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A Search Space Reduction Method for Transmission Expansion Planning Using an Iterative Refinement of the DC Load Flow Model

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***Abstract*—This paper provides a new methodology to compute a reduced but efficient set of candidate lines in a transmission expansion planning (TEP) context. Considering this reduced set of candidate lines should lead to similar investment decisions as if all the possible ac and dc candidate lines that could be installed in the network were considered. A first "hybrid" relaxed TEP problem is solved. Based on this initial solution, a new relaxed TEP problem is iteratively solved in which the dc load flow (DCLF) model is enforced to a certain extent in the partially expanded ac corridors. Once a convergence threshold has been reached, an upper bound of the number of candidate ac and dc lines to install in each corridor can be defined. This process results in a compact search space. Our algorithm has been implemented in General Algebraic Modeling Software (GAMS) and has been tested on a case study based on the European power system. The method produces very promising results and in the considered case study, leads to a very efficient investment.**

***Index Terms*—Dimension reduction, integer linear programming, transmission expansion planning, relaxation methods.**

Nomenclature

Indices and Sets

n	Iteration of the problem $\{0,\ 1,\ 2,\ \cdots\}$
$i,\ \ j$	Node
(i,j)	Corridor
c	Type of circuit (existing or candidate, AC or DC)
(i,j,c)	Set of circuits of the same type in the same corridor
EC	Set of existing AC or DC circuits
CC	Set of candidate AC or DC circuits
AC	Set of existing or candidate AC circuits
s	Snapshot
g	Generator
CONV	Set of conventional generators
RES	Set of Renewable Energy Source (RES) generators
Ω_i^G	Set of conventional or RES generators connected at node *i*
N_B	Total number of nodes
N_S	Total number of snapshots
N_G	Total number of generators

Parameters

Y_{ijc}	Admittance of existing AC circuit (i,j,c) (p.u.)
Y_{ij}^{cand}	Admittance of a single candidate AC circuit in corridor (i,j) (p.u.)
cg_g	Generation cost of generator *g* (€ /MWh)
$d_{i,s}$	Demand level at node *i* during snapshot s (MW)
$\overline{f_{ijc}}$	Power flow capacity of existing circuit (i,j,c) (MW)
$\overline{f_{ijc}^{cand}}$	Power flow capacity of a single candidate circuit of type c in corridor (i,j) (MW)
$\overline{p_{g,s}}$	Generation capacity of RES generator *g* during snapshot *s* (MW)
$\overline{p_g}$	Generation capacity of conventional generator *g* (MW)
ρ_s	Weight of snapshot s (h)
C^{ENS}	Cost of energy non-served (€ /MWh)
C_{ijc}	Investment cost of a single candidate circuit of type c in corridor (i,j) (€)
S_B	Base power (MW)
γ	Fix annual charge rate (p.u.)
$M_{i,j}$	Big-M parameter of corridor (i,j) (MW)
$\overline{x_{n,ijc}}$	Maximum number of candidate circuits of type c that can be installed in corridor (i,j) at iteration n $\{0,\ 1,\ 2,\ \cdots\}\cup\{+\infty\}$

Variables

$x_{n,ijc}$	Number of candidate circuits of type c to invest in the corridor (i,j) at iteration n
$p_{g,s}$	Power production of generator g during snapshot s (MW)
$pns_{i,s}$	Power non-served at node *i* during snapshot s (MW)
$\theta_{i,s}$	Voltage angle at node *i* during snapshot s (p.u.)
$f_{ijc,s}$	Power flow through existing circuit (i,j,c) during snapshot s (MW)
$f_{ijc,s}^{cand}$	Aggregated power flow through all the candidate circuits of type c in corridor (i,j)

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I. Introduction

The aim of Transmission Expansion Planning (TEP) studies is to determine which, where, and when new lines should be constructed at the minimum total cost. The lumpiness of the investment decisions, together with the large size of the problem, make the problem very hard to solve. Moreover, the number of possible combinations of expansion alternatives to potentially explore increases exponentially with the number of candidate grid elements considered. This raises the need for the use of candidate line selection methods to identify the most promising expansion projects. Techniques to select the most appropriate candidate grid assets are called "search space reduction" methods.

In the literature, the list of candidate lines suggested in TEP studies consists most of the times in lines that are parallel to congested lines in the existing network [1]–[3]. Although this method is very intuitive and simple, it has also major drawbacks. On the one hand, this method can only identify necessary reinforcements in existing corridors, and not promising new corridors. On the other hand, this method is not able to provide the potential benefit of reinforcing non-congested corridors. However, nodal prices provide advanced information in this regard. A high nodal price difference between two buses reveals a high potential benefit of installing a new line between these buses, at least for the first MW installed. This information makes it possible to compute a benefit-to-cost ratio[1] between any pair of buses, not only existing corridors, and to select promising candidate lines as those having the highest benefit-to-cost ratio [2], [4], [5]. The main drawback of this sensitivity method, however, is that it estimates candidate lines' benefits based on marginal information, that is, the benefits brought by marginal reinforcement. Because of this, it fails to provide a good estimate of the required additional capacity. On the contrary, authors in [6]–[8] rely on incremental methods that capture more accurately the required capacity expansion in corridors. In [6], authors relax the line capacity constraints when solving an optimal power flow in the existing network and identify promising candidate lines as those in which overflows are occurring and whose reinforcement would not have negative impacts on other congested lines. Yet, similarly to methods based on congested lines [1]–[3], this method only provides information about existing corridors. Moreover, the investment cost of incremental capacity is not taken into account, and this method does not take into account the change in the corridors' equivalent admittance when reinforcing them. Authors in [7] and [8], on the other hand, solve a relaxed "hybrid" TEP problem in which integrality constraints on the investment decision variables are removed and all possible corridors, together with their expansion costs, are taken into account. In [7], the solution obtained indicates which corridors should be reinforced and to which extent. The main drawback of their method, however, is that candidate lines in their relaxed TEP problem obey the transportation load flow model, which is not realistic for AC lines. However, in [8], the solution obtained is used to iteratively select a new candidate line randomly, based on the results produced in the construction phase of a heuristic method called GRASP [9]. In each iteration, a new candidate line is installed and a new relaxed, linear, TEP problem is solved, until the last iteration is reached, when the optimal value of all the investment decision variables is zero. The construction phase algorithm is repeated several times. Authors finally reduce the search space of the TEP problem by limiting, for each corridor, the number of candidate lines to be considered to the maximum number of candidate lines installed across all the times the construction phase algorithm has been run. Even though the problems solved to generate this search space are all linear and, thus, fast to solve, the fact that several occurrences of the construction phase must be run, and the need to go through several iterations in each occurrence of the construction phase, may make the overall search space reduction method computationally burdensome.

[1]The benefit-to-cost ratio of a candidate line is defined as the ratio between the economic benefit brought by the installation of the line and its investment cost. The line's benefit can be estimated thanks to its ends' nodal price. An upper-estimate of the benefit of installing a candidate line is the product of the nodal price difference by the capacity of the candidate line [4].

Authors in [4] and [5] iteratively solve the mixed-integer linear TEP problem and suggest additional candidate lines, based on congestion patterns or nodal prices, in each iteration. Authors in [10], on the other hand, solve the mixed-integer linear TEP problem independently for each stage of the multistage TEP problem, that is, each year of the planning horizon, to find a promising set of candidate lines. The main drawback of these three articles is to include the resolution of smaller, yet complex, mixed-integer linear problem in the search space reduction process, making the reduction technique hard to apply.

Authors in [11]–[13] use constructive heuristic algorithms (CHA) to find good quality TEP solutions. CHA are iterative algorithms that solve a relaxed TEP problem while forcing the installation of new lines at each step of the process. The advantage of this algorithm is that it converges quickly towards a final step in which no new candidate lines improve the quality of the solution. The drawback of this method, however, is that the solution is often a local minimum instead of an optimal global one [11].

Finally, metaheuristic algorithms use a combination of random choices and knowledge of previous results to find a good enough solution of the TEP problem [14]–[16]. Strictly speaking, these methods do not aim to reduce the search space of the TEP problem, but rather to travel through it until a good solution is found. Contrary to all the previous methods above, metaheuristic algorithms do not rely on optimization techniques.

This paper introduces a new efficient search space reduction technique that relies on optimization techniques, instead of metaheuristic algorithms. As discussed above, an interesting option to guide the search space reduction is considering the solution of a "hybrid" relaxed TEP model in which the candidate circuit capacity to be built in each corridor is continuous and unbounded. This TEP model is called "hybrid" because, within it, the flows in the existing AC lines satisfy the DC Load Flow (DCLF) constraints, whereas the flows in candidate AC lines satisfy the Transportation Load Flow (TLF) ones. Given that the TLF model is not accurate enough for AC lines [17], there is a need to find a way to enforce, to the extent possible, the DCLF model in candidate AC lines as well. Moreover, the reduction process should not involve computing a MILP version of the TEP problem, to keep the implementation of the search space reduction method tractable. Finally, avoiding forcing the installation of certain candidate lines during the candidate line selection process is necessary in order not to converge towards local optima, such as in CHA methods.

In order to limit the lack of accuracy resulting from employing the TLF model for candidate AC lines, the method proposed here involves solving a linear relaxation of the TEP problem in which the DCLF model in candidate AC lines is partially enforced. We achieve this by solving, in the first iteration, a "hybrid" relaxed TEP problem in which only flows in existing AC lines satisfy DCLF constraints, while flows in candidate AC and DC lines satisfy TLF ones. Then, in each of the subsequent iterations, making use of the optimal network investment solution computed in the previous iteration, we refine the problem formulation by including additional constraints in order to enforce, to a larger extent, DCLF constraints in relevant candidate AC lines.

The contributions of the proposed method are listed below:

– Most of the previous search space reduction methods make use of information that fail to identify either promising new corridors, such as methods based on line congestion, or the capacity needs of some reinforcements, such as methods based on nodal prices. The present method is based on a linearly-relaxed version of the TEP problem that is able to capture network reinforcement needs of both types.
– Previous research works making use of a relaxed version of the TEP problem to identify promising candidates fully neglect the DCLF constraints in candidate AC lines. The proposed method iteratively refines the relaxed TEP problem to increasingly enforce the DCLF constraints in candidate AC lines, making the representation of the flows in those lines more accurate while identifying promising candidates.
– Contrary to previous works, the problem solved in each iteration is a linear programming (LP) problem, and few iterations are required in practice, making the implementation of the present method more tractable from a computational point of view.
– Contrary to some of the previous works, no candidate line installation is forced during the identification of promising candidate lines, thus avoiding, as much as possible, converging towards a local optimum. However, as well as for existing methods, there is no guarantee that the global optimal solution of the TEP problem belongs to the reduced search space obtained with the proposed method.

The rest of the paper is arranged as follows: Section II introduces the method proposed to reduce the TEP problem's search space. A case study based on the European power system is considered in Section III to assess and validate the proposed method and compare it with alternative search space reduction methods in the literature. Section IV concludes.

II. Methodology

In this section, the method applied to reduce the search space of the TEP problem is described and discussed. The description of this method is divided into the steps that follow:

1) Computation of the relaxed TEP problem solution considering an unbounded number of candidate lines per corridor, and a TLF model in candidate AC lines;

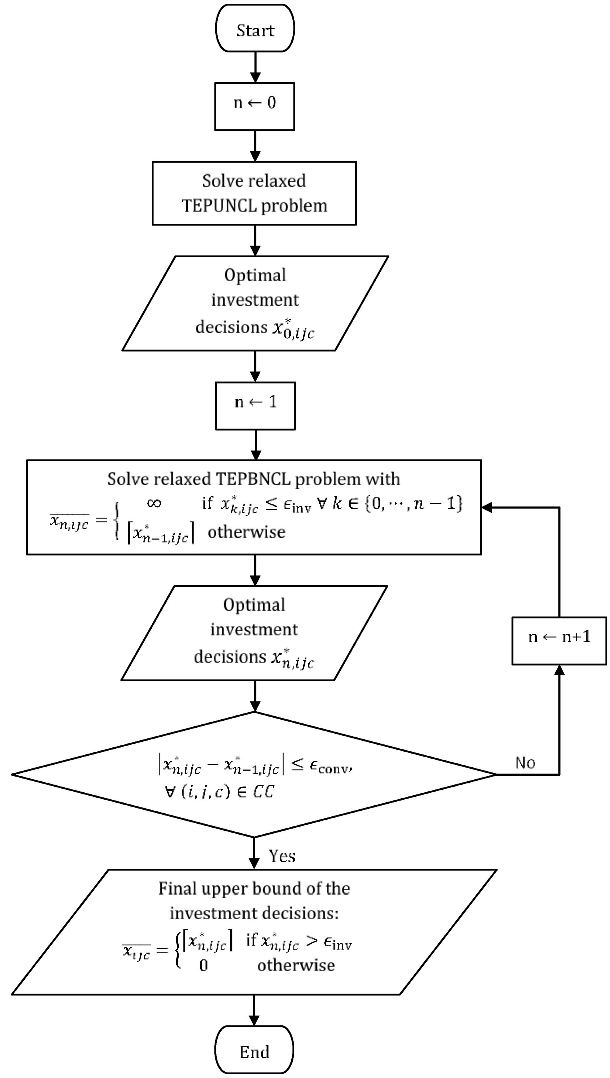

Fig. 1. Flowchart of the methodology applied for search space reduction.

2) Iterative computation, until convergence is achieved, of the relaxed TEP problem solution considering a bounded number of candidate lines per corridor, and a relaxed DCLF model in candidate AC lines;
3) Convergence criterion with proof of convergence, and construction of the reduced search space based on the optimal value of the investment decision variables in the final iteration.

A flowchart of the methodology applied is shown in Fig. 1.

For the sake of simplicity, the N-1 contingency criterion and the representation of losses have not been considered in the problem formulation and the implementation of it. However, the method presented here can be extended to a TEP problem taking into account the N-1 contingency security criterion by modeling the problem as a stochastic one considering many scenarios (one scenario for each potential line outage). As for the losses in candidate lines, a linear representation of them can be used and refined in each iteration of the methodology based on the maximum number of lines $\overline{x_{n,ijc}}$ that can be built.

It is assumed as well that candidate AC lines in the same corridors, i.e., between the same pairs of buses, have the same technical features, that is, the same power flow capacity and the same reactance. Candidate DC lines in the same corridors are also supposed to have the same power flow capacity. However, the method proposed can be easily extended to consider candidates with different technical features within each corridor.

A. *Relaxed TEP Problem With an Unbounded Number of Candidate Lines Per Corridor*

The problem solved in the first iteration of the present method $(n = 0)$ is a TEP with an unbounded number of candidate lines per corridor (TEPUNCL). The mathematical formulation of the TEPUNCL problem is similar to the one used for TEP in previous works by authors [18], [19]. The problem can be expressed as follows:

$$\min \left\{ \sum_{s=1}^{N_S} \rho_s \left(\sum_{g=1}^{N_G} p_{g,s} cg_g \right) + \sum_{s=1}^{N_S} \rho_s \left(\sum_{i=1}^{N_B} pns_{i,s} C^{ENS} \right) + \gamma \left(\sum_{(i,j,c)\in CC} x_{ijc} C_{ijc} \right) \right\} \tag{1}$$

Subject to:

$$\sum\nolimits_{g\in\Omega_i^G} p_{g,s} - d_{i,s} + pns_{i,s} + \sum\nolimits_{jc} f_{jic,s} + \sum\nolimits_{jc} f_{jic,s}^{cand} - \sum\nolimits_{jc} f_{ijc,s} - \sum\nolimits_{jc} f_{ijc,s}^{cand} = 0; \quad \forall i \in [\![1; N_B]\!],\ \forall s \in [\![1; N_S]\!] \tag{2}$$

$$f_{ijc,s} = S_B Y_{ijc} \left(\theta_{i,s} - \theta_{j,s}\right); \quad \forall (i,j,c) \in EC \cap AC,\ \forall s \in [\![1; N_S]\!] \tag{3}$$

$$f_{ijc,s}^{cand} = x_{ijc} S_B Y_{ij}^{cand} \left(\theta_{i,s} - \theta_{j,s}\right) \quad \forall (i,j,c) \in CC \cap AC, \quad \forall s \in [\![1; N_S]\!] \tag{4}$$

$$-\overline{f_{ijc}} \le f_{ijc,s} \le \overline{f_{ijc}};\ \forall (i,j,c) \in EC,\ \forall s \in [\![1; N_S]\!] \tag{5}$$

$$-x_{ijc}\overline{f_{ijc}^{cand}} \le f_{ijc,s}^{cand} \le x_{ijc}\overline{f_{ijc}^{cand}} \quad \forall (i,j,c) \in CC,\ \forall s \in [\![1; N_S]\!] \tag{6}$$

$$0 \le pns_{i,s} \le d_{i,s};\ \forall i \in 1; N_B,\ \forall s \in [\![1; N_S]\!] \tag{7}$$

$$0 \le p_{g,s} \le \overline{p_g};\ \forall g \in \text{CONV},\ \forall s \in [\![1; N_S]\!] \tag{8}$$

$$0 \le p_{g,s} \le \overline{p_{g,s}};\ \forall g \in \text{RES},\ \forall s \in [\![1; N_S]\!] \tag{9}$$

$$x_{ijc} \in \{0,\ 1,\ 2,\ \ldots\};\ \ \forall (i,j,c) \in CC \tag{10}$$

Equation (1) provides the total cost of the system operation and transmission investments, which we aim to minimize. The first two terms of this equation are related to the operation costs and, more specifically, to generation variable production costs and the cost of non-served energy. A weight ρ_s is associated with each snapshot (operation situation) *s* representing its duration in hours. The final term in (1) is related to the network investment costs. In order to obtain an annualized cost that can be directly compared to operational ones, a fixed charge (annualization) rate γ is applied to these investment costs.

Equation (2) represents the power balance at each node. This includes the power produced by a virtual generation plant representing non-served energy.

Equations (3) and (4) correspond to the DC-load flow model used to represent the flow of power through existing and candidate AC lines, respectively.

Equations (5) and (6) refer to the maximum power flow that can go through existing and candidate circuits, respectively.

Equation (7) constrains non-served power in each node to be positive and always lower than the actual demand level in this node. Equations (8) and (9) represent the power generation limits of conventional and RES generators, respectively. The main difference between the two is the time-dependency of RES generators capacity.

Finally, equation (10) refers to the "lumpiness" of investment decisions.

Contrary to previous work [18], [19], equations (3) corresponds to the DCLF model used to represent the flow of power through existing AC circuits, whereas equations (4) correspond to the DCLF model used to represent the total flow of power through a set of candidate AC circuits installed in a given corridor. $Y_{i,j}^{cand}$ is the corresponding admittance of a single candidate AC line in this corridor. Moreover, equations (5) refers to the maximum power flow that can go through existing circuits whereas equations (6) refer to the maximum power flow that can go through a set of candidate circuits installed in a given corridor. $\overline{f_{ijc}^{cand}}$ is the corresponding power flow capacity of a single candidate AC line in this corridor.

For the sake of simplicity, we assume from now on that candidate lines are all AC lines. Since we also assume that all the candidate AC circuits within the same corridor have the same characteristics, we can now refer to each individual candidate AC line by its corridor *(i,j)* instead of *(i,j,c)*. The general case that considers both AC and DC candidate lines can be addressed analogously to this specific case.

Contrary to previous work [18], [19], here the investment variables x_{ij} are not binary. They are integer variables that represent the number of candidate lines installed in the corridor (i, j).

As in [18], [19], we aim to define a linear relaxation of this MILP TEPUNCL problem whose solution provides us with relevant information about its relevant search space considering integer investment variables. For this, we need to linearize the integer variables $x_{i,j}$ and find a convex feasible space that includes the space defined by (1)–(10).

In the linear relaxation of the TEPUNCL problem, the integrality constraint of the investment decisions variables is relaxed; the investment decision variables satisfy equations (11), instead of equation (10):

$$x_{i,j} \in [0; +\infty[\tag{11}$$

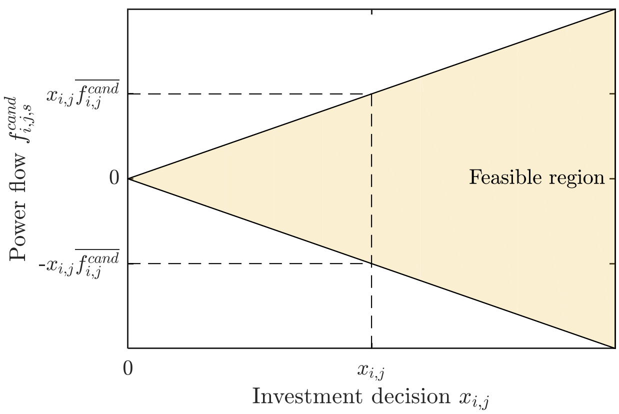

Fig. 2. Convex region delimiting the values of $x_{i,j}$ and $f_{i,j,s}^{cand}$ in the relaxed TEPUNCL problem.

Constraints (1)–(3) and (5)–(9) are linear. Therefore, the space delimited by them is convex and encompasses the feasible space of the TEPUNCL problem. On the other hand, constraint (4), enforcing the DCLF model for candidate AC lines, is not linear because it is defined in terms of the product of an integer variable and a linear variable $x_{i,j}(\theta_{i,s} - \theta_{j,s})$. In [18], [19], we were able to find the smallest convex space delimiting this product of variables by making use of the McCormick envelop. This was previously possible because the investment decision variable was bounded between 0 and 1. Here, the variable $x_{i,j}$ has no upper bound and the McCormick envelop cannot be applied. Therefore, the smallest convex space that encompasses the feasible space of the TEPUNCL problem, which is the feasible space of the relaxed TEP problem, is the space delimited by equations (2)–(3), (5)–(9), and (11).

An illustration of the convex region delimiting the values of the variables $x_{i,j}$ and $f_{i,j,s}^{cand}$ is depicted in Fig. 2.

It can be noticed that the relaxed TEPUNCL is a "hybrid" problem, in which flows in existing AC lines satisfy the DCLF model and flows in candidate AC lines satisfy the TLF model. However, solving the relaxed TEPUNCL problem provides relevant information about the upper bounds of the decision variables in the next iterations of this method, making it possible to refine the flow representation in candidate AC lines.

B. Relaxed TEP Problem With a Bounded Number of Candidate Lines Per Corridor

When solving the relaxed TEPUNCL problem, the optimal investment decisions computed correspond to the best possible tradeoff, for the relaxed problem, between the benefits produced by these investments and their cost. The value of these optimal investment decisions can be used to define, in the following iterations, an upper bound of the number of candidate lines that can be built in each corridor. We shall assume here, as a first approximation, that the number of candidate AC lines that should be considered for a given corridor should be equal to the smallest integer that is greater than the optimal value of the investment variable for this given corridor computed solving the relaxed TEPUNCL problem, except in the case the optimal value of this investment decision variable is below a certain threshold, or null, in all the previous iterations. In this case, the investment decision variables remain unbounded. This exception is made to keep the method open to investment opportunities that are not identified in the first iterations.

With this in mind, we can define a new relaxed TEP problem with a bounded number of candidate lines per corridor (TEPBNCL) in which investment decision variables are bounded by the following upper bound:

$$\overline{x_{n,i,j}} = \begin{cases} +\infty & \text{if } \; x_{k,i,j}^{*} \leq \in_{\text{inv}} \; \forall \, k \in \{0, \, \cdots, n-1\} \\ x_{n-1,i,j}^{*} & \text{otherwise} \end{cases} \tag{12}$$

With $\overline{x_{n,i,j}}$ being the upper bounds of the investment decision variables $x_{n,i,j}$ in the relaxed TEPBNCL problem solved at iteration n, $x_{n-1,i,j}^{*}$ being the optimal value of the investment decision variables for corridors (i, j) computed solving the relaxed TEPUNCL, or TEPBNCL problem, at iteration $n-1$, and $\in_{\text{inv}}$ being the investment threshold above which a candidate corridor is considered to be reinforced.

For those corridors whose investment decision variables $x_{n,i,j}$ have a finite upper-bound, the following constraint is introduced:

$$\begin{aligned} &- M_{i,j}\left(\overline{x_{n,i,j}} - x_{n,i,j}\right) \\ &\leq f_{i,j,s}^{cand} - S_B \overline{x_{n,i,j}} Y_{i,j}^{cand}\left(\theta_{i,s} - \theta_{j,s}\right) \\ &\leq M_{i,j}\left(\overline{x_{n,i,j}} - x_{n,i,j}\right) \end{aligned} \tag{13}$$

Equation (13) is obtained thanks to a generalization of the McCormick envelop used in [18], [19] for any maximum number of candidate lines per corridor $\overline{x_{n,i,j}}$. $M_{i,j}$ is a parameter that must be big enough so the voltage angle difference between buses i and j is not unnecessarily constrained when no line is installed in the corresponding corridor $(x_{n,i,j} = 0)$. An appropriate value for $M_{i,j}$ can be computed by solving a shortest path problem [20]. Equation (13) together with equation (6) represent the convex envelop of the product $x_{i,j}(\theta_{i,s} - \theta_{j,s})$ when $x_{i,j}$ is bounded by:

$$0 \leq x_{i,j} \leq \overline{x_{n,i,j}} \tag{14}$$

Finally, the relaxed TEPBNCL problem can be formulated as (1)–(3), (5)–(9), (13), and (14). An illustration of the convex region delimiting the values of the variables $x_{i,j}$, $f_{i,j,s}^{cand}$ and $(\theta_{i,s} - \theta_{j,s})$ is depicted in Fig. 3.

It can be noticed that the projection of the convex region depicted in Fig. 3 on the two-dimensional subspace $(x_{i,j},\ f_{i,j,s}^{cand})$ corresponds to the sub-region of the one depicted in Fig. 3 that is limited by the constraint (14). Therefore, the feasible space illustrated in Fig. 3 is tighter than the feasible space illustrated in Fig. 2.

The TEPBNCL problem is solved in the second and subsequent iterations of this method $(n = 1,\ 2,\ \ldots,\ n_{final})$. In each iteration, new optimal network investment decisions are computed, and equations (12) are employed to define the new upper bounds of the investment decision variables to be considered in the relaxed TEPBNCL problem to be solved in the next iteration. It should be noted that equation (12) is only

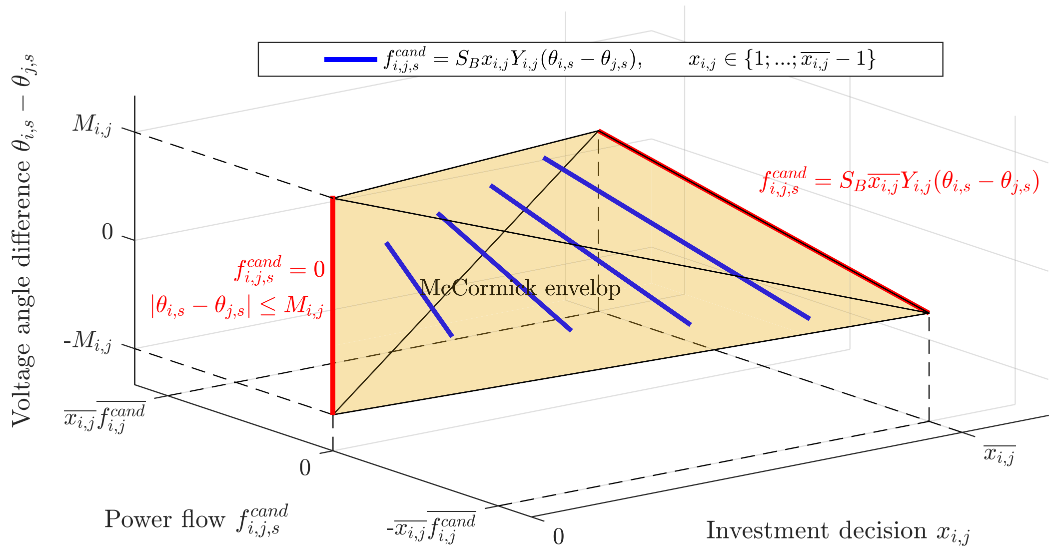

Fig. 3. McCormick envelop delimiting the values of $x_{i,j}$, $f^{cand}_{i,j,s}$ and $(\theta_{i,s} - \theta_{j,s})$ in the relaxed TEPBNCL problem. The envelop, depicted by the orange tetrahedron, is the smallest convex region covering both the feasible region for which no candidate line is installed ($x_{i,j} = 0$) and the feasible region for which the maximum number of candidate lines is installed ($x_{i,j} = \overline{x_{i,j}}$), in red. The feasible regions enforcing the DCLF model for an integer number of candidate lines greater than 0 and lower than $\overline{x_{i,j}}$, are depicted using blue lines. These blue lines all lie in the McCormick envelop.

valid for candidate AC lines. Investment decision variables for candidate DC lines can remain unbounded for all the iterations since the load flow model for those lines (TLF) does not need to be refined.

C. Convergence of the TEPBNCL Problem

The last iteration, n_{final}, is reached when the following convergence criterion is met:

$$\left| x^*_{n_{final},i,j} - x^*_{n_{final}-1,i,j} \right| \leq \epsilon_{\text{conv}}, \quad \forall\ (i,j) \qquad (15)$$

This involves that the difference between the optimal number of new lines to be built in each corridor computed in this and the previous iteration must be lower than a certain threshold, i.e., the development of the network computed in these two subsequent iterations must be similar enough. It can be proven that the method will always converge.

Proof. For each corridor (i, j):

– either the optimal investment decision variable $x^*_{n,i,j}$ is below the threshold ϵ_{inv} for any iteration n, which means that convergence has been achieved. In this case, $\overline{x_{n,i,j}} = +\infty$ for any iteration n.

– or the optimal decision variable $x^*_{n,i,j}$ is greater than the threshold ϵ_{inv} in at least one iteration N. In this case, in the following iteration $N+1$, the investment decision variable $x_{N+1,i,j}$ will be bounded by $\overline{x_{N+1,i,j}} = \lceil x^*_{N,i,j} \rceil$. Because of this, the optimal value $x^*_{N+1,i,j}$ will also satisfy $x^*_{N+1,i,j} \leq \lceil x^*_{N,i,j} \rceil$. Therefore, we have $\lceil x^*_{N+1,i,j} \rceil \leq \lceil x^*_{N,i,j} \rceil$, which involves that $\overline{x_{N+2,i,j}} \leq \overline{x_{N+1,i,j}}$. By recurrence, we can deduce that the sequence of upper bounds $(\overline{x_{n,i,j}})_{n \in \mathbb{N}}$ of this variable is a sequence of decreasing numbers from the rank $N+1$ on. Since this sequence has also a lower bound which is zero, according to the monotone convergence theorem, the sequence converges. Moreover, this sequence is a sequence of integer numbers from the rank $N+1$ on, and thus, converging is equivalent to the value of $\overline{x_{n,i,j}}$ being constant from a certain rank.

Finally, there is a certain rank, or iteration, from which the parameters $\overline{x_{n,i,j}}$ will stay constant for any corridor (i, j). From this iteration on, the TEPBNCL will be defined by the same constraints in all the following iterations. Therefore, the optimal solution will remain the same, and the convergence criterion defined by equation (15) will be met. ■

Once convergence has been achieved, the maximum number of candidate lines to be considered in each corridor is computed as in equation (16). This equation represents the reduced search space to be considered when solving the original, mixed integer non-linear, TEP problem defined by equations (1)–(10).

$$\overline{x_{i,j}} = \begin{cases} \left\lceil x^*_{n_{final},i,j} \right\rceil & \text{if } x^*_{n_{final},i,j} > \epsilon_{inv} \\ 0 & \text{otherwise} \end{cases}, \forall\ (i,j) \qquad (16)$$

A reduced TEP problem can be built by including this set of upper bounds for the investment decision variables $x_{i,j}$ together with equations (1)-(10). The reduced TEP problem can be converted into a mixed integer linear programming (MILP) problem by converting integer variables into binary ones thanks to the binary numerical system [8], and by using a disjunctive model [20].

III. Case Study

A. Systems Description

The proposed method for the reduction of the search space to be considered in TEP has been applied to the 2088-bus case study based on parts of the European power grid described in [21], [22].

TABLE I
CANDIDATE LINE INVESTMENT COSTS

Type of candidate	Investment cost per circuit (M€/km)	Capacity per circuit (MW)
Candidate AC lines	1.21	1,020
Candidate onshore DC lines	0.44	1,000
Candidate submarine DC lines	1.36	1,000
Candidate AC/DC converters (M€)	75.75	1,000

This power system comprises 2088 buses and 2953 lines located in five countries: Portugal, Spain, France, the United Kingdom, and the Republic of Ireland. The total electricity demand for the target year is 1,279 TWh, which is 10% more than the total demand in the five countries in the year 2016. The annualized factor applied to investment costs is $\gamma = 7.39\%$. The investment recovery period is 40 years.

The number of corridors that can be expanded is limited beforehand by the following set of rules:

– AC circuits can be installed in any inland corridor with a length lower than 200 km,
– Onshore DC circuits can be installed in any inland corridor with a length greater than 200 km,
– Offshore DC circuits can be installed in any submarine corridor whose two ends are on a coast,
– Converters can be installed at any 400 kV bus.

DC circuits can only be connected to the AC network through converters. However, two DC circuits can be directly connected to each other, and full controllability of the flow in each DC corridor is assumed.

In total, there are 54,742 candidate corridors, comprising:

– 11,936 corridors for candidate AC circuits,
– 42,198 corridors for candidate DC circuits,
– 84 corridors for candidate Offshore DC circuits,
– and 524 buses with candidate converters.

The investment cost per MW and km of AC and DC circuit, as well as the investment cost per MW of converter, is provided in Table I together with the size of candidate circuits of each type and converters.

For the sake of simplicity, from now on, we will refer to the candidate corridors for AC circuits as "candidate AC corridors", and we will refer to the candidate corridors for onshore and offshore DC circuits and the candidate buses for converters as "candidate DC corridors".

B. Application of the Methodology

The methodology has been applied to two version of the case study described above: one version considering the 54,742 candidate AC corridors and candidate DC corridors, and another version considering the 11,936 candidate AC corridors only.

The investment decision threshold is set at $\in_{\text{inv}} = 10^{-3}$. After several numerical tests, this value turned out to be a good trade-off between the size of the reduced search space and the quality of the solutions it contains. However, values of $\in_{\text{inv}}$ ranging from 10^{-5} to 10^{-2} led to similar results, although not as good.

In the initial step, the relaxed TEPUNCL problem is solved. According to the solution computed, in the case study considering both candidate AC and DC corridors, there are only 175 candidate corridors for which the optimal investment decision variable is greater than this threshold, including 96 candidate AC corridors. This number amounts to 142 candidate AC corridors in the case study considering only candidate AC corridors.

In the following step, when solving the relaxed TEPBNCL problem, the investment decision variables related to these 96, respectively 142, candidate AC corridors are bounded by the nearest integer greater than the optimal value of the decision variable found for those corridors when solving the relaxed TEPUNCL. Thanks to this, we can force the flows in these corridors to meet constraint (13), which involves partially enforcing the DCLF model.

In the case study considering both type of candidate corridors, flows in the remaining 79 candidate DC corridors, on the other hand, do not have to satisfy the DCLF model. Therefore, their investment decision variable can remain unbounded in the TEPBNCL problem.

The optimal investment decision variable is lower than $\in_{\text{inv}}$ for the vast majority of the candidate corridors in the relaxed TEPUNCL problem. The investment decision variables for these candidate corridors are kept unbounded when solving the first iteration of relaxed TEPBNCL problem. They remain unbounded in the following iterations of the relaxed TEPBNCL problem as long as their optimal value in all the previous iterations is lower than $\in_{\text{inv}}$. In this way, the search space is not prematurely reduced. Candidate corridors that were not worth expanding in the relaxed TEPUNCL problem may be worth expanding when adding constraint (13) affecting partially built candidate AC corridors.

The relaxed TEPBNCL problem is solved for as many iterations as needed, updating in each iteration the upper bounds of investment variables, until the maximum change in an optimal investment decision variable over all the candidate corridors is lower than the convergence threshold $\in_{\text{conv}}$. The convergence threshold is set at $\in_{\text{conv}} = 10^{-2}$. For the case study considering both types of candidate corridors, the convergence threshold is reached after 4 iterations involving solving the TEPBNCL problem. On the other hand, 5 iterations are necessary for the case study considering candidate AC corridors only. Solving the TEPUNCL problem and the following iterations of the TEPBNCL problem has taken around 80 minutes when considering candidate AC and DC corridors, and only 20 minutes when considering candidate AC corridors only. A summary of the results obtained is provided in Table II.

Once convergence is reached, the reduced search space to be considered when solving the interger TEP problem is set to comprise all those candidate corridors for which $x^*_{n_{\text{final}},i,j} > \in_{\text{inv}}$ in the final iteration. The maximum number of candidate circuits that can be installed in each of these corridors is set to be equal to $\lceil x^*_{n_{final},i,j} \rceil$. In total, there are 242 candidate circuits to be considered for 191 corridors in the case study considering both types of candidate corridors, and 203 candidate circuits for 191 corridors in the case study considering AC candidate corridors only.

TABLE II
SUMMARY OF THE REDUCTION PROCESS

Case study	Case study with candidate AC and DC corridors					Case study with candidate AC corridors only					
Iteration	0	1	2	3	4	0	1	2	3	4	5
Problem being solved	Relaxed TEP UNCL	Relaxed TEPBNCL				Relaxed TEP UNCL	Relaxed TEPBNCL				
Number of candidate AC corridors with $x^*_{n,i,j} > \epsilon_{\text{inv}}$	96	110	109	112	110	142	164	180	189	190	191
Number of candidate DC corridors with $x^*_{n,i,j} > \epsilon_{\text{inv}}$	79	101	81	102	81	-	-	-	-	-	-
Total number of candidate corridors with $x^*_{n,i,j} > \epsilon_{\text{inv}}$	175	211	190	214	191	142	164	180	189	190	191
Total costs (M€)	33,454	33,464	33,468	33,471	33,471	36,031	36,045	36,053	36,054	36,055	36,056
Computation time (min)	23	12	17	13	14	6	2	3	3	3	3

Finally, the integer investment decision variables to be considered are converted into binary decision variables using the binary numerical system. The number of equivalent candidate lines asscoiated with binary investment decision variables amounts to 222 in the case study considering both types of candidate corridors, and 202 in the case study considering candidate AC corridors only.

C. Validation of Results

In order to assess the efficiency of the search space reduction process here described, the MILP TEP problem is solved taking into account the list of equivalent candidate lines found above. In order to solve the considered problem, the solver Gurobi 8.1 has been run on GAMS using a machine with an Intel Xeon E5-2660 processor running at 2.60 GHz and 144 GB of RAM.

In the case study considering candidate AC and DC corridors, the total costs corresponding to the optimal solution found amount to 33,805 M€ , with an integrality gap of 0.92%. Overall, 102 corridors have been reinforced building 136 new circuits. Moreover, 19 out of the 102 reinforced corridors are associated with investment decision variables whose optimal value in the first iteration (the relaxed TEPUNCL problem) was lower than the investment threshold. In the case study considering candidate AC corridors only, the total costs amount to 36,625 M€ , with an integrality gap of 1.24%. Overall, 88 corridors have been reinforced building 104 new circuits, and 31 out of the 88 reinforced corridors are associated with investment decision variables whose optimal value in the relaxed TEPUNCL problem was lower than the investment threshold.

This proves the relevance of refining, step by step, the set of candidate circuits to consider through the process proposed, which involves updating the convex envelop of the DCLF model to be considered for the flows in the relevant corridors according to the solution computed in each iteration of the TEPBNCL problem. The iterative refinement of the relaxed TEP model helps discovering promising corridors to reinforce that did not appear in the very first iteration, in the solution of the relaxed "hybrid" TEP problem.

Contrary to what happens for the case studies considered when testing the snapshot selection method [18] and the network reduction method [19], here it is not possible to compare the solution of the MILP TEP problem computed when considering the reduced search space with the solution of this problem computed when considering the non-reduced search space, since solving this problem in the latter case is not possible (the problem becomes intractable).

However, the relaxed TEPUNCL computed in the very first iteration of the proposed search space reduction method is a relaxed version of the original TEP problem we aim to solve. Because of this, the total cost of the solution of the relaxed TEPUNCL problem is a lower bound of the total cost of the true solution of the original TEP problem. Moreover, the total cost of the solution of the reduced TEP problem, considering the reduced search space obtained with our method, is an upper bound of the the total cost of the true solution of the original TEP problem (17).

$$\begin{aligned} \text{Total\_Costs}_{\text{relaxed TEPUNCL}} &\leq \text{Total\_Costs}_{\text{original TEP}} \\ &\leq \text{Total\_Costs}_{\text{reduced TEP}} \end{aligned} \tag{17}$$

Therefore, we can conclude that the relative difference in total cost between the true solution of the original TEP problem and the solution of the reduced TEP problem is lower than the relative difference in total cost between the solution of the reduced TEP problem and the solution of the relaxed TEPUNCL problem. This means that, in the case study considering both types of candidate corridors, the total cost of the solution of the reduced TEP problem is less than 1.05% larger than the total cost of the solution of the original TEP problem, whereas it is less than 1.61% larger in the case study considering only candidate AC lines.

We compare the quality of the solution obtained when applying the proposed search space reduction method with the quality of those solutions of the MILP TEP problem resulting from applying other search space reduction methods in the literature. Thus, the proposed method is compared with the method used by the authors in [5], based on nodal prices, the method used by the authors in [7], based on the relaxed hybrid TEP model, and the method used by the authors in [8], based on the GRASP algorithm.

1) Method Based on Nodal Prices [5]: The authors in [5] iteratively enlarge the search space of the TEP problem making use of the information provided by the nodal prices computed in each iteration. In each step (iteration) k of the method they propose, they solve the MILP TEP problem considering a list of candidate lines $\Omega_C^{(k)}$. Then, they compute the benefit-to-cost ratio of every possible new candidate line based on the new nodal prices computed. The candidate lines with the largest benefit-to-cost ratio are added to the list $\Omega_C^{(k)}$ to build the list $\Omega_C^{(k+1)}$. This is carried out time after time until there are no remaining candidate lines with a benefit-to-cost ratio larger than 1. The list of candidate lines considered in the very first step $\Omega_C^{(0)}$ is empty. Then, in this step the authors simply solve an optimal power flow (OPF) problem. As the authors in [5] point out, the number of candidate lines added in each step should strike a trade-off between number of iterations required and the computational burden of each.

We have applied their method to the 2088-bus case study considering both types of candidate corridors. We have chosen a number of candidate lines to add to the list equal to 50, which corresponds to a suitable fraction (not very large, not very small) of the number of suitable candidate lines computed when applying our search space reduction method.

In theory, the benefit-to-cost ratio computed for candidate DC circuits should be based on the nodal price at their ends, as well as for candidate AC circuits. However, since most of the buses in the existing network are not connected yet to an AC/DC converter, the ends of most candidate DC circuits are not connected to the rest of the network. As a result, the nodal prices at the end nodes of most candidate DC circuits are not to be used to calculate their benefit-to-cost ratio. To cope with this issue, we have calculated, instead, the benefit-to-cost ratio of a "DC project" composed of a 1 GW DC circuit together with two 1 GW AC/DC converters at both ends of this circuit. The potential benefit of this DC project is calculated based on the nodal prices at the two buses in the AC network to which the 2 converters would be connected. Then, the benefit-to-cost ratio is computed as the ratio between the aforementioned potential benefit and the sum of the investment cost of the DC circuit and the investment cost of the two converters. Then, if, at a given step, we identify a DC project as one of the 50 most promising candidate lines, we add the DC circuit and the two converters it comprises to the list of candidates.

We have applied the method in [5] as described. We have computed 9 iterations of the algorithm. The main results computed are summarized in Table III. In the first iterations, the total operation-plus-investment cost quickly decreases as the list of candidate lines grows. On the other hand, the computation time required to solve the TEP problem increases with the iteration number, and dramatically increases in iteration 4. From iteration 7 on, it takes more than 24 hours to solve this problem. Even though there are still lines with a benefit-to-cost ratio larger than 1 in iteration 9, we stop the algorithm at this point because of the fact that the decrease in total costs achieved by then in each iteration is insubstantial compared to the computational effort of this iteration.

TABLE III
TEP SOLUTION FOR THE METHOD BASED ON NODAL PRICES

Iteration number	Number of candidate lines considered	Total costs of the MILP TEP solution (M€)	Computation time to solve the TEP problem (min)
1	0	92,681	5
2	50	60,857	6
3	100	45,094	314
4	150	40,738	975
5	200	39,262	238
6	250 + 6cv	37,490	568
7	300 + 6cv	37,034	1,421
8	350 + 12cv	35,839	1,236
9	400 + 12cv	35,620	2,060

Evolution of the number of candidates, the quality of the TEP solution, and the computational effort, with the iteration number when applying the search space reduction method based on nodal prices to the 2088-bus case study. The term "cv" refers to the candidate converters added to the list of candidate lines.

In total, it has taken 79 hours to build a search space composed of 412 candidate lines (time to compute the TEP in the first 8 iterations). The search space computed using the method proposed here is almost 2 times smaller, and it has taken us only 1 hour to compute it. Besides, the best TEP solution computed considering the search space defined based on the method in [5] corresponds to a total cost of 35,620 M€ , which is 5.4% larger than the total cost corresponding to the best TEP solution computed considering the search space defined using the method proposed here.

2) Method Based on the Relaxed Hybrid TEP Model [7]: In [7], the authors solve a relaxed hybrid TEP problem. The investment decisions in it are linearized and the power flows in the candidate AC lines are computed according to the TLF model. As a matter of fact, the relaxed hybrid TEP problem solved by the authors in [7] is exactly the same as the relaxed TEPUNCL problem described in II.A, which is solved in the very first iteration of the proposed search space reduction algorithm.

Authors in [7] assume, and state, that the optimal values of the investment decision variables computed when solving the relaxed hybrid TEP problem correspond to the required additional capacity in each corridor [7]. Contrary to what is carried out in the method proposed in this chapter, they do not use the values of investment decision variables computed when solving the relaxed hybrid problem to reduce the search space considered in the MILP TEP problem. Instead, they directly build an integer TEP solution out of the solution of the relaxed hybrid problem. Given that the optimal values of the investment decision variables in the relaxed hybrid TEP problem are not integer, they are converted into integer values to generate a feasible MILP TEP solution. This can be done in one of the following several ways:

– The optimal values of decision variables $x_{i,j}^*$ are rounded to the next larger integer $\lceil x_{i,j}^* \rceil$,

TABLE IV
TEP SOLUTION FOR THE METHOD BASED ON RELAXED HYBRID TEP MODEL

Rounding method	$x_{i,j} = \lceil x^*_{i,j} \rceil$	$x_{i,j} = \lfloor x^*_{i,j} \rfloor$	$x_{i,j} = [x^*_{i,j}]$
Number of candidate circuits installed	235	60	103
Total costs (M€)	34,684	52,855	36,659

– The optimal values of decision variables $x^*_{i,j}$ are rounded to the next smaller integer $\lfloor x^*_{i,j} \rfloor$,
– The optimal value of decision variables $x^*_{i,j}$ are rounded to the nearest integer $[x^*_{i,j}]$.

To assess the efficiency of the method in [7], the optimal values of the investment decision variables, when solving the relaxed TEPUNCL problem for the case study considering both types of candidate corridors, are rounded according to the 3 rounding options above. Then, for each option, the number of candidate circuits installed in each corridor is set to the corresponding rounded number and an OPF is run for the resulting grid. The number of candidate circuits installed, and the total investment plus operation costs obtained for each rounding method are provided in Table IV.

The best MILP TEP solution computed according to any of the rounding options, obtained by rounding the optimal value of the investment decision variables to the next larger integer, results in total costs that are 2.6% higher than the total costs computed for the TEP solution found with the method proposed here. This MILP TEP solution is a high quality one. Moreover, it has taken us only 14 minutes to find this solution (time required to solve the relaxed TEPUNCL problem), as shown in Table II. However, the TEP solution computed using the method proposed in this chapter to reduce the search space is still noticeably better than the one being discussed.

3) Method Based on the GRASP Algorithm [8]: The authors in [8] compute several occurrences of the construction phase procedure of a heuristic method called GRASP. In each occurrence of the construction phase, an iterative process installs candidate circuits in the system, until a convergence criterion is reached. In each iteration, a candidate circuit is selected randomly from a list of promising candidate lines, and is installed in the system. Then, a new linearly relaxed TEP problem that takes into account the new lines installed is solved. A new list of promising candidate lines is built based on the solution of this problem. The last iteration is reached when the solution of the linearly relaxed TEP problem does not find new promising candidate lines, i.e., when the optimal value of all investment decision variables is zero. The set of lines installed through this procedure represents a possible transmission expansion plan.

Each occurrence of the construction phase procedure results in a different transmission expansion plan with different number of lines installed in each corridor. The reduced search space produced by the method from [8] is defined as the maximum number of lines installed, for each corridor, across all the transmission expansion plans obtained with the construction phase procedure.

We have applied the method in [8] to the 2088-bus case study considering candidate AC corridors only. For a given construction phase procedure, each iteration of the procedure, at the end of which a new candidate circuit is installed, takes in average 10 minutes to be computed. When computing a construction phase procedure, after 100 iterations and around 17 hours of computation time, the convergence is not yet reached. Authors in [8] compute at least 20 occurrences of the construction phase procedure. Doing so with our case study would require more than 300 hours of computation time, which is not tractable. We conclude that the method proposed by the authors is not adapted to our case study. This can be explained by the fact that they apply their method on case studies with at most 183 candidate corridors, whereas our case study considers more than 50 times more candidate corridors.

IV. CONCLUSION

We propose a method for the reduction of the search space to be considered in TEP analyses. This method aims to identify a reduced set of relevant candidate lines by iteratively enforcing the DCLF model for the representation of flows in candidate AC lines. At each iteration, a LP relaxed TEP problem is solved, which makes the method more tractable than other methods based on solving a MILP problem. Moreover, we prove that the proposed method will always converge and, in practice, the number of iterations required to reach convergence is much smaller than for alternative methods. The proposed method has produced promising results when applied to the case studies considered. The reduced search space computed using this method has shown to be more efficient, i.e., more compact and leading to a more efficient TEP solution, than the reduced search spaces computed using the existing methods in the literature. However, there is no guarantee that the optimal TEP solution computed when considering the initial, non-reduced, search space is contained within the reduced search space (selection of candidate lines) computed using the proposed method. Further research works are necessary to identify how close the optimal TEP solution contained in the reduced search space produced by our method can be from the optimal TEP solution computed considering the initial search space.

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