

# ESCUELA TÉCNICA SUPERIOR DE INGENIERÍA (ICAI) INGENIERO INDUSTRIAL 

# Dynamic Pricing in the Retail Sector and their Elasticity Effects 

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# Dynamic Pricing in the Retail Sector and their Elasticity Effects 

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# Dynamic Pricing in the Retail Sector and their Elasticity Effects 


#### Abstract

This project consists on the development of a Dynamic Pricing model in the physical Retail Food Industry. It follows an order process composed by self and cross elasticity curves calculation, a prediction model and an optimization one to maximize the operating margin. The project complexity would make very difficult to extend it to all the products and outlets of the client, for that reason an exhaustive selection has been made. The project expects to have an important Net Present Value, compared to the results of the company, and a payback of 1.32 years. The project is a pilot, and there are many future developments to continue its improvement.


Key words: Dynamic Pricing, Retail, Food Industry, Elasticity, Machine Learning, Prediction, Optimization

## 1. Introduction and State of the Art

Dynamic Pricing is getting well valued in the Retail Sector. This technique is based on updating real-timing prices in order to maximize profits.

The aim of this project is taking this technique to physical supermarkets. As it is implemented in the physical world, the price updating would be daily, to help its implementation.

It started its expansion thanks to ecommerce, where it has been essential for some companies, as airlines or Amazon.

Traditionally, Dynamic Pricing only maximizes each product's margin, but as it has been analysed, it is very important to consider other factors, as cannibalization.

Companies has always given much importance to pricing strategies. Some of those techniques are psychological pricing, where it is better to price at $(x-1) \cdot 99 €$, instead of x.oo€, or competitive pricing, where you price just below your competitors.

There are big Consulting Firms that have started to develop some theoretical models,
and some Start Ups have started implementing pilot projects.

To implement these kind of projects, some complementary technologies need to be adapted by retailers, as smart carts or electronic shelves.

To have an efficient and effective Dynamic Pricing model, it is necessary to develop powerful prediction models to know very well the expected demand. To do so, some Machine Learning algorithms would be very helpful. In this project, models would be mostly supervised learning or Time Series Forecasting.

## 2. Methodology

This project has some fixed objectives, but as it is quite innovative, it is opened to new possibilities that may be found during its development. The main objectives are:

- Dual strategy: With this strategy, the intention is to maximize profits and to be able of reducing waste.
- Knowing the demand: It is very important to know which variables affect the demand, as weather, holidays or competitors' prices.
- Cannibalization impact: Promos in some products may reduce sales of others. In some cases, the global impact of that promo could be negative.
- New opportunities: As it has been explained before, there are many possibilities that can be discovered during the project, and introduced in future developments, as it could be introducing psychological pricing to the model.

A Dynamic Pricing model needs to follow some clear steps.

- Clustering: Group products by categories, fresh or preserved food, or even letting the model cluster.
- Understanding Info: With the groups previously made, being able to understand the demand, see which variables may affect them, computing elasticities, as price-demand.
- Demand Prediction: With the variables previously observed, develop some algorithms to predict the demand.
- Pricing Optimization: With elasticities and predictions, develop an optimization model, which decision variable is the price and the objective is to maximize profits.

The project has been developed during one six months, and the main milestones of the project have been related with the steps previously explained.

The resources used during the execution have been Python libraries, to develop Machine Learning algorithms, to solve Optimization problems and to perform Data Analytics transformations. The platform to develop the coding has been Jupyter Notebooks, which is based on Anaconda.

Finally, to visualize the results, Power BI, developed by Microsoft, has been the tool to
create dynamic graphs containing huge amounts of information.

## 3. Data and Algorithms

The data provided comes from two different competitors of some food industry retailers. That data includes sales, discounts applied, name of products and different categories and subcategories to classify them from one shop of each competitor, located in the same block.

That data is a one year and two months record, and the first year is used to build the prediction models and elasticity curves, and the next two months to compare the optimized results to those that actually happened.

The data has been normalized to compare all products. That means the information treated are discounts and increase in sales.

The procedure to carry out the model has been as follows.

## Self and Cross Elasticity Curves

In this first step it has been calculated the variation of demand depending on the discount.

$$
\Delta \text { Sales }=f(\text { Discount })
$$

It has been calculated as a linear regression, because it has been tried more complex algorithms, as exponential regressions, but the results were very similar.

The self-elasticity curves have been developed for all the products.

The cross-elasticity curves have been developed as stationary and non-stationary for some samples of different categories. The first ones consider a unique relation between products and the second looks into different periods (weekly or monthly) and computes the curves for each period, increasing the complexity.

Non-stationary curves are needed at the beginning of the implementation, because promos are only applied for short periods.

Once the model is correctly working, it could be implemented stationary curves, for products with constants sales over the year, but for products as fruits or vegetables, nonstationary curve would remain necessary.

## Prediction Model

For the prediction model, some different algorithms have been implemented. Firstly an ARIMA to compute the prediction of the demand as a whole, without taking into account subcategories.

After that, a Random Forest Regressor, to predict at a category level. This prediction provides the possibility of developing a Dynamic Pricing not very complex, where the products within a category will change their prices equivalently.

Finally, to develop the prediction at a product level, it has been implemented an Extremely Randomized Trees, which increases complexity but returns a very trustful model.

## Optimization Model

The last step of the project is the optimization. In first place, it has been computed a category and a product optimization without taking into account cannibalization. The model proposed were:
max Profit =
$\max \sum_{\text {Categ }} \sum_{\text {Day }} \sum_{\text {Pt.Curve }} x_{\text {Categ,Day,Pt.Curve }} *$ Dbase $_{\text {Categ }} * \Delta D_{\text {Categ,Pt.Curve }} *\left[\right.$ Pbase $_{\text {Categ }} *$ ( $1-$ Dct $\left._{\text {Pt.Curve }}\right)-$ Cost $\left._{\text {Categ }}\right]$

Subject to

$$
\begin{aligned}
& x_{\text {Categ,Day,Pt.Curve }} * \text { Dbase }_{\text {Categ }} \\
& * \Delta D_{\text {Categ,Pt.Curve }} \\
& \leq 1.1 * \text { Dmax }_{\text {Categ,Day }} \\
& \sum_{\text {Pt.Curve }} x_{\text {Categ,Day,Pt.Curve }} \leq 1
\end{aligned}
$$

Lastly, it has been implemented a model with cannibalization to some products, and the results have been extrapolated, adjusting the previous results.

$$
\begin{aligned}
& \max \text { Profit }= \\
& \max \sum_{\text {Prod } 1} \sum_{\text {Prod }} \sum_{\text {Day }} \sum_{\text {Pt.Curve }} x_{\text {Pro }} \quad \text {,Pro } \quad, \text { Day,Pt.Curve } * \\
& \text { Dbase }_{\text {Prod }} * \Delta D_{\text {Prod1,Prod2,Pt.Curve }} * \\
& {\left[\text { Pbase }_{\text {Prod } 1} *\left(1-\text { Dct }_{\text {Pt.Curve }}\right)-\text { Cost }_{\text {Pro }} \quad\right]}
\end{aligned}
$$

Subject to

$$
\begin{gathered}
\sum_{\text {Prod2 }} x_{\text {Prod1,Prod2,Day,Pt.Curve }} * \text { Dbase }_{\text {Prod } 1} \\
* \Delta D_{\text {Pro ,Prod ,Pt.Curve }} \\
\leq 1.1 * \text { Dmax }{ }_{\text {Prod1,Day }} \\
\sum_{\text {Pt.Curve }} x_{\text {Prod1,Prod2,Day,Pt.Curve }} \leq 1
\end{gathered}
$$

In those two objective functions, the Profit is maximize, which is equal to an average demand multiplied by the increase in sales. This is multiplied by the average price applying a positive or negative discount, and subtracting the costs.

The demand is limited to the prediction and the variables $y_{\text {Pro , Day,Pt.Curve and }}$ $x_{\text {Prod 1,Prod2,Day,Pt.Curve }}$ are binary decision variables to choose only one discount per day and product and the same discount for each product in the model with cannibalization.

## 4. Results and Business Case

In this section, it would be presented the results from each of the models and the results and decisions taken from the Business Case developed.

Model results
The results that will be shown in this abstract will cover elasticity curves, prediction and optimization.

About the elasticity curves, it has been developed self-elasticities at a category and product level. Category curves are more precise and more reliable. However, product ones permit solving the problem at that level.

In Figure 1 it is shown the distribution of the points in the Linear Regression.


Figure 1: Self-elasticity of a category
Additionally to the self-elasticity curves, there are cross-elasticities.

From Table 1 it can be observed a stationary cross-elasticity curve, which would be used once the Dynamic Pricing is stably implemented.

|  | Product 1 | Product 2 |
| :--- | :--- | :--- |
| Product 1 | 5.65 | -2.88 |
| Product 2 | -0.44 | 6.13 |

Table 1: Stationary cross-elasticity curve
Table 2 shows the different seasonal cross elasticity curves, divided by months. The selecte one (with negative slope) shows the period where it has been a promo. This method would be applied at the beginning of the project

| Producto Canibalizado | ${ }^{7} \mathbf{7}$ Mes | $\stackrel{\square}{\square}$ | Pendiente ${ }^{-}$ | R2 1 |
| :---: | :---: | :---: | :---: | :---: |
| CINTA IOMO CERDO TRO7O |  | 12 | 0132 | 01815 |
| CINTA LOMO CERDO TROZO |  | 5 | -0.12338779 | 0.148791 |
| CINTA LOMO CERDO TROZO |  | 1 | 0.12264151 | 0.139505 |
| CINTA LOMO CERDO TROZO |  | 8 | 0.10492333 | 0.125138 |
| CINTA LOMO CERDO TROZO |  | 9 | 0.07041315 | 0.122643 |
| CINTA LOMO CERDO TROZO |  | 4 | -0.12882448 | 0.099096 |
| CINTA LOMO CERDO TROZO |  | 2 | 0.10040161 | 0.089644 |
| CINTA LOMO CERDO TROZO |  | 10 | 0.05191135 | 0.074556 |
| CINTA LOMO CERDO TROZO |  | 7 | 0.05205144 | 0.044392 |
| CINTA LOMO CERDO TROZO |  | 6 | 0.025944 | 0.016275 |
| CINTA LOMO CERDO TROZO |  | 3 | 0.03762542 | 0.015973 |
| CINTA LOMO CERDO TROZO |  | 11 | -0.00615872 | 0.000362 |

Table 2: Seasonal cross-elasticity curves
After the elasticities, the predictions are developed. First, it was made a global demand prediction with an ARIMA model without the expected results. For that reason ML algorithms were implemented

Figure 2 shows one prediction made by a Random Forest Regressor.


Figure 2: Demand prediction of CARNES
Finally, Figure 3 shows how the discount will vary thanks to the optimization model.


Figure 3: Discount evolution of a product

The result of the optimization model would be adjusted due to the seasonality of elasticity curves. This parameter to adjust has been computed with the historical data provided of 2017.

## Business Case

The Business Case contains information about the implementation of the project with the corresponding risks and its financial profitability.

The project will be implemented only for the $30 \%$ of the products' value. Those products would be those with some variations in sales, but not too much erratic and representing an important percentage of those sales.

Moreover, it would be developed in supermarkets where there is an important competition that is to be in big cities, which represents the $20 \%$ of the margin.

The implementation process would be progressive, with a test in a few outlets of different characteristics and with a duration of six months. After the test, it would take part the most important part of the process with the extension to the most affected facilities, during six months. The last step is the extension to the rest of the outlets of the project during three months.

The risks the projects faces are:

- Delay in the different steps of the project
- Software is more expensive
- Software with not enough capacity
- Bad reception from clients
- The algorithms does not give the expected results

Where the most important are the last two ones, because they could become into the failure of the project.

About the financial profitability of the project, the most important aspect is to create value. In Table 3 it is shown that the project has an important Net Present Value
compared to the EBITDA of the project. And a payback of 1.32 years is also very interesting for the client.


Table 3: Financial Results
There is also some collateral non-tangible value as efficiency in the pricing methodology, which would become automated, and some cost saving in personnel. There would be created some qualified jobs to manage the pricing platform and to develop changes from R\&D.

## 5. Conclusions

This project has developed a Dynamic Pricing model combining three different models. The first has been the elasticity curves, with a Linear Regression. Secondly, implementing Machine Learning algorithms, an accurate demand prediction has been made. Finally, the optimization of the operating margin has been carried out with a Linear Programming modelling.

Once the project has been proved to be profitable it would be presented possible future developments that would complement nicely the results achieved in the project.

- Implementation of a complete cannibalization model.
- Introduction of waste reduction in the model.
- Introduction of pricing strategies in the model.
- Usage of demand prediction to optimize the Working Capital.


## Pricing Dinámico en el Sector Retail y los Efectos en la Elasticidad

## Resumen

Este proyecto consiste en el desarrollo de un modelo de Pricing Dinámico en la industria retail físico de alimentos. Sigue un proceso ordenado compuesto por el cálculo de curvas de elasticidad y elasticidad cruzada, un modelo de predicción y una optimización para maximizar el margen operativo. La complejidad del proyecto haría muy difícil extenderlo a todos los productos y puntos de venta del cliente, por lo que se ha realizado una selección exhaustiva. El proyecto espera tener un NPV importante en comparación con los resultados de la compañía, y un payback de 1.32 años. El proyecto es un piloto, y hay muchos desarrollos futuros para continuar su mejora.

Palabras clave: Precios dinámicos, retail, industria alimentaria, elasticidad, Machine Learning, predicción, optimización.

## 1. Introducción y Estado del Arte

El Pricing Dinámico se está valorando positivamente en el sector retail. Esta técnica se basa en la actualización de los precios de tiempo real para maximizar los beneficios.

El objetivo de este proyecto es llevar esta técnica a los supermercados Como se desarrolla en el mundo físico, la actualización de los precios sería diaria, para ayudar a su implementación.

El Pricing Dinámico comenzó su expansión gracias al comercio electrónico, donde ha sido esencial para algunas compañías, como las aerolíneas o Amazon.

Tradicionalmente, los precios dinámicos solo maximizan el margen de cada producto, pero como se ha analizado, es muy importante considerar otros factores, como la canibalización.

Las empresas siempre han dado mucha importancia a las estrategias de fijación de precios. Algunas de esas técnicas son los precios psicológicos, donde es más eficaz un precio de ( $\mathrm{x}-1$ ). $99 €$, en lugar de x.oo $€$, o precios competitivos, donde los precios están justo por debajo de sus competidores.

Hay grandes firmas de consultoría que han comenzado a desarrollar algunos modelos
teóricos, y algunas Start Ups han comenzado a implementar proyectos piloto.

Para llevar a cabo este tipo de proyectos, algunas tecnologías complementarias deben ser adaptadas por los retailers, como carritos inteligentes o estantes electrónicos.

Para tener un modelo de Pricing Dinámico eficiente y efectivo, es necesario desarrollar modelos de predicción potentes para conocer muy bien la demanda esperada. Para hacerlo, los algoritmos de Machine Learning son muy útiles. En este proyecto, los modelos son en su mayoría aprendizaje supervisado o predicción de series temporales.

## 2. Metodología

Este proyecto tiene algunos objetivos fijos, pero como es bastante innovador, está abierto a nuevas posibilidades que se puedan encontrar durante su desarrollo. Los principales objetivos son:

- Estrategia dual: con esta estrategia, la intención es maximizar los beneficios y poder reducir el desperdicio.
- Conocer la demanda: es muy importante saber qué variables afectan la demanda, como el clima,
las vacaciones o los precios de los competidores.
- Impacto de la canibalización: las promociones en algunos productos pueden reducir las ventas de otros. En algunos casos, el impacto de esa promoción podría ser negativo.
- Nuevas oportunidades: como se ha explicado anteriormente, hay muchas posibilidades que pueden descubrirse durante el proyecto e introducirse en desarrollos futuros, como introducir precios psicológicos.

Un modelo de Pricing Dinámico debe seguir algunos pasos claros.

- Clusterización: agrupación de los productos por categorías, como alimentos frescos o en conserva, o incluso dejando que el modelo se clusterice solo.
- Comprensión de la información: con los grupos creados anteriormente, comprender la demanda, ver qué variables pueden afectarla, calcular las elasticidades, como precio-demanda.
- Predicción de la demanda: con las variables observadas anteriormente, desarrollar algunos algoritmos para predecir la demanda.
- Optimización de precios: con elasticidades y predicciones, desarrollo de un modelo de optimización, cuya variable de decisión es el precio y el objetivo es maximizar el margen operativo.

El proyecto se ha desarrollado durante un período de seis meses y los principales hitos del proyecto se han realizado con los pasos explicados anteriormente.

Los recursos utilizados durante la ejecución han sido las bibliotecas de Python, para desarrollar algoritmos de Machine Learning, para resolver problemas de optimización y para realizar transformaciones de análisis de
datos. La plataforma para desarrollar el código ha sido Jupyter Notebooks, que se basa en Anaconda.

Finalmente, para visualizar los resultados, Power BI, desarrollado por Microsoft, ha sido la herramienta para crear gráficos dinámicos que contienen enormes cantidades de información.

## 3. Datos y Algoritmos

Los datos proporcionados provienen de dos importantes competidores, retailers de la industria alimentaria. Los datos incluyen ventas, descuentos aplicados, nombre de productos y diferentes categorías y subcategorías para clasificarlos. Los datos pertenecen a una tienda de cada competidor, ubicada en la misma manzana.

Esa información es un registro de un año y dos meses. El primer año se usa para construir los modelos de predicción y curvas de elasticidad, y los dos meses siguientes para comparar los resultados optimizados con los que realmente sucedieron.

Los datos se han normalizado para comparar todos los productos. Eso significa que la información tratada son descuentos y aumento de ventas.

El procedimiento para llevar a cabo el modelo ha sido el siguiente.

## Curvas de elasticidad

Primero se ha calculado la variación de la demanda en función del descuento.

$$
\Delta V e n t a=f(\text { Descuento })
$$

Se ha calculado como una regresión lineal, porque se han probado algoritmos más complejos, como regresiones exponenciales, pero los resultados fueron muy similares.

Las curvas de auto-elasticidad han sido desarrolladas para todos los productos.

Las curvas de elasticidad cruzada se han desarrollado como estacionarias y no estacionarias para algunas muestras de diferentes categorías. La primera considera una relación única entre productos la segunda analiza diferentes períodos (semanal o mensual) y calcula las curvas para cada período, aumentando la complejidad.

Las curvas no estacionarias son necesarias al comienzo de la implementación, porque las promociones solo aplican a períodos cortos.
Cuando el modelo esté funcionando correctamente, podrían utilizarse curvas estacionarias, para productos con ventas constantes a lo largo del año, pero para productos como frutas o verduras, la curva no estacionaria seguiría siendo necesaria.

Modelo de predicción
Para el modelo de predicción, se han implementado algunos algoritmos diferentes. En primer lugar, un ARIMA para calcular la predicción de la demanda en su conjunto, sin tener en cuenta las categorías.

Después de eso, un Random Forest Regressor, para predecir a nivel de categoría. Esta predicción ofrece la posibilidad de desarrollar un Pricing Dinámico no muy complejo, donde los productos dentro de una categoría cambiarían sus precios de manera equivalente.

Finalmente, para desarrollar la predicción a nivel producto, se ha realizado un Extremely Randomized Tree, con mayor complejidad, pero devuelvolviendo un modelo fiable.

## Modelo de optimización

El último paso del proyecto es la optimización. En primer lugar, se ha calculado una optimización a nivel categoría y una a nivel producto sin tener en cuenta la canibalización. Los modelos han sido:
$\max$ Margen $=$
$\max \sum_{\text {Categ }} \sum_{\text {Día }} \sum_{\text {Pt.Curva }} x_{\text {Categ,Día,Pt.Curva }} *$

$$
\begin{aligned}
& \text { Dbase }_{\text {Categ }} * \Delta D_{\text {Categ,Pt.Curva }} *\left[\text { Pbase }_{\text {Categ }} *\right. \\
& \left.\left(1-\text { Dct }_{\text {Pt.Curva }}\right)-\text { Coste }_{\text {Categ }}\right]
\end{aligned}
$$

Sujeto a:

```
\(x_{\text {Categ,Día,Pt.Curva }} *\) Dbase \(_{\text {Categ }}\)
            \(* \Delta D_{\text {Categ,Pt.Curva }}\)
            \(\leq 1.1 *\) Dmax \(_{\text {Cate ,Día }}\)
        \(\sum_{\text {Pt.Curva }} x_{\text {Categ,Día,Pt.Curva }} \leq 1\)
```

Por último, se ha implementado un modelo con canibalización a algunos productos, y los resultados se han extrapolado, ajustando los resultados anteriores.

```
max Margen =
max }\mp@subsup{\sum}{\mathrm{ Prod1 }}{}\mp@subsup{\sum}{\mathrm{ Prod2 }}{}\mp@subsup{\sum}{\mathrm{ Día}}{}\mp@subsup{\sum}{Pt.Curva }{*}\mp@subsup{x}{\mathrm{ Prod1,Prod ,Día,Pt.Curva *}}{
Dbase Prod1 * 
[Pbase Prod1 * (1 - Dct Pt.Curva})-\mp@subsup{Coste Prod1 }{\mathrm{ Pr}}{
```

Sujeto a:

$$
\begin{gathered}
\sum_{\text {Prod2 }} x_{\text {Prod1,Prod2,Día,Pt.Curva }} * \text { Dbase }_{\text {Prod } 1} \\
* \Delta D_{\text {Prod1,Prod2,Pt.Curva }} \\
\leq 1.1 * \text { Dmax Prod1,Día }
\end{gathered} \sum_{\text {Pt.Curve }} x_{\text {Prod1,Prod2,Día,Pt.Curva }} \leq 18
$$

En esas dos funciones objetivo, la ganancia es máxima, que es igual a una demanda promedio multiplicada por el aumento en las ventas. Esto se multiplica por el precio promedio aplicando un descuento positivo o negativo, y restando los costos.

La demanda está limitada por la predicción.
$y_{\text {Prod2,Día,Pt.Curva. }}$ Y $\quad x_{\text {Prod ,Prod ,Día,Pt.Curva }}$ son variables de decisión binarias para elegir solo un descuento por día y producto y el mismo descuento para cada producto en el modelo con canibalización.

## 4. Resultados

En esta sección, se presentarán los resultados de cada uno de los modelos y las decisiones extraídas del Business Case.

Resultados del modelo
Los resultados que se mostrarán en este resumen cubrirán las curvas de elasticidad, predicción y optimización.

Sobre las curvas de elasticidad, se han desarrollado auto-elasticidades a nivel de categoría y producto. Las curvas de categoría son más precisas y más fiables. Sin embargo, los productos permiten resolver el problema en ese nivel.

En la Figura 1 se muestra la distribución de los puntos en la Regresión Lineal.


Figura 4: Auto-elasticidad de una categoría
Además de las curvas de auto-elasticidad, hay elasticidades cruzadas.

Desde la Tabla 1 se puede observar una curva de elasticidad cruzada estacionaria, que se usaría una vez que se implementa de manera estable el Precio Dinámico.

|  | Product 1 | Product 2 |
| :--- | :--- | :--- |
| Product 1 | 5.65 | -2.88 |
| Product 2 | -0.44 | 6.13 |

Tabla 4: Elasticidad cruzada estacionaria
La Tabla 2 muestra las diferentes curvas de elasticidad cruzada estacionaria, divididas por meses. El dato seleccionado (con pendiente negativa) muestra el período en el
que ha habido una promoción. Este método se aplicaría al inicio del proyecto.

| Producto Canibalizado | ${ }^{-1}$ Mes | $\checkmark$ | Pendiente - | R2 |
| :---: | :---: | :---: | :---: | :---: |
| CINTA IOMO CFRDO TRO7O |  | 12 | 0137 | 01815 |
| CINTA LOMO CERDO TROZO |  | 5 | -0.12338779 | 0.148791 |
| CINTA LOMO CERDO TROZO |  | 1 | 0.12264151 | 0.139505 |
| CINTA LOMO CERDO TROZO |  | 8 | 0.10492333 | 0.125138 |
| CINTA LOMO CERDO TROZO |  | 9 | 0.07041315 | 0.122643 |
| CINTA LOMO CERDO TROZO |  | 4 | -0.12882448 | 0.099096 |
| CINTA LOMO CERDO TROZO |  | 2 | 0.10040161 | 0.089644 |
| CINTA LOMO CERDO TROZO |  | 10 | 0.05191135 | 0.074556 |
| CINTA LOMO CERDO TROZO |  | 7 | 0.05205144 | 0.044392 |
| CINTA LOMO CERDO TROZO |  | 6 | 0.025944 | 0.016275 |
| CINTA LOMO CERDO TROZO |  | 3 | 0.03762542 | 0.015973 |
| CINTA LOMO CERDO TROZO |  | 11 | -0.00615872 | 0.000362 |

Tabla 5: Elasticidad cruzada no estacionaria
Después de las elasticidades, se desarrollan las predicciones. Primero, se hizo una predicción de demanda global con un modelo ARIMA sin los resultados esperados. Así, se implementaron algoritmos de ML.

La Figura 2 muestra una predicción hecha por un Random Forest Regressor.


Figura 5: Predicción de la demanda CARNES
Finalmente, la Figura 3 muestra cómo el descuento variaría, gracias al modelo de optimización.


Figura 6: Evolución del descuento de un producto

El resultado del modelo de optimización se ajustaría debido a la estacionalidad de las curvas de elasticidad. Este parámetro se ha calculado con los datos históricos proporcionados de 2017.

## Business Case

El Business Case contiene información sobre la implementación del proyecto con los riesgos correspondientes y su rentabilidad financiera.

El proyecto se implementará solo para los productos que generan el $30 \%$ del valor. Esos productos serían aquellos con algunas variaciones en las ventas, pero no demasiado erráticos y que representan un porcentaje importante de esas ventas.

Además, se desarrollaría en supermercados donde hay una competencia importante, por lo que se realizará en las grandes ciudades, que representa el 20\% del margen.

El proceso de implementación sería progresivo, con una prueba en algunas tiendas de diferentes características y con una duración de seis meses. Después de la prueba, se llevaría a cabo la parte más importante del proceso con la extensión a las instalaciones más afectadas, durante seis meses. El último paso es la extensión al resto de los puntos de venta del proyecto durante tres meses.

Los riesgos que enfrentan los proyectos son:

- Retraso en los diferentes pasos del proyecto.
- El software es más caro
- Software sin capacidad suficiente.
- Mala recepción de los clientes.
- Los algoritmos no dan los resultados esperados.

Donde los más importantes son los dos últimos, porque podrían convertirse en el fracaso del proyecto.
Sobre la rentabilidad financiera del proyecto, el aspecto más importante es crear valor. En
la Tabla 3 se muestra que el proyecto tiene un NPV importante en comparación con el EBITDA de la empresa. Y un reembolso de 1.32 años también es muy interesante para el cliente.

## NPV \% EBITDA $\mathbf{3 . 7 8 \%}$ <br> Payback Years <br> 1.32

Tabla 6: Resultados financieros
También hay valor colateral no tangible como la eficiencia en la metodología de fijación de precios, que se automatizaría, y un ahorro de costos de personal. Se crearían algunos puestos de trabajo cualificados para administrar la plataforma de precios y desarrollar cambios a partir de la I+D.

## 5. Conclusiones

Este proyecto ha desarrollado un modelo de Pricing Dinámico que combina tres modelos diferentes. El primero ha sido las curvas de elasticidad, con una regresión lineal. En segundo lugar, se han implementando algoritmos de Machine Learning, para realizar una predicción precisa de la demanda. Finalmente, la optimización del margen operativo se ha llevado a cabo con un modelo de programación lineal.

Una vez demostrado que el proyecto es rentable, se presentan posibles desarrollos futuros que complementarían muy bien los resultados alcanzados en el proyecto.

- La implementación de un modelo completo de canibalización.
- Introducción de la reducción de tirado en el modelo.
- Introducción de estrategias de pricing en el modelo.
- Uso de la predicción de la demanda para optimizar el Working Capital.


# Dynamic Pricing in the Retail Sector and their Elasticity Effects Report 

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## Introduction

Nowadays, retail companies are focusing in the potential of data into their companies. They have noticed the possibilities it can open to their business, to optimize their logistics, predicting their demand or minimizing their stocks.


Figure 1: Big Data and the Retail Sector
One of those disruptive innovations, that has been possible thanks to data, is dynamic pricing, which is taking importance in the retail industry. This strategy is based on the variation of prices, depending on many factors as demand, regions or seasonality.

This tendency is not new. It appeared with airline and hotels' pricing methods, which historically has changed their prices depending on demand and seasonal parameters.

Nevertheless, it is taking importance in other fields of e-commerce, promoted by Amazon. Amazon has promoted this pricing strategy in the Retail Sector and it is extending slowly to the whole industry.

Amazon has shown (Ben-David, 2014) that this pricing strategy is successful not only for themselves, but also for customers and sellers. Customers are able to look into the internet and find the cheapest price. They can also wait for the optimal price that would
fit with their needs. And, on the other size, sellers can learn how to optimize their profits depending on the demand and other features.

The aim of this project is to analyze the effects it will have in physical stores, developing models to predict future demand, and optimizing its operation. Those models will be tested with real data provided by actual entities of the retail industry.

To develop this project it will be necessary to make a small review of the state of the industry in this matter. See possible competitors carrying out models with similar purposes. Investigate their success and the problems they might have, and try to notice different aspects where this project would make a difference.

First, it would be developed an elasticity study, to know how demand is affected by changes in prices for the different products. At the same time, a prediction model will be developed to know, depending on external factors, as the weather, how this demand varies over time.

The prediction of the demand will be useful into the optimization model, because it will provide some constraints about the maximum demand expected.

Finally, an elasticity study will be performed, searching the effects of dynamic pricing on supplementary goods. This elasticity study searches possible cannibalization between different products.

The best example is that the retailer may put a promo in 1kg oranges' bag. With this promo, the retailer expects an increase in sales of a $20 \%$, which will produce an increment of $10 \%$ in 1 kg oranges' bag. At this point, the retailer should be very happy, but it is possible that on the other side, the sales of 3 kg oranges' bag has decrease by a $10 \%$ which provokes a decrease in their profit of $5 \%$.

If the total change of profits has been $+2,000 €$ for the first product and $-2,500 €$ for the second one. When the retailer was thinking he was having an increment in profit of $2,000 €$, he was actually having a loss of $-500 €$.

Some companies have already developed dynamic pricing tools and they have shown success, but what makes this project special is the implementation in physical stores, which breaks with traditional promotion mechanisms, and also the last part of cannibalization, which takes care in the consequences that can have some product over other ones.

After this study, a Business Case will be developed to show the advantages derived from Dynamic Pricing and the impact of its implementation. The objective of this last paragraph would be to carry out some sensibility analysis to see the problems this project could have. This Business Case will show that the project is profitable in all the scenarios analyzed, having a Net Present Value of $3.78 \%$ of EBITDA and a Payback of 1.32 years.

This Project will be performed in the Consulting Department of PwC. This department's purpose is to help their clients introducing winning strategies and giving support in each

Dynamic Pricing in the Retail Sector
part of the projects. It would be carried out in the Retail and Consumption Sector, where e-commerce and digital innovation have transformed the companies.

## State of the Art

In this chapter, it would be presented the Retail Sector and the different fields of innovation that could be related with Dynamic Pricing.

In parallel, it would be explained the different developments of Dynamic Pricing that can be found in the sector and the techniques that are necessary to develop successful models and solutions.

This chapter is important in the development of the project because it purposes the main guidelines and inspirations to develop the different models and to provide a critical view to analyze the results.

## The Retail Sector

The Retail Industry is in charge of selling consumer goods to final customers through different channels of distribution. From those sales, they earn their profits, and their costs come from the cost of goods sold and all the logistics behind their supply chain.

Nowadays, the retail sector is suffering a technological revolution, as many other industries. It is living the introduction of data analysis to optimize their margins and maximize sales.

In the sector there are a few trending ideas that all companies need to take into account and act according to them. Those ideas are based on customer experience and the return to tailor-made products.

One of them is store experience (Forbes, 2018). This idea tries to obtain the best customer's experience. In this process is very important the combination of the physical and digital world, getting data from different sources.

Other important aspect is to provide a more tailor-made products. To do that, data is also crucial. Data will let retailers to know their clients and their necessities.

This new store experience, which is getting relevance, is that data needs to be invisible to clients, they need to enjoy their effects, but they do not want to see its complexity. However, in this same line, employees working with data reports are obligated to understand their outputs and results, and with them, be able to make decisions.

Those previous ideas could provoke the idea that new technologies are eliminating jobs, because it is making less necessary blue-collar job. However, this could not be more wrong.

New technologies in the Retail Sector is creating more qualified jobs, increasing efficiency in the system, and therefore those new jobs could be considered as safer.

## Pricing Strategies

As the consumer experience has become so important, it implies designing an industry, where everything is focused on this purpose. This store experience is highly related to the price of the products. Having a nice pricing strategy will help to maximize companies' profits.

For this reason, retailers have always need to pay attention to this fact, and many different strategies have appeared to solve this dilemma.

Some of those well-known strategies (Hudson, 2019) are as follows:

- Markup pricing: It is calculated by fixing a profit margin to the product, and then, summing this margin to the cost of it.
- Vendor pricing: It is very common that manufacturers set a minimum recommended price for the product. Sometimes, they even do not let you sell it at price below it.
- Competitive pricing: This strategy set product's price against competitors. Normally retailer's price below their competitors, but it exists the possibility to price above due to a prestige position of the product.
- Psychological pricing: It is set by introducing prices having an impact on consumer's behavior. That means retailer fix a price at $9.95 €$ instead of $10 €$, because consumers tend to round down.

There are many other strategies, but the ones explained before are the most common ones of the sector. Combining all this kind of strategies in real time, retailers have the possibility to implement dynamic pricing.

## Dynamic Pricing

Dynamic pricing is a strategy based on the possibility of updating the product price during its lifetime. This innovative strategy has appeared caused by the technology development in the Industry.

This strategy is not new. It can be seen in pubs, with the "Happy Hour" or in airlines pricing strategies, as it has been presented in the introduction. Nevertheless, new technology it has been extended to other fields of the sector.

All data available and the speed of processing it, has provoke an increase of the opportunities derived from it. Data is collected, stored and analyzed to make it possible.

Other companies have already developed dynamic pricing, as Competera. This company (Competera, 2018) has developed a tool to get optimal pricing depending on historical data. It defines some targets and limits prices to some constraints.

With all that information, as it is shown in Figure 2, Competera has developed some algorithms that mixes all of it and gets the optimal pricing for each SKU.


Figure 2: Procedure to implement Dynamic Pricing (Competera, 2018)
Other method to propose Dynamic Pricing has been developed by Mckinsey. (Mckinsey, 2017) has been conceptual in their model, and it is more based on giving qualitative information than real results.

Some differentiated steps through the life of the product compose the dynamic pricing strategy. It begins with and introductory module, that prices according to similar products that are already in the market.

Secondly, it updates price accordingly to elasticity. This step is very important, because it shows how the product behaves depending on many inputs.

It continues with the KVI (Key Value Item) module. KVI are products that affects consumers price perception, and in this modules, what is done, is to see how the price of the product affects consumer's perception.

Other important step is the competitive-response, where the price adjusts to competitors prices. In this module, pricing updates follow a strategy similar to the competitive strategy explained before.

Finally, the omnichannel module, at the end of product's life, where sales will try to maximize, to avoid waste.


Figure 3: Dynamic Pricing Strategy (Mckinsey, 2017).

In this field, there has been developed many theoretical models as R\&D projects. For example, (Heien, 1980) models very well the lifetime of products, how they starts getting old and with it, their value starts decreasing following a logarithmic curve.

As an advance in this field, (Dong $\mathrm{Li}, 2006$ ) and ( $\mathrm{Li}, 2012$ ) develop the previous models to optimize supply chain management. They introduce some optimization models to improve the efficiency of the stock management.

Finally, (JUDITH A. CHEVALIER, 2003) has introduced the fact of Dynamic Pricing in the retail sector. This article developed some models to show how elasticity between price and sales is one of the most important variables when applying Dynamic Pricing.

## Dynamic Pricing complementary technologies

Real-time price updating is possible, in physical stores, thanks to new electronic shelf labels. With this technology, pricing updating could be managed in a centralized way.


Figure 4: Electronic Shelf Labels (French Tech Hub, s.f.)
Those new electronics shelfs would be very comfortable to know the real price at each moment. However, if there were implement a Real-Time dynamic pricing, where prices are continuously changing, it could happen that, at a determined moment you take a product at one price and when you arrive to the cash register, the price could have changed, and then you are not interested anymore.

To solve this problem, there are being implemented new technologies, as Caper (Caper, s.f.). Caper is a company that has developed a smart cart. This cart is able to recognize the product and it is linked to the cloud where is registered the price of the product.

That way, the moment a product is in the cart, the price is fixed for that client, and when he is going to pay, the price would be the same as when he took it.

It is very profitable, because it could make unneeded cash registers, because the total price of the shops is saved and it can be paid directly through the mobile device or to the cart.


Figure 5: Smart cart (Caper, s.f.)

## Methodology

## Motivation

Once a small introduction has been made, and the state of the art has been explain, the next step will be to proceed with the exposition of the reasons to carry out this project.

As it has been developed before, there is an increase in participation of the demand in the retail industry. Consumers now, want more than just the product; they are looking for an enjoyable experience.

One important aspect to enjoy the consumption of goods is to know that the price you need to pay is fair, and it is compensated with other prices of the competition.

For this reason, those retailers who want to maximize their profits are interested in introducing Dynamic Pricing. Consumers have been waiting for the development of a technology such as this one for a long time.

However, to explain better the motivation of this project, for now on, the project will be focused on the retailers' side. Dynamic Pricing gives them the opportunity to maximize the profits. They will price accordingly to their competitors, to the season of the year, or to the product's life that is left in a moment given.

All the technology developed, to forecast the demand, will also help retailers to plan their stock and the consumption of products they will have. Even if this project is not going to contemplate those logistics, it is a good point to analyze in future developments.

Just to summarize, the main motivation of this project is to develop a tool that will take profit of the existent technology to help retailers to maximize their profit and increase their knowledge of the demand.

## Objectives

Carrying out this project, there are some objectives that are expected to be achieved. They are aligned with the main motivation of the project and are expected to solve some of the Innovation issues of the Retail Industry.

The objectives the project would try to achieve are as follows:

- A dual Strategy: This dual strategy over the product's life consist in maximizing the profit during the greatest part of it, but at the end of the product's life, try to
maximize sales, in order to avoid waste. This maximizing of sales can be considered as part of the profit maximization, because if the product were not sold, it would derived in a sunk cost.
- Knowing the demand: Now that technologies are increasing their scope, knowing better the demand would let retailers to limit uncertainties, and to react better to different events. It would let them to plan many actions depending on the forecasted demand.
- Knowing cannibalization consequences: As it has been mentioned previously, cannibalization may cause a big problematic when applying dynamic pricing. It is very important to be aware of their possible consequences and prepare some models that considers it.
- New opportunities: As the technology implemented in the project is quite new, at this point, there are not been considered many advantages yet. It is very important to be prepared for new opportunities that Machine Learning could open during the project development.


## Project Procedure

The project follows and ordered scheduled (Caltech, 2013), so the objectives could be achieved systematically. To obtain the desired results, there are four differentiated steps, which need to be done in the indicated order (PwC, n.d.).

- Clustering: As the data provided has a huge amount of products that cannot be treated one by one, it is necessary to group them, following some statement. In Figure 6 it is shown different methods to group data.
- Understanding Info: There some fields related with data that is not know at the beginning of the project. There are mainly related with demand behavior, particularly with its elasticity with respect to different factors.
- Demand Prediction: It is essential to know how demand will evolve in the future. That way, it will be possible to apply the pricing strategy to the data provided. It is one of the most sensible parts of the project because of the Statistical tools that will be applied.
- Pricing Optimization: At the end of the project, the strategy conceived will be applied. The dynamic pricing will be based on the optimal price at each moment. This dynamic pricing will be subject to some constraints. As this project will be applied in physical stores, maybe real-time price update will not be possible, and only a few times per day will be possible to change. However, this project will only be focused on daily changes, because it is the only information available.


Figure 6: Project Procedure Scheme
Once the main steps have been developed, and the results are disposed, the Business Case will be implemented. In this last step, the results obtained will be compared to the pricing strategy the Client has been following recently.

## Machine Learning

Machine Learning (Expert System, n.d.) is a branch of Artificial Intelligence. It enables to analyze data, and acquire knowledge from it.

It searches patterns in data to learn from them, and then predicts the future behavior of the data sets. In order to make predictions, two kind of algorithms are implemented:

- Supervised Machine Learning: It applies what is has been learned from past data, in order to build models that predict future events. It use a training data set, to build the prediction, and a validation data set to compare and see if the output is correct.
- Unsupervised Machine Learning: These algorithms are used when the training data is unlabeled. It explores data trying to describe hidden structures, inferring a function.

Machine Learning programs give the user the power to analyze massive quantity of data and use it to predict future events, and make decisions following the results.

## Time Series Forecasting

As part of Machine Learning tools, Time Series Forecasting (Machine Learning Mastery, 2016) does estimations of the future evolution of the data provided. It provides some instruments to extrapolate the predictions.

It uses classical Time Series Analysis, to obtain some statistical measures, and with them it looks for the following outputs:

- Level: The reference value to start the prediction.
- Trend: The increasing or decreasing behavior of the series over time, considering the big scope.
- Seasonality: The repeating patterns can be found in data, depending, maybe, on the weekday or the season of the year.

Noise: Variations of data that cannot be explained with the model.

## Project Schedule

In order to follow the progress of the project, some milestones have been established during its duration.

Those milestones have been selected as deliverable documents to evaluate the project and some points and to be able to redirect it, if some aspects are not as were desired in the first place.

| Milestone | Milestone Description |
| :--- | :--- |
| M1 | Annex B |
| M2 | Predictive Model Preliminary Presentation |
| M3 | Predictive Model Final Presentation and Results |
| M4 | Optimization Model Preliminary Presentation |
| M5 | Optimization Model Final Presentation and Results |
| M6 | Final Project Presentation and Impact on the Client |

Table 1: Milestones of the Project

Once the milestones and the procedure of the project have been settled, it is necessary to divide the project in different tasks, so it would be possible to prepare a work schedule.

The time scope this project has is six months. The activities have been scheduled in a week scale, in order to be easy to visualize it.

The project has been divided in four main activities, and those who requires more time, have been, at the same time, divided in some subtasks, to make easier and smaller objectives.

In parallel to the project, realization will be the writing of the Memory. This fifth task will be constantly advancing, so all the information achieved during the six months will be collected.

|  | January |  |  |  | February |  |  |  | March |  |  |  | April |  |  |  |  | May |  |  |  | June |  |  |  | $\begin{array}{\|c\|} \hline \text { Jul } \\ \hline 26 \\ \hline \end{array}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 |  |
| Information Research |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Formation |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Predictive Model Preparation |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| - Data Analysis |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| - Data Preparation (Clusterization) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| - Research of parameters influencing in the elasticity |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| - Predictive Model Preparation |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| - Test and Model Evaluation |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Optimization Model Preparation |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| - Model Development |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| - Model Application to the Objective |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| - Objective Impact Evaluation |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Business Case Development |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| - Hypothesis Approach |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| - Economic Analysis |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Project Writing |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| M1 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

## Resources to be applied

To analyze and work with the data provided, it will be necessary a platform to code the needed instructions to obtain the information desired.

The platform selected is Anaconda, which is one of the most popular Python Data Science Platforms.

## 0 ANACONDA.

Anaconda is an open-source distribution, based in Python and R. In this project, the programming language selected will be Python, due, mainly, to the available online information.

The reason why Python has been the language chosen is mainly for the capacity it has for developing data analytics algorithms. In addition, it is almost a universal language (more than R) and the information available on internet, makes it quite easy to learn and solve possible doubts during the process.

Inside Anaconda's platform, Jupyter is a web application, to easy programming.

Within this platform, some Python Libraries will be used to develop a proper code, and to achieve the main objectives.

- Numpy: Is the main package for Scientific Computing. It will be integrated to work with is powerful N -dimensional array object.

- Matplolib: A library to plot high quality 2D figures. Its module pyplot provides an interface like Matlab.


## matpl $\circledast$ tlib

- Pandas: A library provides a high performance and easy-to-use data structures and data analysis tools.

- Scikit learn: A library designed to easy programming Machine Learning algorithms. It has efficient tools to data mining and data analysis, based on clustering, regressions, preprocessing, classification, and others.

- StatsModels: This library provides functions to perform statistical tests and statistical data exploration. It provides very powerful algorithms to develop Time Series Forecasting, and the tests that needs to be done during the process.

$$
\text { SM StatsModels } \begin{aligned}
& \text { Statistics in Python }
\end{aligned}
$$

- H2O: Is a module based on Java. There is a python module to connect both programming languages, and provide access to all the Machine Learning algorithms. In the project, for Machin Learning, it has been used generally Scikit learn, but in punctual moments H 2 O has been useful


## $\mathrm{H}_{2} \mathrm{O}$ ai

- Pyomo: A Python-based open-source software package supports a diverse set of optimization capabilities for formulating, solving, and analyzing optimization models. It is possible to use many solvers within this package, but in this case the chosen solver has been GLPK (GNU Linear Programming Kit), which implements large-scale Linear Programming and Mixed Integer Programming problems to solve the optimization model.

To visualize data, other Software could be applied, as Power BI, developed by Microsoft. Power BI is a business analytics service that delivers insights to enable fast, informed
decisions. Power BI will be used to create some dashboards to visualize the results and to be able to play with the graphs provided.

## Data

In this chapter, there will be presented the data used to develop the project. The quantity of data analyzed has been very big and it has come from different sources independent one from each other.

The data sources have been two of the leading companies in the Food Retail Sector in Spain, which has provided information of their sales for more than one year.

Other information comes from the web (DatosClima, s.f.), which has in their databases weather information from 2013, coming from AEMET.

Finally, there is also holiday's information to be used in the prediction model, coming from the web (Calendarios Laborales, s.f.).

In the following parts of the chapter, the data will be explained deeper, saying the different fields provided, the number of data from each dataset, and other concepts.

This chapter is very important, because it will be explained the changes applied to the datasets in order to develop the project. Those changes are very necessary because it drops the unneeded data and combines information to obtain data that are more selfexplanatory.

The shops selected from each competitors are very near, and for that reason, it is expected that they will be affected by promos.

## Competitor 2

## Competitor 1

Figure 7: Location of the two competitors

## Datasets

There has been analyzed different documents in this project. Depending de quantity of data included in each one, it has been saved in a .csv, or in compressed documents .parquet.gzip.

From the first competitor it has been uploaded a document with 50 columns and $1,152,478$ rows, which represent each sale of the shop during the period studied (2017/01/01-2018/02/28).

The rows represent different fields, as product hierarchy (category, subcategory...), product name, the shop and the region where it belongs, the date of the sale, if there is a promo applied, the discount, the number of sale and the type of sale (promo, normal, liquidation sales...).

From the second competitor, there have been analyzed four different documents. The first one, with 83,124 rows and 71 columns, describes the different products. Each row is a different product and all the columns different features of it (category, fresh or not...). The second document with 11,533 rows and 115 columns describes the different shops, specifying region and other aspects. The third one, with 270,694 rows and 4 columns, explains for the shop analyzed, if a product has a promo and the promo it has in each case, at a day level. Finally, the last document represents the sales of the shop, it indicates the product, the price and the quantity sold per day ( 569,205 rows and 6 columns). All the information has been provided for the same dates than the document of Competitor 1.

Finally, there two simple documents, which are the Weather and the Holiday ones. The first one has information for the needed dates about temperature, rains and sun hours. The second one has information about the free days in the city analyzed.


Figure 8: Data Structure

## Data transformation

Once the data has been explained, it is necessary to explain the changes applied to it. The information firstly provided is enormous, and a lot of information is not needed at all.

This part will start with the changes applied to be able of combining the datasets coming from the two different competitors. In first place, the information coming from them has different names, so it is complicated to combine them.

The solution applied to solve it has been creating a Master document that relates one field of both competitors. The chosen field has been category. There a little bit more than 100 hundred categories in each competitor and most of them are very similar.

That way, has been possible, having for competitor 1 the category "SAL Y ESPECIAS", and for competitor 2 "SAL" and "ESPECIAS", to create the Master Category "SAL Y ESPCECIAS". The Master Category will permit combine both documents, so in the Prediction at a category level, the competitor 2 promo could be a prediction variable.

The second important change has been eliminating unnecessary columns. For competitor 1 it has been passed from 50 columns to 15 . Most of the eliminated columns where codes and some subcategory fields. The final information maintained is some concrete category fields and product names, promo information and sales information.

For competitor 2, firstly, all the documents have been combined into one, and a similar process has been followed to finally get 14 columns.

The last important change applied to data has been creating the sales increment. To do so, firstly has been necessary to group data by product and discount. Once the information has been grouped, it has been needed to create a base quantity sold per day.

This base quantity sold is calculated for the discount equal to 0 , and after that calculating the average quantity sold for the different products and different days.

$$
\begin{gathered}
\text { Avg Sales }=\frac{\sum_{\text {day } \text { Sales }}^{n^{\circ} \text { days }} \forall \text { Promo }=0}{\Delta \text { Sales }\left\{\begin{array}{c}
1 \text { if Promo }=0 \\
\frac{\text { Sales }}{\text { Avg Sales }} \text { if Promo } \neq 0
\end{array}\right.}
\end{gathered}
$$

After this has been applied, for the other discounts, it is calculated also the average quantity sold, and it is compared to the base quantity sold. Once it has been compared to the base, the ratio has been obtained, which is the increment in sales, and this new information needs to be merged with the original dataset.

## Algorithms

During the development of the project, multiple statistical and optimization models have been developed.

Those algorithms could be divided in two. On the first hand, the Machine Learning models, developed to predict the demand (Tryolabs, 2018) to limit the optimization price, and secondly the optimization models to maximize the margin of the client.

In order to prepare the optimization model, there will be needed to calculate some elasticity curves for the different categories and products, and some cross-elasticity curve between products.

## Elasticity curves

In this part of the chapter it is explained how the elasticity curves have been calculated.
This elasticities (PriceBeam, 2017) are needed for the different optimization problems, and will be calculated the elasticity curve for each product and some cross-elasticity ones.

Self-elasticity:

$$
\Delta \text { Sales }_{1}=\beta_{0}+\beta_{1} * \text { Discount }_{1}+\varepsilon
$$

Cross-elasticity:

$$
\Delta \text { Sales }_{1}=\beta_{0}+\beta_{1} * \text { Discount }_{2}+\varepsilon
$$

The elasticity curves will measure how the sales changes with respect to changes in price. Until this point, everything seems very standard, but those elasticities will be normalized in order to compare all the products/categories at the same level.

The normalizing process consist on comparing the increment in demand, depending on the different discounts applied.

This means that for each product, there will be calculated a base price and a base demand. Those values are the average for all the year 2017 when no promo or discount is being applied for the products.

Once the base price and base demand are calculated, there are obtained some ratios, dividing the bases with the real ones. For the price, there is obtained the discount, and for the demand, there is obtained the increase in the demand.

To obtain the equations of the curves, it has been tried two different methods. The first a simple linear regression, and the second one an exponential regression.


Figure 9: Linear vs Exponential regression
The second model has been tried, supposing that the demand increases until one discount, but after the slope of the curves is getting smaller, until the demand grows too little.

Once both models have been applied, it has been seen that the exponential model gives a result very similar to the linear regression and then, for simplicity, it has been decided to apply the linear regression for the category and product's optimization models.

All the curves, as the common sense explain us, should include the point discount $=0$, increment in demand $=1$. In addition, for normal elasticities, the slope must be positive, that means for a positive discount (price reduction) there is an increase in demand, and for negative discounts (price increment), the demand should decrease.

When taking into account cannibalization, it is more difficult to determine the slope. There are products that may be much correlated in there sales, as fruits, but when there are discounts applied they cannibalize a lot. This means that, for example it can be found positive slopes when calculating cross-elasticities between different oranges products, but when it gets down to concrete promo periods, it is seen negative slopes for those small periods of time.

For this reason, cannibalization slopes should be taken for concrete periods, and the complexity that this process gives, makes very difficult to operate cannibalization for the entire portfolio.

## Machine Learning Models

The Machine Learning models, are based on advanced statistics, and are used to predict the demand based on historical data. The data used in this process is a dataset of the sales of one supermarket during one year, for each of the products.

In order to know the possible variables influencing the sales, there has been carried out a logical analysis of the variables provided and external ones.

| Rains |
| :---: |
| Sun hours |
| Maximum temperature |
| Minimum temperature |
| Weekday |
| Monthday |
| Month |
| Holiday eve |
| Christmas |
| Weekend |
| Competitor's discount |

Table 3: Variables to introduce in the prediction model
Those chosen variables are going to be explained hereafter.

- Rains: It may be expected to see a decrease in demand those raining days in some cities.
- Sun hours: When days are longer people are more willing to go shopping.
- Maximum and minimum temperature: Demand varies in different products depending on the temperature. Ice-creams sales increase in warm days, and lentils sales are higher when it is cold.
- Weekday and month: Depending on the day or in the month, demand is expected to change.
- Christmas, Holiday eve and weekend: When there is a period where the supermarket is going to be closed, the demand will increase the day before. Christmas is a special period where demand is expected to do strange things (special products, days with high demand or days with special low demand).
- Competitor's discount: This variable is only used in the category prediction model. The reason is because datasets are only related at a category level. Building a product key is very complicated, because many white-label products are not replicated in the other dataset.

All the data applied in the models have been implemented as numerical data. The main reason of doing this is to avoid categorical variables in the model, which would have increment the complexity of the problem a lot.

The reason why categorical data increments complexity and execution time is because you need to convert them into dummies. This means, create one extra variable for each value of the categorical variable. This is explained Table 4.


Table 4: Conversion to dummies
Once all the variables have been explained, the algorithms applied will be explained.

## ARIMA Model

The ARIMA model has been used to predict the demand as a whole. The intention of using this model was to know the demand was expected to evolve during the first months of 2018.

The ARIMA model (Chatterjee, 2018) is an Autoregressive Integrated Moving Average model. The Autoregression tries to predict using the relation between an observation and a previous one. The Integration, tries to predict using the difference between an observation and a raw of other ones. Finally, the Moving Average uses the relation between an observation and the residual error applied to some previous observations.

In order to implement the ARIMA model, there has been necessary to apply some transformations on data.

First, it was necessary to eliminate the outliers, values equal to o, because that meant days where the shop was closed, and they are not interesting in the model. Secondly, it was also need to apply a logarithmic transformation, to linearize data. After that, differencing to find seasonal trends. Finally, an exponential decay to gives more weight to recent observations.

Once the model is stationary, is the moment to calculate the Autocorrelation Function and the Partial Autocorrelation Function, from which the parameters of the model will be decided. These parameters are the following.

- p: The number of lag observations included in the model, also called the lag order.
- d: The number of times that the raw observations are differenced, also called the degree of differencing.
- q: The size of the moving average window, also called the order of moving average.

Once the model has been developed, the transformations carried out need to be undone.

This model (Hewa, 2018) is based on regression and classification to build Decision Trees, and a technique called Bootstrap Aggregation that consists on training different Decision Trees in different data samples.


Figure 10: A Beginners Guide to Random Forest Regression (Hewa, 2018)
To optimize the results from this model, some hyperparameters have been modified. Firstly, the maximum depth of the trees and the number of estimators, which is the number of trees.

This model has been applied in the case of predicting the demand, dividing the product's portfolio into different categories. There has been used over 90 different categories, as it could been fruit, olive oil and vinegars, fish or deodorants.

## Extremely Randomized Trees

To predict at product level, it has been very difficult to decide the best algorithm to use. There are more than 6000 different products in a supermarket, and each of them should need a different model to fit the best the needs of them.

As it is almost impossible to implement 6000 different models, there has been selected a random sample of products and it has been applied multi tuning to choose the best model.

This library executes the maximum number of models in a determined time, and orders all of them by the AUC (Area Under the Curve). The AUC is used to decide the best model, and it is based on the number of True Positives, False Negatives, False Positives and True Negatives.

| model_id | auc | logloss |
| :--- | :--- | :--- |
| StackedEnsemble_AllModels_0_AutoML_20180119_093938 |  |  |
| StackedEnsemble_BestOfFamily_0_AutoML_20180119_093938 | 0.878817 | 0.399037 |
| GBM_grid_0_AutoML_20180119_093938_model_0 | 0.878523 | 0.400079 |
| GBM_grid_O_AutoML_20180119_093938_model_3 | 0.877985 |  |
| DRF_O_AutoML_20180119_093938 | 0.874259 | 0.535281 |
| GBM_grid_0_AutoML_20180119_093938_model_2 | 0.872272 | 0.418382 |
| GBM_grid_0_AutoML_20180119_093938_model_1 | 0.871849 | 0.419048 |
| GLM_grid_0_AutoML_20180119_093938_model_0 | 0.868148 | 0.416836 |
| DeepLearning_0_AutoML_20180119_093938 | 0.866239 | 0.419353 |
| XRT_0_AutoML_20180119_093938 | 0.866177 | 0.786824 |

Figure 11: Best models of H2o AutoML, ordered by AUC
Once this first method has been applied, the model that has scored the highest for the products in the sample has been the Extremely Randomized Trees.

The main difference between Extremely Randomized Trees and Random Forest is that the first ones use the whole learning sample when training the trees instead of the Bootstrap Aggregation, and it is fully random when splitting the samples.

## Optimization models

Once the prediction models have been developed, and the results have been satisfactory, the optimization models are developed. There will some different models carried out. First, at a category level and the changes in price will be applied to all the products in the same category.

The second model developed is at a product level. It would be the same model, but the information received will be much bigger, and the results more accurate.

Finally, there will be a model considering cannibalization between products.
All the models presented will be more or less simple, and the method applied to solve them will be Linear Programming, where there is an Objective Function and some Constraints.

The model developed, as it will be explained hereafter, has been discretized. This has been done, because it reduces so much the complexity of it. Instead of computing some complex mathematics algorithms, the model only needs to try the different options and choose the optimal between them. It is true that it will not obtain the real optimal, but for this case, it is not needed, it will find a value near to the optimal one, which is enough for the purposes, and it is more realistic when talking about supermarket's discounts.

The models have been solved with the solver gplk in Python, which has been previously explained.

The objective of the following models is to maximize Profit, which reflects the margin at category/product level.

In the following tables it would be explained the Sets, Parameters and Variables for the different models developed.

In Table 5 it is observed the sets of the different models. These are the dimensions of the model. The variables and the different parameters would take some values for each group of sets.

| Sets |  |
| :---: | :--- |
| Categ | It could be also product (Prod), or product 1 and 2 (Prod1, Prod2) <br> depending on the model. It reflects the category, product of set of <br> products (in the cannibalization model) to which a discount will be <br> applied. |
| Day | The model has been developed for a day level. That means that the <br> price chosen for each category/product will be the same within the <br> day. The time scope for the problem is two months (January and <br> February 2018). |
| Pt.Curve | This is the dimensions most difficult to understand. It reflects the <br> different possible discounts. It is the x-axis of the elasticity curve, and <br> once there has been chosen a discount; there will be a corresponding <br> increment in sales. The range of possible discounts is in the interval - <br> 5o\% to 50\%, with a 1\% of increment. That means a 100 different price <br> values for each category/product. |

Table 5: Sets of the optimization model
In Table 6 there are the different parameters taking part in the model. Those parameters are the fixed variables and they depend on some of the sets.

These parameters represent some features of the elasticity curves, which have been discretized for the problem to add some simplicity to it. Other parameters would be included in the constraints, as the results of the prediction models.

| Parameters |  |
| :---: | :---: |
| Dbase $_{\text {Categ }}$ | It is the average demand for each category/product during 2017, when there is no promo/discount applied. |
| $\Delta D_{\text {Categ,Pt.Curve }}$ | It is the increase/decrease applied to the base demand. It takes value 1 when there is not a discount applied. It is calculated with the elasticity curve. |
| Dmax $_{\text {Categ,Day }}$ | It is the result of the previous prediction models |
| Pbase ${ }_{\text {Categ }}$ | It is the average price of each category/product during 2017, when there is no promo/discount applied. |
| Dct Pt.Curve | It is the discount applied to each product. It takes the values of the dimension Point of the Curve, and it will be explained hereafter. |
| Cost $_{\text {categ }}$ | It is the estimated value of the costs of goods. From the P\&L of the client, it has been seen that a reasonable value for the cost is the $60 \%$ of the base price. This value could be more accurate for each category or product, but the client has not provided it. Therefor a $60 \%$ of the base price is accurate enough for the purposes of the project. |

Table 6: Parameters of the optimization models
It can be observed in Table 7 the decision variables of the models. Those variables have a different purpose.

| Variables |  |
| :--- | :--- |
| $x_{\text {Categ,Day,Pt.Curve }}$ | It is the decision variable. It is binary, and it takes 1 in the optimal <br> point, and o in the other ones. |
| $y_{\text {Prodz,Day,Pt.Curve }}$ | The objective of this decision variable is to impose the same discount <br> for a product and day, independently to which other product is <br> comparing. |

Table 7: Variables of the optimization models

## Optimization at Category Level

This model tries to optimize the profit taking into account that each category works as unique product. That means that all the products within a category will vary their prices the same percentage each day, with respect to their average price (without any promotion).

To do so, the profit should be maximize for each day and category. There has been chosen a binary variable to make the decision. This variable would take one as value for the optimal discount (positive or negative) and o for all the other values.

The first constraint shows that the demand for one day and category should be below the maximum demand plus a margin of the $10 \%$. This extra $10 \%$ over the demand estimated reflects a possible increase in the demand due to the introduction of dynamic pricing into the system. As all the prediction is being done without taking into account dynamic pricing, the optimization model reflects the possibility it could have to increase the demand for some days.

The second constraint is built to choose only one point into the discretize elasticity curve per day and category. This means there is only on valid discount for each category-day.
$\max$ Profit $=\max \sum_{\text {Categ }} \sum_{\text {Day }} \sum_{\text {Pt.Curve }} x_{\text {Categ,Day }, \text { Pt.Curve }} *$ Dbase $_{\text {Categ }} * \Delta D_{\text {Categ,Pt.Curve }} *$ $\left[\right.$ Pbase $_{\text {Categ }} *\left(1-\right.$ Dct $\left._{\text {Pt.Curve }}\right)-$ Cost $\left._{\text {Categ }}\right]$
s.t.

$$
\begin{gathered}
x_{\text {Categ,Day,Pt.Curve }} * \text { Dbase }_{\text {Categ }} * \Delta D_{\text {Categ,Pt.Curve }} \leq 1.1 * \text { Dmax }_{\text {Categ,Day }} \\
\sum_{\text {Pt.Curve }} x_{\text {Categ,Day,Pt.Curve }} \leq 1
\end{gathered}
$$

The reason why there has been analyzed, firstly, categories as a whole, has been to simplify the problem and being capable of analyzing if the results are coherent without doing a huge effort.

## Optimization at Product Level

Once the optimization model at category level has been implemented, and it has been seen it works properly, a second model is developed, in order to achieve an optimization result product by product.

The problem is very similar to the category one, but there is an important change between optimizing at product level, due to the increase of time running of this new model.

It is true that the results of this model are much more reliable than the previous one, and that is the reason to implement it.

```
\(\max\) Profit \(=\max \sum_{\text {Prod }} \sum_{\text {Day }} \sum_{\text {Pt.Curve }} x_{\text {Prod,Day,Pt.Curve }} *\) Dbase \(_{\text {Prod }} * \Delta D_{\text {Prod,Pt.Curve }} *\)
\(\left[\right.\) Pbase \(_{\text {Prod }} *\left(1-\right.\) Dct \(\left.\left._{\text {Pt.Curve }}\right)-\operatorname{Cost}_{\text {Prod }}\right]\)
```

s.t.

$$
\begin{gathered}
x_{\text {Prod,Day,Pt.Curve }} * \text { Dbase }_{\text {Prod }} * \Delta D_{\text {Prod,Pt.Curve }} \leq 1.1 * \text { Dmax }_{\text {Prod,Day }} \\
\sum_{\text {Pt.Curve }} x_{\text {Prod,Day,Pt.Curve }} \leq 1
\end{gathered}
$$

## Optimization at Product Level with cannibalization

The last implemented optimization model has been at product level, but considering cannibalization between products.

This model tries to take into account the correlation between different products. For example, you may thing that a discount in oranges should maximize the profit. And probably is true that it maximizes the profit for oranges, but it could have a decrease effect in apple sales and, if you see the system as a whole, the profit, is actually smaller than if you haven't apply any discount.

The problem is solved for each product, and for each of the products, it sees how the related ones are modified. That is why the problem has a second product dimension.

This second product dimension increase a lot the complexity and time of the model.

$$
\begin{aligned}
& \max \text { Profit }=\max \sum_{\text {Prod1 }} \sum_{\text {Prod } 2} \sum_{\text {Day }} \sum_{\text {Pt.Curve }} x_{\text {Prod } 1, \text { Prod ,Day,Pt.Curve }} * \text { Dbase }_{\text {Prod } 1} * \\
& \Delta D_{\text {Prod1,Prod ,Pt.Curve }} *\left[\text { Pbase }_{P r} \quad *\left(1-\text { Dct }_{\text {Pt.Curve }}\right)-\text { Cost }_{\text {Prod } 1}\right]
\end{aligned}
$$

s.t.

$$
\begin{gathered}
\sum_{\text {Prod2 }} x_{\text {Prod1,Prod2,Day,Pt.Curve }} * \text { Dbase }_{\text {Pro }} * \Delta D_{\text {Prod ,Prod2,Pt.Curve }} \leq 1.1 * \text { Dmax }_{\text {Prod1,Day }} \\
\sum_{\text {Pt.Curve }} x_{\text {Prod ,Prod2,Day,Pt.Curve }} \leq 1 \\
y_{\text {Prod2,Day,Pt.Curve }}=x_{\text {Prod1,Prod2,Day,PP.Curve }}
\end{gathered}
$$

As the complexity and time-execution of this model are huge, the model has been decided to be run in some concrete products, and then extrapolate it to the whole product portfolio.

## Results

In this chapter, the results obtained will be shown and explained.
The process to be followed is:

1. Analyze the elasticity curves obtained for all the categories, and after that for all the products' portfolio.
2. The different predictions made will be shown, it will be shown at category/product level, but also at aggregated levels.
3. The optimization problems will be solved. In this last part there will be shown the different how the discounts can evolve for different categories or products, and will be computed the optimal profits for all the optimization models considered.

## Self-Elasticity curves

When calculating the elasticity curves, there has been calculated, at the beginning at category level and secondly at a product level.

Implementing the elasticity curves, some data has been eliminated, considering it outliers. Those outliers are:

- Increase in demand < 10: You may find some periods where applying a discount may provoke an increase in demand over this number, but it is considered that this could be caused by external factors to the discount. It could be cause because it is a peak-demand moment.
- Discount < 80\%: It is true that there are sometimes discounts very large, but those discounts may have been chosen for complicated decisions. Maybe the product is on liquidation, because it is near its caducity date. There are also some promos, as 2x1, which is considered as a $100 \%$ in one of article, and $0 \%$ in the other. This disturbs the real curve, because it should be considered as $50 \%$ discount in both articles.
- Increase in demand > 1: This problem may be caused by similar reasons to the first outlier's option. There can be some periods where the demand is especially low, and even if you apply a discount, it will not have the desired effect. It could happen as well that the shop does not have enough stock, and then there are not selling, as much as they should, or could.

In Table 8, it is shown the ranking of categories with the highest slope, this means with the highest variation of sales, depending on the price variation.

|  | CATEG | CONST | PEND |
| :--- | ---: | ---: | ---: |
| $\mathbf{4 3}$ | ALCOHOLES | 1.010940 | $1.033222 \mathrm{e}+01$ |
| $\mathbf{5 4}$ | BOMBONES | 0.999916 | $8.215795 \mathrm{e}+00$ |
| $\mathbf{2 9}$ | ACEITE-VINAGRES Y ADEREZOS | 0.998057 | $7.885834 \mathrm{e}+00$ |
| $\mathbf{2 3}$ | VINO | 1.003510 | $6.085536 \mathrm{e}+00$ |
| $\mathbf{8 1}$ | VACUNO | 1.002875 | $5.742174 \mathrm{e}+00$ |
| $\mathbf{7 7}$ | PRODUCTOS REFRIGERADOS | 0.999354 | $5.680600 \mathrm{e}+00$ |
| $\mathbf{8 2}$ | CONEJO | 1.002205 | $5.663928 \mathrm{e}+00$ |
| $\mathbf{5 0}$ | PRODUCTOS CUERPO | 1.000934 | $5.544337 \mathrm{e}+00$ |

Table 8: Top categories ordered highest elasticity's slope
For the category "ALOCHOLES", which is the one with the highest slope, it can be seen in Figure 12 the distribution of the different points. As it can be seen, sales may vary quite much when there are small discounts applied. However, it is true that you may find some outliers, as the point discount $=0.5$, increase in demand $=2$.


Figure 12: Increase in Sales depending on Discount for "ALCOHOLES"
Once the top ranking has been shown, it will be represented the bottom categories. Those categories are not exactly the bottom ones, because there are categories that do not have any discount during the period analyzed, and then they are not able to compute any
variation in sales, compared to the price. Those categories are categories with not many sales, as shoes (CALZADOS) or DIY (BRICO).

| $\mathbf{1 9}$ | CEREALES DE DESAYUNO | 1.000986 | $1.872049 \mathrm{e}+00$ |
| :--- | ---: | :--- | :--- |
| $\mathbf{6 0}$ | PRODUCTOS NAVIDAD | 0.999869 | $1.864368 \mathrm{e}+00$ |
| $\mathbf{8 4}$ | BOLLERIA | 0.999327 | $1.815871 \mathrm{e}+00$ |
| $\mathbf{7 4}$ | HUEVOS | 1.000398 | $1.631429 \mathrm{e}+00$ |
| $\mathbf{8 5}$ | VAJILLAS | 1.002127 | $1.473964 \mathrm{e}+00$ |
| $\mathbf{5 9}$ | PAN DE MOLDE | 1.003414 | $1.101494 \mathrm{e}+00$ |
| $\mathbf{4 7}$ | SAL Y ESPECIAS | 1.000000 | $2.708944 \mathrm{e}-14$ |

Table 9: Top categories ordered lowest elasticity's slope
As it can be seen in both Figure 12 and Figure 13 showing the top and bottom slopes' ranking, the intercept is almost one. The small variation from this value is cause by the outliers and the imperfect shape of the curve, but is more than enough satisfactory for the project.

In Figure 13 is represented one of those bottom top categories (BOLLERÍA), and as in the case of "ALCOHOLES", there are some outliers as the point discount $=35 \%$ and increase in demand $=4.2$.


Figure 13: Increase in Sales depending on Discount for "BOLLERIA"
Once it has been followed all the procedure for the different categories, the same process has been applied for the products.

As it is shown in Figure 14 with the product "A.OLUVA V.EXTRA COOSUR 250 ML", calculating elasticity curve for those products you dispose of much less points, because the shop doesn't apply many different discounts to a same product. Nevertheless, it is true that a positive slope with intercept 1 can be defined, and in this case, the shape of the curve is linear.


Figure 14: Increase in Sales depending on Discount for "A.OLUVA V.EXTRA COOSUR 250 ML"
This obtained curves will be used in the optimization model, as it can be seen, the amount of information that is being generated is huge. For example, in the case of the optimization of products' portfolio, there are more than 6,000 curves. With 100 hundred points per curve (after discretization), there are finally, more than 600,000 points.

## Cross-elasticity curves

In this chapter it will be analyzed the results from calculating cross-elasticity curves. These curves relates the sales of one product with the discounts applied to a second one.

To calculate the cross-elasticity curves, it may be considered two different methods, depending on the accuracy-complexity balance desired.

Method 1 considers the curves as stationary. This means that the curve between two products is the same during all the different periods. Method 2 is more accurate, and considers that there are many different curves depending on the period of the year.

Method 1: Stationary curves

First, method 1 will be analyzed, and the results will be studied. To explain this method, the category chosen has been "PESCADOS", due to the expected cannibalization that may be found between similar products.

As in the chapter of self-elasticity, the curves will be accompanied by an estimator showing how much trustful the model is. That estimator is the R2.

The objective is to obtain a correlation matrix, with an R2 matrix, explaining which of the correlations is trustful enough. In Table 10 it is seen the correlation between three different "PESCADOS" products, all of them from hake. As it can be seen in Table 10, the diagonal is the self-elasticity and the other values are the cross-elasticity slopes.

|  | FILETE MERLUZA C/P EROSKI 540G | FILETE MERLUZA EMPDO.340 PESCA | FILETE MERLUZA S/P EROSKI 540G |
| :--- | ---: | ---: | ---: |
| FILETE MERLUZA C/P EROSKI 540G | 5.648140488 | -2.875181302 | -2.97718214 |
| FILETE MERLUZA EMPDO.340 PESCA | -0.436226339 | 6.13242142 | -0.327963201 |
| FILETE MERLUZA S/P EROSKI 540G | -1.662706345 | -3.998236949 | 7.585090777 |

## Table 10: Cross-elasticity curves' slopes for some fishes

In spite of having a negative correlation, which makes a lot of sense, the estimators R2 are very low. They only take acceptable values in the diagonal, which only explains that the self-elasticity curves make sense, which have already been explained in the last chapter.

|  | FILETE MERLUZA C/P ER FILETE MERLUZA EMPDO.340 PESCA | FILETE MERLUZA S/P EROSKI 540G |  |
| :--- | ---: | ---: | ---: |
| FILETE MERLUZA C/P EROSKI 540G | 0.104041604 | 0.014641103 | 0.04247125 |
| FILETE MERLUZA EMPDO.340 PESCA | 0.003910804 | 0.419714046 | 0.003247736 |
| FILETE MERLUZA S/P EROSKI 540G | 0.005940657 | 0.018654677 | 0.181640689 |

Table 11: Cross-elasticity curves' R2 for some fishes
The problem of this method is that if the elasticity curves are considered as stationary, as until today, promos are only applied during certain periods, there would be returned some contradictory information.

For example, if oranges product are looked into, they have the same seasonality. During summer, sales are very low or o, and during winter, they have their peak demand. In these sense, it is applied the cross-seasonality curves as stationary, what it would be seen is that oranges are not cannibalized.

Nevertheless, if concrete period is observed, where one of the two oranges products is having a discount, it is probable to see that there is real cannibalization.

Just to summarize, the stationary curve could be applied if there is already implemented a Dynamic Price system, where prices are constantly changing, and therefor products are constantly cannibalizing other products' sales.

## Method 2: Non-stationary curves

The method 2 is much more complex than method 1 , because it considers different periods within the year, and it needs much calculation.

To first implementing Dynamic Pricing, this method is needed, but it is expected that once the project has been developed, it could be possible to continue with stationary curves.

As it is impossible to show the results for all the different products, in this part, it will be analyzed the results for the following ones: "2-3 PECHUGAS POLLO NATUR" and "CINTA LOMO CERDO TROZO". In this case, the product with the promos is " $2-3$ PECHUGAS POLLO NATUR", and "CINTA LOMO CERDO TROZO" is the one being cannibalized.

The period that has been taken into account is the whole 2017 year.
This case is interesting, because it shows that there is not cannibalization only chicken with chicken. It can be found cannibalization also between different products.

In Table 12, the intention is to show there is a clear cannibalization between the two products analyzed. When there is a 1 in promo applied, it means when promo is applied to the chicken. In the next column, there are the average sales for both products. Finally, the last column shows that chicken increase its sales when applying a promo to it, while, at that same moment, the pork is suffering a decrease in sales.

| Discount on 2-3 PECHUGAS POLLO NATUR |  |  |  |
| :---: | :---: | :---: | :---: |
| Product | Promo applied | Avg Sales | Increment in Sales |
| 2-3 PECHUGAS POLLO NATUR | 0 | 4.32 | $43.7 \%$ |
| 2-3 PECHUGAS POLLO NATUR | 1 | 6.20 |  |
| CINTA LOMO CERDO TROZO | 0 | 0.84 | $-52.4 \%$ |
| CINTA LOMO CERDO TROZO | 1 | 0.40 |  |

Table 12: Comparative of sales with and without PROMO applied for pork and chicken products

In Figure 15 is desired to show the previous graph more graphically, and it is shown that the p romo has been applied during the months of April and May (Weeks 21 and 22).

SALES POLLO VS LOMO


Figure 15: Sales evolution over the year for pork and chicken products
Just in that moment, the chicken sales increase, and pork sales decrease during the two weeks of the promo.

Once the previous information has been shown, in Table 13 it is shown the correlation between the products, dividing it by month. As it is seen, there is only a negative correlation during the months when the promo has been applied. During the rest of the year, there is positive correlation, because people interested in both products have a similar profile, and usually the buy them following the same habitudes.

| Producto Canibalizado | ${ }_{7}{ }^{\text {r }}$ Mes |  | Pendiente - | R2 -1 |
| :---: | :---: | :---: | :---: | :---: |
| CINTA IOMO CFRDO TRO70 |  | 12 | 0132 | 01815 |
| CINTA LOMO CERDO TROZO |  | 5 | -0.12338779 | 0.148791 |
| CINTA LOMO CERDO TROZO |  | 1 | 0.12264151 | 0.139505 |
| CINTA LOMO CERDO TROZO |  | 8 | 0.10492333 | 0.125138 |
| CINTA LOMO CERDO TROZO |  | 9 | 0.07041315 | 0.122643 |
| CINTA LOMO CERDO TROZO |  |  | -0.12882448 | 0.099096 |
| CINTA LOMO CERDO TROZO |  | 2 | 0.10040161 | 0.089644 |
| CINTA LOMO CERDO TROZO |  | 10 | 0.05191135 | 0.074556 |
| CINTA LOMO CERDO TROZO |  | 7 | 0.05205144 | 0.044392 |
| CINTA LOMO CERDO TROZO |  |  | 0.025944 | 0.016275 |
| CINTA LOMO CERDO TROZO |  |  | 0.03762542 | 0.015973 |
| CINTA LOMO CERDO TROZO |  | 11 | -0.00615872 | 0.000362 |

[^0]Therefore, as it has been seen, considering cross-elasticity curves stationary, in this first stage of the project, could lead to some important errors. However, when prices would be changing constantly, stationarity could be acceptable.

## Prediction

In this chapter, the predictions are going to be made. It is going to be followed the same process explained in the Algorithm chapter. First, it will be developed an ARIMA model, to know the evolution of demand.

After that, some Machine Learning models will be developed to compute the predictions at a category or product level.

## Global demand prediction

First, it will be developed an ARIMA model to predict the evolution of the demand during the different months, knowing its seasonality or trends.

To do so, the first step need has been to filter the data, eliminating the zero values of the demand, because those values indicated days where the shop has been closed, but they have been registered into the dataset.

From Figure 16, it can be seen that the moving average has different trends depending on the moment of the year, as well as the standard deviation. It is needed that the data were stationary. To do so, some transformations will be developed during the process to achieve it.

In order to achieve a stationary model, it is needed a low p-value, less than $1 \%$, for the ADCF test, applying the Dickey Fuller test.


Figure 16: Demand evolution, moving average and moving standard deviation
There has been applied the following transformations to achieve stationarity into the model.

Firstly, it has been applied a log-scale transformation, to remove the trend component. After this transformation, it could be seen the model was getting better, but not enough.

So, after that there will be subtracted to the real data the moving average. This is done because both have more or less the same trend component, and it could reduce the effect of it.

Other transformation is an exponential decay, which is because the quantity could be decreasing proportionally to its current value. The transformation equation is the following: $N(t)=N 0 * e^{-\lambda t}$. However, it has been seen that this transformation does not have a positive effect.

The last transformation applied has been a time shift. This means that given the values xo, x1, x2, x3 ...xn, the shifted values are null, xo, x1, x2 ...xn. Then, the new time series would be null, (x1-x0), (x2-x1), (x3-x2), (x4-x3) ... (xn-xn-1).

Given those transformations, the data can be decomposed within its trend, seasonality and residual values, which are shown into Figure 17.


Figure 17: Demand evolution, trend, seasonality and residual
In Figure 18 it is seen the distribution of the residuals of the model. As it is clearly shown in the graph, it is a normal distribution, which indicates the validation of the model.


Figure 18: Residuals distribution
Once there is a good idea from the data, it is proceeded to calculate the ACF and PACF (Autocorrelation and Partial Autocorrelation Functions).

From those functions, it is obtained the parameter for the AR model $\mathrm{Q}=2$ or 4 and for the MA $\mathrm{P}=3$. After several trials, it is found that the best model is ARIMA $(4,2,3)$.


Figure 19: Autocorrelation and Partial Autocorrelation Functions
In Figure 20 it is indicated the real data, and the prediction applying the ARIMA model. It can be seen that some months of the year have been well predicted, but other ones have big errors. For this reason, it has been considered to apply Machine Learning models at category and product levels.


Figure 20: Prediction of the demand

Category demand prediction

For the demand prediction at a category level, it has been implemented a Random Forest Regressor.

When analyzing the predicting capacity of this model, it has been calculated the accuracy of it. It has been obtained the variables influencing more in the prediction for each category.

In Figure 21, it can be seen all the different predicted categories aggregated. As it can be seen, the prediction (blue) is more conservative than the real values (black). This is something very normal, because the predicted values are calculated doing a regression of all the variables.


Figure 21: Aggregated category prediction vs original demand
Once it has been shown the aggregated prediction, it is made a ranking between the best categories that has been predicted the best. As it can be shown, the best categories are some that have great volumes of sales. It could be said, that those categories have liquidity, and therefore it is easier to see patterns into data and have great predictions.

In Table 14 there are three columns. The first one, with the name of the category; the second one with the accuracy of the model developed; the last one with the data that wants to be predicted. The difference between Accuracy and New Accuracy is that, Accuracy is for the test dataset, generated randomly by the model, from the 2017 data, and New Accuracy is the accuracy between the predicted values and the real values for the first two months of 2018.


Table 14: Most accurate category predictions
Only one category is chosen (CARNES) from the top five predicted categories, to analyze the information got from the predictions.

In first place, it is obtained the accuracies, already explained, which let us to understand how good is the model.

Secondly, it is shown in the following Figure 22 the importance of the variables. The information taken from this Figure 22 is how much different variables influence the demand. For example, it is very clear that the day of the week is very important. This could be, because Mondays people tend to buy what they have consume during the weekend.

Additionally, it is curious how Christmas is able to influence "MEAT" demand. As it has been explained before in this report, during Christmas, sales can increase notably, due to new products' offer or to prepare family meals.


Figure 22: Variable's importance for "CARNES"
In Figure 23 of "CARNES" prediction it is reflected the evolution of the demand during the two first months of 2018. In black is represented the real values during that period of time, and in blue there is the predicted ones. As it can be seen, the prediction follows very nicely the real demand, having only two days where there is a significant deviation.


Figure 23: Prediction vs original demand for "CARNES"

## Product demand prediction

In this last part of the prediction chapter, it is going to be explained how the prediction at product level has been conducted.

The model applied in this case has been the Extremely Randomized Tree. It has been proved that is the one getting the best results over many others, as some Deep Learning ones.

In Figure 24, it is shown the prediction of the product "AGUA C/GAS EROSKI 1,25l". As it is notified, the prediction is much more complicated in this case. At a product level, there is not so much liquidity, as it can be in the category level.

Nevertheless, it is very important to predict at a product level, in order to prepare a good optimization model. It is true that maybe the results are not exactly day-by-day for each product, but aggregating many different ones, and within a period large enough, the results obtained are very reliable.


Figure 24: Prediction vs original demand for "AGUA C/GAS EROSKI 1,25l"

## Optimization

Once the elasticities and the demand prediction has been obtained, it is time to start with the optimization problems.

First, it is necessary to remind the objective of these models, and how it is going to be achieved. The model has as purpose to optimize the margin, for each product or category. In order to achieve this objective, it is being implemented a Dynamic Pricing; this means the possibility of changing continuously product prices.

In these models, the pricing changes are limited to once a day. This is because the data proposed for the development of the project is aggregated at this level, and therefore, it is not possible to get deeper into the price variations.

Three main models will be developed. The first one is going to optimize the margin at a category level. This means that the price changes will be applied exactly in the same way for each of the products within a category.

The second model works in a very similar way as the previous one, but there is a main difference: it will be applied at a product level, which means that all the products will change their prices day-to-day. This increase the time execution of the model, but it is still feasible.

The third model is considering cannibalization. This provokes that a product cannot set its price without considering related products and their price variations. This complicates a lot the model, which gets much more complex, and it needs an execution time, which is too high. For those reasons, this model will be developed for some products, and the results will be extrapolated.

## Category level optimization

Once it has been explained how the optimization model would work and the objectives, it will be exposed the results.

In this first case, it is shown in Figure 25 how the discount in prices will evolve. Positive discounts implies decreases in prices, and negative ones increases in it.

As it could be seen in Figure 25, the category selected has been "ACEITUNAS Y ENCURTIDOS", and the discounts are, more or less, periodic varying from $7 \%$ to $-8 \%$.


Figure 25: Discount evolution for "ACEITUNAS Y ENCURTIDOS"
The price increases Fridays and Saturdays, due to a higher demand those days. Otherwise, during the week, prices are lower to encourage clients to buy these products.

Those changes in price will be applied to all the products within the category.

## Product level optimization

After having looked to the category level, it is proceeded to get deeper into the different products of the portfolio. This means to analyze more than 6,000 products and optimizing them.

As it is seen in Figure 26, it is common to find products that changes prices less frequently than at the category level. This is because the demand is less liquid, and you may find very different values in the dataset used to predict. However, with a model that is being used at Real-Time scale, executing each day the prediction for the following one, the data obtained would be much more accurate.

For example, in Figure 26, it has been chosen the product "AQUARIUS LIMON LATA $33 C L$ ", and this product varies its prices from a $10 \%$ discount to a $-17 \%$. This difference against the base price could made you leader in prices when the demand is low, and then you could attract some stricter people. Moreover, when the demand is naturally very high you maximize in those moments the profit sacrificing a small part of those clients.


Figure 26: Discount evolution for "AQUARIUS LIMON LATA 33CL"

Results from the product optimization

The results for the profit in this optimization model are enough important to consider them as a new subpart of the chapter.

Before explaining the results obtained, something that it is not very easy to understand must be explained.

The elasticity curves are not the same over all the time. They may vary depending on the month. For example, Christmas products are more sensible to price changes during Christmas than any other moment of the year.

Nevertheless, the elasticity has been calculated has it was unique for the whole year. As in the project the prediction is only for the first two months of the year 2018, it must be applied a correction for those curves.

Even though the elasticity curves are calculated at a product level, it is being considered that the whole category suffers the same tendencies during time.

In Figure 27 it is shown the procedure to calculate the correction measures for the first two months, for the whole portfolio (light color) and for the category "ALCOHOLES" (dark color).

There are three different graphs: The first one (top left) shows the real profit in $€$, being almost 250,000€ the total profit. In the second graph (top right) it is shown the value that provides the elasticity method calculation, which is quite lower for this period (210,000€). Finally, the third graph at the bottom shows the correction measure, which is a ratio.

For the whole portfolio, the measure tells us that it is necessary to multiply the elasticitycalculated profit by 1.21 during the first two months of the year, in order to obtain the corrected profit. For the case of the category "ALCOHOLES", the correction needed is higher, 1.34 .


Figure 27: Adjustment of the seasonal self-elasticity curve
This correction has been analyzed, and some conclusions have been achieved. The elasticity method is underestimating the profit during the winter months, and overestimating it during the summer months. This means, that people generally need more discounts in winter to buy, and in summer they are more sensible to small price changes.

For the first two months of 2018, the results obtained from the optimization model, for the whole portfolio is $276,000 €$, but this value needs to be corrected due to the variation of the elasticity curve.

Once it has been adjusted, the profit increases to almost $318,000 €$.
In Figure 28, it can be shown the category "CARNE" selected with the rest of the portfolio. In the left graph there is the profit directly obtained from the optimization model, and in the right one, the profit after applying the correction measure. The results given are $17,500 €$ and $20,000 €$ in each of both cases.


Figure 28: Profit optimized adjusted
There is the possibility to compare the results from the models developed with the real results that actually happened during the first two months of 2018.

In those two months, there was a result of almost $309,000 €$. This means, that the Dynamic Pricing applying a Product Optimization model could have an impact of a 2.9\% over the profit. In a shop level, this result may not have an important significance, but it could have a great impact for big companies. The results will better explained in the Business Model chapter, where it will be developed a sensitivity analysis to see the real impact of this revolutionary method.

Results from the product optimization applying cannibalization

In this part of the chapter will be analyzed the results when there is supposed it exits cannibalization between different products.

The two products to analyze are the same that have been presented in the cross-elasticity chapter. That is to be: "2-3 PECHUGAS POLLO NATUR" and "CINTA LOMO CERDO TROZO".

First, it will be presented the figures with the discounts without taking into account cannibalization. Secondly, taking into account cannibalization. And finally, there will be made a comparative about the profits in both cases.

In Figure 29, it is shown the evolution of prices for "2-3 PECHUGAS POLLO NATUR", without considering cannibalization. The prices are expected to be higher than the base price.


Figure 29: Discount of "2-3 PECHUGAS POLLO NATUR" without cannibalization
In Figure 30, "CINTA LOMO CERDO TROZO" price evolution is shown. It presents a very stable price, which could be caused due to a low liquidity of the product sales.


Figure 30: Discount of "CINTA LOMO CERDO TROZO" without cannibalization
Once the non-cannibalization figures have been presented, it is developed the case with cannibalization.

In Figure 31, it can be seen how different the price evolution, in the case of the chicken, is. It has become much flatter, and near to the base price.


Figure 31: Discount of "2-3 PECHUGAS POLLO NATUR" with cannibalization
In the case of the pork, the prices become much more changing. This is caused because now, both prices are able to influence in the demand of the other product.


Figure 32: Discount of "CINTA LOMO CERDO TROZO" with cannibalization
As chicken has a higher demand, it is possible that discounts on this product affects more to the pork one. For that reason, chicken usually needs low changes to produces variations, while pork requires high abrupt changes to be able to influence chicken sales.

If there is only reviewed the price evolution, it is almost impossible to say nothing coherent about the modifications in profits. In consequence, it would be presented the estimated profits generated by the two products, during the first two months of 2018, when it is being studied the Dynamic Pricing.


Figure 33: Profit comparative with and without cannibalization
If both Oranges (NARANJA POSTRE 3KG AND NARANJA DE ZUMO 3KG) are considered, and the chicken and pork products previously analyzed, it can be seen a change in profits of $-18.3 \%$

With Figure 33, it is shown that the pricing strategy may change a lot taking into account cannibalization, and the margin may change about almost a $20 \%$, which would be considered to adjust the increase in profits, from a $2.9 \%$ to a $2.3 \%$.

These results are very grateful in order to proceed to the Business Case. In the Business Case, the prices would not be no longer interesting, but the results of the profit will.

## Business Case

This chapter may be, above all, the most important of the project. In this chapter, the main intention is to collect all the results developed during the project, and build a business case showing the main possibilities of the work carried out.

It is also true that this chapter might be the less "Engineering" one, but it is very interesting to combine both Engineering and Business capabilities to see the possible real impact of it.

In this chapter, nothing is taken for sure (PwC, 2011). All the results obtained will be treated as one of the possible answers to the development of the project. It will be developed some sensitivity analysis to analyze different future scenarios.

This project appears to contribute to digitalization in the Retail Sector. Digitalization offers new possibilities to companies, it helps to be more efficient in business, be more prepared to prevent risks and many other aspects.

This Dynamic Pricing project is helpful in those fields. It has different objectives; first, it gives the possibility of optimizing the product price to maximize profits. Nevertheless, the development of the project opens many collateral objectives that contribute to monitor variables in the business that can be useful in the future.

Those objectives are, having a good prediction of the demand, which could help to have a better stock management, due to a knowledge of future sales, having the possibility of reducing costs.

There is also some advantages from the learning about cannibalization between products. To know how prices can affect in sales between different products is a powerful tool, in order to be able to optimize the margin.

The development of the project would be useful even if it were not implemented at the end. It provides important capabilities and knowledge about the demand and elasticities, which would be incredibly helpful if the traditional promo method is maintained.

The Business Plan will contain the following parts. First, there will be explained how the project will be carried out and introduced into the client, it will be presented also possible alternatives to the project. Within this first part it will be analyzed also the possible risks and how to face them.

Finally, it will be analyzed the profitability of the project, the future increase in income, and the consequent costs of developing the project. This part also will talk about other value creation during the project, as efficiency.

## Implementation

Dynamic Pricing may not have the expected impact. Models are not taking into account some psychological aspects, or differences between products behavior. In this implementation chapter, it would be also necessary to limit the project to some products and outlets, focusing in the probabilities of success.

In a business case, it is very important to know how it will be implemented the project. The process it will follow, the alternatives studied in case the project does not fit correctly within the company's strategy, how will be carried the communication with the employees and the role of each of them.

In this chapter it will also be treated the risks the project will face. As it is very innovative, and can change completely the supermarkets' pricing concept people have, the reception could not be as good as it should.

## Products' selection

In this first part, following the erraticity and a classification of products, according to sales, there has been done a division of the products in 9 parts. This division has previously carried out by PwC in an internal study.

According to sales, there are three categories. These categories are designed following a Pareto diagram and is explained as follows.

- A: This category is composed by the $20 \%$ of the products, representing the $80 \%$ of the sales.
- B: This second category represents the $30 \%$ of the products with the $15 \%$ of sales.
- C: The last category includes the $50 \%$ of products with only the $5 \%$ of sales.

According to erraticity, there another three categories. These categories represents how much constant are sales for different products.

- X: Indicates the products whose sales are the most constant and regular.
- Y: Indicates those products with some erraticity in their sales.
- Z: Indicates products with high variation in there sales, depending on the day or other facts. For example, umbrellas are very erratic because their sales are mostly in rainy days.

Analyzing the distribution of sales depending on the discount applied, which is shown in Figure 34, there will be selected the corresponding categories to be implemented in the Dynamic Pricing project.

According to the categories presented just before, and with some knowledge of the strategy of retail companies, it has been decided to implement the Dynamic Pricing project to the categories AY and BY. The reasons are that they have an important impact on the margin, and as they have some erraticity, but not much, price variation will have some impact in sales. In X products, it is not much likely to see important sales variations depending on discounts, and in Z products, it cannot be predicted the impact of discounts, because they depend in many other variables.

All these other products could be introduced in future phases of the project, where it has been analyzed more deeply these impacts.

Categories AY and BY contribute to the margin with the $30 \%$ of its total value. As it is seen it is an important part of profits.


Figure 34: Erraticity and sales classification of products
Additionally to the products' selection, this project will only be implemented for outlets located in big cities. Following the company's distribution of supermarkets, it can is seen that the $25 \%$ of the profits come from shops located in big cities. The reason why this
selection has been done is that Dynamic Pricing will have a positive impact where there is high competition, and pricing variation will make clients choose the supermarkets.

With those final remarks, the profit increase would be adjusted from a $2.3 \%$ to a $0.17 \%$, which is a realistic result from a project of these dimensions.

## Implementation process

About the implementation process, it needs to be very clear in order to follow it and not get wrong in any point of it.

The project has been decided to be developed during one year and three months, with three different steps clearly differentiated. As the company for which has been designed has more than 35,000 employees and 2,000 outlets extended above different countries, a project of this dimensions, that will affect the entire business needs some time to be implemented correctly.

The three steps are as follows:

- Some shops test: This first step is expected to last six months. In this first six months, dynamic pricing will be tested in some different kind of shops. In this step, it will be needed to promote the project in many different levels to attract people of different profiles and see how they may react to this revolutionary pricing method. The interesting point of this first moment is to be able to stop the project in case of failure, without having done a huge investment.
- Development of establishments with big impact: Not all places are equal in conditions. For example, shops in small villages, where there is no competition are expected to have a worst reception. For that reason, this second step, which is the longest ( 6 months) will be to extend the project to those kind of shops that in the first step has shown success (for example, big cities where there is high competition, and to neighborhoods with a good inclination to new technologies and are opened to innovation).
- Extension to the rest of the outlets: The last step will be carried out in case of success of the previous ones. It is expected to have a duration of three months, and in this point, the project could face two different ways. First, to be increased to the rest of the outlets, and second to be developed only for those shops where the profitability is proved. This last step will also work as a settlement process. Is the moment where all the tasks developed will be standardized, and the company will be completely adapted to Dynamic Pricing.


## Test

Figure 35: Implementation process
During this process, there will be a team in charge of following it and setting the adequate tasks at each moment. They will be also monitoring some variables, as sales or increase in the margin. They will be in charge to give the green light for each of the steps.

## Risks

As it has been previously introduced, this project as any other face different risks that may affect to different fields of the project. There are four different fields, and they will be helpful to classify them (Scope, time, quality and cost). Considering those fields and likelihood and probability levels, the risks of the project will be classified.

| Risk | Cause | Effect risk | Measure |
| :--- | :--- | :--- | :--- |
| Delay in the <br> different steps of <br> the project | The team spends more <br> time in each step due to <br> lack of information | The objective of one <br> year want be achieved, <br> increasing costs | Time and <br> costs |
| Software is more <br> expensive | The capabilities of the <br> software have been <br> underestimated | Investment will be <br> higher | Costs |
| Software with not <br> enough capacity | Cannibalization <br> complicates too much <br> the problem | The main objectives <br> won't be achieved with <br> the desired power. It <br> could need extra time <br> to achieve it | Quality <br> and time |
| Bad reception from <br> clients | People is not ready for <br> such technology | The project will focus <br> to other objectives | Scope |
| The algorithms <br> does not give the <br> expected results | Calculation doesn't <br> correspond to people's <br> psychology | Change of objectives <br> or less results than <br> expected | Scope and <br> quality |

Once it has been explained risks, their causes, effects, and the field to which they are affecting, it is the moment to explain their probability and impact to value them. The scale is the following:

## Probability:

- Rare: 1
- Low: 2
- Medium: 3
- Likely: 4
- Almost certain: 5

| Risk | Probability | Impact | Risk evaluation |
| :---: | :---: | :---: | :---: |
| Delay in the different steps of the project | Medium: <br> objective <br> demanding is $\quad$very | Insignificant: The implementation could take more time without consequences | 3 |
| Software is more expensive | Low: Cost estimation has been estimated accurately | Minor: Cost increase would not be very high | 4 |
| Software with not enough capacity | Low: Technology  <br> available is very <br> powerful   | Significant: It will have important consequences in results quality | 8 |
| Bad reception from clients | Likely: It is very innovative | Moderate: <br> objectives <br> change The <br> should | 12 |
| The algorithms does not give the expected results | Low: Algorithms have been developed taking into account historic data | Significant: The results could be wrong and therefor the conclusions too | 8 |

Table 16: Risks' evaluation

Once the main risks collected have been deeply analyzed, it is taken as a conclusion, that the worst one could be a bad reception from clients. Its probability is very difficult to be reduced, so it is necessary to have some alternatives in case of failure. The project has some secondary objectives, as knowledge of the demand evolution or cannibalization that could be applied with other revenue's sources.

The other two important risks are that the algorithms does not give the expected results and a software with low capacity. Both risks affect to the quality of the results and for that reason, they will be needed much attention to it, increasing investments in those two fields, if needed, to reduce as much as possible the probability. This is because is quite difficult to reduce the impact of both events.

## Value creation

As any project, the main objective is to create value to the company. In this case, the project will be not only financially analyzed, but also it will present collateral value created to the company.

First of all the business model is presented for different scenarios, and secondly it is shown the supplementary value created by the project.

## Financial profitability

First, it will be studied the financial profitability of the project. In this part, some different scenarios will be analyzed taking into account the base case developed during the project, and considering some risks becoming real.

To develop the business model of the project, some of the inputs have been taken from the P\&L of the retail company, some others from the results of the projects, on others have been assumptions made, having a good notion on how it works the retail sector.

The information taken from the results has been the percentage that profits increase due to the development of the project. In the project, this result was calculated for the first two months of a year for only one shop, but it may be extrapolated to the other shops and for the whole year.

This increase in profits has been introduced as the revenues of the project. As it is quite innovative it is considered that the acceptance of the public will be progressive, and it will achieve its maximum at year 4 of the project.

The real profit, to which apply the $0.17 \%$ increase of the results, have been taken from the Audited Accounts of 2016 of the company, and it is the Gross Margin (Revenues - Cost of Goods Sold). This is shown in Table 17.

| Increase in gross margin | \% | o.17\% |
| :--- | :--- | ---: |
| Total Sales 2016 | \%EBITDA | $3,773 \%$ |
| Total Costs 2016 | \%EBITDA | $2,692 \%$ |
| Gorss Margin 2016 | \%EBITDA | $\mathbf{1 , 0 8 1 \%}$ |

This project will be analyzed for a time-scope of 6 years, due to the fact of being a technological project where there are constant changes.

On the other side, the costs of the project have needed to be hypothesized. It has been developed following typical consulting projects in PwC. For the investment, it includes three people during 6 months. The Associate and Senior Associate would work full-time, and the Manager would be working part-time. All that information is collected in Table 18.

| Investment (6 month project development) |  |  |  |
| :---: | :---: | :---: | :---: |
| Employees Price | Associate | \%EBITDA/h | 4.9*10^-5\% |
|  | Senior Associate | \%EBITDA/h | 6.6*10^-5\% |
|  | Manager | \%EBITDA/h | 10^-4\% |
| Work hours | Associate | h | 1040 |
|  | Senior Associate | h | 1040 |
|  | Manager | h | 620 |
| Total Investment |  | \%EBITDA | 0.17\% |

In Table 19, it is shown the Yearly O\&M costs during the project. In this part of the project, there will be an Associate working only one day per week, with a lower remuneration.

| Yearly O\&M (1 day per week) |  |  |  |
| :---: | :---: | :---: | :---: |
| Employees <br> Price | Associate | \%EBITDA/h | $3.8^{* 10^{\wedge}-5 \%}$ |
| Work hours | Associate | h | 208 |
| Total O\&M cost |  |  |  |
| Table 19: O\&M cost calculation | \%EBITDA | $0.01 \%$ |  |

The last cost is the Software. It must be considered the license, that for a project of this dimensions, with the characteristics needed for the calculus with cannibalization, there will go up to $150,000 €$ /year.

According to (KPMG, 2018) the Cost of Capital for the Consumer Goods Industry would be, more or less, $7.2 \%$. This value will be the one in the Net Present Value calculation.

As it is going to be shown in the Business Model, it has been developed as a percentage of the EBITDA. The reason why this has been done is to preserve anonymously the client for which the project has been developed.

| Revenues <br> -Increase in profit | \% EBITDA | $0.18 \%$ | $\mathbf{2 0 2 1}$ | $\mathbf{2 0 2 2}$ | $\mathbf{2 0 2 3}$ | $\mathbf{2 0 2 4}$ | $\mathbf{2 0 2 5}$ |
| :--- | :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Total Revenues | \% EBITDA | $0.18 \%$ | $0.55 \%$ | $1.10 \%$ | $1.84 \%$ | $1.84 \%$ | $1.84 \%$ |
| Costs |  |  |  |  |  |  |  |
| -Project O\&M |  |  |  |  |  |  |  |
| -Software | \% EBITDA | $0.01 \%$ | $0.01 \%$ | $0.01 \%$ | $0.01 \%$ | $0.01 \%$ | $0.01 \%$ |
| Total Costs | \% EBITDA | $0.10 \%$ | $0.10 \%$ | $0.10 \%$ | $0.10 \%$ | $0.10 \%$ | $0.10 \%$ |
| EBITDA | \% EBITDA | $0.07 \%$ | $0.44 \%$ | $0.99 \%$ | $1.72 \%$ | $1.72 \%$ | $1.72 \%$ |
| -Depreciation | \% EBITDA | $0.02 \%$ | $0.02 \%$ | $0.02 \%$ | $0.02 \%$ | $0.02 \%$ | $0.02 \%$ |
| EBIT | \% EBITDA | $0.05 \%$ | $0.42 \%$ | $0.97 \%$ | $1.71 \%$ | $1.71 \%$ | $1.71 \%$ |
| -Taxes | \% EBITDA | $0.01 \%$ | $0.08 \%$ | $0.19 \%$ | $0.34 \%$ | $0.34 \%$ | $0.34 \%$ |
| Net Profit | \% EBITDA | $0.04 \%$ | $0.34 \%$ | $0.78 \%$ | $1.37 \%$ | $1.37 \%$ | $1.37 \%$ |
| -CAPEX | \% EBITDA | $0.17 \%$ |  |  |  |  |  |

## Cash Flow <br> \% EBITDA $-0.11 \% \quad 0.35 \% \quad 0.79 \% \quad 1.38 \% ~ 1.38 \% ~ 1.38 \%$

 Table 20: Business Model
## NPV \% EBITDA 3.78\% <br> Payback <br> Years <br> 1.32

Table 21: Business Model Results
As it is summarized from the Business Model, the project has a Net Present Value of $3.78 \%$ of the EBITDA and the investment is recovered during the second year of its implementation.

| NPV Sensitivity <br> Analysis (M€) | -20\% Revenues | Base Case | +20\% Revenues |
| :---: | :---: | :---: | :---: |
| +20\% Investment | $2.87 \%$ | $3.75 \%$ | $4.62 \%$ |
| +20\% Annual Costs | $2.83 \%$ | $3.69 \%$ | $4.57 \%$ |
| Base Case | $2.91 \%$ | $3.78 \%$ | $4.65 \%$ |

Table 22: NPV Sensitivity Analysis

After presenting the Business Model, it has been developed a Sensitivity Analysis to Revenues, Costs and Investments. Both cost sources have only be considered as an increase compared to the Base Case. This is because, in the best case scenario, the project will have the tariffs presented before. In the other side, Revenues have been considered as they can increase or decrease, because the business case has been developed in a stressed situation, and it could happen that final results would be higher than the Base Case scenario. Considering this analysis, in the best case scenario, the expected value of the project would be $4.65 \%$ of EBITDA, and in the worst case, $2.83 \%$ of EBITDA

## Collateral value

Additionally to the financial viability of the project, the implementation of a Dynamic Pricing tool, within a retail company business plan, it will be created some non-tangible extra value.

The project would contribute to create efficiency in the pricing process. This is because there will be no longer needed to think about temporal promotions where some visual information need to be printed at each time. All this information could be electronically distributed, earning a lot of time and money.

Secondly, automation will contribute to reduce personnel costs. It will create qualified jobs, reducing handwork. All the pricing team will be composed probably by data scientists focus on improving the models and testing their capabilities.

Related with the last advantage, there will come a second one related with R\&D. This new pricing method opens the possibility of continuously improving. In the last chapter of this project, some future developments to this project will be presented, within the options they can open to the business of Dynamic Pricing.

## Conclusions

To finish the project, in this last chapter it will be synthesized the results obtained during the project, the applications they have, and the future developments and improvements to contribute to a better efficiency for Dynamic Pricing.

Dynamic Pricing is the capability of changing prices in real-time. It has been implemented for long time in different industries, but it has had its boom in e-commerce, where airlines or Amazon has introduced this method as their pricing strategy.

To carry out the project, some technologies have been used. All the programming has been developed in Python, were many libraries have been really helpful, designing the prediction and the optimization models. It has been used also Power bi as a tool to visualize the enormous quantity of data that has been used and that has been created in the models.

In this project, it has been found some technologies that may complement very well with this innovative pricing strategy. It is very expensive to change prices by hand, and to solve this, there are electronic shelves that are very helpful. There are also smart carts that would help to save the price if prices changes constantly.

This project has developed some models, based on Machine Learning and Optimization, in order to predict the demand, and with that prediction choose the best price at each moment. In the project, it has been considered that clients are not ready yet to see a constant price changing, and for that reason, the project restrict changes at a level day.

The prediction model has been done at different levels, but it has been considered finally at a product level, considering it the most powerful tool and that provides the possibility to change all products prices at the same time.

For the optimization, it has been taken the demand prediction to restrict sales to it. The optimization has taken elasticity curves to know how product sales may react to different historic changes in prices. It is true, that the model is expected to improve with time, because at this moment it has been developed with historic data where there was not Dynamic Pricing, but traditional promotion periods. Therefore, when Dynamic Pricing will be applied, models will learn from themselves, and the data applied in the models will be more accurate.

In the optimization model, it has been tested different possibilities. First, optimize at a category level, where all the products in a same category would change their prices in the same proportion. Nevertheless, it has been improved, and it has been arrived to a product level, and it has been introduced cannibalization, to see how product prices affect the rest of the portfolio.

With all the information collected, it has been developed a Business Plan, where it has been proved the profitability of the project, but also it has been introduced how the project
will be implemented, once the client and the risks the project will face during the implementation accept it.

This Business Plan has indicated a Net Present Value of $3.78 \%$ of EBITDA and a payback of 1.32 years.

## Future developments

As it has been seen all along the project, implementing Dynamic Pricing in Retail has many opportunities, and some of them have been proved during this report.

Nevertheless, many improvements have been observed, but it has not been possible to develop during the project. Some of the improvements have not been developed, because it needed a highly powerful technology that was not available during the duration of it.

The main future step of the project would be to implement the project into a company. To do that, it would be necessary to follow the steps presented in the Business Plan, respecting and taking into account the risks that the project is facing.

However, some new complementarities can be added to the present project and may add more value to it.

The first one would be to develop completely the cannibalization effect, being able to compare each product with many others, automatically. This step has been develop, and the model is done but, due to the technology available in the project, it has not be possible to implement it into the project.

On the other side, there are other aspects, that has not been introduced, because the complexity of them and the available time of the project, but they may be also helpful to obtain better results.

First, it could be introduced a psychology strategy, incrementing the value of the prices when they are ( $\mathrm{x}-1$ ). 99 instead of x .00 . This strategy would provide the possibility of modelling better the human brain and could give better results than the present models.

Other aspect that could be introduced is the stock management combined with the Dynamic Pricing. If the company has well predicted the demand, they will be able to reduce stock to minimum levels, optimizing their Working Capital, and reducing costs.

A third improvement for the project could be to reduce waste. This would be very difficult to implement but due to the data available during the project, it could not be introduced. It would be needed to add to the optimization equations an extra cost for wasting products, that cost would depend on the lifetime of products, and incrementing as the product is nearer to expire. This extra feature would not only contribute to increase profits, but would also help to sustainability, in a world that people abuse from limited


Figure 36: Future developments
As it can be seen, the possibilities of the project are vast. For that reason, one of the objective was to be open to new opportunities that might be found during this project.

The project has been an introduction to Dynamic Pricing, but there are so many possibilities to discover with it, and the retail business can be develop to points that has not even been thought.

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[^0]:    Table 13: Cross-elasticity seasonal curves'slopes and R2 for chicken and pork products

