MATHEMATICS OF COMPUTATION
Volume 89, Number 322, March 2020, Pages 767–805
https://doi.org/10.1090/mcom/3472
Article electronically published on August 26, 2019

CONDITIONING AND BACKWARD ERRORS OF EIGENVALUES OF HOMOGENEOUS MATRIX POLYNOMIALS UNDER MÖBIUS TRANSFORMATIONS

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Abstract. We present the first general study on the effect of Möbius transformations on the eigenvalue condition numbers and backward errors of approximate eigenpairs of polynomial eigenvalue problems (PEPs). By using the homogeneous formulation of PEPs, we are able to obtain two clear and simple results. First, we show that if the matrix inducing the Möbius transformation is well-conditioned, then such transformation approximately preserves the eigenvalue condition numbers and backward errors when they are defined with respect to perturbations of the matrix polynomial which are small relative to the norm of the whole polynomial. However, if the perturbations in each coefficient of the matrix polynomial are small relative to the norm of that coefficient, then the corresponding eigenvalue condition numbers and backward errors are preserved approximately by the Möbius transformations induced by well-conditioned matrices only if a penalty factor, depending on the norms of those matrix coefficients, is moderate. It is important to note that these simple results are no longer true if a non-homogeneous formulation of the PEP is used.

1. Introduction

Möbius transformations are a standard tool in the theory of matrix polynomials and in their applications. The use of Möbius transformations of matrix polynomials can be traced back to at least [27, 28], where they are defined for general rational matrices which are not necessarily polynomials. Since Möbius transformations change the eigenvalues of a matrix polynomial in a simple way and preserve most of the properties of the polynomial [26], they have often been used to transform a matrix polynomial with infinite eigenvalues into another polynomial with only finite eigenvalues and for which a certain problem can be solved more easily. Recent examples of this theoretical use can be found, for instance, in [13, 36].

A fundamental property of some Möbius transformations, called Cayley transformations, is to convert matrix polynomials with certain structures arising in control applications into matrix polynomials with other structures that also arise in applications. This allows us to translate many properties from one structured class of

Received by the editor October 26, 2018, and, in revised form, April 17, 2019.
2010 *Mathematics Subject Classification.* Primary 65F15, 65F35, 15A18, 15A22.
Key words and phrases. Backward error, eigenvalue condition number, matrix polynomial, Möbius transformation, polynomial eigenvalue problem.
The research of the first author was funded by the "contrato predoctoral" BES-2013-065688 of MINECO.
This work was partially supported by the Ministerio de Economía, Industria y Competitividad (MINECO) of Spain through grants MTM2012-32542, MTM2015-65798-P, and MTM2017-90682-REDT.