CONDITIONING AND BACKWARD ERRORS
OF EIGENVALUES OF HOMOGENEOUS MATRIX
POLYNOMIALS UNDER MÖBIUS TRANSFORMATIONS

LUIZ MIGUEL ANGUAS, MARIA ISABEL BUENO, AND FROIÁN M. DOPICO

ABSTRACT. We present the first general study on the effect of Möbius transforma-
tions on the eigenvalue condition numbers and backward errors of approximate
eigenpairs of polynomial eigenvalue problems (PEPs). By using
the homogeneous formulation of PEPs, we are able to obtain two clear and
simple results. First, we show that if the matrix inducing the Möbius transforma-
tion is well-conditioned, then such transformation approximately preserves
the eigenvalue condition numbers and backward errors when they are defined
with respect to perturbations of the matrix polynomial which are small relative
to the norm of the whole polynomial. However, if the perturbations in each
coefficient of the matrix polynomial are small relative to the norm of that co-
efficient, then the corresponding eigenvalue condition numbers and backward
errors are preserved approximately by the Möbius transformations induced by
well-conditioned matrices only if a penalty factor, depending on the norms of
those matrix coefficients, is moderate. It is important to note that these simple
results are no longer true if a non-homogeneous formulation of the PEP is
used.

1. INTRODUCTION

Möbius transformations are a standard tool in the theory of matrix polynomials
and in their applications. The use of Möbius transformations of matrix polyno-
mials can be traced back to at least [27, 28], where they are defined for general
rational matrices which are not necessarily polynomials. Since Möbius transfor-
mations change the eigenvalues of a matrix polynomial in a simple way and preserve
most of the properties of the polynomial [26], they have often been used to trans-
form a matrix polynomial with infinite eigenvalues into another polynomial with
only finite eigenvalues and for which a certain problem can be solved more easily.
Recent examples of this theoretical use can be found, for instance, in [13, 36].

A fundamental property of some Möbius transformations, called Cayley transforma-
tions, is to convert matrix polynomials with certain structures arising in control
applications into matrix polynomials with other structures that also arise in applica-
tions. This allows us to translate many properties from one structured class of

Received by the editor October 26, 2018, and, in revised form, April 17, 2019.
2010 Mathematics Subject Classification. Primary 65F15, 65F35, 15A18, 15A22.
Key words and phrases. Backward error, eigenvalue condition number, matrix polynomial,
Möbius transformation, polynomial eigenvalue problem.

The research of the first author was funded by the "contrato predoctoral" BES-2013-065688
of MINECO.

This work was partially supported by the Ministerio de Economía, Industria y Competitividad
(MINECO) of Spain through grants MTM2012-32542, MTM2015-65798-P, and MTM2017-90682-
REDT.

©2019 American Mathematical Society

This is a free offprint provided to the author by the publisher. Copyright restrictions may apply.