



## EDITORIAL

### Ten years of the ICMAT

The agreement for the creation of the Institute of Mathematical Sciences was signed ten years ago. This took place on October 29<sup>th</sup>, 2007, after a long process since the decision was first taken in 2005 by the board of the Consejo Superior de Investigaciones Científicas (CSIC), presided at the time by Carlos Martínez Alonso, to set the institute up.

In 2005, the Departamento de Matemáticas del Instituto de Matemáticas y Física Fundamental (IMAFF) tabled an independent proposal for the 2006-2009 Strategic Plan, the aim of which was to bring it up to date and provide it with a more appropriate structure. That was probably the most important initiative undertaken in the history of mathematics at the CSIC. The plans of each institute were later coordinated with those of the areas and those of the central organization in order to draw up a global plan which, after an international and independent evaluation, would lead to a restructuring and the assignment of financial and human resources in accordance with that evaluation.

The plan for the Department of Mathematics was evaluated by an external committee headed by the current chair of the European Research Council (ERC), Jean-Pierre Bourguignon, who pointed out the importance of mathematics within the CSIC; the extraordinary quality of the young researchers we had been able to bring together; the need for a suitable headquarters, and the duty of raising the profile of the discipline at the CSIC by the creation of an institute of mathematics. This centre eventually took the form of a joint institute consisting of the CSIC and the three Madrid universities, with the following objectives (as set out in the introduction to the official agreement):

“... an appropriate channel for the coordination and development of ever-increasing research activity in the Mathematical Sciences and the creation of further channels for the transfer of mathematical knowledge to other sciences as well as to technological, industrial and financial sectors.”

I had the honour of being named acting director charged with carrying the project forward. I began my professional career at the CSIC one morning in January, 1986, when I arrived at the main campus and took up my duties as Scientific Researcher in my office at 123, Serrano Street. Two decades later, that signature in 2007 represented a significant achievement, the result of a struggle to restore mathematics to its natural place in the CSIC. As I said at the time (and still maintain today): “The CSIC cannot live without mathematics, and neither can the Spanish mathematical community be effective without the CSIC”.

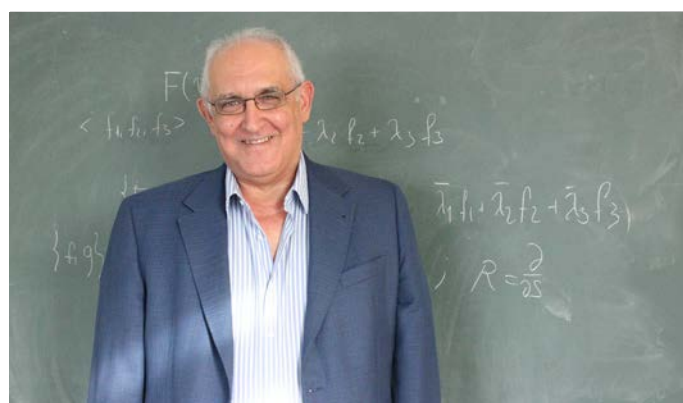


Image: ICMAT

Manuel de León, ICMAT director.

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In terms of the CSIC, the ICMAT had mainly been formed on the basis of Ramón y Cajal contracts consisting of researchers who were subsequently granted up to ten ERC projects, a record for all European research centres. These successes were accompanied by many other achievements, such as the AXA Chair obtained by David Ríos Insúa, one of eight such positions existing in Spain in all scientific fields and the only one held at the CSIC.

Starting in 2011, the ICMAT moved into its own headquarters, a magnificent building located on the UAM campus and which also housed the Institute for Theoretical Physics.

From the time I was named as acting director (a post that was formalized on January 19<sup>th</sup>, 2008), I was continuously at the service of the Institute itself and the other four institutions. I became the full director on July 12<sup>th</sup>, 2012, and I am enormously proud to have been at the helm of this project which, while it has not always been plain sailing, we have navigated with great enthusiasm. And there it remains today, carving a place for itself in international mathematical excellence and enjoying the respect of having achieved in just a few years what other centres have taken decades to accomplish.

These last two years in the lifetime of the ICMAT have been complicated to say the least, perhaps because its growth has been spectacular and its sudden emergence aroused misgivings in the minds of some. Nevertheless, such achievements do not come out of nowhere, but rather from the work of many years full of effort and endeavour, attracting the best researchers and seeking to form a management team. Given the opportunity to prove itself, it is hardly surprising that the Institute flourished so remarkably well, and was the first belonging to the CSIC to be awarded a Severo Ochoa distinction on the very first call.

The future of the ICMAT over the next few years will depend on the commitment of its researchers, the absence of sectarianism, the respect of each and every researcher for his or her colleagues, and providing that the individual and collective bodies of the institute keep up the good work, always bearing in mind that those charged with managerial responsibility should place the work of the collective above that of the personal. If that happens, the ICMAT will be able to sustain its successes, with an institute in which basic research exists side-by-side with applications and knowledge transfer. This is my hope and expectation, and what I also ask of everyone concerned.



## REPORT: Review of the research lines of ICMAT

## Eight lines, nine groups and ten years

Image: ICMAT



Although the ICMAT was founded in 2007, the building in which the center is located was inaugurated on the Cantoblanco campus (Madrid) in 2011.

**It is now ten years since the agreement for the creation of the Instituto de Ciencias Matemáticas (ICMAT) was signed, on October 29th, 2007, following the recommendations by an international scientific committee aimed at strengthening mathematical research at the CSIC. One of the objectives of the Institute was “to establish priorities in consideration of the fact that research in Spain is underdeveloped in some fields, such as number theory, group theory and combinatoric, among others, with the aim of strengthening them”, explains Manuel de León, founder of the ICMAT and its current director.**

**Elvira del Pozo.** From the three research lines established a decade ago at the centre, there are now currently eight on which 138 researchers are working, presently registered in nine groups belonging to the CSIC, and who are undergoing evaluation over these months. This strategy appears to be working, because not only does the Institute publish every year more than one hundred high-impact papers with a large number of citations, it has also obtained and renewed the Severo Ochoa excellence distinction on two occasions, and nine of its researchers have been distinguished with ERC grants, all of which in De León’s words makes the Institute “the leading research centre in mathematics in Spain and among the best in Europe.” In what follows, we outline the development of these research lines, their landmark achievements and the great challenges they still face.

#### [LINE 1: Algebraic Geometry and Mathematical Physics](#)

This area corresponds to [Group 4](#) and is coordinated by [Óscar García-Prada](#). It is a field that currently has close connections with string theory and particle physics. Among the most important results obtained recently by the group, it is worth mentioning the new generalized Teichmüller spaces. Other notable results are those arising from the development of the Kaehler-Yang-Mills theory, introduced by the Group and which combines the Yang-Mills and the Donaldson-Uhlenbeck-Yau theory and the Yau-Tian-Donaldson theory of Kaehler metrics with constant scalar curvature. In addition, the existence of symmetrical solutions has been recently proven for these



equations which, as García-Prada explains, “admit a physical interpretation related with cosmic strings”.

[LINE 2: Differential Geometry, Symplectic Geometry and Geometric Mechanics](#)

This area is being developed by [Group 5](#) coordinated by [Daniel Peralta](#), who received a European Research Council (ERC) Starting Grant and was a plenary speaker at the European Congress of Mathematics in Berlin in 2016. The work is structured around four main lines of research: differential and contact topology, Riemannian geometry, the geometric theory of differential equations, and geometric mechanics and control theory. This research addresses a range of topics, from fundamental problems such as the existence of certain geometric structures and their topological properties, to more applied questions the aim of which is to understand the evolution of mechanical systems and how to control them. The connections with physical theory range from Einstein’s Theory of General Relativity to that of celestial mechanics formulated by Poincaré, as well as including the geometry of quantum systems. The results of greatest impact obtained by this group include the resolution of Arnold’s and Lord Kelvin’s conjectures in topological hydrodynamics, by [Alberto Enciso](#) and Daniel Peralta; the proof of Chern’s conjecture in contact topology, by [Roger Casals](#) and [Francisco Presas](#); and the development of the Hamilton–Jacobi geometric theory, by [Manuel de León](#) and [David Martín](#).

[LINE 3: Mathematical Analysis, Differential Equations and Applications](#)

This area is currently divided into two main lines, each one handled by a research group belonging to the Institute. On the one hand, there is Mathematical Analysis, which is the line investigated by [Group 1](#). It enjoys three ERC projects involving different branches of Harmonic Analysis coordinated by [José María Martell](#), [Javier Parcet](#) and [Keith Rogers](#). Martell’s ERC project, currently in force, has as one of its main objectives the study of certain diffusion problems in irregular bodies using techniques of harmonic analysis. Parcet was awarded the José Luis Rubio de Francia Prize for Research in 2005 and also headed an ERC project focusing on the study of the non-commutative Calderón–Zygmund theory. Rogers studies more classical topics of Fourier Analysis as well as its applications to inverse problems. In addition, [Group 2](#), headed by [Diego Córdoba](#), is focused on Differential Equations and Applications, with another ERC and the Community of Madrid Miguel Catalán Prize, the aim of which is the study of partial differential equations and their applications to fluid mechanics. One of their results with greatest impact is the [construction of](#)

[a mathematical model](#) that explains how waves break – a phenomenon known as splash-type singularities. Furthermore, in this group, under the leadership of [José M. Arrieta](#) and [Aníbal Rodríguez-Bernal](#), infinite-dimensional dynamical systems are also being studied.

[LINE 4: Number Theory](#)

This broad branch of mathematics is addressed by [Group 9](#), with [José Ignacio Burgos](#) at its head. Work in this group is being conducted in several fields, among which it is worth mentioning algebraic theory, which explores connections with number theory in which notable problems exist, such as the Birch and Swinnerton-Dyer conjecture, one of the seven millennium problems. One of the results with greatest impact in this group is the solution to the Sidon problem by Javier Cilleruelo and Carlos Vinuesa, while Javier Fernández de Bobadilla and María Pe arrived at the solution to the problem posed by the mathematician John Nash in singularity theory.

[LINE 5: Group Theory](#)

[Group 8](#), of which [Andrei Jaikin](#) is the principle researcher, is devoted to group theory and focuses on different combinatorial, geometric and algebraic properties of finite and infinite groups, with particular emphasis on asymptotic group theory (for example, L2 invariants, growth functions), geometric group theory (such as non-positively curved groups and mapping class groups), profinite groups and representation theory.

[LINE 6: Statistics, Probability and Operations Research \(SPOR\)](#)

[Group 3](#), known as the SPOR group (Statistics, Probability and Operations Research), is composed of ICMAT researchers interested in stimulating multidisciplinary research and exploring new contexts for the application of Probability, Data Science, Machine Learning, Statistics and Operations Research for modelling processes, making predictions and developing support mechanisms for decision-making in complex problems, including their use in the industrial sector. Activities of note in this group are international and industrial cooperation, competitive doctoral and postdoctoral training, the organization of permanent research international events and outreach activities. The SPOR Group is engaged in projects with companies and public institutions in the spheres of cyber security, fraud detection, precision marketing, medical diagnosis and natural language processing, among others, which are channelled through the [Datalab](#), a support unit for research into data science, and the [AXA Chair](#). This group is currently composed of three researchers from the team ([David Ríos](#), [Antonio Gómez Corral](#) and [David Gómez Ullate](#)), one postdoctoral researcher and seven predoctoral researchers.

Image: ICMAT



José María Martell, Javier Parcet and Diego Córdoba (from left to right) are three of the ICMAT researchers who have been distinguished with ERC projects.

[LINE 7: Mathematics and Quantum Information: Foundations and Applications](#)

[Group 6](#), with [David Pérez García](#) at its head, works on a broad range of mathematical problems underlying quantum technology. On a fundamental level, the objective is to characterize different aspects of the mathematical structure lying behind quantum mechanics. Specifically, this involves applying techniques of functional analysis, among others. As regards applications, great and well defined challenges exist, such as building increasingly larger quantum computers that go far beyond the power of current computers, or putting into practice the protocols of quantum cryptography whose security has already been mathematically demonstrated. This would enable the development of secure communication which would be inviolable: a significant technological breakthrough.

[LINE 8: Mathematical Modelling and Simulation](#)

The research line of [Group 7](#) is mathematical modelling and simulation and is coordinated by [Marco Antonio Fontelos](#), [Ana M. Mancho](#) and [Carlos Escudero](#). Its researchers solve problems in different scientific fields such as engineering, biology, chemistry and earth sciences by using advanced mathematical and computing tools. Their activities are centered on collaboration with multidisciplinary groups that lead to specialized publications concerning the sciences that are the object of their research work, as well as more theoretical work with papers published in the classical journals of applied mathematics. Some important results obtained by the members of this group are the development of formulas for the study of liquid drop impact on solids, which have been used in diverse contexts such as circuit printing using jet printers, the study of the erosion processes of monuments, and also research into forensic medicine, since this work enables the details of a crime to be minutely examined according to the shape and size of bloodstains.

Further results of this group are the development of tools belonging to dynamical systems that assisted emergency services in the control and monitoring of the oil spill caused by the sinking of the Oleg Naydenov fishing vessel off the coast of Gran Canaria, and which can also enable the theory of transition states in chemistry to be addressed with new tools.



Image: ICMAT

The younger members of the SPOR group were protagonists of the Night of the Researchers of the ICMAT in 2017. From left to right: Roi Naveiro, Alberto Redondo, Simón Rodríguez, Víctor Gallego and David Gómez-Ullate.

Image: ICMAT



In 2017 the ICMAT hosted a scientific congress in celebration of the birthday of the mathematician Nigel Hitchin (University of Oxford), organized by Óscar García-Prada.



INTERVIEW: Yakov Sinai, professor at Princeton University, 2014 Abel Prize winner and member of the ICMAT External Advisory Committee

“Many people believe that Chaos Theory should be connected with turbulence, but I’m not so sure”



Sinai, who received the Abel Prize in 2014, is part of the Scientific Advisory Committee of the ICMAT.

**Yakov Sinai (1935, Russia), professor at the Princeton University Department of Mathematics (USA), is one of the members of the ICMAT External Advisory Committee. He is an expert on dynamical systems, mathematical physics and probability theory, and has been a professor at the Moscow State University and the Landau Institute for Theoretical Physics (1971-1993) in Russia. In 2014 he was awarded the Abel Prize for his fundamental contributions to dynamical systems, ergodic theory and mathematical physics. He has also received the following awards: the Boltzmann Medal (1986) from the International Union of Pure and Applied Physics (IUPAP), the Dannie Heineman Prize (1990) from the American Physical Society, the Markov Prize (1990) from the Russian Academy of Sciences, the Dirac Prize (1992) from the ICTP in Trieste, the Wolf Prize (1997) from the Wolf Foundation in Israel, the Moser Prize (2001) from the SIAM, the Nemmers Prize (2002) from the Northwestern University, the Lagrange Prize (2008) from the ISI in Turin, the Henri Poincaré Prize (2009) from the IAMP and the Leroy P. Steele Prize (2013) from the AMS, among others. He is a member of the American Academy of Arts and Sciences (1983), the Russian Academy of Sciences (1991), the London Mathematical Society (1992), the Hungarian Academy of Sciences (1993), the United States National Academy of Sciences (1999), the Brazilian Academy of Sciences (2000) and the Polish Academy of Sciences (2009), among others.**

**Ágata Timón García-Longoria. What’s the relationship between chaos and randomness?**

Randomness is the main notion in probability theory, which is connected with games, noise, etc.. Chaos is a property of dynamics that occurs in situations where there is no randomness at all. The main property of chaos is that it can be found in totally deterministic situations.

**Mathematics tries to find order and structure: What’s the difference between randomness and chaos?**

Probability theory is the part of mathematics that studies randomness. Chaos is the property of dynamics that does not require any randomness.

**Could you explain the concept of entropy?**

It has different meanings. In statistical physics it is related with the size of the system under study. In chaos theory it has to do with some special properties of the deterministic equations of motion. The concept of entropy in dynamics requires the instability of motion. Entropy may be positive only if the dynamics is unstable when nearby points diverge exponentially fast. The value of entropy describes the growth of this instability.

**Together with Andrey Kolmogorov, you were one of the mathematicians who formalized the concept of chaos within chaos theory. Did Claude Shannon’s work in 1940 provide you with inspiration?**

Yes, very much so, in particular the concept of entropy in dynamics based on Shannon’s notion of entropy in information theory.

The concept of entropy in dynamics is based on Shannon’s notion of entropy in information theory

**What are the implications of these ideas for our conception of reality?**

Many people believe that chaos theory should be connected with turbulence, but I’m not so sure.

**More than 50 years later, it’s still a virtually unknown field. What do you think the big challenges are?**

The main challenge in the field is to find many classes of systems with positive entropy. One example is the so-called standard map proposed by the physicists Chirikov and Taylor. Some people say that this very difficult case is the problem for the 22<sup>nd</sup> century.

**One of your models for studying ergodic behaviour is known as the Sinai Billiards, which you introduced in 1960. Could you explain this idea?**

After Kolmogorov proposed his notion of entropy, many people started to think about classes of systems with positive entropy. The first step was taken by the Russian physicist, N. S. Krylov, who discovered the analogy of systems with elastic collisions and the so-called Anosov system, which provide one of the best examples of the model of chaos. My friend suggested another example of this type. Another remarkable example, which is now called the Bunimovich Stadium, was proposed by L. A. Bunimovich and is very popular example in the theory of quantum chaos.

**Could you give us an example?**

The billiards that I studied are called hyperbolic billiards. A typical example is a square from which a circle is removed from

the centre. This case also appears in many problems of quantum chaos. The instability can easily be seen when one considers a bundle of rays coming out of a single point and the size of this bundle grows exponentially in time.

*“The most important were periods of time when I couldn’t solve problems on which I had been working for a long time”*

**What would you say were the most important moments in your career?**

The most important were periods of time when I couldn’t solve problems on which I had been working for a long time. There were many such cases.

REPORT: Heidelberg Laureate Forum 2017

Fifth Anniversary of the Heidelberg Laureate Forum

Image: hiff-mueck\_08



The opening ceremony of this fifth Heidelberg Forum was enlivened by the ‘Balanced Action’ wind quartet.

**Last September in the German city of Heidelberg, young researchers from the world of computation and mathematics met for the fifth successive year with scientists such as Stephen Smale, Martin Hairer, Stephen Cook, Shigefumi Mori and Michael Atiyah. Between keynote speeches, round table discussions and social events with quantum computing as their main theme, those who attended were able to swap ideas and learn about the latest advances in both disciplines.**



**Laura Moreno Iraola.** The German physicist and cofounder of the software corporation [SAP](#) (Systems, Applications and Products in Data Processing), Klaus Tschira (1940-2015), was a great admirer of the [Lindau Forum](#), an annual meeting of Nobel Laureates and emerging young talent in diverse disciplines. Since there are no Nobel Prizes awarded in the category of mathematics and computation, no researchers in these fields had previously attended the meeting. Given this situation, and with the backing of the [Heidelberg Institute for Theoretical Studies](#) (HITS) and the [Klaus Tschira Stiftung](#), Tschira had the idea of adapting the event. So, in 2013 the [Heidelberg Laureate Forum](#) (HLF) was launched, organized by his own foundation and sponsored by the HITS and [Heidelberg University](#).

“Coming into contact with the winners of the most important prizes awarded in the field of mathematics and computing is a dream come true”

Since then, every year in the last week of September, this German city has been the venue for meetings between scientists distinguished with [Fields Medals](#), the [Nevanlinna Prize](#), the [Abel Prize](#), the [ACM Turing Award](#) and the [ACM Prize in Computing](#), and pre- and post-doctoral students and researchers from the five continents. For the majority of these young scientists, “coming into contact with the winners of the most important prizes awarded in the field of mathematics and computing is a dream come true”, says Arti Ramesh, a postdoctoral researcher in computing at New York State University (USA) and one of the 200 young people chosen by the Scientific Committee to attend the [fifth edition of the HLF](#), held from September 24<sup>th</sup> to September 29<sup>th</sup>, 2017.

On this occasion, the forum was attended by laureates such as Sir Michael Atiyah (Fields Medallist 1966 and Abel Prize-winner 2004), Vinton Cerf (ACM A.M. Turing Award 2004), Stephen Cook (ACM A.M. Turing Award 1982), Jeffrey A. Dean (ACM Prize in Computing 2012), Martin Hairer (Fields Medallist 2014), Leslie Lamport (ACM A.M. Turing Award 2013) and Stephen Smale (Fields Medallist 1966), [among others](#). As Smale remarked at a press conference: “While I’ll always maintain that students should be independent and try to educate themselves, not only in science, they also need someone to guide them, and that’s the job that falls to those of us with more experience, and that’s why we’re here at this HLF.” The only drawbacks of the meeting were the cancellations from [Barbara Liskov](#) (ACM A.M. Turing Award 2008), the only female laureate, and [Vladimir Voevodsky](#) (Fields Medallist 2002), both due to health problems. Sadly, [Voevodsky died](#) just one day after the end of the HLF.

“Thanks to the opportunities I’ve had to explain my problem to so many people, I have a clearer idea of where I’m going from now on”

The [inauguration ceremony](#) was accompanied by the wind quartet ‘Balanced Action’ and hosted, among others, by Theresia Bauer, the German Minister for Science, Research and the Arts; Bernhard Eitel, the president of Heidelberg University; Eckart Würzner, the city’s mayor, and Shigefumi Mori, the 1990 Fields Medallist and Chair of the International Mathematical Union (IMU). Quantum computing was the hot topic of the most comprehensive programme yet organized at the HLF, according to members of the organization itself. On this occasion, scientific ac-



Patricia Arias, postdoctoral researcher at the Carlos III University of Madrid and Natalia Díaz Rodríguez, postdoctoral researcher at ENSTA ParisTech (France), were two of the Spanish attendees at the congress.

Image: ICMAT



tivities (talks, workshops and discussion panels) were combined with social events, all of them aimed at promoting interaction to bring the researchers closer together. "I'm very grateful to the laureates, because they have showed great interest in us; they listen to us, and give us advice as well as their perspectives on our work," said Arpita Biswas, a predoctoral student at the [Indian Institute of Science](#), who went on to say that: "In general, thanks to the opportunities I've had to explain my problem to so many people, I have a clearer idea of where I'm going from now on."

Outreach also found a place at the Heidelberg Laureate Forum with the "Math  $\Leftrightarrow$  Art" exhibition that was open to the general public

Spanish participation was not lacking at the congress. In the opinion of [Patricia Arias](#), a postdoctoral researcher at the Carlos III University in Madrid: "The best experience to come out of this HLF was the organization of a workshop, together with Antonio Campello, a postdoctoral researcher at Imperial College. It was based on mathematical foundations for cyber security; in particular, for improving security and privacy using algorithms capable of being resistant in the context of quantum computing. We were fortunate to have [Vinton Cerf](#) as mentor and [Whitfield Diffie](#) for dealing with cyber security issues."

For her part, [Natalia Díaz Rodríguez](#), a postdoctoral researcher at the [ENSTA ParisTech](#) (France), who was attending the congress for the second time (the rules state that one attendance is allowed for pre-doc students and one more as a post-doc), organized in Heidelberg her first workshop. "At first I thought it was going to be a disaster, because I had to change the title, which attracted a very assorted audience, but maybe that helped me to see how I could approach the subject (Machine Learning and the interaction between humans and machines from a mathematical, robotic and psychological perspective) from different points of view, and I had to improvise," she explained.

For Ismael Sierra, who comes from Madrid, this was one of the first opportunities for contact with the world of research. He studies mathematics at [Cambridge University](#) (United Kingdom), and at the age of 19 was the youngest person at this HLF. He was invited by Michael Atiyah, whom he had met by chance one day when having breakfast at the university. "He had come to give a talk and we happened to meet in the cafeteria", said Sierra. "We started chatting and since then we've stayed in touch. He often invites me to attend events related with mathematics."

Outreach also found a place at the Heidelberg Laureate Forum with the "[Math  \$\Leftrightarrow\$  Art](#)" exhibition that was open to the general public. This was organized by the artist and programmer [Aldo Spizzichino](#), who sadly died last June while setting up the exhibition. The works on show were the final result of the process that Spizzichino was developing with his own software, which were designed on the basis of mathematical elements and demonstrations such as fractals, some created by Diffusion Limited Aggregation (DLA); Pythagoras' Theorem and Archimedes' Principle; exotic solids and surfaces; different tessellations; hyperbolic transformations and the cellular automata model, among others. The Italian mathematician, essayist and disseminator, [Piergiorgio Odifreddi](#), participated in the exhibition with a talk on the relation between art and mathematics throughout history, and more recently computing used as an instrument by the artist to extend the scope of his works.

Ismael Sierra, who comes from Madrid and is 19 years of age, was the youngest person to attend this HLF

The 2017 Heidelberg Laureate Forum came to an end at the castle, a characteristic landmark in the city and which will also figure next year in the sixth HFL, scheduled to be held between the 23<sup>rd</sup> and the 28<sup>th</sup> of September. The dates [for applications to attend](#) are now available.



Image: ICMAT



Image: ICMAT

During the forum the open exhibition "Math  $\Leftrightarrow$  Art" was held, developed by the artist and programmer Aldo Spizzichino.

## “Art and science are complementary responses to the universe”

Image: ICMAT



Mario Livio studied mathematics, did research in astrophysics, and has written numerous world-class popularization books.

**In his books aimed at a general public, Mario Livio (1945, Bucharest, Romania) is committed to presenting mathematics as an integral part of culture. In works such as *Is God a Mathematician?*, *The Golden Ratio* and *The Equation that couldn't be solved*, he shows its relations with art, nature and philosophy. In this latter book, he delves into the life of Evariste Galois, whom he regards as the most romantic mathematician in history. A graduate in mathematics, Livio chose to study astrophysics for his Ph.D and his professional career (although he says he uses mathematics a lot in his work). He was invited to participate in the Complutense University of Madrid Summer School course on “From Mathematics to Society”, which was held in July 2017 at El Escorial with the collaboration of the ICMAT.**

**Ágata Timón García-Longoria. In your books you've written about Phi and the Golden Ratio... If you had to choose, what would you say was the most fascinating mathematical concept?**

The most fascinating for me is the central subject of my book *Is God a Mathematician?*, which is not so much a particular notion as the fact that we are able to explain the universe using mathematics.

**Do you think everything can be explained by mathematics?**

It works best for describing fundamental things like the basic laws of nature. It wouldn't work so well if we wanted to describe everything that's happening in this room right now. That involves complex systems with a great number of atoms. We have mathematical models in biology, but we don't have a mathematical theory of biology.

“We can explain the universe using mathematics”

**Where do you think this relation between mathematics and nature comes from?**

In a certain way, mathematics is a language. We need a language to describe nature, and mathematics is the most effective way of doing this. But it's something more than that; we tailor our mathematics to the kind of problems we study. We don't use it blindly, and that's part of its success.

**And is this mathematics already there or do we create it ourselves?**

That's a tough question: Is mathematics invented or do we discover it? I can give you my personal opinion, but a lot of people

wouldn't agree with me. I think mathematics is a very intricate combination of inventions and discoveries. I don't say that because I can't decide between either option, but because it's what I really believe. We invent the concepts; for example, we create the square root of minus one and we call it  $i$ , the imaginary number. Then we discover all kinds of theorems that obey the imaginary numbers. Once we've invented the concept there's no other option for the theorems; they're imposed on us. So I think that mathematics is a combination: we invent the concepts and discover the relations between them.

“We tailor our mathematics to the type of problem we're studying; we don't use it blindly”

**So if we had invented different basic concepts, we'd have a completely different theory of mathematics?**

Yes, a mathematics that's completely different. For example, we could've started with very simple rules, like computer programmes. The mathematician Stephen Wolfram showed that almost all of our mathematics can be generated starting from this different set of concepts.

**Or like what happens with Euclidian and non-Euclidian geometry, which are obtained from some axioms or others. But then how we choose one option or another is simply a question of chance?**

Yes, I think we began with Euclidian geometry for two main reasons. The first is given by our system of perception: we're very good at distinguishing between a straight line and a circle. And

then, we live on a planet with a very weak force of gravity, so our space is very weakly curved; it is almost flat. Had we lived in a place where gravity is so strong that light travelled through curved space, then we might have started with non-Euclidian geometry.

**“Mathematics is a very intricate combination of inventions and discoveries”**

**In the end, mathematics configures the way we think. And what about the way we express ourselves as well? What do you think the relation between mathematics and art is?**

I like to think in a more general way between mathematics and art. Apart from being a scientist, I happen to be an art fanatic. I’ve got hundreds of books on art and I try to visit exhibitions wherever I go. But the truth is I don’t believe that the influences between the two disciplines are very strong. Many people think they are - that Einstein and Picasso are interrelated - but quite honestly that seems a bit forced to me. What I find interesting is that both things, art and science, are attempts to do the same, but from different perspectives. Scientists try to explain the universe, to understand natural phenomena and perhaps to find a mathematical description and so on. Artists try to give an emotional response to the universe around them. So they are both responding to the universe in which we live, but in a complementary way.

**In addition to being a scientist, you’re also a communicator. What would you say is your main aim as a communicator?**

I’d like everyone to see science as an integral part of culture, not as something external to it. Not that everybody thinks they have to read Shakespeare or Cervantes, but how can they not know about the existence of the fundamental laws of nature. That’s also part of culture, and I’d say even more important. It’s beautiful to read a poem by Pablo Neruda, but in the end, apart from enriching your soul, it isn’t particularly helpful. On the other hand, Einstein’s Special and General Theory of Relativity are used in the GPS system on my cellphone, which makes it directly applicable to my daily life. Not everybody has to be a scientist, but we should all understand that science forms part of our everyday life and culture. This ought to be clear to everyone, including those who take political decisions. We’re facing some very big problems (food resources, climate change, etc.) that are informed by scientific knowledge, so the role it has to play is vital.

**“Scientists must not talk down to people. It’s a matter of addressing people at eye level”**

**How can scientists bring their discipline closer to the public at large?**

The first thing is not to talk down to people. It’s a matter of addressing people at eye level, in a way they can understand. Those who are unable to express the essence of what they do in sim-

ple terms, in a way in which people outside their discipline can comprehend, don’t really understand the essence of what they are doing themselves. You have to be able to give a clear and basic idea of what you’re working on. I also think it’s important to stress the notion of science as part of general culture, more than just as a potential application. Applicability is important, but it’s not the reason why we do science; we do it because we’re curious. When Michael Faraday, the father of electromagnetism, was first doing his experiments, someone from the government asked him: “But Mr. Faraday, what is this good for?” To which he replied: “I don’t know yet, but I’m pretty sure that soon you’ll be able to tax it.” Now we can’t live without electromagnetism, but that’s not why Faraday developed it. There’s applied research and there’s research still to be applied. Almost every field of basic science eventually becomes applied science, but you don’t do it because of the applications. We should be able to communicate this curious fact.

**“I don’t think the influence between art and science is very strong”**

**That’s what your latest books is about, human curiosity and its mechanisms. How does that work?**

It’s far more complicated than I thought when I first started researching this. I’m neither a psychologist nor a neuro-scientist, so I had to do an enormous amount of research and talk to lots of experts in order to understand these mechanisms of the mind. It turns out that what we call curiosity comes in different kinds: the kind we feel when we see something that surprises us, which is known as perceptual curiosity, and the curiosity we feel when we investigate something in depth, which is epistemological curiosity. These two types of curiosity are driven by different parts of the brain. If we had known how different they are a long time ago, maybe we wouldn’t have used the same word for both of them. Now we’re stuck with that word and we have to add an adjective.

**Do we experience them in a different way?**

Yes, very differently. Perceptual curiosity is something rather annoying, while the other is a pleasurable thing; we anticipate a reward.

**“It’s important to stress the idea that science forms part of culture, more than just a potential application”**

**But of course they also have things in common.**

Yes, they are evolutionary mechanisms. Understanding the relation between cause and effect was important for survival.

**Do you think it’s something inherent to human beings, or could we make machines with the capacity for curiosity?**

If consciousness is an emerging property of our mental disposition, then it’s possible that curiosity is as well.





Patricia is studying a PhD in the mathematical fundamentals of quantum mechanics at the ICMAT.

**Patricia Contreras (Madrid, 1992) is studying a Ph.D on the mathematical foundations of quantum mechanics under the supervision of Julio de Vicente (UC3M) and Carlos Palazuelos (ICMAT-UCM). For some months she has been working as a predoctoral researcher with a grant at the ICMAT, where she is studying the mathematical structure of nonlocal games and Bell inequalities. Furthermore, she is collaborating with Renato Renner of the Institute for Theoretical Physics in Zurich (Switzerland), with whom she is attempting to solve problems concerning the philosophy of quantum mechanics.**

**Field of research:**

Quantum mechanics, mathematical structure of nonlocal games, Bell inequalities, and the philosophy of quantum mechanics.

## SHE DOES MATHS: Patricia Contreras Tejada

**Elvira del Pozo.** What are the chances of a person throwing a die and getting a three? The classical laws of probability say that it is six to one. And what if another person throws the die also hoping to get a three? The odds would also be six to one, because what comes up in one case does not condition the result of another. In the microscopic world, however, it is not like that: "If these dice were very small, a correlation may exist between both events; that is, according to the value obtained in the first throw, we would know without any doubt what would come out in the second throw", explains enthusiastically [Patricia Contreras](#), a predoctoral researcher at the ICMAT and the Complutense University of Madrid. This is a visual example of what is meant by the entanglement of quantum particles, a unique phenomenon in classical physics that refers to the property of such particles to *communicate* among themselves. It is also the subject that she is investigating in her Ph.D thesis, which is co-supervised by [Carlos Palazuelos](#) (UCM-ICMAT) and [Julio de Vicente](#) (UC3M). One of the things they wish to demonstrate in their research is if, in order to win in a cooperative game, the entanglement between two particles that cooperate must always be the greatest possible. A further question she poses is: "Is it possible to win if the entanglement is less?"

In addition, Contreras is also researching another problem connected with quantum mechanics, "a field that can be described mathematically in just four lines on a blackboard, but when it comes to translating what they mean in words, we get only incoherent results". One example of the difficulty of translating equations in quantum mechanics appears when we want to see what the *state of the system* means. As

Contreras explains: "For *large* systems (tables, chairs...), the state indicates the probability that the system will have one property or another (this probability may represent our ignorance about the properties of the system). On the other hand, in *small* systems (electrons, photons...), we cannot talk about probabilities, but rather we have to consider all the properties that the state represents at the same time in order to understand the behaviour of the system". What is the reason for this change in interpretation of the state? Why does the world behave in two different ways, depending only on size? Why is the boundary where it is? This is at the root of the area of philosophy of quantum mechanics in which she is working together with [Renato Renner](#), of the Institute for Theoretical Physics in Zurich (Switzerland). "How to explain that two different people, in two different laboratories, who are conducting a joint experiment, arrive at conclusions that have nothing to do with each other?"

**"How to explain that two different people, in two different laboratories, who are conducting a joint experiment, arrive at conclusions that have nothing to do with each other?"**

Many are the questions jostling in the head of this young researcher from Madrid. It could be that, apart from "enjoying the work over the next four years of research", she may find answers to some of the questions that were formulated when she was doing her Master in Philosophy of Physics at [Balliol College](#) (Oxford University), which she completed one year ago.

INTERVIEW: Victoria Ley, director of the Coordination, Evaluation and Scientific Monitoring Division of the State Agency for Research

“In the eyes of those who govern us, science does not have the status it deserves as a driver of socio-economic development in Spain”

Image: Gonzalo Merat (Jot Down)



**Victoria Ley is the head of the Coordination, Evaluation and Scientific Monitoring Division of the State Agency for Research. She was the former head of the National Agency for Assessment and Forecasting (ANEP), under the authority of the Secretary of State for Research. Before devoting herself to science management, she conducted research work as a biologist in different institutions, such as the Institut Pasteur in Paris and the U.S. Department of Agriculture. She also writes occasional articles about science for the general public, such as [these](#) that can be found in the online journal *Jot Down*. In order to find out something about the key factors concerning scientific evaluation in Spain, we were able to interview her via email.**

Victoria Ley developed a research career as a biologist before devoting herself to the management of science.

**Ágata Timón García-Longoria. What is the work of the State Agency for Research Coordination, Evaluation and Scientific Monitoring Division?**

The scientific and technical evaluation of all the submissions for funding and human resources managed by the State Agency for Research (AEI), more than 25,000 every year, are dealt with by this Division. In addition, the projects and activities that have received funding are also monitored. All of this work is carried out thanks to our scientific collaborators, who seek the people best qualified to assess every application, as well as writing reports and advising us in other matters. They are vital for the running of the Agency. Lastly, the system works with the collaboration of thousands of researchers, doctors and technologists, who evaluate the submissions and are at the heart of the assessment process. What’s more, since all the calls are grouped together, it’s easy to coordinate them all and make the system more effective.

**What is its structure? Is it composed of thematic panels?**

The structure is basically twofold; first horizontal (evaluation, monitoring, international activities and human resources), and second transversal consisting of scientific areas. In any event, given that the Agency was only set up a year ago, there is still some reorganization to be done to improve the running as well as coordination with the other divisions.

**As regards the previous organization, what changes have taken place with the creation of the Agency in terms of Assessment and Monitoring?**

Previously, scientific and technical evaluation was largely the responsibility of the old ANEP (*Agencia Nacional de Evaluación y Prospectiva*), but not all of it. Now all the evaluation and monitoring activities are managed by the AEI (*Agencia Estatal de Investigación*). The idea is to make things more functional

and efficient, and of course the main point is that decisions about the selection of projects and persons are as fair and objective as possible. The aim is to provide funding for the best projects and research groups; those that advance knowledge, extend frontiers and solve social, health, technological and environmental problems as well as being competitive in the international scientific system. The AEI has a vital role in the advance of science in Spain and its scientific status with respect to other countries.

“Research work is fascinating, and though it’s hard it has very many rewards. The satisfaction of learning, understanding and finding answers to questions is what captivates scientists”

**So what would you say is the goal of the AEI?**

The goal is to select the best projects, institutes, infrastructures and researchers so that the quality of science in Spain attains the highest possible standard of excellence, despite the lower level of funding when compared with other countries.

**What do you regard as the greatest successes of the AEI?**

The greatest merit of the current system is something that the AEI has been doing for some years: its collaboration with the scientific community in the processes of evaluation. It is a generous and honest collaboration, which is essential if the system is to function. It has been one of the mainstays of the system throughout this time, and we have to thank all those who have contributed to this success. In general, researchers trust this system of assessment in which they all participate, bringing independence and objectivity to the whole process.

“The goal is to select the best projects, institutes, infrastructures and researchers so that the quality of science in Spain attains the highest possible standard of excellence, despite the lower level of funding when compared with other countries”

**In what way do you think the present system could be improved?**

It’s necessary for the AEI to have more financing to be able to handle the enormous number of applications it receives, and also to do a good job of monitoring the projects, people and activities that receive funding. The AEI has a first-rate, well-trained and vocational staff, but more personnel is definitely required to carry out these basic tasks, and it is certainly insufficient for conducting studies of trends, impacts and forward planning. State budgets have shown that in the eyes of those who govern us, science does not have the status it deserves as a driver of socio-economic development in Spain, nor is the importance of the AEI in the system reflected there.

There are many other ways in which the AEI can be improved; a very important one would be multi-annual contracts, which

would enable calls to be more effectively coordinated as well as providing stability for the science and technology system, above all for researchers. A better internal coordination and an increase in technical personnel would enable other important functions to be carried out: reflections on what we see in the Division, the impact of the projects we fund; comparative analysis with other countries, and the implementation of improvements suggested by the scientific community, collaborators and advisers, all of which cannot be currently addressed because of deficiencies in personnel and budget.

“It’s worthwhile taking all the time necessary for preparing a good proposal”

**What would you say were the keys to success for a research proposal?**

Such a proposal should be well written and presented. It’s vital to put yourself in the place of the person who is going to read about the project, who will probably be another researcher working in the same field. The project should be clearly understandable, and the idea and the aims should not only arouse interest but also enthusiasm as well as constituting a scientific advance. It’s also important to read the terms of the call and the criteria of evaluation carefully.

It’s worthwhile taking all the time necessary for preparing a good proposal, because several years of funding are at stake and you’re applying for public money. Anyway, in general, good researchers usually write good scientific papers on their results, and those who write good articles should be able to draw up and describe a project very well.

“The chance of working at the ANEP opened up doors for me as well as my vision of science”

**You moved from research to scientific management. Tell us about that transition.**

Research work is fascinating, and though it’s hard it has very many rewards. The satisfaction of learning, understanding and finding answers to questions is what captivates scientists. For me, the years I spent doing research were fantastic and I’m happy to have had the privilege of that background. Even so, when you work as a researcher it’s difficult to interact with researchers from other fields; in the research centre itself, in meetings or at congresses, the people you rub shoulders with work on similar subjects. Personally, I loved working in a research lab and devoting myself to projects on virology, but I was very curious to know what other researchers were doing, not only in biology, but above all in more unrelated areas such as literature, engineering, history, philosophy, physics and so on. The chance of working at the ANEP opened up doors for me as well as my vision of science. It’s a luxury to work with the best researchers, who explain their points of view and put you in the picture about scientific progress in all the disciplines, making you feel that you can contribute to improving the scientific system in the country.



## SCIENTIFIC REVIEW: Models for predicting traffic

**Original title:** “Some general models of traffic flow in an isolated network”.

**Authors:** Ángela Jiménez-Casas (Universidad Pontificia de Comillas),  
Aníbal Rodríguez-Bernal (Universidad Complutense de Madrid-ICMAT).

**Source:** *Mathematical Methods in the Applied Sciences*, Volume 40, Issue 11. Pages 3982–4000.

**Date of publication:** 30th of July 2017

**Summary:** In today’s society, objects, goods, supplies and information are constantly being moved around through distribution networks. These include, for example, internet communication, the delivery of merchandise, the motion of vehicles, aircraft and so on. Yet in spite of the diversity of these examples, they can all be approximated, modelled and dealt with mathematically in a more or less unified way.

In their article “Some general models of traffic flow in an isolated network”, Ángela Jiménez-Casas, a researcher at the *Universidad Pontificia de Comillas* (UPoC) and Aníbal Rodríguez-Bernal, a professor at the *Universidad Complutense de Madrid* (UCM) and ICMAT member, describe these general processes in which *traffic* is carried on *networks* connected to emitter/receiver control centres along *traffic paths*.

The network along which traffic passes is represented by a graph consisting of a series of *nodes* (emitter/receiver traffic control centres) interconnected by means of oriented routes or paths. This means that a route that joins node A with node B can be differentiated from another route linking node B with node A; in mathematical terms they are represented by *oriented edges*. It is along these edges that certain *objects* travel between the nodes. The objects leave some nodes and travel towards other nodes, and they may remain in a node for a certain time before leaving again for another. This motion of objects on the graph is denoted as *traffic*.

The nature of the objects under consideration and the way they travel along the network are certainly different from one case to another, and this difference translates into different types of mathematical models. In the article in question, the authors consider that the traffic on the network obeys the following principles:

- (1) The nodes are connected together by well-defined oriented edges along which the objects travel. Traffic occurs only along these edges.
- (2) There is at most only one oriented edge between two different nodes.
- (3) The path of each object on the graph originates at one of the nodes (departure node) and ends at another node (arrival node). Each object may remain “stored” at one node before travelling to another node.
- (4) Only objects that have previously been at a departure node can travel along one edge of the graph.

These straightforward principles described some types of traffic very well; for example, aircraft travelling between airports, trains between stations, ships between sea ports and supply distribution networks. In other cases, such as cars travelling along streets and roads, it is difficult to accommodate them to these principles because cars are able to stop at any point before arriv-

ing at their destination, and thus do not flow from node to node without stopping. The same may be said of the flow of *bits* on the internet, since new packets of bits may originate in any computer connected to the network and no departure node exists. Furthermore, between two computers connected to the network, there is no single path that carries information from one computer to another. Despite these limitations, these ideas can be used to model a large number of such situations.

To this may be added a fifth principle, which enables the model to be simplified in an initial approximation:

- (5) Any object takes a known and constant amount of time to travel along an edge joining any two nodes. Likewise, the time that any object remains stored at one of the nodes before continuing is fixed.

These principles can be translated directly to a series of integral equations that describe at each moment the number of objects present on any edge and at any node. Only new magnitudes appear in the equations, and they are the rates at which objects at one node leave for another. These departure rates are the number of objects that leave (to each of the other nodes) per unit of time. They are defined at each moment by controllers located at each node. These departure rates being known, the traffic flow on the network can be determined at each moment in time with the sole knowledge of how many objects there are at each node and along each edge.

The next step consists in the assumption that the departure rates are calculated in terms of the traffic values (that is, the number of objects at each node and on each edge), so that a system of supervision (either automatic or human) for the traffic can be designed. This introduces into the problem the *decision operators* at each of the nodes, which on the basis of the traffic information determine the departure rates. This then makes the above-mentioned integral equations implicit. These are equations in which the unknown functions satisfy certain relations that involve them themselves. Thus, they are neither “cleared” nor given by an explicit formula that can be computed.

The existence of solutions to this system is demonstrated in the article. In this model, the decision operators use the traffic information in a period of time to determine the departure rates at a given moment. This transforms the implicit integral equations into delayed functional equations in which the value of an unknown at a given moment is determined by the value of the unknown at a certain time or previous period of time.

The case in which the information used by the decision operator is only the number of objects at that node serves to reduce the number of unknowns in the problem, since the traffic along the edges is explicitly determined by the number of objects at each node.

Certain general conditions (known as Lipschitzianity conditions) for the decision operators are assumed in this article, in which it is shown that the resulting models are well thought out and obey the principle of fleet conservation; that is, that the number of objects travelling through the network does not depend on time. The authors have analyzed in greater detail some cases of linear decision operators in the nodes, for which they have studied some properties of time-independent solutions, the so-called *equilibrium* solutions. They have also demonstrated that if non-updated traffic information is used for decision-making the system may crash in a finite period of time, in the sense that the number of objects at a node becomes negative. On the other hand, if up-to-date traffic information is used for decision-making, solutions exist for all times. This means that traffic flow over the network will function at all times with the chosen set of automatic decision-making rules.

The models outlined in this article differ from previous ones that describe the spatial distribution of objects along the edges of the graph, in which the unknown functions therefore incorporate a spatial as well as a temporal dependence. This means that the resulting models become systems of first-order partial differential equations in the graph, which enables the spatial behaviour of traffic to be described as well as predicting and controlling spatial gridlock (bottlenecks), and thus making it possible to study both the causes and solutions by means of mathematical models.

On the basis of these results, the researchers could consider the design of automated rules of decision-making to enable traffic on the network to behave in a predetermined manner. Operational restrictions could also be introduced into the problem, such as limiting the maximum volume of traffic at each node and/or edge and studying the long-term behaviour of traffic for some reasonable class of decision operators. Likewise, it would be interesting to analyze the case in which the journey time of each edge is not constant, but rather a random variable with small fluctuations on a mean value.

Mathematical Methods in the Applied Sciences

**Research Article**

Received 10 May 2016 Published online 27 January 2017 in Wiley Online Library

(wileyonlinelibrary.com) DOI: 10.1002/mma.4278  
MOS subject classification: 34A12; 34A33; 34A34; 34K31

## Some general models of traffic flow in an isolated network

**Ángela Jiménez-Casas<sup>a</sup> and Aníbal Rodríguez-Bernal<sup>b,c,\*†</sup>**

Communicated by R. Bravo de la Parra

Starting from basic principles, we obtain mathematical models that describe the traffic of material objects in a network represented by a graph. We analyze existence, uniqueness, and positivity of solutions for some implicit models. Also, some linear models and their equilibria are analyzed. Copyright © 2017 John Wiley & Sons, Ltd.

**Keywords:** traffic flow; networks; graphs; delay integral systems

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### 1. Introduction

Nowadays, the displacement of objects, goods, supplies, or information on distribution networks is something that we experience on a daily basis. From Internet communication or good supplies to all sorts of vehicles, including surface motor vehicles, planes, and trains, to mention just a few of illustrating examples. Despite their intrinsic differences, all these phenomena have a common underlying nature that allows to a more or less unified mathematical treatment. First, the spatial network is assimilated to a graph, which is made of a set of 'nodes' connected through oriented, non-intersecting traffic paths or oriented 'edges', along which 'objects' travel between nodes. Some objects could be 'stored' in some node for some time before traveling to another node. The motion of objects along the network is denoted 'traffic'. Second, the nature of the 'objects' and the way the travel on the network differentiate some cases from others. Here, for the description of the traffic, we will assume the following

**Principles of the traffic of material object in a network**

**Principle 1** The nodes are connected through well-defined oriented edges, along which the objects travel. The traffic only occurs along the edges.  
**Principle 2** Between two nodes, there is at most one oriented edge connecting them.  
**Principle 3** The traffic originates at one node (departure node) and ends at another node (arrival node). Traffic can be stored for some time at one node.  
**Principle 4** Along an edge, only objects previously in the departure node can travel.

These principles describe well some types of traffic in networks, which include air traffic between airports, trains traffic between stations, ship traffic between sea ports, or supply distribution networks. Some other types of situations like car traffic or Internet communication packets are more difficult to accommodate to such principles.

The models we present in this paper, derived from the aforementioned principles, are somehow related to previous ones appearing in the literature of air traffic control. For example, the so-called aggregate flow models describe the traffic flow (of airplanes) among connected control centers, which divide the airspace of a given region (e.g. [1–3] and references therein). They give rise to space-time discrete models. On the other hand, the so-called Eulerian models create a spatial grid of surface elements (SEs) covering the region of airspace being modeled. Hence, a typical SE is connected to eight other SEs to which the traffic outflows and inflows. SEs lying on the boundary of the airspace have additional inputs representing traffic entering from outside the network (e.g. [4–8] and references therein). The dynamics of the air traffic flow in a SE also leads to discrete models. See also [6, 9] for comparisons between these two types of models. In our setting, either control centers or SEs can be assimilated to the nodes in the graph and the models in the aforementioned references resemble a time discretization of some of the linear models in Section 4.

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"Some general models of traffic flow in an isolated network", of Ángela Jiménez-Casas and Aníbal Rodríguez-Bernal.

### About the authors

**Ángela Jiménez-Casas** is a professor at the Escuela Técnica Superior de Ingeniería (ICAI) of the Universidad Pontificia de Comillas Department of Applied Mathematics. She graduated in Mathematical Sciences from the Universidad Complutense de Madrid (1980–1985) and obtained her doctorate in Mathematical Sciences from the same university (1990–1996). She was a professor at the Escuela Universitaria de la Universidad Politécnica de Madrid from 1987 to 1998, and since 1985 has been a professor at the ETSI-ICAI de la Universidad Pontificia Comillas Department of Mathematics, where she was also director from 1999 to 2003. From December 2002 to July 2009 she held the post of Deputy Rector of Research, Development and Innovation at the Universidad Pontificia Comillas de Madrid. In addition, she is the coordinator of the Comillas nonlinear dynamics group and a member of Asymptotic Behaviour and Dynamics of Differential Equations (CADEDIF) inter-university group by which she participates in research projects funded by the Spanish Ministry of the Economy and Competitiveness. Since 2002, she has been the Honorary Collaborator at the UCM Department of Applied Mathematics as a researcher in the field of nonlinear partial differential equations. She returned to direct the ETS-ICAI Department of Mathematics until the academic year 2015–2016, where she was director of the UPoC Chair of Technological Science and Religion.

**Aníbal Rodríguez Bernal** is a full professor of Applied Mathematics at the Universidad Complutense de Madrid and has been a member of the ICMAT since its beginnings. He graduated from the UCM in 1986 and obtained his doctorate at the same university in 1990. He has pursued his research career in the field of infinite-dimensional dynamic systems, with emphasis on the study of qualitative properties of solutions to nonlinear dissipative partial differential equations. He has supervised seven PhD theses, headed several research projects as part of the National Plan for Mathematics, and has published some one hundred research papers in international journals. His research work is conducted as part of the UCM CADEDIF (Asymptotic Behaviour and Dynamics of Differential Equations) research team of which he is the founder. He was director of the UCMA Department of Mathematics from 2012 to 2016.

## SCIENTIFIC REVIEW: “Gravitating vortices, cosmic strings, and the Kähler–Yang–Mills equations”

**Title:** “Gravitating vortices, cosmic strings, and the Kähler–Yang–Mills equations”.

**Authors:** Luis Álvarez-Cónsul (ICMAT-CSIC), Mario García-Fernández (ICMAT-UAM), Óscar García-Prada (ICMAT-CSIC).

**Source:** *Communications in Mathematical Physics*, Volume 351, Issue 1, pp. 361–385.

**Date:** April 2017.

**doi:** 10.1007/s00220-016-2728-2

When a body of water cools below freezing and nucleation starts, ice is created about randomly separated nucleation sites, with the water molecules arranged in highly ordered crystal lattices. As the frozen areas grow, adjacent ice crystals join up erratically. The boundaries where the crystal structures are disrupted are topological defects. In general, phase transitions in condensed matter associated with a loss of symmetry, such as the formation of crystals, create similar topological defects.

A combination of ideas from cosmology and the unification of forces in particle physics suggests a sequence of similar symmetry-breaking phase transitions in the very early universe, as it expanded and cooled down. Like the transitions in condensed matter, they may have led to the formation of defects, some of which may have survived up until the present. A special type of topological defects, known as cosmic strings, would be especially interesting from the perspective of cosmology and particle physics. They would be exceedingly narrow filaments of primordial material, left over from the early moments of the universe. Although not observed yet, their existence would explain some exotic astrophysical phenomena, such as an unexpected surplus of high-energy positrons observed flitting through space [1], or the formation of super-massive black holes at high redshifts [2]. They may be observable due to their gravitational effects on the cosmic microwave background or gravitational wave experiments [3].

The basic mathematical structure of a string is a complex scalar field — the Higgs field  $\varphi$  — that winds around the location of the string, where there is a concentration of energy density. The existence of cosmic strings is intimately related with the choice of potential for the Higgs field

$$V(\varphi) = \lambda(|\varphi|^2 - \tau)^2.$$

The minimum energy configuration has  $|\varphi|^2 = \tau$  but the phase of  $\varphi$  is undetermined and labels the points on the manifold of vacua, which is a circle. The choice of ground state in the circle — known as spontaneous symmetry breaking — may vary in space-time and, by continuity,  $\varphi$  can be forced to leave the manifold of vacua producing surfaces where  $\varphi = 0$  (the two-dimensional worldsheet of the string in space-time).

For a class of physical models, B. Linet, and A. Comtet and G. W. Gibbons, showed in the late 1980s that the theoretical existence of cosmic strings reduces to solving a system of partial differential equation on a Riemann surface  $\Sigma$ , known as the Einstein–Bogomol’nyi equations. For non-compact  $\Sigma$ , the analysis of these equations carried out during the early 1990s, mainly by Y. Yang and J. Spruck, led to the construction of families of finite-

energy cosmic strings. For compact  $\Sigma$ , the Einstein–Bogomol’nyi equations were studied by Y. Yang on the 2-dimensional sphere (the only allowed topology), who proved existence of solutions under certain conditions on the relative position of the zeros of the Higgs field.

In a recent publication in *Communications in Mathematical Physics*, L. Álvarez-Cónsul (ICMAT-CSIC), M. Garcia-Fernandez (ICMAT-UAM) and O. García-Prada (ICMAT-CSIC) found interesting relations between the Einstein–Bogomol’nyi equations and several areas of modern geometry. A first link was obtained applying dimensional reduction methods to the Kähler–Yang–Mills equations on the product of the 2-dimensional sphere with a compact Riemann surface. The Kähler–Yang–Mills equations, introduced in a previous paper [4], emerge from a natural extension of two well-studied theories in geometry, respectively devoted to constant scalar curvature Kähler metrics and Hermitian–Yang–Mills connections. The equations obtained by dimensional reduction generalize the Einstein–Bogomol’nyi equations to a Riemann surface of any genus. Since they couple the vortices to a Riemannian metric on the surface, they were called the gravitating vortex equations.

A second link started with the observation that the set of sufficient conditions found by Y. Yang to solve the Einstein–Bogomol’nyi equations have a natural meaning in the Geometric Invariant Theory, as introduced by D. Mumford in the 1960s to construct quotients by group actions in algebraic geometry. These links inspired a conjecture that identifies the moduli spaces parametrizing solutions of the Einstein–Bogomol’nyi equations and the simplest quotients appearing in Mumford’s theory, namely the ones attached to the theory of binary quatics, a topic of the second half of the 19th century in which computational open problems still remain.

Elaborating on the above interplay, L. Álvarez-Cónsul, M. Garcia-Fernandez, O. García-Prada and V. P. Pingali (IIISc–Bangalore) proved in a recent preprint [5] a converse to Yang’s Theorem, establishing in this way a precise correspondence with Geometric Invariant Theory for the Einstein–Bogomol’nyi equations. This provided evidence for the above conjecture about moduli and binary quatics, and furthermore settled in the affirmative a conjecture by Y. Yang about the non-existence of cosmic strings on the Riemann sphere superimposed at a single point [6]. They also showed existence and uniqueness of solutions of the gravitating vortex equations in genus  $g \geq 2$  for an explicit region of coupling parameters. This is a satisfying result that can be compared with the hyperbolic case of the celebrated Uniformization Theorem of Klein, Poincaré, and Koebe from the late 19th and the early 20th centuries, and




the closely related result that a compact Riemann surface of genus  $g \geq 2$  admits a unique metric of constant curvature with fixed volume. The proof requires a combination of modern techniques of symplectic geometry and non-linear analysis on manifolds.

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Commun. Math. Phys. 351, 361–385 (2017)  
Digital Object Identifier (DOI) 10.1007/s00220-016-2728-2

**Communications in  
Mathematical  
Physics**



**Gravitating Vortices, Cosmic Strings,  
and the Kähler–Yang–Mills Equations**

**Luis Álvarez-Cónsul, Mario García-Fernandez, Oscar García-Prada**

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Received: 17 March 2016 / Accepted: 24 May 2016  
Published online: 6 September 2016 – © Springer-Verlag Berlin Heidelberg 2016

**Abstract:** In this paper we construct new solutions of the Kähler–Yang–Mills equations, by applying dimensional reduction methods to the product of the complex projective line with a compact Riemann surface. The resulting equations, which we call gravitating vortex equations, describe abelian vortices on the Riemann surface with back reaction of the metric. As a particular case of these gravitating vortices on the Riemann sphere we find solutions of the Einstein–Bogomol’nyi equations, which physically correspond to Nielsen–Olesen cosmic strings in the Bogomol’nyi phase. We use this to provide a Geometric Invariant Theory interpretation of an existence result by Y. Yang for the Einstein–Bogomol’nyi equations, applying a criterion due to G. Székelyhidi.

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Partially supported by the Spanish MINECO under the ICMAT Severo Ochoa Grant No. SEV-2011-0087, and under Grant No. MTM2013-43963-P. The work of the second author has been partially supported by the Nigel Hitchin Laboratory under the ICMAT Severo Ochoa Grant. The research leading to these results has received funding from the European Union’s Horizon 2020 Programme (H2020-MSCA-IF-2014) under Grant agreement No. 655162, and by the European Commission Marie Curie IRSES MODULI Programme PIRSES-GA-2013-612534.

**About the authors:**

**Luis Álvarez Cónsul** is científico titular (tenured position) at the ICMAT. His research interests lie in pure mathematics, specifically in algebraic geometry and differential geometry and in their interplay with representation theory and theoretical physics. He is especially interested in geometric structures associated to moduli spaces parameterizing objects of various kinds, such as bundles, quiver representations, algebraic varieties and solutions of equations of gauge type.

Álvarez Cónsul carried out his undergraduate studies in Seville and Madrid, and obtained his PhD in physics at Universidad Autónoma de Madrid in 2000. He has held full-time academic positions at the University of Illinois at Urbana-Champaign as a research assistant professor, the University of Bath and CSIC. He has been a recipient of a Marie Curie fellowship and a Ramón y Cajal research contract. Álvarez Cónsul has written articles in *Inventiones Mathematicae*, *Geometry and Topology*, *Crelle* and *Communications in Mathematical Physics*.

**Mario García-Fernández** is an Assistant Professor (Profesor Ayudante Doctor) at the Universidad Autónoma de Madrid, from September 2017. Previously, he has done postdocs at the ICMAT (Marie Curie Fellow, Severo Ochoa Fellow--Nigel Hitchin Laboratory), at the École Polytechnique Fédéral de Lausanne (Switzerland), at the Centre for Quantum Geometry of Moduli Spaces (Denmark) and at the Max Planck Institute for Mathematics (Germany). He obtained his PhD in 2009 at the Universidad Autónoma de Madrid.

His research is in the areas of differential geometry and algebraic geometry. The problems he studies are mainly in complex geometry (Kähler and hermitian), and have strong links with geometric analysis and mathematical physics. The main research line he has carried out is the study of geometric structures and moduli spaces.

Current projects include special metrics and connections in Kähler geometry (constant scalar curvature Kähler, Hermite–Yang–Mills, vortices, Hitchin equations) and their relation to stability conditions in algebraic geometry (GIT), moduli spaces (bundles and varieties, Higgs bundles, wall-crossing), special holonomy and mirror symmetry (Calabi–Yau, G2 geometry, Strominger system, generalized geometry, T-duality, vertex algebras). García-Fernandez is the author of papers in journals such as *Communications in Mathematical Physics*, *Geometry and Topology*, *Mathematische Annalen* y *Proceedings of the London Mathematical Society*.

**Óscar García-Prada** is a CSIC Research Professor at the ICMAT. He obtained a D.Phil. (doctorate) in Mathematics at the University of Oxford in 1991, and had postdoctoral appointments at Institut des Hautes Études Scientific (Paris), University of California at Berkeley, and University of Paris–Sud, before holding positions at Universidad Autónoma de Madrid and École Polytechnique (Paris). In 2002 he joined the Spanish National Research Council (CSIC).

His research interests lie in the interplay of differential and algebraic geometry with differential equations of theoretical physics, more concretely, in the study of moduli spaces and geometric structures. The moduli spaces considered involve objects such as vortices, solutions to general gauge-theoretic equations, Higgs bundles and representations of surface groups and fundamental groups of higher dimensional Kähler manifolds.

Professor García-Prada is the author of a number of papers in journals like *Duke Math. Journal*, *J. Differential Geometry*, *Topology and Math. Annalen*, with collaborators in Spain, USA, UK, France, Germany, Portugal, Italy, Canada and India. He is a member of various international committees and serves as editor in several international journals, including the *International Journal of Mathematics*. He has supervised more than 10 PhD theses as well as several postdoctoral researchers, and has been the Principal Investigator of around 40 national and international research grants.

## PROFILE: Simón Rodríguez

## “Heterogeneity greatly facilitates teamwork”



Image: Simón Rodríguez

Simón Rodríguez.

**Simón Rodríguez (Vega de San Mateo, Gran Canaria, 1993) has always found science attractive, so in 2011 he began his degree in Physical Sciences at the Autonomous University of Madrid (UAM) and completed it in 2015. Throughout that time, his projects involved different branches of physics that ranged from the experimental side to the most theoretical aspects of the subject. It was the latter that exerted the greatest attraction on him and led him to do a Master in Theoretical Physics at the Complutense University of Madrid. However, after completing his final project in this discipline, his interest centered on the machine-learning projects of David Gómez-Ullate (ICMAT-UCM), one of the professors on his master course, who would eventually become his thesis supervisor. In 2016, Rodríguez began his doctorate on online medical diagnosis using image processing, as part of a team at the ICMAT working on Statistics, Probability and Operations Research (SPOR).**

**Elvira del Pozo.** Simón Rodríguez, a Ph.D student with the ICMAT Statistics, Probability and Operations Research (SPOR) group, is a physicist among mathematicians. He gained his degree in Physics at the Autonomous University of Madrid and then went on to complete a Master in Theoretical Physics. “I could already imagine myself doing research in this field; I like it a lot”, he says. However, his interest in artificial intelligence and big data meant that his encounter with one of the professors on his master course caused him to radically change his field of study. This teacher was David Gómez-Ullate (UCM-ICMAT), his current thesis supervisor, who also has a background in physics.

“The official description of the course is not yet definitive, because it’s rather difficult to explain”, says Rodríguez when asked what the work on his Ph.D is about. “We’re trying to develop online medical diagnosis using neural networks”. The point of departure consists of using x-rays of real patients, each one being complex and unique. The first step is to obtain simplified images to identify important data that can be examined by an expert health worker in order to arrive at a diagnosis. Mathematics is then employed to design an algorithm capable of interpreting these indicators autonomously. He goes on to say that: “We have to keep providing this algorithm with examples so it learns to carry out the task properly; the better the examples and the more we provide, the better it can function”.

Processing and classifying the images is no easy task, and working with neural networks is the key to obtaining the best results. As Rodríguez explains: “It is possible for the algorithm to think like a cerebral neural network; each neuron in the brain carries out a very simple operation, such as passing on an electrical impulse, but the structure of the network is able to perform much more complex computations”. In the same way, the algorithms he works with consist of a series of layers capable of carrying out simple operations, products and sums. When all these layers are combined

massively, one can attain a high capacity of calculation. These arrangements can be molded to obtain the results one requires.

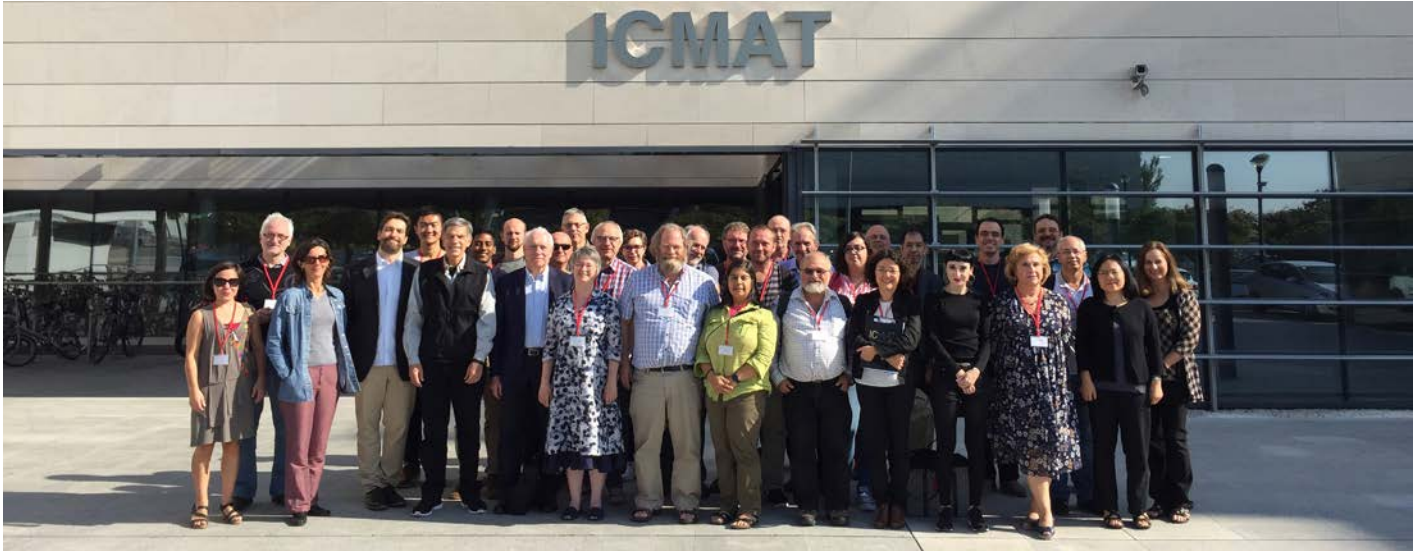
With his background in physics, he recognizes that this research is still in its initial stages, but may perhaps serve to understand certain concepts of lighting and optics that are closely related with the graphic information he is working with – x-ray images. Nevertheless, and above all, he uses statistics, mathematics and programming in his daily work. “It’s a project that requires many disciplines,” he says. This explains why his lab colleagues are not only mathematicians but also computer technicians, and physicists like himself. “Heterogeneity greatly facilitates teamwork, because each member of the group gives the best of what he or she has to offer, as well as helping the others when they need it”.

Ultimately, the “mix” that exists in this small research team is perhaps a reflection of the fact that the highly complicated challenges facing science need to be analyzed from different angles and solved as part of a joint effort. The same philosophy lies behind other projects in which the SPOR group is involved, such as the analysis of feelings and the detection of banking fraud, which are being tackled with a combination of scientists from different branches working in collaboration. They all make use of and implement cutting-edge technologies such as machine learning, big data and artificial intelligence.

Rodríguez has no doubt that he wants to devote himself to research. As he remarks: “I’m very interested in conducting in-depth research in the field of neural networks”. And this in spite of the fact that he is a great believer in basic science and would like to develop techniques that are useful for society. He would even like to incorporate another of his passions into this equation – ecological activism – and “study environmental problems with machine learning”, thereby shedding light on complex and pressing issues such as climate change. When asked if he would ever like to return to his home in the Canary Islands, he replies as follows: “I’d love to, but it’s difficult to do research in Spain, and even more so over there”.

## ICMAT QUESTIONNAIRE: Liz Mansfield and Peter Clarkson

Image: ICMAT



The congress "Integrable systems, symmetries, and orthogonal polynomials" celebrated the 60th birthday of Peter Clarkson and Liz Mansfield.

**Liz Mansfield was born in Sydney, Australia. She studied a Bachelor of Science degree majoring in Mathematics, and obtained her PhD from the University of Sydney, Australia, in 1992 under the supervision of E.D. Fackerell. Currently, she is Professor of Mathematics at the University of Kent, U.K.**

### **Why did you choose mathematics ahead of any other subject?**

I was interested in many things but I could see the difficulties with misogyny, in terms of having a career, in all the fields that interested me. At least with a mathematical problem, you couldn't be told that what you had done was wrong just because you were a girl – things were either right or wrong according to the relevant logic. I also found mathematics to be a wonderfully engaging, calm and clear retreat in a chaotic and difficult world.

### **Besides mathematics, which activities do you like most?**

I play the piano, I enjoy my garden, and I am a 'foodie'!!

### **A movie, book or play you'd recommend?**

I watched 'Harold and Maude' many times when I was a teenager. More recently, I was captivated by the French TV series, 'Les Revenants'. Right now, I am reading Johannes Itten's 'The Art of Color' and listening to Paul Dukas' Piano Sonata in E-flat minor.

### **How was your first encounter with mathematical research?**

I had a job, briefly, in a money market for a bank, and I wrote a short report on a little program I wrote. It became clear that I needed more substantial intellectual content in my job, so I decided to do a PhD when the opportunity came up.

### **What did you like most about your early experiences with mathematical research?**

I liked that what I did was actually useful and that the context and utility could be explained to an audience that might not have been able to read the report itself.

### **Which scientist impressed you most during your career?**

First of all, my father, I could see how much he loved what he could do with his chemistry in his pathology lab, how creative he could be, and how the courage he had, to stand by his own deductions and experiments, paid off. After that, Ariele Iserles: not just the mathematics but the passion for equality and inclusivity.

### **If you could have a one hour blackboard discussion with an ancient mathematician, who would you choose to meet and what would you discuss?**

There isn't an ancient one I'd like to talk to, but I would be interested to talk to Gabrielle Émilie Le Tonnelier de Breuil, Marquise Du Châtelet (1706–1749) about her clarifications of Newton's Principia (still in use today, I am told), and her work on the nature of kinetic energy and the principle of the conservation of energy.

### **Do you have a particular theorem or formula you especially like?**

I love proofs which involve a process, and which can be adapted to nearby problems.

During my PhD, I studied Bachmair's and Buchberger's proof of the 'Diamond Lemma', used to prove that the output of Buchberger's algorithm had the requisite properties of a Groebner basis. I thought it was wonderful – so effective, elegant and so useful!

Later on, I rediscovered for myself, Noether's proof of her second theorem, that the Euler Lagrange equations for an action principle having a pseudogroup symmetry, were not independent. The proof generalises, essentially unchanged, to calculate the dependencies on discrete Euler Lagrange systems.

### **What is your favourite mathematical book?**

I have a long, long list of favourites, from different points of my career. As a student, I absorbed Munkres' text on Topology, also texts by Spanier, and Stakgold. I have come back to "Analysis, Manifolds and Physics" by Choquet-Bruhat and de Witt Morette, many times over the years. Then there is Peter Olver's "Applications of Lie groups to differential equations" which has been a constant companion over 20 years or so. My current favourite 'book' is a collection of three densely written notebooks, taken



during a 90 lecture course on Lie algebras, which I was privileged to attend, given by Georgia Benkart at the University of Madison, Wisconsin.

**How would you describe/sketch your research interest in a few lines?**

My main interest for a while has been to incorporate the Lie group symmetries, which produce Noether’s conservation laws, into any numerical code used to study the relevant physical problems. The motivation is that if you incorporate the symmetry into a bona fide discrete variational problem, then you have, a priori, incorporated the physics. Most numerical methods throw away the physics during the discretisation.

Despite being told this is impossible, in fact, you can do it. The trick is to treat the approximate problem as being exact, in its own right. You work out what are the induced Lie group actions on that class of numerical schemes, and then prove a version of Noether’s theorem for variational problems in that class. Et voilà! You take an invariant discrete Lagrangian and then you have exactly conserved approximate conservation laws obtainable by those symmetries, and most likely, symplectic structures as well.

**Peter Clarkson was born in Westow, North Yorkshire, England in 1957. He studied Mathematics at the University of Oxford, UK, and obtained his DPhil from University of Oxford, UK, in 1983 under the supervision of Professor Bryce McLeod, FRS. Currently, he is Professor of Mathematics at the University of Kent, Canterbury, UK.**

**Why did you choose mathematics ahead of any other subject?**

From an early age, I found Mathematics came to me naturally and it was clearly my best subject throughout my years at school.

**Besides mathematics, which activities do you like most?**

Cooking, travelling and watching sport.

**A movie, book or play you’d recommend?**

“A Beautiful Mind” and “The Man who Knew Infinity”.

**How was your first encounter with mathematical research?**

Writing an essay on “Bessel Functions” as a second-year undergraduate.

**What did you like most about your early experiences with mathematical research?**

Travelling and meeting people from all over the world.

**Which scientist impressed you most during your career?**

Mark Ablowitz.

**If you could have a one hour blackboard discussion with an ancient mathematician, whom would you choose to meet and what would you discuss?**

Paul Painlevé.

**Do you have a particular theorem or formula you especially like?**

Theorem: The Cayley-Hamilton theorem

Formula:  $\exp(i\pi)=-1$

**What is your favourite mathematical book?**

The “NIST Handbook of Mathematical Functions Hardback”, edited by F.W.J. Olver, D.W. Lozier, R.F. Boisvert and C.W. Clark.

**Which recent results in your field would you highlight?**

The understanding and use of the modern definition of Cartan’s moving frame, begun and championed by Peter Olver, is a high-light.

**Which particular mathematical problem do you consider especially challenging?**

Always, the latest one I am working on.

**Which subjects in mathematics outside your field would you like to learn more about?**

My field is rather amorphous and boundary free – I am always learning new things. Right now I am learning about particular infinite dimensional Lie algebras, various numerical schemes, and the geometry of musical harmony.

**In the future, where do you think the interaction between different branches of mathematics may be more fruitful?**

It is important to keep aware of what is happening in fields adjacent to your own, and to communicate using the language of the major branches of mathematics. This means you are ready to make connections when they arise – they are always a surprise.

**How would you describe/sketch your research interest in a few lines?**

My research interests are methods for determining exact and approximate solutions of nonlinear partial differential equations, ordinary differential equations and differential-difference equations of mathematical physics, specifically using soliton theory and symmetry reduction methods involving nonclassical and generalized techniques.

In particular I have made extensive studies of the Painlevé equations, which are six nonlinear ordinary differential equations that may be thought of analogues of the classical special functions. I have studied asymptotics, Bäcklund transformations, connection formulae and exact solutions for these Painlevé equations as well as applications such as nonlinear waves, orthogonal polynomials and random matrices.

**Which recent results in your field would you highlight?**

1. The unified (Fokas) transform for solving integrable initial-boundary value problems.
2. The Deift-Zhou nonlinear steepest descent method.

**Which particular mathematical problem do you consider especially challenging?**

The Painlevé equivalence problem, i.e. given an equation with the Painlevé property, how do we know which Painlevé equation, or Painlevé  $\sigma$ -equation (a second-order, second-degree equation), it is related to?

**Which subjects in mathematics outside your field would you like to learn more about?**

Stochastic differential equations.

**In the future, where do you think the interaction between different branches of mathematics may be more fruitful?**

The development of numerical algorithms for differential equations, in particular nonlinear differential equations, which incorporate various properties of the differential equation concerned.

## TELL ME ABOUT YOUR THESIS: Tania Pernas

**Title:** "On the splat singularity for the Muskat problem".

**Author:** Tania Pernas Castaño.

**Supervisor:** Diego Córdoba Gazolaz.

**Tania Pernas (June 29th, 1988, Viveiro, Galicia)** says in the dedication of her doctoral thesis, "On the splat singularity for the Muskat problem", presented at the ICMAT last February, that "doing a thesis is a time of changes and ups and downs that wouldn't be manageable without the help of people you meet along the way". Her research work focuses on the Muskat problem, a study of the evolution of the interface between two immiscible fluids of different characteristics in a porous medium. Together with her thesis supervisor, Diego Córdoba (CSIC-ICMAT), she analyzed the presence of singularities in this situation. Until these recent contributions, nothing was known of the so-called Splat singularities in the Muskat problem. After work that has lasted four years, she has managed to delve deeper into the study of this type of singularities which, although they exist for other media, had not been demonstrated in the Muskat problem. The most innovative idea in her thesis revolves around this proof, which is based on a reasoning by contradiction. Furthermore, in this research, different cases of porous media are studied: homogeneous and non-homogeneous media, on the way to analyzing the more general problem, or the closest to real physical problems. For the non-homogeneous case, which has received very little attention until now, the solution for a general case is shown to exist, as well as the existence of singularities, which indicates the added difficulty when it comes to the homogeneous case.

It was precisely this result that confronted her with the most difficult moment in her doctorate. As she recalls: "When I thought I'd solved the problem, I sent the result to a journal, and right after that I realized that I'd made a mistake. It was an important flaw that invalidated the entire result. I had to rework the proof from scratch". Happily, she was able to resolve it, just two days before starting her 2016 Christmas vacation. "That was the moment of greatest satisfaction," she says with a chuckle.

This young mathematician, who has already completed a research stay of three months at Princeton University with Fields medallist Charles Fefferman and at present has a contract as assistant professor at the UAM, is now coming to the end of a period in her life that she will remember with special affection. As she says: "After an early period of adaptation, you realize that your place of work has become like your home, and of course your work colleagues as part of your family".

**Tania Pernas.** My thesis is centered on the so-called Muskat problem, a topic in mathematical physics on partial differential equations that come from fluid mechanics, the study of fluids, the forces that give rise to it and their interactions with the boundary that limits them.

The evolution of the fluid over time is represented by partial differential equations. Specifically, in order to approximate the motion of conventional fluids such as water, air or oil, the so-called Navier-Stokes equations are used. They constitute a basic model in different scientific fields such as aeronautics, meteorology, hydraulics and so on. They are regarded as the fundamental law which, together with the laws governing the conservation of mass, enable us to describe the motion of a fluid on the basis of some particular initial and boundary conditions. However, conditions exist in which these equations cannot be applied; for instance, in porous media, where the fluid passes through the interstices of a solid structure. Examples of porous media are rocks, substrates (aquifers and petroleum deposits), zeolites, biological tissue (e.g. bones, wood and cork) and artificial materials such as cement and ceramics. The study of fluid dynamics in porous media is therefore of great interest in areas of applied science and engineering: filtration, mechanics (acoustics, geomechanics, soil mechanics and rock mechanics), engineering (petroleum engineering, bioremediation), geosciences (hydrology, geophysics), biology and biophysics, materials science and so on.

In order to model this dynamic effectively, the resistance provided by the porous medium must be taken into account, which is something the Navier-Stokes equations do not do, and thus they are not sufficient in this case. Darcy's Law is employed instead, the name coming from the civil engineer Henry Darcy (1803-1858), one of the engineers charged with the design and construction of a water distribution system for the French city of Dijon. Darcy established this experimental law around 1850; it adequately describes the flow dynamics of an incompressible fluid in a porous medium.

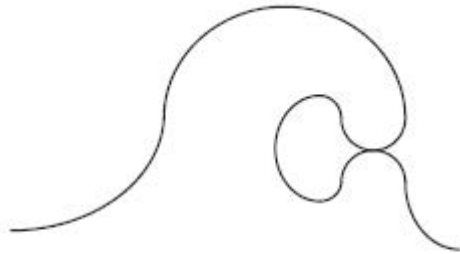
It is this law that generates the equations enabling the Muskat problem to be studied. Morris Muskat (1906-1998) was a North American petroleum engineer who, in collaboration with Milan W. Meres, generalized Darcy's law for the study of the multiphase flow of water, oil and gas in a petroleum reservoir. Specifically, the Muskat problem models the interface between two immiscible fluids of different characteristics in a porous medium. The boundary separating the two fluids – the interface – is caused by the discontinuity of the viscosities and/or densities of the fluids.

This thesis deals mainly with the study of singularities in these equations. Specifically, the presence of two types of singularities is studied; those that appear in the case of water waves. This is the problem in which the evolution of the interface of a region of water in a vacuum is studied.

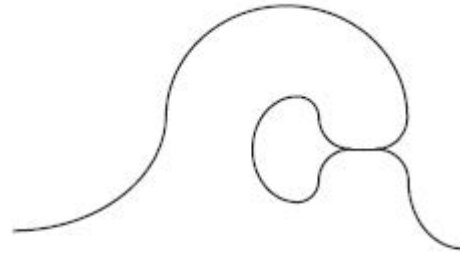
The first type of singularity are "splash" singularities, which correspond to the case in which the interface of the fluid

collapses smoothly in on itself at one point. The second type are known as “splat” singularities and are a variant of the first type, in which the intersection is the arc of a curve instead of a point. This type of singularity also exists for the case of water waves, and also for the case of water waves

with vorticity. However, in the thesis, as the first original result of our research, we prove that splat-type singularities cannot be formed in the Muskat problem under study in the homogeneous case; that is, when the medium presents the same permeability at all its points.



(a) Singularidad de tipo “splash”



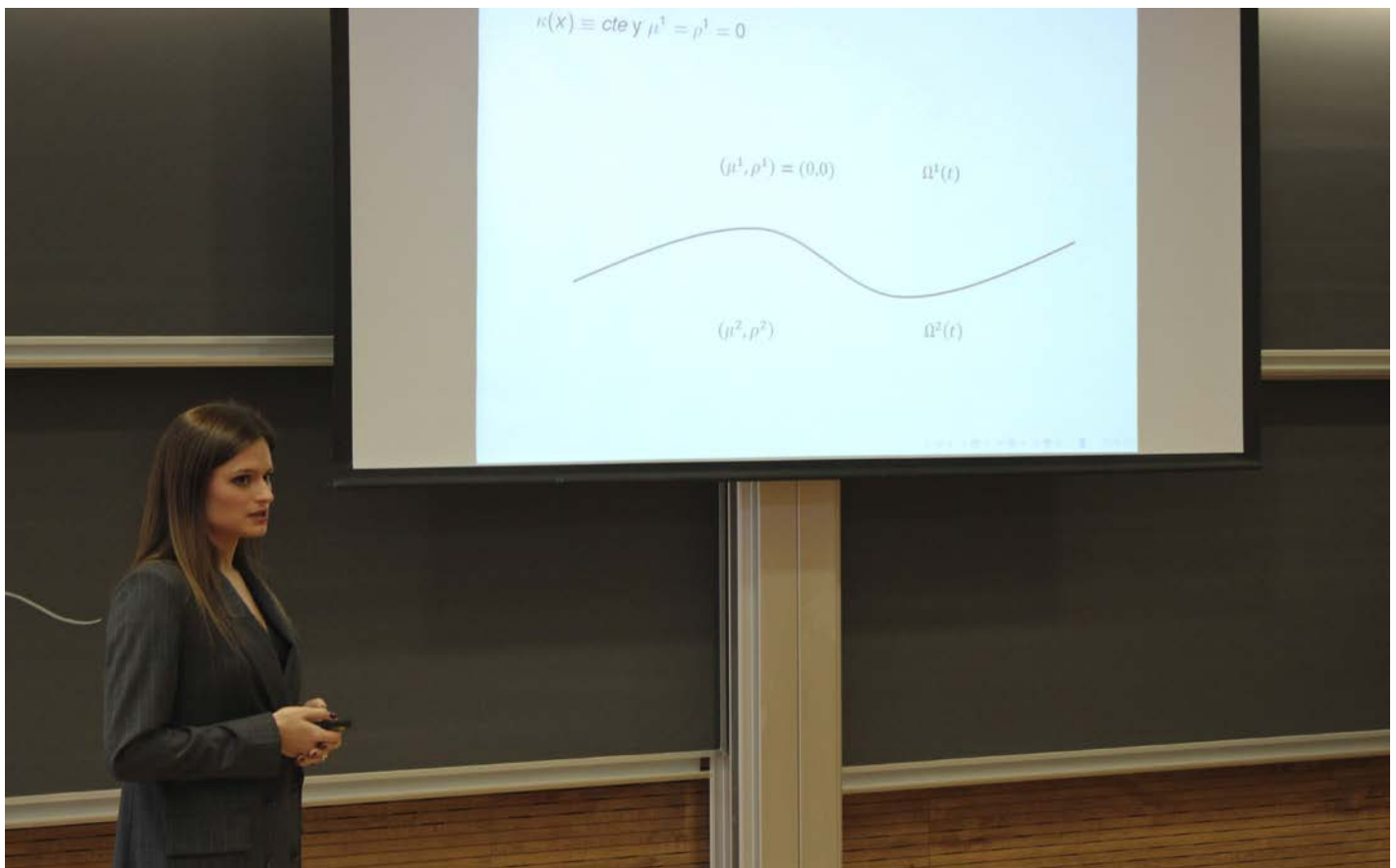
(b) Singularidad de tipo “splat”

In this work we have also addressed the more generic case of the non-homogeneous Muskat problem, when the porous medium has different values of permeability. Furthermore, we prove the local existence in time in Sobolev spaces for the stable regime. Having established the local existence of the non-homogeneous Muskat problem, the next natural step is to conduct a study of finite-time singularities. As a conclusion, we prove the non-exis-

tence of splat-type singularities and the existence of splash-type singularities.

On completion of the thesis, the work to be undertaken now is of a highly technical nature. The idea is to study other cases of the non-homogeneous Muskat problem, ruling out hypotheses of the theorem we proved in my thesis. It would also be interesting to apply these ideas and techniques in other equations.

Image: ICMAT



Tania Pernas defended her doctoral thesis last November at ICMAT.



## The Severo Ochoa and María de Maeztu centres and units of excellence join forces

Image: MINEICO



The representatives of the various institutions met in Madrid in the start-up of the Network.

25 research centres and 16 units credited with the “Severo Ochoa Excellence” and “María de Maeztu Excellence” distinctions by the Spanish Ministry for the Economy, Industry and Competitiveness, including the ICMAT, have come together to form the [Alianza Severo Ochoa y María de Maeztu](#) (SOMMa – Severo Ochoa and

María de Maeztu Alliance). The aim of this platform is to raise the national and international visibility of science in Spain; to promote the exchange of knowledge, technology and best practice among its members, the international scientific community and the main interested parties; to collaborate with the other research centres in Spain in order to strengthen the R+D+i system, and to create a voice in the forming of Spanish and European scientific policy. Over the next two years, Luis Serrano, director of the Centre for Genomic Regulation, will preside over the Alliance, and Teresa Garcia-Milà, director of the Barcelona Graduate School of Economics, will be its vice-president.

SOMMa will consist of different work groups with the aims of improving visibility, collaboration, scientific policy, dissemination of results and the sustainability of the project, among others. One of the most important immediate actions to be undertaken by the Alliance is the launching of a portal that will showcase the science of excellence being done in Spain, and will also provide a resource for the scientific community, journalists and the public at large. A further initiative will be the organization of future editions of the *100xCiencia* conferences, thereby continuing the series that began in 2015 in La Palma, and which this year was held for the second time in Alicante between November 2nd-November 3rd.

## Multimedia Art and Geometry at the *Residencia de Estudiantes*

On December 4th, the ICMAT organized a conference entitled “[Sensory experiences in abstract spaces. Multimedia art from within a curved 3-dimensional space](#)”, which was given by mathematician and artist Pierre Berger (CNRS and Université de Paris 13), as part of the cycle [Mathematics at the Residencia](#), with the collaboration of the CSIC Vice-Presidency and the *Residencia de Estudiantes*.

In his talk aimed at a general public, Berger spoke about the relationship between art and mathematics in the context of his recent exhibition “[Esthétopies](#)”, in which he explored the perception of mathematical objects known as manifolds. These are curved spaces in which perception and acoustics are modified. The collection of art objects resulting from an interdisciplinary research project represent 3-manifolds, which are similar to our own physical space (also of three dimensions) but curved on a larger scale and with different topologies.

The object represented in these works is a mathematical concept; for example, certain abstract mathematical spaces and geometrical operations. “We investigate what feelings they suggest, how they can change our way of perceiving the world”, says Berger. The technique used for creating the art works mixes mathematical, numerical and artistic protocols. “When I created the landscapes of non-visualized, three-dimensional curved spaces, I first worked with a programme for simulating what it would be like to take a photograph inside the landscape”, Berger explains. “Once that is done, I place a particular

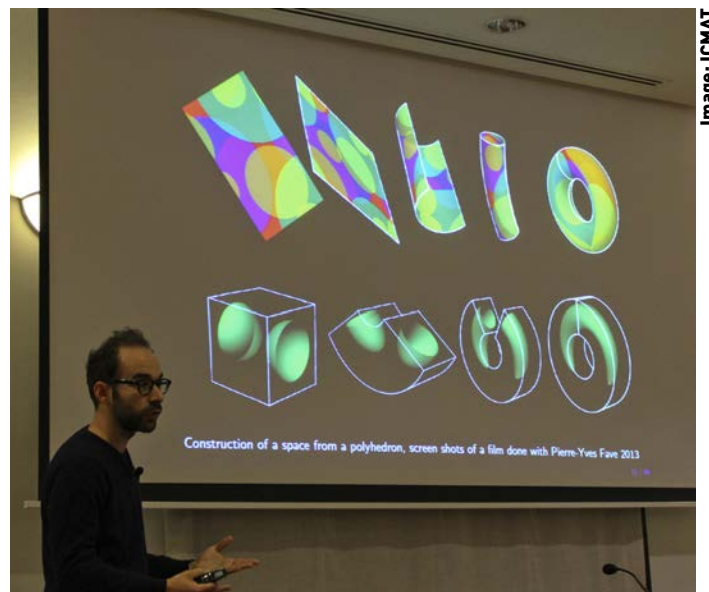


Image: ICMAT

Pierre Berger was the guest speaker for Mathematics in the Residence.

object, a light source in the space, and then choose a point from which to take the photo. In that way, I can completely explore the spaces and determine their mathematical properties and the sensory experiences they produce”.

## Quantum systems capable of detecting their size discovered

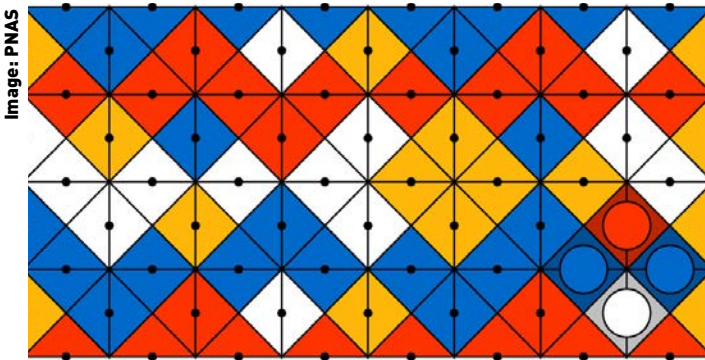


Image: PNAS

The quantum system designed by the authors presents classical or quantum properties depending on the coatings that can be formed using square tiles with colored edges, with the restriction that adjacent tiles have to have the same color on the edges they share.

An international team of researchers, including David Pérez-García (Complutense University of Madrid - ICMAT), is designing a new quantum system able to modify its properties according to its size. This research work, which was published in the journal PNAS this year on December 20th, has important implications for the techniques employed in the study of quantum materials.

Many systems change their properties when one or more of their parameters are modified. For example, water becomes ice when its temperature falls. In systems with zero temperature, and whose physics obeys the laws of quantum mechanics, these phase changes are caused by changes in some of their parameters, such as the intensity of an external magnetic field. However, according to this recent article, the size of the system can also lead to a phase change.

The researchers have proved for the first time the existence of systems that move from a classical behaviour to a quantum behaviour (transformed into a topologically ordered system) when only their extension is modified. Sizes that cause the transition of the system

can be fixed at any order of magnitude, from one microscopically small to one as large as the number of atoms in the universe. Systems with a topological order are interesting because their properties at low temperatures depend on the so-called topological properties of the material. The stability of topologically ordered systems makes them candidates for being quantum memories.

The existence of phase transition caused by size was predicted, in an abstract way, as one of the consequences of the undecidability of the spectral gap, as demonstrated by some of the researchers who authored this article. In the words of David Pérez-García: "Undecidable problems, by definition, cannot be proven experimentally. In this sense, phase transitions due to size can be regarded as a measurable effect of the existence of undecidable questions in quantum mechanics".

Furthermore, this research work has implications for the techniques employed in the study of quantum materials, such as high-temperature superconductors. Current knowledge on the subject is obtained from numerical simulations, but the computing time and the available memory restrict the simulations to a number of particles that is far lower than that normally found in the materials.

In order to overcome this difficulty, an increasing number of particles is analyzed and the numerical data are extrapolated to the limit case of an infinite number of particles, the so-called thermodynamic limit. "Our models provide concrete examples in which this approach may fail, because the properties of the material may change just above the limits of the computer", says Toby Cubitt, a researcher at University College London (UCL). "And the other way round; if the transition occurs in a size greater than the number of particles in the material, then it's possible that the material may show a classical behaviour, very different from the quantum behaviour that would be mistakenly predicted using the thermodynamic limit". He goes on to add that, "This raises the question of what is the right way to make an extrapolation of the numerical data".

## ICMAT researcher David Pérez, 'Miguel Catalán' Prize 2017

For the second time in its history, the Institute of Mathematical Sciences (ICMAT) has been distinguished in the field of sciences with the Community of Madrid "Miguel Catalán" Prize. [David Pérez García](#) has been awarded this prize for his contributions to the field of quantum technologies and mathematical problems associated with this field, which forms the basis of the future quantum computers.

This is the most important prize awarded by the Community of Madrid to outstanding researchers under the age of 40. According to Ignacio Cirac, director of the Max Planck Institute of Quantum Optics (Germany) and of one of the ICMAT Laboratories: "Together with his team, David seeks to develop a new information theory about how to condense, correct and process information, combining the ideas of mathematician Claude Shannon with quantum physics".

His results have had an enormous international impact. As Cirac explains: "I'd like to stress the ideas relating to efficient descriptions of many-particle problems, an issue that appears in many fields (physics, chemistry, mathematics...). David has proved several fundamental theorems in this area".

Pérez García also conducts research into the development of new mathematical techniques in fields such as the physics of matter, complexity theory and quantum information. From



Image: ICMAT

David Pérez, Prize 'Miguel Catalán' 2017, received an ERC project.

these, the most relative are those concerning mathematical analysis as applied to the description and classification of the quantum phases of matter; the study of nonlocality in quantum information, and the discovery of new undecidable problems in the physics of matter and communication theory.

## Manuel de León becomes a numerary member of the Spanish Royal Academy of Sciences

Image: ICMAT



Manuel de León holds the medal number 36 of the Royal Academy of Sciences.

On November 29th, the director of the ICMAT, Manuel de León, was presented with medal nº36 of the Spanish Royal Academy of Exact Physical and Natural Sciences in an official ceremony held at the headquarters of the institution in Madrid. Before an audience of Academy members, De León read his acceptance address entitled “A brief history of geometric mechanics”.

After thanking the Royal Academy “which has always shown me the greatest affection”, and recalling the previous holders of medal nº36, De León outlined the evolution of geometric mechanics, a field in which he is an expert. He concluded with a special mention about research in the area of geometric mechanics, which according to him “is currently in an excellent situation”. He stressed the role of the Geometry, Mechanics and Control Network as well as the protagonism of the Spanish community in the “Journal of Geometric Mechanics”, a leading international publication of which De León is founder and editor.

He also read the reply of the Academy to the physicist and mathematician Pedro Luis García Pérez. After summarizing the career of the new academician, he took his seat in the Academy and the ceremony was concluded. This year, De León has also received the Medal of the *Real Sociedad Matemática Española* (RSME).

## The ICMAT shows the many faces of big data at the European Researchers’ Night

“The use of big data helps decision-making to be accomplished easily and quickly, and to improve efficiency in industrial processes and the transport of people and goods”. So said David Gómez Ullate (ICMAT-UCM), one of the organizers of the conference “[Big data and society: Towards a better world?](#)”, held on September 29th as part of the European Researchers’ Night, coordinated by the *Fundación para el Conocimiento Madri+d* in the Community of Madrid.

In emergency situations, the use of big data becomes indispensable for the organization of humanitarian aid. This technology is also found as part of daily life in everyday applications such as Google Maps, Amazon and the different social networks. All these applications have something in common: they require the management of large amounts of

data by means of increasingly sophisticated modern statistical techniques and algorithms. Furthermore, in the words of Gómez-Ullate: “They enable information to be gathered from sectors such as medicine, public health, social care and so on, to create knowledge on a much broader statistical basis”. However, as with any other type of technology, this also entails risks. These were just some of the subjects debated by Víctor Gallego (ICMAT), David Gómez-Ullate (ICMAT-UCM), Roi Naveiro (ICMAT), Alberto Redondo (ICMAT) and Simón Rodríguez (ICMAT).

With these talks, the ICMAT participated once again this year in the activities of the Autonomous University of Madrid, which took place at the Faculty of Medicine with the overall title of Health and the Environment in the 21<sup>st</sup> Century.

## The art and mathematics of India at the ICMAT

The meeting “The pathways of Indian mathematics”, held on November 8th at the ICMAT, brought together the researchers Nigel Hitchin (Oxford), Mudumbai S. Narasimhan (Bangalore), Sundararaman Ramanan (Chennai) and Antonio Córdoba (ICMAT) to talk about the mathematical relationship between India and Europe. The second part of the day was devoted to Kolam, with the photographic exhibition “Kolam: the ephemeral art by women in the south of India”. The 40 photographs depicted the geometrical compositions drawn by these women every morning at the doorways to their houses using white rice. This is a ritual by which they welcome other beings to their homes, “as a daily offering to harmonious coexistence”, explained Óscar García-Prada, one of the organizers of the meeting. The opening of the exhibition was attended the photographer herself, Claudia Silva, who is also a director of documentary films.

The event coincided with the scientific conference held in honour of the mathematician Ramanan on his 80<sup>th</sup> birthday, which was also celebrated at the ICMAT on November 6-8.



Image: ICMAT

The exhibition “Kolam: an ephemeral art of women in the south of India”, by Claudia Silva, was inaugurated at the ICMAT.



## Antonio Córdoba Barba (UAM-ICMAT) named Doctor Honoris Causa by the University of Murcia

Image: Universidad de Murcia



Antonio Córdoba Barba (UAM-ICMAT), in the act of appointment Doctor Honoris Causa by the University of Murcia, last September.

In September, the University of Murcia conferred an honorary doctorate on Antonio Córdoba Barba, a professor of Analysis at the Autonomous University of Madrid and member and ex-director of the ICMAT. The ceremony was held in the assembly hall of the Faculty of Economics and Business at this university. Ángel Ferrández Izquierdo, professor of Geometry and Topology at the University of Murcia, gave the Laudation of Academic Status, which was followed by the lecture by Antonio Córdoba entitled, "Goldsmiths of ideas, between atoms and stars".

Fernández spoke of Córdoba as being "an ordinary citizen, a citizen of the world from Murcia, a master whose love of mathematics and whose honesty and perseverance working in this field has merited him the highest national distinctions and a recognition enjoyed by few beyond our borders". He also looked back over Córdoba's scientific career in a variety of fields that range from Number Theory to Harmonic Analysis and Partial Differential Equations to Mathematical Physics.

For his part, Antonio Córdoba referred to his "deep roots in Murcia", which made him "feel profoundly moved by the honour conferred on me by the University of Murcia." He went on to recall his childhood in the region, where he first began to feel "fascinated by Quantum Mechanics and Relativity, the universe of atoms and space travel."

## A mathematical algorithm capable of solving jigsaw puzzles and reconstructing archeological remains and fossil bones

This year's Spanish jigsaw puzzle-solving champion, Ángel Heras, took a little more than 40 minutes to fit the 500 pieces of a puzzle together, while the runner-up took almost an hour. A mathematical algorithm developed by [Peter Olver](#), director of the University of Minnesota (USA) School of Mathematics, could cut this time down to 30 minutes. While a human being concentrates on colours and patterns, the mathematical model does so on each fragment without needing to know beforehand either the overall shape or picture. This makes the model applicable to disciplines such as archeology, paleontology and history. These topics were addressed in a talk given by Olver himself at the Spanish Royal Academy of Sciences (RAC) in September.

In his talk, Olver presented the most recent mathematical advances that help in the recomposition of fragments in areas as diverse as the restoration of statues and earthenware in archeology; the reconstruction of damaged tissue in surgery, and the recreation of eggs and fossils in paleontology. "Algorithms similar to those developed by Peter Olver could also be applied to the recomposition of documents destroyed by the Stasi, the secret police of the former German Democratic Republic state security service", said David Gómez-Ullate (ICMAT-UCM), who organized the talk. Lie Groups, reference systems, differential invariants and numerical approximations are the mathematical



Imagen: Peter Olver

Peter Olver presented the latest mathematical advances that help in the recomposition of fragments.

tools employed by Olver to devise the algorithm. This mathematician has also contributed to the search for symmetries in differential equations, which describe an infinite number of physical processes such as fluid dynamics, black holes and hurricanes, among others.

## International experts on number theory meet to commemorate Javier Cilleruelo

The congress "[The Music of Numbers](#)" was held at the ICMAT on September 20th over a period of three days. The event, which constituted a tribute to Javier Cilleruelo one year after his death, consisted of different topics in the field of number theory that were at the heart of the research work conducted by this professor, who belonged to the Autonomous University of Madrid and was also a member of the ICMAT.

Most of the [speakers](#) were mathematicians who had worked with Cilleruelo, such as Harald Helfgott (University of Göttin-

gen, Germany, and the *Centre National de la Recherche Scientifique, Institut de Mathématiques de Jussieu*, France), Antonio Córdoba (UAM-ICMAT), Alain Plagne (*École Polytechnique*, France), Pablo Fernández (UAM), Julia Wolf (University of Bristol, United Kingdom) and Florian Luca (University of Witwatersrand, South Africa), among others. Number theory was addressed from the point of view of combinatorics, analysis and probability.

## AGENDA

### ICMAT scientific activities

[60 Years Alberto Ibert Fest - Classical and Quantum Physics: Geometry, Dynamics and Control](#)

**Date:** 5-9th, March, 2018.

[Research Term on Real Harmonic Analysis and Its Applications to Partial Differential Equations and Geometric Measure Theory](#)

**Date:** 7th May – 9th June, 2018.

[Thematic program `L2-invariants and their analogues in positive characteristic`](#)

**Date:** 19th February – 15th June, 2018.

### ICMAT outreach activity

[4º ESO + Empresa](#)

**Date:** 19th, 20th and 21st, March, 2018.

**International Day of Women and Girls in Science**

**Date:** 12th February, from 10:00.

**Mathematics at the Residencia**

**Date:** 22th March, 19:30.

**Venue:** Residencia de Estudiantes del CSIC (Madrid).

#### **Production:**

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#### **Layout:**

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#### **Translation:**

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#### **Editorial department:**

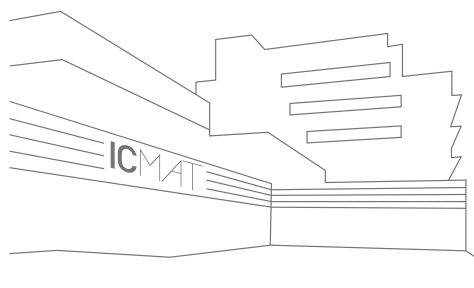
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