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RISK ASSESSMENT OF TRANSMISSION NETWORK FAILURES IN A SINGLE-PRICE ELECTRICITY MARKET USING CONJECTURAL-VARIATION EQUILIBRIUM

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Abstract

Risk Assessment of Transmission Network Faillures in a Single-Price Electricity Market using Conjectural-Variation Equilibrium

by Abdin, Islam Fouad

Risk assessment is a key aspect in power systems' design and planning. Under the new competitive framework of the electricity sector, power systems are found to be more and more strained and operated near their technical limits. On the economic front, problems arising due to transmission lines' limited capacities and reduced reliability becomes an obstacle to perfect competition among the market participants, and negatively affect the electricity market prices. Under these scenarios, it is crucial for the system operator (SO) to have tools that allow the correct assessment of the impact of network failures on the total system's running costs. In this work, we propose a novel risk assessment method that aims to assist the SO in evaluating these costs under different line failure scenarios. The risk is quantified in a single-price market not only in terms of the ENS cost, but also in terms of the so called correction costs arising due to the necessity of correcting the dispatch in case of a line failure. The latter takes into account the effect of exercise of market power by the generation companies (GenCos) if they can anticipate that such failure would occur. The study is conducted on the IEEE 6-bus Reliability Test System (RBTS) by solving a bi-level model consisting of a conjectural-variation equilibrium model, and a DC optimal power flow model. The results shown to offer a guideline for the SO to identify the critical network lines by means of a classification of which line's vulnerability contribute higher costs for the network.

Keywords: Risk Assessment, Network Failures, Equilibrium Modeling, Electricity Markets, Conjectural-Variation, Optimal Power Flow

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Last but -most certainly- not least, to my mother and to my father. I dedicate to you any success I will ever be able to achieve.

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Chapter 1

Introduction and Motivation

There is no need to emphasize the fact that electricity has become the backbone of any industrialized society and economy. Modern countries rely heavily on a steadily uninterrupted supply of electricity for carrying out almost all their economic activities. The ever increasing dependency on continuous power supply related to the wide spread of electronics, the ever increasing industrial production, and all aspects of daily life makes today's society much more vulnerable concerning power supply interruptions. A few minutes of interruptions may already cause some inconvenience for many users, yet a few hours or even days would have a significant impact on the entire economy [Kim et al., 2013], [Bruch et al., 2013]. Other than this increasing dependency which is already straining the operation of the power systems, this industry experienced two major changes during the last two decades: the liberalization and privatization of the electricity systems [Pérez-Arriaga, 2014], and the fast expansion of renewable energy production capacities [International Energy Agency, 2014]. It is well understood that most industrialized countries have many years of experience with dealing with the liberalization of their power systems, yet this separations of activities which have created many additional interfaces, have impacted the coordination activities between all the different stakeholders. This has led to a clear problem of incentive to invest in reliable, and well maintained infrastructures, that most countries are still experimenting with different mixture of policies, regulations and mechanisms to solve. On the other hand, although the expansion of renewable generation in itself is a positive development towards a sustainable energy future, the volatile nature of many of these sources adds to the vulnerability of the system [Flick and Morehouse, 2010], [Nasiruzzaman et al., 2014], [Negeri et al., 2015]. Not only may a scarcity of electricity result in a power outage, an oversupply can also lead to grid instabilities and hinder its secure operation.

Yet a reliable system is not the only challenge confronted; even in instances of secure system operation, significant economic losses can be incurred due to the departure of the competition performance in liberalized systems away from the optimal case of perfect competition, which is the considered as the benchmark in microeconomics analysis. Several causes related to market design and regulatory policies stand behind either a positive or a negative market outcome. Yet for those causes related (even partly) to the technical nature of the system, transmission line constraints would certainly have a leading role in affecting the efficiency of the system's operation as a whole. Several studies have addressed this issue, providing insights especially on the effect of the most commonly faced problem of transmission line congestion on the market performance. So far it has been well documented that regardless of the underlying market clearing scheme implemented (e.g. nodal pricing, zonal pricing or single-price markets), transmission congestion allows a significant chance for market participants to exercise market power, increasing the market prices and decreasing the overall welfare of the society [Gao and Sheble, 2010], [Liu and Wu, 2007], [Delgadillo and Reneses, 2013]. Less explored, however, is the effect of total line contingency on the market outcome and on the behavior of its participants. Perhaps the reason behind that is that the focus on the secure operation of the network considering standards such as the N-1 criterion, may have led to the belief that even in the most severe cases, such contingency would not occur enough times or last long enough to distort the market outcome. Perhaps this is true for cases where investments in the planning and construction phase of the transmission networks have been made to account for an adequate level of reliability in the network. However, this might change as new developments in the power system such as distributed renewable generation and demand side management, may direct the trend towards an opposite direction of decreasing dependency on several parts of the grid, with even less clear incentives for the different stakeholders to maintain its reliable operation. In addition, other studies have pointed out that in many instances, the often neglected effect of extreme weather conditions results in an underestimation of the failure rates in the network, and also an underestimation of its consequences [Rocchetta et al., 2015], [Alvehag and Söder, 2011].

There is a need therefore, to account for the risk of such contingency in operation of the system, and not only in the original planning of the investment. Risk by definition is a probabilistic measure; it assesses the consequence and severance of the occurrence of an undesired effect relative to its probability of occurrence [Zio, 2007], hence providing an insight on its significance. This is especially useful in assessing event commonly perceived as insignificant due to the lack of a quantified measure that verifies this perception. Yet, apart from the typical reliability assessment in terms of the power delivery security aspect often considered, risk assessment methods have been used in a vast variety of studies both related to electric power systems, and electricity markets. Since there are many stakeholders in these systems (generation companies (GenCos), transmission and distribution companies, system operator, consumers, etc.), risk indices have been adopted from different view points including financial, economic and technical assessments [Li, 2014]. From the point of view of the operation of transmission network, these risk assessments have been seen to commonly consider the technical aspect of the system in assessing line contingencies (in terms of risk of energy not being served), and have been found to seldom considers the effect of the power market performance.

Therefore, the present work provides an attempt for proposing a novel risk assessment method which takes into account the impact of network line failures on the performance of the power markets. The aim is to evaluate whether in a specific case of line failure significant distortions of the market performance could occur in such a way so as to give incentive for the network operators to prepare preventive actions for both the market operation and the final dispatch of the network.

Since the nodal pricing market scheme is already designed in such a way that internalizes market inefficiencies in its operation, we consider for this study a single-price market, where GenCos submit bids to maximize their profits in the day-ahead market without taking into consideration any effects for the network constraints. The market is cleared purely on the basis of economic considerations, by aggregating the supply and demand bid offers, and setting the day-ahead market price equal to the marginal unit scheduled for dispatch in the next day. All scheduled GenCos receive this single market price per unit of their dispatch. The system operator (SO) is responsible for the secure, feasible and optimal final dispatch plan which is scheduled taking into account the final received bids. In case of a line failure occurrence, the SO will re-dispatch the system to maintain a secure operation and prevent any potential collapse in other parts of the network. This means that some generation units will be required to increase or decrease their final dispatch relative to the day-ahead market schedule. The re-dispatch schedule is decided while taking into consideration each GenCo's bid offer for an upward or a downward correction of its dispatch.
We consider the ability of the GenCos to anticipate the re-dispatch schedule imposed by the SO in case of a line contingency, and therefore their ability to act strategically to increase their profits from both the day-ahead market operation and the so called correction market combined. This strategic behavior would lead to the need for the SO to correct higher power discrepancy between the day-ahead market schedule and the real time operation after the line failure, leading to a higher efficiency loss (quantified as a cost) in the operation of the network.

The risk assessment proposed combines the typical effect of the cost of energy non-served that could be incurred due to a line failure, with a quantification of the increased correction cost necessary incurred by the SO, due to the strategic behavior of the market participants. This way, the SO can have a new insightful tool for assessing those risks.

The rest of this dissertation is organized as follows:

- In Chapter 2, a comprehensive literature review covering the state of the art in market modeling techniques is provided, with special emphasize on the studies that explicitly considers competition modeling taking into account transmission network constraints. Moreover, a review on some of the most commonly considered reliability and risk assessment methods related to power transmission network is summarized to show the positioning of the proposed assessment method among the existing ones.

- In Chapter 3, the bi-level model used for carrying this study is fully described, consisting of a conjectural-variation equilibrium model for modeling the market competition, and a DC optimal power flow model for simulating the behavior of the SO in finding the feasible schedule. The chapter is concluded by a full description of the solution method for the bi-level model, and the formal quantification of the risk concept and risk assessment method in the context of this study.

- In Chapter 4, the case study chosen for the numerical example is illustrated, and all failure cases considered are fully identified. The results are reported for all the cases and a critical analysis covering all the important results' observations is provided.

- Finally in Chapter 5, an overview of all the milestones achieved is given, and the most important results are summarized, with the main message reached up to this phase of research. The chapter ends with the main suggestions on the future tasks to be considered in enriching and improving this work.

Chapter 2

Literature Review

2.1 Market Modeling in Electric Power Systems

As previously discussed, the deregulation and liberalization of the power sector in many of the systems across the globe, have created an important need and interest for the adequate modeling of the behavior of those systems under the new operating conditions. As well as the studies for the different regulatory schemes and market designs in continuous attempts to optimize the social welfare. This section intends to broadly illustrate the topics that have been covered in the market modeling in electric power systems, the different formulation techniques used to represent these models, and the different solution techniques implemented. Special emphasis is given to summarize the references that address the electricity market modeling taking into consideration the transmission network constraints.

Although it has been around 10 years since Marino Ventosa, Álvaro Baıllo, Andrés Ramos and Michel Rivier published their paper "Electricity Market Modeling Trend" [Ventosa et al., 2005], no sign have been shown during this literature review compilation that indicated that their general description of the modeling trends in terms of structure have undergone any significant change. It remains useful therefore to start with a brief illustration following these major trends.

2.1.1 Structural approaches in Electricity Market Modeling

Ventosa et al. [2005] illustrate that from a structural point of view, the different approaches that have been proposed in the technical literature can be classified according to the scheme shown in Fig 2.1.

In general, research developments follow three main trends: optimization models, equilibrium models and simulation models. Optimization models in electricity markets often focus on the

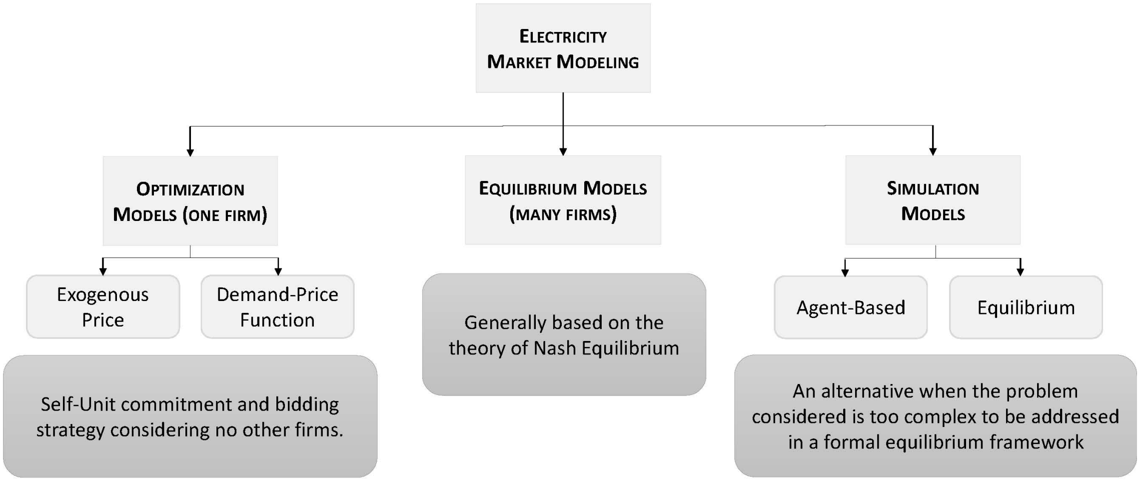

Figure 2.1: Schematic representation of the electricity market modeling trends. [Ventosa et al., 2005]

profit maximization problem of one of the firms competing in the market. Due to its mathematical nature specifically in terms of tractability, these problems can often be highly complex representing high level of details of the system. Equilibrium models on the other hand, represent the overall market behavior taking into consideration competition among all participants, this interaction is often studied based on the notion of "Nash Equilibrium" which will be addressed in details in the next section. Finally Simulation models can be seen as an alternative to equilibrium models when the problem under consideration is too complex to be addressed within a formal equilibrium framework. Although there are many other possible classifications based on more specific attributes, the present classification based on the different mathematical structures of these three modeling trends would help establish a clear division.

Since in this dissertation an equilibrium based model is chosen to undergo the study, I intend to review this type of modeling in more details. Interested readers in any of the two methods (optimization and simulation) could therefore refer to Ventosa et al. [2005] for a more detailed description.

2.2 Equilibrium Modeling in Electric Power Markets

As previously mentioned, the notion of an equilibrium in general is a fundamental concept that can be used in a variety of disciplines. At its core, an equilibrium is a state of the system being modeled for which the system has no "incentive" to change. These incentives are often monetary in the case of electric power systems modeling, since it most often has to do with the profit of the market participants, but it also can be physical or chemical in nature.

From a modeling perspective, identifying the equilibrium state of a system allows one to predict where the system might be in the future under certain conditions. This is not to say that it is a definite state or a completely accurate prediction, in part due to the potential multiple equilibria that might exist [1], but as well because in reality it may take a certain amount of time to transition to one of these equilibria states, especially when we are examining vast complex systems such as the electric power system. Nevertheless, knowledge of such equilibria can be instrumental in determining various characteristics of the system under consideration and could serve as a benchmark in any analysis.

The "equilibrium" notion can be viewed in many different ways, a detailed survey on all of the different conceptual methods addressing the notion of equilibrium is beyond the scope of this thesis. It is however of interest to examine the equilibrium concepts that are relevant for the energy markets context and for the present work. Specifically we can illustrate the model for "Equilibria in Dominant Actions" and the more relaxed concept of "Nash Equilibria".

2.2.1 Nash Equilibria as a relaxation of Equilibria in Dominant Actions

Equilibrium searches for the state where the system (i.e. system participants) has no incentive to change. Clearly if we project this idea on any market context, it will be equivalent to the state at which each market participant receives the best possible profit. Within this framework, it would be easy to see what "Equilibria in Dominant Actions" is referring to. We call an Equilibria in Dominant Actions (EDA) if each market participant can take actions that leads to the best outcome for herself *regardless* of what the other participants do [Shy, 1995]. This is clearly a reasonable concept when trying to predict "players" actions in a competition, but it is not without its practical limitations. This concept can be expressed mathematically as:

$$f_i(x_i^{EDA}, x_{-i}) \geq f_i(x_i, x_{-i}), \forall x_i \in F_i, x_{-i} \in F_{-i}, i = 1 : N \tag{2.1}$$

where i represents the participants "*players*" in a game, N is the number of participants, F_i is the feasible region for the decisions of player i, $f_i(x_i, x_{-i})$ is the objective to be maximized, and x is the decision variable, namely the production quantity if it is in the context of electricity markets.

A numerical example of this type of games can be seen in Shy [1995]. It is clear however that in many competitive contexts, such *dominant actions* may not exist, i.e. it might not be possible

[1] For a comprehensive description and examples of this type of games the reader could refer to Basar et al. [1995] or any documentation on Multiple-equilibria in Non-Cooperative Games.

for one or several players to find a single optimal strategy regardless of what the others do. A relaxation of this is called a Nash Equilibrium (NE). Here, each player decides her best strategy to optimize her payoff while considering what other players' strategies will be. Or in brief, it is only to assert the same sort of inequality shown in the EDA equation 2.1, but only for equilibrium values of the other players [Shy, 1995]. That is, the decision variables (x_i^{NE}, x_{-i}^{NE}) such that:

$$f_i(x_i^{NE}; x_{-i}^{NE}) \geq f_i(x_i; x_{-i}^{NE}), \forall x_i \in F_i, x_{-i} \in F_{-i}, i = 1 : N \tag{2.2}$$

This expression is what implies in the formal definition of Nash Equilibrium that there should be no unilateral incentive to deviate from such a state (x_i^{NE}, x_{-i}^{NE}). Several complications, however, often arise in the handling of equilibrium problems. A problem of this type could possibly have one equilibrium solution, which is the easiest and most straightforwards problem, but it could also have either no equilibrium, multiple equilibria, or an infinite number of equilibria. Especially in the later cases of multiple equilibria, special attention should be taken in analyzing and interpreting the achieved results.

Indeed, there exist various types and variations of a Nash Equilibrium game (e.g. Pure vs Mixed strategy games). However, this level of details might not be of an important significance for the purposes of this dissertation. It is of importance nonetheless, to examine in some details the different type of strategic interactions possibly depicted in an equilibrium model, and to understand the typical nature of this formulation and the computational methods required to solve them.

2.3 Types of Strategic Interaction in Equilibrium Models

When we talk about competition in most industries, we are often referring to strategic choices of production quantities q and selling price p that maximize each company's profit function given a set of technical and shared constraints. This description holds of course in the context of electricity markets. For simplicity moreover, we often consider that the good being traded *(i.e. in this case electricity)* is a homogeneous good; meaning that the buyer *(demand)* will not differentiate between the electricity produced from a certain producer or the other in any particular quality other than the price [2]. In addition, we assume for the purpose of the following definition, that the market operation results in a single market clearing price; this means that the result of the supply and demand interaction result in a single price being paid to all the producers who are participating in the market, regardless of each individual producer's bid. A possible variation from that is the "pay your bid" type of auctions, in which producers who enter in the market receive their bidding price only, regardless of the final market clearing price.

[2]Indeed this could be different in several models to estimate competition for instance between a "green" electricity producer (i.e. renewable sources) and a typical fossil fuel producer.

This later scheme is not covered throughout this text, and interested readers could refer to Pérez-Arriaga [2014] for examples.
We thus proceed to define several types of strategic interaction among electricity suppliers in an electricity market, most of these definitions being familiar concepts from game theory and industrial organization, [Tirole, 1988], [Fudenberg and Tirole, 1991], [Shy, 1995], . They differ in how each generating firm i anticipates that rivals will react to its decisions concerning either prices p or quantities q when submitting their own bids to the market. The main types of these strategic interactions that have been or could be included in power market models include the following:

2.3.1 Perfect Competition

There are three main characteristics that describe what we mean by a perfectly competitive markets:

1. The buyers and sellers are numerous so that no single buyer or seller can influence the market price, they are said to be price takers.
2. All agents have perfect information, with no time lags.
3. Given that all agents have perfect information, it is assumed that they make rational decisions to maximize their self interest.
4. There are no barriers to entry into or exit out of the market.
5. There are assumed to be no externalities, either positive nor negative.

Within this setting, it is assumed that every market agent would like to maximize his profit; that is, for the buyer the aim is to:

$$\underset{q}{Max} \quad \underbrace{U_d(q)}_{\text{Utility of demand}} - \underbrace{p \cdot q}_{\text{Demand expenditure}} \tag{2.3}$$

And for the seller his problem is to:

$$\underset{q}{Max} \; \underbrace{p \cdot q}_{\text{Revenue}} - \underbrace{C(q)}_{\text{Cost}} \tag{2.4}$$

It is important to note that although these are two separate problems, they remain linked by the market price p. The same unitary price is paid/received for each unit purchased/sold. It can be easily shown that taking the first order derivative for each of the equation (2.3, 2.4) and equating with zero to find the optimal solution, we end up with the famous equality:

$$\frac{dU_d(q)}{dq} = p = \frac{dC(q)}{dq} \tag{2.5}$$

or:

$$Marginal\ Revenue = Marginal\ Cost$$

This of course is the same result we obtain if we assume centralized planning of the system (i.e. no competition and the whole system is run by a single entity that does not receive profit, and with the sole aim of reducing production cost subject to certain constraints). Although indeed this is a well known result in microeconomics, it remains important to be illustrated since the perfectly competitive market result serves as the benchmark for the analysis of all other modes of competition.

2.3.2 Generalized Bertrand Strategy (Game in Prices)

We have seen that in the perfectly competitive markets, firms deal with the commodity price (electricity price) as an "exogenous" variable, and they seek to determine the quantities to be produced. This is different in Bertrand strategy, where firms use price p as their strategic variable in maximizing the profit function (2.4) given their quantity q. Although the homogeneous Bertrand case is often considered as a simple problem, it can be difficult to analyze the results obtained. Perhaps we can illustrate this type of strategies by means of a simple example [Ledvina and Sircar, 2012], as follow;

Consider a duopoly where each firm has a constant but different marginal cost, which we denote by MC_i for Firm i, where $MC_1 < MC_2$, and there is no capacity constraint for any of the firms. We illustrate the best response for the two firms, that is to charge a price while your opponent is pricing above your cost, and then to price at your opponent's price minus a small amount ϵ, which we could denote informally as $p_{(1,2)} = (p_2 - \epsilon, p_2)$. Firm 1 in this case captures all of the demand and Firm 2 receives zero profit, this would lead Firm 2 to decrease the price in response and Firm 1 responding accordingly by lowering the price by a small amount and thereby capturing the entire market at a price above cost. Until finally the only equilibrium $p^* = (MC_2 - \epsilon, MC_2)$ is reached. The same concept extends to N firms in the sense that the lowest cost firm is the only one who ever receives demand from the market.

This obviously leads to price being equal to marginal cost in case of identical producers, a result that is shared with the perfectly competitive outcome seen in section 2.3.1. However, the electricity market price almost always deviates from the standard Bertrand price game. The reason is that most often, the electricity producers competing in a market have a maximum production capacity, as well as other forms of capacity limits. This *capacity-constrained* oligopoly would lead to marginal cost pricing unlikely to be an optimal bidding strategy [Armstrong et al., 1994]. Therefore, only a few relevant references can be found in recent publications. Some representative Bertrand competition based models are summarized in Table 2.1.

Cui et al. [2012] present a model of profit maximization for utility companies in an oligopolistic market. They formulate a simulation model to study the effect of the interaction of the supply side and the consumer side in a "smart" electricity grid under dynamic pricing. They model the consumers' preference for the choice of a supplier as a Bertrand game where consumers will initially arbitrarily chose the cheapest utility company, and then both consumer and utility company will attempt to maximize their profits taking into account demand response to electricity prices. In this model, utility companies are considered as non-cooperative, i.e., always making decisions based on their own best solution. A feedback system is utilized based on consumers' reaction to task scheduling and supply selection. The model was implemented and tested on simplified systems, the results showed that all companies achieved significant improvements on their expected profit compared to the solution prior to taking into account the consumers' behavior effect.

Hu et al. [2010] define Bertrand-Edgeworth[3] auctions as a modified version of Bertrand-Edgeworth games where the demand is inelastic and a price cap is set exogenously. Bertrand-Edgeworth auctions are motivated by the discriminatory procurement auctions used in some wholesale electricity markets. They characterize the equilibrium structure for Bertrand-Edgeworth auctions with multiple asymmetric bidding suppliers. Based on a proposed numerical algorithm, it is numerically illustrated that a weak (low-capacity) bidder do not necessarily price more aggressively in an oligopoly market [Hu et al., 2010].

Bompard et al. [2008] present a medium run electricity market simulator based on game theory. The simulator incorporates two different games, one for the unit commitment of thermal units and one for the strategic bidding and hourly market clearing. They differentiate between a Forchheimer (one leader) game and a Bertrand (all players are leaders) one allowing for the simulation of markets with different levels of concentration. The simulator was applied to analyze producers' behavior during the first operative year of the Italian power exchange, while comparing the results with true market results to validate the simulator.

Finally, Federico and Rahman [2003] analyze the effects of changing auction rule from uniform pricing in the wholesale market to pay-as-bid under two polar market structures (i.e., a perfect competition or Bertrand structure and a perfect collusion or monopoly bidding) with demand uncertainty. It is found that under Bertrand structure there is a trade-off between efficiency and consumer surplus while changing to the pay-as-bid rule. Also, a move from uniform pricing to pay-as-bid under monopoly conditions has a negative impact on profits and output, a positive impact on consumer surplus, and ambiguous implications for the welfare and average prices [Federico and Rahman, 2003].

[3] Bertrand Competition but considering firms with maximum output capacity (no single firm can satisfy the whole demand)

TABLE 2.1: Representative Bertrand competition models in electricity markets.

Reference	Model Basis	Assumptions	Applications
Cui et al. [2012]	Bertrand Competition	Small system: 3 Utility companies, 1000 consumers	Modeling the consumers' demand response behavior in a smart grid in the process of determining the prices
Hu et al. [2010]	Bertrand Competition	Inelastic demand and exogenous price cap, simplified system	Profit maximization in a dynamic oligopolistic market
Bompard et al. [2008]	Forchheimer and Bertrand	Realistic representation of a system (technical minima, cost functions, flexibility, maintenance, unavailability)	Simulation model that incorporates two different games, one for the unit commitment of thermal units and one for strategic bidding and hourly market clearing
Federico and Rahman [2003]	Bertrand Competition	Perfect competition or collusion	Market design under the pay-as-bid auction rule.

2.3.3 Cournot Strategy (Games in Quantities)

The classic *Cournot* models are probably the most known and the most used scheme to model competition. In fact, the study of non-cooperative oligopolistic competition originated with the seminal work of Cournot [Cournot, 1838]. His original model assumed firms choose quantities of a homogeneous good to supply and then receive profit based on the single market price as determined through a linear inverse demand function of the aggregate market supply, where marginal costs of production were assumed constant and equal across firms [Ledvina and Sircar, 2012]. The result of Cournot's analysis is what now considered common knowledge: with equal marginal costs across firms, every firm chooses the same quantity to supply and the market price is above cost by an amount that is inversely proportional to one plus the number of firms in the market. Hence, as the number of firms tends to infinity, the price approaches marginal cost, but with a finite number of firms, prices are above cost and firms earn positive profits.

Unlike Bertrand, the Cournot's strategy assumes that firms set their prices p and then compete by choosing their quantities q. This is represented through the inverse demand function $p_i(q)$ for each firm i, which is equal to the change in consumer's response due to the changes in the quantities produced or $\frac{\partial U}{\partial q_i}$ (the rate of change of utility with respect to the rate of change of quantities supplied). Briefly this can be seen formally through: for firm i, revenue =

$$p \cdot q_i = p(q) \cdot q_i = p(q_i + q^*_{-i}) \cdot q_i \tag{2.6}$$

where $p(q)$ is the inverse market demand function discussed above, and q_{-i} is the quantity supplied by all firms other than i. The asterisk in q^*_{-i} means that firm i chooses its strategy while assuming that the quantities chosen by its rivals are known and will not change, and thus it is a fixed value. We can therefore deduct that for each firm to maximize its profit , the first-order optimality conditions will have the following marginal revenue term:

$$\begin{aligned} MR &= \partial(p \cdot q_i)/\partial q_i \\ &= p + (\partial p/\partial q)(1 + \partial q^*_{-i}/\partial q_i)q_i \\ &= p + (\partial p/\partial q)(1 + 0)q_i \\ &= p + (\partial p/\partial q)q_i \end{aligned} \tag{2.7}$$

Although intuitively Cournot competition may not seem the most applicable, in the sense that one would not expect that in a real scenario companies would be setting their quantities instead of their prices, it can be seen through the vast literature produced using both models that indeed Cournot results in a more realistic observation albeit its eccentric assumption [Ledvina and Sircar, 2012] [Dastidar, 1995]. We have seen that following Bertrand's model under the assumption of a single market price leads to very harsh and contradictory results relative to common market observations. If all firms have equal cost, then as long as there are two or more firms in the market, all firms price at cost and have zero profit. This perfectly competitive outcome differs substantially from the Cournot outcome and is commonly referred to as the Bertrand paradox. In general one could think of it that in most settings, the correct set-up of Bertrand leads to the wrong result, while Cournot model gives the right answer for the wrong reason.

Perhaps most importantly after this brief description of Cournot competition is to examine the use of this method for the modeling of competition in electric power systems. There exists a vast literature on this matter that perhaps it would be difficult to comprehensively cover it in this section. Ventosa et al. [2005] presented us with a thorough literature review on the type of issues addressed in electricity market modeling through Cournot competition. In general, they have shown that these models have been extensively used to address *market power analysis*, *hydrothermal coordination*, *effect of network congestions*, and *risk analysis* for competing firms to name a few [Ventosa et al., 2005]. However, the aim here is to focus on the most significant and recent publications, and especially the publication that explicitly consider the effect of the electricity network on the market outcome.

Table 2.2 summarizes some of the most recent and most relevant literature I have been able to find that considers Cournot competition in electricity market modeling. The emphasis was primarily on those publications which explicitly considers the power network flows and its effects on the competing market players. As summarized, Parente et al. [2012] considers a Nash-Cournot equilibrium model in an oligopolistic market. Some key features of their model are short-term planning horizon, pumped storage hydro units representation, and transmission network constraints. They consider a system composed of thermal and pumped-storage hydroelectric power units, sharing a bounded capacity network. They proceed to develop a mathematical model to solve the unit scheduling, the methodology applied is a variable metric proximal decomposition method, they show that this method is effective for small-sized and medium-sized network. Their results show how the production patterns change in water-stressed situations compared to excess water availability especially due to pump-storage units production, as well as due to modifications in network line capacities. Lee [2013] considers a probabilistic Mixed-Strategy Nash Equilibrium problem for analyzing suppliers' strategic generation quantities under transmission line constraints. They present a heuristic method to solve the optimization problem with a set of linear and nonlinear equations arising.

Badri and Rashidinejad [2013] considers a day ahead oligopoly market, whereas a multiperiod auction framework is addressed to simulate market clearing mechanism by means of social welfare maximization, in which the behaviors of market participants are modeled through perceived block curves. The formulation explicitly considers demand eleasticity and transmission security constraints. Likewise, they introduce a heuristic method to solve the model. They model a 24 hours effective-supply curves for the GenCos and show the differences in the strategic bidding under Cournot competition and under perfect competition scenarios. Zhang et al. [2012] and Zhang et al. [2009] addresses the electricity market equilibrium problem in both spot and forward markets. Zhang et al. [2012] addresses the market equilibrium when Gencos sell financial call options or buy financial put options. The Cournot equilibrium model developed takes into account the financial contracts, and the reliability of the generation units (through the forced outage rate). The model also considers the uncertainty in load demand. The results show that call options with relatively low strike prices or put options with relatively high strike prices could be helpful to reduce Gencos' interests to raise market price by strategic behavior. Zhang et al. [2009] on the other hand considers competition in spot and forward markets, where they formulate is as a two-stage game. First level considers the optimal strategies for the Gencos in the spot market, and then the second level is solved considering the GenCos strategies in the forward market. They formulate the problem using two competition representations; namely Cournot and Supply Function Equilibrium, and they propose a co-evolutionary algorithm to solve this two stage problem and apply it on two case studies; one with three GenCos and the other with five GenCos. They conclude that the decision for the companies on whether or not to enter the forward market depends significantly on what type of competition governs the spot market.

Siriruk and Valenzuela [2011] primarily focus on the uncertainty modeling in deregulated electricity markets through Monte Carlo simulations, and its effect on the expected profits for the firms in order to underline its importance for being taken into consideration in firms' decisions. They model a stochastic Cournot Model considering uncertainties in fuel costs and unit outages and present a case study for 3 firms competing owning several hydro and thermal units each.

Ruiz et al. [2008] focus on the computational aspects for a Cournot competition model, their work provides some theoretical results pertaining to the Cournot model applied to short-term electricity market. Price, quantities and profits formulation are presented . These are applied to a simple case study with three identical GenCos, and in another case with only one dominant generation company. They demonstrate the results obtained from their formulation and conduct a sensitivity analysis of the price with respect to the proposed formulation's parameters. Finally, Mookherjee et al. [2008] discuss the dynamic nature of power networks. They propose a more general Nash-Cournot competition model on power networks that explicitly accounts for intra-day dynamics that describe the markets' evolution throughout 24 hour planning horizon, raming constraints and costs for changing the power output of generators, and joint constraints that include variables from other generating companies within the profit maximization problem for individual generators.

Table 2.2: Representative Cournot competition models in electricity markets.

Reference	Model Basis	Assumptions	Applications
Parente et al. [2012]	Classical Nash-Cournot	Short term planning horizon, hydro reservoir and pumping units, representation of network constraints	Scheduling of hydroelectricity production considering all the market players and network constraints. Application on a small-size and medium-size networks.
Lee [2013]	Mixed Strategy Nash-Cournot	Probabilistic Generation outputs and transmission lines constraints.	Two-level hirerchical optimization problem to model competition among multiple participants.
Badri and Rashidinejad [2013]	Cournot based model	Uniform price market. All generators bid with three blocks piecewise increasing curves. All elastic loads bid with three blocks piecewise decreasing curves. DC power flow.	Day ahead optimal bidding strategy of GenCos in oligopolistic power markets taking into consideration the impact of transmission security constraints on paritcipants' market power.
Zhang et al. [2012]	Cournot Competition	Oligopoly in a pool-based electricity wholesale market. Linear demand representation. Consideration of Generation units' forced outages.	Considers the electricity market equilibrium with contractual arrangements (financial options).
Siriruk and Valenzuela [2011]	Cournot Competition	Private contractual agreement for power delivery at future time period. Representation of the operation state of the units through Markov process.	Effect of uncertainty in deregulated electricity markets, related to unit outages and fuel cost.

Downward et al. [2010]	Cournot Competition	Electricity pool markets. Radial transmission networks. Linear demand Function.	Application of a novel methodology to find the optimal power dispatch to the New-Zealand electricity transmission network.
Zhang et al. [2009]	Cournot and Supply Function Equilibrium models	Two-stage game model. Forward market and spot market competition. Risk neutrality on all generation companies.	Novel coevolutionary approach to model the generation companies strategic behavior in choosing among spot and forward markets. Case studies with 3 gencos and with 5 gencos.
Badri et al. [2009]	Cournot Competition	Transmission constraints, limited number of electricity producers. Representation of the independent system operator through solving an optimal power flow.	Strategic bidding for GenCos. Bi-level optimization model proposed. Application on an IEEE-30 bus test system.
Ruiz et al. [2008]	Cournot competition.	Competition in short term electricity market.	Provides closed formulae for price, quantities and profits for a short-term electricity market whose participants rely on Cournot models.
Mookherjee et al. [2008]	Generalized Nash-Cournot	Dynamic formulation for Gencos competition. Ramping constraints and costs for changing the power output of generators. Joint constraints between GenCos.	Bidding strategy for competing Gencos under dynamic Cournot competition.

2.3.4 Cournot Variation (I): Stackelberg models

We have seen in section 2.3.3 how in Cournot models it is assumed that all the competitors in a market have equal opportunity. Meaning that whenever there is a market opportunity (in our case opportunity for electricity production and sales), companies have equal information, equal positioning and in general can equally respond to market demand limited only by their own intrinsic constraints (production capacities, production costs or any other internal factors). However, in some cases, perhaps due to historical, institutional or legal factors, the competitors are placed into a differential or inequitable position in the market. For example, the firm that discovers and develops a new product would have a natural first-mover advantage, or an uneven access to information could also lead to the same situation.

Heinrich von Stackelberg presented an important oligopoly model in 1934 [Von Stackelberg, 1934]. In the Stackelberg model, one firm acts before the others. The leader firm picks its output level and then the other firms are free to choose their optimal quantities given their knowledge of the leader's output. The follower's best response is determined as in the Cournot model that competitors' output is assumed to be fixed. However, the leader's action is different from the followers' actions in Cournot model. Perhaps it is of interest to consider the formulation of the Stackelberg model as follows: consider an oligopoly where firms have generating marginal cost; $MC_i = m_i q_i$, and the inverse demand functions of the market is $p_i(q_i)$ *or* $p_i = a - r(q_i + q_{-i})$ if we consider a linear form. The leader firm is considered to be F_i, while F_{-i} are the followers. The followers' best response condition in terms of profit π is as follows:

$$\frac{\partial \pi_{-i}}{\partial q_{-i}} = \frac{\partial}{\partial q_{-i}}\{p \cdot q_{-i} - C(q_{-i})\}$$

$$= \frac{\partial p}{\partial q_{-i}} \cdot q_{-i} + p - m_{-i} q_{-i} \tag{2.8}$$

This would result in an optimal quantity production for F_{-i} denoted as q^*_{-i}, the leader's firm F_i's revenue in this case is expressed as:

$$\pi_i = p[q_i + q^*_{-i}(p)] \cdot q_i{}^{4} \tag{2.9}$$

To illustrate how this leader-follower problem might result in different outcomes compared to the classical Nash-Cournot formulation, Anderson and Engers [1992] compare the two models considering the Stackelberg leader-follower problem in perhaps its most extreme case; where each firm is preceded by a *leader* firm, in what they refer to as the *Hierarchical Stackelberg Model*. They analyze two distinct interactions in which i firms choose outputs. In the first, hierarchical Stackelberg is considered, where the equilibrium is the sub-game perfect outcome that arises when firms choose their outputs sequentially according to some exogenously determined order of moves. In the second, the equilibrium when firms choose their outputs simultaneously. They show that for a linear demand case, each firm under the Stackelberg scheme earns half the profit of its immediate predecessor, however they highlight that such interpretation for the profit is not straightforward and requires careful analysis.

Stackelberg indicated that generally, history, institution, law or development can all be factors affecting the determination of the leader and the followers in any industry. But we are more interested in understanding how this could arise specifically in the electric power industry. Lee [2014] argues that one of the most important factors for this differentiation in the electric power markets is the transmission network itself. He argues that the physical limits of transmission line can restrict the economic dispatch of the generation power, the generation firms would change their strategies by depending on the site with respect to the congested line. He proceeds to compare the social welfare in terms of surplus analysis by both Stackelberg model and classical Cournot.

Table 2.3 summarizes some of the most recent work implementing Stackelberg model. Nekouei et al. [2014] illustrates a game-theoretic model for demand response from an electricity market

[4] Notice how this result is different than the one obtained in 2.6

perspective. They formulate a Stackelberg game to capture the interplay between a Demand Response Aggregator (DRA) and electricity generators, where the DRA acts as the leader of the game and makes demand reduction bids by taking into account their profitability. The generators are considered as the followers compete for power generation in the wholesale electricity market which is modeled as a strategic game. Based on the results obtained, they have shown that the peak demand period in a highly concentrated market is potentially the only profitable scenario for the demand response. Avila and Behnke [2013] on the other hand considers the problem of generation expansion planning and new entrance in the market under a Stackelberg multi-leader multi-follower approach. The oligopoly is modeled as a group of leader companies, with the first option to invest, facing potential new investors to enter the market. They apply their formulation on a case representing the Chilean power system, and the results suggest that the market power of the oligopoly depends on its ability to control the most profitable expansion technologies.

Campos et al. [2012] study the effect of increasing or decreasing the numbers of leaders and followers in a Stackelberg game. They represent the Stackelberg model in the European Electricity Market (EEM) with price response conjectures for both leaders and followers, and propose a simple convex quadratic optimization problem for solving the model. Bompard et al. [2010b] consider several game theory formulations, namely Cournot, Stackelberg, Conjectur supply function, and supply function equilibrium to assess the performances of network constrained electricity markets. The competitive markets have been analyzed and tested by using the IEEE 30 and 57 bus systems. They measure the market performance using Lerner index and market inefficiency index. They show that Cournot model demonstrated the worst behavior, both under constrained and unconstrained networks. They conclude with an illustration of how the transmission network plays an essential role in determining the market equilibrium. Under constrained network, the simulation results demonstrated that the market clearing price is higher and the cleared demand are lower than the corresponding values under unconstrained network.

Table 2.3: Representative Stackelberg competition models in electricity markets.

Reference	Model Basis	Assumptions	Applications
Nekouei et al. [2014]	Stackelberg, Single-leader Multi-followers	Interaction between Demand Response Aggregators (DRAs) and GenCos. Where DRA acts as the leader and generators act as followers.	Game theoretic model for demand response in electricity market constext. Shows how generators adjust their production levels due to demand response.

Avila and Behnke [2013]	Multi-leader Multi-followers	Generation expansion under imperfections such as indivisibility of projects, risk aversion and oligopolies investing strategically.	Application on the Chilean power system.
Chang et al. [2013]	Single leader, Single follower	Social welfare function is considered composed of a consumer's suplus, producer suplus and the environmental damage function of green house gas. Linear demand function assumed.	Analysis of feed-in-tarifs scheme and its effect on social welfare. Two power plants considered: one that generates power by using traditional fossil fuel, the other generates power by using renewable resources.
Campos et al. [2012]	Multi-leader-follower conjectural Stackelberg equilibrium model.	Considers price-response conjectures.	Proposes a simple convex quadratic optimization problem to analyze how market competition changes with the number of leaders or followers.
Bompard et al. [2010b]	Stackelberg, Cournot, Supply Function Equilibrium (SFE), Conjecture SFE.	Considers the ISO as simplified optimal power flow problem.	Studying the effect of network constraints on generation bidding strategy and market price. Comparison between several game theory representation. Application on the IEEE 30 and 57 Bus test systems.
Lavigne et al. [2000]	Stackelberg, Monopolistic pricing, and Perfect Competition.	Employment of MARKAL (Market Allocation) model to model both supply and demand sectors.	Propose an original heuristic decomposition techniques to compute pure competition and Stackelberg equilibria. A large-scale application to the energy system of Quebes is reported and discussed.

2.3.5 Cournot Variation (II): General Conjectural Variations (CVs)

In all the previous types of competition examined, we have always assumed that when a firm decides its optimal output which would maximizes its profit, the decisions of the other firms are considered *fixed*. This means that the firm indeed assumes that all other rivals will maximize their profits implementing similar logic, but that these decisions or *outputs* would not change particularly due to changes in its own decisions. This might of course seem as an oversimplification of real behavior, albeit -as seen- being fairly a good approximation in many cases especially when considering general outlines for the market agents' behaviors.

Bowley [1924] sought to improve on this point by introducing the concept of *conjectural variations*[5]. The idea is that a firm in an oligopolistic market believes that the quantity (or price) it chooses will affect the quantities (or prices) chosen by its rivals. This belief is taken into account by the firm when selecting the profit maximizing output level. The reactions of the rivals to

[5]Whereas in fact the term *conjectural variation* was coined in Frisch [1933]. [Dockner, 1992]

the output decision of firm i, as subjectively perceived by firm i, is what is called *conjectural variation*. To keep the convention we can simply represent this concept mathematically as:

$$p \cdot q_i = p[q_i + q_{-i}(q_i)] \cdot q_i \tag{2.10}$$

Here we can see that the output $q_{-i}(q_i)$ from all firms other than i is assumed to be a function of q_i, or in other words the rivals are assumed to be reacting to the decision taken by the firm. Again, if we take the first order optimality condition for this revenue function to find the profit maximization expression we get:

$$\begin{aligned} MR &= \partial(p \cdot q_i)/\partial q_i \\ &= p + (\partial p/\partial q)(1 + \partial q_{-i}/\partial q_i)q_i \\ &= p + (\partial p/\partial q)(1 + \theta)q_i \end{aligned} \tag{2.11}$$

The term θ represents the constant conjectural variation that the firm assumes will be the reaction of its rivals. Indeed the estimation of this term is a complicated task and I intend to briefly discuss it in a later part of this section. It is of course easy to see that if θ becomes equal to zero, the marginal revenue in equation 2.11 becomes $= p + (\partial p/\partial q)(1 + 0)q_i$ which is exactly the Cournot result obtained in 2.7, whereas if θ is equal to -1 we obtain $MR = p$ which is the perfectly competitive result we have seen in 2.5. Any value in between [0,-1] gives an oligopoly behavior which is less aggressive than the Cournot model (which is regarded to represent the extreme case of exercise of market power) but also less competitive than the perfect competition model. Therefore, the *conjectural variation* model is very useful if one wants to have the flexibility to represent different extents of market power exercised, although this model also comes with a fair amount of criticism and limitations. The main criticism for the conjectural variation concept, is that when Bowley introduced it he clearly had in mind a dynamic phenomenon, although the analysis is a static one [Dockner, 1992]. Traditionally the main criticism was summarized in the work of Friedman [1983] where he criticizes the conjectural variations analysis in static models and lists several arguments against it:

1. The model is not actually dynamic, thus a dynamic interpretation is not correct.
2. The firms are assumed to maximize one-period profits rather than the discounted stream of profits over a given planning horizon.
3. Firms have incorrect expectations about how their rivals will behave.

The third point especially received a lot of attention in literature, where many researchers have tried to improve on the model in this particular point, giving rise to what is called “consistent conjectural variation”. According to Perry [1982], a conjectural variation is consistent if it is equivalent to the optimal response of the other firms at the equilibrium defined by that conjecture. In his work he includes many references that have addressed this issue, including his own. Cabral [1995] on the other hand presented a view where many authors see that despite the theoretical shortcoming for the conjectural variation model in representing dynamic games, it is still a useful tool for many practical purposes if considered as a reduced form of the real model. He elaborates that conjectural variations are “best interpreted as reduced form parameters that summarize the intensity of rivalry that emerges from what may be complex patterns of behavior”; in particular, from “the equilibrium of an (unmodeled) dynamic oligopolistic game” [Farrell and Shapiro, 1990]. He proceeds to prove this idea himself where he models an explicit dynamic game of which the CV solution is an exact reduced form. The dynamic game is a quantity-setting oligopoly with the equilibrium under the following rules: in each period, each firm produces some designated quantity. If in any period one firm alone deviates from its designated action, then it is minimaxed for T periods (thus receiving a payoff of zero). For some given value of the discount factor δ, there exists a unique solution (optimal equilibrium) that maximizes total profits. His results assert that for each value of δ in a given open set, there exists a value of conjectural variation (in our convention it is designated θ) such that, *for any linear oligopoly structure* each firm's quantity along the optimal equilibrium path is equal to that firm's quantity in the CV solution [Cabral, 1995]. This result although limited to the case of linear oligopolies and for a particular class of equilibria of the dynamic game, still provides a very important insight on the capabilities of conjectural variations models.

Since the conjectural variation model is the basis of this dissertation, it is important to carefully navigate through the recent literature to review what have been done in electricity market modeling using this technique. Díaz et al. [2010] reviews and analyses the main formulations of conjectural equilibria applied to electricity markets. They illustrate five main different types of conjectural variation formulation: (1) Cournot conjecture approach (which is basically the CV model when we assume that the price conjecture is equal to zero), (2) conjectural variation approach, (3) conjectural supply function approach where the conjecture for the firm i is the derivative of the competitors' production with respect to market price (further discussed in subsection 2.3.6, (4) conjectural demand elasticity approach where the conjecture is the relative residual demand curve elasticity, and (5) conjectural price response approach where the conjecture is the derivative of the market price with respect to the changes in production quantities. Díaz et al. [2010] noted that all these formulations (2 to 5) are equivalent, and that the conjecture of any approach can be computed from that of any other. They also summarize the literature on the publications that considers the conjectures as an exogenous variables, and those which provides ways to calculate the conjectures endogenously in their models.

Table 2.4 summarized some of the most recent publications which makes use of the CV method. First, we could see that some conjectural variations models allow for uncertainty analysis through the introduction of stochastic variables, such as the work of Barquin et al. [2005] and Campos et al. [2008]. Barquin et al. [2005] provide a stochastic representation of market equilibrium including its equivalence as an optimization problem. The main sources of stochasticity in their model have been identified to be: uncertainty in hydro inflows, fuel prices, system demand, generating units' failures and competition behavior. They included the stochastic variables through a scenario tree, and solve a case study with stochastic inflows showing the model application. Campos et al. [2008] on the other hand propose a market model to represent agents' strategic behavior in energy markets, by making use of what they refer to as "possibility distributions" to model the agents' residual demand curve (RDC) which is considered as uncertain [6], taking into account the risk-averse attitudes of the agents. The authors argue that the "possibilistic" model is easier to solve than its probabilistic counterpart. The paper also proposes a new and globally convergent Variation Inequality algorithm to solve the complementary equilibrium constraints. They provide a case study including around 200 generation units and more than 100,000 constraints and variables, and show that it can be easily solved using their proposed method.

Most importantly and most relevant to the work presented in this dissertation are the works of Delgadillo et al. [2013], Delgadillo and Reneses [2013], Campos et al. [2014], and Delgadillo and Reneses [2015]. In fact, the model presented in this presentation is primarily extracted from the models proposed in Delgadillo and Reneses [2013] and Campos et al. [2014]. Therefore, an important emphasis is placed on these models which will be fully described in details in the coming chapters. However, it is important to give here an overview of these models, since what they share is that they consider the effect of the network constraints on the market outcomes in different settings. Delgadillo et al. [2013] presents a conjectural-variation based equilibrium model of a single-price electricity market taking into account the effect of network congestion on the behavior of market agents. The single-price market is the one used in most of the European countries, where suppliers are compensated uniformly with the same market price (most often the marginal bidding price of the most expensive committed bid) in each "zone". This differs from the "nodal" pricing system where each zone is further broken down into many network nodes, and each one of these nodes represents a unique price for the generation and demand concerned with this node [7]. This scheme is the one implemented for example in the United States. The authors assume inelastic demand and provide a simple case study of only two areas (zones) to study the effect on network congestion on the bidding strategy of the agents in both zones. An important feature of their model is that the market equilibrium equations are formulated as an equivalent minimization problem taking into account the network congestion.

Since if we only consider two areas power system (a sending area and a receiving area) as in the work of Delgadillo et al. [2013], there is no need to solve a power flow model to represent the network (as it consists of one line only). Delgadillo and Reneses [2013] expands on this

[6]The uncertainty stems from both the uncertainty in the competitors' response and of the demand curve

[7]For a comprehensive explanation for the difference between the two schemes the reader is advised to refer to Pérez-Arriaga [2014]

concept by modeling a DC optimal power flow for the network representation, to allow for the analysis of a model consisting of several sending and receiving areas instead of just one. The main difference is that the network congestion is solved through modeling an optimal power flow problem (which represents the System Operator's control over the network security) to figure out which lines are congested and how this congestion should be solved. The authors also studied the effect of this network congestion on the bidding strategies of the suppliers, yet this time solved on a three-areas network [8]. Campos et al. [2014] provides a similar study to support interconnectivity analysis for weakly connected systems, by considering the effect of network congestion on the exercise of market power for two-zone electricity market. They propose for their analysis a conjectural variation based model solved using Mixed Integer Programming.

TABLE 2.4: Representative Conjectural Variations models in electricity markets.

Reference	Model Basis	Assumptions	Applications
Barquin et al. [2005]	Conjectural Variation	Uncertainty in hydro conditions, demand, generating units' failures and fuel prices.	Medium term generation scheduling planning considering uncertainties in market and system conditions.
Liu et al. [2007]	Consistent conjectural variation	Considers the conjectural variation term as a decision variable and assume that in an oligopoly, supplier's objective is to maximize the discounted stream of profits over an infinite planning period.	Studies the dynamic oligopolistic competition to explore the unique property of consistent conjecture variation equilibrium in electricity markets.
de Haro et al. [2007]	General conjectural variation	Implicit estimation of the conjectural variation term assuming marginal cost consisting of: fossil-fuel prices, thermal efficiencies, hydrothermal water value, unit commitment cost and capacity payments.	Suggesting a parameter inference methodology for conjectural variations based models called "advande inference estimation".
Campos et al. [2008]	General conjectural variation	Considers the nonlinear residual demand curves to be represented by an uncertainty distribution depending on the production of generators such as a probability or a possibility distribution.	Considers the risk averse attitudes of generation companies through uncertainty analysis. Also, proposes a new and globaly convergent Variational Inequality algorithm to solve the equilibrium constraints. Execution on real size problems.

[8] A detailed representation of this model and all the concepts mentioned will be provided in later chapter as they are the basis of this present work.

Vali and Kian [2008]	Conjectural Variation	Dynamic form of quantity setting conjectural variations. Linear inverse demand function.	New model of strategic bidding via learning quantity setting conjectural variations for electricity markets.
Díaz et al. [2010]	Cournot, Conjectural Variation, Conjectural Supply function, Conjectural demand elasticity, Conjectural price response	-	Reviews and compares different types of conjectural variation formulation applied to electricity markets' studies.
Ruiz et al. [2010]	Conjectural Variation	Assumes all producers have identical cost functions. Assumes linear demand.	Analytical method for the analysis of: the influence of changes on the reaction parameters of the producers -in cooperative and non cooperative manners - on the market outcomes, comprehensive analysis of the case in which all producers are identical.
Alikhanzadeh and Irving [2012]	Conjectural Variation	Assumed demand (and supply) curves for the counterparty of the bilateral market.	Bilateral market modeling combining a conjectural variations equilibrium model of an oligopolistic set of generators with a corresponding oligopsonistic equilibrium model of a set of supply companies.
Delgadillo et al. [2013]	Conjectural variation	Effect of congestion in network lines. Two area system with one network line.	Studies the effect of the congestion between areas on the agents' strategic behavior in a single-priced electricity market.
Delgadillo and Reneses [2013]	Conjectural variation	Network representation by a DC optimal power flow. Assumed costs for requirements of increased or decreased generation for solving network congestion.	Conjectural-variation based model of a single-price electricity market considering network congestion's effect on GenCos strategic bidding.
Campos et al. [2014]	Conjectural Variation	Single zone and two zones markets. Linear supply biding curve. Transmission network congestion.	New mathematical model for two-zone electricity dual pricing computations using mixed integer linear programming.
Kalashnikov et al. [2014]	Consistent conjectural variation	Relaxation of the assumption of continuous differentiability of the demand function. Quadratic structure of the agents' cost functions.	Study of the CV equilibrium in a model of mixed oligopoly, establish the existence and uniqueness results for the CV (called *exterior equilibrium* for any set of feasible conjectures and introduce the notion of *interior equilibrium* by developing a consistency criterion for the conjectues.

Delgadillo and Reneses [2015]	Conjectural Variation	Network representation by an AC optimal power flow to examine voltage level requirements in the network.	Studies the voltage requirements on companies' strategic behavior in a single-price electricity market.

Finally, Delgadillo and Reneses [2015] as well studies the effect of network congestion, but this time to analyze the voltage requirements of the network and how it affects the strategic behavior of the suppliers in a single-price electricity market. To represent the voltage requirements, the paper provides an AC optimal power flow for the network modeling. The conjectural-variation equilibrium model is cast as a mixed complementarity problem instead of the equivalent optimization problem previously proposed. A numerical example is provided illustrating three nodes connected by three transmission lines and an analysis is provided for several cases of different generation units' ownership distribution by the competing companies in these three nodes.

2.3.6 Conjectural Supply Function Equilibrium (CSF)

We have seen in subsection 2.3.5 how in conjectural variation competition, it is assumed that the output of rival companies $-i$ respond to the output of firm i according to the function $q_{-i}(q_i)$ in the equation 2.10. I.e. we have assumed that companies' reaction is interpreted in terms of "quantities" produced much like in Cournot competition. Conjectural Supply Function equilibria (CSF) models differ in that they posit a response in terms of "prices", i.e. output by rival companies is anticipated to respond to *price* according to the function $q_{-i}(p)$. As a result, the revenue of firm i in a conjectural supply function model is calculated according to:

$$p \cdot q_i = p[q_i + q_{-i}(p)] \cdot q_i \tag{2.12}$$

Conjectural supply function is thought to have been applied to the study of market power for the first time in the electricity sector in the work of Day et al. [2002]. Day et al. [2002] argue that CSF models give modelers the flexibility to consider more realistic supply responses than any of the previously mentioned market representations. They affirm that CSF suffers from the same limitations of the general conjectural variation approach; that is any particular supply response assumption will be somewhat arbitrary albeit it is possible to empirically estimate it from historical market prices and marginal costs. They further argue that CSF models do not distort the consumption behavior in response to high prices assumption; such as what is observed when we rely on Cournot representations with artificially high and arbitrary elasticities to simulate more intensive competition. In these Cournot models it is noted that demand decreases when prices are high, when in actuality demand would not change, leading to a misrepresentation for the economic and environmental market outcomes [Day et al., 2002].

Table 2.5 summarizes some of the most recent studies that address the effect of network congestion on the electricity markets' agents behavior implementing the CSF approach. Hobbs and Rijkers [2004] and Hobbs et al. [2004] combined represents a long study of the effect of different network pricing schemes for congestion management on the oligopolistic behavior of generation companies. They generalize the CSF model to include each generator's conjectures concerning how the price of transmission services will be affected by the amount of those services that the generator requires. Different transmission system pricing policies are examined to provide insights on their suitability and efficiency in terms of distortions they cause on market power, they include [Hobbs and Rijkers, 2004]:

- Congestion pricing of networks, in which load flows are approximated by a linearized DC model.
- Auctions of capacity along administratively predetermined paths, where these paths can differ from the actual power flows.
- Export fees or other fixed (per megawatt hour) network use tariffs that have no necessary relationship to congestion costs.

Another purpose of these papers, is to illustrate how a network-constrained market equilibrium model formulated as a mixed complementarity problem (MCP) can be derived from models of generator, Transmission System Operators (TSOs), and arbitrager behavior. In [Hobbs et al., 2004], the authors apply the model to the northwestern Europe including the interconnection between Belgium, the Netherlands, France, and Germany. Their application shows that the inefficiencies in allocation of network prices can significantly exacerbate the effects of market power. The second application is to show how generators might expect that transmission prices will change when demands for transmission services are altered, they illustrate that such expectations can lower revenues to transmission providers and weaken effective competition.

Barquín et al. [2009] propose a model to compute CSF equilibrium in an electricity nodal market with limited transmission capacity. They propose an optimization algorithm based on a Gauss-Siedel approach to compute a fixed point, which if it converges to that point then market equilibrium conditions are fulfilled, and therefore a market equilibrium is assumed to be found. The model is applied to a case study consisting of a two-area system and a yearly time horizon is analysed. The case is aimed to resemble the Iberian electricity system. They conclude that the model converges to one equilibrium in some cases, while keeps cycling in others as it is considered the conjectured price response as a function of congestion. Díaz et al. [2012] seek to extend on the type of studies presented in this section, by suggesting a method for the endogenous computation of CSF, instead of the typical consideration as an exogenous parameter, or estimation from historical data. They include the network congestion effect through DC optimal power flow formulation, and propose a method for calculating the parameters of the linear supply function approximation for the generators' supply curve by solving two equilibria at two demand scenarios close enough, consistently with the network line congestion. In this sense, the generators' stratgies adapt to the market structure as well as to the network status, providing more realistic results for long term analysis under different market scenarios. The behavior of the proposed algorithm is analyzed with a simplified version of the Iberian electricity market and a with a meshed three-node electricity market with a close-loop interconnectinon system [Díaz et al., 2012].

TABLE 2.5: Representative Conjectural Supply Function models in electricity markets.

Reference	Model Basis	Assumptions	Applications
Day et al. [2002]	Conjectural Supply Function (CSF)	Afine demand and supply functions. Nodal pricing network scheme. Existense of Arbitrageur to eliminate price difference between nodes. Generators are price-takers with respect to trasmission.	Formulation application on a case of bilateral-contract market with arbitrager, and a case with a PoolCo market. Application to the England-Wales market with and without taking into account trasmission constraints.
Hobbs and Rijkers [2004]	Conjectural Supply Function (CSF)	Considering network representation (nodal scheme) under different assumptions for a mixed transmission pricing system: fixed transmission tariffs, congestion-based pricing, auctions of interface capacity.	Part I: Formulation of the conjectural supply function problem and formulation of the solution technique as a Mixed Complementarity Problem.
Hobbs et al. [2004]	Conjectural Supply Function (CSF)	Considering network representation (nodal scheme) under different assumptions for a mixed transmission pricing system: fixed transmission tariffs, congestion-based pricing, auctions of interface capacity.	Part II: Application of the proposed method to northwest Europe case study.
Barquín et al. [2009]	Conjectural Supply Function (CSF)	CSF assumed to be linear with constant slopes. Conjectured price response assumed to be dependent on the system line's status (congested or not congested).	A procedure based on solving an optimization problem is proposed. Application on a two-area multi-period case study.

Díaz et al. [2012]	Conjectural Supply Function (CSF)	DC optimal power flow for transmission network representation.	Endogenous calculation of CSF response of generators under network congestion. Application on a simplified version of the Iberian electricity market (MIBEL Spain-Portugal).

2.3.7 Supply Function Equilibria (SFE)

Supply Function Equilibrium poses a fundamental difference from both Cournot and Bertrand based models previously discusses. Originally proposed by Klemperer and Meyer [1989], supply function equilibrium (SFE) posit the most general and most natural behavior of firms in a competitive environment. Klemperer and Meyer [1989] argued that an oligopoly facing competition in an uncertain environment (uncertain demand) would prefer to set supply functions (both quantities as a function of price or vice versa), instead of competing only in prices (Bertrand competition) or in quantities (Cournot competition). They observed that under such uncertainty, and regardless of how the other firms decided to behave, the residual demand facing the firm is not known in advance, and hence each firm has a variety of best response actions (profit maximizing points) each corresponding to every possible residual demand. If firms will decide on their strategies in advance of the realization of demand, then they are better off specifying an entire supply curve (price-quantity pairs) rather than a single price or quantity. SFE of course is not without its limitation. I intend to provide a brief overview of the SFE model, its limitations, and some of the most notable application of this model in a context where network constraints are taking into consideration in this section.

It is well known that SFE was introduced by Klemperer and Meyer [1989], and that it was first used in the context of electricity markets by Green and Newbery [1992]. They analyzed competition in the British electricity spot market using SFE approach and built the model to accommodate for the circumstances of the British electricity sector at the time of privatization. They assumed in their work two strategic generators who submit continuously differentiable supply functions to the pool, rather than discrete step functions, and that the market equilibrium was the static one-shot supply function equilibrium [Green and Newbery, 1992] [Von der Fehr and Harbord, 1998]. To illustrate SFE consider the following; we are going to assume a linear demand function, a quadratic convex cost function, and a linear supply function such as: cost function of firm i

$$C_i(q_i) = \alpha_i q_i + \beta_i q_i^2, \qquad \beta_i > 0 \tag{2.13}$$

Also assume firm i to bid a linear increasing supply function with two strategi parameters: intercept θ_i and slope γ_i such that:

$$p = \theta_i + \gamma_i q_i, \qquad \gamma_i > 0 \tag{2.14}$$

And assume a linear decreasing demand function. The profit for firm i :

$$\pi = p \cdot q_i(p) - C_i(q_i) \tag{2.15}$$

Each firm optimizes its parameters, intercept θ_i and slope γ_i to maximize its profit, while the market-clearing condition is satisfied (total production = total demand). The supply function equilibrium implies that no player can increase its profit by unilaterally changing its bid supply function. To solve the profit maximization problem, we set the first-order profit derivative to be zero for all firms:

$$\frac{\partial \pi_i}{\partial p} = q_i(p) + \left[p - \frac{\partial C_i(q_i)}{\partial q_i}\right] \frac{\partial q_i(p_i)}{\partial p} = 0, \qquad \forall i \tag{2.16}$$

Combining this equation with the market clearing condition $Demand(Q) = \sum_{i=1}^{I} q_i$ we obtain:

$$q_i(p) = \left[p - \frac{\partial C_i(q_i)}{\partial q_i}\right] \left(-\frac{\partial Q}{\partial p} + \sum_{v=1, v\neq i}^{I} \frac{\partial q_v(p)}{\partial p}\right) = 0, \qquad \forall i, \forall v = i \tag{2.17}$$

Any solution to these coupled differential equations such that each q_i is non-decreasing over the relevant range of prices is a SFE [Gao and Sheble, 2010]. However, the SFE may not exist since it is a non-linear differential equation system. One important note is that the resulting supply function equilibrium generally represent an intermediate level of competition, lying between the Bertrand (or perfect competition) and the Cournot results. A drawback of SFE is that it is very difficult to solve as it requires solving a set of differential equation. As we have seen the optimization problems for the generators taking into account the network constraints are inherently nonconvex which poses even a bigger challenge for the solution of these problems.

Nonetheless, SFE conjecture makes it attractive for many research purposes both seeking to overcome its limitations or wish to examine the robustness of the results obtained from using these models. Many research work have been done to propose solution methods and to provide procedures to obtain a unique and stable equilibrium solution using the SFE formulation. Some of the recent representative work in include [Liu et al., 2004], [Chen et al., 2005], [Rudkevich, 2005], [Langary et al., 2014], and [Rashedi and Kebriaei, 2014], where each author proposes a solution method and discuss the existence and uniqueness of the solution for the SFE. I intend to focus however on the publications that explicitly consider network representation in the equilibrium solution of the market agents.

Table 2.6 summarizes some representative publications addressing this issue. In general, most of the publications found assumes linearity in both the supply and demand curves, also almost all consider that the pricing scheme for the market is the nodal pricing scheme, which is very reasonable considering that in a uniform pricing scheme, SFE would not necessarily add much benefit for the estimation of the agents' bidding behavior relative to the complication in the calculations. Xian et al. [2004], Bompard et al. [2007], Liu and Wu [2007], Bompard et al. [2010a], Gao and Sheble [2010] and Niknam et al. [2013] all consider a DC representation for the network to take into account the effect of network congestion on the suppliers' behavior in an electricity market. The difference between these publications are mainly; in the solution method they propose (e.g. non-linear complementarity problem, Interior point algorithm, heuristics, etc.), in the size of the case study application, and the time scope (single period vs multi-periods). Petoussis et al. [2007], Petoussis et al. [2013], and Soleymani [2013] on the other hand consider an AC optimal power flow representation of the network, to take into account requirements for the voltage level and the reactive power of the network.

TABLE 2.6: Representative Supply Function Equilibrium models in electricity markets.

Reference	Model Basis	Assumptions	Applications
Xian et al. [2004]	Supply Function Equilibrium (SFE)	DC optimal power flow for network representation. Linear cost functions for suppliers. Linear demand curve. Nodal scheme for the transmission network.	Bidding strategy based on linear supply function cast as a nonlinear complementarity problem and solved by an inexact Levenberg-Marquardt-type algorithm. Application on IEEE 30-bus test system.
Bompard et al. [2007]	Supply Function Equilibrium (SFE)	Nodal pricing scheme. Transmission network representation through DC optimal power flow. Elastic demand.	Modeling the effect of different degrees of demand elasticity on the strategic behavior of generation companies and on network congestions. Application on IEEE-30 bus test system.
Liu and Wu [2007]	Supply Function Equilibrium (SFE)	DC optimal power flow. Nodal pricing system.	Impact of network constraints on electricity market equilibrium. Application on a three-node system.
Petoussis et al. [2007]	Supply Function Equilibrium (SFE)	AC optimal power flow network representation. considers network losses. Considers the requirements for reactive power.	Bidding strategy based on linear supply function cast as a nonlinear complementarity problem and solved by an interior point algorithm. Application on IEEE 14-bus and 30-bus systems.
Bompard et al. [2010a]	Supply Function Equilibrium (SFE)	DC optimal power flow model. Inelastic demand. Nodal pricing.	Proposes and approach to find the SFE in the constrained electricity market assuming the slope of the supply function as a decision variable. Application on IEEE-118 bus test system.

Gao and Sheble [2010]	Supply Function Equilibrium (SFE)	Considers multi-period situation. DC optimal power flow.	Model considers supply function equilibrium applied in a multi-perio and multiple-market situation. The paper proposes a set of Nash Equilibrium conditions based on discrete time optimal control for the proposed model.
Niknam et al. [2013]	Supply Function Equilibrium (SFE)	Linear cost function. One generator per node. Linear demand function. Linear DC optimal power flow.	Proposes a new enhanced bat-inspired algorithm to calculate the SFE of GenCos in a network-constrained electricity market.
Petoussis et al. [2013]	Supply Function Equilibrium (SFE)	AC optimal power flow. Nodal pricing scheme. Linear supply function.	Examines the impact of the choice of parameterization method for the linear SFE model on the market equilibrium solution, considering network congestions. Proposes a primal-dual nonlinear interior point algorithm to find the equilibrium. Application on systems ranging from 3 to 57 buses.
Soleymani [2013]	Supply Function Equilibrium (SFE)	Non linear AC power flow analysis. Assumes both active and reactive poewr markets.	Determination of market equilibrium points as well as a method for GenCos to present their bidding strategies in the ancillary service markets. Application on IEEE 39 bus system.

2.4 Risk and reliability assessment of electricity transmission network

Risk management and quantified risk assessment are very important in the context of power systems. A power system consists of many components, including generators, transmission lines, cables, transformers, breakers, switches, protection and communication devices, and many others. The default of any of them can provide a chance for a failure in the system. Therefore many studies have attempted to quantify the different sources of risk inherent in the system to show their effect on the power system's cost and security levels. Risk in general, is the product of the probability of occurrence of an undesired event and the negative consequence of this occurrence. In the context of power systems, there is a degree of overlap in the concept of "reliability" and that of "risk". For example, the reliability of an electrical grid is the degree of performance of delivering the electricity amount desired by customers within acceptable defined standards. Since there are many chances that different components defaults resulting in these requirements not satisfied; we see that a higher risk might indicate lower reliability. In this section, in order to better illustrate where this current research work is positioned within the existing literature. I attempt to provide a brief overview and examine selected publications that have addressed the issue of risk and reliability assessment in power systems and in particular that of the transmission network.

Table 2.7 summarizes selected publications addressing the risk and reliability assessment in the planning and operation of the transmission network. The publications below can generally be divided in explicitly addressing one of the following:

- Proposing a definition for a suitable risk index to assist in the evaluation of power network operation (e.g. Rocchetta et al. [2015], Arroyo et al. [2010], Henneaux et al. [2012], Mousavi et al. [2012], and Xiao and McCalley [2009]).
- Considering reliability assessment in the analysis of the network's expansion and maintenance (e.g. Hooshmand et al. [2012], Alizadeh and Jadid [2011], Arroyo et al. [2010], Volkanovski et al. [2009], and Zhao et al. [2009]).
- Considering risk analysis for system security in the short term network dispatch (e.g. Zhang et al. [2014], and Wang et al. [2013]).

For risk indexes, there is no certain one defined that is more often used or can be considered more suitable than the others. In fact authors compete in defining different indexes which could be suitable for implementation in different cases and from the point of view of different users and stakeholders. In general, they all follow the same basic definition: that is all indexes consider the probability of an undesired event and the consequence of its occurrence. On the other hand, for reliability assessment it is somewhat different; there exists indexes measuring different aspects of power system reliability; for example Loss of Load Probability (LOLP) quantifying the probability of load loss [Allan et al., 2013], and System Average Interruption Duration Index (SAIDI) measuring the average outage duration for each customer served [Allan et al., 2013]. Generally, the most common indexes are:

- Loss of load expectation (LOLE) (e.g. Hooshmand et al. [2012]).
- Loss of load cost (LOLC) (e.g. da Silva et al. [2010]).
- Energy Expected not Supplied (EENS) (e.g. Zhao et al. [2009]).
- Reliability improvement index (e.g. Chanda and Bhattacharjee [1998]).

Let us first consider the publications in the first category presented. Rocchetta et al. [2015] develop a probabilistic risk assessment for a distributed generation (DG) system, taking into account the effect of severe weather conditions which can impose a major threat and significantly increase the failure rates of the lines. The weather scenarios are accounted for by means of a non-sequential Monte-Carlo simulation, and their effects are included onto the probabilistic failure models of the system components. The risk index is defined as the probability of the severe weather condition multipied by the contingency of the line. The results show that indeed extreme weather conditions leads to an increment in the expected system risk. Moreover, a comparison between a system that includes DG with one that does not confirm the benefits of DG installation in terms of bus voltage, line flows and post-contingency severity. Henneaux et al. [2012] aim to provide a method to identify potential cascading scenarios and better calculate their frequency. They apply their analysis using a Monte Carlo simulation to consider scenarios of coupling between events in cascading failure, and the dynamic response of the grid to stochastic initiating perturbations.

Mousavi et al. [2012] propose a risk assessment method for the effect of cascading outages that could lead to a complete blackout. The cascading outage considers the effects of the active power and the frequency response of the system. The risk index analyzed is based on the Expected Load Not Served (ELNS) and Complementary Cumulative Density Function (CCDS) which is calculated from the lost load data. Xiao and McCalley [2009] develop a probabilistic risk index to assess real-time power system security level. It explicitly considers security levels associated with low voltage and line overload withing a multi-objective framework using Non-Sorting Geneatic Algorithm optimization method. Risk indices are defined as the low voltage risk and overload risk for the assessment of the post contingency severity. They show that the multi-objective approach results in a less risky and less costly operating conditions, and provides a practical algorithm for implementation.

Arroyo et al. [2010] as shown spans over two of the aforementioned categories; it presents a risk-based approach for the transmission network expansion planning; therefore explicitly proposing a risk index definition within a an expansion problem. The risk in this work is defined as the risk of deliberate outages, i.e. intentional attacks which expose network planners to non-random uncertainty of outages, having the need to plan in such a way that mitigates those outages within budgetary limits. Risk index used in association with this type of low frequency and high impact phenomena is modeled by a regret function, that is the regret felt by the network planner after verifying that the selected decision is not optimal. The risk aversion of the network planners is characterized by the minimax optimization of this regret function.

For the second group, Hooshmand et al. [2012] differs from the aforementioned studies in that it considers reliability assessment of transmission network in deregulated power system. Which means that the cost faced by the planner is not only the investment cost in new lines, but also the congestion management cost, congestion surplus, and some others. Although no explicit risk analysis is done, the study considers a reliability assessment for the transmission system based on minimizing the EENS and the LOLE by a Monte-Carlo simulation approach. It shows through an AC optimal power flow modeling of the problem how their proposed method significantly improves the planning cost of the system.

Alizadeh and Jadid [2011] also proposes an expansion planning optimization problem considering system reliability through hierarchical indices consisting of the Loss of Load Probability (LOLP) and Expected Loss of Load (ELOL). It is suggested that for a large power system, the system planner can use it as a tool to thoroughly eliminate the probable unsafe states of the system (in terms of the reliability indices) by reinforcement of the system. A risk assessment in this case would typically consider the economic quantification of the reliability indices with respect to cost.

Volkanovski et al. [2009] integrate fault tree analysis and power flow model for the assessment of power system reliability. The results identify the reliability measures related to a particular network node and the reliability measures connected to the power system as a whole. The study proposes the so called “importance measures” which are two risk measures; network risk achievement worth (NRAW) and network risk reduction worth (NRRW) characterized by the impact on the power system reliability, to identify systems deficiencies.

For the third group, Zhang et al. [2014] and Wang et al. [2013] as mentioned consider the short term operation of the network. Zhang et al. [2014] considers the effect of increased uncertainty in the operation of the power systems on the cost of operation. The sources of increased uncertainties are considered to be due to the high penetration of renewables and depend on several factors including market design, performance of renewable generation forecasting and the specific dispatch procedure. They propose the usage of a Risk Limiting Dispatch (RLD) model to study the effect of these uncertainties as well as network congestion on the dispatch procedures and the optimal cost. The RLD is formulated as a two stage optimization problem where one stage is the real time optimal power flow, while the other is the day ahead stochastic power flow. The latter model takes into account the expectation of the forecast and enforces a constrain that limits the purchase of generation power within the acceptable range (risk range).

Wang et al. [2013] propose a concept for the modeling of risk-based security-constrained economic dispatch. The risk is modeled as a product of contingency probability and overload severity, three levels of severity are considered each considering a trade off between the system security level and the system cost (chosen arbitrarily). They argue that including risk assessment to the traditional (SCED) problem improves the economic performance of the power system while enhancing the system's overall security level. A case study test was applied to the IEEE 9-bus system, and another one validated on a real power system (New England system).

Finally, although it has been shown in this review that many studies addressed the issue of exercise of market power in an electricity market context, and especially due to transmission network line congestion, as well as all the different studies that sought to quantify the risk faced by the network operator in both the planning and control of the network, and specifically due to reliability requirements, to the best of the author's knowledge, no studies have considered the calculation of the risk index for a network failures taking into account a combined measure of energy not served, and market operation inefficiency. More specifically the well-fare loss considered, quantified by the so called correction cost used in this dissertation.

Table 2.7: Selected publications considering risk and reliability of the transmission network.

Reference	Application
Rocchetta et al. [2015]	Develops a probabilistic risk assessment and risk-cost optimization framework for distributed power generation systems , and taking the effects of extreme weather conditions into account using non-sequential Monte-Carlo algorithm.
Zhang et al. [2014]	Studies a two-stage stochastic economic dispatch problem to provide an analytic quantification and an intuitive understanding of the effects of uncertainties and network congestion on the dispatch procedure and the optimal cost.
Wang et al. [2013]	Proposes a Risk-Based Security-Constrained Economic Dispatch (RB-SCED) applied on a case study of the IEEE 9-bus system and a real power system (New England system).
Hooshmand et al. [2012]	Proposes an AC model of transmission expansion planning, associated with Reactive Power Planning using Monte Carlo simulation to determine system reliability considered as the Expected Energy Not Supplied (EENS) and Loss of Load Expectation (LOLE).

Henneaux et al. [2012]	Proposes a Monte-Carlo simulation model specifically developed to identify dangerous cascading scenarios and better calculate their frequency.
Mousavi et al. [2012]	Proposes a risk assessment method for power outages considernig the effect of cascading network failure. The risk of blackout is quantified through a complementary cumulative density function and expected load not served (ELNS).
Alizadeh and Jadid [2011]	DC power flow model for the transmission expansion planning including environmental constraints and fuel supply limitations.
Arroyo et al. [2010]	Proposes a risk-based approach for the transmission expansion planning considering deliberate outages. The risk aversion attitude is characterized by the minimax weighted regret criterion.
Xiao and McCalley [2009]	Proposes a risk based decision support model for secure network operation, formulated as a multiobjective optimization for finding optimal trade off between the security level and the cost, and solved using evolutionary algorithm.
Volkanovski et al. [2009]	Proposes a method for power system reliability analysis using the fault tree analysis approach. It shows that through the quantitative evaluation of the fault trees it is possible to identify the most important elements in the power system.
Zhao et al. [2009]	Proposed method for transmission expansion planning using mixed integer nonlinear programming so that conflicting objectives can be optimized simultaneously.
Hamoud [2008]	Describes a probabilistic approach for assessing the criticality of bulk transmission system components in the de-regulated electricity market.
Yamin et al. [2004]	Considers the impact of transmission failure on a GenCos' expected profit in a competitive environment.

Chapter 3

Model Assumptions and Formulation

3.1 Overview

We proceed to describe in details the model formulation. As mentioned in Chapter 1, the aim of this present study is to quantify the risk that the system operator is facing when operating the network to ensure secure, reliable and economic dispatch of power. I extend on the notion that has been previously and extensively studied which is that the quantitative risk the System Operator is facing is in terms of "cost of energy not served (ENS)", to also attempt to examine the more subtle effects that such failures can induce in terms of strategic behavior of generators and the chance they have to exercise market power. It is well understood (and will be further discussed), that typically the strategic behavior modeling methods covered in Chapter 2 represent medium to long term equilibrium behaviors of GenCos, that is at least from several weeks to several years long. Line failures of course occur at a much less frequent pace and lasts much shorter time periods (maximum could be one or two days [Billinton et al., 1989]) due to the critical importance of those line remaining operational. Gaming behavior in this sense is ought to give a general guideline for the quantification of the risk of different line failures, and is not intended to represent real market prices nor propose what would be exactly the real final generation scheduling. This "comparative" analysis could thus prove useful to identify the effect of network security, not only in terms of cost for the energy not supplied, but also taking into account the strategic behavior of market agents.

As seen in the literature review, almost all of the models that consider the GenCos bidding behavior taking into account network constraints, are done so as bi-level models; a market equilibrium model which examines the strategic behavior of GenCos within the notion of Nash Equilibrium, and an optimization model which represents the network and the decisions facing the network's System Operator. These bi-level representation have shown to be the mostly used not only while incorporating a specific market modeling technique, but generally in utilizing any of them. The bi-level representation indeed makes sense, the problem that the SO is facing in the operation of the network on real time basis is to maximize social-welfare. This "welfare" for the society as mentioned, is to ensure secure, uninterrupted, most economical, and least polluting dispatch possible among other things. This problem in essence is an optimization problem; a set of exogenous parameters which allows the SO to minimize or maximize one of the decision variables involved (e.g. cost, security, emission) subject to system constraints. The alternative for solving a bi-level model would be to combine both models in a single optimization or equilibrium problem. Although if modeled correctly this would perhaps provide better and more tractable result. One could argue however, that the ambiguity inherent in a two-stage solution method in fact better resembles what we observe in reality; where there is no closed-loop solutions or completely instantaneous exchange of information between the market agents and the System Operator, leading to results that perhaps could be better simulated as separate problems as presented here.

In this chapter, the model formulation is discussed in details. Two separate problems are modeled; a market equilibrium model based on Conjectural-Variation technique, and a DC optimal power flow. The equilibrium model is borrowed from the work of Delgadillo and Reneses [2015], and the solution technique used is an adaptation of the work of Delgadillo et al. [2013] and Delgadillo and Reneses [2013]. The proposed model's main feature is that it considers the market to be a "single-price" market, where all scheduled generators are paid the same market price, regardless of their location on the network. This is one of three most common market designs for electricity trading; the other two being the "zonal" pricing scheme widely adopted in Europe, and the last being the nodal-price markets widely adopted in the United States and Canada. It is perhaps of interest to first illustrate the general characteristics and differences among those three schemes.

Nodal-Price, Zonal-price, and Single-Price electricity markets

There exist different regulatory options to deal with the allocation of limited transmission capacity for trading among the market participants under normal market conditions. Typically, these schemes can be combined in one of two groups: the regulatory schemes which involves algorithms that solves a detailed representation of the transmission network (such as the nodal pricing scheme), and those other which only consider a simplified one (such as the zonal pricing and the single pricing ones) [Pérez-Arriaga, 2014].

Nodal Pricing applies security constrained economic dispatch to calculate a bus by bus Locational Marginal Prices (LMP), it reflects the value of energy at a specific location at the time that it is delivered. Nodal pricing provides an accurate description of the technical and economic effects of the grid on the cost of electricity by internalizing the losses and congestion effects in a single value that varies at each system node.

Zonal Pricing consists of using a single market price except where signigicant grid constraints arise frequently between a limited number of sufficiently well-defined zones of the power system. This pricing mechanism distinguishes energy prices by zone instead of nodes, and the same price prevails at all nodes within a given zone.

Single Pricing completely ignores the transmission system (in terms of losses and congestions) when the electricity market is cleared. This is typically in systems where supposedly no systematic or structural congestions or failures occur. In the few cases in which such constraints are detected, the System Operator re-dispatches the system, determining which generation units must withdraw from the system and which are to be included [Pérez-Arriaga, 2014].

3.2 Market Equilibrium Model: Conjectural-Variation based Equilibrium

In electricity market, competing generation companies who wish to produce, have to participate in what is called the *day ahead market*, by offering bids that consists of quantities and prices pairs for next day production schedule to the market operator. The market operator aggregates all these supply bids, and also collects and aggregates all the demand bids to construct the supply-demand curve. The market operator rearranges all the bids received from the supply in an ascending order (each generation unit considered separately), and each bid received from the demand in a descending order, until the total generation equals the demand. Thus, the market marginal price is set to the bid price of the last unit dispatched. In a single price market, this price will be the same price used for the remuneration of all the units committed. If we do not take into account the network constraints, equilibrium models are enough to study the type of competition that could take place between the different companies; they can range in the degree of complexity of representation of the competing firms (e.g. taking into account ramp rates, start-up and shut-down costs, maintenance cost, etc.), by taking into account several sources of uncertainties (e.g. stochastic demand, reliability of generation units, hydro inflow, etc.), or by taking into consideration the dynamic nature of the problem allowing it to span over multiple periods (hours, days, or even months).

3.2.1 General formulation of the equilibrium problem

We proceed to formulate the conjectural-variation equilibrium model proposed by Delgadillo and Reneses [2015]. Let us assume in the simplest situations, that in an electricity market competition, the profit π_i of the generation company i is equal to the revenues minus the costs of the company; or

$$\pi = \lambda \cdot q_i - C_i(q_i) \qquad \forall i \tag{3.1}$$

Where λ is the single market clearing price resulting from the aggregate supply and demand curves, q_i is the production quantity of firm i, and C_i is the cost function of firm i as a function of the quantity produced. The equilibrium point is found by expressing the first-order profit-maximization condition for each generation company which yields

$$\frac{\partial \pi_i}{\partial q_i} = \lambda + q_i \cdot \frac{\partial \lambda}{\partial q_i} - \frac{\partial C_i(q_i)}{\partial q_i} = 0, \qquad \forall i \tag{3.2}$$

The term $\partial\lambda/\partial q_i$ is the *conjecture price response* or it is how each firm assumes that its quantity decisions affect the market price as shown in the description of the conjectural variation method in the literature survey. If we assume that the conjecture price response is equal to $\theta = -\partial\lambda/\partial q_i \geq 0 \forall i$ and substitute into (3.2) we obtain:

$$\lambda - \theta \cdot q_i = \frac{\partial C_i(q_i)}{\partial q_i}, \qquad \forall i \tag{3.3}$$

In this manner, the market equilibrium is reached when the marginal revenue MR_i [the left-hand side of equation 3.3] is equal to the marginal cost MC_i [the right-hand side of equation 3.3] for each company i. Furthermore, in electric power systems, it is mandatory that the generation and demand are balanced:

$$\sum_i q_i = D \tag{3.4}$$

The inverse demand curve, $\lambda(D)$, is the relationship between market price and demand D. To ensure the existence of the equilibrium, the inverse demand curve must satisfy certain properties. This function is continuous, differentiable, monotone and strictly decreasing. In this simplest form, the optimization problem of each firm would be to maximize its profit 3.1 taking into consideration the conjecture price response for its quantities decisions, if we assume that each company i owns more than just one generation unit j this can be expressed as:

$$\max_{\lambda_i, q_i} \quad \lambda_i \cdot \sum_{j \in j_i} q_j - \sum_{j \in j_i} C_j(q_j) \tag{3.5}$$

Subject to:

$$\lambda_i = \lambda - \theta_i \cdot \left(\sum_{j \in j_i} q_j - \sum_{j \in j_i} q_j^* \right) \tag{3.6}$$

$$q_j \leq q_j^{max} \qquad \forall j. \tag{3.7}$$

$$q_j \geq 0 \qquad \forall j. \tag{3.8}$$

It should be noted that here we changed the market equilibrium price λ with the market price λ_i representing company i's *belief* about what the market price is going to be. Hence it is necessary to add constraint (3.6) which reflects the relationship between this believed price λ_i and the final market price λ that results from the difference of the chosen supply quantity q_j with respect to the optimal quantity q_j^*, multiplied by the conjectured price response θ_i. Constraint (3.7) ensures that the production quantity of every unit j of firm i does not exceed its maximum limit q_j^{max}, and constraint (3.8) ensures that no unit can produce negative quantities. It is important to note than in the equilibrium state, this maximization problem needs to be solved for all firms simultaneously, an important constraint in this case would be the demand balance equation (3.4) for all generation units for all firms, this is constraint will be shared by all generation companies and will be the link between all optimization problems. The market price λ will be the dual variable of this demand balance equation. This will be further illustrated in details in the following sections.

3.2.2 Effect of Network Representation and Line Failure on the Equilibrium Formulation

We proceed to derive the formulation of the equilibrium problem if in this single-price market we want to represent the network effect as proposed in Delgadillo and Reneses [2015]. If we do not take into consideration the network representation, it is very probable that the schedule resulting from the solution of the equilibrium problem may not be feasible or may exceed the maximum capacity in lines. Moreover, in the case of line failures, it is assumed that the System Operator will need to re-dispatch the generators to minimize the energy not served due to this failure, and to ensure the system stability, in the sense that no other line becomes overloaded perhaps leading to a cascade network failure. Although the model of Delgadillo and Reneses [2015] is originally developed with the idea that the network representation has the aim of eliminating overflows leading to congestions, it can be seen that the same principle proposed could be used in a similar sense in which line failures would also lead to the need for re-dispatch of the generators. Generation companies can anticipate the reaction of the system operator in cases of network failures, allowing them the chance to exercise strategic behavior during these periods, even though these failure typically would not last long due to maintenance activities.

The re-dispatch strategy assumed in the present work, is similar to the congestion management mechanism discussed in Delgadillo et al. [2013]. A brief description of this mechanism is as follows: network failures occurs unexpectedly, typically in real-time operation after the day-ahead market clearing process. The SO in the day-ahead market receives price and quantity bids from the agents about the production schedule, and other bids about what the firms are willing to increase or reduce for the production of each unit with respect to the result of the day-ahead market. The system operator would typically solve an optimal power flow to perform a security analysis of the system, taking into account the schedule proposed in the day-ahead market. Normally this analysis would have the aim of identifying and eliminating network congestion, however, in addition to this aim, if a network failure occurs, the system operator will have to increase the production of some units and decrease the production of other, to ensure the optimal and secure operation of the system. For those units that will have to increase their production, the difference between the real time production and the day-ahead market schedule will be paid at the price that the agents offered in his participation in the congestion management mechanism (the secondary market after the day-ahead market, as described above). On the other hand, if a unit is required to decrease its production, and finally will not be producing the total quantity originally scheduled, it will be considered as a *charge* that the company is paying equal to the difference in the real time quantity and the bid quantity, multiplied by the day-ahead market price. Or in other words, the opportunity cost that the company could have earned but finally did not. Obviously, the quantities increased and decreased by the units are planned according to the lowest cost solution, which will have to be represented in the optimization problem solved by the SO.

To illustrate this mathematically, the profit π_i of firm i in a single-price market with a correction market mechanism as described above will look like:

$$\pi_i = \lambda \cdot \sum_{j \in J_i} q_j + \gamma \cdot \sum_{j \in J_i} x_j - \lambda \cdot \sum_{j \in J_i} z_j - \sum_{j \in J_i} C_j \left(q_j + x_j - z_j \right), \qquad \forall i \in I \tag{3.9}$$

where:

- λ is the day-ahead market clearing price.
- γ is the correction-market (secondary market) clearing price.
- q_j are the scheduled quantities for each unit j in the day ahead market.
- x_j are the increased quantities for each unit j requested by the system operator with respect to the original scheduled quantities q_j, due to network constraints.
- z_j are the decreased quantities for each unit j requested by the system operator with respect to the original scheduled quantities q_j, due to network constraints.
- C_j is the cost for unit j as a function of real-time production quantities.

The first term represents the total revenue obtained from being scheduled in the day-ahead market such as; λ is the day ahead market price multiplied by the total quantity produced q_j for all scheduled units j that belong to firm i. The second term represents the revenue earned from the request of the system operator from firm i to increase any production quantities x_i. As discussed, this production increase x_j will be paid at a different price γ corresponding to the correction market price. The third term represents the *opportunity cost* that company i

incurs as a result of decreasing its production quantities by an amount z_j, this opportunity cost is equal to this reduced amount multiplied by the day-ahead market price. Finally, the last term represents the firm's production cost function for every unit C_j, the cost function could be linear, quadratic or a step-function and depends on the quantity scheduled in the day-ahead market q_j plus the increased quantities x_j minus the decreased quantities z_j; or in other words what firm i is actually going to produce in real time based on the system operator's decision.

Now we need to consider how the system operator will decide which units should decrease their quantities to solve the network problem, especially if several units exist on the same bus where it is required that the production decreases. Delgadillo et al. [2013] explain one of the possible mechanisms, which is the one used in their model which is borrowed here, and represents how this mechanism works in Spain: in the case of network congestion, the quantity reduced by the units depends on what is called the *contribution factor* to this congestion, commonly referred to as Generator Shift Factor. This factor expresses the change in the flow over the interconnection line that results from increasing the generation of each unit at the exporting area [Delgadillo et al., 2013]. In general, the unit with the highest contribution factor to the network problem is reduced first, and the following units with highest factors will be reduced until the network problem disappears. If we assume the simplest case, we can consider that all units in the same bus which can solve the network problem have the same contribution factor. This would be indeed logical in the case of a network problem induced by a line failure, in which is not straightforward to attribute the cause of this failure to a specific unit on a specific bus. If we assume this uniform contribution factor for all units, then the quantities reduced z_j can be expressed as a percentage of the original scheduled quantities q_i such as:

$$z_j = m_j \cdot q_j \tag{3.10}$$

where m_j in this case will be the *reduction factor* which is the ratio of the quantity reduced z_j for unit j with respect to the original quantity scheduled. Thus equation (3.9) can be expressed as:

$$\pi_i = \lambda \cdot \sum_{j \in J_i} (1 - m_j) \cdot q_j + \gamma \cdot \sum_{j \in J_i} x_j - \sum_{j \in J_i} C_j \left((1 - m_j)\, q_j + x_j \right), \qquad \forall i \in I \tag{3.11}$$

Equation 3.11 is the profit function for each firm considering its bid in the day-ahead market and in the correction-market discussed. In order to express for each firm i its strategy of maximizing its profit, we need to calculate the first-order optimality condition of this profit equation. It is important to note however, that firm i needs to maximize its profit taking into consideration both the revenue from the day ahead market and what it expects to have as a revenue from the correction market. That is we need to calculate the first order optimality with respect to production quantity q_j and increased production quantity x_j. Thus we obtain:

$$\begin{aligned}\frac{\partial \pi_i}{\partial q_j} &= (1-m_j)\cdot\lambda + \frac{\partial \lambda}{\partial q_j}\cdot\sum_{k\in J_i}(1-m_k)\cdot q_k \\ &\quad -(1-m_j)\cdot\frac{\partial C_j\left((1-m_j)\,q_j+x_j\right)}{\partial\left((1-m_j)\,q_j+x_j\right)} = 0, \quad \forall i\in I, \forall j\in J_i\end{aligned} \tag{3.12}$$

$$\frac{\partial \pi_i}{\partial x_j} = \gamma + \frac{\partial \gamma}{\partial x_j}\cdot\sum_{k\in J_i}x_k - \frac{\partial C_j\left((1-m_j)\,q_j+x_j\right)}{\partial\left((1-m_j)\,q_j+x_j\right)} = 0, \quad \forall i\in I, \forall j\in J_i \tag{3.13}$$

The term $[\partial C_j\left((1-m_j)\,q_j+x_j\right)/\partial\left((1-m_j)\,q_j+x_j\right)]$ is the Marginal Cost of unit j evaluated at the respective real time production, or $MC_j\left((1-m_j)\,q_j+x_j\right)$. If the cost function C_j is linear, then the marginal cost will be a constant value. However, the model gives the flexibility for different representation of the cost function other than the linear representation. Important to note are the terms $[\partial\lambda/\partial q_j]$ and $[\partial\gamma/\partial x_j]$ which are the conjectured-price-response for company i in the day-ahead market and that of the correction-market respectively. This is how, in the model, the conjectural-variation is incorporated, or how firm i conjectures that its choices in quantities will affect the market price and hence the decisions of the other firms. The conjectured price responses for firm i are herein represented as:

$$\theta_i = -\frac{\partial \lambda}{\partial q_j}, \qquad \forall i\in I, \forall j\in J_i \tag{3.14}$$

$$\beta_i = -\frac{\partial \gamma}{\partial x_j}, \qquad \forall i\in I, \forall j\in J_i \tag{3.15}$$

θ_i being the conjectured-price response in the day-ahead market, and β_i being that of the correction market. As discussed in Chapter 2, the values of these parameters are difficult to be endogenously calculated and are most often arbitrarily chosen to represent different levels of competition. These conjectures can take continuous values between 0 and 1; where 0 represents no conjecture assumption (or perfect competition behavior), and 1 represents the highest form of exercise of market power (or Cournot competition behavior). We can see here that both conjecturd-price responses θ_i and β_i are assumed to be the same for all the units of the agent regardless of their location, this is because as discussed, when we consider a single-price market, the market clearing process does not take into account the power network, and the day-ahead price will be the same for all areas. Therefore, the agent is indifferent to the location of its units because modifying the production of any of them, the market price will be effected in the same way. Hence, the constant conjectured-price response value.

Substituting (3.14) and (3.15) in equations (3.12) and (3.13) respectively, and rearranging we obtain:

$$\lambda = MC_j\left((1-m_j)\cdot q_j + x_j\right) + \frac{\theta_i}{(1-m_j)} \cdot \sum_{k\in J_i}(1-m_k)\cdot q_k, \quad \forall i \in I, \forall j \in J_i \tag{3.16}$$

$$\gamma = MC_j\left((1-m_j)\cdot q_j + x_j\right) + \beta_i \cdot \sum_{k\in J_i} x_k, \quad \forall i \in I, \forall j \in J_i \tag{3.17}$$

These very important equations (3.16) and (3.17) clearly show us how the day-ahead market price λ and correction price γ are affected by the firm's decisions in quantities q_j and x_j, by the level of competition θ_i and β_i, but above all, by the correction decision of real-time production imposed by the SO due to system security and reliability constraint, which is represented here primarily by the *reduction factor* m_j. Of course these equations arranged in that manner calculates the values of λ and γ for each unit j, thus only the *marginal unit* or in other words the most expensive unit committed in the day-ahead market, and the one committed in the correction market, are the ones which are setting the single market prices λ and γ respectively. However, calculating those prices for each individual unit still gives a very important insight; for any unit other than the *marginal unit*, the calculated λ and γ from equations (3.16) and (3.17), result in what is called the *apparent cost* of the respective unit. This apparent cost corresponds to an equivalent marginal cost that is perceived by the system participants when this unit produces a determined quantity in the day-ahead market [Delgadillo and Reneses, 2013].

To understand this concept, consider for example how a line constraint, congestion or failure, modifies the conjectured-price response of the agent and consequently the unit's *apparent cost* and the market price. If we take for example a case in the day-ahead market price equation (3.16), where unit j does not affect and therefore can not correct the flow-gate's congestion, the factor m_j is equal to 0. Leading to the apparent cost of this unit to be the marginal cost MC_j plus the conjectured-price response $\theta_i \cdot \sum_{k\in J_i}(1-m_k)\cdot q_k$. If however the unit is required to reduce its production, the factor m_j is greater than 0. Thus, the conjectured-price response θ_i is modified by the factor $[1/(1-m_j) > 1]$, leading to this unit's increased apparent cost as perceived by the firm. The unit seems more expensive because the agent anticipates that it will have to reduce its production due to the system reliability requirement as imposed by the system operator, and hence it foresees that he will lose a part of the planned profit resulting from this unit's production. This translation into the apparent cost of the unit could give the agent incentive not to bid quantities with this unit in the day-ahead market if it is more profitable to do so, resulting in a different bid portfolio and potentially a different marginal unit scheduling leading to a different (and obviously higher) market price.

Finally, in the electricity market, the constraint that links all the optimization problems of all market participants is the demand-balance equation. The total generation scheduled from all units belonging to all agents must meet the total demand in the system:

$$\sum_{j \in J} q_j = \sum_{a \in A} D_a + \text{losses} \tag{3.18}$$

$$\sum_{j \in J} x_j = \sum_{j \in J} m_j \cdot q_j. \tag{3.19}$$

where D_a is the demand at bus a. Equations (3.18) and (3.19) are the power balance constrains in the day-ahead market and in the correction market respectively. Notice that in the correction-market, the balance is between the total production increase requested (x_j) and the total production decrease imposed $(m_j \cdot q_j)$ *or* (z_j) to ensure that the total demand remains served even after implementing those changes.

Individual firm's optimization problem

We proceed to gather all the concepts described above in order to formulate the individual firm's profit maximization problem. Each firm i will need to solve the following optimization problem [Delgadillo and Reneses, 2015]:

$$\max_{\lambda_i, \gamma_i, q_i, x_i} \lambda_i \cdot \sum_{j \in J_i} (1 - m_j) \cdot q_j + \gamma_i \cdot \sum_{j \in J_i} x_j - \sum_{j \in J_i} C\left((1 - m_j) \cdot q_j + x_j\right) \tag{3.20}$$

Subject to:

$$\lambda_i = \lambda^* - \theta_i \cdot \left(\sum_{j \in j_i} q_j - \sum_{j \in j_i} q_j^* \right) \tag{3.21}$$

$$\gamma_i = \gamma^* - \beta_i \cdot \left(\sum_{j \in j_i} x_j - \sum_{j \in j_i} x_j^* \right) \tag{3.22}$$

$$\overline{q_j} - q_j \geq 0 : \quad (\overline{\mu_j}) \qquad \forall j \tag{3.23}$$

$$\overline{q_j} \cdot u_j - x_j \geq 0 : \quad (\overline{\nu_j}) \qquad \forall j \tag{3.24}$$

$$\overline{q_j} - q_j - x_j \geq 0 : \quad (\overline{\xi_j}) \qquad \forall j \tag{3.25}$$

$$q_j \geq 0, \quad x_j \geq 0 \qquad \forall j \tag{3.26}$$

$$\{m_j, u_j\} \in \arg \Xi \tag{3.27}$$

where *Indices*:

- i is the firm index.
- j is the production unit index.

Sets:

- J is the set of indices of production units.
- J_i is the set of indices of production units owned by Firm i.
- Ξ is the set of decision variables of the optimization problem solved by the system operator (DC optimal power flow problem), described in details in the next section.

Parameters:

- C is the cost for unit j as a function of real-time production quantities.
- θ_i is the conjectured price response of firm i in the day ahead market.
- β_i is the conjectured price response of firm i in the correction market.
- $\overline{q_j}$ is the maximum production capacity for unit j.
- m_j is the reduction factor for the units j that will need to reduce their production relative to their original schedule to solve the network problem.
- u_j is a binary-variable which is equal to 1 if unit j has to increase its production in the correction mechanism, and 0 otherwise.

Variables

- λ_i is the *assumption* made by firm i for the day-ahead market clearing price.
- γ_i is the *assumption* made by firm i for the correction market clearing price.
- q_j are the scheduled quantities for each unit j in the day ahead market.
- x_j are the incremented quantities for each unit j requested by the system operator in the correction mechanism.
- $\overline{\mu_j}, \overline{\nu_j}$, and $\overline{\xi_j}$ are the dual-variables for constraints (3.23), (3.24) and (3.25) respectively.
- λ^* is the real single-market price in equilibrium.
- γ^* is the real correction market price in equilibrium.
- q_j^* are the optimal quantities produced in the equilibrium state.
- x_j^* are the final production increments in the equilibrium state.

Equation (3.20) is the objective function to be maximized for each firm, described in full-details in the previous section. Constraints (3.21) and (3.22) represent how the firm conjectures that the electricity prices will change if the company changed its production; each company i has an estimation of the prices in the day-head and in the correction market λ_i and γ_i. This estimation λ_i is different (less) than the real market price λ^* as long as the firm does not produce its optimal quantity q_j^*. The firm seeks to maximize its revenue by increasing the price as much as it could, therefore this constraint models the incentive that the firm produces at the optimal quantity q_j^* so as to reach that the price estimated λ_i or γ_i is equal to the equilibrium market prices. Since all firms will behave in the same manner looking for optimal values while satisfying those constraints, and since the prices λ^* and γ^* are the same and shared by all companies, then the choice for the production for any unit belonging to any company affecting these market prices will also affect the decisions reached for other firms, until an equilibrium is reached. Constraints (3.23)-(3.26) are the boundaries of the decision variables. Important to note is how in constraint (3.24), unit j is only allowed to increase its production in the correction mechanism based on the recommendation of the system operator, which is communicated through the binary variable u_j. This variable u_j is a binary decision variable from the subsequent problem of real time network operation, and is equal to 1 only when the corresponding generation unit is required to increase the production. Finally, equation (3.27) indicates that both the input parameters for this profit maximization problem m_j and u_j are decision variables in the system operator's DC optimal power flow problem described in the next section.

Now that the optimization problem for each firm is defined, the next step is to try to find the market equilibrium solution. The equilibrium as described in chapter 2 in which all firms decide on their quantity bids such as no firm has an incentive to deviate unilaterally from this decision, or what we mentioned as the *Nash Equilibrium*. As a concept, in order to achieve this mathematically in the model, we need to solve all the optimization problems for all firms i simultaneously. This can be achieved through various methods; Delgadillo and Reneses [2013] proposes a method in which all the optimization problems are expressed as a single quadratic optimization problem possessing some special characteristics. On the other hand, in Delgadillo and Reneses [2015], the authors proposed that the simultaneous solution for all the profit maximization problem to be expressed and solved as a *Mixed Complementarity Problem (MCP)*. In essence, this is done by gathering together the first-order optimality conditions for all companies and then adding the market-clearing conditions in a single problem which defines the mixed complementarity problem.

The constraints of the proposed formulation represents the Karush-Kuhn-Tucker (KKT) conditions of the maximization problem (3.20)-(3.27). I chose to express the problem as the MCP proposed in Delgadillo and Reneses [2015] since there exist ready solvers to solve such problems (such as the PATH solver in GAMS language) and because of reported problems of convergence in the other equivalent quadratic optimization formulation proposed. A detailed representation follows in the next section.

Equilibrium Problem formulation

Mixed-Complementarity Problem (MCP)

Equations (3.28)-(3.34) define the MCP which solves the profit maximization problems for all agents simultaneously. An MCP problem is typically read as follows. **Find (λ^*, γ^*, q_j^*, x_j^*) such that:**

$$\sum_{j \in J} q_j = D \quad : \quad (\lambda \ unrestricted) \tag{3.28}$$

$$\sum_{j \in J} x_j = Y \quad : \quad (\gamma \ unrestricted) \tag{3.29}$$

$$0 \leq \overline{\mu_j} \perp \overline{q_j} - q_j^* \geq 0, \qquad \forall j \in J_i, \forall i \in I \tag{3.30}$$

$$0 \leq \overline{\nu_j} \perp \overline{q_j} \cdot u_j - x_j^* \geq 0, \qquad \forall j \in J_i, \forall i \in I \tag{3.31}$$

$$0 \leq \overline{\xi_j} \perp \overline{q_j} - q_j^* - x_j^* \geq 0, \qquad \forall j \in J_i, \forall i \in I \tag{3.32}$$

$$\begin{aligned} 0 \leq q_j^* \perp &- (1 - m_j) \cdot \lambda^* + \theta_i \cdot \sum_{j \in J_i} (1 - m_j) \cdot q_j^* \\ &+ (1 - m_j) \cdot MC_j \left((1 - m_j) \cdot q_j^* + x_j^*\right) + \overline{\mu_j} + \overline{\xi_j} \geq 0, \qquad \forall j \in J_i, \forall i \in I \end{aligned} \tag{3.33}$$

$$\begin{aligned} 0 \leq x_j^* \perp -\gamma^* + \beta_i \cdot \sum_{j \in J_i} x_j^* + MC_j \left((1 - m_j) \cdot q_j^* + x_j^*\right) \\ + \overline{\nu_j} + \overline{\xi_j} \geq 0, \qquad \forall j \in J_i, \forall i \in I \end{aligned} \tag{3.34}$$

Constraints (3.28) and (3.29) represents the supply-demand balance equations in the day-ahead market and in the correction market respectively. Constraint (3.28) ensures that the total production of all units j regardless of which firm is owning which unit is equal to the total market demand D. This constraint links all the production of all generation units together and the market price λ is the dual variable (or shadow price) of this constraint. Similarly for (3.29), the total increment in production imposed by the system operator in the correction market should be equal to the total decrements required Y, to ensure that the total generation and demand remains balanced after the correction. The correction market price γ is the dual-variable resulting from this constraint. The dual-variables λ and γ represents how much it costs the system to supply an extra unit of electricity (or more accurately a *marginal* unitary increase in the right-hand side of the equation) if there is a unitary increase in demand D (or a marginal unitary increase in the left-hand side of the equation. This extra unit production cost is the *marginal cost* of the most expensive unit scheduled, and hence it represent the market clearing prices. Important to note is that the dual-variables λ and γ are *unrestricted*, which means that no bounds should be imposed on these variables for the calculation. Constraints (3.30)-(3.34) are the Karush-Kuhn-Tucker (KKT) conditions of the maximization problem (3.20)-(3.27) for each company i. The *"perp"* operator ($\perp$) denotes the inner product of two vectors equal to zero. It is important to note that in constraints (3.33) and (3.34), the variables λ_i and γ_i are substituted by constraints (3.21) and (3.22) respectively. Moreover, since the solution of the MCP corresponds to the equilibrium, production variables q_j and x_j in the maximization problem are replaced by the equilibrium variables q_j^* and x^*j in the MCP problem, respectively. The dual variables $\overline{\mu_j}, \overline{\nu_j}$, and $\overline{\xi_j}$ from the maximization problem are incorporated into constraints (3.33) and (3.34) of the MCP. For the generation units j whose optimal productions q_j^* and x_j^* are less than the maximum values $\overline{q_j}$, constraints (3.23)-(3.25) are not binding and therefore those dual-variable $\overline{\mu_j}, \overline{\nu_j}$, and $\overline{\xi_j}$ are equal to zero. If we remove those variables from constraints (3.33) and (3.34) and rearrange them we obtain:

$$\lambda^* = MC_j\left((1-m_j)\cdot q_j^* + x_j^*\right) + \frac{\theta_i}{(1-m_j)}\cdot\sum_{k\in J_i}(1-m_k)\cdot q_k^*, \quad \forall i \in I, \forall j \in J_i. \tag{3.35}$$

$$\gamma^* = MC_j\left((1-m_j)\cdot q_j^* + x_j^*\right) + \beta_i\cdot\sum_{k\in J_i} x_k^*, \quad \forall i \in I, \forall j \in J_i. \tag{3.36}$$

Which are exactly the same equations (3.16) and (3.17) obtained in the previous section. Those equations follows the same description provided above regarding how they represent the *apparent cost* of the unit. Or the unit cost perceived by the system (the firms and the system operator) due to the anticipated change in production relative to the original schedule.

To summarize, the above equations represent the conjectural-variation equilibrium problem of an electricity market. Where generation firms decide their supply bids in the day-ahead market, while taking into consideration the network constraints through the decisions of the system operator in the correction market. Firms in the formulation can undergo strategic behavior modeled through the conjectured-price response in the day-ahead market as θ_i and that in the correction market as β_i. It has been mentioned that the decision of the system operator is communicated via the input parameters m_j and u_j which are the active power reduction factor, and the chosen unit commitment decision respectively. We proceed to illustrate how those input parameters are obtained as decision variables from the following DC optimal power flow problem.

3.3 Direct-Current (DC) Optimal Power Flow Model

As previously discussed, the technical representation of the electricity grid is modeled through a DC optimal power flow problem. This problem illustrates the decision that the system operator needs to take shortly before real-time dispatch, and after receiving the final supply bids and demand bids from the market participants. The SO is required to take many aspects in his operational decisions for the network; essentially the supply security, network reliability, and economic operation. The security of supply is of course one of the most important aspects and its aim is to ensure that all demands are served. The reliability of the network is also critical to ensure that the network is operated under both its thermal limits and its voltage limits, and finally the operation of the network must be accomplished under minimum cost to maximize the social welfare the society. Each of these aspects is represented in the SO's problem through several constraints. Since the SO possesses most information about every aspect of the system (supposedly having all the information about the supply as well about the demand), the optimal power flow problem can have many details and many constraints to reflect all these information in the SO's decisions. Constraints for example taking into account -in addition to the production and active power flows in lines- parameters such as the upward and downward ramp-rate limits for the generation units, the unit commitment of these units, the temporal relationship in units' scheduling, the spatial relationship between hydro-reservoirs, the uncertainties in demand, and many more can be included in this problem. In this work however, we consider only the simplest case for the representation of the network operation, which is a deterministic DC power flow taking into account active power production only, similar to what is proposed by Delgadillo and Reneses [2013]. One of the reasons of this simplified model in this phase of the research work, is to be able to clearly and easily make sense of the results and identify a meaningful reliability index solely emphasized on the strategic behavior of the generation companies, without other distortions (more on that in the results chapter).

We formulate a Mixed-Integer Linear programming problem, to obtain the solution for the decisions-set faced by the SO such as:

$$\min_{\Xi} \sum_{j \in J} ACX_j \cdot x_j^{\Omega} + (K - ACZ_j) \cdot z_j^{\Omega} \tag{3.37}$$

Subject to:

$$\sum_{j \in J_a} q_j^{\Omega} + \sum_{a' \in N} F_{(a,a')} - D_a = 0, \quad \forall a, a' \in N, (a,a') \in L \tag{3.38}$$

$$q_j^{\Omega} = q_j + x_j^{\Omega} - z_j^{\Omega}, \quad \forall j \in J \tag{3.39}$$

$$z_j^{\Omega} = m_j \cdot q_j, \quad \forall j \in J \tag{3.40}$$

$$0 \leq q_j^{\Omega} \leq \overline{q_j}, \quad \forall j \in J \tag{3.41}$$

$$0 \leq x_j^{\Omega} \leq \overline{q_j} \cdot u_j, \quad \forall j \in J \tag{3.42}$$

$$0 \leq z_j^{\Omega} \leq \overline{q_j} \cdot (1 - u_j), \quad \forall j \in J \tag{3.43}$$

$$F_{(a,a')} = mc_{(a,a')} B_{(a,a')} \left(\delta_a - \delta_{a'}\right), \quad \forall a, a' \in N, (a,a') \in L \tag{3.44}$$

$$mc_{(a,a')} B_{(a,a')} \left(\delta_a - \delta_{a'}\right) \leq V \times Amp_{(a,a')}, \quad \forall a, a' \in N, (a,a') \in L \tag{3.45}$$

$$-mc_{(a,a')}B_{(a,a')}\left(\delta_a - \delta_{a'}\right) \leq V \times Amp_{(a,a')}, \quad \forall a, a' \in N, (a,a') \in L \tag{3.46}$$

$$m_j \in [0,1], \quad \forall j \in J \tag{3.47}$$

$$u_j \in \{0,1\}, \quad \forall j \in J \tag{3.48}$$

where *Indices*:

- a is the electricity bus index.
- N is the bus number.

Sets:

- Ω The decision set of the optimal power flow problem.
- L Set containing the existing flowgate between the bus combination (a, a').

Parameters:

- K is a constant that is higher than the maximum value of ACZ_j.
- D_a is the demand at bus a [MW].
- q_j is the net production of unit j obtained from the market-equilibrium problem [MW].
- ACX_j is the apparent cost of units j which will be required to increase their productions with respect to the original supply bids [€/ MW].
- ACZ_j is the apparent cost of units j which will be required to decrease their productions with respect to the original supply bids [€/ MW].
- $mc_{(a,a')}$ is the mechanical state of existing flow-gates [0,1].
- $B_{(a,a')}$ is the line susceptance [1 / Ohm].
- V Nominal voltage of the network (constant and is equal to 1 per unit) [kV].
- $Amp_{(a,a')}$ is the ampacity of the flow-gates (a, a') [A].

Variables:

- x_j^{Ω} is the increments required in the power production of unit j in the optimal power flow problem [MW].
- z_j^{Ω} is the decrements required in the power production of unit j in the optimal power flow problem [MW].
- q_j^{Ω} is the optimal net production of unit j (after taking into account the increments x_j and the decrements z_j in the optimal power flow problem [MW].
- m_j is the reduction factor of units j.
- u_j is a binary-variable that is equal to 1 if unit j is required to increase its production (or if $x_j > 0$) and is equal to 0 otherwise.

- $F_{(a,a')}$ is the active power flow from bus a to bus a' if $(a, a') \neq 0$ [MW].
- δ_a is the voltage phase angle at bus a [rad].

First, let us consider the objective function for the optimal power flow problem. As previously mentioned, there are several aims the System Operator seeks to achieve in terms of system security, reliability and cost. Any of these aims could formulate the objective function of the problem at hand, and the others being the constraints that should have values that remains within certain limits. Typically, system cost is the aspect that is sought to be minimized, given that the security and reliability aspects remain within certain permissible tolerances. As such, the cost that is much more often minimized in the OPF problems, is the "cost of power dispatch"; the S.O. would typically (and correctly so) seek to run the optimization problem while scheduling the generators in the increasing cost order in a way that is feasible to the network. Minimizing the "marginal cost" or the "bid price" of the generation units therefore seems to be the most logical and straightforward problem to consider. However, the minimization problem that Delgadillo and Reneses [2015] proposes -and which is considered here- does not consider the units marginal cost, but rather it consider the *apparent cost* of those units as described in a previous section. For my own purposes in using this model, the reason why I used the apparent cost as the objective function parameter is explained in the "Solution Method" section 3.4, as it is more related to the philosophy of the solution implemented. For now, I proceed to explain how the objective function (3.37) is constructed, and what it means.

The objective function (3.37) is to find the values for the decision variables set $\Xi = (q_j^{\Omega}, x_j^{\Omega}, z_j^{\Omega}, m_j, u_j, F_{(a,a')}, \delta_a)$ by minimizing the ***total cost of changes in the production quantities due to network constraints, with respect to the original schedule***. Obviously, the amount of change in quantities is represented by the variable x_j and z_j which are the increments and the decrements respectively. The meaning of the apparent cost of increasing generation units ACX_j, and that of the apparent cost of the decreasing generation units ACZ_j have been previously explained in section (3.2.2), where they are the same as equations (3.17) and (3.16) respectively. Or for a more clear illustration consider the following:

$$ACX_j = MC_j\left((1-m_j)\cdot q_j + x_j\right) + \beta_i \cdot \sum_{k\in J_i} x_k, \quad \forall i \in I, \forall j \in J_i. \tag{3.49}$$

$$ACZ_j = MC_j\left((1-m_j)\cdot q_j + x_j\right) + \frac{\theta_i}{(1-m_j)} \cdot \sum_{k\in J_i} (1-m_k)\cdot q_k, \quad \forall i \in I, \forall j \in J_i. \tag{3.50}$$

It can be easily seen that both apparent costs will always be equal or higher than the marginal cost MC_j of the generation units. This is an obvious observation since we consider strategic behavior of firms to exercise market power expressed by the conjectured-price response θ_i and β_i. Yet, how does the change in the apparent cost affects the system operator's minimization problem? In fact, those units whose apparent cost ACX_j is *lowest*, will be the ones selected to increase their production if needed. This is straightforwardly seen if we consider that ACX_j is basically the marginal cost of each unit. On the other hand for those units whose apparent cost ACZ_j is *highest*, will be the ones selected to decrease their production, again a straightforward conclusion. However, this is not straightforwardly calculated in the minimization problem; since the problem minimizes z_j^{Ω} which is the *decrease in production*, then the units with the highest cost should be the ones allowed to have positive values of z_j^{Ω}, this is ensured through subtracting ACZ_j from the constant K which is higher in value, to make the more expensive units space in the algorithm to obtain the highest reduction values. Other less obvious implication for the use of the apparent cost, however, will be discussed in the next section.

Now that we have set the optimal power flow problem to minimize the changing cost of generation, we formulate the technical constraints of the network. Constraint (3.38) is the supply-demand balance equation considering the power flows in lines $F_{(a,a')}$ which are either entering (positive) or leaving (negative) bus a. Constraint (3.39) ensures that the final production q_j^{Ω} of generation unit f in the optimal power flow problem, is equal to the the original production bid q_j plus or minus whatever amount which needs to be increased or decreased respectively. Hence, ensuring the logical consistency of the OPF problem. Constraint (3.40) calculates the reduction factor m_j as the ratio between the production decrease decision z_j^{Ω} of each unit with respect to its original production schedule. Constraints (3.41)-(3.43) are the boundaries for the decision variable. Worthy to note that here the binary variable u_j ensures that any generation unit j can not both increase and decrease its production simultaneously. The variable u_j will only be equal to 1 if the unit is required to increase its production based on the solution of the optimal power flow problem. Constraints (3.43)-(3.45) describe the technical limits for the transmission lines. These equation link the susceptance and bus voltage angles with the maximum capacity for the lines. The binary variable $mc_{(a,a')}$ represents the mechanical state of the transmission lines, where 1 means the line is operation and 0 means that the line is not, and that is how the failure of lines is introduced in the model. Finally, constraints (3.47) and (3.48) set the limits for the variables m_j and u_j respectively; m_j being a *factor* can have any continuous values between 0 and 1, while the binary variable u_j can only have values of either 0 or 1.

The simplified OPF is thus defined, as well the Market-Equilibrium problem. Since now we have the tools to represent the decision making of both the generation companies and the system operator, we proceed to illustrate how those models combined can be solved.

3.4 Model Solution Method

First we consider each problem separately; for the Mixed Complementarity Problem (3.28)-(3.34), the problem is formulated on the GAMS (General Algebraic Modeling System) software

[GAMS Development Corporation, 2014], and is solved using the PATH solver [1] [Dirkse et al., 2013]. For the DC optimal power flow, the problem is formulated on Matlab [MATLAB, 2013]. Since the optimal power flow problem contains the binary decision variable u_j, it has to be solved as a Mixed Integer Linear Programming (MILP) to ensure that the variable only takes the values of [1,0]. The ILOG - CPLEX solver is used to solve this MILP in Matlab.

Now we consider the solution for the general problem; we have seen that m_j and u_j are considered input parameters with respect to the equilibrium problem (3.28)-(3.34), representing the anticipated decision the the SO is taking regarding the decrease or increase of a certain generation unit j. Since these values m_j and u_j are decision variables in the optimal power flow problem (3.37)-(3.48), an iterative method is then needed to be implemented in order to find the general optimal solution of both problems. Delgadillo and Reneses [2013] and Delgadillo and Reneses [2015] propose a simple iterative method that have been implemented in the present work with some modifications, the solution method is as follows:

1. Set a smoothing factor α such as $\alpha \in [0, 1]$.
2. Set an iteration counter (υ) and initialize it with 1.
3. Initialize the variables $m_j^{(\upsilon)} = 0$, $u_j^{(\upsilon)} = 0$, $D^{(\upsilon)} = \sum_a Load_a$, $Y^{(\upsilon)} = 0$. These values corresponds to the case with no technical constraints (without taking into account the system operator's decisions).
4. Solve the Mixed Complementarity Problem (3.28)-(3.34). We obtain a solution for q_j^*, x_j^*, λ^*, γ^*.
5. Update the active power $q_j^{(\upsilon)}$, the power increment $x_j^{(\upsilon)}$, the day-ahead market price $\lambda^{(\upsilon)}$ and the correction-market price $\gamma^{(\upsilon)}$ following:

$$q_j^{(\upsilon)} = \alpha \cdot q_j^* + (1 - \alpha) \cdot q_j^{(\upsilon-1)} \tag{3.51}$$

$$x_j^{(\upsilon)} = \alpha \cdot x_j^* + (1 - \alpha) \cdot x_j^{(\upsilon-1)} \tag{3.52}$$

$$\lambda^{(\upsilon)} = \alpha \cdot \lambda^* + (1 - \alpha) \cdot \lambda^{(\upsilon-1)} \tag{3.53}$$

$$\gamma^{(\upsilon)} = \alpha \cdot \gamma^* + (1 - \alpha) \cdot \gamma^{(\upsilon-1)} \tag{3.54}$$

6. Evaluate the apparent cost for the units required to increase their production ACX_j, and the apparent cost for the units required to decrease their productions ACZ_j at the current

[1] The PATH solver relies on a generalization of the Newton's numerical method for solving a square system of non-linear equations, it relies on a non-smooth reformulation of the complementarity problem [Ferris and Munson, 2000].

iteration (υ) such as:

$$ACX_j^{(\upsilon)} = MC\left(\left(1 - m_j^{(\upsilon)}\right) \cdot q_j^{(\upsilon)} + x_j^{(\upsilon)}\right) + \beta_i \cdot \sum_{j \in J_i} x_j^{(\upsilon)} \tag{3.55}$$

$$ACZ_j^{(\upsilon)} = MC\left(\left(1 - m_j^{(\upsilon)}\right) \cdot q_j^{(\upsilon)} + x_j^{(\upsilon)}\right) + \frac{\theta_i}{\left(1 - m_j^{(\upsilon)}\right)} \cdot \sum_{j \in J_i} \left(1 - m_j^{(\upsilon)}\right) \cdot q_j^{(\upsilon)} \tag{3.56}$$

7. Solve the DC-OPF (3.37)-(3.48). This gives a solution for q_j^{Ω}, x_j^{Ω}, z_j^{Ω}, m_j, u_j, $F_{(a,a')}$, δ_a.

8. Update the optimal power flow total active power $q_j^{\Omega(\upsilon)}$, power increments $x_j^{\Omega(\upsilon)}$ and power decrements $z_j^{\Omega(\upsilon)}$ as following:

$$q_j^{\Omega(\upsilon)} = \alpha \cdot q_j^{\Omega} + (1 - \alpha) \cdot q_j^{\Omega(\upsilon-1)} \tag{3.57}$$

$$x_j^{\Omega(\upsilon)} = \alpha \cdot x_j^{\Omega} + (1 - \alpha) \cdot x_j^{\Omega(\upsilon-1)} \tag{3.58}$$

$$z_j^{\Omega(\upsilon)} = \alpha \cdot z_j^{\Omega} + (1 - \alpha) \cdot z_j^{\Omega(\upsilon-1)} \tag{3.59}$$

9. Calculate an updated value for the variables $D^{(\upsilon)}$ and $Y^{(\upsilon)}$ such as:

$$D^{(\upsilon)} = \sum_{j \in J} q_j^{\Omega(\upsilon)} \tag{3.60}$$

$$Y^{(\upsilon)} = \sum_{j \in J} z_j^{\Omega(\upsilon)} \tag{3.61}$$

10. If the difference in the values of the variables in iteration (υ) and iteration $(\upsilon - 1)$ is lower than a tolerance value (ϵ), the algorithm stops; otherwise the iteration counter increases by one and go to step 4.

The smoothing factor α is in fact a critical component for this solution method; it can be considered as a *learning rate* and is primarily used to achieve a smooth convergence in the value of the variables. This is especially difficult in problems involving iterative solutions and more specifically when it is highly non-linear as in the present case. Of course if α is equal to zero, then that means that the iteration will just keep solving the original problem with the initialized values over and over with no change at all in the solution obtained in the first iteration. If α is equal to 1, however, the solution is obtained using only the information given in the previous iteration.

I have previously mentioned, that an explanation for the reason why the apparent costs ACX_j and ACZ_j are specifically used in this model, rather than any other cost definition (which would also provide a solution to the problem). In fact, in order to illustrate the reason, it is important to consider the iterative nature of the solution proposed. If we examine the problem from the point of view of the SO, we see that since he/she is simulating the reactions (behavior) of the market agents with respect to his decisions in the network operations, then in each iteration we consider that the GenCos are *learning* what the SO would typically do, and thus reacting accordingly. This reaction changes in each iteration as they anticipate the next step, until both equilibrium problem solution and OPF solution no longer update their values in the iterative interaction, indicating that this would be the final equilibrium (learning) state if this interaction occurred enough number of times to allow it. In this case, considering a fixed *marginal cost* in the OPF problem for each unit would fail to capture the effect of this learning procedure that the market agents are undergoing which for example, make them value less the units anticipated to be required to reduce their production (and therefore provide an opportunity cost (loss) for the GenCo). The apparent cost provides therefore, a very important *"behavioral"* link between the two problems that better captures the rational behavior of real time agents. It provides the system operators' OPF problem with the insights to see how those firms value their units based on his/her decisions, and hence be able to solve the network problem accordingly.

3.5 Risk Index and Assessment Method

Risk is a composite index meant to reflect both the probability and the severity of a negative outcome in certain operating conditions. To adopt a quantitative definition of risk, we refer to the most commonly used one as the product of the probability of occurrence of an undesired event (e.g. transmission network line failure) and the related consequence [Zio, 2007]. To take into account the negative effect of several undesired events, the definition is extended by summing all the relevant contribution. More formally we can express the risk definition as:

$$Risk(R) = \sum_{n} p(E_n) \cdot Sev(E_n) \tag{3.62}$$

where n is the event index, $p(E_n)$ is the probability of occurrence of the undesired event E_n, and $Sev(E_n)$ is the severity of the related consequences. Typically in the context of power systems, system contingency is considered as the unexpected loss of one or more of several elements: distribution line, transformer, or generator. In addition, commonly severity function is related to the cost of energy not served (ENS). Below I proceed to define those functions for the purpose of our study.

3.5.1 Probability Model

Very simply, the undesired event considered for this case study is the transmission line failure. There can be different considerations for the calculation of the probability of occurrence of such failure; for instance one could consider failures related to the active power overflows in the networks, those could result from dispatch errors or the default of a critical generator leading to power flows instabilities. On the other hand, one could consider the probability of extreme weather conditions which endangers the power lines, or even the intentional power outages discussed in the literature review. Moreover, the calculation of probability of a line failure can be related to voltage stability for instance due to high dependence on the stochastic renewable resources generation. Each of these cases leads to different definitions of the probability model.

Here, we adhere to the intrinsic failure characteristics of the power lines themselves, relying on extrapolating their historical calculation in terms of of the permanent outage rate for each line, and their respective outage duration in hours. Formally, the probability model for the risk assessment in this work is defined as:

$$p(E_l) = \frac{OR_l \cdot T_l}{Hrs}, \qquad \forall l \tag{3.63}$$

where l is the transmission line index, OR_l is the outage rate per year per line, T_l is the average outage duration for transmission line l in hours, and Hrs is the total number of hours per year. It should be noted that we changed the general index n in the former equation to the line index l as we are concerned with calculating the undesired event being the line failure probability, if no line failure is considered then the probability function is simply equal to zero.

3.5.2 Severity calculation

We consider an *economic* severity function, meaning that the risk impact is considered to influence the system cost. Since we are aiming at adopting the SO's point of view for running the system. We refer to his/her main objective which is operating the system in a way that maximizes the social welfare. Of course the major target for the SO is to ensure that all network demand is served. In an electricity market context, this means all demand which can be "economically" served (i.e. bidding higher than the final market clearing price). We consider the typical reliability index for not meeting all demand in the network that is the amount of Energy Not Served (ENS) quantified through the cost of ENS in (€/ MWh) as being one of the consequences the SO is facing. Moreover, since we consider oligopolistic market behavior, many economic inefficiencies arise due to the exercise of the market power by the market agents. In the previous sections we discussed about those agents' strategic behaviors in anticipating the re-dispatch of the network as a consequence of line failures supported by a correction market, and the objective of the SO to make sure that these changes are implemented as efficiently as possible. We thus define another severity factor, which is the cost of correcting the dispatch schedule in the case of line failure. The reason we consider this cost, is that this is an inefficiency that arises in the power system when run in a market context. In a centralized system, no correction costs would exist as the SO has full control on the generators. We formally define the schedule correction cost as:

$$ACZ_j \cdot z_j + ACX_j \cdot x_j \tag{3.64}$$

These terms have been discussed in details in section 3.3. The first one is the cost of decreasing the scheduled generation relative to the original day-ahead market clearing, while the second is the cost of increasing them. Both upwards and downwards corrections incurred to ensure feasible system dispatch. It should be noted that these costs do not necessarily reflect real monetary transactions in running the power system[2], i.e. the SO does not in fact pay the generators which will be required to reduce their productions (only those required to increase are paid the correction market prices), however, we still consider this cost for the SO as it reflects a reduction in the social-welfare in the system in general. Formally, the severity cost that the SO is facing in case of a line failure is considered to be:

$$Sev(E_l) = (cens_a \cdot ENS_{a,l}) + (ACZ_{j,l} \cdot z_{j,l}) + (ACX_{j,l} \cdot x_{j,l}), \qquad \forall a, \forall j, \forall l \tag{3.65}$$

where ($cens_a$) is the cost of energy not served at network bus a, and $ENS_{a,l}$ is the amount of energy not served at bus a in the case of failure of line l. $ACZ_{j,l}$ and $ACX_{j,l}$ are the apparent cost of units required to decrease their production and the apparent cost of those required to increase their production respectively, in case of line l failure. Finally $z_{j,l}$ and $x_{j,l}$ are the production amount decreases or increases respectively, in case of line l failure.

The risk assessment index considered is thus calculated as:

$$Risk(R) = \sum_l \frac{OR_l \cdot T_l}{Hrs} \cdot \left[(cens_a \cdot ENS_{a,l}) + (ACZ_{j,l} \cdot z_{j,l}) + (ACX_{j,l} \cdot x_{j,l})\right], \qquad \forall a, \forall j \tag{3.66}$$

It should be noted that the severity function definition provided is the main focus of this study. For the probability function; several factors can be considered within the forced outage rates (e.g. probability of failure due to extreme weather conditions). In the next chapter, we apply the model formulated and the risk index defined on a numerical example, and provide a critical analysis of the results obtained.

[2] Cost allocation in case of failures is a complex topic and highly depend on the regulations designed for each system, therefore we will not be addressing them here, only considering the cost as an inefficiency that need to be accounted for.

Chapter 4

Numerical Example, Results and Analysis

In this Chapter, I proceed to illustrate the numerical example built for the analysis of the model, and discuss the results obtained. In the first section, all the characteristics of the network constructed are shown and discussed, and in the second section the model results under different line failures are portrayed. In the last section, a critical analysis is provided with emphasis on the difficulties and limitations of the method used.

4.1 Numerical Example

The electricity system under study is a simple 6-bus system adapted from the IEEE 6-bus Reliability Test System [Billinton et al., 1989]. This system although being small (to permit the conduct of a large number of reliability studies with reasonable solution time) provides sufficient details to reflect the actual complexities involved in a practical reliability analysis. In general, the method applied for the study on this system, and the algorithm developed is reported to be suitable for applications including larger and more complex systems [Delgadillo and Reneses, 2013]. I adhere however to this simple network in order to provide concise explanation of the results, and to remain able to provide intuitive analysis, with more complex case studies to follow in later phases of the research.

4.1.1 Overview of the Reliability Test System (RBTS)

Figure 4.1 shows the single line diagram of the adapted RBTS system. As shown the system has 2 PV buses (generator buses) containing 11 generation units, 4 PQ buses (load buses), and

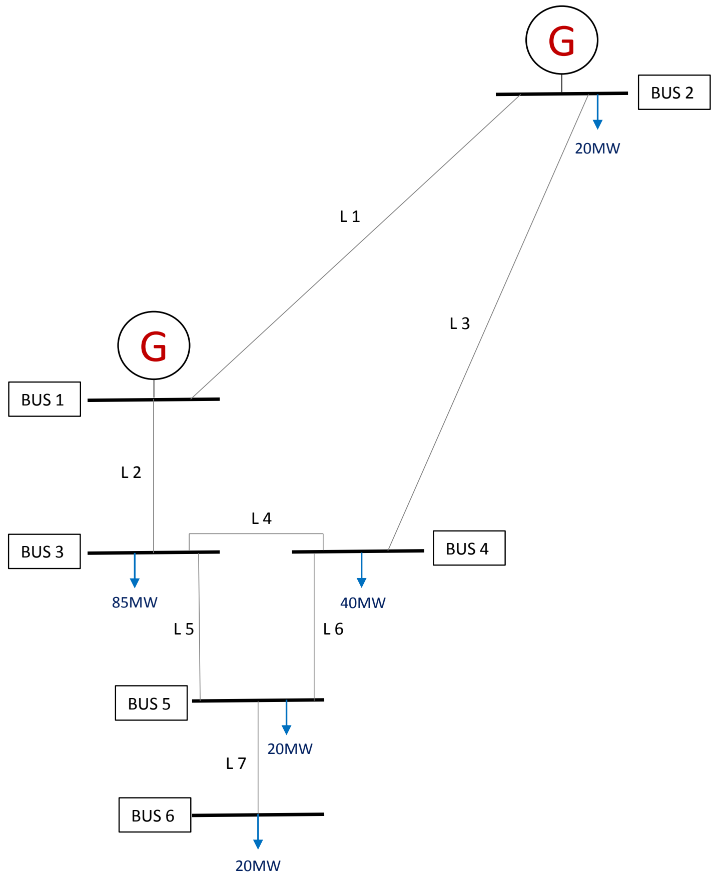

FIGURE 4.1: Single line diagram of the Reliability Test System Modeled.

7 transmission lines. The minimum and the maximum ratings of the generating units are 5 MW and 40 MW respectively. The voltage level of the transmission system is 230 kV. The system has a peak load of 185 MW and the total installed capacity amounts to 240 MW. The transmission network's line lengths in Figure 4.1 are shown in proportion to their actual lengths summarizes in Table 4.3.

Table 4.1 summarizes the percentage of the generation mix per technology, 46% of the network generation capacity are assumed to be coming from thermal units (Coal, Gas, fuel, etc.), while 54% are assumed to be generated from hydro units. Table 4.2 illustrates the breakdown of the total available capacity and demand per network bus. Demand is distributed from bus 2 to bus 6, and the values shown in the table represent the peak hour demand values. Generation however, is concentrated along two network buses only, which is a logical and typical representation for the positioning of traditional fuel and hydro units, since hydro resources might often be concentrated in certain locations on the grid, as well as there might be social requirements for the highly polluting thermal units to be located away from the consumers. Finally, since I do not consider reactive power requirements and reactive power flows in the network, it is assumed that bus voltages magnitude is constant and is equal to 1 per unit. In addition the power factor at each bus is equal to 1.

TABLE 4.1: Generation Mix in the Electricity System.

	MW	%
Thermal	110	46
Hydro	130	54

TABLE 4.2: Bus Power Capacity and Bus Demand.

Bus	Total Available Capacity (MW)	Demand (MW)
1	110	0
2	130	20
3	0	85
4	0	40
5	0	20
6	0	20
Total	240	185

TABLE 4.3: Transmission lines Characterization

	Buses					
Line	From	To	Line Length (Km)	Resistance R (p.u.)	Reactance X (p.u)	Susceptance B (p.u)
1	1	2	200	0.0912	0.480	2.010
2	1	3	75	0.0342	0.180	5.362
3	2	4	250	0.1140	0.600	1.608
4	3	4	50	0.0228	0.120	8.043
5	3	5	50	0.0228	0.120	8.043
6	4	5	50	0.0228	0.120	8.043
7	5	6	50	0.0228	0.120	8.043

100 MVA base
230 kV base

With regards to the transmission lines, Table 4.3 summarizes the technical characteristics of the transmission flow-gates. Lines resistances, reactances and line susceptances are reported in the table as “per unit” values. They are calculated on the basis of the system’s base power rating being 100 MVA and voltage rating being 230 kV.

4.1.2 Detailed Generation Units’ Breakdown in the Network

We further breakdown the network to illustrate the detailed topology of the system, and the positioning of each generation unit along the network. Figure 4.2 shows the individual generation units’ placement on the electricity grid. It is worthy to note that although previously mentioned that the total number of units in the network grid are 11 units, the single line diagram 4.2 is

TABLE 4.4: Generation Units Capacity and Cost Data.

			Variable costs, €/MWh		
Unit Num.	Technology	Capacity (MW)	Fuel Cost	Operation Cost	Total Cost
1	Thermal	10	10	3.5	13.5
2	Thermal	20	9.75	2.75	12.5
3	Thermal	40	9.75	2.5	12.25
4	Thermal	40	9.50	2.5	12
5	Hydro	5	0.65	0.15	0.8
6	Hydro	5	0.65	0.15	0.8
7	Hydro	20	0.45	0.05	0.5
8	Hydro	20	0.45	0.05	0.5
9	Hydro	20	0.45	0.05	0.5
10	Hydro	20	0.45	0.05	0.5
11	Hydro	40	0.45	0.05	0.5

showing 17 units. This is because 6 of these units (units 12 to unit 17) are *virtual* generators which do not actually produce active power. Their presence in the network is for calculating the amount of energy not served in their respective demand bus.
Table 4.4 shows the technical capacities and the cost data for each of the generation unit (the real units providing active power). The maximum unit capacities as seen range from 5 MW to 40 MW, where we do not consider losses due to the efficiency of the units, or in other words the efficiency factor is assumed to be equal to 1. The cost data represent the units' variable costs in (€/ MWh), it consists of the fuel cost and other operation cost such as the maintenance cost. The major component of the units' variable cost is the fuel cost as seen in Table 4.4, being the cost directly associated with energy production. This is clearly understood for the thermal units as the cost of different types of oil, fuel or gas. For hydro units however, the fuel cost represents the "water value" or "water rental charges". This is a concept that links the temporal aspect for the usage of water as a source of electricity, since the way that a firm values its water content in stored in its reservoirs depends on many uncertainties such as the forecast for hydro inflows, the uncertainties in future market prices, regulation requirements in certain countries, and many others. A comprehensive discussion about the calculation of water value is beyond the scope of this dissertation, especially given the static nature of the model in this phase of the research work, which eliminates the need for a detailed consideration of this concept. The water charge is considered here as an exogenous parameter with the values shown in the table.

Next, we consider the *"virtual generators"* which are added to the network buses. The aim of these generators is to provide an easy and convenient way for modeling the load that is not served due to network failure. In case a certain bus load can not be satisfied with the real (and cheaper) active power coming from generators 1 to 11, then one or several of the virtual generators 12 to 13 will have to produce a *"virtual"* quantity to compensate for the difference

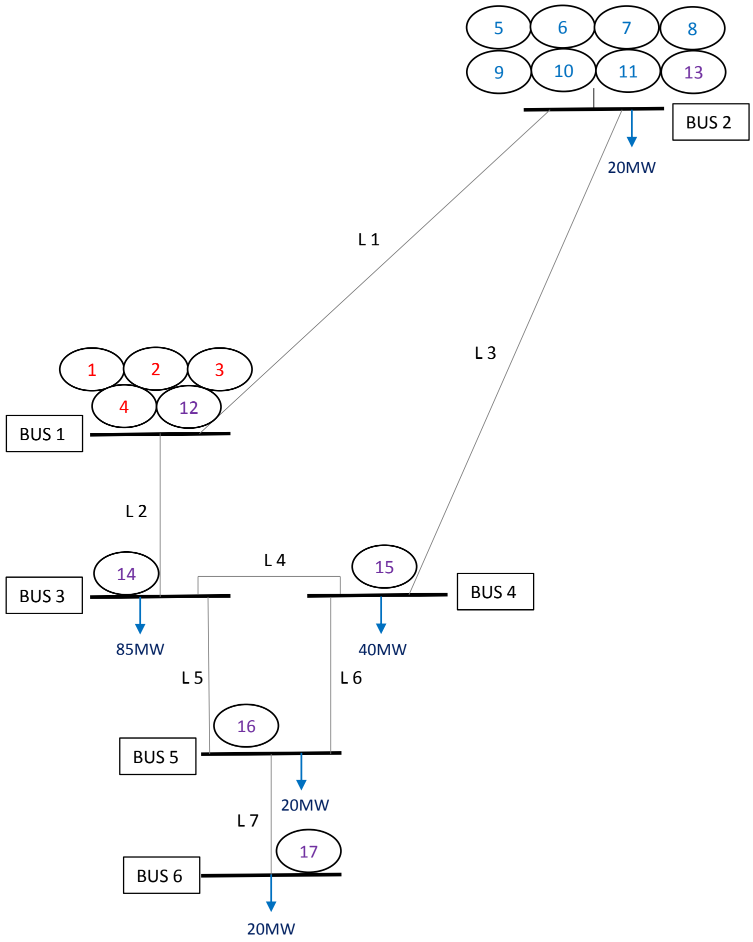

FIGURE 4.2: Generation units' placement on the RBTS Single line diagram.

and keeps the supply-demand balance equation (3.38) in the OPF problem satisfied. The virtual production value of these units are thus the amount of energy not served in each bus; Table 4.5 illustrates the capacity limits and cost data for those virtual units. First, we discuss how the cost of these units, or rather the cost of energy not served (ENS) is obtained. There are many ways to calculate the ENS cost as it differs from one system to another and from one country to another, depending on the policy implemented. In Billinton et al. [1989] it can be found a well detailed guideline for the cost of interruption of power per sector (industrial, household, commercial) as a function of duration. In this work, however, quite simply we calculate the ENS cost per MW as having a constant value for all network demand, dependent only on the distribution of the load among the network buses. The ENS value is multiplied by the percentage of the demand in a certain bus with respect to the total network demand, resulting in the values shown in Table 4.5.

Regarding the capacity of those generators, although being virtual units they can possibly have unlimited capacity, the limits are set to the maximum amount of load in each network bus to ensure that no virtual generator compensates for loads located in any other place in the network other than the bus where it is located. This is especially crucial when we consider the cost of

Table 4.5: Load Shedding (Virtual Generators) cost data.

Unit Num.	Technology	Capacity (MW)	ENS Cost €/MWh
13	Virtual Generator	20	132.973
14	Virtual Generator	85	175.135
15	Virtual Generator	40	145.946
16	Virtual Generator	20	132.973
17	Virtual Generator	20	132.973

Table 4.6: Transmission lines Capacities and outage data.

	Buses				
Line	From	To	Maximum Line Capacity (MW)	Permanent Outage rate (per year)	Outage duration (hours)
1	1	2	40	4.0	15.0
2	1	3	95	1.5	15.0
3	2	4	70	5.0	15.0
4	3	4	20	1.0	15.0
5	3	5	20	1.0	15.0
6	4	5	30	1.0	15.0
7	5	6	20	2.0	15.0

ENS due to the failures of different lines, which leads to load shedding in different locations in the network. Finally, Table 4.6 illustrates the transmission lines maximum capacity assumptions, and the outage data expressed as the number of complete line outage for each line per year, and finally the duration of this outage in hours. Important to note is that the maximum lines capacities are not chosen arbitrarily, rather they represent the maximum stable loading of these lines under the current load and supply distributed on the network. The logic behind that, is the attempt to simulate the behavior of the network taking into consideration that lines are operated close to their maximum technical capacity. This is a reasonable assumption when we consider a market context where suppliers have incentives to earn as much profit as possible, as well as the long term nature of the transmission network expansion, that could pertain a certain network capacity for long time although both demand and supply are constantly increasing. Therefore, it is of interest to study the risk assessment of this network close to its maximum capacity.

4.1.3 Market Agents Representation

Since we consider a market representation for the electricity system operation and power dispatch, we need to characterize the competition aspect in the network. Typically, we have seen in Chapter 3 that we assume i firms each owning $j \in J_i$ generation units competing in the single

TABLE 4.7: Characterization of market agents.

Agent i	θ_i $\left[\frac{€/MWh}{MW}\right]$	β_i $\left[\frac{€/MWh}{MW}\right]$	Unit j	Bus a	Marginal Cost [$€/MWh$]	$\overline{q_j}$ [MW]
1	0.02	0.1	3	1	12.25	40
			4	1	12.00	40
			5	2	0.80	5
2	0.02	0.1	8	2	0.50	20
			10	2	0.50	20
			11	2	0.50	40
3	0.01	0.1	1	1	13.50	10
			2	1	12.50	20
			6	2	0.80	5
4	0.02	0.1	7	2	0.50	20
			9	2	0.50	20

price market. Table 4.7 illustrates how the different generation units are distributed among the different market agents. In this case study, it is assumed that 4 generation companies (GenCos) are competing. As shown, each one of the four companies owns a different generation mix and has in its possession different total production capacities divided differently along the network buses. The important thing to note is the conjecture-price response terms θ_i and β_i for the competition in the day-ahead market, and for the correction market respectively. For the day-ahead market, firms are assumed to have the ability to exercise market power as most often electricity market are governed by an oligopoly behavior rather than a perfectly competitive behavior. As previously mentioned the value of θ_i can range from 0 to 1; 0 simulating the perfect competition, and 1 simulating the Cournot competition. The value for θ_i in this work is chosen arbitrarily, although in different existing publications there are methods to calculate it endogenously. In this work θ_i is chosen to be equal to 0.02 for firms 1, 2 and 4, and equal to 0.01 for firm 3. Since we assume that a firm having the smallest capacity and the most expensive units (such as firm 3) would typically have less chances to exercise market power than the firms which have "cheaper" units more often scheduled. For the correction market, firms are also assumed to be excising market power, the conjecture-price response (β_i) for this market is set to be equal to 0.1 for all firms. Finally, for simplicity it is assumed that the cost functions for the generation units are linear, although as seen in the problem formulation, it can accommodate non-linear functions.

For the case studies considered; it is assumed in each of them that a line failure occurs independently; i.e. no more than one line fails simultaneously. Line failure is modeled by setting the mechanical state parameter for the respective line equal to zero in equations (3.45) and (3.46) in the optimal power flow problem. Both the equilibrium model (3.28)-(3.36) and the optimal power flow model (3.37)-(3.48) are solved iteratively, for each case of the line failures (line 1 to line 7). The benchmark for the study is the case solved with no line failure, which is the normal operating scenario for peak hour in our system. In the next section, I start by showing the model results for the base case of no line failure, then proceed to present and analyze the results of all other failure cases considered and subsequently propose a definition for the risk index. The different cases solved are enumerated from "Case I" to "Case VI", where "Case I" represents the solution for the instance of line 1 failure up to "Case VI" which represents that of line 7 failure. Finally, it is important to note that the smoothing factor "α" discussed in the "solution method" section 3.4 is set throughout this whole study at a fixed value equal to "0.1".

4.2 Results and Analysis I - Reduced bids in case of load shedding requirement

There is one important note regarding the results of the different case studies. We separate the results reported below in two sets; the first is solved assuming that in the case of the anticipation of a definite load shedding in the network resulting from a line failure, market agents are allowed to submit reduced bids that matches only the remaining demand that can be reached in case of this failure. This means that constraint (3.28) in the equilibrium problem considers the total demand minus the load that is not possible to be reached. The other results set relaxes this assumption, and regardless of the load shedding reason or amount in the network, market agents must keep bidding for the total demand. A discussion is provided regarding these assumptions at the end of the results section.

4.2.1 Base Case: No Network Lines Failures

We start with the basic case of no network failures under oligopolistic behavior. The network is originally characterized in a manner that allows for the operation of the network with no lines congestion if line failure occurs, albeit being operated near their capacity limits. In this case we expect that only the market forces (price competition for supply and demand) will determine the dispatched schedule in the network; as there is no technical interventions necessary. Moreover, since the total installed generation capacity in the network exceeds the total demand even in the peak hour modeled; the only logical power dispatch will be based on the least cost solution (merit order) even as competition is modeled through the conjecture-price response assumption. The solution for the base case is reached by solving both equilibrium model and the optimal power flow model iteratively leaving all network lines operational. Table 4.8 summarizes the most important results obtained in this case. As shown, the results are arranged by the units' location on the network and then further arranged in decreasing marginal cost order. Table 4.8a shows the active power production by the generation units, while Table 4.8b illustrates the amount of load not served in each demand bus due to any technical limitations in the network.

Let us first consider the results illustrated in Table 4.8a. The "Final Schedule" results are the results of the optimal power flow problem as seen by the system operator and taking into account the network constraints. The SO receives the initial agents' quantity bids, run the optimal power flow with the objective of minimizing the cost of changes (if needed) in the original bids, and obtain the final feasible schedules as those seen in the table. The "Agents' Bids" column summarizes the quantity bidding decision of the market agents (electricity suppliers) *after* they have anticipated the SO's possible reaction in case of network limitations and the need of re-dispatch. ACZ and ACX illustrated are the apparent costs of the units in the day-ahead market and in the correction market respectively, and finally the reduction factor results are shown.

TABLE 4.8: Model solution for Base Case

(A) Active power production

Bus a	Unit j	Marginal Cost [€/ MWh]	Final Schedule q_j^{Ω}[MWh]	Agents' Bids q_j[MWh]	ACZ [€/ MWh]	ACX [€/ MWh]	Reduction Factor
1	1	13.50	0.00	0.00	13.75	13.50	0.00
	2	12.50	20.00	20.00	12.75	12.50	0.00
	3	12.25	0.00	0.00	13.05	12.25	0.00
	4	12.00	35.00	35.00	12.80	12.00	0.00
2	5	0.80	5.00	5.00	1.60	0.80	0.00
	6	0.80	5.00	5.00	1.05	0.80	0.00
	7	0.50	20.00	20.00	1.30	0.50	0.00
	8	0.50	20.00	20.00	2.10	0.50	0.00
	9	0.50	20.00	20.00	1.30	0.50	0.00
	10	0.50	20.00	20.00	2.10	0.50	0.00
	11	0.50	40.00	40.00	2.10	0.50	0.00

Oligopoly: $\theta_i \neq 0$, $\beta_i \neq 0$
Market Price $\lambda = 12.80$ €/ MWh
Correction Price $\gamma = 0$ €/ MWh

(B) Load Shedding

Bus a	Unit j	Final Schedule [MWh]	ENS cost [€/ MWh]
2	13	0.00	132.973
3	14	0.00	175.135
4	15	0.00	145.946
5	16	0.00	132.973
6	17	0.00	132.973

As expected, since there is no line failures, only the market equilibrium solution dictates the power dispatch in the network, the SO does not face any technical constraints and therefore accepts the agents' bids as received, resulting in exactly the same final schedule. The market agents are modeled as competing, and therefore only the lowest cost units are dispatched. This is consistent with the results obtained where only the units with lowest costs are committed. Important to note however, is that since we assume an oligopoly market, agents' are modeled as exercising market power (and therefore bidding higher than their marginal costs); this means that the bidding prices are not those illustrated in the "marginal cost" column, but rather those shown in the (ACZ) one which considers the conjecture-price response. We see then that all the cheap hydro units located on bus 2 are dispatched at maximum production capacities, whereas for thermal units located on bus 1, only the cheapest one (unit 2) is scheduled at full capacity. The second cheapest (unit 4) is partially dispatched and is therefore considered the "marginal unit" setting the day-ahead market price (λ), this value is obtained as being that of the dual-variable of the supply-demand balance equation (3.28) in the equilibrium problem.

Regarding the subsequent correction market, it is quite straightforward to see that since there is no need for any unit to increase its generation beyond what it was originally bid, the correction market price (γ) is equal to zero. Moreover, the agents are not competing at all in this market and this is the reason why the apparent cost of the units in this market (ACX) is the same as the marginal costs of the units. Same applies for units required to decrease their production, since there is no reduction requirements, the reduction factors are equal to zero. This ensures that there is no incentive for the market participants to alter their behavior from what they would have originally bid by not taking into account the network constraints. Finally, Table 4.8b confirms that there is no demand which will not be served. The virtual units (13 to 17) are not producing indicating that there is no need to compensate for any energy loss.

Base Case: Comparison to Perfect Competition

I find it perhaps of interest before departing from the base case shown above, to consider how the model outcome would differ if we assume perfectly competitive behavior (i.e. no conjectured-price response $\theta_i,\ \beta_i = 0$) in the day-ahead market and in the correction market. The aim for this illustration is to validate the logical consistency of the model, in the sense that we should expect to see lower market prices if agents are perfectly competitive than if they de-facto exercise market power. Table 4.9 summarizes the most important results under the assumption of perfect competition (4.9a) and under the previous assumption of oligopoly behavior (4.9b). Here however, the units are rearranged according to the agents' ownership to better illustrate how each agent's behavior is specifically different for each competition level.

It is interesting to see in the scenario of perfect competition, how the apparent cost of the units in both the day-ahead market and the correction market (ACZ and ACX) are the same as the marginal costs of the units; i.e. agents are bidding their marginal costs. This indeed leads

TABLE 4.9: Comparison of model solution for the base case under perfect competition and under oligopolistic behavior

(A) Perfect Competition $\theta_i, \beta_i = 0$

Agent i	Unit j	Final Schedule q_j^{Ω} [MWh]	Agents' Bids q_j[MWh]	ACZ [€/ MWh]	ACX [€/ MWh]
1	3	15.00	15.00	12.25	12.25
	4	40.00	40.00	12.00	12.00
	5	5.00	5.00	0.80	0.80
2	8	20.00	20.00	0.50	0.50
	10	20.00	20.00	0.50	0.50
	11	40.00	40.00	0.50	0.50
3	1	0.00	0.00	13.50	13.50
	2	0.00	0.00	12.50	12.50
	6	5.00	5.00	0.80	0.80
4	7	20.00	20.00	0.50	0.50
	9	20.00	20.00	0.50	0.50

Market Price $\lambda = 12.25$ €/ MWh

(B) Oligopoly $\theta_i,\ \beta_i \neq 0$

Agent i	Unit j	Final Schedule q_j^{Ω} [MWh]	Agents' Bids q_j[MWh]	ACZ [€/ MWh]	ACX [€/ MWh]
1	3	0.00	0.00	13.05	12.25
	4	35.00	35.00	12.80	12.00
	5	5.00	5.00	1.60	0.80
2	8	20.00	20.00	2.10	0.50
	10	20.00	20.00	2.10	0.50
	11	40.00	40.00	2.10	0.50
3	1	0.00	0.00	13.75	13.50
	2	20.00	20.00	12.75	12.50
	6	5.00	5.00	1.05	0.80
4	7	20.00	20.00	1.30	0.50
	9	20.00	20.00	1.30	0.50

Market Price $\lambda = 12.80$ €/ MWh

to a lower day-ahead market price of ($\lambda = 12.25$ €/MWh) as opposed to the higher oligopoly price of ($\lambda = 12.80$ €/MWh). However we notice that the merit order of the units is different (leading to different dispatch of units 2, 3, and 4) in both scenarios, especially if we consider that the relative apparent cost change ought to have remained constant, resulting in the same dispatch despite at a higher market price. The key to that is the different assumption for the conjectured-price response value θ_i for agent 1 (who owns units 3 and 4), and for agent 3 (who owns unit 2). Agent 3 is assumed to have a smaller θ value than the rest of the market participants, resulting in his/her inability to raise her bid prices as high as the other agents would do. This is since he/she owns the least total generation capacity (distributed among two of the most expensive generators) and therefore less incentive to raise his prices in order to mitigate the risk of not being committed. The consequence is that in the oligopoly scenario, agent 3 bids a lower price for unit 2 (12.75 €/ MWh) than that for unit 3 or unit 4 (13.05 and 12.80 €/ MWh, respectively) allowing her to capture the entire demand up to the maximum capacity of this unit.
The resulting agents profits in both cases are illustrated in Table 4.10 showing the percentile change in profit between both competition scenarios. The logical consistency of the model is validated in this case, as it is clearly shown that the profit under oligopolistic behavior is higher than that under perfect competition assumption.

4.2.2 Case I: Failure of Line 1

We proceed to consider the more advance cases of simulating line failures. We start by simulating the failure of line 1 and observe the model behavior and outcome. First, if one refers to Figure 4.2 to visualize line 1's failure, it could be deducted that the failure of this line would result in the

TABLE 4.10: Profit of market agents considering different competition levels considering no line failures (Base Case)

Agent i	Profit when $\theta_i = 0$ €	Profit when $\theta_i \neq 0$ €	Percentage Change %
1	67.50	88.00	+30.37
2	940.00	984.00	+4.68
3	57.25	66.00	+15.28
4	470.00	492.00	+4.680

inability of all the capacity of the cheaper hydro units in bus 2 to be exported to the rest of the network serving as much demand as economically as possible. Hydro units located in bus 2 can only export their production using line 3 whose maximum capacity is less than the total capacity required to be transported. This in a way imitates the consideration of line 3 as being *congested*, which indeed is expected to have an effect on competition and therefore on the market price. On the other hand, the most expensive thermal units in bus 1 are now required to compensate for this shortage in capacity for the rest of the network, which we expect could lead to a higher market price than that we have seen in the base case.

The bi-level model is solved iteratively simulating the result in case of the ability of market agents to anticipate the occurrence of the failure of line 1 and therefore preparing their bids accordingly. The results obtained are reported in Table 4.11, and again arranged according to the units' location on the network to better assist with the following illustration. First, we see in Table 4.11b that despite of the line failure, all loads are served and there are no load shedding. This is because bus 1 possess enough spare generation capacity previously too expensive to be committed that would compensate for the power lost in the network. More importantly are those results combined in Table 4.11a representing the summary of the solution for the active power production. First of all, we notice the differences between what the market agent's are considered to be bidding q_j and what the final schedule q_j^{Ω} is ought to be. In addition, we observe that the final schedule in this case is different than the one seen in the base case. We start by exploring the first observation; agents who have units located at bus 2 foresee that in case of line 1 failure some of their units will have to reduce production, this results that the apparent cost of those units is not only affected by the conjecture-price response θ_i as in the base case, but also with the reduction factor (m_j) that those units are facing, by the amount $1/(1-m_j)$ as seen in equation (3.50). The agents solve the equilibrium problem again and check if their profit is maximized with respect to the anticipated changes that would be imposed by the optimal power flow problem, until a *meta* equilibrium is reached where there is no better bidding proposal made that would result in a higher profit after those final changes. Same procedure is undergone from the SO point of view with respect to the correction costs incurred.

Indeed the final schedule in that case shows that units in bus 2 have to reduce their productions

Table 4.11: Model solution for Case I (Line 1 failure)

(a) Active power production

Bus a	Unit j	Marginal Cost [€/ MWh]	Final Schedule q_j^{Ω}[MWh]	Agents' Bids q_j[MWh]	ACZ [€/ MWh]	ACX [€/ MWh]	Reduction Factor
1	1	13.50	0.00	0.00	13.62	14.783	0.00
	2	12.50	20.00	7.17	12.62	13.783	0.00
	3	12.25	35.00	25.16	13.307	14.467	0.00
	4	12.00	40.00	27.66	13.06	14.217	0.00
2	5	0.80	0.00	0.00	1.857	3.017	0.00
	6	0.80	5.00	5.00	0.92	2.083	0.00
	7	0.50	20.00	20.00	1.30	0.50	0.00
	8	0.50	20.00	20.00	1.40	0.50	0.00
	9	0.50	20.00	20.00	1.30	0.50	0.00
	10	0.50	20.00	20.00	1.40	0.50	0.00
	11	0.50	5.00	40.00	7.70	0.50	0.875

Oligopoly: $\theta_i \neq 0$, $\beta_i \neq 0$
Market Price $\lambda = 13.307$ €/ MWh
Correction Price $\gamma = 14.467$ €/ MWh

(b) Load Shedding

Bus a	Unit j	Final Schedule [MWh]	ENS cost [€/ MWh]
2	13	0.00	132.973
3	14	0.00	175.135
4	15	0.00	145.946
5	16	0.00	132.973
6	17	0.00	132.973

due to network constraints and the opposite for those in bus 1 compared to the base case. In addition, in the base case we explained how the apparent cost in the correction market (ACX) was the same as the marginal cost due to no exercise of market power in this market as there are no units committed. In this case however, agents behave strategically by withdrawing quantities in the day-ahead market so that they would be required to increase them in the correction market at higher prices and therefore earn higher profits. This results in (ACX) being higher than the marginal cost, and that the marginal unit which will be required to increase its production (in this case unit 3) is the one that sets the correction market price γ.

The final result reached from solving the model is one that maximizes the agents' profits and minimizes the SO's market correction costs; to translate this behavior from the output of the model let us compare our current case with the base one, considering the re-arranged units in the order of agents' ownership as depicted in Table 4.12. We can see that in the final agents' bid in Case I (table 4.12b), the only unit which will be required to decrease its production after the modified proposed bids is unit 11 (located in bus 2). Even though this unit will finally reduce its production by 87.5%, agent 2 still bids full capacity for this unit exactly as she did in the base case, except that this time he/she perceives that the cost of producing with this unit is significantly higher than that of the base case (7.70 €/ MWh as opposed to 2.10 €/ MWh). On the other hand, agent 1 who owns more thermal units in bus 1 (which can solve the loss in production) acts strategically by increasing his/her bid prices relative to the ones in the base case (as shown in Table 4.12b). In this case however, he/she also considers that since unit 5 is going to reduce its production that it is more profitable not to bid with this unit. Last significant observation is regarding unit 2; here agent 3 anticipates that the correction market price would be higher than the day-ahead market and accordingly bid less capacity for this unit in the day-ahead market, benefiting from the anticipation that it will be required to increase the production to correct the power shortages at a higher price.

Table 4.12: Comparison of day-ahead market bidding behavior (Base Case & Case I)

(a) Base Case: No Failure in Lines

Agent i	Unit j	Final Schedule q_j^{Ω}[MWh]	Agents' Bids q_j[MWh]	ACZ [€/ MWh]	Reduction Factor
1	3	**0.00**	**0.00**	**13.05**	0.00
	4	**35.00**	**35.00**	**12.80**	0.00
	5	**5.00**	**5.00**	**1.60**	**0.00**
2	8	20.00	20.00	2.10	0.00
	10	20.00	20.00	2.10	0.00
	11	**40.00**	**40.00**	**2.10**	**0.00**
3	1	0.00	0.00	13.75	0.00
	2	20.00	20.00	12.75	0.00
	6	5.00	5.00	1.05	0.00
4	7	20.00	20.00	1.30	0.00
	9	20.00	20.00	1.30	0.00

Oligopoly: $\theta_i \neq 0$, $\beta_i \neq 0$
Market Price $\lambda = 12.80$ €/ MWh
Correction Price $\gamma = 0$ €/ MWh

(b) Case I: Failure of Line 1

Agent i	Unit j	Final Schedule q_j^{Ω}[MWh]	Agents' Bids q_j[MWh]	ACZ [€/ MWh]	Reduction Factor
1	3	**35.00**	**25.16**	**13.307**	0.00
	4	**40.00**	**27.66**	**13.06**	0.00
	5	**0.00**	**0.00**	**1.857**	0.00
2	8	20.00	20.00	1.40	0.00
	10	20.00	20.00	1.40	0.00
	11	**5.00**	**40**	**7.70**	**0.875**
3	1	0.00	0.00	13.62	0.00
	2	20.00	7.17	12.62	0.00
	6	5.00	5.00	0.92	0.00
4	7	20.00	20.00	1.30	0.00
	9	20.00	20.00	1.30	0.00

Oligopoly: $\theta_i \neq 0$, $\beta_i \neq 0$
Market Price $\lambda = 13.307$ €/ MWh
Correction Price $\gamma = 14.467$ €/ MWh

The resulting day-ahead market price (λ) and correction market price (γ) are 13.307 €/ MWh and 14.467 €/ MWh, respectively. We calculate the agents' profit from the previously defined maximization function and we obtain the results shown in Table 4.13. Compared to the base case, all agents except for agent 2 are making higher profits, either due to the higher day-ahead market price (agent 4), to their participation in the higher price correction market (agent 3), the scheduling of units previously out of the market competition (agent 1), or a combination of the three. For agent 2, the decreased profit is essentially due to the reduced dispatch of his/her unit.

TABLE 4.13: Comparison of agents' profit (Base Case & Case I)

Agent i	Profit if no line fails (Base Case) €	Profit if line 1 fails (Case I) €	Percentage Change %
1	88.0	115.004	+30.69%
2	984.0	576.315	-41.43%
3	66.0	93.558	+41.75%
4	492.0	512.280	+4.12%

Oligopoly profit: $\theta_i \neq 0$ for both cases.

4.2.3 Case VII: Failure of Line 7

So far, both failure cases discussed did not result in load shedding in the network. It is important perhaps before moving on to present the summary of the results for all cases to briefly analyze a failure case that results in energy not served. Case VII would be an interesting illustrative case since the failure of line 7 would undoubtedly leads to non served energy since it is the only line connecting bus 6 (which has 20 MW of demand at peak hour) to the rest of the network. In addition, since we assume that in the case of a probable loss of load in some scenario having certain probability, agents are allowed to submit a reduced bid (which is required to meet only the definite load remaining after the failure), then this case is expected to illustrate only the model's behavior due to load shedding and isolate it from the effect of network congestion in any part of the network.

We refer to the load shedding results in Table 4.14b to show that indeed in this case all demand in bus 6 can not be serve due to line 7 failure, this means that in this particular scenario, with a certain probability, market agents will submit reduced bids in terms of quantities to supply only the remaining demand in the network. Market agents find a bidding strategy that maximizes their profit which will be accepted by the SO with no requirements for any unit to change its production (neither increasing nor decreasing) as shown in Table 4.14a. Similar to the base case, the apparent cost of the units in the correction market (ACX) are equal to the marginal costs of the units since no unit will be required to increase its production if this schedule is implemented. Moreover, the results obtained for the apparent cost of the units in the day-ahead market; in the comparison summarized in Table 4.15 between the base case and case VII, it is seen that ACZ has in fact decreased compared to the base case, specifically for the units owned by the agents who are required to reduce their total production due to the shedding. This of course results in a lower market price (λ) of 12.63 €/MWh compared to initial value of 12.80 €/MWh. Leading to all agents receiving a lower profit as shown in Table 4.16.

Table 4.14: Model solution for Case VII (Line 7 failure)

(a) Active power production

Bus a	Unit j	Marginal Cost [€/ MWh]	Final Schedule q_j^{Ω}[MWh]	Agents' Bids q_j[MWh]	ACZ [€/ MWh]	ACX [€/ MWh]	Reduction Factor
1	1	13.50	0.00	0.00	13.63	13.50	0.00
	2	12.50	8.33	8.33	12.63	12.50	0.00
	3	12.25	0.00	0.00	12.88	12.25	0.00
	4	12.00	26.67	26.67	12.63	12.00	0.00
2	5	0.80	5.00	5.00	1.43	0.80	0.00
	6	0.80	5.00	5.00	0.93	0.80	0.00
	7	0.50	20.00	20.00	1.30	0.50	0.00
	8	0.50	20.00	20.00	2.10	0.50	0.00
	9	0.50	20.00	20.00	1.30	0.50	0.00
	10	0.50	20.00	20.00	2.10	0.50	0.00
	11	0.50	40.00	40.00	2.10	0.50	0.00

(b) Load Shedding

Bus a	Unit j	Final Schedule [MWh]	ENS cost [€/ MWh]
2	13	0.00	132.973
3	14	0.00	175.135
4	15	0.00	145.946
5	16	0.00	132.973
6	17	20.00	132.973

Table 4.15: Comparison of day-ahead market bidding behavior (Base Case & Case VII)

(a) Base Case: No Failure in Lines

Agent i	Unit j	Final Schedule q_j^{Ω}[MWh]	Agents' Bids q_j[MWh]	ACZ [€/ MWh]	Reduction Factor
1	3	0.00	0.00	13.05	0.00
	4	35.00	35.00	12.80	0.00
	5	5.00	5.00	1.60	0.00
2	8	20.00	20.00	2.10	0.00
	10	20.00	20.00	2.10	0.00
	11	40.00	40.00	2.10	0.00
3	1	0.00	0.00	13.75	0.00
	2	20.00	20.00	12.75	0.00
	6	5.00	5.00	1.05	0.00
4	7	20.00	20.00	1.30	0.00
	9	20.00	20.00	1.30	0.00

Oligopoly: $\theta_i \neq 0$, $\beta_i \neq 0$
Market Price $\lambda = 12.80$ €/ MWh
Correction Price $\gamma = 0.00$ €/ MWh

(b) Case VII: Failure of Line 7

Agent i	Unit j	Final Schedule q_j^{Ω}[MWh]	Agents' Bids q_j[MWh]	ACZ [€/ MWh]	Reduction Factor
1	3	0.00	0.00	12.88	0.00
	4	26.67	26.67	12.63	0.00
	5	5.00	5.00	1.43	0.00
2	8	20.00	20.00	2.10	0.00
	10	20.00	20.00	2.10	0.00
	11	40.00	40.00	2.10	0.00
3	1	0.00	0.00	13.63	0.00
	2	8.33	8.33	12.63	0.00
	6	5.00	5.00	0.93	0.00
4	7	20.00	20.00	1.30	0.00
	9	20.00	20.00	1.30	0.00

Oligopoly: $\theta_i \neq 0$, $\beta_i \neq 0$
Market Price $\lambda = 12.633$ €/ MWh
Correction Price $\gamma = 0.00$ €/ MWh

TABLE 4.16: Comparison of agents' profit (Base Case & Case VII)

Agent i	Profit if no line fails (Base Case) €	Profit if line 7 fails (Case VII) €	Percentage Change %
1	88.00	76.047	-13.58 %
2	984.00	970.640	-1.36 %
3	66.00	60.273	-8.68 %
4	492.00	485.320	-1.36 %

Oligopoly profit: $\theta_i \neq 0$ for both cases.

4.2.4 Summary of results for all failure cases

Now that some detailed analysis have been presented spanning different possible results, we proceed to provide a summary for the model solution for all aforementioned cases (Base Case to Case VI) modeling no line failure and line failure 1 to line failure 7 respectively. As discussed these results are all considering oligopolistic behavior parametrized according to the conjectured-price response values (θ_i and β_i) defined in the model formulation section. Tables 4.17 - 4.22 illustrate the final feasible schedule, final agents' bids, energy not served, apparent cost in the day-ahead market and in the correction market, reduction factor, market prices and agents' profits respectively; for all generation units, for all agents, in all network buses and for all 8 cases.

Except for the base case, case I and case IV; all other failure scenarios result in energy not served in the network. The reason for that in the case of line 1 have been discussed. Regarding line 4, it is important to note in general it is a redundant line that is mainly used when there is limited capacity in either line 5 or line 6. Therefore when simulating the failure of this line independently it does not affect the operation of the network in any way; leading to the final solution being exactly similar to that of no line failure. Another important remark is that the reduction factors for every generation unit (m_j) summarized in Table 4.20 is reduced to show only the cases that have non zero values; meaning that any reduction factor not shown in the table has a value equal to zero.

In general the analysis of each case separately follows the same logic for those discussed in the previous section. We focus here on the most important observations that can be obtained from this aggregated picture of the network. First one being the agents' profits. In Table 4.22 it is shown that the line failure almost always results in a reduced profit for all market agents compared to the base case, again with the exception of case I and IV. The reason for the no change in profits due to line 4 failure has been discussed. For line 1 also, we have seen that this line's failure does not result in any load shedding. In fact, the only effect that this particular failure has is forcing congestion in all the other lines. So in principle this case is modeling the agents' exercise of market power in case of line congestion and therefore follows the same analyses undergone in the work of Delgadillo et al. [2013] and Delgadillo and Reneses [2013]. The results of line 1 failure show an opposite trend than all other cases; some agents are able to make higher profits due to the failure being one of those trends, and the other significant one is that it is the only case where the day-ahead market price (λ) would increase due to the failure (Table 4.21). This confirms the results concluded in Delgadillo and Reneses [2013] regarding how network congestions modify the agents' strategic behavior in the day-ahead market.

TABLE 4.17: Model solution for day-ahead market bid schedule and final schedule (all cases)

(A) Day-ahead market bid quantities by market agents (MWh)

		Feasible Schedule q_j^{Ω} [€/MWh]							
Agent i	Unit j	No Failure	Case I	Case II	Case III	Case IV	Case V	Case VI	Case VII
	3	0.00	35.00	0.00	0.00	0.00	0.00	0.00	0.00
1	4	35.00	40.00	0.00	40.00	35.00	30.00	26.67	26.67
	5	5.00	0.00	5.00	0.00	5.00	5.00	5.00	5.00
	8	20.00	20.00	13.33	20.00	20.00	20.00	20.00	20.00
2	10	20.00	20.00	13.33	15.00	20.00	20.00	10.00	20.00
	11	40.00	5.00	13.33	0.00	40.00	40.00	40.00	40.00
	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
3	2	20.00	20.00	0.00	15.00	20.00	15.00	18.33	8.33
	6	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00
4	7	20.00	20.00	20.00	0.00	20.00	20.00	20.00	20.00
	9	20.00	20.00	20.00	20.00	20.00	20.00	20.00	20.00

(B) Final feasible schedule as imposed by the system operator (MWh)

		Agents' Bids q_j [€/MWh]							
Agent i	Unit j	No Failure	Case I	Case II	Case III	Case IV	Case V	Case VI	Case VII
	3	0.00	25.16	0.00	0.00	0.00	0.00	0.00	0.00
1	4	35.00	27.66	0.00	35.00	35.00	30.00	26.67	26.67
	5	5.00	0.00	5.00	0.00	5.00	5.00	5.00	5.00
	8	20.00	20.00	13.33	20.00	20.00	20.00	20.00	20.00
2	10	20.00	20.00	13.33	20.00	20.00	20.00	20.00	20.00
	11	40.00	40.00	13.33	0.00	40.00	40.00	40.00	40.00
	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
3	2	20.00	7.17	0.00	15.00	20.00	15.00	8.33	8.33
	6	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00
4	7	20.00	20.00	20.00	0.00	20.00	20.00	20.00	20.00
	9	20.00	20.00	20.00	20.00	20.00	20.00	20.00	20.00

As we have solved the model for most major line failure cases, and have reached a solution

Table 4.18: Amount of energy not served (all cases)

	ENS [MWh]							
Bus	No Failure	Case I	Case II	Case III	Case IV	Case V	Case VI	Case VII
2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
3	0.00	0.00	45.00	0.00	0.00	0.00	0.00	0.00
4	0.00	0.00	10.00	30.00	0.00	0.00	0.00	0.00
5	0.00	0.00	20.00	20.00	0.00	0.00	0.00	0.00
6	0.00	0.00	20.00	20.00	0.00	10.00	20.00	20.00

representing what would be the equilibrium outcome in case both market agents and SO can anticipate the actions of each other, we can now adopt the point of view of the SO and identify what costs he/she is facing in operating the network under the different failure scenarios. We know that in the economic sense, the SO aims to operate the network while maximizing the social welfare; that is the benefits for *both* the consumers and the producers. There is obviously no doubt that any scenario which leads to power cuts and demand not being served will represent a significant welfare loss for the SO. One major element to consider is therefore the cost of non served energy reported in Table 4.23.
The other important cost that the SO is facing is what we have been referring to as the "correction cost"; that is the cost of correcting the power dispatch in the system to ensure the most reliable (subject to the technical constraints) and most economic dispatch. This correction cost is in fact the objective function we have illustrated in the optimal power flow problem simulating the role of the SO. We have seen that failure cases themselves as well as their effect on the congestion of other lines impose a need for these corrections to occur. We calculate the cost of these corrections as the sum of the apparent cost of day-ahead market for each unit required to reduce its production times the quantity to be reduced, and the apparent cost of the correction market for each unit required to increase its production times the quantity to be increased. Or:

$$ACZ_j \cdot z_j + ACX_j \cdot x_j \tag{4.1}$$

It should be noted however, that this correction cost is a *conceptual* cost that the SO is facing, since in fact he/she would not be paying the generation units to decrease their productions from the one they bid. Yet it can be argued that this cost represent the inefficiencies arising in running a system characterized by an oligopoly where many transaction costs and exercise of market power hinder the smooth and optimal operation of the system. The so called correction cost of this study based on the above definition is calculated and illustrated in Table 4.23 for all cases. It can be seen that this correction cost is not correlated to load shedding in the network; there might be a need to correct the dispatch in a case where there is no load shedding (Case I) or in a case where there is (Case III, VI), and vice versa. In fact it follows no rule in this regards except for the objective of the agents to maximize their profits by adjusting their

TABLE 4.19: Apparent cost of generation units (all cases)

(A) Apparent cost of the units in the day-ahead market bidding ACZ_j (€/ MWh)

		ACZ [€/MWh]							
Agent	Unit	No Failure	Case I	Case II	Case III	Case IV	Case V	Case VI	Case VII
1	3	13.05	13.31	12.35	12.95	13.05	12.95	12.88	12.88
	4	12.08	13.06	12.10	12.70	12.08	12.70	12.63	12.63
	5	1.60	1.856	0.90	1.44	1.60	1.50	1.43	1.43
2	8	2.10	1.40	1.30	1.20	2.10	2.10	1.90	2.10
	10	2.10	1.40	1.30	1.43	2.10	2.10	3.30	2.10
	11	2.10	7.70	1.30	1.257	2.10	2.10	1.90	2.10
3	1	13.75	13.62	13.55	13.70	13.75	13.70	13.63	13.63
	2	12.75	12.62	12.55	12.70	12.75	12.70	12.63	12.63
	6	1.05	0.92	0.85	1.00	1.05	1.00	0.93	0.93
4	7	1.30	1.30	1.30	0.94	1.30	1.30	1.30	1.30
	9	1.30	1.30	1.30	0.90	1.30	1.30	1.30	1.30

(B) Apparent cost of the units in the correction market bidding ACX_j (€/ MWh)

		ACX [€/MWh]							
Agent	Unit	No Failure	Case I	Case II	Case III	Case IV	Case V	Case VI	Case VII
1	3	12.25	14.467	12.25	12.75	12.25	12.25	13.00	12.25
	4	12.00	14.217	12.00	12.50	12.00	12.00	12.75	12.00
	5	0.80	3.017	0.80	1.30	0.80	0.80	1.55	0.80
2	8	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
	10	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
	11	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
3	1	13.50	14.783	13.50	13.50	13.50	13.50	13.75	13.50
	2	12.50	13.783	12.50	12.50	12.50	12.50	12.75	12.50
	6	0.08	2.083	0.80	0.80	0.80	0.80	1.05	0.80
4	7	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
	9	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50

TABLE 4.20: Generation units' reduction factor m_j (all cases)

		Reduction Factor m_j		
Agent	Unit	Case I	Case III	Case VI
2	10	0.00	0.25	0.50
	11	0.875	0.00	0.00

TABLE 4.21: Market Price (all cases)

	Market Price [€/ MWh]							
Market	No Failure	Case I	Case II	Case III	Case IV	Case V	Case VI	Case VII
λ	12.80	13.307	1.30	12.70	12.80	12.70	12.633	12.633
γ	0.00	14.467	0.00	12.50	0.00	0.00	12.75	0.00

TABLE 4.22: Agents' Profit in different cases of line failures

(A) Agents' profits (€)

	Profit [€]							
Agent	No Failure	Case I	Case II	Case III	Case IV	Case V	Case VI	Case VII
1	88.00	115.00	2.50	27.00	88.00	80.50	76.05	76.05
2	984.00	576.32	32.00	427.00	984.00	976.00	849.31	970.64
3	66.00	93.56	2.50	62.50	66.00	62.50	62.77	60.27
4	492.00	512.28	32.00	244.00	492.00	488.00	485.32	485.32

(B) Percentile change in agents profits with respect to the base scenario

Agent	No Failure	Case I	Case II	Case III	Case IV	Case V	Case VI	Case VII
1	0.00%	+30.69%	-97.16%	-69.32%	0.00%	-8.52%	-13.58%	-13.58%
2	0.00%	-41.43%	-96.75%	-56.61%	0.00%	-0.81%	-13.69%	-1.36%
3	0.00%	+41.75%	-96.21%	-5.30%	0.00%	-5.30%	-4.89%	-8.68%
4	0.00%	+4.12%	-93.50%	-50.41%	0.00%	-0.81%	-1.36%	-1.36%

bidding behaviors.

The SO thus can be considered as facing both these costs illustrated in Table 4.23 in assessing the economic risks associated with operating the network. Yet these line failures do not have the same probability of occurrence, it is therefore necessary to quantify them in terms of their respective probabilities so as to provide a risk assessment tool for the failures of these lines in the network. For the purpose of this study, we have opted for the probability of the forced outages defined in the RBTS system data as seen in the "Overview of the Reliability test System" section. For each line, we multiply the permanent outage rate value by the outage duration to get the total amount of outage hours per line per year. Dividing these values by the total number of operational hours per year ($24\text{hrs} \cdot 365\text{days} = 8760\text{hrs}$) gives us the probabilities of occurrence of those events illustrated in Table 4.23. Multiplying each probability by the relevant cost would result in the "load shedding risk index" and the "schedule correction risk index" summarized in Table 4.24. The "combined index" is merely the summation of the other two and is the index suggested to be used for assessing the risk of different lines failures. For instance, notice how the index for Case I is higher than those of Case V and Case VI, meaning that it posits a higher cost on the system although in this case there is no expensive load shedding but only cost of market correction. There are of course many factors, assumptions and parameters that could

Table 4.23: Different system operating costs face by the SO (all cases)

	No Failure	Case I	Case II	Case III	Case IV	Case V	Case VI	Case VII
Probability of occurrence	97.35%	0.68%	0.26%	0.86%	0.17%	0.17%	0.17%	0.34%
ENS Cost [€]	0.00	0.00	14659.60	9697.30	0.00	1329.70	2659.40	2659.40
Correction Cost [€]	0.00	764.00	0.00	69.67	0.00	0.00	160.50	0.00

affect this result; most obviously is the cost of non served energy assumed and the probability of occurrence of the failures. In the last chapter, I attempt to provide a critical view on which parameters could have a high effect on the solution.

Up to this point, an attempt has been made to quantify the risk faced in running an electricity power system in a single-price market facing various line failure scenarios. The behavior of the system participants have been analyzed under oligopolistic behavior, assuming that in case of high expectation of the inability to serve all demand, agents are permitted to deposit reduced bids matching only the remaining demand. Based on these assumptions a risk index definition have been proposed.
In the next section, I provide a sample solution for relaxing the assumption of the permission of submitting reduced bids by the market agents even if they can anticipate the inevitability of load shedding, comparing it to the solutions achieved here.

Table 4.24: Risk Index for system operator

	Risk Index							
Risk	No Failure	Case I	Case II	Case III	Case IV	Case V	Case VI	Case VII
Load shedding risk index	0.00	0.0	38.11	83.40	0.00	2.26	4.52	9.04
Schedule correction risk index	0.00	5.20	0.00	0.60	0.00	0.00	0.27	0.00
Combined index	0.00	5.20	38.11	84.00	0.00	2.26	4.79	9.04

4.3 Results and Analysis II (Non reduced Bids)

It might be argued that in sudden failure cases, market agents would not be allowed to reduce their bidding quantities in the day-ahead market to match less demand than that which can be economically achieved in the system. In such case, market agents would anticipate the effect of a particular line failure on the final dispatch taking into account the exact same supply/demand balance constraint, while still finding an optimal bidding solution that maximizes their profits. At first instance, this basically mean that there are more bid quantities that will be reduced in the final dispatched schedule. Yet how exactly would this affect the market prices, the agents profits the load shedding and the final correction cost?

We consider in our analysis three cases namely: Case V, Case VI and Case VII which are line 5, 6 and 7 failures respectively. Those three cases are already shown having load shedding in the final dispatch. We solve the model in these cases, yet this time forcing constraint (3.28) to remain equal to the total demand in the network. Obviously, the most apparent expectation is that load shedding would not be different; those line failure de-facto hinder the possibility of serving all demand in the network. Table 4.25b confirms this expectation showing the exact same final loads shed in the network under the different cases. Table 4.25b in general summarizes all the important results regarding the feasible schedule and the agents' bids. We are more interesting however in the analysis of these results.

TABLE 4.25: Model solution - Non reduced demand (Cases V, VI, and VII)

(A) Day-ahead market bid quantities by market agents (MWh)

		Agents' bids q_j [MWh]		
Agent i	Unit j	Case V	Case VI	Case VII
1	3	0.0	0.00	0.00
	4	35.00	35.00	35.00
	5	5.00	5.00	5.00
2	8	20.00	20.00	20.00
	10	20.20	20.00	20.00
	11	40.00	40.00	40.00
3	1	0.00	0.00	0.00
	2	20.00	20.00	20.00
	6	5.00	5.00	5.00
4	7	20.00	20.00	20.00
	9	20.00	20.00	20.00

(B) Load Shedding

	ENS [MWh]		
Bus	Case V	Case VI	Case VII
2	0.00	0.00	0.00
3	0.00	0.00	0.00
4	0.00	0.00	0.00
5	0.00	0.00	0.00
6	10.00	20.00	20.00

First important observation is that in all three cases, the day-ahead market prices (shown in table 4.26) in the case of not permitting reduced bids are higher than those in the case where it is permitted. There is more room for market agents to exercise higher market power as the system *virtually* assumes that there is more demand offer in the system. With higher market prices and similar final dispatch; the agents receive higher profits in their real time operations. The ENS cost the system is facing remains the same, yet it is shown that now the SO operator is facing "correction costs" in cases that was not previously expected to have any cost for correction under equilibrium. Specifically as seen for line 5 and line 7 failures in Table 4.28b.

Although under the assumptions presented in this study, this increased correction cost did not lead to any significant changes in terms of risk ranking of the different failure cases, it shows that there are increased inefficiencies if a policy is designed in such a way that does not give enough flexibility for the electricity market participants to adapt their behaviors according to their evaluation of the risk. Although this inversely-proportional relationship between risk

Table 4.26: Market Price comparison (Reduced bids vs Non reduced bids)

(a) Reduced bids permitted

	Market Price [€/ MWh]		
	Case V	Case VI	Case VII
Day-ahead (λ)	12.70	12.633	12.633
Correction (γ)	0.00	12.75	0.00

(b) Reduced bids not permitted

	Market Price [€/ MWh]		
	Case V	Case VI	Case VII
Day-ahead (λ)	12.80	13.050	13.030
Correction (γ)	0.00	0.00	0.00

Table 4.27: Agents' Profit comparison (Reduced bids vs Non reduced bids)

(a) Reduced bids permitted

	Profit [€]		
Agent	Case V	Case VI	Case VII
1	80.50	76.05	76.05
2	976.00	849.31	970.64
3	62.50	62.77	60.27
4	488.00	485.32	485.32

(b) Reduced bids not permitted

	Profit [€]		
Agent	Case V	Case VI	Case VII
1	88.00	85.543	76.795
2	984.00	878.50	1002.40
3	63.00	72.25	71.75
4	492.00	502.00	501.20

Table 4.28: Risk Index comparison (Reduced bids vs Non reduced bids)

(a) Reduced bids Permitted

	Case V	Case VI	Case VII
Probability of occurrence	0.17%	0.17%	0.34%
ENS Cost [€]	1329.70	2659.40	2659.40
Correction Cost [€]	0.00	160.50	0.00
Load shedding risk index	2.26	4.52	9.04
Schedule correction risk index	0.00	0.27	0.00
Combined index	2.26	4.79	9.04

(b) Reduced bids not Permitted

	Case V	Case VI	Case VII
Probability of occurrence	0.17%	0.17%	0.34%
ENS Cost [€]	1329.70	2659.40	2659.40
Correction Cost [€]	128.00	161.68	260.53
Load shedding risk index	2.26	4.52	9.04
Schedule correction risk index	0.218	0.27	0.89
Combined index	2.478	4.79	9.93

and flexibility is well known, a quantification of its negative effects could provide incentive for adjusting the market design if proven that it could provide enough benefit.

Chapter 5

Conclusion and Future Work

In the present work, a novel risk assessment method has been proposed considering the operation of the electricity network in a liberalized market, under different cases of transmission line failures. The risk assessment index is a probabilistic quantification of the combined effect of energy not served due to a line failure and the effect of the exercise of market power by the GenCos, if they can anticipate the consequence of this failure on their final dispatch, and hence bid strategically to collect profits from participating in the correction market. The former effect have been calculated as the product of the amount of energy not supplied and its cost, while the latter have been calculated as the production quantity required to be increase or decreased in the final dispatch, multiplied by the respective cost of this increase or decrease. The combined risk index is the product of the total cost incurred due to the failure of the line and the probability of occurrence of this failure.

The market model adopted in this study is a single-price market where GenCos do not consider network constraints in their day-ahead market bidding and receive the same price for all power dispatched from their committed generation units, equal to the bidding price of the system's marginal unit. Any corrections necessary in the final dispatch due to network failures is undergone by the system operator according to the bidding prices in the so called "correction market".

The activities performed during this research work include:

1. A comprehensive literature survey covering the existing electricity market modeling methods, focusing on the equilibrium modeling techniques, and discussing their main underlying conceptual and mathematical formulation differences. For each equilibrium modeling technique, a summary of selected publications have been provided, primarily focused on those studied which explicitly consider transmission network constraints' effect on market competition. Moreover, the literature survey covered some of the most recent publications

on risk and reliability assessment in power systems, summarizing the most commonly used reliability indexes in network assessment, and discussing some of the risk indexes proposed for the risk assessment of the transmission network. It has been concluded that to best of the author's knowledge, no work have previously proposed the risk assessment method proposed in this work.

2. A suitable model have been identified for the calculation of the desired market interaction. For this purpose, a bi-level model proposed in the the work of Delgadillo and Reneses [2013] and Delgadillo and Reneses [2015] have been adopted. Consisting of a conjectural-variation equilibrium (CVE) model for modeling the oligopolistic behavior of market agents in both the day-ahead market and in the correction market, and a DC-optimal power flow model for modeling the SO's optimal decision for the final dispatch of power in the network. The CVE model is cast as a mixed complementarity problem and is solved using the PATH solver in GAMS, while the DC-optimal power flow model is solved using mixed integer linear programming in Matlab. The bi-level model is finally solved iteratively until a convergent solution is achieved, simulating the interaction among both the SO and the GenCos. Finally, the risk concept formulation is given, and it follows the definition given at the beginning of this chapter.

3. An example system has been constructed for application of the study, it consists of a network adapted from the IEEE 6 bus Reliability Test System (RBTS), consisting of 11 generation units with a total capacity of 240 MW distributed along two buses, and total network demand of 180 MW distributed along 5 buses. Furthermore, the generation units are assumed to be owned by 4 competing GenCos having different conjectured-price response values in order to simulate the oligopolistic behavior. The network comprises 7 transmission lines, and their characterization is given including maximum line capacities, line outage rates and outages duration.

4. The numerical example is solved for each independent failure of all 7 lines, with the benchmark case considered as the model solution in case of no line failure. The case study have been solved twice considering two different underlying market design assumptions: the first is assuming that market agents are allowed to submit reduced bids in terms of production quantities in the day-ahead market, if they anticipate that a line failure would undoubtedly result in load shedding, and the second is assuming that reduced bids will not be allowed.

In general, the validation and logical consistency of the model's results have been discussed, and a detailed explanation for understanding them have been provided. Under the first market design assumption, it has been shown that even though the cases of line failures that lead to ENS, have caused the reduction in the profits of the market agents, and the reduction of the final market prices (since agents are only meeting reduced demand), in many cases the SO still faces correction costs due to the inefficiency imposed by the agents' strategic behaviors. Such inefficiency could have a significant impact on the total cost, and their assessment would assist in mitigating their occurrence if it provides enough benefits. If a line failure does not lead to ENS in the network, then its effect has shown to be exactly similar to that of solving the model assuming congested network line: which means a final higher market prices in addition to the correction costs.

If in case of a line failure agents are not allowed to submit reduced bids, it is shown that on one hand market prices will inevitably be higher. Yet more importantly, that in all cases leading to energy not served, the SO will be facing higher schedule correction costs, making the index even more relevant in assessing those cases.

Future Work

Currently, the possibility of including the effect of modeling demand-response on the risk assessment proposed is being investigated. By demand-response we mean the ability of demand to actively participate in the market by bidding its willingness for being considered for load shedding, in cases where the system is facing network constraints such as line failures. This can be done via representing the demand in each bus by a virtual generator and find its optimal bidding strategy by including it in the CVE problem. It is already being experimented with this concept, however, formal results would require a more rigorous definition of the pricing system and bidding rules for these new participants to be added as constraints in the model.

An important development to be carried on, is the inclusion of stochastic renewable power sources in the network, and the examination of their effect on its risk assessment index proposed. Methods such as Monte Carlo simulation would be required to simulate the different states of the network and provide a probabilistic overview of its performance. Moreover, more analytic and simulation techniques will be investigated for the ability to modeling a multi-period analysis of the system.

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