a), k);
save('Ca.mat');
%REPRESENTATION 4
figure(4)
colorstring2 = 'ybcr';
for(i=1:1:k)
plot([1:1:J],Ca(i,:), 'Color', colorstring2(i))
title('SCALED DATA CLUSTERING')
xlabel('scaled time') %units may vary from season to season, could be
1h or 2h...
ylabel('a=DNI [pu]')
grid('on')
hold on
    stairs([1:1:J],Ca(i,:), 'Color', colorstring2(i))
2
%
   hold on
end
%DE-SCALING FUNCTION CALLING
Y=zeros(H,D);
for d=1:1:D
Y(:,d)=function_descaling(a(:,d),ETRN(:,d)); %function [output]
=descaling(input)
end
save('Y.mat');
%REPRESENTATION 5
figure(5)
for(d=1:1:D)
stairs([1:1:H],Y(:,d),'Color', colorstring(d));
holdon;
title('RE-SCALED DATA')
xlabel('time [h]')
ylabel('Y=DNI [W/m^2]')
grid('on')
end
```

Function Data

```
%TAKING DATA FROM EXCEL TO MATLAB
%inputs: -
%outputs: U[Wh/m^2] ETRN[Wh/m^2]
function [U,ETRN]=function_data%function [output] =scaling(input)
global J H D
fileName='Lubbock_2008_solar.xls';
data=xlsread(fileName);
Udata=data(:,9);
ETRNdata=data(:,5);
U=zeros(H,D);
ETRN=zeros(H,D);
i=1;
for d=1:1:D
for h=1:1:H
U(h,d)=Udata(i);
ETRN(h,d)=ETRNdata(i);
i=i+1;
end
end
U;
ETRN;
end
```

Function Scaling

```
%SCALE ALGORITHM
%inputs: U[W/m^2] ETRN[Wh/m^2]
%outputs: a[pu]
function a=function_scaling(U,ETRN) %function [output]
=scaling(input)
global J H D
%definitions
B=max(ETRN); %B=Base or Benchmark
%tr&ts computation
tr=0;
ts=0;
for h=2:1:H
if ETRN(h-1)==0 && ETRN(h)~=0 &&tr==0
tr=h;
end
if ETRN(h-1)~=0 && ETRN(h)==0 &&ts==0
ts=h;
end
end
tr;
ts;
u=[0;U(tr:ts)/B;0];
%J=equal duration subperiods in a season
j=[1:1:J]; %J scaled subperiods sunrise-sunset
DeltaJ=(ts-tr)/J;
t=zeros(J+1,1);
for j=1:1:J+1
t(j)=tr+(j-1)*DeltaJ;
end
% M=sunrise-sunset periods-2 CHECK.... BASED ON TR TS
M=ts-tr+1;
%T=tau's
T=zeros(M,1);
for m=1:M
T(m) = tr + m - 1;
end
Delta = (T(M) - T(1) + 1) / M;
%Τ;
%t;
%u;
```

```
&ACTUAL ALGORITHM
a=zeros(J,1);
m=1;
p=1;
for j=1:1:J
if t(j) <= T(m)
    p=m;
while(t(j+1)>T(p))
       p=p+1;
end
if m==p
a(j)=u(m)*(t(j+1)-t(j))/(t(j+1)-t(j));
else
       a(j) = (u(m) * (T(m) - t(j)) + sum(u(m+1:p-
1))*Delta+u(p)*(t(j+1)-T(p-1)))/(t(j+1)-t(j));
end
    m=p;
end
end
end
```

Function De-scaling

```
%DESCALE ALGORITHM
%inputs: a[pu] ETRN[Wh/m^2]
%outputs: Y[W/m^2]
function Y=function_descaling(a, ETRN) %function [output]
=scaling(input)
global J H D
%definitions
B=max(ETRN); %B=Base or Benchmark
%tr&ts computation
tr=0;
ts=0;
for h=2:1:H
if ETRN(h-1)==0 && ETRN(h)~=0 &&tr==0
tr=h;
end
if ETRN(h-1)~=0 && ETRN(h)==0 &&ts==0
ts=h;
end
end
tr;
ts;
%J=equal duration subperiods in a season
j=[1:1:J]; %J scaled subperiods sunrise-sunset
DeltaJ=(ts-tr)/J;
t=zeros(J+1,1);
for j=1:1:J+1
t(j)=tr+(j-1)*DeltaJ;
end
% M=sunrise-sunset periods-2 CHECK.... BASED ON TR TS
M=ts-tr+1;
%T=tau's
T=zeros(M,1);
for m=1:M
T(m) = tr + m - 1;
end
Delta=(T(M)-T(1)+1)/M;
%T;
%t;
%u;
```

```
%ACTUAL ALGORITHM
%similar to scaling algorithm changing:
%a->y
%u->a
%m->j
%T->t
y=zeros(M,1);
for m=1:1:M
p1=0;p2=0;
for j=1:J
if T(m)<=t(j) && p1==0
          p1=j;
end
if T(m)+Delta>t(j)
          p2=j;
end
end
     j=p1;p=p2;
if j==0
        j=p;
end
if j==p
y(m)=a(j);
else
           y(m) = (a(j)*(t(j)-T(m))+sum(a(j+1:p-
1))*DeltaJ+a(p)*(T(m)+Delta-t(p-1)))/Delta;
end
end
%we have to give the value with units
Y = [zeros(tr-1,1); y*B; zeros(24-ts,1)];
ok=1;
end
```

Program 2

```
clearall
 clc
 closeall
 $$$$$$$$$$$$$$$$$$
global J H D
J=50;
H = 24;
 D = 365;
 ୧୧୧୧୧
୧୫୫୫୫୫୫୫୫୫୫୫୫୫୫୫
 %PROGRAMME 2
 %DATA FROM BEFORE
 load('Y.mat'); %Y=DNI [W/m^2]
Y=Y*10^-3;
                                                 %Y=DNI [kW/m^2]
 %OPTIMIZATION CALLING
P=zeros(H,D);
 for d=1:1:D
P(:,d)=function_optimization(Y(:,d)); %function [output]
 =optimization(input)
 end
 save('P.mat');
 %REPRESENTATION 6
colorstring =
 'ymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrq
bwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcrqbwkymcq
rgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwky
mcrqbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbw
kymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgbwkymcrgb
bwkymcrgbwkymcrg';
 figure(6)
 for(d=1:1:D)
plot([1:1:H],P(:,d),'Color', colorstring(d));
holdon;
title('POWER OUTPUT SCHEDULE')
xlabel('time [h]')
ylabel('P [MW]')
grid('on')
 end
 %CLUSTERING THE POWER OUTPUT
%K-MEANS SCALED DATA
k=4;
[IDX, CP] = kmeans(transpose(P), k);
save('CP.mat');
 %REPRESENTATION 7
figure(7)
colorstring2 = 'ybcr';
for(i=1:1:k)
plot([1:1:H],CP(i,:), 'Color', colorstring2(i))
title('POWER OUTPUT CLUSTERING')
xlabel('time [h]') %units may vary from season to season, could be 1h
or 2h...
ylabel('P [MW]')
 grid('on')
holdon
                stairs([1:1:J],CP(i,:), 'Color', colorstring2(i))
 %
```

39

```
% hold on
end
%REPRESENTATION 8
d=119; %chosing a random day (i.e.119, 29th April)
figure(8)
plot([1:1:H],P(:,d),'r');
title('POWER OUTPUT at random day')
xlabel('Ime [h]')
ylabel('P [MW]')
grid('on')
```

Function Optimization

```
%MAXIMIZATION ALGORITHM
%inputs: Y [kW/m2]
%outputs:P [MW]
function P=function optimization(Y)
qlobal J H D
Y=1000*[0;0;0;0;0;0;0;0;0.09490909090912;0.440498181818181;0.3755290
90909091;0.0257018181818177;0.20788363636363636;0.0637272727272726;0.264
754545454546;0.4150254545456;0.066316363636363636;0.081999999999996;0
.08200000000000;0;0;0;0;0;0]
%constant definitions
delta=1; %subperiod duration [hours]
eff charging=0.97; %eta, (rendimiento)
eff discharging=0.98; %mu
loss factor=0.05;%Psi
%subperiod charging min/max limits rates [MW]
K_min=0;
K max=70;
K MIN=K min*ones(H,1);
K MAX=K max*ones(H,1);
%subperiod discharging min/max limits rates [MW]
Q min=0;
O max=70;
O MIN=O min*ones(H,1);
O MAX=O max*ones(H,1);
%Thermal Energy storage limits [MW]
hours=5; %number of hours CSP can produce output at rated capacity of
70MW only from TES
E min=0;
E max=70*hours;
E_MIN=E_min*ones(H,1);
E_MAX=E_max*ones(H,1);
%Thermal Energy to electricity limits [MW]
Z_min=0;
Z_max=999;
Z_MIN=Z_min*ones(H,1);
Z_MAX=Z_max*ones(H,1);
%Power Output limits [MW]
P_min=0;
P_max=30;
P_MIN=P_min*ones(H,1);
P_MAX=P_max*ones(H,1);
%variables inizialization and definition
K=zeros(H,1); %subperiod charging rate [MW]
Q=zeros(H,1); %subperiod discharging rate [MW]
E=zeros(H,1); %stored thermal energy at END of subperiod [MW]
Z=zeros(H,1); %thermal energy to elctricity [MW]
P=zeros(H,1); %electrical power output [MW]
X0=[K;Q;E;Z;P]; %starting point for optimization>fmincon
```

%Lower Bounds Limits
LB=[K_MIN;Q_MIN;E_MIN;Z_MIN;P_MIN];
%Upper Bounds Limits
UB=[K_MAX;Q_MAX;E_MAX;Z_MAX;P_MAX];

```
%objective funtion from past system marginal price data [$/kWh]
%gama=[0.10073, 0.09758,0.09321, 0.09442, 0.09432, 0.09501, 0.10212,
0.10287, 0.10582, 0.10568, 0.10254, 0.09947, 0.09945, 0.09977,
0.09896, 0.09986, 0.09911, 0.09993, 0.09928, 0.09876, 0.11039,
0,11365, 0,10238, 0,09910] %$/kWh
gama=ones(1,H); %in this case we want to maximize power not profits
%T=function_beta(Y) solar to thermal non-linear maping
%elec=function_alfa(z) thermal to electricity non-linear maping
%Matrixes inizialization and definition
A=[];
B=[];
Aeq=[];
Beq=[];
%
    Constraints
2
   E(h)-(1-loss_factor)*E(h-1)-eff_charging*K(h)*delta+Q(h)*delta=0;
%(1)
%
    Z(h)-function_beta(Y(h))+K(h)*delta-eff_discharging*Q(h)*delta=0;
%(2)
8
   P(h)-function_alfa(Z(h))=0;
%(3)
%Building Aeq
%Fisrt constraint
Kml=eye(H)*(-eff_charging*delta);
Qml=eye(H)*(delta);
Eml=eye(H)*(1);
for h=1:1:H-1
Eml(h+1,h) = -(1-loss_factor);
end
Zml=eye(H)*0;
Pml=eye(H)*0;
%Second constraint
Km2=eye(H)*delta;
Qm2=eye(H)*(-eff_discharging*delta);
Em2=eye(H)*0;
Zm2=eye(H)*1;
Pm2=eye(H)*0;
%Third constraint
Km3=eye(H)*0;
Qm3=eye(H)*0;
Em3=eye(H)*0;
Zm3=-(3/7)*eye(H); %P=function_alfa(Z)
Pm3=eye(H)*1;
%A
A=[Km1,Qm1,Em1,Zm1,Pm1;Km2,Qm2,Em2,Zm2,Pm2;Km3,Qm3,Em3,Zm3,Pm3];
%Building Beg
%Fisrt constraint
Bml=ones(H,1)*0;
%Second constraint
Bm2=ones(H,1);
for h=1:1:H
Bm2(h,1)=0.14*Y(h); %T=function_beta(Y)
```

```
end
%Third constraint
Bm3=ones(H,1)*0;
%B
B=[Bm1;Bm2;Bm3];
```

%Maximization problem>help>function browser>fmincon
[X,FVAL] = fmincon(@FUN,X0,A,B,Aeq,Beq,LB,UB);

P=X(97:120,1);

Function FUN

```
%OBJECTIVE FUNCTION FUN
functionoptimP = FUN(X)
global J H D;
H=24;
%gama=[0.10073, 0.09758,0.09321, 0.09442, 0.09432, 0.09501, 0.10212,
0.10287, 0.10582, 0.10568, 0.10254, 0.09947, 0.09945, 0.09977,
0.09896, 0.09986, 0.09911, 0.09993, 0.09928, 0.09876, 0.11039,
0,11365, 0,10238, 0,09910] %$/kWh
gama=ones(1,H);
delta=1;
%we want to MAXIMIZE it so multiply by (-1)
optimP=-gama*X(97:120,1)*delta;
%note that P=X(97:120,1)
end
```

Function alfa

Function beta

Appendix C. Explanatory drawings

First drawing explains the scaling and de-scaling process done to the DNI data for different values of J. First with a small value of 9 and later for J=50.



Second explanation stands for the matrixes entered in Matlab due to the optimization problem.

$$Aeq = \begin{bmatrix} (Km1) & (Qm1) & (Em1) & (Zm1) & (Pm1) \\ (Km2) & (Qm2) & (Em2) & (Zm2) & (Pm2) \\ (Km3) & (Qm2) & (Em3) & (Zm3) & (Pm3) \end{bmatrix}$$

$$\operatorname{Km1} = \begin{pmatrix} -\eta * \Delta & 0 & \cdots & 0 \\ 0 & -\eta * \Delta & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & -\eta * \Delta \end{pmatrix}$$

$$\operatorname{Qm1} = \begin{pmatrix} \Delta & 0 & \cdots & 0 \\ 0 & \Delta & 0 \\ \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \Delta \end{pmatrix}$$

$$\operatorname{Em1} = \begin{pmatrix} -(1 - \psi) & 1 & 0 & 0 & 0 \\ 0 & -(1 - \psi) & 1 & 0 \\ \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{pmatrix}$$

$$\operatorname{Zm1} = \operatorname{Pm1} = \operatorname{Em2} = \operatorname{Pm2} = \operatorname{Km3} = \operatorname{Qm3} = \operatorname{Em3} = \begin{pmatrix} 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \end{pmatrix}$$

$$Qm2 = \begin{pmatrix} -\mu * \Delta & 0 & \dots & 0 \\ 0 & -\mu * \Delta & & 0 \\ \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & -\mu * \Delta \end{pmatrix}$$

$$Zm2 = Pm3 = \begin{pmatrix} 1 & \cdots & 0\\ \vdots & \ddots & \vdots\\ 0 & \cdots & 1 \end{pmatrix}$$

$$Zm3 = \begin{pmatrix} -\alpha(\cdot) & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & -\alpha(\cdot) \end{pmatrix}$$

$$Beq = \begin{bmatrix} (Bm1) \\ (Bm2) \\ (Bm3) \end{bmatrix}$$
$$Beq = \begin{bmatrix} \begin{pmatrix} 0 \\ \cdots \\ 0 \end{pmatrix} \\ \beta(Y(d, h = 1)) \\ \cdots \\ \beta(Y(d, h = H)) \end{pmatrix} \\ \begin{pmatrix} 0 \\ \cdots \\ 0 \end{pmatrix} \end{bmatrix}$$

Figure 25: Matlab's *fmincon* explanation

```
>> help fmincon
fmincon finds a constrained minimum of a function of several variables.
    fmincon attempts to solve problems of the form:
    min F(X) subject to: A*X <= B, Aeq*X = Beq (linear constraints)
                            C(X) \leq 0, Ceq(X) = 0 (nonlinear constraints)
     х
                               LB <= X <= UB
                                                    (bounds)
   fmincon implements four different algorithms: interior point, SQP, active
    set, and trust region reflective. Choose one via the option Algorithm:
    for instance, to choose SQP, set OPTIONS = optimset('Algorithm','sqp'),
   and then pass OPTIONS to fmincon.
   X = fmincon(FUN, X0, A, B) starts at X0 and finds a minimum X to the
   function FUN, subject to the linear inequalities A*X <= B. FUN accepts
    input X and returns a scalar function value F evaluated at X. X0 may be
    a scalar, vector, or matrix.
   X = fmincon(FUN, X0, A, B, Aeq, Beq) minimizes FUN subject to the linear
    equalities Aeq^*X = Beq as well as A^*X \leq B. (Set A=[] and B=[] if no
    inequalities exist.)
   X = fmincon(FUN, X0, A, B, Aeq, Beq, LB, UB) defines a set of lower and upper
   bounds on the design variables, X, so that a solution is found in
   the range LB <= X <= UB. Use empty matrices for LB and UB
   if no bounds exist. Set LB(i) = -Inf if X(i) is unbounded below;
```

set UB(i) = Inf if X(i) is unbounded above.