Spatial and temporal synchronization of water and energy systems: Towards a single integrated optimization model for long-term resource planning (Supplementary Material)

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Abstract

Feedback between the water and energy sectors exist across system life-cycles and link the resources both spatially and temporally. Tracking the impacts of policies made in one sector on the other can thus be complicated and several nexus methodologies have been developed to address these issues. However, the different physical, temporal and spatial characteristics of the water and energy systems present several hurdles including identifying which of the many links between the two systems to model, with what detail to represent each system and how to synchronize the temporal and spatial differences while simultaneously dealing with data scarcity and large uncertainties. This paper addresses some of these issues and presents a fully integrated hard-linked water-energy linear optimization model. Keeping in mind the varying needs of different stakeholders the model is deliberately made flexible, allowing users to modify objective function sub-component weights as well as providing adjustable spatial and temporal scales. Additional system processes and end-user technologies can be added to the model while existing representations can be further refined depending on the detail required. The capabilities of the fully integrated model are investigated in an example case study for Spain. The performance of the model run in an integrated mode is compared to that of the model run in a non-integrated mode without any inter-sector links. An integrated approach is shown to have higher initial costs when planning for future scenarios as a result of the additional water-energy nexus constraints taken into consideration. However, the performance of an integrated plan is shown to have several benefits during simulations of future scenarios including lower total costs, better resource efficiency and improved robustness in the face of various sources of uncertainty.

Keywords: Water-energy nexus, integrated planning, optimization modeling

Contents

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SM.1. Introduction

This Supplementary Material expands on the information presented in the original paper by providing further details on the water sub-module formulation. For a detailed description of the energy sub-module refer to López-Peña et al. [1] [2].

A summary of the overall water sub-module from the main paper is repeated here and the following sections then expand on this introduction.

"The water sub-module can be conceptualized as presented in Figure SM.1 showing the flow of water through different processes. Each node represents a mass-balance equation with the different colored lines representing parameters and variables. All flows into a node must equal all flows out of the node. Water can be of different qualities such as saline, potable, untreated, waste or recycled water.

This system is applied to each spatial sub-division over the chosen temporal sub-divisions. In Figure SM.1 the different boxes represent water entering or leaving the chosen spatial boundary. Yellow boxes represent exogenous parameters which define water entering the system and comprise of precipitation and ocean water. Green boxes represent water leaving the spatial boundary as runoff, environmental flows or waste water. Final demand consumption and non-served water are represented by the dashed-line box. At each node water can also leave the system either as evapotranspiration indicated by red lines or as leakages (green lines representing process leaks and pink lines representing distribution leaks). Certain nodes also have storage capabilities indicated by a blue line. Storage capabilities include snow and soil moisture at the "Precipitation Balance" node, ground water aquifer storage at the "Ground Water" node, reservoir storage at the "Reservoir" node and rainwater harvesting storage at the "Rainwater Harvesting" direct and central nodes. As seen in the figure a distinction is made between "Direct" users, who use water directly from the system and "Central" users, who are provided water by a central administration. Purification, waste water treatment and reclaimed water redistribution is included as a service provided by the central administration.

For each spatial and temporal unit the mass-balance is checked according to Equation SM.1. Changes in storage for every temporal sub-unit occur as a result of the difference between water entering the system (from precipitation and desalination as well as transfers and runoff from other regions) and water leaving the system (as evapotranspiration as well as transfers and runoff leaving each region). Evapotranspiration is composed of interception evaporation, snow sublimation, plant transpiration, surface evaporation, soil evaporation and water consumed or evaporated as part of different conversion, distribution, treatment and end-use processes."

$$\delta S(b,p)/\delta p = P(b,p) + D(b,p) + I_{in}(b,p) + Q_{in}(b,p) - V(b,p) - I_{out}(b,p) - Q_{out}(b,p) \tag{SM.1}$$

Where..

b : Spatial sub-unit, p : Temporal sub-unit, S : Storage, P : Precipitation

D : Desalination, I_{in} : Inter-basin transfers in, Q_{in} : Runoff in

V : Evapotranspiration, I_{out} : Inter-basin transfers out, Q_{out} : Runoff out

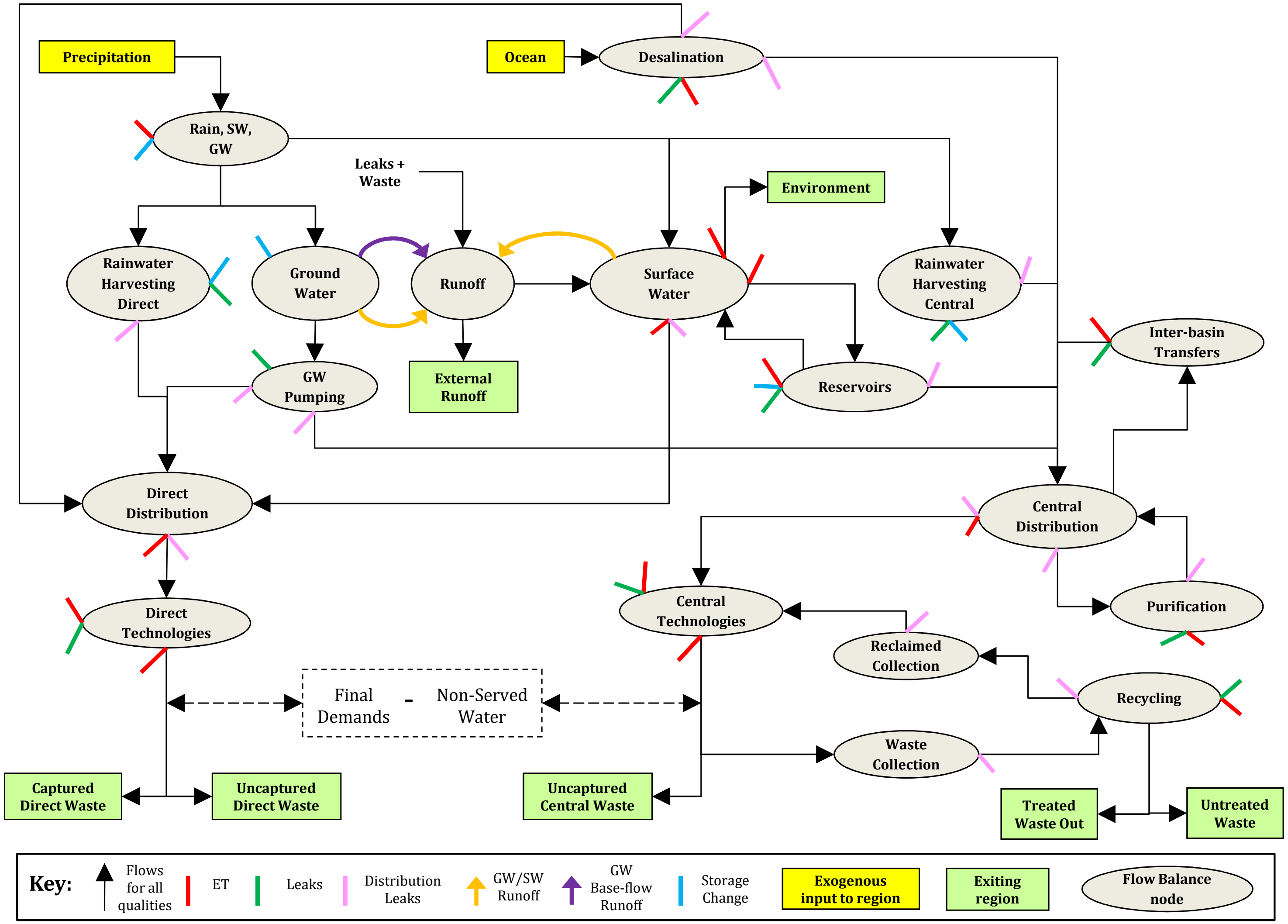

Figure SM.1: Water sub-module conceptual framework showing the flow of water volume tracked through different water processes.

SM.2. Water Model Flow Balance Equations

The following sections describe the equations used to model each water process shown in Figure SM.1. The model is programmed in GAMS (General Algebraic Modeling System) [3] and the original GAMS syntax is maintained. A guide to some of the symbols and prefixes used in the equations is provided in Table SM.1. Sets in the water sub-module are defined as shown in Table SM.2. A few elements for each set are provided as examples. Depending on the case study and needs of the user elements can be added or removed from each set. Brief descriptions of each set of equations are provided to guide the reader. All parameters and scalars are exogenous inputs to the model.

Table SM.1: Guide to symbols in equations

Symbol	Description
$e\_$	Prefix indicating an equation
$s\_$	Prefix indicating a scalar
$p\_$	Prefix indicating a parameter
$v\_$	Prefix indicating a variable
$=e=$	Symbol for "Equal to"
$=l=$	Symbol for "Less than"
$=g=$	Symbol for "Greater than"

Table SM.2: Sets in water sub-module.

Set	Description
q	Water quality (e.g. Potable, saline, waste etc.)
dsal	Desalination technologies (e.g. Reverse osmosis, Multi-stage flash)
pur	Purification technologies (e.g. Primary, secondary or tertiary treatment)
rcyl	Recylcing technologies (e.g. Primary, secondary of tertiary treatment)
wsec	Water Sectors (e.g. Agriculture, Industry, Energy etc.)
tech	Water provision technologies (e.g. Drip irrigation)
b	Spatial sub-unit (e.g. River basins)

SM.2.1. Precipitation Flows

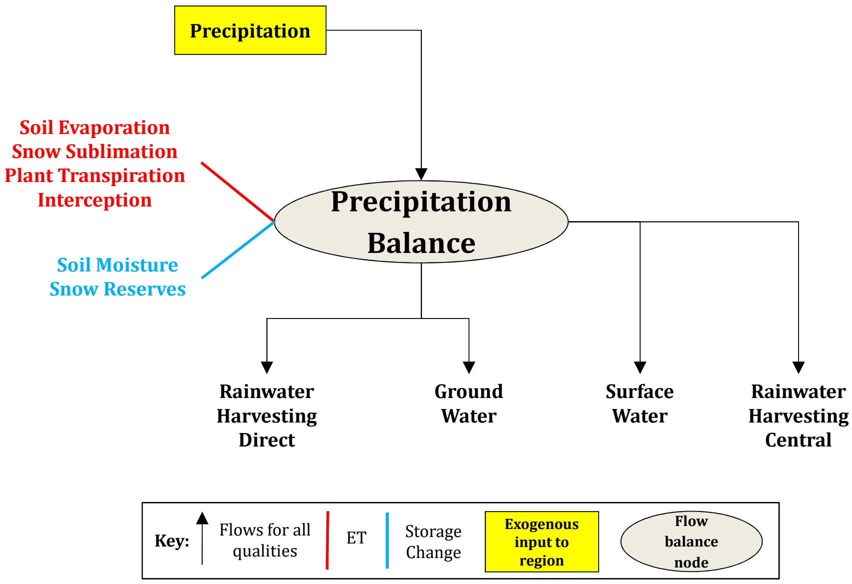

Figure SM.2: Precipitation flow balance

Precipitation Balance: Figure SM.2 shows the flows of water for the precipitation balance. The flows in the figure are modeled as shown in Equation SM.2a which starts with an exogenous rainfall parameter as the input. Part of the rainfall entering the system is lost as surface evaporation, interception evaporation, soil evaporation and snow sublimation all captured by the aggregated term $p\_f\_precip2ET\_Calc$. Part of the precipitation is captured by changes in snow reserve storage and soil moisture capacity. The remaining rainfall is then either captured by centralized or distributed/direct rainwater harvesting. Part of the uncaptured water then flows to groundwater as groundwater recharge defined in Equation SM.2b. The remaining water flows to surface water.

$$
\begin{aligned}
&e\_precipBal(b,q,p)..\\
&p\_rainfall\_hm3(b,q,p) &&= e = v\_f\_precip2rwharvC(b,q,p)\\
& &&+ \quad v\_f\_precip2rwharvD(b,q,p)\\
& &&+ \quad p\_f\_precip2ET_{C}alc(b,q,p)\\
& &&+ \quad v\_f\_precip2surf(b,q,p)\\
& &&+ \quad v\_f\_precip2GW(b,q,p)\\
& &&+ \quad v\_strChng\_soilSnow(b,q,p);
\end{aligned} \tag{SM.2a}
$$

$$
\begin{aligned}
&e\_f\_precip2GW(b,q,p)..\\
&v\_f\_precip2GW(b,q,p) &&= e = p\_gwRchrg\_hm3(b,q,p);
\end{aligned} \tag{SM.2b}
$$

SM.2.2. Direct Rainwater Harvesting Flows

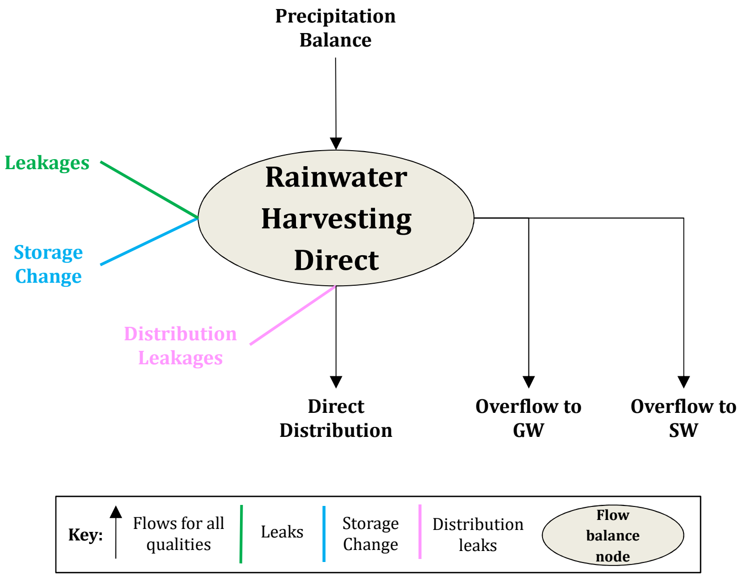

Figure SM.3: Direct rainwater harvesting flow balance

Direct rainwater harvesting Figure SM.3 shows the flows of water for direct rainwater harvesting by distributed users. The flows in the figure are defined as shown in Equation SM.3a where changes in rain water harvesting storage volume are equal to new water being captured from precipitation less any water removed from the storage for different uses, any leaks within the harvesting processes, any water overflows, as well as any leaks during distribution from the harvesting site. Equation SM.3b and Equation SM.3c initialize the model by limiting the flow of rainwater entering to less than the initial and invested capacity less any initial volume already present in storage. Equation SM.3d to Equation SM.3f track the changes in rainwater storage for each time period as well as limit the volume to less than available capacity. Equation SM.3g allows users to set a maximum limit to the amount of rainwater that can be harvested.

$$\begin{aligned}
&e_rwharvDBal(b,q,p)\\
&v_strChng_rwharvD(b,q,p) \;=e=\; v_f_precip2rwharvD(b,q,p)\\
&\quad - v_f_rwharvD2distD(b,q,p)\\
&\quad - v_wleak_rwharvD(b,q,p)\\
&\quad - v_wleakD_rwharvD2distD(b,q,p)\\
&\quad - v_f_rwharvD2overflow(b,q,p);
\end{aligned} \tag{SM.3a}$$

$$\begin{aligned}
&e_f_precip2rwharvDInit(b,p)\$(ord(p)eq1)..\\
&sum(q,v_f_precip2rwharvD(b,q,p)) \;=l=\; p_cap_rwharvD_initCap_hm3(b)\\
&\quad + v_cap_rwharvD_InvCap_hm3(b)\\
&\quad - sum(q,p_rwharvDlevel_init(b,q));
\end{aligned} \tag{SM.3b}$$

$$\begin{aligned}
&e_f_precip2rwharvD(b,p)\$(not(ord(p)eq1))..\\
&sum(q,v_f_precip2rwharvD(b,q,p)) \;=l=\; p_cap_rwharvD_initCap_hm3(b)\\
&\quad + v_cap_rwharvD_InvCap_hm3(b)\\
&\quad - sum(q,v_rwharvD_vol(b,q,p-1));
\end{aligned} \tag{SM.3c}$$

$$\begin{aligned}
&e_rwharvDVolInit(b,q,p)\$(ord(p)eq1)..\\
&v_rwharvD_vol(b,q,p) \;=e=\; p_rwharvDlevel_init(b,q)\\
&\quad + v_strChng_rwharvD(b,q,p);
\end{aligned} \tag{SM.3d}$$

$$\begin{aligned}
&e_rwharvDVolBal(b,q,p)\$(not(ord(p)eq1))..\\
&v_rwharvD_vol(b,q,p) \;=e=\; v_rwharvD_vol(b,q,p-1)\\
&\quad + v_strChng_rwharvD(b,q,p);
\end{aligned} \tag{SM.3e}$$

$$\begin{aligned}
&e_rwharvDCapBal(b,p)..\\
&sum(q,v_rwharvD_vol(b,q,p)) \;=l=\; p_cap_rwharvD_initCap_hm3(b)\\
&\quad + v_cap_rwharvD_InvCap_hm3(b);
\end{aligned} \tag{SM.3f}$$

$$\begin{aligned}
&e_rwharvDCapMax(b,p)..\\
&p_rwharvD_MaxCap_hm3(b) \;=g=\; p_cap_rwharvD_initCap_hm3(b)\\
&\quad + v_cap_rwharvD_InvCap_hm3(b);
\end{aligned} \tag{SM.3g}$$

SM.2.3. Centralized Rainwater Harvesting Flows

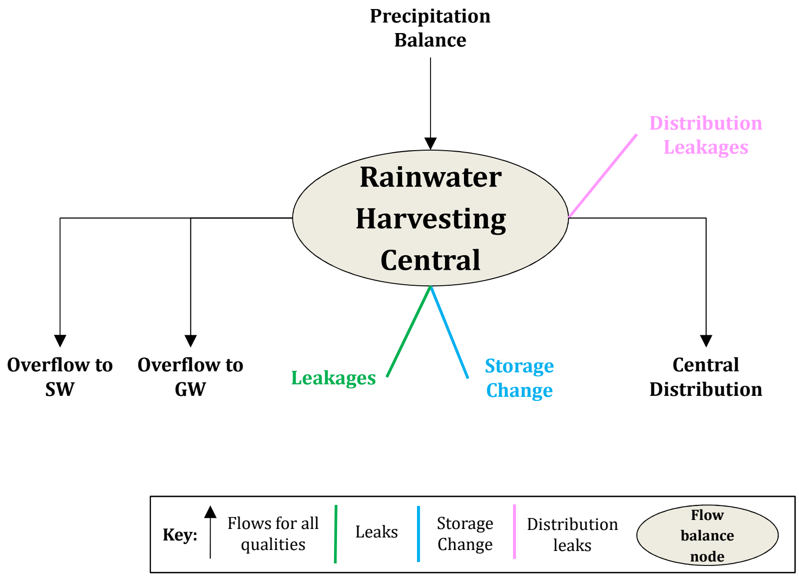

Figure SM.4: Central rainwater harvesting flow balance

Centralized rainwater harvesting Figure SM.4 shows the flows of water for centrally controlled rainwater harvesting. The flows in the figure are defined as shown in Equation SM.4a where changes in rain water harvesting storage volume are equal to new water being captured from precipitation less any water removed from the storage for different uses, any leaks within the harvesting processes, any water overflows, as well as any leaks during distribution from the harvesting site. Equation SM.4b and Equation SM.4c initialize the model by limiting the flow of rainwater entering to less than the initial and invested capacity less any initial volume already present in storage. Equation SM.4d to Equation SM.4f track the changes in rainwater storage for each time period as well as limit the volume to less than available capacity. Equation SM.4g allows users to set a maximum limit to the amount of rainwater that can be harvested.

$$
\begin{aligned}
&e\_rwharvCBal(b,q,p)..\\
&v\_strChng\_rwharvC(b,q,p) &&= e = v\_f\_precip2rwharvC(b,q,p)\\
& &&\quad - \quad v\_f\_rwharvC2distC(b,q,p)\\
& &&\quad - \quad v\_wleak\_rwharvC(b,q,p)\\
& &&\quad - \quad v\_wleakD\_rwharvC2distC(b,q,p)\\
& &&\quad - \quad v\_f\_rwharvC2overflow(b,q,p); \qquad \text{(SM.4a)}
\end{aligned}
$$

$$
\begin{aligned}
&e\_f\_precip2rwharvCInit(b,p)\$(ord(p)eq1)..\\
&sum(q,v\_f\_precip2rwharvC(b,q,p)) &&= l = p\_cap\_rwharvC\_initCap\_hm3(b)\\
& &&\quad + \quad v\_cap\_rwharvC\_InvCap\_hm3(b)\\
& &&\quad - \quad sum(q,p\_rwharvClevel\_init(b,q)); \qquad \text{(SM.4b)}
\end{aligned}
$$

$$
\begin{aligned}
&e\_f\_precip2rwharvC(b,p)\$(not(ord(p)eq1))..\\
&sum(q,v\_f\_precip2rwharvC(b,q,p)) &&= l = p\_cap\_rwharvC\_initCap\_hm3(b)\\
& &&\quad + \quad v\_cap\_rwharvC\_InvCap\_hm3(b)\\
& &&\quad - \quad sum(q,v\_rwharvC\_vol(b,q,p-1)); \qquad \text{(SM.4c)}
\end{aligned}
$$

$$
\begin{aligned}
&e\_rwharvCVolInit(b,q,p)\$(ord(p)eq1)..\\
&v\_rwharvC\_vol(b,q,p) &&= e = p\_rwharvClevel\_init(b,q)\\
& &&\quad + \quad v\_strChng\_rwharvC(b,q,p); \qquad \text{(SM.4d)}
\end{aligned}
$$

$$
\begin{aligned}
&e\_rwharvCVolBal(b,q,p)\$(not(ord(p)eq1))..\\
&v\_rwharvC\_vol(b,q,p) &&= e = v\_rwharvC\_vol(b,q,p-1)\\
& &&\quad + \quad v\_strChng\_rwharvC(b,q,p); \qquad \text{(SM.4e)}
\end{aligned}
$$

$$
\begin{aligned}
&e\_rwharvCCapBal(b,p)..\\
&sum(q,v\_rwharvC\_vol(b,q,p)) &&= l = p\_cap\_rwharvC\_initCap\_hm3(b)\\
& &&\quad + \quad v\_cap\_rwharvC\_InvCap\_hm3(b); \qquad \text{(SM.4f)}
\end{aligned}
$$

$$
\begin{aligned}
&e\_rwharvCCapMax(b,p)..\\
&p\_rwharvC\_MaxCap\_hm3(b) &&= g = p\_cap\_rwharvC\_initCap\_hm3(b)\\
& &&\quad + \quad v\_cap\_rwharvC\_InvCap\_hm3(b); \qquad \text{(SM.4g)}
\end{aligned}
$$

SM.2.4. *Surface Water Flows*

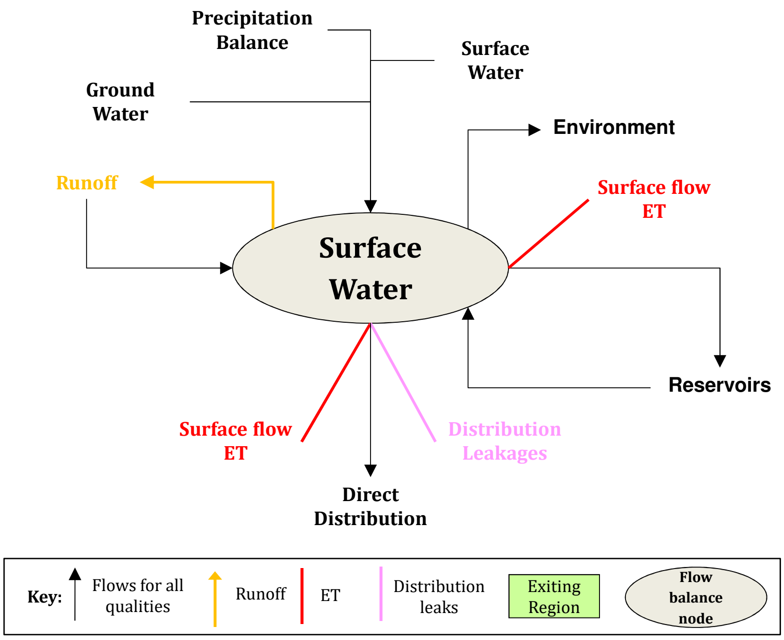

Figure SM.5: Surface flow balance

Surface to distribution: Figure SM.5 shows the flows for surface water balance. The flows in the figure are defined as shown in Equation SM.5a where water entering surface water flows is equal to the sum of part of the water from precipitation, water released from reservoirs back to surface waters, part of the overflow from rainwater harvesting, as well as any runoff from upstream basins with drainage outlets into the existing basin. Water exiting surface water flows is equal to water extracted for reservoir storage, water extracted directly by any individual users, water lost during the extraction process, water lost to evapotranspiration and water leaving the basin as runoff. Water that is needed for environment as per local regulations is also indicated by a separate variable so that it cannot be used for other uses. Equation SM.5b defines the water lost to evapotranspiration for surface water processes. These processes are assumed to be exposed to the open air and a scalar is used to define the evaporation from the surface. The scalar defines the percentage of water of the total flow volume for each process which is lost to the atmosphere. This scalar can be refined to reflect the impacts of temperature and climate in future developments.

$$
\begin{aligned}
&e\_surfBal(b,q,p)..\\
&v\_f\_precip2surf(b,q,p) =e= v\_f\_surf2res(b,q,p)\\
&\quad + \; v\_f\_surf2distD(b,q,p)\\
&\quad - \; v\_f\_res2surf(b,q,p)\\
&\quad + \; v\_f\_surf2env\_hm3(b,q,p)\\
&\quad + \; v\_f\_surf2ET(b,q,p)\\
&\quad + \; v\_f\_surf2runoff(b,q,p)\\
&\quad + \; v\_wleakD\_surf2distD(b,q,p)\\
&\quad - \; sum((bf), v\_f\_b2b\_runoff\_hm3(bf,b,q,p))\\
&\quad - \; v\_f\_rwharvD2overflow(b,q,p) * (1 - p\_gwRchrg\_prcnt(b))\\
&\quad - \; v\_f\_rwharvC2overflow(b,q,p) * (1 - p\_gwRchrg\_prcnt(b));
\end{aligned} \tag{SM.5a}
$$

$$
\begin{aligned}
&e\_f\_surf2ET(b,q,p)..\\
&v\_f\_surf2ET(b,q,p) =e= v\_f\_surf2distD(b,q,p) * s\_ET\_surf\\
&\quad + \; v\_f\_surf2res(b,q,p) * s\_ET\_surf\\
&\quad + \; v\_f\_surf2env\_hm3(b,q,p) * s\_ET\_surf;
\end{aligned} \tag{SM.5b}
$$

SM.2.5. Environmental Water Flows

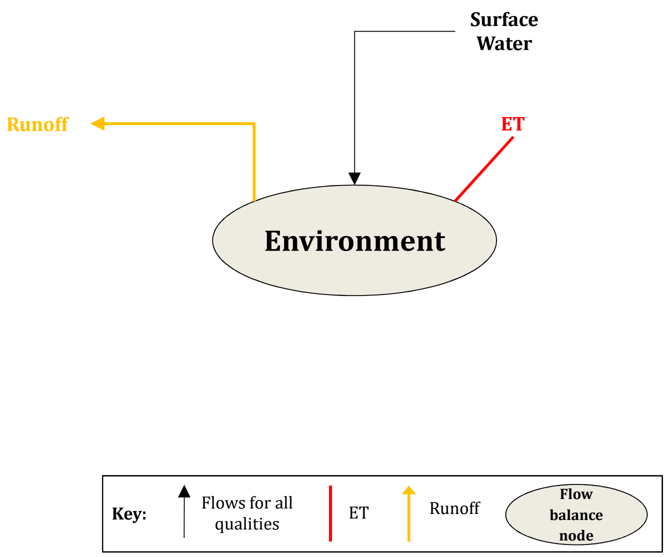

Figure SM.6: Environmental water flow balance

Environmental flow balance: Figure SM.6 shows the flows for environmental water flow balance. The flows in the figure are defined as shown in Equation SM.6a which defines the amount of water entering the environment equal to the sum of the water lost as evapotranspiration and the remaining water flowing to runoff. Equation SM.6b defines the amount of water which needs to be released for environmental purposes. This is currently defined by a scalar as a percentage of the natural precipitation flows to surface waters. The value can be refined to reflect local regulations and can be replaced by absolute values or by relationships with other parameters or variables. Equation SM.6c defines the amount of water which is lost to evapotranspiration in the environment, based on a parameter which defines the evapotranspiration potential per sub-region and sub-period.

$$
\begin{aligned}
&e\_f\_envBal(b,q,p)..\\
&v\_f\_surf2env(b,q,p) &&= e = v\_f\_env2ET(b,q,p)\\
& &&+ \quad v\_f\_env2runoff(b,q,p);
\end{aligned} \tag{SM.6a}
$$

$$
\begin{aligned}
&e\_f\_surf2env\_hm3(b,q,p)..\\
&v\_f\_surf2env\_hm3(b,q,p) &&= e = v\_f\_precip2surf(b,q,p) * s\_f\_env;
\end{aligned} \tag{SM.6b}
$$

$$
\begin{aligned}
&e\_f\_env2ET(b,q,p)..\\
&v\_f\_env2ET(b,q,p) &&= e = v\_f\_surf2env(b,q,p)\\
& &&* \quad p\_ET\_surfX(b,p);
\end{aligned} \tag{SM.6c}
$$

SM.2.6. Reservoir Flows

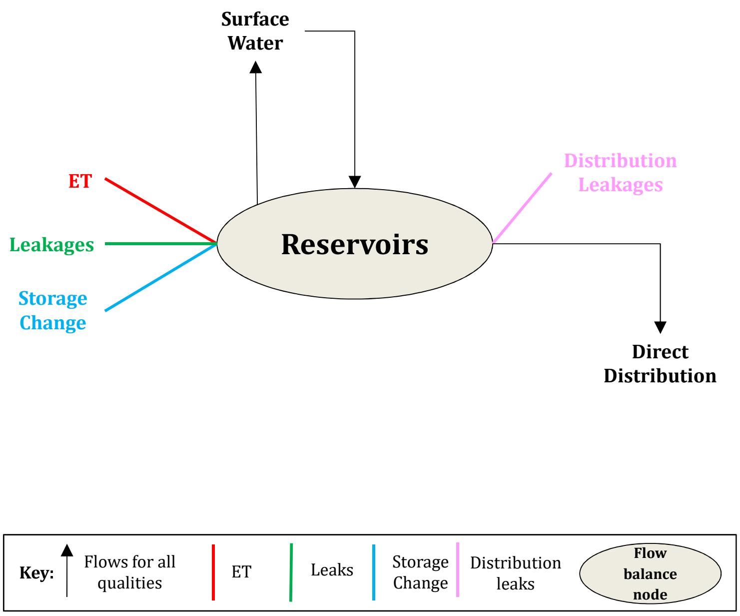

Figure SM.7: Reservoir flow balance

Reservoir flow balance: Figure SM.7 shows the flows for reservoir water balance. The flows in the figure are defined as shown in Equation SM.7a where water entering reservoirs from surface waters is equal to the change in reservoir volume, water leaving the reservoirs to central distribution networks, losses incurred during distribution, water released from reservoirs to downstream surface water, water lost to evapotranspiration and water leakages from reservoirs themselves. Equation SM.7b to Equation **??** constrain the volume of water in the reservoirs based on the initial volume stored, the available capacity, new capacity installed and any regulations stipulating the maximum or minimum storage volumes to maintain within the reservoirs. Equation SM.7d defines the relationship of hydro-energy production with water released from reservoirs. The equation uses a parameter for each sub-region to define the percentage of hydro-electric reservoirs (in contrast to other multi-purpose reservoirs without hydroelectric production capabilities). Water released from these reservoirs is then related to electricity production by a linear relationship whose parameters are defined for each sub-region based on historical records. This relationship can be further refined or the parameters adjusted based on local knowledge and availability of data. Equation SM.7e defines the amount of water lost to evapotranspiration based on the volume of water in the reservoirs and the potential evapotranspiration parameters defined for each sub-region and sub-period.

$$
\begin{aligned}
&e\_resBal(b,q,p).. \\
&v\_f\_surf2res(b,q,p) &&= e = v\_strChng\_res(b,q,p) \\
& &&+ \quad v\_f\_res2distC(b,q,p) \\
& &&+ \quad v\_f\_res2surf(b,q,p) \\
& &&+ \quad v\_f\_res2ET(b,q,p) \\
& &&+ \quad v\_wleak\_res(b,q,p) \\
& &&+ \quad v\_wleakD\_res2distC(b,q,p); \qquad \text{(SM.7a)}
\end{aligned}
$$

$$
\begin{aligned}
&e\_resVolBalInit(b,p)\$(ord(p)eq1).. \\
&sum(q,v\_res\_Vol(b,q,p)) &&= e = sum(q,p\_res\_strVol\_init(b,q)) \\
& &&+ \quad sum(q,v\_strChng\_res(b,q,p)); \qquad \text{(SM.7b)}
\end{aligned}
$$

$$
\begin{aligned}
&e\_resVolBal(b,p)\$(not(ord(p)eq1)).. \\
&sum(q,v\_res\_Vol(b,q,p)) &&= e = sum(q,v\_res\_Vol(b,q,p-1)) \\
& &&+ \quad sum(q,v\_strChng\_res(b,q,p)); \qquad \text{(SM.7c)}
\end{aligned}
$$

$$
\begin{aligned}
&e\_res\_hydronrgprod(b,p).. \\
&v\_res\_hydronrgprod(b,p) &&= e = p\_hydroPot\_prcnt(b) * (sum(q,v\_f\_res2surf(b,q,p)) \\
& &&\quad *p\_hydroPot\_linrel\_a(b) \\
& &&+ \quad p\_hydroPot\_linrel\_b(b)); \qquad \text{(SM.7d)}
\end{aligned}
$$

$$
\begin{aligned}
&e\_f\_res2ET(b,q,p).. \\
&v\_f\_res2ET(b,q,p) &&= e = v\_res\_Vol(b,q,p) * p\_ET\_resX(b,p); \qquad \text{(SM.7e)}
\end{aligned}
$$

SM.2.7. Groundwater Flows

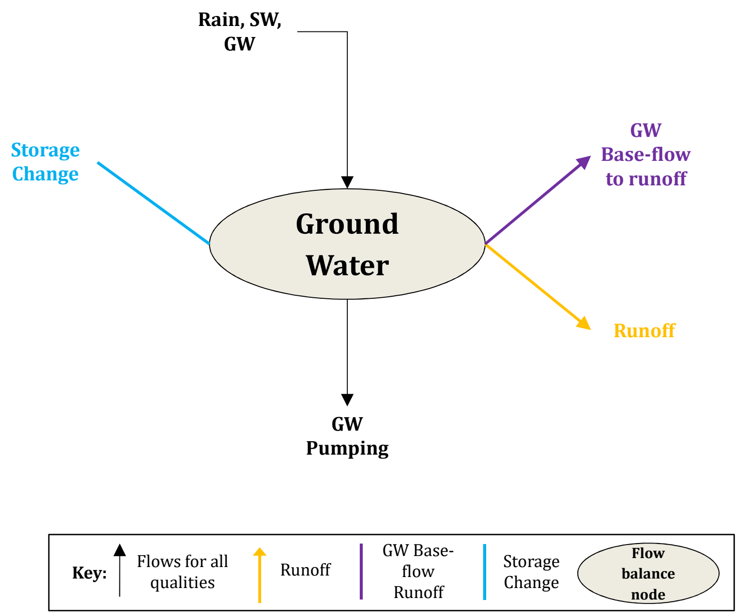

Figure SM.8: Groundwater flow balance

Groundwater flow balance: Figure SM.8 shows the flows for ground water flow balance. The flows in the figure are defined as shown in Equation SM.8 where water entering ground water aquifers is equal to a pre-defined groundwater recharge based on historical percentage of rainfall per sub-region plus part of the water overflows from rainwater harvesting tanks, less water leaving ground water aquifers comprising changes in ground water storage, water being pumped out, water exiting as base-flow and any water from precipitation which can't be held in the aquifer due to full saturation as ground water runoff.

$$
\begin{aligned}
&e\_gw\_strChng\_hm3(b,q,p)..\\
&p\_gwRchrg\_hm3(b,q,p) \quad = e = v\_f\_ground2gwpump(b,q,p)\\
&\qquad + \; v\_strChng\_gw(b,q,p)\\
&\qquad + \; v\_f\_gw2runoff\_hm3(b,q,p)\\
&\qquad + \; p\_gw2runoffBaseflow\_hm3(b,q,p)\\
&\qquad - \; v\_f\_rwharvD2overflow(b,q,p) * (p\_gwRchrg\_prcnt(b))\\
&\qquad - \; v\_f\_rwharvC2overflow(b,q,p) * (p\_gwRchrg\_prcnt(b));
\end{aligned}
\tag{SM.8a}
$$

SM.2.8. *Groundwater Pumping Flows*

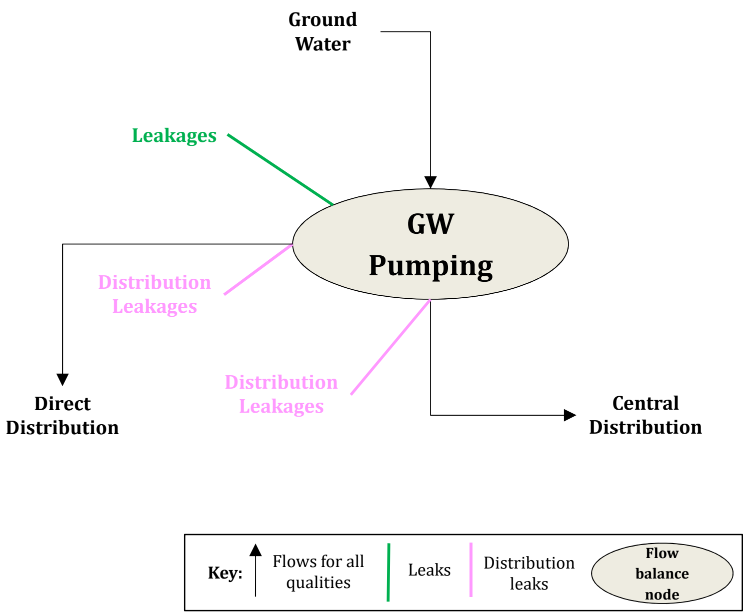

Figure SM.9: Groundwater pumping flow balance

Groundwater pumping balance: Figure SM.9 shows the flows for ground water pumping balance. The flows in the figure are defined as shown in Equation SM.9a where water that is pumped out from ground water aquifers is equal to the sum of water distributed to central or direct users as well as any leakages in the pumping and distribution processes. Equation SM.9b and Equation SM.9c track the changes in groundwater depth, based on an initial ground water depth and any subsequent changes in storage volume from Equation SM.8. Equation SM.9d constrains the amount of water that can be pumped to less than the recharge for that basin. This constraint can be turned off which would allow over-exploitation of non-renewable ground water resources. Equation SM.9e constrains the water that can be pumped to less than the existing and any new installed pumping capacity.

$$e\_gwpumpBal(b,q,p)..$$

$$\begin{aligned} v\_f\_ground2gwpump(b,q,p) \quad &= e = v\_f\_gwpump2distC(b,q,p) \\ &+ \quad v\_f\_gwpump2distD(b,q,p) \\ &+ \quad v\_wleak\_gwpump(b,q,p) \\ &+ \quad v\_wleakD\_gwpump2distC(b,q,p) \\ &+ \quad v\_wleakD\_gwpump2distD(b,q,p); \end{aligned} \tag{SM.9a}$$

$$e\_gw\_depthVol\_hm3Init(b,q,p)\$(ord(p)eq1)..$$

$$\begin{aligned} v\_gw\_depthVol\_hm3(b,q,p) \quad &= e = p\_gw\_depthVolSpare\_Init\_hm3(b,q) \\ &- \quad v\_strChng\_gw(b,q,p); \end{aligned} \tag{SM.9b}$$

$$e\_gw\_depthVol\_hm3(b,q,p)\$(not(ord(p)eq1))..$$

$$\begin{aligned} v\_gw\_depthVol\_hm3(b,q,p) \quad &= e = v\_gw\_depthVol\_hm3(b,q,p-1) \\ &- \quad v\_strChng\_gw(b,q,p); \end{aligned} \tag{SM.9c}$$

$$e\_gwRchrg2pump(b,q,p)..$$

$$p\_gwRchrg\_hm3(b,q,p) \quad = g = v\_f\_ground2gwpump(b,q,p); \tag{SM.9d}$$

$$e\_cap\_gwPump\_Inv(b,p)..$$

$$\begin{aligned} sum(q, v\_f\_ground2gwpump(b,q,p)) \quad &= l = p\_cap\_gwPump\_init\_hm3(b) \\ &+ \quad v\_cap\_gwPump\_Inv(b); \end{aligned} \tag{SM.9e}$$

SM.2.9. Desalination Flows

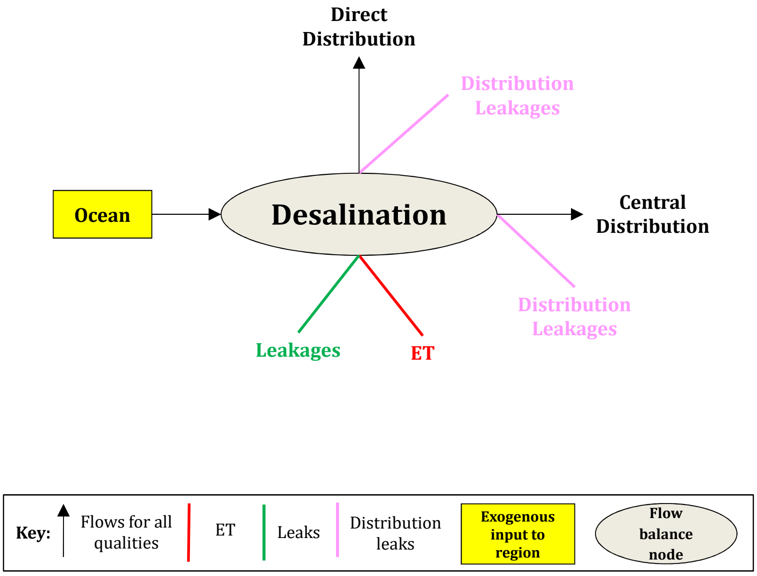

Figure SM.10: Desalination flow balance

Desalination balance: Figure SM.10 shows the flows for desalinated water balance. The flows in the figure are defined as shown in Equation SM.10a where saline water extracted from the ocean is equal to water processed and distributed through central and direct desalination plants as well as any leakages in the desalination process it self or leakages in the distribution processes. Water consumed during the desalination process is also tracked in the variable for evapotranspiration. Equation SM.10b is used to define which basins have access to ocean or brackish water. Equation SM.10c limits the amount of water extracted for desalination to less than the initial capacity plus any additional installed capacity.

$$e\_dsalBal(b, dsal, p)..$$

$$\begin{aligned}
sum(q, v_f_ocean2dsal(b, dsal, q, p)) =e= \; & sum(q, v_f_dsal2distC(b, dsal, q, p) \\
+ \quad & v_f_dsal2distD(b, dsal, q, p) \\
+ \quad & v_wleak_dsal(b, dsal, q, p) \\
+ \quad & v_wleakD_dsal2distC(b, dsal, q, p) \\
+ \quad & v_wleakD_dsal2distD(b, dsal, q, p) \\
+ \quad & v_f_dsal2ET(b, dsal, q, p)); \qquad \text{(SM.10a)}
\end{aligned}$$

$$e\_ocean2dsal(b, q, p)..$$

$$p\_oceanAvail(b, q) =g= sum(dsal, v\_f\_ocean2dsal(b, dsal, q, p)); \qquad \text{(SM.10b)}$$

$$e\_cap\_dsal\_Inv(dsal, b, p)..$$

$$\begin{aligned}
sum(q, v_f_ocean2dsal(b, dsal, q, p)) =l= \; & p_cap_dsal_init_hm3(dsal, b) \\
+ \quad & v_cap_dsal_Inv(dsal, b); \qquad \text{(SM.10c)}
\end{aligned}$$

SM.2.10. Inter-basin Flows

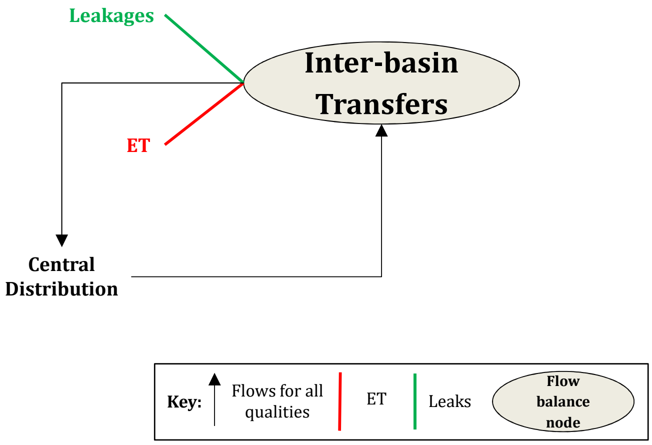

Figure SM.11: Interbasin in flow balance

Interbasin Flow Balance: Figure SM.11 shows the flows for inter-basin water balance. The flows in the figure are defined as shown in Equation SM.11a where total volume of water flowing into each sub-region (b) is equal to the sum of all water flowing from all other sub-regions (bf) to the particular sub-region less any losses as leakages and evapotranspiration during the transfer. Equation SM.11b defines the total volume of water flowing into each basin out from all the other sub-regions. Equation SM.11c and Equation SM.11d limit the water flow between sub-regions to less than the existing transfer capacity plus any new installed capacity. Water transfers between the same sub-regions is not allowed and is fixed to 0.

$$e\_trnsf\_IN(b,q,p)..$$

$$\begin{aligned} v\_f\_b2b\_vol\_in(b,q,p) \quad &= e = sum(bf, v\_f\_b2b\_vol(bf,b,q,p)) \\ &- \quad v\_wleak\_IB(b,q,p) \\ &- \quad v\_f\_IB2ET(b,q,p); \end{aligned} \tag{SM.11a}$$

$$e\_trnsf\_OUT(b,q,p)..$$

$$v\_f\_b2b\_vol\_out(b,q,p) \quad = e = sum(bt, v\_f\_b2b\_vol(b,bt,q,p)); \tag{SM.11b}$$

$$e\_trnsf\_cap\_max(bf,bt,p)..$$

$$\begin{aligned} sum(q, v\_f\_b2b\_vol(bf,bt,q,p)) \quad &= l = p\_trnsfCap\_vol\_Init(bf,bt) \\ &+ \quad v\_trnsfCap\_vol\_Inv(bf,bt); \end{aligned} \tag{SM.11c}$$

$$e\_trnsf\_capback\_max(bf,bt,p)..$$

$$\begin{aligned} sum(q, v\_f\_b2b\_vol(bt,bf,q,p)) \quad &= l = p\_trnsfCap\_vol\_Init(bf,bt) \\ &+ \quad v\_trnsfCap\_vol\_Inv(bf,bt); \end{aligned} \tag{SM.11d}$$

SM.2.11. Direct Distribution Flows

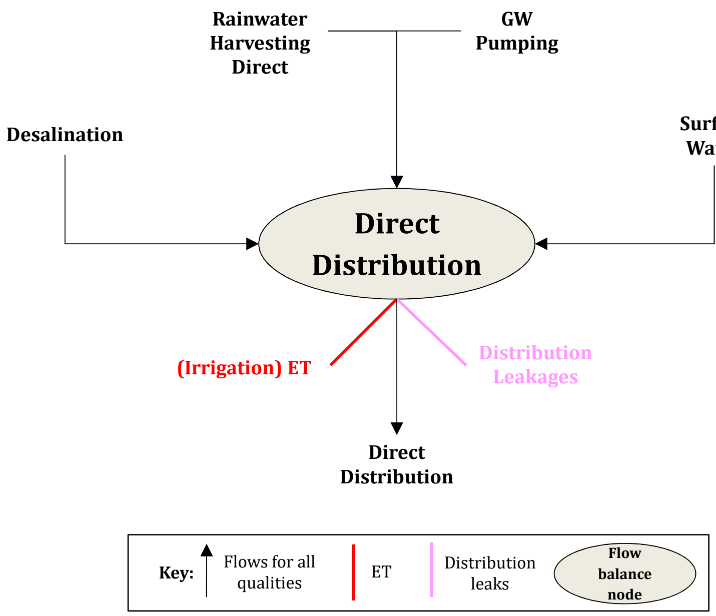

Figure SM.12: Direct distribution flow balance

Direct distribution flows: Figure SM.12 shows the flows balance for distribution to direct users. The flows in the figure are defined as shown in Equation SM.12a where the sum of the water extracted and distributed to different technologies is equal to the sum of water taken from rainwater harvesting units, desalination units, directly from surface water and from groundwater pumps less any water lost in leakages during the distribution process as well as water lost to evapotranspiration in open distribution channels to technologies such as traditional irrigation. Equation SM.12b limits the total distribution to less than the existing and new installed capacity.

$$
\begin{aligned}
&e\_distDBal(b,q,p)..\\
&sum((tech), v\_f\_distD2tech(b,tech,q,p)) = e = v\_f\_rwharvD2distD(b,q,p)\\
&\quad + \quad v\_f\_surf2distD(b,q,p)\\
&\quad + \quad v\_f\_gwpump2distD(b,q,p)\\
&\quad + \quad sum(dsal, (v\_f\_dsal2distD(b,dsal,q,p)))\\
&\quad - \quad sum(tech, v\_wleakD\_distD2tech(b,tech,q,p))\\
&\quad - \quad sum(tech, v\_f\_distD2techET(b,tech,q,p));
\end{aligned}
\tag{SM.12a}
$$

$$
\begin{aligned}
&e\_cap\_distD(b,p)..\\
&sum(q, v\_f\_distDFlows(b,q,p)) = l = p\_cap\_distD\_init(b)\\
&\quad + \quad v\_cap\_distD\_Inv(b);
\end{aligned}
\tag{SM.12b}
$$

SM.2.12. *Central Distribution Flows*

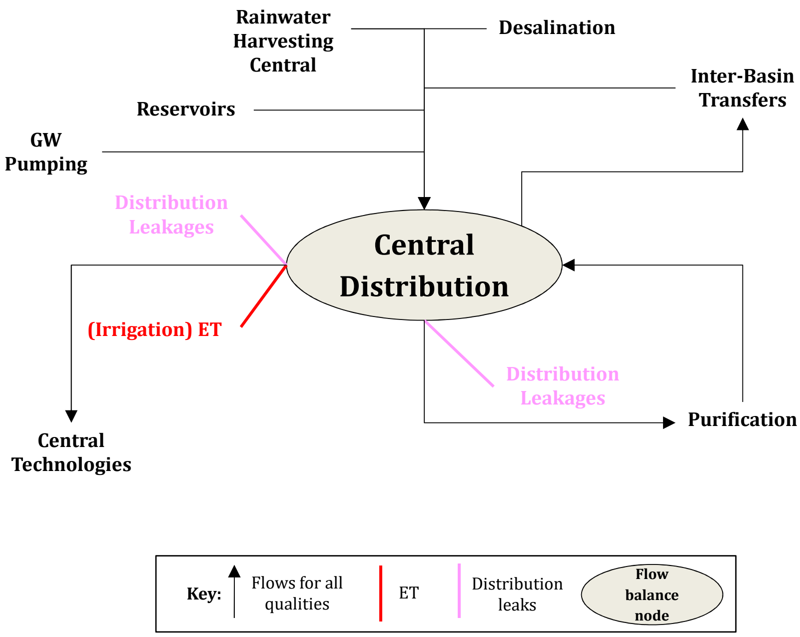

Figure SM.13: Central distribution flow balance

Central distribution flows: Figure SM.13 shows the flows for reservoir water balance. The flows in the figure are defined as shown in Equation SM.13a where where the sum of the water extracted and distributed to different central technologies is equal to the sum of water taken from rainwater harvesting units, desalination, reservoirs, groundwater pumping, purified water and inter-basin transfers into the region less any water sent for purification, water transferred to other regions, as well as water lost in leakages during the distribution process and water lost to evapotranspiration in open distribution channels to technologies such as traditional irrigation. Equation SM.13b limits the total distribution to less than the existing and new installed capacity.

$$
\begin{aligned}
&e\_distCBal(b,q,p)..\\
&sum((tech), v\_f\_distC2tech(b,tech,q,p)) = e = sum(pur, v\_f\_pur2distC(b,pur,q,p))\\
&\quad + \quad v\_f\_res2distC(b,q,p)\\
&\quad + \quad v\_f\_gwpump2distC(b,q,p)\\
&\quad + \quad sum(dsal, v\_f\_dsal2distC(b,dsal,q,p))\\
&\quad + \quad v\_f\_b2b\_vol\_in(b,q,p)\\
&\quad + \quad v\_f\_rwharvC2distC(b,q,p)\\
&\quad - \quad sum(pur, v\_f\_distC2pur(b,pur,q,p))\\
&\quad - \quad v\_f\_b2b\_vol\_out(b,q,p)\\
&\quad - \quad sum(tech, v\_wleakD\_distC2tech(b,tech,q,p))\\
&\quad - \quad sum(pur, v\_wleakD\_distC2pur(b,pur,q,p))\\
&\quad - \quad sum(tech, v\_f\_distC2techET(b,tech,q,p));
\end{aligned}
\tag{SM.13a}
$$

$$
\begin{aligned}
&e\_cap\_distC(b,p)..\\
&sum(q, v\_f\_distCFlows(b,q,p)) = l = p\_cap\_distC\_init(b)\\
&\quad + \quad v\_cap\_distC\_Inv(b);
\end{aligned}
\tag{SM.13b}
$$

SM.2.13. Direct Technologies Flows

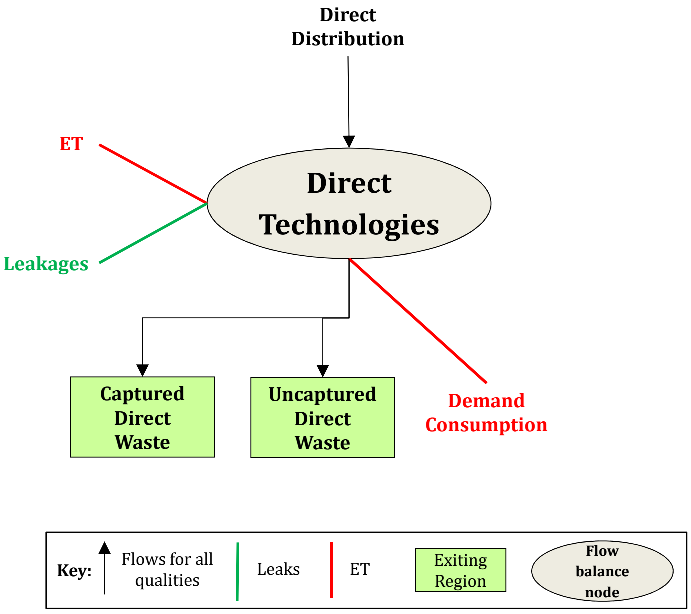

Figure SM.14: Direct technologies flow balance

Direct technology flows: Figure SM.14 shows the flow balance for water transferred to direct user technologies. The flows in the figure are defined as shown in Equation SM.14a where water required from the direct distribution system for different technologies is equal to the water consumed by the final water uses, any waste water captured or uncaptured after the water has been used as well as any water lost to leakages and evapotranspiration. Equation SM.14b limits the water processed through technologies to less than the existing and any new installed technology capacities.

$$
\begin{aligned}
&e\_techDBal(b, tech, q, p)\$(tech\_dir(tech))..\\
&v\_f\_distD2tech(b, tech, q, p) \quad = e = sum(wsec, v\_f\_tech2demCons(b, tech, wsec, q, p))\\
&\quad + \; sum((wsec), v\_wleak\_tech(b, tech, wsec, q, p))\\
&\quad + \; sum((wsec), v\_ET\_tech(b, tech, wsec, q, p))\\
&\quad + \; sum((wsec), v\_wwstCaptD\_IN(b, tech, wsec, q, p))\\
&\quad + \; sum((wsec), v\_wwstUnCaptIN\_techD(b, tech, wsec, q, p));
\end{aligned} \tag{SM.14a}
$$

$$
\begin{aligned}
&e\_techDCapBal(b, tech, q, p)\$(tech\_dir(tech))..\\
&v\_f\_distD2tech(b, tech, q, p) \quad = l = p\_cap\_tech\_init(tech, b)\\
&\quad + \; v\_cap\_tech\_Inv(tech, b);
\end{aligned} \tag{SM.14b}
$$

SM.2.14. Central Technologies Flows

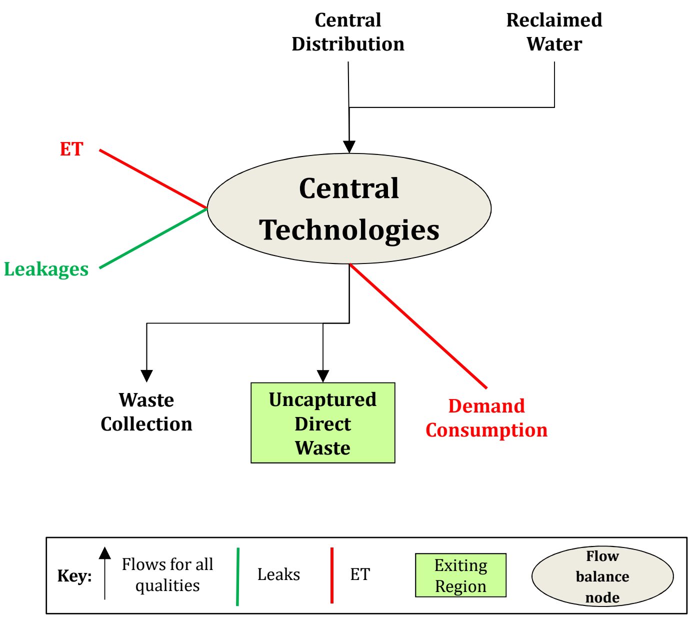

Figure SM.15: Central technologies flow balance

Central technology flows: Figure SM.15 shows the flow balance for water transferred to centrally connected user technologies. The flows in the figure are defined as shown in Equation SM.15a where water required from reclaimed water and the central distribution system for different technologies is equal to the water consumed by the final water uses, any waste water captured or uncaptured after the water has been used as well as any water lost to leakages and evapotranspiration. Equation SM.15b limits the water processed through technologies to less than the existing and any new installed technology capacities.

$$
\begin{aligned}
&e\_techCBal(b, tech, q, p)\$(tech\_cnt(tech))..\\
&v\_f\_distC2tech(b, tech, q, p) \quad = e = sum(wsec, v\_f\_tech2demCons(b, tech, wsec, q, p))\\
&\quad + \quad sum((wsec), v\_wleak\_tech(b, tech, wsec, q, p))\\
&\quad + \quad sum((wsec), v\_ET\_tech(b, tech, wsec, q, p))\\
&\quad + \quad sum((wsec), v\_wwstCaptC\_IN(b, tech, wsec, q, p))\\
&\quad + \quad sum((wsec), v\_wleakD\_techC2distWIN(b, tech, wsec, q, p))\\
&\quad + \quad sum((wsec), v\_wwstUnCaptIN\_techC(b, tech, wsec, q, p))\\
&\quad - \quad v\_f\_distR2tech(b, tech, q, p);
\end{aligned}
\tag{SM.15a}
$$

$$
\begin{aligned}
&e\_techCCapBal(b, tech, q, p)\$(tech\_cnt(tech))..\\
&v\_f\_distC2tech(b, tech, q, p) \quad = l = p\_cap\_tech\_init(tech, b)\\
&\quad + \quad v\_cap\_tech\_Inv(tech, b);
\end{aligned}
\tag{SM.15b}
$$

SM.2.15. Purification Flows

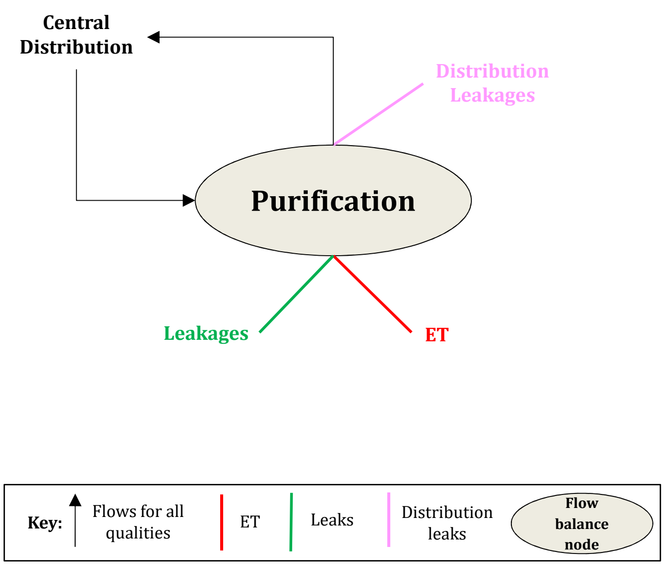

Figure SM.16: Purification flow balance

Purification balance: Figure SM.16 shows the flows for reservoir water balance. The flows in the figure are defined as shown in Equation SM.16a where water sent to purification from central distribution is equal to the purified water sent back to central distribution, any water lost during the transfer as well as water lost to leakages and evapotranspiration during the purification processes. Equation SM.16b limits the water sent for purification to less than the existing capacity and any newly installed capacity.

$$
\begin{aligned}
&e\_purBal(b, pur, p)..\\
&sum(q, v\_f\_distC2pur(b, pur, q, p)) = e = sum(q, v\_f\_pur2distC(b, pur, q, p))\\
&\qquad + \quad sum(q, v\_wleak\_pur(b, pur, q, p))\\
&\qquad + \quad sum(q, v\_wleakD\_pur2distC(b, pur, q, p))\\
&\qquad + \quad sum(q, v\_f\_pur2ET(b, pur, q, p));
\end{aligned} \tag{SM.16a}
$$

$$
\begin{aligned}
&e\_cap\_pur\_Inv(pur, b, p)..\\
&sum(q, v\_f\_distC2pur(b, pur, q, p)) = l = p\_cap\_pur\_init(pur, b)\\
&\qquad + \quad v\_cost\_pur\_Inv(b, pur);
\end{aligned} \tag{SM.16b}
$$

SM.2.16. Final Demand Flows

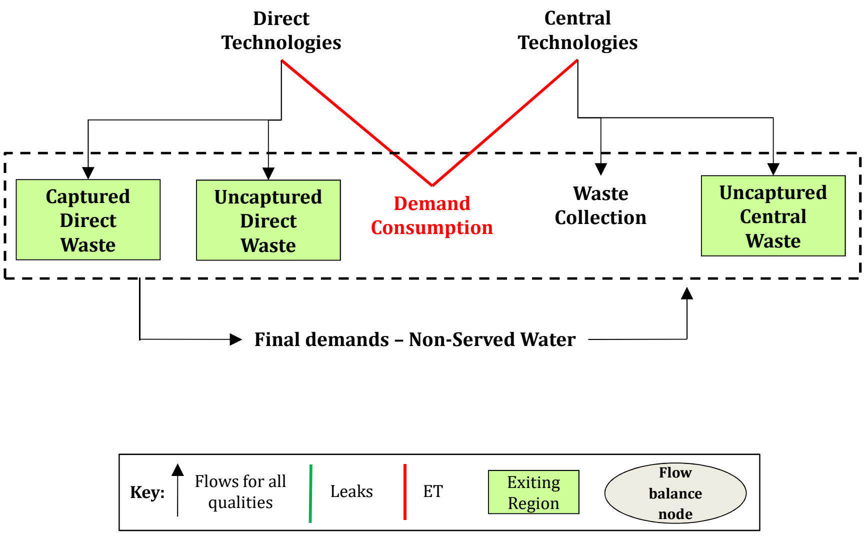

Figure SM.17: Demand flow balance

Demand Balance: Figure SM.17 shows the flows for final water balance. The flows in the figure are defined as shown in Equation SM.17a where water processed through both direct and central technologies to serve final demands are equal to the water consumed during the different uses as well as the captured and uncaptured waste in each technology. A parameter ($p\_reducDem\_tech(tech)$) is also introduced which allows defining the ability of a technology to reduce traditional water consumption. This can be used to characterize new technologies. Equation SM.17b introduces the non-served water term while Equation SM.17c limits the volume of water processed through the different technologies to less than the existing and any new capacity installed. Equation SM.17d and Equation SM.17e define the amount of waste captured per technology based on the pre-defined parameter $p\_WasteCapPrcnt(tech)$. For each technology part of the water used is consumed as evapotranspiration while part of the remaining water is either captured as waste or released as uncaptured waste. Equation SM.17f to Equation SM.17j are used to conserve mass balance as water is processed through technologies and changes from different qualities to waste water.

$$
\begin{aligned}
&e\_tech2demBal(b, tech, wsec, p)..\\
&sum(q, v\_f\_tech2dem(b, tech, wsec, p)) &&= e = sum(q, (v\_f\_tech2demCons(b, tech, wsec, q, p)\\
& &&+\quad v\_wwstCaptC\_IN(b, tech, wsec, q, p)\\
& &&+\quad v\_wwstCaptD\_IN(b, tech, wsec, q, p)\\
& &&+\quad v\_wwstUnCaptIN\_techC(b, tech, wsec, q, p)\\
& &&+\quad v\_wwstUnCaptIN\_techD(b, tech, wsec, q, p)))\\
& &&*\quad (1 + p\_reducDem\_tech(tech)); \qquad \text{(SM.17a)}\\
&e\_demBal(b, wsec, p)..\\
&v\_WdemIN(b, wsec, p) &&= e = sum((tech, q), v\_f\_tech2dem(b, tech, wsec, q, p))\\
& &&+\quad v\_nswOUT(b, wsec, p); \qquad \text{(SM.17b)}\\
&e\_techCapBal(b, tech, p)..\\
&sum((q, wsec), v\_f\_tech2dem(b, tech, wsec, q, p)) &&= l = p\_cap\_tech\_init(tech, b)\\
& &&+\quad v\_cap\_tech\_init(tech, b); \qquad \text{(SM.17c)}\\
&e\_demConsBal(b, tech, wsec, p)..\\
&sum((q), v\_f\_tech2demCons(b, tech, wsec, q, p)) &&= e = sum((q), v\_f\_tech2dem(b, tech, wsec, q, p))\\
& &&*\quad p\_ET\_tech(tech); \qquad \text{(SM.17d)}\\
&e\_UncapWasteBalC(b, tech, wsec, p)\$(tech\_cnt(tech))..\\
&sum((q), v\_wwstUnCaptIN\_techC(b, tech, wsec, q, p)) &&= e = (sum((q), v\_f\_tech2dem(b, tech, wsec, q, p))\\
& &&-\quad sum((q), v\_f\_tech2demCons(b, tech, wsec, q, p)))\\
& &&*\quad (1 - p\_WasteCapPrcnt(tech)); \qquad \text{(SM.17e)}\\
&e\_UncapWasteBalD(b, tech, wsec, p)\$(tech\_dir(tech))..\\
&sum((q), v\_wwstUnCaptIN\_techD(b, tech, wsec, q, p)) &&= e = (sum((q), v\_f\_tech2dem(b, tech, wsec, q, p))\\
& &&-\quad sum((q), v\_f\_tech2demCons(b, tech, wsec, q, p)))\\
& &&*\quad (1 - p\_WasteCapPrcnt(tech)); \qquad \text{(SM.17f)}\\
&e\_wwstUncapCconvert(b, p)..\\
&sum((tech, wsec, q),\\
&v\_wwstUnCaptIN\_techC(b, tech, wsec, q, p)) &&= e = sum((tech, wsec, q),\\
& &&\quad v\_wwstUnCaptOUT\_techC(b, tech, wsec, q, p)); \qquad \text{(SM.17g)}\\
&e\_wwstUncapDconvert(b, p)..\\
&sum((tech, wsec, q),\\
&v\_wwstUnCaptIN\_techD(b, tech, wsec, q, p)) &&= e = sum((tech, wsec, q),\\
& &&\quad v\_wwstUnCaptOUT\_techD(b, tech, wsec, q, p)); \qquad \text{(SM.17h)}\\
&e\_wwstCconvert(b, p)..\\
&sum((tech, wsec, q), v\_wwstCaptC\_IN(b, tech, wsec, q, p)) &&= e = sum((q), v\_wwstCaptCOUT(b, q, p)); \qquad \text{(SM.17i)}\\
&e\_wwstDconvert(b, p)..\\
&sum((tech, wsec, q), v\_wwstCaptD\_IN(b, tech, wsec, q, p)) &&= e = sum((q), v\_wwstCaptDOUT(b, q, p)); \qquad \text{(SM.17j)}
\end{aligned}
$$

SM.2.17. *Waste Water Collection Flows*

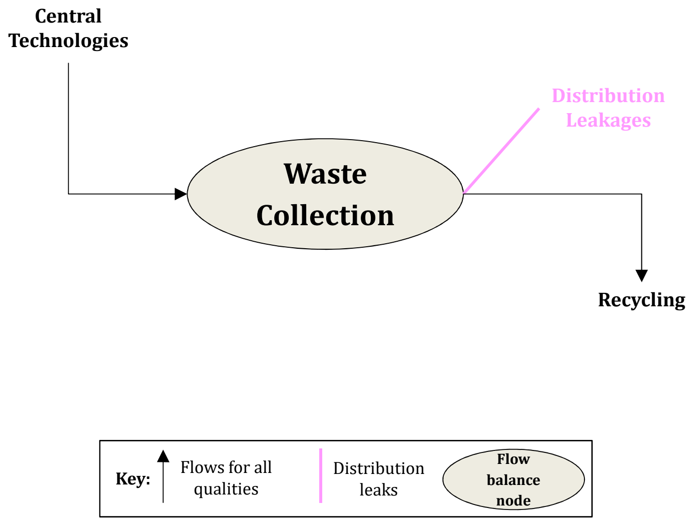

Figure SM.18: Waste water flow balance

Waste Water Balance: Figure SM.18 shows the flows for centrally captured waste water balance. The flows in the figure are defined as shown in Equation SM.18a where water captured from centrally connected technologies is equal to the water collected and sent to the recycling plants less any water lost during the transfer. Equation SM.18b limits the waste water transported to the recycling system to less than the existing waste water collection capacity and any additional capacity installed.

$$\begin{aligned}
&e_distWBal(b,p)..\\
&sum((rcyl,q),\\
&v_wwstCaptCOUT(b,q,p)) \quad = e = sum((rcyl,q), v_wwstCaptCOUT2rcyl(b,rcyl,q,p))\\
&\qquad + \quad sum((rcyl,q), v_wleakD_wwstCaptCOUT2rcyl(b,rcyl,q,p));
\end{aligned} \tag{SM.18a}$$

$$\begin{aligned}
&e_cap_distW(b,p)..\\
&sum((q), v_wwstCaptCOUT(b,q,p)) = l = p_cap_distW_init(b)\\
&\qquad + \quad v_cap_distW_Inv(b);
\end{aligned} \tag{SM.18b}$$

SM.2.18. Recycling Flows

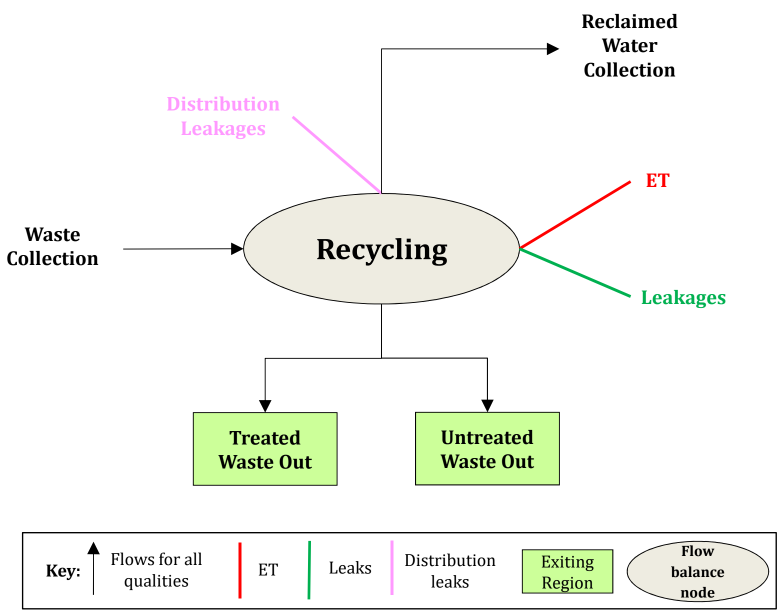

Figure SM.19: Recycling flow balance

Recycling balance: Figure SM.19 shows the flows for recycling water balance. The flows in the figure are defined as shown in Equation SM.19a where waste water captured sent to recycling plants is equal to treated water leaving the recycling plant, untreated water, leakages during the recycling process, water lost during the recycled water transportation and water lost to evapotranspiration during the recycling process. Equation SM.19b limits the water entering the recycling plants to be less than the existing capacity and any additional recycling capacity installed less any water that will not be treated.

$$\begin{aligned}
&e_rcylBal(b,p)..\\
&sum((rcyl,q),v_wwstCaptCOUT2rcyl(b,rcyl,q,p)) = e = sum((rcyl,q),v_f_rcyl2distR(b,rcyl,q,p))\\
&\quad + sum((rcyl,q),v_rcyl_untreat(b,q,p))\\
&\quad + sum((rcyl,q),v_wleak_rcyl(b,rcyl,q,p))\\
&\quad + sum((rcyl,q),v_wleakD_rcyl2distR(b,rcyl,q,p))\\
&\quad + sum((rcyl,q),v_ET_rcyl(b,rcyl,q,p));
\end{aligned} \tag{SM.19a}$$

$$\begin{aligned}
&e_cap_rcyl_Inv(rcyl,b,p)..\\
&sum((q),v_wwstCaptCOUT2rcyl(b,rcyl,q,p)) = e = p_cap_rcyl_init(rcyl,b)\\
&\quad + v_cap_rcyl_Inv(rcyl,b)\\
&\quad + sum((q),v_rcyl_untreat(b,rcyl,q,p));
\end{aligned} \tag{SM.19b}$$

SM.2.19. *Water Reuse Distribution Flows*

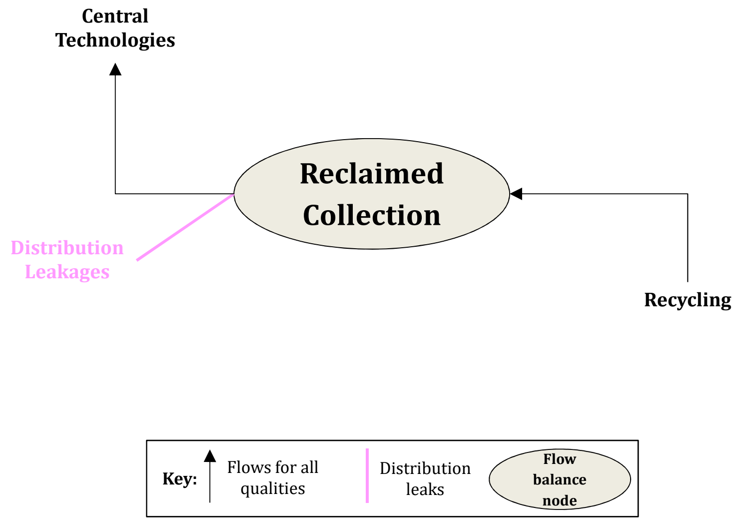

Figure SM.20: Water reuse flow balance

Reuse balance: Figure SM.20 shows the flows for recycled water distribution flows balance. The flows in the figure are defined as shown in Equation SM.20a where recycled water leaving the recycling plants to the distribution system is equal to waste water sent to central technologies less any water lost in the distribution system. Equation SM.20b limits the recycled water sent to the distribution system to less than the existing and any additional recycled water distribution capacity.

$$
\begin{aligned}
&e\_distRBal(b,q,p)..\\
&sum(rcyl, v\_f\_rcyl2distR(b,rcyl,q,p)) &&= e = sum(tech, v\_f\_distR2tech(b,tech,q,p))\\
& &&+ \quad sum(tech, v\_wleakD\_distR2tech(b,tech,q,p))
\end{aligned}
\tag{SM.20a}
$$

$$
\begin{aligned}
&e\_cap\_distR(b,p)..\\
&sum((rcyl,q), v\_f\_rcyl2distR(b,rcyl,q,p)) &&= l = p\_cap\_distR\_init(b)\\
& &&+ \quad v\_cap\_distR\_Inv(b);
\end{aligned}
\tag{SM.20b}
$$

SM.2.20. Runoff Flows Balance

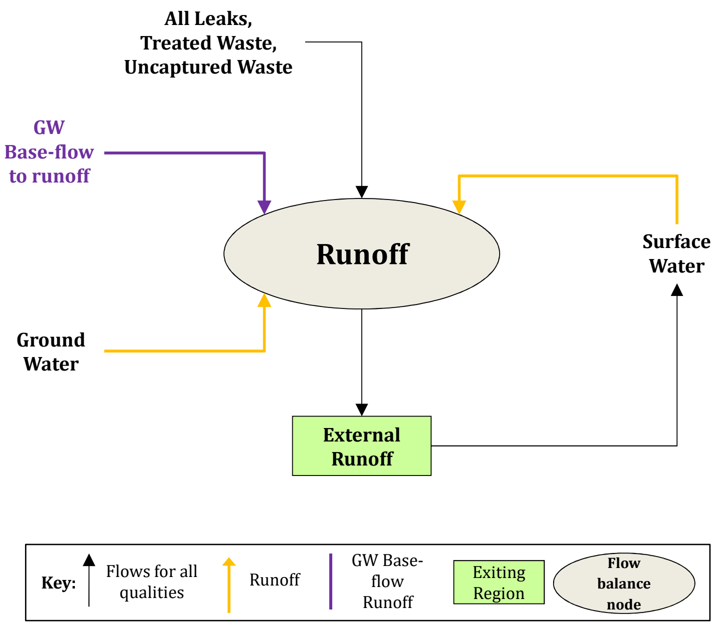

Figure SM.21: Demand flow balance

Demand Balance: Figure SM.21 shows the flows for reservoir water balance. The flows in the figure are defined as shown in Equation SM.21a which captures all the leakages from processes and distribution, treated and untreated captured waste water, uncaptured waste water, water released to the environment as well as runoff from groundwater and surface water. Equation SM.21b and Equation SM.21c define what percentage of runoff from each sub-region flow into other sub-regions and what percentage flow into external regions or water bodies.

$$
\begin{aligned}
&e\_f\_internal\_runoff\_hm3(b,q,p)..\\
&v\_f\_internal\_runoff\_hm3(b,q,p) \quad =e= v\_f\_surf2runoff(b,q,p)\\
&\quad + \; v\_f\_gw2runoff\_hm3(b,q,p)\\
&\quad + \; p\_gw2runoffBaseflow\_hm3(b,q,p)\\
&\quad + \; sum(rcyl, v\_rcyl\_untreat(b,rcyl,q,p))\\
&\quad + \; sum(rcyl, v\_rcyl\_treatedWaste2runoff(b,rcyl,q,p))\\
&\quad + \; v\_wleak\_distC(b,q,p)\\
&\quad + \; v\_wleak\_distD(b,q,p)\\
&\quad + \; sum((rcyl), v\_wleakD\_rcyl2distR(b,rcyl,q,p))\\
&\quad + \; sum((tech), v\_wleakD\_distR2tech(b,tech,q,p))\\
&\quad + \; sum((rcyl), v\_wleakD\_wwstCaptCOUT2rcyl(b,rcyl,q,p))\\
&\quad + \; sum((rcyl), v\_wleakD\_techC2distWOUT(b,rcyl,q,p))\\
&\quad + \; sum((tech,wsec), v\_wleak\_tech(b,tech,wsec,q,p))\\
&\quad + \; v\_wleak\_rwharvC(b,q,p)\\
&\quad + \; v\_wleak\_rwharvD(b,q,p)\\
&\quad + \; sum((pur), v\_wleak\_pur(b,pur,q,p))\\
&\quad + \; sum((rcyl), v\_wleak\_rcyl(b,rcyl,q,p))\\
&\quad + \; sum((dsal), v\_wleak\_dsal(b,dsal,q,p))\\
&\quad + \; v\_wleak\_gwpump(b,q,p)\\
&\quad + \; v\_wleak\_res(b,q,p)\\
&\quad + \; v\_f\_surf2env\_hm3(b,q,p)\\
&\quad + \; sum((tech,wsec), v\_wwstUnCaptIN\_techC(b,tech,wsec,q,p))\\
&\quad + \; sum((tech,wsec), v\_wwstUnCaptIN\_techD(b,tech,wsec,q,p));
\end{aligned} \tag{SM.21a}
$$

$$
\begin{aligned}
&e\_f\_b2b\_runoff\_hm3(bf,bt,q,p)..\\
&v\_f\_b2b\_runoff\_hm3(bf,bt,q,p) \quad =e= v\_f\_internal\_runoff\_hm3(bf,q,p)\\
&\quad * \; p\_interbasin\_runoff\_prcnt(bf,bt);
\end{aligned} \tag{SM.21b}
$$

$$
\begin{aligned}
&e\_f\_external\_runoff\_hm3(bf,q,p)..\\
&v\_f\_external\_runoff\_hm3(bf,q,p) \quad =e= v\_f\_internal\_runoff\_hm3(bf,q,p)\\
&\quad - \; sum(bt, v\_f\_b2b\_runoff\_hm3(bf,bt,q,p));
\end{aligned} \tag{SM.21c}
$$

SM.2.21. Overall Mass Balance

Overall Mass Balance: As introduced in Section SM.1 in Equation SM.1 overall mass balance is ensured by constraining water entering the system (rainfall, inter-basin transfers and desalinated water) to equal water leaving the system (evapotranspiration, runoff and interbasin transfers out of the basin). Equation SM.22a and Equation SM.22b define this balance in the model and account for the evapotranspiration, runoff, water leaving the system as interbasin transfers, water entering the system as interbasin transfers in, water entering the system from the ocean after desalination, water captured by direct users as well as any changes in storage capacity.

$$\begin{aligned}
&e_watBal(b,p)..\\
&v_watbalCheck(b,p) = e = sum(q, p_f_precip2ET_Calc(b,q,p))\\
&\qquad - \quad sum(q, p_rainfall_hm3(b,q,p))\\
&\qquad + \quad sum(q, v_f_surf2ET(b,q,p))\\
&\qquad + \quad sum(q, v_f_env2ET(b,q,p))\\
&\qquad + \quad sum(q, v_f_res2ET(b,q,p))\\
&\qquad + \quad sum((dsal,q), v_f_dsal2ET(b,dsal,q,p))\\
&\qquad + \quad sum(q, v_f_IB2ET(b,q,p))\\
&\qquad + \quad sum((tech,q), v_f_distC2techET(b,tech,q,p))\\
&\qquad + \quad sum((pur,q), v_f_pur2ET(b,pur,q,p))\\
&\qquad + \quad sum((tech,wsec,q), v_ET_tech(b,tech,wsec,q,p))\\
&\qquad + \quad sum((rcyl,q), v_f_rcyl2distRET(b,rcyl,q,p))\\
&\qquad + \quad sum(q, v_f_internal_runoff_hm3(b,q,p))\\
&\qquad - \quad sum((bf,q), v_f_b2b_runoff_hm3(bf,b,q,p))\\
&\qquad + \quad sum((tech,wsec,q), v_wwstCaptD_IN(b,tech,wsec,q,p))\\
&\qquad + \quad sum(q, v_strChng_soilSnow(b,q,p))\\
&\qquad + \quad sum(q, v_strChng_rwharvC(b,q,p))\\
&\qquad + \quad sum(q, v_strChng_rwharvD(b,q,p))\\
&\qquad + \quad sum(q, v_strChng_gw(b,q,p))\\
&\qquad + \quad sum(q, v_strChng_res(b,q,p))\\
&\qquad + \quad sum((rcyl,q), v_rcyl_treatedWaste2runoff(b,rcyl,q,p))\\
&\qquad + \quad sum(q, v_f_b2b_vol_out(b,q,p))\\
&\qquad - \quad sum(q, v_f_b2b_vol_in(b,q,p))\\
&\qquad - \quad sum((q,dsal), v_f_ocean2dsal(b,dsal,q,p)); \qquad \text{(SM.22a)}
\end{aligned}$$

$$\begin{aligned}
&e_watBal(b,p)..\\
&v_watbalCheck(b,p) = e = 0; \qquad \text{(SM.22b)}
\end{aligned}$$

SM.2.22. Total Direct Extraction Flows

Equation SM.23 shows the flows of all water extracted by direct users from surface waters, groundwater, desalination and rainwater harvesting. Equation SM.23 limits the volume of extracted water to less than existing extraction capacity plus any additional installed extraction capacity. Extraction refers to the process of transferring source water to the distribution network supplying final user technologies.

$$\begin{aligned}
&e_f_extract2distDFlows(b,q,p)..\\
&v_f_extract2distDFlows(b,q,p) &&= e = v_f_surf2distD(b,q,p)\\
& &&+ \quad v_f_gwpump2distD(b,q,p)\\
& &&+ \quad sum(dsal, v_f_dsal2distD(b,dsal,q,p))\\
& &&+ \quad v_f_rwharvD2distD(b,q,p); \qquad \text{(SM.22c)}\\
&e_cap_extract2distD(b,p)..\\
&sum((q), v_f_extract2distDFlows(b,rcyl,q,p)) &&= l = p_cap_extract2distD_init(b)\\
& &&+ \quad v_cap_extract2distD_Inv(b); \qquad \text{(SM.22d)}
\end{aligned}$$

SM.2.23. Total Centralized Extraction Flows

Equation SM.23 shows the flows of all water extracted by the central water management authority from reservoirs, groundwater, desalination and rainwater harvesting. Equation SM.23 limits the volume of extracted water to less than existing extraction capacity plus any additional installed extraction capacity. Extraction refers to the process of transferring source water to the distribution network supplying final user technologies.

$$\begin{aligned}
&e_f_extract2distCFlows(b,q,p)..\\
&v_f_extract2distCFlows(b,q,p) &&= e = v_f_res2distC(b,q,p)\\
& &&+ \quad v_f_gwpump2distC(b,q,p)\\
& &&+ \quad sum(dsal, v_f_dsal2distC(b,dsal,q,p))\\
& &&+ \quad v_f_rwharvD2distC(b,q,p); \qquad \text{(SM.22e)}\\
&e_cap_extract2distC(b,p)..\\
&sum((q), v_f_extract2distCFlows(b,rcyl,q,p)) &&= l = p_cap_extract2distC_init(b)\\
& &&+ \quad v_cap_extract2distC_Inv(b); \qquad \text{(SM.22f)}
\end{aligned}$$

SM.2.24. Total Direct Distribution Flows

Equation SM.23 shows the flows of all water distributed to final direct user technologies and to waste water collection sites.

$$\begin{aligned}
&e_f_distDFlows(b,q,p)..\\
&v_f_distDFlows(b,q,p) &&= e = sum((tech), v_f_distD2tech(b,tech,q,p))\\
& &&+ \quad sum((tech,wsec), v_wwstCaptD_IN(b,tech,wsec,q,p)); \qquad \text{(SM.23)}
\end{aligned}$$

SM.2.25. Total Centralized Distribution Flows

Equation SM.24 shows the flows of all water distributed to final centrally connected user technologies. The central distribution system also accounts for water distributed to and from purification plants as well as captured waste water.

$$
\begin{aligned}
&e\_f\_distCFlows(b,q,p)..\\
&v\_f\_distCFlows(b,q,p) \quad = e = sum((tech), v\_f\_distC2tech(b,tech,q,p))\\
&\qquad + \quad sum((pur), v\_f\_pur2distC(b,pur,q,p))\\
&\qquad + \quad sum((pur), v\_f\_distC2pur(b,pur,q,p))\\
&\qquad + \quad sum((tech,wsec), v\_wwstCaptC\_IN(b,tech,wsec,q,p)); \qquad \text{(SM.24)}
\end{aligned}
$$

SM.3. Water Model Costs, Energy, Evapotranspiration and Leakage Equations

This section lists the equations used to calculate various variables such as investment costs, operation costs, evapotranspiration and leakages in the different water processes. For most of the processes the calculation is the volume of water processed times a parameter defining the parameter value per unit volume of water. When data is available the parameter is defined per sub-region and sub-period to reflect regional and seasonal differences. In other cases it is defined as a simple scalar. Investment costs are calculated as an amortized annuity. The annuity is calculated by multiplying the expected total investment by an amortization parameter ($s\_amortMulti\_$) based on the interest rate and expected life span of each technology.

SM.3.1. Direct Rainwater Harvesting Variables

$$\begin{aligned} &e\_cost\_rwharvD\_OnM(b,q,p)..\\ &v\_cost\_rwharvD\_OnM(b,q,p) \quad =e= v\_f\_rwharvD2distD(b,q,p) * s\_cost\_rwharv\_OnM; \end{aligned} \tag{SM.25a}$$

$$\begin{aligned} &e\_cost\_rwharvD\_Inv(b)..\\ &v\_cost\_rwharvD\_Inv(b) \quad =e= v\_cap\_rwharvD\_InvCap\_hm3(b)\\ &\qquad * \quad s\_amortMulti\_rwharv * s\_cost\_rwharv\_Inv; \end{aligned} \tag{SM.25b}$$

$$\begin{aligned} &e\_wleak\_rwharvD(b,q,p)..\\ &v\_wleak\_rwharvD(b,q,p) \quad =e= v\_f\_rwharvD2distD(b,q,p) * s\_wleak\_rwharv; \end{aligned} \tag{SM.25c}$$

$$\begin{aligned} &e\_wleakD\_rwharvD2distD(b,q,p)..\\ &v\_wleakD\_rwharvD2distD(b,q,p) \quad =e= v\_f\_rwharvD2distD(b,q,p) * s\_wleak\_distD; \end{aligned} \tag{SM.25d}$$

SM.3.2. Central Rainwater Harvesting Variables

$$
\begin{aligned}
&e\_cost\_rwharvC\_OnM(b,q,p)..\\
&v\_cost\_rwharvC\_OnM(b,q,p) \quad =e= v\_f\_rwharvC2distC(b,q,p) * s\_cost\_rwharv\_OnM;
\end{aligned}
\tag{SM.26a}
$$

$$
\begin{aligned}
&e\_cost\_rwharvC\_Inv(b)..\\
&v\_cost\_rwharvC\_Inv(b) \quad =e= v\_cap\_rwharvC\_InvCap\_hm3(b)\\
&\qquad * \quad s\_amortMulti\_rwharv\\
&\qquad * \quad s\_cost\_rwharv\_Inv;
\end{aligned}
\tag{SM.26b}
$$

$$
\begin{aligned}
&e\_wleak\_rwharvC(b,q,p)..\\
&v\_wleak\_rwharvC(b,q,p) \quad =e= v\_f\_rwharvC2distC(b,q,p) * s\_wleak\_rwharv;
\end{aligned}
\tag{SM.26c}
$$

$$
\begin{aligned}
&e\_wleakD\_rwharvC2distC(b,q,p)..\\
&v\_wleakD\_rwharvC2distC(b,q,p) \quad =e= v\_f\_rwharvC2distC(b,q,p) * s\_wleak\_distC;
\end{aligned}
\tag{SM.26d}
$$

SM.3.3. Groundwater Pumping Variables

For groundwater pumping the investment cost in Equation SM.27b and the pumping energy in Equation SM.27c are based on the depth of the groundwater levels ($p\_gwlevel\_invDepth\_m$) from which the water needs to be pumped.

$$\begin{aligned} &e\_wcost\_gwpump\_OnM(b,q,p)..\\ &v\_cost\_gwpump\_OnM(b,q,p) \quad = e = v\_f\_ground2gwpump(b,q,p) * s\_cost\_gwpump\_OnM; \end{aligned} \tag{SM.27a}$$

$$\begin{aligned} &e\_wcost\_gwpump\_Inv(b)..\\ &v\_cost\_gwpump\_Inv(b) \quad = e = v\_cap\_gwpump\_Inv(b)\\ &\quad * \; s\_amortMulti\_gwpump\\ &\quad * \; p\_gwlevel\_InvDepth\_m(b)\\ &\quad * \; s\_cost\_gwpump\_Inv; \end{aligned} \tag{SM.27b}$$

$$\begin{aligned} &e\_wnrg\_gwpump(b,q,p)..\\ &v\_nrgGWh\_gwpump(b,q,p) \quad = e = v\_f\_ground2gwpump(b,q,p)\\ &\quad * \; p\_gwlevel\_invDepth\_m(b)\\ &\quad * \; s\_gravity\_mPerSec2\\ &\quad * \; s\_watDensity\_KgPerM3\\ &\quad * \; s\_gwpump\_Effic * 1000000\\ &\quad * \; s\_Joule2GWh; \end{aligned} \tag{SM.27c}$$

$$\begin{aligned} &e\_wleak\_gwpump(b,q,p)..\\ &v\_wleak\_gwpump(b,q,p) \quad = e = v\_f\_ground2gwpump(b,q,p) * s\_wleak\_gwpump; \end{aligned} \tag{SM.27d}$$

$$\begin{aligned} &e\_wleakD\_gwpump2distC(b,q,p)..\\ &v\_wleakD\_gwpump2distC(b,q,p) \quad = e = v\_f\_gwpump2distC(b,q,p) * s\_wleak\_distC; \end{aligned} \tag{SM.27e}$$

$$\begin{aligned} &e\_wleakD\_gwpump2distD(b,q,p)..\\ &v\_wleakD\_gwpump2distD(b,q,p) \quad = e = v\_f\_gwpump2distD(b,q,p) * s\_wleak\_distD; \end{aligned} \tag{SM.27f}$$

SM.3.4. Reservoir Variables

$$\begin{aligned}
&e_wcost_res_OnM(b,q,p)..\\
&v_cost_res_OnM(b,q,p) \quad = e = v_f_res2surf(b,q,p) * s_cost_res_OnM; \qquad \text{(SM.28a)}
\end{aligned}$$

$$\begin{aligned}
&e_wcost_res_Inv(b)..\\
&v_cost_res_Inv(b) \quad = e = v_cap_res_Inv(b)\\
&\qquad * \quad s_amortMulti_res\\
&\qquad * \quad s_cost_res_Inv; \qquad \text{(SM.28b)}
\end{aligned}$$

$$\begin{aligned}
&e_wleak_res(b,q,p)..\\
&v_wleak_res(b,q,p) \quad = e = v_f_res2surf(b,q,p) * s_wleak_res; \qquad \text{(SM.28c)}
\end{aligned}$$

$$\begin{aligned}
&e_wleakD_res2distC(b,q,p)..\\
&v_wleakD_res2distC(b,q,p) \quad = e = v_f_res2distC(b,q,p) * s_wleak_distC; \qquad \text{(SM.28d)}
\end{aligned}$$

$$\begin{aligned}
&e_wleakD_surf2distD(b,q,p)..\\
&v_wleakD_surf2distD(b,q,p) \quad = e = v_f_surf2distD(b,q,p) * s_wleak_distD; \qquad \text{(SM.28e)}
\end{aligned}$$

SM.3.5. Desalination Variables

$$\begin{aligned}
&e_wcost_dsal_OnM(b, dsal, q, p)..\\
&v_cost_dsal_OnM(b, dsal, q, p) &&= e = (v_f_dsal2distC(b, dsal, q, p)\\
& &&+ \quad v_f_dsal2distD(b, dsal, q, p)) * p_cost_dsal_OnM(dsal);
\end{aligned} \tag{SM.29a}$$

$$\begin{aligned}
&e_wcost_dsal_Inv(b, dsal)..\\
&v_cost_dsal_Inv(b, dsal) &&= e = v_cap_dsal_Inv(dsal, b)\\
& &&* \quad p_amortMulti_dsal(dsal)\\
& &&* \quad p_cost_dsal_Inv(dsal);
\end{aligned} \tag{SM.29b}$$

$$\begin{aligned}
&e_wnrg_dsal(b, dsal, q, p)..\\
&v_nrgGWh_dsal(b, dsal, q, p) &&= e = (v_f_dsal2distC(b, dsal, q, p)\\
& &&+ \quad v_f_dsal2distD(b, dsal, q, p)) * p_nrg_dsal(dsal);
\end{aligned} \tag{SM.29c}$$

$$\begin{aligned}
&e_wleak_dsal(b, dsal, q, p)..\\
&v_wleak_dsal(b, dsal, q, p) &&= e = (v_f_dsal2distC(b, dsal, q, p)\\
& &&+ \quad v_f_dsal2distD(b, dsal, q, p)) * p_wleak_dsal(dsal);
\end{aligned} \tag{SM.29d}$$

$$\begin{aligned}
&e_wleakD_dsal2distC(b, dsal, q, p)..\\
&v_wleakD_dsal2distC(b, dsal, q, p) &&= e = v_f_dsal2distC(b, dsal, q, p) * s_wleak_distC;
\end{aligned} \tag{SM.29e}$$

$$\begin{aligned}
&e_wleakD_dsal2distD(b, dsal, q, p)..\\
&v_wleakD_dsal2distD(b, dsal, q, p) &&= e = v_f_dsal2distD(b, dsal, q, p) * s_wleak_distD;
\end{aligned} \tag{SM.29f}$$

$$\begin{aligned}
&e_f_dsal2ET(b, dsal, q, p)..\\
&v_f_dsal2ET(b, dsal, q, p) &&= e = (v_f_dsal2distC(b, dsal, q, p)\\
& &&+ \quad v_f_dsal2distD(b, dsal, q, p)) * p_ET_dsal(dsal);
\end{aligned} \tag{SM.29g}$$

SM.3.6. Interbasin Variables

Interbasin transfer variables such as the investment costs and pumping energy is based on the distances between central distribution points in each basin and the head between these two points. Either the net head, assuming energy is regained when water is descending or the cumulative head, assuming no energy is regained during descent, or a combination of the two can be used.

$$\begin{aligned}
&e_trnsf_NRG(bf, bt, p)..\\
&v_trnsf_Nrg(bf, bt, p) && = e = s_watDensity_KgPerM3\\
&&& * \quad sum(q, v_f_b2b_vol(bf, bt, q, p))\\
&&& * \quad s_gravity_mPerSec2\\
&&& * \quad p_b2b_NetHead_km(bf, bt);
\end{aligned} \tag{SM.30a}$$

$$\begin{aligned}
&e_trnsf_invspipeCost(bf, bt)..\\
&v_trnsf_cost_Inv(bf, bt) && = e = s_cost_trnsfCap_Inv\\
&&& * \quad v_trnsfCap_vol_Inv(bf, bt)\\
&&& * \quad p_b2b_Dist_km(bf, bt);
\end{aligned} \tag{SM.30b}$$

$$\begin{aligned}
&e_wleak_IB(b, q, p)..\\
&v_wleak_IB(b, q, p) && = e = sum(bf, v_f_b2b_vol(bf, b, q, p)) * s_wleak_IB;
\end{aligned} \tag{SM.30c}$$

$$\begin{aligned}
&e_f_IB2ET(b, q, p)..\\
&v_f_IB2ET(b, q, p) && = e = sum(bf, v_f_b2b_vol(bf, b, q, p)) * s_ET_IB;
\end{aligned} \tag{SM.30d}$$

$$\begin{aligned} &e\_wcost\_extract2distD\_Inv(b,q,p)..\\ &v\_cost\_extract2distD\_Inv(b,q,p) &&= e = v\_cap\_extract2distD\_Inv(b)\\ & &&\quad * \quad s\_amortMulti\_extract2distD\\ & &&\quad * \quad s\_cost\_extract2distD\_Inv; \end{aligned} \tag{SM.31a}$$

$$\begin{aligned} &e\_wcost\_extract2distD\_OnM(b,q,p)..\\ &v\_cost\_extract2distD\_OnM(b,q,p) &&= e = v\_f\_extract2distDFlows(b,q,p)\\ & &&\quad * \quad s\_cost\_distD\_OnM; \end{aligned} \tag{SM.31b}$$

$$\begin{aligned} &e\_wnrg\_extract2distD(b,q,p)..\\ &v\_nrgGWh\_extract2distD(b,q,p) &&= e = v\_f\_extract2distDFlows(b,q,p)\\ & &&\quad * \quad s\_nrg\_extract2distD; \end{aligned} \tag{SM.31c}$$

$$\begin{aligned} &e\_wleak\_extract2distD(b,q,p)..\\ &v\_wleak\_extract2distD(b,q,p) &&= e = v\_wleakD\_rwharvD2distD(b,q,p)\\ & &&\quad + \quad sum(dsal, v\_wleakD\_dsal2distD(b,dsal,q,p))\\ & &&\quad + \quad v\_wleakD\_surf2distD(b,q,p)\\ & &&\quad + \quad v\_wleakD\_gwpump2distD(b,q,p; \end{aligned} \tag{SM.31d}$$

SM.3.8. Central Extraction Variables

$$
\begin{aligned}
&e\_wcost\_extract2distC\_Inv(b,q,p)..\\
&v\_cost\_extract2distC\_Inv(b,q,p) \quad =e= v\_cap\_extract2distC\_Inv(b)\\
&\qquad * \quad s\_amortMulti\_extract2distC\\
&\qquad * \quad s\_cost\_extract2distC\_Inv;
\end{aligned}
\tag{SM.32a}
$$

$$
\begin{aligned}
&e\_wcost\_extract2distC\_OnM(b,q,p)..\\
&v\_cost\_extract2distC\_OnM(b,q,p) \quad =e= v\_f\_extract2distCFlows(b,q,p)\\
&\qquad * \quad s\_cost\_distC\_OnM;
\end{aligned}
\tag{SM.32b}
$$

$$
\begin{aligned}
&e\_wnrg\_extract2distC(b,q,p)..\\
&v\_nrgGWh\_extract2distC(b,q,p) \quad =e= v\_f\_extract2distCFlows(b,q,p)\\
&\qquad * \quad s\_nrg\_extract2distC;
\end{aligned}
\tag{SM.32c}
$$

$$
\begin{aligned}
&e\_wleak\_extract2distC(b,q,p)..\\
&v\_wleak\_extract2distC(b,q,p) \quad =e= v\_wleakD\_rwharvC2distC(b,q,p)\\
&\qquad + \quad sum(dsal, v\_wleakD\_dsal2distC(b,dsal,q,p))\\
&\qquad + \quad v\_wleakD\_res2distC(b,q,p)\\
&\qquad + \quad v\_wleakD\_gwpump2distC(b,q,p;
\end{aligned}
\tag{SM.32d}
$$

SM.3.9. Direct Distribution Variables

$$\begin{aligned}
&e_wcost_distD_Inv(b,q,p)..\\
&v_cost_distD_Inv(b,q,p) &&= e = v_f_distDFlows(b,q,p)\\
& &&\quad * \quad s_amortMulti_distD\\
& &&\quad * \quad s_cost_distD_Inv; \qquad \text{(SM.33a)}
\end{aligned}$$

$$\begin{aligned}
&e_wcost_distD_OnM(b,q,p)..\\
&v_cost_distD_OnM(b,q,p) &&= e = v_f_distDFlows(b,q,p) * s_cost_distD_OnM; \qquad \text{(SM.33b)}
\end{aligned}$$

$$\begin{aligned}
&e_wnrg_distD(b,q,p)..\\
&v_nrgGWh_distD(b,q,p) &&= e = v_f_distDFlows(b,q,p) * s_nrg_distD; \qquad \text{(SM.33c)}
\end{aligned}$$

$$\begin{aligned}
&e_wleakD_distD2tech(b,tech,q,p)..\\
&v_wleakD_distD2tech(b,tech,q,p) &&= e = v_f_distD2tech(b,tech,q,p) * s_wleak_distD; \qquad \text{(SM.33d)}
\end{aligned}$$

$$\begin{aligned}
&e_wleak_distD(b,q,p)..\\
&v_wleak_distD(b,q,p) &&= e = sum(tech, v_wleakD_distD2tech(b,tech,q,p)); \qquad \text{(SM.33e)}
\end{aligned}$$

$$\begin{aligned}
&e_f_distD2techET(b,tech,q,p)..\\
&v_f_distD2techET(b,tech,q,p) &&= e = v_f_distD2tech(b,tech,q,p)\\
& &&\quad * \quad p_ET_distX(tech,b,p); \qquad \text{(SM.33f)}
\end{aligned}$$

268 *SM.3.10. Central Distribution Variable*

$$
\begin{aligned}
&e\_wcost\_distC\_Inv(b,q,p)..\\
&v\_cost\_distC\_Inv(b,q,p) && = e = v\_f\_distCFlows(b,q,p)\\
& && \quad * \quad s\_amortMulti\_distC\\
& && \quad * \quad s\_cost\_distC\_Inv; && \text{(SM.34a)}
\end{aligned}
$$

$$
\begin{aligned}
&e\_wcost\_distC\_OnM(b,q,p)..\\
&v\_cost\_distC\_OnM(b,q,p) && = e = v\_f\_distCFlows(b,q,p) * s\_cost\_distC\_OnM; && \text{(SM.34b)}
\end{aligned}
$$

$$
\begin{aligned}
&e\_wnrg\_distC(b,q,p)..\\
&v\_nrgGWh\_distC(b,q,p) && = e = v\_f\_distCFlows(b,q,p) * s\_nrg\_distC; && \text{(SM.34c)}
\end{aligned}
$$

$$
\begin{aligned}
&e\_wleakD\_distC2tech(b,tech,q,p)..\\
&v\_wleakD\_distC2tech(b,tech,q,p) && = e = v\_f\_distC2tech(b,tech,q,p) * s\_wleak\_distC; && \text{(SM.34d)}
\end{aligned}
$$

$$
\begin{aligned}
&e\_wleakD\_distC2pur(b,pur,q,p)..\\
&v\_wleakD\_distC2pur(b,pur,q,p) && = e = v\_f\_distC2pur(b,pur,q,p) * s\_wleak\_distC; && \text{(SM.34e)}
\end{aligned}
$$

$$
\begin{aligned}
&e\_wleak\_distC(b,q,p)..\\
&v\_wleak\_distC(b,q,p) && = e = sum(tech, v\_wleakD\_distC2tech(b,tech,q,p))\\
& && \quad + \quad sum(pur, v\_wleakD\_distC2pur(b,pur,q,p))\\
& && \quad + \quad sum(pur, v\_wleakD\_pur2distC(b,pur,q,p)); && \text{(SM.34f)}
\end{aligned}
$$

$$
\begin{aligned}
&e\_f\_distC2techET(b,tech,q,p)..\\
&v\_f\_distC2techET(b,tech,q,p) && = e = v\_f\_distC2tech(b,tech,q,p)\\
& && \quad * \quad p\_ET\_distX; && \text{(SM.34g)}
\end{aligned}
$$

SM.3.11. Technology Variables

$$\begin{aligned}
&e_wcost_tech_OnM(b, tech, wsec, q, p)..\\
&v_cost_tech_OnM(b, tech, wsec, q, p) \quad = e = v_f_tech2dem(b, tech, wsec, q, p) * p_cost_tech_OnM(tech);
\end{aligned} \tag{SM.35a}$$

$$\begin{aligned}
&e_wcost_tech_Inv(b, tech)..\\
&v_cost_tech_Inv(b, tech) \quad = e = v_cap_tech_Inv(tech, b)\\
&\qquad * \quad p_amortMulti_tech(tech)\\
&\qquad * \quad p_cost_tech_Inv(tech);
\end{aligned} \tag{SM.35b}$$

$$\begin{aligned}
&e_wnrg_tech(b, tech, wsec, q, p)..\\
&v_nrgGWh_tech(b, tech, wsec, q, p) \quad = e = v_f_tech2dem(b, tech, wsec, q, p) * p_nrg_tech(tech);
\end{aligned} \tag{SM.35c}$$

$$\begin{aligned}
&e_wleak_tech(b, tech, wsec, q, p)..\\
&v_wleak_tech(b, tech, wsec, q, p) \quad = e = v_f_tech2dem(b, tech, wsec, q, p) * p_wleak_tech(tech);
\end{aligned} \tag{SM.35d}$$

$$\begin{aligned}
&e_ET_tech(b, tech, wsec, q, p)..\\
&v_ET_tech(b, tech, wsec, q, p) \quad = e = v_f_tech2demCons(b, tech, wsec, q, p);
\end{aligned} \tag{SM.35e}$$

SM.3.12. Purification Variables

$$\begin{aligned}
&e_wcost_pur_OnM(b,pur,q,p)..\\
&v_cost_pur_OnM(b,pur,q,p) &&= e = v_f_pur2distC(b,pur,q,p) * p_cost_pur_OnM(pur);
\end{aligned} \tag{SM.36a}$$

$$\begin{aligned}
&e_wcost_pur_Inv(b,pur)..\\
&v_cost_pur_Inv(b,pur) &&= e = v_cost_pur_Inv(b,pur)\\
& && \quad * \quad p_amortMulti_pur(pur)\\
& && \quad * \quad p_cost_pur_Inv(pur);
\end{aligned} \tag{SM.36b}$$

$$\begin{aligned}
&e_wnrg_pur(b,pur,q,p)..\\
&v_nrgGWh_pur(b,pur,q,p) &&= e = v_f_pur2distC(b,pur,q,p) * p_nrg_pur(pur);
\end{aligned} \tag{SM.36c}$$

$$\begin{aligned}
&e_wleak_pur(b,pur,q,p)..\\
&v_wleak_pur(b,pur,q,p) &&= e = v_f_pur2distC(b,pur,q,p) * p_wleak_pur(pur);
\end{aligned} \tag{SM.36d}$$

$$\begin{aligned}
&e_wleakD_pur2distC(b,pur,q,p)..\\
&v_wleakD_pur2distC(b,pur,q,p) &&= e = v_f_pur2distC(b,pur,q,p) * s_wleak_distC;
\end{aligned} \tag{SM.36e}$$

$$\begin{aligned}
&e_f_pur2ET(b,pur,q,p)..\\
&v_f_pur2ET(b,pur,q,p) &&= e = v_f_pur2distC(b,pur,q,p) * p_ET_pur(pur);
\end{aligned} \tag{SM.36f}$$

SM.3.13. Waste Water Collection Variables

$$\begin{aligned}
&e_wcost_distW_Inv(b)..\\
&v_cost_distW_Inv(b) &&= e = v_cap_distW_{I}nv(b)\\
& && \quad * \quad s_amortMulti_distW\\
& && \quad * \quad s_cost_distW_Inv));
\end{aligned} \tag{SM.37a}$$

$$\begin{aligned}
&e_wcost_distW_OnM(b,q,p)..\\
&v_cost_distW_OnM(b,q,p) &&= e = sum((tech,wsec),(v_wwstCaptC_IN(b,tech,wsec,q,p)\\
& && \quad * \quad s_cost_distW_OnM));
\end{aligned} \tag{SM.37b}$$

$$\begin{aligned}
&e_wnrg_distW(b,q,p)..\\
&v_nrgGWh_distW(b,q,p) &&= e = sum((tech,wsec),(v_wwstCaptC_IN(b,tech,wsec,q,p)\\
& && \quad * \quad s_nrg_distW));
\end{aligned} \tag{SM.37c}$$

SM.3.14. Waste Water Treatment Variables

$$\begin{aligned}
&e_wcost_wasteTreat_OnM(b,rcyl,q,p)..\\
&v_cost_wasteTreat_OnM(b,rcyl,q,p) &&=e= v_rcyl_treatedWaste2runoff(b,rcyl,q,p)\\
& && \quad * \quad p_cost_rcyl_OnM(rcyl);
\end{aligned} \tag{SM.37d}$$

$$\begin{aligned}
&e_wnrg_distW(b,q,p)..\\
&v_nrgGWh_distW(b,q,p) &&=e= v_rcyl_treatedWaste2runoff(b,rcyl,q,p)\\
& && \quad * \quad p_nrg_rcyl(rcyl);
\end{aligned} \tag{SM.37e}$$

$$\begin{aligned}
&e_wleakD_distW2rcyl(b,p)..\\
&sum((rcyl,q),\\
&v_wleakD_wwstCaptCOUT2rcyl(b,rcyl,q,p)) &&=e= sum((rcyl,q), v_wwstCaptCOUT2rcyl(b,rcyl,q,p))\\
& && \quad *s_wleak_distW;
\end{aligned} \tag{SM.37f}$$

SM.3.15. Recycling Variables

$$\begin{aligned}
&e_wcost_rcyl_OnM(b,rcyl,q,p)..\\
&v_cost_rcyl_OnM(b,rcyl,q,p) &&=e= v_f_rcyl2distR(b,rcyl,q,p) * p_cost_rcyl_OnM(rcyl);
\end{aligned} \tag{SM.38a}$$

$$\begin{aligned}
&e_wcost_rcyl_Inv(b,rcyl)..\\
&v_cost_rcyl_Inv(b,rcyl) &&=e= v_cap_rcyl_Inv(rcyl,b)\\
& && \quad * \quad p_amortMulti_rcyl(rcyl)\\
& && \quad * \quad p_cost_rcyl_Inv(rcyl);
\end{aligned} \tag{SM.38b}$$

$$\begin{aligned}
&e_wnrg_rcyl(b,rcyl,q,p)..\\
&v_nrgGWh_rcyl(b,rcyl,q,p) &&=e= v_f_rcyl2distR(b,rcyl,q,p) * p_nrg_rcyl(rcyl);
\end{aligned} \tag{SM.38c}$$

$$\begin{aligned}
&e_wleak_rcyl(b,rcyl,q,p)..\\
&v_wleak_rcyl(b,rcyl,q,p) &&=e= v_f_rcyl2distR(b,rcyl,q,p) * p_wleak_rcyl(rcyl);
\end{aligned} \tag{SM.38d}$$

$$\begin{aligned}
&e_ET_rcyl(b,rcyl,q,p)..\\
&v_ET_rcyl(b,rcyl,q,p) &&=e= v_f_rcyl2distR(b,rcyl,q,p) * s_ET_rcyl;
\end{aligned} \tag{SM.38e}$$

SM.3.16. Water Reuse Variables

$$\begin{aligned}
&e_wcost_distR_Inv(b)..\\
&v_cost_distR_Inv(b) && = e = v_cap_distR_Inv(b)\\
& && * \quad s_amortMulti_distR\\
& && * \quad *s_cost_distR_Inv)); \qquad \text{(SM.39a)}
\end{aligned}$$

$$\begin{aligned}
&e_wcost_distR_OnM(b,q,p)..\\
&v_cost_distR_OnM(b,q,p) && = e = sum((rcyl),(v_f_rcyl2distR(b,rcyl,q,p)\\
& && * \quad s_cost_distR_OnM)); \qquad \text{(SM.39b)}
\end{aligned}$$

$$\begin{aligned}
&e_wnrg_distR(b,q,p)..\\
&v_nrgGWh_distR(b,q,p) && = e = sum((rcyl),(v_f_rcyl2distR(b,rcyl,q,p)\\
& && * \quad s_nrg_distR)); \qquad \text{(SM.39c)}
\end{aligned}$$

$$\begin{aligned}
&e_wleak_distR(b,q,p)..\\
&v_wleak_distR(b,q,p) && = e = sum((rcyl), v_wleakD_rcyl2distR(b,rcyl,q,p)); \qquad \text{(SM.39d)}
\end{aligned}$$

$$\begin{aligned}
&e_wleakD_rcyl2distR(b,rcyl,q,p)..\\
&v_wleakD_rcyl2distR(b,rcyl,q,p) && = e = v_f_rcyl2distR(b,rcyl,q,p) * s_wleak_distR; \qquad \text{(SM.39e)}
\end{aligned}$$

SM.3.17. Non-Served Water Variables

$$\begin{aligned}
&e_cost_nsw(b,wsec,q,p)..\\
&v_cost_nsw(b,wsec,q,p) && = e = v_nswOUT(b,wsec,p) * p_cost_nsw(wsec); \qquad \text{(SM.40)}
\end{aligned}$$

SM.4. Sub-Region Annual Summary Equations

This section summarizes the various variables by summing over different parameters and time-periods to give the total annual values.

SM.4.1. Sub-Region Flow Balance

$$\begin{aligned} &e\_dsalIN\_b(q,b)..\\ &v\_dsalIN\_b(q,b) \quad = e = sum(p, v\_f\_ocean2dsal(b,q,p)); \end{aligned} \tag{SM.41a}$$

$$\begin{aligned} &e\_purIN\_b(q,b)..\\ &v\_purIN\_b(q,b) \quad = e = sum((pur,p), v\_f\_distC2pur(b,pur,q,p)); \end{aligned} \tag{SM.41b}$$

$$\begin{aligned} &e\_rcylIN\_b(q,b)..\\ &v\_rcylIN\_b(q,b) \quad = e = sum((tech,wsec,rcyl,p), v\_wwstCaptC\_IN(b,tech,wsec,q,p)); \end{aligned} \tag{SM.41c}$$

$$\begin{aligned} &e\_dsalOUT\_b(q,b)..\\ &v\_dsalOUT\_b(q,b) \quad = e = sum(p, v\_f\_dsal2distC(b,q,p)); \end{aligned} \tag{SM.41d}$$

$$\begin{aligned} &e\_purOUT\_b(q,b)..\\ &v\_purOUT\_b(q,b) \quad = e = sum((pur,p), v\_f\_pur2distC(b,pur,q,p)); \end{aligned} \tag{SM.41e}$$

$$\begin{aligned} &e\_rcylOUT\_b(q,b)..\\ &v\_rcylOUT\_b(q,b) \quad = e = sum((rcyl,p), v\_f\_rcyl2distR(b,rcyl,q,p)); \end{aligned} \tag{SM.41f}$$

$$\begin{aligned} &e\_gwpumpIN\_b(q,b)..\\ &v\_gwpumpIN\_b(q,b) \quad = e = sum(p, v\_f\_ground2gwpump(b,q,p)); \end{aligned} \tag{SM.41g}$$

$$\begin{aligned} &e\_gwpumpOUT\_b(q,b)..\\ &v\_gwpumpOUT\_b(q,b) \quad = e = sum(p, v\_f\_gwpump2distC(b,q,p)); \end{aligned} \tag{SM.41h}$$

$$
\begin{aligned}
&e\_cost\_nsw\_b(b)..\\
&v\_wcost\_nsw\_b(b) \quad =e= sum((wsec,q,p), v\_cost\_nsw(b,wsec,q,p)); && \text{(SM.42a)}
\end{aligned}
$$

$$
\begin{aligned}
&e\_wcost\_OnM\_b(b)..\\
&v\_wcost\_OnM\_b(b) \quad =e= sum((tech,wsec,q,p), v\_cost\_tech\_OnM(b,tech,wsec,q,p))\\
&\quad + \quad sum((q,p), v\_cost\_extract2distC\_OnM(b,q,p))\\
&\quad + \quad sum((q,p), v\_cost\_extract2distD\_OnM(b,q,p))\\
&\quad + \quad sum((q,p), v\_cost\_distC\_OnM(b,q,p))\\
&\quad + \quad sum((q,p), v\_cost\_distD\_OnM(b,q,p))\\
&\quad + \quad sum((q,p), v\_cost\_distW\_OnM(b,q,p))\\
&\quad + \quad sum((q,p), v\_cost\_distR\_OnM(b,q,p))\\
&\quad + \quad sum((q,pur,p), v\_cost\_pur\_OnM(b,pur,q,p))\\
&\quad + \quad sum((q,rcyl,p), v\_cost\_rcyl\_OnM(b,rcyl,q,p))\\
&\quad + \quad sum((q,rcyl,p), v\_cost\_wasteTreat\_OnM(b,q,p))\\
&\quad + \quad sum((q,p), v\_cost\_dsal\_OnM(b,q,p))\\
&\quad + \quad sum((q,p), v\_cost\_gwpump\_OnM(b,q,p))\\
&\quad + \quad sum((q,p), v\_cost\_res\_OnM(b,q,p))\\
&\quad + \quad sum((q,p), v\_cost\_rwharvC\_OnM(b,q,p))\\
&\quad + \quad sum((q,p), v\_cost\_rwharvD\_OnM(b,q,p)); && \text{(SM.42b)}
\end{aligned}
$$

$$
\begin{aligned}
&e\_wcost\_Inv\_b(b)..\\
&v\_wcost\_Inv\_b(b) \quad =e= sum((tech), v\_cost\_tech\_Inv(b,tech))\\
&\quad + \quad sum((q,pur), v\_cost\_pur\_Inv(b,pur))\\
&\quad + \quad sum((q,rcyl), v\_cost\_rcyl\_Inv(b,rcyl))\\
&\quad + \quad v\_cost\_dsal\_Inv(b)\\
&\quad + \quad v\_cost\_extract2distC\_Inv(b)\\
&\quad + \quad v\_cost\_extract2distD\_Inv(b)\\
&\quad + \quad v\_cost\_distD\_Inv(b)\\
&\quad + \quad v\_cost\_distC\_Inv(b)\\
&\quad + \quad v\_cost\_distW\_Inv(b)\\
&\quad + \quad v\_cost\_distR\_Inv(b)\\
&\quad + \quad v\_cost\_res\_Inv(b)\\
&\quad + \quad v\_cost\_gwpump\_Inv(b)\\
&\quad + \quad v\_cost\_rwharvC\_Inv(b)\\
&\quad + \quad v\_cost\_rwharvD\_Inv(b); && \text{(SM.42c)}
\end{aligned}
$$

$$
\begin{aligned}
&e\_wnrg\_b(b)..\\
&v\_nrgGWh\_b(b) \quad = e = sum((tech, wsec, q, p), v\_nrgGWh\_tech(b, tech, wsec, q, p))\\
&\quad + \quad sum((q, p), v\_nrgGWh\_extract2distC(b, q, p))\\
&\quad + \quad sum((q, p), v\_nrgGWh\_extract2distD(b, q, p))\\
&\quad + \quad sum((q, p), v\_nrgGWh\_distC(b, q, p))\\
&\quad + \quad sum((q, p), v\_nrgGWh\_distD(b, q, p))\\
&\quad + \quad sum((q, p), v\_nrgGWh\_distW(b, q, p))\\
&\quad + \quad sum((q, p), v\_nrgGWh\_distR(b, q, p))\\
&\quad + \quad sum((q, pur, p), v\_nrgGWh\_pur(b, pur, q, p))\\
&\quad + \quad sum((q, rcyl, p), v\_nrgGWh\_rcyl(b, rcyl, q, p))\\
&\quad + \quad sum((q, p), v\_nrgGWh\_dsal(b, q, p))\\
&\quad + \quad sum((q, p), v\_nrgGWh\_gwpump(b, q, p)); \qquad \text{(SM.42d)}
\end{aligned}
$$

$$
\begin{aligned}
&e\_wnrg\_b\_p(b, p)..\\
&v\_wnrg\_b\_p(b, p) \quad = e = sum((tech, wsec, q), v\_nrgGWh\_tech(b, tech, wsec, q, p))\\
&\quad + \quad sum((q), v\_nrgGWh\_extract2distC(b, q, p))\\
&\quad + \quad sum((q), v\_nrgGWh\_extract2distD(b, q, p))\\
&\quad + \quad sum((q), v\_nrgGWh\_distC(b, q, p))\\
&\quad + \quad sum((q), v\_nrgGWh\_distD(b, q, p))\\
&\quad + \quad sum((q), v\_nrgGWh\_distW(b, q, p))\\
&\quad + \quad sum((q), v\_nrgGWh\_distR(b, q, p))\\
&\quad + \quad sum((q, pur), v\_nrgGWh\_pur(b, pur, q, p))\\
&\quad + \quad sum((q, rcyl), v\_nrgGWh\_rcyl(b, rcyl, q, p))\\
&\quad + \quad sum((q), v\_nrgGWh\_dsal(b, q, p))\\
&\quad + \quad sum((q), v\_nrgGWh\_gwpump(b, q, p)); \qquad \text{(SM.42e)}
\end{aligned}
$$

$$
\begin{aligned}
&e\_wleak\_b(b)..\\
&v\_wleak\_b(b) && = e = sum((tech, wsec, q, p), v\_wleak\_tech(b, tech, wsec, q, p))\\
& && + \quad sum((q, p), v\_wleak\_extract2distC(b, q, p))\\
& && + \quad sum((q, p), v\_wleak\_extract2distD(b, q, p))\\
& && + \quad sum((q, p), v\_wleak\_distC(b, q, p))\\
& && + \quad sum((q, p), v\_wleak\_distD(b, q, p))\\
& && + \quad sum((q, p), v\_wleak\_distW(b, q, p))\\
& && + \quad sum((q, p), v\_wleak\_distR(b, q, p))\\
& && + \quad sum((q, pur, p), v\_wleak\_pur(b, pur, q, p))\\
& && + \quad sum((q, rcyl, p), v\_wleak\_rcyl(b, rcyl, q, p))\\
& && + \quad sum((q, p), v\_wleak\_dsal(b, q, p))\\
& && + \quad sum((q, p), v\_wleak\_gwpump(b, q, p))\\
& && + \quad sum((q, p), v\_wleak\_rwharvC(b, q, p))\\
& && + \quad sum((q, p), v\_wleak\_rwharvD(b, q, p));
\end{aligned}
\tag{SM.42f}
$$

$$
\begin{aligned}
&e\_wwstC\_b(b)..\\
&v\_wwstC\_b(b) && = e = sum((tech, wsec, q, p), v\_wwstCaptC\_IN(b, tech, wsec, q, p))\\
& && + \quad sum((tech, wsec, q, p), v\_wwstUnCaptIN\_techC(b, tech, wsec, q, p));
\end{aligned}
\tag{SM.42g}
$$

$$
\begin{aligned}
&e\_wwstD\_b(b)..\\
&v\_wwstD\_b(b) && = e = sum((tech, wsec, q, p), v\_wwstCaptD\_IN(b, tech, wsec, q, p))\\
& && + \quad sum((tech, wsec, q, p), v\_wwstUnCaptIN\_techD(b, tech, wsec, q, p));
\end{aligned}
\tag{SM.42h}
$$

$$
\begin{aligned}
&e\_nsw\_b(wsec, q, b)..\\
&v\_nsw\_b(wsec, b) && = e = sum(p, v\_nswOUT(b, wsec, p));
\end{aligned}
\tag{SM.42i}
$$

$$
\begin{aligned}
&e\_surf2distC\_b(q, b)..\\
&v\_surf2distC\_b(q, b) && = e = sum(p, v\_f\_surf2distC(b, q, p));
\end{aligned}
\tag{SM.42j}
$$

$$
\begin{aligned}
&e\_surf2distD\_b(q, b)..\\
&v\_surf2distD\_b(q, b) && = e = sum(p, v\_f\_surf2distD(b, q, p));
\end{aligned}
\tag{SM.42k}
$$

SM.5. Combined Regions Annual Summary Equations

This section further summarizes the sub-region variables to get aggregated values for the entire regional scope.

SM.5.1. Combined Regions Flow Balance

$$\begin{aligned}&e\_dsalIN(q)..\\&v\_dsalIN(q) \quad = e = sum(b, v\_dsalIN\_b(q,b));\end{aligned} \tag{SM.43a}$$

$$\begin{aligned}&e\_purIN(q)..\\&v\_purIN(q) \quad = e = sum(b, v\_purIN\_b(q,b));\end{aligned} \tag{SM.43b}$$

$$\begin{aligned}&e\_rcylIN(q)..\\&v\_rcylIN(q) \quad = e = sum(b, v\_rcylIN\_b(q,b));\end{aligned} \tag{SM.43c}$$

$$\begin{aligned}&e\_dsalOUT(q)..\\&v\_dsalOUT(q) \quad = e = sum(b, v\_dsalOUT\_b(q,b));\end{aligned} \tag{SM.43d}$$

$$\begin{aligned}&e\_purOUT(q)..\\&v\_purOUT(q) \quad = e = sum(b, v\_purOUT\_b(q,b));\end{aligned} \tag{SM.43e}$$

$$\begin{aligned}&e\_rcylOUT(q)..\\&v\_rcylOUT(q) \quad = e = sum(b, v\_rcylOUT\_b(q,b));\end{aligned} \tag{SM.43f}$$

$$\begin{aligned}&e\_gwpumpIN(q)..\\&v\_gwpumpIN(q) \quad = e = sum(b, v\_gwpumpIN\_b(q,b));\end{aligned} \tag{SM.43g}$$

$$\begin{aligned}&e\_gwpumpOUT(q)..\\&v\_gwpumpOUT(q) \quad = e = sum(b, v\_gwpumpOUT\_b(q,b));\end{aligned} \tag{SM.43h}$$

SM.5.2. Combined Regions Variables

$$e\_Wcost\_OnM\_GEUR..$$
$$v\_Wcost\_OnM\_GEUR \quad =e= sum(b, v\_wcost\_OnM\_b(b))/(1E+9); \tag{SM.44a}$$

$$e\_Wcost\_Inv\_GEUR..$$
$$v\_Wcost\_Inv\_GEUR \quad =e= sum(b, v\_wcost\_Inv\_b(b))/(1E+9); \tag{SM.44b}$$

$$e\_Wcost\_b2b\_Inv\_GEUR..$$
$$v\_Wcost\_b2b\_Inv\_GEUR \quad =e= sum((bf, bt), v\_trnsf\_cost\_Inv(bf, bt))/(2E+9); \tag{SM.44c}$$

$$e\_Wcost\_nsw\_GEUR..$$
$$v\_Wcost\_nsw\_GEUR \quad =e= sum(b, v\_wcost\_nsw\_b(b))/(1E+9); \tag{SM.44d}$$

$$e\_Wcost..$$
$$\begin{aligned} v\_Wcost \quad &=e= v\_Wcost\_OnM\_GEUR \\ &+ \quad v\_Wcost\_Inv\_GEUR \\ &+ \quad v\_Wcost\_b2b\_Inv\_GEUR \\ &+ \quad v\_Wcost\_nsw\_GEUR; \end{aligned} \tag{SM.44e}$$

$$e\_nrg..$$
$$v\_nrg \quad =e= sum(b, v\_nrgGWh\_b(b)); \tag{SM.44f}$$

$$e\_wleak..$$
$$v\_wleak \quad =e= sum(b, v\_wleak\_b(b)); \tag{SM.44g}$$

$$e\_wwstC..$$
$$v\_wwstC \quad =e= sum(b, v\_wwstC\_b(b)); \tag{SM.44h}$$

$$e\_wwstD..$$
$$v\_wwstD \quad =e= sum(b, v\_wwstD\_b(b)); \tag{SM.44i}$$

$$e\_nsw(wsec, q)..$$
$$v\_nsw(wsec) \quad =e= sum(b, v\_nsw\_b(wsec, b)); \tag{SM.44j}$$

$$e\_res2distC(q)..$$
$$v\_res2distC(q) \quad =e= sum(b, v\_res2distC\_b(q, b)); \tag{SM.44k}$$

$$e\_surf2distD(q)..$$
$$v\_surf2distD(q) \quad =e= sum(b, v\_surf2distD\_b(q, b)); \tag{SM.44l}$$

SM.6. Integrating Water and Energy Equations

SM.6.1. Water system Energy Feedback

In Equation SM.45a, QACTESST is the variable from the energy model which defines the demand from each "Energy Service Supply Technology" ($esst$). In these equations the energy required for the water system ($esst_wat$) is defined as that calculated in the water model Equation SM.44f. Equation SM.45b allows the user to specify a fixed demand from the water system thus unlinking the water and energy models.

$$
\begin{aligned}
& e\_l\_EdemWat(esst, p, s, l)\$(sameas(esst, "esst\_wat")).. \\
& QACTESST(esst, p, s, l) \qquad = e = sum(b, v\_wnrg\_b\_p(b, p)) \\
& \qquad * \quad D(p, s, l)/sum((as, al), D(p, as, al));
\end{aligned}
\tag{SM.45a}
$$

$$
\begin{aligned}
& e\_l\_EdemWatFix(esst, p, s, l)\$(sameas(esst, "esst\_wat")).. \\
& QACTESST(esst, p, s, l) \qquad = e = p\_wnrg\_p\_FX(p)* \\
& \qquad D(p, s, l)/sum((as, al), D(p, as, al));
\end{aligned}
\tag{SM.45b}
$$

SM.6.2. Energy System Water Feedback

Equation SM.46a, Equation SM.46b and Equation SM.46c define the water demands from the energy sector. Equation SM.46a allows the user to pre-define water demands from the energy sector in case an unlinked model is used. Equation SM.46b defines the water demands from all the other water sectors $wsec$, based on the exogenous parameter p_WdemIN. Equation SM.46c defines the water demands from the energy sector "nrg", given by the variables $v\_WatCons\_PE$ and $v\_WatCons\_CE$ for water consumption in primary energy and conversion energy technologies. Equation SM.46d defines the distribution of energy capacity by river basin and Equation SM.46e through Equation SM.46h define the water consumption and withdrawal for each primary and conversion energy technology per sub-region and per time period.

$$e\_l\_demBal\_notlinked(b, wsec, p)..$$
$$v\_WdemIN(b, wsec, p) \quad = e = p\_WdemIN(b, wsec, p); \tag{SM.46a}$$

$$e\_l\_demBal(b, wsec, p)\$(notsameas(wsec, "nrg"))..$$
$$v\_WdemIN(b, wsec, p) \quad = e = p\_WdemIN(b, wsec, p); \tag{SM.46b}$$

$$e\_l\_demBalNRG(b, wsec, p)\$(sameas(wsec, "nrg"))..$$
$$v\_WdemIN(b, wsec, p) \quad = e = v\_WatCons\_CE(ce, b, p) + v\_WatCons\_PE(pe, b, p); \tag{SM.46c}$$

$$e\_l\_nrgdist\_CEOUT(ce, b, p)..$$
$$v\_nrgdist\_CEOUT(ce, b, p) \quad = e = sum((s, l), DPRD\_CEOUT(b, ce, p, s, l)); \tag{SM.46d}$$

$$e\_l\_watConsCE(ce, b, p)..$$
$$v\_WatCons\_CE(ce, b, p) \quad = e = sum((s, l), DPRD\_CEOUT(b, ce, p, s, l)) * p\_WatCons\_CE(ce); \tag{SM.46e}$$

$$e\_l\_watConsPE(pe, b, p)..$$
$$v\_WatCons\_PE(pe, b, p) \quad = e = sum((s, l), DPRD\_PEOUT(b, pe, p, s, l)) * p\_WatCons\_PE(pe); \tag{SM.46f}$$

$$e\_l\_watDrawCE(ce, b, p)..$$
$$v\_WatDraw\_CE(ce, b, p) \quad = e = sum((s, l), DPRD\_CEOUT(b, ce, p, s, l)) * p\_WatDraw\_CE(ce); \tag{SM.46g}$$

$$e\_l\_watDrawPE(pe, b, p)..$$
$$v\_WatDraw\_PE(pe, b, p) \quad = e = sum((s, l), DPRD\_PEOUT(b, pe, p, s, l)) * p\_WatDraw\_PE(pe); \tag{SM.46h}$$

SM.6.3. Hydropower Potential

Equation SM.47a assigns the hydropower production from reservoir outflows as defined in Equation SM.7d to the electricity produced from the energy system technology for reservoirs "CEHYRSCAP". Equation SM.47b defines the run-of-river electricity production based on the reservoir production and an assumption that the run-of-river production is a percentage of the total hydroelectricty production for each basin.

$$
\begin{aligned}
&CONSTR\_CEBALANCE\_RSCAP\_P123\\
&('TEELECE','CEHYRSCAP',p)..\\
&sum((s,l),QPWR('CEHYRSCAP','TEELECE',p,s,l)\\
&\qquad\qquad * \quad D(p,s,l)) * xGWhtoEJ\\
&\qquad = e = sum(b,v\_res\_hydronrgprod(b,p))\\
&\qquad\qquad * \quad xGWhtoEJ;
\end{aligned}
\tag{SM.47a}
$$

$$
\begin{aligned}
&CONSTR\_CEBALANCE\_RURIV\_P123\\
&('TEELECE','CEHYRURIV',p,s,l)..\\
&QPWR('CEHYRURIV','TEELECE',p,s,l)\\
&\qquad = e = (sum(b,v\_res\_hydronrgprod(b,p))\\
&\qquad\qquad / \quad (1 - RURIVNRGSHARE(p)))*\\
&\qquad\qquad\quad RURIVNRGSHARE(p)\\
&\qquad\qquad / \quad sum((as,al),D(p,as,al));
\end{aligned}
\tag{SM.47b}
$$

SM.7. Water Sector Demand Priorities

Equation **??** defines the water allocation priorities by constraining the non-served water from particular sectors to be less than that in other sectors. The current order of priorities insures that the residential sector will be the last sector to have non-served water. This order can be changed to reflect the local policies.

$$
\begin{aligned}
&e\_prior\_res(b,wsec,p)..\\
&v\_nswOUT(b,"res",p) \quad = l = v\_nswOUT(b,"agri",p); \qquad \text{(SM.48a)}\\
&e\_prior\_agri(b,wsec,p)..\\
&v\_nswOUT(b,"agri",p) \quad = l = v\_nswOUT(b,"nrg",p); \qquad \text{(SM.48b)}\\
&e\_prior\_nrg(b,wsec,p)..\\
&v\_nswOUT(b,"nrg",p) \quad = l = v\_nswOUT(b,"ind",p); \qquad \text{(SM.48c)}
\end{aligned}
$$

SM.8. Objective Function

Equation SM.49 defines the multiple objective function. $OFVA\_TOTSUPCOSTOPERINVST\_P23$ is the total costs from the energy system model and includes operation, investments and non-served energy costs as well as revenues from exports. These are summed with the water operation and maintenance, Investment, interbasin investments and any non-served water costs. The total costs are then multiplied by a scalar defining the desired weight for costs. The objective function also consists of other terms such as emissions, water consumption and water withdrawal each with its own weight. Depending on the needs of the user the existing variables may be removed or other variables such as losses or leakages can be added with their appropriate weights.

$$
\begin{aligned}
& e\_l\_tot\_cost.. \\
& v\_cost \quad = e = (OFVA\_TOTSUPCOSTOPERINVST\_P23 \\
& \quad + \; v\_Wcost\_OnM \\
& \quad + \; v\_Wcost\_Inv \\
& \quad + \; v\_Wcost\_b2b\_Inv \\
& \quad + \; v\_Wcost\_nsw) * s\_ObjWght\_Cost \\
& \quad + \; TOTEM * s\_ObjWght\_Emissions \\
& \quad + \; v\_nrg * s\_ObjWght\_EnergyConsByWat \\
& \quad + \; (sum((ce, b, p), v\_WatCons\_CE(ce, b, p)) \\
& \quad + \; sum((pe, b, p), v\_WatCons\_PE(pe, b, p))) * s\_ObjWght\_WaterConsByNrg \\
& \quad + \; (sum((ce, b, p), v\_WatDraw\_CE(ce, b, p)) \\
& \quad + \; sum((pe, b, p), v\_WatDraw\_PE(pe, b, p))) * s\_ObjWght\_WaterDrawByNrg;
\end{aligned}
\tag{SM.49}
$$

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