

# Review on generation and transmission expansion co-planning models under a market environment

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**Abstract:** This study presents a review of the state-of-the-art on the coordination of generation and transmission expansion planning. First, the authors present the different investment and operation modelling approaches, with an emphasis on the centralised co-optimisation problem. Second, a comprehensive review of co-planning hierarchical equilibrium models, under a market environment, is carried out. The authors categorise the distinctive market approaches that usually represent the lower level of co-planning problems. They offer an updated and detailed classification of multilevel equilibrium models, based on their hierarchical and regulatory structure versus their equivalent reduced structure. Finally, the authors identify research gaps in the literature of each one of the mentioned model categories.

## 1 Introduction

Power systems were conventionally structured under a centralised environment, where a cost minimising vertically-integrated utility was in charge of deciding, among other matters, both transmission expansion planning (TEP) and generation expansion planning (GEP). However, due to computational limitations, in the past, TEP and GEP were usually solved as independent problems. A great amount of research has been published on these separate problems, with the focus shifting recently to contemplating bigger networks, the more detailed operation of conventional units, renewable generation, batteries, distributed generation, and their corresponding support schemes. For a complete review of these separate problems, please refer to [1].

With the development of computational capability, the joint consideration of TEP and GEP became possible, allowing the important links between generation dispatch and transmission supply along with their siting and sizing decisions. These models, usually known as GEP and TEP co-optimisation models (GEPTEP co-optimisation models), take into consideration the links between generation and transmission, resulting in a lower system cost compared to a separate GEP/TEP optimisation approach, as shown in [2–5]. This cost minimisation framework is equivalent to maximising the total welfare of the system when three assumptions are made: (i) perfect competition, (ii) simultaneous operating and investment decisions, and (iii) perfect information. However, this co-optimisation framework does not address sequential and strategic decisions that emerge in a decentralised market environment.

The vast majority of power systems today are structured in a liberalised market, in which private companies compete with each other (in generation and retail). Therefore, the investment and operation decisions in this market environment are quite different from those in a centralised environment. In a liberalised market, generation expansion and operation are decided in a competitive way where every generation company (GENCO) makes its own decisions aiming to maximise total profits, while the transmission planning remains centralised.

In this sense, the liberalisation of electricity markets has introduced new dynamics that lead to conflicting interests between the different decision makers in the electric power system. The behaviour of GENCOs can be modelled by means of game theory to represent their strategic interactions as Nash equilibrium. Moreover, if we consider the sequence between investment and

operation for these strategic agents, hierarchical models (bi-level) allow us to represent such structures. Bearing this in mind, hierarchical equilibrium models represent an adequate tool to study how different agents in GEPTEP problems behave under a market environment.

Once the context is set, we can move on from the term ‘co-optimisation’ to the more accurate ‘co-planning’. We introduce this term, given that these models do not address, in essence, a single optimisation problem. Thus, co-planning models help us to understand how transmission companies (TRANSCOs) and GENCOs take strategic and sequential decisions. For example, consider the case where transmission expansion decisions are made first, and subsequently, under a market framework, GENCOs make their expansion decisions. In this sense, equilibrium models, in particular, bi-level or multi-level problems help us model this kind of interaction.

It is important to note that strategic behaviour does not only occur in operation but also in investment decisions. This is particularly true nowadays because of the shorter construction span of generation units (mainly for renewables technologies) and longer times for transmission lines (because of stricter environmental restrictions or more demanding communities) allow GENCOs to site their units strategically in such a way that they induce congestions in the network, leading to higher operational incomes for GENCOs.

This review aims to present an overview of the current state of GEPTEP models: in Section 2, the classical network and generation modelling are presented. We update the references presented in Krishnan *et al.* [3] for co-optimisation models and we extend it to the co-planning models. In Section 3, the properties of co-planning models and their solutions are analysed. As a novelty, we classify hierarchical GEPTEP papers based on their general hierarchical structures and sequential decisions.

Therefore, we contribute to the relevant literature by classifying both the hierarchical structure and the solution techniques of the co-planning models, offering a direct comparative of co-planning models based on consistent parameters. Additionally, we describe and classify the most commonly used techniques to solve equilibrium models, and we consider the different investment and operation modelling options and their impact on both the equilibrium structure and the type of mathematical problem. Lastly, in Section 4, we conclude.

## 2 GEPTEP modelling approaches

There are two general approaches to GEPTEP modelling. On the one hand, GEPTEP models can be used to assess national and regional energy issues (including transportation, electricity or gas sectors) to provide guidelines to policy makers. Some examples of such policy-oriented models are MARKAL/TIMES (market allocation developed by US International Energy Agency) [5], National Energy Modeling Systems developed by the US Energy Information Administration [6] and integrated planning model by US Environment Protection Agency [7]. More recently, Cole *et al.* [8] conducted a detailed comparison of US policy analysis models, including [6, 7, 9, 10]. On the other hand, there are some GEPTEP models focused more closely on the electricity sector and, therefore, offer a more detailed representation of technical constraints. In this group, the Regional Energy Deployment System developed by the National Renewable Energy Laboratory model is worth noting [9–24].

When discussing GEPTEP models in general, it is important to understand how they represent the complex reality of the decision-making process in mathematical format, as modelling simplifications can potentially have a great impact on model results. The purpose of this section is to point out the most important modelling questions in GEPTEP models and to discuss the pros and cons of different modelling approaches. The remainder of the section will discuss what we consider to be the most important modelling topics. The representation of the transmission network in GEPTEP models in Section 2.1. The representation of generation investment and operation in GEPTEP models in Section 2.2. How to deal with end effects regarding the temporal horizon in Section 2.3. How to reduce the model size by employing size reduction techniques to the network and the time horizon in Section 2.4; and finally, how to deal with the most recent developments in the field, with the representation of storage and renewable technologies along with the uncertainty GEPTEP models in Section 2.5.

Please note that this discussion is aimed at providing the necessary background on basic modelling issues in the GEPTEP realm, which will be fundamental to understand the subsequent analysis of Section 3. Let us now analyse the modelling topics previously pointed out, and extend the work of Krishnan *et al.* [3]

by classifying the relevant works in the literature according to each modelling category. This classification, as well as an updated list of references on co-optimisation models, is presented in Table 1.

Please note that the classification on co-planning models contains the modelling categories from the co-optimization models; however, given that co-planning models are more general, their classification is presented separately in Table 2 (see Section 3).

### 2.1 Network representation

The way the network is represented is a key issue for TEP and, as a consequence, for GEPTEP problems. The transmission network is usually represented as a pipeline (the most simplified approach), as a DC lossless network (the most used approach) or as an AC network (the most accurate approach). In the case of the transportation model (also known as transshipment or pipeline), the network is represented by pipelines in which the flows can be decided to ignore the physical laws that govern power flows in an electrical network. In several long-term models [5, 6, 9, 12], the network is represented in such a way because its mathematical formulation is very simple and linear. Apart from having an overly simplistic network representation, these models also consider continuous transmission investment variables (by disregarding investment lumpiness [lumpiness of investments refers to the discrete nature of the investment decisions, for instance, half transmission line cannot be built]), which allows them to solve large-scale systems while remaining under a linear formulation.

Conversely, in order to successfully represent the lumpiness of transmission investment, binary variables should be used. For instance, Spyrou *et al.* [25] claim that a transshipment model with binary decisions approximates well real operation. This is shown in [47] by comparing a DC model with binary investment variables versus both a DC model with continuous investment and a transportation model with discrete investment variables. Munoz *et al.* [47] show that, for a 2–11% renewable portfolio standard target, disregarding lumpiness creates more distortion than disregarding a DC network approximation. However, given that a lossless DC approximation would be a better approximation (while still maintaining linearity), it is found that most of the existing detailed transmission planning models implement it. Additionally, we can also find TEP models that consider DC network losses [48],

**Table 1** Classification of co-optimisation models according to their modelling approaches

References		[4]	[5, 7]	[9]	[11]	[12]	[13]	[14]	[16, 17] <sub>1</sub>
Year		2012	2009	2012	2016	2012	2016	2011	2018/19
network represents model	pipe line		X	X		X	X		
	DC	X			X			X	
	AC								X
network investment	binary	X			X	X	X	X	X
	integer								
generation investment	continuous		X	X					
	binary	X		X	X	X	X	X	X
	continuous		X						
end effect	rec./pres value							X	
	annual value	X	X	X	X	X	X		X
	extended period								
time represents	load level	X		X	X	X	X	X	X
	represents periods		X						
dynamicity	static	X		X			X	X	X
	dynamic		X		X	X			
storage modelling	short term	X	X	X		X* simplified			
	long term		X simplified	X*		X			
	none				X		X	X	X
uncertainty	deterministic	X	X	X	X	X	X		X
	probabilistic								
	stochastic								X <sub>1</sub>
test system		Brazil-46 flexible data	Flex Zonal-5	WECC 50	Eastern interconnection (EI)-25	Garver-6	IEEE-24		

References		[18]	[19]	[20]	[21]	[22]	[23]	[24]	[25]
Year		2015	2018	2014	2017	2013	2018	2017	2017
network represents model	pipe line								X
	DC	X	X	X	X	X	X		
network investment	AC							X	
	binary	X	X	X	X	X	X	X	
generation investment	integer								X
	continuous								
end effect	binary	X	X	X		X	X	X	
	continuous				X				X
time represents	rec./pres value			X		X	X		
	annual value	X	X		X			X	
dynamicity	extended period								X
	load level	X	X	X		X	X	X	X
storage modelling	represents periods				X				
	static	X	X	X			X	X	X
uncertainty	dynamic				X	X			
	short term	X			X	X*	X*		
test system	long term						X		X
	none		X	X				X	
test system	deterministic		X scenario analysis		X		X	X	X
	probabilistic			X		X ex-post simulation			
	stochastic	X							
		IEEE-14	IEEE-118	IEEE-118	IEEE-24	IEEE-118	Chile-27	IEEE-118	EI-24

however, they have not been applied to mathematical-based GEPTEP problems. In particular, some heuristics models such as in [49] have considered DC losses in their planning frameworks.

Finally, the AC power flow is the most accurate representation of the network, even though it includes highly non-linear constraints that yield more complicated models such as mixed integer non-linear programming (MINLP)s. Some linear approximations have been applied to the AC-TEP problem. The authors of [50, 51] proposed a non-scalable linear approximation that reaches a global optimum under certain conditions. Additionally, Camponogara *et al.* [52] propose a piece-wise linear approximation that proves global optimality for small instances, but only feasibility for large instances. Some other techniques have been developed, such as second-order cone programming and semi-definite programming that formulate convex approximations for the AC power flow [53, 54]. In fact, the authors of [16, 17, 24] propose an AC GEPTEP co-optimisation problem including a second-order relaxation of the AC power flow.

The authors of [16, 24] show that central processing unit (CPU) time decreases up to 10 times compared to the traditional mixed integer conic programming. It is important to note that AC formulation makes it possible to integrate flexible alternating current transmission system (FACTS) [24] technology in the co-optimisation problems by assessing the load shedding caused by reactive power [17].

We would like to emphasise that all the manuscripts reviewed on GEPTEP co-planning (see Table 2) consider a DC power flow network representation (which implies that the only difference between co-planning models is whether transmission investment is binary or continuous). In particular, in [55], the application of an AC power flow for a bi-level model in the context of transmission system operator (TSO)-distribution system operator (DSO) coordination is implemented. This framework can be used as reference for future research on how to represent the AC network in the co-planning context.

## 2.2 Generation representation

The aspect of the representation of generation and generation expansion that interests us here is the use of binary or discrete variables. While using discrete variables instead of continuous ones might represent reality more accurately in many cases (i.e. investment decisions, start-up/shut-down decisions), it also greatly

impacts the computational complexity of the resulting GEPTEP model. In general, the representation of unit commitment constraints is not included in GEPTEP problems mainly due to CPU limitations. To the best of our knowledge only [21] has considered a detailed unit commitment formulation. Additionally, in terms of investment decisions, generation expansion can be modelled either as continuous variables [7, 47] integer variables such as the approach followed by the authors of [13, 14, 25] or as binary decisions taken in [22–25, 30, 47–55]. The alternative of using continuous variables instead of binary variables decreases the search space and computation time, but it reduces model accuracy. However, given that economies of scale in generation are much lower than in transmission investment, generation lumpiness can be sometimes disregarded. Therefore, under certain circumstances, the binary generation investment variables can be relaxed [27] and finally adopt different rated capacities for each generator and achieve accurate results. More recently, some reliability models have been developed to tackle units' availability. These models are solved either by optimisation [19, 20] or by meta-heuristics [47, 56].

## 2.3 End effect

As a consequence of computational limitations, the planning horizon for GEPTEP models is usually lower than the real lifetime of generation and transmission assets. Consequently, in GEPTEP models, the value in use of the investments can be usually distorted at the end of the planning periods. Therefore, modelling investment recovery is a key point in the generation and TEP approaches. This issue can be solved by including recovery values for the assets at the end of the study horizon as shown in [57]. Additionally, an extended simulation can be run as shown in [25], where authors consider a 40 years horizon by duplicating the results of the first 20 years of operation.

The annualised value of investments can be implemented to internalise the value of money over time, as shown in [18]. For a multiyear model, the annual value of investments contemplates not only the value of money but also the optimal building time of the facilities (as opposed to static approaches). All these approaches have some pros and cons, as shown in [58]; either choice represents a trade-off between the CPU time and accuracy. However, among the papers reviewed here (see Table 1), modelling an annualised value is by far the most used option, because it easily

introduces the value of money over time in the whole optimisation horizon.

Finally, it is important to note that the mathematical-based algorithms usually consider a single target year because they are not suitable for large-scale problems. Consequently, most of the work done for multi-year programming has been tackled with some alternative meta-heuristics algorithms [49]. However, recent advances in computational speed, by properly considering the most relevant assumptions, have allowed tackling multistage programming as seen in [21], making it possible to determine not only the optimal siting but also the optimal time of construction of the investments. Additionally, the advancements in algorithms to represent uncertainty, that also considers large-scale problems (see Section 3.4.2), has also permitted to tackle the multistage programming [46].

## 2.4 Size reduction techniques

Long-term models have to deal with the trade-off between the representation of short-term operation constraints and the representation of long-term investment decisions. This implies that hourly operating constraints cannot be retained for several years in a large system because the model becomes computationally intractable. This concern has increased because of the high penetration of renewable technologies that make the impact of ramping constraints, and the capability of storage technologies to balance them, more relevant.

Consequently, current research is focused on reducing either the network size or the time steps representation. The actual transmission network can be reduced so that an equivalent resulting network renders the same or approximate solution. Some of these techniques [59–62] have been applied only for TEP problems. On the other hand, time-steps can be reduced by, for example, using load levels or representative days. As seen in Table 1, most of the models used a traditional load level approach and only a few of them used a representative period approach.

**Table 2** Classification of GEPTep co-planning models

Authors	[26]/[27]	[28]/[29]	[30]	[31]	[32]/[33]	[34]	[35]/[36]/[37]
publication year	2006/2007	2013/2018	2009	2011/2012	2007/2012	2010	2013/14/17
type	optimistic	optimistic	optimistic	optimistic	optimistic	optimistic	optimistic/ pessimistic
network investment	fix investment	binary new lines	binary new lines	binary new lines	binary new lines	binary new lines	CONT/binary new lines
TSO objective function	max social welfare	max SW/max PR-IC (FB)	max PR – IC with FB	max welfare-IC	max profits/min CP-IC	several planning criteria	min total cost/min OC min–max
generation investment	CONT.	CONT.	CONT.	CONT.	binary new generation (NG)	binary NG	CONT/binary NG
GENCOs objective	max profit	max profit	max PR-IC	max profit	max PR & CapPay	max PR-IC	max profit
ISO objective	max social welfare/RD	max social welfare	min CP	max SW	min P. mis. min OC	max SW	min cost operation/
dynamicity regulatory framework	static proactive versus reactive	static proactive	static other see Table 5	static other	static other (GEPTep in the upper level and MO in the lower level)	static proactive	static proactive
hierarchical structure	MLMF	one leader multiple followers (OLMF)	one level	OLMF	multi leader one follower (MLOF)/MLMF	MLMF	MLMF
operation competition	strategic	perfect	strategic	strategic	strategic/perfect	strategic	perfect
uncertainty	no	no	no	no	yes	no	no/deterministic (DET)/yes
demand elasticity	elastic	inelastic	inelastic	elastic	elastic/inelastic	inelastic	inelastic
time represent. end effect	1 h annual	load level annual	1 h annual	1 h annual	load level net present value (NPV)/annual	load level NPV	blocks annual
storage represent.	no	no/short-term	no	no	no	no	no
test cases (when several cases are tested only the biggest one has been referenced)	Chilean 32 /Cornell 30	IEEE-21 Garver-6	IEEE 14 bus	6-bus	6-bus/IEEE-118	5- bus	4-bus/Chilean 34/24 Node
solution technique	sequential quadratic programming- linear complementarity problem (first level)	MIP	quadratically constrained program	MIP (authors compare NLP, mixed integer quadratic program (MIQP) and MIP approaches)	iterative (holistic simulation by using benders and the Lagrange relaxation)/MIP	iterative agent based	MIP (generation strategies are enumerated and finally a MIP is solved)/CG (CG and disjunctive Cutting plane)

Authors	[38]	[39]/[40]	[41]	[42]	[43]/[44]	[32] <sub>1</sub> , [34] <sub>2</sub> , [45] <sub>1</sub> , [46] <sub>2</sub>
publication year	2014	2014/2017	2017	2018	2015/2019	18/17/19/18
type	optimistic	pessimistic	pessimistic	optimistic	optimistic	robust
network investment	binary new lines	binary new lines	binary exp./CONT.	binary upgrades	cont./binary new lines.	binary new lines
TSO objective function	min weighted sum CC-gencos profit (GP)	min IC–SW	min LIC + GIC + OC	min LIC + Exp. OC	min IC + OC	min IC + OC
generation investment	binary NG	CONT	binary expansion	CONT	CONT	CONT
GENCOs objective	max profit	max profit	max exp PR-IC	max exp PR-IC	max profit	—
ISO objective	min accep. bids	max social welfare	min OC	max social welfare	max congestion rent (CR)	min OC
dynamicity	multi-period	static	static	static	static	stat. <sub>1</sub> /dyn <sub>2</sub>
regulatory framework	proactive	proactive	proactive–reactive	reactive	proactive	other see Table 5
hierarchical structure	MLMF	MLMF	OLMF/MLOF	MLMF	OLMF	OLMF
operation competition	perfect	strategic	perfect	perfect	perfect/strategic	perfect
uncertainty	no	no	yes	no	no	yes
demand elasticity	inelastic	elastic	inelastic	inelastic	elastic	inelastic
time represent.	1 h	1 h/blocks	blocks	represen. periods	1 h/ERP	represen. periods
end effect	NPV	annual	annual	annual	annual	annual
storage represent.	no	no	no	short term	short/short & long-term	no
test cases (when several cases are tested only the biggest one has been referenced)	Garver-6	IEEE-118/IEEE-14	IEEE-118	WECC-240	4-bus	118/Chile 20/118/118
solution technique	Kth best algorithm	diagonalization (DG) and complementarity reformulation (CF)/mixed integer linear programming (MILP)	MBA/MILP	MIP/iterative CG	MIP	CG <sub>2</sub>

SW, social welfare; CC, costumer cost; CP, consumer payment; IC, investment cost; OC, operation cost, ERP, enhanced representative periods; CONT, continuous; LIC, line investment cost; GIC, generation investment cost.

In general, detailed clustering approaches for time reduction [63–66] are proposed for GEP problems when intraday constraints are needed to be modelled, as in the case of battery operation. However, only some of these techniques have been applied to GEPTEP problems. For instance, Bustos *et al.* [23] applied a load level approach with a square-mean-error clustering technique in a GEPTEP problem with batteries deployment. Additionally, they characterise wind and solar availability profiles of each hour before clustering load levels, but they disregard transitions between clusters. Some of these techniques have been applied to GEPTEP co-planning problems, as will be discussed in 3.4.

### 2.5 Most recent developments

The major recent developments in GEPTEP have been the introduction of renewables generation, which brings along a high uncertainty in renewables resources, and the utilisation of storage technologies that help manage the intermittency introduced by renewables.

**2.5.1 Uncertainty representation:** There are multiple sources of uncertainty for GEPTEP problems; some of them are long-term uncertainties such as climate variables (i.e. hydro seasons), fuel availability, and demand growth; some are short terms, such as daily weather for renewables, units availability, daily demand, and transmission capacity factor. The representation of uncertainty was initially approached by probabilistic methods, in which the availability of either generation units or lines is simulated to take into account reliability measures [14, 20]. Later, stochastic programming has been considered in a few cases [17, 21] applied to the traditional co-optimisation model. Finally, other techniques

such as robust optimisation have appeared, mainly applied to co-planning models in a market environment context, as will be discussed in Section 3.4.2.

**2.5.2 Storage modelling:** Co-optimisation of transmission and storage investments can be found most notably in [18, 21], both studies achieve a lower cost system compared to a separate optimisation. Energy storage systems (ESS) sizing and siting optimisation are also presented in [67, 68]. Wogrin and Gayme [67] demonstrate that the conditions of siting are dependent on the type of ESS technology; Fernández-Blanco *et al.* [68] conclude that a minimum profit constraint should be included in order to guarantee recovery of investment.

Additionally, Hu *et al.* [4] show that investment in ESS reduces transmission investment costs. In [69], the authors consider ESS and a DC transmission loss approximation, the conclusion is that ESS reduces transmission costs and add flexibility to the system. The general inference of the previous studies is that ESS is a substitute for transmission, however, Bustos *et al.* [23] show that ESS can also be complementary to transmission, depending on the system characteristics and the level of distribution of the ESS deployment.

### 2.6 Modelling approaches gaps

Cole *et al.* [8] research the challenges for renewables generation modelling in policy analysis models and compare results for a US study case. They conclude that active areas for modelling enhancement are (i) spatial and temporal resolution, (ii) resource adequacy, and (iii) economics of energy production. Additionally, lower times of constructions for a renewable generation (wind,

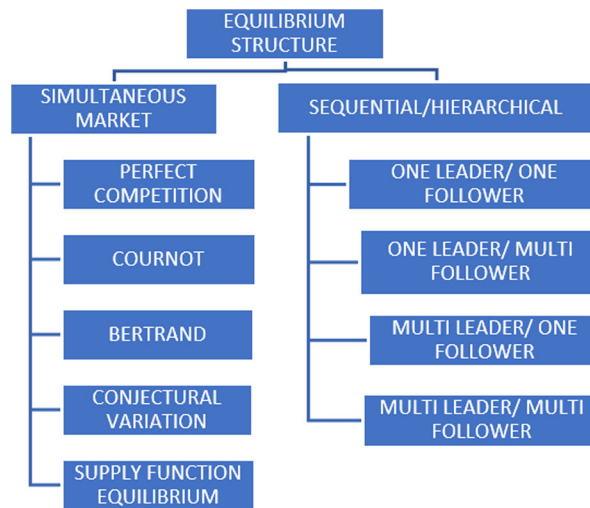


Fig. 1 General mathematical structure of equilibrium models

solar), and longer construction time for transmission allow GENCOs to exercise more market power in response to transmission siting, making the analysis of strategic decisions more relevant. Finally, considering strategic reactive power in the co-planning problem could help reduce load shedding and overall cost of the system, through joint allocation of transmission lines, conventional units, and reactive power sources [17].

### 3 Co-planning equilibrium models

In this section, we present a literature review on equilibrium co-planning GEPTEP models (as opposed to co-optimisation models) with a particular focus on multi-level models. We will consider four different categories in the analysis of co-planning GEPTEP models: equilibrium structure, regulatory framework, solution techniques, and the most recent development on the modelling of storage and uncertainty in a market environment. We will provide a detailed literature review and a classification of the existing equilibrium GEPTEP models, as well as the individual discussion on each category.

In Section 3.1, we classify the different GEPTEP co-planning models depending on their equilibrium structure. In order to do so, we first introduce market-only (no investment) equilibrium models, as they usually constitute the lower level of multi-level co-planning models. We then present the possible different structures of equilibrium models and we classify co-planning models accordingly.

In Section 3.2, we examine the possible regulatory frameworks where co-planning GEPTEP models are applied, and the corresponding hierarchy of decisions and the degree of competition considered. Section 3.3 explores the most common solutions techniques for hierarchical equilibrium models and classifies the literature in the corresponding categories. Finally, Section 3.4 contains a review of storage and uncertainty approaches under a co-planning market framework. Table 2 summarises the properties of GEPTEP co-planning models explored in the whole Section 3.

#### 3.1 Equilibrium structure

When discussing the equilibrium structure of GEPTEP models, several different cases have to be considered, as depicted in Fig. 1.

The first distinction is whether the nature of the game is simultaneous or sequential. Simultaneous games are equilibrium structures where all decision variables are assumed to be taken simultaneously, i.e. TEP investment, GEP investment, and market operating decisions happen all at once. In a sequential or hierarchical structure, one set of decisions is taken before the other in a Stackelberg manner. Section 3.1.1 introduces simultaneous equilibrium models, both market and co-planning. In Section 3.1.2, we continue with sequential equilibrium markets and conclude with the characterisation of sequential co-planning models.

**3.1.1 Simultaneous one-level structure:** We present simultaneous market models, to then cite some simultaneous co-planning models, despite the latter being scarce in the literature (given that co-planning models are usually more interesting when studying the sequence between investment and operation decisions).

*Simultaneous market models:* There exists a wide range of research on simultaneous equilibrium models that simulate the electricity market functioning, mainly to represent oligopolistic behaviour among decentralised GENCOs. In this sense, the following modelling approaches are usually studied: (a) perfect competition: no market power, (b) Cournot: firms decide on quantity, (c) Bertrand: firms decide on price, (d) conjectural variations: a generalisation that over a ‘conjecture’, can result in models (a) & (b), and (e) supply function equilibrium (SFE): firms decide a price-quantity bid. A description of simultaneous equilibrium models applied to electricity markets when the network is disregarded is shown in [70]. Additionally, a review of oligopolistic network-constrained models (ONCMs) is presented in [71]. The seminal paper on ONCMs by Hashimoto [72] introduces the network equilibrium model to study systematically the oligopolistic behaviour of producers in a simplified transportation network. Under this framework, we identify two decision makers: GENCOs and ISO (or TSO). It is important to distinguish the two main features that affect the way prices are created in ONCMs (please refer to Table 3 for the summary)

(a) GENCOs reaction to transmission prices. As shown in [73], even if generation operation is competitive, GENCOs can exercise market power if transmission rights are passive. As an alternative, in [73], a parallel market that is proposed for transmission rights that affect generators' bids and leads to optimal pricing. Subsequently, in [74, 75], the authors consider two different ways for modelling transmission prices, stamp (uniform), and marginal prices. This is obtained by considering that GENCOs react *a la Cournot* to transmission prices. Authors claim that multiple equilibria can arise under stamp pricing; while uniqueness can only be guaranteed for marginal.

(b) Nodal price difference: a concern with previous models [73–75] is that differences in nodal prices might not necessarily be explained by GENCOs' marginal costs. Hobbs [76] proposes adding an arbitrager to the network. With this in mind, Hobbs [76] proposes solving the bilateral market with a quadratic optimisation problem (that copes with large-scale systems). Under this framework, GENCOs compete *a la Cournot*; while they react to transmission prices *a la Bertrand* (GENCOs cannot affect transmission prices). Hobbs [76] also shows that a bilateral market with an arbitrager is equivalent to a POOLCO market (where players react *a la Cournot* to transmission prices).

**Table 3** ONCM models characteristics

	TSO/ISO* objective function	GENCO objective function	Reaction to prices	Arbitrage	Structure
[73]	maximise: social welfare	maximise: profits – TP	Cournot	no	SIM
[74]	i) min: investment cost ii) fixed rule	maximise: profits – TP	Cournot	no	SIM
[75]	fixed rule	maximise: profits	Bertrand	no	SIM
[76]	maximise: congestion rents	maximise: profits – TP- ABP	Bertrand	both	SIM
[77]	maximise: social welfare	i) max: profit + nodal premium ii) anticipate	i) Bertrand ii) Stackelberg	no	i) SIM ii) SEQ
[78]	maximise congestion rents + ABP	maximise: profits – TP- ABP	i) Cournot ii) Stackelberg	yes	i) SIM ii) SEQ
[79]	maximise: social welfare	maximise: profits – TP	Stackelberg	yes	SEQ
[80]	maximise: utility	maximise: benefits	Stackelberg	no	SEQ
[81]	maximise congestion rents + ABP	maximise: profits	i) Bertrand ii) Stackelberg	yes	i) SIM ii) SEQ

SIM, simultaneous; SEQ, sequential; TP, transmission price; ABP, arbitrated payments (nodal price × traded quantities).

**Table 4** Classification of hierarchical multilevel models

	Leaders	Followers
OLOF	one	one
OLMF	one	multiple
MLOF	multiple	one
MLMF	multiple	multiple

*Simultaneous co-planning models:* As mentioned before, our search only found two papers that deal with simultaneous co-planning models. On the one hand, Wei and Smeers [74] model both transmission and GEP. However, computation limitations at the time of publication prevent the study of a real size model. Additionally, in [30], the authors formulate an equilibrium model to study the strategic interactions between TRANSCOs and GENCOs; then they transform the resulting mixed complementarity problem (MCP) to a quadratic programming problem, allowing them to solve big size problems. While a simultaneous decision-making structure leads to simpler models, it is also a simplification of reality that can potentially lead to a distortion of optimal planning results.

**3.1.2 Hierarchical multi-level structure:** Contrary to simultaneous games, the sequential games model a decision-making hierarchy *a la Stackelberg* [82].

Von Stackelberg [82] simulates a market with a leading firm and multiple followers. This game is defined as an equilibrium, where the decisions of the leader are made considering the best reaction of the followers that, simultaneously, make their decisions knowing how the leader would react anticipating their own decisions. In this sense, bi-level programming generalises the Stackelberg model by extending the number of players (and the type of decision) in the game. Table 4 classifies these models following [83].

It is important to note that the different levels in a multi-level framework can be either represented by different actors (e.g. TRANSCO, system operator or GENCOs) or by different types of decisions (e.g. investment and operation). As mentioned in Section 3.1.1, the market is generally considered as a simultaneous game, where GENCOs and system operator decisions are taken. Nevertheless, other models consider a sequence between the decisions of GENCOs and the clearing process made by the system operator. These models are called ‘hierarchical market models’, and some of their properties will be discussed because they are relevant for the subsequent review of hierarchical GEPTEP models.

*Hierarchical market models:* In Table 3, the main characteristics of simultaneous ONCMs (see in 3.1.1) and hierarchical ONCMs (as an alternative to improve transmission pricing) are presented.

In this respect, Metzler [78] extends the work on simultaneous ONCM done in [76]. They propose a sequential Stackelberg model where GENCOs anticipate TSO decisions (leader). The main

contribution of [78] is to demonstrate that their proposal is a generalisation of [76]. Additionally, Neuhoff *et al.* [84] make a comparison of three large-scale hierarchical market models. Neuhoff *et al.* [84] make a comparison of the model comprehensive market power in electricity transmission and energy simulator [85], Cambridges I, II [79], and the Madrid model [80].

Neuhoff *et al.* [84] find that, when the perfect competition is considered, production and pricing results are the same for all models. However, when the oligopolistic competition is considered, pricing results might highly differ; prices could be twice as much in one case compared to others. Therefore, whenever a GEPTEP model is analysed, a strong emphasis must be made on how the market is represented, given that the resulting prices can highly affect investment decisions.

*Hierarchical GEPTEP models:* In this sub-section and the following sections (Sections 3.1.2, 3.2, 3.3, and 3.4), we characterise co-planning equilibrium models. Table 2 summarises this information.

Bi-level models can represent the sequence between investment and operation decisions in either GEP or TEP problems separately. For instance, Garcés *et al.* [57] consider a bi-level TEP by modelling market competition in the lower level and transmission expansion in the upper level. Some other authors develop a bi-level GEP by considering either perfect or imperfect competition in the lower level [86–89]. In order to properly classify and understand the existing hierarchical structure of GEPTEP problems, the difference between decisions and decision-makers must be pointed out.

Decision-makers can be typically classified as GENCOs, TRANSCO(s), and market operators. On the other hand, decisions can be classified as GEP, TEP, and market operation (MO). MO could also be split into a market clearing (MC) and GENCOs operation (GO). In reality, there is an inherent sequence in GEPTEP decision making: the investment stage before the market stage. Now, as we have pointed out previously in Table 3, there even exist different sequential stages within the market stage. Conceptually, a complete GEPTEP model consists of multiple concatenated hierarchical stages; however, when decisions are assumed to be simultaneous, they are reduced mathematically to one single stage.

In the remainder of this section, and summarised in Table 5, we characterise GEPTEP co-planning models according to their conceptual sequence and their corresponding mathematical sequence.

**Table 5** GEPTEP model classification depending on hierarchical structure (mathematical levels 1–4 versus GEPTEP options I–VIII)

	I	II	III	IV	V	VI	VII	VIII
1	GEP TEP MO	GEP	TEP	GEP TEP	GEP	GEP	TEP	TEP
2	—	TEP MO	GEP MO	MO	TEP	MO	GEP	GEP
3	—	—	—	—	MO	TEP	MO	MC
4	—	—	—	—	—	—	—	GO
	[30] [74]	—	[28, 29] [43] [44] [92]	[32] [31] [33] [93]	[41] [42]	[90]	[38, 91] [26, 41] [35, 37] [36, 39] [27, 40]	[34]

In Table 5, we classify the GEPTEP models considering the investment and operation decision hierarchy. To that purpose, Table 5, lists eight (I–VIII) different options, which are shown in the column space. The row space of Table 5 represents the separate mathematical levels of each model. Please note that simultaneous decisions are represented in Table 5 when decisions appear together on a single level. The simplest GEPTEP option, *option I*, is a single-level equilibrium model, which considers GEP, TEP and MO decisions simultaneously. Options II, III, and IV represent bi-level models, in which mathematically speaking there are only two levels.

This means that conceptually speaking two of the three decision levels (GEP, TEP, and MO) are considered to be taken simultaneously. Options V, VI, and VII represent three-level co-planning models with the following structure: some investments are made in the upper level (GEP/TEP) given some other investments in the middle level (TEP/GEP), subject to MO in the lower level. Finally, model VIII is a four-level model with the same structure as the previous three-level models; but also considers the market model is itself hierarchical. Additionally, some techniques can be applied to reduce the initial three-level model to a two-level structure as shown in [26, 33].

Anyhow, reduction techniques are applied only for solving purposes, and therefore the underlying hierarchical structure remains an MLMF, which is much more complex than an OLMF structure (when no anticipation of the market and only one leader is considered). This fact would imply to have, instead of a mathematical programme with equilibrium constraints (MPEC), an equilibrium programme with equilibrium constraints (EPEC), whose solution technique is more complex (see Section 3.3).

**3.1.3 Equilibrium structure gaps:** In terms of the commented structures of GEPTEP equilibrium models, we can identify some issues for potential further research. For instance, in Table 3, we have identified a potential model II that has not been proposed in the literature yet. This model could represent an electricity market with generation investment in the upper level and merchant transmission investments and operation at the lower level. Additionally, most of the research has focused on proactive models (see Table 5) type VII, but a large field of research on reactive model types V and VI still remain almost unexplored. Finally, the most realistic framework would be similar to structure VIII (four levels), where there is a sequence between TEP and GEP, while investment decisions anticipate the market outcome and, at the same time, MC anticipates generation operation (as in hierarchical market models). However, this framework is intractable from an equilibrium point of view, and therefore iterative algorithms can help to simulate the real operation of the market.

### 3.2 Regulatory framework

From a regulatory point of view, GEPTEP co-planning models can be classified depending on the decision maker considered to be the leader (which implies which investment decisions are assumed to be taken first). Depending on whether GEP or TEP is considered to

be first, the regulatory framework can be proactive or reactive. Moreover, co-planning models can also be classified according to the level of competition in the market: markets with an oligopolistic structure versus those closer to perfect competition, both will be discussed below.

**3.2.1 Proactive versus reactive planning approaches:** A key issue in generation and TEP is the choice of which investment decision is considered to be taken first. Does the transmission planner take its decisions after the generation has been sited or do the GENCOs plan their investments after transmission assets have been decided? What comes first, the chicken or the egg?

These choices are proactive and reactive transmission planning approaches. Sauma and Oren [27] propose a proactive planning approach as a framework in which the network planner has the ability to influence generation investment and spot market behaviour. In terms of the hierarchy, it means that TRANSCO is the leader and the GENCOs that anticipate market outputs are the followers.

Respectively, under a reactive planning approach, the network planner assumes that generation capacities are given, and then makes an optimisation based only on the spot market equilibrium. Reactive planning is thus represented by a model with multi-leaders GENCOs and one or several TRANSCOs as followers. Sauma and Oren [27] consider an oligopoly structure and demonstrate theoretically how proactive planning leads to greater social welfare in comparison to reactive planning.

Some alternative approaches are available like presented in [31] (please see Table 5). Here, a two-level model is defined, where the upper level represents the investment decisions (both GEP and TEP), while the lower level represents the MO. Jin and Ryan [31] additionally consider fuel supply as another investment variable. In practice, as mentioned in [25] most of the TRANSCO companies in the world follow a reactive approach, and, to the best of our knowledge, no institution has applied a strictly proactive approach as the one proposed in [27]. However, as mentioned in [37] there are some other approaches that are close to proactive planning. For example, the US government approved a regulation that includes the concept of anticipative (proactive) transmission planning to obtain a higher social welfare [94]. In Chile, a regulation that enforces the consideration of coordination between transmission and generation has also been approved [95]. Additionally, in the current European context, ENTSOE plays the role of a centralised agent that proposes future planning pathways, coordinated regionally, and then GENCOs can react to its decisions. Thus, under this regulatory context, a proactive planning approach would make more sense.

**Proactive planning approach:** Most of the literature in co-planning equilibrium models for GEPTEP has used a proactive planning approach. Proactive planning research can be summarised as follows. On the one hand, Sauma and Oren [26] extend the work done in [27]; they analyse different objective functions and consider a spot market where the distinctive ownership structures are reflected (one GENCO can own several units), as proposed in [77]. Pozo *et al.* [35] extend the theoretical work done in [26, 27]



where the complete multi-level model was not solved and only a set of different fixed transmission investment plans were evaluated. The work done in [35] proposes the first complete model; however, they relax the Cournot assumption and consider only perfect competition in the spot market.

On the other hand, Pozo *et al.* [36] also define three levels and assume perfect competition in the market in the lower level, strategic generation expansion in the middle level and transmission expansion in the upper level. Compared to [35, 36] adds uncertainty in the demand and applies the model to a real-size case study. The same authors in [36] extend their work in [37] by proposing a pessimistic and optimistic network planner (the pessimistic case is used to eliminate multiple equilibria by considering the worst generation expansion case) to describe all possible outcomes of the EPEC in the lower level. The authors conclude that in practice, if multiple generation expansion exists in the equilibrium, proactive planning does not always yield the best welfare results, and it can even reduce social welfare.

Additionally, Jin and Ryan [39] extend this approach and propose a model with Cournot strategic decisions in the market. Finally, Motamedi *et al.* [34] relax the Cournot assumption (in reaction to transmission prices) and propose a two-level approach in the market outcome by considering the interaction between ISO MC problem and GENCOs optimal bidding strategies (please see Section 3.1.2 and Table 5), resulting in a four-level model. This model is solved by means of an agent-based methodology.

Apart from the three-level proactive approach, there are two-level approaches where transmission investment decisions are taken first and then generation investment and operation decisions are taken simultaneously. On the one hand, the work in [28] models the lower level based on the work of [76]. Jenabi *et al.* [28] consider perfect competition in the lower level and define different objective functions in the upper level that are compared with a vertically integrated one-level approach. Additionally, they consider a network fee so the TRANSCO can recover investments in the case of a flow-based fee regulation, typically used in the US. Later, Weibelzahl and März [29] extend the work in [28], choosing a pessimistic TRANSCO and demonstrating some subsequent uniqueness properties, battery expansion is also considered in their framework.

On the other hand, Maurovich-Horvat *et al.* [43] consider a stochastic bi-level model with merchant (for details on modelling of merchant TRANSCOs refer to [96, 97]) investors of transmission in the upper level and wind expansion and MO in the lower level, considering Cournot competition. Finally, the authors of [44, 92] apply the same structure, but consider storage expansion and Cournot competition in the lower levels. Additionally, Gonzalez-Romero *et al.* [92] compare the bi-level model with the traditional inelastic cost minimisation approach. Both works [43, 44] find counterintuitive results when considering Cournot competition in the lower level compared to a perfect competition case.

**Reactive planning approach:** As mentioned above, the reactive planning approach was first proposed in [26], where some theoretical properties were shown and some other practical results were presented for fixed transmission plans. Unfortunately, subsequent research is limited. In general, under this approach, several GENCOs are considered as the leaders and a single TRANSCO as a follower. However, it could also be the case that only one GENCO is the leader and the rest GENCOs and TRANSCO(s) are followers, this would represent an OLMF structure, which, as mentioned above, is simpler to solve.

There are only three subsequent studies on reactive planning. In [41], the authors propose a new comparison between proactive and reactive approaches. In contrast to the work done by Pozo *et al.* [36], Tohidi *et al.* [41] do not consider anticipation of market outcomes by GENCOs and propose the elimination of the multiple Nash equilibria by considering a pessimistic or optimistic TRANSCO. Additionally, Dvorkin *et al.* [42] propose a real size reactive planning approach in which merchant storage is decided in the upper level, while transmission investment and MO are decided in the middle and lower level, respectively. The authors conclude that the co-planning of storage and transmission lead to greater cost

savings than an independent storage planning. Akbari *et al.* [90] propose a four-level with merchant TRANSCOs.

Finally, in terms of transmission modelling, it is important to note that for all proactive planning models, capacity expansion of new lines is represented by binary variables, while, in reactive planning, line expansions are represented by continuous variables to keep the convexity of the lower level, as shown in [41]. However, in [42], transmission expansions are represented by binary variables by applying the dual formulation of the lower level and using the strong duality condition as it will be discussed in Section 3.3

### 3.2.2 Perfectly competitive versus oligopolistic GEPTEP approaches:

In the usual proactive planning approach, a centralised TEP and several GENCOs are considered. Thus, given the three-level structure of GEPTEP problems (see Section 3.1.2); there could be strategic decisions either on GO decisions or on GENCOs investment decisions. As mentioned in Section 3.1.1, the competition between generators in the spot market, and their reaction to transmission prices, can be modelled as either Cournot, Bertrand or SFE. We will discuss the different approaches found in the literature for GEPTEP problems.

**Competitive market:** Most of the GEPTEP hierarchical models consider perfect competition in the spot market (please see Table 2). This simplifies the solution techniques, and more importantly, guarantees that under certain conditions, the uniqueness of the solution is achieved.

If perfect competition (both in GENCOs investment and market functioning) and cost minimisation objective is considered in both levels, there would not be a difference between a proactive bi-level decentralised GEPTEP problem and a centralised vertically integrated co-optimisation problem, as shown in [98]. In other words, if the objective function of both levels is aligned and a perfectly competitive market is considered, both approaches will render the same solution.

**Oligopolistic market:** As mentioned above, there could be strategic behaviour either in the investment or in the generation operation decisions. If only strategic investment decisions are considered and no anticipation of the competitive spot market is assumed (simultaneous generation and perfectly competitive MO), the problem will have an MPEC structure. However, even in this case, multiple Nash equilibria can arise. As mentioned in Section 3.1.2 no anticipation of the spot market is assumed in [15, 28, 41]. In [29, 41], multiple equilibria are eliminated considering a pessimistic TRANSCO approach.

If the only anticipation of the spot market is considered, there might still exist multiple Nash equilibria (as pointed out in [36]) given that an EPEC problem would be tackled. As mentioned in Section 3.2.1, in the context of proactive planning some models consider strategic decisions in both investment and operation levels, as shown in [34, 39] (Cournot or SFE). However, they represent a MLMF structure, whose solution method does not guarantee a globally optimum solution.

### 3.2.3 Regulatory framework gaps:

In spite of the fact that proactive planning has been proven to lead to the most efficient investment and operation results (and most of the research has focused on its analysis), in practice, few jurisdictions have strictly applied this approach. Therefore, it is important to compare different regulatory contexts to be able to propose additional formulations of the GEPTEP problem for understanding the operation and investment strategies in an imperfect market structure. Additionally, the lower construction times for generation and the higher construction times for transmission lines allow GENCOs to exercise strategic investment and operation decisions more easily. This scenario leaves an open field for future research on novel regulatory structures to model the entrance of new merchant GENCOs in the market.

## 3.3 Solution techniques

In this section, we present the techniques used to solve hierarchical GEPTEP co-planning problems. Depending on the initial

$$\begin{aligned}
& \text{Max } F(x, y) \\
& \text{Subject to:} \\
& \text{Max } f(x_1, y) \quad \text{Max } f(x_2, y) \quad \dots \quad \text{Max } f(x_n, y) \\
& \text{subject to} \quad \text{subject to} \quad \dots \quad \text{subject to} \\
& g_1(x_1, y) \leq 0 \quad g_2(x_2, y) \leq 0 \quad \dots \quad g_n(x_n, y) \leq 0 \\
& \quad \quad \quad x_1, y \geq 0 \quad \quad \quad x_2, y \geq 0 \quad \dots \quad \quad \quad x_n, y \geq 0
\end{aligned}$$

Fig. 2 Bi-level problem with an OLMF structure

$$\begin{aligned}
& \text{Max } y F(x, y) \\
& \text{Subject to:} \\
& \text{Dual} \left\{ \begin{array}{l} \partial L / \partial x_1 = 0 \quad \partial L / \partial x_2 = 0 \quad \dots \quad \partial L / \partial x_n = 0 \\ \text{CC} \left\{ \begin{array}{l} \lambda_1 g_1(x_1, y) = 0 \quad \lambda_2 g_2(x_2, y) = 0 \quad \dots \quad \lambda_n g_n(x_2, y) = 0 \\ \text{Primal} \left\{ \begin{array}{l} g_1(x_1, y) \leq 0 \quad g_2(x_2, y) \leq 0 \quad \dots \quad g_n(x_n, y) \leq 0 \\ x_1, y \geq 0 \quad x_2, y \geq 0 \quad \dots \quad x_n, y \geq 0 \end{array} \right. \end{array} \right. \end{array} \right.
\end{aligned}$$

Fig. 3 OLMF problem with lower level KKT conditions

hierarchical structure (please see Section 3.1.2), the techniques might be different. Therefore, in order to solve problems with a single leader, i.e. an one leader one follower (OLOF) or OLMF structure, the solution techniques for the arising mathematical programmes with equilibrium constraints (MPECs) described in Section 3.3.1 are to be used. Alternatively, if there are multiple leaders in the upper level, i.e. an MLOF or MLMF structure, the techniques explained in Section 3.3.2 to solve EPEC are to be used.

**3.3.1 Mathematical programmes with equilibrium constraints:** When a bi-level problem is defined as an OLMF game (as defined in Section 3.1.2), its mathematical structure is seen as an MPEC or as a simple bi-level programming problem. As seen in Fig. 2, the OLMF is a type of mathematical structure in which a single optimisation problem (upper level) is constrained by several simultaneous optimisation problems (lower level) that represent an equilibrium. In Fig. 2,  $x$  and  $y$  represent lower and upper level decision variables, respectively.

As explained in Section 3.1.2, in some cases, this lower-level equilibrium can be converted into a single optimisation problem. Nevertheless, even if this is possible, the resulting complete problem cannot be solved directly by classical optimisation techniques, because an optimisation problem is constrained by another optimisation problem.

Therefore, in order to solve a bi-level problem with an OLMF structure, we can follow the next steps. First, the set of lower-level optimisation problems can be converted into a set of non-linear and non-convex constraints by applying the Karush–Kuhn–Tucker (KKT) (if the optimisation problem is convex) conditions. As seen in Fig. 3, the resulting optimisation problem is constrained by the primal feasibility constraints, the dual feasibility constraints and the complementarity conditions (CC).

The resulting set of constraints is non-linear and non-convex, given the CC of the problem. This lower level has the structure of a MCP, and therefore the whole problem has the structure of an MPEC. Please note that bi-level (OLMF) problems are particular cases of MPECs.

Now we list the most used techniques to solve this kind of problem. As mentioned in [99], these techniques can be divided into dedicated (efficient algorithms that ensure global optimality but require significant additional coding) and non-dedicated algorithms. We explain here the non-dedicated algorithms (that can be implemented directly using commercial software): NLP/MPEC, regularisation, penalisation, MIP KKT-DUAL.

**NLP/MPEC:** The only non-convexity in Fig. 3 is the one introduced by the CC. Therefore, this problem can be solved directly using an ordinary NLP solver. However, given that this is a specific NLP structure embedded in an MPEC, specific solvers,

such as PATH, that tackle directly this problem more efficiently can be used. Unfortunately, both non-linear and MPEC solvers cannot guarantee a globally optimal solution to the MPEC, given that all feasible points are non-regular [99], and consequently, solution methods can easily get stuck in a local optimum or not even find a feasible solution.

**Regularisation:** This method [99] relaxes the complementarity condition of the MPEC problem. Instead, the set of equations for  $g_n(x_n, y) \leq t$  are solved. Then the NLP problem for small values of  $t$  is iteratively solved. The solution of each iteration will be the initial point of the following iteration; this process is faster but only provides a local optimum point solution for the MPEC.

**Penalisation:** The penalisation approach [99] is similar to regularisation. Conversely, in this case, the CCs are penalised in the objective function by a parameter that is reduced along with the iterations until a sufficiently small value of the parameter is reached. As before, the solution of each iteration will be the initial point of the following iteration.

**MIP KKT-DUAL:** As an alternative, the non-linear problem described in Fig. 3 can be converted into a MILP (when the upper level objective function is linear) by linearising the CC. This linearisation can be achieved by applying the methodology proposed by Fortuny-Amat and McCarl [100] or by the discretisation method proposed in [101]. In the first case, a disjunctive formulation is applied to transform complementarity constraints into binary constraints. This is done by splitting the original constraint into two disjunctive constraints limited by a large enough parameter. This is usually known as the Big M constraints.

This method is, by far, the most used method to solve bi-level problems. However, most of the papers that use it do not explicitly mention a method to determine the Big-M values. In fact, as mentioned in [102] if these values are small, suboptimal solutions can appear, and conversely, too large Big-Ms can lead to numerical issues (when different variable magnitudes are reflected in dual variables), such as unstable solutions or large execution times. In [99], a method is proposed to define Big M values by mixing the regularisation and KKT-MIP previously commented methods. The authors show that this method is more efficient computationally speaking and it reaches the optimal solution in more cases compared to other methods. This method is proposed for linear bi-level problems, but it seems to be also efficient for convex problems in general.

Additionally, instead of applying the whole set of KKT conditions, the CCs can be replaced by the strong-duality conditions (where the objective function of the dual problem equals the objective function of the primal problem), which together with its primal and dual feasibility conditions, leads to an equivalent primal–dual formulation.

In [103], a comparison of the KKT and the primal–dual formulation is presented and applied to a vulnerability analysis of the power system. The authors find that the primal–dual approach is more efficient because the size of the problem is highly reduced. This is the result of the lower number of Big Ms (alternatively index constraints or SOS1 variables can be used to programme disjunctive constraints [104]) needed to linearise the strong duality conditions compared to those needed to linearise the CCs (it depends on the ratio #variables/#constraints).

**3.3.2 Equilibrium problem with equilibrium constraints (EPEC):** In case of structures with multiple leaders and one

follower or multiple followers, it will be more difficult to solve the resulting problem, given that the resulting formulation consists of several optimisation problems (equilibrium) subject to several optimisation problems (equilibrium). This problem can be seen as a collection of several MPECs. As shown in [83], in order to solve this problem, the following techniques are available: (a) *diagonalisation algorithms*, the MPEC of every agent is solved sequentially one after the other. (b) *Simultaneous solution method*, all problems are solved simultaneously by defining the strong stationary condition. (c) *System of inequalities with equilibrium constraints* used when the problem has finite strategies.

Unfortunately, the solution of multilevel equilibrium problems can only be guaranteed for the case of OLMF models (MPECs). For MLOF and MLMF (EPEC) problems, there is no guarantee of the existence of the equilibrium. As mentioned in [83], there is still a lack of understanding of the existence of EPEC solutions, thus, only simulation models and approximation algorithms are applied. Additionally, we can have hybrid methods as the *one level reformulation of bi-level games*. In this case, the lower level is reformulated by its equivalent KKT conditions and then it is inserted into every optimisation problem in the upper level. Then the KKT conditions of the whole problem are formulated and solved again. However, the resulting solution might not be an optimum. Ex-post validation should be carried out to verify its optimality. An example of this approach can be found in [40].

Albeit the difficulty of solving these mathematical structures, in the literature, there are several models that tackle more than two levels, by trying to reduce the multi-level problems into a two or one level equivalent problem.

For instance, Motamedi *et al.* [34] propose a coordination framework to take into account the reaction from GENCOs by adopting generation expansion decisions within a four-level problem. They solve the coordination problem iteratively using agent-based modelling and a search-based optimisation technique. In [105], they further extend the model and develop an iterative process by simulating the interaction between TRANSCOs and the ISO and they propose a multi-leader–multi-follower agent-based model. They consider both several TRANSCOs and GENCOs. Additionally, we can find an EPEC problem when only one TRANSCO is considered in the upper level and multiple investing GENCOs in the middle level to anticipate the market outcome of the lower level. In this case, the middle and lower level results in a MLMF structure and thus an EPEC formulation is solved as presented in [36].

Finally, it is important to note that, in most co-planning models, generation expansion is modelled as a continuous variable, as shown in [25, 30, 35, 91]. This assumption responds to the need to obtain convexity conditions in the lower levels and implies that (in most cases) only repowering of existing units is represented.

Alternatively, other authors represent the expansion decisions with integer variables, as [13, 33, 38], but in these cases, only the GEP model is solved for new wind generators. Finally, to the best of our knowledge, only [37, 41] consider binary variables in both investment levels of the GEPTEP problem, which yields non-convex sub-problems that can only be solved by using complex algorithms such as column generation (CG) or the Moore–Bard Algorithm (MBA). Authors in [37] propose a CG and cutting plane algorithm to solve a three-level proactive problem. The CG algorithm is close to the usual diagonalisation technique, but it considers a master problem that creates a meaningful solution to the sub problem that is solved by a diagonalisation-like procedure. This algorithm guarantees a global solution and efficient computation times.

**3.3.3 Solution technique gaps:** In the case of MPEC problems, there is still an active field of research for finding efficient methods to solve these optimisation problems. As mentioned before in [102], Big M is the most common technique to solve the one-level reformulation of bi-level programs. However, small values of Big Ms can lead to suboptimal solutions and large constants can lead to numerical issues (if different orders of magnitude are present in the dual variables of the lower level). Additionally, some authors have tackled the consideration of binary variables in the lower level but

all the solutions imply the implementation of complex dedicated algorithms [37, 41, 42]. Finally, even though some progress has been done in the resolution of EPECs [37] there is even more space for research in this area, given that its application to real size cases is still an immature area of knowledge.

### 3.4 Most recent developments

In this section, we introduce the most recent research on co-planning equilibrium models. This research focuses on modelling the detailed operation of storage technologies and on representing renewable uncertainty using novel hierarchical structures.

**3.4.1 Representation of storage in GEPTEP co-planning models:** Only [41] is the only study that considers long-term hydro storage in equilibrium models, but a simplified version that does not consider reservoir management. To the authors' knowledge, two reviewed papers have addressed short-term storage modelling in GEPTEP co-planning models.

On the one hand, Weibelzahl and März [29] consider storage expansion and perfect competition in the spot market simultaneously formulated at the lower level. Weibelzahl and März [29] show that adequate storage investment can reduce the line investment cost of the TSO. They also show that investment in a zonal market can be suboptimal compared to a nodal market.

On the other hand, in [42] investment in merchant storage resources is considered in the upper level. The authors use a representative period approach to simulate the time steps in which the period of study is divided. The authors demonstrate that merchant storage is economically feasible under the case study considered.

More recently, authors in [44] propose a co-planning model that includes Cournot competition in the market and the representation of short-term (batteries) and long-term (hydro) storage resources with a representative-period formulation that includes a transition matrix and cluster indices as proposed in [63]. Additionally, authors in [44] find counterintuitive results when a proactive approach is considered with a Cournot competition in the market.

**3.4.2 Representation of uncertainty in investment and operating decisions:** Given the complexity of GEPTEP hierarchical models, most of the papers reviewed do not consider the modelling of uncertainty (as seen in Table 2) in their formulations. Accordingly, given that the correct implementation of renewables depends mainly on the introduction of the uncertain availability of the resources, renewables are usually not included in detail in these models (additionally, determining support schemes is an important field of research that has not been fully assessed by GEPTEP models but has been assessed separately in bi-level GEP and network constrained GEP [106, 107].) are usually not included in detail in these models.

As mentioned in Section 2.5, probabilistic and stochastic approaches were the most common way to represent uncertainty. For instance, Baringo and Conejo [33] considers a stochastic approach with scenarios for wind levels and demand. However, it considers traditional load blocks and therefore it is not suitable for adequately simulating storage operation. Baringo and Conejo [33] study how different wind subsidies affect the total welfare of the system. They conclude that transmission investments highly condition the investment in wind. They consider different hydro seasons, limiting the maximum energy produced at each season, and consider a Weibull distribution to introduce stochasticity in wind speed that limits the maximum generation capacity of each winding unit. They also consider a load block approach.

Recent developments in uncertainty representations have introduced other techniques such as robust optimisation, mainly by the application of adaptive robust optimisation, which has proved to be computationally efficient and to represent properly the long-term uncertainties [46].

In this sense, the most recent work on robust GEPTEP considers a min–max–min approach in which simultaneous GEP and TEP decision is taken in the upper level, uncertainty realisation in the middle level and operation in the lower level [45, 46]. In

particular, Baringo and Baringo [93] consider stochastic programming and robust optimisation to deal with both long- and short-term uncertainties. Finally, some other authors additionally consider a certain type of reliability criteria [108, 109]. Please note that the computation efficiency of the algorithms used to solve robust problems has permitted to consider multistage dynamic planning approaches, which had been previously of limited application [45, 46].

A different approach for representing uncertainty is presented in [37]. The authors introduce uncertainty in generation investment, by using similar techniques to those used in robust optimisation. Therefore, Pozo *et al.* [37] take into account the possible multiple generation investment equilibria resulting from a hierarchical model (see Section 9). Therefore, instead of considering the parameters as the uncertainty set, the authors consider the multiple investment equilibria (resulting from the middle level) as their uncertainty set.

It is important to highlight the recent prolific research in robust optimisation. The theoretical background to solve robust optimisation problems is close to the dual theory and the techniques used to solve hierarchical models. However, in hierarchical models, the levels considered represent either different agents or decisions. Conversely, in robust optimisation, the levels considered typically represent different instances of uncertainty realisation. For instance, in [93], both the GEP and TEP expansion decisions are made in the second level where the worst operational case is simulated and in the third level corrective measures are taken to minimise operational costs. This robust optimisation framework can be an important field of research that, together with stochastic programming, is able to couple long and short-term uncertainties in capacity expansion planning.

**3.4.3 Gaps in storage and uncertainty modelling approaches:** Renewable uncertainty and storage operation are still wide fields of research in co-planning equilibrium models. Given the properties of equilibrium models, there is an interesting field of research to study and compare extreme competition cases, where the uncertainty can come not only from the fuel and sources availability but also on the multiple equilibria that can arise from imperfect competition. Additionally, detailed time representation and novel solution techniques can permit us to model more complex markets.

## 4 Concluding remarks

In this study, we addressed the GEPTEP problem. First, we considered the GEPTEP co-optimisation problem in a centralised environment in which a vertically integrated utility takes investment and operation decisions. Then, we focused on the GEPTEP co-planning problem in a market environment, where strategic behaviour and sequential decisions of decentralised agents was studied.

The main findings of this literature review are two-fold:

(i) Given the usual tractability trade-off in planning problems, it is difficult to determine the best modelling options to represent GEPTEP problems, however, we found that: (a) in general, considering lumpy transmission investments might be more important than representing a detailed network. (b) In contrast, given that the economies of scale in generation investment are much lower than in transmission investment, lumpy generation investment can be sometimes disregarded. (c) Finally, as shown by Xu and Hobbs [110], a thorough uncertainty representation can be more important than representing generation operating constraints in a detailed manner.

(ii) For the case of GEPTEP co-planning problems in a market environment, we found that it is a very useful framework to model more realistic market structures. In general, the most studied proactive approach, which renders higher welfare, is still not spread around countries. We found that even if there is perfect competition in the operation, considering the strategic sequential investment decisions between transmission and generation can highly change the planning results. Additionally, the consideration

of merchant investors helps to give insights on how to define optimal support schemes. Finally, some counterintuitive results arise when considering imperfect competition in the MO, i.e. under Cournot competition, allowing trade between areas (by building more lines) can decrease total welfare [26, 43, 44].

We found the following gaps in the literature:

(i) Modelling approaches: there is an active field of research on spatial and temporal resolution, resource adequacy and economics of energy production. (ii) Equilibrium structure: most of the equilibrium structures studied considers a two-, three- or even four-level traditional proactive approach, however, some proactive structures and most reactive structures remain unexplored. (iii) Regulatory structure: in spite of the fact that proactive planning has been proven to lead to the most efficient investment and operation results, in practice, few jurisdictions have strictly applied this approach. Therefore, it is important to compare different regulatory contexts in order to understand the optimal operation and investment strategies in imperfect markets. (iv) Solution technique: in the case of MPEC problems, there is still an active field of research for finding efficient and standard methods to solve these equilibrium problems. Additionally, even though some progress has been achieved in the resolution of EPECs, there is even more space for research in this area, given that its application to real-size cases is still an immature area of knowledge. (v) Most recent developments: renewable uncertainty and storage operation are still wide fields of research in co-planning equilibrium models. On the one hand, more studies on the complementarity between transmission and storage investment are necessary, as well as the joint consideration of both short-term and long-term storage. On the other hand, given the properties of equilibrium models, there is an interesting field of research to study and compare extreme competition cases, where the uncertainty can come not only from the fuel prices and the availability of generating units but also from the multiple equilibria that can arise from imperfect competition. Finally, given that perfect information is a strong assumption, including imperfect information theory in the GEPTEP problems, can make these models more useful for real applications.

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## 6 References

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