



COMILLAS
UNIVERSIDAD PONTIFICIA

ICAI

BACHELOR'S DEGREE IN ENGINEERING FOR
INDUSTRIAL TECHNOLOGY (GITI)

END-OF-DEGREE PROJECT

STUDY OF CONTROL STRATEGIES FOR AN
INVERTED PENDULUM USING NUMERICAL
SIMULATION TOOLS.

Author: Miguel Rodríguez González

Director: Enrique Simón Gutiérrez Wing

Madrid

June 2019

AUTHORIZATION FOR DIGITALIZATION, STORAGE AND DISSEMINATION IN THE NETWORK OF END-OF-DEGREE PROJECTS, MASTER PROJECTS, DISSERTATIONS OR BACHILLERATO REPORTS

1. Declaration of authorship and accreditation thereof.

The author Mr. /Ms. Miguel Rodríguez González

HEREBY DECLARES that he/she owns the intellectual property rights regarding the piece of work: STUDY OF CONTROL STRATEGIES FOR AN INVERTED PENDULUM USING NUMERICAL SIMULATION TOOLS that this is an original piece of work, and that he/she holds the status of author, in the sense granted by the Intellectual Property Law.

2. Subject matter and purpose of this assignment.

With the aim of disseminating the aforementioned piece of work as widely as possible using the University's Institutional Repository the author hereby **GRANTS** Comillas Pontifical University, on a royalty-free and non-exclusive basis, for the maximum legal term and with universal scope, the digitization, archiving, reproduction, distribution and public communication rights, including the right to make it electronically available, as described in the Intellectual Property Law. Transformation rights are assigned solely for the purposes described in a) of the following section.

3. Transfer and access terms

Without prejudice to the ownership of the work, which remains with its author, the transfer of rights covered by this license enables:

- a) Transform it in order to adapt it to any technology suitable for sharing it online, as well as including metadata to register the piece of work and include "watermarks" or any other security or protection system.
- b) Reproduce it in any digital medium in order to be included on an electronic database, including the right to reproduce and store the work on servers for the purposes of guaranteeing its security, maintaining it and preserving its format.
- c) Communicate it, by default, by means of an institutional open archive, which has open and cost-free online access.
- d) Any other way of access (restricted, embargoed, closed) shall be explicitly requested and requires that good cause be demonstrated.
- e) Assign these pieces of work a Creative Commons license by default.
- f) Assign these pieces of work a HANDLE (*persistent* URL). by default.

4. Copyright.

The author, as the owner of a piece of work, has the right to:

- a) Have his/her name clearly identified by the University as the author
- b) Communicate and publish the work in the version assigned and in other subsequent versions using any medium.
- c) Request that the work be withdrawn from the repository for just cause.
- d) Receive reliable communication of any claims third parties may make in relation to the work and, in particular, any claims relating to its intellectual property rights.

5. Duties of the author.

The author agrees to:

- a) Guarantee that the commitment undertaken by means of this official document does not infringe any third party rights, regardless of whether they relate to industrial or intellectual property or any other type.

- a) Guarantee that the content of the work does not infringe any third party honor, privacy or image rights.
- b) Take responsibility for all claims and liability, including compensation for any damages, which may be brought against the University by third parties who believe that their rights and interests have been infringed by the assignment.
- c) Take responsibility in the event that the institutions are found guilty of a rights infringement regarding the work subject to assignment.

2. Institutional Repository purposes and functioning.

The work shall be made available to the users so that they may use it in a fair and respectful way with regards to the copyright, according to the allowances given in the relevant legislation, and for study or research purposes, or any other legal use. With this aim in mind, the University undertakes the following duties and reserves the following powers:

- a) The University shall inform the archive users of the permitted uses; however, it shall not guarantee or take any responsibility for any other subsequent ways the work may be used by users, which are non-compliant with the legislation in force. Any subsequent use, beyond private copying, shall require the source to be cited and authorship to be recognized, as well as the guarantee not to use it to gain commercial profit or carry out any derivative works.
- b) The University shall not review the content of the works, which shall at all times fall under the exclusive responsibility of the author and it shall not be obligated to take part in lawsuits on behalf of the author in the event of any infringement of intellectual property rights deriving from storing and archiving the works. The author hereby waives any claim against the University due to any way the users may use the works that is not in keeping with the legislation in force.
- c) The University shall adopt the necessary measures to safeguard the work in the future.
- d) The University reserves the right to withdraw the work, after notifying the author, in sufficiently justified cases, or in the event of third party claims.

Madrid, on the 3rd. of June, 2019.....,

HEREBY ACCEPTS

Signed..........

Reasons for requesting the restricted, closed or embargoed access to the work in the Institution's Repository

I hereby declare that I am the only author of the project report with title:

A STUDY OF CONTROL STRATEGIES FOR AN INVERTED PENDULUM USING NUMERICAL SIMULATION TOOLS

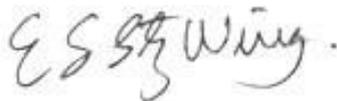
that has been submitted to ICAI School of Engineering of Comillas Pontifical University in the academic year 2018/19. This project is original, has not been submitted before for any other purpose and has not been copied from any other source either fully or partially. All information sources used have been rightly acknowledged.



Fdo.: Miguel Rodríguez González

Date: 28/05/2019

I authorize the submission of this project.



Fdo.: Dr. Enrique Simón Gutiérrez Wing

Date: 28/05/2019



COMILLAS
UNIVERSIDAD PONTIFICIA

ICAI

**BACHELOR'S DEGREE IN ENGINEERING FOR
INDUSTRIAL TECHNOLOGY (GITI)**

END-OF-DEGREE PROJECT

**STUDY OF CONTROL STRATEGIES FOR AN
INVERTED PENDULUM USING NUMERICAL
SIMULATION TOOLS.**

Author: Miguel Rodríguez González

Director: Enrique Simón Gutiérrez Wing

Madrid

June 2019

ESTUDIO DE ESTRATEGIAS DE CONTROL PARA UN PÉNDULO INVERTIDO UTILIZANDO HERRAMIENTAS DE SIMULACIÓN NUMÉRICA.

Autor: Rodríguez González, Miguel.

Director: Gutiérrez Wing, Enrique Simón.

Entidad Colaboradora: Boston University.

RESUMEN DEL PROYECTO

El proyecto a realizar surge de la necesidad y la búsqueda de implementar procesos de aprendizaje que se ajusten más a la realidad y que sirva de utilidad a los futuros ingenieros. Esta nueva metodología busca implementarse en la asignatura Senior Design I, la cual sirve de introducción para la realización del proyecto final de grado.

El objetivo principal de esta iniciativa es introducir el uso de softwares de diseño y desarrollo que permita a los estudiantes observar los comportamientos de distintos sistemas mecánicos y que, a su vez, este les permita realizar cambios con relativa facilidad para estudiar los efectos que estos tienen en el sistema.

Por este motivo, el estudio propuesto se realiza como un ejemplo que podría implementarse en esta futura metodología. Este es el caso del estudio de estrategias de control para un péndulo invertido utilizando herramientas de simulación numérica en el programa de SolidWorks, a través de la herramienta de SolidWorks Motion.

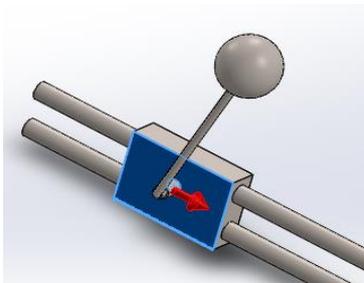
Los objetivos de este estudio, de forma resumida, son los siguientes:

- Realizar distintas simulaciones en un sistema de péndulo invertido con un control de lazo cerrado que mantenga la estabilidad vertical del péndulo.
- Realizar un estudio numérico estableciendo una estrategia de control.
- Realizar un diseño del sistema.
- Proponer y construir dicho sistema.

En primer lugar, se realizó un análisis de las ecuaciones que rigen el comportamiento de un péndulo invertido con el objetivo de comprender el sistema a estudiar. Posteriormente, mediante sencillos modelos de péndulos invertidos, se estuvieron realizando diferentes simulaciones con las que llegar a resultados cercanos a los pretendidos. Tras esto, se llegó a una estrategia de control compuesta por cinco variables:

$$a = IF(ABS(\{Angular Displacement4\}) - 60:$$

$$(-2000 * \{Angular Displacement4\} - \{Angular Velocity3\} * 200 + \{Velocity2\} * 20 + 20 * (\{Linear Displacement2\} + 200)), (-2000 * \{Angular Displacement4\} - \{Angular Velocity3\} * 200 + \{Velocity2\} * 20 + 20 * (\{Linear Displacement2\} + 200)), 3000 * \sin(2 * PI * 0.833 * Time))$$



La fórmula de control rige la aceleración otorgada a la base a través de un motor lineal con el objetivo de mantener el péndulo en la posición vertical deseada.

Cabe destacar que en dicha fórmula podemos encontrar un comando IF, así como cinco variables distintas usadas para controlar el comportamiento del sistema.

El comando IF se utiliza para diferenciar distintos casos que podemos encontrar en el movimiento del péndulo. En primer lugar, se necesita una fórmula que busque mover el péndulo desde la posición inferior a una posición que se acerque a la vertical superior deseada. Esta fórmula es:

$$3000 * \sin(2 * PI * 0.833 * Time)$$

En la que podemos encontrar una constante (3000) que busca potenciar el efecto del resto de la ecuación, la cual usa la frecuencia natural del péndulo para comenzar el movimiento, cuya amplitud se ve aumentada a medida que pasa el tiempo.

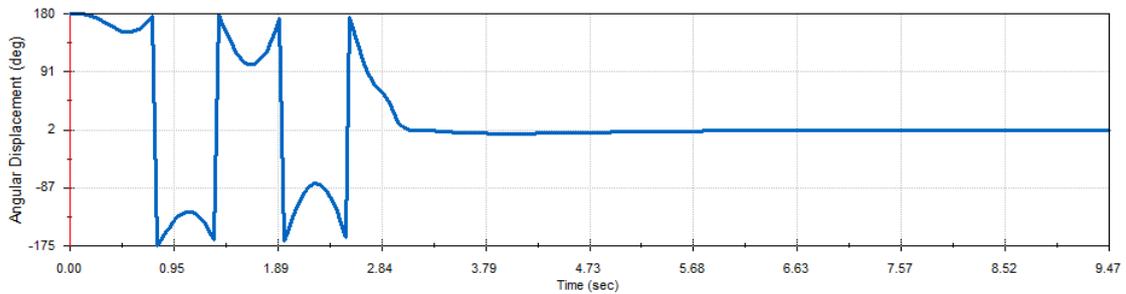
Una vez que hemos conseguido establecer un movimiento en el péndulo, es necesario establecer el límite a partir del cual entrará en vigor la ecuación de control para establecer el péndulo en equilibrio. Esta ecuación es:

$$ABS(\{Angular Displacement4\}) - 60$$

Dicha ecuación limita la primera ecuación hasta que el péndulo alcanza 60° respecto la vertical. En este momento, la ecuación de control diseñada es capaz de corregir el movimiento y alcanzar la posición deseada. Esta ecuación es:

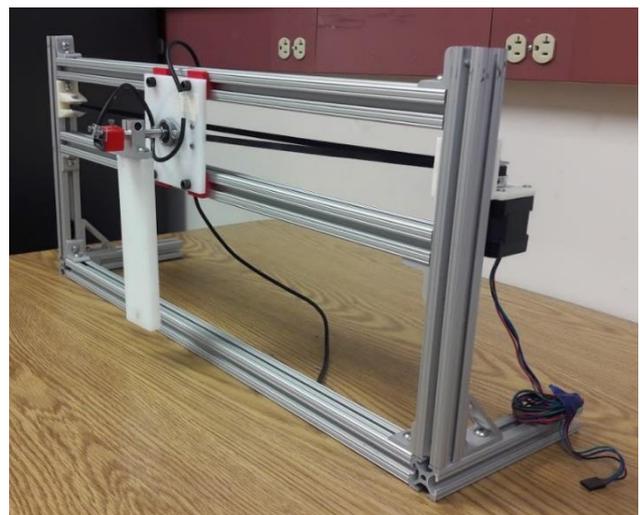
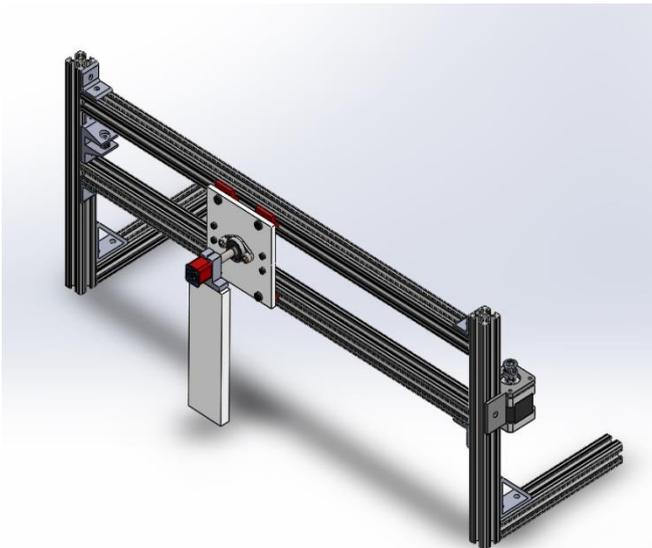
$$(-2000 * \{Angular Displacement4\} - \{Angular Velocity3\} * 200 + \{Velocity2\} * 20 + 20 * (\{Linear Displacement2\} + 200))$$

En esta, se utilizan cinco variables distintas para establecer el control de lazo cerrado. Estas variables son la del desplazamiento lineal y angular, velocidad angular y lineal, y la del tiempo. Manejando estas variables, el control es capaz de establecer los valores finales de los desplazamientos y velocidades a un valor nulo, consiguiendo pues la posición de equilibrio del péndulo invertido. Un ejemplo del resultado se puede observar en la siguiente gráfica, que muestra el desplazamiento angular del péndulo.



Una vez que este objetivo principal se cumplió, la siguiente propuesta era la de construir un modelo al cual poder implementar un sistema de control que garantizase el mismo resultado que el obtenido mediante el software de diseño.

Para ello, se tuvieron en cuenta distintas limitaciones del sistema, siendo el más representativo y crítico el del tamaño del péndulo, el cual limitaba el recorrido que podía realizar la deslizadora. Una vez estudiados estos casos, con el fin de poder tener una mayor libertad de diseño, se decidió ampliar el rango de deslizamiento que podía realizar la base deslizadora mediante el uso de unas estructuras denominadas “frames”. Tras esto, se realizó el diseño y se construyeron distintas partes, tanto con impresión 3D como mecanizándolas manualmente. Otras partes también fueron compradas. Al final, se obtuvo el siguiente diseño, y, por tanto, el siguiente resultado.



Para el movimiento, se instaló un stepper que a través de un cinturón (timing belt) estableciera el movimiento necesario. Para el control de las variables mencionadas, se instaló en el eje de giro del péndulo un acelerómetro, el cual es capaz de medir en los 3 ejes dimensionales.

STUDY OF CONTROL STRATEGIES FOR AN INVERTED PENDULUM USING NUMERICAL SIMULATION TOOLS.

Author: Rodríguez González, Miguel.

Director: Gutiérrez Wing, Enrique Simón.

Collaborating Entity: Boston University.

PROJECT SUMMARY

The project surges of the necessity and the search to implement learning processes that fit more to the reality and that are useful to the future engineers. This new methodology seeks to be implemented in the subject Senior Design I, which serves as an introduction to the completion of the capstone project.

The main objective of this initiative is to introduce the use of design and development software that allows students to observe the behaviors of different mechanical systems and that, in turn, allows them to make changes with relative ease to study the effects they have on the system.

For this reason, the proposed study is carried out as an example that could be implemented in this future methodology. This is the case of the study of control strategies for an inverted pendulum using numerical simulation tools in the SolidWorks software, through the SolidWorks Motion tool.

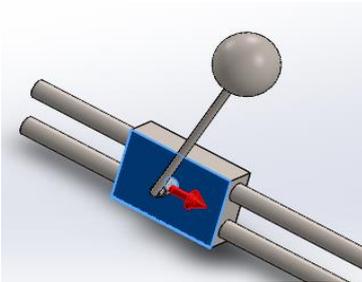
The objectives of this study, summarized, are as follows:

- Perform different simulations in an inverted pendulum system with a closed loop control that maintains the vertical stability of the pendulum.
- To carry out a numerical study establishing a control strategy.
- Perform a system design.
- Proposing and building such system.

First, an analysis of the equations that govern the behavior of an inverted pendulum was carried out in order to understand the system to study. Afterwards, using simple models of inverted pendulums, different simulations were carried out to reach similar results to the desired ones. Following this, a control strategy consisting of five variables was reached:

$$a = IF(ABS(\{Angular Displacement4\}) - 60:$$

$$(-2000 * \{Angular Displacement4\} - \{Angular Velocity3\} * 200 + \{Velocity2\} * 20 + 20 * (\{Linear Displacement2\} + 200)), (-2000 * \{Angular Displacement4\} - \{Angular Velocity3\} * 200 + \{Velocity2\} * 20 + 20 * (\{Linear Displacement2\} + 200)), 3000 * \sin(2 * PI * 0.833 * Time))$$



The control formula governs the acceleration granted to the base through a linear motor with the aim of keeping the pendulum in the desired upright vertical position.

It should be noted that in this formula we can find an IF command, as well as five different variables used to control the behavior of the system.

The IF command is used to differentiate different cases that we can find in the pendulum movement. First, a formula is needed to seek the movement of the pendulum from the lower position to a position that approaches the desired top vertical. This formula is:

$$3000 * \sin(2 * PI * 0.833 * Time)$$

In which we can find a constant (3000) that seeks to enhance the effect of the rest of the equation, which uses the natural frequency of the pendulum to begin the movement, whose amplitude increases as time passes.

Once we have managed to establish a movement in the pendulum, it is necessary to establish the boundary from which the control equation will enter into force to establish the pendulum in equilibrium. This equation is:

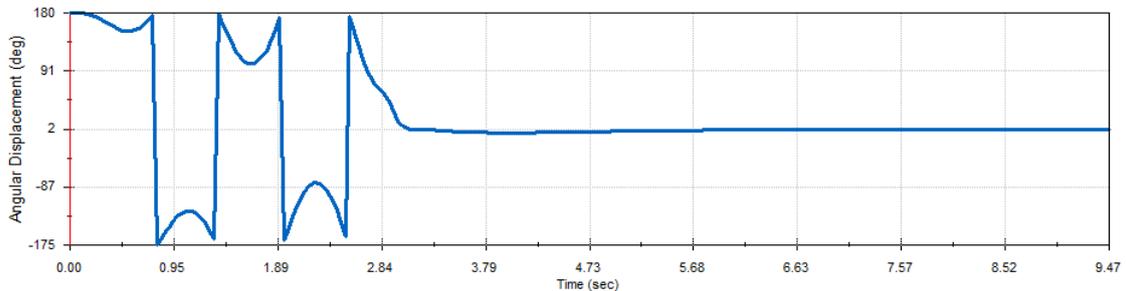
$$ABS(\{Angular Displacement4\}) - 60$$

This equation limits the first equation until the pendulum reaches 60° with respect to the vertical axis. At this time, the designed control equation is able to correct the movement and reach the desired position. This equation is:

$$(-2000 * \{Angular Displacement4\} - \{Angular Velocity3\} * 200 + \{Velocity2\} * 20 + 20 * (\{Linear Displacement2\} + 200))$$

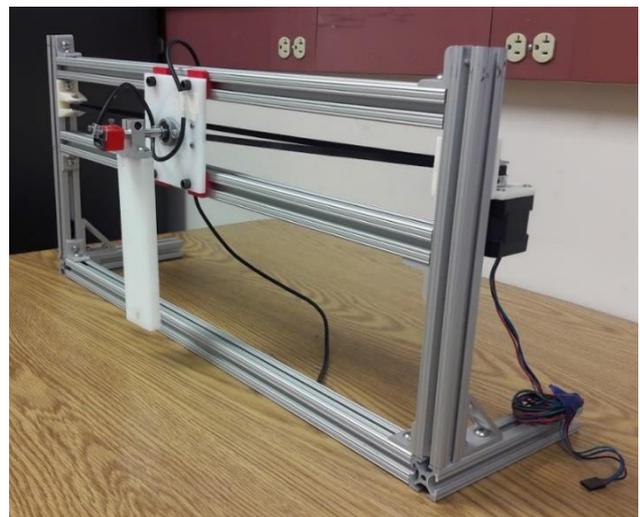
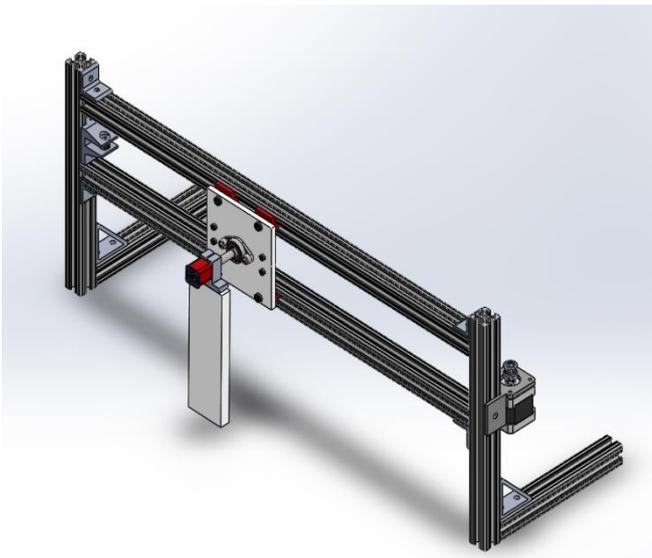
In this, five different variables are used to establish the closed loop control. These variables are linear and angular displacement, angular velocity and line, and time.

Handling these variables, the control is able to set the final values of the displacement and velocities to a null value, thus obtaining the equilibrium position of the inverted pendulum. An example of the result can be seen in the following graph, which shows the angular displacement of the pendulum.



Once this main objective was fulfilled, the following proposal was to build a model to be able to implement a control system that would guarantee the same result as the one obtained through the design software.

To this end, we took into account different limitations of the system, being the most representative and critical of the size of the pendulum, which limited the route that could make the slide. Once these cases were studied, in order to be able to have a greater freedom of design, it was decided to extend the sliding range that could make the sliding base, by using frames as the rails. After this, the design was made and different parts were built, both with 3D printing and machining them manually. Other parts were also purchased. In the end, the following design was obtained, and therefore the following result.



For the movement, a stepper was installed, and through a timing belt, it established the necessary movement. For the control of the mentioned variables, an accelerometer was installed in the rotation axis of the pendulum, which is able to measure in the 3 dimensional axes.

Index

1. Introduction and the approach of project.....	1
1.1 .Motivation.	1
2. Description of technologies.....	3
2.1. Introduction to the object of study.	3
2.1.1. Inverted Pendulum System.	3
2.1.2. Applications.	4
3. Description of the developed model.....	5
3.1. Objective of the project.	5
3.1.1. Methodology of work.	5
3.1.2. Resources to use.....	6
3.2. Equations analysis of an Inverted Pendulum System.	6
3.3. Algorithms developed for the study.	8
3.4. Numerical implantation.	9
4. Results analysis.	21
4.1. Initial case results.....	21
4.2. Analysis of potential final design.	23
4.3. Sensitivity analysis.	24
4.3.1. Effect of pendulum dimensions in the IPC.	24
5. Description of the built model.....	29
5.1. Building process.	29
5.1.1. Phase I.....	29
5.1.2. Phase II.	31
5.1.3. Phase III.	34
5.1.Drawings for the system.....	38
5.2. Economic analysis.	56
6. Conclusions.	60
6.1. Conclusions on the methodology.	60
6.2. Conclusions on the results.	60
6.3. Recommendations for future studies.	61
7. References.	62
8. Appendix.....	64

Index of figures

Figure 1: Inverted Pendulum hinged to a cart (IPC). [5].....	6
Figure 2: SolidWorks Motion Analysis location in the software.	8
Figure 3: Initial design of an IPC.....	9
Figure 4: Motion options	9
Figure 5: Application of the Linear Motor.....	9
Figure 6: Expression options 1	10
Figure 7: Expression options 2	10

Figure 8: Detail of the first approach and variables created	11
Figure 9: Situation 1 of the motion of the system	12
Figure 10: Situation 2 of the motion of the system	13
Figure 11: Angular Displacement of the IPC.....	14
Figure 12: Angular Velocity of the IPC.....	14
Figure 13: Linear Displacement of the IPC	15
Figure 14: Linear Velocity of the IPC	15
Figure 15: Initial position	15
Figure 16: Position at $t = 0.1$ s.....	15
Figure 17: Position at $t = 0.15$ s.....	15
Figure 18: Position at $t = 0.2$ s.....	15
Figure 19: Position at $t = 0.4$ s.....	16
Figure 20: Position at $t = 0.5$ s.....	16
Figure 21: Position at $t = 1$ s.....	16
Figure 22: Position at $t = 0.2$ s.....	16
Figure 23: Worst case scenario to start the movement of the IPC.	17
Figure 24: Linear Displacement of the IPC	18
Figure 25: Linear Velocity of the IPC	18
Figure 26: Angular Displacement of the IPC.....	18
Figure 27: Angular Velocity of the IPC.....	18
Figure 28: Initial position	19
Figure 29: Time at $t = 0.1$ s.....	19
Figure 30: Time at $t = 0.5$ s.....	19
Figure 31: $t = 1$ s. *At the same time it's moving to the right. Problem: the rails end at a certain point	19
Figure 32: Time at $t = 0.1$ s.....	19
Figure 33: Time at $t = 3$ s. At this point, the desired position for the angular displacement is satisfied.....	19
Figure 34: Linear Displacement of the IPC	21
Figure 35: Linear Velocity of the IPC	21
Figure 36: Angular Displacement of the IPC.....	21
Figure 37: Angular Velocity of the IPC.....	21
Figure 38: $t = 1$ s. *At the same time it's moving to the right. Problem: the rails end at a certain point	22
Figure 39: Time at $t = 0.6$ s.....	22
Figure 40: Time at $t = 1$ s.....	22
Figure 41: Time at $t = 1.7$ s.....	22
Figure 42: Time at $t = 2.15$ s.....	22
Figure 43: Time at $t = 2.6$ s.....	22
Figure 44: Time at $t = 3$ s. At this point, the correcting equation takes action.	22
Figure 45: Time at $t = 3.3$ s.....	22
Figure 46: Time at $t = 3.6$ s. At this point, the correcting equation takes action.	22
Figure 47: Time at $t = 4.35$ s.....	22
Figure 48: Time at $t = 6.65$ s. At this point, the correcting equation has stabilized the system.....	22
Figure 49: Final design, ISO view.....	23
Figure 50: Final design, TOP view.....	23
Figure 51: Final design, FRONT view.....	23
Figure 52: Final design, SIDE view.....	23
Figure 53: Final design, BACK view.....	23
Figure 54: Initial size of IP.	24
Figure 55: Displacement of IPC for initial size.	24
Figure 56: Size of 2.5 inches.....	25
Figure 57: Displacement of IPC for 2.5 inches.....	25
Figure 58: Size of 2 inches.....	25
Figure 59: Displacement of IPC for 2 inches.....	26
Figure 60: Size of 2 inches.....	26
Figure 61: Displacement of IPC for 2.5 inches.....	26
Figure 62: T-Slotted Profile Frame [8].....	30
Figure 63: Structure with frames, ISOMETRIC view	30
Figure 64: Structure with frames, TOP view	30
Figure 65: Structure with frames, FRONTAL view	30
Figure 66: Structure with frames, SIDE view	30

Figure 67: Base of the IPC	31
Figure 68: Slider	31
Figure 69: Flange Bearing [9]	31
Figure 70: Initial design with the shape of the.....	32
Figure 71: Second design that would be attached	32
Figure 72: Linear Rail Shaft Guide Support Bracket, 8 mm Diameter	32
Figure 73: Pendulum	32
Figure 74: IPC, ISOMETRIC view	33
Figure 75: IPC, TOP view	33
Figure 76: IPC, FRONTAL view	33
Figure 77: IPC, SIDE view.....	33
Figure 78: IPC after Phase II	34
Figure 79: Stepper [10]	34
Figure 80: Pulley for the stepper [11]	35
Figure 81: Motor holder.....	35
Figure 82: Second pulley	35
Figure 83: Pulley holder	35
Figure 84: Timing belt [12]	36
Figure 85: Accelerometer [13]	36
Figure 86: Accelerometer holder	36
Figure 87: Final Design, ISOMETRIC view	37
Figure 88: Final Design, TOP view	37
Figure 89: Final Design, FRONTAL view	37
Figure 90: Final Design, SIDE view.....	37
Figure 91: IPC, ISOMETRIC view	54
Figure 92: IPC, TOP view	54
Figure 93: IPC, FRONTAL view	54
Figure 94: IPC, SIDE view.....	54

Index of drawings

Drawing 1 - Structure.....	39
Drawing 2 – Base pendulum	40
Drawing 3 – Slider.....	42
Drawing 4 – Belt holder	44
Drawing 5 – Pendulum	46
Drawing 6 – Motor holder	48
Drawing 7 – Pulley holder.....	50
Drawing 8 – Accelerometer holder.....	52
Drawing 9 – Frame profile	65
Drawing 10 – Flange bearing KFL 08	65
Drawing 11 – Linear rail shaft guide support.....	65

Index of tables

Table 1: GANT table of the planned schedule.....	5
Table 2: Symbology and brief description of the variables of the IPC system. [GARC16]	6
Table 3: Building phase description.	29
Table 4: Costs table for the structure.	56
Table 5: Costs table for corner brackets.	56
Table 6: Costs table for crafted parts in Phase II	56
Table 7: Costs table for commercial parts in Phase II	57
Table 8: Costs table for crafted parts in Phase III.....	57
Table 9: Costs table for commercial parts in Phase III	57
Table 10: Costs table of screws.....	58
Table 11: Total costs	58

Index of equations

$x_{cg} = x + l \sin \theta$ Eq. 1.....	6
$y_{cg} = l \cos \theta$ Eq. 2.....	6
$Mx + Fx = u(t)$ Eq. 3.....	6
$I\theta = Fy l \sin \theta - Fx l \cos \theta$ Eq. 4.....	6
$Fx = m x_{cg}$ Eq. 5.....	7
$Fx = mx + ma$ Eq. 6.....	7
$a = (\cos \theta)\theta - (\sin \theta)\theta^2$ Eq. 7.....	7
$\rightarrow Fx = mx + m l (\cos \theta)\theta - m l (\sin \theta)\theta^2$ Eq. 8.....	7
$Fy = m y_{cg} + mg$ Eq. 9.....	7
$Fy = m l b + mg$ Eq. 10.....	7
$a = (-\sin \theta)\theta - (\cos \theta)\theta^2$ Eq. 11.....	7
$\rightarrow Fy = mg - m l (\sin \theta)\theta - m l (\cos \theta)\theta^2$ Eq. 12.....	7
$I\theta = mg - m l (\sin \theta)\theta - m l (\cos \theta)\theta^2 l \sin \theta - [mx + m l (\cos \theta)\theta - m l (\sin \theta)\theta^2] l \cos \theta$ Eq. 13.....	7
$m g l \sin \theta = m l^2 \theta + l \sin \theta - m g x \cos \theta$ Eq. 14.....	7
$M + mx - m l (\sin \theta)\theta^2 + m l \theta \cos \theta = u(t)$ Eq. 15.....	7
$m l m g \cos \theta m l \cos \theta (M + m)\theta x = m g \sin \theta u t + m l (\sin \theta)\theta^2$ Eq. 16.....	7
$m g \theta = m l \theta + m x$ Eq. 17.....	8
$M + mx + m l \theta = u(t)$ Eq. 18.....	8
$x = -m M g \theta + 1 M u(t)$ Eq. 19.....	8
$\theta = M - m M l g \theta - 1 M l u(t)$ Eq. 20.....	8
$a = K1 (\{\text{Linear displacement}\} - x_0) + K2 \text{Angular displacement} + K3 \text{Angular velocity} + K4 \{\text{Linear velocity}\}$ Eq. 21.....	11
- In Figure 9: a due to angular velocity = $-K3 \text{Angular velocity} > 0 < 0$ Eq. 22.....	13
- In Figure 10: a due to angular velocity = $-K3 \text{Angular velocity} < 0 > 0$ Eq. 23.....	13
$a = 20 \text{Linear displacement} + 20 - 2000 \text{Angular displacement} - 200 \text{Angular velocity} + 20 \{\text{Linear velocity}\}$ Eq. 24.....	14
$IF(e1: e2, e3, e4)$ Eq. 25.....	16
$e1 \rightarrow ABS \text{Angular displacement} - 60$ Eq. 26.....	16
$e4 \rightarrow K5 \sin(2\pi\omega n t)$ Eq. 27.....	17
$e4 \rightarrow 3000 * \sin(2 * PI * 0.833 * Time))$ Eq. 28.....	17
$-2000 * \text{Angular Displacement}^4 - \text{Angular Velocity}^3 * 200 + \text{Velocity}^2 * 20 + 20 * \text{Linear Displacement}^2 + 200, -2000 * \text{Angular Displacement}^4 - \text{Angular Velocity}^3 * 200 + \text{Velocity}^2 * 20 + 20 * \text{Linear Displacement}^2 + 200, 3000 * \sin(2 * PI * 0.833 * Time))$ Eq. 29.....	19

1. Introduction and the approach of project.

The project to be carried out is directed by Professor Gutiérrez, from Boston University. It is a project that seeks to be applied in one of the current subjects offered by the institution. This subject is Senior Design I, which offers engineering students the opportunity to undertake a final project, based on continuous work guided by a university professor.

The proposal of the project seeks to increase the contribution of the subject to the training of future engineers. In this way, it seeks to provide support for the Electromechanical control module with the aim of using different tools to learn control concepts, simulate, design and predict the operation of a system with closed loop control.

That is why the suggested project is based on the concept of an inverted pendulum, from which it will show the relative speed with which the behavior of the system can be evaluated taking into account its geometry and distribution of mass, architecture and control system. Models with idealized components are currently used (mass-free bars, point concentrated masses, rigid components, straight trajectories, etc.). For this purpose, a simulation tool will be used to study the effect of real geometries; include effects such as friction, change in trajectory or other irregularities; quickly assess the effects of changes in design; and to corroborate the analytical results obtained for idealized systems and to evaluate the ability of analytical methods to predict the behavior of non-real or unidealized systems.

Finally, as a brief introduction to the concept of inverted pendulum, an inverted pendulum is a pendulum that has its center of mass above its point of support. This is an unstable system, to which a control system can be applied to monitor the angle of the bar and in turn, be able to move the point of support of the pendulum. Also, a certain torque could be applied to the bar that kept the pendulum in a vertical position, implying that there are many ways to achieve the same goal. This is a problem of dynamics and control theory. [1]

1.1. Motivation.

Given the situation described, a change to the current system of analysis is proposed, to convert it to a more practical use through the use of a design software available at the College of Engineering of BU, in order to provide a teaching through numerical simulation of models that serves as a link between theory and practice during teaching. a way to teach students to use their knowledge of CAD, mechanics, and electromechanical systems design is needed.

The main motivation of applying this new form of teaching is that students can apply the theory of real system design; being able to build mechanical systems with closed loop control in a virtual environment; being able to predict their behavior; being able to alter their designs to work properly; allowing them to design with real non-idealized components; and allowing them to quickly evaluate the effects of mechanical changes (geometry, architecture, material) and control (coefficients, control schemes).

In this way, future students will have the possibility to access a more visual software, more real and close to the working world, that allows a deeper and more interactive analysis of the various cases under study. Not only that, but also this new methodology will allow the possibility of presenting new systems very different from the previous ones, also being able to increase the complexity.

2. Description of technologies.

The Senior Subject Design I is a course for mechanical engineers that is based on the teaching of a control module on an electromechanical system. The module consists of three sessions in which the theory and application of control modules are explained. This is made up of different objectives throughout the semester, including planning and executing a project that represents the culmination of the mechanical engineering program. To this end, different themes, works and simulations are presented that allow the student to gain experience in systems and components design, project planning and communication in engineering. [2]

For most of the students, this is the first contact with control systems, to which, added to the large number of students there is per class, it is very difficult to illustrate different control concepts using only physical models.

In order to provide a more practical and visual learning to the student, the course presents an exercise that is based on the analysis of a system and its behavior in different stimuli. Currently, for this type of exercise, the Excel program is used through which a series of data is obtained from the system to be analyzed and its behavior is studied through them.

Due to this type of analysis, the complexity of the systems to be studied is limited by the difficulty that this form of study implies. There is currently no connection between simulation and design. There is no way to check if the calculations are correct. A physical prototype is needed, and at a more fundamental level that incorporates a method for designing physical prototypes and testing or predicting their behavior.

2.1. Introduction to the object of study.

The aim of the study will be to stabilize an inverted pendulum over a carriage (IPC). This concept will be explained in detail, as well as the different parts that took place during the development of the project, from the software simulations carried on to the construction of a prototype.

2.1.1. Inverted Pendulum System.

Trying to balance a broom-stick on your index finger or devices such as the Segway or the hoverboard are some examples of applications of the concept of an inverted pendulum. In the broom-stick example, when trying to balance it, a constant movement of the hand is needed to keep the stick in a vertical position. An inverted pendulum system is based on the same idea, but its movement is limited to one dimension, unlike the movement of the hand.

An Inverted Pendulum (IP) could be analyzed as a body or multi-body system whose center of mass is above its pivot point. It is based on the use of a feedback control system to stabilize an open-loop unstable system. [3] In order to balance the IP, an external force must be applied to keep the system in an equilibrium state.

Several models and ways to balance the inverted pendulum are possible: from using a servo motor or stepper to apply a certain torque to the pendulum, to the use of a carriage that will move to keep the inverted pendulum in a vertical position. For the purpose of this study, the analyzed system will be formed by an inverted pendulum hinged to a moving platform, which resembles to an inverted pendulum over a carriage. In order to abbreviate, the term IPC (Inverted Pendulum on a Carriage) will be used.

2.1.2. Applications.

It is also important to know why the study of an inverted pendulum system is important to engineering. This kind of system is considered to be “one of the fundamental engineering problems” [IOAN17]. The importance of this system arises both from the theoretical and practical applications.

On the first side, the theoretical value of this system comes from its nature as a nonlinear system, meaning that the equations of motion are nonlinear differential equations. This makes it a difficult task to compute an appropriate control to the system. In addition, in some circumstances, this model can be simplified into a linear system around the equilibrium located on the upright vertical position, being able to apply linear system methods. This variety makes it a great example for teaching and studying these two kinds of nonlinear and linear methods.

On the other side, the practical one, the numerous practical applications makes this study even more important. For example, in robotics, where balancing systems are developed based on this inverted pendulum system, as well as in object transport using drones. As another example, in bioengineering, it is the base of the systems that support walking for patients. Not only that but also large scale constructions such as buildings are modeled as inverted pendulums. As a clear example of a direct application of this system is the Segway, a transport device [IOAN17].

3. Description of the developed model.

3.1. Objective of the project.

The objective of the project consists of different components.

- Make different numerical simulations on a multi-body dynamic model of an inverted pendulum with closed-loop control to maintain vertical stability.
- To carry out a numerical study of strategies of control of an inverted pendulum.
- Perform a design of the control system.
- Propose and build a design of the system.

3.1.1. Methodology of work.

In order to carry out the project, the following work methodology was proposed.

- Theoretical study: Analysis and understanding of the equations that govern the behavior of an inverted pendulum.
- Introduction to the software: Making contact with the software to be used (SolidWorks) and initiation with the Module of work Motion Analysis.
- Inverted pendulum design: Design of the mechanical system with the different components that will be made up.
- Control system: Based on the characteristics of the dynamic system, devise a pendulum control system.
- Calibrating: Optimizing and calibrating the created design.
- Prototype: Build a prototype and apply the design control to a real system.
- More real systems: Based on the acquired base, study the effects of different geometries, control schemes and materials.
- Simulations: Applying and calibrating the new module control to the new mechanical system.
- Analysis of results: Study of the behavior of the system from the results obtained.
- Memory Writing.

Planning	January				February				March				April				May			
	W1	W2	W3	W4	W1	W2	W3	W4	W1	W2	W3	W4	W1	W2	W3	W4	W1	W2	W3	W4
Theoretical study																				
Introduction to software																				
Inverted pendulum design																				
Control system																				
Calibrating																				
Prototype																				
Simulations																				
Analysis of results																				
Memory writing																				

Table 1: GANT table of the planned schedule

3.1.2. Resources to use.

For the development of the project, the following resources will be available:

- SolidWorks design software.
- Construction workshop: Epic.
- 3D printers to perform parts of the system.
- Materials of control for a linear system.

3.2. Equations analysis of an Inverted Pendulum System.

The first step of the project was to understand the concept of an inverted pendulum and to be familiar with the equations that govern its behavior. As it was mentioned before, this study will be focused on an inverted pendulum hinged to a moving platform (IPC), which can move the pendulum in the same plane as the carriage. See *Figure 1*.

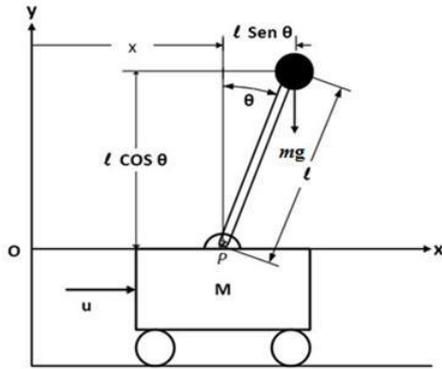


Figure 1: Inverted Pendulum hinged to a cart (IPC). [GARCl6]

Symbology	Description
M	Mas of the car.
m	Mass of the center of gravity.
l	Bar length.
I	Inertia of the pendulum.
u	Force applied to the car.
x	Car position.
θ	Pendulum angle from the vertical top.
g	Gravitational constant.
P	Pivot point.

Table 2: Symbology and brief description of the variables of the IPC system. [GARCl6]

This section will explain the result of applying of the laws of Newtonian dynamics to the proposed model.

The equations for the coordinate system are:

$$x_{cg} = x + l \sin \theta \quad \text{Eq. 1}$$

$$y_{cg} = l \cos \theta \quad \text{Eq. 2}$$

For the equation related to the movement of the system in the horizontal axis:

$$M\ddot{x} + F_x = u(t) \quad \text{Eq. 3}$$

Where F_x represents the movement in the horizontal axis and $u(t)$ represents the input force.

For the equation related to the rotational movement referred to the center of gravity C_{cg} :

$$I\ddot{\theta} = F_y l \sin \theta - F_x l \cos \theta \quad \text{Eq. 4}$$

Where I is the moment of inertia of the system referred to C_{cg} , F_x the movement in the horizontal axis and F_y the movement in the vertical axis.

The movement F_x is:

$$F_x = m x_{cg} \quad \text{Eq. 5}$$

Combining Eq. 1 and Eq. 5,

$$F_x = m\ddot{x} + m\ddot{a} \quad \text{Eq. 6}$$

Where $\ddot{a} = \frac{d^2}{dt^2} \sin \theta$, which leads to

$$a = (\cos \theta) \ddot{\theta} - (\sin \theta) \dot{\theta}^2 \quad \text{Eq. 7}$$

$$\rightarrow F_x = m\ddot{x} + m l (\cos \theta) \ddot{\theta} - m l (\sin \theta) \dot{\theta}^2 \quad \text{Eq. 8}$$

The movement F_y :

$$F_y = m\ddot{y}_{cg} + mg \quad \text{Eq. 9}$$

Combining Eq. 2 and Eq. 9,

$$F_y = m l \ddot{b} + mg \quad \text{Eq. 10}$$

Where $\ddot{b} = \frac{d^2}{dt^2} \cos \theta$, which leads to:

$$a = (-\sin \theta) \ddot{\theta} - (\cos \theta) \dot{\theta}^2 \quad \text{Eq. 11}$$

$$\rightarrow F_y = mg - m l (\sin \theta) \ddot{\theta} - m l (\cos \theta) \dot{\theta}^2 \quad \text{Eq. 12}$$

Combining Eq. 4, Eq. 8 and Eq. 12, we can represent the rotational movement of the pendulum:

$$I\ddot{\theta} = [mg - m l (\sin \theta) \ddot{\theta} - m l (\cos \theta) \dot{\theta}^2] l \sin \theta - [m\ddot{x} + m l (\cos \theta) \ddot{\theta} - m l (\sin \theta) \dot{\theta}^2] l \cos \theta \quad \text{Eq. 13}$$

As the mass m is placed on the end of the pendulum, the equation can be simplified by assuming $I\ddot{\theta} = 0$, leading to:

$$mgl \sin \theta = m l^2 \ddot{\theta} + l \sin \theta - mg\ddot{x} \cos \theta \quad \text{Eq. 14}$$

The Eq. 14 represents the rotational movement of the pendulum. In order to get the translational movement of the IPC, Eq. 3 and Eq. 8 will be combined:

$$(M + m)\ddot{x} - m l (\sin \theta) \dot{\theta}^2 + m l \ddot{\theta} \cos \theta = u(t) \quad \text{Eq. 15}$$

The movement of the system has two degrees of freedom. The Eq. 16 represents Eq. 14 and Eq. 15 in matrix form:

$$\begin{bmatrix} ml & mg \cos \theta \\ ml \cos \theta & (M + m) \end{bmatrix} \begin{bmatrix} \ddot{\theta} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} mg \sin \theta \\ u(t) + m l (\sin \theta) \dot{\theta}^2 \end{bmatrix} \quad \text{Eq. 16}$$

The main purpose of the control loop is to keep the pendulum in a vertical position where, assuming θ and $\dot{\theta}$ to be small, we can simplify by $\sin \theta = \theta$ and $\cos \theta = 1$, and neglecting $\theta \dot{\theta}^2$ from Eq. 15, the next equations were obtained:

$$mg\theta = m l \ddot{\theta} + m \ddot{x} \quad \text{Eq. 17}$$

$$(M + m)\ddot{x} + m l \ddot{\theta} = u(t) \quad \text{Eq. 18}$$

Finally, we get \ddot{x} and $\ddot{\theta}$:

$$\ddot{x} = -\frac{m}{M} g\theta + \frac{1}{M} u(t) \quad \text{Eq. 19}$$

$$\ddot{\theta} = \frac{M-m}{Ml} g\theta - \frac{1}{Ml} u(t) \quad \text{Eq. 20}$$

3.3. Algorithms developed for the study.

The well-known design software used for the purpose of this study was SolidWorks. The software was not only used because of the wide variety of tools and functions that it offers, but also because of the Motion Analysis that it is provided with, which is very convenient for the purpose of this project.



Figure 2: SolidWorks Motion Analysis location in the software.

This function is an Add-In that we can find in the software, shown in Figure 2. This is a way created to accurately simulate and analyze the motion of an assembly, in this case, the IPC, while incorporating different effects of the Motion Study elements, which includes gravity, different forces, springs, dampers and friction. Consequently, motion constraints, material properties, mass and component contact can be included in the SOLIDWORKS Motion kinematic solver calculations. [3].

3.4. Numerical implantation.

Once the understanding of the software and the different applications of the mentioned function, the next step followed was to create a first design with which some ideas and feedback could be obtained as a start.

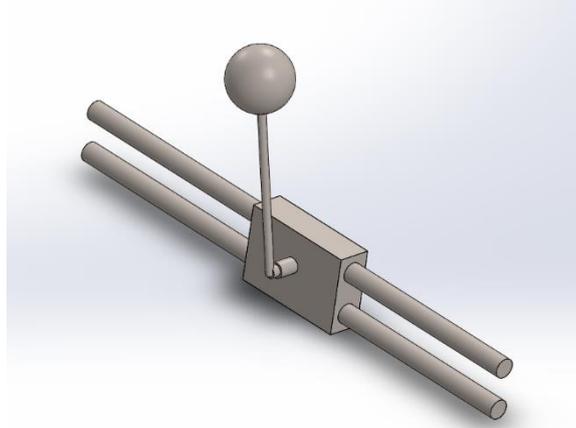


Figure 3: Initial design of an IPC.

This first model shown in *Figure 3* was created to test the different parameters and forces that could be applied to an IPC, at the same time as reaching a better understanding on how the system behaves.

The first approach to achieve the desired result was to simulate the movement of the base of the inverted pendulum. In order to do this, the tool “Linear Motor” was chose, being applied to the base in the X direction, as it can be seen in *Figure 4*.

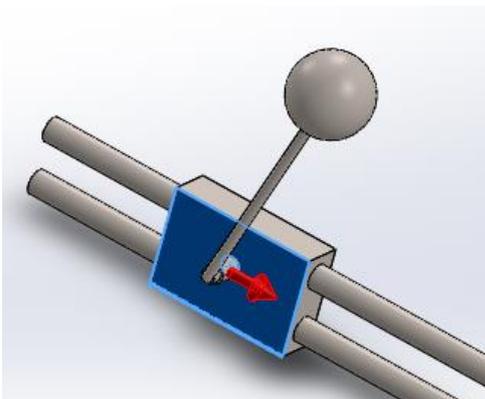


Figure 5: Application of the Linear Motor

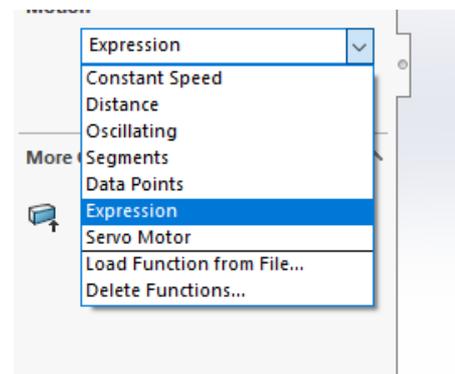


Figure 4: Motion options

Once chosen, the next step is to select the correct kind of motion. As the movement of the base would be related to the movement of the pendulum, the trajectory of this base would move as a response to different variables of the system. Therefore, the option “Expression”, shown in *Figure 5*, is the motion that would suit the best the desired movement.

This section includes a wide variety of options and mathematical expressions that could be applied. First of all, the value (y) to be applied in the linear motor. In our case, acceleration was chosen to control the movement of the system, as it can be seen in *Figure 6*. In terms of the expression itself, there are several functions that can be applied, as it can be seen in the section “Mathematical Functions” in *Figure 7*. Not only that, but also, some variables and constants such as time or PI can be applied. However, the most useful section for this project was “Motion Study Results”, where different variables measured in the system itself could be created, such as Linear and Angular Displacement, Angular Velocity, and Linear Acceleration.

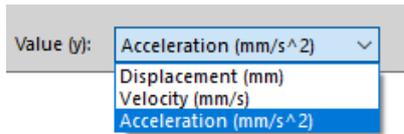


Figure 6: Expression options 1

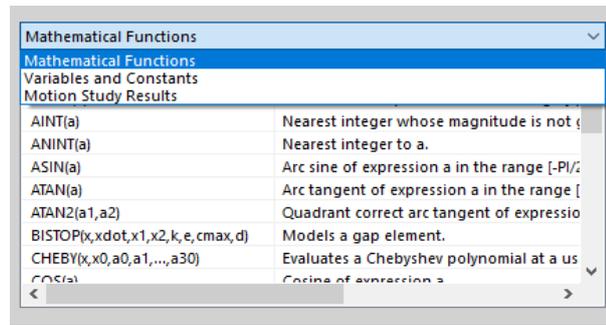


Figure 7: Expression options 2

The first part of the analysis and motion study was to introduce a function that could keep the inverted pendulum in a vertical position from a position close to this point. See *Figure 8* for more details. For that, after some iterations and some tryouts, the testing equation will be based on four main variables:

- Linear displacement: with the purpose that at the end, the variable has the value of the final desired position.
- Angular displacement: with this variable, the main goal is to reach the zero value, so the pendulum stays in the upright vertical position.
- Angular velocity: aiming to reduce this velocity to zero, or at least, to the smallest value possible to keep the mentioned position in equilibrium.
- Linear velocity: once the inverted pendulum is in the upright vertical position, the carriage should stop so the whole system rests in this new point of equilibrium.

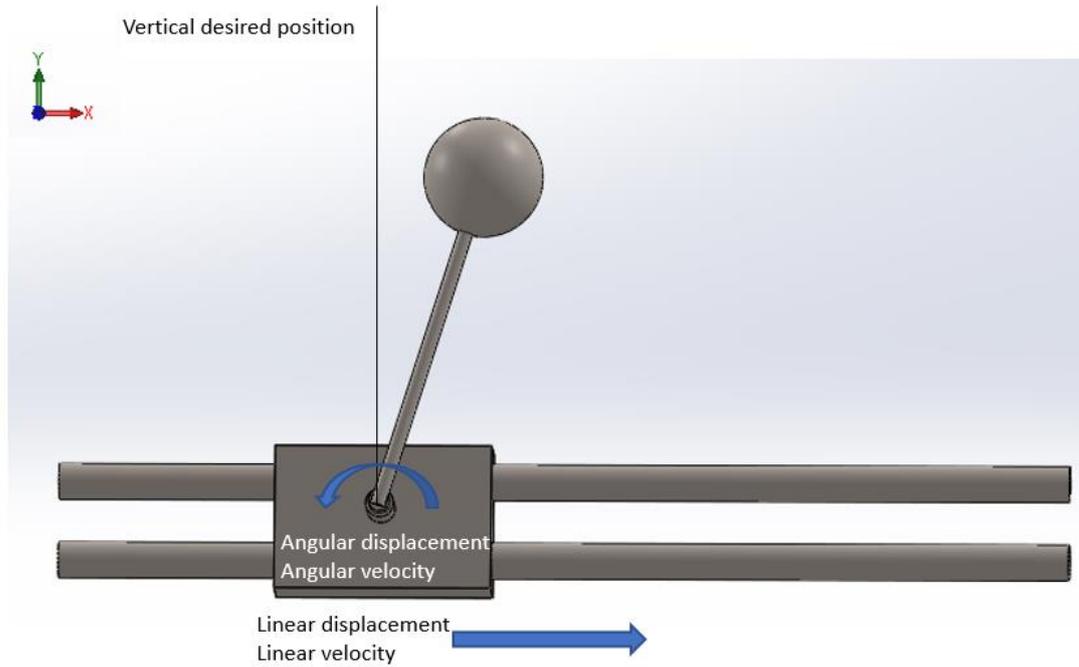


Figure 8: Detail of the first approach and variables created

The proposed expression would follow the same idea as a PID control, following a formula as follows:

$$a = K_1 (\{\text{Linear displacement}\} - x_0) + K_2 \{\text{Angular displacement}\} + K_3 \{\text{Angular velocity}\} + K_4 \{\text{Linear velocity}\} \quad \text{Eq. 21}$$

Where a is the acceleration applied by the linear motor, measured in mm/s^2 ; $K_1 [s^{-2}]$, $K_2 [mm/s^2]$, $K_3 [mm/s]$, $K_4 [s^{-1}]$ are the constants applied to the different variables with which the IPC will be controlled; and x_0 is the desired final position of the pendulum, which depends on the reference system in the model, measured in mm .

The signs correspond, not only to the coordinate system, but also to the movement that the carriage needs to follow in response to the movement of the inverted pendulum. See *Figure 9* and *Figure 10*.

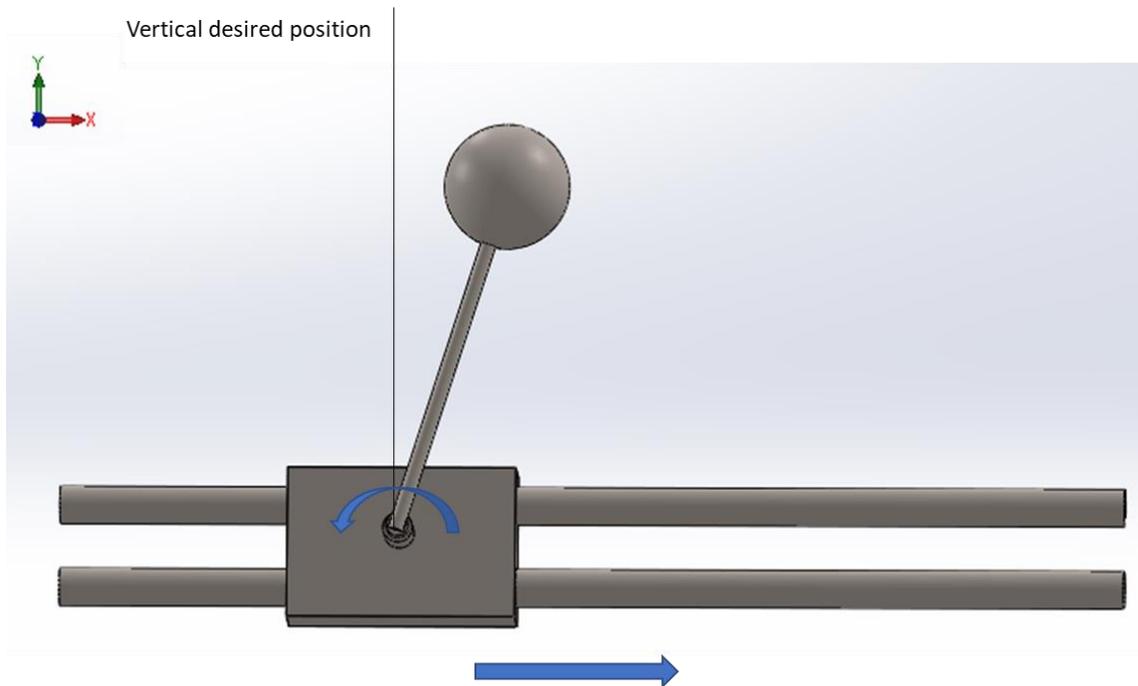


Figure 9: Situation 1 of the motion of the system

In this case, $\{\text{Angular displacement}\} < 0$ and $\{\text{Angular velocity}\} > 0$ when the pendulum is getting closer to the desired position. The expected movement of the base will be to move to the right, in order to help the system reach that position. As we can see, the *Angular displacement* and *Linear velocity* will have the opposite sign.

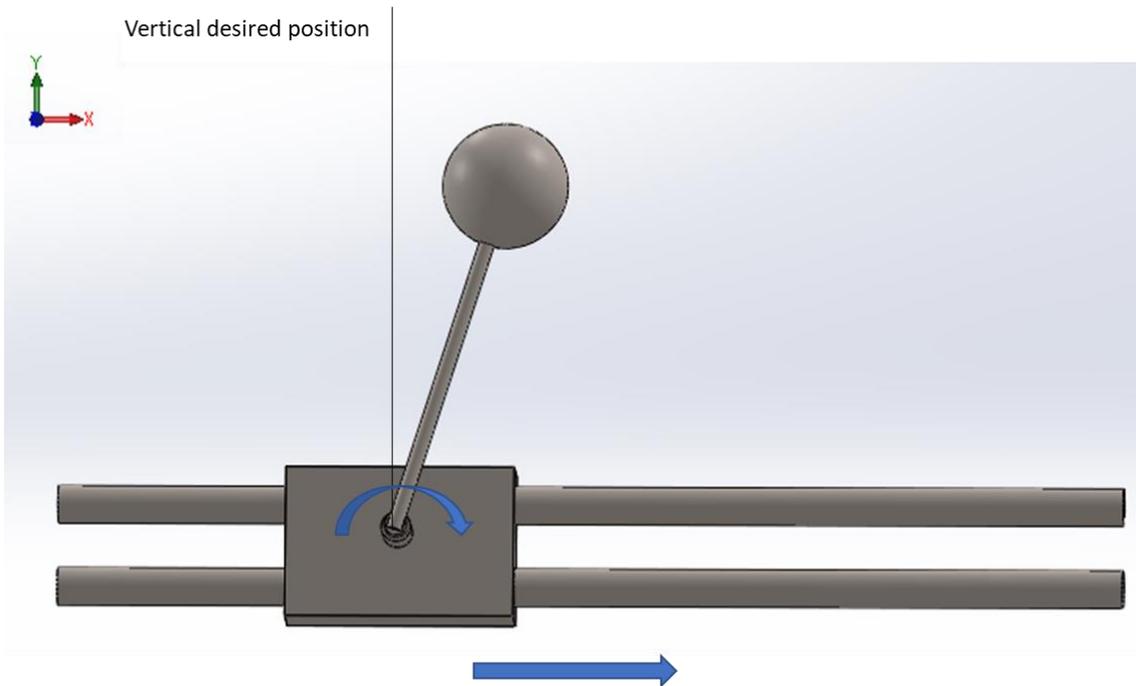


Figure 10: Situation 2 of the motion of the system

In this case, $\{\text{Angular displacement}\} < 0$ and $\{\text{Angular velocity}\} < 0$ when the pendulum rotates away from the desired position. The expected movement of the base will be to move to the right, in order to help the system go back to that position. As we can see, the *Angular displacement* and *Linear velocity* will have the opposite sign.

What we can conclude from these two situations is that the variables *Angular displacement* and *Linear velocity* have always opposite signs.

As for the sign of the constant of *Angular velocity*, it is directly related to the acceleration needed to be applied. In *Figure 9*, the angular velocity is greater than zero, which means that it is already moving to the desired position, meaning that the acceleration applied to the carriage will not be as large as in *Figure 10*, where the angular velocity is less than zero and therefore, rotating away from the upright vertical position. For this purpose, K_3 in *Eq. 21* will need to be negative. In other words, following the situation in *Figure 10*, when the angular velocity is negative, the acceleration due to this variable will be positive, being the overall acceleration applied greater than in the situation in *Figure 9*, as it can be seen in *Eq. 22* and *Eq. 23*. Physically, this makes sense, as the expected acceleration of the carriage is higher in *Figure 10* than in *Figure 9*.

- In *Figure 9*: a (due to angular velocity) = $-|K_3| \{\text{Angular velocity}(> 0)\} < 0$ *Eq. 22*

- In *Figure 10*: a (due to angular velocity) = $-|K_3| \{\text{Angular velocity}(< 0)\} > 0$ *Eq. 23*

In the opposite way it will happen for the situations in which the inverted pendulum is on the left side of the line shown in *Figure 8*, but the motion will still aim to balance the pendulum to the upright vertical position.

For the variable *Linear displacement*, when the value of *Linear velocity* is positive, the *Linear displacement* must have the same sign, as it will move in the same direction, providing more acceleration to the carriage and obtaining a quicker response.

After doing some tests, the following conclusions were obtained:

- The variables *Angular displacement* and *Angular velocity* were the ones that would have the most control over the system, being *Angular displacement* on top, as the main purpose of the whole system is to set the angle to zero, as detailed in *Figure 8*.
- The larger these two variables were, the quicker the system would respond and would try to stabilize the IPC.
- Due to the differences in the constants, the system would establish the equilibrium position of the pendulum first, and afterwards, it would reduce the velocity of the carriage to zero at the same time it moves the platform to the desired x_0 position.

With these considerations, the final expression that would be able to keep the desired equilibrium position of the system from the initial position showed in *Figure 8* was the following:

$$a = 20 (\{\text{Linear displacement}\} + 20) - 2000 \{\text{Angular displacement}\} - 200 \{\text{Angular velocity}\} + 20 \{\text{Linear velocity}\} \quad \text{Eq. 24}$$

To show how it works, the next example represents the motion of the system when the initial angle is -30° :

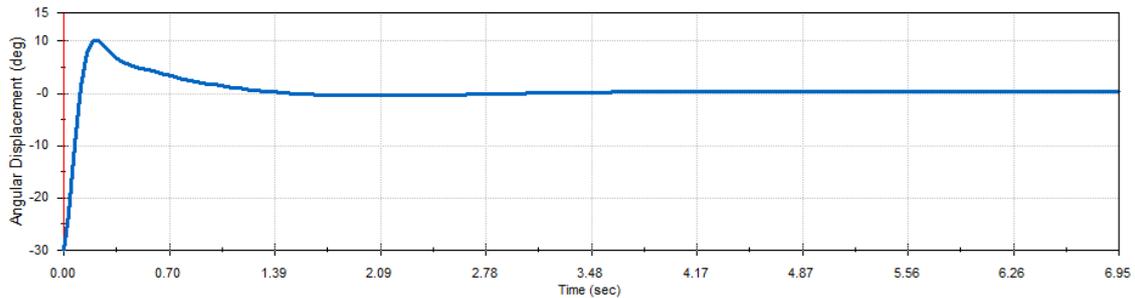


Figure 11: Angular Displacement of the IPC

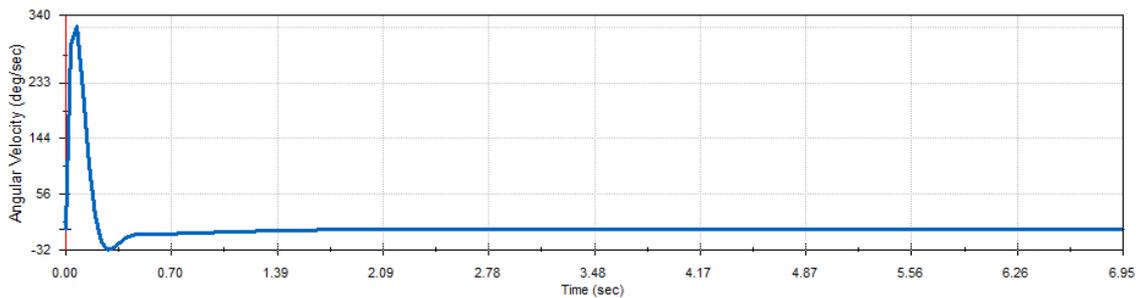


Figure 12: Angular Velocity of the IPC

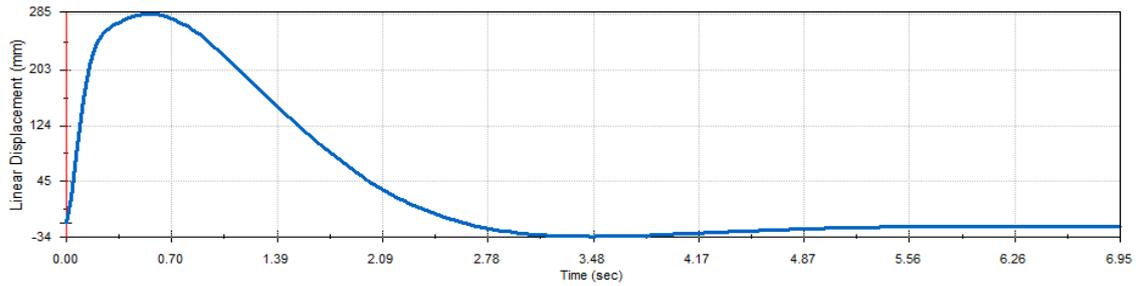


Figure 13: Linear Displacement of the IPC

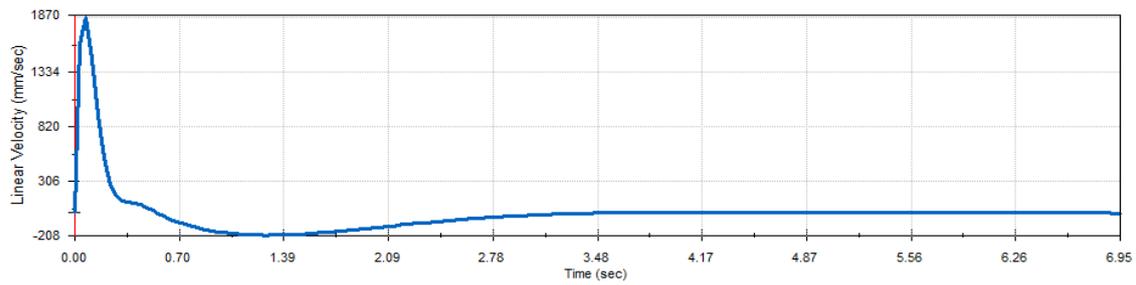


Figure 14: Linear Velocity of the IPC

As it can be appreciated, the control takes not more than 4 seconds to balance the inverted pendulum and return it to the desired position x_0 . This case is relatively fast due to the small initial variation in the angular displacement.

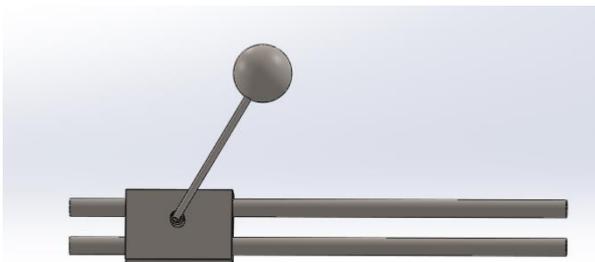


Figure 15: Initial position

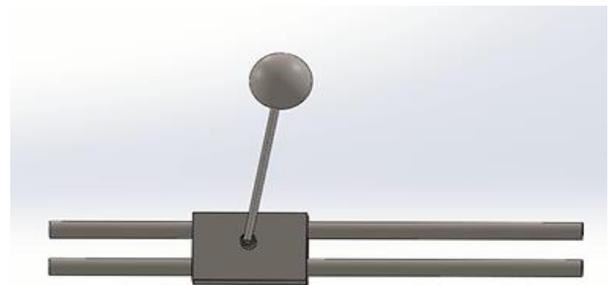


Figure 16: Position at $t = 0.1$ s

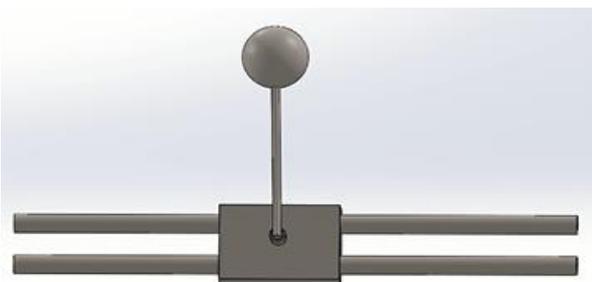


Figure 17: Position at $t = 0.15$ s

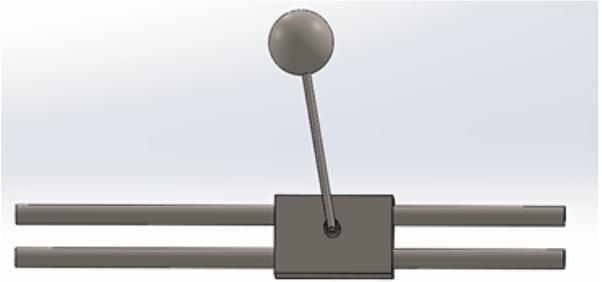


Figure 18: Position at $t = 0.2$ s

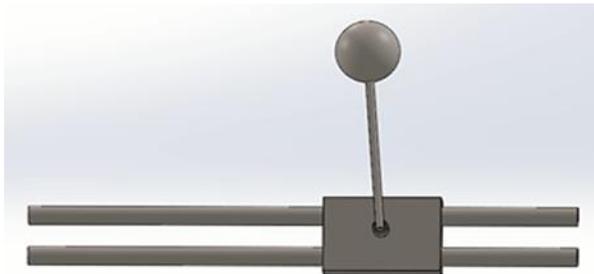


Figure 19: Position at $t = 0.4$ s

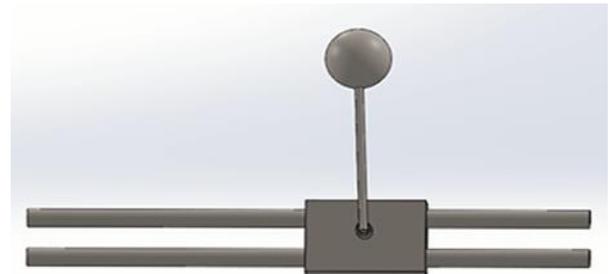


Figure 20: Position at $t = 0.5$ s

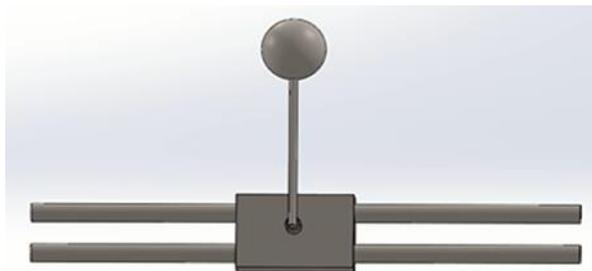


Figure 21: Position at $t = 1$ s

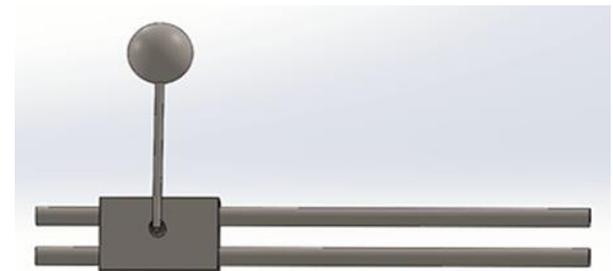


Figure 22: Position at $t = 0.2$ s

However, this formula would be useful only for those cases where the *Angular displacement* were relatively small. In the simulation, no absolute angles larger than 65° would be able to be corrected with the formula, due to the gigantic acceleration required by the linear motor, making SolidWorks to fail in the simulation for these cases. Therefore, another process was required.

After some research, SolidWorks Motion Study offers the possibility to apply several programming commands to the expressions of motion. One of these was the command *IF*, which is extremely helpful in this case [4]. This command works as shown below:

$$IF(e1: e2, e3, e4)$$

Eq. 25

This command analyzes the expression 1 ($e1$) and proceeds in the following way:

- If the value for $e1 < 0$, it returns $e2$.
- If the value for $e1 = 0$, it returns $e3$.
- If the value for $e1 > 0$, it returns $e4$.

The expression 1 would need to be such an expression that would identify the initial angle and estimate if the acceleration required was possible or not. For this, the expression chosen for $e1$ was:

$$e1 \rightarrow ABS(\{\text{Angular displacement}\}) - 60$$

Eq. 26

Where $ABS(expression)$ is a command to get the absolute value of the expression; and 60 is the angle chosen to be the limit where the Eq. 24 can be applied. A margin of 5° was chosen to prevent possible failures in the simulation. Therefore, the expected expression for e2 and e3 will be the one shown in Eq.24, however, the expression for e4 was still unknown.

When designing and choosing the formula expression for e4, the main purpose of the project was taken into consideration. To create a successful system that balances an IPC, the worst case scenario is when the pendulum is in equilibrium in the position showed in Figure 11.

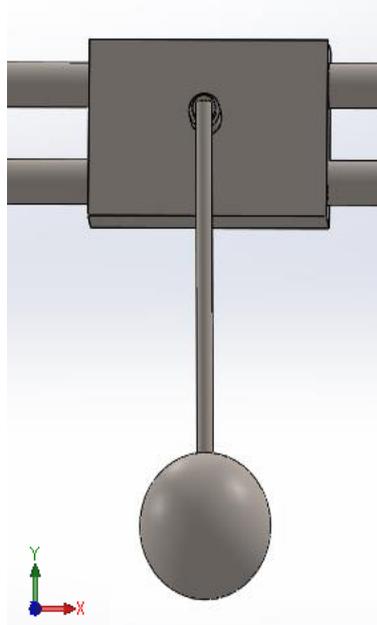


Figure 23: Worst case scenario to start the movement of the IPC.

From this position, the best and smoothest way to start the movement of the pendulum consists on starting the motion slowly, increasing the amplitude of the pendulum (which will be controlled by the amplitude of the displacement of the carriage) and the velocities, both linear and angular velocities. For this purpose, the expression will have the following structure:

$$e4 \rightarrow K_5 \sin(2\pi\omega_n t) \quad \text{Eq. 27}$$

Where $K_5 [mm/s^2]$ is the constant that will represent the magnitude and velocity with which the IPC will move in its first steps; $\omega_n [rad/s]$ is the natural frequency of the system; $t[s]$ is the time.

For the system created in first place, the ω_n was close to $0.833 rad/s$. The value for K_5 was choosing after doing some testing, selecting the one that fitted best the design. The final expression applied in the motion analysis was:

$$e4 \rightarrow 3000 * \sin(2 * PI * 0.833 * Time)) \quad \text{Eq. 28}$$

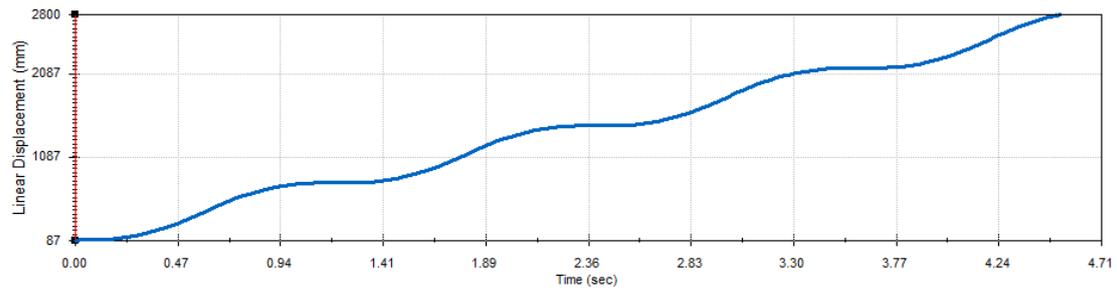


Figure 24: Linear Displacement of the IPC

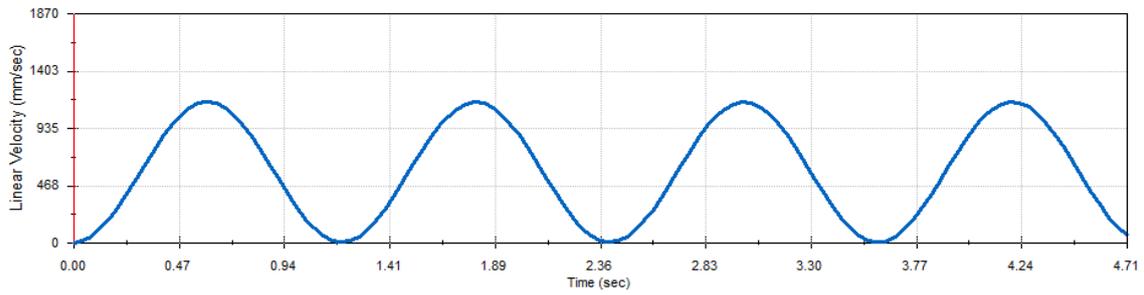


Figure 25: Linear Velocity of the IPC

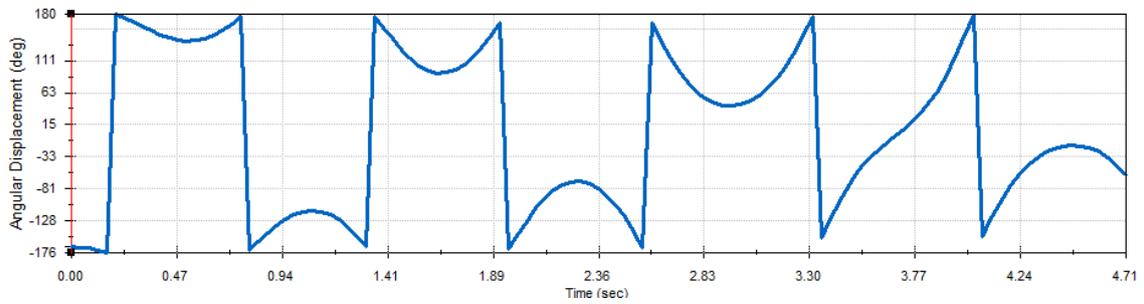


Figure 26: Angular Displacement of the IPC

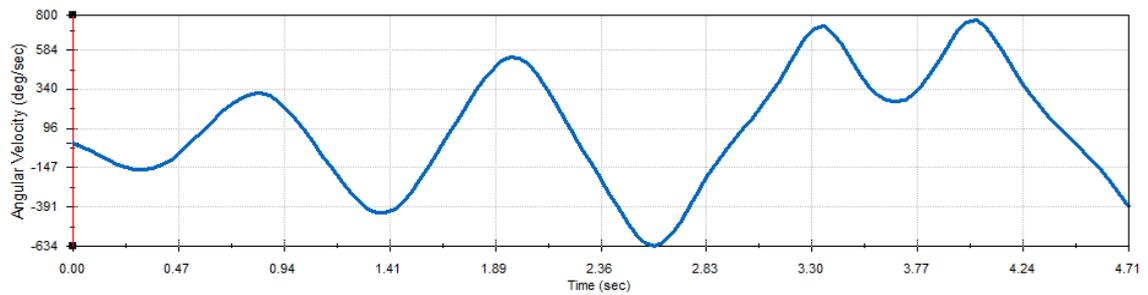


Figure 27: Angular Velocity of the IPC

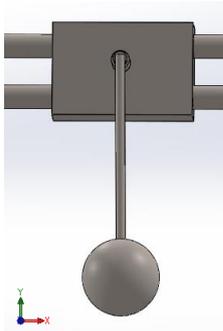


Figure 28: Initial position

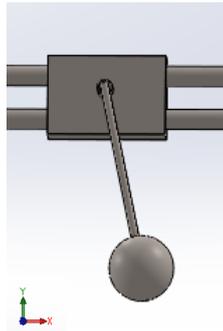


Figure 29: Time at t = 0.1 s.

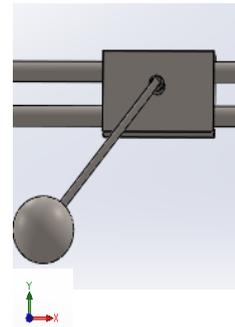


Figure 30: Time at t = 0.5 s.

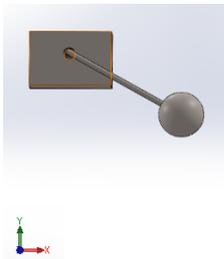


Figure 31: t = 1s. *At the same time it's moving to the right. Problem: the rails end at a certain point

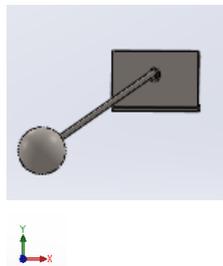


Figure 32: Time at t = 0.1 s.

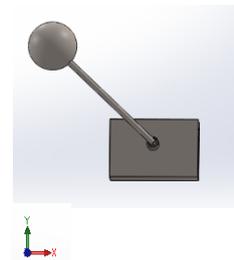


Figure 33: Time at t = 3 s. At this point, the desired position for the angular displacement is satisfied.

At the last point, the pendulum has surpassed the limit of 60° mentioned before, where the previous formula in Eq. 24 would take place and would balance the IPC. With this, the final formula was completed, having the following expression:

$$a = IF(ABS(\{Angular\ Displacement4\}) - 60:$$

$$\begin{aligned} & (-2000 * \{Angular\ Displacement4\} - \{Angular\ Velocity3\} * 200 + \{Velocity2\} * 20 + \\ & 20 * (\{Linear\ Displacement2\} + 200)), (-2000 * \{Angular\ Displacement4\} - \\ & \{Angular\ Velocity3\} * 200 + \{Velocity2\} * 20 + 20 * (\{Linear\ Displacement2\} + \\ & 200)), 3000 * \sin(2 * PI * 0.833 * Time)) \end{aligned} \quad Eq. 29$$

4. Results analysis.

4.1. Initial case results.

Applying this complete equation, the results were the following:

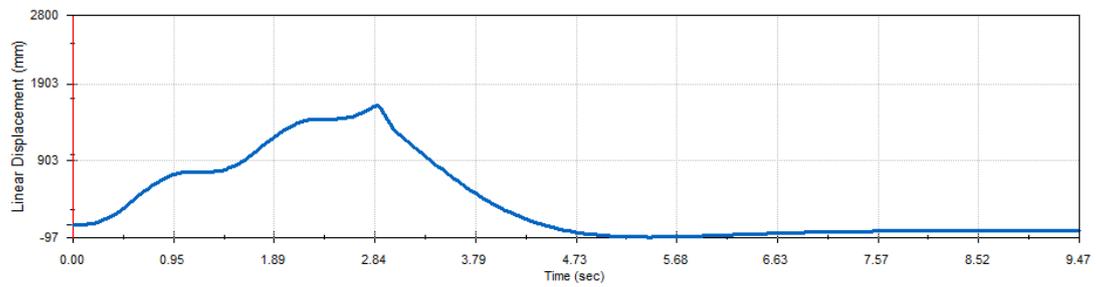


Figure 34: Linear Displacement of the IPC

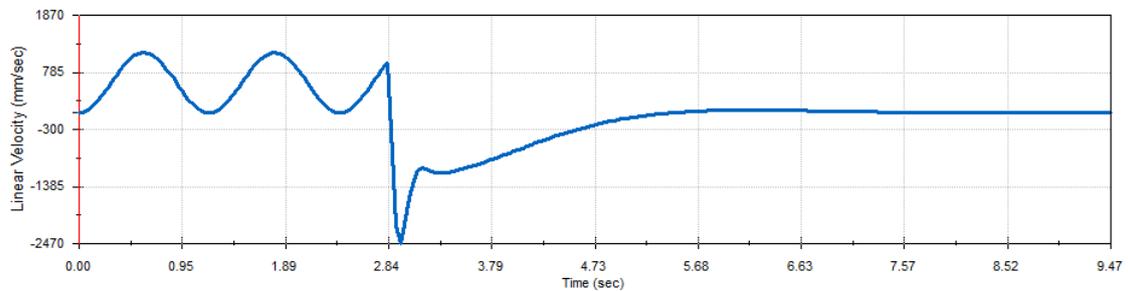


Figure 35: Linear Velocity of the IPC

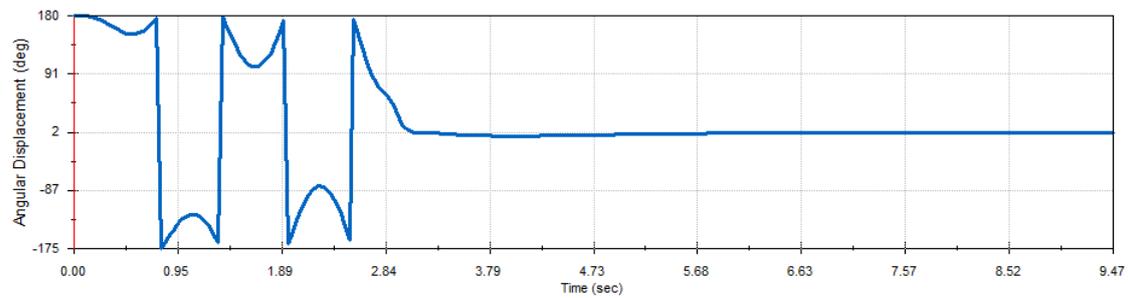


Figure 36: Angular Displacement of the IPC

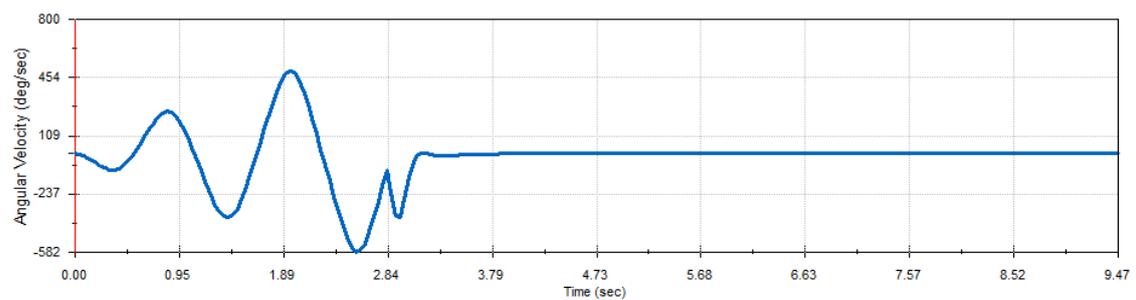


Figure 37: Angular Velocity of the IPC



Figure 38: $t = 1s$. *At the same time it's moving to the right. Problem: the rails end at a certain point

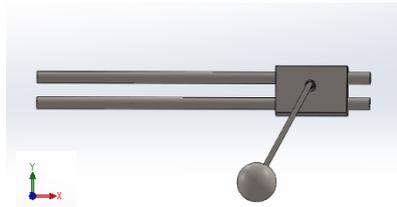


Figure 39: Time at $t = 0.6 s$.

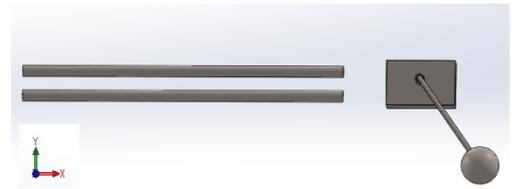


Figure 40: Time at $t = 1 s$.



Figure 41: Time at $t = 1.7 s$

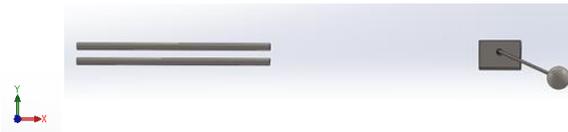


Figure 42: Time at $t = 2.15 s$.

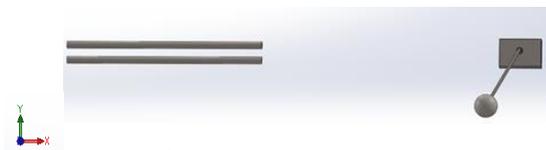


Figure 43: Time at $t = 2.6 s$.



Figure 44: Time at $t = 3 s$. At this point, the correcting equation takes action.



Figure 45: Time at $t = 3.3 s$.

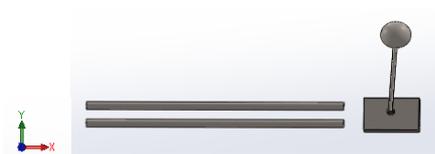


Figure 46: Time at $t = 3.6 s$. At this point, the correcting equation takes action.

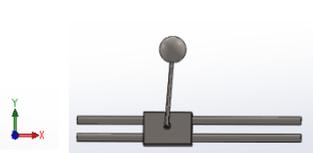


Figure 47: Time at $t = 4.35 s$.

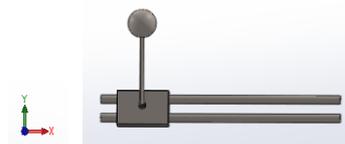


Figure 48: Time at $t = 6.65 s$. At this point, the correcting equation has stabilized the system.

As it can be seen in the *Figure 36*, the pendulum reached the angle 0 and stays in that position, meaning that the equilibrium point has been reached. Once proved this expression (Eq.29) works, the next step was to create and design the real system for the IPC.

4.2. Analysis of potential final design.

Once the testing was complete, the next step was to create a definitive and realistic design of what the system would look like. For that purpose, and after analyzing the available materials and parts that could be used in one of these systems, the next design was created:

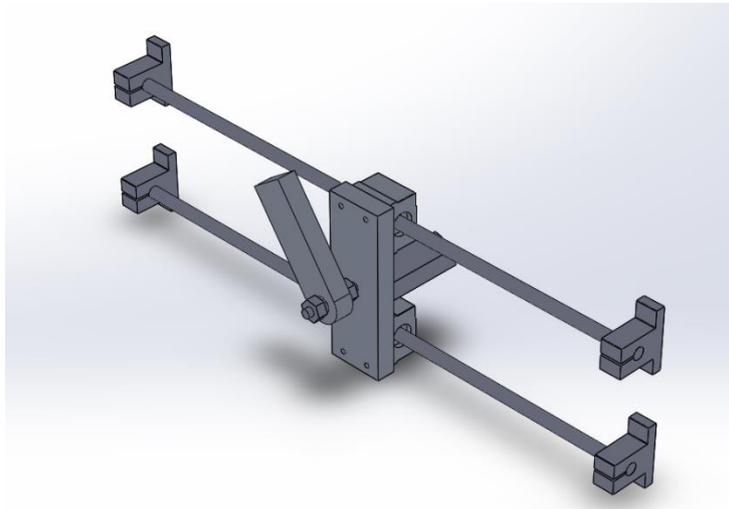


Figure 49: Final design, ISO view.

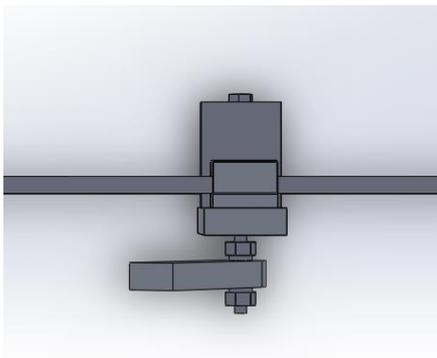


Figure 50: Final design, TOP view.

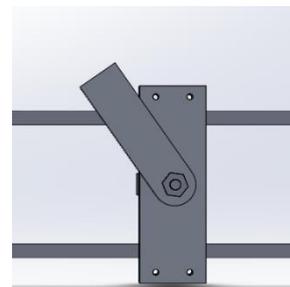


Figure 51: Final design, FRONT view.

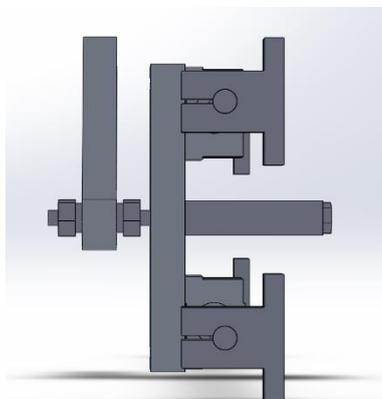


Figure 52: Final design, SIDE view.

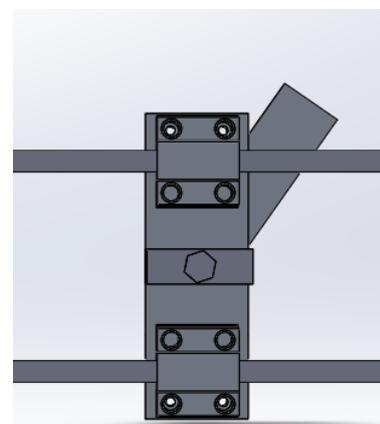


Figure 53: Final design, BACK view.

4.3. Sensitivity analysis.

Although the design was feasible, after some testing on the new design, one problem and inconvenient was found: The size of the pendulum directly affected the distance covered by the whole system, needing larger rails for large pendulums. As the available bars used as rails were 16 inches long, we needed to test if this distance was large enough to stabilize the pendulum before reaching the limit of the rail.

These tests were carried out by doing some modifications on the dimensions of the pendulum, specifically on the length from the center of the curve feature to the end of the pendulum, as shown in *Figure 54*; and simulating it with its correspondent natural frequency to test if the rail constraints were satisfied. The width and thickness remained constant.

4.3.1. Effect of pendulum dimensions in the IPC.

Testing the initial design, shown in *Figure 49*, whose dimensions are shown in the *Figure 54*, the purpose was to get an estimation on the length traveled by the IPC, from its initial position to the final position where the IPC is balanced.

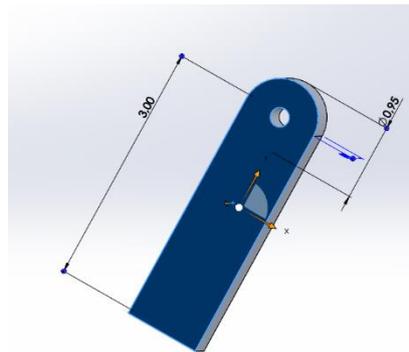


Figure 54: Initial size of IP.

For these dimensions, the natural frequency for the pendulum was 1.8 Hz. With this, the distance travel was close to 17.4 inches, as shown in *Figure 55*.

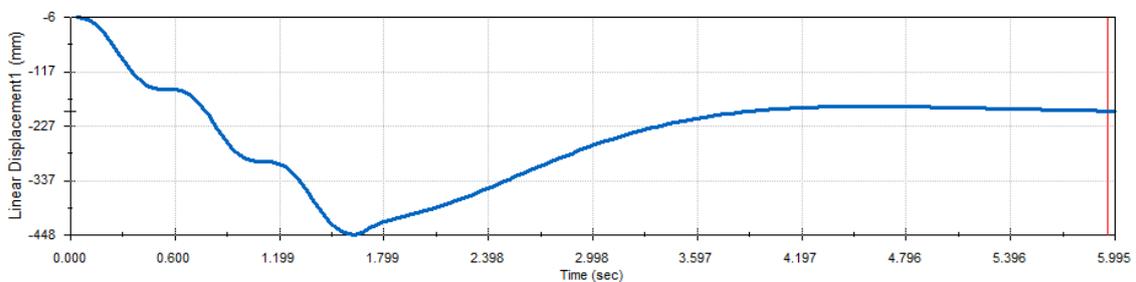


Figure 55: Displacement of IPC for initial size.

When changing the dimensions (the dimension of 3 inches is reduced to 2.5 inches, *Figure 56*), not only the new covered distance changes, but also the natural frequency.

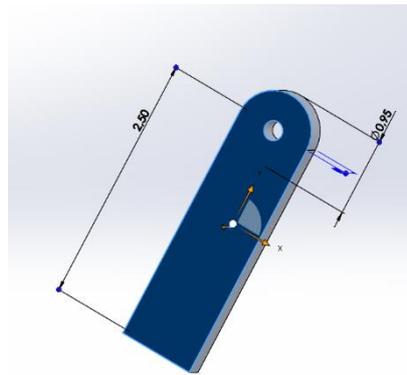


Figure 56: Size of 2.5 inches.

Once again, obtaining the natural frequency, with a value of 1.95 Hz, the result was a total distance of 14.8 inches, as shown in *Figure 57*.

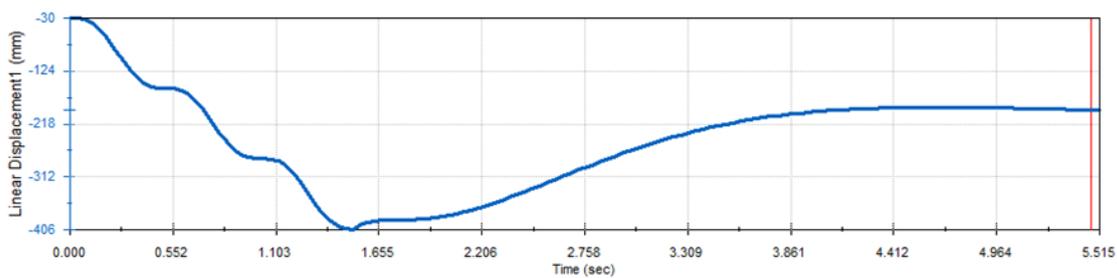


Figure 57: Displacement of IPC for 2.5 inches.

When changing the dimensions (the dimension of 2.5 inches is reduced to 2 inches, *Figure 58*), not only the new covered distance changes, but also the natural frequency.

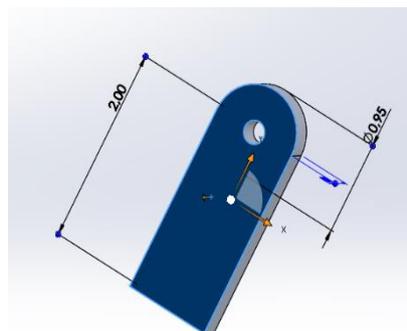


Figure 58: Size of 2 inches.

After obtaining the natural frequency, with a value of 2.3 Hz, the result was a total distance of 12.5 inches, as shown in *Figure 59*.

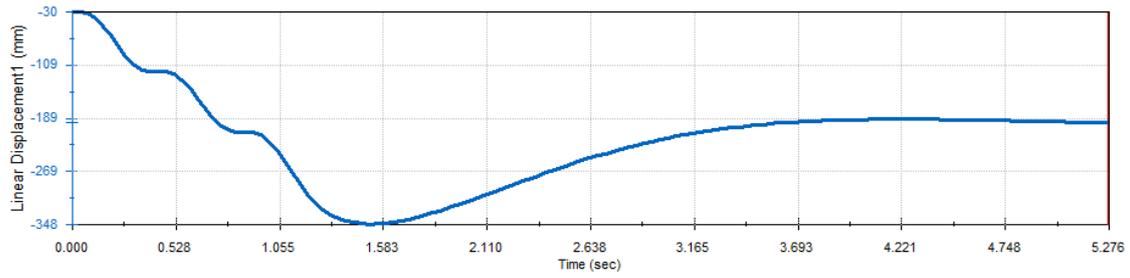


Figure 59: Displacement of IPC for 2 inches.

After these few tests, it seemed to be an inverse relationship between the natural frequency and the travelled distance. At the same time, it seems that the larger the pendulum is, the smaller the natural frequency is, which leads to a clear statement. In order to reduce the distance travelled by the IPC, the size of the pendulum must be reduced. In this case, to optimize the design, the pendulum will be preferably smaller than the initial design, so the rail constraints are satisfied.

The best ratio size – distance travelled will be chosen. Although the design for the dimension of 2 inches in Figure 58 was good enough, this pendulum may be too short for the beauty of the design. Another test was carried out by changing the dimension to 2.25 inches.

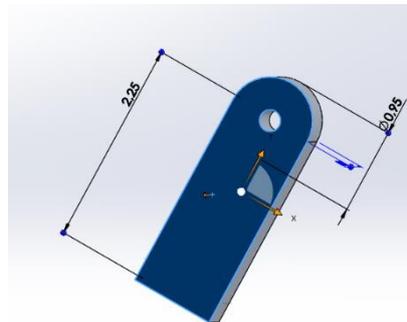


Figure 60: Size of 2 inches.

Once again, obtaining the natural frequency, with a value of 2.4 Hz, the result was a total distance of 10 inches, as shown in Figure 61.

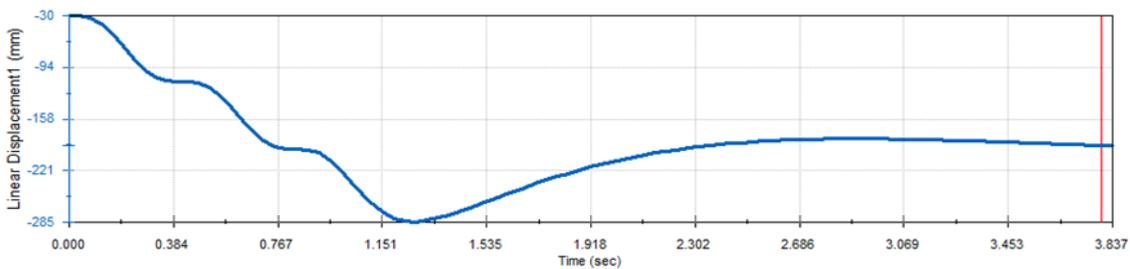


Figure 61: Displacement of IPC for 2.5 inches.

The conclusion and solution to the problem of the design was solved after these tests. However, no final size was chosen, as it was decided that the real size would be specifically chosen according to the definitive design and the characteristics of the motor of the whole system, which would determine which natural frequencies would be possible.

The design of these components of the system (final pendulum, motor, where to place the motor, how to move the system, etc.) was postponed until the first two phases of the building process was completed.

5. Description of the built model.

5.1. Building process.

The building process could be divided in three main phases, as showed in *Table 3*.

Phases	Concept	Description	Processes
Phase I	Rail system Support of the system	Create the main structure of the system, including the rails and the supporting system	Assembling
Phase II	Inverted Pendulum	Create the inverted pendulum, as well as the carriage that slides along the rails	3D printing Milling Cutting Drilling and tapping
Phase III	Control System	Add the element that will control the motion of the system such as the motor, the timing belt, etc.	3D printing ASsemble

Table 3: Building phase description.

5.1.1. Phase I.

For Phase I there was a drastic change in terms of the main structure. As described in the *Section 4.3.1*, there was still a solution to take in terms of design, as the available rails would be too short for the initial designs of the IPC. This is why, when starting the first building phase, a new idea of structure came on the scene: instead of creating a system of rails, it would be better to create a structure that would already have incorporated a railing system. For this purpose, the structure proposed was to use the frames in *Figure 62*, which were already available in the lab.



© 2020 Inc. All Rights Reserved

Figure 62: T-Slotted Profile Frame [5]

The main concept of the system remained the same. However, the new rail system was changed to a sliding system through the T-Slotted profile frame. The final main structure was, as shown in *Figure 63*, built with different sizes of this frame and put together by inside corner brackets, as it can be seen in the next figures.

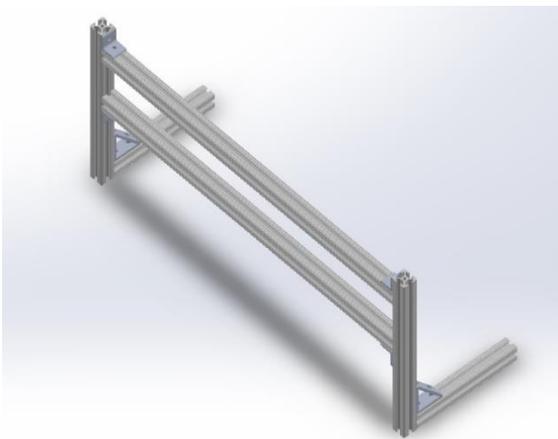


Figure 63: Structure with frames, ISOMETRIC view

- **Total length:** 26 inches
- **Total width:** 8 inches
- **Total height:** 11 inches



Figure 64: Structure with frames, TOP view



Figure 65: Structure with frames, FRONTAL view

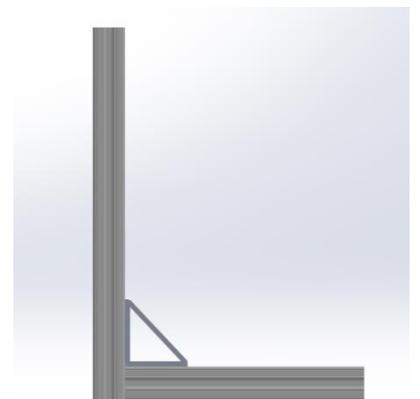


Figure 66: Structure with frames, SIDE view

5.1.2. Phase II.

For the Phase II, the next step was to add the inverted pendulum system in the structure. The pendulum would be formed by several parts:

- The base of the pendulum, the one that was attached to the sliders, was made out of HDPE, with a shape of 4x4 inches and with several holes to attach other parts, as shown in *Figure 67*.

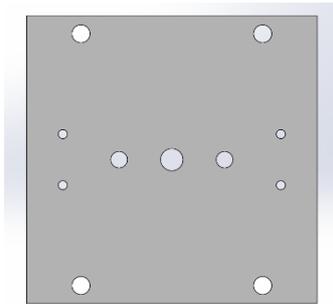


Figure 67: Base of the IPC

- The sliders, 4 in total, were attached to the base and could slide through the frames.

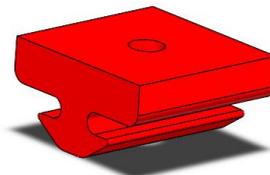


Figure 68: Slider

- The flange bearings were the ones that would hold the shaft of the pendulum and let it rotate. Two of these were used, one at each face of the base.

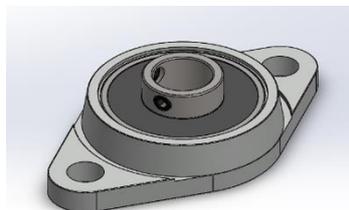


Figure 69: Flange Bearing "KFL 08" [6]

- The belt holders were parts designed to retain the timing belt that would move the base. There was a first design, that once attached, didn't have enough strength to hold the timing belt. Therefore, a new and simpler design was made. In the final design, two of these (Figure 71) were used.

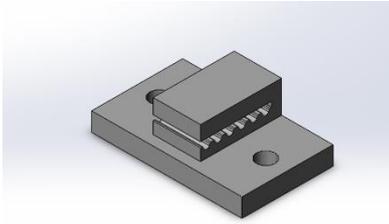


Figure 70: Initial design with the shape of the timing belt to hold it when moving.

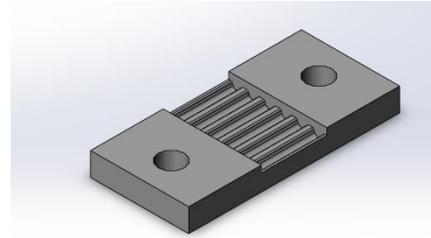


Figure 71: Second design that would be attached against the surface of the base to hold the belt.

- The shaft was an 8 mm aluminum solid rod.
- The part that attached the pendulum to the shaft was the linear rail shaft guide support bracket.

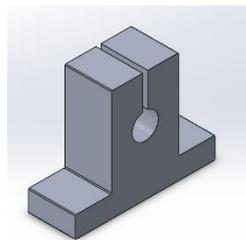


Figure 72: Linear Rail Shaft Guide Support Bracket, SK8, 8 mm Diameter

- The pendulum was a rectangular part made out of an HDPE sheet.

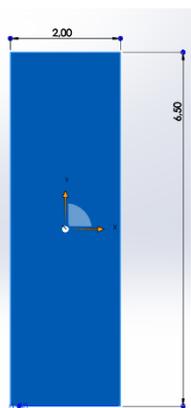


Figure 73: Pendulum

The results of these parts attached together are shown in the next Figures:

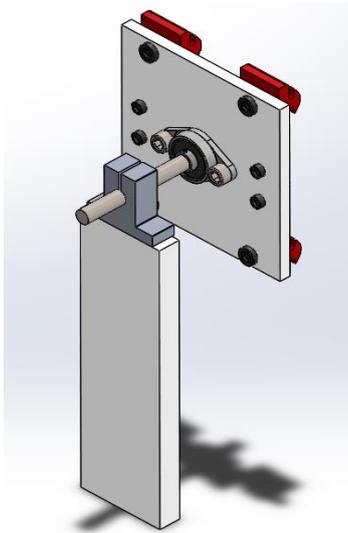


Figure 74: IPC, ISOMETRIC view

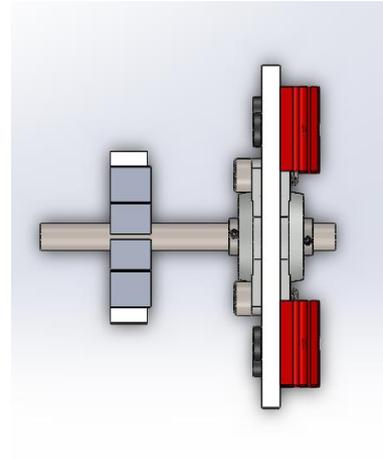


Figure 75: IPC, TOP view

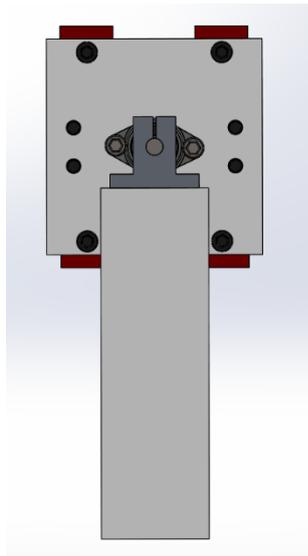


Figure 76: IPC, FRONTAL view

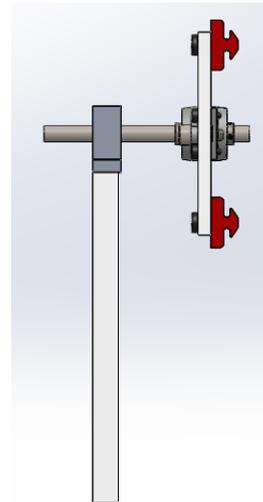


Figure 77: IPC, SIDE view

At this point, the structure would look like the following:

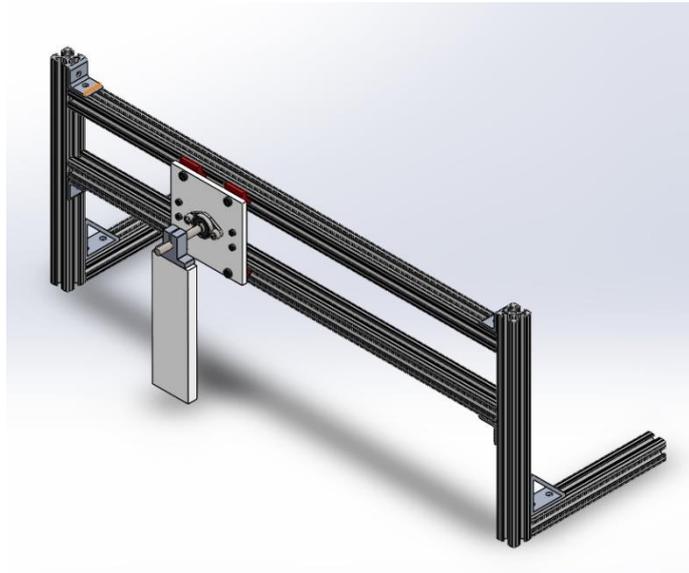


Figure 78: IPC after Phase II

5.1.3. Phase III.

The final phase of the construction consisted on adding to the system those parts that would provide the movement and control to the inverted pendulum. For this, the next parts were included:

- A stepper motor 17hs16-2004s1 as responsible for the movement of the carriage (the one called "Base", *Figure 67*).

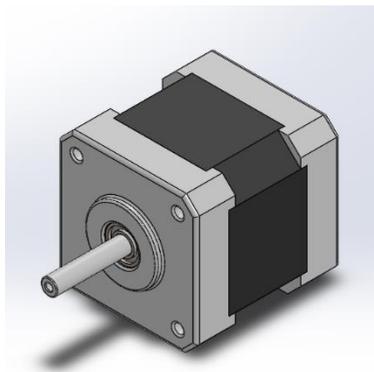


Figure 79: Stepper [7]

- A pulley for the stepper motor that could be attached to a timing belt.

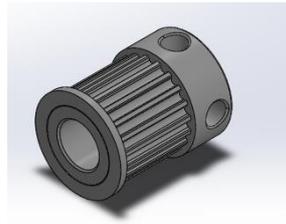


Figure 80: Pulley for the stepper [8]

- A 3D printed part to attach the motor to the frame.

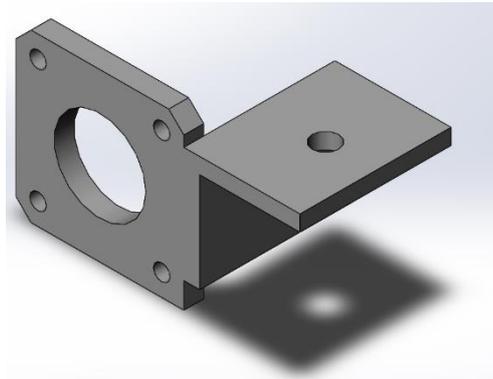


Figure 81: Motor holder

- A pulley to attach to the opposite site of the structure.

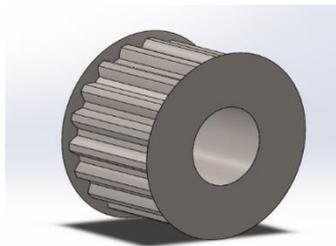


Figure 82: Second pulley

- A pulley holder made by a 3D printer.

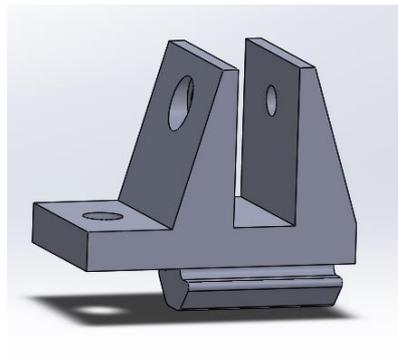


Figure 83: Pulley holder

- A timing belt that would be attached to the base by the belt holders and would pass by the pulley and the motor, providing the movement required.



Figure 84: Timing belt [9]

- An accelerometer “Adafruit triple-axis accelerometer - $\pm 2/4/8g$ @ 14-bit - mma8451” that would measure the movement in the axis X and Y, which would let the system to obtain the required variables to apply the control system.



Figure 85: Accelerometer [10]

- A 3D printed part that holds the accelerometer to the shaft.

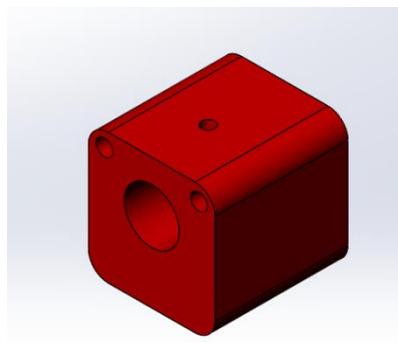


Figure 86: Accelerometer holder

The results of these parts attached together is shown in the next Figures:

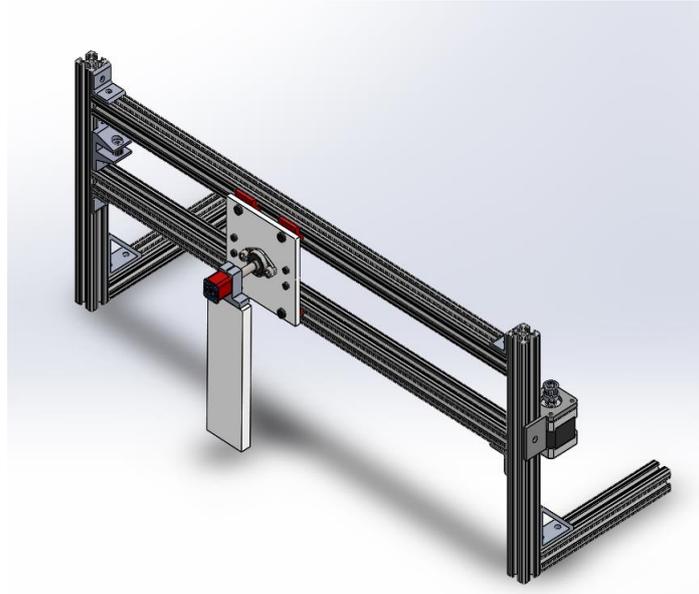


Figure 87: Final Design, ISOMETRIC view

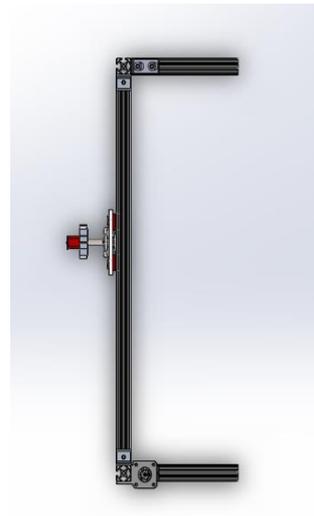


Figure 88: Final Design, TOP view

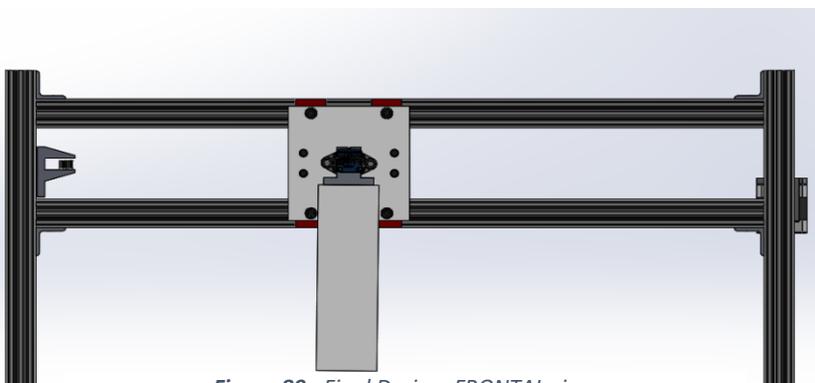


Figure 89: Final Design, FRONTAL view

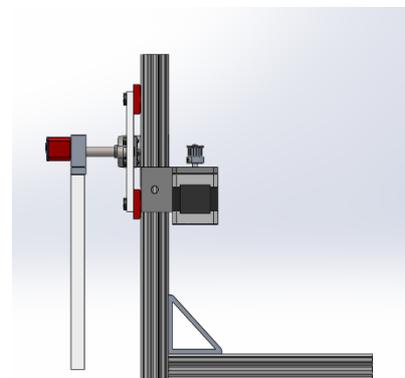


Figure 90: Final Design, SIDE view

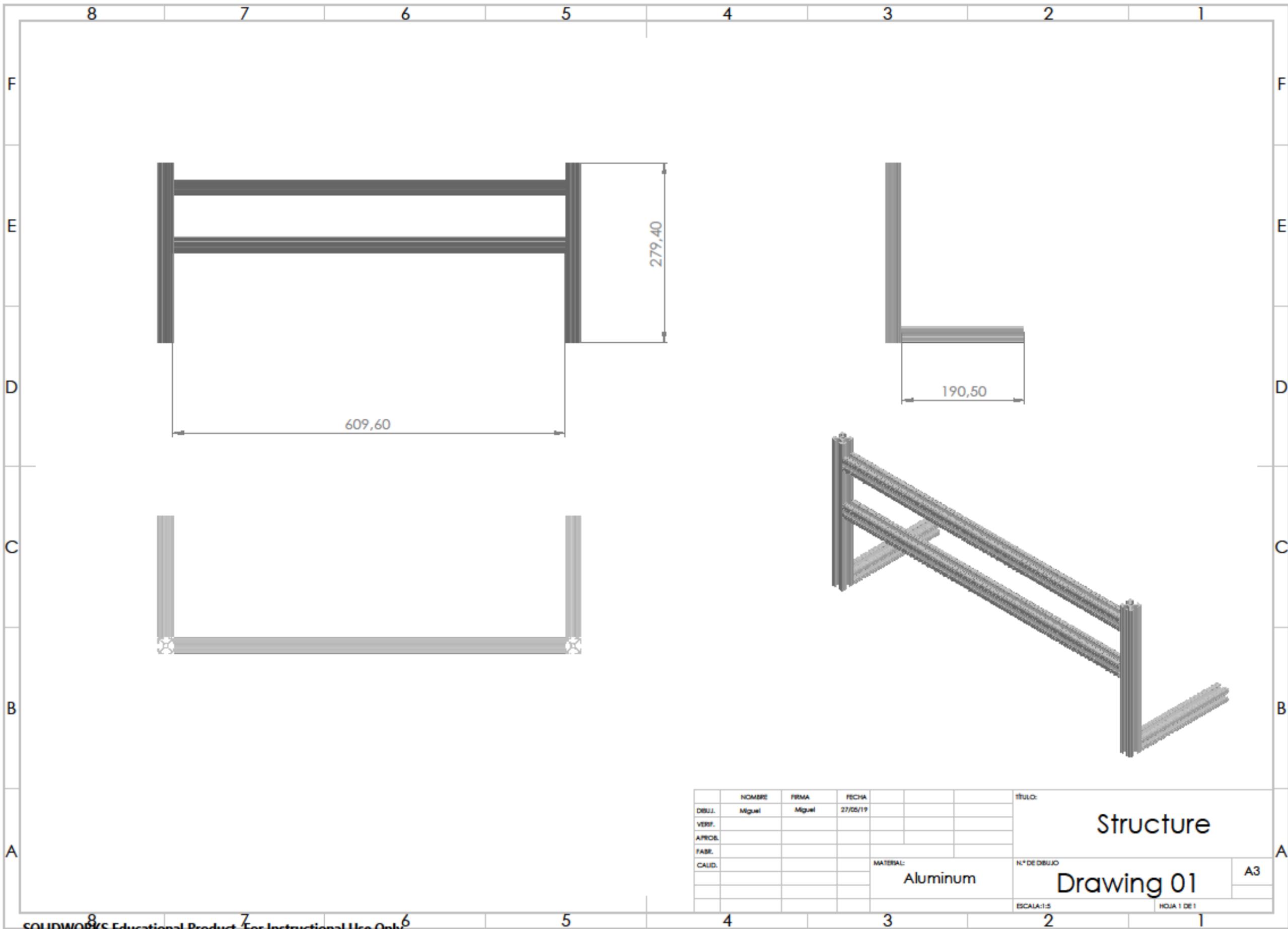
5.1. Drawings for the system

Once again,

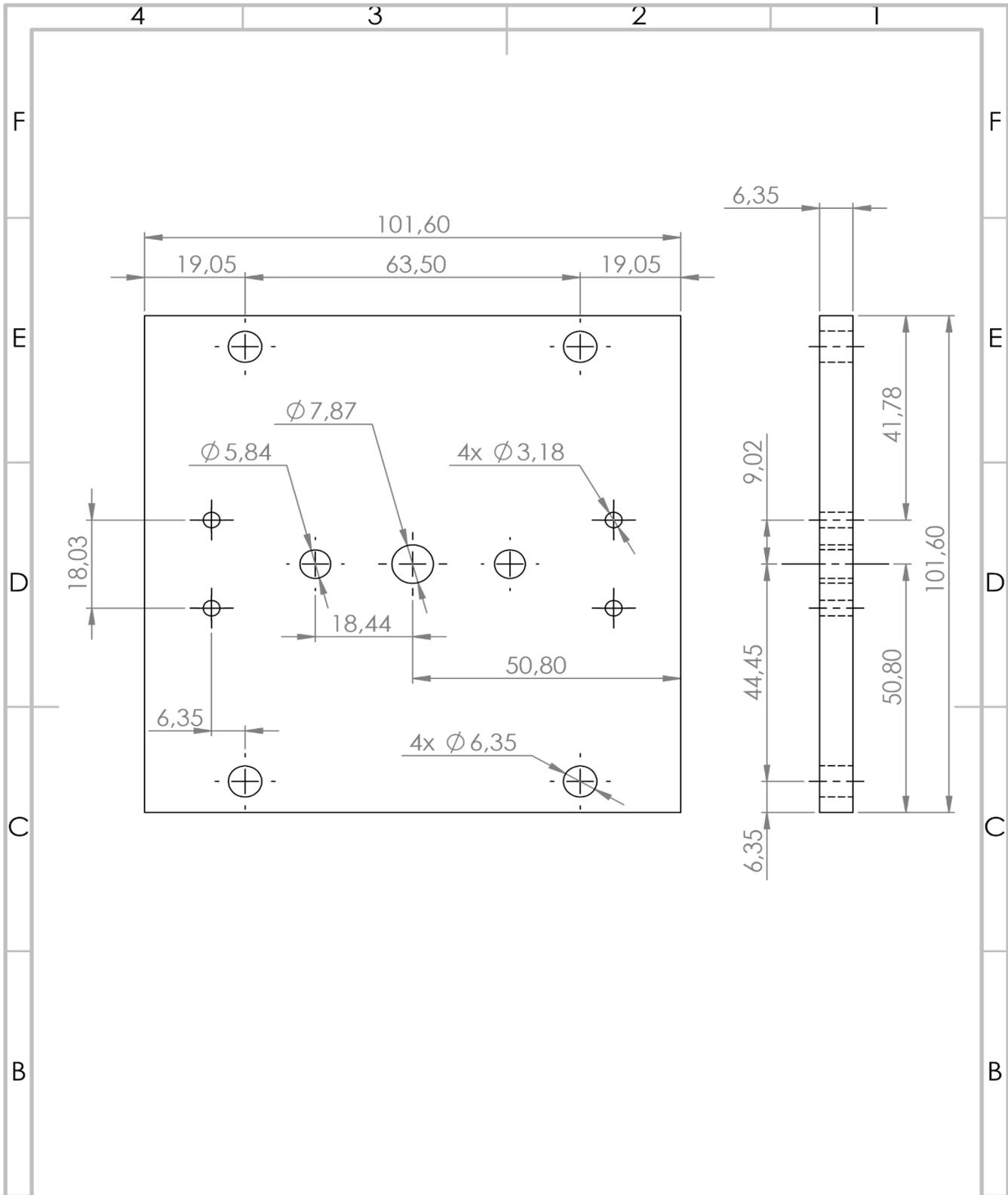
Index of drawings

Drawing 1 - Structure.....	39
Drawing 2 – Base pendulum.....	40
Drawing 3 – Slider.....	42
Drawing 4 – Belt holder.....	44
Drawing 5 – Pendulum.....	46
Drawing 6 – Motor holder.....	48
Drawing 7 – Pulley holder.....	50
Drawing 8 – Accelerometer holder.....	52
Drawing 9 – Frame profile.....	65
Drawing 10 – Flange bearing KFL 08.....	65
Drawing 11 – Linear rail shaft guide support.....	65

The first 8 drawings shown in the next pages correspond to those parts created and designed specifically for the purpose of this project. The rest of the parts correspond to commercial parts whose drawings can be easily found online. For the purpose of this project, in order to save space, some commercial drawings will be shown in the appendix and for the rest of them, a link to the online drawing will be provided.



	NOMBRE	FIRMA	FECHA	TÍTULO:	
DEJ.	Miguel	Miguel	27/05/19	Structure	
VERF.					
APROB.					
FABR.					
CAUD.				MATERIAL:	N.º DE DIBUJO
				Aluminum	Drawing 01
				ESCALA:1:5	HOJA 1 DE 1
					A3



	NOMBRE	FIRMA	FECHA
DIBUJ.	Miguel	Miguel	27/05/19
VERIF.			
APROB.			
FABR.			
CALID.			

TÍTULO:
Base pendulum

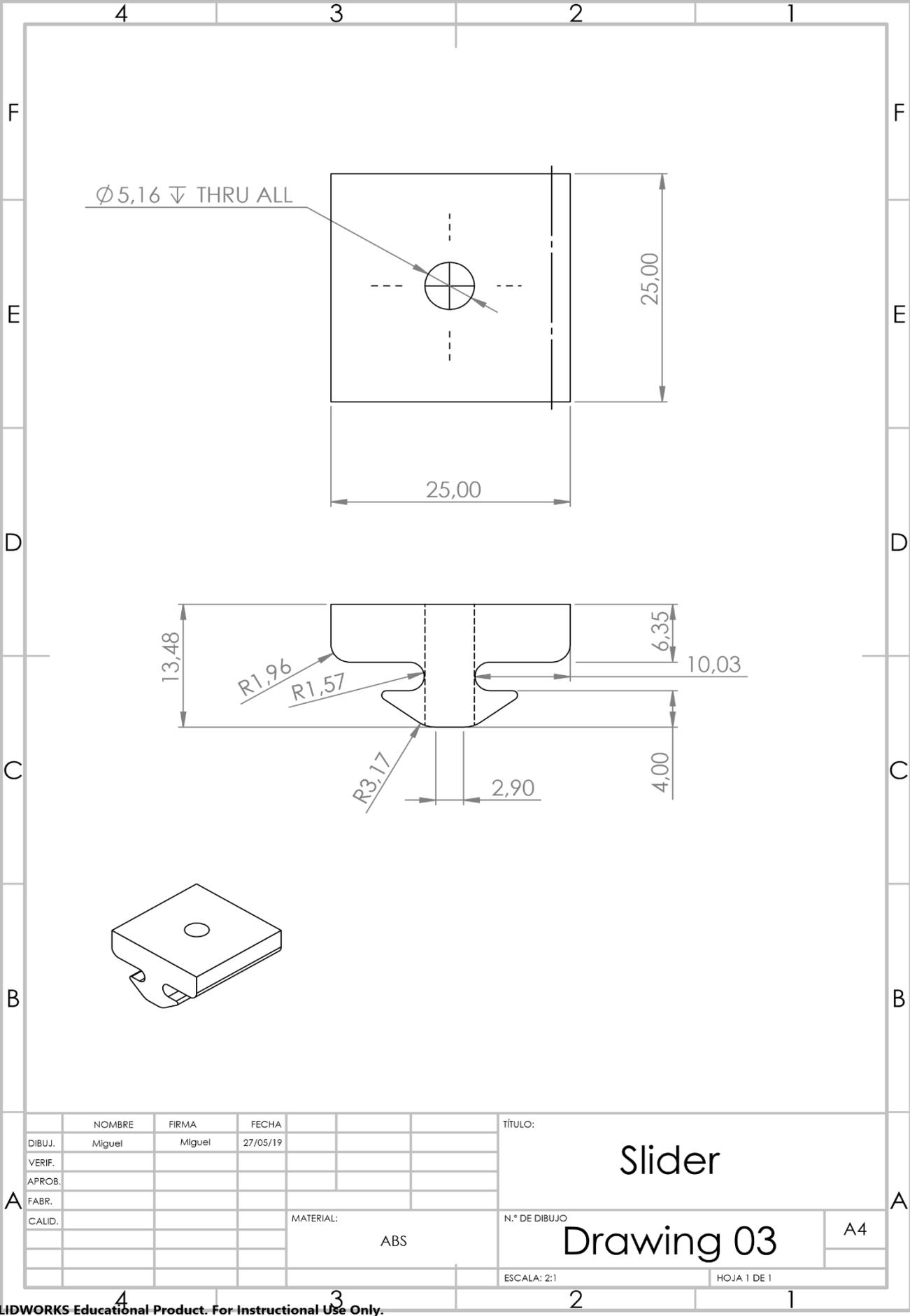
N.º DE DIBUJO
Drawing 02

ESCALA:1:1

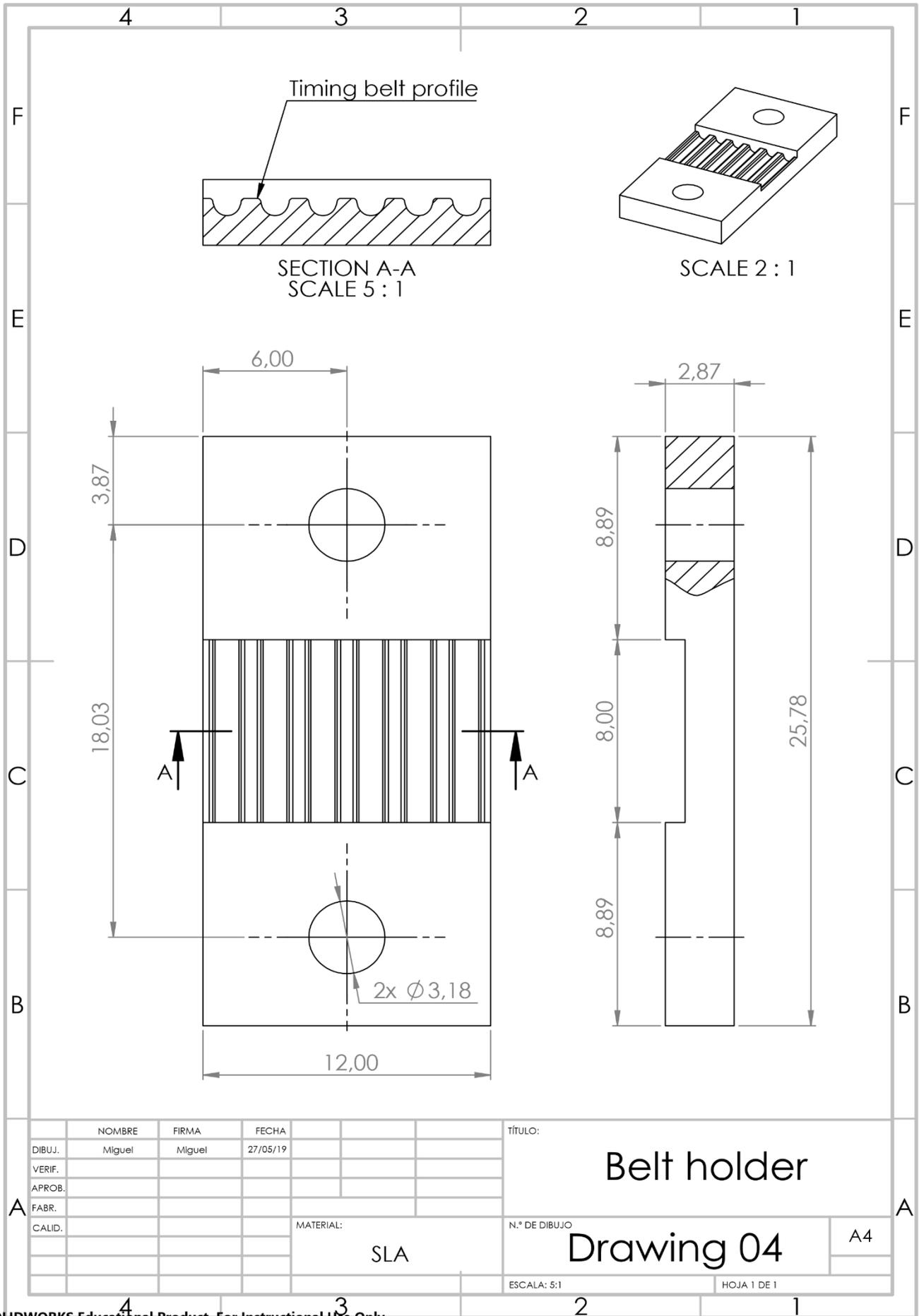
HOJA 1 DE 1

MATERIAL:
HDPE

A4



	NOMBRE	FIRMA	FECHA		TÍTULO:
DIBUJ.	Miguel	Miguel	27/05/19		Slider
VERIF.					
APROB.					
FABR.					
CALID.				MATERIAL:	N.º DE DIBUJO
				ABS	Drawing 03
					A4
				ESCALA: 2:1	HOJA 1 DE 1



	NOMBRE	FIRMA	FECHA
DIBUJ.	Miguel	Miguel	27/05/19
VERIF.			
APROB.			
FABR.			
CALID.			

TÍTULO:

Belt holder

N.º DE DIBUJO

Drawing 04

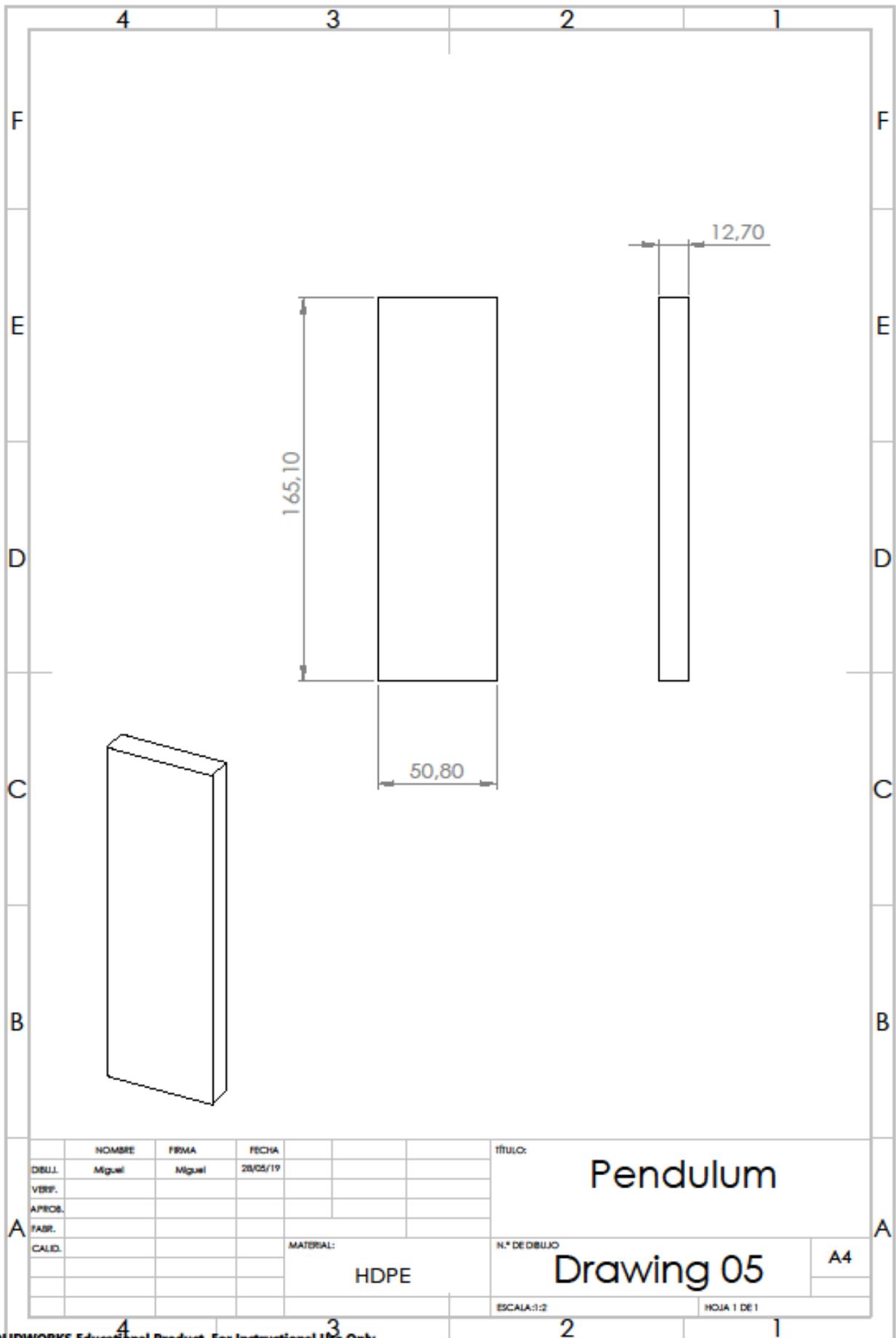
ESCALA: 5:1

HOJA 1 DE 1

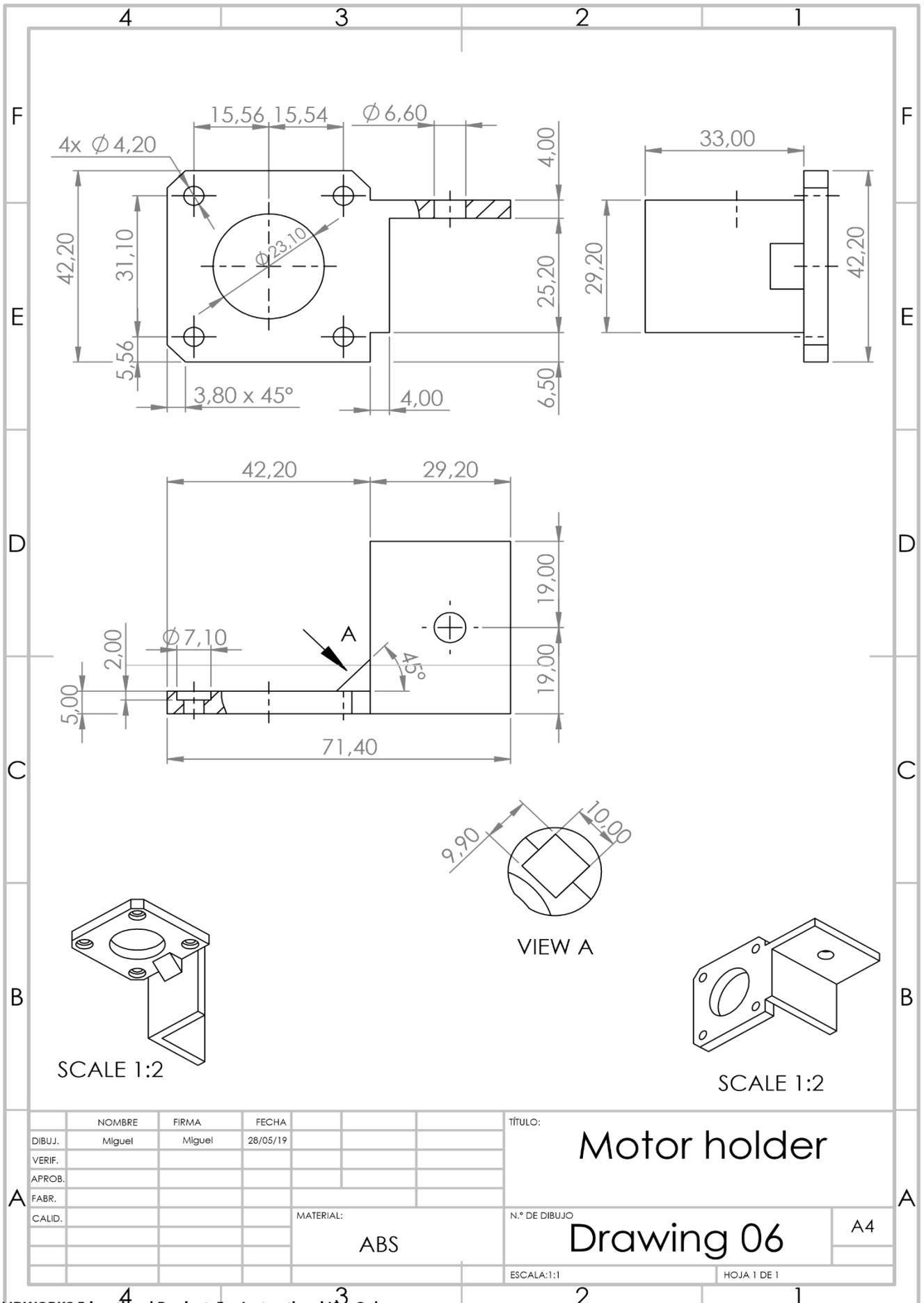
MATERIAL:

SLA

A4

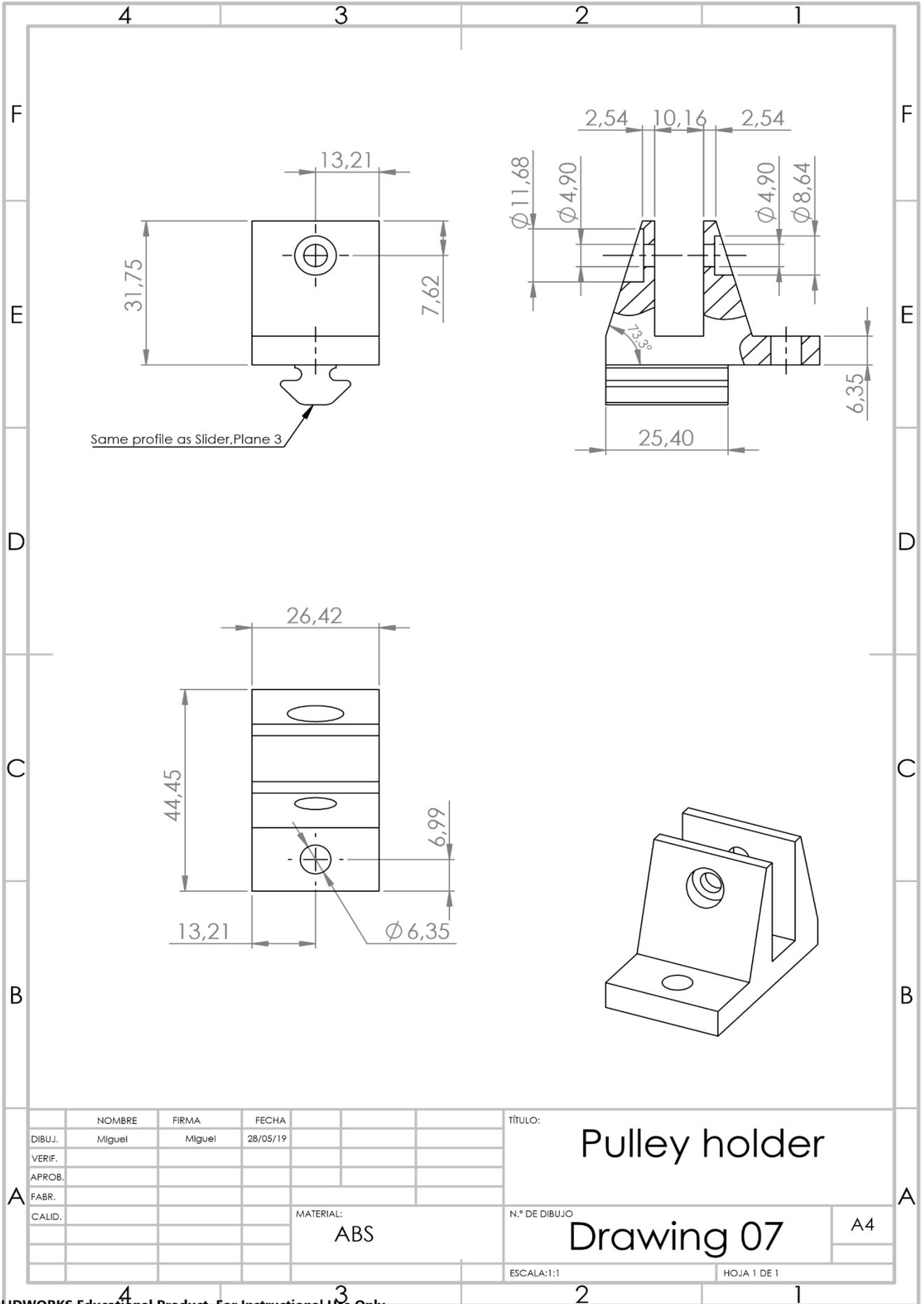


SOLIDWORKS Educational Product. For Instructional Use Only.



	NOMBRE	FIRMA	FECHA
DIBUJ.	Miguel	Miguel	28/05/19
VERIF.			
APROB.			
FABR.			
CALID.			
MATERIAL:			
ABS			

TÍTULO:	Motor holder	
N.º DE DIBUJO	Drawing 06	A4
ESCALA:1:1	HOJA 1 DE 1	



	NOMBRE	FIRMA	FECHA
DIBUJ.	Miguel	Miguel	28/05/19
VERIF.			
APROB.			
FABR.			
CALID.			

TÍTULO: **Pulley holder**

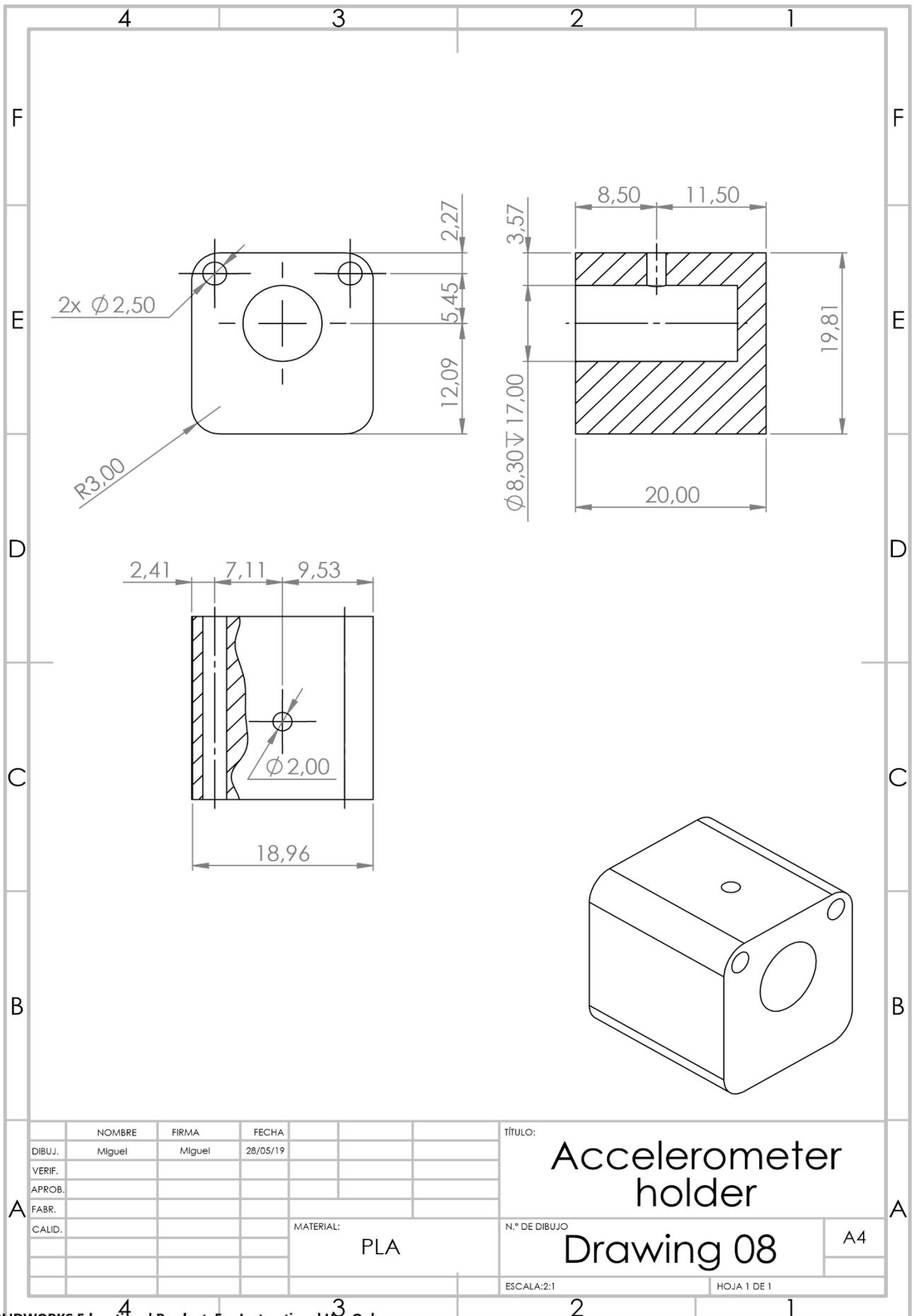
N.º DE DIBUJO: **Drawing 07**

MATERIAL: **ABS**

ESCALA: 1:1

HOJA 1 DE 1

A4



	NOMBRE	FIRMA	FECHA
DIBUJ.	Miguel	Miguel	28/05/19
VERIF.			
APROB.			
FABR.			
CALID.			

TÍTULO: **Accelerometer holder**

N.º DE DIBUJO: **Drawing 08**

ESCALA: 2:1

HOJA 1 DE 1

MATERIAL: **PLA**

A4

After a few weeks, the result was the following:

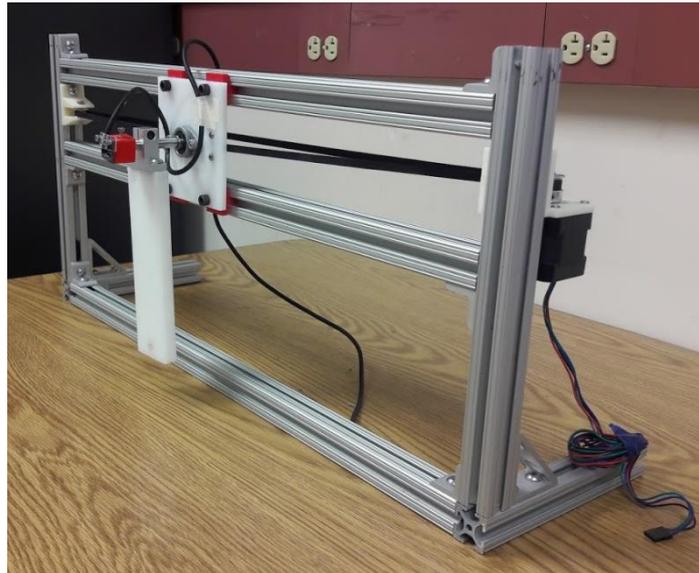


Figure 91: IPC, ISOMETRIC view



Figure 92: IPC, TOP view

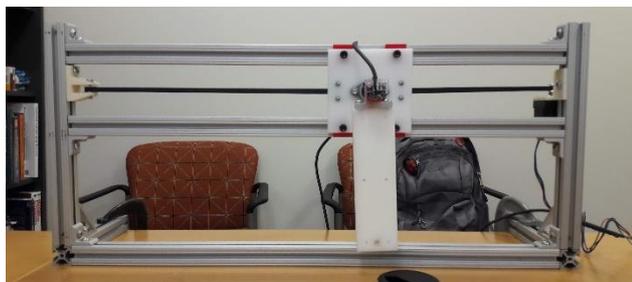


Figure 93: IPC, FRONTAL view



Figure 94: IPC, SIDE view

5.2. Economic analysis.

In this section, a simple analysis of the economic part will be described. It is important to clarify that in this case, the whole project was created with materials that the university already had, using their 3D printers and their laboratories. However, when analyzing the costs, the initial hypothesis will be that they will only include the price of the different parts, no workforce will be taken into consideration.

Following the same structure as the building process, the costs of creating this system could be divided in three parts.

- Phase I:
 - o Structure:

Supplier:	Length of frame	Units	Cost per frame (\$)	Cost (\$)
80/20.net	7,5 inches	2	3,68	7,36
	11 inches	2	4,48	8,96
	24 inches	2	7,47	14,94
Total cost in \$ =				\$31,26

Table 4: Costs table for the structure.

- o Corner brackets:

Supplier:	Holes in bracket	Units	Cost per corner bracket (\$)	Cost (\$)
80/20.net	1 per side	4	2,75	11
	2 per side	2	5,65	11,3
Total cost in \$ =				\$22,3

Table 5: Costs table for corner brackets.

- Phase II:

In this phase, the costs were divided between crafted parts and commercial parts.

- o Crafted parts:

Supplier:	Part	Units	Cost per part (\$)	Cost (\$)
University BU Source of prices is BU.	Base of pendulum (HDPE)	1	3	3
	Slider (PLA, 3D printed)	4	0,8	3,2
	Pendulum (HDPE)	1	3	3
Total cost in \$ =				\$9,2

Table 6: Costs table for crafted parts in Phase II

- Commercial parts:

Supplier: Different for each part	Part	Units	Cost per part (\$)	Cost (\$)
	Flange bearing [6]	2	2,9	5,8
	Shaft * [11]	1	0,54	0,54
	Linear rail shaft guide [12]	1	3,95	3,95
Total cost in \$ =				\$10,29

Table 7: Costs table for commercial parts in Phase II

- Phase III:

Once again, the costs can be divided in crafted parts and commercial parts:

- Crafted parts:

Supplier: University BU Source of prices is BU.	Part	Units	Cost per part (\$)	Cost (\$)
	Motor holder (ABS, 3D printed)	1	4,7	4,7
	Pulley holder (ABS, 3D printed)	1	4,2	4,2
	Accelerometer holder (PLA, 3D printed)	1	4	4
Total cost in \$ =				\$12,9

Table 8: Costs table for crafted parts in Phase III

- Commercial parts:

Supplier: Different for each part	Part	Units	Cost per part (\$)	Cost (\$)
	Stepper [7]	1	9,32	9,32
	Timing belt and pulleys [13]	1	6,75	6,75
	Accelerometer [10]	1	7,95	7,95
Total cost in \$ =				\$24,02

Table 9: Costs table for commercial parts in Phase III

Cost of screws and related parts:

Supplier:	Screw	Units per bag	Cost per bag (\$)	Cost per screw (\$)	Units needed	Cost (\$)
McMaster [14]	S.Allen 1/4" x 20 x 1/4" *	50	10,18	0,20	25	5,09
	S.Allen 5/40" x 20 x 1/4" *	100	10,51	0,11	4	0,42
	M6	1	3,33	3,33	2	6,66
Total cost in \$ =						\$12,17

Table 10: Costs table of screws

The total cost of the system was:

Phase	Kind of part	Cost
Phase I	Structure	31,26
	Coirner brackets	22,3
Total Phase I (\$)		53,56
Phase II	Crafted parts	9,2
	Commercial parts	10,29
Total Phase II (\$)		19,49
Phase III	Crafted parts	12,9
	Commercial parts	24,02
Total Phase III (\$)		36,92
Screws (\$)		12,17
Total cost		\$122,14

Table 11: Total costs

- * The parts marked with "*" means that they are sold in packages with more than one unit, which is why, for the purpose of this study, the cost of the package was divided between each part to get the unit price of each part.

6. Conclusions.

6.1. Conclusions on the methodology.

In this section, the methodology for the new desired system of teaching in Senior Design I will be analyzed.

First of all, after the complete study of the pendulum system, it can be concluded that using this kind of software seems to be very useful. Not only it is a very intuitive tool to work with, but it also provides very fast results compared to some other methodologies. This methodology has introduced new ways of learning, being able to test different ideas and simulations in no more than a few minutes, not only saving time but also giving quick feedback in what would work and what would not.

As it is explained in the introduction of this essay, the subject Senior Design I is an introduction to the subject Senior Design II, which is where the students will prepare their capstone projects. For this matter, this kind of methodology could be very helpful for future students, as designing and building devices will be two of the main phases for these capstone projects, and ultimately, for their career.

Once it can be concluded that the methodology could have a great impact in the education of the senior students, the study itself will be analyzed.

6.2. Conclusions on the results.

After testing different simulations, different designs, different ideas of how this system could work, a formula and an understanding of the system was developed. As it has been explained throughout the study, the final simulation has produced great results in achieving the main objective of the system, to set the pendulum in the equilibrium point thanks to the movement of the carriage.

Not only that, but it has also been seen how different variations in some components could change drastically the final results. This is why, after some thinking, it could be concluded that the results are efficient and are good enough for the matter of this project. However, as it will be explained in the next section, future changes could be done in order to get an optimum result.

As for the construction part, although the whole project followed a reasonable structure of how to build it, due to the restrictions of budget, some parts lack of the required efficiency that would improve the overall efficiency of the whole project. Future changes will be suggested afterwards. Despite this, the efficiency achieved was good enough for the purpose of this project.

6.3. Recommendations for future studies.

In future studies following a similar pattern as this one, some recommendations would be suggested.

In first place, when designing and simulating a specific system, it is very helpful to understand and comprehend how the different dimensions of the components of the system work and affect the behavior of the system. It would be recommended to test different dimensions of the different components before trying to get the final simulation of the device. Following this, it would allow us to obtain an optimum simulation of the system and a better understanding of the studied device.

As for the construction part, some changes would be recommended. The first one would be regarding the accelerometer. Not only because the way it is attached to the system creates some doubts on how it will measure the displacements, but also because the precision that this device offers is not good enough when trying to set the pendulum into its equilibrium point. For this matter, the recommended device to implement would be an encoder, which may be more expensive, but it would offer greater data about how the system behaves.

Some other changes suggested would be in the motion system. It was checked that the designed system worked, however, in some parts of the motion, greater accuracy was needed, not only in terms of the power required (related to the stepper) but also with the sensitivity of the movement (related to the timing belt system). This is why, another kind of motion system would be suggested, possibly a smoother and more sophisticated one, although this would mean higher costs.

7. References.

[IOAN17] Ioannis Kafetzis, Lazaros Moysis, “Inverted Pendulum: A system with innumerable applications”, Aristotle University of Thessaloniki, 2017.

[KENT02] Kent Lundberg, “The Inverted Pendulum System”, MIT, 1994 – 2002.

[KHAL03] Khalil Sultan, “Inverted Pendulum, Analysis, Design and Implementation”, IIEE.

[GARC16] García, Ramírez, Siordia y Martínez, “Las leyes de Newton en el modelado y control del péndulo invertido sobre un carro”, Revista Tecnología e Innovación, 2016.

[1] Senior Design I, BU courses, ENG ME 360, 11/02/19,
<https://www.bu.edu/academics/eng/courses/eng-me-460/>

[2] Inverted Pendulum, Wikipedia, 19/04/19, https://en.wikipedia.org/wiki/Inverted_pendulum

[3] Motion Analysis Overview, SOLIDWORKS Help, 19/04/19,
http://help.solidworks.com/2018/English/SolidWorks/motionstudies/c_Motion_Analysis.htm

[4] IF, SOLIDWORKS Help, 20/04/19,
http://help.solidworks.com/2017/english/solidworks/motionstudies/r_if_function_swmotion.htm

[5] T-Slotted Profile Frame, “80/20 Inc. The Industrial Erector Set”, 26/04/19,
<https://8020.net/25-2525.html>

[6] CAD:

Flange Bearing, “KFL08 8mm Zinc-Aluminum Alloy Flange Pillow Block Bearing”, GrabCAD, 26/04/19, <https://grabcad.com/library/kfl08-8mm-zinc-aluminum-alloy-flange-pillow-block-bearing-rolamento-com-mancal-de-aluminio-1>

PRICE:

Flange bearing, eBay 30/05/19, <https://www.ebay.es/i/273148615810?chn=ps>

[7] CAD:

Stepper motor, “Stepper motor 17hs16-2004s1”, GrabCAD, 26/04/19,
<https://grabcad.com/library/nema-17-stepper-motor-17hs16-2004s1-3d-printer-1>

PRICE:

Stepper motor, stepperonline, 30/05/19, <https://www.omc-stepperonline.com/nema-17-bipolar-45ncm-64ozin-2a-42x40mm-4wires-w-1m-cable-and-connector-17hs16-2004s1.html>

[8] Stepper motor pulley, “GT2 Timing Gear Alumium For Belt Width 10mm - 20 teeth - Bore 6.35mm”, GrabCAD, 26/04/19, <https://grabcad.com/library/stepper-motor-nema-17>

[9] Timing belt, “376XL Rubber Timing Belt Synchronous Closed Loop Belt Timing Pulley Tools 10mm Width”, 26/04/19, https://www.amazon.com/uxcell-Rubber-Timing-Synchronous-Closed/dp/B07HBLMS79/ref=sr_1_11_sspa?keywords=timing+belt&qid=1556589836&s=gate-way&sr=8-11-spons&psc=1

[10] Accelerometer, "AdaFruit triple-axis accelerometer - $\pm 2/4/8g$ @ 14-bit - mma8451", AdaFruit Products, 30/05/19, https://www.adafruit.com/product/2019?gclid=Cj0KCQjwkoDmBRCcARIsAG3xzl-UgJhAx3ioDtWril3y5HG8GLAp9KSTjRRUp91mFVOvstaT8EIRvDoaAn5EEALw_wcB

[11] Aluminum rod, Amazon, 30/05/19,

https://www.amazon.com/Swpeet-Assorted-Aluminum-Diameter-Miniature/dp/B07BVTQNDZ?ref=Oct_BSellerC_11260297011_2&pf_rd_p=50cf16db-7f18-5f16-8864-9b7d44c76754&pf_rd_s=merchandise-search-6&pf_rd_t=101&pf_rd_i=11260297011&pf_rd_m=ATVPDKIKX0DER&pf_rd_r=FN93ZK3K8GWA BGPDVVQV&pf_rd_r=FN93ZK3K8GWABGPDVVQV&pf_rd_p=50cf16db-7f18-5f16-8864-9b7d44c76754

[12] Linear rail shaft guide, Adafruit, 02/06/19, <https://www.adafruit.com/product/1182>

[13] Timing + pulleys 30/05/19

[https://www.automationdirect.com/adc/shopping/catalog/power_transmission_\(mechanical\)/synchronous_drives_\(timing_belts_-a-_pulleys\)](https://www.automationdirect.com/adc/shopping/catalog/power_transmission_(mechanical)/synchronous_drives_(timing_belts_-a-_pulleys))

[14] McMaster, 02/06/19, <https://www.mcmaster.com/>

8. Appendix.

As for the commercial parts used, here are some of the drawings of these parts. In some cases, due to the length of these, a link will be provided.

The drawing shown correspond to the structure, flange bearing and the linear rail shaft guide.

For the stepper, visit the following link:

<https://www.omc-stepperonline.com/nema-17-bipolar-45ncm-64ozin-2a-42x40mm-4wires-w-1m-cable-and-connector-17hs16-2004s1.html>

For the accelerometer, visit the following link:

<https://learn.adafruit.com/adafruit-mma8451-accelerometer-breakout/downloads>

For more specification, see the index of drawings.

3D PDF DATASHEET	 eCATALOG solutions	 BIM catalogs.net	 PART solutions
Create Your Individual 3D PDF Datasheet ▶	Learn How to Create Your Mechanical Product Catalog ▶	Learn How to Create Your BIM Product Catalog ▶	Reduce Your Costs in Engineering and Purchasing ▶

1010-11-CL

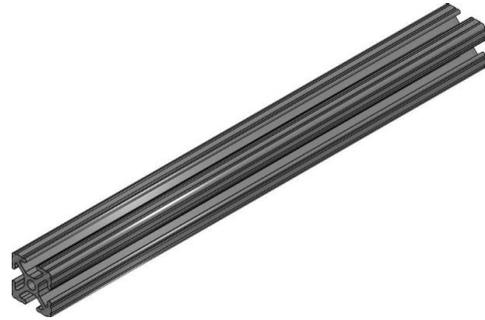
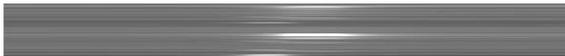
T-Slotted Profile - 10 Series

80/20' Inc.

The *Industrial Erector Set*®

2D derivation

•SO View



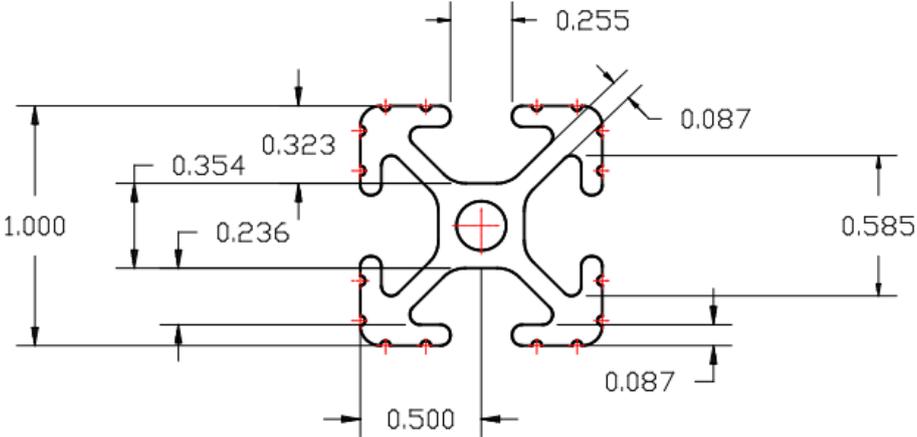
3D PDF DATASHEET	 eCATALOG solutions	 BIM catalogs.net	 PART solutions
Create Your Individual 3D PDF Datasheet ▶	Learn How to Create Your Mechanical Product Catalog ▶	Learn How to Create Your BIM Product Catalog ▶	Reduce Your Costs in Engineering and Purchasing ▶

1010-11-CL

T-Slotted Profile - 10 Series 80/20' Inc.

The *Industrial Erector Set*®

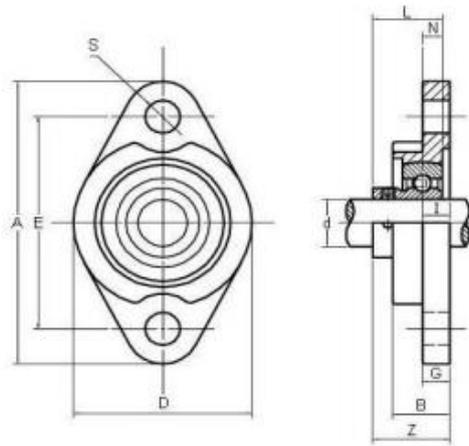
Technical drawings





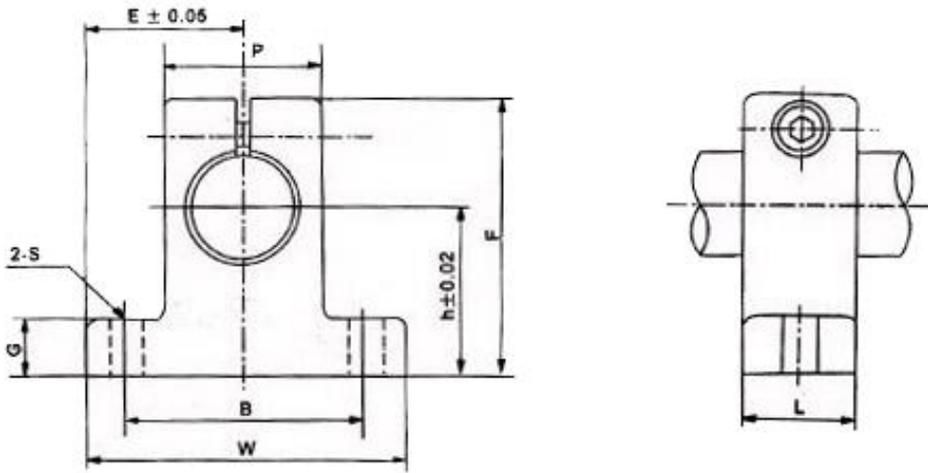
KFL 000 - 2-Bolt Flanged Housing Series

Silver Series, rustproof zinc aluminium housing, high carbon chromium bearings, set screw locking, solid base.



Type	Dimensions (mm)											Bolt Size	Mass
	d	A	E	I	G	B	S	D	Z	L	N		
KFL 08	8	48	37	4,5	4,5	8,5	4,8	27	11,5	10,5	3,5	M 4	0,08
KFL 000	10	60	45	5,5	5,5	11,5	7	36	15,5	14,0	4,0	M 6	0,10
KFL 001	12	63	48	5,5	5,5	11,5	7	38	16,0	14,5	4,0	M 6	0,10
KFL 002	15	67	53	6,5	6,5	13,0	7	42	18,6	16,5	4,5	M 6	0,10
KFL 003	17	71	56	7,0	7,0	14,0	7	46	19,5	17,5	5,0	M 6	0,20
KFL 004	20	90	71	8,0	8,0	16,0	10	55	23,0	21,0	6,0	M 8	0,20
KFL 005	25	95	75	8,0	8,0	16,0	10	60	24,5	22,5	6,0	M 8	0,20
KFL 006	30	112	85	9,0	9,0	18,0	13	70	27,0	24,5	6,5	M 10	0,30

Linear rail shaft guide.



	Rod Dia.	h	E	W	L	F	G	P	B	S	Set Screw	Attach Screws	Weight (kg)
SK8	8	20	21	42	14	32.8	6	18	32	5.5	M4	M5	0.024