UNIVERSIDAD PONTIFICIA COMILLAS DE MADRID
ESCUELA TÉCNICA SUPERIOR DE INGENIERÍA (ICAI)
(Instituto de Investigación Tecnológica)

GENERATION EXPANSION PLANNING IN ELECTRICITY MARKETS WITH BILEVEL MATHEMATICAL PROGRAMMING TECHNIQUES

Tesis para la obtención del grado de Doctor

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Madrid 2013

CONSTANCIA REGISTRAL DEL TRIBUNAL DEL ACTO DE LA DEFENSA DE TESIS DOCTORAL

TÍTULO: Generation Expansion Planning in Electricity Markets with Bilevel Mathematical Programming Techniques

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Fecha de lectura: 26 de junio de 2013

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To my family.

Acknowledgments

First of all, I want to thank my supervisors Efraim Centeno and Julián Barquín for all their advice, their continuous support and contributions throughout the entire PhD. Apart from introducing me to the amazing world of power systems and electricity markets, they are also responsible for proposing this wonderful thesis topic which, like myself, is a bit of a mixture of mathematics and engineering and could not have been more perfect. I can honestly say that I would do my PhD again tomorrow, and I have the two of them to thank for that.

I am grateful to the Electricity Policy Research Group at the University of Cambridge and the Department of Geography and Environmental Engineering at Johns Hopkins University for hosting my research visits and especially to Benjamin Hobbs and Daniel Ralph for their incredibly insightful comments, suggestions and most of all for making EPECs that much fun.

At this point I would like to express my gratitude towards the Instituto de Investigación Tecnológica for providing the economic support and infrastructure that made this thesis possible. I also want to thank all "IITianos" in general for creating such a special work environment, but in particular, Javier Reneses, Andrés Ramos and Mariano Ventosa for their helpful suggestions to guide this thesis in the right direction. I am also much obliged to Luiz Barroso from PSR, Steven Gabriel from the University of Maryland and Mort Webster from MIT for their interesting comments and ideas. I am thankful for the opportunity to collaborate with Endesa on several projects and most of all for their willingness to share their invaluable expertise. I am also indebted to my friends Christophe, Rhea, Verena and Pablo for all their advice and their kind words when in need.

Finally, my gratitude goes out to David who always encourages me to be the best I can possibly be, and to my family, Melanie and Lilli, my grandmother and my parents who have always supported my decisions and endeavors. Without them this thesis would not have been possible.

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Notation

Indices:

i, i^*	Index for generation companies (GENCOs).
$-i^*$	Index for all GENCOs except GENCO i^*.
j, j^*	Index for generation technologies.
k	Index for discretization intervals.
l	Index for load levels per year.
m	Index of intersection points.
$s, \widetilde{s}$	Index for spot market scenarios.
y	Index for years in time horizon.

Parameters:

C^x	Suitably large constant used in discretization.
C^μ	Suitably large constant used in discretization.
CP_{ijy}	Capacity payment [M€/GW].
D^0_{yl}	Demand intercept [GW] of demand curve for year y and load period l.
E_{iys}	Annual maximum hydro production [GWh].
F	Discount rate [p.u.].
G^0_y	Maximum possible capacity payment [M€/GW].
H_{iyls}	Upper bound on hydro production [GW].
I	Total number of generation companies.
IS_{iylm}	Intersection points in discretization of quadratic term [GW].

K_{ijy}	Existing generation capacity [GW].
LB_i	Active set of load blocks at capacity of firm i.
M_{iyl}	Amount used for financial hedging [GW].
$MaxIt$	Maximum number of iterations in diagonalization.
P^0_{yl}	Price intercept [€/MWh] of demand curve.
S_m	Slope of tangent lines in discretization [GW].
T_{yl}	Duration of the load period l in year y [kh].
V_i	Weights of classification function [p.u.].
W_s	Probability of scenario s [p.u.].
$X_{\text{-}i^*jys}$	Capacity investment data of competitors [GW].
α_{yl}	Demand slope [GW/(€/MWh)] of demand curve.
β_{ijy}	Annual investment cost [(M€/GW)/year].
$\hat{\beta}_{ijy}$	Total investment cost [M€/GW].
γ_y	Slope of capacity payment curve [M€/GW^2].
δ_{ij}	Production cost [€/MWh].
ε	Convergence tolerance.
$\underline{\zeta^\mu}$	Lower bound in discretization of $\zeta^\mu_{i^*ijyl}$.
η_{iyl}	Price of contract for differences for financial hedging [€/MWh].
θ_{iyl}	Conjectured-price response parameter [(€/MWh)/GW].
Φ_{iyl}	Normalized conjectured-price-response parameter [p.u.].

Variables:

b^μ_{iyls}	Binary variable needed to linearize complementarity conditions.
b^x_{kijy}	Binary variable needed to discretize x.
b^{dist}_{iylm}	Auxiliary binary variables used in discretization of quadratic term.
d_{yls}	Quantity demanded [GW].
$dist_{iylm}$	Auxiliary variables used in discretization of quadratic term [GW].
h_{iyls}	Hydro quantity [GW] produced in the market.

p_{yls} Market clearing price [€/MWh].

q_{ijyls} Quantity [GW] produced in the market.

$\bar{q}_{iyl}$ Auxiliary variables representing q^2 [GW^2].

x_{ijy} New capacity [GW] of firms i in technology j and year j.

$\tilde{x}_{ijy}$ Approximation of new capacity investments [GW].

z^x_{kijy} Variable used to discretize x.

$z^\zeta_{ki^*ijyl}$ Variable used to discretize $\zeta^\mu_{i^*ijyl}$.

$\zeta^\lambda_{i^*ijyl}$ Dual variables of complementarity constraints.

$\zeta^\mu_{i^*ijyl}$ Dual variables of complementarity constraints.

λ_{ijyls} Dual variable of upper bound on thermal production.

λ^H_{iyls} Dual variable of upper bound on hydro production.

λ^E_{iys} Dual variable of maximum annual hydro energy.

μ_{ijyls} Dual variable of lower bound on thermal production.

μ^H_{iyls} Dual variable of lower bound on hydro production.

Symbols:

Δ Step size.

$\mathcal{L}$ The Lagrangian function.

∇ Nabla symbol representing a vector differential operator.

∂ Partial derivative notation.

Acronyms:

AP Approximation.

BBOM Basic bilevel optimization model.

BBEM Basic bilevel equilibrium model.

BEM Bilevel equilibrium model.

BL Bilevel.

BMEM	Basic market equilibrium model.
BMOM	Basic market equilibrium as optimization model.
BOM	Bilevel optimization model.
BPP	Bilevel programming problem.
BSEM	Basic single-level equilibrium model.
CC	Combined cycle gas turbine.
CO	Coal technology.
CS	Consumer surplus.
EPEC	Equilibrium problem with equilibrium constraints.
FERC	Federal energy regulatory commission.
GT	Gas turbine.
ISO	Independent system operator.
KKT	Karush-Kuhn-Tucker conditions.
K-S	Kreps and Scheinkman.
MCP	Mixed complementarity problem.
ME	Market efficiency.
MILP	Mixed integer linear program.
MPEC	Mathematical program with equilibrium constraints.
NPV	Net present value.
NU	Nuclear technology.
SBOM	Stochastic bilevel optimization model.
SEOM	Single-level equilibrium as optimization model.
SL	Single level.

Chapter 1

Introduction

1.1 Motivation

Sustainable and high-quality supply of electric energy is a key ingredient of every modern-day society. All our lives critically depend in one way or another on the smooth functioning of this energy supply, be it for indispensable services such as health care, telecommunications, water supply, heating or lighting, or simply for the sake of charging our smartphones or watching the television. Most consumers are unaware of the complex set of tasks that are involved in switching on the light and, in general, do not give a lot of thought to what it would mean for their lives if one day this well-oiled machinery, the energy sector, stopped working. Sufficient and adequate generation capacity will have to be installed in order to prevent this from happening and in order to meet society's future electricity demand under emerging environmental constraints.

Within the last decade alone, the total world electricity consumption and corresponding total installed generation capacity have grown by around 50% worldwide, an enormous figure which would translate to around 500 new combined cycle plants per year. The problem of generation expansion planning is one of the most important within the electricity sector with good reason and this is why this thesis focuses on finding a better solution for this problem.

The supply of energy not only requires thorough planning in terms of the transmission network - which is usually a regulated activity - but also in terms of planning the expansion of generation capacity in order to meet future electricity demand. In regulated electricity systems this task is usually carried out by a centralized planner who minimizes total cost while meeting a future demand forecast, reliability constraints and environmental requirements identified by the government. Under this type of framework, expansion planning can be regarded as stable, relatively predictable and essentially risk-free for generation companies. However, the liberalization of the electricity sector and the introduction of electricity markets, which first emerged in the 1980s in countries like Chile, the United Kingdom and New Zealand and nowadays have spread to the majority of all developed countries, have greatly complicated the organization of the electricity sector, especially for generation companies.

Expansion decisions under a centralized framework are usually regulated activities and investment costs of generation companies are recognized. However, under a liberalized framework investment decisions are no longer regulated but to a large extent up to the responsibility of the generation companies. This has the effect that generation companies are not only exposed to uncertainties stemming from demand, hydro inflows and fuel costs - uncertainties which have already existed in regulated systems - but also to uncertainty stemming from competition in imperfect markets and from uncertain market prices. On top of that, generation expansion investments are extremely capital-intensive and have very long amortization periods, which makes them vulnerable to credit risk and long-term uncertainties such as unexpected regulatory changes. In summary, while the liberalization process might have succeeded in increasing efficiency of the sector and decreasing prices, there is no doubt that the role and tasks of generation companies have been complicated immensely.

In this context, the general goal of this thesis is to contribute to the smooth functioning of the electricity sector under a liberalized setting by developing bilevel models, which will be defined below, that assist generation companies to take generation expansion decisions in a highly competitive framework. There are several improvements of existing models that can be made and limitations that we plan to overcome with the work contained in this thesis, which has also been presented in five articles published in internationally recognized journals. There exist several methods to address the generation expansion problem in liberalized electricity markets, however, in this thesis a game-theoretic modeling approach is adopted due to the close analogy between the competitive situation of generation companies in electricity markets and the concept of "games" in game theory.

Bilevel models - which have first been used in the electricity sector to formulate electricity markets for example by Cardell et al. [23], Berry et al. [12], Weber and Overbye [114], Hobbs et al. [61] or Ramos et al. [100] just to name a few - allow us to represent a sequential decision making process as opposed to single-level models where all decisions are considered to be taken simultaneously, which can be a gross simplification of reality and distort model outcomes. As a matter of fact, one of the main objectives of this thesis is to characterize the exact difference between single-level generation expansion models, where investment and production decisions are represented to be taken at the same time, and bilevel generation expansion models, where first investments are decided and then the market equilibrium takes place. Once that this difference has been established, two different types of bilevel models for generation expansion planning will be defined: a bilevel optimization model that assists one generation company in particular and that is formulated as a Mathematical Program with Equilibrium Constraints (MPEC); and a bilevel equilibrium model where capacity decisions of all generation companies are represented and that is formulated as an Equilibrium Problem with Equilibrium Constraints (EPEC). In both models the market is represented via a conjectured-price response equilibrium formulation, which gives us the advantage to explore the impact that the strategic behavior of market agents in the spot market has on investment decisions.

The analysis of these types of models strongly supports the decisions of market agents, decisions that are being taken under uncertainty and are therefore subject to high risks. Therefore this thesis aims at developing relevant numerical bilevel models for generation expansion planning in a liberalized framework, that are sufficiently large in size to be realistic and that can be solved in a timely manner in order to carry out relevant studies and to investigate the impact of strategic behavior of the agents on the market. Solving these types of problems adequately has become an extremely important issue ever since the liberalization of the electricity sector, because generation companies have to face tough decisions every day that not only involve large amounts of money but also have a strong influence on the smooth functioning of the supply of electric energy.

Section 1.2 presents the objectives that we aim to achieve with this thesis and finally, in section 1.3, a thesis outline is presented which gives an overview of the organization of this document and establishes what individual topic will be covered in which chapter.

1.2 Thesis Objectives

In this section we give an outline of the objectives set to achieve by this doctoral thesis. We start by presenting the main objective of this thesis in section 1.2.1 and then in sections 1.2.2 and 1.2.3 more detailed information on specific objectives are stated.

1.2.1 Main Objective

The general objective of this thesis is to advance research in generation expansion planning for liberalized electricity markets by using bilevel mathematical programming techniques. In particular, we want to analyze in depth the impact of bilevel formulations on the generation planning problem and to investigate and study market behavior within proposed bilevel formulations and draw relevant conclusions. Moreover, we want to maintain an adequate balance between (I) models that are sufficiently realistic in size and design to represent a liberalized electricity market in order to carry out relevant studies, the bilevel formulation of these problems that allows game-theoretic studies on strategic behavior of market agents and (II) models that can be solved in a timely manner.

Therefore, the specific objectives of this thesis can be divided in two different topics: methodological objectives and computational objectives, which are described in detail in sections 1.2.2 and 1.2.3 respectively.

1.2.2 Methodological Objectives

The overall methodological goal is to propose and formulate bilevel models that can shed light on taking investments decisions influenced by the strategic behavior in liberalized electricity markets and the specific goals are stated below:

1. Answer the question of how the results of a bilevel generation expansion model differ compared to a more simplified model. And in particular, we analyze the differences of a single-level (or open loop) investment problem, inspired by the problem presented in work done by Ventosa et al. [113], where capacity investment and production decisions are taken simultaneously and a bilevel (closed loop) model proposed in this thesis where investment and production decisions are taken sequentially.

2. Propose and formulate bilevel generation expansion models that extend existing bilevel approaches in the literature. Design models that take the point of view of one generation company in particular and assist this company to take its capacity decisions (MPECs), but also models with a more general point of view that take into account investment decisions of all generation companies competing in the market (EPECs).

   (a) We intend to extend the work of, for example, Garcia-Bertrand et al. [54], Kazempour and Conejo [69] or Murphy and Smeers [84], which assume either perfect competition or Cournot competition in the spot market - to capture intermediate strategic behavior using a conjectured-price response market representation. This allows us to model a range of oligopolistic market behavior and to observe its impact on investment decisions.

   (b) The proposed models yield an investment schedule over the entire time horizon, as opposed to a static investment decision for a future target year, as done by static approaches like the one in Kazempour et al. [70].

(c) In order to take investment decisions many existing bilevel approaches resort to cost minimization like the work of Baringo and Conejo [3], which is not sufficiently representative of what happens in liberalized electricity markets where companies rather maximize profits as opposed to minimize costs. The models proposed in this thesis, however, assume that each generation company maximizes its profits (rather than minimizing costs).

3. Extend and improve the proposed models by introducing some aspects that make them more realistic: capacity mechanisms; incorporate hydro power; model financial hedging through contracts for differences; discretize investment decisions; introduce a methodology to handle different types of uncertainty, i.e., demand uncertainty, fuel price uncertainty or uncertainty regarding competitors' investments and their strategic market behavior. Illustrate and validate the newly proposed models and methodologies in case studies.

1.2.3 Computational Objectives

The computational objectives contain everything that is related to the process of being able to solve realistic bilevel generation expansion models satisfactorily. Bilevel problems however have the tendency to be very hard to solve, particular cases have even be proven to be NP-hard as shown by Jeroslow [67], especially when they increase in size. We therefore state the following specific computational objectives of this thesis below:

1. Propose a single-level generation expansion equilibrium model, inspired by the work of Ventosa et al. [113], which captures intermediate strategic behavior and formulate it as an equivalent convex quadratic optimization problem, which can be solved efficiently.

2. Since the complexity of the EPEC complicates commonly known solution processes, propose a methodology of a single-level approximation scheme for bilevel generation expansion equilibria which reduces the computational time and al-

lows us to solve bilevel equilibrium models reasonably well under certain circumstances.

3. Explore and compare different formulations and corresponding solution methods for the arising bilevel problems. In particular, for bilevel optimization problems formulated as MPECs explore nonlinear programming methods, mixed integer programming methods where the MPECs are converted to mixed integer programs (MIPs), and decomposition techniques. For bilevel equilibrium problems formulated as EPECs explore iterative methods like diagonalization which in turn resort to nonlinear programming methods, mixed complementarity problem (MCP) approaches, mixed integer programming and finally, approximation schemes.

1.3 Organization of the Document and Thesis Outline

In this thesis we develop and analyze mathematical bilevel models for generation expansion planning in liberalized electricity markets.

In particular, in chapter 2 we first provide a literature review on the subject which puts our work in context to the existing state of the art and then we discuss the model hypotheses and other basic concepts which are useful to better understand the rest of the thesis. These introductory sections are followed by the formulation of a basic version of existing single-level models and the newly developed bilevel generation expansion MPEC and EPEC models in order to motivate the question whether the additional modeling effort with respect to the corresponding single-level models actually pays off.

In chapter 3 we provide an answer to this question by carrying out a theoretical analysis of single-level and bilevel generation expansion equilibrium models. The obtained results, which have been proven in a theorem, show that in general the bilevel model is more realistic than the single-level model because it more adequately represents investment behavior of generation companies in liberalized electricity markets. However, we also demonstrate that under certain circumstances, single-level and bilevel results can coincide and we characterize when this happens.

Since the theoretical framework of chapter 3 underlines that bilevel models are more realistic for generation expansion planning, we want to derive large-scale versions of such bilevel models. The first step towards formulating a large-scale bilevel equilibrium model, is a bilevel generation expansion optimization model, i.e., an MPEC, which represents the investment decision of one generation company and which is discussed in detail in chapter 4 of this thesis. This model is particularly useful from the point of view of a generation company because it allows to decide and assess investment decisions under an uncertain and highly competitive framework. Moreover, the MPEC model can be extremely useful for solving EPEC models.

In chapter 5 we arrive at the bilevel generation expansion equilibrium model, formulated as an EPEC, which consists of the MPEC models that have been introduced in the previous chapter. Since these EPEC models are very hard to solve, we also propose an approximation scheme which allows us to arrive at a good solution two orders of magnitude faster than with standard EPEC methods. Chapter 5 concludes the methodological contributions of this thesis.

Chapter 6 summarizes the numerical techniques that have been applied to solve the MPEC and EPEC models that have arisen throughout this thesis, and chapter 7 contains some additional numerical examples of interest including real-size case studies. Finally, chapter 8 presents the conclusions, the thesis contributions and future research.

Chapter 2

Generation Expansion Planning Models in Liberalized Electricity Markets

This chapter first gives an overview of existing techniques to analyze capacity expansion in electricity markets, describes the state of the art of this topic and provides a detailed literature review in section 2.1. Then in section 2.2 all model hypotheses, that are used later in order to define the models proposed in this thesis, and simplifications as well as the necessary basic concepts are presented. Section 2.3 contains one of the main pillars of our generation expansion models - the conjectured-price market equilibrium - and shows how to extend this approach to a single-level generation expansion model. Finally, section 2.4 introduces two basic versions of bilevel models, which are an original contribution of this thesis, and raises the question if and how bilevel models differ from single-level models, a question which is answered in chapter 3. Moreover, possible model extensions are briefly discussed. These possible extensions will be addressed in detail in the following chapters.

2.1 Literature Review and State of the Art

This section is dedicated to a literature review and a revision of the state of the art of the use of bilevel programming for analyzing the generation expansion problem in liberalized electricity markets. First, in section 2.1.1, a brief general overview of different methods that can be used for generation expansion planning is provided, and then game theory is emphasized as the approach chosen in this thesis. Subsequently, game-theoretic approaches are divided into single-level and bilevel approaches, the latter being the main focus of this thesis. Finally, section 2.1.2 is dedicated to pointing out the most relevant existing mathematical programming techniques that are used to solve bilevel problems in this thesis.

2.1.1 Generation Expansion Planning Techniques

The liberalization of the electricity sector which began in the 1980s in pioneering countries like Chile, Great Britain and Norway was a re-structuring process of the entire electricity industry leading away from centralized, vertically integrated monopolies and going towards the introduction of competition and electricity markets. One of the main reasons for this deregulation, which led to the privatization and vertical de-integration of the industry, was that the introduction of competition would lead to an increase of efficiency in the sector, thereby ultimately leading to lower electricity prices. Nowadays, liberalized electricity markets are in place worldwide ranging from Latin and Central America, the United States and Canada to Europe and Australia. In Spain, the liberalization of the electricity sector took place in 1998.

Due to the liberalization of electricity markets the task of taking generation capacity investment decisions has become an even more complex problem than it already had been under a centralized framework. In regulated electricity systems, generation expansion planning is usually carried out by a centralized planner who in general minimizes total system costs subject to environmental constraints and reliability constraints, some of which can be found in Billinton and Allan [14]. For this reason generation expansion decisions are generally easier to predict in centralized systems and therefore relatively stable. Under such a framework, generation companies are guaranteed a certain return on their investments and therefore hardly face any risk at all when taking investment decisions. The main techniques to model these kinds of capacity investment decisions under a regulated framework are multi-criteria decision methods, as proposed by Merrill and Schweppe [82] and optimization methods, which often boil down to a cost minimization problem.

Liberalized systems pose a more complex task to generation companies, as they make generation expansion decisions at their own risk, in a highly uncertain framework which is no longer driven by mere cost minimization. Generation companies are now exposed to a higher level of risk, having to deal with the strategic behavior of competitors in imperfect markets and coping with the uncertainty due to regulatory decisions, fuel prices, demand and hydro inflows among others.

For generation expansion planning in liberalized markets, let us classify the different techniques according to Sánchez [108] where available generation expansion techniques are separated into methods that place emphasis on uncertainty treatment and methods that focus on the analysis of markets and the behavior of its competitors. Let us now point out some examples of these techniques applied to the energy sector. When the focus lies on treating uncertainty, then typically the following methods are employed: scenario analysis such as the work by Ghanadan and Koomey [55] where the UK and Californian system are studied; decision theory as applied by Mosquera et al. [83] analyzing medium-term risks faced by electrical generation companies in competitive environments; risk management as for example applied by Cabero et al. [20] in the context of the electricity sector; and real options theory, as applied to generation expansion planning by Botterud [15] or to investment in distributed generation by Siddiqui and Marnay in [109].

However, these techniques usually disregard the effects of strategic behavior of market agents. Representing these effects often requires the use of methods such as: system dynamics as applied to the electric power industry by Ford [48] and to generation expansion by Kadoya et al. [68] and Sánchez [108]; agent-based simulation applied to investment in electricity markets by Costa and Oliveira [29]; and game theory, which is the technique used in this thesis and which will therefore be analyzed in greater detail as follows.

Game Theory in Generation Expansion Planning

Game theory is particularly useful in the energy sector because it represents a natural way to model various agents - in our case generation companies - who are competing in the market where the decision of each company has an influence on the outcome of the other agents. This interactive and competitive behavior of generation companies can be defined as a game where each player has its set of strategies and where the optimal decision of each player depends on how the rest of the players behave. The strong analogy between the concepts of games in game theory and the actual situation of generation companies in electricity markets is the reason why in this thesis we adopt a game-theoretic approach to generation expansion planning. This methodology allows us to analyze electricity markets in depth and in particular, it allows us to focus on the impact that the strategic behavior in the spot market has on investment decisions. It is pointed out here that in liberalized electricity markets there are many situations that could be modeled as a game, such as the market equilibrium problem, or providing optimal bids for the market, or the generation capacity expansion problem, which is the main focus of this thesis. The generation expansion problem can be interpreted as a game among generation companies where each firm maximizes its total profits, deciding capacities and subsequently productions.

Given the above definition of games in liberalized electricity markets, the concept of a Nash equilibrium [86] can be introduced as the point where no market agent can improve its profits by unilaterally changing its strategy, thereby moving away from this equilibrium point. There exist different types of equilibrium formulations, the most important ones being: the Bertrand equilibrium [13], where the decision variables are prices; the Cournot equilibrium [31], where the decision variables are quantities; and equilibrium formulations with conjectural variations, which under certain circumstances includes the above and moreover, allows to represent intermediate situations of competitive market behavior or even collusion. The conjectural variations approach is the one employed in the models of this thesis and is therefore discussed in more detail in section 2.2.2.

Let us now revisit another type of equilibrium, i.e., the Stackelberg equilibrium [110], which similar to Cournot also is an equilibrium in quantities; however, in the Stackelberg equilibrium two different types of players are considered: the leader and the followers. The leader firm is the first to set quantity levels, and then the followers decide quantities depending on the leader's previously made decision. In comparison to the Cournot equilibrium concept, mentioned in the previous paragraph where all players take decisions simultaneously, the Stackelberg game represents a sequential decision making process, where a leader moves first and then the others follow. It is commonly known that Cournot and Stackelberg equilibria yield different quantity solutions and hence lead to different profits. This is due to the fact that the Stackelberg leader can change the Cournot outcomes in its favor because of the privilege of moving before everybody else does. It is therefore stressed here that simultaneous decision making, as in the Cournot game, and sequential decision making, as in the Stackelberg game, are not only conceptually different but can lead to different outcomes.

With this in mind, we now consider the generation expansion planning problem. Generation expansion planning has an innate sequential structure because first capacity decisions are taken (power plants have to be built) and then, production decisions can be taken in the market. If we draw a parallel to the Stackelberg game, then capacity decisions would be considered the "leader" since they move first, and production decisions in the market would be the "follower" decisions since they depend on what has previously been decided in the investment stage. From this analogy it becomes apparent that there are two separate levels in the generation expansion planning problem: the investment level and the production level. Therefore, all approaches that model this type of two-level structure of the generation expansion problem are referred to as bilevel approaches (also known as closed-loop or two-stage), which will be the focus of this thesis. On the other hand, approaches where capacity and production decisions are considered to be taken simultaneously will be referred to as single-level (also known as open-loop or one-stage) approaches. In the remainder of the literature review single-level and bilevel modeling approaches to generation expansion planning are going to be distinguished distinctly.

Strictly speaking both capacity and production decisions are taken in an alternating manner repeatedly for all the years of the time horizon. However, it is a common simplification, at least when modeling electricity systems, to consider the capacity investments of the entire time horizon first, and then all the production decisions of the entire time horizon. This simplification allows to keep the arising models tractable and moreover, it yields a useful and reasonable representation of how electricity markets work.

Single-Level Generation Expansion Planning Models

Within the game-theoretic framework, one approach for the capacity expansion problem is to extend medium-term models to longer terms, by considering that investment and production decisions are taken at the same time. The medium term corresponds to a time horizon ranging from one month to a couple of years and is usually dedicated to tactical decisions such as maintenance management, hydro reservoir management or more market-based analyses. Medium-term market models, as for example the one presented by Barquín et al. [5], can be extended to the longer term by introducing investment decisions as additional variables. Such an approach has been adopted in the Cournot-based equilibrium model by Ventosa et al. [113]. Let us now mention some of these modeling approaches.

This corresponds to the perfectly-competitive case and the so called open-loop equilibrium conditions presented in work of Murphy and Smeers [84], where the authors theoretically explore different generation capacity expansion models. Since the focus of this work lies on the theoretical analysis of the models, the actual problem size is kept small, i.e., the time horizon is one year, two generators and investments in one technology are considered. Murphy and Smeers prove that under the assumptions made in their paper, the open-loop Cournot equilibrium always exists and is unique. They also compare their open-loop Cournot model to a closed-loop (two-stage) Cournot model, which will be discussed in more detail in the bilevel model section of this literature review.

Another example of a single-level generation expansion model is the Cournot-based model of Ventosa et al. [113]. This equilibrium model is more realistic than the previous one since it accommodates a time horizon of various years, subperiods within this year, e.g., months, and load periods within the subperiods. Moreover, hydro-power constraints and pumped-hydro storage among other realistic details are considered. The model is formulated as a Mixed Complementarity Problem (MCP) and solved using the standard solver PATH [38]. The model analyzed by Centeno et al. [25] extends the market model of Reneses, Centeno and Barquín [101] and improved by the same authors [5], to simultaneously consider continuous capacity investments. The continuous capacity investments are then discretized for each year in an algorithm using a static expansion for a single year in a future time horizon.

However, the single-level approach has its drawbacks, as it may overly simplify the dynamic nature of the problem, as expansion and operation decisions are taken simultaneously. In this case, the problem requires taking into account the fact that expansion and operation decisions are taken sequentially instead of simultaneously. Even when considering a perfectly competitive spot market, the fact that agents decide their investments before the spot market takes place, creates the opportunity for the players to benefit from spot market reactions to investment decisions.

Bilevel Generation Expansion Planning Models

In this section we focus on bilevel game-theoretic approaches to generation expansion planning in liberalized electricity markets. In general, the term “bilevel” before generation expansion planning, refers to the sequentiality of decision making - first capacity decisions and second production decisions - and more concretely speaking, a mathematical programming problem is classified as a bilevel programming problem when one of the constraints of an optimization problem is also an optimization problem. Among existing bilevel approaches we furthermore distinguish between: optimization-based bilevel approaches, which correspond to problems that are closely related to bilevel programs, i.e., Mathematical Programs with Equilibrium Constraints (MPECs); and equilibrium bilevel approaches, which correspond to models formulated as Equilibrium Problems with Equilibrium Constraints (EPECs) [111]. Both MPEC and EPEC approaches have been applied in various fields like engineering, economics and finance. An MPEC is an optimization problem in which the essential constraints are defined by a parametric variational inequality, see Facchinei and Pang [43], or a complementarity system as mentioned by Cottle et al. [30], which typically model a certain equilibrium phenomenon. In the electricity sector, MPECs have first been used to formulate electricity markets for example by Cardell et al. [23], Berry et al. [12], Weber and Overbye [114], Hobbs et al. [61] or Ramos et al. [100]. An EPEC is an equilibrium problem where several market agents simultaneously face an MPEC. These problems (MPECs and EPECs) are more complicated than single-level problems, but they fulfill the purpose of translating the sequentiality of decision making into model formulations thereby differentiating capacity and production decisions. Considering expansion and operation decisions separately leads to bilevel modeling.

Let us first present the optimization-based bilevel approaches in the capacity framework, which focus on the investment decisions of one market agent (usually being a GENCO) in particular and are usually modeled as an MPEC. In the capacity framework there exist leader-follower games like the Stackelberg game as the MPEC given by Ventosa et al. [113], where a leader firm takes capacity decisions in the first stage and then in the second stage the market is cleared a la Cournot also taking into account hydro constraints and pumped-hydro storage. This work focuses on comparing numerical results of this Stackelberg model and a single-level Cournot model. Other contributions take a more centralized approach to expansion planning like the transmission and wind power investment MPEC of Baringo and Conejo [3] where investments are decided in the upper level by minimizing total costs, subject to a lower level that represents the market clearing under different load and wind power conditions. The subsequent work of the authors [4] corresponds to the Benders decomposition approach of the previous model. All of the following examples are optimizing the capacity expansion decisions of only one GENCO. Garcia-Bertrand et al. [54] present a linear bilevel model that determines the optimal investment decisions of one generation company. They consider uncertainty in the demand and in the capacity decisions of the competition; however, the market is considered to be perfectly competitive. In [107], Sakellaris applies a two-stage model in which firms choose their capacities under demand uncertainty prior to competing in prices and presents regulatory conclusions. In [70] Kazempour et al. present a stochastic static bilevel generation capacity expansion model, i.e., an MPEC, where investment decisions and strategic production actions are taken in the upper level for a single target year in the future, while the lower level represents the market clearing. In [69], Kazempour and Conejo apply Benders decomposition to this stochastic MPEC. The extension of this model to incorporate futures markets is presented in the work of Kazempour et al. [71].

Let us now discuss existing equilibrium bilevel approaches in the literature. An instance of this modeling approach is the closed-loop Cournot game described in the work of Murphy and Smeers [84]. As previously mentioned, in their paper Murphy and Smeers theoretically analyze generation expansion models with a time horizon of one year and Cournot behavior in the market and establish existence and uniqueness results. In the case of the closed-loop Cournot model, the existence and uniqueness of this kind of equilibrium are not guaranteed in most situations (see Ralph and Smeers [99] for a discussion on the topic). Another example of bilevel equilibria in generation expansion is the work of Kazempour, Conejo and Ruiz [72, 73]. The authors revisit their previously mentioned MPEC model for each generation firm thereby obtaining an EPEC which is then linearized and solved as a Mixed Integer Linear Problem (MILP). Similar to the corresponding MPEC model, the EPEC model also computes a static expansion for one future year. In the PhD thesis of Özdemir [92] two bilevel equilibrium models for the generation expansion problem are presented: one with perfectly competitive market behavior and another one assuming Cournot behavior. In her thesis Özdemir shows that it is possible under certain circumstances to recast the bilevel perfectly competitive equilibrium problem as a convex but nonlinear optimization problem. However, in general convexity properties are lost when market power is introduced. Moreover existence and uniqueness results are discussed and proven.

Thesis Scope

In general, in this thesis we would like to address some of the shortcomings of existing approaches in the literature. First of all, even though existing open-loop approaches such as the work of Centeno et al., Murphy and Smeers or Ventosa et al. [25, 84, 113] are adequate and useful to approximate the generation capacity problem, they do not model the significant temporal separation between when capacity decisions are taken and when energy is produced with that capacity. We overcome this problem by proposing a bilevel model. Furthermore, existing bilevel approaches in the literature assume either perfectly competitive, for example in work of Garcia-Bertrand et al. and Özdemir [54, 92], or Cournot behavior in the spot market, see Haurie et al., Murphy and Smeers or Ventosa et al. [59, 84, 113]. We want to extend these approaches to also capture intermediate oligopolistic behavior in order to explore how capacity decisions would change if competitive behavior in the spot market changed. The models presented in this thesis represent the market via a conjectured-price response formulation, thereby allowing us to model a range of oligopolistic market behavior. Finally, the models proposed in this thesis yield an investment schedule over the entire time horizon, as opposed to a static investment decision for a future target year, as done in the work of Kazempour et al. [69, 70].

In particular, the following optimization-based bilevel models have been developed: in Centeno, Wogrin et al. [26], which corresponds to a contribution of this thesis, we have presented a deterministic MPEC, which decided capacity investments in the upper level maximizing profits of one generation company, while the lower level represented a conjectured-price response market equilibrium. The capacity decisions of the competing market agents were assumed fixed. The extension of this work to include stochasticity in terms of competitors' investments and strategic spot market behavior is presented in Wogrin et al. [116].

As a part of this thesis the following bilevel equilibrium models have been developed: in Wogrin et al. [119], which also forms part of the contributions of this thesis, the authors present and theoretically analyze a closed-loop single year capacity expansion model with a conjectured-price response market equilibrium allowing for strategic behavior to range between perfect competition and Cournot competition. One result of this comparison is a Kreps-Scheinkman-like [74] result, which established that the closed-loop (bilevel) model always yields Cournot capacities. Some examples of more realistically-sized bilevel generation expansion equilibrium models are given by the work of Wogrin et al. [115], which presents a novel way to model the generation capacity expansion problem in a liberalized framework via a multi-year bilevel equilibrium model. In the upper level the competing generation companies maximize their individual profits, while the lower level represents the market using a conjectured-price response approach, which allows us to vary the strategic spot market behavior, to see how much the reigning competitive behavior impacts investment decisions. This EPEC model represents part of the contributions of this thesis.

The purpose of Figure 2-1 is to establish how the work of this thesis compares to other existing approaches in the literature in terms of four criteria that have been established as the main focus of this thesis. The criteria that were considered as most important in this thesis are: the market representation which we classify in perfect competition or Bertrand behavior, Cournot behavior or conjectural variations; the size of the model which can either be small as used for theoretical purposes or realistically-sized; the type of investment decision where we distinguish between linear objective functions (such as cost minimization for example) and more complex nonlinear objective functions (as obtained under profit maximization for example) and within both cases we furthermore classify whether the approach is a static or a dynamic one; and finally, the number of investing agents considered in the model where we distinguish between models that decide investments of only one agent and models that have multiple investing agents.

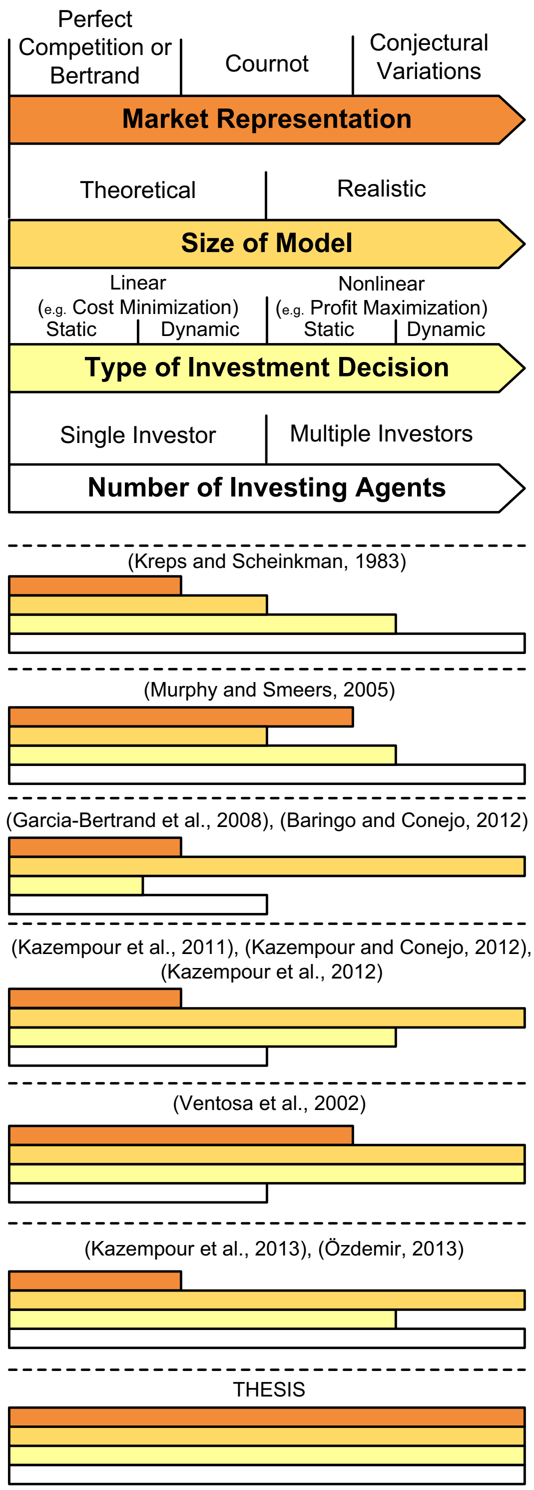

Figure 2-1: Comparison of the level of detail of different bilevel generation expansion models in the literature with respect to the criteria emphasized in this thesis.

2.1.2 Bilevel Programming Techniques

In this subsection of the literature review we mention some of the most relevant techniques for solving bilevel problems and their most relevant applications to electricity markets. In particular, in the remainder of this section, we distinguish between methods for MPECs and methods for EPECs. Note that this section contains a list of methods for MPECs and EPECs most relevant to this thesis and not a complete list of all existing numerical methods.

Bilevel Programming Problems (BPPs) were introduced in the operations research literature in the early 1970s by Bracken and McGill in a series of papers [16, 17, 18]. From a general point of view, a BPP can be classified as a mathematical optimization problem which is constrained by another optimization problem. A BPP is a hierarchical programming problem where the constraints of a problem (the upper level problem) are in part defined by another optimization problem (the lower level problem). A Mathematical Program with Equilibrium Constraints (MPEC) [80] is an optimization problem in which the essential constraints are defined by a parametric variational inequality or a complementarity system, which typically model a certain equilibrium phenomenon. Under certain circumstances MPECs are equivalent to bilevel problems. Finally, an Equilibrium Problem with Equilibrium Constraints (EPEC) [111] is a problem of finding an equilibrium point that solves several MPECs simultaneously.

MPECs have non-convex constraints and in some cases even nonlinear objective functions and may therefore have multiple equilibria. For this reason in some cases EPECs might not even have pure strategy equilibria as mentioned by Berry et al. or Hu and Ralph [12, 63]. Under some circumstances bilevel problems have been proven to be NP-hard as shown by Ben-Ayed and Blair [10]. For a discussion on MPEC resolution algorithms the reader is referred to Luo et al. [80].

In the electricity sector, MPECs, bilevel problems, and EPECs were first used to represent short-run bidding and production games among power producers with existing capacity, e.g.,[12, 23, 61, 114, 120]. EPECs belong to a recently developed class of mathematical programs that often arise in engineering and economics applications and can be used to model electricity markets as shown by Ralph and Smeers [99].

Let us now revisit some of the existing MPEC approaches in literature. The first, most straight-forward approach, as adopted by Centeno, Wogrin et al. [26] is to convert the MPEC into a nonlinear program and to apply standard nonlinear solvers to the problem. Some solvers such as NLPEC [46] take advantage of the complementarity problem formulation within the constraints of an MPEC to address it more efficiently. Since the EPEC is an equilibrium problem, under certain circumstances, it can be modeled as a complentarity problem and solved as such using standard solver as for example PATH [38]. For a detailed discussion on complementarity problems and their application in energy markets the reader is referred to the book of Gabriel et al. [51]. Once the MPEC has been formulated as a nonlinear program it is also a possibility to apply heuristic methods, as done by Fampa et al. [45] for the optimal bidding problem.

Another approach to solve an MPEC is to linearize the nonlinear parts (such as the complementarity conditions for example), thereby transforming the MPEC into a MILP. Such an approach is adopted in work of Kazempour et al. or Wogrin et al. [70, 71, 116] for the generation expansion problem, in work of Baringo and Conejo [3] for the transmission expansion problem or in work of Fampa et al. [45, 104] for the optimal bidding problem. The advantage of MIP approaches with respect to the MPEC is that the obtained solution is globally valid as opposed to local solutions which are yielded by nonlinear solvers. These MIP approaches to MPECs can be extended to EPECs. In particular, Ruiz et al. [105] suggest to linearize all nonlinearities of the EPEC for the optimal bidding problem and then they propose a methodology which allows them to choose among the different equilibrium solutions. The same methodology has also been applied to the generation expansion planning problem by Wogrin et al. [115] (presented in chapter 4 of this thesis) and by Kazempour et al. [72, 73]. There also exist problems with three levels that can be transformed into a MILP, such as the static generation and transmission planning model by Pozo et al. [97]. In [96], Pozo and Contreras not only take a MILP approach to modeling a stochastic EPEC, but they also propose a procedure to find all Nash equilibria to their problem by creating "holes" in the feasibility region.

Decomposition methods have also been successfully applied to MPECs, as for example Kazempour and Conejo [69] for the generation expansion MPEC or Baringo and Conejo [4] for transmission expansion.

Another method to solve EPECs is via an iterative procedure called diagonalization, which solves the EPEC by iteratively solving MPECs. For more details on methods to solve EPECs, i.e., diagonalization, the reader is referred to Hu, Hu and Ralph or Leyffer and Munson [62, 63, 77]. The advantage of the diagonalization method is that it only requires solving a less complicated model, i.e., an MPEC. However, the disadvantages of diagonalization, as stated by Su [111], are that even if an equilibrium exists, diagonalization might not find it and moreover, no convergence results exist for this method. In the PhD thesis of Su [111], the author also develops another method to solve EPECs, i.e., a sequential nonlinear complementarity method, however, this type of method has not been applied in the scope of this thesis.

Finally, in order to solve a generation expansion EPEC, Wogrin et al. [117] (presented in chapter 5 of this thesis) have developed a single-level approximation scheme, which approximates bilevel equilibria well if strategic spot market behavior is closer to Cournot than to perfect competition. This methodology forms part of the contributions of this thesis.

2.2 Modeling Hypotheses and Basic Concepts

In this section we first - in section 2.2.1 - discuss the hypotheses regarding the demand representation that have been made in our models. Then, in section 2.2.2 we introduce the concept of conjectural variations, which is indispensable for our representation of the market equilibrium, followed by a brief discussion about the electricity network in section 2.2.3. Since this thesis focuses on bilevel generation expansion models, in section 2.2.4 provides a formal definition of bilevel programming and section 2.2.5 introduces the concept of complementarity problems, which also appear frequently in the model formulations of this thesis.

Some aspects of our models have been simplified in order to achieve the maximum clarity in the formulation of the models. Hence, in this section we point out all the hypotheses taken in the development of our generation expansion models and analyze the corresponding limitations of the models. Moreover, we describe some basic concepts and prerequisites that are necessary in order to formulate and solve the proposed models.

In general, the models that are developed in this thesis do not incorporate a direct representation of reliability measurements, nor do they directly represent non-supplied energy. Instead we consider an indirect treatment of reliability measures, see Billinton and Allan [14], by assuming that measures such as the loss of load probability or the equivalent expected forced outage rate have already been incorporated into all system variables. For example, in the following the variable q which we will refer to as (net) production decisions, really corresponds to the probabilistic term representing average production minus average expected forced outage rates.

All the models presented in this thesis consider a load duration curve that is approximated by a step function. Demand is represented by an affine inverted price curve, which means that we consider an affine relation between market price and demand. This is discussed in more detail in section 2.2.1. When formulating the spot market we do not consider each generator's individual stepwise supply offer curves, instead we approximate these curves by introducing a conjectured-price response which represents the strategic behavior of each generator. The definition and interpretation of this conjectured-price response parameter are presented in section 2.2.2. The network has not been incorporated into the models developed in this thesis. This fact will be discussed in more detail in section 2.2.3 and future research will be motivated. Finally, in section 2.2.4 and section 2.2.5 we define the concept of a bilevel programming problem - since these types of models are the core contribution of this thesis - and introduce the concept of complementarity problems, which will serve as an alternative way to formulate bilevel problems.

2.2.1 Demand Representation

In most electricity markets each day can be divided into several periods per day. The number of these daily periods depends on the design of each individual electricity market. In the Spanish system there is a market clearing for each hour of the day, which yields 24 pairs of market price and corresponding demand. In England even half-hourly market prices are considered.

When trying to adequately represent what is going on in the electricity market, the most realistic approach would be (for the example of Spain) an hourly representation of price and demand. As a matter of fact, in models whose time horizon is short term, for example the unit commitment problem, demand and prices are usually represented on an hourly basis (or even less). Otherwise, operational details such as ramping constraints, start-up or shut-down decisions, could not be adequately modeled. However, when considering the generation capacity expansion problem, the time horizon that is explored can be up to several decades and an hourly representation of multiple decades is simply not tractable computationally and might not even be crucial taking into account that many operational details of the market, as for example start-up and shut-down decisions, that might be crucial in a unit commitment framework, become less important and more negligible in a long-term framework. It is therefore desirable to represent demand in a more efficient way. A common approach is to approximate demand by a step function by introducing what we will refer to as load periods, load blocks or load levels.

In order to define these load periods, we start off with the annual hourly load curve of the system that we are trying to model. In Figure 2-2 we present such an annual load curve for the Spanish system. Then, we derive the monotonic annual load curve, given in Figure 2-3, by ordering the previous curve in a descending manner. In this process we lose information of sequentiality of individual hours. The goal is to approximate this curve by introducing L load blocks, where each load block is defined by a demand level and a duration. In order to achieve this goal we solve an optimization problem that partitions the monotonic load curve into L clusters.

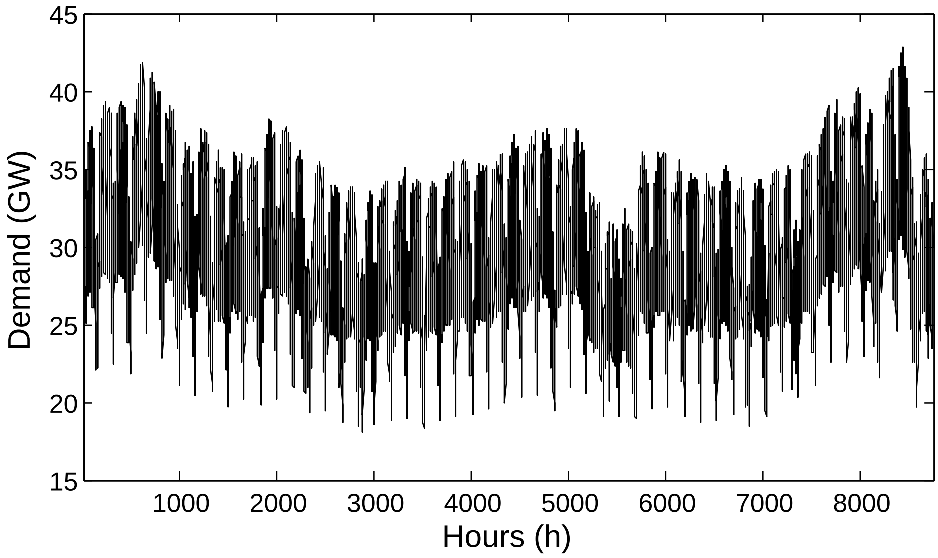

Figure 2-2: Annual load curve of the Spanish system in 2007.

This partition minimizes the sum, over all clusters, of the within-cluster sums of point-to-cluster-centroid distances. The result of this process yields a representation of demand that looks like a step function, as given in Figure 2-4 where d represents demand. Note that this approach is very common in the electricity sector and has been adopted frequently to represent the demand curve, e.g., Barquín et al. or Murphy and Smeers [5, 84]

The advantage of this type of representation is that instead of 8760 individual one-hour blocks per year, demand can be represented by only L blocks, where $L \ll 8760$. This reduction in model size allows us to expand the time horizon to multiple years (decades) while still representing demand in an adequate fashion. The disadvantage is that we have lost some detail as for example sequentiality of individual hours, however, this particular detail might be of less importance in the long run than in the short run.

Finally, demand in each load level l is defined by an affine demand function which links demand d_l with price p_l of the same load period. This demand function is given by $d_l = D_l^0 - \alpha_l p_l$, and is defined by the data $D_l^0 > 0$ representing the demand intercept and $\alpha_l > 0$ representing the demand slope. An example of such a demand function of load period l is presented in Figure 2-5.

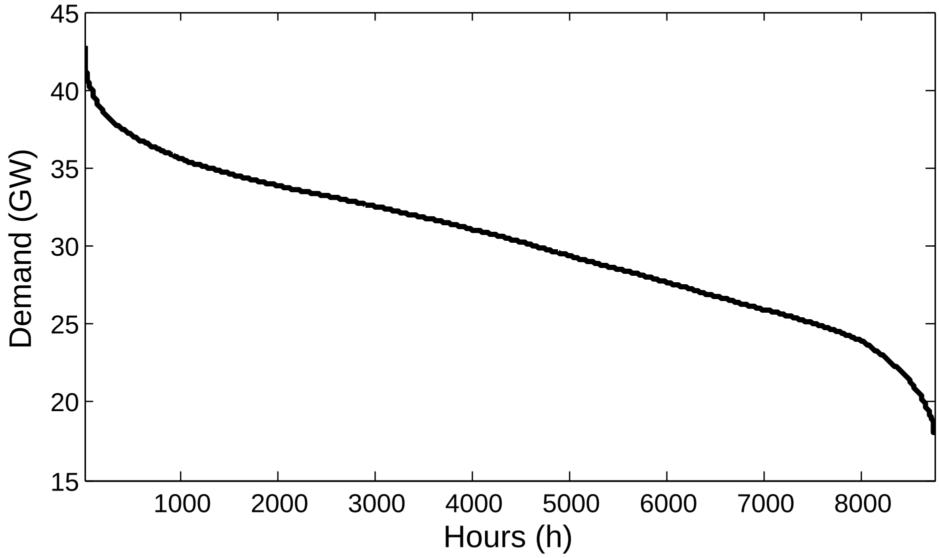

Figure 2-3: Monotonic annual load curve of the Spanish system in 2007.

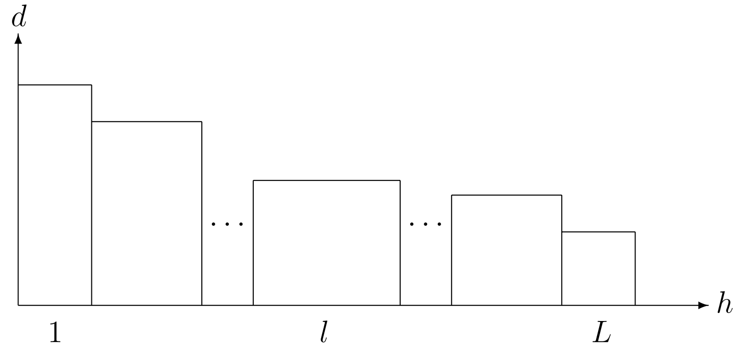

Figure 2-4: Annual demand decomposition into L load periods.

2.2.2 Conjectured-Price Response in Expansion Models

One of our thesis objectives (in particular methodological objective 2a) is to extend existing generation expansion models which assume either perfect competition or Cournot competition in the spot market to capture various degrees of strategic behavior in the spot market because this allows us to observe the impact of market behavior on investment decisions.

We achieve this goal by introducing conjectural variations into the short-run energy market formulation. The conjectural variations development can be related to standard industrial organization theory, see Fudenberg and Tirole [50]. In particular, we introduce a conjectured-price response parameter that can easily be translated

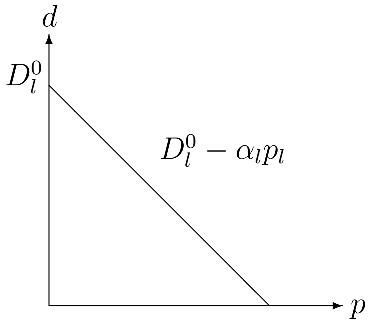

Figure 2-5: Affine demand function (inverted price function).

into conjectural variations with respect to quantities, and vice versa if we consider demand to be linear.

Definition of Conjectured-Price Response

Let us now introduce conjectural variations as will be used in this thesis. Therefore, let us consider several firms ($i = 1, \ldots, I$) with perfectly substitutable products, for which we furthermore assume an affine relation between demand and price, i.e., $d = D^0 - \alpha p$, where d is the quantity demanded, α is the demand slope and $D^0 > 0$ the demand intercept as previously mentioned in section 2.2.1. Demand d and quantities produced q_i, with i being the index for the market agents, are linked by the market clearing condition $\sum_i q_i = d$. Price can be written as a function of demand, i.e., $p(d) = (D^0 - d)/\alpha$, which can furthermore be transformed into $p(d) = (D^0 - \sum_i q_i)/\alpha$ using the market clearing condition. Hence, we will refer to price as a function of quantities, i.e., $p(q_1, \ldots, q_i, \ldots, q_I)$. Furthermore, let i^* be an alias index of i.

Then we define the conjectural variation parameters as $\Phi_{i^*,i}$. These represent agent i's belief about how agent i^* changes its production in response to a change in i's production. Therefore:

$$\Phi_{i^*,i} = \frac{dq_{i^*}}{dq_i}, \quad i \neq i^*, \tag{2.1}$$

$$\Phi_{i,i} = 1. \tag{2.2}$$

Note that the expression in (2.1) does not represent the actual derivative but agent i's belief of what this derivative yields. And hence using (2.1)-(2.2) and our assumed $p(q_1, \ldots, q_i, \ldots, q_I)$, we obtain:

$$\frac{dp(q_1, \ldots, q_i, \ldots, q_I)}{dq_i} = -\frac{1}{\alpha} \sum_{i^*} \Phi_{i^*,i} = -\frac{1}{\alpha}(1 + \sum_{i^* \neq i} \Phi_{i^*,i}) \tag{2.3}$$

It may not always be practical to consider the conjectural variation $\Phi_{i^*,i}$ for each individual competitor. Instead, a global conjectural variation Φ can be considered which represents the reaction of all competitors combined. Therefore relation (2.3) simplifies to:

$$\frac{dp(q_1, \ldots, q_i, \ldots, q_I)}{dq_i} = -\frac{1}{\alpha}(1 + \Phi) \tag{2.4}$$

Now let us define the conjectured-price response parameter θ_i as company i's belief concerning its influence on price p as a result of a change in its output q_i:

$$\theta_i = -\frac{dp(q_1, \ldots, q_i, \ldots, q_I)}{dq_i} = \frac{1}{\alpha}(1 + \Phi) \geq 0, \tag{2.5}$$

which immediately shows how to translate a conjectural variations parameter into the conjectured-price response and vice versa. Throughout the thesis we will formulate all models using the conjectured-price response parameter as an alternative to the conjectural variations parameter, because its depiction of the firms' influence on price is more convenient for the derivations, as opposed to a firm's influence on production by competitors. For a review and analysis of the main formulations of conjectural variations equilibria applied to electricity markets, the reader is referred to Diaz et al. [37].

Special Cases of Conjectured-Price Response

As has been proven by Daxhelet [33], the conjectural variation (conjectured-price response) representation allows us to express special cases of oligopolistic behavior such as perfect competition, the Cournot oligopoly, or collusion. A general formulation of each firm's profit objective would state that $p = p(q_1, \ldots, q_i, \ldots, q_I)$, with the firm anticipating that price will respond to the firm's output decision. For the remainder of the thesis we term this a conjectured-price response model. Let us now derive how to express perfect competition, the Cournot oligopoly and collusion using the conjectured-price response framework.

- If the firm takes p as exogenous (although it is endogenous to the market), the result is the price-taking or perfect competition, similar to the Bertrand conjecture [74] under certain circumstances. Then the conjectured-price response parameter θ_i equals 0, which means that none of the competing firms believes it can influence price.

- If instead $p = p(q_1, \ldots, q_i, \ldots, q_I)$ is the inverse demand curve which is given by $(D^0 - q_i - \sum_{i^* \neq i} q_{i^*})/\alpha$, with q_{i^*} being rival firms' outputs which are taken as exogenous by firm i, then the model is a Nash-Cournot [31] oligopoly[1]. In the Cournot case, θ_i equals $1/\alpha$, which would translate to $\Phi = 0$ in the conjectural variations framework.

- We can also express collusion[2] (or quantity matching) as $\theta_i = I/\alpha$, which translates to $\Phi = I - 1$ because it is considered that $\Phi_{i^*,i} = 1$. Note that I is the number of firms in the market.

- Apart from these special cases we also express values between the extremes of perfect competition and the Cournot oligopoly.

Representation of Dynamic Games via Single-Level Conjectured-Price Response Games

Some more complex dynamic games can be reduced to a one-stage game with intermediate values for Φ (or θ_i respectively). For example, the well-known leader-follower Stackelberg game. Let us now show in Lemma 2.1 how this two-stage game can be

[1]The Cournot-Nash model is the simplest oligopoly model which assumes that there are two equally positioned firms that compete on the basis of quantity (as opposed to price as in the Bertrand model). Each firm makes an output decision assuming that the other firm's behavior is fixed.

[2]In economics, collusion can be defined as the cooperation of rival companies for their mutual benefit, which most often takes place in an oligopolistic framework.

reduced to a one-stage conjectural variations based game, as shown in Centeno et al. [26].

Lemma 2.1. ***(Stackelberg as Single-Level Conjectural Variations Game)*** *The bilevel leader-follower Stackelberg game can be reduced to a one-stage conjectural variations game where the leader firm has conjecture* $\theta = 1/(\alpha(1{+}I))$ *and the followers have conjectures* $\theta = 1/\alpha$*, where* I *is the number of followers and* α *is the demand slope of the affine demand curve.*

Proof. Let us define q as the production decision of a leader firm and let q_i be the production decisions of a set of I followers. First, the leader chooses its production, and then, followers compete for the rest of production "a la Cournot". In order to solve this model, we start off by defining the followers' profits π_i in (2.6) as market revenues minus total production cost, where δ_i represents company i's variable production cost. Again, demand is given by $d = D^0 - \alpha p$ and the market clearing condition reads $d = q + \sum_i q_i$. Assuming that there is Cournot competition between the followers, individual profit maximization leads to (2.7)-(2.9).

$$\begin{aligned}
\pi_i &= (p - \delta_i)q_i && (2.6)\\
\frac{d\pi_i}{dq_i} &= p - \delta_i + \frac{dp}{dq_i}q_i && (2.7)\\
&= p - \delta_i - \theta_i q_i = 0 && (2.8)\\
&= p - \delta_i - q_i/\alpha = 0 && (2.9)
\end{aligned}$$

The system of equations consisting of (2.9), the market clearing condition and the inverse demand function, yields an expression of price that - apart from data - only depends on the leader's output q and is given in (2.10).

$$p = \frac{D^0 - q + \alpha \sum_i \delta_i}{\alpha(1 + I)} \quad (2.10)$$

Similar to the followers' profit, we define the leader's profit in (2.11). If we take the derivative of (2.11) with respect to q, this yields (2.12)-(2.14).

$$\begin{aligned} \pi &= (p-\delta)q && (2.11) \\ \frac{d\pi}{dq} &= p-\delta+\frac{dp}{dq}q && (2.12) \\ &= p-\delta-\theta q = 0 && (2.13) \\ &= p-\delta-q/(\alpha(1+I)) = 0 && (2.14) \end{aligned}$$

The system of equations (2.9) and (2.14) together with the inverted demand function and the market clearing conditions, corresponds to a conjectured-price response market with the leader's conjecture being $\theta = 1/(\alpha(1+I))$ and the followers' conjectures being $\theta = 1/\alpha$. □

In the case of electricity markets, production decisions undertaken by power producers result from a complex dynamic game within multi-settlement markets. Typically, bids in the form of supply functions are submitted in two or more successive markets at different times prior to operation, where the second and successive markets account for the commitments made in previous markets. Conjectural variations models can represent a reduced form of a dynamic game as pointed out in Figurières [47]. This kind of reinterpretation has been proposed by several authors: in the context of the private provision of a public good as done by Itaya [65, 66], where steady state conjectures in a dynamic game are interpreted as conjectural variations in the corresponding static game; in the context of the oligopoly, conjectural variations have been presented as the reduced form of a quantity-setting repeated game as done by Cabral [21], or for example as the reduced form of a differential games model with adjustment costs as presented by Dockner and Driskill [39, 40].

The two-stage forward contracting/spot market Allaz-Vila [2] game can also be reduced to a one-stage conjectural variations model, as shown by Murphy [85], assuming $\Phi = -1/2$ (or $\theta = 1/(2\alpha)$). In the remainder of this thesis, when we refer to "Allaz-Vila" market behavior, we suppose spot market behavior of $\theta = 1/(2\alpha)$. Therefore, conjectural variations can be used to capture very complex games in a computationally tractable way. This is a major reason why many econometric industrial organization studies estimate oligopolistic interactions using model specifications based on the assumption of constant conjectural variations, see Perloff et al. [94]. Our discussion of these references is only to state that in general conjectural variations can represent more complicated games and that bilevel games, as the Stackelberg game, can be equivalent to a single-level game in conjectural variations as has been shown by Lemma 2.1. In this thesis we do not consider the problem of estimating or calculating conjectural variations, which can be a very complicated process and would depend on the nature of the particular game that is reduced. Reasonable values for these parameters, representing different degrees of oligopoly, will be considered as known. The estimation of the conjectured-price response is often based on historical data and there exist implicit methods as presented by López de Haro [79] (adjusting past market prices) and explicit methods as presented by Bunn [19]. For a summary of conjectural variations estimation methods the reader is referred to Diaz et al. [37].

2.2.3 The Electricity Network

In general, the electricity network is an important factor to consider when designing a generation capacity expansion plan. That being said, it should be pointed out that in the models developed in this thesis, the electricity network has not been taken into account. The reason for this is the following. When considering generation expansion planning on a European scale, then including network constraints becomes very important since they may influence generation expansion significantly. On the other hand, when analyzing systems whose networks are meshed enough, the effect of the network (and its arising constraints) is not as relevant. In such systems network constraints are not of much physical relevance and are therefore not considered as that critical in terms of investment behavior.

Moreover, considering a detailed network formulation in our models might make the arising bilevel problems very complicated and even intractable. Since the main purpose of this thesis was to understand the impact of a conjectured-price response market equilibrium formulation on generation expansion decisions in a bilevel setting, which even without the network can still be regarded as a very challenging topic per se, introducing the network into our models will be part of future research.

There exist some market equilibrium frameworks with conjectural variations that also consider a formulation of the electricity network, as for example the work of Barquín et al. [6, 7] and Delgadillo et al. [35]. These articles could be considered as a starting point for future research to formulate the lower level (market equilibrium) with network constraints.

2.2.4 Bilevel Programming Problem

Bilevel Programming Problems (BPPs) were introduced in the operations research literature in the early 1970s by Bracken and McGill in a series of papers [16, 17, 18]. From a general point of view, a BPP can be classified as a mathematical optimization problem which is constrained by another optimization problem. A BPP is a hierarchical programming problem where the constraints of a problem (the upper level problem) are in part defined by another optimization problem (the lower level problem). A BPP can be formulated mathematically as follows:

$$\min_{x,y} \; F(x,y) \tag{2.15}$$

$$\text{s.t. } G(x,y) \leq 0, \;\; H(x,y) = 0 \tag{2.16}$$

$$\min_{y} \; f(x,y) \tag{2.17}$$

$$\text{s.t. } g(x,y) \leq 0, \;\; h(x,y) = 0 \tag{2.18}$$

Now let us give a simple example of a bilevel problem - *a toll-setting problem* - which was taken from Colson et al. [28]. Note that this is not an example from the electricity sector. Let us assume we have a traffic transportation network, where there is a toll set on some of the network links (for instance certain parts of the freeway) which has to be paid by each user of this link. The network manager has the objective to maximize the revenues raised from the tolls (upper level problem), the network users however have the objective of minimizing their traveling costs (lower level problem).

In this framework we can easily observe the hierarchical relationship between the two decision makers, i.e., the network manager and the network users, who in this case have conflicting interests, i.e., maximizing revenues and minimizing costs. Once the network manager has set the values of the tolls, the users react to these tolls and choose their traveling route such that their traveling costs, which may depend on time or distance as well, are minimized. This example problem can easily be related to the Stackelberg leader-follower problem, where in this case the leader would be the network manager and the followers would be the network users. For further details on BPPs the reader is referred to Candler and Norton or Colson et al. [22, 28].

In section 2.4 we introduce how the generation capacity expansion problem, that will be studied in this thesis, can be formulated as a BPP.

2.2.5 Complementarity Problem

Informally, a complementarity problem is a problem that includes complementarity conditions, which require that the product of two or more non-negative quantities should be zero. Now let us define a generalized complementarity problem mathematically. Given a mapping $F(y) : \mathbb{R}^n \to \mathbb{R}^n$, find a $y \in \mathbb{R}^n$ satisfying:

$$y \geq 0, \quad F(y) \geq 0, \quad y^T F(y) = 0. \tag{2.19}$$

Let us now present how the Karush-Kuhn-Tucker (KKT) conditions of an optimization problem can be written as a complementarity problem. Therefore, we consider the following optimization problem:

$$\begin{aligned} & \max_x f(x) && && (2.20)\\ \text{s.t. } & F(x) && \geq 0 \quad && : \lambda \quad (2.21)\\ & x && \geq 0 \quad && : \mu, \quad (2.22) \end{aligned}$$

where λ and μ represent the dual variables of the constraints in (2.21) and (2.22) respectively. The KKT conditions of the optimization problem (2.20)-(2.22) are given by:

$$\nabla_x f(x) + \lambda^T \nabla_x F(x) + \mu = 0 \tag{2.23}$$

$$F(x) \geq 0, \quad x \geq 0 \tag{2.24}$$

$$\lambda^T F(x) = 0, \quad \mu^T x = 0 \tag{2.25}$$

$$\lambda \geq 0, \quad \mu \geq 0 \tag{2.26}$$

Using the positivity of μ given in (2.26) and the derivative of the Lagrangian, given in (2.23), we obtain:

$$\nabla_x f(x) + \lambda^T \nabla_x F(x) \leq 0 \tag{2.27}$$

$$F(x) \geq 0, \quad x \geq 0, \quad \lambda \geq 0 \tag{2.28}$$

$$\lambda^T F(x) = 0, \quad (\nabla_x f(x) + \lambda^T \nabla_x F(x))^T x = 0 \tag{2.29}$$

Finally, we transform (2.27)-(2.29) into (2.30)-(2.31), which yields a complementarity problem.

$$x \geq 0 \qquad \perp \qquad \nabla_x f(x) + \lambda^T \nabla_x F(x) \leq 0 \tag{2.30}$$

$$\lambda \geq 0 \qquad \perp \qquad F(x) \geq 0 \tag{2.31}$$

As demonstrated, complementarity conditions arise for instance in the KKT conditions but they also appear in the study of equilibrium problems and are therefore related to Mathematical Problems with Equilibrium Constraints and Equilibrium Problems with Equilibrium Constraints, which are the models that have been developed in this thesis.

2.3 Market Equilibrium and Single-Level Generation Expansion Models

In this section we introduce the basic version of the conjectured-price response spot market equilibrium in section 2.3.1, which will serve as the basis of our generation expansion models. Power market oligopolies, like the one presented here, have been proposed before based on conjectural variations by Centeno et al. [24] and conjectured-price responses by Day et al. [34], but only for short-term markets where capacity is fixed.

Previously, in section 2.2.2, we have mentioned that in electricity markets, production decisions undertaken by power producers result from a complex dynamic game within multi-settlement markets where usually, bids in the form of supply functions are submitted in two or more successive markets at different times prior to operation. The second and successive markets account for the commitments made in previous markets. Representing this complex dynamic process mathematically is very difficult. From section 2.2.2 we also recall that sometimes it is possible to reduce a complex dynamic game to a single-level game by introducing conjectural variations, as shown in Lemma 2.1 for the Stackelberg game. Therefore in the proposed market models, the introduced conjectured-price response parameter can be interpreted not only as the strategic spot market behavior reigning in the market, but also as a parameter representing the reduced version of the more complex multi-settlement market game.

As will be presented in section 2.3.2, the conjectured-price response market equilibrium problem can also be formulated as an equivalent quadratic optimization problem, which has been proven by Barquín et al. [5]. Finally, we show a straight-forward approach to the generation expansion problem by extending the medium-term market equilibrium model to longer terms, by considering that investment and production decisions are taken at the same time. This approach is referred to as "single-level" (also as open loop, one-shot or one-stage) investment equilibrium model because investment and production decisions are carried out simultaneously, and is presented in section 2.3.3.

This model is closely related to the open-loop equilibrium conditions presented by Murphy and Smeers [84], the Cournot-based model presented by Ventosa et al. [113], which is solved using a Mixed Complementarity Problem (MCP) scheme, and the model analyzed in Centeno et al. [25], which is solved using an equivalent optimization problem. Note that the model that is presented here considers an arbitrary conjectured-price response, which allows us to change the strategic behavior in the spot market instead of considering just one fixed value. This is an advantage since it makes the model more flexible and allows us to try out different values of strategic behavior to explore how this behavior affects market outcomes. However, this approach also has its drawbacks, as it may overly simplify the dynamic nature of the problem, as expansion and operation decisions are taken simultaneously. Differentiating expansion and operation decisions leads to the more complex bilevel modeling, which we will introduce in section 2.4.

Definition of Indices

Before we formulate the models, let us first define all the indices that will be used throughout this section. The index y corresponds to the set of years of the time horizon that is considered; l corresponds to the load level with duration T_{yl} of each year in the time scope; i is the index of all generation companies and j corresponds to the different technologies of generation capacity.

2.3.1 Conjectured-Price Response Market Equilibrium

In this section, we formulate the basic version of the market equilibrium model (BMEM), which consists of several simultaneously considered optimization problems, which represent each generation firm's individual market profit maximization problem. This system of optimization problems, together with a market clearing condition and an affine function that links price and demand, form the market equilibrium problem as graphically depicted in Figure 2-6, where q_i are the variables that represent the production decisions, x_i represents the newly-installed capacity and K_i is data representing already existing capacity.

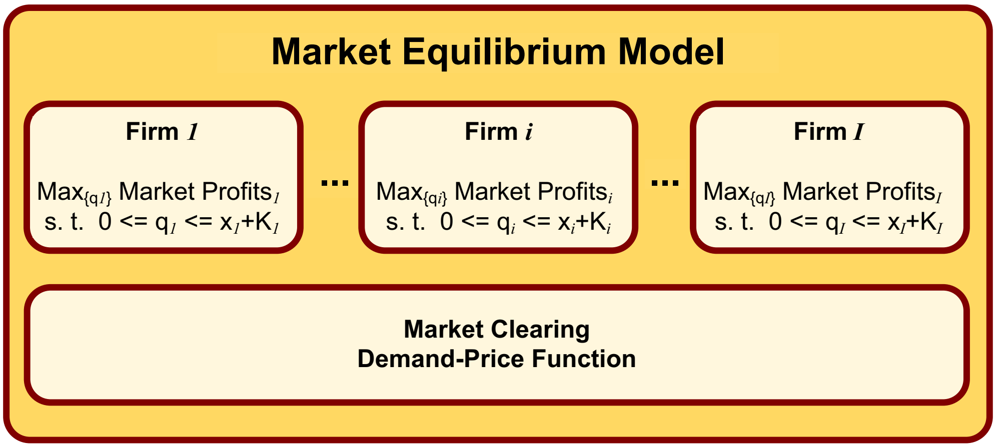

Figure 2-6: Graphic representation of the market equilibrium problem.

Formally, the market equilibrium is formed by equations (2.32), (2.33) and (2.34). In (2.32) all GENCOs individually maximize the net present value of their total market profits as the difference between their market revenues minus their production costs, deciding their production subject to the constraint that production will not exceed capacity. These maximization problems are linked by the market clearing condition (2.33) and the affine relation between price and demand (2.34) and together they form the market equilibrium. Thus, the market equilibrium problem can be written as:

Basic Market Equilibrium Model (BMEM):

$$\forall i \left\{ \begin{array}{lll} \max_q \sum_y \frac{1}{(1+F)^y} \left\{ \sum_{jl} T_{yl}(p_{yl} - \delta_{ij}) q_{ijyl} \right\} & & \\ \text{s.t.} \quad q_{ijyl} \leq x_{ijy} + K_{ijy} & \forall jyl & : \lambda_{ijyl} \\ \qquad\qquad 0 \leq q_{ijyl} & \forall jyl & : \mu_{ijyl} \end{array} \right. \tag{2.32}$$

$$d_{yl} - \sum_{ij} q_{ijyl} = 0 \quad \forall yl \tag{2.33}$$

$$d_{yl} - D^0_{yl} + \alpha_{yl} p_{yl} = 0 \quad \forall yl \tag{2.34}$$

The parameters of the equilibrium problem above as well as possible parameter units are defined as follows: F [p.u.] corresponds to the discount rate; T_{yl} [kh] corresponds to the duration of load period l in year y; δ_{ij} [€/MWh] corresponds to the production cost of technology j and generation company i; K_{ijy} [GW] is the already existing capacity of generation company i in technology j and year y; as before, D^0_{yl} and α_{yl} are the demand intercept and the slope of the demand curve. The new capacity investments of GENCO i in technology j and year y are denoted by x_{ijy} [GW]. In the market equilibrium these capacity investments are considered to be parameters, however, they are decision variables in generation expansion models. The variables of the market equilibrium problem are: the production decisions q_{ijyl} [GW] of firm i in technology j, year y and load period l; the demand d_{yl} [GW] and the resulting price p_{yl} [€/MWh] in each year y and load period l; μ_{ijyl} and λ_{ijyl} represent the dual variables of the lower and upper bounds on production. Note that both demand and price are not decision variables of each individual generation company's profit maximization problem given by (2.32), but are variables that are defined by the joint production of all generation companies. It can be easily verified that the objective function is measured in M€.

In the optimization problem given above in (2.32) for each i, the conjectured-price-response parameter θ_{iyl} [(€/MWh)/GW], that we have defined in section 2.2.2, is not explicitly visible, and hence we re-write the market equilibrium, and substitute the optimization problem in (2.32) by its KKT conditions for all companies. In the resulting equilibrium problem, which is presented below, the conjectured-price response is explicit and can be found in equation (2.35). Considering the convexity and continuity of all cost functions, the market equilibrium can be written as an equivalent convex optimization problem, which was proven by Barquín et al. [5]. We present this equivalent optimization problem in 2.3.2. Hence the equations (2.35)-(2.43) are also the optimality conditions.

$$\frac{\delta_{ij}T_{yl}}{(1+F)^y} + \frac{\sum_j \theta_{iyl}q_{ijyl}T_{yl}}{(1+F)^y} - \frac{T_{yl}p_{yl}}{(1+F)^y} + \lambda_{ijyl} - \mu_{ijyl} = 0 \quad \forall ijyl \tag{2.35}$$

$$\mu_{ijyl}q_{ijyl} = 0 \quad \forall ijyl \tag{2.36}$$

$$\lambda_{ijyl}(K_{ijy} + x_{ijy} - q_{ijyl}) = 0 \quad \forall ijyl \tag{2.37}$$

$$q_{ijyl} \leq x_{ijy} + K_{ijy} \quad \forall ijyl \tag{2.38}$$

$$0 \leq q_{ijyl} \quad \forall ijyl \tag{2.39}$$

$$0 \leq \mu_{ijyl} \quad \forall ijyl \tag{2.40}$$

$$0 \leq \lambda_{ijyl} \quad \forall ijyl \tag{2.41}$$

$$d_{yl} - \sum_{ij} q_{ijyl} = 0 \quad \forall yl \tag{2.42}$$

$$d_{yl} - D^0_{yl} + \alpha_{yl} p_{yl} = 0 \quad \forall yl \tag{2.43}$$

Above we present the first order conditions of the nonlinear program given by (2.32). In general, the first order conditions or a nonlinear program characterize local maxima, local minima or saddle points. In order to guarantee that the obtained solution actually is a local maximum - since (2.32) represents a profit maximization problem - the second derivatives of the objective function with respect to the decision variables need to be non-negative. The second order conditions of each GENCO's individual market profit maximization given in (2.32) are as follows:

$$\frac{\partial^2 \mathcal{L}}{\partial q^2_{ijyl}} = \frac{\theta_{iyl} T_{yl}}{(1+F)^y} \geq 0 \quad \forall ijyl, \tag{2.44}$$

where $\mathcal{L}$ is the Lagrangian of problem (2.32). This condition is satisfied because θ_{iyl} is defined as a non-negative parameter.

The market equilibrium problem, as given by (2.35)-(2.43), is a nonlinear problem due to complementarity conditions (2.36) and (2.37). It could therefore be solved as a nonlinear problem using a nonlinear solver. However, this might not be the most efficient way to tackle this problem. As has been mentioned in section 2.2.5, the system of KKT conditions can be rewritten as a complementarity system and hence, the market equilibrium problem can also be formulated as an MCP and solved as such using a standard solver, e.g., PATH [38].

2.3.2 Market Equilibrium as Optimization Problem

In this section we present an optimization problem, given in (2.45)-(2.48), which first has been introduced in work of Barquín et al. [5] and which represents a basic version of the market equilibrium as an optimization model (BMOM). In this work the term "extended cost" is introduced. The extended costs refer to the first two sums in (2.45) and contain the total discounted production costs and the quadratic term including θ. The demand utility is defined as the negative term in (2.45). The objective function given in (2.45) resembles the expression of social welfare and in particular, if market behavior were considered perfectly competitive, i.e., $\theta = 0$, then the arising objective function would be the same as a maximization of social welfare. However, if market behavior is not exactly perfect competition, then the total costs reflected in the objective function correspond to the sum of marginal costs plus a certain mark-up which is defined by the term involving θ. Therefore, the objective function could be interpreted as a kind of social welfare maximization but with generation companies' extended costs, as opposed to their marginal costs. This optimization problem is equivalent to the market equilibrium problem. Note that market price p_{yl} is the dual variable of the balance equation (2.48).

It is easy to verify that the KKT conditions of the optimization problem below and the market equilibrium conditions given by (2.35)-(2.43) coincide. In equation (2.48) the multiplication by parameters $T_{yl}/(1+F)^y$ is only done in order for the KKT conditions to coincide exactly with the previously presented conditions. Furthermore, this optimization problem is a convex problem, which has been proven and thoroughly discussed by Barquín et al. [5]. This implies that a local solution of this optimization problem is also a global one and moreover, considering convexity of the cost functions, it can be said that this solution will be unique. In [5] it has also been shown that the solution of this optimization problem actually is a solution of the market equilibrium. It is easy to verify that the second order conditions of the optimization problem below coincide with (2.44) and $T_{yl}/(\alpha_y l(1+F)^y)$.

Basic Market Equilibrium as Optimization Model (BMOM):

$$\begin{aligned}\min_{q,d} \quad & \sum_{ijyl} \frac{\delta_{ij} q_{ijyl} T_{yl}}{(1+F)^y} + \frac{1}{2} \sum_{iyl} \frac{\theta_{iyl} T_{yl} (\sum_j q_{ijyl})^2}{(1+F)^y} \\ & - \sum_{yl} \frac{T_{yl}}{\alpha_{yl}(1+F)^y} (D^0_{yl} d_{yl} - \frac{d^2_{yl}}{2})\end{aligned} \tag{2.45}$$

$$\text{s.t.} \quad q_{ijyl} \leq x_{ijy} + K_{ijy} \quad \forall ijyl \quad : \lambda_{ijyl} \tag{2.46}$$

$$0 \leq q_{ijyl} \quad \forall ijyl \quad : \mu_{ijyl} \tag{2.47}$$

$$\frac{T_{yl}}{(1+F)^y}(d_{yl} - \sum_{ij} q_{ijyl}) = 0 \quad \forall yl \quad : p_{yl} \tag{2.48}$$

2.3.3 Single-Level Investment Equilibrium

The single-level (or open loop) capacity expansion equilibrium model is depicted in Figure 2-7 and formed by equations (2.49), (2.50) and (2.51). Throughout this thesis we refer to single-level results as (SL). Its formulation is similar to the previously presented market equilibrium problem (2.32)-(2.34), with the only difference that now all GENCOs i are maximizing total profits as the difference between their market revenues minus their production costs minus their investment costs, simultaneously deciding capacity investments x_{ijy} as well as production decisions q_{ijyl}.

Basic Single-Level Equilibrium Model (BSEM):

$$\forall i \begin{cases} \max_{x,q} \sum_y \frac{1}{(1+F)^y} \{ \sum_{jl} T_{yl}(p_{yl} - \delta_{ij}) q_{ijyl} - \sum_j \beta_{ijy} x_{ijy} \} \\ \text{s.t.} \quad q_{ijyl} \leq x_{ijy} + K_{ijy} \quad \forall jyl \quad : \lambda_{ijyl} \\ \quad 0 \leq q_{ijyl} \quad \forall jyl \quad : \mu_{ijyl} \end{cases} \tag{2.49}$$

$$d_{yl} - \sum_{ij} q_{ijyl} = 0 \quad \forall yl \tag{2.50}$$

$$d_{yl} - D^0_{yl} + \alpha_{yl} p_{yl} = 0 \quad \forall yl \tag{2.51}$$

The parameters of the single-level capacity equilibrium problem above are the same as defined for the market equilibrium problem in section 2.3.1 and the annual investment costs β_{ijy} which could be measured in [(M€/GW)/year]. The variables of the open-loop capacity equilibrium problem and possible units are: the capacity investments x_{ijy} [GW], the production decisions q_{ijyl} [GW], the demand d_{yl} [GW] and the resulting price p_{yl} [€/MWh]; again, μ_{ijyl} and λ_{ijyl} are the dual variables of the lower and upper bounds on production. The objective function is measured in [M€].

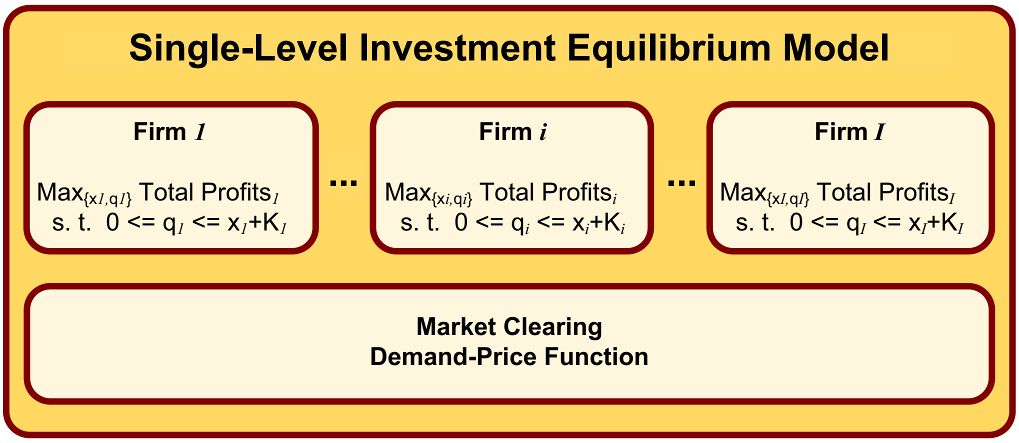

Figure 2-7: Graphic representation of the single-level investment equilibrium problem.

In the formulation above, the conjectured-price-response parameter θ_{iyl} given in (€/MWh)/GW is not explicitly visible. Hence, we re-write the open-loop capacity equilibrium conditions, and substitute the optimization problem in (2.49) by its KKT conditions for all companies, which are given in (2.52)-(2.53).

$$\frac{\beta_{ijy}}{(1+F)^y} - \sum_l \lambda_{ijyl} = 0 \quad \forall ijy \tag{2.52}$$

$$(2.35) - (2.43) \tag{2.53}$$

Note that the majority of the constraints coincide with the KKT conditions representing the market equilibrium. As (2.49) is linear in x, there are no second order conditions for x, and second order conditions for q coincide with (2.44).

Since, regarding its formulation, the single-level investment equilibrium model is very similar to the market equilibrium model, it is not surprising that it can also be formulated as an MCP and solved as such. Another way to solve this problem, which is an original contribution of this thesis and a simple extension of the work of Barquín et al. [5] and Ventosa et al. [113], is to formulate an equivalent quadratic optimization problem, which has been proposed for a "Cournot" market by Centeno et al. [25]. This alternative will be presented and discussed in detail in chapter 5 of this thesis.

2.4 Bilevel Generation Expansion Models

This section represents an original contribution of this thesis, where we introduce the basic versions of two types of newly developed generation capacity expansion models: one that can be formulated as a Mathematical Program with Equilibrium Constraints (MPEC), which is presented in section 2.4.1, and another one that is an Equilibrium Problem with Equilibrium Constraints (EPEC), which is introduced in section 2.4.2. Both of these types of models are bilevel problems where the upper level (or first stage) corresponds to the investment stage where capacity expansion decisions are taken, and the lower level (or second stage) represents the spot market level in which productions and prices are decided. Finally, in section 2.4.3 we raise some aspects that would need to be included in the models in order to make them more realistic.

Both types of models, MPECs and EPECs, are analyzed and discussed in great detail in separate chapters of this thesis, i.e., in chapter 4 and 5, however, a basic version of both models is introduced at this point to motivate the main research questions of this thesis. First of all, when comparing model formulations, it becomes apparent that the complexity of bilevel models exceeds the complexity level of single-level models by far. One cannot help but wonder whether all this additional modeling effort - let alone the additional effort when it comes to solving bilevel models - actually pays off. The arising research question is whether single-level and bilevel models yield different solutions, and under what circumstances. Finding the answer to these questions is one of the thesis objectives (in particular methodological objective 1) and has been achieved by chapter 3.

Second, since MPEC and EPEC models are conceptually different, it remains to characterize where these differences lie, what impact they have on the results and finally to give a recommendation as to how and where to employ these models.

Therefore, in chapter 4 and 5, we give a detailed mathematical formulation and analysis of both of the newly developed MPEC and EPEC generation expansion models. In these chapters we also discuss how to realize possible model extensions, which have been pointed out in 2.4.3.

As in the previous section, y corresponds to the set of years of the time horizon that is considered, l corresponds to the load periods, i and i^* are alias indices of all generation companies and j corresponds to the different technologies of generation capacity.

2.4.1 Mathematical Program with Equilibrium Constraints

In this section we define the term Mathematical Program with Equilibrium Constraints (MPEC) and propose a new generation expansion model, which is formulated as an MPEC, to assist a generation company in making its long-term generation capacity investment decisions. The purpose of introducing this newly developed bilevel model in this section is merely as a contrast to the already existing single-level models. For the detailed mathematical formulation as well as the theoretical and numerical analysis of this model and the conducted case studies, the reader is referred to chapter 4. Let us state here that the bilevel formulation allows for the uncoupling of investment and generation decisions, as investment decisions of the single investing generation company are taken in the upper level with the objective to maximize profits, and in the lower level generation decisions by all companies are considered. The lower level represents the oligopolistic market equilibrium via a conjectured-price response formulation, which can capture various degrees of strategic market behavior like perfect competition, the Cournot oligopoly and intermediate cases.

In order to derive such a model, we first need to define the concept of a Mathematical Program with Equilibrium Constraints (MPEC). An MPEC is an optimization problem in which the essential constraints are defined by a parametric variational inequality [1] as in Facchinei and Pang [43, 44] or a complementarity system as in

[1] A variational inequality is an inequality involving a functional, which has to be solved for all the values of a given variable, usually belonging to a convex set.

Cottle et al. [30], which typically model a certain equilibrium phenomenon. Under certain circumstances MPECs are equivalent to bilevel problems, which have been introduced in section 2.2.4, providing an alternative way to formulate problems. In the electricity sector, MPECs have first been used to formulate electricity markets by Hobbs et al. [61] and Ramos et al. [100]. The general formulation of an MPEC taken from Pieper [95], where f, g and F are twice continuously differentiable functions, is presented below. Note how the constraints (2.56)-(2.58) correspond to the definition of a complementarity problem in (2.19).

$$\min_{x,y} \quad f(x,y) \tag{2.54}$$

$$\text{s.t.} \quad g(x,y) \geq 0 \tag{2.55}$$

$$y \geq 0 \tag{2.56}$$

$$F(x,y) \geq 0 \tag{2.57}$$

$$y^T F(x,y) = 0 \tag{2.58}$$

Let us now introduce an MPEC in the generation expansion framework. Let i^* represent a specific generation firm that we would like to assist to take their capacity decisions. For this purpose, we consider two different decision stages in our problem setting: the investment stage or upper level, where firm i^* takes capacity decisions x_{i^*} by maximizing its total profits, consisting of the gross margin from the lower level (revenues minus variable production costs) minus total investment costs (unitary annual investment cost β_{ijy} [(M€/GW)/year] times capacity investments); and the production stage or lower level represented by the conjectured-price response market equilibrium, which yields production decisions of all firms competing in the market and market price. Graphically, this problem is represented in Figure 2-8.

Let us now concretize the basic formulation of this problem, which we will refer to as basic bilevel optimization model (BBOM). In the upper level or investment stage, the generation company i^* maximizes its total profits and chooses its capacities subject to the lower level equilibrium response. We furthermore impose the constraint that capacity can only increase in time. The fact that capacity can only increase

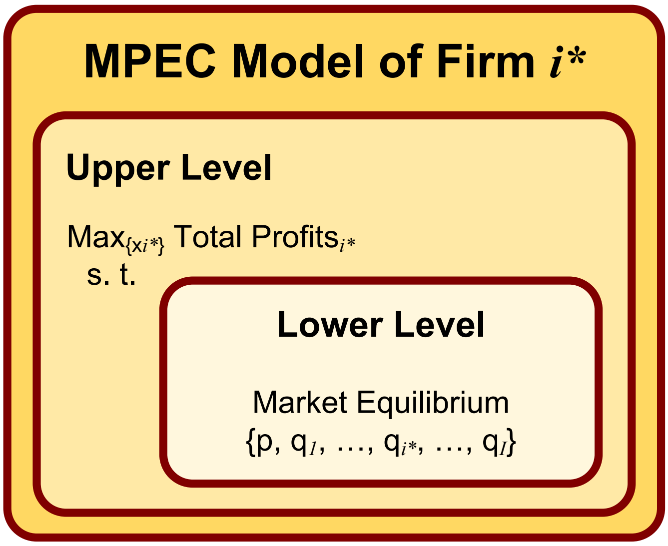

Figure 2-8: Graphic representation of the MPEC generation expansion problem faced by firm i^*.

over time is only a weak hypothesis if a sustained demand increase is reflected in the demand data. Increasing demand over time is a realistic assumption in light of historic observations. This constraint has been introduced in order to facilitate the numerical solution process, however, if this constraint were omitted, most likely the optimal solution would not change. This (BBOM) model of the generation expansion problem can be written as follows.

Basic Bilevel Optimization Model (BBOM):

$$\max_{x_{i^*jy}} \sum_y \frac{1}{(1+F)^y} \{ \sum_{jl} T_{yl}(p_{yl} - \delta_{i^*j}) q_{i^*jyl} - \sum_j \beta_{i^*jy} x_{i^*jy} \} \tag{2.59}$$

$$\text{s.t.} \qquad 0 \le x_{i^*jy} \le x_{i^*j(y+1)} \quad \forall jy \tag{2.60}$$

$$\text{s.t.} \qquad \text{Market Equilibrium Formulation} \tag{2.61}$$

Equation (2.61) is a placeholder for a mathematical formulation of the conjectured-price response market equilibrium problem. As we have mentioned in previous sections, there are several ways to formulate this equilibrium problem, for example, as the optimization problem given by (2.45)-(2.48). Replacing (2.61) with the market equilibrium optimization problem (2.45)-(2.48) in the generation expansion problem of firm i^*, leads to a problem formulation that coincides exactly with the definition of a bilevel programming problem, i.e., (2.15)-(2.18), previously presented in section 2.2.4. Therefore, considering two separate decision stages, i.e., investment and production stage, in the generation expansion problem, yields formulations that mathematically belong to the class of bilevel problems.

Furthermore, in section 2.3.1 we have derived the formulation of the conjectured-price response market equilibrium problem as the system of KKT conditions of all market agents, given by (2.35)-(2.41), connected by the market clearing condition (2.42) and the affine demand function (2.43). Using this particular representation of the market equilibrium, literally yields a mathematical problem with constraints that represent an equilibrium (MPEC). Moreover, in section 2.2.5 we have introduced complementarity problems and have shown that an optimization problem can also be re-written as a complementarity problem, by first deriving its KKT conditions and then condensing them using the complementarity conditions. If we apply this methodology to the market profit maximization problems given in (2.32), this yields the following complementarity problem $\forall ijyl$:

$$q_{ijyl} \geq 0 \quad \perp \quad \frac{\delta_{ij} T_{yl}}{(1+F)^y} + \frac{\sum_j \theta_{iyl} q_{ijyl} T_{yl}}{(1+F)^y} - \frac{T_{yl} p_{yl}}{(1+F)^y} + \lambda_{ijyl} \geq 0 \tag{2.62}$$

$$\lambda_{ijyl} \geq 0 \quad \perp \quad x_{ijy} + K_{ijy} - q_{ijyl} \geq 0 \tag{2.63}$$

If instead of representing the market equilibrium via an optimization problem, i.e., (2.45)-(2.48), we represent the market equilibrium as the complementarity problem (2.62)-(2.63), the market clearing condition and the demand function and add these constraints to the generation expansion problem, then the arising formulation of the entire generation expansion problem coincides with our definition of an MPEC, given in (2.54)-(2.58).

2.4.2 Equilibrium Problem with Equilibrium Constraints or Bilevel Investment Equilibrium

In this section we define the term Equilibrium Problem with Equilibrium Constraints (EPEC) and propose a new generation expansion model, which is formulated as an EPEC, to assist all generation companies, that are participating in the market, with their long-term generation capacity expansion decisions. Similarly to the previous section, the purpose of introducing this EPEC model at this point is merely to compare it to the existing single-level (SL) equilibrium model presented in section 2.3.3. Again, for the detailed theoretical and numerical analysis of this model as well as the conducted case studies and possible model extensions, the reader is referred to chapters 3 and 5. In general it can be said that while single-level equilibrium models represent simultaneous decision making, the bilevel equilibrium model presented here represents sequential decision making, i.e., first capacity is decided and then, production decisions and prices follow. At first glance, this might seem as only a slight difference, however, it has a very significant impact on results. Chapter 3 is dedicated entirely to pointing out the differences between these two types of equilibrium models. Throughout the thesis we will refer to all bilevel results as (BL).

An EPEC belongs to class of mathematical programs that often arise in engineering and economics applications, as in Berry et al., Cardell et al. or Hobbs et al. [12, 23, 61]. Formally, we could describe an EPEC as finding a Nash equilibrium between I players that are all facing an MPEC, i.e., resolving {MPEC(1), $\ldots$, MPEC(i^*),$\ldots$, MPEC(I)}, where each MPEC is the type of problem that has been defined in the previous section 2.4.1. For further information on EPECs the reader is referred to Su [111]. Incorporating this definition to the generation capacity expansion framework, we define the generation capacity EPEC as the equilibrium problem where each firm i^* for $i^* = 1, \ldots, I$ faces the generation capacity MPEC defined in the previous section, which is a bilevel problem where the firm i^* decides its capacity investments subject to the conjectured-price response market equilibrium. This EPEC represents a two-stage equilibrium in capacity investments

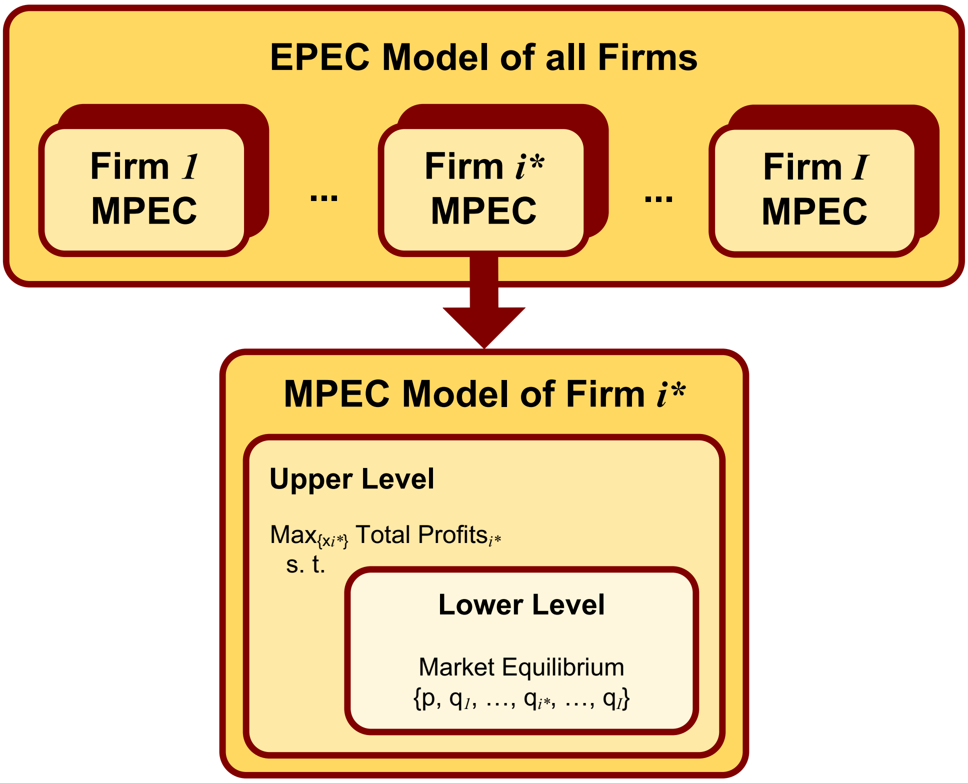

Figure 2-9: Graphic representation of the EPEC generation expansion problem faced by all firms.

and in production decisions. Figure 2-9 graphically represents basic version of the bilevel equilibrium model for generation capacity expansion planning formulated as an EPEC.

Basic Bilevel Equilibrium Model (BBEM):

$$\forall i^* \begin{cases} \max_{x_{i^*jy}} \sum_y \frac{1}{(1+F)^y} \left\{ \sum_{jl} T_{yl}(p_{yl} - \delta_{i^*j}) q_{i^*jyl} - \sum_j \beta_{i^*jy} x_{i^*jy} \right\} \\ \text{s.t.} \qquad 0 \leq x_{i^*jy} \leq x_{i^*j(y+1)} \quad \forall jy \\ \text{s.t.} \qquad \text{Market Equilibrium Formulation} \end{cases} \tag{2.64}$$

We will see later on, in chapter 3, that single-level (or one-stage) equilibrium models and bilevel (or two-stage) equilibrium models in general yield different results. The bilevel model captures the sequential decision making process, where investment and generation decisions are separated. The single-level model does not consider sequential decision making and is therefore a simplification, which tends to lead to results which are overly confident in terms of capacity investment. The bilevel model is therefore more realistic because it captures that GENCOs know that their capacity investments will influence the market. The proof and detailed comparison of single-level and bilevel models for capacity expansion can be found in chapter 3, where we show that it is not the same to take investments and generation decisions simultaneously maximizing total profits, or to first, decide investments maximizing total profits and then take generation decisions maximizing market profits.

2.4.3 Model Extensions

In the previous section 2.4.1 and 2.4.2 we have introduced the basic versions of the MPEC and EPEC models. The purpose of this section is to point out the aspects that would have to be added to these models in order to make them more realistic. In the list below, we enumerate certain topics, which will be formulated and discussed in detail in the following chapters 4 and 5.

- First of all, in the basic version of the proposed models we only consider thermal production q_{ijyl} in different thermal technologies j, however, it is also important to model hydro-technology in order to represent a realistic electricity system. Since hydro has different characteristics than other technologies, hydro-specific additional constraints will also have to be added to the models.

- The investment decision variables x_{ijy} are considered to be continuous variables, which obviously is a simplification of reality. It is, however, a reasonable approach because when analyzing the long term, continuous investment decisions can be interpreted as an approximation of "real" investment decisions. Moreover, when considering investment in for example renewable energy technologies with small individual plant size, then continuous variables might be a sensible approximation. A desirable extension would be to actually consider discrete decision variables x_{ijy}, however, it is clear that this drastically complicates the resolution of the models. In chapter 4 we will discuss how this issue can be tackled for the MPEC models.

- In many liberalized markets it has become apparent that an "energy only" market might not offer enough incentives for new capacity investments. In order to guarantee the security of supply, capacity mechanisms have to be introduced to the system. Therefore we will consider capacity mechanisms in our models and assess their impact on investment decisions.

- Generation expansion planning is a long-term problem whose time horizon can stretch over various decades, which makes it very vulnerable to all kinds of uncertainties. For example, fuel prices or demand-induced uncertainty caused by renewable energy sources like solar or wind. In chapter 3 we will develop a new methodology to cope with uncertainty in the generation expansion problem by introducing different scenarios of the market equilibrium. This methodology is also useful to model different scenarios of strategic behavior of competing firms in order to assess its impact on optimal investment decisions.

- More realistic details have to be added to the models, like for example financial hedging through the consideration of long-term contracts.

2.5 Conclusions

In this chapter we have presented a literature review on generation expansion planning in section 2.1 in order to emphasize how our work differs from the existing literature. Then, we have introduced some basic concepts and modeling hypotheses in section 2.2 for a better understanding of the thesis. In sections 2.3 and 2.4 we have presented a basic version of single-level and bilevel generation expansion models and pointed out that they are different in terms of formulation, which immediately raises the question whether their results will be very different as well. Chapter 3 provides an answer to this question.

Chapter 3

Single-Level Versus Bilevel Capacity Equilibria in Electricity Markets under Perfect and Oligopolistic Competition

This chapter provides an answer to the question whether and under what circumstances single-level and bilevel generation expansion models may yield the same solutions by carrying out a theoretical analysis of a single-level and a bilevel conjectured-price response investment equilibrium model. In particular, in this chapter, which constitutes a key contribution of this thesis accepted for publication in Wogrin et al. [119], we consider two game-theoretic models of the generation capacity expansion problem in liberalized electricity markets. The first is a single-level equilibrium model, where generation companies simultaneously choose capacities and quantities to maximize their individual profit. The second is a bilevel model, in which companies first choose capacities maximizing their profit anticipating the market equilibrium outcomes in the second stage. This problem is an Equilibrium Problem with Equilibrium Constraints. In both models, the intensity of competition among producers in the energy market is represented using conjectural variations. Considering one load period, we show that for any choice of conjectural variations ranging from perfect competition to Cournot, the bilevel equilibrium coincides with the Cournot single-level equilibrium, thereby obtaining a "Kreps and Scheinkman"-like result [74] and extending it to arbitrary strategic behavior. In the bilevel model the investment stage is Cournot-like. Therefore similarities arise in the results of the bilevel model, which yields Cournot outcomes even when the market stage is perfectly competitive. When expanding the model framework to multiple load periods, the bilevel equilibria for different conjectural variations can diverge from each other and from single-level equilibria. We also present and analyze alternative conjectured-price response models with switching conjectures. Surprisingly, the rank ordering of the bilevel equilibria in terms of consumer surplus and market efficiency (as measured by total social welfare) is ambiguous. Thus, regulatory approaches that force marginal cost-based bidding in spot markets may diminish market efficiency and consumer welfare by dampening incentives for investment. We also show that the bilevel capacity yielded by a conjectured-price response second stage competition can be less or equal to the bilevel Cournot capacity, and that the former capacity cannot exceed the latter when there are symmetric agents and two load periods.

3.1 Introduction

The purpose of this chapter is to compare game-theoretic models that can be used to analyze the strategic behavior of companies facing generation capacity expansion decisions in liberalized electricity markets. In particular, we seek to characterize the difference between single and bilevel models of investment. The obtained results provide the theoretical basis for a single-level approximation scheme of the bilevel generation expansion model which is presented later on in section 5.4.

Single-level models, similar to the one presented in this chapter, extend short-term models to a longer time horizon by modeling investment and production decisions as being taken at the same time. This corresponds for example to the single-level Cournot equilibrium conditions presented by Murphy and Smeers [84], the Cournot-based model presented in Ventosa et al. [113], which is solved using a Mixed Complementarity Problem (MCP) scheme, and the model analyzed in Centeno et al. [25], which is solved using an equivalent optimization problem. However, this approach may overly simplify the dynamic nature of the problem, as it assumes that expansion and operation decisions are taken simultaneously.

The reason to employ more complicated bilevel formulations is that the generation capacity expansion problem has an innate two-stage structure: first investment decisions are taken followed by determination of energy production in the spot market, which is limited by the previously chosen capacity. A two-stage decision structure is a natural way to represent how many organizations actually make decisions. One organizational subunit is often responsible for capital budgeting and anticipating how capital expenditures might affect future revenues and costs over a multi-year or even multi-decadal time horizon, whereas a different group is in charge of day-to-day spot market bidding and output decisions. This type of bilevel model is in fact an Equilibrium Problem with Equilibrium Constraints (EPEC), see Leyffer and Munson or Su [77, 111], arising when each of two or more companies simultaneously faces its own profit maximization problem modeled as a Mathematical Program with Equilibrium Constraints (MPEC). In reality, the generation expansion problem is even more complicated and has more than just two levels, however, representing and solving multi-level games computationally is almost intractable at this point in time. In the literature, there are some recent three-level approaches as for example the transmission and generation expansion problem by Pozo et al. [97], but currently it seems like three levels are the limit.

Solving large-scale bilevel models can be very challenging, sometimes even not tractable. Therefore, in real-world applications there is a strong incentive to resort to easier single-level models, simply because the corresponding bilevel model cannot be solved (yet). The results presented in this chapter indicate that when practical considerations motivate adoption of easier, less complicated single-level models, the results may be very different from (possibly more realistic) bilevel formulations.

3.1.1 Review of Literature

Several bilevel approaches to the generation capacity expansion problem have been proposed. The papers most relevant for our work are Murphy and Smeers [84] and Kreps and Scheinkman [74], which will be discussed below. With their paper [74], Kreps and Scheinkman (K-S) tried to combine Cournot's [31] and Bertrand's [13] theory by constructing a two-stage game, where first firms simultaneously set capacity and second, after capacity levels are made public, there is price competition. They find that when assuming two identical firms and an efficient rationing rule (i.e., the market's short-run production is provided at least cost), their two-stage game yields Cournot outcomes. Davidson and Deneckere [32] formulate a critique of K-S, where they say that results critically depend on the choice of the rationing rule. They claim that if the rationing rule is changed, the equilibrium outcome need not be Cournot. In defense of K-S, Paul Madden proved [81] that if it is assumed that demand functions are of the constant elasticity form and that all costs are sunk, then the K-S two-stage game reduces to the Cournot model for any rationing mechanism between the efficient and proportional extremes. However, Deneckere and Kovenock [36] find that the K-S result does not necessarily hold if costs are asymmetric.

More recently, Hartmann and Lepore [58, 75] address the extension of the K-S model to uncertainty of marginal costs. Hartmann [58] shows that due to uncertainty of marginal costs, equilibria were necessarily asymmetric. Reynolds and Wilson [102] address the issue of uncertain demand in a K-S like model, which is related to our extension to multiple load periods. They discover that if costs are sufficiently high, the Cournot outcome is the unique solution to this game. However, they also find that if costs are lower, no pure strategy equilibria exists. Lepore [76] also demonstrates that, under certain assumptions, the K-S result is robust to demand uncertainty. Our results extend this literature by considering generalizations of K-S-like models to conjectural variations other than competitive (or Bertrand) as well as multiple load periods or, equivalently, stochastic load which is a different concept but which mathematically can be modeled the same way.

In [84] Murphy and Smeers present and analyze three different models: a single-level perfectly competitive model, a single-level Cournot model and a bilevel Cournot model. Each considers several load periods which have different demand curves and two firms, one with a peak load technology (low capital cost, high operating cost) and the other with a base load technology (high capital cost, low operating cost). They analyze when single and bilevel Cournot models coincide and when they are necessarily different. Moreover, they demonstrate that the bilevel Cournot equilibrium capacities fall between the single-level Cournot and the single-level competitive solutions. Our work differs by considering a range of conjectural variations between perfect competition and Cournot. Our formal results are for symmetric agents but they extend to asymmetric cases. We derive certain equivalency results that can also be extended to asymmetric firms. Moreover, in our models we consider a constant second stage conjectural variation rather than a situation in which the conjectural variation switches to Cournot when rival firms are at capacity. We consider this alternative conjectural variation in section 3.3.5.

Existing generation capacity expansion approaches in the literature assume either perfectly competitive, for example Garcia-Bertrand et al. [54], or Cournot behavior in the spot market, for example Centeno et al. or Ventosa et al. [25, 113]. Power market oligopoly models have been proposed before based on conjectural variations, see Centeno et al. [24] and conjectured-price responses, see Day et al. [34], but only for short-term markets in which capacity is fixed. The proposed single and bilevel models of this chapter extend previous approaches by including a generalized representation of market behavior via conjectural variations, in particular through an equivalent conjectured-price response. This allows us to represent various forms of oligopoly, ranging from perfect competition to Cournot.

3.1.2 Outlook on Results

In this chapter, we consider two identical firms with perfectly substitutable products - in our case electricity - each facing either a one-stage or a two-stage competitive situation. The one-stage situation, represented by the single-level model, describes the one-shot investment operation market equilibrium. The bilevel model, which is an EPEC, describes the two-stage investment-operation market equilibrium and is similar to the well-known K-S game [74]. Considering one load period, we find that the bilevel equilibrium for any strategic market behavior between perfect competition and Cournot yields the single-level Cournot outcomes, thereby obtaining a K-S-type result and extending it to any strategic behavior between perfect competition and Cournot. As previously mentioned, Murphy and Smeers [84] have found that under certain conditions the single and bilevel Cournot equilibria coincide. Our result furthermore shows that considering one load period, all bilevel models assuming strategic spot market behavior between perfect competition and Cournot coincide with the single-level Cournot solution. In the multiple load period case we define some sufficient conditions for the single and bilevel capacity decisions to be the same. However, this result is parameter dependent. When capacity is the same, outputs in non-binding load periods are the same for single and bilevel models when strategic spot market behavior is the same, otherwise outputs can differ.

When the bilevel capacity decisions differ for different conjectural variations, then the resulting consumer surplus and market efficiency (as measured by social welfare, the sum of consumer surplus and profit) will depend on the conjectural variation. It turns out that which conjectural variation results in the highest efficiency is parameter dependent. In particular, under some assumptions, the bilevel model considering perfect competition in the energy market can actually result in lower market efficiency, lower consumer surplus and higher prices than Cournot competition. This surprising result implies that regulatory approaches that force marginal cost-based bidding in spot markets may decrease market efficiency and consumer welfare and may therefore actually be harmful. For example, the Irish spot market rules [98] require bids to equal short-run marginal cost. Meanwhile, local market power mitigation procedures in several US organized markets reset bids to marginal cost (plus a small adder) if significant market power is present in local transmission-constrained markets, see O'Neill et al. [91]. These market designs implicitly assume that perfect competition is welfare superior to more oligopolistic behavior, such as Cournot competition. As our counter-example will show, this is not necessarily so.

In [57], Grimm and Zoettl have arrived at a similar result, however, they only examine the polar cases of perfect competition or Cournot-type competition. In our work we generalize strategic behavior using conjectural variations and look at a range of strategic behavior, from perfect competition to Cournot competition and we also observe that an intermediate solution between perfect and Cournot competition can lead to even larger social welfare and consumer surplus.

The results obtained are suggestive of what might occur in other industries where storage is relatively unimportant and there is time varying demand that must be met by production at the same moment. Examples include, for instance, industries such as airlines or hotels.

The remaining chapter is organized as follows. In section 3.2 we formulate symmetric single and bilevel models for one load period and establish that our K-S-like result also holds for arbitrary strategic behavior ranging from perfect to Cournot competition. This is followed by section 3.3, which extends the symmetric K-S-like framework to multiple load periods. We furthermore analyze alternative models in which the second stage conjectural variation switches depending on whether rivals' capacity is binding or not, instead of being constant. In section 3.4 we first show that the bilevel capacity yielded by a conjectured-price response second stage competition can be less or equal to the bilevel Cournot capacity, and that the former capacity cannot exceed the latter for symmetric agents and two load periods. Also in that section we show by example that under the bilevel framework, more competitive behavior in the spot market can lead to less market efficiency and consumer surplus. Finally, section 3.5 concludes this chapter.

3.2 Generalization of the K-S-like Single Load Period Result to Arbitrary Oligopolistic Conjectures

In this section we consider two identical firms with perfectly substitutable products, facing either a one-stage or a two-stage competitive situation. The one-stage situation is represented by the single-level model presented in 3.2.1 and describes the one-shot investment-operation market equilibrium. In this situation, firms simultaneously choose capacities and quantities to maximize their individual profit, while each firm conjectures a price response to its output decisions consistent with the conjectured-price response model. The bilevel model given in 3.2.2 describes the two-stage investment-operation market equilibrium, where firms first choose capacities that maximize their profit while anticipating the equilibrium outcomes in the second stage, in which quantities and prices are determined by a conjectured-price response market equilibrium. We furthermore assume that there is an affine relation between price and demand and that capacity can be added in continuous amounts.

The main contribution of this section is Theorem 3.1, in which we show that for two identical agents, one load period and an affine non-increasing inverse demand function, the one-stage model solution assuming Cournot competition is a solution to the bilevel model independent of the choice of conjectured-price response within the perfect competition-Cournot range. When the conjectured-price response represents perfect competition, then this result is very similar to the findings of Kreps and Scheinkman [74]. As a matter of fact, Bertrand competition boils down to perfect competition when there is no capacity constraint as shown by Tirole [112], in which case our results would be equivalent to the K-S result. However, considering that we have a capacity limitation, Bertrand competition and perfect competition are not equivalent. For this reason our results are not exactly K-S, however, taking into account the similarities, they can be labeled as K-S-like. Thus, Theorem 3.1 extends the “Kreps and Scheinkman”-like result to any conjectured-price response within a range. Later in the chapter we show how this result can be generalized to the case of multiple load periods.

Throughout this section we will use the notation presented below. Note that in parenthesis we give an example of coherent units.

- q_i denotes the quantity [MW] produced by firms $i = 1, 2$.
- x_i denotes the capacity [MW] of firms $i = 1, 2$.
- d denotes quantity demanded [MW].
- p [€/MWh] denotes the clearing price. Moreover $p(d) = (D^0 - d)/\alpha$ where D^0 and α are positive constants, and P^0 denotes D^0/α.
- T [h/year] corresponds to the duration of the load period per year.
- β [€/MW/year] corresponds to the annual investment cost.
- δ [€/MWh] is the variable production cost.
- θ is a constant in $[0, 1/\alpha]$, that is the conjectured-price response corresponding to the strategic spot market behavior for each i, see (2.2.2).

Furthermore we will make the following assumptions:

- Both cost parameters, δ, β, are nonnegative.
- The investment cost plus the variable cost will be less than the price intercept times duration T, i.e., $\delta T + \beta < P^0 T$, which is an intuitive condition as it simply states that the maximum price P^0 is high enough to cover the sum of the investment cost and the operation cost. Otherwise there is clearly no incentive to participate in the market.
- The same demand curve assumptions are made as in section 2.2.1.
- We consider one year rather than a multi-year time horizon, and so each firm attempts to maximize its annualized profit.

3.2.1 The Single-Level Model

In the single-level model, every firm i faces a profit maximization problem in which it chooses capacity x_i and production q_i simultaneously. When firms simultaneously compete in capacity and quantity, the single-level investment-operation market equilibrium problem consists of all the firms' profit maximization problems plus market clearing conditions that link together their problems by $d = D^0 - \alpha p(q_i, q_{\text{-}i})$. Note that -$i$ is the index corresponds to the all firms except i. Conceptually, the resulting equilibrium problem can be written as (3.1)-(3.3):

$$\forall i \begin{cases} \max_{x_i,q_i} & T(p(q_i, q_{\text{-}i}) - \delta)q_i - \beta x_i \\ s.t. & q_i \leq x_i \end{cases} \tag{3.1}$$

$$d = q_i + q_{\text{-}i} \tag{3.2}$$

$$d = D^0 - \alpha p(q_i, q_{\text{-}i}) \tag{3.3}$$

In (3.1) we describe i's profit maximization as consisting of market revenues $Tp(q_i, q_{\text{-}i})q_i$ minus production costs $T\delta q_i$ and investment costs βx_i. The non-negativity constraints can be omitted in this case.[1]

Although (3.1)'s constraint is expressed as an inequality, it will hold as an equality in equilibrium, at least in this one-period formulation. That $x_i = q_i$ for $i = 1, 2$ will be true in equilibrium, can easily be proven by contradiction. Let us assume that at the equilibrium $x_i > q_i$; then firm i could unilaterally increase its profits by shrinking x_i to q_i (assuming $\beta > 0$), which contradicts the assumption of being at an equilibrium.

In this representation the conjectured-price response is not explicit. Therefore we re-write the single-level equilibrium stated in (3.1)-(3.3) as a Mixed Complementarity Problem (MCP) by replacing each firm's profit maximization problem by its first order Karush-Kuhn-Tucker (KKT) conditions. The objective function in (3.1) is concave

[1] For completeness, let us consider the explicit non-negativity constraint $0 \leq q_i$ in the optimization problem (3.1) and let us define $\mu_i \geq 0$ as the corresponding dual variable. Then, due to complementarity conditions arising from the KKT conditions, we can separate two cases, the one where $\mu_i = 0$ and the other where $\mu_i > 0$. The first case will lead us to the solution presented in this chapter, and case $\mu_i > 0$ will lead us to a solution where $\mu_i = T(\delta - P^0)$. Considering that we assumed $P^0 > \delta$, this yields a contradiction to the non-negativity of μ_i. Hence, this cannot be the case and therefore we omit the non-negativity constraint.

for any value of θ in $[0, 1/\alpha]$. Note that the parameter θ represents strategic spot market behavior and its value can vary depending on who is taking decisions. Then, due to linearity of $p(d)$, (3.1)-(3.3) is a concave maximization problem with linear constraints, hence its solutions are characterized by its KKT conditions. Therefore let $\mathcal{L}_i$ denote the Lagrangian of company i's corresponding optimization problem, given in (3.1) and let λ_i be the Lagrange multiplier of constraint $q_i \leq x_i$. Then, the single-level equilibrium problem is then given in (3.4)-(3.6).

$$\forall i \begin{cases} \frac{\partial \mathcal{L}_i}{\partial q_i} = Tp(q_i, q_{\text{-}i}) - T\theta q_i - T\delta - \lambda_i = 0 \\ \frac{\partial \mathcal{L}_i}{\partial x_i} = \beta - \lambda_i = 0 \\ q_i \leq x_i \\ \lambda_i \geq 0 \\ \lambda_i(x_i - q_i) = 0 \end{cases} \tag{3.4}$$

$$d = q_i + q_{\text{-}i} \tag{3.5}$$

$$d = D^0 - \alpha p(q_i, q_{\text{-}i}) \tag{3.6}$$

Due to the fact that $\lambda_i = \beta > 0$, the complementarity condition yields $x_i = q_i$ in equilibrium. In this formulation we can directly see the conjectured-price response parameter θ in $\frac{\partial \mathcal{L}_i}{\partial q_i}$. Solving the resulting system of equations yields:

$$q_i = \frac{D^0T - \alpha(\beta + \delta T)}{T(\alpha\theta + 2)} \quad \forall i \tag{3.7}$$

$$p = \frac{D^0T\theta + 2(\beta + \delta T)}{T(\alpha\theta + 2)}. \tag{3.8}$$

In the single-level model we have not explicitly imposed $q_i \geq 0$, however, from (3.7) we obtain that the single-level model has a non-trivial solution (i.e., each quantity is positive at equilibrium) if parameters are chosen such that $D^0T - \alpha(\beta + \delta T) > 0$ is satisfied. This condition is equivalent to $\delta T + \beta < P^0T$ using the fact that $\alpha = D^0/P^0$ and $D^0 > 0$, which has already been stated in the assumptions above.

A special case of the conjectured-price response is the Cournot oligopoly. In order to obtain the single-level Cournot solution, we just need to insert the appropriate value of the conjectured-price response parameter θ, which for Cournot is $\theta = 1/\alpha$. This solution is unique, see Murphy and Smeers [84]. Then (3.7)-(3.8) yield:

$$q_i = \frac{D^0 T - \alpha(\beta + \delta T)}{3T} \quad \forall i \tag{3.9}$$

$$p = \frac{D^0 T + 2\alpha(\beta + \delta T)}{3T\alpha}. \tag{3.10}$$

3.2.2 The Bilevel Model

We now present the bilevel conjectured-price response model describing the two-stage investment-operation market equilibrium. In this case, firms first choose capacities maximizing their profit anticipating the equilibrium outcomes in the second stage, in which quantities and prices are determined by a conjectured-price response market equilibrium. We stress that the main distinction of this model from the equilibrium model described in section 3.2.1 is that now there are two stages in the decision process, i.e., capacities and quantities are not chosen at the same time. Then we present Theorem 3.1 which establishes a relation between the single-level and the bilevel models for the single demand period case.

The Production Level - Second Stage

The second stage (or lower level) represents the conjectured-price-response market equilibrium, in which both firms maximize their market revenues minus their production costs, deciding their production subject to the constraint that production will not exceed capacity. The argument given above shows, at equilibrium, that this constraint binds if there is a single demand period. These maximization problems are linked by the market clearing condition. Thus, the second stage market equilibrium problem can be written as:

$$\forall i \begin{cases} \max_{q_i} & T(p(q_i, q_{\text{-}i}) - \delta)q_i \\ s.t. & q_i \leq x_i \end{cases} \tag{3.11}$$

$$d = q_i + q_{\text{-}i} \tag{3.12}$$

$$d = D^0 - \alpha p(q_i, q_{\text{-}i}) \tag{3.13}$$

As in the single-level case, p may be conjectured by firm i to be a function of its output q_i. Using a justification similar to that in the previous section, we now substitute firm i's KKT conditions for (3.11) and arrive at the conjectured-price response market equilibrium conditions given by:

$$\forall i \begin{cases} \frac{\partial \mathcal{L}_i}{\partial q_i} = Tp(q_i, q_{\text{-}i}) - T\theta q_i - T\delta - \lambda_i = 0 \\ q_i \leq x_i \\ 0 \leq \lambda_i \\ \lambda_i(x_i - q_i) = 0 \end{cases} \tag{3.14}$$

$$d = q_i + q_{\text{-}i} \tag{3.15}$$

$$d = D^0 - \alpha p(q_i, q_{\text{-}i}) \tag{3.16}$$

The Investment Level - First Stage

In the first stage, both firms maximize their total profits, consisting of the gross margin from the second stage (revenues minus variable production costs) minus investment costs, and choose their capacities subject to the second stage equilibrium response. This can be written as the following equilibrium problem:

$$\forall i \begin{cases} \max_{x_i} & T(p(q_i, q_{\text{-}i}) - \delta)q_i - \beta x_i \\ s.t. & \text{Second Stage, } (3.14) - (3.16) \end{cases} \tag{3.17}$$

We know that at equilibrium, production will be equal to capacity. As in the single-level model, this can be shown by contradiction. Since there is a linear relation between price and demand, it follows that price can be expressed as $p = \frac{D^0 - d}{\alpha}$. Substituting $x_i = q_i$ in this expression of price, yields $p = \frac{D^0 - x_1 - x_2}{\alpha}$. Then expressing the objective function and the second stage in terms of the variables x_i yields the following simplified bilevel equilibrium problem:

$$\forall i \begin{cases} \max_{x_i} & T(\frac{D^0 - x_1 - x_2}{\alpha} - \delta)x_i - \beta x_i \\ s.t. & \frac{D^0 - x_1 - x_2}{\alpha} - \theta x_i - \delta \geq 0 \qquad : \gamma_i \end{cases} \tag{3.18}$$

where γ_i are the dual variables to the corresponding constraints. Writing down the bilevel equilibrium conditions (assuming a nontrivial solution $x_i > 0$) then yields:

$$\forall i \begin{cases} T(\frac{D^0 - x_1 - x_2}{\alpha} - \delta) - Tx_i/\alpha - \beta + \gamma_i(-\theta - 1/\alpha) = 0 \\ \hfill \frac{D^0 - x_1 - x_2}{\alpha} - \theta x_i - \delta \geq 0 \\ \hfill \gamma_i(\frac{D^0 - x_1 - x_2}{\alpha} - \theta x_i - \delta) = 0 \\ \hfill \gamma_i \geq 0 \end{cases} \tag{3.19}$$

When solving the system of equations given by (3.19) we distinguish between two separate cases: $\gamma_i = 0$ and $\gamma_i > 0$. The first case, i.e., $\gamma_i = 0$, yields the following solution for the bilevel equilibrium, where λ_i has been obtained from (3.14):

$$x_i = \frac{D^0 T - \alpha(\beta + \delta T)}{3T} \quad \forall i. \tag{3.20}$$

$$p = \frac{D^0 T + 2\alpha(\beta + \delta T)}{3T\alpha} \tag{3.21}$$

$$\lambda_i = \frac{D^0 T + \alpha^2(\beta + \delta T)\theta + \alpha(2\beta - T(\delta + D^0\theta))}{3\alpha} \quad \forall i. \tag{3.22}$$

Moreover, it is easy to show that for $\theta \in [0, 1/\alpha]$ $\lambda_i \geq 0$ will be satisfied,[2] which shows that x_i is indeed the optimal value of q_i in (3.14), confirming the validity of (3.18) for $\gamma_i = 0$. As in the previous section, the solution is nontrivial due to the assumption that $\delta T + \beta < P^0 T$.

As for uniqueness of the bilevel equilibrium, Murphy and Smeers [84] have proven for the Cournot bilevel equilibrium that if an equilibrium exists, then it is unique. We will investigate uniqueness issues of the bilevel conjectured-price response model in

[2]Case $\theta = 0$: from (3.22) we get $D^0 T + 2\alpha\beta - \alpha\delta T = D^0 P^0 T + 2\alpha\beta P^0 - D^0\delta T \geq 2\alpha\beta P^0 \geq 0$; Case $\theta = 1/(k\alpha)$ with $k \geq 1$: $D^0 T + \alpha^2(\beta + \delta T)/(k\alpha) + \alpha(2\beta - T(\delta + D^0(k\alpha))) = (k-1)D^0 T/k + 2\alpha\beta + \alpha\beta/k - (k-1)D^0 T\delta/(kP^0) \Rightarrow (k-1)D^0 TP^0/k + 2\alpha\beta P^0 + \alpha\beta P^0/k - (k-1)D^0 T\delta/k \geq 2\alpha\beta P^0 + \alpha\beta P^0/k \geq 0$.

future research. Comparing (3.20) and (3.21) with the single-level equilibrium (3.7) and (3.8) we see that this is exactly the single-level solution considering Cournot competition, i.e., (3.9) and (3.10).

Now let us consider the second case, i.e., $\gamma_i > 0$. Then (3.19) yields the following values for capacities and γ_i:

$$x_i = \frac{D^0 - \alpha\delta}{2 + \alpha\theta} \quad \forall i. \tag{3.23}$$

$$\gamma_i = \frac{-(D^0 T + \alpha^2(\beta + \delta T)\theta + \alpha(2\beta - T(\delta + D^0\theta)))}{(\alpha\theta + 1)(\alpha\theta + 2)} \quad \forall i. \tag{3.24}$$

In the formulation of γ_i in (3.24), the numerator of the right hand side is the negative of numerator on the right hand side of the formula (3.22), where we know that latter is nonnegative for $\theta \in [0, 1/\alpha]$. That is, it is impossible for $\gamma_i > 0$. Hence the only solution to the bilevel equilibrium is the single-level Cournot solution that results when $\gamma_i = 0$.

3.2.3 Theorem: Comparison of Single and Bilevel Equilibria

In Theorem 3.1 presented below, we compare the single-level generation expansion equilibrium problem with the bilevel generation expansion equilibrium problem in terms of investment decisions.

Theorem 3.1. *Let there be two identical firms with perfectly substitutable products and one load period. Let the affine price $p(d)$ and the parameters needed to define the single-level equilibrium problem* (3.4)-(3.6) *be as described at the start of section 3.2. When comparing the single and bilevel competitive equilibria for two firms, we find the following: The single-level Cournot solution, see* (3.9)-(3.10)*, is a solution to the bilevel conjectured-price response equilibrium* (3.20)-(3.22) *for any choice of the conjectured-price response parameter θ from perfect competition to Cournot competition.*

Proof. Sections 3.2.1 and 3.2.2 above prove this theorem. As in the single-level model, the bilevel model has a non-trivial solution if data is chosen such that $P^0 T > \beta + \delta T$

is satisfied. □

Theorem 3.1 extends to the case of asymmetric firms but we omit the somewhat tedious analysis which can, however, be found in working paper Wogrin et al. [118].

What we have proven in Theorem 3.1 is that as long as the strategic behavior in the market (which is characterized by the parameter θ) is more competitive than Cournot, then in the bilevel problem firms could decide to build Cournot capacities. Even when the market is more competitive than the Cournot case (e.g., Allaz-Vila or perfect competition), firms can decide to build Cournot capacities. Hence Theorem 3.1 states that the Kreps and Scheinkman-like result holds for any conjectured-price response more competitive than Cournot (e.g., Allaz-Vila or perfect competition), not just for the case of perfect second stage competition. When referring to the "Allaz-Vila" case, we refer to a strategic spot market behavior of $\theta = 1/(2/\alpha)$.

Note that Theorem 3.1 describes sufficient conditions but they are not necessary. This means that there are cases where Theorem 3.1 also holds for $\theta > 1/\alpha$. For example Theorem 3.1 may hold under collusive behavior ($\theta = 2/\alpha$) when the marginal cost of production (δ) is sufficiently small.[3]

In the following section we will extend the result of Theorem 3.1 to the case of multiple demand periods. In particular, under stringent conditions, the Cournot single-level and bilevel solutions can be the same, and the Cournot single-level capacity can be the same as the bilevel capacity for more intensive levels of competition in the second stage of the bilevel game. But this result is parameter dependent, and in general, these solutions differ. Surprisingly, for some parameter assumptions, more intensive competition in the second stage can yield economically inferior outcomes compared to Cournot competition, in terms of consumer surplus and total market surplus.

[3]Let $D^0 = 1, T = 1, \alpha = 1, \beta = 1/2$ and $\delta = 0$, then the single-level Cournot solution is $p = 2/3$, with $x = 1/6$ for each firm. In this case, with these cost numbers, the single-level Cournot equals the bilevel equilibrium with $\theta = 2/\alpha$ (collusion, $\Phi = 1$).

3.3 Extension of K-S-like Result for Multiple Load Periods

In this section we extend the previously established comparison between the single-level and the bilevel model to the situation in which firms each choose a single capacity level, but face time varying demand that must be met instantaneously. This characterizes electricity markets in which all generation capacity is provided by dispatchable thermal plants and there is no significant storage (e.g., reservoirs associated to hydraulic plants). We also do not consider intermittent nondispatchable resources (such as wind); however, if their capacity is exogenous, their output can simply be subtracted from consumer quantity demanded, so that d represents effective demand. In particular, this extension will be characterized by Proposition 3.2. We start out by introducing some definitions and conditions, followed by the statement of Proposition 3.2 in which we compare single-level and bilevel equilibrium models with multiple load periods. In the remainder of this section we then introduce the single and the bilevel model for multiple load periods in sections 3.3.1 and 3.3.2. In section 3.3.3 we present the proof of Proposition 3.2. Section 3.3.4 contains a numerical example of the theoretical results obtained in this section. Finally, in section 3.3.5 we introduce and briefly analyze alternative conjectured-price response models with switching instead of constant second stage behavior, which is arguably more realistic.

We adopt the following assumptions:

- We still consider two identical firms and a linear demand function.
- Additionally let us define l as the index for distinct load periods. Now production decisions depend on both i and l.
- Furthermore let us define the active set of load blocks (or load periods) LB_i as the set of load periods in which equilibrium production equals capacity for firm i, i.e., $LB_i := \{l | q_{il} = x_i\}$.
- θ_l is a constant in $[0, 1/\alpha]$, that is the conjectured-price response in each load period l for each i.

- Both cost parameters, δ, β, are nonnegative.

- The same demand curve assumptions are made as in section 2.2.1 and an affine relation between demand and price is assumed, i.e., $d_l = D_l^0 - \alpha_l p_l$ or $p_l(d) = (D_l^0 - d_l)/\alpha_l$ where D_l^0 and α_l are positive constants, and P_l^0 denotes D_l^0/α_l.

- In addition, let $P_l^0 > \delta$ be true for $l \in LB_i$, which means that the maximum price that can be attained in the market has to be bigger than the production cost, otherwise there would be no investment or production.

- We also assume $P_l^0 \geq \delta$ for $l \notin LB_i$, which is similar to the condition above and guarantees non-negativity, however, it allows production to be zero in non-binding load periods.

- Similarly to the assumption made in the previous section, we also assume that $\sum_{l \in LB_i} P_l^0 T_l > \beta + \delta \sum_{l \in LB_i} T_l$, which states that if maximum price P_l^0 is paid for the durations T_l then the resulting revenue must be more than the sum of the investment cost and the operation cost, otherwise there is no incentive to participate in the market.

- We consider one year rather than a multi-year time horizon, and so each firm is maximizing its annualized profit.

Let us now present Proposition 3.2, in which single-level and bilevel multi-period equilibrium models are compared in terms of their capacity solutions. Moreover, we point out when single-level and bilevel equilibria coincide. The proof of Proposition 3.2 can be found a little later in this chapter, in particular, in section 3.3.3. This proposition provides the theoretical foundation for the approximation scheme of the bilevel model that we derive in section 5.4 and which is an original of this thesis.

Proposition 3.2. *(a) If the bilevel solutions for different θ between perfect competition and Cournot competition exist and have the same active set of load periods (i.e., firm i's upper bound on production is binding for the same load periods l) and the second stage multipliers, corresponding to the active set, are positive at equilibrium,* *then capacity x_i is the same for those values of θ. (b) Furthermore, if we assume that the single-level Cournot equilibrium, i.e., $\theta = 1/\alpha$, has the same active set, then the Cournot open and bilevel equilibria are the same.*

Perhaps the most difficult assumption of Proposition 3.2 is the existence of bilevel equilibria, since in general, EPECs may not have pure strategy equilibria as shown by Hobbs and Helman [60] and example 4 of the work of Hu and Ralph [63].

3.3.1 The Single-Level Model for Multiple Load Periods

The purpose of this section is to develop the stationary conditions for the single-level model for general θ and multiple load periods and thereby characterize the equilibrium capacity x_i. Therefore, we write the single-level investment-operation market equilibrium as:

$$\forall i \begin{cases} \max_{x_i, q_{il}} & \sum_l T_l(p_l(q_{il}, q_{\text{-}il}) - \delta)q_{il} - \beta x_i \\ s.t. & \qquad q_{il} \leq x_i \qquad \forall l \end{cases} \tag{3.25}$$

$$d_l = q_{il} + q_{\text{-}il} \quad \forall l \tag{3.26}$$

$$d_l = D_l^0 - \alpha_l p_l(q_{il}, q_{\text{-}il}) \quad \forall l \tag{3.27}$$

As previously mentioned in section 3.2.1, the non-negativity constraints can be omitted in this case.[4] Let us now derive the investment-operation market equilibrium conditions distinguishing load levels where capacity is binding from when capacity is slack. We can omit the complementarity between λ_{il} and $q_{il} < x_i$, because $\lambda_{il} = 0$ for

[4]As in section 3.2.1, let us consider the explicit non-negativity constraints $0 \leq q_{il}$ in the above optimization problem (3.25)-(3.27) and let us define $\mu_{il} \geq 0$ as the corresponding dual variables. Then, due to complementarity conditions arising from the KKT conditions, we can separate two cases, the one where $\mu_{il} = 0$ and the other where $\mu_{il} > 0$. First, let us consider the non-binding load periods: the case $\mu_{il} = 0$ leads us exactly to what we have presented in this chapter; when $\mu_{il} > 0$ then this simply leads us to zero production in the non-binding load periods. Now let us consider the binding load periods: again, the case $\mu_{il} = 0$ leads us exactly to the capacity presented in this chapter; case $\mu_{il} > 0$ immediately leads us to the trivial solution of zero capacity. Therefore we omit the explicit non-negativity constraint.

$l \notin LB_i$ and $x_i = q_{il}$ for $l \in LB_i$.

$$\forall i \begin{cases} \frac{\partial \mathcal{L}_i}{\partial q_{il}} = T_l p_l(q_{il}, q_{\text{-}il}) - T_l \theta_l q_{il} - T_l \delta - \lambda_{il} = 0 & l \in LB_i \\ \frac{\partial \mathcal{L}_i}{\partial q_{il}} = T_l p_l(q_{il}, q_{\text{-}il}) - T_l \theta_l q_{il} - T_l \delta = 0 & l \notin LB_i \\ \frac{\partial \mathcal{L}_i}{\partial x_i} = -\beta + \sum_{l \in LB_i} \lambda_{il} = 0 & \\ q_{il} = x_i & l \in LB_i \\ q_{il} < x_i & l \notin LB_i \\ 0 \leq \lambda_{il} & \forall l \end{cases} \tag{3.28}$$

$$d_l = q_{il} + q_{\text{-}il} \quad \forall l \tag{3.29}$$

$$d_l = D_l^0 - \alpha_l p_l(q_{il}, q_{\text{-}il}) \quad \forall l \tag{3.30}$$

For the non-binding load periods $l \notin LB_i$ we can obtain the solution to the equilibrium by solving the system of equations given by (3.28)-(3.30), which yields:

$$q_{il} = \frac{D_l^0 - \alpha_l \delta}{2 + \alpha_l \theta_l} \quad \forall i, l \notin LB_i \tag{3.31}$$

$$p_l = \frac{D_l^0 \theta + 2\delta}{2 + \alpha_l \theta_l} \quad \forall l \notin LB_i. \tag{3.32}$$

In order to obtain the solution for load levels when capacity is binding, we sum $\frac{\partial \mathcal{L}_i}{\partial q_{il}}$ over all load periods $l \in LB_i$, substitute $q_{il} = x_i$ and use the $\frac{\partial \mathcal{L}_i}{\partial x_i} = 0$ condition:

$$\sum_{l \in LB_i} \frac{\partial \mathcal{L}_i}{\partial q_{il}} = \sum_{l \in LB_i} (T_l p_l(q_{il}, q_{\text{-}il}) - T_l \delta - T_l \theta_l q_{il}) - \sum_{l \in LB_i} \lambda_{il} \tag{3.33}$$

$$= \sum_{l \in LB_i} (T_l p_l(q_{il}, q_{\text{-}il}) - T_l \delta - T_l \theta_l x_i) - \beta = 0 \tag{3.34}$$

If we express price as a function of capacity ($q_i = x_i$) and we solve the system of equations (3.30), together with (3.34) $\forall i$, this yields:

$$x_i = \frac{\sum_{l \in LB_i} (D_l^0 T_l \prod_{n \neq l \in LB_i} \alpha_n) - \prod_{l \in LB_i} \alpha_l (\beta + \delta \sum_{l \in LB_i} T_l)}{\sum_{l \in LB_i} (T_l (2 + \alpha_l \theta_l) \prod_{n \neq l \in LB_i} \alpha_n)} \quad , \forall i \tag{3.35}$$

We know that for $\theta_l \in [0, 1/\alpha_l]$, q_{il} will be a continuous function of x_i and hence from (3.28) we get that λ_{il} will also be a continuous function of x_i. Having obtained capacities x_i, the prices p_l and demand d_l for $l \in LB_i$ follow. We furthermore observe that (3.7) is a special case of (3.35) in which we only have one binding load period.

Above it has not been explicitly stated that q_i and x_i are positive variables, but it can be easily seen that this is satisfied at the equilibrium point. In non-binding load periods, production levels q_{il} given in (3.31) will be nonnegative due to the assumption that $P_l^0 \geq \delta$. Capacity x_i given in (3.35) will be positive as long as $\sum_{l \in LB_i}(D_l^0 T_l \prod_{n \neq l \in LB_i} \alpha_n) > \prod_{l \in LB_i} \alpha_l(\beta + \delta \sum_{l \in LB_i} T_l)$ holds, which is true due to the assumption $\sum_{l \in LB_i} P_l^0 T_l > \beta + \delta \sum_{l \in LB_i} T_l$.[5]

3.3.2 The Bilevel Model for Multiple Load Periods

Let us now derive the stationary conditions for the bilevel problem for multiple load periods which then yields an expression for the equilibrium capacity. First, we state the second stage production game for the bilevel game with multiple load periods in (3.36)-(3.38) and define Lagrange multipliers λ_{il} for the constraint $q_{il} \leq x_i$.

$$\forall i \begin{cases} \max_{q_{il}} & \sum_l T_l(p_l(q_{il}, q_{\text{-}il}) - \delta) q_{il} \\ s.t. & \qquad\qquad q_{il} \leq x_i \quad \forall l \end{cases} \tag{3.36}$$

$$d_l = q_{il} + q_{\text{-}il} \quad \forall l \tag{3.37}$$

$$d_l = D_l^0 - \alpha_l p_l(q_{il}, q_{\text{-}il}) \quad \forall l \tag{3.38}$$

Now we derive the market equilibrium conditions, assuming that each firm holds the same conjectured-price response θ_l in each load period l. θ_l can differ among periods. The complementarity between λ_{il} and $q_i < x_i$ for $l \notin LB_i$ implies that $\lambda_{il} = 0$ for $l \notin LB_i$. Hence we omit that complementarity condition for those load periods in the market equilibrium formulation of (3.39)-(3.41). Moreover, we assume that multipliers λ_{il} for $l \in LB_i$ will be positive at equilibrium. (If any multipliers are zero,

[5] Let us consider the numerator of (3.35). Dividing the numerator by $\prod_{l \in LB_i} \alpha_l$ yields: $\sum_{l \in LB_i}(D_l^0 T_l \prod_{n \neq l \in LB_i} \alpha_n) / \prod_{l \in LB_i} \alpha_l - \beta - \delta \sum_{l \in LB_i} T_l = \sum_{l \in LB_i}(D_l^0 T_l / \alpha_l) - \beta - \delta \sum_{l \in LB_i} T_l$ $= \sum_{l \in LB_i}(P_l^0 T_l) - \beta - \delta \sum_{l \in LB_i} T_l > 0$ due to assumption.

then Proposition 3.2 may not hold.)

$$\forall i \left\{ \begin{array}{rl} \frac{\partial \mathcal{L}_i}{\partial q_{il}} = T_l p_l(q_{il}, q_{\text{-}il}) - T_l\theta_l q_{il} - T_l\delta - \lambda_{il} = 0 & l \in LB_i \\ \frac{\partial \mathcal{L}_i}{\partial q_{il}} = T_l p_l(q_{il}, q_{\text{-}il}) - T_l\theta_l q_{il} - T_l\delta = 0 & l \notin LB_i \\ q_{il} = x_i & l \in LB_i \\ q_{il} < x_i & l \notin LB_i \\ 0 \leq \lambda_{il} & \forall l \end{array} \right. \tag{3.39}$$

$$d_l = q_{il} + q_{\text{-}il} \quad \forall l \tag{3.40}$$

$$d_l = D_l^0 - \alpha_l p_l(q_{il}, q_{\text{-}il}) \quad \forall l \tag{3.41}$$

For the non-binding load periods $l \notin LB_i$ we can obtain the solution to the conjectured-price response market equilibrium by solving the system of equations given by (3.39)-(3.41), which yields:

$$q_{il} = \frac{D_l^0 - \alpha_l\delta}{2 + \alpha_l\theta_l} \quad \forall i, l \notin LB_i \tag{3.42}$$

$$p_l = \frac{D_l^0\theta + 2\delta}{2 + \alpha_l\theta_l} \quad \forall l \notin LB_i. \tag{3.43}$$

We cannot yet solve the market equilibrium for the binding load periods $l \in LB_i$ depending as they do upon the x_i's. Hence we move on to the investment equilibrium problem to obtain those x_i's, which is formulated below:

$$\forall i \left\{ \begin{array}{ll} \max_{x_i} & \sum_l T_l p_l(q_{il}, q_{\text{-}il}) q_{il} - \sum_l T_l \delta q_{il} - \beta x_i \\ s.t. & (3.39) - (3.41) \end{array} \right. \tag{3.44}$$

After recalling that $q_i = x_i$ for l belonging to LB_i and then re-arranging terms, we can rewrite the objective function as:

$$\sum_{l \in LB_i} (T_l p_l x_i - T_l \delta x_i) + \sum_{l \notin LB_i} (T_l p_l q_{il} - T_l \delta q_{il}) - \beta x_i \tag{3.45}$$

Note that we have separated the terms of the objective function that correspond to inactive capacity constraints ($l \notin LB_i$) which do not involve x_i's at all, and the terms that correspond to the active capacity constraints ($l \in LB_i$). We furthermore know that for load periods $l \in LB_i$ the price $p_l = \frac{D_l^0 - d_l}{\alpha_l} = \frac{D_l^0 - \sum_i x_i}{\alpha_l}$. Replacing p_l for $l \in LB_i$ in (3.45), yields:

$$\sum_{l \in LB_i} (T_l \frac{D_l^0 - x_1 - x_2}{\alpha_l} x_i - T_l \delta x_i) + \sum_{l \notin LB_i} (T_l p_l q_{il} - T_l \delta q_{il}) - \beta x_i \tag{3.46}$$

We now show that (3.46) is smooth for small perturbations of x_i around its equilibrium level. In other words, (3.46) is a local description of the MPEC (3.44). Barquín et al. have proven in [5] that the second stage problem, i.e., the conjectured-price response spot market equilibrium, has an equivalent strictly concave optimization problem. Hence the solution q_{il} is unique, see Nocedal and Wright [87]. This yields that q_{il} is a continuous function of x_i. Therefore it follows from uniqueness of multipliers as a function of the optimal second stage quantity (due to the linear independence constraint qualification [87]) that λ_{il} is also a continuous function of x_i. Hence, for small changes in x_i the active set will not change and we obtain smoothness of objective function (3.46). Finally, it is obvious that the only nonlinear term in (3.46) is quadratic in x_i with a negative coefficient, $\sum_{l \in LB_i} T_l / \alpha_l$, thus (3.46) is concave in x_i.

Therefore all we need to do is take the derivative of the objective function (3.46) with respect to x_i, set it to zero and solve for x_i, which yields:

$$x_i = \frac{\sum_{l \in LB_i} (D_l^0 T_l \prod_{n \neq l \in LB_i} \alpha_n) - \prod_{l \in LB_i} \alpha_l (\beta + \delta \sum_{l \in LB_i} T_l)}{3 \sum_{l \in LB_i} (T_l \prod_{n \neq l \in LB_i} \alpha_n)} \quad \forall i \tag{3.47}$$

We observe that the capacity given by (3.47) is independent of θ_l. This means that for any other bilevel equilibrium whose active set coincides with LB_i and whose λ_{il} are positive at equilibrium, the capacity at equilibrium will also be described by (3.47), even though strategic spot market behavior may be different. We furthermore observe that (3.20) is a special case of (3.47) in which we only have one binding load period.

Now that we have obtained the values for x_i, the values for q_{il} as well as prices p_l and demand d_l with $l \in LB_i$ follow. As we have already shown in the single-level section, our assumptions imply that the solution will be non-trivial.

3.3.3 Proof of Proposition: Comparison Single and Bilevel Capacity Equilibria

In this section we present the proof of the previously stated Proposition 3.2, which compares single-level and bilevel capacity equilibria in a multiple load period setting.

Proof. (Part a - Bilevel capacity solutions) First, we observe that the bilevel capacity, given by (3.47), does not depend on the conjectured-price responses θ_l, for $l = 1, \ldots, L$, and in particular this means that two bilevel equilibria with different θ_l's have the exact same capacity solution as long as their active sets are the same with λ_{il} positive for $l \in LB_i$.

(Part b - Equivalence of single and bilevel model) comparing the bilevel capacity (3.47) with the single-level capacity (3.35) we note that the single-level capacity does depend on the strategic behavior θ_l in the market whereas the bilevel capacity does not. Moreover we observe that if single-level and bilevel models have the same active set at equilibrium, then their solutions are exactly the same under Cournot competition ($\theta_l = 1/\alpha_l$). If single-level and bilevel equilibria have the same active set and their θ_l coincide but are not Cournot, then in general their capacity will differ. However, their production q_{il} for $l \notin LB_i$ will be identical, as can be seen by comparing (3.31)-(3.32) and (3.42)-(3.43). □

In general, prices will be lower in the second stage under perfect competition than under Cournot competition for periods other than LB_i. In that case, consumers will be better off (and firms worse off) under perfect competition than Cournot competition. The numerical example in section 3.3.4 illustrates this point. However, this result is parameter dependent as will be demonstrated by the example in section 3.4.2, where we will show that in some cases Cournot competition can yield more capacity and higher market efficiency than perfect competition. This can only occur for cases where either the binding sets LB_i differ, or the LB_i are the same but the λ_{il} are zero for some l. Note that for one load period, Proposition 3.2 reduces to Theorem 3.1.

Proposition 3.2, like Theorem 3.1, can be extended to asymmetric firms. As the details are tedious we refer the reader to the general proof presented in [118].

3.3.4 Example with Two Load Periods: Same Binding Periods for all Values of Conjectural Variations

Let us now consider an illustrative numerical example where two firms both consider an investment in power generation capacity with the following data:

- Two load periods l, with durations of $T_1 = 3760$ and $T_2 = 5000$ [h/year]
- Demand intercepts D_l^0 given by $D_1^0 = 2000$ and $D_2^0 = 1200$ [MW]
- Demand slopes α_l equal to $\alpha_1 = D_1^0/250$ and $\alpha_2 = D_2^0/200$
- Annualized capital cost $\beta = 46000$ [€/MW/year]
- Operating cost $\delta = 11.8$ [€/MWh]

Having chosen the demand data for the two load levels such that capacity will not be binding in both periods in any solution, we solve the single-level and the bilevel model and compare results. In Figure 3-1 we present the solution of one firm (as the second firm - being identical - will have the same solution). First we depict the capacity that was built, then we compare production for both load periods and finally profits. Note that for both firms, LB_i will be the same for all θ and will include only period $l = 1$. Later we will present another example where this is not the case, and the results differ in important ways.

As demonstrated in Proposition 3.2 part (a), the bilevel capacity does not depend on behavior in the spot market. However we will see that profits do depend on the competitiveness of short-run behavior, and unlike the single demand period case, are not the same for all conjectural variations θ between perfect competition and Cournot. We refer to the binding load period as “peak” and to the non-binding load period as “base”. The bilevel production in the peak load level is the same for all θ, as long as the competitive behavior in the spot market is at least as competitive as Cournot. However, base load production depends on the strategic behavior in the spot market. This can be explained as follows: as long as the strategic behavior in the spot market is at least as competitive as Cournot, peak load outputs are independent of θ because

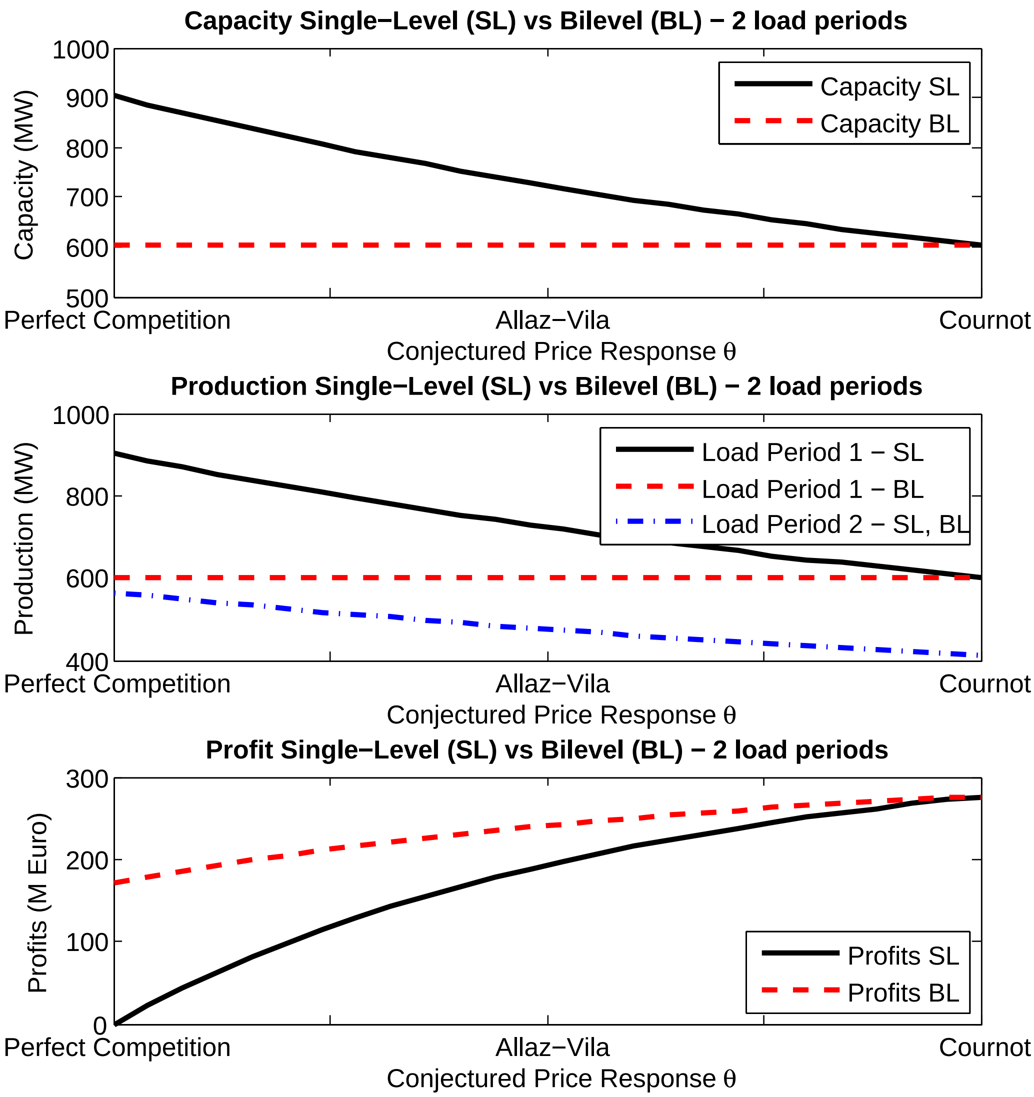

Figure 3-1: Built capacity, production and profit of one firm in the two load period numerical example.

Table 3.1: Bilevel equilibrium solution perfect competition ($\theta_l = 0$), Allaz-Vila ($\theta_l = 1/(2\alpha_l)$) and Cournot ($\theta_l = 1/\alpha_l$) second-stage competition.

l		**Peak**	**Base**
q_{il}[MW]	$\theta_l = 0$	602.6	564.6
q_{il}[MW]	$\theta_l = 1/(2\alpha_l)$	602.6	451.7
q_{il}[MW]	$\theta_l = 1/\alpha_l$	602.6	376.4
p_l [€/MWh]	$\theta_l = 0$	99.4	11.8
p_l [€/MWh]	$\theta_l = 1/(2\alpha_l)$	99.4	49.4
p_l [€/MWh]	$\theta_l = 1/\alpha_l$	99.4	74.5

agents are aware that building Cournot capacities will cause the peak period capacity constraint to bind and will limit production on the market to the Cournot capacity. However, given our demand data we also know that capacities will not be binding in the base period and as a consequence outputs will not be limited either. Hence during the base periods the closed loop model will find it most profitable to produce the equilibrium outcomes resulting from the particular conjectured-price response.

On the other hand, when considering the single-level model, the capacity (peak load production) does depend on θ. In particular, the single-level capacity will be determined by the spot market equilibrium considering the degree of competitive behavior specified by θ. We observe that for increasing θ between perfect competition and Cournot in the single-level model, less and less capacity is built until we reach the Cournot case, at which point the single and bilevel results are exactly the same. Comparing single and bilevel models for a given θ reveals that while their base load outputs are identical, see Tables 3.1 and 3.2, capacity and thus peak load production differs depending on θ. Figure 3-1 also shows that profits obtained in the bilevel model equal or exceed the profits of the single-level model. This gap is largest assuming perfect competition and becomes continuously smaller for increasing θ until the results are equal under Cournot. This means that the further away that spot market competition is from Cournot, the greater the difference between model outcomes.

In standard single-level oligopoly models, see Fudenberg and Tirole [50], without capacity constraints, perfect competition gives lower prices and total profits of firms, and greater consumer surplus, and market efficiency compared to Cournot compe-

Table 3.2: Single-level equilibrium solution perfect competition ($\theta_l = 0$), Allaz-Vila ($\theta_l = 1/(2\alpha_l)$) and Cournot ($\theta_l = 1/\alpha_l$) second-stage competition.

l		**Peak**	**Base**
q_{il}[MW]	$\theta_l = 0$	903.9	564.6
q_{il}[MW]	$\theta_l = 1/(2\alpha_l)$	723.1	451.7
q_{il}[MW]	$\theta_l = 1/\alpha_l$	602.6	376.4
p_l [€/MWh]	$\theta_l = 0$	24.0	11.8
p_l [€/MWh]	$\theta_l = 1/(2\alpha_l)$	69.2	49.4
p_l [€/MWh]	$\theta_l = 1/\alpha_l$	99.4	74.5

Table 3.3: Market efficiency (ME), consumer surplus (CS) and total profit in bilevel solutions.

	Perfect competition	**Allaz-Vila**	**Cournot**
ME [10^9€]	1.21	1.19	1.15
CS [10^9€]	0.87	0.68	0.57
Total profit [10^9€]	0.34	0.51	0.58

tition.[6] We observe that this occurs for this particular instance of the single and bilevel models, see Tables 3.2 and 3.3. Note that in Table 3.3 the units are given in billion or giga €, i.e., [10^9 €]. It can be readily proven more generally that market efficiency, consumer surplus, and average prices are greater for lower values of θ (more competitive second stage conditions) if LB_i are the same for those θ (and multipliers are positive), and capacity is not binding in every l.[7] However, we will also demonstrate by counter-example that this result does not necessarily apply when LB_i differ for different θ. In particular, in section 3.4.2 we will present an example in which Cournot competition counterintuitively yields higher market efficiency than perfect competition.

[6] Total Profit is defined as $\sum_l T_l(p_l - \delta)(q_{il} + q_{\text{-}il}) - \beta(x_i + x_{\text{-}i})$. Consumer Surplus (CS) is defined as the integral of the demand curve minus payments for energy, equal here to $\sum_l T_l(P_l^0 - p_l)(q_{il} + q_{\text{-}il})/2$. Market Efficiency (ME) is defined as CS plus Total Profits.

[7] This is proven by demonstrating that for smaller θ, the second stage prices will be lower and closer to marginal operating cost in load periods for those periods that capacity is not binding

3.3.5 Conjectured-Price Response Models with Switching Conjectures

In this section we consider and analyze alternative conjectured-price response models that are a variant of the previously presented models. In particular, we propose models in which a firm always has a Cournot conjectured-price response with respect to the output of a rival in periods when their capacity is binding, and an arbitrary conjectured-price response θ between perfect competition and Cournot when the rival's capacity is not binding. This type of model is arguably more realistic because producers in the second stage will recognize the times when rivals are at their capacity constraint and cannot increase output. This argument has been thoroughly discussed in [85]. In general, when solving models with switching conjectures, one has to have in mind that in a multi-player game, some generation companies may have binding capacity, but others might not in the same load period. In this case, the conjecture of the generation company at capacity would be θ and the rival's conjecture would be the Cournot conjecture. Such models are more difficult to solve than the previously presented models as some kind of iterative process has to be adopted and moreover, pure strategy equilibria might not exist. Hence, for the sake of simplicity of this analysis, we assume that a pure strategy equilibrium exists and we furthermore assume that the equilibrium is a symmetric one. A detailed analysis for asymmetric equilibria, which is more complicated and extensive than the case discussed in this section, can be found in work of Murphy and Smeers [85]. In this thesis we do not explore these alternative models outside of this section, however, we recognize their interest and we may address this topic in future research.

Let us now introduce this alternative type of model first for the single-level case. Note that, due to the symmetry assumption we get $LB_i = LB_{\text{-}i}$. Then, the alternative single-level model is almost identical to the single-level model introduced in section 3.3.1, with the only difference that now there are two different parameters for the conjectured-price response: $\hat{\theta}_l$ for load periods $l \in LB_i$ when capacity is binding; and θ_l for the non-binding load periods. Moreover, while θ_l can represent any strategic behavior between perfect competition and Cournot, the conjectured-price response in load periods when capacity is binding is chosen to always correspond to Cournot behavior, i.e., $\hat{\theta}_l = 1/\alpha_l$. With this in mind the single-level investment-market equilibrium conditions, which are given in (3.48)-(3.50), only differ from the previously presented single-level model (3.28)-(3.30) in its expression of $\frac{\partial \mathcal{L}_i}{\partial q_{il}}$ when $l \in LB_i$.

$$\forall i \begin{cases} \frac{\partial \mathcal{L}_i}{\partial q_{il}} = T_l p_l(q_{il}, q_{\text{-}il}) - T_l \hat{\theta}_l q_{il} - T_l \delta - \lambda_{il} = 0 & l \in LB_i \\ \frac{\partial \mathcal{L}_i}{\partial q_{il}} = T_l p_l(q_{il}, q_{\text{-}il}) - T_l \theta_l q_{il} - T_l \delta = 0 & l \notin LB_i \\ \frac{\partial \mathcal{L}_i}{\partial x_i} = -\beta + \sum_{l \in LB_i} \lambda_{il} = 0 & \\ q_{il} = x_i & l \in LB_i \\ q_{il} < x_i & l \notin LB_i \\ 0 \leq \lambda_{il} & \forall l \end{cases} \tag{3.48}$$

$$d_l = q_{il} + q_{\text{-}il} \quad \forall l \tag{3.49}$$

$$d_l = D_l^0 - \alpha_l p_l(q_{il}, q_{\text{-}il}) \quad \forall l \tag{3.50}$$

If we assume that a pure strategy equilibrium exists, then we can analyze this alternative single-level model in a manner paralleling the analysis in section 3.3.1. As a result, we can see that this model yields the capacity given in equation (3.52), which coincides with the expression of the bilevel capacity previously given in (3.47), once we substitute that $\hat{\theta}_l = 1/\alpha_l$.

$$x_i = \frac{\sum_{l \in LB_i}(D_l^0 T_l \prod_{n \neq l \in LB_i} \alpha_n) - \prod_{l \in LB_i} \alpha_l(\beta + \delta \sum_{l \in LB_i} T_l)}{\sum_{l \in LB_i}(T_l(2 + \alpha_l \hat{\theta}_l) \prod_{n \neq l \in LB_i} \alpha_n)} \quad , \forall i \tag{3.51}$$

$$= \frac{\sum_{l \in LB_i}(D_l^0 T_l \prod_{n \neq l \in LB_i} \alpha_n) - \prod_{l \in LB_i} \alpha_l(\beta + \delta \sum_{l \in LB_i} T_l)}{3 \sum_{l \in LB_i}(T_l \prod_{n \neq l \in LB_i} \alpha_n)} \quad , \forall i \tag{3.52}$$

Similar to the alternative single-level model, we can derive the alternative bilevel model by replacing θ_l with $\hat{\theta}_l$ when $l \in LB_i$, yielding market equilibrium equations identical to (3.39)-(3.41) with the only exception that in $\frac{\partial \mathcal{L}_i}{\partial q_{il}}$ for $l \in LB_i$ we now use the Cournot conjecture $\hat{\theta}_l = 1/\alpha_l$. Paralleling the bilevel analysis of section 3.3.2 it is easy to see that this alternative type of model yields the same closed form expression for capacity as the bilevel model previously presented in (3.47).

The comparison between these alternative single and bilevel models yields the result that if their sets of active load periods LB_i coincide and a pure strategy solution exists, both models (single and bilevel) yield the same capacity independent of the choice of strategic behavior θ_l in the spot market when capacity is not binding. Moreover, the same can be said when comparing two instances of the alternative bilevel model with different strategic behavior (i.e., different assumed values of θ_l) in load periods when capacity is not binding, i.e., if the active sets of load periods are the same at equilibrium, then they both yield the same capacity solution. The proof of the comparison of two bilevel models is a simple extension of the Allaz-Vila analysis in [85], which furthermore provides numerical examples. Moreover, when comparing the bilevel model with switching conjectures to the bilevel model previously presented in this chapter, it can be said that if they are at capacity in the same load periods, then their capacity will be the same; however, when the binding load periods are different, then the alternative model where strategic behavior switches can yield a different capacity than the model where the second stage behavior is constant.

Asymmetric equilibria may exist, even if the firms themselves are symmetric. This may happen, for example, when the conjectural variation assumed for the production game is greater than Cournot. The character of the equilibrium in this case is that one generation company is at capacity and sees the conjectural variation in the rival while the generation company below capacity sees the Cournot conjectural variation. If there is no symmetric equilibrium because the production in one load period exceeds capacity when using θ but falls below capacity when using $\hat{\theta}$, one firm is at capacity and the other is below. Total capacity is larger than with the Cournot conjecture. If there is a symmetric equilibrium with a load period near capacity but below, there may be an asymmetric equilibrium in which one firm reduces capacity to cause the capacity to bind because that causes a discrete drop in production by the other firm in that load period. For a detailed analysis of asymmetric equilibria and numerical examples that show this can happen with the Allaz-Vila conjectural variation, the reader is referred to Murphy and Smeers [85].

3.4 Ranking of Bilevel Equilibria: Capacity and Market Efficiency

In this section we make some observations concerning capacity results in bilevel equilibria. We also discuss the ambiguities that occur regarding social welfare when comparing two bilevel equilibrium solutions with different strategic behavior in the spot market. In section 3.4.1 we prove that the capacity of a bilevel model with competitive behavior between perfect competition and Cournot can be lower or equal to the bilevel Cournot second stage capacity, depending on the choice of data, and moreover, that it cannot be higher for symmetric players in the two period case. In 3.4.2 we prove by counter-example that the ranking of bilevel conjectured-price response equilibria, in terms of market efficiency (aggregate consumer surplus and market surplus) and consumer welfare, is parameter dependent.

3.4.1 Comparisons of Capacity from Bilevel Equilibria

In this section we analyze the effects of the strategic behavior in the spot market on capacity in the bilevel model. This work is an extension of the work of Murphy and Smeers [85], in which they compare a bilevel Cournot model to a model with an additional forward market stage, i.e., a bilevel Allaz-Vila model with capacity decisions. They find that, depending on the data, the capacity yielded by the bilevel Allaz-Vila model can either be more, less or equal to the capacity given by the bilevel Cournot model in a market with asymmetric players. We extend their results to general conjectural variations considering symmetric companies, and compare our bilevel model with Cournot second stage competition to a bilevel model with arbitrary second stage competition between perfect competition and Cournot. We show that in this comparison the capacity yielded by conjectured-price response second stage competition can be less (decreasing) or equal to the bilevel Cournot capacity, which is shown in section 3.4.1. Further, in section 3.4.1, we prove a stronger result: that the former capacity cannot exceed the latter for symmetric agents and two load periods.

Conjectured-Price Response Can Yield Same or Less Capacity

Part (a) of Proposition 3.2 proves that if two bilevel solutions for different θ between perfect competition and Cournot competition have the same active set of load periods, then capacity is the same for those values. The corresponding numerical example has been presented in section 3.3.4. This demonstrates that it is possible for two different bilevel models to yield the same capacity.

When the active sets of two solutions with different strategic behavior coincide, we know that their capacity must be equal. However, from the closed form expression for capacity, given in (3.47), we also know that when active sets of load periods do not coincide, then the solutions will generally not be the same. We show that the relationship between capacities resulting from different θ is ambiguous.

For an example in which the bilevel Cournot second stage capacity is strictly above the capacities yielded by other bilevel models with more competitive strategic behavior, we revisit the numerical example in section 3.3.4 and increase the base demand intercept D_2^0 to 1650 MW. In Table 3.4 we present the corresponding bilevel results for second stage perfect competition, Allaz-Vila, and Cournot second stage competition. It can be observed that for second stage Cournot competition, the capacity of 602.6 MW is only binding in the peak period. This fact does not change for more competitive strategic spot market behavior until we reach a certain threshold, θ around $1/(1.7\alpha)$, when base load production exceeds the capacity of 602.6 MW. At this point the set of active load periods at equilibrium changes and capacity is binding in both peak and base load period and the new capacity is 554.7 MW.

In Table 3.5 we present the market efficiency, the consumer surplus and the total profits of the decreasing capacity solution in billion or giga €, i.e., [10^9€]. We observe that Allaz-Vila second stage competition yields a lower market efficiency than Cournot second stage competition. In section 3.4.2 we will demonstrate that this market efficiency result is ambiguous, as we present a counter-example where market efficiency is higher under Allaz-Vila than under Cournot second stage competition, even though capacity is less in the Allaz-Vila case than in the Cournot case.

Table 3.4: Bilevel equilibrium less capacity under more competition: perfect competition ($\theta_l = 0$), Allaz-Vila ($\theta_l = 1/(2\alpha_l)$), and Cournot ($\theta_l = 1/\alpha_l$) second-stage competition.

l		**Peak**	**Base**
q_{il}[MW]	$\theta_l = 0$	554.7	554.7
q_{il}[MW]	$\theta_l = 1/(2\alpha_l)$	554.7	554.7
q_{il}[MW]	$\theta_l = 1/\alpha_l$	602.6	517.6
p_l [€/MWh]	$\theta_l = 0$	111.3	65.5
p_l [€/MWh]	$\theta_l = 1/(2\alpha_l)$	111.3	65.5
p_l [€/MWh]	$\theta_l = 1/\alpha_l$	99.4	74.5

Table 3.5: Market efficiency (ME), consumer surplus (CS) and total profit of bilevel solutions in which more competition yields less capacity.

	Perfect competition	**Allaz-Vila**	**Cournot**
ME [10^9€]	1.32	1.32	1.33
CS [10^9€]	0.66	0.66	0.66
Total profit [10^9€]	0.66	0.66	0.67

Conjectured-Price Response Can Yield More Capacity

When presenting the case in which conjectured-price response yields more capacity than Cournot in the bilevel game in [85], Murphy and Smeers mainly restrict their discussion to a case with two load periods, peak and base. We will do the same here. However, we also demonstrate that in the case of symmetric agents and two load periods the conjectured-price response assumption cannot yield more capacity. This will be proven in Lemma 3.3.

Lemma 3.3. *In the case of symmetric agents and two load periods, the solution of the conjectured-price response bilevel generation expansion game can never yield more capacity than the Cournot bilevel game.*

Proof. Due to Proposition 3.2 we know that this increasing capacity case could only happen when the active sets of bilevel solutions with different strategic behavior do not coincide. Hence we consider two separate cases: case one where the bilevel Cournot capacity is binding only in the peak period; and case two where it is binding in both load periods.

Case 1: Let therefore x_{peak} denote the bilevel Cournot second stage capacity solution, which is only binding in the peak period and let x_{both} denote the capacity of the bilevel conjectured-price response equilibrium, where capacity is binding in both load periods. Then, from (3.47) we obtain the values for both terms and they are given below.

$$x_{\text{peak}} = \frac{D_1^0 T_1 - \alpha_1(\beta + \delta T_1)}{3T_1} \tag{3.53}$$

$$x_{\text{both}} = \frac{D_1^0 T_1 \alpha_2 + D_2^0 T_2 \alpha_1 - \alpha_1 \alpha_2(\beta + \delta(T_1 + T_2))}{3(T_1\alpha_2 + T_2\alpha_1)} \tag{3.54}$$

In Table 3.4 we have presented an example where x_{peak} was larger than x_{both}. However, in order for the opposite case to be possible and feasible, the following two conditions have to hold. First, production in the base period, given by (3.42), for the Cournot case cannot exceed its capacity x_{peak}, which is formulated in (3.55) and second, x_{both} has to be strictly larger than x_{peak}, which is expressed in (3.56).

$$\frac{D_2^0 - \alpha_2\delta}{3} \leq x_{\text{peak}} \tag{3.55}$$

$$x_{\text{peak}} < x_{\text{both}} \tag{3.56}$$

If we insert the expression of x_{peak} given in (3.53) into (3.55) and simplify the resulting inequality[8], we obtain a lower bound on the peak demand intercept D_1^0, which is given in (3.57). Similarly, if we insert expressions (3.53) and (3.54) into (3.56) and simplify the resulting inequality[9], we obtain a strict upper bound on D_1^0,

[8] The inequality given by (3.55) reads $\frac{D_2^0-\alpha_2\delta}{3} \leq \frac{D_1^0 T_1 - \alpha_1(\beta+\delta T_1)}{3T_1}$. First, we multiply both sides by 3, then we multiply the resulting inequality by T_1, add $\alpha_1(\beta + \delta T_1)$ and finally we divide by T_1. The resulting inequality then reads $\frac{(D_2^0-\alpha_2\delta)T_1+\alpha_1(\beta+\delta T_1)}{T_1} \leq D_1^0$.

[9] The inequality given by (3.56) reads $\frac{D_1^0 T_1 - \alpha_1(\beta+\delta T_1)}{3T_1} < \frac{D_1^0 T_1\alpha_2 + D_2^0 T_2 \alpha_1 - \alpha_1\alpha_2(\beta+\delta(T_1+T_2))}{3(T_1\alpha_2+T_2\alpha_1)}$. Again we multiply both sides by 3, and then we multiply the numerator of each side with the denominator of the other side. As both sides now have the same denominator we only compare resulting numerators, which yield $(D_1^0 T_1 - \alpha_1(\beta + \delta T_1))(T_1\alpha_2 + T_2\alpha_1) = D_1^0 T_1^2 \alpha_2 + D_1^0 T_1 T_2 \alpha_1 - \alpha_1(\beta + \delta T_1)(T_1\alpha_2 + T_2\alpha_1)$ $< D_1^0 T_1^2 \alpha_2 + D_2^0 T_1 T_2 \alpha_1 - T_1\alpha_1\alpha_2(\beta + \delta(T_1 + T_2))$. Now we bring all terms that include D_1^0 to the left side of the inequality and the remaining terms to the right. Then $D_1^0(T_1^2\alpha_2 + T_1 T_2 \alpha_1 - T_1^2\alpha_2)$ $= D_1^0 T_1 T_2 \alpha_1 < D_2^0 T_1 T_2 \alpha_1 - T_1\alpha_1\alpha_2(\beta + \delta(T_1 + T_2)) + \alpha_1(\beta + \delta T_1)(T_1\alpha_2 + T_2\alpha_1) = \alpha_1(D_2^0 T_1 T_2 -$ $T_1\alpha_2(\beta + \delta(T_1 + T_2)) + (\beta + \delta T_1)(T_1\alpha_2 + T_2\alpha_1))$. Dividing both sides by $T_1 T_2 \alpha_1$ yields that $D_1^0 <$ $\frac{D_2^0 T_1 T_2 + (\beta+\delta T_1)(T_1\alpha_2+T_2\alpha_1) - T_1\alpha_2(\beta+\delta(T_1+T_2))}{T_1 T_2}$.

which is given in (3.58).

$$\frac{(D_2^0-\alpha_2\delta)T_1+\alpha_1(\beta+\delta T_1)}{T_1} \leq D_1^0 \tag{3.57}$$

$$D_1^0 < \frac{D_2^0 T_1 T_2+(\beta+\delta T_1)(T_1\alpha_2+T_2\alpha_1)-T_1\alpha_2(\beta+\delta(T_1+T_2))}{T_1 T_2} \tag{3.58}$$

It is easy to verify that both the lower bound in (3.57) and the strict upper bound in (3.58) of D_1^0 yield the same value, which is a contradiction to the strict inequality and hence to the assumption that $x_{\text{peak}} < x_{\text{both}}$. Thus the capacity under more intensive competition (x_{both}) cannot exceed the Cournot capacity (x_{peak}) under the assumption that Cournot binds only in the peak period while more intensive competition binds in both.

Case 2: We furthermore show that the case of increasing capacity under more intensive competition can never happen when the active set of load periods is reversed, i.e., when the bilevel Cournot second stage capacity is binding in both load periods and the bilevel conjectured-price response capacity is only binding in the peak period. Let us assume we had such a case, then the conjectured-price response production in the base period, given by (3.42), will be strictly less than its capacity x_{peak} because we know that capacity is only binding in the peak load period. Moreover, from (3.42) it is easy to see that unrestricted Cournot base production would be less than (3.42) for all $\theta_2 \leq 1/\alpha_2$, all of which is shown in (3.59). As we assumed that the Cournot bilevel capacity is binding in both load periods, it follows that x_{both} will be less or equal to the unrestricted Cournot base production, which is expressed in (3.60).

$$\frac{D_2^0-\alpha_2\delta}{3} \leq \frac{D_2^0-\alpha_2\delta}{2+\alpha_2\theta_2} < x_{\text{peak}} \tag{3.59}$$

$$x_{\text{both}} \leq \frac{D_2^0-\alpha_2\delta}{3} \tag{3.60}$$

Similarly to before we insert the expression of x_{peak} and x_{both} into (3.59) and (3.60) and simplify the resulting inequalities to obtain a lower and upper bound on the base demand intercept D_2^0, which are given in (3.61) and (3.62). It is easy to verify that both the lower bound in (3.61) and the strict upper bound in (3.62) of D_2^0

yield the same value, which is a contradiction to the strict inequality and hence to the assumption that x_{both} (Cournot capacity) $< x_{\text{peak}}$ (capacity under more intensive competition).

$$\tfrac{D_1^0 T_1\alpha_2 - \alpha_1\alpha_2(\beta+\delta(T_1+T_2)) + \alpha_2\delta(T_1\alpha_2+T_2\alpha_1)}{T_1\alpha_2} \leq D_2^0 \tag{3.61}$$

$$D_2^0 < \tfrac{D_1^0 T_1 - \alpha_1(\beta+\delta T_1) + \alpha_2\delta T_1}{T_1} \tag{3.62}$$

Hence, we have demonstrated that, for two load periods, the case in which capacity increases with increasing competition (which occurred for an asymmetric case in Murphy and Smeers [85]) cannot happen for symmetric agents. □

Our result therefore shows that, for the two load period case by Murphy and Smeers [85], asymmetry is a necessary condition in order for the capacity of the bilevel conjectured-price response solution to be larger than in the bilevel second stage Cournot equilibrium. We raise the hypothesis that in the case of symmetric agents this might generally be true for multiple load periods as well. However, proving this hypothesis or finding a counterexample is out of the scope of this thesis and will be a topic of future research.

3.4.2 Ambiguity in Ranking of Bilevel Equilibria when Binding Load Periods Differ for Different Values of Conjectural Variations

In this section we show by counter-example that the ranking of the bilevel conjectured-price response equilibria, in terms of market efficiency and consumer welfare, is parameter dependent. An interesting result we obtain is that it is possible for the bilevel model that assumes perfectly competitive behavior in the market to actually result in lower market efficiency (as measured by the sum of surpluses for all parties and load periods), lower consumer surplus, and higher average prices than when Cournot competition prevails. This counter-intuitive result implies that contrary to the common belief that requiring marginal cost bidding is enough to protect consumers - a belief underlying some regulatory market rules - it can actually be harmful. Perfect markets in both the investment and the operation stage protect consumers, however, partially imperfect markets (as in our case in the investment stage) can lead to the effects described in this section. In [57] Grimm and Zoettl have arrived at a similar result comparing perfectly competitive and Cournot spot market behavior, however, they only look at the polar cases of perfect competition or Cournot-type competition. In our work we generalize strategic behavior using conjectural variations and look at a range of strategic behavior, from perfect competition to Cournot competition and we furthermore observe that an intermediate solution between perfect and Cournot competition can lead to even larger social welfare and consumer surplus despite yielding a level of installed capacity intermediate between the perfect competition and the Cournot cases. In particular: *The ranking of conjectured-price response equilibria in terms of market efficiency and consumer welfare is parameter dependent.* This occurs because in general the LB_i differ among the solutions. It does not occur when LB_i are the same for all θ and multipliers are positive as proven (and illustrated) in the previous section.

A counter-example: Let us now consider two firms both making an investment in generation capacity using the following data:

- Twenty equal length load periods l, so $T_l = 438$ [h/year] for $l = 1, \ldots, 20$
- Demand intercept D_l^0, obtained by $D_l^0 = 2000 - 50(l-1)$ [MW] for $l = 1, \ldots, 20$
- Demand slope α_l, obtained by $D_l^0/250$ for $l = 1, \ldots, 20$
- Capital cost $\beta = 46000$ [€/MW/year]
- Operating cost $\delta = 11.8$ [€/MWh]

First we will assume perfect competition, i.e., $\theta_l = 0$. We solve the resulting bilevel game by diagonalization, see Hu and Ralph [63], which is an iterative method in which firms take turns updating their first-stage capacity decisions, each time solving a two-stage MPEC while considering the competition's capacity decisions as fixed. The bilevel equilibrium solution assuming perfect competition in stage two is shown in Table 3.6. Second, we assume Cournot competition in the spot market, i.e., $\theta_l = 1/\alpha_l$. Again we solve the bilevel game by diagonalization, yielding the results shown in Table 3.7. We observe that under second stage perfect competition, the capacity of 456.2 MW is binding in every load period and prices never fall to marginal operating cost. Moreover, the total installed capacity of 912.4 MW is significantly lower than that installed under Cournot, which is 1101.2 MW. On the other hand, under Cournot competition, each firm's capacity of 550.6 MW is binding only in the first six load periods and the firms exercise market power by restricting their output to below capacity in the other fourteen periods. Furthermore considering that the Cournot capacity is well above the perfectly competitive capacity, it follows that during the six peak load periods, perfectly competitive prices will be higher than Cournot prices. The single-level equilibrium solutions assuming Cournot competition, perfectly competitive behavior and Allaz-Vila competition are presented in Tables 3.7, 3.10 and 3.11 respectively. Under Cournot competition, single and bilevel equilibrium solutions coincide. Meanwhile the system optimal plan, which is obtained by central planning under a maximization of social welfare objective, yields the same solution as the single-level equilibrium under perfect competition, which is presented in Table 3.10. As expected, this solution exhibits the highest total installed capacity of 1651.8 MW, the lowest prices and the greatest market efficiency.

This bilevel investment game can be viewed as a kind of prisoners' dilemma among multiple companies. An individual company might be able to unilaterally improve its profit by expanding capacity, with higher volumes making up for lower prices. But if all companies do that, then everyone's profits could be lower than if all companies instead refrained from building. (Of course, in this prisoners' dilemma metaphor we have not taken into account another set of players that is better off when the companies all build. These are the consumers, who enjoy lower prices and more consumption; as a result, overall market efficiency as measured by total market surplus may improve when firms "cheat".)

Standard (single stage) oligopoly models, see Fudenberg and Tirole [50], with-

Table 3.6: Bilevel equilibrium solution under second-stage perfect competition ($\theta_l = 0$) with capacity $x_i = 456.2$ MW.

l	1	2	3	4	5	6	7	8	9	10
q_{il} [MW]	456.2	456.2	456.2	456.2	456.2	456.2	456.2	456.2	456.2	456.2
p_l [€/MWh]	135.9	133.0	129.9	126.7	123.3	119.7	115.8	111.8	107.4	102.8

l	11	12	13	14	15	16	17	18	19	20
q_{il} [MW]	456.2	456.2	456.2	456.2	456.2	456.2	456.2	456.2	456.2	456.2
p_l [€/MWh]	97.9	92.7	87.1	81.0	74.5	67.5	59.9	51.6	42.6	32.8

out capacity constraints find that perfect competition gives lower prices and greater market efficiency than Cournot. Considering that standard result, our results seem counter-intuitive, but they are due to the two-stage nature of the game. In particular, less intensive competition in the commodity market can result in more investment and more consumer benefits than if competition in the commodity market is intense (price competition a la Bertrand). In terms of the prisoners' dilemma metaphor, higher short run margins under Cournot competition provide more incentive for the "prisoners" to "cheat" by adding capacity. Note that in order to get these counter-intuitive results, firms do not need to be symmetric, as shown in a numerical example in Wogrin et al. [118].

Finally, we solve the bilevel game assuming Allaz-Vila as competitive behavior between perfect competition and Cournot, i.e., $\theta_l = 1/(2\alpha_l)$. This yields the equilibrium given in Table 3.8. Comparing the market efficiency (ME) and the consumer surplus (CS) that we obtain in the perfectly competitive, Cournot, Allaz-Vila and the social welfare maximizing solutions in Table 3.9 in billion €, we observe that, surprisingly, apart from the welfare maximizing solution the highest social welfare and the highest consumer surplus is obtained under the intermediate Allaz-Vila case. Even more surprising is that the capacity obtained under Allaz-Vila competition is lower than the Cournot capacity, but still yields a higher social welfare. This is because the greater welfare obtained during periods when capacity is slack (and Allaz-Vila prices are lower and closer to production cost) offsets the welfare loss during peak periods when the greater Cournot capacity yields lower prices.

Table 3.7: Bilevel equilibrium solution under Cournot second-stage competition ($\theta_l = 1/\alpha_l$) and single-level Cournot equilibrium solution, both with capacity $x_i = 550.6$ MW.

l	1	2	3	4	5	6	7	8	9	10
q_{il} [MW]	550.6	550.6	550.6	550.6	550.6	550.6	539.9	524.0	508.2	492.3
p_l [€/MWh]	112.4	108.8	105.1	101.2	97.1	92.7	91.2	91.2	91.2	91.2
l	11	12	13	14	15	16	17	18	19	20
q_{il} [MW]	476.4	460.5	444.6	428.8	412.9	397.0	381.1	365.2	349.4	333.5
p_l [€/MWh]	91.2	91.2	91.2	91.2	91.2	91.2	91.2	91.2	91.2	91.2

Table 3.8: Bilevel equilibrium solution assuming Allaz-Vila second-stage competition ($\theta_l = 1/(2\alpha_l)$) with capacity $x_i = 515.2$ MW.

l	1	2	3	4	5	6	7	8	9	10
q_{il} [MW]	515.2	515.2	515.2	515.2	515.2	515.2	515.2	515.2	515.2	515.2
p_l [€/MWh]	121.2	117.9	114.4	110.8	106.9	102.8	98.5	93.9	89.0	83.8
l	11	12	13	14	15	16	17	18	19	20
q_{il} [MW]	515.2	515.2	515.2	514.5	495.5	476.4	457.3	438.3	419.2	400.2
p_l [€/MWh]	78.3	72.3	66.0	59.4	59.4	59.4	59.4	59.4	59.4	59.4

Another surprise is that not only market efficiency but also profits are non-monotonic in θ. Both perfect competition and Cournot profits are higher than Allaz-Vila profits; the lowest profit thus occurs when market efficiency is highest, at least under these parameters. However, higher profits do not always imply lower market efficiency, as a comparison of the perfect competition and Cournot single-level cases shows. Cournot shows higher profit, consumer surplus, and market efficiency than perfect competition. That is, Cournot is pareto superior to perfect competition under these parameters because all parties are better off under the Cournot equilibrium.

Finally, we observe the market efficiency (ME), the consumer surplus (CS) and total profits of the single-level model assuming perfect competition, Allaz-Vila and Cournot competition in the market. As opposed to the bilevel case, market efficiency in the single-level solutions increases monotonically with the level of competition in the market and is therefore highest under perfect competition.

Table 3.9: Market efficiency (ME), consumer surplus (CS) and total profit in bilevel solutions and social welfare maximizing solution.

	Perfect competition	**Allaz-Vila**	**Cournot**	**Social welfare**
ME [10^9€]	1.24	1.30	1.28	1.47
CS [10^9€]	0.62	0.72	0.64	1.44
Total profit [10^9€]	0.62	0.58	0.64	0.03

Table 3.10: Single-level equilibrium solution assuming perfect competition and system optimal plan solution with capacity $x_i = 825.9$ MW.

l	1	2	3	4	5	6	7	8	9	10
q_{il} [MW]	825.9	825.9	825.9	825.9	825.9	825.9	809.9	786.1	762.2	738.4
p_l [€/MWh]	43.5	38.2	32.7	26.8	20.6	14.0	11.8	11.8	11.8	11.8
l	11	12	13	14	15	16	17	18	19	20
q_{il} [MW]	714.6	690.8	667.0	643.1	619.3	595.5	571.7	547.9	524.0	500.2
p_l [€/MWh]	11.8	11.8	11.8	11.8	11.8	11.8	11.8	11.8	11.8	11.8

Table 3.11: Single-level equilibrium solution assuming Allaz-Vila competition ($\theta_l = 1/(2\alpha_l)$) with capacity $x_i = 660.7$ MW.

l	1	2	3	4	5	6	7	8	9	10
q_{il} [MW]	660.7	660.7	660.7	660.7	660.7	660.7	647.9	628.8	609.8	590.7
p_l [€/MWh]	84.8	80.6	76.1	71.4	66.5	61.2	59.4	59.4	59.4	59.4
l	11	12	13	14	15	16	17	18	19	20
q_{il} [MW]	571.7	552.6	533.6	514.5	495.5	476.4	457.3	438.3	419.2	400.2
p_l [€/MWh]	59.4	59.4	59.4	59.4	59.4	59.4	59.4	59.4	59.4	59.4

Table 3.12: Market efficiency (ME), consumer surplus (CS) and total profit in single-level solutions.

	Perfect competition	**Allaz-Vila**	**Cournot**
ME [10^9€]	1.47	1.39	1.28
CS [10^9€]	1.44	0.92	0.64
Total profit [10^9€]	0.03	0.47	0.64

3.5 Conclusions

In this chapter we compare two types of models for modeling the generation capacity expansion game: a single-level model describing a game in which investment and operation decisions are made simultaneously, and a bilevel equilibrium model, where investment and operation decisions are made sequentially. The purpose of this comparison is to emphasize that when resorting to easier, less complicated single-level models, instead of solving the more realistic but more complicated bilevel models, the results may differ greatly and to characterize when results are similar. In both models the market is represented via a conjectured-price response, which allows us to capture various degrees of oligopolistic behavior. Setting out to characterize the differences between these two models, we have found that for one load period, the bilevel equilibrium equals the single-level Cournot equilibrium for any choice of conjectured-price response between perfect competition and Cournot competition - a generalization of Kreps and Scheinkman-like [74] findings. In the case of multiple load periods, this result can be extended. In particular, if bilevel models under different conjectures have the same set of load periods in which capacity is constraining and the corresponding multipliers are positive, then their first stage capacity decisions are the same, although not their outputs during periods when capacity is slack. Furthermore, if the Cournot single and bilevel solutions have the same periods when capacity constrains, then their solutions are identical. We also explore alternative conjectured-price response models in which the strategic second stage competition switches to Cournot in load periods in which rivals' capacity is binding. When capacity does not bind, the strategic behavior can range from perfect competition to Cournot. Such alternative models may be more realistic, however, pure strategy equilibria might not exist and they are more difficult to solve.

A first numerical example indicates that when having market behavior close to Cournot competition, the additional effort of computing the bilevel model (as opposed to the simpler single-level model) does not pay off because the outcomes are either exactly the same or very similar. But if behavior on the spot market is far from

Cournot and approaching perfect competition, the additional modeling effort might be worthwhile, as the bilevel model is capable of depicting a feature that the single-level model fails to capture, which is that generation companies would not voluntarily build all the capacity that might be determined by the spot market equilibrium if that meant less profits for themselves. Thus the bilevel model could be useful to evaluate the effect of alternative market designs for mitigating market power in spot markets and incenting capacity investments in the long run, e.g., capacity mechanisms, in Sakellaris [107]. Extensions could also consider the effect of forward energy contracting (as in Murphy and Smeers [85]). These policy analyses will be the subject of future research. The second numerical example shows that when the sets of active load periods do not coincide for bilevel solutions with different strategic spot market behavior, then the bilevel conjectured-price response capacity can be less than the bilevel Cournot second stage capacity. We also prove that the former capacity cannot exceed the latter when there are symmetric agents and two load periods.

The third numerical example demonstrates that depending on the choice of parameters, more competition in the spot market may lead to less market efficiency and less consumer surplus in the bilevel model. This surprising result indicates that regulatory approaches that encourage or mandate marginal pricing in the spot market in order to protect consumers may actually lead to situations in which both consumers and generation companies are worse off.

In future research we may address the issue of existence and uniqueness of solutions, as has been done for the Cournot case by Murphy and Smeers [84], who found that a pure-strategy bilevel equilibrium does not necessarily exist but if it exists it is unique. We may also address the question concerning under what *a priori* conditions the active sets of single-level and bilevel equilibria coincide. There may be further investigation of games in which the conjectural variation is endogenous, resulting from the possibility that power producers might adopt the Cournot conjecture in binding load periods since they may be aware that their rivals cannot expand output at such times. The extension of these games to multi-year games with sequential capacity decisions will be presented in the following chapters.

Chapter 4

Bilevel Generation Expansion Optimization Models

Since chapter 3 has established that bilevel generation expansion models are more realistic than single-level models, we now want to extend the theoretically-sized models from chapter 3 to more realistic, large-scale, multi-year, multi-load period and multi-technology models. In order to model large-scale bilevel equilibrium models formulated as EPECs, the first step is to formulate the corresponding MPECs. Therefore this chapter is dedicated entirely to bilevel models representing the generation expansion problem, i.e., building new power stations, of one generation company in particular, which is formulated as an MPEC and represents an original contribution of this thesis.

The bilevel formulation allows for the differentiation of investment and operating decisions. Investment decisions of only one generation company are taken in the upper level with the objective to maximize profits and generation decisions by all companies are considered in the lower level which is constituted by the conjectured-price response market equilibrium. If one (and only one) firm decides investments, then this yields a bilevel optimization problem which is the type of model we discuss in this chapter. If instead all firms compete in capacities, this yields a bilevel equilibrium problem, which is the type of model that will be discussed in the following chapter 5.

In particular, we start out by an introduction of this topic in section 4.1 and continue by modeling and discussing the bilevel expansion optimization model in detail in section 4.2, which can be formulated as an MPEC and whose basic version has already been introduced in chapter 2. Apart from the theoretical methodology, a case study is presented and results are analyzed thoroughly. Then, in section 4.3 we extend the presented framework to incorporate stochasticity of competitors' investment decisions and strategic behavior, derive an extended stochastic version of the MPEC and verify the model in a numerical example. Section 4.4 describes how to formulate the possible model extensions, which have been raised in section 2.4.3 and which allow for a more realistic representation of the generation expansion model. Finally, section 4.5 concludes this chapter.

4.1 Introduction

Due to the liberalization of electricity markets the task of taking generation capacity investment decisions has become an even more complex problem than it already has been under a centralized framework. Generation companies are now exposed to a higher level of risk, having to deal with the strategic behavior of competitors in imperfect markets and coping with the uncertainty due to fuel prices, demand and hydro inflows among others.

In this thesis we focus on game-theoretic methods in liberalized frameworks, in particular on bilevel formulations since - as we have established in chapter 3 - bilevel models are more realistic than single-level models because they embody a sequential decision-making process. Game theory is particularly useful in the energy sector because it allows us to analyze the strategic behavior of agents - in our case generation companies - whose interests are opposing and whose decisions influence the outcome of other agents.

With the newly developed model in this chapter we would like to address some of the shortcomings of existing approaches in the literature. First of all, even though existing single-level approaches like in Centeno et al., Murphy and Smeers or Ventosa et al. [25, 84, 113] are adequate and useful to approximate the generation capacity problem, they do not model the significant temporal separation between when capacity decisions are taken and when energy is produced with that capacity. We overcome this problem by proposing a bilevel model. Furthermore, existing bilevel approaches in the literature assume either perfectly competitive, see Garcia-Bertrand et al. [54], or Cournot behavior in the spot market, see Haurie et al. or Ventosa et al. [59, 113] . We want to extend these approaches to also capture intermediate oligopolistic behavior in order to explore how capacity decisions would change if competitive behavior in the spot market changed. The model presented in this chapter represents the market via a conjectured-price response formulation like the one presented in 2.3.1, thereby allowing us to model a range of oligopolistic market behavior. Finally, the model yields an investment schedule over the entire time horizon, as opposed to a static investment decision for a future target year, as done by Kazempour and Conejo [70].

In particular, in this chapter a bilevel optimization model - formulated as an MPEC - is proposed which takes the point of view of a single investing agent faced with generation capacity expansion decisions. This model represents an original contribution of this thesis and is presented in section 4.2, which includes a basic version of the MPEC model and a corresponding numerical example, both of which have previously been published in an international journal [27]. In the upper level the investing agent maximizes its profit deciding its generation capacity. The lower level problem corresponds to a conjectural variations market equilibrium closely inspired by Centeno et al. [24], which can be formulated as a convex optimization problem corresponding to Barquín et al. [5]. Replacing the lower level by its KKT conditions, yields an MPEC.

Since this MPEC is a deterministic model, and there are several sources of uncertainty in the generation expansion problem, in section 4.3 we extend this basic model formulation to a stochastic optimization problem. This newly developed model can be formulated as a stochastic MPEC, where the investing agent maximizes its expected profit in the upper level subject to various scenarios of the lower level equilibrium. The scenarios of the particular case considered in section 4.3 are dependent on the uncertainty regarding the investment decisions of the competition and a corresponding strategic behavior on the market. As investment decisions crucially depend on the strategic behavior of the generation companies in the market, which we model using a conjectured-price-variation approach, we incorporate a set of scenarios of conjectured-price responses into the generation capacity expansion problem, each representing a different realization of possible market behavior. The proposed model, as well as the presented case study, represent another original contribution of this thesis and have been published in an international journal [116].

In order to keep the model formulation simple, uncertainty corresponding to competition is being considered as the most driving factor in section 4.3. Additional sources of uncertainty, such as fuel prices or demand growth, are not considered in the stochastic MPEC model of section 4.3, however, in section 4.4 we show how to approach these other important sources of uncertainty. Furthermore, in section 4.4 we also address possible model extensions, such as introduction of hydro energy, capacity mechanisms, financial hedging or how to discretize investment decisions in the model formulation. Such extensions have been stated as very desirable in section 2.4.3 as they improve the representation of reality of the proposed MPEC models. These extensions provide the tools to explore and analyze the electricity market using mathematical models.

It is important to keep in mind that the type of model presented in this chapter prioritizes the capacity investment decision of only one market agent, which could be seen as a limitation of the model. But if we consider that we have one large generation company, and the rest of the firms are followers, then the large company has a pretty good idea about how the followers are going to respond to certain actions. In this case the MPEC approach presented in this chapter is a suitable approach to assist this large generation company when taking capacity investment decisions since we are representing a Stackelberg-type situation; however, when the scope of the study is different, for example when the regulator or the government want to study how the generation capacity is going to evolve over the next decades, then models like the ones by Kazempour et al. or Wogrin et al. [70, 116] or Kazempour and Conejo [69] are not that suitable considering that there is clear favoritism towards only one generation company. Therefore when the purpose of the model is to study the general evolution of the generation capacity of a country or region, then a different type of model is needed which does not prioritize any generation company in particular and which allows analyzing the strategic investment trends when all GENCOs are competing against each other. We propose such a model in the following chapter 5 of this thesis and remind the reader that all models presented in chapter 4 have to be interpreted as a tool whose purpose it is to be employed by a single GENCO exploring its capacity investment options. Moreover, the ability to solve MPEC models will be extremely useful when solving more complicated EPEC models in chapter 5.

4.2 Generation Expansion Planning: An MPEC Approach

This section describes the basic version of the bilevel generation expansion optimization model that assists one generation company in taking its capacity investment decisions. First, in section 4.2.1 the lower level or production stage of the bilevel problem, which models the strategic spot market equilibrium among all market participants, is introduced, followed by a description of the upper level or investment stage in section 4.2.2 where capacity decisions are being taken. Merging the formulations of upper and lower level, leads to the formulation of the entire problem as an MPEC which is given by section 4.2.3. This type of model embodies a sequential decision making process (similar to the Stackelberg game). Finally, section 4.2.4 presents a numerical example of the presented MPEC model which contains a detailed analysis of results.

4.2.1 Formulation of the Lower Level (or Production Stage)

The lower level or production stage of the MPEC generation expansion model corresponds to the conjectured-price response market equilibrium which has been introduced in chapter 2. We recall that the market equilibrium can be written as a system of nonlinear equations given by (2.35)-(2.43), which arise from the KKT conditions of each individual generation company, or an equivalent convex quadratic optimization problem which has been presented in (2.45)-(2.48) and which we remember below:

$$\begin{aligned}\min_{q,d} \quad & \sum_{ijyl} \frac{\delta_{ij} q_{ijyl} T_{yl}}{(1+F)^y} + \frac{1}{2} \sum_{iyl} \frac{\theta_{iyl} T_{yl} (\sum_j q_{ijyl})^2}{(1+F)^y} \\ & - \sum_{yl} \frac{T_{yl}}{\alpha_{yl}(1+F)^y} (D^0_{yl} d_{yl} - \frac{d^2_{yl}}{2})\end{aligned} \tag{4.1}$$

$$\text{s.t.} \quad q_{ijyl} \leq x_{ijy} + K_{ijy} \quad \forall ijyl \quad : \lambda_{ijyl} \tag{4.2}$$

$$0 \leq q_{ijyl} \quad \forall ijyl \quad : \mu_{ijyl} \tag{4.3}$$

$$\frac{T_{yl}}{(1+F)^y} (d_{yl} - \sum_{ij} q_{ijyl}) = 0 \quad \forall yl \quad : p_{yl} \tag{4.4}$$

Since in this chapter we focus on assisting one generation company, GENCO i^*, in taking its investments decisions, let us rewrite the market equilibrium (as a system of nonlinear equations) (2.35)-(2.43) making a distinct separation between agent i^* whose investment decisions are considered variables and the rest of the agents -i^* whose investment decisions are considered constants. This distinction is made only in constraints that involve investment decisions x_{ijy}, i.e., constraints that correspond to (2.37) (a complementarity condition) and (2.38) (the upper bound on production). When considering only the lower level, then there is absolutely no difference between (2.35)-(2.43) and (4.5)-(4.14). The difference only comes into existence when adding an upper level and will therefore be pointed out in the next sections.

$$\frac{\delta_{ij} T_{yl}}{(1+F)^y} + \frac{\sum_j \theta_{iyl} q_{ijyl} T_{yl}}{(1+F)^y} - \frac{T_{yl} p_{yl}}{(1+F)^y} + \lambda_{ijyl} - \mu_{ijyl} = 0 \qquad \forall ijyl \tag{4.5}$$

$$\mu_{ijyl} q_{ijyl} = 0 \qquad \forall ijyl \tag{4.6}$$

$$\lambda_{i^*jyl}(K_{i^*jy} + x_{i^*jy} - q_{i^*jyl}) = 0 \qquad \forall jyl \tag{4.7}$$

$$\lambda_{\text{-}i^*jyl}(K_{\text{-}i^*jy} + X_{\text{-}i^*jy} - q_{\text{-}i^*jyl}) = 0 \qquad \forall \text{-}i^*jyl \tag{4.8}$$

$$q_{i^*jyl} \leq x_{i^*jy} + K_{i^*jy} \qquad \forall jyl \tag{4.9}$$

$$q_{\text{-}i^*jyl} \leq X_{\text{-}i^*jy} + K_{\text{-}i^*jy} \qquad \forall \text{-}i^*jyl \tag{4.10}$$

$$0 \leq q_{ijyl} \qquad \forall ijyl \tag{4.11}$$

$$0 \leq \mu_{ijyl}, \quad 0 \leq \lambda_{ijyl} \qquad \forall ijyl \qquad (4.12)$$

$$d_{yl} - \sum_{ij} q_{ijyl} = 0 \qquad \forall yl \qquad (4.13)$$

$$d_{yl} - D^0_{yl} + \alpha_{yl} p_{yl} = 0 \qquad \forall yl \qquad (4.14)$$

As a reminder, the parameters (and possible parameter units) of the equilibrium problem above are: F [p.u.] the discount rate; T_{yl} [kh] the duration of each load period; δ_{ij} [€/MWh] the unitary production cost; K_{ijy} [GW] the already existing capacity; D^0_{yl} the demand intercept; α_{yl} the demand slope and θ_{iyl} [(€/MWh)/GW] the conjectured-price-response parameter. Note that at the market stage, the new capacity investments x_{ijy} [GW] are considered to be parameters as well, however, at the investment stage some are considered variables. In particular, since in this chapter we are considering the MPEC generation expansion model where only company i^* is taking investment decisions, only x_{i^*jy} will be considered variables at the investment stage. The competitors' investment variables $X_{\text{-}i^*jy}$ on the other hand will be considered parameters even at the investment stage.

The decision variables (and possible variable units) of the market equilibrium problem are: the production decisions q_{ijyl} [GW]; the demand d_{yl} [GW]; the resulting market price p_{yl} [€/MWh]; μ_{ijyl} and λ_{ijyl} which represent the dual variables of the lower and upper bounds on production.

4.2.2 Formulation of the Upper Level (or Investment Stage)

The upper level or investment stage determines the value of the maximum plant capacity x for each year, with the objective of maximizing one agent's, that is, i^*, net present value (NPV) considering the discount rate F, consisting of the gross margin from the lower level (revenues minus production costs) minus total investment costs (total investment cost $\hat{\beta}_{ijy}$ [M€/GW] or unitary annual[1] investment cost β_{ijy} [(M€/GW)/year] times capacity investments), by choosing its capacities x_{i^*jy} subject

[1]The unitary annual investment cost can be calculated as for example $\beta_{ij} = \hat{\beta}_{ij} / \sum_{y=1}^{Y} 1/(1+F)^y$, where F is the discount rate and Y corresponds to the number of years in the life cycle of a plant of technology j.

to the lower level equilibrium response. It is considered that agent i^* is the only one investing and thus the capacity built by the rest of the companies, which is referred to as -i^* , is considered as exogenous. The NPV is a parameter maximized in the long term as a measurement of a firm's value. It is used in this work, but includes some simplifications to ease the implementation and to keep it country independent. Taxes, as well as tax savings as a result of assets depreciation, have not been included. We furthermore impose the constraint that capacity can only increase in time ((4.16) and (4.18)), as annual investments have to be positive and always greater or equal to its value in the previous year. The fact that capacity can only increase over time is only a weak hypothesis if a sustained demand increase is reflected in the demand data. Increasing demand over time is a realistic assumption in light of historic observations. This constraint has been introduced in order to facilitate the numerical solution process, however, if this constraint were omitted, most likely the optimal solution would not change.

Essentially we consider two different versions to model the NPV, one that considers the total investment costs $\hat{\beta}_{ijy}$ which could have the unit [M€/GW] and another one where instead we consider the unitary annual investment costs β_{ijy} which can have the units [(M€/GW)/year]. This difference in concept of how to interpret investment costs, leads to two different formulations of the upper level, the first one being:

$$\max_{x_{i^*jy}} \sum_y \frac{1}{(1+F)^y} \big\{ \sum_{jl} T_{yl}(p_{yl} - \delta_{i^*j}) q_{i^*jyl} - \sum_j \hat{\beta}_{i^*jy}(x_{i^*jy} - x_{i^*j(y-1)}) \big\} \quad (4.15)$$

$$\text{s.t.} \qquad 0 \leq x_{i^*jy} \leq x_{i^*j(y+1)} \quad \forall jy \quad (4.16)$$

Considering total investment costs $\hat{\beta}_{ijy}$ means that the entire cost of investment is paid at once, exactly when the investment is taken. Since the variable x_{i^*jy} represents the total new capacity available in a certain year, the investment cost corresponds to the increment in investments with respect to the previous year. The disadvantage of this formulation is that, when considering a relatively short time horizon, base-load technologies like nuclear or coal are not profitable, which is why some sort of residual value would have to be considered. Base-load technologies are characterized by very high investment costs and low production costs. Since these technologies need a longer time horizon to recover their capital investments, the corresponding models have to consider some extra years in the time horizon in order to estimate companies' residual value.

The second formulation considers annual investment costs β_{ijy} instead of total investment costs. Instead of paying the total investment cost at once, with this type of formulation, the generation company i^* pays a certain amount $\beta_{i^*jy}x_{i^*jy}$ during a fixed number of years, which are considered the years of the life cycle of a plant with technology j. This allows us to consider the total amount of new capacity, instead of annual capacity increments. Moreover, with this type of formulation the length of the time horizon that is considered in the model does not distort the optimal technology spectrum in the same way that the previous formulation does. This means that even when considering a shorter time horizon, of for example five years, base-load technologies can and do appear in the optimal technology mix in comparison to the previous formulation where base-load technologies are not considered profitable unless the time horizon is at least as long as the life cycle of the power plant in question. The arising upper level formulation is given below:

$$\max_{x_{i^*jy}} \sum_y \frac{1}{(1+F)^y} \{ \sum_{jl} T_{yl}(p_{yl} - \delta_{i^*j}) q_{i^*jyl} - \sum_j \beta_{i^*jy} x_{i^*jy} \} \tag{4.17}$$

$$\text{s.t.} \qquad 0 \leq x_{i^*jy} \leq x_{i^*j(y+1)} \quad \forall jy \tag{4.18}$$

The formulation of the upper level that can be found in (4.17)-(4.18) corresponds to the "leave it to the market" approach. The "leave it to the market" approach refers to a type of market in which no regulatory capacity mechanism is in place and where capacity decisions are solely driven by market prices and productions. The presented methodology could be adapted in order to incorporate a different capacity mechanism, e.g., capacity markets or capacity payments, by adding a new term corresponding to the capacity revenues in the objective function. This possible extension will be discussed later on in this chapter.

4.2.3 Formulation of the MPEC

The bilevel generation expansion optimization model which has been developed as a part of this thesis, is a Mathematical Program with Equilibrium Constraints and consists of an upper level introduced in section 4.2.2, where investment decisions of firm i^* are taken, subject to the lower level introduced in 4.2.1 which represents the conjectured-price response spot market equilibrium where all GENCOs compete in the market. The formulation of the arising MPEC for company i^* is given in (4.19)-(4.30), where equations (4.19) and (4.20) correspond to the upper level or investment stage and equations (4.21)-(4.30) correspond to the lower level or production stage.

Bilevel Optimization Model (BOM):

$$\max_{\Omega_{i^*}} \sum_y \frac{1}{(1+F)^y} \{ \sum_{jl} T_{yl}(p_{yl} - \delta_{i^*j}) q_{i^*jyl} - \sum_j \beta_{i^*jy} x_{i^*jy} \} \tag{4.19}$$

$$\text{s.t.} \qquad 0 \le x_{i^*jy} \le x_{i^*j(y+1)} \qquad \forall jy \tag{4.20}$$

$$\frac{\delta_{ij} T_{yl}}{(1+F)^y} + \frac{\sum_j \theta_{iyl} q_{ijyl} T_{yl}}{(1+F)^y} - \frac{T_{yl} p_{yl}}{(1+F)^y} + \lambda_{ijyl} - \mu_{ijyl} = 0 \qquad \forall ijyl \tag{4.21}$$

$$\mu_{ijyl} q_{ijyl} = 0 \qquad \forall ijyl \tag{4.22}$$

$$\lambda_{i^*jyl}(K_{i^*jy} + x_{i^*jy} - q_{i^*jyl}) = 0 \qquad \forall jyl \tag{4.23}$$

$$\lambda_{\text{-}i^*jyl}(K_{\text{-}i^*jy} + X_{\text{-}i^*jy} - q_{\text{-}i^*jyl}) = 0 \qquad \forall \text{-}i^*jyl \tag{4.24}$$

$$q_{i^*jyl} \le x_{i^*jy} + K_{i^*jy} \qquad \forall jyl \tag{4.25}$$

$$q_{\text{-}i^*jyl} \le X_{\text{-}i^*jy} + K_{\text{-}i^*jy} \qquad \forall \text{-}i^*jyl \tag{4.26}$$

$$0 \le q_{ijyl} \qquad \forall ijyl \tag{4.27}$$

$$0 \le \mu_{ijyl}, \quad 0 \le \lambda_{ijyl} \qquad \forall ijyl \tag{4.28}$$

$$d_{yl} - \sum_{ij} q_{ijyl} = 0 \qquad \forall yl \tag{4.29}$$

$$d_{yl} - D^0_{yl} + \alpha_{yl} p_{yl} = 0 \qquad \forall yl \tag{4.30}$$

The set Ω_{i^*}, i.e., $\Omega_{i^*} = \{x_{i^*jy}, p_{yl}, d_{yl}, q_{ijyl}, \mu_{ijyl}, \lambda_{ijyl}\}$ $\forall ijyl$, contains the variables of the presented MPEC. Remember that only firm i^*'s capacity investments are considered variables of this problem. The capacity decisions of the competitors $X_{\text{-}i^*jy}$

are parameters of this problem. In the previous section 4.2.1, we have separated some lower level equations in equations for agent i^* and the rest -i^*. This has been done in order to emphasize now that while in equation (4.25) the right-hand side of the inequality contains a variable of the MPEC, i.e., x_{i^*jy}, in equation (4.26) the right-hand side is a parameter of the problem. In the market stage, all GENCOs decide their productions q_{ijyl} in the market, which furthermore yields market price p_{yl} and demand d_{yl} and implicitly yields dual variables μ_{ijyl} and λ_{ijyl}. However, in the bilevel optimization model given by this MPEC, only one GENCO decides its capacity investments, while the competitors investments are considered exogenous.

Note that in this formulation β_{ijy} is the unitary annual investment costs in units of [(M€/GW)/year]. By replacing (4.19) with (4.15) and changing the annual investment costs β_{ijy} to total investment costs $\hat{\beta}_{ijy}$, the other previously discussed formulation can be considered.

The MPEC model presented above is conceptually quite different from a single-level model. The conceptual difference is that capacity decisions of firm i^* are decided knowing that they can and will influence the market outcome. This has been thoroughly discussed in chapter 3. In particular, this bilevel model embodies sequential decision making as opposed to simultaneous decision making. By just looking at the formulation of the MPEC given by (4.19)-(4.30) one might be led to believe that all variables given by the set Ω_{i^*} are decided simultaneously, which numerically speaking would be true, however conceptually speaking it is not. It is important to remember that the set of lower level constraints, i.e., (4.21)-(4.30), are the KKT conditions of an equivalent optimization problem (4.1)-(4.4) which has been pointed out in section 4.2.1. This means that the above MPEC could also be written as the BPP given by equations (4.31)-(4.37).

In the BPP format it is quite obvious that not all problem variables are conceptually decided at the same time. When looking at the above model formulation, the parallel to a Stackelberg-type model becomes apparent and in a Stackelberg model we have leader decisions and follower decisions, which conceptually speaking are not taken at the same time. We therefore draw the conclusion that - even by just looking at the formulation of the MPEC model - it is clear that this model represents sequential decision making and in particular, first capacity decisions are taken and then quantities and the corresponding demand are decided.

$$\max_{x_{i^*jy}} \sum_{y} \frac{1}{(1+F)^y} \{ \sum_{jl} T_{yl}(p_{yl} - \delta_{i^*j}) q_{i^*jyl} - \sum_{j} \beta_{i^*jy} x_{i^*jy} \} \tag{4.31}$$

$$\text{s.t.} \qquad 0 \leq x_{i^*jy} \leq x_{i^*j(y+1)} \quad \forall jy \tag{4.32}$$

$$\begin{aligned} \min_{q,d} \quad & \sum_{ijyl} \frac{\delta_{ij} q_{ijyl} T_{yl}}{(1+F)^y} + \frac{1}{2} \sum_{iyl} \frac{\theta_{iyl} T_{yl} (\sum_j q_{ijyl})^2}{(1+F)^y} \\ & - \sum_{yl} \frac{T_{yl}}{\alpha_{yl}(1+F)^y} (D^0_{yl} d_{yl} - \frac{d^2_{yl}}{2}) \end{aligned} \tag{4.33}$$

$$\text{s.t.} \qquad q_{i^*jyl} \leq x_{i^*jy} + K_{i^*jy} \quad \forall jyl \quad : \lambda_{i^*jyl} \tag{4.34}$$

$$q_{\text{-}ijyl} \leq X_{\text{-}ijy} + K_{ijy} \quad \forall\text{-}i^* jyl \quad : \lambda_{\text{-}i^*jyl} \tag{4.35}$$

$$0 \leq q_{ijyl} \quad \forall ijyl \quad : \mu_{ijyl} \tag{4.36}$$

$$\frac{T_{yl}}{(1+F)^y} (d_{yl} - \sum_{ij} q_{ijyl}) = 0 \quad \forall yl \quad : p_{yl} \tag{4.37}$$

Due to the complementarity conditions (4.22)-(4.24) and the bilinear term $p_{yl} q_{i^*jyl}$ in the objective function describing the market revenues, the MPEC model becomes a nonlinear and in particular a non-convex optimization problem. There are several ways to tackle the resolution of this type of problem, which will be discussed in great detail in chapter 6 and in particular in section 6.1.

4.2.4 Numerical Example of Generation Expansion MPEC

This numerical example has previously been presented and published in a journal article [27], which forms part of the original contributions of this thesis.

System Description

The case study represents a stylized electric power system with the objective of assessing the performance of the MPEC generation expansion model. There are three generation companies (i_1 the investing company, i_2 and i_3) in the market, and four

Table 4.1: Installed generation capacity [MW] of firm 2 and firm 3.

	Firm 2	**Firm** 3
Nuclear	2500	2500
Coal	1250	1250
CC	750	750
Gas turbine	500	500

Table 4.2: Production and investment cost for each technology.

	Production cost δ [€/MWh]	**Investment cost** $\hat{\beta}$ [M€/GW]
Nuclear	11.8	1380
Coal	16.2	915
CC	24.0	460
Gas turbine	34.0	240

different technologies, that is, nuclear (NU), coal (CO), combined cycle gas turbine (CC) and gas turbine (GT). The capacity plans for companies i_2 and i_3 are considered fixed and are given in Table 4.1. The first company will be the only company who can change its investments unilaterally. As previously mentioned, this represents a dominant position for i_1 at the investment level because it implies that company i_1 knows what the competition is going to do. The known capacity plan $x_{\text{-}ijy}$ is given in Table 4.1 and we assume a 0.5% increase in generation capacity every year for companies i_2 and i_3. Note that K_{ijy} is considered to be zero in this case study.

The production costs of each technology, as well as its investment costs, are given in Table 4.2 and were based on data given by the International Energy Agency [89]. Since we consider total investment costs $\hat{\beta}$ (instead of annual investment costs), the MPEC model will be of the first version, which we have mentioned in section 4.2.2. We furthermore assume that production costs are the same for every firm. Nuclear and coal technologies have a high investment cost and a low production cost. Hence, they represent base-load plants. The remaining technologies, i.e., CC and gas turbine, on the other hand, have low investment costs but high production costs, and therefore correspond to peak technologies.

Tables 4.3 and 4.4 represent the demand data of the system. The scope is split into 15 years, and six load levels are considered for each year (see Table 4.3). Table 4.4 shows the intercept D^0 of the demand curve in the first year. Each year a sustained demand increase of 3% is considered. Additionally, the demand slope α is 0.23 GW/(€/MWh) for each level, which was based on Garcia-Alcalde et al. [53]. A discount rate F of 2% is considered. Four different market situations have been considered at the lower level (Table 4.5 shows values of conjectured-price responses for each one) ranging from perfect competition to Cournot oligopoly. Two intermediate oligopoly situations have been considered. A period of 10 years has been analyzed. Five extra years (11 - 15) have been included to estimate companies' residual value.

This MPEC was modeled using General Algebraic Modeling System (GAMS) and solved using the solver CONOPT [41]. Owing to the non-convexities, which are stemming from complementarity conditions and the bilinear market revenue term, the solver only guarantees to find a local optimum. In this application a good local solution may be very interesting for the investing company i_1, as they were starting from a given plan trying to improve their profits by unilaterally changing their investment. A local solution may not be as good as the global one, but it is still useful when improving a known solution. A gradient-based method, like the one used by CONOPT, may therefore be useful when initialized smartly. Hence, to obtain a meaningful initial solution to the MPEC, we first solve a simultaneous optimization problem, that is, investment and production decisions are taken simultaneously, such as done by Ventosa in Appendix B of [113] or the one introduced in section 2.3.3, which is formulated as a MCP. The computational time to solve the MCP in order to obtain a good initial guess, and then to solve the MPEC for one set of conjectured-price responses on an Intel(R) Core(TM) 2 Quad Processor with 3.21 GB RAM is 6.4 seconds.

Table 4.3: Annual load period duration [h].

Working day			**Weekend**		
Peak	**Off-peak** 1	**Off-peak** 2	**Peak**	**Off-peak** 1	**Off-peak** 2
300	3000	3000	300	1080	1080

Table 4.4: Demand intercept D^0 in year 1 [GW].

Working day			Weekend		
Peak	**Off-peak** 1	**Off-peak** 2	**Peak**	**Off-peak** 1	**Off-peak** 2
31.5	27.2	17.9	23.8	19.7	15.1

Table 4.5: Conjectured-price responses θ of firms [(€/MWh)/GW].

	Firm 1	**Firm** 2	**Firm** 3
Perfect competition	0.0	0.0	0.0
Intermediate case 1	1.3	1.3	1.3
Intermediate case 2	4.1	4.1	4.1
Cournot	4.35	4.35	4.35

Analysis of Results

In this section, we analyze the strategic behavior one company has to adopt when deciding its investments in generation capacity by optimizing its generation portfolio. For doing so, the firm assumes rivals' investment decisions fixed and known. Thus, in this case study we are assuming that i_1 is the only firm deciding on its investment. Generation companies i_2 and i_3 have a fixed investment determined by their initial capacity of 5 GW total, given in Table 4.1, and a 0.5% increase in this capacity in each subsequent year.

Regardless of the strategic behavior of the spot market, the case study considers a strategic investor who benefits from two mechanisms to manipulate spot prices. On the one hand, the power producer can reduce investments to obtain higher spot prices. On the other hand, when there are opportunities to exercise market power in the spot market, the power producer may prefer higher investment levels, and use the strategy of withholding production in the spot market to raise the price.

In order to demonstrate the application of the first strategy, in Figure 4-1 we will compare the total installed capacity of two models: the proposed bilevel model assuming perfect competition in the spot market; and a one-stage cost minimization problem - which will be referred to as "cost-based case" - also assuming perfect competition. As shown in Figure 4-1, when the investor takes into account the increase in future spot prices resulting from low investment levels, the optimal amount of capacity installed in the system is significantly lower. The reason for this result is that strategic investors (bilevel case) use their ability of withholding investment to benefit from higher spot prices.

Another effect given by this analysis is that, if power producers are forced to bid their true production costs, the only remaining strategy to raise the spot price is to under-invest. Therefore if there is strategic behavior in the investment level, the lower the market power opportunities in the spot market, the higher the incentive to under-invest. Thus, under the model's assumptions, one should expect higher investment levels under imperfect spot competition.

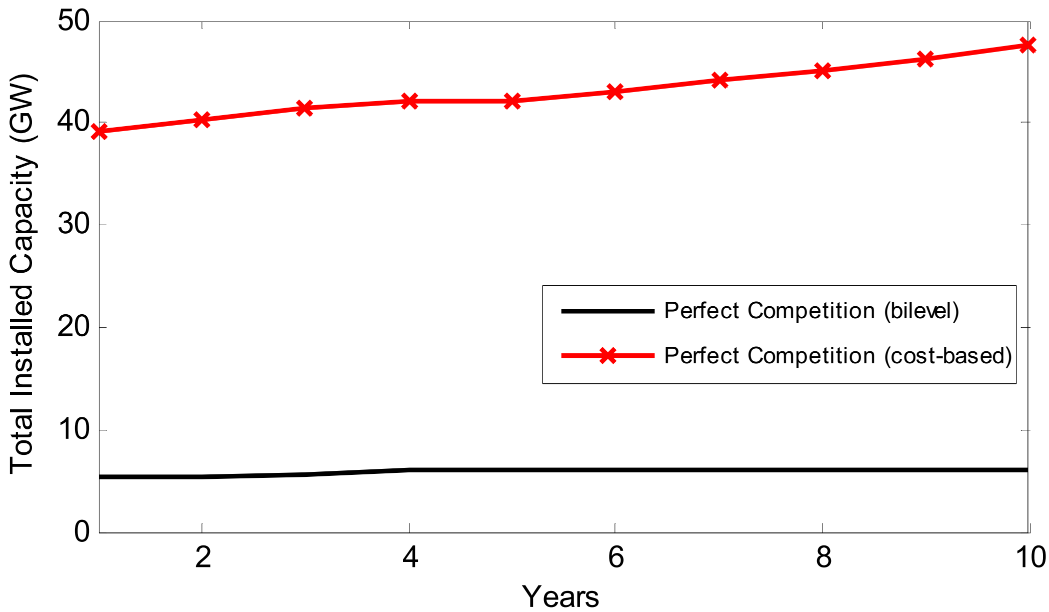

Figure 4-1: Total capacity in perfect competition (bilevel case) versus perfect competition (cost-based case).

To investigate this effect, we first present the results of our model under four different assumptions of the strategic behavior of the firms in the spot market: perfect competition; a Cournot model as the extreme case for oligopolistic imperfect competition; and two intermediate cases modeled by conjectural variations. Note that perfect competition, Cournot and intermediate cases refer only to the behavior of companies in the lower level, while at the upper level, i_1 always has a dominant position. In Figure 4-2, we can observe the total installed capacity corresponding to the strategic behavior. We can observe that the total installed capacity increases when the strategic behavior in the market becomes increasingly oligopolistic. When considering perfect competition, company i_1 invests 6 GW in new capacity; however, in the most extreme oligopolistic case - the Cournot case - company i_1 invests 7 GW.

The interaction between both strategies (withholding investment or production) also has an effect on the generation portfolio. The advantage of withholding production instead of investment is that, in the former case, the power producer can choose the hours in which to exercise market power. For instance, it is likely that prices during peak hours are high enough to make the producer prefer producing at the maximum output than raising the price even more. Conversely, low prices during off-peak hours likely make it more profitable to raise the price by withholding production. However, when spot markets are perfectly competitive, the only strategy to raise the price is to lower the investment level.

From this viewpoint, the optimal generation portfolio in the case of oligopolistic spot market would be made up of some base-load plants (high investment cost, low operation cost), intended to produce in every period, and some peak-load plants (low investment cost, high operation cost) that can be used only in the periods when there is little incentive to withhold production. On the other hand, if the spot market is perfectly competitive, all investments are aimed at producing in every period, and so the percentage of base-load technologies should be higher.

To show this effect in the generation portfolio, in Figure 4-4 we show the investment decisions of company i_1 for all four cases of different strategic behavior in the spot market and we compare the obtained solutions. As the conjectural variation cases represent intermediate steps between perfect competition and the Cournot case, we directly compare perfect competition with the intermediate case 1 (in the first subplot of Figure 4-4), the two intermediate cases (in the second subplot) and finally the intermediate case 2 with the Cournot case.

Analyzing the investments in intermediate case 1 with respect to perfect competition, we can observe a lower investment in nuclear (a base-load technology) which

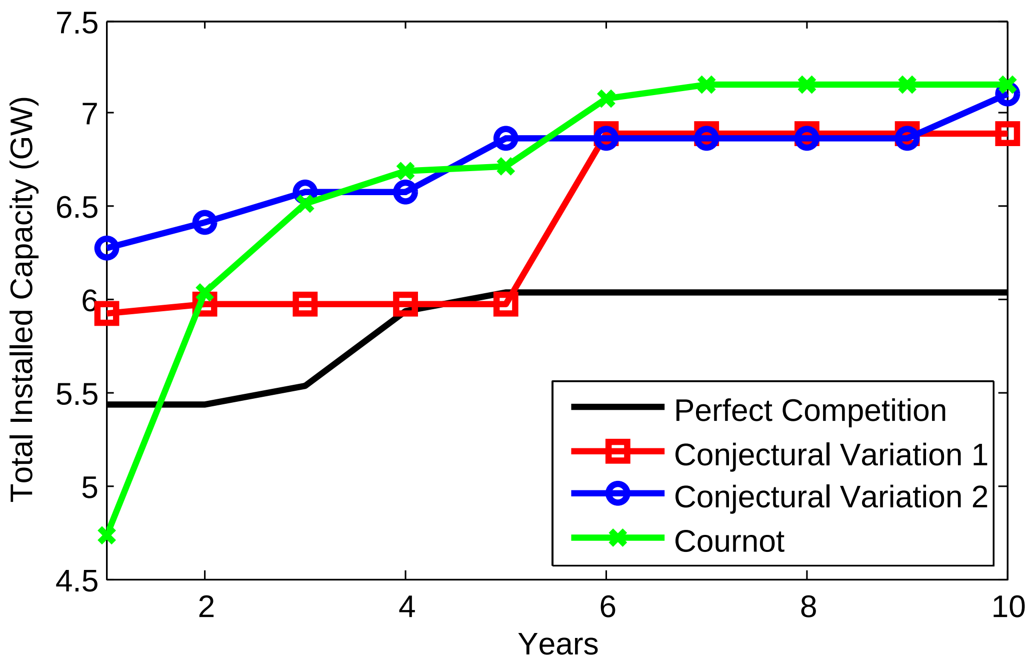

Figure 4-2: Total installed capacity for different cases of strategic spot market behavior.

is compensated for by a higher investment in technologies coal and CC. With respect to the two previous cases, the observed shift in technologies continues in the intermediate case 2. Capacity in nuclear decreases even more, capacity in coal and CC increases, and for the first time there is an investment in gas turbines, the most extreme peak-load technology in the model. In the Cournot case, capacity in coal and CC decreases slightly with respect to the intermediate case 2 and is compensated for by an increased investment in gas turbines.

Turning to the analysis of spot prices, when the firm plays both the under-investment and the withholding-production strategies, the incentive for obtaining extremely high peak prices is reduced with respect to the case where only the under-investment strategy is played (the case with a cost-based spot market). The logic for this is that the former investor can raise off-peak prices without the need for withholding capacity in peak hours. To put it in another way, peak prices will be higher in the cost-based spot market than in the Cournot spot market, which may seem surprising at first sight but which becomes clear with the reasoning explained above. On the other hand, cost-based spot market will have lower off-peak prices

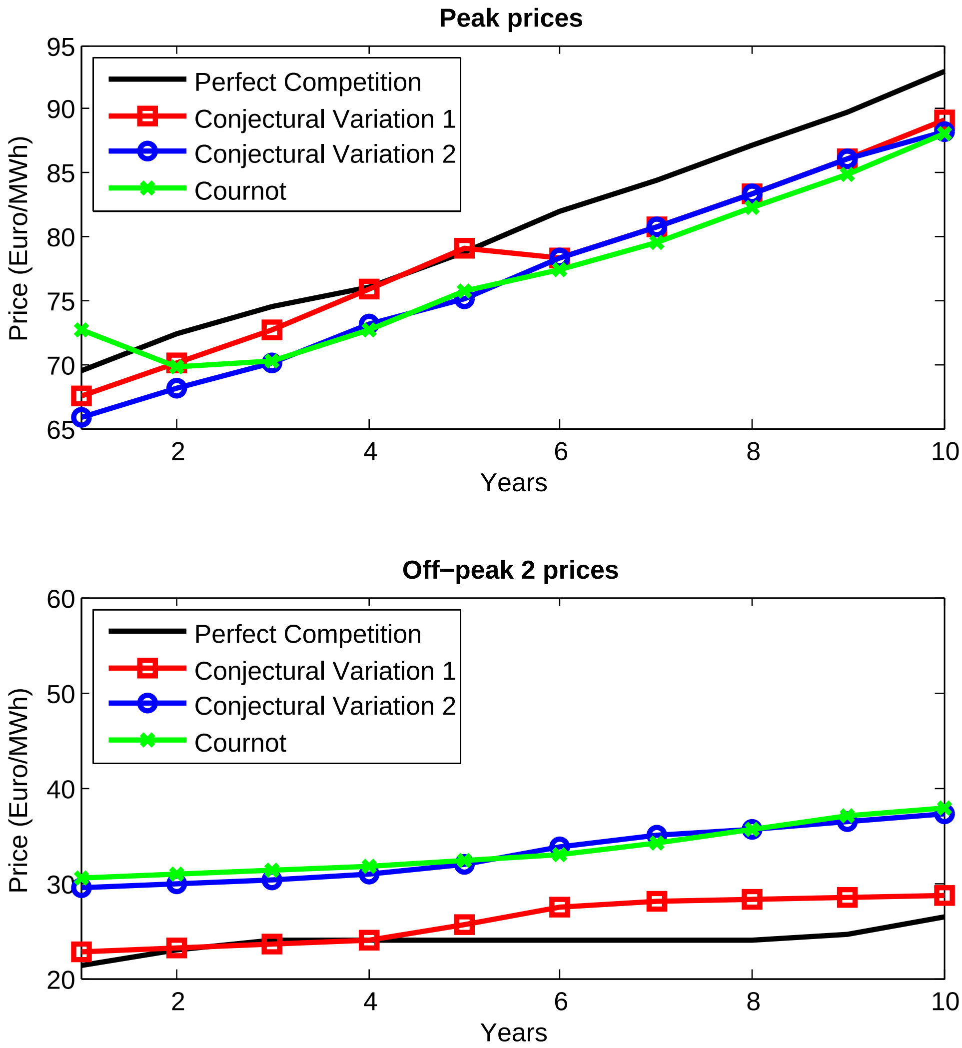

Figure 4-3: Price evolution for different cases of strategic spot market behavior.

than the Cournot spot market. Figure 4-4 shows this effect. When the competition in the spot market is closer to Cournot's, the peak prices fall whereas the off-peak two prices increase. In fact, Figure 4-4 shows that the lowest peak prices correspond to a Cournot investor in most of the years.

The evolution in capacity and prices is closely related to the generation company's profits. In Table 4.6, we present the NPV of company i_1 for the four different strategic situations. Note that the profit in the intermediate case 1 is only 7.3% higher than the value obtained under perfect competition. The profit in the Cournot case is significantly different from the perfect competition case, that is, 24.6% higher.

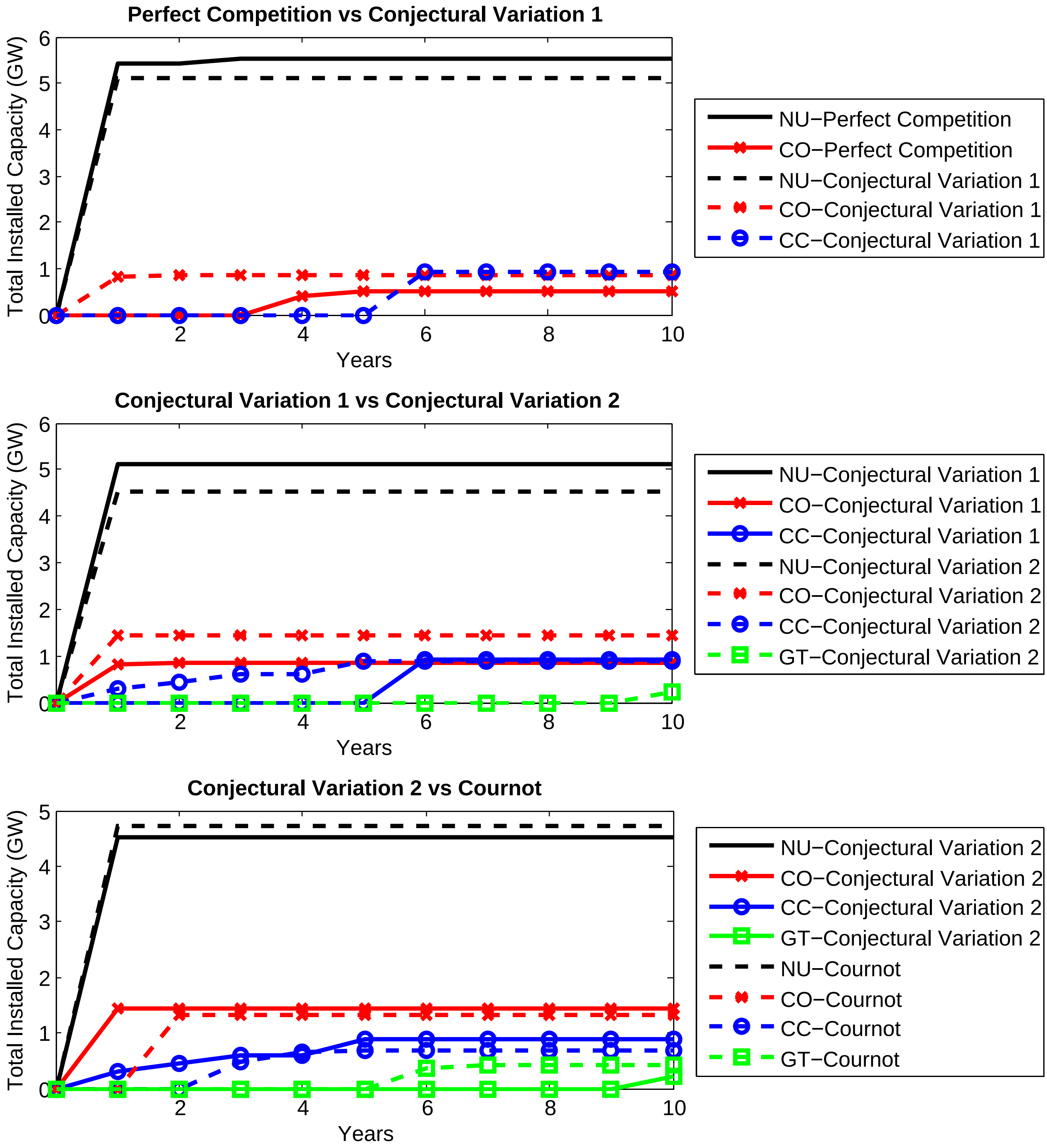

Figure 4-4: Capacity evolution comparing all cases of strategic behavior.

The limitations of the presented model are that the investment decisions are assumed to be continuous and that the investment decisions of the competition is fixed. Moreover, the obtained solution is only local, even though a local solution is useful

Table 4.6: Net present value of investor firm i_1 [M€].

	NPV
Perfect competition	12019
Intermediate case 1	12893
Intermediate case 2	14854
Cournot	14977

when parting from an existing capacity plan that should be improved. All these topics will be addressed in the remainder of this chapter.

4.3 Stochastic Generation Expansion MPEC Approach

Generation expansion planning is a long-term problem that is subject to uncertainties. This section presents a methodology to handle uncertainty in the generation expansion problem by introducing stochasticity into the model formulation yielding a stochastic MPEC. Obviously, there are various sources of uncertainty, however, in this chapter we focus on uncertainty stemming from competitors' investment decisions. This methodology can be extended to other uncertain factors, e.g., fuel prices, demand etc, will be addressed and discussed in chapter 7.

Similar to section 4.2, in this section we start out by discussing lower and upper level formulations in sections 4.3.1 and 4.3.2, followed by the stochastic MPEC formulation in section 4.3.3 and a case study given in 4.3.4. The idea behind this stochastic model is that capacity decisions in the upper level are taken considering several scenarios of the lower level, which is depicted in Figure 4-5. As previously mentioned, a variety of parameters can be considered stochastically with this type of methodology. However, in order to derive a concrete model formulation, in this section, we consider competitors' investments as stochastic parameter.

In our approach, the choice of capacity investments of the investing generation company mainly depends on two factors: the investment of the competing generation companies and the strategic behavior in the market, which determines prices

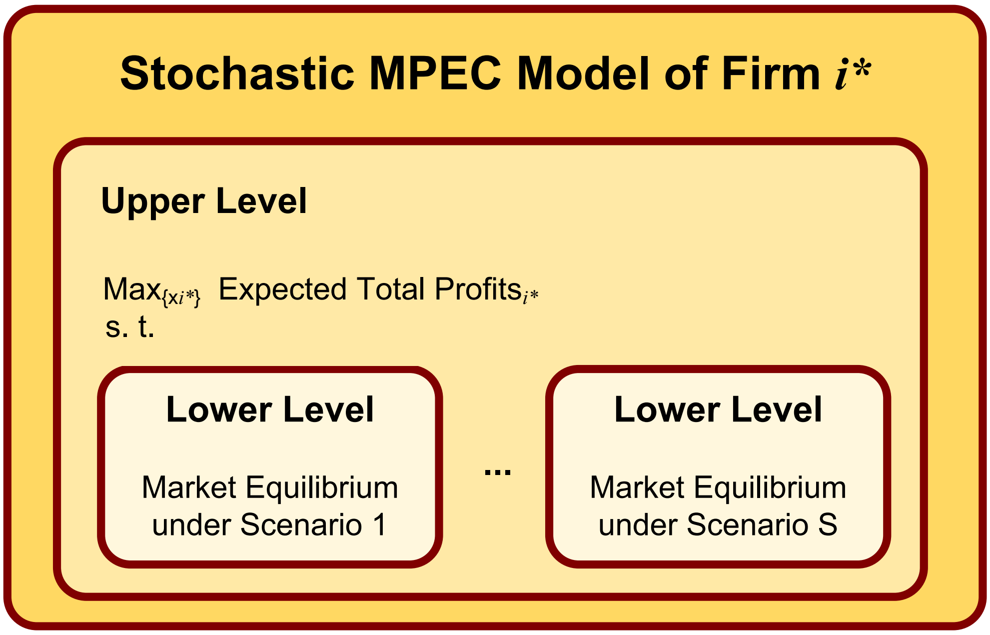

Figure 4-5: Graphic representation of the stochastic MPEC generation expansion problem faced by firm i^*.

and productions. As these two factors are by no means independent of each other, uncertainty has been introduced through a set of scenarios s of investment capacities of competitors coupled with corresponding conjectured-price responses. Note that the capacity investment can also depend on financial limitations of the generation company, however, this factor is not addressed here.

Then these scenarios are introduced in the formulation of the market equilibrium. Hence all lower level variables like productions q_i, demand d and price p will be stochastic variables and therefore depend on the chosen scenario. The upper level variables, which correspond to the installed generation capacity of the investing agent, however will not be stochastic variables, because a generation company can only make one investment decision as it is impossible to know which scenario is going to occur in reality. In order to incorporate the uncertainty in the upper level of our bilevel model, the criterion of maximizing the expected value to the upper level objective function is applied, i.e., the investing generation company will now be maximizing the expected profits considering all given scenarios. This implies that the investing firm is assumed to be risk neutral. Introducing risk aversion in our models might be an interesting extension for future work.

Definition of Indices

Before we formulate the model, let us define all the indices that will be used throughout this section. The index y corresponds to the set of years of the time horizon for which we make our investment decisions; l corresponds to the load level with duration T of each year in the time scope; i is the index of all generation companies; by i^* we denote the investing generation company and by -i^* we denote all generation companies excluding the investing company; j corresponds to the different technologies of generation capacity; s is the set of all scenarios and finally let $\widetilde{s}$ denote one of these scenarios.

4.3.1 Formulation of Stochastic Lower Level

For one scenario $\widetilde{s}$, the lower level is presented in (4.38)-(4.41) and represents the conjectured-price response market equilibrium mentioned in section 2.3.1, formulated as an optimization problem as detailed in section 2.3.2 and re-introduced in section 4.2.1. In this section the lower level model will be presented for one scenario, however the reader should keep in mind that in the resulting bilevel model the lower level model has to be solved simultaneously for every scenario s.

Let us now describe the lower level model under scenario $\widetilde{s}$ in detail. The decision variables of the lower level are given by:

- Production $q_{ijyl\widetilde{s}}$ [GW] of each agent i, of each technology j, in each load level l of each year y in scenario $\widetilde{s}$.

- Demand $d_{yl\widetilde{s}}$ [GW] in each load level l and year y depending on the current scenario $\widetilde{s}$.

$$\begin{aligned} \min_{q,d} \quad & \sum_{ijyl} \frac{\delta_{ij} q_{ijyl\widetilde{s}} T_{yl}}{(1+F)^y} + \frac{1}{2} \sum_{iyl} \frac{\theta_{iyl\widetilde{s}} T_{yl} (\sum_j q_{ijyl\widetilde{s}})^2}{(1+F)^y} \\ & - \sum_{yl} \frac{T_{yl}}{\alpha_{yl}(1+F)^y} (D^0_{yl} d_{yl\widetilde{s}} - \frac{d^2_{yl\widetilde{s}}}{2}) \end{aligned} \tag{4.38}$$

$$\text{s.t. } 0 \leq q_{i^*jyl\widetilde{s}} \leq x_{i^*jy} + K_{i^*jy} \quad : \mu_{i^*jyl\widetilde{s}}, \lambda_{i^*jyl\widetilde{s}} \tag{4.39}$$

$$0 \leq q_{\text{-}i^*jyl\widetilde{s}} \leq X_{\text{-}i^*jy\widetilde{s}} + K_{\text{-}i^*jy} \quad : \mu_{\text{-}i^*jyl\widetilde{s}}, \lambda_{\text{-}i^*jyl\widetilde{s}} \tag{4.40}$$

$$\frac{T_{yl}}{(1+F)^y} \sum_{ij} (d_{yl\widetilde{s}} - q_{ijyl\widetilde{s}}) = 0 \quad : p_{yl\widetilde{s}} \tag{4.41}$$

The lower level objective function, which corresponds to the one in (4.1), is stated in (4.38) and yields the market equilibrium by minimizing the difference between the extended costs minus the demand utility, i.e., solving the market clearing. The extended costs don't correspond to a real-life measure but represent an auxiliary function in order to capture the market outcomes. The extended costs are computed using the constants δ_{ij} [€/MWh] which correspond to unitary production costs of each technology j, the duration of each load level T_{yl} [kh] and the conjectured-price responses $\theta_{iyl\widetilde{s}}$ [(€/MWh)/GW] of each company i in each load level l of each year y in scenario $\widetilde{s}$.

The constraints that are linking the lower and the upper level can be found in (4.39) and (4.40), which correspond to the lower and upper bounds of production $q_{ijyl\widetilde{s}}$. For the investing agent, the production is limited from above by the upper level variable x_{i^*jy} plus the already installed capacity data K_{i^*jy}, see (4.39), which is the generation capacity in technology j and year y. For the non-investing companies however, the productions are limited from above, see (4.40), by the constants $X_{\text{-}i^*jy\widetilde{s}}$, which depend on the scenario $\widetilde{s}$ plus the already installed capacity constants plus $K_{\text{-}i^*jy}$. Equation (4.41) represents the power-demand balance equation that we have also seen in problem (4.4). Note that the dual variable of this equation $p_{yl\widetilde{s}}$ corresponds to the system's marginal price that clears the market. Similarly, $\mu_{ijyl\widetilde{s}}, \lambda_{ijyl\widetilde{s}}$ are dual variables corresponding to the constraints given in (4.39)-(4.40).

Since the previously presented optimization problem is continuous and convex, it can be replaced by its KKT conditions. The optimization problem given by (4.38)-(4.41) represents the conjectural variations market equilibrium under one particular scenario $\widetilde{s}$. However, the MPEC model that we set out to develop in this section is a stochastic model and therefore has to be able to handle various scenarios. Therefore, the spot market equilibrium for a set of scenarios s, using the KKT conditions to represent each individual market equilibrium, is given by:

$$\frac{\delta_{ij} T_{yl}}{(1+F)^y} + \frac{\sum_j \theta_{iyls} q_{ijyls} T_{yl}}{(1+F)^y} - \frac{T_{yl} p_{yls}}{(1+F)^y} + \lambda_{ijyls} - \mu_{ijyls} = 0 \qquad \forall ijyls \quad (4.42)$$

$$\mu_{ijyls} q_{ijyls} = 0 \qquad \forall ijyls \quad (4.43)$$

$$\lambda_{i^*jyls}(K_{i^*jy} + x_{i^*jy} - q_{i^*jyls}) = 0 \qquad \forall jyls \quad (4.44)$$

$$\lambda_{\text{-}i^*jyls}(K_{\text{-}i^*jy} + X_{\text{-}i^*jys} - q_{\text{-}i^*jyls}) = 0 \qquad \forall \text{-}i^*jyls \quad (4.45)$$

$$q_{i^*jyls} \leq x_{i^*jy} + K_{i^*jy} \qquad \forall jyl \quad (4.46)$$

$$q_{\text{-}i^*jyls} \leq X_{\text{-}i^*jys} + K_{\text{-}i^*jy} \qquad \forall \text{-}i^*jyls \quad (4.47)$$

$$0 \leq q_{ijyls} \qquad \forall ijyls \quad (4.48)$$

$$0 \leq \mu_{ijyls}, \quad 0 \leq \lambda_{ijyls} \qquad \forall ijyls \quad (4.49)$$

$$d_{yls} - \sum_{ij} q_{ijyls} = 0 \qquad \forall yls \quad (4.50)$$

$$d_{yls} - D^0_{yl} + \alpha_{yl} p_{yls} = 0 \qquad \forall yls \quad (4.51)$$

4.3.2 Formulation of Stochastic Upper Level

In the upper level, which can be found in (4.52)-(4.53), we maximize the expected value of profit, i.e., net present value of company i^* which is the only company deciding its investment in generation capacity x_{i^*jy} of each technology j in each year y. When considering the expected NPV we assume that firm i^* is risk neutral, however, a firm that is risk averse or prone could adopt a different approach. Note that the capacities of the rest of the companies $X_{\text{-}i^*jys}$ are not decision variables of this problem, and are incorporated via the scenarios s. In (4.52) we sum over the net present value, discounted with discount rate F , obtained in each scenario s multiplied by the probability of this scenario W_s.

In general the profit of company i^* corresponds to the market revenues minus the arising costs. The market revenues are given by the product between market price p_{yls} and productions q_{i^*jyls}. The costs consist of production costs and investment costs. The production costs correspond to the term $\delta_{i^*j} q_{i^*jyls}$ and the investment costs are given by the term $\beta_{i^*jy} x_{i^*jy}$, where β_{i^*jy} [(M€/GW)/year] is the unitary annual investment cost. An additional monotonicity of generation capacity is fulfilled in (4.53). Monotonicity of generation capacity is a weak hypothesis considering that demand usually increases over time. This assumption has been made in order to assist the solver in the search for a solution, however, if this constraint were omitted, most likely the optimal solution would not change.

$$\max_{x_{i^*jy}} \sum_{sy} \frac{W_s}{(1+F)^y} \big[\sum_{jl} T_{yl}(p_{yls} - \delta_{i^*j}) q_{i^*jyls} - \sum_{j} \beta_{i^*jy} x_{i^*jy} \big] \tag{4.52}$$

$$\text{s.t.} \qquad 0 \leq x_{i^*j(y-1)} \leq x_{i^*jy} \tag{4.53}$$

4.3.3 Formulation of Stochastic MPEC

The proposed stochastic MPEC modeling the stochastic generation capacity expansion problem is given in (4.54)-(4.65), where (4.54) and (4.55) is the part corresponding to the former upper level and (4.56)-(4.65) represents the lower level for each scenario s. The difference between this model and the one previously presented in section 4.2.3, is that while previously only one deterministic case was considered, the stochastic MPEC provides the methodology to consider various scenarios of competitors' investments (or production costs or demand data).

Equations (4.56) and (4.65) are the derivatives of the Lagrangian of the lower level, (4.57)-(4.59) are the complementarity conditions and (4.63) contains the positivity constraints of the dual variables from the lower level. Finally, (4.60)-(4.62) and (4.64) are the constraints of the lower level. The set of variables of the stochastic MPEC is $\Omega_{i^*} = \{x_{i^*jy}, q_{ijyls}, p_{yls}, d_{yls}, \mu_{ijyls}, \lambda_{ijyls},\} \; \forall ijyls$. The nonlinearities or the arising MPEC are the complementarity conditions (4.57)-(4.59) and the market revenue term in the objective function.

Stochastic Bilevel Optimization Model (SBOM):

$$\max_{\Omega_{i^*}} \sum_{sy} \frac{W_s}{(1+F)^y} \big[\sum_{jl} T_{yl}(p_{yls} - \delta_{i^*j}) q_{i^*jyls} - \sum_{j} \beta_{i^*jy} x_{i^*jy} \big] \tag{4.54}$$

$$\text{s.t.} \qquad 0 \leq x_{i^*j(y-1)} \leq x_{i^*jy} \qquad \forall jy \tag{4.55}$$

$$\frac{\delta_{ij}T_{yl}}{(1+F)^y} + \frac{\sum_j \theta_{iyls}q_{ijyls}T_{yl}}{(1+F)^y} - \frac{T_{yl}p_{yls}}{(1+F)^y} + \lambda_{ijyls} - \mu_{ijyls} = 0 \qquad \forall ijyls \quad (4.56)$$

$$\mu_{ijyls}q_{ijyls} = 0 \qquad \forall ijyls \quad (4.57)$$

$$\lambda_{i^*jyls}(K_{i^*jy} + x_{i^*jy} - q_{i^*jyls}) = 0 \qquad \forall jyls \quad (4.58)$$

$$\lambda_{\text{-}i^*jyls}(K_{\text{-}i^*jy} + x_{\text{-}i^*jys} - q_{\text{-}i^*jyls}) = 0 \qquad \forall\text{-}i^*jyls \quad (4.59)$$

$$q_{i^*jyls} \leq x_{i^*jy} + K_{i^*jy} \qquad \forall jyl \quad (4.60)$$

$$q_{\text{-}i^*jyls} \leq x_{\text{-}i^*jys} + K_{\text{-}i^*jy} \qquad \forall\text{-}i^*jyls \quad (4.61)$$

$$0 \leq q_{ijyls} \qquad \forall ijyls \quad (4.62)$$

$$0 \leq \mu_{ijyls}, \quad 0 \leq \lambda_{ijyls} \qquad \forall ijyls \quad (4.63)$$

$$d_{yls} - \sum_{ij} q_{ijyls} = 0 \qquad \forall yls \quad (4.64)$$

$$d_{yls} - D^0_{yl} + \alpha_{yl}p_{yls} = 0 \qquad \forall yls \quad (4.65)$$

4.3.4 Numerical Example of Stochastic MPEC

In this section we present a numerical example of the stochastic MPEC to demonstrate how the new stochastic methodology can be applied. In particular, we start out by briefly discussing two different solution methods of the stochastic MPEC, one involving nonlinear programming and the other one involving mixed integer programming. Then the test system and necessary data is described. Finally, the results are presented and analyzed paying particular attention to the performance of the different resolution methods and a comparison between the stochastic and the deterministic cases. This case study forms part of the journal article [116] and is an original contribution of this thesis.

Resolution Methods

Here we will present two ways to solve the presented stochastic MPEC: via nonlinear programming and via mixed integer programming. Nonlinear programming has the advantage of solving realistically-sized problems in reasonable time but fails to yield a global solution whereas mixed integer programming yields a global solution but takes longer to solve and in general only allows us to solve smaller problem instances.

The presented MPEC is a nonlinear non-convex problem, due to the complementarities and the bilinear product of prices and quantities in the objective function. Therefore employing a nonlinear solver does not guarantee to find a global optimum. However, smart initialization may assist the nonlinear solver to find a satisfactory solution. In order to obtain a meaningful initial solution to the MPEC, we first solve a simultaneous optimization problem, i.e., investment and production decisions are taken simultaneously, such as is done by Ventosa in appendix B of [113], which is formulated as a Mixed Complementarity Problem (MCP).

As a second resolution approach we linearize the presented MPEC to obtain a MILP. As stated above the nonlinearities of the MPEC are due to the complementarities and the product of prices and quantities in the objective function. We take care of the complementarities (4.57)-(4.59) by replacing them by their linear equivalent, as done by Fortuny-Amat [49]:

$$\begin{aligned} C^{\mu} b^{\mu}_{ijyls} &\geq \mu_{ijyls}, && (4.66)\\ C^{\mu}(1 - b^{\mu}_{ijyls}) &\geq q_{ijyls}, && (4.67)\\ C^{\lambda} b^{\lambda}_{i^*jyls} &\geq \lambda_{i^*jyls}, && (4.68)\\ C^{\lambda}(1 - b^{\lambda}_{i^*jyls}) &\geq (K_{i^*jy} + x_{i^*jy} - q_{i^*jyls}), && (4.69)\\ C^{\lambda} b^{\lambda}_{\text{-}i^*jyls} &\geq \lambda_{\text{-}i^*jyls}, && (4.70)\\ C^{\lambda}(1 - b^{\lambda}_{\text{-}i^*jyls}) &\geq (K_{\text{-}i^*jy} + X_{\text{-}i^*jys} - q_{\text{-}i^*jyls}), && (4.71) \end{aligned}$$

for C^{μ}, C^{λ} suitably large constants and $b^{\mu}_{ijyls}, b^{\lambda}_{ijyls}$ binary variables.

As for the bilinear terms $p_{yls} q_{i^*jyls}$ in the objective function, we apply a method called binary expansion to the variable price p_{yls}, as done by Pereira in [93]:

$$p_{yls} = \underline{p}_{yls} + \Delta_{p_{yls}} \sum_{k} 2^k b^{p}_{kyls}, \tag{4.72}$$

where $\underline{p}_{yls}$ is the lower bound, $\Delta_{p_{yls}}$ is the step size of price, k the set of discretization intervals and b^{p}_{kyls} are binary variables. Then the bilinear terms $p_{yls} q_{i^*jyls}$ of the objective can be replaced by $\underline{p}_{yls} q_{i^*jyls} + \Delta_{p_{yls}} \sum_k 2^k z_{ki^*jyls}$, where z_{ki^*jyls} symbolizes the product of prices with quantities and is defined by the following constraints, which also have to be added to the problem:

$$0 \leq \quad z_{ki^*jyls} \quad \leq C^p b^p_{kyls}, \tag{4.73}$$

$$0 \leq \quad q_{i^*jyls} - z_{ki^*jyls} \quad \leq C^p(1 - b^p_{kyls}), \tag{4.74}$$

for C^p a suitably large constant.

System Description

In this case study we present a stylized electric power system consisting of three generation companies i_1, i_2, i_3, where i_1 will be the only firm deciding its investment in generation capacity while the capacity investments of the other two firms will be incorporated via three alternative scenarios s_1, s_2, s_3. It will be assumed that generation companies i_2 and i_3 are identical. The installed capacity of these two firms in the first year is given in Table 4.7 for each of the investment scenarios. The initial investments are assumed to increase 0.5% with every passing year. Note that K_{ijy} is considered to be zero in this case study.

Three different scenarios of the competitors' investments and corresponding conjectured-price responses θ are incorporated: perfect competition (s_1), an intermediate case (s_2) which lies between perfect competition and the Cournot oligopoly, and finally the Cournot oligopoly (s_3). Table 4.8 provides the probability W_s as well as the corresponding conjectured-price response θ of each scenario.

Company i_1 will have the choice between two different technologies, which correspond to Nuclear and CC, where Nuclear represents a base-load technology as it has high investment costs and low variable costs and CC represents a peak-load technology because it has lower investment but higher production costs. Investment costs and production costs of each technology are presented in Table 4.9 and were estimated based on data given by the International Energy Agency [89]. We assume these costs are the same for every company.

Table 4.11 provides the duration of each of the two load levels and Table 4.10

Table 4.7: Installed generation capacity [MW] of firm 2 and firm 3 for all scenarios.

	s_1	s_2	s_3
Nuclear	8000	6210	4520
CC	2670	2070	1510

Table 4.8: Probability [p.u.] and conjectured-price response θ [(€/MWh)/GW] of scenarios.

	s_1	s_2	s_3
W_s	0.2	0.5	0.3
θ	0.0	2.175	4.35

the intercept of the demand curve in the first year. Each year a sustained 3% increase of the demand is considered. The demand slope α was chosen to be 0.23 [GW/(€/MWh)] based on Garcia-Alcalde et al. [53]. Moreover we consider a time scope of five years and a discount rate F of 2%.

Analysis of Results

All models have been formulated in GAMS. The computational time to solve the stochastic MPEC of the case study - a problem of 643 variables - by initializing smartly and by using the solver CONOPT on an Intel(R) Core(TM) 2 Quad Processor with 3.21 GB RAM was 0.8 seconds. Using CPLEX to solve the case study problem as a MILP with a step size $\Delta_{p_{yls}} = 0.15$ €/MWh yields a model of about 1850 variables and took about 11.5 hours. For a MILP with a finer step size of $\Delta_{p_{yls}} = 0.04$ €/MWh the number of variables increases to about 2050 and the resolution time increases to about 20 hours.

In Table 4.12 we compare the expected profits of the investing firm obtained by solving the MPEC using the nonlinear solver CONOPT and by solving the MILP

Table 4.9: Production and investment cost for each technology.

	Production cost δ [€/MWh]	**Annual investment cost β** [M€/GW]
Nuclear	11.8	46,000
CC	24.0	15,000

Table 4.10: Demand intercept D^0 in year 1 [GW].

Peak	**Off-peak**
55.213	36.036

Table 4.11: Annual load level durations [h].

Peak	**Off-peak**
3300	5460

with two different step sizes using CPLEX. In this particular case we observe that the MPEC yields expected profits that are higher than the profits obtained by the coarser MILP but lower than the profits of the finer MILP, implying that the choice of $\Delta_{p_{yls}}$ is critical.

In Figure 4-6 we present the corresponding capacity investment decisions over the entire time horizon, as well as the starting point that was used to compute the MPEC, which is referred to as MCP and was obtained by solving a simultaneous optimization problem, as the MCP given in Ventosa et al. [113]. The upper plot depicts capacity investments in nuclear and the lower plot depicts the capacity investments in CC. Comparing the investment decisions of the two MILPs, we observe that even though there only is a 4% difference in the objective function value, the investment decisions are very different, i.e., under the coarser MILP there is more than double the investment in CC than under the finer MILP and there are 6 GW less of nuclear investments. This shows that the solution of the MILP depends considerably on the choice of the step size and hence it has to be chosen carefully.

Moreover we observe that for this case study the investment decisions of the MPEC are not too different from the decisions taken by the finer MILP, i.e.,$\Delta_p = 0.04$

Table 4.12: Total expected net present value of investing firm in [M€].

	MPEC	**MILP** $\Delta_p = 0.15$ [€/MWh]	**MILP** $\Delta_p = 0.04$ [€/MWh]
NPV [M€]	30921	30691	31877

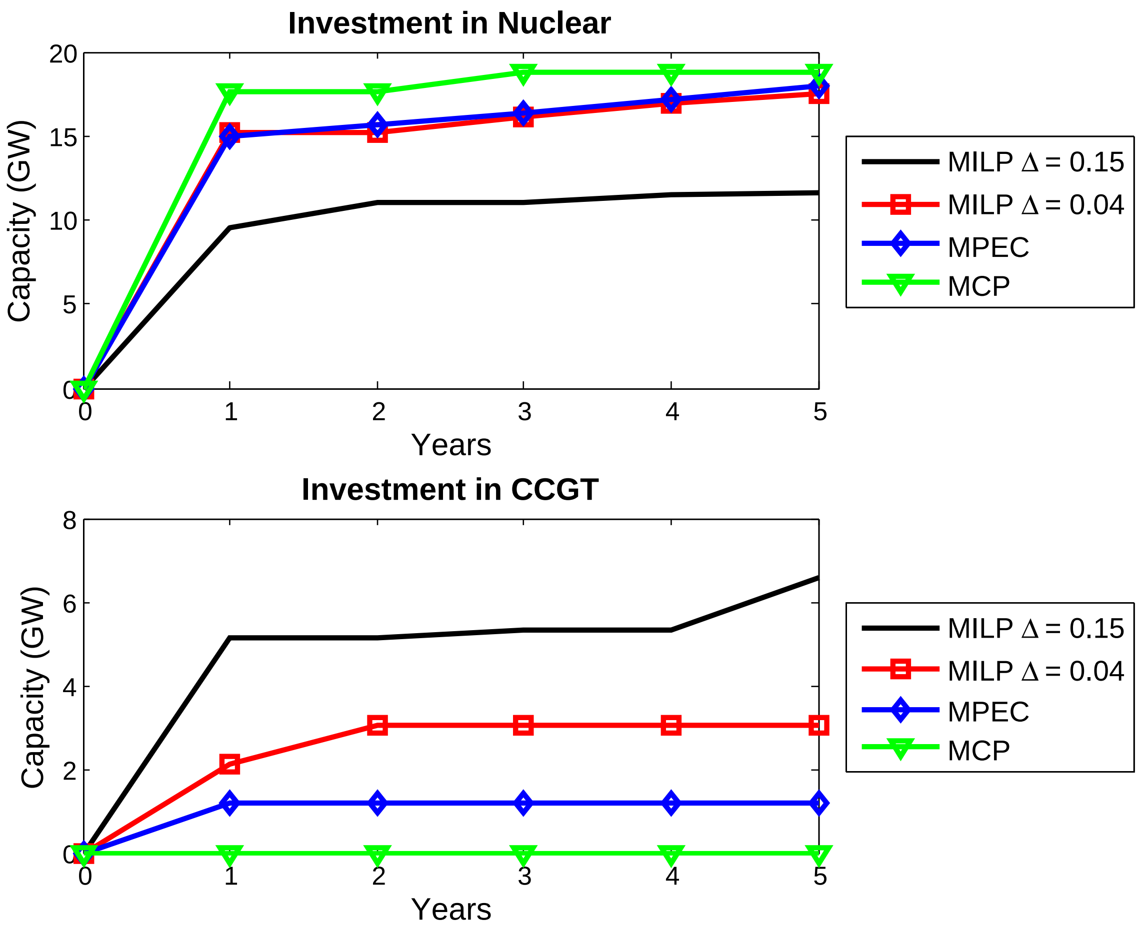

Figure 4-6: Capacity investment results obtained by MILPs, MPEC and MCP.

€/MWh, while only taking a small fraction of its computational time. In particular, when comparing the MPEC and the finer MILP, the investment in nuclear is almost exactly the same - in the last year there only is a difference of about 0.5 GW out of 18 GW installed. Under the finer MILP the investment in CC is about 1.9 GW higher than under the MPEC. In terms of total capacity the difference amounts to 1.4 GW, which yields a relative difference of only 7%.

Figure 4-7 gives the investment results of the stochastic MILP with $\Delta_{p_{yls}} = 0.15$ €/MWh and compares them to three deterministic MILPs, each representing one of the three scenarios individually. In particular we solve three deterministic MILPs, each considering a fixed generation capacity of the competition, given by Table 4.7, and a corresponding strategic behavior in the spot market (lower level), which is characterized by θ, given in Table 4.8. As scenario s_1 assumes a perfectly competitive market, we will refer to its deterministic results as "Perfect Competition". The deterministic results of scenario s_2 will be referred to as "Intermediate" and the deterministic results of scenario s_3 shall be referred to as "Cournot" as the strategic behavior in the market is the Cournot oligopoly. Finally, note that each deterministic MILP was discretized using the same step size.

The first observation we make is that the investment in the stochastic case lies between the deterministic scenarios as expected. The stochastic investment in nuclear almost exactly coincides with the Cournot solution, while the stochastic investment in CC lies about halfway between the Cournot and the Intermediate solution. We furthermore notice that when the strategic behavior in the spot market becomes more competitive, the investments in the peak load technology CC decrease while investments in the base load technology nuclear seem to increase. This can be explained as follows. When the spot market is perfectly competitive, it is not lucrative for the investing agent to build peaking units because these units do not yield any profits in off-peak hours. In off-peak hours capacity will not be binding in the market and hence the perfectly competitive solution in the market yields the market price as the marginal cost of the most expensive unit needed. Even if this unit is a CC peaking unit and is hence dispatched during off-peak hours, it would only recover its variable cost but not the investment cost. In peak hours capacity will be binding and the market price will be higher than the marginal cost of a peaking unit as there is under-investment in capacity even in the perfectly competitive case. This is due to the fact that in a two-stage model the investing generation company is aware that investments influence market outcomes and that if there were over-investment there would be no profits at all. This is often observed for two-stage models. However, in this case study the peak period seems to be too short for the peaking units to be profitable under perfect competition.

On the other hand, when the strategic behavior in the spot market is oligopolistic then the market price will be above marginal costs, hence yielding profits for peaking units in both peak and off-peak periods. Therefore, investment in peaking units

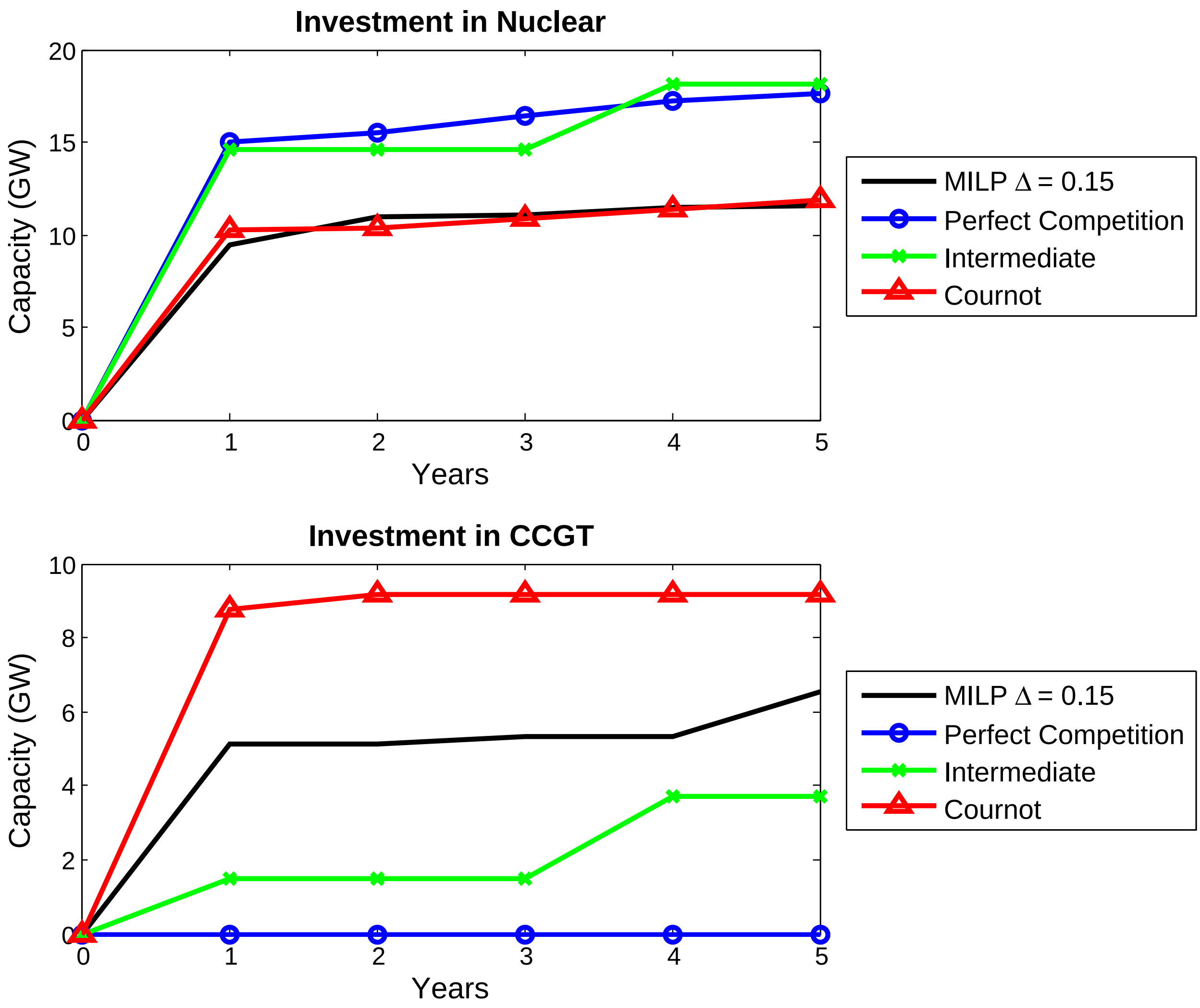

Figure 4-7: Capacity investment results obtained by stochastic MILP and deterministic MILPs.

becomes more attractive under oligopolistic behavior than under perfect competition. Moreover, under oligopolistic behavior the investing generation company can exert market power and hence raises prices by decreasing its amount of base load capacity. Both effects can be observed in Figure 4-7.

In the following section 4.4 we discuss how to formulate possible model extensions, which have previously been motivated in section 2.4.3. The corresponding case studies can be found in chapter 7.

4.4 Generation Expansion Model Extensions

In chapter 2, the generation expansion optimization model, which is formulated as an MPEC, has been introduced in a basic version. In the section 2.4.3, we have pointed out that there are certain aspects of the model which could be extended in order to make the model more realistic. After having discussed the MPEC model in detail in this chapter, we now present how to model the motivated extensions mentioned in section 2.4.3. In particular, in section 4.4.1 we introduce hydro energy, show how investment decisions can be discretized in section 4.4.2, how capacity mechanisms can be approached in section 4.4.3 and other instances of uncertainty treatment in fuel prices or hydro inflows in section 4.4.4, and finally in section 4.4.5 we briefly talk about other details in the model formulation.

In order to keep the formulation of the extended model as simple as possible, in all subsections of this section 4.4 we part from the same basic version of the MPEC that has been introduced previously in (4.19)-(4.30), however, it is also possible to consider an MPEC that is a combination of various of the below presented features.

4.4.1 Introduction of Hydro Power

As pointed out in section 2.4.3, in the basic version of our MPEC model we only consider thermal technologies. However, in order to represent a realistic power system, hydro power has to be introduced. Hence, in this section we first formulate the lower level with introduced hydro power constraints, then we show how to adapt the upper level formulation in order to include hydro power, and finally, we merge the two levels to present the new MPEC model which also represents the hydro technology.

Lower Level with Hydro Production

First of all, let us denote h_{iyl}, which is a variable of the lower level, as the power production of hydro plants of firm i in year y and load period l. Then the conjectured-price response market equilibrium including hydro energy, can be written as an equivalent optimization problem, which is given by (4.75)-(4.79). Note that the only change in the extended cost term in the objective function (4.75) is that now total production is considered as the sum of hydro and thermal production. While the lower and upper bound on thermal production given in (4.76) have not changed, there are several new constraints considered in this problem. The remaining constraints are related to hydro production: upper (and lower) (4.77) hydro level bounds with H, maximum company hydro power output considered as constant throughout each year; (4.78) maximum yearly hydro energy production represented as E_{iy}; and (4.79) the power balance, whose dual variable is price p; It can be shown that the lower level optimization problem is convex. The model is oriented to study thermal expansion and hydro capacity, included to make the model more realistic, is taken as known. With this representation, each agent dispatches its hydro production in order to maximize profit.

$$\begin{aligned}\min_{q,h,d} \quad & \sum_{ijyl} \frac{\delta_{ij} q_{ijyl} T_{yl}}{(1+F)^y} + \frac{1}{2}\sum_{iyl} \frac{\theta_{iyl} T_{yl}(h_{iyl} + \sum_j q_{ijyl})^2}{(1+F)^y} \\ & - \sum_{yl} \frac{T_{yl}}{\alpha_{yl}(1+F)^y}(D^0_{yl} d_{yl} - \frac{d^2_{yl}}{2})\end{aligned} \tag{4.75}$$

$$\text{s.t.} \quad 0 \le q_{ijyl} \le x_{ijy} + K_{ijy} \quad \forall ijyl \quad : \mu_{ijyl}, \lambda_{ijyl} \tag{4.76}$$

$$0 \le h_{iyl} \le H_{iyl} \quad \forall iyl \quad : \mu^H_{iyl}, \lambda^H_{iyl} \tag{4.77}$$

$$\sum_l T_{yl} h_{iyl} \le E_{iy} \quad \forall iy \quad : \lambda^E_{iy} \tag{4.78}$$

$$\frac{T_{yl}}{(1+F)^y}(d_{yl} - \sum_i h_{iyl} - \sum_{ij} q_{ijyl}) = 0 \quad \forall yl \quad : p_{yl} \tag{4.79}$$

Since the equivalent optimization problem in (4.75)-(4.79) is a convex problem, it can be replaced by its KKT conditions which are:

$$\frac{\delta_{ij} T_{yl}}{(1+F)^y} + \frac{\theta_{iyl} T_{yl}(h_{iyl} + \sum_j q_{ijyl})}{(1+F)^y} - \frac{T_{yl} p_{yl}}{(1+F)^y} + \lambda_{ijyl} - \mu_{ijyl} = 0 \qquad \forall ijyl \tag{4.80}$$

$$\frac{\theta_{iyl} T_{yl}(h_{iyl} + \sum_j q_{ijyl})}{(1+F)^y} - \frac{T_{yl} p_{yl}}{(1+F)^y} + \lambda^H_{iyl} - \mu^H_{iyl} + \lambda^E_{iy} T_{yl} = 0 \qquad \forall iyl \tag{4.81}$$

$$\mu_{ijyl} q_{ijyl} = 0, \quad \lambda_{ijyl}(K_{ijy} + x_{ijy} - q_{ijyl}) = 0 \qquad \forall ijyl \tag{4.82}$$

$$\mu^H_{iyl} h_{iyl} = 0, \quad \lambda^H_{iyl}(H_{iyl} - h_{iyl}) = 0 \qquad \forall iyl \tag{4.83}$$

$$\lambda^E_{iy}(E_{iy} - \sum_l T_{yl}h_{iyl}) = 0 \qquad \forall iy \quad (4.84)$$

$$0 \leq q_{ijyl} \leq x_{ijy} + K_{ijy} \qquad \forall ijyl \ (4.85)$$

$$0 \leq h_{iyl} \leq H_{iyl} \qquad \forall iyl \quad (4.86)$$

$$0 \leq \mu_{ijyl}, \quad 0 \leq \lambda_{ijyl} \qquad \forall ijyl \ (4.87)$$

$$0 \leq \mu^H_{iyl}, \quad 0 \leq \lambda^H_{iyl} \qquad \forall iyl \quad (4.88)$$

$$0 \leq \lambda^E_{iy} \qquad \forall iy \quad (4.89)$$

$$d_{yl} - \sum_i h_{iyl} - \sum_{ij} q_{ijyl} = 0 \qquad \forall yl \quad (4.90)$$

$$d_{yl} - D^0_{yl} + \alpha_{yl}p_{yl} = 0 \qquad \forall yl \quad (4.91)$$

Upper Level with Hydro Production

The upper level considering hydro production is similar to the already known version of the upper level given in previous sections of this chapter. The only change is that now market revenues in the objective function (4.92) are calculated as the product of market price and total productions, given as the sum of hydro and thermal production. Note that for hydro energy, a variable production cost of zero is considered and hence there are no corresponding production costs. Moreover, since in many countries, for example in Spain, the maximum capacity for hydro technology has already been reached, no further capacity investment decisions in hydro technology are being considered.

$$\begin{aligned} \max_{x_{i^*jy}} \sum_y \frac{1}{(1+F)^y} \{ \sum_l T_{yl}p_{yl}(h_{i^*yl} + \sum_j q_{i^*jyl}) \\ - \sum_{jl} T_{yl}\delta_{i^*j}q_{i^*jyl} \sum_j \beta_{i^*jy}x_{i^*jy} \} \end{aligned} \quad (4.92)$$

$$\text{s.t.} \qquad 0 \leq x_{i^*jy} \leq x_{i^*j(y+1)} \quad \forall jy \qquad (4.93)$$

MPEC with Hydro Production

Finally, the bilevel optimization generation expansion model considering hydro energy is given by the MPEC below, where the set of MPEC variables Ω_{i^*} is given as $\Omega_{i^*} = \{x_{i^*jy}, p_{yl}, d_{yl}, q_{ijyl}, h_{iyl}, \mu_{ijyl}, \lambda_{ijyl}, \mu^H_{iyl}, \lambda^H_{iyl}, \lambda^E_{iy}\}$ $\forall ijyl$. Note that equation (4.96) represents the market equilibrium.

$$\max_{\Omega_{i^*}} \sum_y \frac{1}{(1+F)^y} \{ \sum_l T_{yl} p_{yl} (h_{i^*yl} + \sum_j q_{i^*jyl}) - \sum_{jl} T_{yl} \delta_{i^*j} q_{i^*jyl} \sum_j \beta_{i^*jy} x_{i^*jy} \} \tag{4.94}$$

$$\text{s.t.} \quad 0 \leq x_{i^*jy} \leq x_{i^*j(y+1)} \quad \forall jy \tag{4.95}$$

$$\text{Constraints: } (4.80) - (4.91) \tag{4.96}$$

4.4.2 Discrete Investment Decisions

One of the simplifications of the basic generation expansion bilevel optimization model is that investment decisions x_{ijy} were assumed to be continuous variables. However, in reality capacity decisions are discrete and it would therefore be desirable to model these variables as such. In a first, naive approach variables x_{ijy} could be defined as integer variables, which would yield a mixed integer nonlinear program, which can be quite complicated to solve. However, when taking a closer look at the basic MPEC model, given by (4.19)-(4.30), it can be shown that this nonlinear program can be transformed into a MILP by discretizing investment decisions.

First of all, let us recall that the nonlinearities of the MPEC were formed by the complementarity conditions that arise when replacing the lower level by its KKT conditions, and the nonlinear market revenue term of the objective function. As has been mentioned in section 4.3.4, there is a linear equivalent of complementarity conditions which can be obtained introducing large constants and binary variables in the model formulation. In this way, all complementarity conditions constraints of the MPEC, like $\mu q = 0$, can be replaced by the following expression, where C^μ is a sufficiently large constant and b^μ is a binary variable.

$$C^\mu b^\mu \geq \mu \tag{4.97}$$

$$C^\mu (1 - b^\mu) \geq q \tag{4.98}$$

The only nonlinearity left to deal with is the bilinear term of the market revenues in the objective function of the MPEC. In the previously presented case study in section 4.3.4, this nonlinearity has been linearized by discretizing the market price p_{yl}. While this approach has the desired effect of transforming the MPEC into a MILP, it does not solve the issue of having discrete investment decisions, and moreover, market prices are lower level variables, which should be continuous since they correspond to the dual variable of the demand balance equation. By discretizing prices we might loose valuable possible solutions to the problem. For example, the globally optimal solution might occur at a market price of 43.21 €/MWh, however, if due to the discretization the market price is composed of increments of 4 or 15 c€, then this solution can never be achieved by the discretized MILP. Therefore, due to discretization, the optimal solution of the original problem and the MILP might not coincide, leading to the loss of the actual optimum. Hence, another approach to discretize the market revenue term has to be considered. For this purpose, we recall the basic MPEC model with $\Omega_{i^*} = \{x_{i^*jy}, p_{yl}, d_{yl}, q_{ijyl}, \mu_{ijyl}, \lambda_{ijyl}\}$ $\forall ijyl$:

$$\begin{aligned} \max_{\Omega_{i^*}} \sum_y \frac{1}{(1+F)^y} \Big\{ \sum_{jl} T_{yl} p_{yl} q_{i^*jyl} - \sum_{jl} T_{yl} \delta_{i^*j} q_{i^*jyl} \\ - \sum_j \beta_{i^*jy} x_{i^*jy} \Big\} \end{aligned} \tag{4.99}$$

$$\text{s.t.} \qquad 0 \leq x_{i^*jy} \leq x_{i^*j(y+1)} \qquad \forall jy \tag{4.100}$$

$$\frac{\delta_{ij} T_{yl}}{(1+F)^y} + \frac{\sum_j \theta_{iyl} q_{ijyl} T_{yl}}{(1+F)^y} - \frac{T_{yl} p_{yl}}{(1+F)^y} + \lambda_{ijyl} - \mu_{ijyl} = 0 \qquad \forall ijyl \tag{4.101}$$

$$\mu_{ijyl} q_{ijyl} = 0 \qquad \forall ijyl \tag{4.102}$$

$$\lambda_{ijyl}(K_{ijy} + x_{ijy} - q_{ijyl}) = 0 \qquad \forall ijyl \tag{4.103}$$

$$q_{ijyl} \leq x_{ijy} + K_{ijy} \qquad \forall ijyl \tag{4.104}$$

$$0 \leq q_{ijyl} \qquad \forall ijyl \tag{4.105}$$

$$0 \leq \mu_{ijyl}, \quad 0 \leq \lambda_{ijyl} \qquad \forall ijyl \tag{4.106}$$

$$d_{yl} - \sum_{ij} q_{ijyl} = 0 \qquad \forall yl \tag{4.107}$$

$$d_{yl} - D^0_{yl} + \alpha_{yl} p_{yl} = 0 \qquad \forall yl \tag{4.108}$$

From equation (4.101) we obtain an expression for price p_{yl}, which is given in (4.109). Multiplying both sides of (4.109) with q_{ijyl} yields (4.110). Due to the complementarity conditions (4.102) and (4.103), the term $\mu_{ijyl}q_{ijyl}$ is zero and the term $\lambda_{ijyl}q_{ijyl}$ can be replaced by $\lambda_{ijyl}(K_{ijy} + x_{ijy})$. Considering that (4.110) is true for all j, we sum (4.110) over j, which in turn yields (4.111).

It is clear to see that the left-hand side of (4.111) represents the nonlinear term of the classification function. The terms on the right-hand side of (4.111) are linear except the term including $\lambda_{ijyl}x_{ijy}$ and the quadratic term including $(\sum_{j^*} q_{ij^*yl})^2$. The first term can be linearized by discretizing investment decisions and the quadratic term can be represented by a piecewise linear function. In the remainder of this section we describe in detail how to linearize both of these terms which then yields the linear formulation of the objective function.

$$\frac{T_{yl}p_{yl}}{(1+F)^y} = \frac{\delta_{ij}T_{yl}}{(1+F)^y} + \frac{\theta_{iyl}T_{yl}\sum_{j^*} q_{ij^*yl}}{(1+F)^y} + \lambda_{ijyl} - \mu_{ijyl} \tag{4.109}$$

$$\frac{T_{yl}p_{yl}q_{ijyl}}{(1+F)^y} = \frac{\delta_{ij}T_{yl}q_{ijyl}}{(1+F)^y} + \frac{q_{ijyl}\theta_{iyl}T_{yl}\sum_{j^*} q_{ij^*yl}}{(1+F)^y} + \lambda_{ijyl}q_{ijyl} - \mu_{ijyl}q_{ijyl} \tag{4.110}$$

$$\frac{\sum_j T_{yl}p_{yl}q_{ijyl}}{(1+F)^y} = \frac{\sum_j \delta_{ij}T_{yl}q_{ijyl}}{(1+F)^y} + \frac{\theta_{iyl}T_{yl}(\sum_{j^*} q_{ij^*yl})^2}{(1+F)^y} + \sum_j \lambda_{ijyl}(K_{ijy} + x_{ijy}) \tag{4.111}$$

Discretize Approximated Investments

The bilinear term $\lambda_{ijyl}x_{ijy}$ in (4.111) can be linearized easily by discretizing investment decisions x_{ijy}. We discretize x_{ijy} as presented in (4.112), where Δ_x is the chosen step size, k the set of discretization intervals and b^x_{kijy} are binary variables. Then the bilinear terms $\lambda_{ijyl}x_{ijy}$, can be replaced by $\Delta_x \sum_k 2^k z^x_{kijy}$, where z^x_{kijy} symbolizes the product of the two variables and is defined by the following constraints, which also have to be added to the problem:

$$x_{ijy} = \Delta_x \sum_k 2^k b^x_{kijy}, \tag{4.112}$$

$$0 \leq z^x_{kijy} \leq C^x b^x_{kijy}, \tag{4.113}$$

$$0 \leq \lambda_{ijyl} - z^x_{kijy} \leq C^x(1 - b^x_{kijy}), \tag{4.114}$$

for C^x a suitably large constant. With this formulation we achieve that investment decisions are converted into discrete variables, which is more realistic than considering them continuous variables.

Piecewise Linear Function

In order to linearize the quadratic $(\sum_j q_{ijyl})^2$ in (4.111), we formulate a piecewise linear function. This is done in the following fashion. At several points the curve $(\sum_j q_{ijyl})^2$ is approximated by the tangent lines. Let us refer to the x-axis value of the intersection points of these lines as IS_{iylm}, where m is the index of intersection points and S_{iylm} represents the slope of the tangent lines. Then the variables dist_{iylm}, that are supposed to sum up to $\sum_j q_{ijyl}$, refer to the part of $\sum_j q_{ijyl}$ between the intersection points m and $m-1$. The binary variables b^{dist}_{iylm} are one if the variable dist_{iylm} is at its upper bound. Then, in order to approximate the function $(\sum_j q_{ijyl})^2$ linearly, we define the variable $\bar{q}_{iyl}$ as follows:

$$(\mathrm{IS}_{iylm} - \mathrm{IS}_{iylm-1}) b^{\mathrm{dist}}_{iylm} \leq \mathrm{dist}_{iylm} \quad \forall iylm \tag{4.115}$$

$$(\mathrm{IS}_{iylm} - \mathrm{IS}_{iylm-1}) b^{\mathrm{dist}}_{iylm-1} \geq \mathrm{dist}_{iylm} \quad \forall iylm \tag{4.116}$$

$$\sum_j q_{ijyl} = \sum_m \mathrm{dist}_m \quad \forall iyl \tag{4.117}$$

$$\bar{q}_{iyl} = \sum_m \mathrm{dist}_m \mathrm{S}_m \quad \forall iyl \tag{4.118}$$

Discrete Investment MPEC

Finally, applying this to the basic version of the MPEC that we started out with, yields a MILP formulation of this MPEC, which is formulated as:

$$\max \sum_{iy} \frac{1}{(1+F)^y} \Big\{ \sum_{l} T_{yl}\theta_i \bar{q}_{iyl} - \sum_{j} \beta_{ijy} x_{ijy} + \sum_{lj} \lambda_{ijyl} K_{ijy} (1+F)^y + \sum_{jl} (1+F)^y \Delta_x \sum_{k} 2^k z^x_{kijy} \Big\} \tag{4.119}$$

$$\text{s.t. Linear complementarity conditions:} \qquad (4.97) - (4.98) \tag{4.120}$$

$$\text{s.t. Rest of lower level constraints:} \qquad (4.101), (4.104) - (4.108) \tag{4.121}$$

$$\text{s.t. Discretized approximated investments:} \qquad (4.112) - (4.114) \tag{4.122}$$

$$\text{s.t. Piecewise linear function:} \qquad (4.115) - (4.118) \tag{4.123}$$

4.4.3 Capacity Mechanisms

One of the main advantages of the type of model that we propose in this thesis is that it is useful to assess the functionality and effectiveness of certain regulatory measures before introducing them into the market. For a thorough discussion of this topic the reader is referred to Battle and Rodilla [8, 103]. In this section we discuss the very important topic of capacity mechanisms and how they can be introduced into the presented bilevel optimization model.

First of all, the basic version of the MPEC model which has been introduced in section 4.2 does not consider any kind of regulatory intervention in the market in order to guarantee security of supply. The regulatory instrument of no intervention can be referred to as "leave it to the market" or "energy-only" approach. This is the simplest capacity mechanism, which is based on the principle that the market itself will provide the appropriate price signals to incentivize capacity investments and that market agents will learn about the importance of long-term hedging. Theoretically speaking, this instrument has its merits but for it to work there have to be blackouts and non-served energy - causing energy prices to go through the roof - which furthermore leads to the recuperation of the investments of peaking plants. In practice however, this method ignores the existence of market failures as for example the risk aversion of investors or the passiveness of consumers. Moreover, in many electricity systems there are price caps which distort the economic signals of the market. Therefore in the majority of electricity systems there exists some form of regulatory intervention in the market in order to guarantee security of supply. In this section we motivate how to introduce two different regulatory capacity instruments in our model - capacity payments and some form of locational capacity market - and in section 7.1 we carry out a corresponding case study. Other capacity mechanisms as for example capacity options are more complicated to model and their introduction into our models would be an interesting line of future research.

Capacity Payments

The capacity payment is a price-based mechanism which assigns payments to individual generators based on their contribution to system reliability. The motivation of this regulatory mechanism is to incentivize more capacity investments and reducing the risk aversion of investors by guaranteeing them a stable income. This type of payment is usually linked to the firm capacity installed by all generators. The amount of the payment depends on the technology of the individual plant because 1 MW of hydro technology contributes differently to the system reliability as 1 MW of installed CC depending on reservoir management strategies or contracts etc. From a regulatory point of view, this mechanism has two weak points: first, the lack of a clear definition of a reliability product and second, they have to fix a certain price. For a detailed discussion on capacity payments, we refer the reader to Battle and Rodilla [8]. In our models we therefore introduce the capacity payment parameter CP_{ijy} [M€/GW] and we furthermore assume for simplicity of the formulation that the newly installed capacity x_{ijy} and the already existing capacity K_{ijy} are firm. Then the arising MPEC with $\Omega_{i^*} = \{x_{i^*jy}, p_{yl}, d_{yl}, q_{ijyl}, \mu_{ijyl}, \lambda_{ijyl}\}$ $\forall ijyl$ can be formulated as follows.

$$\max_{\Omega_{i^*}} \sum_y \frac{1}{(1+F)^y} \Big\{ \sum_{jl} T_{yl}(p_{yl} - \delta_{i^*j}) q_{i^*jyl} - \sum_j \beta_{i^*jy} x_{i^*jy} + \sum_j CP_{i^*jy}(x_{i^*jy} + K_{i^*jy}) \Big\} \tag{4.124}$$

$$\text{s.t.} \qquad 0 \leq x_{i^*jy} \leq x_{i^*j(y+1)} \quad \forall jy \tag{4.125}$$

$$\text{s.t. Market Equilibrium:} \qquad (4.21) - (4.30) \tag{4.126}$$

In the upper level objective function, given in (4.124), the first term corresponds to market revenues minus production costs, the second term corresponds to investment costs and the new third term represents the capacity payments that firm i^* receives for the total installed capacity in technology j for each year y. We omit the full formulation of the lower level at this point since it is equivalent to a previous formulation. The difference between this model and the basic MPEC (without regulatory intervention) is that here the generation company also takes into account the additional revenues that it obtains for its installed capacity. The purpose of capacity payments, obviously, is to increase capacity investments. This model can be employed to assess how much a certain amount of capacity payments CP_{ijy} actually increases investments in comparison to the basic MPEC which considered a “leave it to the market” approach. Exactly this type of comparison will be carried out in chapter 7.

Locational Capacity Markets

Capacity markets are a specific example of a regulatory measure known as capacity obligations, whose goal is to guarantee a regulated adequacy target for the system and define commitments of the individual agents. The system operator or the regulator determine certain levels of capacities - MW of installed capacity - and then load entities have to contract this capacity. In capacity markets, demand has the obligation to contract capacity required to supply future consumption. For more detail, the reader is referred to Batlle and Rodilla [8].

Capacity prices stemming from systems with a capacity market are often volatile, since they are either zero if there is excess capacity or an extremely high penalty if there is lack of capacity. In order to reduce spikes in capacity prices, the ISO New England proposed a different approach to FERC, i.e., the locational capacity market (LICAP) [42] . In this method instead of considering a zero/penalty capacity price step curve a “demand-for-generation-capacity” curve was proposed. Let us revisit this approach in a very simplified manner. Essentially we propose here to define the capacity price CP_y in year y as a function of total system capacity $\bar{x}_y$ in this year, i.e., $CP_y = CP_y(\bar{x}_y)$. With this definition let us formulate the corresponding MPEC in (4.127)-(4.132).

$$\max_{\Omega_{i^*}} \sum_y \frac{1}{(1+F)^y} \Big\{ \sum_{jl} T_{yl}(p_{yl} - \delta_{i^*j}) q_{i^*jyl} - \sum_j \beta_{i^*jy} x_{i^*jy} + CP_y \sum_j (x_{i^*jy} + K_{i^*jy}) \Big\} \tag{4.127}$$

$$\text{s.t.} \quad 0 \leq x_{i^*jy} \leq x_{i^*j(y+1)} \quad \forall jy \tag{4.128}$$

$$\bar{x}_y = \sum_{ij} (x_{ijy} + K_{ijy}) \quad \forall y \tag{4.129}$$

$$CP_y = G^0_y - \gamma_y \bar{x}_y \quad \forall y \tag{4.130}$$

$$0 \leq CP_y \quad \forall y \tag{4.131}$$

$$(4.21) - (4.30) \tag{4.132}$$

This MPEC is similar to the one previously defined when discussing capacity payments. However, the main difference is that now the actual capacity price CP_y, which appears in the upper level objective function (4.127), is not a constant but a variable function which depends on total system capacity $\bar{x}_y$. Total system capacity is defined in (4.129) as the sum of already existing capacity and new capacity investments of all firms and over all technologies. In theory, the capacity price could be defined as any function in $\bar{x}_y$, but here we will assume that $CP_y(\bar{x}_y)$ is linear in $\bar{x}_y$. In particular, this affine relation between total system capacity and the resulting capacity payment is formulated in (4.130), where G^0_y [M€/GW] corresponds the maximum possible capacity payment and γ_y [M€/GW^2] is the slope of this "demand-for-generation-capacity" curve. Note that the point G^0_y/γ_y [GW] represents the maximum remunerated capacity. Capacity price CP_y is furthermore defined as positive in (4.131) which guarantees that the maximum remunerated capacity is never exceeded. The market equilibrium is given in (4.132). The set of variables of this MPEC is given by $\Omega_{i^*} = \{x_{i^*jy}, \bar{x}_y, CP_y, p_{yl}, d_{yl}, q_{ijyl}, \mu_{ijyl}, \lambda_{ijyl}\} \; \forall ijyl$.

4.4.4 Sources of Uncertainty

As previously pointed out in section 2.4.3, the long-term generation expansion problem is subject to various sources of uncertainty. In section 4.3 of this chapter we have introduced a methodology to handle uncertainty when designing expansion planning models. In particular, we have adapted this methodology to handle uncertainty stemming from competitors' investment decisions and their corresponding strategic spot market behavior. However, there are other factors that have an important impact on capacity expansion planning, the most significant being: uncertainty in fuel prices and demand uncertainty. In this section we want to formalize how these types of uncertainty can be addressed in the stochastic MPEC setting that we have introduced in this chapter. Another factor that can influence investments is future hydro inflows, for which we also propose an adequate model alternative. Other possible sources of uncertainty in the generation expansion planning are not explicitly modeled here; however, for some of them the presented methodology can be adapted correspondingly. An important part of the problem, which the model by itself does not reflect, is to adequately characterize the random variables and construct appropriate scenarios.

Fuel Price Uncertainty

Similar to the uncertainty in hydro inflows or demand uncertainty, presented below, it is possible to model uncertainty stemming from varying fuel prices. In this setting, the variable production costs would be considered under different fuel price scenarios, which would lead to a stochastic parameter δ_{ijys}. In order to model this type of uncertainty it makes sense to have production costs that can vary each year y. Since we are considering various scenarios of variable costs in the lower level, all lower level variables become stochastic and depend on s. The arising lower level is similar to the stochastic lower level with hydro inflow uncertainty or other stochastic lower levels that have been presented in this section and will therefore be omitted here. However, the upper level objective function for firm i^* can be formulated as:

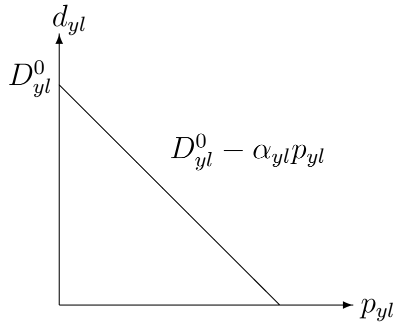

Figure 4-8: Affine demand function in year y and load period l.

$$\max_{x_{i^*jy}} \sum_y \frac{W_s}{(1+F)^y} \Big\{ \sum_{jl} T_{yl}(p_{yls} - \delta_{i^*js}) q_{i^*jyls} - \sum_j \beta_{i^*jy} x_{i^*jy} \Big\} \tag{4.133}$$

Demand Uncertainty

The introduction of renewable energy sources, like wind or solar energy, in modern electricity systems, has caused a rising level of uncertainty of net demand (real demand minus available non-dispatchable renewable generation). In a country like Spain, where more than 20 GW of wind capacity is installed for a peaking power of about 45 GW, the fluctuations in net demand depending on whether or not there is a lot of wind energy dispatched in the system can be quite large. Naturally, this additional source of uncertainty has an impact on the optimal investment mix. For this purpose we present a methodology that allows us to handle wind-induced (or non-dispatchable renewable-induced) uncertainty in the bilevel optimization generation expansion problem.

When introducing the basic version of the MPEC model in section 4.2, demand d_{yl} in year y and load period l has been taken as elastic, which is a realistic assumption when considering the long term. Moreover, demand is defined by an affine expression involving market price p_{yl}, i.e., $d_{yl} = D^0_{yl} - \alpha_{yl} p_{yl}$, where D^0_{yl} has been defined as the demand intercept and α_{yl} the demand slope. This relation is presented graphically in Figure 4-8.

We recall from section 2.2.1 where we introduced this demand representation, that the load periods l of a certain year y are based on the monotonic annual load curve, like the one presented in Figure 2-3. Instead of considering the actual hourly demand, let us consider the hourly net demand, calculated as demand minus wind production, then this would yield the monotonic annual net demand curve, which in general is a much flatter curve than the monotonic annual demand curve. Following the same methodology as in section 2.2.1, load periods can be carved out of this monotonic annual net demand curve. Note that in terms of formulation there is no difference between considering actual demand and net demand, however, there is a conceptual difference. Therefore, when considering net demand in order to form load blocks, the arising data of the demand intercept D_{yl}^0 and the demand slope α_{yl} depend on the hourly wind profile that has been used to calculate the net demand. We emphasize here that different wind profiles or different wind scenarios lead to a different monotonic annual net demand curve and therefore to a different representation of the net demand in a certain year, which in turn leads to different values of demand intercept and demand slope. In Figure 4-9 we have depicted possible scenarios of the affine demand curve in year y and load period l stemming from different scenarios of wind production. Note that introducing renewables into the power system, changes the representation of demand and most likely it changes they way in which load periods are constructed. In a system with a high penetration of renewable energy sources other more technical details of the markets (as for example start-ups, shut-downs and ramping constraints) become more important drivers of the investment decision. Introducing these types of constraints into our generation expansion models poses an interesting task for future research. We have taken a first step in this direction in the conference paper by Nogales et al. [88].

Taking into account different scenarios s of net demand, originating from different scenarios of renewable energy sources like wind energy, the bilevel generation expansion optimization problem can be derived correspondingly. The arising stochastic MPEC is given below, where (4.134)-(4.135) corresponds to the upper level and (4.136)-(4.141) represents the spot market equilibrium for each scenario. The set of variables is characterized by $\Omega_{i^*} = \{x_{i^*jy}, p_{yls}, d_{yls}, q_{ijyls}, \mu_{ijyls}, \lambda_{ijyls}\} \; \forall ijyls$.

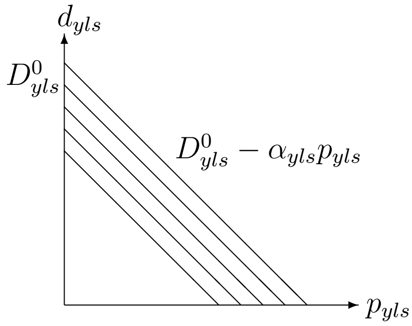

Figure 4-9: Scenarios of the affine demand function in year y and load period l.

$$\max_{\Omega_{i^*}} \sum_y \frac{W_s}{(1+F)^y} \Big\{ \sum_{jl} T_{yl}(p_{yls} - \delta_{i^*j}) \sum_j q_{i^*jyls} - \sum_j \beta_{i^*jy} x_{i^*jy} \Big\} \tag{4.134}$$

$$\text{s.t.} \qquad 0 \le x_{i^*jy} \le x_{i^*j(y+1)} \qquad \forall jy \tag{4.135}$$

$$\frac{\delta_{ij} T_{yl}}{(1+F)^y} + \frac{\theta_{iyl} T_{yl} \sum_j q_{ijyls}}{(1+F)^y} - \frac{T_{yl} p_{yls}}{(1+F)^y} + \lambda_{ijyls} - \mu_{ijyls} = 0 \qquad \forall ijyls \tag{4.136}$$

$$\mu_{ijyls} q_{ijyls} = 0, \quad \lambda_{ijyls}(K_{ijy} + x_{ijy} - q_{ijyls}) = 0 \qquad \forall ijyls \tag{4.137}$$

$$0 \le q_{ijyls} \le x_{ijy} + K_{ijy} \qquad \forall ijyls \tag{4.138}$$

$$0 \le \mu_{ijyls}, \quad 0 \le \lambda_{ijyls} \qquad \forall ijyls \tag{4.139}$$

$$d_{yls} - \sum_{ij} q_{ijyls} = 0 \qquad \forall yls \tag{4.140}$$

$$d_{yls} - D^0_{yls} + \alpha_{yls} p_{yls} = 0 \qquad \forall yls \tag{4.141}$$

Hydro Inflow Uncertainty

In the previous section 4.4.1 we have presented how the basic MPEC model can be extended to accommodate hydro production, which is an essential part of realistic energy systems. The model that has been introduced in 4.4.1, and in particular in (4.94)-(4.96), is a deterministic model which assumes one value of maximum hydro production H_{iyl} [GW] and one fixed value for maximum annual hydro energy E_{iy} [GWh]. The maximum hydro production H_{iyl} is data which mainly depends on the capacity of the turbine of hydro plants, but it is also related to the water level in reservoirs which in turn depends on hydro inflows of a particular year. The maximum annual hydro energy E_{iy} completely depends on the hydro inflows that have taken place in year y. Hence, different scenarios s of possible hydro inflows are introduced and the corresponding hydro data is defined as dependent on the scenarios s yielding H_{iyls} and E_{iys}.

When introducing stochasticity in the lower level of the MPEC, all the lower level variables become stochastic variables and hence depend on s. The lower level variables are: market price p_{yls}; thermal production q_{ijyls}; hydro production h_{iyls}; demand d_{yls}; and all dual variables of lower level constraints, them being upper and lower bounds on thermal production $\mu_{ijyls}, \lambda_{ijyls}$, upper and lower bounds on hydro production $\mu^H_{iyls}, \lambda^H_{iyls}$, and the maximum annual hydro energy constraint with dual variable λ^E_{iys}. With this notation in mind, the corresponding market equilibrium problem, written as the system of KKT conditions, can be formulated as:

$$\frac{\delta_{ij}T_{yl}}{(1+F)^y} + \frac{\theta_{iyl}T_{yl}(h_{iyls} + \sum_j q_{ijyls})}{(1+F)^y} - \frac{T_{yl}p_{yls}}{(1+F)^y} + \lambda_{ijyls} - \mu_{ijyls} = 0 \quad \forall ijyls \tag{4.142}$$

$$\frac{\theta_{iyl}T_{yl}(h_{iyls} + \sum_j q_{ijyls})}{(1+F)^y} - \frac{T_{yl}p_{yls}}{(1+F)^y} + \lambda^E_{iys}T_{yl} + \lambda^H_{iyls} - \mu^H_{iyls} = 0 \quad \forall iyls \tag{4.143}$$

$$\mu_{ijyls}q_{ijyls} = 0, \quad \lambda_{ijyls}(K_{ijy} + x_{ijy} - q_{ijyls}) = 0 \quad \forall ijyls \tag{4.144}$$

$$\mu^H_{iyls}h_{iyls} = 0, \quad \lambda^H_{iyls}(H_{iyls} - h_{iyls}) = 0 \quad \forall iyls \tag{4.145}$$

$$\lambda^E_{iys}(E_{iy} - \sum_l T_{yl}h_{iyls}) = 0 \quad \forall iys \tag{4.146}$$

$$0 \le q_{ijyls} \le x_{ijy} + K_{ijy} \quad \forall ijyls \tag{4.147}$$

$$0 \le h_{iyls} \le H_{iyls} \quad \forall iyls \tag{4.148}$$

$$0 \le \mu_{ijyls}, \quad 0 \le \lambda_{ijyls} \quad \forall ijyls \tag{4.149}$$

$$0 \le \mu^H_{iyls}, \quad 0 \le \lambda^H_{iyls} \quad \forall iyls \tag{4.150}$$

$$0 \le \lambda^E_{iys} \quad \forall iys \tag{4.151}$$

$$d_{yls} - \sum_{i} h_{iyls} - \sum_{ij} q_{ijyls} = 0 \qquad \forall yls \tag{4.152}$$

$$d_{yls} - D^0_{yl} + \alpha_{yl} p_{yls} = 0 \qquad \forall yls \tag{4.153}$$

Since we have introduced stochasticity in the lower level, the upper level of the MPEC has to be adapted to (4.154)-(4.155) for example if we assume that firm i^* is risk neutral. The parameter W_s represents the probability of each scenario s. Note that the arising stochastic MPEC is formed by the upper level (4.154)-(4.155) subject to all the scenarios of the lower level given by (4.142)-(4.153). This formulation allows us to study how different hydro inflow scenarios can impact the generation expansion decisions. In section 7.1 a corresponding case study is presented.

$$\begin{aligned} \max_{x_{i^*jy}} \sum_{y} \frac{W_s}{(1+F)^y} \{ \sum_{l} T_{yl} p_{yls} (h_{i^*yls} + \sum_{j} q_{i^*jyls}) \\ - \sum_{jl} T_{yl} \delta_{i^*j} q_{i^*jyls} \sum_{j} \beta_{i^*jy} x_{i^*jy} \} \end{aligned} \tag{4.154}$$

$$\text{s.t.} \qquad 0 \leq x_{i^*jy} \leq x_{i^*j(y+1)} \quad \forall jy \tag{4.155}$$

4.4.5 More Details

Additionally to the model extensions that have been presented in section 4.4, there are other details that can be modeled in the generation expansion problem in order to make it more realistic. One of these examples would be financial hedging (or price hedging). Let us define the parameter financial coverage M_{iyl} as the quantity in [GW] of a firm i which is not exposed to the market price. This can be obtained, for example, through contracts for differences where quantity M_{iyl} is offered at a certain price η_{iyl} in [€/MWh]. Both M_{iyl} and η_{iyl} can be considered as data for the generation expansion problem. With this in mind we can formulate the concept of the market equilibrium problem below. The only difference to the formulation introduced in section 2.3.1 is the lower level objective function representing the market net present value.

$$\forall i \begin{cases} \max_q \sum_y \frac{1}{(1+F)^y}\big\{ \sum_{jl} T_{yl}(p_{yl}-\delta_{ij})q_{ijyl} + \sum_l T_{yl}M_{iyl}(\eta_{iyl}-p_{yl})\big\} \\ \text{s.t.} \qquad q_{ijyl} \le x_{ijy} + K_{ijy} \quad \forall jyl \quad : \lambda_{ijyl} \\ \qquad 0 \le q_{ijyl} \quad \forall jyl \quad : \mu_{ijyl} \end{cases} \tag{4.156}$$

$$d_{yl} - \sum_{ij} q_{ijyl} = 0 \quad \forall yl \tag{4.157}$$

$$d_{yl} - D^0_{yl} + \alpha_{yl}p_{yl} = 0 \quad \forall yl \tag{4.158}$$

By rearranging terms the lower level objective function can be rewritten as (4.159). The last term is a constant and does hence not influence the optimality conditions. The term $(\sum_j q_{ijyl} - M_{iyl})$ represents the power that is still exposed to the market and is therefore subject to price risk. With respect to this term the situation of the firm is usually characterized by the expressions “short” or “long”. When the term is positive the firm is “long”, which means that they produce more power than they have hedged by a contract for differences, then it is interested in high market prices which would increase profits. On the other hand, when the firm is “short” then they produce less than what is covered by the contract for differences and hence they are interested in low market prices.

$$\begin{aligned} \max_q \sum_y \frac{1}{(1+F)^y}\big\{ \sum_l T_{yl}p_{yl}(\sum_j q_{ijyl} - M_{iyl}) - \sum_{jl} T_{yl}\delta_{i^*j}q_{ijyl} + \\ \sum_l T_{yl}M_{iyl}\eta_{iyl}\big\} \end{aligned} \tag{4.159}$$

The conceptual market equilibrium given by (4.156)-(4.158) can be replaced by its KKT conditions, given by (4.160)-(4.165), since the convexity of the problem is not compromised by adding the linear term representing the contract for differences.

$$\begin{aligned} \frac{\delta_{ij}T_{yl}}{(1+F)^y} + \frac{T_{yl}\theta_{iyl}(\sum_j q_{ijyl} - M_{iyl})}{(1+F)^y} - \frac{T_{yl}p_{yl}}{(1+F)^y} \quad \forall ijyl \\ +\lambda_{ijyl} - \mu_{ijyl} = 0 \end{aligned} \tag{4.160}$$

$$\mu_{ijyl}q_{ijyl} = 0, \quad \lambda_{ijyl}(K_{ijy} + x_{ijy} - q_{ijyl}) = 0 \quad \forall ijyl \tag{4.161}$$

$$0 \leq q_{ijyl} \leq x_{ijy} + K_{ijy} \qquad \forall ijyl \tag{4.162}$$

$$0 \leq \mu_{ijyl}, \quad 0 \leq \lambda_{ijyl} \qquad \forall ijyl \tag{4.163}$$

$$d_{yl} - \sum_{ij} q_{ijyl} = 0 \qquad \forall yl \tag{4.164}$$

$$d_{yl} - D^0_{yl} + \alpha_{yl} p_{yl} = 0 \qquad \forall yl \tag{4.165}$$

Finally, the corresponding MPEC which contains a formulation for price hedging through a contract for differences is presented below. The set of variables remains the same as in the basic model, i.e., $\Omega_{i^*} = \{x_{i^*jy}, p_{yl}, d_{yl}, q_{ijyl}, \mu_{ijyl}, \lambda_{ijyl}\}$ $\forall ijyl$.

$$\begin{aligned} \max_{\Omega_{i^*}} \sum_y \frac{1}{(1+F)^y} \Big\{ \sum_{jl} T_{yl}(p_{yl} - \delta_{i^*j}) \sum_j q_{i^*jyl} \\ - \sum_j \beta_{i^*jy} x_{i^*jy} + \sum_l T_{yl} M_{iyl}(\eta_{iyl} - p_{yl}) \Big\} \end{aligned} \tag{4.166}$$

s.t.

$$0 \leq x_{i^*jy} \leq x_{i^*j(y+1)} \qquad \forall jy \tag{4.167}$$

$$\begin{aligned} \frac{\delta_{ij} T_{yl}}{(1+F)^y} + \frac{T_{yl}\theta_{iyl}(\sum_j q_{ijyl} - M_{iyl})}{(1+F)^y} - \frac{T_{yl} p_{yl}}{(1+F)^y} \\ + \lambda_{ijyl} - \mu_{ijyl} = 0 \end{aligned} \qquad \forall ijyl \tag{4.168}$$

$$\mu_{ijyl} q_{ijyl} = 0, \quad \lambda_{ijyl}(K_{ijy} + x_{ijy} - q_{ijyl}) = 0 \qquad \forall ijyl \tag{4.169}$$

$$0 \leq q_{ijyl} \leq x_{ijy} + K_{ijy} \qquad \forall ijyl \tag{4.170}$$

$$0 \leq \mu_{ijyl}, \quad 0 \leq \lambda_{ijyl} \qquad \forall ijyl \tag{4.171}$$

$$d_{yl} - \sum_{ij} q_{ijyl} = 0 \qquad \forall yl \tag{4.172}$$

$$d_{yl} - D^0_{yl} + \alpha_{yl} p_{yl} = 0 \qquad \forall yl \tag{4.173}$$

4.5 Conclusions

In this chapter we have discussed bilevel optimization models for generation expansion planning. In particular, we have introduced a bilevel optimization problem in section 4.2 that models the behavior of an investing firm when facing generation capacity investment decisions. The lower level represents the market using a conjectured-price variation, which allows to consider market situations between perfect competition and Cournot oligopoly. In the upper level, a simplified version of the NPV of the investor firm is maximized. The model shows that, even if the spot market is perfectly competitive, strategic behavior at the investment stage, for example, caused by significant entry barriers, reduces the efficiency of the market, and that such effects can be higher than oligopoly in spot markets. A numerical example has been presented in section 4.2.4 to confirm the relevance of the proposed model formulation. We furthermore observe different investment portfolios, that is, a shift from base-load to peak-load technologies, when the representation of the spot market is changed from perfect competition to Cournot oligopoly, considering different values for conjectural variations as intermediate situations. The obtained capacity in the perfect competition situation is smaller than in the oligopolistic cases.

Then, in section 4.3 we extend the previously proposed model to a stochastic framework, where we introduce a stochastic bilevel model to assist a generation company to decide its investment schedule assuming different investment scenarios of the competition, however, the same methodology could be applied to different production cost scenarios or demand uncertainty, as will be shown in section 7. In the upper level the investing agent maximizes its expected net present value. The lower level corresponds to several scenarios of a conjectural variations market equilibrium, which allows us to cope with uncertainty in the generation expansion problem. The proposed bilevel methodology has been applied to a case study in section 4.3.4, where two resolution methods, a nonlinear method and a linearization method, have been compared. The nonlinear method quickly resolves the exact problem but can not guarantee global optimality. The linearization method guarantees a global optimum, but it takes much longer to solve and the choice of the step size is critical. We furthermore compare the stochastic solution to several instances of the deterministic MPEC and observe that the investment decisions vary drastically.

Finally, in section 4.4 we have discussed how to formulate possible model extensions, which help to make our models more realistic and help address the issue of modeling different kinds of uncertainty in the generation expansion problem.

Chapter 5

Bilevel Generation Expansion Equilibrium Models

In this chapter a bilevel generation expansion equilibrium model is introduced and formulated as an EPEC. In contrast to the previous chapter 4 where the presented MPEC model only considered one generation company to decide its capacity investments, the EPEC model of this chapter takes into account investment decisions of all generation companies. Therefore, this chapter extends the bilevel equilibrium model of chapter 3 to a multi-year and multi-technology framework which is necessary when trying to tackle real-world problems. First of all, an introduction to this problem and a literature review are presented in section 5.1. Then, in section 5.2 the formulation of the EPEC is introduced and a case study is presented from which it becomes apparent that EPECs are quite hard to solve. In section 5.3 we revisit the single-level expansion equilibrium and show how to formulate it as an equivalent quadratic convex optimization problem, which can be solved very efficiently, which will be taken advantage of in the following section. As an alternative, in section 5.4 a single-level approximation scheme of the bilevel equilibrium problem is proposed and validated in a numerical example. Due to this newly developed scheme, bilevel equilibria can - under certain conditions - be approximated two orders of magnitude faster than with a standard EPEC solving technique like diagonalization. Finally, we draw some conclusions in section 5.5.

5.1 Introduction

Generation capacity expansion planning in liberalized electricity markets is a very complex task and poses a great challenge to both generation companies (GENCOs) and regulators. Planning capacity expansion is a long-term problem whose time horizon can range up to several decades and involves the investment of huge quantities of money.

As mentioned, there exist various approaches to generation expansion planning, including scenario analysis, decision theory, real options, system dynamics or game theory. A natural way of representing the generation expansion planning problem is using a game-theoretic approach because it allows us to define the generation capacity expansion problem as a game among generation companies where each GENCO is explicitly maximizing total profits deciding capacity investments. Arising profits do not only depend on capacity investments of the individual GENCO but also on the decisions taken by the competitors which allows to represent this problem as a game, with an equilibrium point a la Nash [86].

Within the game-theoretic framework we particularly want to differentiate between single-level and bilevel generation expansion equilibrium approaches. In previous work, presented in chapter 3 or published in Wogrin et al. [119], we have compared two different capacity equilibrium models that represent either a one-stage or a two-stage competitive situation. The one-stage situation is represented by a single-level model and describes the one-shot investment-operation market equilibrium. In this situation, firms simultaneously choose capacities and quantities to maximize their individual profit, while each firm conjectures a price response to its output decisions. The bilevel model describes the two-stage investment-operation market equilibrium, where firms first choose capacities that maximize their profit while anticipating the equilibrium outcomes in the second stage, in which quantities and prices are determined by a conjectured-price-response market equilibrium. When comparing these single-level and bilevel equilibrium models, we have found that the bilevel model is more realistic than the single-level model. One-stage models can fail to capture realistic features, as for example the under-investment for strategic reasons, and that is why we think a two-stage problem is better suited for representing the generation expansion problem. We adopt this modeling approach because it captures the innate temporal and hierarchical separation between the time when investment decisions are taken and the time when energy is dispatched.

Within the last decade, bilevel models have been increasingly applied to problems arising from the energy sector and in particular from electricity markets. Some examples of operational bilevel equilibrium approaches can be found in the work of Pozo and Contreras [96], Ralph and Smeers [99] and Ruiz et al. [105]. Considering that the focus of this thesis is bilevel approaches to capacity expansion, we discuss the existing literature in more detail.

As previously mentioned, bilevel generation capacity models represent sequential decision making, which is more realistic than single-level approaches. Among the bilevel approaches we differentiate between the two-stage optimization models which lead to mathematical programs with equilibrium constraints (MPECs) [80], and two-stage equilibrium models which lead to equilibrium problems with equilibrium constraints (EPECs) [111]. Since bilevel MPEC approaches to generation expansion planning have been discussed in chapter 4, they will not be repeated here. Instead we focus on bilevel EPEC approaches.

A classic example of EPECs or bilevel equilibrium models can be found in [74], where Kreps and Scheinkman construct a two-stage game, where first firms simultaneously set capacity and second, after capacity levels are made public, there is price competition. They find that when assuming two identical firms and an efficient rationing rule their two-stage game yields Cournot outcomes. Another example is the bilevel Cournot model of Murphy and Smeers [84], where in the upper level firms choose capacity and in the lower level they sell production. The authors furthermore demonstrate that the bilevel Cournot equilibrium capacities fall between the single-level Cournot and the single-level competitive solutions. In a working paper by Grimm and Zoettl [56], the authors examine bilevel equilibrium models considering only the polar cases of perfect or Cournot-type competition, where firms choose capacities under uncertainty. Gabszewicz and Poddar [52] study the impact of uncertain demand on firms' capacity decisions when they operate in an oligopolistic environment. The authors define a two-stage game where firms choose capacity in the first stage subject to a second stage with demand uncertainty.

The contributions in the methodology and formulation of the generation expansion EPEC in this chapter are as follows: first of all, we extend existing bilevel approaches - as for example the work of Kazempour and Conejo [69] or Murphy and Smeers [84], which assume either perfect competition or Cournot competition in the spot market - to capture intermediate strategic behavior using a conjectured-price response market representation. Second, the model presented in this chapter extends the single-year, single-technology bilevel equilibrium presented in chapter 3 to a multi-year framework, and it yields an investment schedule over the entire time horizon, as opposed to a static investment decision for a future target year. Third, instead of considering to cost minimization as the driver of investment decisions, which is not representative of what happens in liberalized electricity markets, we rather maximize profits as opposed to minimize costs. Furthermore we extend MPEC approaches, that emphasize on investment decisions of one generation company in particular while competitors play a secondary role, to EPEC approaches where each generation company maximizes its profits (rather than minimizing costs) subject to the conjectured-price response market equilibrium.

From a computational point of view it should be stated that solving large-scale bilevel equilibrium models can be challenging and potentially take a very long time, as pointed out in the works of Hu [62], Garcia-Bertrand [63], Leyffer [78] and Ruiz [105]. In the numerical example, presented in section 5.2.4, this also becomes apparent. Since the complexity of the model formulation complicates commonly known solution processes, an improved technique - an approximation scheme - to solve the generation expansion EPEC is developed and has been published in Wogrin et al. [117].

For this purpose, the following computational contributions have been made: in section 5.3 we show how to formulate the single-level capacity expansion equilibrium problem as an equivalent convex quadratic optimization problem, which can be solved very efficiently. Second, we propose a newly developed single-level approximation scheme for the bilevel capacity expansion equilibrium problem in section 5.4, which is based on the theoretical results obtained in chapter 3 and relies on the alternative formulation of the single-level equilibrium presented in section 5.3. Third, in section 5.4.3 we carry out a small numerical example to test the proposed approximation scheme and draw conclusions of when the approximation works best. A large-scale case study, which confirms our initial conclusions of when the approximation is most accurate, is presented later on in section 7.3.

5.2 Bilevel Generation Expansion Equilibria Formulated as an EPEC

This section is dedicated to the presentation of the bilevel generation expansion equilibrium model, formulated as an EPEC, in a multi-year and multi-technology framework, as opposed to the single-year, single technology model that has previously been discussed in chapter 3. The bilevel model results will be referred to as (BL) throughout this section. This model also yields an investment schedule over the entire time horizon, as opposed to a static investment decision for a future target year, as done by static approaches like the one in Kazempour et al. [70]. This model formulation has been published in Wogrin et al. [115]. Unlike approaches that resort to cost minimization when deciding investments, such as Baringo and Conejo [3], in our model we consider profit maximization. Other approaches, like the ones presented in chapter 4, emphasize on investment decisions of one generation company in particular, while competitors play a secondary role as in Kazempour et al. [69, 70] or Wogrin et al. [116]. We remedy this by proposing an EPEC where each generation company maximizes its profits (rather than minimizing costs) subject to the conjectured-price response market equilibrium.

We start out by recalling the concepts of bilevel equilibria in section 5.2.1 which have been introduced in chapter 2 and which present the MPEC as a part of the

EPEC. Since the MPEC is a necessary ingredient to arrive at the EPEC, in section 5.2.2 we recall the formulation of the MPEC of chapter 4. Finally, in section 5.2.3 the EPEC is formulated. This formulation as well as the following numerical example of section 5.2.4 are an original contribution of this thesis and have been published in Wogrin et al. [115]. In this section we do not focus on EPEC solution techniques. For this topic the reader is referred to chapter 6.

5.2.1 Revisiting Concepts of Bilevel Expansion Equilibria

The concept of the bilevel generation expansion equilibrium models have first been introduced in section 2.4.2 of this thesis, where we mentioned that they were the type of problems where each player was facing a bilevel generation expansion optimization problem. This bilevel generation expansion optimization problem can be formulated as an MPEC and has been discussed and analyzed in detail in the previous chapter of this thesis. It can therefore be said that the corresponding MPEC of a certain player, i.e., a generation company participating in the market, is a key ingredient of the bilevel equilibrium model that we derive in this chapter.

A rather conceptual formulation of such a bilevel equilibrium model has already been made in section 2.4.2 of this thesis and shall be repeated here for the purpose of clarity. In the upper level, firms i^*, where i and i^* are alias indices, maximize total profits deciding capacity investments x_{i^*jy} in all technologies j and all years y, subject to the market equilibrium conditions, which yields production decisions q_{ijyl}, demand d_{yl} and prices p_{yl}.

$$\forall i^* \begin{cases} \max_{x_{i^*jy}} \sum_y \frac{1}{(1+F)^y} \left\{ \sum_{jl} T_{yl}(p_{yl} - \delta_{i^*j}) q_{i^*jyl} - \sum_j \beta_{i^*jy} x_{i^*jy} \right\} \\ \text{s.t.} \qquad 0 \leq x_{i^*jy} \leq x_{i^*j(y+1)} \quad \forall jy \\ \text{s.t.} \qquad \text{Market Equilibrium Formulation} \end{cases} \tag{5.1}$$

From the above formulation it becomes apparent that this type of model actually is an equilibrium problem in two levels - just as the name “bilevel equilibrium” motivates. As a matter of fact, in this type of problem, firms are competing on both levels, the upper level representing the investment stage, and the lower level representing the market stage. This stands in contrast to the MPEC models of the previous chapter. Even though MPECs are also bilevel models, they only consider an equilibrium in the lower level but the upper level consists of an optimization problem. This is exactly the reason why the MPEC is referred to as a "bilevel optimization" model.

Since we have established the MPEC as a key module of the bilevel capacity equilibrium model, this problem can be written as $\{\text{MPEC}(i^*)\}_{i^*=1}^{I}$, where i^* is the index of generation firms and I is the total number of firms. In section 2.4.2 we have previously mentioned that this type of problem can be formulated as an EPEC. Finally, in the remainder of section 5.2 we show how to come by such a formulation.

5.2.2 The MPEC as Part of the EPEC

It has been established, in section 2.4.2 such as in section 5.2.1, that the generation expansion bilevel optimization MPEC, like the one presented in the previous chapter in section 4.2.3 of this thesis, form a part of the bilevel capacity equilibrium model that we wish to derive in this thesis chapter. Therefore, we revisit the previously proposed MPEC, representing the generation expansion bilevel problem of one generation company i^*.

Let us now write the corresponding MPEC for company i^* whose primal variables are given by the set $\Omega_{i^*} = \{x_{i^*jy}, q_{ijyl}, p_{yl}, d_{yl}, \mu_{ijyl}, \lambda_{ijyl}\}$. The dual variables of each constraint can be found after the colon or that same constraint. Note that the notation of the dual variables is slightly different here (as opposed to previous instances) due to their increased amount in the subsequent formulation.

$$\max_{\Omega_{i^*}} \sum_y \frac{1}{(1+F)^y} \Big\{ \sum_{jl} T_{yl}(p_{yl} - \delta_{i^*j}) q_{i^*jyl} - \sum_j \beta_{i^*jy} x_{i^*jy} \Big\} \tag{5.2}$$

$$\text{s.t.} \qquad 0 \le x_{i^*jy} \quad \forall jy \qquad : \quad \xi^{x_{min}}_{i^*jy} \tag{5.3}$$

$$x_{i^*j(y-1)} \le x_{i^*jy} \quad \forall jy \qquad : \quad \xi^{x_{max}}_{i^*jy} \tag{5.4}$$

$$q_{ijyl} \le x_{ijy} + K_{ijy} \quad \forall ijyl \qquad : \quad \xi^{q_{max}}_{i^*ijyl} \tag{5.5}$$

$$0 \le q_{ijyl} \quad \forall ijyl \qquad : \quad \xi^{q_{min}}_{i^*ijyl} \tag{5.6}$$

$$0 \leq \mu_{ijyl} \quad \forall ijyl \qquad : \; \omega^{\mu}_{i^*ijyl} \tag{5.7}$$

$$0 \leq \lambda_{ijyl} \quad \forall ijyl \qquad : \; \omega^{\lambda}_{i^*ijyl} \tag{5.8}$$

$$\begin{aligned} \frac{\delta_{ij} T_{yl}}{(1+F)^y} + \frac{\sum_j \theta_{iyl} q_{ijyl} T_{yl}}{(1+F)^y} - \frac{T_{yl} p_{yl}}{(1+F)^y} \\ + \lambda_{ijyl} - \mu_{ijyl} = 0 \quad \forall ijyl \end{aligned} \qquad : \; \zeta^{q}_{i^*ijyl} \tag{5.9}$$

$$\mu_{ijyl} q_{ijyl} = 0 \quad \forall ijyl \qquad : \; \zeta^{\mu}_{i^*ijyl} \tag{5.10}$$

$$\lambda_{ijyl}(K_{ijy} + x_{ijy} - q_{ijyl}) = 0 \quad \forall ijyl \qquad : \; \zeta^{\lambda}_{i^*ijyl} \tag{5.11}$$

$$D^0_{yl} - \alpha_{yl} p_{yl} - d_{yl} = 0 \quad \forall yl \qquad : \; \zeta^{d}_{i^*yl} \tag{5.12}$$

$$\sum_{ij} q_{ijyl} - d_{yl} = 0 \quad \forall yl \qquad : \; \zeta^{Bal}_{i^*yl} \tag{5.13}$$

5.2.3 Formulation of the EPEC

Finally, we arrive at the desired bilevel (BL) equilibrium model when all companies i face the MPEC described in the previous section 5.2.2. We formulate the bilevel equilibrium as an EPEC by combining the KKT conditions of each GENCO's MPEC, which is presented below. A graphic representation of the EPEC consisting of each generation company's MPEC is provided in Figure 5-1.

Note that $\perp$ denotes a complementarity between a constraint and its dual variable. Equations (5.14)-(5.19) correspond to the derivative of the Lagrangian with respect to the decision variables. In particular, equation (5.14) corresponds to the derivative of the Lagrangian with respect to the investment variables. Depending on the year y, the variable x_{i^*jy} can appear on either one or both sides of the constraint (5.4). Let us consider that $y = 1, \ldots, Y$, then we have to differentiate three cases: if $1 < y < Y$ then x_{i^*jy} appears on both sides of (5.4), which leads to terms like $\xi^{x_{max}}_{i^*jy} - \xi^{x_{max}}_{i^*jy-1}$ in the derivative; if $y = Y$ then x_{i^*jy} only appears once, on the right hand side of (5.4), which leads to $-\xi^{x_{max}}_{i^*jy-1}$; similarly, if $y = 1$, then x_{i^*jy} also only appears once, on the left hand side of (5.4) which leads to $\xi^{x_{max}}_{i^*jy}$ in the derivative. In order to show that the derivative of x_{i^*jy} in (5.14) depends on the index y, we have put the corresponding terms in between curly braces $\{\cdot\}$. These terms are only included if the condition, which is indicated on the lower right corner of the closing bracket, is

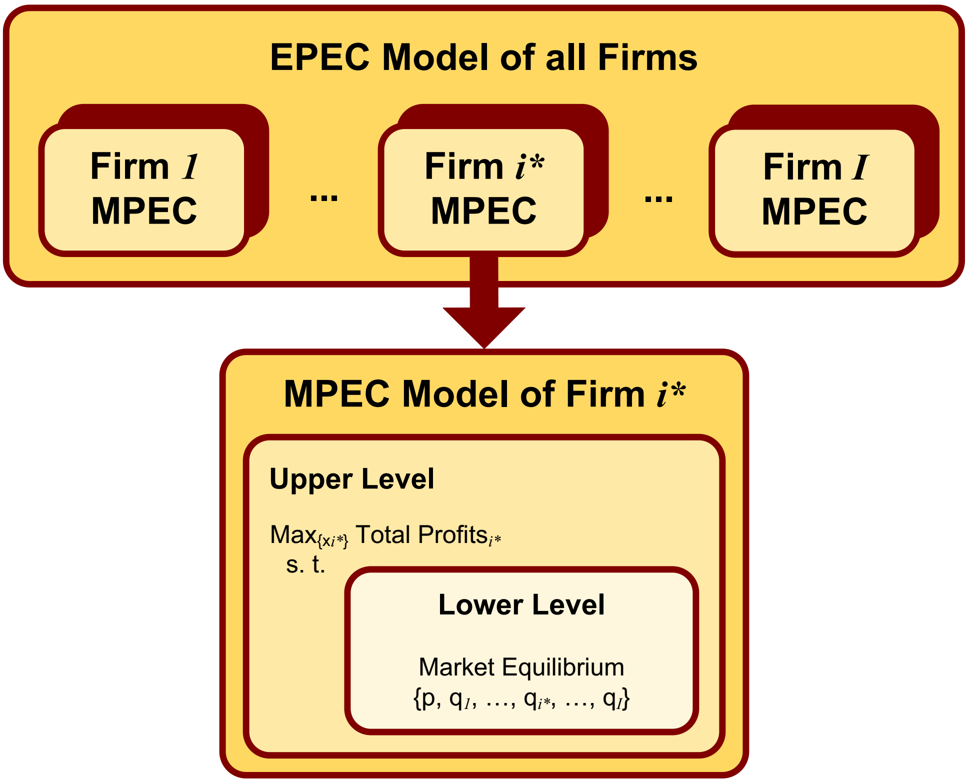

Figure 5-1: The EPEC generation expansion as a system of MPEC problems faced by all firms.

true. The same is also true for (5.15), where the term in curly braces is only included when $i = i^*$. (5.20)-(5.25) represent the inequality constraints and the corresponding complementarity conditions. Finally, (5.26) corresponds to the equality conditions stemming from the lower level.

Bilevel Equilibrium Model (BEM):

$$\begin{aligned}\partial L_{i^*}/\partial x_{i^*jy} = \{\xi^{x_{max}}_{i^*j(y+1)}\}_{y=1} + \{\xi^{x_{max}}_{i^*j(y+1)} - \xi^{x_{max}}_{i^*jy}\}_{1<y<Y}& \\ +\{-\xi^{x_{max}}_{i^*jy}\}_{y=Y} + \frac{\beta_{i^*jy}}{(1+F)^y} - \xi^{x_{min}}_{i^*jy}& \quad \forall i^*jy \quad (5.14) \\ -\sum_l \xi^{q_{max}}_{i^*i^*jyl} + \sum_l \lambda_{i^*jyl}\zeta^{\lambda}_{i^*i^*jyl} = 0&\end{aligned}$$

$$\begin{aligned}\partial L_{i^*}/\partial q_{ijyl} = \{-\frac{T_{yl}p_{yl}}{(1+F)^y} + \frac{T_{yl}\delta_{ij}}{(1+F)^y}\}_{i=i*} + \frac{\theta_{iyl}T_{yl}\zeta^{q}_{i^*ijyl}}{(1+F)^y}& \quad \forall i^*ijyl \quad (5.15) \\ +\zeta^{Bal}_{i^*yl} + \xi^{q_{max}}_{i^*ijyl} - \xi^{q_{min}}_{i^*ijyl} + \zeta^{\mu}_{i^*ijyl}\mu_{ijyl} - \zeta^{\lambda}_{i^*ijyl}\lambda_{ijyl} = 0&\end{aligned}$$

$$\partial L_{i^*}/\partial \mu_{ijyl} = -\zeta^{q}_{i^*ijyl} - \omega^{\mu}_{i^*ijyl} + q_{ijyl}\zeta^{\mu}_{i^*ijyl} = 0 \qquad \forall i^*ijyl \quad (5.16)$$

$$\partial L_{i^*}/\partial \lambda_{ijyl} = \zeta^{q}_{i^*ijyl} - \omega^{\lambda}_{i^*ijyl} + (K_{ijy} + x_{ijy} - q_{ijyl})\zeta^{\lambda}_{i^*ijyl} = 0 \qquad \forall i^*ijyl \quad (5.17)$$

$$\partial L_{i^*}/\partial p_{yl} = -\frac{T_{yl}\sum_j q_{i*jyl}}{(1+F)^y} - \frac{\sum_{ij}\zeta^{q}_{i^*ijyl}T_{yl}}{(1+F)^y} - \zeta^{d}_{i^*yl}\alpha_{yl} = 0 \qquad \forall i^*yl \quad (5.18)$$

$$\partial L_{i^*}/\partial d_{yl} = -\zeta^{d}_{i^*yl} - \zeta^{Bal}_{i^*yl} = 0 \qquad \forall i^*jy \quad (5.19)$$

$$0 \le \xi^{x_{max}}_{i^*jy} \quad \perp \quad x_{i^*j(y-1)} \le x_{i^*jy} \qquad \forall i^*jy \quad (5.20)$$

$$0 \le \xi^{x_{min}}_{i^*jy} \quad \perp \quad 0 \le x_{i^*jy} \qquad \forall i^*jy \quad (5.21)$$

$$0 \le \xi^{q_{max}}_{i^*ijyl} \quad \perp \quad q_{ijyl} \le x_{ijy} + K_{ijy} \qquad \forall i^*ijyl \quad (5.22)$$

$$0 \le \xi^{q_{min}}_{i^*ijyl} \quad \perp \quad 0 \le q_{ijyl} \qquad \forall i^*ijyl \quad (5.23)$$

$$0 \le \omega^{\mu}_{i^*ijyl} \quad \perp \quad 0 \le \mu_{ijyl} \qquad \forall i^*ijyl \quad (5.24)$$

$$0 \le \omega^{\lambda}_{i^*ijyl} \quad \perp \quad 0 \le \lambda_{ijyl} \qquad \forall i^*ijyl \quad (5.25)$$

$$(5.9) - (5.13) \qquad (5.26)$$

From the mathematical point of view, the optimization problem of one GENCO that we describe in section 5.2.2 is a nonlinear problem. Moreover, due to the market revenue term in the objective function, the entire objective function is non-concave and we have nonlinear, non-convex constraints as well, which are the complementarity conditions. Hence, global optimality is not guaranteed. This means that when applying a nonlinear solver, we might end up in a local optimum and there might be multiple local optima. This problem does not disappear when forming the EPEC, quite the contrary. EPECs are known to be very complicated problems which could have multiple equilibria (including an infinite number of them) or even none at all.

There exist various methods to solve an EPEC and section 6.2 of chapter 6 has been dedicated to discuss and analyze these methods in more detail. As a preview let us mention methods like diagonalization, see Hu [62] or writing the EPEC as a Complementarity Problem and solving it using, for example, PATH [63]. A disadvantage of the aforementioned methods is that - even when terminated properly - one can only guarantee to find a local equilibrium. However, the advantage of these methods is that the computational time can be quite fast. An EPEC may have multiple (even infinite) equilibria, which could be very different from each other. For this reason, another resolution method would be a combination between diagonalization and the linearization approach as proposed in Ruiz et al. [105], where a linear version of the EPEC is formulated and a linear classification function is defined, which characterizes the equilibrium that one wants to find and then diagonalization is employed to verify that the obtained point actually is an equilibrium.

5.2.4 Numerical Example of Bilevel Expansion Equilibria

This case study has been taken from Wogrin et al. [115] and its purpose is threefold: first, we apply the presented EPEC methodology to a numerical example; second, we show that even for a simple instance of the EPEC, there can be multiple equilibria, which is why it is so important to be able to choose among them; third, we want to demonstrate that equilibrium solutions of the EPEC can differ in terms of the optimal technology mix - another argument for the necessity of exploring the solution space of the EPEC. First, we briefly describe the methodology we apply to be able to choose among different equilibria of the EPEC, then we describe the stylized electric power system that serves as our case study and finally we present the corresponding numerical results.

Classification of Equilibria

As above, we want to be able to choose a particular equilibrium among all the possible equilibria of the EPEC. In order to achieve this goal, we employ a linearization approach which is discussed in detail in section 6.2. In this approach the EPEC is formulated as a MILP with a naive objective function and then a linear classification function which serves as the objective function of this MILP whose constraints consist of the linear version of the EPEC. This is an additional objective function which is neither the objective function of the lower nor of the upper level of the EPEC.

There is a wide range of classification functions that would make sense in the generation capacity expansion framework and different market agents might be interested in different types of equilibria. However, considering that our EPEC is designed to facilitate capacity investment decisions in a liberalized framework, it makes sense to choose the equilibrium that maximizes total profits. The desired classification function representing weighted total profits of all GENCOs is given in (5.27), where V_i represents the weight that we assign to each GENCO's profit. Changing these weights allows us to explore the solution space of the EPEC.

$$\sum_i V_i \sum_y \frac{1}{(1+F)^y} \{ \sum_{jl} T_{yl} p_{yl} q_{ijyl} - \sum_{jl} T_{yl} \delta_{ij} q_{ijyl} - \sum_j \beta_{ijy} x_{ijy} \} \tag{5.27}$$

As we are trying to build a MILP, the classification function has to be linear. The annual market revenues of GENCO i (price times quantities) in the classification function presented above, however, are nonlinear. Therefore, this particular classification function has to be approximated linearly in order for us to use it in a MILP setting. The details of this linearization/approximation approach can be found in section 6.2. For the remainder of this case study we assume that such a linear approximation has been achieved.

System Description

The stylized electric power system in this case study consists of two generation companies i_1, i_2, each of which has the choice to invest in two different technologies, them being coal and combined cycle gas turbine (CC). The respective unitary production and investment costs are given in Table 5.1. The time horizon considered is two years with two load levels each, i.e., peak and off-peak. In Table 5.6 we provide the duration T_{yl} of each of the two load levels and the demand intercept D^0_{yl} of the affine price-demand function. The demand slope α_{yl} for all years and load levels is considered to be 0.23 GW/(€/MWh) based on Garcia-Alcalde [53]. In this case study we do not consider initial capacity. Moreover the discount rate F is set at 9%. As for the weights V of the classification function and the strategic spot market behavior θ_{iyl}, we try out several different cases to explore the solution space of the EPEC and the corresponding results are presented as follows.

Table 5.1: Production and investment cost for each technology.

	Production cost δ [€/MWh]	**Annual investment cost** β [(M€/GW)/year]
Coal	33.0	50.3
CC	39.0	25.4

Table 5.2: Demand intercept D^0_{yl} and load level durations T_{yl}.

			Peak l_1	**Off-peak** l_2
D^0_{yl}	[GW]	y_1	27.61	18.03
		y_2	28.22	18.42
T_{yl}	[h]	y_1	3300	5460
		y_2	3300	5460

Analysis of Results

The bilevel equilibrium capacity expansion model presented in this case study translates to the optimization of the linear classification function subject to the linear version of the EPEC and is modeled as a MILP, formulated in GAMS, and solved using CPLEX on an Intel(R) Core(TM) i5-2410M running at 2.30 GHz 4 GB RAM. The numerical model of this case study consists of 2609 continuous and 1036 binary variables.

We consider several different cases of strategic behavior and weights in this case study. In the first four cases we present how different weights can lead to different equilibria. In the first two cases for example, the strategic behavior we adopt is $\theta_{iyl} = 0.25/\alpha_{yl}$, and we vary the weights of the classification function. In particular, in the first case, the classification function maximizes the approximated profits of i_1 only, i.e., $V_i = (1, 0)$; in the second case the classification function regards both GENCOs as equally important, i.e., $V_i = (0.5, 0.5)$. The same is done assuming $\theta_{iyl} = 0.5/\alpha_{yl}$. The average computational time with weights $V_i = (1, 0)$ is about 10 hours.

In the rest of the cases we compare results with the same weights, i.e., $V_i = (0.5, 0.5)$ and vary the strategic spot market behavior in order to compare how it affects investment decisions. In particular we change the reigning spot market behavior from more competitive cases starting at $\theta_{il} = 0.25/\alpha_{yl}$ to Cournot, i.e., $\theta_{il} = 1/\alpha_{yl}$. The average computational time with weights $V_i = (0.5, 0.5)$ is about 24 hours.

Table 5.3: Investments [GW] and total capacity [GW].

Case	**Year**	**Investments Coal**		**Investments CC**		**Total capacity**
		i_1	i_2	i_1	i_2	
$\theta_{il} = 0.25/\alpha_{yl}$	y_1	4.32	3.79	1.33	1.79	11.68
$V = (1, 0)$	y_2	4.34	4.09	1.46	1.79	
$\theta_{il} = 0.25/\alpha_{yl}$	y_1	4.03	4.02	1.60	1.60	11.65
$V = (0.5, 0.5)$	y_2	4.20	4.20	1.63	1.63	
$\theta_{il} = 0.5/\alpha_{yl}$	y_1	3.91	3.42	1.73	2.17	11.67
$V = (1, 0)$	y_2	3.91	3.69	1.90	2.17	
$\theta_{il} = 0.5/\alpha_{yl}$	y_1	3.62	3.62	2.00	2.00	11.65
$V = (0.5, 0.5)$	y_2	3.78	3.78	2.04	2.05	
$\theta_{il} = 0.6/\alpha_{yl}$	y_1	3.48	3.48	2.14	2.14	11.65
$V = (0.5, 0.5)$	y_2	3.64	3.63	2.19	2.20	
$\theta_{il} = 1/\alpha_{yl}$	y_1	3.40	3.40	2.22	2.22	11.65
$V = (0.5, 0.5)$	y_2	3.54	3.54	2.29	2.29	

Table 5.4: Profits and classification function value [M€].

Case	**Profits**		**Classification function value**
	i_1	i_2	
$\theta_{il} = 0.25/\alpha_{yl}$ $V = (1, 0)$	1213.74	1178.78	1223.43
$\theta_{il} = 0.25/\alpha_{yl}$ $V = (0.5, 0.5)$	1201.94	1201.53	1201.80
$\theta_{il} = 0.5/\alpha_{yl}$ $V = (1, 0)$	1306.25	1262.40	1315.07
$\theta_{il} = 0.5/\alpha_{yl}$ $V = (0.5, 0.5)$	1291.21	1290.63	1292.82
$\theta_{il} = 0.6/\alpha_{yl}$ $V = (0.5, 0.5)$	1315.59	1314.85	1318.98
$\theta_{il} = 1/\alpha_{yl}$ $V = (0.5, 0.5)$	1328.33	1328.33	1328.39

In Table 5.3 we present the total capacity and the individual capacity investment decisions of both GENCOs for all the different cases of the weights of the classification

Table 5.5: Market prices [€/MWh].

Case	Year	Load levels	
		l_1	l_2
$\theta_{il} = 0.25/\alpha_{yl}$	y_1	71.24	43.12
$V = (1, 0)$	y_2	71.92	43.44
$\theta_{il} = 0.25/\alpha_{yl}$	y_1	71.14	43.37
$V = (0.5, 0.5)$	y_2	72.03	43.56
$\theta_{il} = 0.5/\alpha_{yl}$	y_1	71.21	46.44
$V = (1, 0)$	y_2	71.96	47.02
$\theta_{il} = 0.5/\alpha_{yl}$	y_1	71.14	46.87
$V = (0.5, 0.5)$	y_2	72.03	47.21
$\theta_{il} = 0.6/\alpha_{yl}$	y_1	71.14	48.08
$V = (0.5, 0.5)$	y_2	72.03	48.47
$\theta_{il} = 1/\alpha_{yl}$	y_1	71.45	49.33
$V = (0.5, 0.5)$	y_2	72.35	50.61

function and the strategic spot market behavior, while Table 5.4 contains the corresponding total profits of each GENCO and the value of the classification function of each case. Table 5.5 contains the market prices for all different cases.

Comparing the investment decisions of the first four cases, we observe that depending on the choice of weights in the classification function, the equilibrium solution is different. This means that the EPEC does not have a unique solution but that there exist several equilibria. For example, in the first case with asymmetric weights $V_i = (1, 0)$ we obtain an asymmetric equilibrium while weights $V_i = (0.5, 0.5)$ yield a symmetric equilibrium where both GENCOs build the same amount of capacity in each technology. Moreover, in the first case the GENCOs build 11.68 GW, 8.43 GW in coal and 3.25 GW in CC, while in case two, we get slightly less total capacity 11.65 GW. This decrease is happening only in coal capacity. Total CC capacity actually increases slightly to 3.26 GW, which indicates that different equilibrium solutions can yield a different technology mix.

In the cases where weights are $V_i = (1, 0)$, GENCO i_1 builds more coal and less CC capacity than GENCO i_2, which seems to be more profitable as can be seen in Table 5.4. We furthermore observe in Table 5.4 that the more weight we assign to one GENCO, the higher the corresponding profits. This is not surprising, considering that the weight determines whose approximated profits are more important when searching the solution space of the EPEC.

In the cases where weights stay the same, fixed at $V_i = (0.5, 0.5)$, and the strategic behavior θ_{il} changes from very competitive ($\theta_{il} = 0.25/\alpha_{yl}$) to Cournot ($\theta_{il} = 1/\alpha_{yl}$) behavior, there is a clear trend in investments: coal capacity decreases and CC capacity increases. Having more CC capacity in the systems leads to higher market prices and therefore to higher profits, which increase when the market behavior is less competitive. In general, the more market power, the higher the profits for the GENCOs. It seems intuitive that more competition in the spot market leads to lower market prices, as can be observed in Table 5.5.

Finally, the weighted approximation of the profits, which we used as classification function, is very close to the real profits. This can be verified by taking the weighted sum of the real profits and comparing it to the classification function value. For the cases presented, the relative error between the two is never more than 0.8%.

5.3 Alternative Formulation of the Single-Level Investment Equilibrium

In this section we propose an alternative formulation of the single-level conjectured-price response generation expansion equilibrium model, which has been initially introduced and formulated in section 2.3.3, where it has been presented as a straightforward extension of the spot market equilibrium problem. We recall the formulation of the single-level (SL) equilibrium problem in section 2.3.3 as a system of optimization problems in (2.49)-(2.51), and as a system of arising KKT conditions in (2.52)-(2.53). In this section we will propose an alternative formulation of the single-level investment-production problem as a quadratic optimization problem, which will be useful later on in section 5.4.

A simplified version of this model has also been considered previously in chapter 3 where it has been compared to a bilevel equilibrium model. The conclusions about this type of single-level equilibrium model that can be drawn from chapter 3 is that they are not as realistic as bilevel models since they assume that investment and production decisions are taken simultaneously, which is a gross simplification of reality. On the other hand, single-level models are mathematically easier to formulate and a lot easier to solve, which is demonstrated in this section. For this purpose we will present an alternative formulation of the single-level generation expansion equilibrium model as an equivalent quadratic optimization problem.

In equations (5.28)-(5.31) we present the equivalent optimization problem of the single-level capacity expansion equilibrium of section 2.3.3. We refer to this model as Single-Level Equilibrium as Optimization Model (SEOM). To our knowledge, this model has not been proposed before and therefore is an original contribution of this thesis, which has been published in Wogrin et al. [117]. This is a simple extension of Barquín et al. [5] where we extend the market equilibrium problem (and its equivalent optimization problem) to include investment decisions as well. In particular, starting from the equivalent optimization problem of the market equilibrium, given in section 2.3.2, we include an additional linear term in the objective function of (2.45) representing total investment costs and declare x_{ijy} as variables of the problem. This arising problem is very similar to the Cournot model presented by Ventosa et al. [113] with the difference that instead of assuming a fixed strategic behavior, we introduce conjectural variations into the model. These alterations do not compromise convexity properties of the optimization problem, which has been proven below in Lemma 5.1. Moreover, it can be verified that the KKT conditions of the equivalent optimization problem coincide with the single-level equilibrium conditions given in (2.52)-(2.53).

Single-Level Equilibrium as Optimization Model (SEOM):

$$\begin{aligned} \min_{x,q,d} \quad & \sum_{ijyl} \frac{\delta_{ij} q_{ijyl} T_{yl}}{(1+F)^y} + \sum_{ijy} \frac{\beta_{ijy} x_{ijy}}{(1+F)^y} + \frac{1}{2} \sum_{iyl} \frac{\theta_{iyl} T_{yl} (\sum_j q_{ijyl})^2}{(1+F)^y} \\ & - \sum_{yl} \frac{T_{yl}}{\alpha_{yl}(1+F)^y} (D^0_{yl} d_{yl} - \frac{d^2_{yl}}{2}) \end{aligned} \tag{5.28}$$

$$\text{s.t.} \quad q_{ijyl} \leq x_{ijy} + K_{ijy} \quad \forall ijyl \quad : \lambda_{ijyl} \tag{5.29}$$

$$0 \le q_{ijyl} \quad \forall ijyl \quad : \mu_{ijyl} \tag{5.30}$$

$$\frac{T_{yl}}{(1+F)^y}(d_{yl} - \sum_{ij} q_{ijyl}) = 0 \quad \forall yl \quad : p_{yl} \tag{5.31}$$

Due to the convexity assumptions of the cost functions, it holds that the solution to this optimization problem is unique which furthermore yields that the single-level capacity equilibrium solution is unique as well. It can also be verified that the second order conditions for a local optimum of (5.28)-(5.31), given below in Lemma 5.1, hold. Since Lemma 5.1 furthermore proves convexity of the given optimization problem, the obtained local solution, automatically is a global solution.

Lemma 5.1. *The quadratic minimization problem given by equations* (5.28)-(5.31) *is a convex optimization problem.*

Proof. In order to prove the convexity of the equivalent optimization problem of the single-level investment model presented above, we revisit the formulation given by equations (5.28)-(5.31) and make a simple variable transformation, i.e., $Q_{iyl} = \sum_j q_{ijyl}$. This constraint is added to the optimization problem, which then reads:

$$\min_{x,q,d,Q} \quad \sum_{ijyl} \frac{\delta_{ij} q_{ijyl} T_{yl}}{(1+F)^y} + \sum_{ijy} \frac{\beta_{ijy} x_{ijy}}{(1+F)^y} + \frac{1}{2} \sum_{iyl} \frac{\theta_{iyl} T_{yl} Q_{iyl}^2}{(1+F)^y} - \sum_{yl} \frac{T_{yl}}{\alpha_{yl}(1+F)^y}(D_{yl}^0 d_{yl} - \frac{d_{yl}^2}{2}) \tag{5.32}$$

$$\text{s.t.} \quad q_{ijyl} \le x_{ijy} + K_{ijy} \quad \forall ijyl \tag{5.33}$$

$$0 \le q_{ijyl} \quad \forall ijyl \tag{5.34}$$

$$\frac{T_{yl}}{(1+F)^y}(d_{yl} - \sum_{ij} q_{ijyl}) = 0 \quad \forall yl \tag{5.35}$$

$$\sum_j q_{ijyl} - Q_{iyl} = 0 \quad \forall iyl \tag{5.36}$$

In the optimization problem presented above all constraints are linear. The objective function is quadratic, however, we will show that the objective function is a convex function. To this purpose we form its Hessian matrix, which consists of the second order derivatives of the objective function with respect to the variables x, q, d

and Q. First of all we note that the objective function is linear in variables x and q, which implies that its second order derivatives will be zero. Furthermore we observe that there are no bilinear terms in the objective function, which implies that the Hessian matrix will be a diagonal matrix. Therefore, we only have to look at the second derivatives $\partial^2/\partial Q_{iyl}^2$ and $\partial^2/\partial d_{yl}^2$, which are given by (5.37) and (5.38) respectively and are both nonnegative due to the assumptions that $\theta_{iyl} \geq 0$ and $\alpha_{yl} \geq 0$, which have been stated in sections 2.2.1 and 2.2.2. Therefore all eigenvalues of the Hessian are nonnegative as well, which finally yields that the objective function is convex.

$$\frac{\partial^2}{\partial Q_{iyl}^2} = \frac{\theta_{iyl} T_{yl}}{(1+F)^y} \geq 0 \tag{5.37}$$

$$\frac{\partial^2}{\partial d_{yl}^2} = \frac{T_{yl}}{\alpha_{yl}(1+F)^y} \geq 0 \tag{5.38}$$

□

5.4 Approximation of Bilevel Equilibria by Single-Level Equilibria

In section 5.4 we propose a new methodology to approximate bilevel capacity equilibria using only single-level capacity equilibrium models. This newly developed methodology represents an original contribution of this thesis which has been published in Wogrin et al. [117]. In the bilevel model, generation companies choose capacities that maximize their individual profit in the first stage while the second stage represents the conjectured-price-response market equilibrium. In the single-level model, firms simultaneously choose capacities and quantities to maximize their individual profit, while each firm conjectures a price response to its output decisions. The bilevel equilibrium model is an equilibrium problem with equilibrium constraints, which belongs to a class of problems that is very hard to solve. The single-level equilibrium model is much easier to solve, however, it is also less realistic in some situations. With the approximation scheme proposed in this thesis, we are able to solve the bilevel model reasonably well by smartly employing single-level models which reduces the computational time by two orders of magnitude. We achieve this by transforming the single-level equilibrium problem into an equivalent convex quadratic optimization problem which can be solved efficiently. The theoretical basis that sparked the idea of the approximation scheme is the comparison between single and bilevel equilibrium models from chapter 3. In section 5.4.1 we recall the most relevant points of this theoretical analysis. Then, in section 5.4.2 the actual approximation scheme is presented. In section 5.4.3 a small case study is presented in order to validate the proposed approximation scheme. A large-scale case study is carried out in section 7.3 where we find that for multi-year, multi-load period cases the approximation scheme works well when market behavior is closer to oligopoly than to perfect competition.

5.4.1 Recalling Comparison of Single and Bilevel Capacity Equilibria

In chapter 3 of this thesis, which has been accepted for publication in Wogrin et al. [119], we have theoretically analyzed and compared single and bilevel generation capacity equilibrium models for a single-year framework and for strategic spot market behavior ranging between perfect competition and Cournot. The single-level model describes the one-shot investment-operation market equilibrium, where firms simultaneously choose capacities and quantities to maximize their individual profit. The bilevel model describes the two-stage investment-operation market equilibrium, where firms first choose capacities that maximize their profit and then decide market outcomes in the second stage. In both models, the strategic market behavior is represented by a conjectured-price response. We now summarize the most relevant results of this work, since they have great relevance in this section.

First of all, the capacity yielded by the bilevel model is independent of the strategic (or oligopolistic) spot market behavior. For the case of one load period, this result has been proven in Theorem 3.1 of chapter 3. In particular, in chapter 3 we showed that the capacity yielded by the bilevel model, given by equation (3.20), is always the same and does not depend on the strategic behavior.

Second, the single-level solution, which in general depends on the strategic spot market behavior as can be seen in equation (3.7), considering Cournot behavior is given by equation (3.9) and coincides with the bilevel solution for any strategic behavior. This is why we use the term "Cournot" capacities for bilevel models. The fact that the bilevel model always yields Cournot capacities is an extension to the result of Kreps and Scheinkman [74], which states that capacity precommitment followed by Bertrand competition yields Cournot capacities. We have shown that capacity precommitment followed by any market behavior between perfect competition and Cournot, yields Cournot capacities. The result of Theorem 3.1 can be extended to the multiple load period case, which we have proven in Proposition 3.2, but the proof gets a little more complicated. When considering only one load period, it is obvious that - no matter what the strategic behavior - capacity is always going to be binding in this load period otherwise it would not be an equilibrium. When we have multiple load periods a priori we do not know in which load periods capacity is going to be binding. As a matter of fact when considering two different degrees of strategic behavior, the resulting equilibria could be binding in different load periods. However, if two equilibria happen to be binding in the same load periods, then we can prove something about the resulting capacity. In particular, in part (a) of Proposition 3.2 we show that if the bilevel solutions for different strategic behavior have the same active set of load periods, then they yield the same capacity. The bilevel capacity, given by equation (3.47), does not depend on strategic behavior. And moreover, if we assume that the active sets coincide then the single-level Cournot solution coincides with the bilevel solution, which has been proven in part (b) of Proposition 3.2.

Third, we have shown that if the single and bilevel solutions are at capacity in the same load periods and if they consider the same conjectured-price response, then their production decisions and prices in load periods that are not at capacity are identical. In particular, this has been shown in chapter 3 by comparing single-level production decisions in not binding load periods, given by (3.31)-(3.32), and the corresponding bilevel production decisions, given by (3.42)-(3.43).

One conclusion of chapter 3 is that the bilevel model is more realistic than the single-level model because it captures that GENCOs would never voluntarily build all the capacity that might be determined by the spot market if that meant less (or none at all) profits for themselves. The bilevel model depicts how strategic underinvestment can improve profits. The single-level model on the other hand fails to capture this feature.

As the bilevel model is more complicated to solve than the single-level model, it would be desirable to approximate the equilibrium solution of the bilevel model by solving the simpler single-level model and thereby avoiding having to solve a complicated problem which is known as EPEC. In section 5.4.2 we propose such an approximation scheme and we motivate how the findings of chapter 3 have sparked the idea of this approximation scheme.

5.4.2 Single-Level Approximation Scheme of Bilevel Model

In section 5.4.1 as well as in chapter 3, we have already discussed that in general, the bilevel model is more realistic than the single-level model because the single-level model tends to yield over-investment as it does not consider the sequentiality of decision making. However, the bilevel model is also much more difficult to solve. Considering that the single-level model is comparably easy to solve, employing the equivalent convex quadratic optimization problem defined in 5.3, it would be desirable to be able to approximate the complicated but more realistic bilevel model with the single-level model. In this section we present such an approximation scheme.

A first idea of a single-level approximation scheme could go along the lines of section 2.2.2, where we have shown that conjectural variations can be used to reduce multi-level games into single-level games, as for example the Allaz-Vila game or Stackelberg game as shown in Lemma 2.1. A straight forward idea could be to approximate the bilevel generation expansion equilibrium by a single-level model using a specific conjectural variation which embodies the reduction of the bilevel game. We have not employed this method for two reasons. First, because finding such a conjecture can be very complicated and second, even if by change we might find a conjecture which yields a good approximation for capacity investments, the arising market prices and production decisions might not resemble the bilevel results at all and hence not be useful. Therefore, the approximation scheme we propose here not only succeeds in yielding accurate capacity values, but also good approximations of market prices, NPVs and production decisions - that is if the strategic spot market behavior is closer to Cournot than to perfect competition.

In order to motivate the proposed approximation scheme, let us recall three points from section 5.4.1 which have been proven in chapter 3: First, the bilevel capacity solutions are the same for different degrees strategic spot market behavior θ as long as the active sets of load periods of the solutions coincide. In the special case of only one load period, this means that no matter what the strategic behavior, the bilevel capacity solution is the same. Second, if the active sets of load periods coincide, then the single-level model assuming Cournot behavior in the market yields the same solution as the bilevel model for arbitrary θ. Third, for the same value of the strategic spot market behavior θ, single and bilevel models yield the same production decisions in load periods where capacity is not binding. We have already recalled these properties in section 5.4.1 and refer to either chapter 3 or to Wogrin et al. [119] for the detailed proof.

From these properties we draw the conclusion that in the bilevel model the capacity decisions seem to be "Cournot" no matter what the strategic spot market behavior θ. However, in load periods where capacity is not binding, the specific market behavior θ determines production decisions. Since solving a bilevel model can be quite hard, we would like to approximate the bilevel solutions using a single-level model. From what we have learned in chapter 3, capacity decisions and production decisions in the bilevel model follow a different trend and need to be approximated separately. Since we know that the single-level Cournot capacities coincide with the bilevel capacities when the active set of load periods is the same, we propose to approximate bilevel capacities by solving the single-level Cournot model. However, we also know that in load periods where capacity is not binding, production decisions depend on θ and therefore we fix the obtained capacities and solve the single-level model again, but this time assuming behavior θ instead of Cournot behavior. This leads to the following approximation scheme of the bilevel capacity equilibrium problem with strategic spot market behavior θ_{iyl}:

Single-Level Approximation Scheme of Bilevel Model

1. Solve the single-level equilibrium model, (5.28)-(5.31), assuming Cournot behavior in the market. This yields capacity decisions x_{ijy}.

2. Fix the capacity decisions x_{ijy} which have been decided in the previous step.

3. For fixed capacity decisions, solve the single-level equilibrium model again but this time with strategic spot market behavior θ_{iyl}, which yields market prices p_{yl}, demand d_{yl} and production decisions q_{ijyl}. Note that it is the same to solve the single-level generation expansion equilibrium, given by (5.28)-(5.31), for fixed capacity decisions and to solve the market equilibrium, (2.45)-(2.48), for the same value of fixed capacity.

It is easy to see that this approximation scheme is exact for one load period and a time horizon of one year, since the single-level Cournot capacity is the same as the bilevel capacity for any strategic behavior, as has been discussed in section 5.4.1. For a bilevel model with multiple load periods and a time horizon of one year, the approximation would also be exact but only if the active set of load periods of the bilevel model and the single-level Cournot model coincided, which a priori cannot be predicted. If the active sets coincided, then the bilevel capacity - independent of θ - would be the same as the Cournot capacity as has been proven by Proposition 3.2.

Since the approximation tries to imitate bilevel behavior and is exact in very simplified cases as we pointed out previously in chapter 3, in this section we wanted to verify from a practical point of view whether the approximation also works for cases with multiple load periods where active sets of load periods do not coincide. Since there is no guarantee that at equilibrium the single-level Cournot model and the bilevel model are going to have the same set of active load levels, it cannot be guaranteed that the capacity obtained by the single-level Cournot model will actually coincide with the bilevel capacity. Therefore in the approximation we expect to commit an error. The purpose of this the next section, i.e., a case study, is to quantify this error and to validate whether the newly proposed approximation scheme can be successfully applied to multi-load-period cases. The extension of this numerical validation to a multi-year, multi-load-period and multi-technology case is presented in section 7.3 of this thesis.

In order to evaluate the quality of the solution obtained with the proposed approximation scheme we compare it to the actual bilevel solution which we obtain via diagonalization, which is a technique where MPECs are solved iteratively until convergence. For more details on this method, the reader is referred to section 6.2. If this algorithm converges, then it yields an equilibrium point of the bilevel problem. The question we want to answer is whether there exists a bilevel equilibrium which is close to the solution yielded by the single-level approximation scheme. In fact, numerical evaluation in the small case study of section 5.4.3 shows that indeed results are not very different which is partly due to the size of the case study. Hence, we extend the case study to a large-scale numerical example presented in section 7.3 where we conclude that the quality of the approximation for large-scale models depends on the level of competition in the market: the closer the market behavior is to oligopoly, the more accurate the approximation.

5.4.3 Numerical Example of Approximation Scheme

The purpose of this numerical example is to validate the approximation scheme numerically. Moreover, we want to show that even in cases where active sets of load periods of the bilevel and the single-level Cournot solution do not coincide (and capacity solutions therefore differ), the approximation still works well. Hence, for simplicity in this case study we consider a time horizon of only one year and one technology. Note that a large-scale numerical example with a time horizon of multiple years, multiple load periods and various technologies is presented in section 7.3. In this large-scale case study we also carry out an analysis yielding the result that for a multi-year, multi-technology case, approximation results are most accurate and thus useful when the strategic spot market behavior is closer to Cournot than to perfect competition.

The electricity system we use in this case study is very simple and consists of two generation companies only. Furthermore we consider investment in only one technology, i.e., combined cycle gas turbine (CC), since for the last decades this has been the prominent technology for new capacity investments in Spain. For the sake of simplicity, we consider a time horizon of one year and six load periods within this year. The discount rate is 9%. The demand slope α is assumed to be 0.23 GW/(€/MWh), while the corresponding demand intercepts and durations for each of the six load periods are presented in Table 5.6. Variable costs δ are 39 €/MWh and investment costs are considered as 113 (M€/GW)/year. The strategic spot market behavior θ is $0.3/\alpha$. Therefore, the objective is to solve the bilevel investment equilibrium model, which has been introduced in this chapter, assuming strategic behavior as given above. Note that we refer to results corresponding to the bilevel equilibrium model as (BL) for bilevel.

Table 5.6: Demand intercept D_l^0 and load level durations T_l.

		WD Peak l_1	**WD Shoulder** l_2	**WD Off-peak** l_3	**WE Peak** l_4	**WE Shoulder** l_5	**WE Off-peak** l_6
D_l^0	[GW]	94.46	54.44	35.8	47.6	39.4	30.1
T_l	[h]	300	3000	3000	300	1080	1080

First of all we solve the original BL model using diagonalization, an iterative method which is discussed in more detail in section 6.2.1. The convergence tolerance (for definition refer to section 6.2.1) for the diagonalization method is of 10^{-4}. Note that the computational time of this method depends on the initial point and can therefore vary. Without any prior knowledge of the optimal investment solutions, an arbitrary point was chosen as initial solution. On an Intel(R) Core(TM) 2 Quad Processor with 3.21 GB RAM the diagonalization converged after 180 iterations and a computational time of 144 seconds. The obtained capacity investment for one GENCO in CC is 13.67 GW. Note that since both GENCOs consider the exact same data, their solutions are identical. In Table 5.7 production decisions and market prices are presented. We observe that capacity is binding in three load periods, i.e., l_1, l_2 and l_4. The obtained BL net present value for each GENCO is 3507.62 M€.

Table 5.7: Bilevel (BL) equilibrium solution under intermediate second-stage competition ($\theta = 0.3/\alpha$) with capacity $x_i = 13.67$ GW.

l	1	2	3	4	5	6
q_l [GW]	13.67	13.67	11.66	13.67	13.24	9.19
p_l [€/MWh]	291.82	117.81	54.21	87.96	56.27	50.99

Let us now present the obtained results of the approximation scheme (AP), step by step. As pointed out in section 5.4.2, the first step of the approximation scheme is to solve the single-level investment equilibrium model assuming Cournot conjectures. This first step in the approximation scheme aims at obtaining a realistic solution for the investment capacity, as theoretical results - provided in chapter 3 - establish that in bilevel models capacity decisions behave in a “Cournot” manner. The first step of the AP yields results presented in Table 5.8 with investment capacities of 13.74 GW in CC for one GENCO. The first observation we make is that in the solution of the single-level Cournot model, capacity is only binding in two load periods, as opposed to the bilevel solution. In section 5.4.2, we mention that for a multiple-load period case, the approximation scheme is exact but only if the active sets of load periods of the bilevel model and the single-level Cournot model coincided. This has been proven theoretically by Proposition 3.2. However, when active sets do not coincide, which is the case here, then the AP capacity solution does not necessarily have to coincide with the BL capacity. Our numerical results show that since active sets of load periods are not identical, the obtained capacity solutions differ. However, we also observe that they do not differ by much. The absolute difference between BL and AP capacity investment solutions is 7 MW, which translates to a relative error of only 0.5%. Step 1 of the approximation scheme therefore fulfills its purpose in obtaining a capacity solution very close to the bilevel one. The following steps of the AP scheme assume capacity levels to be fixed to 13.74 GW and try to approximate market prices and production quantities more accurately.

Table 5.8: Solution after first step of approximation (AP) scheme with capacity $x_i = 13.74$ GW.

l	1	2	3	4	5	6
q_l [GW]	13.74	13.74	8.94	12.87	10.15	7.05
p_l [€/MWh]	291.19	117.18	77.88	94.94	83.14	69.64

Once capacity levels are fixed, the approximation scheme proceeds by solving the conjectured-price response market equilibrium but this time assuming strategic behavior $\theta = 0.3/\alpha$, which is the conjecture that has been used in the bilevel model. The results after the final step of the AP scheme are presented in Table 5.9.

Table 5.9: Solution after final step of approximation (AP) scheme with capacity $x_i = 13.74$ GW.

l	1	2	3	4	5	6
q_l [GW]	13.74	13.74	11.66	13.74	13.24	9.19
p_l [€/MWh]	291.19	117.18	54.21	87.32	56.27	50.99

When comparing the AP solution after the first step, given in Table 5.8, and the final step, given in Table 5.9, we note that in the final solution capacity is binding in l_4 as well. This is due to the fact that in Table 5.8 the market behavior is assumed to be Cournot, i.e., $\theta = 1/\alpha$, but in the final solution market behavior is more competitive than Cournot, i.e., $\theta = 0.3/\alpha$, which leads to higher production decisions in general, and in particular, in l_4 it has led to production reaching capacity limits. We furthermore observe that since capacity in the final AP solutions is binding in the same load periods as in the BL solution, production decisions in non-binding load periods are identical. The same is true for market prices in these load levels. When comparing AP and BL solutions for binding load periods, it should be noted that since capacities differ slightly, market prices as well as production decisions are also slightly different. The average relative error in prices[1] is only around 0.2%. The NPV obtained by the AP is 3493.13 M€, which is differs from the BL solution by only 0.4%.

In summary, the AP scheme decently approximates the BL solution. In particular, the relative error in capacity investments is 0.5%, the maximum relative error in market prices is 0.7% and the relative error in production decisions is zero for non-binding load periods and 0.5% for binding load periods. The computational time of the AP on an Intel(R) Core(TM) 2 Quad Processor with 3.21 GB RAM was only 0.5 seconds. In comparison, the diagonalization method with arbitrary initial point was 144 seconds, so the AP scheme is more than two orders of magnitude faster than diagonalization. Note that the performance of the diagonalization can be improved by using a better initial point. For example, when using the solution obtained by the AP scheme as initial point for diagonalization, then the computational time of this method can be reduced to 6.5 seconds and converges in only 7 iterations. However, this improved performance is entirely due to the accuracy with which the AP scheme approximates the BL solution.

Let us now briefly explore the solution space of the bilevel equilibrium problem. For this purpose the market equilibrium problem is solved for many different combinations of capacity investments and the corresponding net present values of each firm are observed. In Figure 5-2 the net present value of GENCO i_1 is presented for different values of investments of GENCO i_2. The black continuous lines in the figure correspond to the NPV that GENCO i_1 obtains depending on its capacity investments when assuming that the capacity of firm i_2 is fixed to a certain value, and in particular, to values ranging from 0 GW to 20 GW in 2.5 GW increments. The blue dotted line represents the NPV of GENCO i_1 if we fix the capacity of GENCO i_2 to the equilibrium capacity of 13.67 GW. The red continuous line represents the NPV of GENCO i_1 for the cases where both i_1 and i_2 have the same amount of capacity

[1]The average relative error in prices is calculates as $\sum_l \frac{T_l |p_l^{AP} - p_l^{BL}|}{8760\ p_l^{BL}}$.

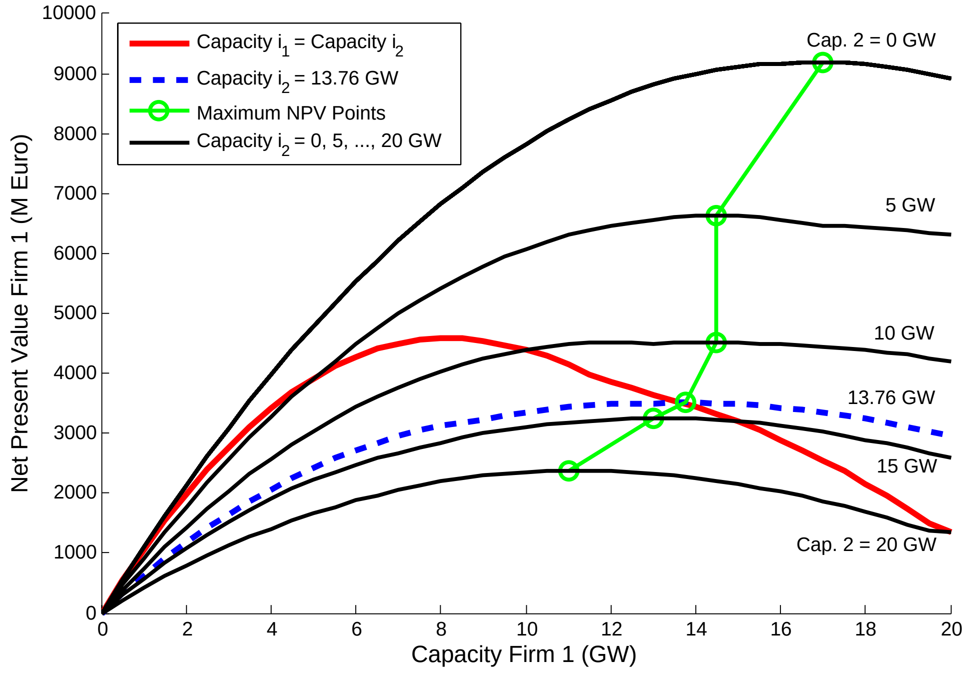

Figure 5-2: Net present value of firm 1 for different fixed values of capacity of firm 2.

(which is indicated by the x-axis). Since in this case study the parameters of both market agents are assumed to be identical, it makes sense to observe the points where capacity investments of both agents are identical, i.e., the red line. For example, if we observe the point where both firms invest in 5 GW, where the red curve intersects with the black line (in which firm i_1 varies its capacity while firm i_2 is fixed to 5 GW), then it is clear to see that i_1 has an incentive to move away from this point along the black line since it can increase its NPV. In particular, this means that the gradient of the black line at the intersection point with the red line is not zero, which furthermore means that this point cannot be an equilibrium. As a matter of fact, the same is true for each of the intersection points of the red with the black lines depicted in the figure. The only point where the gradient disappears, i.e, where there is no incentive to unilaterally change its capacity, is at the intersection of the red and the blue dotted line, which is exactly where both i_1 and i_2 build 13.67 GW of capacity. This is the equilibrium point and - as can be see in Figure 5-2 - it is the only symmetric equilibrium point. The green line connects the maximum NPV points of each of the black continuous lines and also the blue dotted line. At the equilibrium point, the red, blue and green lines intersect which means that in this point both agents build the same amount of capacity, which is 13.67 GW, and that for i_1 this is the NPV maximum point, which means there is no incentive to move away.

In Figure 5-3 the net present value curves of both GENCOs are presented at once. Since both market agents assume symmetric data it is not surprising that the equilibrium occurs at the intersection of both curves. Furthermore it can be observed that - due to the curvature of the net present value curves - the other points on the intersection curve allow an increase of the net present value by unilaterally changing capacity, which means that these points do not qualify as local equilibria. Note that the curve where both NPVs intersect in Figure 5-3, actually corresponds to the red curve previously depicted in Figure 5-2. For this very simplified numerical example it can therefore be concluded that there is only one symmetric equilibrium, which is the one presented in Table 5.7.

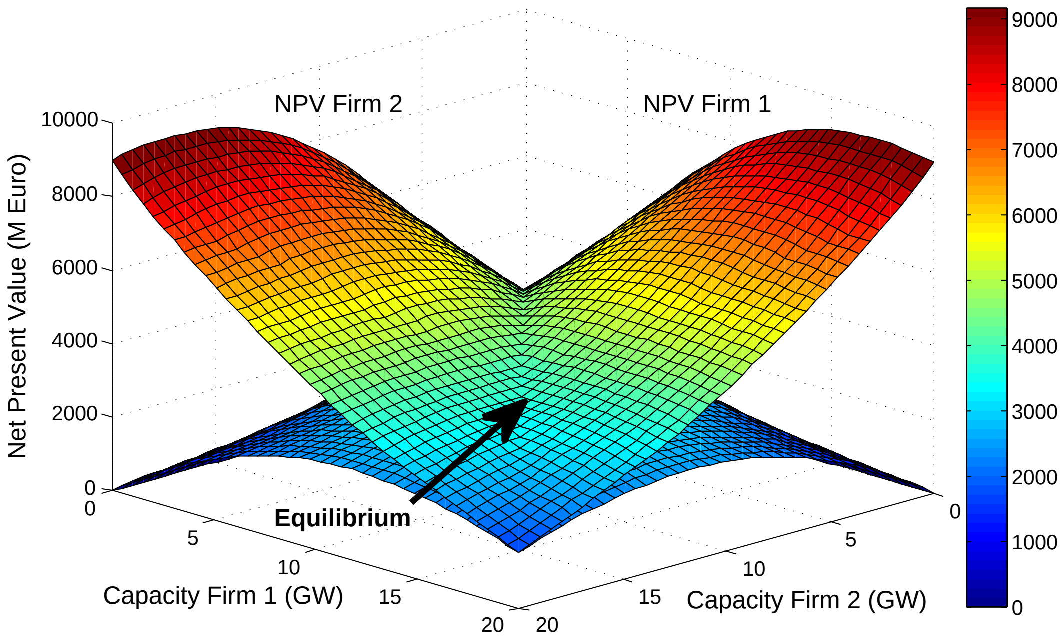

Figure 5-3: Net present value of both firms for different range of capacity investments.

In summary, it can be said that for the assumed strategic behavior of $\theta = 0.3/\alpha$, the approximation scheme succeeds in approximating the results of the bilevel equilibrium in capacities as well as in market prices and production decisions. Let us now analyze whether the success of the AP depends on the particular value of θ.

To that purpose, we repeat this case study with different strategic behavior ranging from perfect competition $\theta = 0$ (corresponds to $\Phi = 0$ on a normalized scale) to the Cournot oligopoly $\theta = 1/\alpha$ (corresponds to $\Phi = 1$ on a normalized scale). In Figure 5-4 we present the average relative errors in capacities, prices and NPVs between the AP and the BL solution for a range of strategic market behavior. It can be observed that for market behavior between $\Phi = 0.81$ and Cournot ($\Phi = 1$) the AP and the BL solutions coincide exactly. Between $\Phi = 0.24$ and $\Phi = 0.8$ the average relative errors of NPVs, prices and capacities are still very small and only go up to 0.5%, which indicates that the AP is working extremely well. Between $\Phi = 0.1$ and $\Phi = 0.23$ the average relative errors increase notably to 4.1% for prices, to 6.4% for capacities, and 7.3% to for the NPV, however, they can still be considered acceptable. In the interval between $\Phi = 0$ and $\Phi = 0.09$ on the other hand, the average relative errors shoot up significantly to 17.4% in capacities, 20.5% in prices and 27.6% in NPVs. For this particular range the average errors are so high that the AP cannot be considered successful.

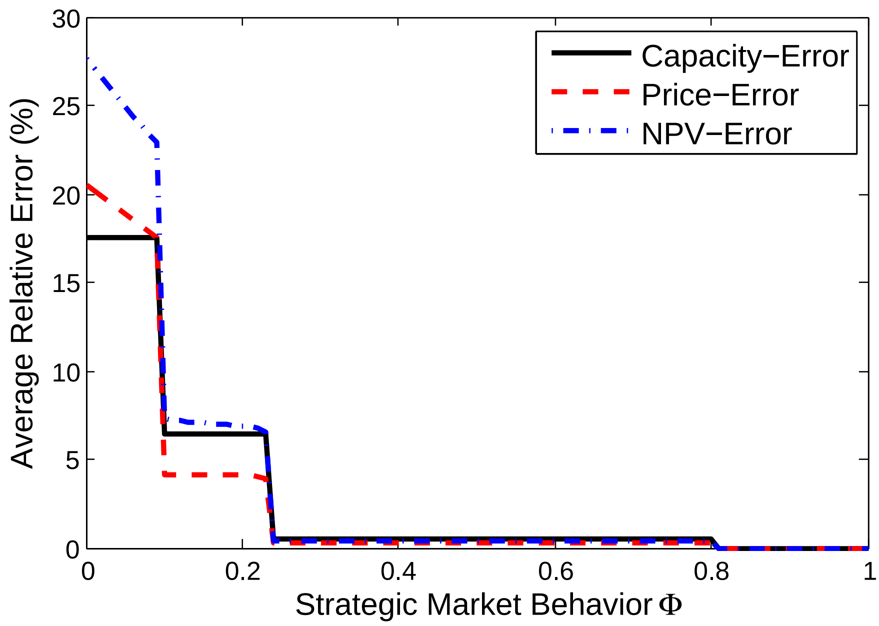

Figure 5-4: Average relative errors between approximation and bilevel solution for a range of strategic market behavior.

For this small case study we observe the overall trend that the closer we get to Cournot, the more accurate the approximation scheme. In a case study, presented in section 7.3, we verify that this trend is also true for large-scale numerical examples. It can be concluded that the developed approximation scheme works well when market behavior is closer to Cournot than to perfect competition.

5.5 Conclusions

In this chapter we have discussed bilevel equilibrium models for generation expansion planning. In section 5.2 we introduced an EPEC model to tackle the generation capacity investment problem in liberalized electricity markets. In the upper level of this bilevel equilibrium problem, firms choose capacities maximizing their profit anticipating the equilibrium outcomes of the lower level, in which quantities and prices are determined by a conjectured-price response market equilibrium, which allows for an assessment of the impact of the strategic spot market behavior on investment decisions. Moreover, this model yields an annual investment schedule over the entire time horizon. In a case study in section 5.2.4 we have proven by example that even a small numerical example can yield multiple equilibrium solutions, and that the corresponding investment decisions can vary in terms of the optimal technology mix.

In section 5.4 we have proposed a single-level approximation scheme of a bilevel capacity expansion equilibrium problem and furthermore we have presented a quadratic optimization problem that is equivalent to the single-level capacity expansion equilibrium problem. The bilevel equilibrium more accurately represents reality than the single-level problem, however, it is also more complicated to formulate and solve. A numerical example, presented in section 5.4.3, shows that our approximation scheme yields a good, unique solution in computational times two orders of magnitude smaller than the computational times of the bilevel problem, which allows us to apply this technique to real-size problems although with some constraints as has been shown in section 5.4.3 and will be confirmed in section 7.3. In particular, we have observed a trend that the closer market behavior is to Cournot, the better the approximation.

In the large-scale case study, presented in section 7.3, the approximation scheme is validated and we furthermore explore the impact of the model parameters on the quality of the approximation scheme. A detailed study on strategic spot market behavior leads to confirm the conclusion that when strategic spot market behavior moves away from perfect competition, then the proposed approximation scheme works very well and yields accurate price and capacity investment results.

Chapter 6

Solution Techniques for Bilevel Generation Expansion Models

This chapter summarizes all the numerical techniques that have been applied when solving the MPEC and EPEC models of this thesis representing the generation expansion problem. Since bilevel problems are a class of problems that can be very hard to solve, there is no such thing as a perfect solution method. Hence, several different solution techniques have been explored and evaluated. In particular, section 6.1 is dedicated to the methods that solve bilevel generation expansion optimization problems, formulated as MPECs. Solving MPEC models by itself is an interesting and challenging endeavor, but on top of that it also provides the basis for EPEC solution methods as some of these methods resort to iteratively solving a series of MPECs. Section 6.2 focuses on the techniques that have been applied to tackle bilevel generation expansion equilibrium problems, formulated as EPECs. Finally, section 6.3 provides an overview of the explored methods and their advantages as well as disadvantages are pointed out. It is emphasized here that this chapter does not represent a complete overview of all available methods to solve MPECs and EPECs, as for example the method presented by Su [111], but simply addresses the methods that have been employed in this thesis. All of the presented models and methods have been formulated in GAMS.

6.1 Solution Techniques for the Generation Expansion MPECs

This section is dedicated to the solution techniques that have been applied to solve the proposed MPECs in this thesis. Apart from discussing computational performance, we also point out advantages and disadvantages of the techniques. Section 6.1.1 discusses the nonlinear programming approach to the MPEC, while in section 6.1.2 linearization techniques are analyzed. In section 6.1.3 we briefly raise the topic of decomposition methods and in particular Benders decomposition. It is discussed that the standard approach of Benders decomposition cannot be applied to the type of MPECs that are proposed in this thesis. However, decomposition methods pose a promising topic for future research.

6.1.1 Nonlinear Programming

The bilevel generation expansion optimization model (BOM), which has been analyzed in detail in chapter 4 of this thesis, is formulated as an MPEC in section 4.2.3 and in particular in equations (4.19)-(4.30). It is clear to see from this formulation that the presented optimization problem is a nonlinear program due to the numerous complementarity conditions (4.22)-(4.24) and the bilinear market revenues term in the objective function (4.19). Apart from being nonlinear, the nonlinearities are also non-convex which complicates the solution techniques of this problem because globality of the solution cannot be guaranteed. In general, non-convex functions can have multiple local optima. Therefore, a gradient-based solution method, which searches the solution space along the descent (or ascent if we are maximizing) lines, terminates when they arrive at a point where the first derivative is zero. Since there can be many such points (multiple local optima), the final point that is yielded by the algorithm can depend greatly on the initial solution. This implies that a priori one cannot say which of the local optima the algorithm is going to yield, nor how many there are in total, nor - and this is the most important fact - whether the obtained solution is the best among all local optima, i.e., whether the solution is global.

Since the problem in question is a nonlinear program, the most straight-forward way to tackle the MPEC is to employ a nonlinear solver. In particular, the MPEC models in this thesis have been solved (if not otherwise stated) using the NLPEC [46] solver which reformulates the MPEC into a NLP and then calls the solver CONOPT [41]. In our experience this technique is very efficient computationally speaking. For example, the case study presented in section 4.3.4 led to an MPEC of 643 variables (5 years, 2 load periods, 3 firms, 3 scenarios, 2 technologies) and only took 0.8 seconds to solve on an Intel(R) Core(TM) 2 Quad processor with 3.21 GB RAM. On the same machine, a larger case study (10 years, 6 load periods, 3 firms, 4 technologies), presented in section 4.2.4 only took 6.4 seconds.

The advantage of this method is the computational efficiency which allows us to tackle large-scale problems in very reasonable time (the order of seconds) and moreover memory does not pose a problem.

The clear disadvantage of applying a nonlinear solver like NLPEC is that when the algorithm terminates correctly it only yields a local solution. The problem is that a priori we do not know how many local optima the MPEC has. Moreover, with the solution obtained by NLPEC there is no way of saying how good the solution is globally speaking. As a matter of fact, the local optimum heavily depends on the initial point that is given to the solver. Therefore, one possible approach to remedy the downside of this method is to try out many different initial points in order to search the solution space of the MPEC for possible local optima. Another option could be a heuristic search, however, in the proposed problem, heuristic methods also fail to provide information about global optimality and take considerably longer to solve and might not even yield a feasible solution, which may make this method less desirable in this context.

There are other nonlinear solvers, like BARON [106], that when given appropriate bounds on variables solve a nonlinear problem to global optimality, even if the problem is non-convex. This solver has been applied to our generation expansion MPEC and we have drawn the following conclusions. For very small case studies (with a total size of 34 variables, which translates to 3 firms, 2 technologies, 1 scenario, 1 years and 1 load period) the solver yields the globally optimal solution after 61 seconds on an Intel(R) Core(TM) 2 Quad processor with 3.21 GB RAM. Note that the purpose of this very small case study is to compare computational techniques and therefore the investment and other results are not presented here. When comparing the obtained solution with the solution yielded by NLPEC in only 0.5 seconds, it turns out the both solutions coincide. This means that NLPEC, even though global optimality could not be guaranteed, yielded the global optimum. The potential issue with BARON is that a large amount of memory is required and that even slightly larger case studies (for example the 60 variable case study - which is similar to the previous one with the only difference that now it considers 2 load periods) could not be solved on the available machine and ran out of memory after three hours, even after having augmented the workspace available to the solver. Therefore, it seems that for now BARON is not going to be successful for large-scale case studies.

6.1.2 Linearization Methods

Another solution technique for the MPEC, which has previously been mentioned briefly in the numerical example in section 4.3.4, is mixed integer programming. The core idea of this method is that the nonlinear MPEC is transformed into a MILP and solved as such using numerical solver like CPLEX [64]. As we pointed out in the previous section, the nonlinearities of the MPEC stem from the complementarity conditions (4.22)-(4.24) and the bilinear term of market revenues in the objective function (4.19).

The complementarity conditions can be "linearized" by applying a trick by Fortuny-Amat [49] where we introduce binary variables and replace each complementarity equality by two inequalities as follows. For example, in order to linearize the nonlinear complementarity $\mu_{ijyls} q_{ijyls} = 0$.

$$C^{\mu} b^{\mu}_{ijyls} \geq \mu_{ijyls}, \tag{6.1}$$

$$C^{\mu}(1 - b^{\mu}_{ijyls}) \geq q_{ijyls}, \tag{6.2}$$

for C^μ a suitably large constant and b^μ_{ijyls} binary variables. Note that all other complementarity conditions can be approached equivalently.

The only nonlinearity left to deal with in the MPEC is the market revenue term in the objective function. So far, we have identified two effective ways to deal with this term. The first approach involves the discretization of the lower level variable price p by binary expansion [93] as has been mentioned in section 4.3.4. To that purpose, price is written as:

$$p_{yls} = \underline{p}_{yls} + \Delta_{p_{yls}} \sum_k 2^k b^p_{kyls}, \tag{6.3}$$

where $\underline{p}_{yls}$ is the lower bound, $\Delta_{p_{yls}}$ is the step size of price, k the set of discretization intervals and b^p_{kyls} are binary variables. Usually, in our numerical examples, the lower bound $\underline{p}_{yls}$ of price has been set to be zero. The most accurate choice of step size $\Delta_{p_{yls}}$ would be 1 c€/MWh, however, reasonable results might also be obtained with a slightly higher step size, for example, 5 c€/MWh, but would not lead to such a high number of binary variables. Then the bilinear terms $p_{yls}q_{i^*jyls}$ of the objective can be replaced by $\underline{p}_{yls}q_{i^*jyls} + \Delta_{p_{yls}} \sum_k 2^k z_{ki^*jyls}$, where z_{ki^*jyls} symbolizes the product of prices with quantities and is defined by the following constraints, which also have to be added to the problem:

$$0 \leq \quad z_{ki^*jyls} \quad \leq C^p b^p_{kyls}, \tag{6.4}$$

$$0 \leq \quad q_{i^*jyls} - z_{ki^*jyls} \quad \leq C^p(1 - b^p_{kyls}), \tag{6.5}$$

for C^p a suitably large constant.

In the lower level, market price has been considered as a continuous variable. Therefore, when discretizing this variable, the solution space of the lower level problem, i.e., the market equilibrium, might be reduced and in particular, this can lead to situations were the actual solution of the initial continuous market equilibrium is not a feasible solution when considering market prices as discrete. This means that in the attempt to obtain a global solution to the MPEC instead of only a local one (which could be obtained by nonlinear programming techniques), we might have lost the actual global solution due to the discretization of the market price. Due to this reason, when applying this approach, one has to be careful when choosing the step size of price. For example, if the step size were 1 c€/MWh, then the discretization would not lead to a loss of realism of the model, however, this small step size might lead to a high number of binary variables which furthermore complicates the solution process of the MILP. The reader is referred the case study in section 4.3.4 where the performance of this discretization method is compared to the method of the previous section 6.1.1 (nonlinear programming).

The second approach to deal with the bilinear market revenue term is by discretizing capacity investment decisions. This method has been derived and discussed in detail in section 4.4.2. When discretizing price, as mentioned in the first approach, realistic solutions of the problem might be lost, as we discretize a continuous lower level variable. However, investment decisions are inherently discrete and therefore we actually increase realism of the model when discretizing capacity decisions. Therefore, the discretization of investments is superior to the approach of discretized prices. In any case, both of the previously mentioned approaches to linearize the market revenue term, lead to the desired MILP formulation of the MPEC.

In general, the obvious advantage of the arising MILP is that the obtained solution is a global optimum. On the other hand, the disadvantage is that due to the binary variables that we have introduced in the problem formulation, the computational time of the MILP increases greatly with respect to a standard nonlinear solver. For example, the small numerical example of section 4.3.4 which took only 0.8 seconds to solve using NLPEC, now leads to a MILP of 1850 variables (with a step size of 15 c€/MWh) and takes about 11.5 hours on the same machine. When considering a smaller step size of 4 c€/MWh, then computational time increases to 20 hours. With the MILP approach, memory also becomes an issue.

6.1.3 Decomposition Methods

In this section we briefly discuss the potential of decomposition methods and in particular the Benders decomposition [11] for the proposed MPEC models (BOM) and (SBOM). This type of method has not yet successfully been applied to model (BOM) and (SBOM); however, such an approach and its challenges are outlined below and remain to be addressed in future research.

First of all, let us recall that the MPEC is a type of bilevel programming problem (BPP) where the upper level is the investment stage and the lower level represents the market equilibrium problem. Once the investment variables are fixed, the market equilibrium is very easy to solve, for example as its equivalent quadratic optimization problem. In section 4.3 we introduced a stochastic MPEC model (SBOM) where capacity decisions were subject to many different scenarios of what could happen in the market. Once the capacity decision is clear, all these different scenarios of the market equilibrium are independent from each other and can be solved separately. At first glance, this might seem like a suitable problem to apply the standard Benders decomposition, but actually due to the bilevel nature of the problem, the application of Benders is not straight forward.

In particular, the BPPs we propose in this thesis consist of two levels: the upper level and the lower level, which forms part of the constraints of the upper level problem. The lower level is another optimization problem with a different objective function than the upper level objective function. When considering the formulation of a bilevel problem, it becomes clear that a standard Benders approach cannot be applied to a BPP, which has two different objective functions, one in the upper level and one in the lower level. First of all, the arising BPP has to be transformed into a regular optimization problem consisting of only one objective function and constraints. Therefore, the lower level optimization problem - representing the market equilibrium - is replaced by its KKT conditions. This transformation yields a nonlinear optimization problem - the MPEC (BOM) or the stochastic MPEC (SBOM). In particular, the arising optimization problems are non-convex due to the complementarity conditions of the lower level. For the Benders decomposition to work successfully, the objective function expressed as a function of capacity investment variables has to have a convex envelope. Since our entire problem is inherently non-convex, the standard Benders approach is not guaranteed to converge. It is therefore clear that with an MPEC we cannot apply the standard Benders decomposition. In any case, let us now consider applying an adapted Benders decomposition, such as proposed by Kazempour and Conejo [69] to this problem and discuss the arising issues further.

As previously mentioned, the complicating variables are the investment decisions. Once these decisions are known, the rest of the problem, i.e., the market equilibrium, can be solved efficiently and separately for each scenario. Therefore, when separating the MPEC into a Master and a subproblem, the investment decisions and the corresponding constraints will be part of the Master problem, and the production decisions, market prices and demand will form part of the subproblem. The complication that occurs is that the subproblem is non-convex due to the complementarity conditions. In order to tackle this problem, the subproblem can be re-written as a convex problem by linearizing the complementarity conditions using Fortuny-Amat [49]. In total, we consider two options to model the subproblem. First, considering the continuous KKT conditions, which are nonlinear and non-convex and second, linearizing the complementarity conditions which convexifies the formulation of the constraints, however, it also introduces binary variables into the subproblem formulation.

A subproblem containing the nonlinear but continuous formulation of the lower level provides accurate sensitivities which are needed in a Benders approach; however, the nonlinearities may lead to non-robust behavior. On the other hand, a subproblem containing the linearized formulation of the lower level provides the optimal solution but does not provide useful sensitivities. The combination of the two subproblems - one for providing sensitivities and the other one for providing an optimal solution - can be used to supply the Master problem with useful information. In some cases it can be shown as for example by Kazempour and Conejo [69] that with an increasing number of scenarios, the objective function expressed as a function of capacity investment variables becomes increasingly convex which makes a Benders approach possible.

In current work in progress, we have implemented this approach, however, the obtained results have not been satisfactory so far. When comparing the solutions of the Benders approach as described above and the global solution of the MPEC formulated as a MILP, then the performance of the Benders approach was poor since solutions were very different. This poor performance might be due to the small number of scenarios that were considered in the case studies, however, the MILP cannot be solved for a large-scale scenario and as a first step, we wanted to test the performance of the Benders approach on a small case study before applying it to a large-scale problem. In future research this should to be analyzed in more detail. It also remains to be shown that the objective function of our proposed MPEC as a function of capacity investments has a convex envelope and whether some type of Benders decomposition can be applied. Successful Benders approaches to the investment problem in energy markets, as done by Baringo, Kazempour and Conejo [3, 69], should be considered as references in the literature.

6.2 Solution Techniques for the Generation Expansion EPECs

In general, EPECs are known to be very complicated problems which could have multiple equilibria or even none at all, due to inherent nonlinearities and non-convexities (due to the complementarity conditions). There exist a variety of methods to solve EPECs, and in this section we mention some of them - as well as their advantages and disadvantages - that we have applied when solving the EPEC models which have been proposed in this thesis. In particular, we present an iterative method called diagonalization in section 6.2.1 and then in section 6.2.2 we briefly talk about the complementarity problem formulation of the EPEC. In section 6.2.3 the linearization method is presented and finally in section 6.2.4 an approximation scheme, which is an original contribution of this thesis, is mentioned.

6.2.1 Diagonalization

Solving an EPEC directly can be very complicated, therefore a straight-forward way to solve an EPEC is by iteratively solving a series of MPECs which are simpler to solve than the EPEC. This method is called diagonalization. Diagonalization, as mentioned by Hu, Hu and Ralph or Leyffer and Munson [62, 63, 77], is a kind of fixed-point iteration where each player, in our case each GENCO, updates its investment strategy (by solving an MPEC) while considering the competition's investment decisions to be fixed. This type of method has been previously applied in energy bilevel models by Ahn and Hogan [1], Cardell et al. [23] and more recently by Hu and Ralph [63].

Let us now concretize this method for the bilevel generation expansion equilibrium model that we aim to solve. We therefore define the following algorithm:

Solving the Generation Expansion EPEC of Section 5.2.3 by Diagonalization

1. **Initialization**: Set iteration counter $count = 0$; define a maximum number of iterations *MaxIt* and define a convergence tolerance ε; provide initial point $x^{(0)} = \{x^0_{1jy}, \ldots, x^0_{ijy}, \ldots, x^0_{Ijy}\}$.

2. **Iteration**: For $i = 1$ to I: given the current iterate $x^{(count)}$, solve GENCO i's bilevel generation expansion problem [MPEC$_i$ given by (4.19)-(4.30)] which yields firm i's optimal investment decision x^*_{ijy}. Update the current iterate by replacing $x^{(count)}_{ijy}$ with the obtained optimal solution x^*_{ijy}.

3. **Stopping Condition**: If $\| x^{(count+1)} - x^{(count)} \| < \varepsilon$ then we have converged to point $x^{(count+1)}$ and stop the algorithm; else if *count* $<$ *MaxIt* then set count = count+1 and go to Step 2; else if *count* $=$ *MaxIt* then we stop the algorithm without having converged.

Step 1 of the diagonalization method is the initialization step, where we set the convergence tolerance ε, the maximum allowed number of iterations *MaxIt*, the initial capacity investment solution given by vector $x^{(0)} = \{x^0_{1jy}, \ldots, x^0_{ijy}, \ldots, x^0_{Ijy}\}$ and the iteration counter *count*. Step 2 of the algorithm is referred to as the iterative step because this is repeated in each iteration of the algorithm. For the current point of capacity investments, i.e., $x^{(count)}$, we loop through all GENCOs i and solve the MPEC$_i$ of the current GENCO i (while keeping the other GENCOs' investments fixed to $x^{(count)}$). MPEC$_i$ yields a new solution for the optimal investment decision of GENCO i which we refer to as x^*_{ijy}. We now update the current vector of capacity investments $x^{(count)}$ by replacing $x^{(count)}_{ijy}$ with the optimal solution x^*_{ijy} just obtained from MPEC$_i$. We repeat this for all GENCOs. Then we move on to the third step of the algorithm in which we check the stopping criterion. In particular, if the difference between two consecutive solutions $x^{(count)}$ and $x^{(count+1)}$ is sufficiently small, i.e., $\| x^{(count+1)} - x^{(count)} \| < \varepsilon$, then the algorithm has converged and we can stop. However, if the difference is greater than the tolerance ε then we have not converged yet. If we have not reached the maximum number of iterations yet, then the counter is therefore incremented by one and we go to step 2 of the algorithm, otherwise the algorithm terminates without having converged.

One of the advantages of this method is that it yields a bilevel equilibrium solution - that is if it converges - without having to actually solve an EPEC. Instead, this bilevel equilibrium solution is obtained by cyclically solving MPECs, which is a much easier task. In particular, as mentioned in section 6.1.1, the corresponding MPEC can be formulated in GAMS and solved using the solver NLPEC [46] which transforms the MPEC into a nonlinear program and furthermore calls another solver, i.e., CONOPT [41]. Another advantage of this method is that the computational time is relatively small, even for larger case studies. The case study which has been presented in section 5.4.3 (2 firms, 4 technologies, 6 load periods and time horizon of 15 years) led to an MPEC that consisted of 2400 variables. The arising bilevel equilibrium model has also been solved using diagonalization and on an Intel(R) Core(TM) i5-2410M running at 2.30 GHz 4 GB RAM, the computational time was 68 seconds.

The disadvantage of the diagonalization technique is that, depending on the initial point, we can end up at different solutions. This is not surprising, since an EPEC can have multiple solutions and therefore the bilevel generation expansion problem can have multiple equilibrium solutions. The diagonalization, if it converges, yields one of these equilibrium points but a priori we cannot say which one, nor can we actively choose among these equilibria. Moreover, the diagonalization method does not necessarily always converge. If it does not converge, this might point to the lack of a pure strategy equilibrium and the existence of an equilibrium in mixed strategies. A pure strategy provides a complete definition of how a player will play a game and determines the move this player will make for any situation he or she could face. There are games where there exist no Nash equilibria in pure strategies, however, they exist in mixed strategies. A mixed strategy is an assignment of a probability to each pure strategy. This allows for a player to randomly select a pure strategy with a certain probability. As for the bilevel generation expansion equilibrium problem, that we have proposed in this thesis, so far the diagonalization method has always converged to a pure strategy equilibrium.

Diagonalization can also be useful to check certain points for their equilibrium status - points that might have been obtained through other EPEC solution methods. Such a point is then used as initial solution for diagonalization. If the algorithm converges immediately, then the initial solution actually was an equilibrium point, otherwise it was not. It might have been just a saddle point of the EPEC or no special point in particular.

6.2.2 Mathematical Complementarity Problem

Another option to solve EPECs is via the complementarity problem formulation. In section 2.2.5 we have introduced the definition of a mixed complementarity problem (MCP). Moreover, in section 2.2.5 we have shown that an optimization problem such as the one presented below can be formulated as an MCP by using its KKT conditions.

$$
\begin{aligned}
&\max_{x} f(x) && && (6.6)\\
&\text{s.t. } F(x) \geq 0 && : \lambda && (6.7)\\
&\qquad\quad x \geq 0 && : \mu && (6.8)
\end{aligned}
$$

In particular, the MPEC problem (BOM) given by equations (5.2)-(5.13) can be interpreted as an optimization problem as the one presented above. As has been shown in section 2.2.5, the KKT conditions of such an optimization problem can be transformed into the following MCP:

$$
\begin{aligned}
&x \geq 0 \quad \perp \quad \nabla_x f(x) + \lambda^T \nabla_x F(x) \leq 0 && (6.9)\\
&\lambda \geq 0 \quad \perp \quad F(x) \geq 0 && (6.10)
\end{aligned}
$$

Since the generation expansion EPEC model (BEM), presented in (5.14)-(5.26), is a set of KKT conditions of each GENCO's MPEC, it can also be written as a complementarity problem. In fact, the formulation of the EPEC model (BEM) in (5.14)-(5.26) already is a complementarity problem formulation.

This type of problem can be addressed using specific MCP solvers like PATH [38]. For our particular generation expansion equilibrium problem, the complementarity approach has not worked very well. In the majority of cases the solver terminated yielding an infeasible solution, even if we could prove (by using other methods like diagonalization) that equilibria existed. In some sporadic cases and only for very small problem instances, the solver actually yielded a feasible solution. Another disadvantage of the MCP formulation is that feasibility does not necessarily imply that the obtained solution is an equilibrium of the EPEC, it might just be a stationary point of the EPEC as observed by Hu and Ralph [63]. Therefore, the MCP approach - while working well for single-level equilibrium problems - has not been employed further for the EPEC model in this research. However, this type of approach has successfully been applied in the literature. In particular, the reader is referred to the book of S. Gabriel et al. [51] for theory and applications of complementarity modeling in energy markets.

6.2.3 Linearization Methods

In the numerical example of the EPEC presented in section 5.2.4 we have already emphasized that the EPEC has multiple equilibria and that some of them may be more interesting than others depending on the scope of the study. The set of equations and inequalities as formulated in (5.14)-(5.26) in section 5.2.3 representing the EPEC can therefore have several solutions and it would be desirable to be able to choose among the different equilibria. It is therefore necessary to define an additional criterion to choose a specific equilibrium of interest. This criterion is embodied by the classification function introduced herein.

Therefore, let us interpret the EPEC model, which consists of a set of equations and inequalities, as the constraints of an optimization problem, whose objective function fulfills the purpose to classify the type of equilibrium that we would like to choose. This methodology allows to choose a specific equilibrium (among possibly many existing equilibria) which for example maximizes total market profits. In order for the obtained solution to be globally valid, the proposed optimization problem should be transformed into a MILP. We hence define a MILP, as proposed below, where the constraints correspond to a linear version of the EPEC and the objective function corresponds to a classification function. This allows us to choose the equilibrium that most appropriately satisfies the criterion of the classification function.

$$\begin{aligned} \max \quad & \text{Linear classification function} \\ \text{s.t.} \quad & \text{Linear version of the EPEC} \end{aligned} \tag{6.11}$$

Moreover, points that satisfy the previously defined EPEC conditions are called Nash stationary points and are not necessarily always an equilibrium. Therefore, after having solved (6.11) we carry out a post-optimality check in order to identify whether the obtained stationary point is also an equilibrium. To this end, we apply a diagonalization technique, which has previously been introduced in section 6.2.1, where the investment decisions obtained by (6.11) serve as the initial point. If it turns out that the obtained point is not an equilibrium then we solve (6.11) again adding an upper bound on the classification function value that is strictly less than the one previously obtained. This procedure allows us to search the solution space for different equilibria. Furthermore we employ another check where we manually change the obtained investment decisions and solve the market equilibrium and calculate profits. If unilateral changes in investments lead to a decrease in profit, then the obtained points actually are equilibria. However, it has to be stated that the process of solving the linear version of the EPEC various times can be quite slow.

In the remainder of the section we discuss how to linearize the nonlinear program that is the EPEC, given by (5.14)-(5.26), and we propose a suitable classification function and put all of the parts together.

Linearization of EPEC

The nonlinearities of the EPEC model (BEM), given by (5.14)-(5.26), are due to the complementarity conditions and the bilinear terms arising from the product of $\zeta^{\mu}_{i^*ijyl}$ and $\zeta^{\lambda}_{i^*ijyl}$ with other variables. We take care of the complementarities (5.10), (5.11) and (5.20)-(5.25) by replacing them by their linear equivalent, as presented by Fortuny-Amat [49]. An example for the linear equivalent of complementarity (5.10) can be seen below:

$$\begin{aligned} C^{\mu} b^{\mu}_{ijyl} &\geq \mu_{ijyl} && (6.12)\\ C^{\mu}(1 - b^{\mu}_{ijyl}) &\geq q_{ijyl}, && (6.13) \end{aligned}$$

for C^{μ} suitably large constants and b^{μ}_{ijyl} binary variables.

As for the other bilinear terms, arising in (5.14)-(5.17), we apply a binary expansion, as proposed by Pereira et al. [93] to the variables $\zeta^{\mu}_{i^*ijyl}$ and $\zeta^{\lambda}_{i^*ijyl}$. The formulation of the binary expansion for $\zeta^{\mu}_{i^*ijyl}$ is given below:

$$\zeta^{\mu}_{i^*ijyl} = \underline{\zeta^{\mu}} + \Delta_{\zeta^{\mu}} \sum_k 2^k b^{\zeta}_{ki^*ijyl}, \tag{6.14}$$

where $\underline{\zeta^{\mu}}$ is the lower bound, $\Delta_{\zeta^{\mu}}$ is the chosen step size, k the set of discretization intervals and $b^{\zeta}_{ki^*ijyl}$ are binary variables. Then the bilinear terms $\zeta^{\mu}_{i^*ijyl}q_{ijyl}$, for example, can be replaced by $\underline{\zeta^{\mu}}q_{ijyl} + \Delta_{\zeta^{\mu}} \sum_k 2^k z^{\zeta}_{ki^*ijyl}$, where $z^{\zeta}_{ki^*ijyl}$ symbolizes the product of the two variables and is defined by the following constraints, which also have to be added to the problem:

$$0 \leq \quad z^{\zeta}_{ki^*ijyl} \quad \leq C^{\zeta} b^{\zeta}_{ki^*ijyl} \tag{6.15}$$

$$0 \leq \quad q_{ijyl} - z^{\zeta}_{ki^*ijyl} \quad \leq C^{\zeta}(1 - b^{\zeta}_{ki^*ijyl}), \tag{6.16}$$

for C^{ζ} a suitably large constant.

In this way we transform the nonlinear EPEC into a system of linear equations and inequalities, which we refer to as the linear version of the EPEC.

Classification Function

In section 5.2.4 we have established that an interesting classification function would be the weighted sum of overall profits, given by equation (5.27) and presented below, where V_i represent the weight of firm i's total profit. The total profits consist of the market revenue term ($T_{yl}p_{yl}q_{ijyl}$), minus total production cost ($T_{yl}\delta_{ij}q_{ijyl}$), minus investment costs ($\beta_{ijy}x_{ijy}$):

$$\sum_i V_i \sum_y \frac{1}{(1+F)^y} \{ \sum_{jl} T_{yl}p_{yl}q_{ijyl} - \sum_{jl} T_{yl}\delta_{ij}q_{ijyl} - \sum_j \beta_{ijy}x_{ijy} \} \tag{6.17}$$

The variables of that appear in this classification function are: capacity investments x_{ijy}, market prices p_{yl} and production decisions q_{ijyl}. Since the market revenue term ($T_{yl}p_{yl}q_{ijyl}$) of the total profits is nonlinear, this function needs to linearized in order to serve as a classification function in a MILP framework. Similar to the derivations previously carried out in section 4.4.2 when introducing a discretized version of the MPEC, we rewrite the market revenue term of the classification function as equation (6.18). For the details of this derivation the reader is referred to section 4.4.2.

$$\begin{aligned}\frac{\sum_j T_{yl} p_{yl} q_{ijyl}}{(1+F)^y} &= \frac{\sum_j \delta_{ij} T_{yl} q_{ijyl}}{(1+F)^y} + \frac{\theta_{iyl} T_{yl} (\sum_{j^*} q_{ij^*yl})^2}{(1+F)^y} \\ &+ \sum_j \lambda_{ijyl}(K_{ijy} + x_{ijy})\end{aligned} \tag{6.18}$$

In the expression of market revenues given by (6.18), there are still two nonlinearities: a bilinear term stemming from the product $\lambda_{ijyl} x_{ijy}$ and a quadratic term $(\sum_{j^*} q_{ij^*yl})^2$. The quadratic term can be approximated by a piece-wise linear function, as has been previously shown in equations (4.115)-(4.118). Let us now focus on the other nonlinear term.

The bilinear term $\lambda_{ijyl} x_{ijy}$ in the classification function could be linearized easily by discretizing investment decisions x_{ijy}. However, even though this was a desirable approach for the MPEC, in the EPEC investment decisions have been considered continuous variables, which was necessary in order to take derivatives. Therefore discretizing x_{ijy} in the classification function would mean reducing the solution space of a primal variable of the EPEC and is therefore not desirable in this case because it might lead to the loss valid solutions of the EPEC. Instead we introduce a new variable $\tilde{x}_{ijy}$ which represents the approximation of investment decisions x_{ijy} and we replace $\lambda_{ijyl} x_{ijy}$ with $\lambda_{ijyl} \tilde{x}_{ijy}$ in the classification function. Then we discretize $\tilde{x}_{ijy}$ using a binary expansion as presented in (6.19), where Δ_x is the chosen step size, k the set of discretization intervals and b^x_{kijy} are binary variables. Then the bilinear terms $\lambda_{ijyl} \tilde{x}_{ijy}$, can be replaced by $\Delta_x \sum_k 2^k z^x_{kijy}$, where z^x_{kijy} symbolizes the product of the two variables and is defined by the following constraints, which also have to be added to the problem:

$$\tilde{x}_{ijy} = \Delta_x \sum_k 2^k b^x_{kijy}, \tag{6.19}$$

$$0 \leq z^x_{kijy} \leq C^x b^x_{kijy}, \tag{6.20}$$

$$0 \leq \lambda_{ijyl} - z^x_{kijy} \leq C^x(1 - b^x_{kijy}), \tag{6.21}$$

for C^x a suitably large constant. We also add constraints (6.22) and (6.23) in order to guarantee that the difference between x and its approximation is at most the step size.

$$\tilde{x}_{ijy} - x_{ijy} \leq \Delta_x \tag{6.22}$$

$$x_{ijy} - \tilde{x}_{ijy} \leq \Delta_x \tag{6.23}$$

Finally, applying this to (6.17) yields a linear classification function that represent the approximation of profits and completes the MILP, that we solve in the case study presented in section 5.2.4:

$$\begin{aligned} \max \sum_{iy} \frac{V_i}{(1+F)^y} \Big\{ \sum_l T_{yl}\theta_i \bar{q}_{iyl} - \sum_j \beta_{ijy} x_{ijy} + \sum_{lj} \lambda_{ijyl} K_{ijy} (1+F)^y \\ + \sum_{jl} (1+F)^y \Delta_x \sum_k 2^k z^x_{kijy} \Big\} \end{aligned} \tag{6.24}$$

$$\text{s.t. Discretize approximated investments:} \qquad (6.19) - (6.23) \tag{6.25}$$

$$\text{s.t. Piecewise linear function:} \qquad (4.115) - (4.118) \tag{6.26}$$

$$\text{s.t. Linear version of the EPEC} \tag{6.27}$$

The advantage of this method is that it allows us to search the solution space of the EPEC and find different equilibria. As the numerical example presented in section 5.2.4 shows, even for a relatively small case of an EPEC multiple equilibria exist and they yield a different technology mix in capacities. Moreover, by introducing the classification function, we are able to choose particular equilibria that might be interesting depending on the study that is carried out. However, when converting the EPEC into a MILP many binary variables are required, which makes this a particularly difficult model to solve. As mentioned in the numerical example in section 5.2.4, on an Intel(R) Core(TM) i5-2410M running at 2.30 GHz 4 GB RAM, depending on the choice of weights, the computational time could range from 10 to 24 hours and it is a very memory intensive process, which for the time being makes the resolution of large-scale examples intractable.

6.2.4 Approximation Scheme

An alternative method to solve the EPEC generation expansion model is the single-level approximation scheme that has been presented previously in section 5.4 and that has been published in an international journal, see Wogrin et al. [117]. In this scheme the EPEC is approximated by solving the single-level expansion equilibrium model twice, first assuming Cournot behavior in the market, fixing the obtained investment decisions and then solving the single-level model a second time assuming the actual strategic behavior θ.

In section 5.4.1 it has been discussed in detail how the results from chapter 3 have sparked the idea for this approximation scheme and how they are the theoretical basis for the functioning of this scheme. Essentially, what has been found in chapter 3 is that the generation expansion EPEC for a single-year, single-period situation yields Cournot capacities despite of the strategic behavior in the spot market and that the single-level and bilevel model coincide when assuming Cournot behavior. Under certain circumstances these results extend to the multiple-period case. Therefore, instead of solving the bilevel model, we solve the single-level model assuming Cournot behavior in the market, which hopefully yields investment decisions similar to what the bilevel model would have yielded. This is the first step of the approximation procedure which provides us with investment decisions, however, the single-level Cournot model does not yield appropriate market prices and productions since the bilevel model considered different strategic behavior θ in the market. Hence, we fix the investment decisions that we have obtained in the first step and solve the market equilibrium problem again, this time assuming strategic behavior θ instead of Cournot behavior. This provides us with a good approximation of prices and production decisions. All in all, the first step of the approximation yields the investment decisions and the rest yields prices and production decisions. For further detail the reader is referred to section 5.4.

The advantages of this methodology are numerous. First of all, we have developed an alternative way to formulate the single-level expansion equilibrium as an equivalent quadratic convex optimization problem, see section 5.3, which is a very efficient way to solve this problem computationally, even for very large-scale problems. As a reference we refer to the numerical example presented in section 5.4.3, where the computational time of a two player, four technology, six load level and 15 year EPEC on an Intel(R) Core(TM) i5-2410M running at 2.30 GHz 4 GB RAM using the CPLEX solver was only 0.6 seconds. Let us compare this computational time to other methods, for example diagonalization. The same problem, when solved using diagonalization, where one MPEC consisted of 2400 variables and the stopping tolerance was set to 0.0001 was 68 seconds. This means that the approximation scheme by two orders of magnitude faster than the diagonalization methods, which demonstrates the promise that the approximation method has when wanting to tackle real-life large-scale problems. Let us also recall from 5.2.4 that the linearization approach for a much smaller numerical example of two firms, two technologies, two load periods and two years, yields a computational time of about 10 hours, which would be five orders of magnitude larger than the approximation scheme. Not to mention that with the available computational power, a numerical example of the size considered in 5.4.3 could not have been solved to optimality using the linearization approach.

On the other hand, the disadvantages of employing the approximation scheme to solve the generation expansion EPEC is that in the only one unique solution is provided, which is independent from starting points etc. But we know from the previous section 6.2.3 that the EPEC has multiple equilibria. It might be interesting to explore the space of possible equilibria in order to analyze in what kind of market situation a certain equilibrium might occur and/or how to avoid this from happening. Since the approximation scheme only yields one solution, even though this solution might be very well motivated, this technique does not allow for further exploration of the solution space of the EPEC. Moreover, it has to be considered that the solution yielded by the approximation scheme - as the name indicates - is an approximation of an EPEC solution which does not necessarily have to coincide with an actual EPEC solution. However, the numerical example in 5.4.3 has shown that under certain circumstances, this approximation is very accurate.

6.3 Summary of Solution Methods and Conclusions

Table 6.1 contains a summary of the solution methods for MPECs and EPECs that have been discussed in this chapter. In Table 6.1 we present the type of bilevel problem that has been discussed (MPEC or EPEC), the solution method applied, a relevant reference where to find such an approach, and a summary of the advantages and disadvantages of such an approach. Note that the comparison only refers to methods that have in one way or another been used in this thesis and is not a complete list of all existing solution methods for MPECs and EPECs.

As a summary, for MPEC problems, NLP has allowed us to solve realistic case studies very efficiently, however, since the MPEC problem is non-convex, the obtained solution is only local and there is no way of saying how far this solution might be from the global optimum. MILP methods on the other hand provide this type of information. The obtained solution is a global maximum, however, since many binary variables are introduced into the problem, so far only moderately-sized problem instances could be solved. Decomposition methods could solve this problem, however, since MPEC problems are inherently non-convex, straight-forward decomposition methods, such as Benders decomposition, are not guaranteed to work.

For EPEC problems, there are methods such as diagonalization which relies on the iterative solution of MPEC problems. This method can handle large-scale problems and if it converges, then it yields an equilibrium solution. However, this method is not guaranteed to converge even if an equilibrium exists. Moreover, diagonalization yields one equilibrium solution, but there might be many local equilibria to choose from. The proposed MILP methods provide the methodology to choose a specific equilibrium among all the existing ones, however, only relatively small problem instances can be solved. Moreover, the solution might not even be an equilibrium but only a stationary point. MCP methods can handle large-scale problems, even though for our problem they have not worked well, but they also yield only stationary points, which would have to be checked for their equilibrium status. Finally, the proposed approximation scheme yields good solutions very fast, but only works well when the strategic spot market behavior is closer to Cournot than to perfect competition.

It is clear to see that each of the explored approaches has its strong and its weak points and that among these methods there is no such thing as the perfect solution method with only advantages and no drawbacks. Therefore, as a conclusion in terms of computational methods for MPECs and EPECs and as a recommendation it can only be said that the choice of method greatly depends the purpose of the analysis.

Table 6.1: Comparison of advantages and disadvantages of different solution methods.

Problem Type	Solution Method	References	Advantages	Disadvantages
MPEC	NLP	Centeno [27]	Realistic cases Small CPU time	Local solution
	MILP	Wogrin [116]	Global solution	Small cases High CPU time
	Decomposition	Kazempour [69]	Realistic cases Reasonable CPU time	Only works under certain circumstances[1]
EPEC	Diagonalization	Hu [63]	Realistic cases Reasonable CPU time Easy (uses MPECs)	Convergence not guaranteed Cannot classify equilibria[2]
	MCP	Gabriel [51]	Realistic cases Reasonable CPU time	Cannot classify equilibria Yields stationary points[3] Did not work in our case
	MILP	Ruiz [105] Wogrin [115]	Classifies equilibria	Small cases High CPU time Yields stationary points
	Approximation	Wogrin [117]	Realistic cases Small CPU time	Only works under certain circumstances[4]

[1]For a Benders approach to work, the problem needs to fulfill certain convexity requirements, which in general MPECs do not fulfill. In some cases (Kazempour and Conejo [69]) the objective function expressed as a function of capacity investment variables becomes increasingly convex with an increasing number of scenarios, which makes a Benders approach possible. However, this is not generally true for all MPECs and would have to be verified for each particular problem.

[2]This refers to the fact that such a method does not allow to choose a particular equilibrium solution from among the existing ones.

[3]Both MCP and MILP versions of the EPEC incorporate the first order conditions of all MPECs. Hence, it is not guaranteed that the obtained solution actually is an equilibrium point but only a stationary point, such as a saddle point for example, which is not the solution of the EPEC we are looking for. Therefore, once a solution is obtained an additional verification has to be carried out in order to check the obtained point for its equilibrium status.

[4]The approximation scheme, proposed in section 5.4, works well when strategic behavior is closer to Cournot than to perfect competition.

Chapter 7

Additional Case Studies

This chapter is dedicated to some additional case studies that have been referred to throughout the thesis. In total, three different case studies are presented here. First, in section 7.1 a large-scale stochastic MPEC with demand uncertainty is introduced. Furthermore, in this numerical example, capacity payments, financial hedging and hydro energy are considered, which makes this the most realistic case study presented in this thesis. Then, in section 7.2 a stochastic MPEC which considers discrete investment decisions is considered. Finally, in the case study of section 7.3 the single-level approximation scheme of the bilevel generation expansion equilibrium model - an original contribution of this thesis - is applied to a large-scale numerical example in order to verify its validity. In section 5.4.3 the approximation scheme has been validated numerically for a very small case study. Section 7.3 aims at applying the approximation scheme to a multi-year, multi-technology, multi-load period numerical example in order to characterize the quality of the scheme with respect to realistic case studies. We draw the conclusion that the bilevel model can be approximated reasonably well for cases where the market behavior is closer to oligopoly than to perfect competition. The reduction in computational time by two orders of magnitude still applies even for the large-scale numerical example. This case study answers one of the main research questions that have been posed in this thesis, which was to quantify if and when bilevel generation expansion models can be accurately approximated by single-level models. Moreover, a sensibility analysis on how model parameters affect the quality of the approximation scheme is carried out. Finally, section 7.4 contains the conclusions and an overview of the case studies of this thesis.

7.1 Large-Scale Stochastic MPEC with Demand Uncertainty

The purpose of this first case study is to present a large-scale numerical example in which several of the proposed model extensions, motivated in section 4.4, are included. In particular, in the following we present a stochastic MPEC which considers stochastic demand, which also incorporates hydro energy, capacity payments and contracts for differences. The data of this case study is calibrated in order to resemble the Spanish power system.

In particular, we consider six generation companies i_1 to i_6 where the first five correspond to five large GENCOs of Spain and the sixth GENCO i_6 has been designed as an aggregate of the rest of the players in the market. Four different thermal technologies are considered for investment capacity, i.e., nuclear (NU), coal (CO), combined cycle gas turbines (CC) and open cycle gas turbines (GT). Production and annual investment costs of those technologies are based on the updated document of the International Energy Agency [90] and are presented in Table 7.1, which also contains values of a capacity payment which has been inspired by the Spanish system. The discount factor F is assumed to be 9% and the demand slope is taken as 0.26 GW/(€/MWh). Due to space limitations, not all data is presented here but can be found in the appendix A. Tables A.4, A.5 and A.6 contain the already existing generation capacity K_{ijy} of each of the six GENCOs, which has roughly been based on the Spanish power system. Note that the expected equivalent forced outage rates and self-consumption of power real plants have been incorporated and hence Tables A.4, A.5 and A.6 have to be interpreted as an approximation of the net existing capacity (capacity minus expected equivalent forced outage rates and self-consumption of generators) in Spain.

Table 7.1: Production and investment cost for each technology.

	Production cost δ [€/MWh]	**Annual investment cost** β [(M€/GW)/year]	**Capacity payment** CP [(M€/GW)/year]
NU	6.41	386.77	0.00
CO	20.78	229.60	12.00
CC	43.57	70.90	3.15
GT	65.10	39.00	3.15

Since the main focus of this numerical example is to study the impact of demand uncertainty on investments, both the hydro data and the competitors' investment and strategic data is assumed deterministic. Total annual hydro generation for each market agent has been estimated as follows. Parting from an average annual hydro energy value of 26 TWh, this energy has been divided among the individual market agents according to realistic shares of Spanish market agents. The individual hydro energy data for each generation company can be found in the appendix in Table A.1. Maximum hydro output per load period H_{iyl} is given in Table A.2. Strategic spot market behavior (characterized by the conjectured-price response) for each firm is given in Table A.7. Contracts for differences have been considered at 40 €/MWh and an annual quantity M_{iyl} is presented in Table A.8. No new investment of the competitors' X_{ijy} is considered, however, in the solution they still have production since they part from some existing capacity K_{ijy}.

Since Spain is a country with a considerable amount of non-dispatchable renewable energy, which is not explicitly modeled in the stochastic MPEC, we incorporate this as follows: the term demand d_{yls} that is considered in this model does not represent the real demand of the stylized Spanish power system, but the net demand (calculated as demand minus wind) of a stylized Spanish power system. In order to derive corresponding demand intercept data, we therefore assume two different scenarios of annual demand growth and one scenario of wind production growth. The obtained values are subtracted and ultimately lead to the demand intercept data presented in Tables A.9 and A.10, which are both considered equally likely demand scenarios, i.e., a high-demand scenario and a low-demand scenario. In the high-demand scenario, demand intercept values range from 17.5 GW in off-peak periods to 62.7 GW in peak periods in the first year of the time horizon. In the last year of the time horizon these values range from 14.7 GW to 90.2 GW. On average, a 2% demand intercept increase is considered in peak periods. In the low-demand scenario, the demand intercept ranges from 15.6 GW in off-peak periods to 55.8 GW in peak periods during the first year of the time horizon. In the last year of the time horizon the demand intercept ranges from 2.2 GW to 59.5 GW and on average, a 0.3% demand intercept increase is considered in peak periods. For further detail, the reader is referred to Tables A.9 and A.10. It should be kept in mind that when we refer to "demand" in the remainder of this section, we actually mean net system demand, where wind production has already been subtracted.

The considered time horizon of this case study is 20 years and we consider 12 load periods in each year. Annual load period durations of the 12 annual load periods considered are presented in Table A.3. These 12 periods can be separated into four peak periods (l_1 to l_4), four shoulder periods (l_5 to l_8) and four off-peak periods (l_9 to l_{12}). The arising stochastic MPEC has been formulated in GAMS and is made up of 30483 variables and 30459 equations - a problem size which could not be tackled so far with the existing MIP methods. Solving this model using NLPEC [46] on an Intel(R) Core(TM) 2 Quad Processor with 3.21 GB RAM takes around 7 minutes.

In Figure 7-1 we present the solution for capacity investments in coal technology of firm i_1 obtained by the stochastic MPEC. Note that in the optimal solution there are no investments in other technologies. Moreover, we include the capacity investment plans that we obtain if we solve two deterministic MPECs, one for each demand scenario, the high and the low-demand scenario. Due to the existing over-capacity that is currently reigning in Spain, it is not surprising that until the year 16 of the time horizon, the are no new investments in capacity yielded by the stochastic MPEC. At the end of the explored time horizon, the stochastic MPEC yields a capacity of 1.6 GW in coal. In comparison, in the high-demand scenario capacity investments become more attractive at an earlier time and in particular, capacity investments in coal start in year 13 and yield a total of 3.3 GW. On the other hand, in the low

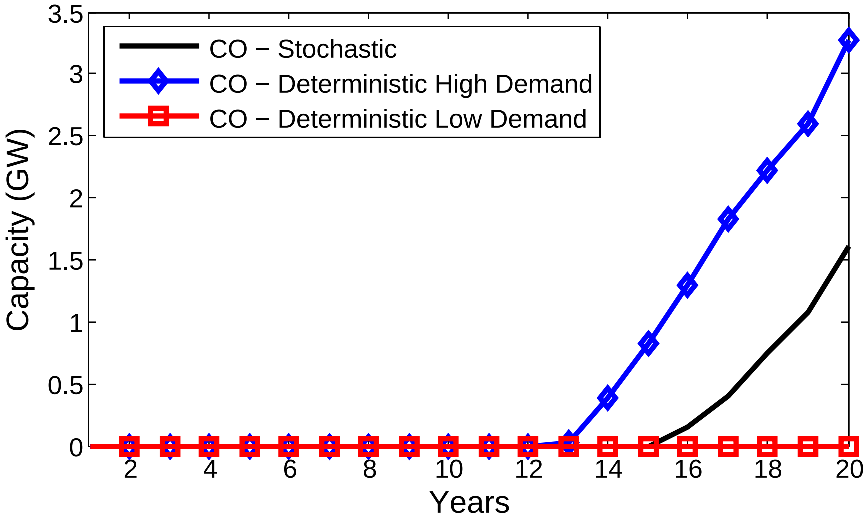

Figure 7-1: Capacity investment results obtained by stochastic MPEC and by the deterministic high-demand scenario.

deterministic demand scenario, capacity investments are not profitable for the entire time horizon, since demand can be covered by already existing capacity and there are therefore no new capacity investments in this scenario. The investment plan determined by the stochastic MPEC yields an expected NPV of 21360 M€, while in the deterministic cases the obtained NPVs in the high and low demand cases are 26054 and 16775 M€, respectively. As a summary it can be said that, since both scenarios are considered equally likely, the new capacity investments yielded by the stochastic MPEC are quite low, simply because if more capacity were built and a low-demand scenario occurred, then firm i_1 would not recover its investments. It is therefore more profitable to invest less capacity.

Annual net demand (demand minus wind) and average annual spot market prices of the stochastic MPEC for each demand scenario are presented in Figure 7-2. Under the high-demand scenario we obtain an annual average price of 46.1 €/MWh in the first year, which ranges from 21.9 to around 75.2 €/MWh depending on the load period. At the end of the explored time horizon prices range from 22.3 to 180.0 €/MWh with an annual average of 81.0 €/MWh. Thus in the last year of the time horizon, peak prices can go as high as 180 €/MWh which is the value of the price cap in the Spanish system. It seems that the investments are not enough to lower

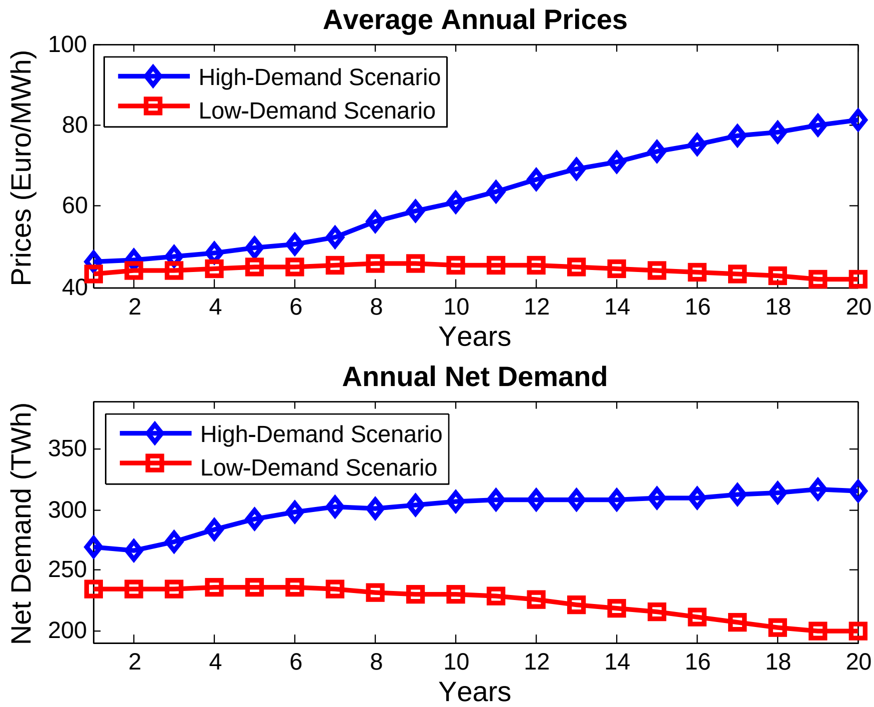

Figure 7-2: Average annual prices [€/MWh] and annual net demand [TWh] under both scenarios.

peak prices more. Let us now observe the low-demand scenario prices. Due to the lower values of assumed demand intercept and actual demand, given in the second subplot of Figure 7-2, prices in this scenario are lower than the ones in the high-demand scenario. This is due to the fact that in this scenario the assumed annual increase in demand intercept is considered very small. As a matter of fact, due to the increased integration of wind power, actual net system demand decreases over time in the low-demand scenario. In the first year of the time horizon prices range from about 21.2 to 66.5 €/MWh with an annual average of 43.4 €/MWh, and in the last year prices range from 5.5 to 75.1 €/MWh with an average of 42.1 €/MWh, which is lower compared to the first year because off-peak prices have dropped more drastically in future years since a higher wind generation (which leads to a lower net demand level) has been assumed. The detailed results of demand in each load period, year and scenario can be found in Tables A.11 and A.12.

Let us now analyze the return on investments, given in Table 7.2, of our investing generation company starting in year 16 of the time horizon. The return on invest-

Table 7.2: Return on investments [%] for investing market agent i_1 over time horizon.

	Stochastic	High Demand	Low Demand
y_{16}	0.9	24.0	-22.2
y_{17}	2.4	32.7	-27.9
y_{18}	2.2	37.1	-32.8
y_{19}	4.5	40.3	-31.2
y_{20}	3.9	41.7	-34.0

ments has been calculated as annual net profits of investments divided by annual investment costs. Note that the annual net profits of investments have been obtained as the difference of total annual profits minus total annual profits in the case where no investments have been assumed. In Table 7.2 we present the return on investments yielded by the stochastic model, but also for each of the demand scenarios individually. It is interesting to observe that the high-demand scenario leads to a return on investments as high as 41.74%, whereas under the low-demand scenario the return on investments is negative. This means that if the low-demand scenario occurs then the investing company is not going to recover its investments. However, since the possible return on investments under the high-demand scenario is slightly higher then the possible losses under the low-demand scenario, on average we still obtain a positive return on investments of up to 4.54% in the stochastic model. These results would dissuade a risk-averse firm from making this investment.

In conclusion it can be stated that in a power system resembling the Spanish one, the possibility of the low-demand scenario delays possible capacity investments and the current capacity payments are not enough to counter this effect. If the capacity payments were to be doubled, then new investments in coal would already emerge one year earlier, i.e., in year 15, and in total around 480 MW more would be built. This shows us that the presented stochastic MPEC model can be employed to analyze the impact of regulatory measures.

7.2 Comparison between Continuous and Discrete Investment Decisions

In this case study we revisit the numerical example of section 4.3.4, where we presented a stochastic MPEC. The stochastic parameters were the competitors' investment decisions and their corresponding strategic spot market behavior. In the case study in section 4.3.4 we also compared two different solution techniques of the MPEC: the first was to employ the nonlinear solver NLPEC, and the second option was to transform the MPEC to a MILP by discretizing market prices. Since market prices are lower level variables, this particular discretization may lead to the loss of realism of the representation, apart from the fact that computational times increase tremendously (NLPEC takes 0.8 seconds while the MILP depending on the step size used in the discretization can take up to 20 hours). Therefore, in this case study we apply the discretization of investment variables, as proposed in section 4.4.2, instead. Since investment decisions are inherently discrete, this type of discretization even increases the state of realism of the model. The purpose of this case study is to show the difference in terms of optimal capacity mix yielded by the MPEC solved by NLPEC and the MILP with discrete capacity decisions and analyze the results. We also compare the obtained results to a static expansion, which optimizes investments for one target year in the future, and analyze its differences.

All the data that has been assumed in this case study is identical to the data in the numerical example of section 4.3.4 and is therefore not repeated here. However, let us define all the additional data that has been used in the MILP model with discretized capacity investment decisions. The discrete step size for each power plant is 1000 MW for nuclear and 400 MW for combined cycles.

Let us now analyze the obtained results. In Table 7.3 the total expected NPV is presented for the original MPEC with continuous variables, the stochastic MILP with discrete investment decisions, a static MILP which also assumed discrete investment decisions but only for one target year in the future, and for three deterministic MILPs considering discrete investments and different strategic spot market behavior, i.e.,

Table 7.3: Total expected net present value [M€] of investing firm.

	NPV
MPEC	30921
Stochastic MILP	30893
Static MILP	30877
Deterministic MILP (Perfect competition)	19610
Deterministic MILP (Intermediate competition)	29732
Deterministic MILP (Cournot competition)	40980

perfect competition, Cournot behavior and intermediate behavior. Note that the NPV for the static case has been calculated as follows: the discrete MILP is solved for a future target year, i.e., the last year of the time horizon. All the data considered - the discount factor, the demand intercept and the competitors' investments - have to be adapted to this future target year. Once we obtain the optimal total investment capacity of each technology in this target year, we solve the stochastic MILP again but this time fixing the capacity of year 5 to the static capacity. We do this in order to obtain an investment schedule over the entire time horizon, however, we stick to the total capacity yielded by the static MILP. We carry out such a procedure because the computational time is only a small fraction of the computational time of the complete stochastic MILP. Therefore this could be a promising approach to tackle large-scale case studies. The resulting NPV is presented in Table 7.3 and referred to as "Static MILP". Comparing the resulting NPVs our first observation is that the difference between the stochastic and the static MILP is relatively small, only 0.05%. It is also interesting that the MPEC yields a slightly higher NPV than the stochastic MILP, which means that even though global optimality of the MPEC cannot be guaranteed, the relaxation of the investment variables to continuous variables leads to an increase of the NPV with respect to the MILP solution, which we know to be globally optimal. From the NPVs of the deterministic cases and the stochastic MILP we obtain the expected value of perfect information, which is the amount which GENCO i_1 would be willing to pay in order to obtain a perfect forecast of competitors' investments

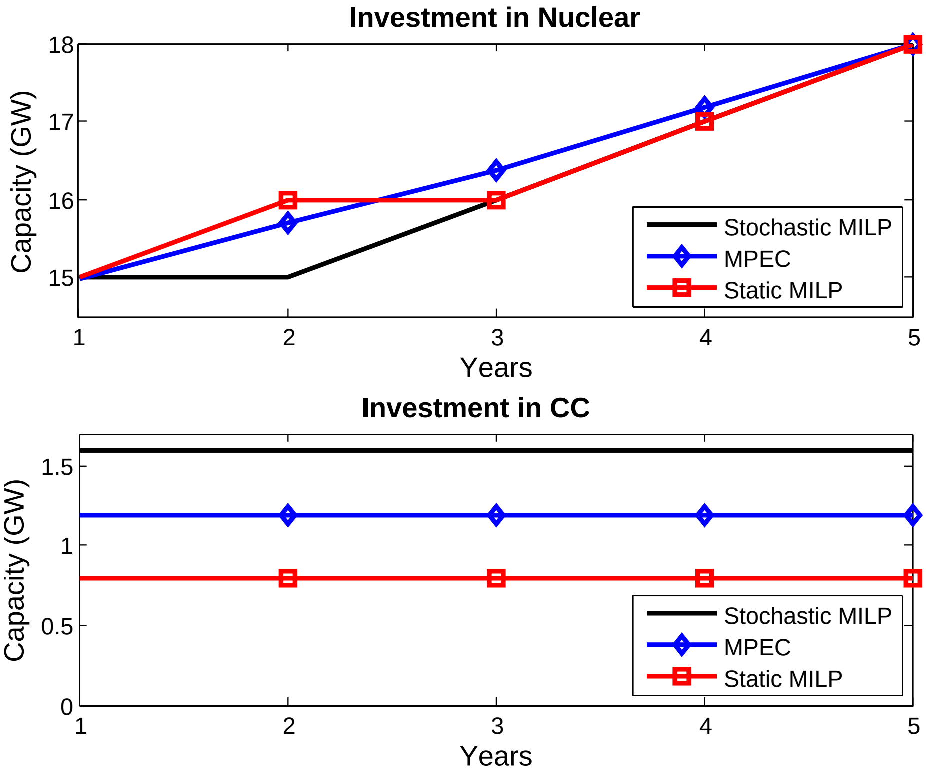

Figure 7-3: Capacity investment results obtained by MPEC, stochastic and static MILP with discrete capacity decisions.

and behavior. The expected value of perfect information is 189M€, which means that with perfect information, GENCO i_1 would be able to increase its NPV for only around 0.6%. This value depends on the choice of scenarios.

In Figure 7-3 the investment decisions of the stochastic MILP, the MPEC and the static MILP, for which we have calculated an investment plan over the entire time horizon as mentioned in the previous paragraph. The capacity invested in nuclear technology is very similar in all three cases. As a matter of fact the total nuclear capacity at the end of the time horizon is 18 GW in all cases and 15 GW in the first year. In terms of nuclear investments, the static and the stochastic MILP only differ by one plant of 1000 MW, which the static MILP builds in year 2 and the stochastic MILP builds in year 3 instead. When comparing the stochastic MILP with the MPEC solution for nuclear investments it is clear to see that the difference in capacity is never greater than 1000 MW, which is the step size that has been chosen. It can therefore

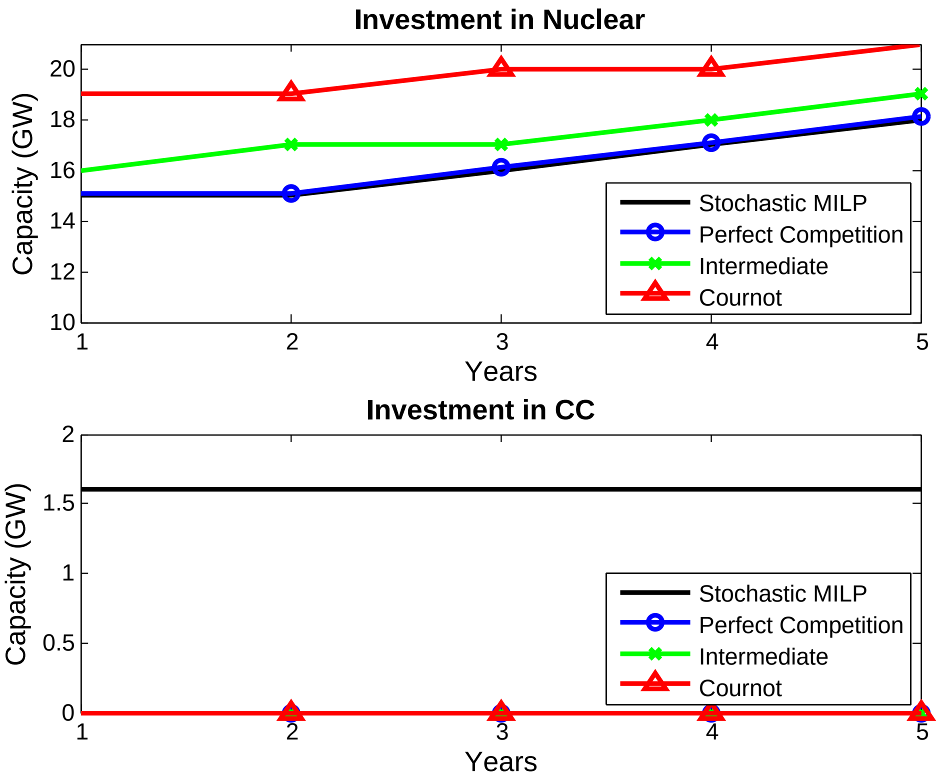

Figure 7-4: Capacity investment results obtained by stochastic MILP and deterministic MILPs with discrete capacity decisions.

be said that the MPEC and the stochastic MILP are fairly similar. When looking at the investment in combined cycle gas turbines (CC), it can be observed that the static and the stochastic MILP differ by two CC plants of 400 MW each. Under the stochastic framework it is profitable to invest 800 MW more in CC. The MPEC solution lies in between the other two solutions and in particular around 1.2 GW of CC, which is one plant more than in the static solution and one plant less than in the stochastic solution.

Figure 7-4 contains a comparison of the investment decisions made by the stochastic MILP and three separate deterministic cases of the discrete MILP assuming either perfect competition, Cournot behavior or intermediate strategic spot market behavior. The investments in nuclear technology of the stochastic MILP coincide with the perfectly competitive solution. The more oligopolistic the spot market behavior, the higher the investments in nuclear technology. This is due to the fact that, the closer to Cournot, the higher the market prices, the higher the obtained NPVs and therefore the higher the incentive to invest. On the other hand, as for investments in CC plants, in neither of the deterministic MILPs there is investment in this technology, however, there is in the stochastic solution. If strategic behavior and capacity investment of the competition are certain, then it seems more profitable to cover the entire new demand with nuclear capacity which in comparison to CC might be cheaper over the entire time horizon. However, if strategic behavior is uncertain, then it makes sense to also have a certain amount of CC capacity installed.

7.3 Large-Scale Numerical Example of Approximation Scheme

In this case study we apply the single-level approximation scheme of bilevel generation expansion equilibria, as proposed in section 5.4 of this thesis, to a large-scale numerical example. With the approximation scheme proposed in this thesis, we are able to solve large-scale bilevel models reasonably well when market behavior is closer to oligopoly than to perfect competition by smartly employing single-level models which reduces the computational time by two orders of magnitude. This is achieved by transforming the single-level equilibrium problem into an equivalent convex quadratic optimization problem which can be solved efficiently. This case study validates the proposed approximation scheme for large-scale models for a certain range of parameters. Moreover a sensibility analysis on model parameters is carried out. From the results it can be concluded that strategic spot market behavior is one of the main drivers of the quality of the approximation scheme.

In the following, we first describe the stylized electric power system that serves as our case study in section 7.3.1 and then in section 7.3.2 we present and analyze the results.

7.3.1 System Description for Approximation Scheme

We consider a system with four different thermal technologies them being nuclear (NU), coal (CO), combined cycle gas turbine (CC) and gas turbine (GT) that can be built by two generation companies i_1, i_2. We consider a time horizon of 15 years and each year is represented by six individual load periods, which are separated into peak, shoulder and off-peak of the weekdays (WD) and peak, shoulder and off-peak of weekends (WE). Demand is affine and defined by the demand slope α_{yl} which is equal to 0.23 GW/(€/MWh) based on [53] and the demand intercept D^0_{yl} [GW], which is defined in Table A.13. In Table 7.4 we present the durations T_{yl} [h] of each load level l in each year y. The assumed discount rate F is 9%.

Both generation companies consider capacity investments in each of the four available technologies. Table 7.5 shows both production and investment costs of each thermal technology. If not otherwise specified, the strategic spot market behavior θ, i.e., the conjectured-price response variation, is assumed to be $0.7/\alpha_{yl}$, which lies between perfect competition $\theta_{iyl} = 0$ and Cournot $\theta_{iyl} = 1/\alpha_{yl}$.

Table 7.4: Annual load level durations T_{yl} [h].

	WD Peak	**WD Shoulder**	**WD Off-peak**	**WE Peak**	**WE Shoulder**	**WE Off-peak**
T_{yl}	300	3000	3000	300	1080	1080

Table 7.5: Production and investment costs.

Technology	**Production cost** δ [€/MWh]	**Investment cost** β [(M€/GW)/year]
NU	8.0	230
CO	33.0	113
CC	39.0	57
GT	55.0	12

7.3.2 Results of Approximation Scheme

This section contains the results of the case study and in particular it contains the comparison between the bilevel equilibrium solution considering a strategic market behavior of $\theta_{iyl} = 0.7/\alpha_{yl}$ and its approximation under the previously presented scheme. In the approximation scheme we first solve the single-level equilibrium assuming Cournot market behavior, then fix investment decisions obtained under Cournot, and finally solve the single-level equilibrium again assuming the same market behavior as in the bilevel model. In the following we denote single-level by (SL), bilevel by (BL) and approximation by (AP).

All models have been formulated in GAMS and solved on an Intel(R) Core(TM) i5-2410M running at 2.30 GHz 4 GB RAM. The SL equilibrium model has been formulated as a quadratic program and solved using CPLEX. Since the AP scheme solves the SL model twice, it has also been solved using the solver CPLEX. The BL capacity equilibrium problem has been solved using diagonalization. Each individual MPEC has been solved using the non-linear solver CONOPT. The numerical model of the SL equilibrium consists of 930 primal continuous variables, while each MPEC consists of 2400 variables. The computational time of the AP scheme is 0.6 seconds whereas the BL model (with a convergence tolerance of 10^{-4}) takes 68 seconds. The AP scheme is more than two orders of magnitude faster than the BL model.

We present the demand yielded by the approximation scheme in Table A.14, and in Figure 7-5 we compare the investments yielded for one generation company by the BL model (dashed line), the SL model (dash-dot line) and by the AP (continuous line). The first subplot of Figure 7-5 contains investments in NU and the second corresponds to investments in CC. Neither CO nor GT plants are built and are hence omitted in the Figure. It can be observed that with respect to the BL solution, the AP yields a little more capacity in CC and a little less in NU. In particular, the relative errors of investments between the BL solution and the AP can be found in Table 7.6 and are calculated as $RE_{ijy} = (x_{ijy}^{BL} - x_{ijy}^{AP})/x_{ijy}^{AP}$. Since neither CO nor GT plants are built, the corresponding error is zero and is hence omitted in Table 7.6. The total average relative error (calculated as the sum of absolute value of relative errors for all technologies divided by the number of values, i.e., $\sum_{jy} |RE_{ijy}|/30$) in capacities is 2.26%. It can furthermore be observed that the SL solution is overly conservative and yields investments that are way beyond the obtained BL investments. The total average relative error between SL and BL solutions in capacities is 11.54% which is five times larger than the total relative error obtained with the AP scheme.

Table 7.6: Relative errors of investments [%].

Year	**NU**	**CC**
y_1	1.14	-6.11
y_2	1.02	-5.88
y_3	1.00	-5.68
y_4	0.88	-5.22
y_5	0.77	-5.04
y_6	0.67	-4.85
y_7	0.66	-4.72
y_8	0.56	-4.59
y_9	0.55	-4.25
y_{10}	0.38	-4.12
y_{11}	0.30	-3.99
y_{12}	0.29	-3.68
y_{13}	0.22	-0.56
y_{14}	0.14	-0.37
y_{15}	0.07	-0.18

Figure 7-6 contains the market prices of the AP and the BL model, separated into weekdays and weekends. Relative errors between the BL and the AP solution are presented in Table 7.7. We can see that BL peak prices are slightly above the AP prices during peak and shoulder periods on weekdays and during the peak on weekends. This is most likely due to the fact that in the BL solution less peak capacity, i.e., CC, is built. In the off-peak period on weekdays BL prices are slightly below AP prices, which is due to the fact that in the BL solution more NU capacity is available. We furthermore observe that prices coincide exactly for the shoulder and off-peak periods on weekends. Relative errors in prices are quite small and never exceed 1.61%, while the total average relative error is only 0.52%. Since the SL model yields investments that are very different from the BL solution, as can be seen

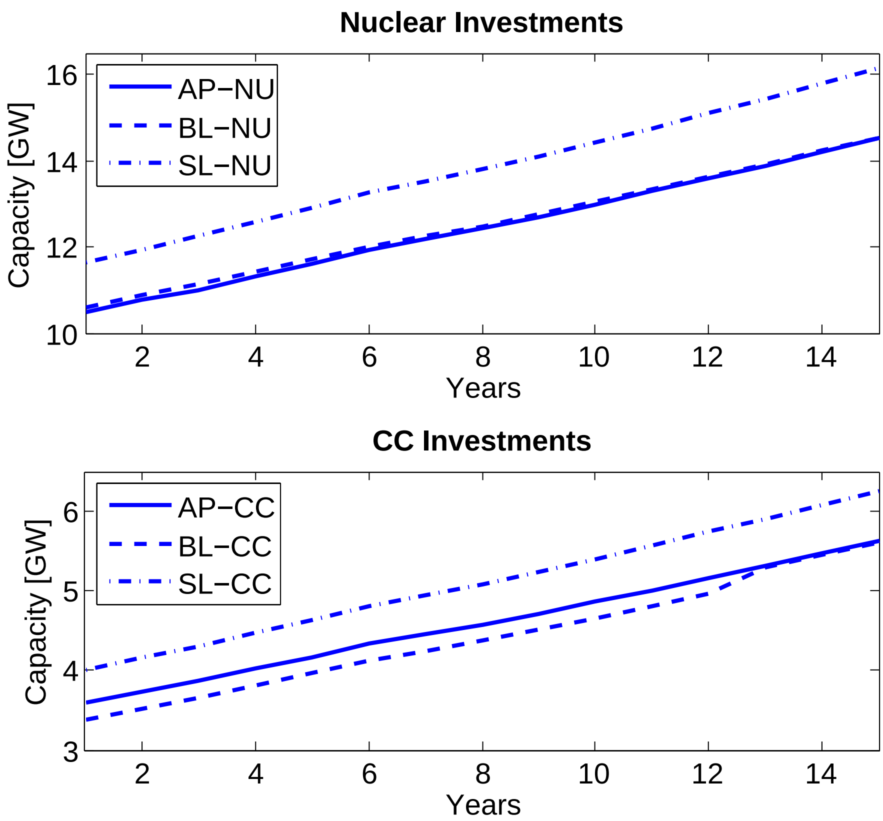

Figure 7-5: Capacity investment results of one firm in CC and NU technologies obtained by the single-level model (SL), the bilevel model (BL) and its approximation (AP).

in Figure 7-5, it is not surprising that also the prices given by the SL model are quite different from the BL solution. As a comparison, the total average relative error between the SL and the BL prices is 8.38%. In summary, the approximation only yields a 2.26% error in capacity investments and a 0.52% error in market prices which is acceptable.

We carry out a study to see what parameters of the model most affect the quality of the AP and hence we present total average relative errors between BL and AP results in prices and investments, as defined above, in Table 7.8 for different cases. The first case is the base case and contains the previously mentioned total average relative errors. In the subsequent cases we change one parameter with respect to the base case. The parameter change is pointed out in the first column of Table 7.8. Overall, we observe that the AP prices are very accurate, considering that total

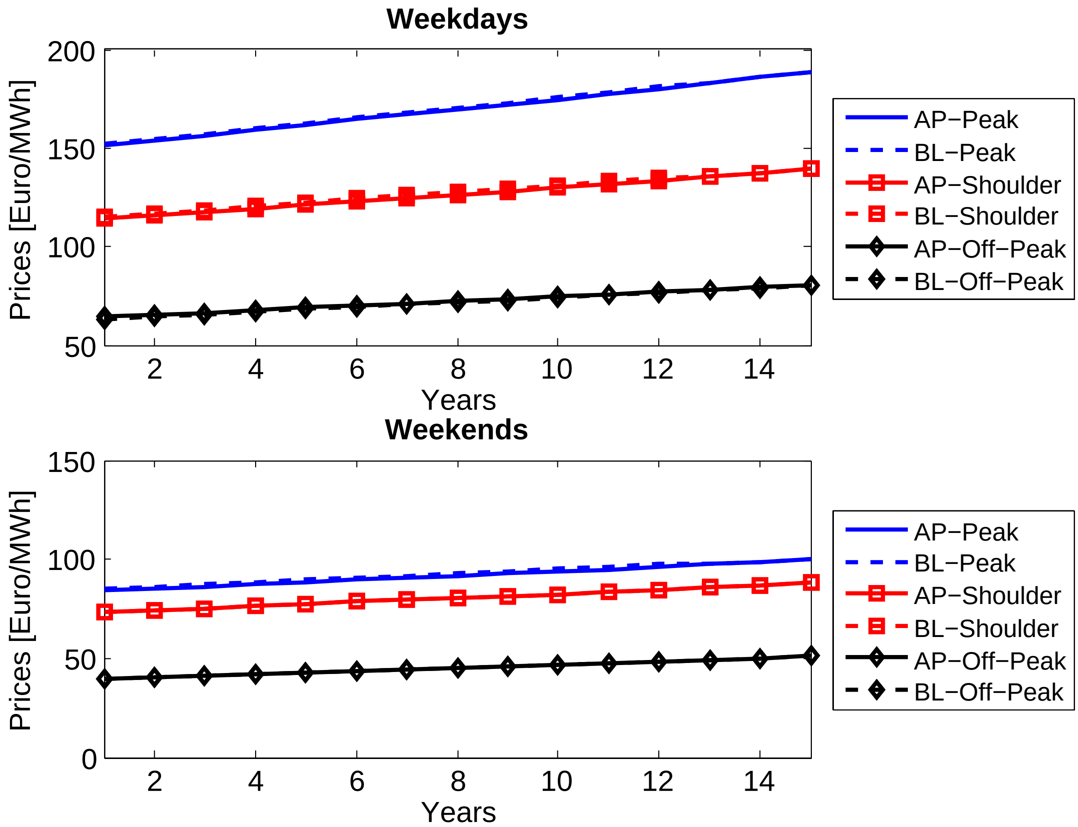

Figure 7-6: Market prices of the bilevel model (BL) and its approximation (AP) during weekdays and weekends.

average relative errors never exceed 1.61%. Relative errors in capacity are higher than relative errors in prices, however, still relatively small.

Comparing the results to the base case, we observe that the demand intercept D^0 and demand slope α do not seem to be driving factors of the quality of AP as relative errors remain more or less the same. Production costs seem to have a considerable impact on the solution. Changes of the NU production cost can lead to an average relative error of 5.01%, however, this seems still quite reasonable when taking into account the considerable decrease in computational time of the AP scheme. In order to assess the impact of the strategic spot market behavior θ on the results, we carry out a more detailed analysis.

In order to fully capture the impact of the strategic market behavior on the quality of AP results, we calculate the total average relative error in investments and prices

Table 7.7: Relative errors of prices [%].

Year	**WD Peak**	**WD Shoulder**	**WD Off-peak**	**WE Peak**	**WE Shoulder**	**WE Off-peak**
y_1	0.59	0.78	-1.61	1.04	0.00	0.00
y_2	0.60	0.79	-1.51	1.09	0.00	0.00
y_3	0.62	0.83	-1.38	1.12	0.00	0.00
y_4	0.64	0.85	-1.27	1.15	0.00	0.00
y_5	0.66	0.87	-1.13	1.18	0.00	0.00
y_6	0.67	0.89	-1.02	1.22	0.00	0.00
y_7	0.68	0.91	-0.93	1.26	0.00	0.00
y_8	0.70	0.93	-0.83	1.29	0.00	0.00
y_9	0.70	0.95	-0.74	1.32	0.00	0.00
y_{10}	0.72	0.97	-0.64	1.35	0.00	0.00
y_{11}	0.74	0.99	-0.54	1.38	0.00	0.00
y_{12}	0.76	1.01	-0.44	1.39	0.00	0.00
y_{13}	0.00	0.00	-0.35	0.00	0.00	0.00
y_{14}	0.00	0.00	-0.25	0.00	0.00	0.00
y_{15}	0.00	0.00	-0.15	0.00	0.00	0.00

between the AP and the BL solution considering a large variety of strategic spot market behavior ranging from perfect competition ($\theta = 0$) to Cournot behavior ($\theta = 1/\alpha$). Even though it is not very likely that in a market with only two players the strategic behavior will be perfect competition or even close to perfect competition, the results assuming perfectly competitive behavior are included in this case study for completeness and to validate when the approximation scheme is most accurate. In order to benchmark these results we also calculate the total average relative error between the SL and the BL solution. The obtained results of the total average relative errors are presented in Figure 7-7 on a normalized scale of strategic spot market behavior ranging from perfect competition ($\Phi = 0$) to Cournot ($\Phi = 1$).

The difference between the AP and the SL solution is that when deciding capacities the approximation considers Cournot behavior, while the SL solution considers the same θ as the BL solution. In the first subplot of Figure 7-7 we present the total average relative error of the AP and the SL solution in investments and in the second subplot the total average relative error in prices. In general it can be said that the closer θ is to Cournot behavior, the more accurate the AP and the SL equilibrium,

Table 7.8: Total average relative errors [%].

Case		**Average relative error in prices**	**Average relative error in investments**
Base		**0.52**	**2.26**
D^0	$+5\%$	0.46	2.82
	-5%	0.69	2.77
α	0.25	0.73	2.81
	0.21	0.31	1.22
θ	$1/\alpha$	0.00	0.00
	$0.75/\alpha$	0.14	1.10
	$0.65/\alpha$	1.12	2.62
	$0.6/\alpha$	0.97	4.16
δ_{NU}	8.5	1.58	5.01
	7.5	0.60	3.18
δ_{CC}	41	0.71	4.29
	37	0.70	3.81
β_{NU}	240	0.93	1.92
	220	0.66	3.41
β_{CC}	60	0.64	2.89
	55	0.46	1.65

both in investments and prices. If strategic spot market behavior is close to perfect competition, then the AP error in investments can become up to 46% which means that the investment decisions are not accurate. However, in the range between $\Phi = 0.4$ and Cournot, i.e., ($\Phi = 1$), the AP in capacity investments is quite good and the error does not exceed 10%. As for the AP error in prices, between $\Phi = 0.2$ and Cournot the relative error is less than 10% and even very close to perfect competition the error in prices is relatively small and only goes up to 29%. On the other hand, the SL error in prices skyrockets up to 175%. Finally, it should be observed that even though the AP might not be very accurate in capacity investments close to perfect competition, it still outperforms the SL solution. The error yielded by the AP is always below the error given by the SL solution for prices and in the majority of cases also for investments. We observe that the AP quite accurately reproduces BL prices when market behavior is not close to perfect competition while the SL equilibrium fails to capture BL prices when θ is not very close to Cournot behavior.

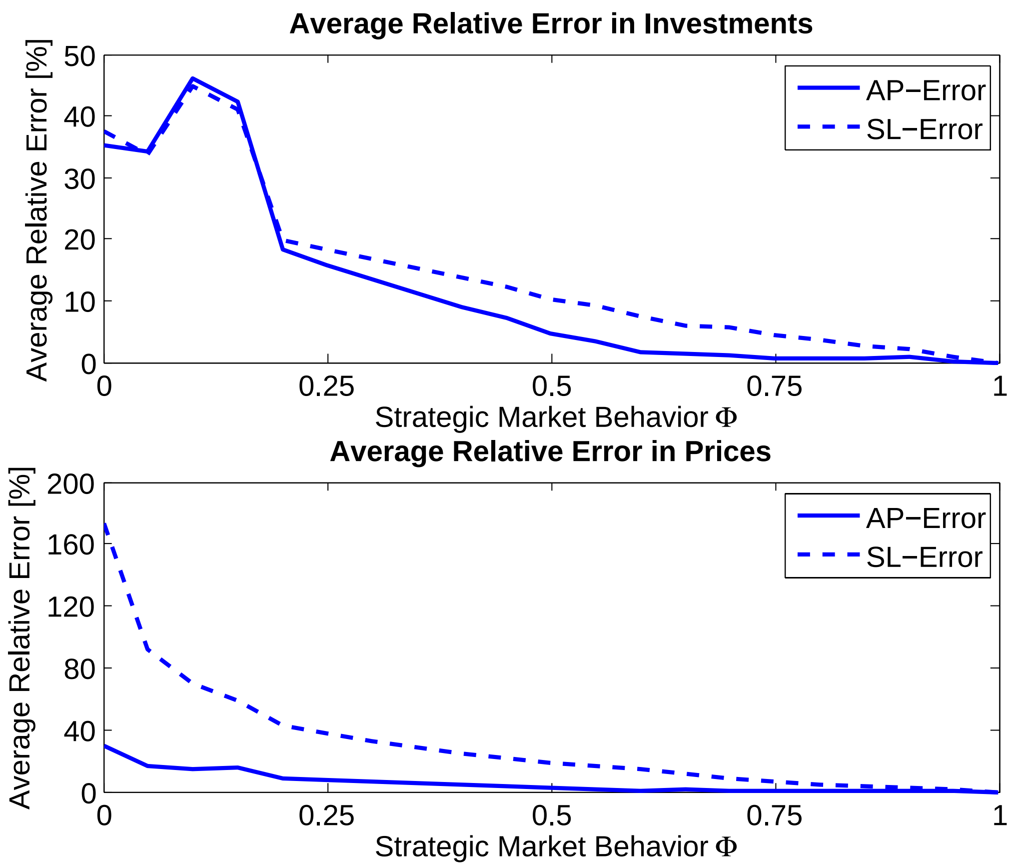

Figure 7-7: Total average relative error in investments and prices for θ ranging from perfect competition to Cournot behavior.

7.4 Conclusions of Case Studies

In this chapter we carry out three case studies. The first numerical example presented in section 7.1 is a large-scale stochastic MPEC designed to resemble the Spanish power system. In this model the possible model extensions of introducing uncertainty in demand, capacity payments, financial hedging and hydro power are considered, as motivated previously in section 4.4. The second case study in section 7.2 explores the possibility of considering discretized investment decisions in an MPEC framework, as proposed in section 4.4.2. Finally, section 7.3 contains a case study in which the single-level approximation scheme of bilevel generation expansion equilibria is applied to a large-scale, multi-year, multi-technology, multi-load-period numerical example. Carrying out a sensitivity analysis on model parameters yields that the quality of the approximation scheme greatly depends on the choice of strategic spot market behavior. In particular, it can be concluded that the approximation scheme works well when strategic behavior is closer to oligopoly than to perfect competition. In the presented numerical example it is validated that the approximation scheme yields a solution in computational times two orders of magnitude smaller than the computational times of standard bilevel problem techniques such as diagonalization. This makes the approximation scheme a suitable approach for real-size problems provided that strategic spot market behavior is not close to perfect competition.

This result provides an answer to one of the main research questions of this thesis, namely whether and how single-level models can approximate bilevel generation expansion models accurately. The case study presented in section 7.3 of this chapter not only affirms that large-scale bilevel models can be approximated efficiently by single-level models, but it also provides a precise characterization of when this approximation works and when it does not. In particular, by observing the arising error between the approximation and the real bilevel solution, it can be concluded that when the strategic spot market behavior is closer to Cournot than to perfect competition, then the approximation works well and can be computed two orders of magnitude faster than the bilevel solution.

In Table 7.9 we provide a summary and comparison of all the case studies carried out in this thesis, the corresponding models, the problem type that has been solved, model size, solution method and CPU time. Note that model size is defined by stating the maximum number of each index, i.e., the number of firms considered i, the technologies j, the years of the time horizon y, the load periods per year l and, if stated, the scenarios s.

Table 7.9: Comparison of case studies of this thesis.

Model[1]	**Problem Type**	**Case Study Objective**	**Case Study Section**	**Size**[2] i	j	y	l	s	**Solution Method**	**CPU Time**
BBEM	EPEC	Theoretical Analysis	3.3.4	2	1	1	2		Analytical[3] (Diagonalization)	-
BBEM	EPEC	Theoretical Analysis	3.4.2	2	1	1	20		Analytical[3] (Diagonalization)	-
BOM	MPEC	Study Impact of θ on Investments	4.2.4	3	4	10	6		NLP	6.4 s
SBOM	MPEC	Study Stochasticity of Competitors' Investments	4.3.4	3	2	5	2	3	NLP MILP	0.8 s 11.5 h - 20 h
BEM	EPEC	Classify Multiple Equilibria	5.2.4	2	2	2	2		MILP	10 h - 24 h
BEM	EPEC	Introduce Approximation	5.4.3	2	1	1	6		Diagonalization[4] Approximation	6.5 s - 144 s 0.5 s
SBOM	MPEC	Large-Scale Example of Spanish System	7.1	6	4	20	12	2	NLP	7 min
SBOM	MPEC	Discrete Capacity Decisions	7.2	3	2	5	2	3	NLP MILP	0.8 s 20 h
BEM	EPEC	Large-Scale Validation of Approximation	7.3	2	4	15	6		Approximation Diagonalization[5]	0.6 s 68 s

[1] The models in this table correspond to: Basic Bilevel Equilibrium Model (BBEM), Bilevel Optimization Model (BOM), Stochastic Bilevel Optimization Model (SBOM), Bilevel Equilibrium Model (BEM).

[2] The indices in this table are: i for GENCOs, j for technologies, y for years, l for load periods per year and s for scenarios.

[3] Since these models can be solved analytically no CPU time has been included in the table.

[4] The CPU time of diagonalization depends on the initial solution. For a random initial solution this case study took 144s. When initialized with the approximation solution, the CPU time of diagonalization could be reduced to 6.5s.

[5] This corresponds to the CPU time of diagonalization when initialized with the approximation solution.

Chapter 8

Conclusions

This chapter summarizes the work presented in this dissertation and points out its main conclusions in section 8.1. Then, in section 8.2 the most relevant contributions of this work are stated and contrasted to the original thesis objectives. Finally, some possible lines of future research are mentioned in section 8.3.

8.1 Thesis Summary and Conclusions

This section contains a summary of the main results of this thesis and the most relevant conclusions that can be drawn from the presented research.

Ever since the liberalization of the electricity sector, generation expansion planning has moved away from a centralized planner and has become the responsibility of generation companies, which has greatly increased the complexity of the investment decision process as stated in chapter 1 of this thesis. Therefore generation companies require the most adequate tools to assist them to take and evaluate generation expansion decisions while coping with an uncertain and highly competitive environment. Bilevel models allow generation companies to represent a sequential decision making process where first capacity decisions are taken and then there is the market clearing, as opposed to simplified approaches which assume simultaneous decision making using single-level models where both capacity and production decisions are taken at the same time. This thesis aims at proposing such bilevel models to tackle the generation expansion planning problem in liberalized electricity markets, to point out the difference and advantages that such an approach has with respect to single-level models, and to solve realistic instances of such bilevel models satisfactorily, which can be challenging since bilevel models are known to be complicated.

In chapter 2 we first present a literature review of generation expansion planning in liberalized electricity markets and of computational methods used in bilevel programming in order to emphasize how our work differs from the existing literature. Then we introduce the hypotheses made in our models and discuss the basic concepts necessary for a better understanding of the thesis. Among other topics, we cover the concepts of conjectural variations and bilevel programming. Everything discussed until this point provides the background necessary to understand the single-level models which are introduced subsequently, i.e., a single-level conjectured-price response market equilibrium model and its simple extension to a single-level investment equilibrium model. Once these single-level models have been established, we introduce a basic version of the newly developed bilevel models that are proposed in this thesis. Among bilevel models we distinguish between a bilevel optimization model that represents the investment problem of one generation company which is formulated as an MPEC, and a bilevel equilibrium model representing the generation capacity investment problem of all generation companies in the market which is formulated as an EPEC. The MPEC model can be seen as a crucial intermediate step on the way to formulate an EPEC. Comparing the previously mentioned single-level models to the newly introduced bilevel models it becomes obvious that the bilevel models are a lot more complicated - in terms of formulation and as we will see later on also in terms of solution techniques - than the single-level models, which immediately raises the question whether this additional modeling effort actually pays off. In particular, it raises the question whether single-level and bilevel generation expansion models sometimes yield the same solutions and if they do, under what circumstances, and moreover, it raises the question whether the more complicated bilevel model provides us with additional valuable information which justifies the additional modeling effort.

Chapter 3 provides an answer to the questions raised by chapter 2 by carrying out a theoretical analysis of a single-level and a bilevel conjectured-price response investment equilibrium model for a time horizon of one year. The single-level model describes a game in which investment and operation decisions are made simultaneously, and in the bilevel equilibrium model investment and operation decisions are made sequentially. The purpose of this comparison is to emphasize that when resorting to easier, less complicated single-level models, instead of solving the more realistic but more complicated bilevel models, the results may differ greatly and to characterize when results are similar. Let us now point out the main conclusions of this analysis.

(a) This comparison shows that independent of the strategic spot market behavior, the bilevel model always yields Cournot capacities, which is proved in a theorem and a subsequent proposition. This result is an extension of the findings of Kreps and Scheinkman [74] which have shown that a bilevel equilibrium game where first companies compete in capacities and then in prices a la Bertrand yields Cournot capacities. Our results show that this is not only true for a Bertrand second stage, but also for any second-stage behavior between perfect competition and Cournot behavior. These findings underline that bilevel models yield more realistic results than single-level models since they capture the effect that generation companies know that even if the market is perfectly competitive they can influence the market outcomes by building less capacity than needed in the market, which leads to a substantial increase in prices which furthermore makes profits go up as well. The capacity solution of the single-level model on the other hand depends on the strategic behavior and therefore tends to unrealistic over-investment when compared to the bilevel results. Thus the bilevel model could be useful to evaluate the effect of alternative market designs for mitigating market power in spot markets and incenting capacity investments in the long run.

(b) Another conclusion we obtain is that under certain circumstances the single-level and the bilevel model indeed yield the same result, and in particular, this happens when both models consider Cournot market behavior and when their active sets of load periods coincide. However, the further away the strategic behavior is from Cournot the more the results between single-level and bilevel model differ. This indicates that the further away market behavior moves from Cournot competition, the more the additional effort of computing the bilevel model (as opposed to the simpler single-level model) pay off because the outcomes are very different. On the other hand, when market behavior is close to Cournot then single-level and bilevel results are either exactly the same or very similar depending on the data.

(c) In addition to this theoretical analysis, we also prove by counter-example that depending on the choice of parameters, more competition in the spot market may lead to less market efficiency and less consumer surplus in the bilevel model. This surprising, counter-intuitive result implies that contrary to the common belief that marginal cost bidding protects consumers - a belief underlying some regulatory market rules - it can actually be harmful and it may lead to situations in which both consumers and generation companies are worse off.

Since chapter 3 has established that bilevel generation expansion models are more realistic than single-level models, we now want to extend the theoretically-sized models from chapter 3 to more realistic, large-scale, multi-year, multi-load period and multi-technology models. In order to model large-scale bilevel equilibrium models formulated as EPECs, the first step is to formulate the corresponding MPECs. Therefore chapter 4 is dedicated entirely to the bilevel model representing the generation expansion problem of one generation company, which is formulated as an MPEC. First of all, the basic version of this model is introduced and a case study is presented which shows how the conjectured-price response market representation allows to study how strategic behavior impacts capacity decisions. Then, we extend this model to incorporate stochasticity in order to handle certain sources of uncertainty that are common to the generation expansion problem and thereby improve the decision making process. Finally, we show how to introduce desirable model extensions into the MPEC. Let us now point out the main conclusions of this chapter.

(a) From numerical examples it can be verified that indeed the strategic spot market behavior has a great impact on investment decisions, because it greatly affects the capacity solution in terms of total capacity and moreover depending on the assumed strategic spot market behavior, the optimal technology mix also changes. Thus, it can be concluded that when taking generation expansion decisions, the assumed strategic market behavior has to be studied carefully since it can change the optimal investment decisions for the generation company in question.

(b) The proposed MPEC models are nonlinear, non-convex optimization problems and therefore straight-forward NLP methods can only guarantee a local solution of a problem that has multiple equilibria. On the other hand, these methods allow for large-scale problem instances to be solved at least locally.

(c) When transforming the MPECs into MILPs, then the global solution can be obtained, however, at the cost of only being able to solve moderately-sized problems.

In chapter 5 the focus lies on the bilevel equilibrium generation expansion model formulated as an EPEC. The generation expansion EPEC model consists of each generation company's MPEC model, which has been discussed in great detail in chapter 4 and which provides the theoretical basis for the EPEC models further analyzed in chapter 5. This type of model takes into account all market agents and yields a capacity plan for each of them, which makes it useful from the point of view of a regulator for example. First, the EPEC formulation of the bilevel generation expansion model is derived and then a case study is presented which underlines how difficult it can be to solve EPECs. Then, an alternative formulation of the single-level generation expansion model is presented which allows solving this model very efficiently. Finally, an approximation scheme for the EPEC is presented which only makes use of single-level models. In a small case study it is derived that the approximation scheme works well when strategic behavior is not close to perfect competition. The main conclusions of this chapter are as follows:

(a) From the first numerical example it becomes apparent that even for a small problem instance, the generation expansion EPEC can have multiple equilibria, each yielding a different optimal technology mix.

(b) Since there exist multiple equilibrium solutions for the EPEC, it is desirable to be able to classify them in order to explore the solution space of the EPEC. To this purpose a MIP methodology is presented.

(c) The advantage of the MIP approach for solving EPECs is that it allows us to classify equilibria, but so far only small problem cases could be solved. Diagonization on the other hand can handle large-scale problem sizes, however, there is no a priori way of knowing in which of the multiple equilibria we are going to end up at.

(d) Another alternative to solve EPECs is the proposed approximation scheme which works very well for problems with strategic spot market behavior close to Cournot and yields a solution in computational time two orders of magnitude faster than standard EPEC techniques such as diagonalization. This algorithm is therefore very promising for realistically-sized problems.

Even though chapter 5 concludes the methodological contributions of this thesis, it still remains to discuss the numerical solution techniques of MPECs and EPECs that have been employed in several numerical examples throughout the thesis. Therefore, chapter 6 summarizes all the numerical techniques that have been applied when solving the MPEC and EPEC models of this thesis representing the generation expansion problem. We individually discuss and analyze methods for both MPECs and EPECs and point out advantages and disadvantages. It is emphasized here that the following conclusions only refer to the generation expansion models of this thesis.

(a) When facing a large-scale bilevel generation expansion optimization model formulated as an MPEC, then NLP methods are the most successful even though they cannot guarantee globality of the solution. To that purpose many different starting points are employed to search the solution space for different local optima. If on the other hand, the globality of the solution is the main focus, then MIP methods are more adequate, however, so far we have only been able to solve moderately-sized problems. In order to still represent a long-term time horizon one could consider clusters of years instead of individual years.

(b) When solving EPECs and the main focus is to study the solution space of the EPEC, then MIP methods provide a framework to search the solution space, however, one has to keep in mind the limitation of this method in terms of problem size. On the other hand, if the emphasis lies on solving a large-scale EPEC then methods like diagonalization or the newly proposed approximation scheme work best.

In chapter 7 some additional case studies of interest are presented: a realistic case study representing the Spanish power system considering demand uncertainty, hydro power and other realistic market details is considered; a stochastic MIP considering discrete investment decisions is also presented; finally, the previously presented approximation scheme of chapter 5 is applied to a large-scale model. The following conclusions can be obtained from these case studies:

(a) The proposed model extensions of section 4.4, as discrete capacity decisions, hydro power and demand uncertainty, can be successfully introduced in our generation expansion models.

(b) The single-level approximation scheme of bilevel generation expansion equilibria also works well for large-scale, multi-year, multi-technology and multi-load period cases as long as strategic spot market behavior is closer to Cournot than to perfect competition. This means that under certain circumstances the bilevel generation expansion model actually can be approximated by simplified single-level models.

8.2 Original Contributions

In this section the original contributions of this thesis are pointed out. As follows, first we revisit the methodological contributions and then the computational contributions of this thesis and finally, we mention the contributions in terms of journal articles that are directly related to this dissertation.

Methodological Contributions

Let us now revisit the specific methodological thesis contributions and explain what exactly has been done to cover the corresponding methodological thesis objectives given in section 1.2.2 :

1. One of the key contributions of this thesis is the theoretical comparison of single-level and bilevel generation expansion equilibrium models carried out in chapter 3 which has quantified the difference between model outcomes thereby establishing the impact that a simultaneous modeling approach (single-level model) has on investment decisions compared to a sequential decision making process (bilevel model). The results have been proven in a theorem and a proposition, which furthermore show that single-level and bilevel models sometimes coincide, and in particular that this happens when Cournot market behavior is considered. This work, which has also been accepted for publication in Mathematical Programming [119], more than fulfills the second methodological thesis objective, which raises the question how the results of a bilevel generation expansion model differ compared to a more simplified single-level model. We furthermore show by counter-example that in bilevel models more competition can lead to less market efficiency and less consumer surplus which implies that contrary to the common belief that marginal cost bidding protects consumers it can actually be harmful.

2. Since we have proven that bilevel models are more realistic than single-level models, we propose and formulate two novel bilevel generation expansion models: the first model assists one generation company in particular to take capacity

investment decisions and is modeled as an MPEC; the second model takes into account all generation companies in the market and is formulated as an EPEC. Chapter 4 is dedicated entirely to the detailed formulation of the MPEC model which is an intermediate step necessary to formulate the EPEC. Moreover, this type of model also provides valuable insight for a generation company since it focuses only on one market agent. The introduction of stochasticity to this modeling framework allows one generation company to assess optimal capacity decisions under uncertainty, which provides a useful modeling tool for generation companies. We propose and analyze the EPEC in chapter 5 which decides capacity investments of all agents and takes a more general point of view. This type of model can be very useful for the regulator in order to assess the impact of new regulatory measures on the market and its agents. The proposed models cover the second thesis objective, which proposed the development of bilevel generation expansion models assisting either one (MPEC) or all (EPEC) generation companies.

3. The proposed MPEC and EPEC models have been extended and made more realistic. In particular, section 4.4 contains the detailed derivation of the desired model extensions for incorporating hydro power, discrete capacity decisions, stochastic treatment of uncertain parameters as demand or competitors' investments, the introduction of capacity mechanisms and other realistic details like financial hedging parameters. Throughout the thesis there have been carried out several case studies to evaluate the proposed models, thereby fulfilling the third methodological thesis objective in which specific model extensions are requested.

Computational Contributions

Let us now revisit the computational contributions and compare them to the computational objectives of this thesis given in section 1.2.3:

1. An alternative formulation of the single-level generation expansion equilibrium

model as a convex quadratic optimization problem is proposed in section 5.3, published in [117], which allows us to solve this problem very efficiently. This model is a straight-forward extension of works of Barquín [5] and Ventosa [113] to consider investment decisions in a conjectured-price response market formulation. This contribution corresponds to the first computational objective.

2. A key contribution of this thesis is the single-level approximation scheme for large-scale bilevel generation expansion equilibria which is proposed in section 5.4 of this thesis. Due to the fact that this approximation scheme only solves the alternative formulation of the single-level model, the computational time of this method is by two orders of magnitude faster than a standard EPEC technique like diagonalization. The large-scale numerical case study provided in section 7.3 confirms that this approximation scheme works well when strategic spot market behavior is closer to Cournot than to perfect competition. Both the approximation scheme and the numerical case study have been published in an international journal [117]. This work covers the second computational objective which requested a methodology to approximate generation expansion EPECs.

3. In chapter 6 we present, analyze and compare the solution techniques and numerical methods that have been applied to solve the bilevel generation expansion models of this thesis. Pros and cons of each method are discussed. This chapter addresses the third computational objective in which an exploration and comparison of different existing solution techniques is requested.

Publications

The following five papers are directly related to this dissertation and have been published or accepted in relevant SCI-indexed international journals. Note that appendix B contains a list of all conference presentations.

- S. Wogrin, B. F. Hobbs, D. Ralph, E. Centeno, and J. Barquín. Open versus closed loop capacity equilibria in electricity markets under perfect and oligopolistic competition. *Mathematical Programming (Series B)*, 2012, *accepted for publication.* (Results of reference [119] appear in chapter 3.)

- E. Centeno, S. Wogrin, A. López-Peña, and M. Vázquez. Analysis of investments in generation capacity: A bilevel approach. *Generation, Transmission Distribution*, *IET*, 5(8):842-849, 2011. (Results of reference [27] appear in chapter 4.)

- S. Wogrin, E. Centeno, and J. Barquín. Generation capacity expansion in liberalized electricity markets: A stochastic MPEC approach. *IEEE Transactions on Power Systems*, 24(4):2526-2532, 2011. (Results of reference [116] appear in chapter 4.)

- S. Wogrin, J. Barquín, and E. Centeno. Capacity expansion equilibria in liberalized electricity markets: An EPEC approach. *IEEE Transactions on Power Systems*, 28(2):1531-1539, 2013. (Results of reference [115] appear in chapter 5.)

- S. Wogrin, E. Centeno, and J. Barquín. Generation capacity expansion analysis: Open loop approximation of closed loop equilibria. *IEEE Transactions on Power Systems*, PP(99):1, 2013. (Results of reference [117] appear in chapters 5 and 7.)

8.3 Future Research

To conclude this thesis we point out some interesting topics for future research which have arisen throughout this document. Let us divide these lines of future work in four different topics: improvement of the formulation of current bilevel generation expansion models; theoretical analysis; computational improvements and development of new generation expansion models.

- Improvement of the formulation of current bilevel generation expansion models

- In the models proposed in this thesis the generation companies are considered risk-neutral, since they decide investments maximizing their net present value. Risk measures could be introduced in order to capture for example risk aversion of generation companies.
- In this thesis we have considered the formulation of capacity payments, however, it might be interesting to consider other capacity mechanisms as reliability options (as in Batlle et al. [9]) or capacity markets.
- A CO_2 emissions market could be incorporated into the generation expansion models.
- The representation of pump storage facilities or other storage devices in the models might be considered.
- The electricity network could also be introduced into the presented model formulations.
- Model extensions could also consider the effect of forward energy contracting (as in Murphy and Smeers [85]).

• Theoretical analysis

- In future research we may address the issue of existence and uniqueness of bilevel conjectured-price response models, as has been done for the Cournot case by Murphy and Smeers [84], who found that a pure-strategy bilevel equilibrium does not necessarily exist but if it exists it is unique.
- We may also address the question concerning under what *a priori* conditions the active sets of single-level and bilevel generation expansion equilibria coincide.
- Extend Lemma 3.3 to the multiple load period case, which would prove the hypothesis that for symmetric agents the conjectured-price response bilevel equilibrium solution never yields more capacity than the Cournot bilevel equilibrium solution.

- Computational improvements
  - Extend the standard Benders decomposition methods to tackle the non-convex MPEC problem with binary variables in the subproblem, or apply other decomposition techniques to the stochastic generation expansion MPEC models of this thesis.
- Development of new generation expansion models
  - There may be further investigation of games in which the conjectural variation is endogenous, resulting from the possibility that power producers might adopt the Cournot conjecture in binding load periods since they may be aware that their rivals cannot expand output at such times. The conjectural variation could be calculated as a function of the capacity mix.
  - In a system with a high penetration of renewable energy sources, other more technical details of the market (as for example start-ups, shut-downs, secondary reserve requirements, minimum power constraints and ramping constraints) become more important drivers of the investment decision. Introducing these types of constraints into generation expansion models poses an interesting task for future research. We have taken a first step in this direction in the conference paper by Nogales et al. [88].

Appendix A

Additional Tables

Table A.1: Total annual hydro energy E_{iy} [TWh] of firm i in section 7.1.

i_1	i_2	i_3	i_4	i_5	i_6
6.0	13.0	4.0	0.5	1.0	2.0

Table A.2: Maximum hydro output H_{iyl} [GW] of firm i, year y and load period l in section 7.1.

	l_1	l_2	l_3	l_4	l_5	l_6	l_7	l_8	l_9	l_{10}	l_{11}	l_{12}
i_1	1.50	1.50	1.31	1.31	0.87	0.87	0.00	0.00	0.28	0.28	0.00	0.00
i_2	3.25	3.25	2.85	2.85	1.90	1.90	0.00	0.00	0.60	0.60	0.00	0.00
i_3	0.98	0.98	0.86	0.86	0.57	0.57	0.00	0.00	0.18	0.18	0.00	0.00
i_4	0.20	0.20	0.17	0.17	0.11	0.11	0.00	0.00	0.04	0.04	0.00	0.00
i_5	0.20	0.20	0.17	0.17	0.11	0.11	0.00	0.00	0.04	0.04	0.00	0.00
i_6	0.46	0.46	0.40	0.40	0.27	0.27	0.00	0.00	0.08	0.08	0.00	0.00

Table A.3: Annual duration [h] of load period l used in section 7.1.

l_1	l_2	l_3	l_4	l_5	l_6	l_7	l_8	l_9	l_{10}	l_{11}	l_{12}
60	180	560	960	1190	1350	1190	1190	1120	680	220	60

Table A.4: Existing net nuclear generation capacity [GW] of firm i and year y.

	i_1	i_2	i_3	i_4	i_5	i_6
y_1	2.77	2.57	0.46	0.12	0.00	0.00
y_2	2.77	2.57	0.46	0.12	0.00	0.00
y_3	2.77	2.57	0.46	0.12	0.00	0.00
y_4	2.77	2.57	0.46	0.12	0.00	0.00
y_5	2.77	2.57	0.46	0.12	0.00	0.00
y_6	2.77	2.57	0.46	0.12	0.00	0.00
y_7	2.77	2.57	0.46	0.12	0.00	0.00
y_8	2.60	2.40	0.46	0.12	0.00	0.00
y_9	2.60	2.40	0.46	0.12	0.00	0.00
y_{10}	2.60	2.40	0.46	0.12	0.00	0.00
y_{11}	2.60	2.40	0.46	0.12	0.00	0.00
y_{12}	2.60	2.40	0.46	0.12	0.00	0.00
y_{13}	2.60	2.40	0.46	0.12	0.00	0.00
y_{14}	2.60	2.40	0.46	0.12	0.00	0.00
y_{15}	2.60	2.40	0.46	0.12	0.00	0.00
y_{16}	2.60	2.40	0.46	0.12	0.00	0.00
y_{17}	2.60	2.40	0.46	0.12	0.00	0.00
y_{18}	2.60	2.40	0.46	0.12	0.00	0.00
y_{19}	2.60	2.40	0.46	0.12	0.00	0.00
y_{20}	2.31	1.97	0.37	0.12	0.00	0.00

Table A.5: Existing net coal generation capacity [GW] of firm i and year y.

	i_1	i_2	i_3	i_4	i_5	i_6
y_1	3.79	0.71	1.49	1.14	1.07	0.24
y_2	3.79	0.54	1.37	0.95	0.79	0.24
y_3	3.79	0.54	1.16	0.95	0.79	0.24
y_4	3.68	0.54	1.16	0.95	0.67	0.24
y_5	3.68	0.54	1.16	0.95	0.67	0.00
y_6	3.68	0.54	1.16	0.95	0.67	0.00
y_7	3.68	0.54	1.16	0.42	0.67	0.00
y_8	3.59	0.27	0.42	0.42	0.67	0.00
y_9	3.59	0.00	0.42	0.42	0.67	0.00
y_{10}	3.59	0.00	0.42	0.42	0.67	0.00
y_{11}	3.59	0.00	0.42	0.42	0.67	0.00
y_{12}	3.59	0.00	0.42	0.42	0.43	0.00
y_{13}	3.59	0.00	0.42	0.42	0.43	0.00
y_{14}	3.59	0.00	0.42	0.42	0.43	0.00
y_{15}	3.59	0.00	0.42	0.42	0.43	0.00
y_{16}	3.59	0.00	0.42	0.42	0.43	0.00
y_{17}	3.59	0.00	0.42	0.42	0.43	0.00
y_{18}	3.59	0.00	0.42	0.42	0.43	0.00
y_{19}	3.59	0.00	0.42	0.42	0.43	0.00
y_{20}	3.59	0.00	0.42	0.42	0.43	0.00

Table A.6: Existing net combined cycle generation capacity [GW] of firm i and year y.

	i_1	i_2	i_3	i_4	i_5	i_6
y_1	4.25	4.96	3.00	1.19	1.50	10.26
y_2	4.25	4.96	3.00	1.19	1.50	10.26
y_3	4.25	4.96	3.00	1.19	1.50	10.26
y_4	4.25	4.96	3.00	1.19	1.50	10.26
y_5	4.25	4.96	3.00	1.19	1.50	10.26
y_6	4.25	4.96	3.00	1.19	1.50	10.26
y_7	4.25	4.96	3.00	1.19	1.50	10.26
y_8	4.25	4.96	3.00	1.19	1.50	10.26
y_9	4.25	4.96	3.00	1.19	1.50	10.26
y_{10}	4.25	4.96	3.00	1.19	1.50	10.26
y_{11}	4.25	4.96	3.00	1.19	1.50	10.26
y_{12}	4.25	4.96	3.00	1.19	1.50	10.26
y_{13}	4.25	4.96	3.00	1.19	1.50	10.26
y_{14}	4.25	4.96	3.00	1.19	1.50	10.26
y_{15}	4.25	4.96	3.00	1.19	1.50	10.26
y_{16}	4.25	4.96	3.00	1.19	1.50	10.26
y_{17}	4.25	4.96	3.00	1.19	1.50	10.26
y_{18}	4.25	4.96	3.00	1.19	1.50	10.26
y_{19}	4.25	4.96	3.00	1.19	1.50	10.26
y_{20}	4.25	4.96	3.00	1.19	1.50	10.26

Table A.7: Conjectured price responses θ [(€/MWh)/GW] of firms i and load period l.

	l_1	l_2	l_3	l_4	l_5	l_6	l_7	l_8	l_9	l_{10}	l_{11}	l_{12}
i_1	3.5	2.0	1.5	1.0	1.0	1.0	1.0	1.0	1.0	1.0	0.5	0.5
i_2	4.0	2.0	1.5	1.5	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
i_3	3.5	1.5	0.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
i_4	3.0	1.0	0.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
i_5	3.5	1.0	0.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
i_6	0.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Table A.8: Total amount of contracts for differences M_{iyl} [GW] for each market agent i and year y in section 7.1.

	i_1	i_2	i_3	i_4	i_5	i_6
y_1	2.16	1.65	0.99	0.49	0.51	2.10
y_2	2.16	1.61	0.97	0.45	0.46	2.10
y_3	2.16	1.61	0.92	0.45	0.46	2.10
y_4	2.14	1.61	0.92	0.45	0.43	2.10
y_5	2.14	1.61	0.92	0.45	0.43	2.05
y_6	2.14	1.61	0.92	0.45	0.43	2.05
y_7	2.14	1.61	0.92	0.35	0.43	2.05
y_8	2.09	1.53	0.78	0.35	0.43	2.05
y_9	2.09	1.47	0.78	0.35	0.43	2.05
y_{10}	2.09	1.47	0.78	0.35	0.43	2.05
y_{11}	2.09	1.47	0.78	0.35	0.43	2.05
y_{12}	2.09	1.47	0.78	0.35	0.39	2.05
y_{13}	2.09	1.47	0.78	0.35	0.39	2.05
y_{14}	2.09	1.47	0.78	0.35	0.39	2.05
y_{15}	2.09	1.47	0.78	0.35	0.39	2.05
y_{16}	2.09	1.47	0.78	0.35	0.39	2.05
y_{17}	2.09	1.47	0.78	0.35	0.39	2.05
y_{18}	2.09	1.47	0.78	0.35	0.39	2.05
y_{19}	2.09	1.47	0.78	0.35	0.39	2.05
y_{20}	2.03	1.39	0.76	0.35	0.39	2.05

Table A.9: Demand intercept D^0 [GW] of load period l and year y used in high-demand scenario in section 7.1.

	l_1	l_2	l_3	l_4	l_5	l_6	l_7	l_8	l_9	l_{10}	l_{11}	l_{12}
y_1	62.72	59.54	56.23	52.61	48.92	45.20	41.68	38.23	33.92	28.72	23.33	17.49
y_2	62.66	59.48	56.16	52.54	48.82	45.09	41.58	38.12	33.82	28.60	23.18	17.33
y_3	64.08	60.85	57.47	53.78	50.02	46.23	42.65	39.11	34.74	29.44	23.93	17.98
y_4	65.75	62.44	59.02	55.26	51.45	47.59	43.94	40.35	35.90	30.53	24.95	18.92
y_5	67.43	64.07	60.57	56.74	52.86	48.94	45.20	41.53	36.99	31.51	25.82	19.66
y_6	68.95	65.50	61.91	58.00	54.00	49.99	46.17	42.41	37.77	32.14	26.31	20.00
y_7	70.51	66.96	63.25	59.22	55.11	50.96	47.03	43.18	38.38	32.58	26.57	20.06
y_8	72.02	68.39	64.58	60.41	56.16	51.90	47.87	43.89	38.98	32.99	26.79	20.07
y_9	73.78	70.02	66.09	61.79	57.39	53.00	48.85	44.76	39.67	33.49	27.08	20.14
y_{10}	75.15	71.31	67.30	62.88	58.36	53.85	49.60	45.42	40.22	33.88	27.28	20.16
y_{11}	76.77	72.84	68.71	64.17	59.51	54.89	50.54	46.24	40.88	34.35	27.57	20.24
y_{12}	78.35	74.30	70.04	65.33	60.50	55.72	51.24	46.78	41.26	34.49	27.45	19.87
y_{13}	80.14	75.94	71.52	66.60	61.54	56.56	51.90	47.26	41.51	34.42	27.06	19.16
y_{14}	81.55	77.23	72.67	67.56	62.32	57.17	52.38	47.56	41.63	34.27	26.64	18.46
y_{15}	83.20	78.74	74.03	68.73	63.26	57.93	52.97	47.97	41.82	34.15	26.21	17.75
y_{16}	84.88	80.27	75.41	69.89	64.21	58.68	53.55	48.35	41.94	33.98	25.70	16.95
y_{17}	86.84	82.08	77.01	71.26	65.35	59.60	54.26	48.82	42.16	33.85	25.23	16.13
y_{18}	88.35	83.47	78.25	72.32	66.18	60.27	54.74	49.11	42.21	33.59	24.67	15.32
y_{19}	90.17	85.14	79.76	73.61	67.25	61.15	55.44	49.59	42.45	33.52	24.28	14.64
y_{20}	90.24	85.21	79.83	73.68	67.32	61.22	55.51	49.66	42.52	33.59	24.35	14.71

Table A.10: Demand intercept D^0 [GW] of load period l and year y used in low-demand scenario in section 7.1.

	l_1	l_2	l_3	l_4	l_5	l_6	l_7	l_8	l_9	l_{10}	l_{11}	l_{12}
y_1	55.86	53.03	50.09	46.86	43.58	40.26	37.13	34.04	30.21	25.58	20.78	15.59
y_2	56.13	53.28	50.31	47.07	43.76	40.42	37.26	34.16	30.32	25.65	20.82	15.59
y_3	56.40	53.54	50.55	47.28	43.95	40.59	37.41	34.29	30.42	25.73	20.87	15.60
y_4	56.70	53.82	50.82	47.54	44.19	40.83	37.64	34.50	30.62	25.92	21.03	15.75
y_5	56.97	54.06	51.05	47.75	44.37	40.98	37.77	34.62	30.71	25.96	21.05	15.72
y_6	57.20	54.27	51.21	47.85	44.43	41.00	37.76	34.56	30.60	25.79	20.79	15.38
y_7	57.39	54.42	51.30	47.88	44.37	40.88	37.58	34.34	30.30	25.37	20.25	14.73
y_8	57.59	54.57	51.41	47.90	44.31	40.74	37.41	34.10	29.99	24.96	19.73	14.09
y_9	57.78	54.72	51.50	47.91	44.22	40.59	37.20	33.83	29.64	24.48	19.13	13.37
y_{10}	57.99	54.89	51.61	47.96	44.21	40.52	37.08	33.63	29.39	24.12	18.66	12.79
y_{11}	58.20	55.05	51.74	48.00	44.18	40.41	36.92	33.42	29.09	23.70	18.13	12.17
y_{12}	58.36	55.16	51.77	47.93	43.98	40.14	36.56	32.94	28.50	22.95	17.22	11.12
y_{13}	58.49	55.20	51.72	47.75	43.64	39.67	35.99	32.24	27.63	21.87	15.90	9.65
y_{14}	58.64	55.28	51.72	47.61	43.40	39.33	35.52	31.64	26.88	20.92	14.77	8.37
y_{15}	58.77	55.35	51.69	47.46	43.10	38.92	34.98	30.95	26.03	19.88	13.53	7.01
y_{16}	58.91	55.40	51.63	47.27	42.77	38.46	34.39	30.20	25.10	18.74	12.20	5.58
y_{17}	59.04	55.43	51.54	47.03	42.38	37.94	33.72	29.34	24.03	17.46	10.74	4.16
y_{18}	59.24	55.50	51.48	46.83	42.05	37.50	33.14	28.61	23.12	16.38	9.56	3.08
y_{19}	59.49	55.58	51.38	46.59	41.69	37.02	32.52	27.84	22.19	15.29	8.42	2.19
y_{20}	59.49	55.58	51.38	46.59	41.69	37.02	32.52	27.84	22.19	15.29	8.42	2.19

Table A.11: Demand d [GW] of load period l and year y obtained in high-demand scenario in section 7.1.

	l_1	l_2	l_3	l_4	l_5	l_6	l_7	l_8	l_9	l_{10}	l_{11}	l_{12}
y_1	43.03	43.56	41.70	39.11	36.10	33.78	30.26	26.81	22.51	17.31	14.35	11.74
y_2	42.78	43.37	41.54	38.97	35.93	33.32	29.72	26.71	22.41	17.19	13.60	11.55
y_3	43.67	44.42	42.60	40.01	36.98	33.76	30.53	27.70	23.33	18.02	13.38	12.12
y_4	44.53	44.88	43.82	41.27	38.22	34.80	31.50	28.94	24.48	19.11	13.53	12.93
y_5	44.63	44.64	43.83	42.28	39.44	35.96	32.54	29.54	25.57	20.10	14.40	12.91
y_6	44.63	44.64	43.83	43.18	40.45	36.89	33.40	30.24	26.35	20.73	14.90	12.91
y_7	44.10	44.10	43.29	43.29	41.26	37.69	34.10	30.74	26.97	21.17	15.15	12.38
y_8	42.66	42.66	41.85	41.85	39.94	38.42	34.65	31.22	27.56	21.58	15.37	10.94
y_9	42.39	42.39	41.58	41.58	39.67	39.26	35.37	31.98	27.85	22.07	15.66	10.67
y_{10}	42.39	42.39	41.58	41.58	39.67	39.59	35.83	32.57	28.28	22.46	15.87	10.67
y_{11}	42.39	42.39	41.58	41.58	39.67	39.59	35.83	33.29	28.80	22.94	16.16	10.67
y_{12}	42.15	42.15	41.34	41.34	39.43	39.35	35.59	33.68	29.05	23.08	16.04	10.43
y_{13}	42.15	42.15	41.34	41.34	39.43	39.35	35.59	34.06	29.25	23.01	15.65	10.43
y_{14}	42.15	42.15	41.34	41.34	39.43	39.35	35.59	34.30	29.35	22.86	15.22	10.43
y_{15}	42.15	42.15	41.34	41.34	39.43	39.35	35.59	34.62	29.49	22.74	14.80	10.43
y_{16}	42.31	42.31	41.50	41.50	39.58	39.51	35.74	34.92	29.62	22.57	14.29	10.58
y_{17}	42.55	42.55	41.74	41.74	39.82	39.75	35.98	35.30	29.85	22.43	13.81	10.18
y_{18}	42.90	42.90	42.09	42.09	40.17	40.10	36.33	35.53	29.96	22.18	13.25	9.46
y_{19}	43.24	43.24	42.43	42.43	40.51	40.44	36.67	35.90	30.22	22.11	12.86	8.86
y_{20}	42.96	42.96	42.16	42.16	40.24	40.17	36.40	35.84	30.29	22.18	12.93	8.86

Table A.12: Demand d [GW] of load period l and year y obtained in low demand scenario in section 7.1.

	l_1	l_2	l_3	l_4	l_5	l_6	l_7	l_8	l_9	l_{10}	l_{11}	l_{12}
y_1	38.42	38.39	36.67	35.44	32.16	28.84	25.71	22.62	18.79	15.53	14.35	10.04
y_2	38.40	38.46	36.66	35.31	32.34	29.01	25.84	22.74	18.90	14.77	13.60	9.97
y_3	38.51	38.62	36.82	35.09	32.53	29.17	25.99	22.87	19.00	14.56	13.38	9.96
y_4	38.66	38.82	37.02	34.86	32.77	29.41	26.22	23.08	19.20	14.50	13.15	10.07
y_5	38.76	38.96	37.17	34.81	32.70	29.56	26.35	23.20	19.29	14.55	12.91	10.02
y_6	38.91	39.12	37.31	34.89	32.70	29.58	26.34	23.14	19.18	14.37	12.91	9.71
y_7	38.87	39.13	37.30	34.80	32.17	29.46	26.16	22.92	18.88	13.95	12.38	9.08
y_8	38.71	39.07	37.24	34.68	31.81	29.32	25.99	22.68	18.57	13.54	10.94	8.40
y_9	38.83	39.18	37.31	34.69	31.73	29.17	25.78	22.41	18.22	13.06	10.67	7.76
y_{10}	38.97	39.31	37.41	34.73	31.72	29.10	25.66	22.21	17.97	12.70	10.67	7.26
y_{11}	39.11	39.44	37.51	34.77	31.69	28.99	25.50	22.00	17.67	12.28	10.67	6.72
y_{12}	39.14	39.47	37.49	34.67	31.49	28.72	25.14	21.52	17.08	11.60	10.43	5.67
y_{13}	39.22	39.51	37.45	34.51	31.22	28.26	24.57	20.82	16.21	11.60	9.98	5.57
y_{14}	39.32	39.57	37.45	34.40	31.03	27.91	24.10	20.22	15.46	11.60	8.98	5.57
y_{15}	39.41	39.62	37.43	34.27	30.79	27.51	23.56	19.53	14.61	11.60	7.88	5.18
y_{16}	39.50	39.66	37.38	34.10	30.56	27.04	22.98	18.78	13.68	11.76	6.75	3.90
y_{17}	39.59	39.68	37.30	33.89	30.61	26.52	22.30	17.92	12.61	11.02	5.57	2.56
y_{18}	39.73	39.74	37.25	33.72	30.63	26.08	21.72	17.19	12.35	10.17	5.57	1.57
y_{19}	39.90	39.80	37.17	33.56	30.27	25.60	21.10	16.42	12.68	9.30	5.57	0.76
y_{20}	39.82	39.75	37.13	33.58	30.27	25.60	21.10	16.42	12.41	9.18	4.77	0.74

Table A.13: Demand intercept D^0 [GW] for entire time horizon in section 7.3.

Year	**WD Peak**	**WD Shoulder**	**WD Off-peak**	**WE Peak**	**WE Shoulder**	**WE Off-peak**
y_1	63.0	54.4	35.8	47.6	39.4	30.1
y_2	64.4	55.6	36.6	48.6	40.3	30.8
y_3	65.7	56.8	37.3	49.6	41.1	31.4
y_4	67.2	58.1	38.2	50.8	42.1	32.1
y_5	68.8	59.5	39.1	52.0	43.1	32.9
y_6	70.4	60.9	40.0	53.2	44.1	33.7
y_7	71.6	61.9	40.7	54.1	44.8	34.2
y_8	72.9	63.1	41.5	55.1	45.7	34.9
y_9	74.3	64.2	42.2	56.1	46.5	35.5
y_{10}	75.8	65.6	43.1	57.3	47.5	36.3
y_{11}	77.4	66.9	44.0	58.4	48.4	37.0
y_{12}	78.9	68.2	44.9	59.6	49.4	37.7
y_{13}	80.5	69.6	45.8	60.8	50.4	38.5
y_{14}	82.1	71.0	46.7	62.0	51.4	39.3
y_{15}	83.7	72.4	47.6	63.3	52.4	40.0

Table A.14: Approximation demand d [GW] in section 7.3.

Year	**WD Peak**	**WD Shoulder**	**WD Off-peak**	**WE Peak**	**WE Shoulder**	**WE Off-peak**
y_1	28.2	28.2	21.0	28.2	22.6	20.9
y_2	29.0	29.0	21.5	29.0	23.2	21.4
y_3	29.8	29.8	22.0	29.8	23.8	21.9
y_4	30.7	30.7	22.6	30.7	24.5	22.5
y_5	31.6	31.6	23.2	31.6	25.3	23.0
y_6	32.5	32.5	23.9	32.5	26.0	23.6
y_7	33.2	33.2	24.3	33.2	26.6	24.0
y_8	34.0	34.0	24.8	34.0	27.2	24.5
y_9	34.8	34.8	25.4	34.8	27.8	24.9
y_{10}	35.7	35.7	26.0	35.7	28.5	25.5
y_{11}	36.6	36.6	26.6	36.6	29.2	26.0
y_{12}	37.5	37.5	27.2	37.5	30.0	26.6
y_{13}	38.4	38.4	27.8	38.4	30.7	27.1
y_{14}	39.4	39.4	28.4	39.3	31.4	27.7
y_{15}	40.3	40.3	29.1	40.2	32.2	28.3

Appendix B

Publications and Conference Presentations

Publications

The following five papers are directly related to this dissertation and have been published or accepted in relevant SCI-indexed international journals.

- S. Wogrin, B. F. Hobbs, D. Ralph, E. Centeno, and J. Barquín. Open versus closed loop capacity equilibria in electricity markets under perfect and oligopolistic competition. *Mathematical Programming (Series B)*, 2012, *accepted for publication.*

- E. Centeno, S. Wogrin, A. López-Peña, and M. Vázquez. Analysis of investments in generation capacity: A bilevel approach. *Generation, Transmission Distribution, IET*, 5(8):842-849, 2011.

- S. Wogrin, E. Centeno, and J. Barquín. Generation capacity expansion in liberalized electricity markets: A stochastic MPEC approach. *IEEE Transactions on Power Systems*, 24(4):2526-2532, 2011.

- S. Wogrin, J. Barquín, and E. Centeno. Capacity expansion equilibria in liberalized electricity markets: An EPEC approach. *IEEE Transactions on Power Systems*, 28(2):1531-1539, 2013.

- S. Wogrin, E. Centeno, and J. Barquín. Generation capacity expansion analysis: Open loop approximation of closed loop equilibria. *IEEE Transactions on Power Systems*, PP(99):1-10, 2013.

Conference Presentations

The following conference presentations are related to the work carried out in this thesis.

- S. Wogrin, E. Centeno and J. Barquín. Impact of renewable energy sources on generation capacity investments: a stochastic MPEC approach. *INFORMS 2012 Annual Meeting*, Phoenix, USA, October 2012.

- E. Centeno, S. Wogrin, J. Barquín. Study of strategic behavior impact in the spot market on capacity expansion using a linearised EPEC. *INFORMS 2012 Annual Meeting*, Phoenix, USA, October 2012.

- S. Wogrin, B.F. Hobbs, D. Ralph, E. Centeno and J. Barquín. Market power and investment decisions in electricity markets: open vs closed loop equilibria. *INFORMS 2011 Annual Meeting*, Charlotte, USA, November 2011.

- S. Wogrin, J. Barquín and E. Centeno. How capacity payments influence investment decisions in electricity markets. *12th Centre for Competition and Regulatory Policy Workshop - CCRP*, Paris, France, July 2011.

- E. Centeno, J. Reneses, S. Wogrin and J. Barquín. Representation of Electricity Generation Capacity Expansion by Means of Game Theory Models. *European Energy Markets 2011*, Zagreb, Croatia, May 2011.

- S. Wogrin, E. Centeno and J. Barquín. Analysis of investments in electricity plants with Stochastic Bilevel programming techniques. *INFORMS 2010 Annual Meeting*, Austin, USA, November 2010.

- S. Wogrin, E. Centeno and J. Barquín. Advantages of Stochastic Bilevel programming in the generation capacity expansion problem. *4th Annual Trans-*

Atlantic INFRADAY Conference on Applied Infrastructure Modeling and Policy Analysis, Washington, USA, November 2010.

- S. Wogrin, E. Centeno and J. Barquín. Analysis of strategic investments in electricity generation capacity under uncertainty. *8th Young Energy Economists & Engineers Seminar - YEEES*, Cambridge, UK, April 2010.

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